#### ORIGINAL RESEARCH



# Ramsey's conditionals

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#### **Abstract**

In this paper, we propose a unified account of conditionals inspired by Frank Ramsey. Most contemporary philosophers agree that Ramsey's account applies to indicative conditionals only. We observe against this orthodoxy that his account covers subjunctive conditionals as well—including counterfactuals. In light of this observation, we argue that Ramsey's account of conditionals resembles Robert Stalnaker's possible worlds semantics supplemented by a model of belief. The resemblance suggests to reinterpret the notion of conditional degree of belief in order to overcome a tension in Ramsey's account. The result of the reinterpretation is a tenable account of conditionals that covers indicative and subjunctive as well as qualitative and probabilistic conditionals.

**Keywords** Frank Ramsey · Ramsey test · Stalnaker semantics · Degrees of belief · Unified account of conditionals

### 1 Introduction

You believe 'if p then q' when supposing p makes you believe q. This simple idea for when to believe a conditional has come a long way. It originated from a footnote in the paper "General Propositions and Causality" (GPC), written by Frank Ramsey in 1929.

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The idea has become known as the Ramsey Test and served as a point of departure for a great deal of work. Ernest Adams (1965), for example, took the idea to develop a semantics for indicative conditionals according to which you accept a conditional if the corresponding conditional probability is high. Robert Stalnaker (1968) worked out another Ramsey Test semantics of conditionals using the concept of a possible world. These works, in turn, have become highly influential.<sup>1</sup>

Over the years, an orthodoxy has emerged, viz. that the Ramsey Test is for indicative conditionals only. Here, we will see that Ramsey's own account of conditionals runs against this orthodoxy: Ramsey's account applies to both indicative and subjunctive conditionals. The textual evidence in GPC for this claim is supplemented by a note titled "The Meaning of Hypothetical Propositions" (MHP), which Ramsey wrote in 1928. The textual evidence suggests that Ramsey's account of conditionals is supposed to be general. It is meant to cover indicative and subjunctive conditionals, including counterfactuals. If our interpretation of GPC and MHP is correct, Ramsey aimed at a unified account of conditionals.

We will argue that Ramsey's account resembles the analyses of variably strict conditionals more than has been acknowledged so far. In fact, we think Ramsey's conditionals may be seen as an agent-relative predecessor of the variably strict conditionals put forth by Stalnaker (1968) and David Lewis (1973a). The resemblance of Ramsey's account to the aforementioned possible worlds semantics supplemented by a model of belief calls for a re-interpretation of the "degree of belief in q given p". Indeed, Ramsey's interpretation of this notion as conditional probability leads to a tension within his account. We thus re-interpret the concept of conditional degree of belief and thereby propose our own unified account of conditionals that overcomes the tension.

We proceed as follows. In Sect. 2, we outline where the currently orthodox interpretation of the Ramsey Test comes from. In Sect. 3, we lay out Ramsey's account of conditionals. We will see that this account applies also to subjunctive and counterfactual conditionals. Yet we will point out a tension within Ramsey's account. In Sect. 4, we compare Ramsey's account to Stalnaker's possible world semantics and develop a qualitative Ramsey Test. This helps us, in Sect. 5, to overcome the tension arrived at in Sect. 3 and to propose our unified account of conditionals.

# 2 The orthodox interpretation

Ramsey's footnote in GPC on page 247 goes as follows:

If two people are arguing 'if p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense 'if p, q' and 'if p,  $\overline{q}$ ' are contradictories. We can say they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered *void*. If either party believes  $\overline{p}$  for certain, the question

<sup>&</sup>lt;sup>1</sup> The Ramsey Test plays a major role in the literature of belief revision as well. See, for instance, Gärdenfors (1978, 1986), Alchourrón et al. (1985), and Hansson (1992). More recent contributions on the Ramsey Test include Bradley (2007), Rott (2017), and Andreas and Günther (2019, 2020, 2021).



ceases to mean anything to him except as a question about what follows from certain laws or hypotheses.

The first mention of Ramsey's footnote is in Chisholm (1946), a paper titled "The Contrary-to-Fact Conditional". There, Chisholm analyses conditionals whose antecedents are (believed to be) 'contrary to the facts' or false. Such conditionals have become known as 'counterfactuals'. Chisholm's analysis "proceeds on the basis of certain suggestions made by F.P. Ramsey", including the footnote. To use the Ramsey Test as a basis to analyse counterfactuals is almost ironic in the light of the current orthodoxy implying that the Ramsey Test does not apply to counterfactuals. Where does the orthodoxy come from?

Adams (1965, 1966, 1975) takes the phrase 'degrees of belief' in Ramsey's footnote seriously. To evaluate a conditional you fix your degree of belief in the consequent given the antecedent. He develops a logic for indicative conditionals based on Ramsey's idea that your degrees of belief can be represented by a probability function. Your degree of belief in q given p is equated to the probability of q conditional on p. Hence, you accept the indicative conditional 'if p then q' if the conditional probability of q given p is high, that is close to 1. Adams (1975, pp. 3, 10) follows Ramsey (GPC, pp. 180-1) in stipulating the notion of conditional probability as follows: for all non-conditional propositions expressed by the formulas  $\psi$  and  $\phi$ , the conditional probability of  $\psi$  given  $\phi$  is defined by

$$P(\psi \mid \phi) = \frac{P(\psi \land \phi)}{P(\phi)}$$
, if  $P(\phi) > 0$ .

A degree of belief in q given p is thus undefined when the probability of p is 0.

Following Adams, Edgington (1986, p. 19) claims that an indicative conditional is acceptable just in case the corresponding conditional probability—in the sense of the ratio definition—is high. Edgington (1995, pp. 264-5) thinks indicative conditionals are used only when you judge the probability of the antecedent to be greater than 0. On this view, the intolerance to zero probability antecedents is a defining feature of indicatives rather than a bug. By contrast to this putative feature of indicatives, the antecedents of counterfactuals are (believed to be) false. Adams and Edgington seem to think that this entails a zero probability for the antecedent. Hence, Adams's acceptability condition cannot apply to counterfactuals, at least prima facie.<sup>3</sup>

<sup>&</sup>lt;sup>3</sup> It should be noted that Adams (1966, p. 273) deviates from the usual Kolmogorov axioms for probability measures. If  $P(\phi) = 0$ , he stipulates that  $P(\psi \mid \phi) = 1$  for all non-conditional  $\psi$ ,  $\phi$ . Hence, according to him, you accept *any* conditional whose antecedent you judge to have probability zero—you accept any counterfactual. There are several ways to save the idea that the acceptability of a counterfactual is determined by a conditional probability. Rényi (1955) and Popper (1959), for example, propose alternative axiomatic systems that take conditional probability as primitive and define absolute probabilities in terms



<sup>&</sup>lt;sup>2</sup> It was Goodman (1947) who named contrary-to-fact conditionals 'counterfactuals'. Most often, or so it seems, counterfactuals are expressed in the subjunctive mood. Yet there seem to be exceptions, such as 'If you are Cristiano Ronaldo, you have the urge to win another Ballon d'Or.' Similarly, there are subjunctive conditionals which do not express counterfactuals. A doctor who investigates Jones's death may say: if Jones had taken arsenic, he would have shown just exactly those symptoms which he does in fact show. Anderson (1951, p. 37) observes that the truth of this subjunctive conditional does not imply that the antecedent is (believed to be) false.

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The 'two people' in Ramsey's footnote are initially 'both in doubt as to p'. Stalnaker (1968, p. 101) interprets this to say that the Ramsey Test "covers only the situation in which you have no opinion about the truth value of the antecedent". He ignores the phrases starting with 'If p turns out false' and 'If either party believes  $\overline{p}$  for certain'. Overlooking that Ramsey explicitly treats the case, where the antecedent is believed to be false, Stalnaker seeks to extend the Ramsey Test to cover the cases, where the antecedent is believed to be true or false, respectively. Ever since, Ramsey's account has been more and more considered to be restricted to indicative conditionals implicating that there is no theory of counterfactuals to be found there. This is a clear misinterpretation, as we will see below.

In line with Adams, Edgington, and Stalnaker, Morton (2004, p. 294) remarks that Ramsey's procedure is commonly taken to apply to indicative conditionals only. Lewis (1973a, p. 70, fn. 2) states that Ramsey tackles "assertability conditions for indicative conditionals rather than counterfactuals." Gibbard (1981, p. 239) simply writes that "the Ramsey test does not apply to subjunctive conditionals". This orthodoxy might be explained by the influence of the works of Adams and Stalnaker. It is so predominant that "A Philosophical Guide to Conditionals" by Bennett (2003, pp. 29-30) explicitly states that the "Ramsey test thesis does not hold for subjunctive conditionals". In light of this orthodoxy, it may come as a surprise that GPC and MHP contain an account that covers subjunctive and counterfactual conditionals. We turn to this account next.

# 3 Ramsey's account of conditionals

In this section, we will present Ramsey's account of conditionals. On this account, conditionals are supported by certain rules Ramsey calls 'variable hypotheticals'. First, we will exemplify what variable hypotheticals are by taking recourse to Ramsey's cake example. This example illustrates the role of variable hypotheticals by presenting a situation, where two people disagree about the merely possible. We then move on to Ramsey's analysis of conditionals. It has an inferential and a probabilistic aspect, to say the least. We will treat the inferential part before we return to Ramsey's footnote. Finally, we will observe that there is a tension within Ramsey's account: the inferential

Footnote 3 continued

of conditional probabilities. This promising line of work has been philosophically defended by Hájek (2003) and may give us acceptability conditions for counterfactuals. However, such acceptability conditions require at the very least some modifications to Kolmogorov's by now standard probability theory. Some of those modifications have been put forth and discussed by Stalnaker (1970), Harper (1975), van Fraassen (1976), and Spohn (1986). Another way to save acceptability conditions for counterfactuals is due to Skyrms (1981) who circumvents the non-applicability of standard probabilities to counterfactuals by taking recourse to prior propensities instead of present epistemic probabilities. Similarly, to arrive at a probabilistic semantics of counterfactuals Leitgeb (2012a, b) takes Popper measures as primitive and interprets these conditional probabilities as objective single case chances. However, to say the least, it seems to be a stretch to convey Ramsey's degrees of belief an objective interpretation.

<sup>&</sup>lt;sup>5</sup> Stalnaker's version of the Ramsey Test applies to both indicative and counterfactual conditionals. Yet, as we will see in Sect. 4.1, his interpretation of Ramsey's original procedure neglects that Ramsey's test applies to counterfactuals as well. And Stalnaker's interpretation of Ramsey's procedure seems to be the predominant one nowadays.



<sup>&</sup>lt;sup>4</sup> Of course, there are some heretics. For an interesting dissenting view, see Levi (2007, pp. 46-50).

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analysis applies to counterfactuals, whereas the degree of belief formulation does not. This internal tension sheds doubt on whether Ramsey provides a unified account of conditionals.

# 3.1 Variable hypotheticals

Ramsey's account of conditionals in GPC centers on what he calls 'variable hypotheticals'. A variable hypothetical is a rule Ramsey expresses by "If I meet a  $\phi$  I shall regard it as a  $\psi$ " (p. 241). Such rules express your inferential habits, that is inferences you are anytime prepared to make. For example, 'all men are mortal' is a variable hypothetical that expresses the belief 'whenever you encounter a man, you judge him to be mortal'. To believe this variable hypothetical means to believe, for any x that turns up, if x is a man, x is mortal. Ramsey describes the logical form of a variable hypothetical by  $\forall x(\phi(x) \rightarrow \psi(x))$  (see p. 249). In this section, we follow Ramsey by characterising variable hypotheticals in contrast to finite conjunctions and material implications.

A variable hypothetical goes beyond our finite experience. There is, for instance, no way to conclusively establish that all women—past, present, and future—are mortal. In particular, we cannot foretell the future with certainty, and so we cannot conclusively know whether or not any future woman will be mortal. Formally speaking, a variable hypothetical ranges over an infinite domain, that is over an infinite number of individuals or events. Ramsey distinguishes variable hypotheticals from "the other kind" of "general propositions" (p. 237). The latter but not the former range over a finite domain and can be thought of as conjunctions. The sentence 'Everyone in Cambridge voted for this motion', for instance, does not express a variable hypothetical. The sentence is meant to express a finite list or class of people who voted for the motion: Moore, Russell, Braithwaite, and Wittgenstein voted for it. The sentence is meant to range over a rather restricted domain, viz. a voting event at a certain time and place. 'Everyone in Cambridge voted for this motion' expresses that everyone present voted yes. By contrast, the sentence 'Everyone in Cambridge votes for this motion' understood as a variable hypothetical would express that anyone who ever has voted, is voting, or will be voting in Cambridge is voting for this motion. It would support the conditional 'if she had been present, she would have voted yes'. But this is, of course, not a viable common sense interpretation of the initial sentence. Upon hearing the sentence, we naturally and automatically restrict the domain to a finite class of people in Cambridge.

<sup>&</sup>lt;sup>7</sup> The rejection of generalisations as infinite conjunctions marks Ramsey's turn towards finitism and intuitionism in his philosophy of mathematics (see Majer (1989, 1991), Misak (2020)).



<sup>&</sup>lt;sup>6</sup> Ramsey speaks of *believing* variable hypotheticals and conditionals and we will follow suit. We are aware that the attitude of belief is one of the ingredients in Gärdenfors's (1986) triviality result. Not only since then, many authors talk about *accepting* conditionals, e.g. Levi (1977). However, unlike a common conception of belief, Ramsey's does not imply that believed variable hypotheticals and conditionals have truth values (cf. Gärdenfors (1986, p. 90)). Moreover, Ramsey does not develop his account within the formal framework of belief revision theory. It is thus at the very least not clear whether Ramsey's account is susceptible to Gärdenfors's triviality results.

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Variable hypotheticals can, however, be applied to finite classes. All people are mortal can be applied to the finite class of people in a spatio-temporal region such as Cambridge. If we are inclined to do so, we obtain the inferential result that Moore, Russell, Braithwaite, *and* Wittgenstein are mortal. Importantly, variable hypotheticals go beyond any single application to a limited domain. They rather express beliefs that also apply to future, not yet existing instances. A variable hypothetical you believe encodes a general expectation you have. If you believe the variable hypothetical that all women are mortal, then you expect any woman you will meet to be mortal. The expectations encoded in variable hypotheticals guide how a believer, or epistemic agent, will change her beliefs. The set of believed variable hypotheticals is thus "the system with which the speaker meets the future" (GPC, p. 241).

Let us say the motion was in fact passed unanimously. It follows that 'if she was there, she voted for it' in the sense of the material implication: either she was not there, or she voted for it. On the other hand, it is sensible to say 'if she had been there, she would have voted against it.' Interestingly, this conditional implies that she was not there given the fact that the motion was passed unanimously. What is expressed goes beyond what actually happened in the limited domain and touches on a mere possibility. To entertain the merely possible belief that she was there makes the agent believe that she would have voted against the motion. An agent may believe that the character of this particular woman was so that she would have voted so. To utter the conditional reveals thus a part of the system with which the agent meets the future.

Ramsey (GPC, pp. 246-7) illustrates what variable hypotheticals express and how they relate to conditionals using the following situation. Suppose you decide not to eat the piece of cake in front of you, because you believe doing so will upset your stomach. You act on the belief 'if I eat the cake, I will have a stomach ache'. Let us also suppose, after you did not eat the cake, that I disagree with you by thinking 'if you had eaten the cake, you would not have had a stomach ache'. This situation illustrates that the conditionals cannot be understood as material implications. We both know that you did not eat the cake. So we think the proposition expressed by the material implication 'if you eat the cake, you will have a stomach ache' is true. As you have no factually false belief from my point of view, on what do I disagree with you?

The cake example shows that we can disagree about the merely possible, about what would have happened had the facts been different from how they actually are. We both agree on the facts of the matter, that is you did not eat the cake and you had no stomach ache. The difference we have is not primarily about the facts we believe. Our disagreement comes from the different variable hypotheticals we adopt. You believe 'if I had eaten the cake, I would have a stomach ache', whereas I believe 'if you had eaten the cake, you would not.' According to Ramsey (GPC, p. 247), these "assertions about unfulfilled conditions" reflect that our beliefs are guided by different variable hypotheticals. If our systems with which we meet the future are not equivalent, it is logically possible that the systems imply different expectations for the future. Hence,

<sup>&</sup>lt;sup>8</sup> As an anonymous referee correctly pointed out, the agent is here strictly speaking a *doxastic* agent. This being said, we continue to use epistemic agent understanding the "epistemic" broadly so as to include attitudes like beliefs, credences, being certain, and so on. This is a common usage of "epistemic" nowadays (Rendsvig and Symons, 2019; Goldman and O'Connor, 2021; Edgington, 2020).



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we may have a disagreement, even if the actual future agrees with both of our non-equivalent belief systems.

There are two different ways in which we might disagree. In the example, you say 'If I eat the cake, I will have a stomach ache', and I say 'No, you will not'. Thereby, I am not negating the conditional understood as material implication. The root of our disagreement is that you adopt certain variable hypotheticals (and background facts) which I do not believe. Technically, there are two ways in which I can disagree with you. Let us assume you adopt the variable hypothetical  $\forall x (\phi(x) \to \psi(x))$  that supports your conditional. First, I can disagree with you by believing the antithetical  $\forall x (\phi(x) \to \neg \psi(x))$ . Second, I can disagree with you by believing none of the mentioned variable hypotheticals. Thereby, I neither infer  $\psi$  nor  $\neg \psi$  from  $\phi$ . I simply do not comply to make any inference as to  $\psi$ 's status. To deny the inferability of  $\psi$  from  $\phi$  is one way to deny the conditional. We will return to this issue below. In particular, we will explain how variable hypotheticals 'support' conditionals on Ramsey's view.

A final remark on variable hypotheticals is in order. According to GPC, pp. 248-9, laws have the form of variable hypotheticals. A variable hypothetical  $\forall x (\phi(x) \rightarrow \psi(x))$  is a causal law if any instance of  $\psi$  denotes events no earlier than the respective instantiation of  $\phi$ . For what follows, it is important to keep in mind the close link between variable hypotheticals and laws.

#### 3.2 Conditionals

In this section, we present Ramsey's account of conditionals and illustrate how it works, before we note two properties of Ramsey's conditionals. In GPC, p. 248, he summarizes his account as follows:

'If p then q' means that q is inferrible from p, that is, of course, from p together with certain facts and laws not stated but in some way indicated by the context.

The meaning of Ramsey's 'if p then q' is relative to a rational agent who believes certain facts and variable hypotheticals. Believing a Ramsey conditional implies that the corresponding material implication should be believed. To see this, assume Ramsey's 'if p then q'. Hence, the agent can infer q from the assumption of p together with her contextual information she believes to be true. There are two cases: if the assumption is true, she should believe p and so q as well; if the assumption is false, she should believe  $\neg p$ . Hence, believing Ramsey's conditional implies that in one of the two cases  $\neg p$  should be believed to be true, and in the other  $p \land q$  should be believed to be true. But this implies that the material implication  $p \rightarrow q$  should be believed to be true whenever Ramsey's conditional is believed.

However, the meaning of 'if p then q' is not exhausted by the material implication. This becomes evident when considering the two paradoxes of the material implication,



<sup>&</sup>lt;sup>9</sup> Laws are thus a kind of variable hypothetical. As such laws go beyond our finite experience and our factual beliefs. They cannot be true in the sense that a finite conjunctive fact can be. However, a genuine law, or as Ramsey (GPC, p. 244) says a "true" law, never misleads us. In other words, our inferences guided by a 'true' law are never disconfirmed.

<sup>&</sup>lt;sup>10</sup> See GPC, p. 248 and MHP, p. 238.

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that is cases where p is known to be false and cases where q is known to be true. The first case has already been exemplified by the disagreement about whether eating a particular cake would induce a stomach ache. In the cake example, what you and I disagree on is not covered by a mere material implication. In the second case, where the consequent q is known to be true, uttering 'if p then q' may, for example, indicate an explanation for q. Suppose you and I know that he has a stomach ache. I ask you: "But why?" "Well," you answer, "if he ate this cake, no wonder!" In both cases, the point to state a conditional is to convey more than a material implication is able to.

As we have just seen, conditionals can be used to express more than material implications. <sup>11</sup> To capture what conditionals express, in GPC, pp. 248-9, and in a more detailed way in MHP, pp. 237-9, Ramsey gives the following analysis of conditionals. 'If p then q' means that there is an r such that  $(p \land r) \to q$  is an instance of a variable hypothetical  $\forall x (\phi(x) \to \psi(x))$ . That is, for some individual constant  $a, p \land r$  is an instance of  $\phi$ , in symbols  $p \land r = \phi(a)$ , and q an instance of  $\psi$ , in symbols  $q = \psi(a)$ , and while p may be merely supposed and q merely inferred, r must be believed to be true. There are some further conditions on r. It must be a conjunction of propositions which do not contain  $\lor$ ,  $\to$ ,  $\exists$ ,  $\neg$ , but may contain  $\forall$ , and there are certain temporal restrictions on which events it can describe (see below). <sup>12</sup> r can be conceived of as information imported from the actual context. <sup>13</sup> And, finally, the assumption of p and r must be compatible in the sense that  $\neg(p \land r)$  is not an instance of any law. <sup>14</sup> This is Ramsey's inferential analysis of conditionals in a nutshell.

Let us illustrate how Ramsey's analysis works. An agent believes the conditional 'If she had eaten the cake, she would have had a stomach ache' if the agent believes a specific woman and a bad cake to be co-present, and the agent believes the variable hypothetical 'if a woman eats a bad cake, she will get a stomach ache'. Believing this variable hypothetical is tantamount to infer that she will get a stomach ache upon supposing that the woman eats the cake (whether or not she actually does), at least in the situation described by r, viz. the co-presence of a woman and a bad cake. <sup>15</sup> The example illustrates how variable hypotheticals support conditionals. They do so by providing an inferential connection from the antecedent, in a certain context, to

 $<sup>^{15}</sup>$  Of course, an agent might believe the conditional on other grounds. The conjunction r might be different, and so might the variable hypothetical. Furthermore, there could be more variable hypotheticals involved, for example 'if a person eats a bad cake, they will get a stomach ache' and 'if x is a woman, x is a person'.



<sup>&</sup>lt;sup>11</sup> Ramsey (GPC, p. 246) observes, however, that 'if p then q' may simply mean the material implication  $p \to q$ . In MHP, p. 237, he adds that one meaning of the conditional is that q follows logically from p. There, he even claims that a conditional "often means just" the material implication.

<sup>&</sup>lt;sup>12</sup> Here, an atomic proposition is a limiting case of a conjunction.

<sup>&</sup>lt;sup>13</sup> See GPC, p. 248, where Ramsey refers to Mill (1843), and MHP, p. 238. The idea of importing information from actuality has recently been revived by Priest (2018).

<sup>&</sup>lt;sup>14</sup> We may see now more clearly why believing the conditional implies to believe true the corresponding material implication. r is required to be true, and r and  $(p \wedge r) \rightarrow q$  imply  $p \rightarrow q$ . Let us make the epistemic agent explicit. You believe 'if p then q'. This means you believe some r to be true such that you believe q under the assumption of p in virtue of your variable hypotheticals. There are two cases: the assumption of p is true, in which case you should believe p and q due to the variable hypotheticals you believe; or else the assumption of p is false, in which case you should believe  $\neg p$ . Hence, you should infer from your belief 'if p then q' that  $\neg p \lor q$  using reasoning by cases. But note that believing this disjunction does not mean that you believe  $\neg p$  or you believe q: you may believe neither and yet believe the disjunction, for example, if you believe none of p,  $\neg p$ , q, and  $\neg q$ , but you believe the conditional.

the consequent. And this works if the antecedent is already believed to be true or merely supposed. Notice the role of the contextual information r: if the agent does not believe that the cake is bad, the variable hypothetical 'if a woman eats a bad cake, she will get a stomach ache' is not triggered. Whether the conditional is believed thus depends on the contextual beliefs. If the beliefs about the actual context were different in the example, the conditional would not be believed. Even if some variable hypotheticals would support a conditional, the conditional is only believed when there is some appropriate information r imported from the context.

The r is 'in some way indicated by the context.' It can be made explicit though (maybe only to a limited degree). Let us try. To believe 'If Catherine had eaten the cake, she would have had a stomach ache' requires to believe a variable hypothetical  $\forall x(\phi(x) \to \psi(x))$ , let us say 'every woman who eats a bad cake will have a stomach ache', and to believe a certain r. Let r be given by  $r_1 \wedge r_2$ , where  $r_1$  stands for Catherine is a woman and  $r_2$  for a bad cake is present. Someone who disagrees might simply believe  $r_1 \wedge r_3$  instead of  $r_1 \wedge r_2$ , where  $r_3$  stands for a good cake is present. Most disagreements seem to be more complex. What makes a bad cake? Well, you might believe that 'all cakes that contain salmonella bacteria are bad cakes' ( $r_4$ ), and that 'This cake contains salmonella bacteria' ( $r_5$ ). A disagreement similar to the more simple one arises when two agents believe the variable hypothetical under consideration and one believes  $r_1 \wedge r_4 \wedge r_5$ , while the other believes  $r_1 \wedge r_4 \wedge \neg r_5$ . Of course, the disagreement might be due to other factors, or concern the variable hypotheticals involved. In any case, let us keep in mind that whether or not a certain conditional is believed depends on the information imported by r. <sup>16</sup>

The compatibility of the antecedent p and r deserves further attention.  $\neg(p \land r)$  is logically equivalent to  $p \to \neg r$ . Hence, the condition that  $\neg(p \land r)$  is not an instance of any law requires that the assumption of p does not lawfully imply the negation of r. We may illustrate this requirement by means of another example. Let p stand for 'Sally can jump four meters high' and q for 'Sally jumps on the roof to fix it.' Furthermore, let r be the conjunction 'Sally weighs about 50kg and is within the human range of strength, she lives on Earth and the surface gravity of Earth is about 9.807  $^m/s^2$ '. Now, p and r are incompatible. For, if you assume p, you can infer, using common-sensical laws of how physical bodies behave, that  $\neg r$ . In other words,  $p \to \neg r$  is instantiated by your system of variable hypotheticals.

What should you do when p and r are incompatible? Well, you just need to find  $some\ r'$ . One strategy would be that you shrink the content of r to find r'. While the conjuncts of r are required to be believed true, it is not required that any conjunct that is believed to be true is contained in r. Ramsey does not forbid one to assume an antecedent that violates our understanding of, for instance, mechanical laws. Rather the content of r is restricted by the believed laws. To be explicit, you could simply take away some of the components of r. Assuming that Sally can jump four meters high

 $<sup>^{17}</sup>$  Note that Ramsey's compatibility of antecedent p and contextual information r can be seen as a precursor to Goodman's (1947) notion of *cotenability*: p is cotenable with r if it is not the case that r would not be true if p were. Like Ramsey, Goodman thinks that counterfactuals must be supported by laws. Unlike Ramsey, Goodman discusses at length the challenge of specifying r in a non-circular way.



 $<sup>^{16}</sup>$  In other words, a Ramsey conditional is believed only under certain antecedent conditions r and not others. In this respect, Ramsey conditionals behave like *ceteris paribus* laws (Reutlinger et al., 2019).

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might then trigger a believed variable hypothetical or another that lets you infer that Sally is particularly strong or that the surface gravity of Earth is lower, or some other proposition that you actually do not believe, but which you would believe under the assumption. When assuming an antecedent that violates your variable hypotheticals, you encounter the problem that it is underdetermined which conjuncts of r you are supposed to give up, and also what exactly you are supposed to infer. It seems that Ramsey gives no answer to this problem of underdetermination, leaving the actual reasoning to the agent under consideration.  $^{18}$ 

In the Sally example, you assume to be true p, while you believe  $\neg p$  due to your beliefs in r and the common-sensical variable hypotheticals you believe. Nevertheless, p is assumed to be true, and so other beliefs incompatible with p must be removed from r. It is a tricky question which beliefs must go, but it depends on the variable hypotheticals believed by the epistemic agent. The content of r is thus restricted by the meaning of the assumption p, the believed laws, and, of course, the facts believed to be true in the first place.

We have said that there are certain temporal restrictions on which events r can describe. To clarify these restrictions, Ramsey (MHP, pp. 237-9) puts forth an example similar to the following. You know that it didn't rain and that Mary came to a meeting. Consider the two conditionals:

- (a) If it had rained, she wouldn't have come.
- (b) If it had rained, she would have come all the same.

Both conditionals have an antecedent p you believe to be false. The consequents are believed to be false and true, respectively. For conditionals of type (a), Ramsey (MHP, p. 238) says that "no further limitation on r is necessary" beyond being a true conjunction that is compatible with p. By contrast, there be a temporal restriction on r for conditionals of type (b). In these cases, r must be restricted to describing events no later than the ones described in the antecedent. r must be about events previous to Mary's "actual or possible setting out" (p. 239). It can, for instance, not include her actual presence at the meeting now. Otherwise you would believe the consequent of (b) merely due to r without any need of the antecedent. You would trivially believe (b) in absence of the temporal restriction. Notice that the temporal restriction applies only when you believe the consequent to be true, or when you have a belief that lets you infer the consequent without using the antecedent.

Ramsey notes two properties of his conditionals. First, contraposition fails. 'If she came to the meeting, she would have left it by now' does not entail that 'If she had not left the meeting by now, she would not have come to it.' The issue here is that you can infer q from p given a suitable context and your variable hypotheticals, and at the same time p from  $\neg q$  together with some different variable hypotheticals. If Mary did not leave the meeting by now, she is still there, which implies that she came to it. p and  $\neg q$ 

<sup>&</sup>lt;sup>18</sup> One way to solve the problem of underdetermination parallels a solution in belief revision theory that defines belief contraction based on an order of *epistemic entrenchment* (see, e.g., Gärdenfors and Makinson (1988), Gärdenfors (1988, Ch. 4), and Rott (2001, Ch. 8)). In our context, the rough idea would be to rank the variable hypotheticals according to epistemic importance. Upon assuming an antecedent that violates your system of variable hypotheticals, you would then give up a conjunct that is only supported by the comparatively lowest-ranked variable hypothetical. However, this and similar procedures would go beyond Ramsey's account.



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might trigger different variable hypotheticals and they might be lawfully compatible with quite different information imported from the context. Although contraposition fails in general, there is an interesting observation to be made. For the same true r,  $p \land r \rightarrow q$  is logically equivalent to  $\neg q \land r \rightarrow \neg p$ . Without contextual shifts a Ramsey conditional thus implies its contrapositive material implication. Hence, contraposition holds for a Ramsey conditional if the r remains the same and  $\neg q$  is lawfully compatible with it. By contrast, contraposition may or may not hold for a Ramsey conditional when the context shifts. <sup>19</sup>

Ramsey (GPC, p. 240) states the second property as follows: "the ordinary hypothetical [...] asserts something for the case when its protasis is true: we apply the Law of Excluded Middle not to the whole thing but to the consequence only". A Ramsey conditional asserts its consequent under the assumption of the antecedent, independently of whether the antecedent is believed to be true or not. Supposing the antecedent either makes you infer the consequent, or else it does not. It depends on your system of variable hypotheticals whether or not you infer the consequent, its negation, or you simply infer nothing from the supposed antecedent.

The above means that Ramsey's conditional does not validate the Law of Conditional Excluded Middle: it is not the case that you infer q or  $\neg q$  from an arbitrary supposition p; you may infer neither. But this principle holds for any non-contradictory p: if you infer q from a supposition  $p \land r$ , then you cannot consistently infer  $\neg q$  from the same supposition. As a consequence, it is not the case that you can consistently believe both "if  $p_r$  then q" and "if  $p_r$  then  $\neg q$ ", where  $p_r$  is short for  $p \land r$ . By contrast, you can, for instance, consistently believe both 'If Cesar had entered the Vietnam War, he would have used catapults' and 'If Cesar had entered the Vietnam War, he would have used the atomic bomb (and no catapults)'. The reason is that the respective r differs for the two conditionals: the first antecedent says something like 'If Cesar had entered the Vietnam War and it is the time of Cesar and …'  $(p \land r_a)$ , whereas the second antecedent says something like 'If Cesar had entered the Vietnam War and it is the time of the Vietnam War and …'  $(p \land r_b)$ . Where  $p_{r_a}$  abbreviates  $p \land r_a$  and similarly for  $p_{r_b}$ , 'if  $p_{r_a}$  then q' and 'if  $p_{r_b}$  then q' are not the same conditional, and so you may consistently believe 'if  $p_{r_a}$  then q' and 'if  $p_{r_b}$  then  $\neg q$ '.

# 3.3 Ramsey's footnote reconsidered

Let us return to the footnote in GPC and its interpretation. The context of the footnote is the cake example, where we adopt counterfactuals that are contrary "in a sense". Before you decide not to eat the cake in front of you, we ask 'If you eat the cake, will you get a stomach ache?' Ramsey's test requires to add (in a way to be specified) the antecedent hypothetically to our certain beliefs, or "stock of knowledge", to see whether or not the consequent follows. If you infer the consequent and I infer its negation upon supposing the antecedent, we contradict each other 'in a sense', viz. the consequents contradict each other under the assumption of the antecedent.

<sup>&</sup>lt;sup>19</sup> In general, the failure of contraposition seems to be linked to context shifts, as has been argued by Tichý (1984) and is more recently discussed by Gomes (2019).



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The footnote continues with what sounds like a characterisation of its first sentence. In the cake example, we fix our degrees of belief in the consequent q given the antecedent p. "If p turns out false", you have not eaten the cake, "these degrees are rendered *void*. If either party believes  $\overline{p}$  for certain, the question ceases to mean anything to him except as a question what follows from certain laws or hypotheses." (p. 247) The last two sentences of the footnote raise at least one issue: what does it mean that degrees of belief are rendered 'void'? If we interpret the degree of belief in q given p by the conditional probability  $P(q \mid p)$ , as Adams and Edgington suggest, then this degree of belief is undefined when  $\neg p$  is believed for certain.<sup>20</sup> However, after we both know that you did not eat the cake, the question 'if you had eaten the cake, would you have come to have a stomach ache?' barely ceases to mean anything. After all, we can have a disagreement about this question. Of course, the degrees of belief are rendered void in the sense that nothing about actuality follows from your zero-probability 'belief' that you have eaten the cake and your belief if you had, you would have had a stomach ache. Importantly, this does not mean that nothing possible follows from the mere assumption of p. The question becomes "a question about what follows from certain" variable hypotheticals. On this reading, 'void' cannot mean undefined; after all, it is not undefined what follows from the variable hypotheticals. In Sect. 5, we will propose how to reconcile this tension induced by cases, where  $\neg p$  is believed for certain, but the suppositional degree of belief in q given p is still meaningful.

The last two sentences of the footnote put the focus on counterfactuals. Above we stipulated with Chisholm that counterfactuals are conditionals whose antecedents are contrary to the facts. Ramsey's account of conditionals is relative to an epistemic agent. Here counterfactuals are thus best understood as conditionals, where the agent believes the antecedent to be false. Ramsey (GPC, p. 247) calls such 'counterdoxasticals' "assertions about unfulfilled conditions", or "unfulfilled conditionals" (p. 246) for short. As should be clear by now, Ramsey's account covers counterfactuals. In fact, he does not make a substantial difference between indicative, subjunctive, and counterfactual conditionals. And in his prime example, the cake scenario, the unfulfilled conditionals are formulated in the subjunctive mood.

Ramsey (MHP, pp. 242-3) gives even three reasons why we are interested in what follows from unfulfilled conditions: counterfactual reasoning is a "fiction of peculiar interest because near to reality", it is a "way of apportioning praise and blame", and a "way of stating laws". Let us consider the last reason. According to GPC, p. 249 'If p had happened, q would have happened' is a way to state laws. If the conditional is supported by a causal law, q describes events no earlier than p. In this case, Ramsey (MHP, pp. 240-1) speaks of "causal implications" which can be indicated by unfulfilled conditions.

<sup>&</sup>lt;sup>21</sup> As to the second reason: "We cannot blame a man", says Ramsey (GPC, p. 246), "except by considering what would have happened if he had acted otherwise, and this kind of unfulfilled conditional cannot be interpreted as a material implication, but depends essentially on variable hypotheticals."



<sup>&</sup>lt;sup>20</sup> Under the standard definition of conditional probability, the degree of belief  $P(q \mid p)$  is mathematically undefined whenever P(p) = 0.  $P(q \mid p)$  does not exist if p 'turns out false' in the sense that ' $\neg p$  is believed for certain'. You cannot be certain that p is false and still assign it a positive degree of belief.

In the cake example, we may even disagree *before* you do no eat the cake. The reason for this disagreement is that your degree of belief in q given p differs from mine (see GPC, p. 247). We have different variable hypotheticals of the form 'If  $\phi(x)$ , then probability a for  $\psi(x)$ ' (see p. 251). These probabilistic variable hypotheticals are certain conditional degrees of belief Ramsey (GPC, p. 207) calls "chances". They do not have the logical form of  $\forall x(\phi(x) \to P(\psi(x)) = a)$ , or  $P(\phi(x) \to \psi(x)) = a$  for all x and some number  $a \in [0, 1]$ . They have rather the form of a conditional probability, that is  $P(\psi \mid \phi) = a$ . In Ramsey's terms, a probabilistic variable hypothetical "clearly" does not express a degree of belief in  $\neg \phi \lor \psi$ , but it does express a degree of belief in  $\psi$  given  $\phi$  (see pp. 246, 251). To adopt different probabilistic variable hypotheticals gives rise to different degrees of expectation. In brief, you judge it very likely that you will get a stomach ache given that you eat the cake, whereas I think it is rather unlikely. Here we see that these "degrees of hypothetical belief" influence how we behave, whether or not we eat the cake for instance (p. 246).

After you have decided not to eat the cake, you believe for certain that you do not eat the cake. By interpreting your degree of belief in having a stomach ache given that you eat the cake as conditional probability, your degree of belief is mathematically undefined. Supposing that you ate the cake, when in fact you didn't, has no meaning with respect to an actual stomach ache. You simply cannot change the past. After your decision not to eat the cake, the mere supposition of eating it is thus meaningless in a sense: whatever follows from your hypothetical action does not actually follow from that action. After all, you decided not to eat the cake. Counterfactuals are practically meaningless insofar the consequences of the merely supposed antecedents are not actual, at least not in virtue of those antecedents. Yet what follows from the believed variable hypotheticals (and some compatible facts) is of course not meaningless. The mere supposition is meaningful in the sense that it still informs us about the variable hypotheticals that guide your beliefs, and possibly, if your hypotheticals are not misleading, about what would have happened if the state of affairs had been different. So, if you believe the antecedent to be false, the counterfactual is not entirely meaningless. We can still have a dispute about it.

We have just argued that, after the decision not to eat the cake, the counterfactual becomes irrelevant for your practical deliberation about what actually follows from your merely hypothetical action. In general, your degree of belief in the consequent given the antecedent is 'rendered void' in the sense of becoming irrelevant for what actually follows from the antecedent. But your degree of belief is still relevant for what would have followed from the antecedent.<sup>22</sup> Or so we propose to read Ramsey.

<sup>&</sup>lt;sup>22</sup> Not acting based on a believed counterfactual is—in general—not practically meaningless. You might be criticised for adopting certain counterfactuals (based on believing certain variable hypotheticals) instead of others (based on other variable hypotheticals). A modification of the cake example due to Misak (2016, p. 196) nicely illustrates this point. Say you know that I baked the cake, that I used excellent ingredients, that I know your food sensitivities and allergies, and that I wish you only well. But you still refuse to eat the cake, thinking it will make you ill. Here, we have grounds to judge your belief in the counterfactual as mistaken and irrational. Of course, after you have decided not to eat the cake, it does not actually follow—from the supposition that you did—that you will have a stomach ache. An actual stomach ache still does not follow from your actually not eating the cake. Yet it does follow from believing the whole counterfactual that your belief is judged mistaken and irrational. So there are actual consequences for your merely hypothetical beliefs. And these actual consequences might indeed be consequences impacting your future behaviour.



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In GPC, Ramsey indicates a connection between variable hypotheticals and probabilistic variable hypotheticals. He states: "A law is a chance unity" (p. 251). Suppose a law  $\forall x(\phi(x) \to \psi(x))$ . Let us abbreviate  $p \land r$  by  $p_r$ . Hence, for all  $p_r$  and q instantiating  $\phi$  and  $\psi$ , respectively,  $P(p_r \to q) = 1$ . From this it follows that  $P(q \mid p_r) = 1$ , if  $P(p_r) > 0$ . A law implies that the corresponding conditional probability equals 1, under the assumption that the probability of the antecedent does not equal 0. However, generalisations and conditional probabilities do in general not perfectly fit together. Aside the limiting cases of  $P(p_r \land \neg q) = 0$  and/or  $P(p_r) = 1$ , the probability of a material implication does, in general, not equal the corresponding conditional probability. This foreshadows a tension within Ramsey's account of conditionals to which we turn next.

#### 3.4 An internal tension

Consider the situation of the second half of the footnote: you believe for certain that 'you did not eat the cake'—that is you believe for certain that  $\neg p$ . We are interested in the counterfactual 'If you had not eaten the cake, you would have had a stomach ache'. On Ramsey's inferential account, you believe 'if p then q' if there is a properly constrained r such that  $(p \land r) \rightarrow q$  instantiates one of your variable hypotheticals  $\forall x(\phi(x) \rightarrow \psi(x))$ . As outlined above, the inferential Ramsey Test applies straightforwardly to this situation. By contrast, the putative redescription of Ramsey's inferential account in terms of degrees of belief cannot handle the present situation. On this degrees of belief account, you believe 'if p then q' if your degree of belief in q given  $p_r$  is appropriate. Now, if you believe  $\neg p$  for certain,  $P(p_r) = 0$ . And so your conditional degree of belief  $P(q \mid p_r)$  is undefined. As outlined above, the degrees of belief account does not apply to the present situation.<sup>25</sup>

<sup>&</sup>lt;sup>25</sup> The degrees of belief account does at least not apply without modification. As pointed out in footnote 3, using Rényi or Popper measures instead of the standard Kolmogorov measures may render the degree of belief account applicable. But the former probability measures have only been developed after Ramsey's life time. It would, furthermore, be implausible that  $P(p_r \to q) = P(q \mid p_r)$  in general—even if P is a Rényi or Popper measure. After all, the material implication is true when  $\neg p_r$ , whereas this case seems irrelevant for the conditional probability of q given p.



From Eq. (2) Here is a proof. Since  $p_r \to q$  is logically equivalent to  $\neg(p_r \land \neg q)$ ,  $P(\neg(p_r \land \neg q)) = 1$ . By the probability calculus, we obtain  $1 - P(\neg(p_r \land \neg q)) = P(p_r \land \neg q) = 0$ . Under the assumption that  $P(p_r) > 0$ , we obtain  $\frac{P(p_r \land \neg q)}{P(p_r)} = P(\neg q \mid p_r) = 0$ . As  $P(\neg q \mid p_r) + P(q \mid p_r) = 1$ , we may conclude that  $P(q \mid p_r) = 1$ .

<sup>&</sup>lt;sup>24</sup> The proof has already been carried out by, for instance, Edgington (2005). In general,  $P(p_r \to q) \ge P(q \mid p_r)$ . For  $P(q \mid p_r)$  to be defined, we have  $1 \ge P(p_r) > 0$ . But then  $\frac{P(p_r \land \neg q)}{P(p_r)} \ge P(p_r \land \neg q)$ . Hence,  $P(\neg q \mid p_r) \ge P(p_r \land \neg q)$ . By  $P(\neg q \mid p_r) + P(q \mid p_r) = 1$  and  $P(p_r \land \neg q) + P(\neg (p_r \land \neg q)) = 1$ , we obtain  $P(\neg (p_r \land \neg q)) \ge P(q \mid p_r)$ . But this is just a way to state  $P(p_r \to q) \ge P(q \mid p_r)$ . As we can easily observe, the  $\ge$  sign in  $\frac{P(p_r \land \neg q)}{P(p_r)} \ge P(p_r \land \neg q)$  turns into a = sign iff  $P(p_r \land \neg q) = 0$  or  $P(p_r) = 1$ . Therefore, if  $P(p_r \land \neg q) \ne 0$  and  $P(p_r \land \neg q) \ne 0$  and  $P(p_r \land \neg q) \ne 0$  and  $P(p_r \land \neg q) \ne 0$  is much greater than  $P(p_r \land q)$ . Suppose  $P(p_r) = .1$ ,  $P(p_r \land q) = .01$ ,  $P(p_r \land \neg q) = .09$ , and  $P(q \mid p_r) = .1$ . It follows that  $P(p_r \to q) = .91$ , because  $P(p_r) \Rightarrow P(p_r) \Rightarrow P(p_r \land \neg q) \Rightarrow P(p_r) \Rightarrow P(p_r \land \neg q) \Rightarrow P(p_r \land$ 

Is it a problem for Ramsey's account that the inferential account applies to the considered situation, whereas the degrees of belief account does not? There are at least two answers. The first says yes. Ramsey does not have a unified account for conditionals. Rather he proposes two accounts that do not fit together. The degrees of belief account does not merely refine the inferential account to include cases of uncertainty. As shown above, the probability of a material implication and the conditional probability come apart when the probability of the antecedent is non-extreme and the probability of the antecedent and the negation of the consequent is not zero. Hence, even if both accounts are applicable, they will have a different result in many cases that involve uncertainty. We could not find any textual evidence supporting that Ramsey put forth two unrelated accounts. Indeed, he thinks that degrees of belief "in the imaginary case" can have meaning (GPC, p. 248). And his inferential account is meant to apply to "hypotheticals in general" (ibid.). Ramsey's test in the footnote seems to be part of his more general account of conditionals.

The second answer is no, there is no problem for Ramsey's account of conditionals. There is only one account of conditionals to be found. However, in cases where you believe  $\neg p$  for certain, you only use the inferential account. In cases of uncertainty, you use the degrees of belief account only. Since the two accounts do not give the same results in general, we need to decide on one. Ramsey seems to 'clearly' favour the degrees of belief account for cases of uncertainty. In brief, Ramsey has but one account of conditionals consisting of two sub-accounts that share the workload.

Even if you think there are two sub-accounts that share the work, an issue arises. We may, and sometimes do, utter conditionals that make a probabilistic statement, such as 'If you had eaten the cake, it would have been very likely that you would have had a stomach ache', or even 'If you had eaten the cake, it is four times more likely than not that you would have had a stomach ache'. The issue is simple to express: which of the two accounts is meant to apply? The answer is tricky. It seems that Ramsey's inferential account does not apply, because the conditionals express uncertainty.<sup>26</sup> Ramsey's degrees of belief account cannot apply, because you assign the antecedent probability zero. Both sub-accounts do not apply. A unified account of conditionals, however, ought to apply to such cases. So either Ramsey does not provide a unified account for qualitative and quantitative conditionals after all, or there is a mistake in Ramsey's account. Observe that the whole tension between the two sub-accounts arises only due to the artefact that a conditional probability is undefined for zero-probability antecedents. Perhaps, there is only a technical problem while the spirit of Ramsey's account is dead-on. In Sect. 5, we will argue for this claim, in particular, that Ramsey's interpretation of conditional degrees of belief by conditional probabilities is mistaken with respect to his inferential account of conditionals. First, however, we will compare Ramsey's account of conditionals to Stalnaker's possible worlds semantics.

<sup>&</sup>lt;sup>26</sup> Perhaps, one could modify Ramsey's inferential account such that it applies to cases of uncertainty iff the antecedent is false. The variable hypotheticals needed would be of the logical form  $\forall x (\phi(x) \to P(\psi(x)) = a)$ , or  $P(\phi(x) \to \psi(x)) = a$  for all x and some number  $a \in [0, 1]$ . However, we could not find any trace of such a modification in Ramsey's works.



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# 4 Comparison to possible worlds semantics

Ramsey's work has been a source of inspiration. His idea to relate degrees of belief to conditionals has become influential in the tradition of Adams (1965) that seeks to give a semantics for conditionals in terms of conditional probabilities. Ramsey's inferential account has been taken up by Stalnaker (1968). He develops a possible worlds semantics of conditionals based on "a suggestion made some time ago by F.P. Ramsey" (p. 101). Here, we compare Ramsey's account of conditionals to Stalnaker's possible worlds semantics. Based on the comparison, we develop a novel qualitative Ramsey Test which we dub "Stalnaker's Ramsey Test".

# 4.1 Stalnaker's possible worlds semantics

Stalnaker (1968, p. 101) describes Ramsey's suggestion for the evaluation of conditionals as follows:

add the antecedent (hypothetically) to your stock of knowledge (or beliefs), and then consider whether or not the consequent is true. Your belief about the conditional should be the same as your hypothetical belief, under this condition, about the consequent.

This quote resembles Ramsey's footnote indeed. In a nutshell, you should believe a conditional if you come to believe the consequent to be true under the assumption of the antecedent.<sup>27</sup>

As we have seen in Sect. 3, Ramsey's inferential account of conditionals covers the cases where you believe the antecedent to be false. Stalnaker (1968, p. 101) remarks that "Ramsey's suggestion covers only the situation in which you have no opinion about the truth value of the antecedent". It seems as if Stalnaker is limiting 'Ramsey's suggestion' in the footnote by interpreting the phrase 'in doubt as to p' as having no belief about p's truth value. Stalnaker goes on to generalize 'Ramsey's suggestion' to cover cases, where the antecedent is believed to be true or false, respectively. Stalnaker's 'generalization' of the Ramsey Test reads as follows:

First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true. (p. 102)

Based on this coinage of the Ramsey Test, Stalnaker develops a possible worlds semantics. A possible world is a way the world might be, might have been, could be, or could

<sup>27</sup> Ramsey switches in his footnote between knowledge and belief. Similarly, Stalnaker's paraphrase of Ramsey's evaluation procedure leaves it open whether the antecedent is added to the agent's knowledge or merely her beliefs. Like knowledge and certain belief, suppositions are characterized by certainty. For Ramsey, it seems that knowledge is true full belief acquired by a reliable method. A thorough investigation of the difference between knowledge, supposition, and belief would go beyond the confines of this paper. However, we think it is crucial that the stock of knowledge contains only certain beliefs. And so we proceed upon the assumption that the important aspect of Ramsey's adding the antecedent to the 'stock of knowledge' is that the antecedent is treated as if it were certain.



have been. <sup>28</sup> Stalnaker says that a possible world may figure as the "ontological analogue of a stock of hypothetical beliefs." (p. 102) In later work, he represents an agent's belief state by a set of possible worlds (Stalnaker, 1996, 2006). The idea is that an agent believes to be true what is true in all possible worlds she cannot exclude to be the actual world. To believe p to be true is modelled by p being true in all worlds the agent cannot exclude to be the actual world. The belief in a proposition p is thus represented by a set of possible worlds that satisfy p.

We may model Stalnaker's version of the Ramsey Test as follows. Your stock of beliefs is a set of worlds you cannot exclude to be actual. Now, supposing the antecedent is modelled by moving from any world w in your stock of beliefs to the respective possible world  $w_p$  that satisfies the antecedent and is otherwise minimally different from w. According to Stalnaker's paraphrase of the Ramsey Test, it remains to consider whether or not the consequent is true in all the possible worlds  $w_p$ . If so, Stalnaker's Ramsey Test is satisfied; if not, not.

Stalnaker uses Kripke's (1963) framework for modal logic to implement his possible worlds semantics. According to this semantics, a conditional is true at a world w just in case its consequent is true in the possible world that satisfies the antecedent and is otherwise most similar to w. For brevity, let us call a possible world that satisfies p a 'p-world'. The semantic clause of the Stalnaker conditional can then be expressed as follows: p > q is true at a possible world w iff q is true in the p-world most similar to w. By this truth condition relative to a possible world we can model when an agent believes a conditional. An agent believes 'if p then q' understood as p > q iff p > q is true at each possible world w the agent cannot exclude to be the actual.<sup>29</sup> In this sense, we may model the belief condition for a conditional by its truth condition at possible worlds.

The evaluation of a Stalnaker conditional requires an assessment of similarity between the considered possible worlds. For this purpose, we use a Lewisian order of similarity. The similarity order  $\leq$  specifies how similar the possible worlds are to the considered world. That is, the similarity order assigns each world w an order  $\leq_w. v \leq_w u$  means that v is at least as similar to the point of evaluation w than w. For convenience, we define the strict relation < induced by  $\leq: w' <_w w''$  iff  $w' \leq_w w''$  and  $w'' \not\leq_w w'$  for all elements w, w', w'' in the set of (accessible) worlds w. Stalnaker imposes a constraint on the similarity order  $\leq:$  any world is uniquely most similar to itself. This strong reflexivity constraint, or so-called 'strong centering', can be expressed as follows: for all  $w, w' \in w$ ,  $w <_w w'$ . Furthermore, Stalnaker's uniqueness assumption translates in our framework into the following condition: there is a unique most similar p-world, if there is one at all. That is, for any  $\phi$  such that

<sup>&</sup>lt;sup>30</sup> Stalnaker's original semantics uses a world selection function. For Stalnaker's presentation of his semantics see Stalnaker and Thomason (1970). For the correspondence between selection functions and similarity orderings, consult, e.g. Lewis (1973a), in particular Sects. 2.7 and 3.4.



<sup>&</sup>lt;sup>28</sup> See Stalnaker (2003) for details on what he takes a possible world to be.

<sup>&</sup>lt;sup>29</sup> Using epistemic logic, we can summarize our Ramsey Test as follows: believe 'if p then q' iff B(p > q), where B is a belief operator (van Benthem, 2006; Baltag and Renne, 2016).

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there is a  $\phi$ -world in the model:  $w' <_w w''$  for some world w' of all  $\phi$ -worlds w''. We say that w' is the unique most similar  $\phi$ -world under  $\leq_w$  and write  $w' = \min_{\leq_w} [\phi]$ . <sup>31</sup>

### 4.2 Ramsey and Stalnaker

Above, we have claimed that Stalnaker's possible worlds semantics supplemented by a model of belief resembles Ramsey's inferential account. His inferential account specifies when an agent believes a conditional. You believe 'if p then q' if there is an r such that  $(p \land r) \to q$  instantiates a variable hypothetical. Stalnaker's semantics, by contrast, specifies when a conditional is true at a possible world. p > q is true at a possible world w iff q is true in the most similar p-world  $\min_{\leq_w} [p]$  (or there is no p-world). In the last section, we have supplemented Stalnaker's semantics by a simple belief model. We dub the result Stalnaker's  $Ramsey\ Test$ : you believe 'if p then q' iff p > q is true at each world in your stock of beliefs. On a high level of abstraction, both Ramsey's inferential account and Stalnaker's Ramsey Test are a specification of the suppositional scheme: you believe a conditional 'if p, then q' when you believe q upon supposing p. In this section, we argue that the accounts have much more in common.

The first point of comparison concerns the supposition of the antecedent p. In order to believe a conditional both accounts require to suppose p. Ramsey's inferential account models the supposition of p by treating it as if it were true. The conditional is believed only when this supposition, together with other beliefs in r, triggers a variable hypothetical to infer the consequent q. On Stalnaker's Ramsey Test, the supposition of p is modelled by moving to the most similar p-worlds. This moving implements that the antecedent p is treated as if it were true. The conditional is only believed when the consequent q is true in each p-world that is uniquely most similar to the respective candidate for the actual world in the stock of beliefs.

The second point of comparison concerns the structure of the possible. On Ramsey's inferential account, the space of possibilities is structured by variable hypotheticals or laws. This role is played by the similarity order between possible worlds on Stalnaker's Ramsey Test. A similarity order between possible worlds can be defined in terms of laws. Suppose you believe 'If I had eaten the cake, I would have had a stomach ache'. The variable hypothetical that supports the conditional can be translated into the following similarity judgment: possible worlds where you ate the cake and you have a stomach ache are more similar to actuality than possible worlds where you ate the cake but you do not have a stomach ache. If you are equipped with enough variable hypotheticals, you can summarize the induced similarity judgments in an order. Sa

Lewis (1973b) proposes such a possible worlds semantics—even though he interprets the worlds as metaphysical and not epistemic possibilities.



<sup>&</sup>lt;sup>31</sup> Further conditions are usually imposed on the similarity order, such as transitivity and connectivity. We omit a discussion of these conditions, as we will not need them explicitly in what follows. The technically-minded reader may conceive of each  $\leq_w$  as a total well-order over the considered possible worlds.

<sup>&</sup>lt;sup>32</sup> See, for instance, Lewis (1973a, Sect. 3.3), Lewis (1979) and Halpern (2013).

<sup>&</sup>lt;sup>33</sup> Here the question arises whether Stalnaker's assumption that there is always a unique most similar world is plausible. On Ramsey's account, the question would be whether an agent has always sufficiently many and sufficiently expressive variable hypotheticals such that one most similar world is singled out. If not, we need to consider a set of most similar possible worlds as opposed to just a single possible world. (Footnote 33 continued)

The third point of comparison concerns which information is imported from actuality. On Ramsey's account, the imported information is denoted by r. r is constrained by (i) the meaning of the supposition p, (ii) the believed variable hypotheticals or laws, and (iii) the facts and laws believed to be true in the first place. Now, r corresponds to the set of most similar p-worlds on Stalnaker's account. Each most similar world  $\min_{\leq_w} [p]$  is determined by (i') the meaning of p, (ii') the similarity order  $\leq$  between worlds as a proxy for the variable hypotheticals, and (iii') the candidate w for the actual world.

Ramsey's context  $p \wedge r$  for evaluating the consequent is constrained by the requirement that r must be true and a conjunction of propositions. Hence, r imports information from the actual context which ties the hypothetical situation back to actuality. Under the supposition of p, r renders the context of evaluation similar to the actual situation. Recall that p and r must be compatible in the sense that the supposition of p does not lawfully imply  $\neg r$ . The r is thus constrained in the sense that  $p \to \neg r$  does not instantiate any believed variable hypothetical. This is how variable hypotheticals determine the context of evaluation for the consequent, viz.  $p \wedge r$ .<sup>34</sup> If anything, r imports actual 'laws and facts' compatible with p and relevant to the consequent q. In fact, r may import as many actual laws and facts as compatible with p. Such an extensive r results in an ample context of evaluation  $p \wedge r$  very much like a set of most similar p-worlds from the respective 'not-excluded-to-be-actual' worlds. Like the ample  $p \wedge r$ , any most similar p-world satisfies p and otherwise differs minimally from 'its' actual world. Each most similar p-world ensures that p is supposed and, due to the similarity order, determines a context of evaluation for q that is as much like actuality as possible. Hence, moving to the most similar p-worlds corresponds to supposing p and importing contextual information, viz. the information that is shared by all most similar p-worlds. The most similar p-worlds figure in Stalnaker's Ramsey Test just like the conjunction  $p \wedge r$  in Ramsey's inferential account.

For illustrative purposes, let us assume that our agent can exclude any possible world to be actual except one. Her belief state contains only one world  $w_@$  she thinks to be actual. Hence, she has an opinion on any proposition. She believes any proposition to be either true, or else false. Our opinionated agent thinks she knows it all. She is a *Besserwisser* par excellence. By contrast, if an agent cannot exclude any but one world, he might be undecided about particular propositions. For instance, if he cannot exclude from being actual a world that satisfies p and a world that satisfies p, he believes neither p nor p.

For a *Besserwisser*, the belief condition of p > q collapses to the truth condition of p > q at  $w_{@}$ . Hence, she believes p > q or  $p > \neg q$  (due to the assumption that there is a unique most similar p-world if there is any). Either p is a contradiction and so she believes both conditionals trivially; or there is exactly one most similar p-world and this world either makes q true, or else  $\neg q$ . The Stalnaker semantics validates Conditional Excluded Middle: at least one of p > q or  $p > \neg q$  is true at any world. Except for *Besserwissers*, belief states contain more than one possible world. Relative to the worlds in the stock of belief, if one most similar p-world satisfies q and another

 $<sup>^{34}</sup>$  It remains open whether the variable hypotheticals determine the context of evaluation uniquely. Ramsey's account demands only that there is some r indicating that he does not impose the requirement of a unique context of evaluation.



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satisfies  $\neg q$ , the agent believes neither p > q nor  $p > \neg q$ . But if the agent believes p > q in a non-trivial way, she does not believe  $p > \neg q$ . This fits with Ramsey's second property: either the agent believes p > q, or she does not. The latter case has two subcases, either she believes  $p > \neg q$ , or she simply does not believe p > q.

Let us consider some cases to see more clearly that Ramsey's context of evaluation  $p \wedge r$  and Stalnaker's most similar p-world play a similar role. Assume 'if p then q'. Then r imports contextual information that allows to infer the consequent q from p. Similarly, the most similar p-world provides a context in which q is true. In cases, where you believe p to be the case, r contains information from the actual context; in this case, the most similar p-world from any w you cannot exclude to be actual is w itself, that is  $\min_{\leq_w} [p] = w$ . This follows from Stalnaker's constraint that any world is uniquely most similar to itself. Hence, like on Ramsey's account, these most similar p-worlds import only information that is believed to be actual.

In cases, where you believe  $\neg p$  is actual, r must be restricted. It cannot contain any proposition that is believed to be true but would contradict the antecedent p. Hence, r cannot contain  $\neg p$  which is believed to be true. In the considered case, all the worlds you cannot exclude to be actual are  $\neg p$ -worlds. Hence, the set of most similar p-worlds is disjoint from the set of worlds that represents part of your belief state (the other part being the similarity order). Your hypothetical belief state, represented by the set of most similar p-worlds, does not contain any proposition that contradicts p. When you explore the consequences of an antecedent you believe to be false, you keep using the variable hypotheticals and the similarity order, respectively. Thereby, the conditionals given by Ramsey's inferential account and Stalnaker's Ramsey Test both go beyond the actual, that is they can express mere possibilities.

There are further similarities between Ramsey's inferential account and Stalnaker's Ramsey Test. Like believing the Ramsey conditional, believing the Stalnaker conditional p > q implies to believe the material implication  $p \to q$ . For, if p is true in any candidate for the actual world, the Stalnaker conditional requires q to be true there as well; and if p is not true there, this world satisfies  $\neg p$ , which is sufficient to make the material implication true.

Furthermore, like Ramsey's conditional, Stalnaker's Ramsey Test conditional invalidates Contraposition. Suppose the Stalnaker conditional 'If she came to the meeting, she would have left it by now' is believed. This means that the respective most similar worlds, where she came to the meeting, are worlds in which she has left it by now. However, the previous sentence says nothing whatsoever about the most similar world, where she did not leave the meeting by now. In fact, these most similar worlds satisfy that she came to the meeting. This provides the same counterexample to Contraposition we have discussed for Ramsey's conditional in Sect. 3.2.<sup>35</sup> We have seen that the two accounts show striking similarities in the cases, where the antecedent is believed to be false.

Somewhat surprisingly, Ramsey's conditional validates and-to-if presupposed the agent believes logical truths as variable hypotheticals (as is rational). If you believe p and q to be true, then you believe if p then q. For, then, there is an r that contains p

<sup>&</sup>lt;sup>35</sup> Interestingly, Ramsey's account and Stalnaker's Ramsey Test each seem to validate the following form of Modus Tollens. If the conditional 'If p, then q' is believed, then believing  $\neg q$  entails that you do not believe p. For, if you were to believe p, you would believe q.



and q such that  $(p \land r) \to q$  is an instance of the tautology  $\forall x (\phi(x) \to \phi(x))$ . It is well known that the Stalnaker semantics validates and-to-if. So if you believe p and q, you believe p > q as well.

Perhaps even more surprisingly, Ramsey's conditional does not validate Antecedent Strengthening: if you believe if p then q, then you do not necessarily believe if  $p \wedge p'$ then q. The reason is that, for if p then q to be believed, the antecedent p and r must be compatible in the sense that  $p \to \neg r$  is not an instance of any law. But strengthening p to  $p \wedge p'$  requires then that  $(p \wedge p') \rightarrow \neg r$  is not an instance of any law. Even if  $p \to \neg r$  is not an instance of any law, and so if p then q may be believed, it may be the case that  $(p \land p') \rightarrow \neg r$  is an instance of some law. And so if  $p \land p'$ then q is not believed. You believe, for example, "if you regularly practice Yoga, you will see some health benefits", where r contains r' "you don't injure yourself". Now, you do not believe "if you regularly practice Yoga and you injure yourself, you will see some health benefits". For the strengthened antecedent is not compatible with r any more:  $(p \land \neg r') \rightarrow \neg r$  is an instance of the tautological variable hypothetical  $\forall x(\phi(x) \to \phi(x))$ : if you injure yourself, you injure yourself. (Note that  $\neg r'$  implies  $\neg r$ .) To sustain compatibility you need to shrink your r and thereby you may lose certain consequences. The invalidity of Antecedent Strengthening is of course one of the hallmarks of the Stalnaker semantics and any other variably strict analysis of conditionals.

The resemblance of Ramsey's inferential account and Stalnaker's Ramsey Test can be further illustrated by relating them to each other. As we have seen above, one has three options as regards the adoption of a variable hypothetical. One can believe  $\forall x(\phi(x) \to \psi(x)), \forall x(\phi(x) \to \neg \psi(x))$ , or simply neither. In the latter case, one denies to make an inference regarding  $\psi$  when a  $\phi$  turns up. Let us exemplify the three cases by toy models of possible worlds. In all of them, an agent believes none of  $\phi$ ,  $\neg \phi$ ,  $\psi$ ,  $\neg \psi$ .

In the first Stalnaker model, the agent comes to believe the variable hypothetical  $\forall x (\phi(x) \to \psi(x))$ . This variable hypothetical can be translated into the following constraint: for all worlds the agent cannot exclude to be the actual, the most similar  $\phi$ -world is a  $\psi$ -world.<sup>36</sup> Abstracting away from other propositions, the belief state can be represented by Fig. 1.

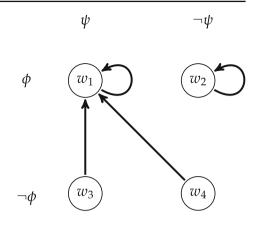
Initially, the agent modelled in Fig. 1 cannot exclude  $w_1, w_2, w_3$  and  $w_4$  to be the actual world. Under the constraint derived from the variable hypothetical  $\forall x (\phi(x) \rightarrow \psi(x)), w_1$  is the most similar  $\phi$ -world as seen from  $w_3, w_4$ , and  $w_1$  itself. Due to the reflexivity constraint Stalnaker imposes on his semantics,  $w_2$  must be its most similar  $\phi$ -world. The Stalnaker conditional  $\phi > \psi$  is true at  $w_1, w_3$  and  $w_4$ . It is false at  $w_2$ . Believing  $\forall x (\phi(x) \rightarrow \psi(x))$  entails that  $w_2$  must be excluded to be the actual world. And this makes sense: if the agent meets a  $\phi$  she ought to regard it as a  $\psi$ . Hence, any  $\phi \land \neg \psi$ -world is judged to be non-actual.

The variable hypothetical or law  $\forall x (\phi(x) \rightarrow \psi(x))$  can be translated into the above constraint. If the above constraint is satisfied, an agent believes the Stalnaker conditional  $\phi > \psi$ . If the Stalnaker conditional gives the form of a law, the material

<sup>&</sup>lt;sup>36</sup> We move here from a variable hypothetical—a rule expressing 'If I meet a  $\phi$  I shall regard it as a  $\psi$ '—to a quantification over possible worlds. This translation does not retain the universal quantifier ranging over individuals or objects in Ramsey's explicit symbolisation  $\forall x (\phi(x) \to \psi(x))$ .



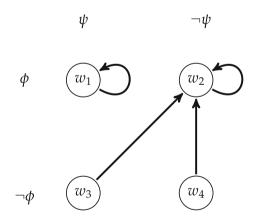
Fig. 1 A four-worlds Stalnaker model illustrating the belief in the variable hypothetical  $\forall x (\phi(x) \rightarrow \psi(x))$ . The arrows indicate which world is the respective most similar  $\phi$ -world from the world, where the respective arrow originates. The variable hypothetical can be conceived of as the constraint imposing on the belief state that, for all candidate worlds, the most similar  $\phi$ -world is a  $\psi$ -world.  $w_2$  does not satisfy this constraint and is thus excluded from the candidates for the actual world



implication  $\phi \to \psi$  gives the form of an accidental generalization, like 'Everyone in Cambridge voted for the motion'. They make the same claim about the actual world: any actual  $\phi$  is a  $\psi$ . However, the Stalnaker conditional makes a stronger claim: any  $\neg \phi$  would be a  $\psi$  if it were a  $\phi$ . The Stalnaker conditional makes a claim about mere possibilities, just like variable hypotheticals.

Let us model the situation where our agent comes to believe the variable hypothetical  $\forall x(\phi(x) \to \neg \psi(x))$ . This variable hypothetical can be translated into the following constraint: for all candidate worlds, the most similar  $\phi$ -world is a  $\neg \psi$ -world. The epistemic situation is symmetric to the situation modelled in Fig. 1. This time the  $\phi \land \psi$ -world  $w_1$  is excluded from the candidate worlds upon believing  $\forall x(\phi(x) \to \neg \psi(x))$ . The belief state can be represented by Fig. 2.

Fig. 2 A four-worlds Stalnaker model illustrating the belief in the variable hypothetical  $\forall x (\phi(x) \rightarrow \neg \psi(x))$ . The variable hypothetical can be conceived of as the constraint imposing on the belief state that, for all candidate worlds, the most similar  $\phi$ -world is a  $\neg \psi$ -world.  $w_1$  does not satisfy this constraint and is thus excluded from the candidates for the actual world

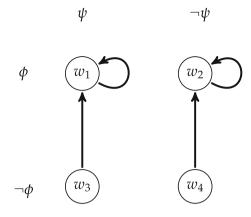


Let us model a belief state, where neither  $\forall x(\phi(x) \to \psi(x))$  nor  $\forall x(\phi(x) \to \neg \psi(x))$  is believed. The constraint derived from  $\forall x(\phi(x) \to \psi(x))$  is violated: there is a most similar  $\phi$ -world from  $w_4$  that is a  $\neg \psi$ -world. Similarly, the constraint derived from  $\forall x(\phi(x) \to \neg \psi(x))$  is violated: there is a most similar  $\phi$ -world from  $w_3$  that is a  $\psi$ -world.



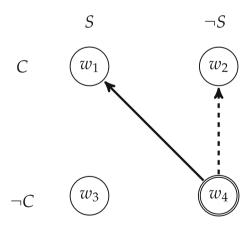
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Fig. 3 A four-worlds Stalnaker model, where neither the variable hypothetical  $\forall x (\phi(x) \rightarrow \psi(x))$  nor the variable hypothetical  $\forall x (\phi(x) \rightarrow \neg \psi(x))$  is believed



In the above cake example, we agree that you did not eat the cake  $(\neg C)$  and that you do not have a stomach ache  $(\neg S)$ . However, we adopt different variable hypotheticals. By the translational constraint, we believe different conditionals. You believe C > S, whereas I believe  $C > \neg S$ . In Fig. 3, you think that  $w_1$  would be actual if C were true, whereas I think  $w_2$  would be actual under this supposition. Hence, although we both believe  $\neg C$  and  $\neg S$ , we have a disagreement about the merely possible. Notice that a less pronounced disagreement occurs in case you would believe neither variable hypothetical. This case would correspond to the situation, where you deny the inferability of the consequent from the antecedent.

Fig. 4 A four-worlds Stalnaker model illustrating the cake example.  $w_4$ , so we agree, is the actual world. The arrow indicates that you think the most similar C-world is a S-world. The dashed arrow indicates that I think the most similar C-world is a  $\neg S$ -world



We may conclude that Ramsey's inferential account and Stalnaker's Ramsey Test bear rather strong resemblances. To the best of our knowledge, no one has pointed out that Ramsey's inferential account is a close relative—at least in spirit—to Stalnaker's account of conditionals supplemented by a belief state.<sup>37</sup> Admittedly, the

<sup>&</sup>lt;sup>37</sup> One could make a case that Stalnaker (1968) himself pointed out that his account is in Ramsey's spirit.
Yet Stalnaker does not compare the two accounts in detail. In fact, Stalnaker overlooked that Ramsey's



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non-existence of a link between the two accounts might just be a by-product of a more general neglect of Ramsey's inferential account. In any case, we have established the resemblance between the accounts.

As an aside to our comparison, consider the remarks by Sahlin (1990, p. 123):

Ramsey's theory [of conditionals and general propositions] also shows us how wrong most of the 'possible world' semantics are. The similarity relation that, according to these theories, exists between different possible worlds is assumed to be quite disconnected from whichever general proposition we accept for the moment. But the above reasoning suggests that to obtain a reasonable system this similarity relation must be linked with a system of laws.

We have illustrated above how general propositions in the sense of Ramsey's variable hypotheticals can be related to the similarity relation between possible worlds. In fact, we have shown how to ground an agent's similarity order between worlds in terms of the variable hypotheticals the agent believes. To do this, we only needed Stalnaker's semantics of conditionals and our model of an agent's belief state in terms of possible worlds. Pace Sahlin, we have demonstrated that the similarity relation need not be disconnected from the believed variable hypotheticals. Rather the similarity relation can be linked to (a set of) variable hypotheticals.

### 5 A unified account of conditionals

The time has come to propose a tenable unified account of conditionals. We will briefly review the tension within Ramsey's account. A diagnosis of the tension suggests that Ramsey committed a mistake by interpreting conditional degrees of belief in terms of conditional probabilities. We will give the concept of conditional degree of belief a new interpretation. This re-interpretation reveals a unified account of conditionals in Ramsey's spirit.

In Sect. 3.4, we have seen that there is a tension in Ramsey's account of conditionals. His inferential account is not fully in line with his degrees of belief account. He formalises the conditional degree of belief in q given p by the conditional probability  $P(q \mid p)$ . Now,  $P(q \mid p)$  is undefined when the agent believes  $\neg p$  for certain—that is the agent has the degree of belief P(p) = 0. Ramsey's notion of degrees of belief cannot apply to zero probability antecedents. By contrast, his inferential account applies to such counterfactuals, or better counterdoxasticals, without further ado. But the inferential account does not give the same results in cases of uncertainty; and in these cases, Ramsey insists that we apply the uncertain variable hypotheticals that have the form of conditional probabilities, that is  $P(q \mid p) = a$  with  $a \in (0, 1)$ . Hence, it seems as if Ramsey's account cannot handle counterfactuals that express an uncertain relation between its antecedent and its consequent.

account applies to antecedents believed to be false, as we have already mentioned in the Introduction. It is plausible that he overlooked Ramsey's inferential account as well, focusing instead exclusively on parts of Ramsey's footnote.



<sup>(</sup>Footnote 37 continued)

The tension arises only because the conditional probability  $P(q \mid p)$  is undefined when P(p) = 0. The supposition of p as modelled by the 'given p' of a conditional probability is undefined whenever  $P(\neg p) = 1$ . The whole probabilistic mass is associated to the  $\neg p$ -worlds. Conditioning on p cuts off all the  $\neg p$  worlds, and so there is no probabilistic mass left (to be renormalized). It is thus implausible to model the supposition of a mere possibility by the notion of a conditional probability. After all, we can suppose what we certainly believe not to be actual. Even after we know that you have not eaten a cake, we can suppose you had. Hence, the culprit for the tension is to model a supposition by the notion of conditional probability, or so we propose.

The tension suggests that Ramsey commits a mistake by using the notion of conditional probability to model the supposition of the antecedent. To be precise, Ramsey's mistake is to formally implement the supposition of an antecedent by conditioning on this antecedent. And this mistake turned out to be highly influential. Inspired by Ramsey's degrees of belief account, Adams stipulates the probability of a simple indicative conditional as the conditional probability of the consequent given its antecedent. Let A and C be propositions that contain no conditional. Adams (1975, p. 3) states the following thesis:<sup>38</sup>

$$P(\text{if } A \text{ then } C) := P(C \mid A).$$

Lewis (1976) has shown that this stipulation does not hold for Stalnaker's conditional. Apart from trivial cases, the probability of a Stalnaker conditional does not equal the corresponding conditional probability. That is, in general, we have  $P(A > C) \neq P(C \mid A)$ . This is bad news for Ramsey's account of conditionals, as we have shown that his inferential characterisation of conditionals is related to the Stalnaker semantics.<sup>39</sup> Of course, at the time Ramsey was developing his theory of conditionals, Lewis's triviality result has not yet been shown.

In light of Lewis's (1976) triviality result, it is dubious whether conditional probabilities are the right tool to analyse conditionals. Luckily, an alternative to Bayesian conditionalization on the antecedent is not far to seek. In the same paper, Lewis has found a probabilistic updating rule, which he named imaging. Imaging on some proposition A transfers the probability shares associated to  $\neg A$ -worlds to the respective most similar A-worlds. We may interpret imaging on A as another way to come to believe A with certainty. Notably, the probability of a Stalnaker conditional equals the probability

<sup>&</sup>lt;sup>39</sup> One might argue that the situation is even worse. A number of authors have generalized Lewis's triviality result showing that there is *no* proposition that corresponds to the conditional probability. See Hájek (2011) and Fitelson (2015) for a recent state of the art on triviality results. Williams (2012) proposes a generalization of Lewis's triviality results to counterfactual conditionals. And Raidl (2019) has shown that Lewis's triviality results hold also for "quasi probabilities", certain plausibility measures that are weaker than probabilities.



 $<sup>^{38}</sup>$  Adams thinks that conditionals do not express propositions. The probability on the left-hand side is thus not interpreted as the probability that the conditional is *true*. P(if A then C) is rather interpreted as the degree to which the conditional is *assertible*. Hence, Adams's conditional lacks truth compositionality: it cannot be combined with propositions by truth-functional connectives. It would be unclear how to evaluate the truth of the combination. On the positive side, Adams's thesis is not susceptible to Lewis's triviality results because it does not speak of propositions. For details, see Adams (1965, 1966, 1975).

of its consequent after imaging on the antecedent. That is:

$$P(A > C) = P^{A}(C)$$
, provided A is possible.

By replacing the updating rule of conditionalization with imaging, Adams's stipulation becomes a theorem for Stalnaker's conditional.

Lewis (1976, p. 310) defines imaging as follows.

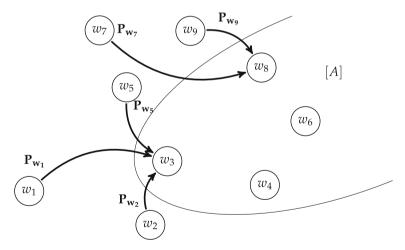
# **Definition 1 Imaging**

For each probability function P over a finite set of worlds, and each formula A, there is a probability function  $P^A$  such that, for each world w', we have:

$$P^{A}(w') = \sum_{w \in W} P(w) \cdot \begin{cases} 1 & \text{if } w_{A} = w' \\ 0 & \text{otherwise} \end{cases}$$

We say that we obtain  $P^A$  by imaging P on A, and call  $P^A$  the image of P on A.<sup>40</sup>

Figure 5 illustrates what happens when imaging some probability function P on A. The arrows represent the transfer of probability shares from the respective  $\neg A$ -worlds to their most similar A-world. Observe that each probability share remains 'as close as possible' to the world at which it has been previously located.



**Fig. 5** A set of possible worlds on which a probability distribution is defined. The similarity between worlds is represented by the distance between them. Imaging on *A* transfers the probability share of each world to its 'closest' *A*-world

<sup>&</sup>lt;sup>40</sup> Günther (2018, 2017) generalized Lewis's imaging in analogy to Jeffrey's generalisation of Bayesian conditionalization. He shows that his Jeffrey imaging is applicable to the learning of uncertain conditional information and solves Douven's (2012) benchmark examples as well as Van Fraassen's (1981) Judy Benjamin Problem. This set of problems is unresolved for Bayesians who rely on Jeffrey conditionalization or minimization of probabilistic distances (see Douven (2016)).



We have argued that Stalnaker's semantics supplemented by a simple model of belief is rather similar to Ramsey's account of conditionals. Hence, we propose to interpret the notion of suppositional degree of belief by imaged probabilities (rather than conditional probabilities). Ramsey's basic idea is that 'If p then q would probably result' is a degree of belief in q given p. According to him, the 'given' is meant to express a supposition. Yet conditional probabilities are inadequate to model counterfactual suppositions. This indicates that the 'given' should rather be modelled by imaging than conditionalization.

The resemblance of Ramsey's conditional and Stalnaker's Ramsey Test conditional suggests the following unified account of conditionals. An agent believes 'If A then C' iff A > C is true at each world the agent cannot exclude to be actual. The agent's suppositional degree of belief in C if A is thus given by  $P^A(C)$ .

To illustrate our unified account, consider the cake example in Fig. 4 again. You believe the Stalnaker conditional C > S. Hence, imaging on C yields:  $P^C(S) = P^C(w_1) = P(w_4) = 1$ . Your degree of belief that S would be if C were the case equals one. Notice that your degree of belief is suppositional, and even counterfactual: you believe for certain that  $\neg C$  and  $\neg S$ , and so your degree of belief is  $P(\neg C \land \neg S) = 1$ . By contrast, my counterfactual degree of belief that S would be if C were the case equals zero. In my case, under the supposition of C, imaging transfers the probability associated to  $w_4$  to  $w_2$ . Thereby, 'If C then S' and 'If C then  $\neg S$ ' are 'contradictories' in a sense.

Let us dwell on the cake example for a moment. 'Either party believes  $\neg C$  for certain' and yet the two people can have a sensible disagreement. Although their suppositional degrees of belief in C are 'rendered void' for what follows from C in the actual world, they are not undefined. On the contrary, it is specified what would follow under the supposition of C. All of this sounds like a description of Ramsey's account and at the same time like the outlined unified account based on the Stalnaker conditional and imaging. And, by contrast to Ramsey's original account, there is no tension arising on our account. Crucially, due to the additive definition of imaging, it can be applied when the antecedent has probability zero.

Recall that it was unclear which of Ramsey's sub-accounts applies to counterfactuals that express a relation of uncertainty. Consider, for example, 'If you had eaten the cake, it is four times more likely than not that you would have had a stomach ache.' On our account, this issue resolves. Suppose an agent believes none of C,  $\neg C$ , S,  $\neg S$ , and believes C > S. The agent then believes the uncertain counterfactual above iff  $P^C(S) = 4/5$ . Some of the most similar worlds (to the worlds you cannot exclude to be actual), in which you ate the cake, are worlds where you got no stomach ache.

We have seen that our account can handle zero probability antecedents and counterfactuals that express uncertainty. Furthermore, our account can handle nested conditionals. Unlike conditionalization, imaging can be applied to nested conditionals. For instance, the image on the Stalnaker conditional A > C,  $P^{A>C}(E)$ , is well-defined and equals the probability of the nested conditional A > C, A > C, we hence, our account seems to be a promising candidate to provide a rather general account of conditionals.

<sup>&</sup>lt;sup>41</sup> In case there are 'uncertain suppositions', and we are not sure whether there are such cases, we may generalize the proposed unified account by replacing the image on *A* by Günther's (2018) Jeffrey image on *A*.



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It seems as if the tension of Ramsey's account dissolves once conditionalization is replaced by imaging.<sup>42</sup> In Ramsey's life time, imaging and modern possible worlds semantics had not yet been developed. The account we just proposed thus cannot be attributed to Ramsey. However, if one is convinced that Ramsey's inferential account resembles Stalnaker's Ramsey Test for conditionals, and one thinks that his inferential and degree of belief accounts should match, one should consider our account as a unified theory of conditionals in Ramsey's spirit.

#### 6 Conclusion

We have started out with the current orthodoxy that the Ramsey Test is for indicative conditionals only. The Ramsey Test is at the core of Ramsey's account of conditionals. Yet it seems that this account offers two versions of the Ramsey Test, an inferential and a probabilistic one. While the inferential Ramsey Test applies to counterfactual conditionals without further ado, the same cannot be said of the degrees of belief version.

We have established that the inferential account resembles Stalnaker's (1968) possible worlds semantics of conditionals supplemented by a model of belief. From a metaphysical perspective, Ramsey's variable hypotheticals or laws can be used to ground the similarity order of a possible worlds semantics. From a technical point of view, Ramsey's inferential account of conditionals can be seen as an agent-relative predecessor to a possible worlds semantics.

We have observed a mismatch in Ramsey's theory of conditionals between the inferential and degrees of belief aspects. This problem can be repaired by interpreting the concept of conditional degrees of beliefs by imaged probabilities rather than conditional probabilities. Thereby, an account emerges that applies to indicative, subjunctive, and counterfactual conditionals, and independently of whether they express a strict or probabilistic relation between antecedent and consequent. The result is a unified and tenable account of conditionals.

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<sup>42</sup> Of course, Ramsey's account can be rescued in other ways, for instance by introducing two functions for an agent, a degrees of belief function and a objective chance function.



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