REVIEW OF PARTICLE PHYSICS*

Particle Data Group

Abstract

The *Review* summarizes much of particle physics and cosmology. Using data from previous editions, plus 2,717 new measurements from 869 papers, we list, evaluate, and average measured properties of gauge bosons and the recently discovered Higgs boson, leptons, quarks, mesons, and baryons. We summarize searches for hypothetical particles such as supersymmetric particles, heavy bosons, axions, dark photons, etc. Particle properties and search limits are listed in Summary Tables. We give numerous tables, figures, formulae, and reviews of topics such as Higgs Boson Physics, Supersymmetry, Grand Unified Theories, Neutrino Mixing, Dark Energy, Dark Matter, Cosmology, Particle Detectors, Colliders, Probability and Statistics. Most of the 120 reviews are updated, including many that are heavily revised.

The *Review* is divided into two volumes. Volume 1 includes the Summary Tables and 97 review articles. Volume 2 consists of the Particle Listings and contains also 23 reviews that address specific aspects of the data presented in the Listings.

The complete *Review* (both volumes) is published online on the website of the Particle Data Group (pdg.lbl.gov) and in a journal. Volume 1 is available in print as the *PDG Book*. A *Particle Physics Booklet* with the Summary Tables and essential tables, figures, and equations from selected review articles is available in print, as a web version optimized for use on phones, and as an Android app.

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HIGHLIGHTS OF THE 2024 EDITION OF THE REVIEW OF PARTICLE PHYSICS

All PDG data and review articles are available online at **pdg.lbl.gov**.

869 new papers with 2,717 new measurements

- 320 new measurements from *LHC* experiments (ATLAS, CMS and LHCb).
- Extensive *Higgs boson* coverage from 231 new papers and 557 measurements with latest results on mass, couplings, decay width and branching ratios, plus searches for other neutral and charged Higgs bosons.
- *Supersymmetry*: 36 new papers with 148 measurements with major exclusions.
- *Top quark*: 35 new papers provide latest results on mass, coupling, and the discovery of four top quark production at LHC.
- *W mass*: CDF published a new measurement of the W mass. It is more precise than the world average at the time, but the two results disagree. See comments in Particle Listings and our W mass review referencing the extensive studies done by the *LHC* TeV M_W working group.
- *c-, b-hadron physics*: 161 papers and 597 measurements provide world-best data on lifetimes, mixing, CP violation, CKM angles, and constraints on new physics.
- *Neutrino Physics*: 10 new papers with measurements on neutrino mass, mixing, and CP-violation.
- *Meson resonances*: Substantial improvements, including new cross particle fits involving $J/\psi(1S)$, $\psi 2S)$ and $\eta_c(1S)$, merging of $\pi_1(1400)$ and $\pi_1(1600)$ with non-$q\bar{q}$ quantum numbers into same node, listing of complex pole positions when available, and inclusion of recently observed states. Several mesons now considered established added to Summary Tables.
- New *naming scheme* for the many recently discovered mesons and baryons with c or b quarks that are not compatible with conventional $q\bar{q}$ or qqq structures. Listed in “Other mesons” and “Exotic baryons” sections.

120 reviews (most are revised)

- Major revision of *Higgs Physics* review summarizes latest LHC results and constraints on new physics.
- Major revision of *Cosmic rays* review, now covering all components of cosmic rays including neutrinos (which were previously discussed in Neutrino Telescopes in Particle Detectors for Non-Accelerator Physics).
- Major revison of *Large time-projection chambers for rare event detection* in Particle Detectors for Non-Accelerator Physics.
- Major revision of *Top Quark* review summarizes latest LHC results and constraints on new physics from top quark.
- Major revision of *Heavy Non-$q\bar{q}$ Mesons* review follows new naming scheme for states not compatible with $q\bar{q}$ or qqq structures.
- *Semiconductor detectors* section of Particle Detectors at Accelerators review includes recent developments.
- New section on goodness-of-fit tests and expanded discussion of statistical tests in *Statistics* review.
- Updated EWK fit in *Electroweak Model and Constraints on New Physics* review.
- *Semileptonic B Hadron Decays, Determination of V_{cb} and V_{ub}* includes latest results and discusses long-standing issues with inclusive and exclusive measurements.
- Updated cross section results in the *Neutrino Cross Section Measurements* review.
- Updated τ decay parameters and branching ratio (BR) fits, including new treatment of BR$(a1(1260) \to \pi\gamma)$.

VOLUME 1: SUMMARY TABLES AND REVIEWS

SUMMARY TABLES

REVIEWS, TABLES, AND PLOTS

Constants, Units, Atomic and Nuclear Properties

Standard Model and Related Topics

Astrophysics and Cosmology

Experimental Methods and Colliders

Mathematical Tools

Kinematics, Cross-Section Formulae, and Plots

Particle Properties

Gauge Bosons

Charged Leptons

Quarks

(Continued on next page.)

Mesons

Baryons

Hypothetical Particles and Concepts

VOLUME 2: PARTICLE LISTINGS
(available online only)

Illustrative key and abbreviations

Gauge and Higgs bosons

Leptons

Quarks

Mesons

Baryons

Searches not in Other Sections

INTRODUCTION

ONLINE PARTICLE PHYSICS INFORMATION

Introduction

1 Overview

The *Review of Particle Physics* is a comprehensive review of the field of Particle Physics and of related areas in Cosmology. It is divided into two volumes. Volume 1 includes the "Summary Tables" and "Reviews, Tables, and Plots". Volume 2 consists of the "Particle Listings".

The *Review* is updated each year and made available on the PDG website (pdg.lbl.gov). In even-numbered years, the *Review* is also published in a journal and printed as the *PDG Book* together with an abridged *Particle Physics Booklet* containing Summary Tables and essential tables, figures, and equations from selected review articles. This edition is an updating through January 2024.

The Summary Tables give our best values and limits for particle properties such as masses, widths or lifetimes, and branching fractions, as well as an extensive summary of searches for hypothetical particles and a summary of experimental tests of conservation laws.

The 95 review articles in Reviews, Tables and Plots cover a wide variety of theoretical and experimental topics. Together with the Summary Tables they provide an exhaustive reference for the practicing particle physicist. Two more review articles, Online Particle Physics Information and Tests of Conservation Laws, can be found in the introduction and Summary Tables, respectively.

The Particle Listings are a compilation/evaluation of data on particle properties. They contain all the data used to get the values given in the Summary Tables. They also give information on unconfirmed particles and particle searches. In this edition, the Particle Listings include 2,717 new measurements from 869 papers, in addition to the 46,838 measurements from 12,909 papers that first appeared in previous editions [1]. 23 review articles are part of the Particle Listings and address specific aspects of the data presented in the Listings. Because of the large quantity of data, the Particle Listings are not an archive of all published data on particle properties. We refer interested readers to earlier editions for data now considered to be obsolete.

We organize the particles into six categories:

- Gauge and Higgs bosons
- Leptons
- Quarks
- Mesons
- Baryons
- Searches not in other sections

The last category only includes searches for particles that do not belong to the previous groups. For example, it includes searches for supersymmetric particles, compositeness and extra dimensions, while searches for heavy charged leptons are with the leptons.

In Sec. 2 of this Introduction, we list the main areas of responsibility of the authors of the Particle Listings. Our many consultants, without whom we would not have been able to produce this *Review*, are acknowledged in Sec. 3. In Sec. 4, we mention briefly the naming scheme for hadrons, and in Sec. 5, we discuss our procedures for choosing among measurements of particle properties and for obtaining best values of the properties from the measurements.

The accuracy and usefulness of this *Review* depend in large part on interaction between its users and the authors. We appreciate comments, criticisms, and suggestions for improvements of any kind. Please send them to pdg@lbl.gov from where they will be forwarded to the appropriate author according to the list of responsibilities in Sec. 2 below.

In addition to the online publication at pdg.lbl.gov, the *Review* is available in different formats:

- The printed *PDG Book* includes volume 1 only, *i.e.* it contains the Summary Tables and most review articles. Since the 2016 edition [2] the detailed tables from the Particle Listings are no longer printed.
- The *Particle Physics Booklet* includes the Summary Tables plus essential tables, figures, and equations from selected review articles. Starting with the Booklets of the 2018 edition, we have excluded most text and explanations in order to revert back to a more pocket-sized format. The Booklet is available in print, as a web version optimized for use on phones, and as an Android app (see pdg.lbl.gov/booklet).
- *pdgLive* (pdgLive.lbl.gov) is a web application giving more interactive access to PDG data than the static web pages and PDF files that are also available.
- Files that can be downloaded from the PDG website include a table of masses, widths, and PDG Monte Carlo particle ID numbers; PDF files of volume 1 (PDG Book), volume 2 (Particle Listings) and Booklet; individual review articles; all figures; and an archive file containing the complete PDG website (except for pdgLive).
- Starting with the 2024 edition, the data published in the *Review of Particle Physics* is available in machine-readable format through a new PDG API (Application Programming Interface). See pdg.lbl.gov/api for details.

Copies of the *PDG Book* or the *Particle Physics Booklet* can be ordered from our website or directly at pdg.lbl.gov/order. For special requests only, please email pdg@lbl.gov in North and South America, Australia, and the Far East, and pdg-products@cern.ch in all other areas.

This *Review* is considered to be a single comprehensive review of particle physics and related areas. Therefore we prefer that it be cited as a whole, rather than citing *e.g.* an individual review article that is part of this *Review*. For the 2024 edition, the proper citation is:

S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024).

If you wish to refer to a specific part of the *Review*, for example to the Higgs boson review article, the following form should be used:

Status of Higgs Boson Physics in S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024).

2 Particle Listings responsibilities

* Asterisk indicates the people to contact with questions or comments about Particle Listings sections. Please contact them by e-mail to pdg@lbl.gov.

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Graviton	A. Bettini,* D.E. Groom
W, Z	M. Grünewald,* A. Gurtu*
Higgs bosons	S. Heinemeyer,* K. Hikasa, J. Tanaka
Heavy bosons	R. Bonventre,* K.A. Olive, M. Tanabashi
Axions	C. O'Hare, K.A. Olive, G. Raffelt,* F. Takahashi

- Leptons

Neutrinos	M. Goodman, C.-J. Lin,* K. Nakamura, K.A. Olive, A. Piepke
Double-β decay	A. Bettini*, A. Piepke
e, μ	A. Bettini,* C. Grab
τ	A. Lusiani, K. Mönig*

- Quarks

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- Mesons

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D (stable)	G. Casarosa, J. Rademacker, D. Robinson*
B (stable)	A. Cerri,* P. Eerola, M. Kreps, Y. Kwon, W.-M. Yao*

Unstable mesons	C. Amsler,* T. Gutsche, C. Hanhart, J.J. Hernández-Rey, C. Lourenco, A. Masoni, M. Mikhasenko, R.E. Mitchell, S. Navas, C. Patrignani, C. Schwanda, S. Spanier, G. Venanzoni, C.Z. Yuan

- Baryons

Stable baryons	C. Grab, D. Robinson*
Unstable baryons	V. Burkert, V. Crede, U. Thoma, L. Tiator, R.L. Workman*
Charmed baryons	G. Casarosa, J. Rademacker, D. Robinson*
Bottom baryons	A. Cerri,* P. Eerola, M. Kreps, Y. Kwon, W.-M. Yao*

- Miscellanoues searches

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Supersymmetry	H.K. Dreiner,* A. de Gouvêa, I.-A. Melzer-Pellmann, K.A. Olive, I. Vivarelli
Technicolor	K. Agashe,* K.A. Olive, M. Tanabashi
Compositeness	M. Tanabashi, J. Terning*
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3 Consultants

The Particle Data Group benefits greatly from the assistance of hundreds of physicists who are asked to referee review articles and verify every piece of data entered into this *Review.* Of special value is the advice of the PDG Advisory Committee, which meets biennially and thoroughly reviews all aspects of our operation. The members of the 2022 committee are:

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4 Naming scheme for hadrons

We introduced in the 1986 edition [3] a new naming scheme for the hadrons. Changes from older terminology affected mainly the heavier mesons made of u, d, and s quarks. Otherwise, the only important change to known hadrons was that the $F^\pm$ became the $D_s^\pm$. None of the lightest pseudoscalar or vector mesons changed names, nor did the $c\overline{c}$ or $b\overline{b}$ mesons (we do, however, now use χ_c for the $c\overline{c}$ χ states), nor did any of the established baryons. The Summary Tables give both the new and old names whenever a change has occurred.

In the 2018 edition [4], the naming scheme was extended to address the naming of charmonium and bottomonium states that were commonly referred to as X, Y, or Z states in the literature. The further discovery of various exotic hadron states — including in particular tetraquarks and pentaquarks containing $qq\bar{q}\bar{q}$ and $q\bar{q}qqq$ minimal quark content, respectively — has rendered the 2018 extension insufficient. In this edition, the naming scheme is revised and extended to cover all experimentally-known states. The current scheme is described in "Naming Scheme for Hadrons" (p. 1) of this *Review*. A table details the correspondence between the names newly adopted by the PDG and those that have previously appeared in the literature.

We give here our conventions on typesetting style. Particle symbols are italic (or slanted) characters: e^-, p, Λ, π^0, K_L, D_s^+, b. Charge is indicated by a superscript: B^-, Δ^{++}. Charge is not normally indicated for p, n, or the quarks, and is optional for neutral isosinglets: η or η^0. Antiparticles and particles are distinguished by charge for charged leptons and mesons: τ^+, K^-. Otherwise, distinct antiparticles are indicated by a bar (overline): $\overline{\nu}_\mu$, $\overline{t}$, $\overline{p}$, $\overline{K}^0$, and $\overline{\Sigma}^+$ (the antiparticle of the Σ^-).

5 Procedures

5.1 *Selection and treatment of data*

The Particle Listings contain all relevant data known to us that are published in journals. With very few exceptions, we do not include results from preprints or conference reports. Nor do we include data that are of historical importance only (the Listings are not an archival record). We search every volume of 30 journals through our cutoff date for relevant data. We also include later published papers that are sent to us by the authors (or others).

In the Particle Listings, we clearly separate measurements that are used to calculate or estimate values given in the Summary Tables from measurements that are not used. We give explanatory comments in many such cases. Among the reasons a measurement might be excluded are the following:

- It is superseded by or included in later results.
- No error is given.
- It involves assumptions we question.
- It has a poor signal-to-noise ratio, low statistical significance, or is otherwise of poorer quality than other data available.
- It is clearly inconsistent with other results that appear to be more reliable. Usually we then state the criterion, which sometimes is quite subjective, for selecting "more reliable" data for averaging. See Sec. 5.4.
- It is not independent of other results.
- It is not the best limit (see below).
- It is quoted from a preprint or a conference report.

In some cases, *none* of the measurements is entirely reliable and no average is calculated. For example, the masses of many of the baryon resonances, obtained from partial-wave analyses, are quoted as estimated ranges thought to probably include the true values, rather than as averages with errors. This is discussed in the Baryon Particle Listings.

For upper limits, we normally quote in the Summary Tables the strongest limit. We do not average or combine upper limits except in a very few cases where they may be re-expressed as measured numbers with Gaussian errors.

As is customary, we assume that particle and antiparticle share the same spin, mass, and mean life. The Tests of Conservation Laws table, following the Summary Tables, lists tests of CPT as well as other conservation laws.

We use the following indicators in the Particle Listings to tell how we get values from the tabulated measurements:

- OUR AVERAGE —From a weighted average of selected data.
- OUR FIT —From a constrained or overdetermined multiparameter fit of selected data.
- OUR EVALUATION —Not from a direct measurement, but evaluated from measurements of related quantities.
- OUR ESTIMATE —Based on the observed range of the data. Not from a formal statistical procedure.
- OUR LIMIT —For special cases where the limit is evaluated by us from measured ratios or other data. Not from a direct measurement.

An experimentalist who sees indications of new a particle will of course want to know what has been seen in that region in the past. Hence, we include in the Particle Listings all reported states that, in our opinion, have sufficient statistical merit and that have not been disproved by more reliable data. However, we promote to the Summary Tables only those states that we feel are well-established. This judgment is, of course, somewhat subjective and no precise criteria can be given. For more detailed discussions, see the reviews section on Particle Properties.

5.2 *Averages and fits*

We divide this discussion on obtaining averages and errors into three sections: (1) treatment of errors; (2) unconstrained averaging; (3) constrained fits.

5.2.1 *Treatment of errors*

In what follows, the "error" δx means that the range $x \pm \delta x$ is intended to be a 68.3% confidence interval about the central value x. We treat this error as if it were Gaussian. Thus, when the error *is* Gaussian, δx is the usual one standard deviation (1σ). Many experimenters now give statistical and systematic errors separately, in which case we usually quote both errors, with the statistical error first. For averages and fits, we then add the two errors in quadrature and use this combined error for δx.

When experimenters quote asymmetric errors $(\delta x)^+$ and $(\delta x)^-$ for a measurement x, the error that we use for that measurement in making an average or a fit with other measurements is a continuous function of these three quantities. When the resultant average or fit $\overline{x}$ is less than $x - (\delta x)^-$, we use $(\delta x)^-$; when it is greater than $x + (\delta x)^+$, we use $(\delta x)^+$. In between, the error we use is a linear function of x. Since the errors we use are functions of the result, we iterate to get the final result. Asymmetric output errors are determined from the input errors assuming a linear relation between the input and output quantities.

In fitting or averaging, we usually do not include correlations between different measurements, but we try to select data in such a way as to reduce correlations. Correlated errors are, however, treated explicitly when there are a number of results of the form $A_i \pm \sigma_i \pm \Delta$ that have identical systematic errors Δ. In this case, one can first average the $A_i \pm \sigma_i$ and then combine the resulting statistical error with Δ. One obtains, however, the same result by averaging $A_i \pm (\sigma_i^2 + \Delta_i^2)^{1/2}$, where $\Delta_i = \sigma_i \Delta[\sum(1/\sigma_j^2)]^{1/2}$. This procedure has the advantage that, with the modified systematic errors Δ_i, each measurement may be treated as independent and averaged in the usual way with other data. Therefore, when appropriate, we adopt this procedure. We tabulate Δ and invoke an automated procedure that computes Δ_i before averaging, and we include a note saying that there are common systematic errors.

Another common case of correlated errors occurs when experimenters measure two quantities and then quote the two and their

58. τ Branching Fractions

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58.1 τ Branching Fractions

The τ Listings contains 252 entries that correspond to either a τ partial decay fraction into a specific decay mode (branching fraction) or a ratio of two τ partial decay fractions (branching ratio). Experimental measurements provide values for 148 of these quantities, upper limits for 67 branching fractions to Lepton Family number, Lepton number, or Baryon number violating modes, and 37 additional upper limits for other modes. A total of 129 τ branching fractions and branching ratios are determined with a fit of 170 measurements. 85 quantities have at least one measurement in the fit.

58.2 The constrained fit to τ branching fractions

The τ branching fractions fit uses the reported values, uncertanties and statistical correlations of the τ branching fractions and branching ratios measurements. Asymmetric uncertainties are symmetrized as $\sigma^2_{\text{symm}} = (\sigma^2_+ + \sigma^2_-)/2$. If only a few measurements are correlated, the correlation coefficients are listed in the footnote for each measurement (see for example $\Gamma(\text{particle}^- \geq 0\,\text{neutrals} \geq 0\,K^0\nu_\tau\ (\text{“1-prong”}))/\Gamma_{\text{total}}$). If a large number of measurements are correlated, then the full correlation matrix is listed in the footnote to the measurement that first appears in the τ Listings. Footnotes to the other measurements refer to the first one. For example, the large correlation matrices for the branching fraction or ratio measurements contained in Refs. [1] [2] are listed in Footnotes to the $\Gamma(e^-\overline{\nu}_e\nu_\tau)/\Gamma_{\text{total}}$ and $\Gamma(h^-\nu_\tau)/\Gamma_{\text{total}}$ measurements respectively. Additionally, the most precise experimental inputs are treated according to how they depend on external parameters on the basis of their documentation [3]. The τ measurements may depend on parameters such as the τ pair production cross-section in e^+e^- annihilations at the $\Upsilon(4S)$ peak. In some cases, measurements reported in different papers by the same collaboration may depend on common parameters like the estimate of the integrated luminosity or of particle identification efficiencies. For all the significant detected dependencies, the τ measurements and their uncertainties are updated to account for the updated values of the external parameters. The dependencies on common systematic effects are also determined in size and sign, and all the common systematic dependencies of different measurements are used together with the published statistical and systematic uncertainties and correlations in order to compute an updated all-inclusive variance and covariance matrix of the experimental inputs of the fit.

The fit procedure parameters correspond to all measured τ branching fractions and ratios, to some non-measured branching fractions and ratios like for instance $\mathcal{B}(\tau^- \to \pi^- K^0_L K^0_L \nu_\tau)$ and to one nuisance variable. When discussing the fit results in the following, the fit χ^2, the number of degrees of freedom, the residuals and pulls all refer to the subset of fit parameters that correspond to τ branching fractions and ratios, excluding nuisance variables. The fit parameters are optimized while respecting relations described by a series of constraint equations. All the experimental inputs and all the constraint equations are reported in the τ Listings section that follows this review. In some cases, constraints describe approximate relations that nevertheless hold within the present experimental precision. For instance, the constraint $\mathcal{B}(\tau^- \to K^-K^-K^+\nu_\tau) = \mathcal{B}(\tau^- \to K^-\phi\nu_\tau) \times \mathrm{B}(\phi \to K^+K^-)$ is justified within the current experimental evidence. The constraint equations between the τ branching fractions and ratios include quantities other than τ branching fractions and branching ratios, like for instance the branching fractions of the η and ω mesons. We neglect the uncertainties on these values for all quantities except $\mathcal{B}(a_1^- \to \pi^-\gamma)$, whose value and uncertainty were estimated by ALEPH [1] to be $(0.21 \pm 0.08)\cdot 10^{-2}$, relying on a measurement of $\Gamma(a_1^- \to \pi^-\gamma)$ [4]. This quantity is included in the fit parameters as nuisance variable, with a χ^2 term corresponding to its estimate. We assume that $\mathcal{B}(\tau^- \to a_1^-\nu_\tau) = \mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau\ (\text{ex.}K^0,\omega)) + \mathcal{B}(\tau^- \to \pi^-2\pi^0\nu_\tau\ (\text{ex.}K^0)) + \mathcal{B}(\tau^- \to a_1^-(\pi^-\gamma)\nu_\tau)$, neglecting the observed but negligible branching fractions to other modes, and that $\mathcal{B}(\tau^- \to a_1^-(\pi^-\gamma)\nu_\tau) = \mathcal{B}(\tau^- \to a_1^-\nu_\tau)\cdot\mathcal{B}(a_1^- \to \pi^-\gamma)$. The values of all other quantities in the constraint equations are taken from the 2023 edition of the Review of Particle Physics.

In the fit, uncertainty scale factors are applied to the published uncertainties of measurements only if significant inconsistency between different measurements remain after accounting for all relevant uncertainties and correlations. When performing the fit with no scale factors, the two measurements of $\mathcal{B}(\tau^- \to K^-K^-K^+\nu_\tau)$ have pulls exceeding $5\,\sigma$ from the fit values. There are 170 pulls, one per measurement. They are partially correlated, and the effective number of independent pulls is equal to the number of degrees of freedom of the fit, 125. The probability of getting pulls equal or larger than either one of the two very large pulls in a sample of 125 is smaller that the probability of a $3\,\sigma$ deviation for a Normal variable. Therefore, it has been decided to apply an uncertainty scale factor of 5.4 on all measurements of $\mathcal{B}(\tau^- \to K^-K^-K^+\nu_\tau)$ (one by BaBar and one by Belle). The scale factor has been computed according to the standard PDG procedure. After applying the scale factor, the pull distribution of the measurements in figure 58.1 is reasonably Normal and the pull probability distribution in figure 58.2 is reasonably flat.

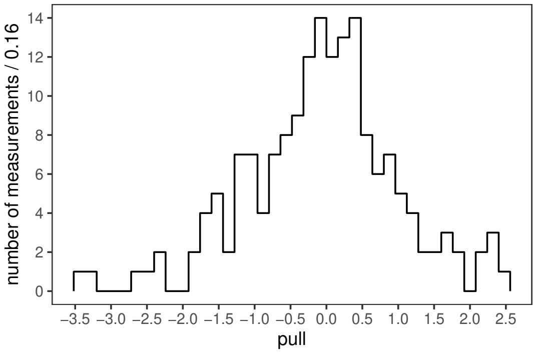

Figure 58.1: Pulls of individual measurements against the respective fitted quantity.

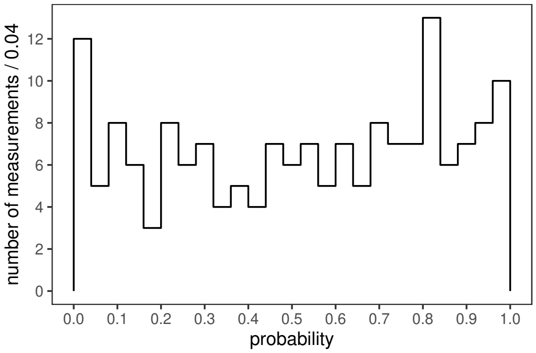

Figure 58.2: Probability of individual measurement pulls against the respective fitted quantity.

Considering only the residuals with respect to τ branching fractions and ratios measurements, the constrained fit has a χ^2 of 135 for 125 degrees of freedom, corresponding to a χ^2 probability of 26.0%. We use 170 measurements and 84 constraints on the branching fractions and ratios to determine 129 quantities, consisting of 112 branching fractions and 17 branching ratios. The constraints include the unitarity constraint on the sum of all the exclusive τ decay modes, $\mathcal{B}_{\text{all}} = 1$. If the unitarity constraint is released, the fit result for $\mathcal{B}_{\text{all}}$ is consistent with unitarity with $1 - \mathcal{B}_{\text{all}} = (0.07 \pm 0.11)\%$.

For the convenience of summarizing the fit results, we list in the following the values and uncertainties for a set of 46 “basis” decay modes, from which all remaining branching fractions and ratios can be obtained using the constraints. The basis decay modes are not intended to sum up to 1. Since some basis quantities represent multiple branching fractions that are related by constraint equations, the unitarity constraint corresponds to a linear combination of branching fractions, with the coefficients listed in the following. The coefficients of $\mathcal{B}(\tau^- \to \pi^- 2\pi^0 \nu_\tau\ (\text{ex.}K^0))$ and $\mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau\ (\text{ex.}K^0, \omega))$ depend on the fitted value of the nuisance variable, $\mathcal{B}(a_1^- \to \pi^-\gamma) = 0.0022 \pm 0.0008$. The correlation matrix between the basis modes is reported in the τ Listings.

decay mode	fit result	coeff.
$\mathcal{B}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$	0.1739 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)$	0.1782 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to \pi^-\nu_\tau)$	0.1082 ± 0.0005	1.0000
$\mathcal{B}(\tau^- \to K^-\nu_\tau)$	0.00696 ± 0.00010	1.0000
$\mathcal{B}(\tau^- \to \pi^-\pi^0\nu_\tau)$	0.2549 ± 0.0009	1.0000
$\mathcal{B}(\tau^- \to K^-\pi^0\nu_\tau)$	0.00433 ± 0.00015	1.0000
$\mathcal{B}(\tau^- \to \pi^-2\pi^0\nu_\tau\ (\text{ex.}K^0))$	0.0926 ± 0.0010	1.0022
$\mathcal{B}(\tau^- \to K^-2\pi^0\nu_\tau\ (\text{ex.}K^0))$	$(0.65 \pm 0.22) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-3\pi^0\nu_\tau\ (\text{ex.}K^0))$	0.0104 ± 0.0007	1.0000
$\mathcal{B}(\tau^- \to K^-3\pi^0\nu_\tau\ (\text{ex.}K^0, \eta))$	$(0.48 \pm 0.21) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to h^-4\pi^0\nu_\tau\ (\text{ex.}K^0, \eta))$	0.0011 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to \pi^-\bar{K}^0\nu_\tau)$	0.00838 ± 0.00014	1.0000
$\mathcal{B}(\tau^- \to K^-K^0\nu_\tau)$	0.001486 ± 0.000034	1.0000
$\mathcal{B}(\tau^- \to \pi^-\bar{K}^0\pi^0\nu_\tau)$	0.00382 ± 0.00013	1.0000
$\mathcal{B}(\tau^- \to K^-K^0\pi^0\nu_\tau)$	0.00150 ± 0.00007	1.0000
$\mathcal{B}(\tau^- \to \pi^-\bar{K}^02\pi^0\nu_\tau\ (\text{ex.}K^0))$	$(0.26 \pm 0.23) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-K_S^0K_S^0\nu_\tau)$	$(235 \pm 6) \cdot 10^{-6}$	2.0000
$\mathcal{B}(\tau^- \to \pi^-K_S^0K_L^0\nu_\tau)$	0.00108 ± 0.00024	1.0000
$\mathcal{B}(\tau^- \to \pi^-K_S^0K_S^0\pi^0\nu_\tau)$	$(18.2 \pm 2.1) \cdot 10^{-6}$	2.0000
$\mathcal{B}(\tau^- \to \pi^-K_S^0K_L^0\pi^0\nu_\tau)$	$(0.32 \pm 0.12) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \bar{K}^0h^-h^-h^+\nu_\tau)$	$(0.25 \pm 0.20) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau\ (\text{ex.}K^0, \omega))$	0.0899 ± 0.0005	1.0022
$\mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau\ (\text{ex.}K^0, \omega))$	0.0274 ± 0.0007	1.0000
$\mathcal{B}(\tau^- \to h^-h^-h^+2\pi^0\nu_\tau\ (\text{ex.}K^0, \omega, \eta))$	0.0010 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to \pi^-K^-K^+\nu_\tau)$	0.001435 ± 0.000027	1.0000
$\mathcal{B}(\tau^- \to \pi^-K^-K^+\pi^0\nu_\tau)$	$(61 \pm 18) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-\pi^0\eta\nu_\tau)$	0.00139 ± 0.00007	1.0000
$\mathcal{B}(\tau^- \to K^-\eta\nu_\tau)$	$(155 \pm 8) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^-\pi^0\eta\nu_\tau)$	$(48 \pm 12) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-\bar{K}^0\eta\nu_\tau)$	$(94 \pm 15) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\eta\nu_\tau\ (\text{ex.}K^0))$	$(220 \pm 13) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^-\omega\nu_\tau)$	$(0.41 \pm 0.09) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to h^-\pi^0\omega\nu_\tau)$	0.0041 ± 0.0004	1.0000
$\mathcal{B}(\tau^- \to K^-\phi\nu_\tau)$	$(44 \pm 16) \cdot 10^{-6}$	0.8300
$\mathcal{B}(\tau^- \to \pi^-\omega\nu_\tau)$	0.0195 ± 0.0006	1.0000
$\mathcal{B}(\tau^- \to K^-\pi^-\pi^+\nu_\tau\ (\text{ex.}K^0, \omega))$	0.00293 ± 0.00007	1.0000
$\mathcal{B}(\tau^- \to K^-\pi^-\pi^+\pi^0\nu_\tau\ (\text{ex.}K^0, \omega, \eta))$	$(0.39 \pm 0.14) \cdot 10^{-3}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-2\pi^0\omega\nu_\tau)$	$(72 \pm 16) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau\ (\text{ex.}K^0, \eta, \omega, f_1))$	$(14 \pm 27) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 3\pi^-2\pi^+\nu_\tau\ (\text{ex.}K^0, \omega, f_1))$	$(775 \pm 30) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^-2\pi^-2\pi^+\nu_\tau\ (\text{ex.}K^0))$	$(0.6 \pm 1.2) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 2\pi^-\pi^+\omega\nu_\tau\ (\text{ex.}K^0))$	$(84 \pm 6) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau\ (\text{ex.}K^0, \eta, \omega, f_1))$	$(38 \pm 9) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to K^-2\pi^-2\pi^+\pi^0\nu_\tau\ (\text{ex.}K^0))$	$(1.1 \pm 0.6) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-f_1(2\pi^-2\pi^+)\nu_\tau)$	$(52 \pm 4) \cdot 10^{-6}$	1.0000
$\mathcal{B}(\tau^- \to \pi^-2\pi^0\eta\nu_\tau\ (\text{ex.}K^0))$	$(0.19 \pm 0.04) \cdot 10^{-3}$	1.0000

In defining the fit constraints and in selecting the modes that sum up to one we made some assumptions and choices. We assume that some channels, like $\tau^- \to \pi^-K^+\pi^- \geq 0\pi^0\nu_\tau$ and $\tau^- \to \pi^+K^-K^- \geq 0\pi^0\nu_\tau$, have negligible branching fractions as expected from the Standard Model, even if the experimental limits for these branching fractions are not very stringent. The 95% confidence level upper limits are $\mathcal{B}(\tau^- \to \pi^-K^+\pi^- \geq 0\pi^0\nu_\tau) <$ 0.25% and $\mathcal{B}(\tau^- \to \pi^+K^-K^- \geq 0\pi^0\nu_\tau) <$ 0.09%, values not so different from measured branching fractions for allowed 3-prong modes containing charged kaons. For decays to final states containing one neutral kaon we assume that the branching fraction with the K_L^0 are the same as the corresponding one with a K_S^0. On decays with two neutral kaons we assume that the branching fractions with $K_L^0K_L^0$ are the same as the ones with $K_S^0K_S^0$.

58.3 BaBar and Belle measure on average lower branching fractions and ratios.

We compare the BaBar and Belle measurements with the results of a fit where all the B-factories measurements have been excluded. We restrict the comparison to the measurements that are used in the fit, omitting two measurements that are superseeded with other results, and one measurement that is fully correlated with other measurements. We find that BaBar and Belle measure on average lower τ branching fractions and ratios than the other experiments. Figures 58.3 and 58.4 show histograms of the 25 pulls of the differences between B-factory measurements and the respective non-B-factory fit results. The average pull between the two sets of measurements is -0.8σ (-0.7σ for the 14 Belle measurements and -0.9σ for the 11 BaBar measurements).

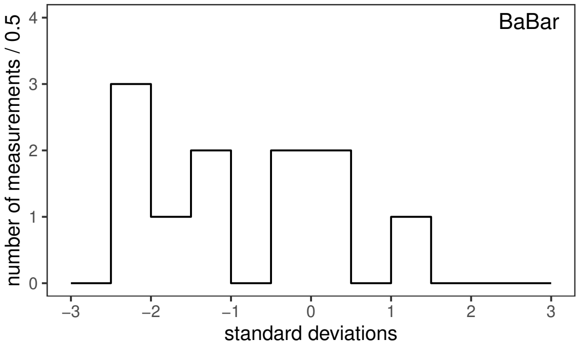

Figure 58.3: Distribution of the normalized difference between 11 measurements of branching fractions and ratios published by the BaBar collaboration and the respective averages computed using only non-B-factory measurements.

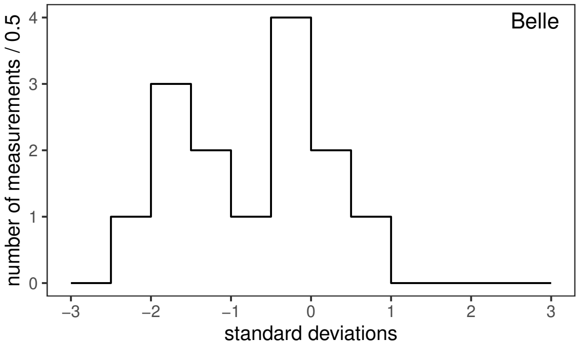

Figure 58.4: Distribution of the normalized difference between 14 measurements of branching fractions and ratios published by the Belle collaboration and the respective averages computed using only non-B-factory measurements.

58.4 Overconsistency of Leptonic Branching Fraction Measurements.

As observed in the previous editions of this review, measurements of the leptonic branching fractions are more consistent with each other than expected from the quoted uncertainties on the individual measurements. The χ^2/n.d.o.f. are 0.34/4 for $\mathcal{B}_e$ and 0.08/4 for $\mathcal{B}_\mu$. The probability of getting a smaller χ^2 is 1.3% for B_e and 0.08% for B_μ.

58.5 Technical implementation of the fit

The fit computes a set of quantities denoted with q_i by minimizing a χ^2 while respecting a series of equality constraints on the q_i. The quantities q_i represent τ branching fractions and branching ratios, and nuisance variables. The fit minimization procedure is equivalent to choosing a set of basis fit variables, using functions of these basis variables to predict measurements, and determining these basis variables by minimizing the measurements' χ^2. The χ^2 is computed using the measurements m_i and their covariance matrix E_{ij} as $\chi^2 = (m_i - A_{ik}q_k)^t E_{ij}^{-1}(m_j - A_{jl}q_l)$, where the model matrix A_{ij} is used to get the vector of the predicted measurements m'_i from the vector of the fit parameters q_j as $m'_i = A_{ij} q_j$. There is one fit variable for each of the modes that have measurements, therefore $A_{ij} = 1$ when q_j corresponds to the τ branching fraction or branching ratio of the measurement m_i, and $A_{ij} = 0$ otherwise. The constraints are equations involving the fit parameters. The fit does not impose limitations on the functional form of the constraints. In summary, the fit requires:

$$\min\left[\chi^2(q_k)\right] = \min\left[(m_i - A_{ik}q_k)^t E^{-1}_{ij}(m_j - A_{jl}q_l)\right] \ , \quad (58.1)$$

$$\text{subjected to} \quad f_r(q_s) - c_r = 0 \ , \quad (58.2)$$

where the left term of Eq. 58.2 defines the constraint expressions. Using the method of Lagrange multipliers, a set of equations is obtained by taking the derivatives with respect to the fitted quantities q_k and the Lagrange multipliers λ_r of the sum of the χ^2 and the constraint expressions multiplied by the Lagrange multipliers λ_r, one for each constraint:

$$\begin{aligned}\min\left[(A_{ik}q_k - m_i)^t E^{-1}_{ij}(A_{jl}q_l - m_j) + 2\lambda_r(f_r(q_s) - c_r)\right] &= \\ = \min\left[\tilde{\chi}^2(q_k, \lambda_r)\right] \ , \\ (\partial/\partial q_k, \partial/\partial\lambda_r)\left[\tilde{\chi}^2(q_k, \lambda_r)\right] = 0 \ . \quad (58.3)\end{aligned}$$

Eq. 58.3 defines a set of equations for the vector of the unknowns (q_k, λ_r), some of which may be non-linear, in case of non-linear constraints. An iterative minimization procedure approximates at each step the non-linear constraint expressions by their first order Taylor expansion around the current values of the fitted quantities, $\bar{q}_s$:

$$f_r(q_s) - c_r = f_r(\bar{q}_s) + \left.\frac{\partial f_r(q_s)}{\partial q_s}\right|_{\bar{q}_s} (q_s - \bar{q}_s) - c_r \ ,$$

which can be written as

$$B_{rs} q_s - c'_r \ ,$$

where c'_r are the resulting constant known terms, independent of q_s at first order. After linearization, the differentiation by q_k and λ_r is trivial and leads to a set of linear equations

$$A^t_{ki} E^{-1}_{ij} A_{jl} q_l + B^t_{kr} \lambda_r = A^t_{ki} E^{-1}_{ij} m_j \ , \quad (58.4)$$

$$B_{rs} q_s = c'_r \ , \quad (58.5)$$

which can be expressed as

$$F_{ij} u_j = v_i \ , \quad (58.6)$$

where $u_j = (q_k, \lambda_r)$ and v_i is the vector of the known constant terms running over the index k and then r in the right terms of Eq. 58.4 and Eq. 58.5, respectively. Solving the equation set in Eq. 58.6 by matrix inversion gives the the fitted quantities and their variance and covariance matrix, using the measurements and their variance and covariance matrix. The fit procedure starts by computing the linear approximation of the non-linear constraint expressions around the quantities seed values. With an iterative procedure, the unknowns are updated at each step by solving the equations and the equations are then linearized around the updated values, until the variation of the fitted unknowns is reduced below a numerically small threshold.

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