In Search of Cosmic Time: Complete Observables and the Clock Hypothesis

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This article considers a new and deeply challenging aspect of the problem of time in the context of cosmology, drawing on the work of Thiemann. Thiemann argues for a radical response to the cosmic problem of time that requires us to modify the classical Friedmann equations. By contrast, we offer a conservative proposal for the solution of the problem, by bringing together ideas from the contemporary literature regarding reference frames, complete observables, and the model-based account of time measurement. On our approach, we must reinterpret our criteria of observability in light of the clock hypothesis and the model-based account of measurement in order to preserve the Friedmann equations as the dynamical equations for the universe.

1. Introduction

1.1. Précis

A cosmic problem: In every causally stable spacetime of general relativity there exists a symmetry invariant cosmic time function. However, it is unclear whether we should treat such cosmic times as 'observables' according to the standard 'Dirac criterion' of observability deployed by scientists in analysing the theory. A radical proposal: Taking the problem of the observability of cosmic time seriously in the context of the Friedmann equations for cosmology leads to potentially significant modifications to the equations. In particular, one can show that consistently applying the standard Dirac criterion may modify the form of the equations in a manner that can have empirical consequences. Our alternative approach: A different option is to reinterpret the criteria of observability in light of the clock hypothesis. On the assumption that there are clocks that measure proper time along any given world-line, one is able to treat cosmic time as an observable and preserve the form of the Friedmann equations. The conservative's dilemma: Whatever option we take, something must change for things to stay as they are.

1.2. The problem of cosmic time

Spacetime symmetry and time evolution are not straightforward to reconcile in the context of the diffeomorphism invariance of general relativity. In particular, the ability to 're-slice' relativistic spacetimes into arbitrary decompositions of space-like hypersurfaces indicates that time is 'many-fingered' within the theory.[1] In the context of cosmology, however, it is standard to make reference to a notion of 'cosmic time'. One application of cosmic time is, of course, in discussions of the age of the universe. Most vividly, in simple FLRW cosmological models (named for Friedmann, Lemaître, Robertson, and Walker), the cosmic time function labels isotropic and homogenous spatial hypersurfaces and corresponds to the temporal length of geodesic paths that run from the big bang until the present day.[2] Cosmic times can be shown to be well defined in any relativistic cosmology with stable causal structure (Hawking [1969]) and are consistent with the spacetime symmetries of general relativity since they can be defined in a manifestly spacetime diffeomorphism invariant manner.[3]

There are, however, plausible formal and physical reasons to doubt the status of a cosmic time as an independent physical observable. First, on very general grounds, we might demand of an observable that it corresponds to a measurable quantity of the theory and it is not immediately clear how one would measure cosmic time since there is no natural 'clock' that we would understand to read off cosmic time nor clear methodology to construct one. Second, we might also demand that genuine observables are relational in the sense that they correspond to correlations between measurable quantities. On such an understanding of what is to be an observable we would not expect that cosmic time is observable since it is does not appear to have the right form to be a correlation between two measurable quantities.

This potential challenge to the observable status of cosmic time becomes all the more acute in the context of the canonical formalism for general relativity. A remarkable achievement of the canonical gravitational formalism, pioneered by Dirac and Arnowitt,

[1] The implications of diffeomorphism invariance and the related notions of general covariance and background independence are much debated; see (Pooley [2017]; Read [2023]). Many-fingered time and the decomposition into space-like hyperspaces requires relativistic spacetimes to be globally hyperbolic (Geroch [1970]; Misner et al. [2017]).

[2] An important connected notion of time in general relativity is 'York time'. This corresponds to the time that labels spatial slices of constant mean curvature and has important connections with conformal invariant approaches to the initial value problem of general relativity (York [1972], [1973]; Gomes et al. [2011]; Gryb and Thébault [2016c]). York time corresponds to cosmic time for homogenous spatial slices, meaning that in FLRW models the constant mean curvature slices are the same as the slices of constant cosmic time. However, in general, the two are distinct (see Roser and Valentini [2014]).

[3] Formally, a cosmic time function on a spacetime manifold, M, is a function that assigns to any $p \in M$ the supremum of the durations of all future-directed continuous timelike curves ending at p (Andersson et al. [1998]; Fletcher [2024]). For philosophical overviews of time in cosmology, see (Smeenk [2013]; Callender and McCoy [2021]). For a discussion focused on cosmic time, see (Rugh and Zinkernagel [2009]).

Deser, and Misner (ADM), was to show that one can represent spacetime symmetries within a $(3\mathrm{D}+1)$-space and time formalism in constrained Hamiltonian terms (see Dirac [1958b]; Arnowitt et al. [1959], [1962]). In this formulation, the spacetime diffeomorphism symmetry of the four-dimensional covariant theory is encoded in the hypersurface deformation algebroid of constraints. This algebroid is a mathematical structure encoding how the constraints governing the dynamics of the three-dimensional spatial slices transform under infinitesimal normal and tangential deformations of the slices (Teitelboim [1973]; Gryb and Thébault [2016a]). These deformations correspond to 'shifts' in time and space and ensure that the canonical Dirac formalism preserves the underlying covariance of general relativity, even within the $(3\mathrm{D}+1)$ decomposition. Although it is restricted to spacetimes admitting globally hyperbolic topology, there is no expectation that such a limitation will conflict with the application of the formalism to cosmology (the black hole case is more subtle due to the Kerr solutions not being globally hyperbolic).[4] A further attractive feature of the canonical formulation of general relativity is that it comes equipped with a formal criterion, known as the Dirac criterion, which identifies a quantity as an observable if it has (weakly) vanishing Poisson bracket with all first-class constraints of the theory. Furthermore, in a groundbreaking and highly influential analysis, it was shown that a particular approach to relational conception of an observable, as a correlation between two partial observables leading to a complete observable (Rovelli [2002b]), provides a construction of objects that satisfy the Dirac criterion. An unambiguous criterion for functions to be observables is of course indispensable staging material needed towards the pursuit of canonical quantization. It is the algebra of classical observables that one seeks to faithfully represent as operators on a physical Hilbert space. Such a criterion is also useful for settling questions of observability in the classical formalism. The problem, however, is that it is not at all clear that canonical representations of cosmic time would in fact satisfy the Dirac criterion. In particular, *prima facie*, cosmic time does not appear to be the right kind of relational object to fit with the complete observables approach. It is not at all clear that cosmic time is either a partial or complete observable in Rovelli's terms. This is, ultimately, a cosmological manifestation of the infamous problem of time, which we name the cosmic problem of time and about which we will have more to say later.

The acuteness of the cosmic problem of time can be illustrated most clearly in the context of the Friedmann equations that are the cornerstone of the standard model of cosmology. These equations describe the expansion of the universe in terms of the dynamics of the spatial scale factor, a, with respect to the cosmic time parameter, t. These equations can be straightforwardly represented in canonical terms since they turn out to

[4] The general expectation that all physically realistic spacetimes must be globally hyperbolic is based upon assumptions regarding the instability of features such as Cauchy horizons that lead to a breakdown in global hyperbolicity; see (Penrose [1980], sec. 12; Wald [1984], p. 202).

simply be equivalent to the Hamilton equations. In this case, it is simple to observe that neither the cosmic time nor the scale factor are Dirac observables. As per our earlier discussion, by the Dirac criterion, such observables must be phase space functions that have (weakly) vanishing Poisson bracket with the Hamiltonian constraint that generates the dynamics of the Friedmann equation. However, clearly neither a nor t can satisfy such a criterion. This is because, in the first case, a evidently has non-zero Poisson bracket with the Hamiltonian for any non-trivial cosmological dynamics by the definition of the Friedmann equations. In the second case, not only is t not naturally understood as a phase space variable, rather it is a parameter of a phase space flow, but any phase space function that 'marches in step' with t will have non-zero Lie drag with the Hamilton vector field of the Hamiltonian, and thus non-vanishing Poisson bracket. Though simple to articulate and difficult to ignore once it is recognized, this problem has received almost no discussion in physics or philosophy.

Isolation of the problem crucially depends upon how strict one is regarding the application of the Dirac criterion in the context of 'partial observables'—both in general and in the cosmological case. Important discussions of this issue in general and in the cosmological context can be found in the work of the noted theoretical physicist Thomas Thiemann. The general question of observability of partial observables is discussed in his textbook, where he argues that 'a measurable quantity is always a complete observable, even pointers of a clock are observables and not partial observables. Now complete observables are defined with respect to non-measurable quantities [. . .] which we will simply call non-observables' (Thiemann [2007], p. 78). Thiemann's view implies that both the scale factor and the cosmic time parameter are non-observables and that the Friedman equations therefore do not describe the evolution of observable quantities. Indeed, an analysis along precisely these lines can be found in (Thiemann [unpublished]). Thiemann explicitly argues there that 'it is incorrect to interpret the FLRW equations as evolution equations of observable quantities'. Rather, he suggests, we should follow the complete observables procedure and re-write the equations relationally. The true evolution equations can then shown for a simple model to acquire empirical modifications when compared to the Friedmann equations. We are thus lead from a formal–conceptual problem regarding time and observability to a physical proposal for a new approach to cosmology with empirical consequences. One of the major goals of this article is to consider this chain of argument leading to Thiemann's radical cosmological revisionism.

The second major goal of this article is to formulate and defend a conservative approach to finding cosmic time based upon the application of the clock hypothesis in the context of the concept of Hubble flow.[5] The development of this alternative approach

[5] The idea of 'operationalizing' cosmic time via Hubble flow is a well-known standard approach in the literature on FLRW cosmology; see, for example, (Misner et al. [2017], sec. 27.4; Ryden [2016]).

will lead us, in turn, to reconsider the methodological status of the clock hypothesis in cosmology and the meaning of 'measurability' in the context of the complete observables programme. Our analysis draws crucially on the different types of reference frames in physical theory and the insights that this can deliver for clarifying foundational questions regarding the construction of 'complete observables' as correlations between 'partial observables'. Our goal is to synthesize key ideas from the contemporary literature regarding reference frames (Bamonti [unpublished]; Bamonti and Gomes [unpublished-a]), complete observables (Gryb and Thébault [2016b], [2023]), and the model-based account of time measurement (Tal [2016]), while drawing attention to a new and deeply challenging aspect of the problem of time in the context of cosmology. The cosmic problem of time leads to a dilemma: we can apply a conservative understanding of Dirac observables, downplay the significance of the clock hypothesis, and modify the Friedmann equations; or we can reinterpret our criteria of observability in light of the clock hypothesis and the model-based account of measurement, and preserve the Friedmann equations. Whatever option we take, something must change for things to stay as they are.

1.3. Roadmap

Section 2 provides a brief overview of the complete and partial observables programme as a response to the problem of time. We then apply recent work on reference frames to disambiguate two important details in the definition of a partial observable and better understand what it means for a physical variable to play the role of a clock in the context of a complete observable. Section 3 considers the status of time in the Friedmann equations, poses Thiemann's challenge to the standard interpretation of these equations as dynamical equations and reviews his solution in terms of a de-parametrization via a phantom field through the well-known Brown and Kuchař ([1995]) mechanism. Section 4 reframes comic time as a proper time parameter, τ, along the Hubble flow. In this context, we consider the question of the observability and measurability of Hubble parameter (that is, $H = {}^{\dot{a}}/_{a}$) and the question of whether we can consider $H(\tau)$ to be a complete observable whose dynamics is described by the unmodified Friedman equations. Problematically, there is no physical system that even approximately measures proper time along the Hubble flow. Section 5 articulates our proposed solution: the introduction of a more liberalized sense of 'measurement procedure' in the context of cosmic time. This more liberalized notion draws upon the model-based account of time measurement developed in the context of atomic clocks by Tal ([2016]).

2. Complete Observables and Real Reference Frames

The problem of time is best understood as a cluster of formal, physical and conceptual challenges to the isolation of the physical degrees of freedom in theories that display temporal diffeomorphism symmetry.[6] In the canonical representation, many of the challenges stem from lack of an unambiguous phase space representation of re-foliation symmetry and the implications that this has for quantization. However, the problem is not restricted to canonical representations, and reoccurs in covariant form, for example, in terms of the challenge of finding an appropriate measure in path-integral approaches. One particular pressing aspect of the problem is the tension between the standard definition of a gauge-invariant observable and the seemingly obvious fact that observable quantities change. In particular, in the context of constrained Hamiltonian theories, following Dirac ([1950], [1958a], [1964]), the criterion to be an observable is to have (weakly) vanishing Poisson bracket with first class (primary) constraints.[7] Even for simple theories, temporal diffeomorphism symmetry immediately leads to a problem of time since, in such theories, we have that the Hamiltonian is a sum of first class constraints. Application of the Dirac criterion then immediately implies that observables are condemned to be frozen as constants of the motion. In general relativity this problem recurs in a more complex fashion but with essentially the same elements. There is an infinite family of Hamiltonian constraints and if we insist that Dirac observables commute with them, then the observables of general relativity are frozen.

The standard approach to the problem of reconciling the Dirac definition of observables with the necessity to describe dynamics is the partial and complete observables approach. This approach was pioneered by Rovelli ([1991a], [1991b], [2002b], [2004], [unpublished-a]) and later formalized by Dittrich ([2006], [2007]).[8] The essence of this approach is to designate a subset of measurable quantities or 'partial observables' as internal clocks, and then use correlations between the remaining variables and these clocks to construct 'complete observables' that are both predicable and measurable, and which correspond

[6] For a scientific overview, see (Kuchař [1992]; Isham [1993]; Anderson [2017]). Casadio et al. ([2024]) provides an overview of a family of alternative approaches in which first-class phase-space constraints may be relaxed based on an interpretation of them as fixing the values of new degrees of freedom. Technically informed discussion in the philosophical literature include (Belot and Earman [2001]; Belot [2007]; Thébault [2021b]). A hybrid formal and philosophical monograph-length treatment of the global problem of time is (Gryb and Thébault [2023]). Further references will be given where relevant.

[7] This idea also traces back to the discussions of Bergmann ([1956], [1961a], [1961b]; Bergmann and Komar [1960]; [1962]) and so one might plausibly use the term Bergmann–Dirac observables. However, Bergmann changed his view at various points. For discussion, see (Pitts [2019]).

[8] For a detailed overview, see (Thiemann [2007]; Tambornino [2012]). Important developments of the approach include (Gambini and Porto [2001]; Gambini et al. [2009]; Bojowald et al. [2011a], [2011b]; Höhn [2019]). Critical responses include (Kuchař [1991], [1992], [1999]; Dittrich et al. [2017]). For a review of the various notions of observable, that includes discussion of the limitations of the partial and complete observables approach, see (Anderson [2014]; Anderson [2017]). For analysis of relationship between the partial and complete observables approach and earlier influential work by Page and Wootters ([1983]), see the analysis of Höhn et al. ([2021])—this work demonstrates the equivalence between the two approaches by putting the latter on a more rigours footing and resolving problems within its presentation as highlighted by Kuchař ([1992]). For philosophical analysis of the ontological implications of the partial and complete observables approach an excellent extended discussion can be found in (Rickles [2007], pp. 161–71). For a further overview, with references and discussion, see (Thébault [2021b]).

to Dirac observables. More specifically, the formal application of the approach requires one to consider, for each Hamiltonian constraint, one physical variable to play the role of a physical clock. One first constructs parametrized flow expressions for the 'evolution' of the clock and non-clock variables under the Lie flow of the vector field associated with the constraint; the simplest way to do this is via the relevant Hamilton–Jacobi equation (Rovelli [2004]; Gryb and Thébault [2016b], [2023]). One next inverts the flow equation for the clock variable and substitutes it into flow equations for non-clock variables to construct an algebraic expression for their correlation that is parameter free. Finally, one considers the correlation between the clock variable and the other variables at a particular value of the clock variable. This is a complete observable and corresponds to the value of the non-clock partial observables when the clock partial observables takes a particular value.

Let us provide the simplest possible physical example so the reader can conceptualize clearly how the procedure works. Consider two free particles moving in one dimension and described by a theory with a single Hamiltonian constraint. We can write an expression for the integral of motion in terms of the time parameter of the flow of the vector field associated with constraint, by solving the Hamilton–Jacobi equation. This will give us an expression for the position of each particle, q_i, as a functions of the constants of motion (that is, initial position and momenta), Q_i and P_i, and parameter time, t. This takes the form:

$$q_i(t) = Q_i + \frac{P_i}{m_i} t, \tag{1}$$

for $i = 1, 2$. These variables are partial observables and do not commute with the Hamiltonian constraint $\mathcal{H}$, since we have that $\{\mathcal{H}, q_i\} = \dot{q}_i \neq 0$. However, we can combine the two expressions for for $i = 1, 2$ to describe the correlation between the values of the position of each particle. We do this by inverting the expression for one variable such that we obtain t as a function of (q_i, Q_i, P_i), and then inserting this expression into the expression for the other. The first variable is then playing the role of a physical clock and we evaluate the second variable for a given value of the second variable, say $s \in \mathbb{R}$. In this way we get a family of complete observables, one for each value of s.[9] For even slightly complicated physical systems the inversion step may run into significant obstacles and is typically such that we can only define the relevant expressions for restricted values of the time parameter. Dittrich ([2007]) provides a detailed treatment of such a case. In our case, by contrast, since the physical dynamics is trivial and we are able to solve Hamilton equations for the considered system, the inversion is simply given by:

[9] Note that this is also the construction of the so-called evolving constant of motions (Rovelli [1991a]). In fact, a complete observable formally coincides with an evolving constant. The difference between the two concepts lies mainly in the fact that for evolving constants, the focus is on the evolution of the quantity with respect to the parameter, s, that serves as 'internal time'.

$$t = \frac{m_1}{P_1}(q_1 - Q_1).$$

Re-inserting this into equation 1, we get:

$$q_2(q_1) = Q_2 - \frac{P_2}{m_2}\frac{m_1}{P_1}(q_1 - Q_1).$$

Finally, we evaluate our expression $q_2(q_1)$ at $q_1 = s$ to get the parametrized family of complete observables:

$$q_2(q_1)|_{q_1=s} = Q_2 - \frac{P_2}{m_2}\frac{m_1}{P_1}(q_1 - Q_1)|_{q_1=s}.$$

This is a complete observable constructed according to the Rovelli–Dittrich procedure. It is also a Dirac observable since for any specification of s we have $q_2(q_1)|_{q_1=s} : \Gamma \to \mathbb{R}$ and $\{\mathcal{H}, q_2(q_1)|_{q_1=s}\} = 0$, where Γ is four dimensional phase space $(q_i, p_i) \in \Gamma$ for $i = 1, 2$.

There is a specific tension within the physics literature regarding the interpretation of the partial observables. This tension will prove crucial in the context of application of the approach to cosmology. Consider, in particular, that according to the original approach of Rovelli ([2002b], p. 2), by definition, a partial observable is 'a physical quantity with which we can associate a (measuring) procedure leading to a number'.[10] By contrast, following Thiemann ([2007], p. 78), we have that 'a measurable quantity is always a complete observable, even pointers of a clock are observables and not partial observables. Now complete observables are defined with respect to non-measurable quantities [...] which we will simply call non-observables'. A third view is advocated by Gryb and Thébault ([2016b], [2023]) in the context of theories with a single Hamiltonian constraint. On this approach, one can think of the complete and partial observables programme as allowing us de-parametrize evolution purely in terms of observable quantities. However, this evolution is fundamentally controlled by the evolution equations generated by the Hamiltonian constraint and is always well defined, even when a particular deparametrization breaks down. On this approach, even if one wishes to use parameter-free complete observable expressions, one is still required to retain the full partial observables representation. This supports the Rovelli ([2002b]) perspective, in which partial observables are measurable quantities, rather than the Thiemann ([2007]) perspective, where the partial observables are understood as non-measurable.

A further important disambiguation can be made based upon the connection between partial and complete observables and reference frames. The role of reference frames in

[10] In his original definition Rovelli makes clear that the definition should not be understood in operationalist terms. In particular, he notes: 'The operational tone of the [partial observable] definition does not imply any adherence to operationalism here [...] the reference to measuring procedures is just instrumental for clarifying a distinction' (Rovelli [2002b]). Crucially, the partial observables are well-defined theoretical quantities whose definition does not require specification of a measurement procedure. For more discussion of operationalism, see (Chang [2021]; Fankhauser and Dürr [2021]).

general relativity has an extensive philosophical literature.[11] Most relevant to our analysis is the distinction made by Bamonti ([unpublished]) between: idealized reference frames (IRFs), in which any dynamical interaction of the material system represented by the reference frame is ignored; dynamical reference frames (DRFs), in which the set of equations that determine the dynamics of the matter field is included but the the stress–energy tensor of the matter field used as reference frame is neglected; and real reference frames (RRFs) in which both the dynamics of the chosen material system and its stress–energy tensor are taken into account. This distinction allows us to disambiguate two important details in the definition of a partial observable, which is often not stressed in the relevant literature. Following Bamonti and Gomes ([unpublished-a]), we should understand partial observables to be relational but gauge-variant quantities that are nevertheless associated with a measuring procedure. We can understand this seeming contradiction in terms of the fact that partial observables are defined relative to an IRF. In particular, the parameter of flow equation acts as an IRF and, as such, partial observables are relational in the sense that they describe the correlation between a physical variable and the IRF. Consider our expression for the partial observables in equation 1. The variables $q_i(t)$ are measurable quantities of the theory, but they are not measurable independently of a specification of the value of the flow parameter, t. Furthermore, since 'all measurements are comparisons between different physical systems' (Anderson [1967], p. 128),[12] Here, t represents a physical system, whose dynamics is neglected as a result of approximations. The second relevant remark is that not every pair of physical quantities to which measuring instruments can be associated can play the role of partial observables. *Bona fide* partial observables must be dynamically coupled to each other, in order for their relation to constitute a *bona fide* complete observable (Bamonti and Gomes [unpublished-a]).

Let us then consider the status of the complete observables. In this context, the partial observable that is chosen as the clock observable is playing the role of a reference frame. The earleir point suggests that to construct a complete observable, we must use DRFs or RRFs. Since we are considering finite dimensional particle mechanics there is no stress–energy tensor to consider. However, the distinction between DRFs and RRFs can still be made. That is, a clock variable is always a DRF since its dynamics is always relevant via the flow equation. However, it is only an RRF when the coupling between the clock variable and the other non-clock partial observables is included. Our simple system with free particles is thus an implementation of the complete observables programme in terms of a DRF rather than an RRF.[13] However, in cases where the coupling is included,

[11] See (Earman and Friedman [1973], [1974]; Norton [1989], [1993]; DiSalle [2020]). Recently, a community of scholars has also emerged in the field of the so-called quantum reference frames, see (Giacomini [2021]; Kabel et al. [2025]), and reference therein.

[12] See also: 'In physics, when we talk about measurement, we refer to an interaction between a measured system S and a measuring apparatus O' (Rovelli [2014], p. 99).

[13] This connection also points to the sense in which the idea of inertial reference frames as discussed

complete observables admit an interpretation in terms of an RRF. This is precisely the application of the complete observables approach that we will consider in the context of cosmology in the following section.

3. Time and the Friedmann Equations

The universe is estimated to be 13.7 billion years old. This estimation is made based upon the standard model of cosmology—the so-called ΛCDM model—in which the spacetime structure of the universe is described via general relativity with a cosmological constant, Λ (Weinberg [1972]).[14] The base-model of modern cosmology, upon which more sophisticated models are built, is one in which the spatial structure is extremely simple. The cosmological principle is defined by the condition that the universe is spatially homogeneous and isotropic on large scales ($\geq 10^2$ MPc). This means that, from any location, the distribution of matter and energy appears the same and without any preferred direction. FLRW spacetimes are the class of generally relativistic spacetimes that are spatially isotropic and homogenous and satisfy certain physically motivated energy conditions or equations of state.[15] FLRW spacetimes are the formal basis for modelling the large-scale structure and dynamics of the universe with realistic models involving perturbations about an FLRW metric.

In the context of the FLRW metric and a perfect fluid model of matter, the Einstein field equations take on the remarkably simple form given by the Friedmann equations. These equations describe the evolution of a single geometric variable—the scale factor, $a(t)$—and take the form:

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}, \end{aligned}$$

where t is the cosmological time, $\rho(t)$ and $p(t)$ are the density and pressure of the matter, $k = 0, -1, +1$ is the spatial curvature parameter, and the constants have their usual meaning.[16] For a given specification of matter–energy, we can solve these equations to

in the late-nineteenth century by Lange, Neumann, Tait, and others are examples of DRFs and not RRFs (see Barbour [2000], pp. 101–4; Barbour [2001], sec. 12). This is one way of thinking about Mach's criticisms (see Thébault [2021a]).

[14] 'CDM' here stands for 'cold dark matter', which is a form of matter that does not interact with electromagnetic radiation (hence 'dark') and moves slowly compared to the speed of light (hence 'cold'). Cold dark matter is not universally regarded as being composed of material particles within an 'extended standard model'. For a detailed discussion, see, for example, (Turner [2000]).

[15] The original papers are (Friedman [1922]; Lemaître [1931]; Robertson [1935]; Walker [1937]). A formally precise overview is given in (Malament [2012a], sec. 2.11).

[16] The value of k does not fix the overall topology. In fact, different topological choices are possible for the same k. For example, a hyperplane (closed topology) is characterized by curvature parameter

get dynamical expressions for the scale factor. For the matter–energy mix that we take to correspond to our universe (including dark energy and dark matter), the relevant expressions describe an expanding universe that matches our observational data to a remarkable degree—although there are existing challenges (Smeenk [2022]; Perivolaropoulos and Skara [2022]; Ó Colgáin et al. [unpublished]). This model provides a standard, textbook-level story of the expansion of the universe that is assumed by almost all cosmologists to be unproblematic, at least back to the (presumed) inflationary epoch.

Remarkably, however, when the story regarding the Friedman equations and the expansion of the universe is combined with the Dirac criterion for observables, we run into an immediate and deeply problematic conflict. As just noted, the Friedmann equations describe the evolution of the scale factor, a, and this appears to provide a clear description of the time evolution of the spatial geometric structure of the universe. However, the Friedmann equations are equivalent to those generated by a Hamiltonian constraint. So, if the evolution equations generated by a constraint are interpreted as gauge transformations, then we should understand the Friedmann equations not as dynamical equations, but as gauge equations.[17] Quantities such as a might appear to evolve over time in cosmology. However, they are not gauge-invariant, and so this evolution is not to be understood as physical. Remarkably, this conflict between the Friedman equations and the Dirac criterion for observables has received almost not detailed discussion in the physics or philosophy literature.

The major exception is Thiemann ([unpublished]), who explicitly argues that 'it is incorrect to interpret the FLRW equations as evolution equations of observable quantities' although he does 'not doubt the validity the Einstein equations' he wants to 'stress that their interpretation as physical evolution equations of observables is fundamentally wrong'. Moreover, on this view: 'All textbooks on classical [general relativity] incorrectly describe the Friedmann equations as physical evolution equations rather than what they really are, namely gauge transformation equations. The true evolution equations acquire possibly observable modifications to the gauge transformation equations whose magnitude depends on the physical clock that one uses to deparametrise the gauge transformation equations'. A simple approach to formalizing this idea is to note that $a(t)$ does not Poisson commute with the Hamiltonian constraint of the theory. The time derivative of the scale factor is given by: $\dot{a}(t) = \{\mathcal{H}, a\} \neq 0$, where $\mathcal{H}$ is the Hamiltonian constraint in canonical

$k = 0$, like a hyperplane (open topology).

[17] We might, of course, simply reject Dirac's argument connecting gauge transformations to Hamiltonian constraints. In particular, his theorem that first class constraints generate gauge transformations does not apply to Hamiltonian constraints. For details, see (Barbour and Foster [unpublished]; Gryb and Thébault [2023], sec. 7.3). Moreover, rigorous formal analysis of these constraints indicate that there are distinct gauge generating and dynamics generating roles that can be explicitly disentangled in the case of theories with a single Hamiltonian constraint. For details, see (Gryb and Thébault [2023], sec. 13).

general relativity and $\{,\}$ denotes the Poisson bracket. When the theory is understood in these terms, it is indisputable that the scale factor is not a Dirac observable. This leads to an apparent contradiction with the physical observations of the universe's expansion. The natural response to this problem is to apply the complete and partial observable scheme to construct Dirac observables based upon the Friedmann system of equations. This is precisely what Thiemann suggests. His explicit proposal involves introducing a scalar field as a clock, capable of de-parametrizing the theory through the Brown and Kuchař ([1995]) mechanism. This approach allows for the construction of a physical Hamiltonian, which generates the evolution of gauge-invariant Dirac observables. Since the dynamics of the chosen material system and its stress–energy tensor are taken into account, it is also to explicitly implement a de-parametrization in terms of a RRF as per the discussion of the last section.

We give a sketch of the construction. Let us introduce a spatially homogeneous scalar field, ϕ, that acts as a 'phantom' field, which is not directly observable in modern cosmology but can have significant dynamical consequences leading to observable effects.[18] The key innovation of this approach is to de-parametrize the Hamiltonian constraint of general relativity, transforming it from a constraint equation into a physical Hamiltonian. The Hamiltonian constraint $\mathcal{H}$ is rewritten as $\mathcal{H} = \pi + \mathtt{h}$, where π is the conjugate momentum to the scalar field and $\mathtt{h}$ is called the physical Hamiltonian generating the temporal physical evolution of observables. Using this scalar field Thiemann constructs a framework in which the universe's time evolution is generated by the physical Hamiltonian rather than the Hamiltonian constraint. This reformulation allows for the de-parametrization of the theory, with $\mathtt{h}$ now acting as a physical Hamiltonian that generates time evolution for gauge-invariant observables. The crucial difference here is that $\mathtt{h}$ is not constrained to vanish, as is the case with the traditional Hamiltonian constraint in general relativity.

Once the theory is de-parametrized, the time evolution of observables such as the scale factor, $a(\phi)$, can be computed using the physical Hamiltonian:

$$\frac{da(\phi)}{d\phi}|_{\phi = s} = \Big\{\mathtt{h},\, a(\phi)\Big\}|_{\phi = s} \neq 0,$$

for $s \in \mathbb{R}$. Of course,

$$\Big\{\mathcal{H},\, a(\phi)\Big\}|_{\phi = s} = 0.$$

This evolution is now consistent with the Dirac criterion, as $a(\phi)$ is a Dirac observable that Poisson commutes with all constraints, unlike the original scale factor $a(t)$, which

[18] It is worth nothing here that there are two importantly different senses of phantom that coincide in Thiemann's ([unpublished]) usage. First, 'phantom' is used in the sense of 'missing physics' that is not directly observable in modern cosmology. Second, 'phantom' is used in the more formal sense of cosmologists, to indicate a field with a first order kinetic term in the Lagrangian with a coefficient that has a sign opposite to the sign in the Klein–Gordon Lagrangian.

did not Poisson commute with the Hamiltonian constraint. Crucially, however, in this framework, the evolution of $a(\phi)$ is governed by a modified version of the Friedmann equations, which includes additional terms due to the presence of the scalar field. In particular, the first Friedmann equations reads as:

$$\left(\frac{da/d\phi}{a(\phi)}\right)^2 = \left[\frac{8\pi G}{3}[\rho_m(\phi) + \rho_{\text{phantom}}(\phi)] + \frac{\Lambda}{3}\right]\left(1 + \frac{1}{x}\right),$$

where $x = {E^2}/{\alpha^2 a(\phi)^6}$ is a deviation parameter, used to quantify how much the dynamics of the universe, governed by the modified Friedmann equation, differs from the standard cosmological model; E is a constant of motion, representing the energy of the universe; α is a model parameter characterizing the influence of the phantom field. We chose $k = 0$ to adhere to Thiemann's formalism.

We thus arrive at an observationally distinct formulation of the theory that implements the Dirac observable prescription. The implication is then that we can either have the standard Friedmann equation and give up on our formalism for gauge-invariant observables or we can keep our formalism for observables gauge-invariant observables and modify the Friedmann equations. We cannot have both. Thiemann emphasizes the gravity of this problem, stating that either the mathematical formalism of general relativity is inappropriate for cosmology, or we are missing some new physics. In the following section, we seek to extricate ourselves from Thiemann's dilemma based upon the use of Einstein's famous clock hypothesis: that physical clocks measure proper time along their world-lines.

4. Hubble Flow and the Clock Hypothesis

Let us return to the derivation of the Friedman equations and seek an alternative physical interpretation of the t in the equations. One crucial aspect of the model we have not yet explicitly considered is the idea of Hubble flow. This 'flow' describes the large-scale motion of matter, driven by the expansion of spacetime itself. More formally, Hubble flow is the component of recessional velocity of matter due to the expansion, separating it from peculiar velocities caused by local gravitational interactions.

One way of understanding the derivation of the FLRW metric is via the adoption of the so-called synchronous reference frame (Landau and Lifshitz [1987]). In this frame there is a common, global cosmological time for all observers comoving with the Hubble flow. Crucially, this means that the synchronous frame is a geodetic reference frame. Consequently, time-like trajectories orthogonal to space-like three-dimensional hypersurfaces are geodesics of spacetime and the four-velocity of each observer $U^\mu = (1, \vec{0})$ automatically satisfy the geodesic equation.

From this perspective the Friedmann equations are not gauge transformation equations. Rather, they are evolution equations in a specific gauge. We can see this as follows: Recall that in the ADM formalism (Arnowitt et al. [1960]) for canonical general relativity the metric is expressed in terms of the lapse function $N(t)$, the shift vector $N^i(t)$, and a spatial metric h_{ij}.[19] The lapse function N formalizes the temporal separation between two infinitesimally close hypersurfaces, measured in the normal direction to the first hypersurface. The shift vector N^i measures the displacement between the spatial coordinates x^i of a point $P \in \Sigma_t$ and its orthogonal projection $Q \in \Sigma_{t+dt}$. The connection between N and temporal diffeomorphisms and N^i and three-diffeomorphisms emerges. It is specified that in order to have a future-directed foliation, the lapse function N must be positive. In this formalism the general line element becomes:

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right).$$

In the specific case of FLRW cosmology, the lapse function $N(t) = 1$ and the shift vector $N^i(t) = 0$.

These choices defines the synchronous gauge, reflecting the absence of preferred locations and directions and simplifying the FLRW metric. Since in the synchronous gauge we can assume the coordinate time t to coincide with the proper time τ measured by observers comoving with the Hubble flow, we will have that the t in the equations will coincide with the proper time of the relevant bundle of geodesics following the Hubble flow. This means we can re-write the equations in terms of proper time τ simply by equating $t = \tau$.

It is worth noting that it is not necessary to show that $\left\{\mathcal{H},\, a(\tau)\right\} = 0$, since we already 'gauge-fixed' to the synchronous gauge. This means that $a(\tau)$ can be seen as a gauge-fixed observable and, given this, is gauge-invariant. A gauge-fixed observable is defined in that particular gauge and need not commute with the constraints because there are no more gauge transformations to refer to. In other words, the gauge freedom has already been eliminated and we no longer have any gauge constraints left, because we have chosen a specific reference frame (on this, see, for example, Wallace [2024]). We can then clarify the claim of Dittrich ([2007]) that complete observables and gauge-fixed observables are the same, since a choice of reference frame can be seen as a gauge choice (a choice of section on a fibre bundle). Or, at least, it is always true that a gauge-fixed observable is a complete observable and only the reverse is less immediately obvious. For our current purpose, the key insight is recognizing that the Friedmann equations, when considered in a synchronous gauge, function like dynamical evolution equations.[20]

[19] From Geroch's ([1970]) theorem it follows that a globally hyperbolic spacetime can be foliated, that is, decomposed into spatial slices parametrized by a global parameter, t. The ADM formalism can thus be applied to any globally hyperbolic spacetime.

[20] A more detailed discussion of the status of gauge-fixings, reference frames and gauge invariant observables in the context of cosmic proper time, within the fibre bundle formalism, is provided in in

To show this, consider that the Hubble flow should be understood to be the flow of a perfect fluid whose stress–energy tensor in the synchronous gauge is $T_{\mu\nu} = \mathrm{diag}\big(\rho(\tau),\ -p(\tau)\big)$, where $\rho(\tau)$ is the fluid's energy density, $p(\tau)$ its pressure. The Friedmann equations are then understood to describe how the expansion rate changes with the proper time of the observers comoving with the fluid. They can be written in terms of the Hubble parameter $H(\tau) = {}^{\dot{a}(\tau)}/_{a(\tau)}$ as:

$$\begin{aligned} H(\tau)^2 &= \frac{8\pi G}{3}\rho(\tau) - \frac{kc^2}{a(\tau)^2} + \frac{\Lambda c^2}{3}, \\ \dot{H}(\tau) + H(\tau)^2 &= -\frac{4\pi G}{3}\left(\rho(\tau) + \frac{3p(\tau)}{c^2}\right) + \frac{\Lambda c^2}{3}, \end{aligned}$$

where we have used the fact that ${}^{\ddot{a}}/_{a} = \dot{H} + H^2$ and assumed differentiation with respect to proper time. For a given matter model we can then write the density and pressure in terms of the scale factor allowing us to for example re-write the first equation as:

$$H^2(\tau) = H_0^2\left[\Omega_{m,0}\frac{1}{a(\tau)^3} + \Omega_{r,0}\frac{1}{a(\tau)^4} + \Omega_\Lambda + \Omega_k\frac{1}{a(\tau)^2}\right],$$

where H_0 is the Hubble constant and we have introduced experimentally measurable density parameters at the current time, $\Omega_{0,R}$ for the radiation density, $\Omega_{0,M}$ for the matter (dark plus baryonic) density, $\Omega_{0,k}$ for the spatial curvature density, and $\Omega_{0,\Lambda}$ for the cosmological constant density.[21] The question is then whether and in what sense Hubble parameter $H(\tau)$ and the proper time along the Hubble flow can be understood to be measurable quantities. Let us consider each in turn.

First, the Hubble parameter: Experimentally, we cannot of course directly measure the Hubble parameter with a 'H-metre'. However, we can surely measure it indirectly, using astronomical observations that allow us to trace the expansion of the universe at different cosmic epochs. Let's just consider one of the various experimental possibilities. Type Ia supernovae are considered standard candles in cosmology because they have a well-known absolute luminosity. By measuring the apparent luminosity of a supernova, we can estimate its distance (luminosity distance, d_L), while the redshift, z, is a direct indicator of how much the universe has expanded from the time of light emission of an object to the present day, τ. The luminosity distance, d_L, is related to the Hubble parameter, $H(z)$, through the following relation:

(Bamonti and Thébault [unpublished-b], appendix A). It is worth noting that conceptualizing the Friedmann equations as gauge-fixed equations can be made consistent with the arguments of Gryb and Thébault ([2023], sec. 13.2), which imply a dynamical view of global Hamiltonian constraints in which the flow along a solution of the Friedmann equations would not formally be a gauge symmetry. This is because one may view the symmetries of the theory as acting on entire histories and take their action on the boundary to be fixed and then recover the 'gauge-fixed' perspective where the Friedmann equations in synchronous gauge describe evolution of the true degrees of freedom.

[21] The measurement of H_0 suffers from the so-called Hubble-tension. For a review, see (Smeenk [2022]).

$$d_{\mathrm{L}}(z) = (1+z)\int_0^z \frac{c\,dz'}{H(z')}. \tag{2}$$

By measuring z through spectrometers and d_{L} through large-field telescopes such as the Hubble space telescope, we can infer $H(z)$. However, notice that this procedure requires 'fitting' noisy discrete data, introducing mild model dependence. From $H(z)$, the value of Hubble parameter, $H(\tau)$, at different times, τ, can be derived. In fact, using the definition of redshift in terms of scale factor a (namely, $a(\tau) = {}^{1}/_{1+z}$) and the fact that $H = {}^{\dot{a}}/_{a}$, we can derive the following expression:

$$H(z) = -\frac{\dot{z}}{1+z}.$$

This equation highlights that since it is impractical to directly measure $\dot{z}$, we must infer its behaviour using a specific cosmological model. The construction of $H(\tau)$ requires combining observations with a theoretical model that relates cosmic time τ to redshift z.[22] Thus, any inference of H from redshift measurements likely involves some degree of model dependence. The procedure to transition from $H(z)$ to $H(\tau)$ can be summarized as follows: first, infer $H(z)$ using equation 2; second, compute $\tau(z)$ according to the chosen theoretical model (see section 5); third, convert relation $\tau(z)$ to obtain $z(\tau)$; finally, combine $H(z)$ with $z(\tau)$ to construct $H(\tau)$. This procedure underscores the fact that H and τ are not completely independent: knowledge of one requires knowledge of the other. Their (dynamical) interdependence supports the claim that $H(\tau)$ is a complete observable, in line with the remark outlined in section 2, proposed by Bamonti and Gomes ([unpublished-a]). For completeness, we note that model dependence is not unique to the measurement of $H(z)$. Many cosmological parameters rely on similar assumptions.

Second, and more subtle, is the question of whether we can measure the proper time along the Hubble flow. Again, we evidently cannot measure directly the proper time, τ, of our galaxy following the Hubble flow. If we say that we use whatever periodic physical system as a clock 'attached to the galaxy', it will not follow the Hubble flow. Actually, the concept of Hubble flow can be valid only at cosmological scales, so even for our Galaxy we should account for the effects of peculiar velocities. Completely eliminating peculiar motions from measurements of galaxy recession velocities is not possible, but it is possible to correct them in an approximate way. Therefore, strictly speaking, experimentally we cannot measure directly with a clock the proper time of any object following the Hubble flow. What we can do is appeal to the clock hypothesis. This amounts to the hypothetical assumption there is a clock that measures proper time along any given world-line. Proper time can be rigorously defined in the context of a relativistic spacetime, (M, g_{ab}), following (Malament [2012b], sec. 2.3): Let $\gamma : [s_1, s_2] \to M$ be a smooth future-directed timelike

[22] See equation 4 in section 5, where a way to calculate $\tau(z)$ is provided.

curve in the manifold M with tangent ξ^a. Then the proper time associated with the curve relative to the metric g_{ab} is given by:

$$||\gamma|| = \int_{s_1}^{s_2} \left(g_{ab}\, \xi^a\, \xi^b\right)^{1/2} ds, \tag{3}$$

where ds is the line element. Since the clock hypothesis applies also to the world-lines of observers following the Hubble flow, we seem to have solved the problem by stipulation. Since we can associate measurements with partial observables and partial observables with reference frames, we can wonder: what kind of reference frame the cosmic proper time is? And it is a *bona fide* partial observable? Given the clock hypothesis, we can stipulate a clock that measures cosmic time. Furthermore, cosmic time will always be dynamically coupled with the expansion rate of the Universe, parametrized by $H(\tau)$, since both quantities depend on the same FLRW metric. The cosmic time value, being a proper time, is a structural property of the gravitational field given by equation 3. This is very similar to the sense in which the Hubble parameter measures the rate of expansion of the volume of the universe as a geometric quantity, which is derived from the gravitational field: $dV = \sqrt{-g}\, dx^\mu$. Here, τ is a geodesic reference clock, since the four-velocity of the cosmological fluid is associated to the geodesic dynamics of a dust fluid, whose energy–momentum tensor $T_{ab} = \rho\, U_a\, U_b$ is source of the EFEs and give rise to the FLRW solution. Thus, τ is an RRF in Bamonti's ([unpublished]) classification. The nature of the RRF clock comes from the fact that τ is the proper time of the cosmological fluid, whose back-reaction on gravity is taken into account and gives rise to the FLRW metric that in turn determines the proper time, τ (this is the essence of the non-linear feedback of EFEs). This RRF clock provides the privileged representation in which the cosmic microwave background radiation is represented as perfectly homogeneous and isotropic, in absence of small inhomogeneities of the primordial universe. We thus have that cosmic proper time, first, is an RRF since it involved back-reaction; second, is a *bona fide* partial observable according to Bamonti and Gomes ([unpublished-a]); and, third, corresponds to a measurable quantity by the clock hypothesis. Is this enough for us to conclude that $H(\tau)$ is a complete observable and thus have solved Thiemann's dilemma? Almost.

As already stated, the problem is that on a practical level, it is not possible to have an experimentally accessible clock (that is, with which we can exchange signals), that follows the Hubble flow. In general, distant galaxies are considered to follow the Hubble flow, as for very distant galaxies, the contribution of their peculiar motions is negligible compared to the recession velocity due to the expansion of the universe. Thus, let's consider a galaxy in our past light cone as a satellite sending radiation towards us. In this way, we would use it as a kind of Rovelli's ([2002a]) 'GPS clock' that would allow us to define local quantities, such as the Hubble parameter, in its proper time.[23] Again, however, the cosmic time

[23] Note also that clusters of matter represent inhomogeneities that are assumed to evolve following the

would be the time measured by some stipulated and not further defined clock 'attached' to that galaxy-satellite and broadcasting the measured value to the experimenter, via light signals. Apart from the experimental problem to construct a valuable experimental setting, and the need to take into account the galaxy's peculiar velocities, there remains the problem of defining an origin of such time measured by the clock-galaxy. One solution would be to conveniently place the origin as the 'zero time' of the formation event of such a galaxy. Or also, analogous to the construction of GPS coordinates in (Rovelli [2002a]), as the time of the galaxy's encounter with another galaxy. In any case, the proper time of the galaxy will never be a global time. As Rovelli ([unpublished-b]) states: 'Our Galaxy and Andromeda are heading towards a collision: when they will meet, the times elapsed from the Big Bang will be different in the two galaxies. None of the two will have any claim of being more of a "true" time than the other'. It is practically impossible to have a clock measuring the proper time parametrizing the Hubble flow.[24] In this context, one might view the clock hypothesis as constituting the definition of clocks as objects that measure proper time. The important point is that if the clock hypothesis holds, then we are able to treat the τ in the Friedmann equations as a measurable quantity just like the Hubble parameter. However, in the context of cosmology and the Hubble flow, there is a tension between applying the clock hypothesis and the idea of a clock as a real, experimentally accessible physical system. There do not exist real physical systems that approximate clocks that measure proper time along the Hubble flow. In the following section, we return to the ideas of partial and complete observable and RRFs to better understand both this challenge and the comparative merits of the approach of Thiemann described in the previous section.

5. Finding Cosmic Time

Let us recap. One the one hand, the widely used and accepted criterion for an observable in a theory with temporal diffeomorphism symmetry is that such observables should be Dirac observables and therefore have (weakly) vanishing Poisson bracket with all first class constraints. On the other hand, the widely used and empirically established Friedmann equations describing the dynamics of the scale factor can be understood to correspond to those generated by a first class constraint in a theory with temporal diffeomorphism symmetry. It seems like we must either give up on the Dirac criterion for observables or

underlying FLRW background structure. So, their evolution does not influence the global FLRW evolution: 'More precisely, it is assumed that effects from the small scale inhomogeneities onto the largest scales can be neglected, i.e. there is no substantial backreaction' (Schander and Thiemann [2021]). Thus, galaxies, clusters and other agglomerates of matter are treated as test particles.

[24] In a similar vein Brown and Read ([2016]) note: 'For any given clock, no matter how ideal its performance when inertial, there will in principle be an acceleration-producing external force, or even tidal effects inside the clock, such that the clock "breaks", in the sense of violating the clock hypothesis. Might it not be more appropriate to call it the clock condition?'.

modify our understanding cosmological dynamics. We have considered two alternative responses to this dilemma as follows: First, Thiemann argues that we should adopt the second option and demonstrates how we might explicitly reconstruct Friedmann cosmology as a de-parametrized theory based upon a phantom matter field acting as a physical clock that measures cosmic time. A deviation parameter then quantifies how much the dynamics of the universe, governed by the modified Friedmann equation, differs from the standard model of cosmology. Second, we have constructed an alternative approach that re-interprets Friedmann equations as evolution equations parametrized by proper time, rather than coordinate time. On this approach we understand the equations as describing the correlation between two independently measurable 'partial observables' given by the Hubble parameter and proper time along the Hubble flow. The Hubble parameter is a measurable quantity within modern cosmology. Furthermore, following the clock hypothesis, we have that since clocks measure proper time along world-lines, a clock following the Hubble flow will necessarily measure cosmic time. The problem, however, is that the proper time along the Hubble flow does not correspond to a physical quantity associated with a measuring procedure by a clock leading to a number, and so it seems we are no longer implementing the partial and complete observables procedure in the spirit of Rovelli ([2002b]). The crucial issue is to define an experimental measure of cosmic time consistent with the clock hypothesis.

In this context, it is worth noting again that Thiemann holds a different understanding of the partial and complete observables approach to Rovelli. In particular, for Thiemann partial 'observables' are not observables at all and so there is no sense in which they need to be associate to a measuring procedure. We can thus understand the Thiemann approach to complete observables and cosmic time as built upon abandoning not one by two conventionally accepted aspects of the formalism, namely, the clock hypothesis and the distinction between partial and complete observability. An observable *simpliciter* is defined as a Dirac observable and there is no requirement that such observables are built out of independently measurable functions, which, however, are not (Dirac) observables. On this way of thinking, $H(\tau)$ will not be an observable *simpliciter*, and should not be expected to commute with the Hamiltonian constraint. In fact, since the clock hypothesis is abandoned, τ is not observable, and for that $H(\tau)$ is not a (Dirac) observable either. Plausibly, it is precisely the abandonment of the clock hypothesis that led Thiemann to use a phantom scalar clock as the physical, observable clock of the theory. In any case, the crucial point is that adopting this perspective does not amount simply to an alternative interpretation of the theory. Rather, it is to reformulate the classical theory of cosmology such in a way that modifies the empirical consequences. What modifications are made will depend upon the clock choice. However, there is no choice that corresponds to a strict preservation of the Friedmann equations: 'whatever matter is used for deparametrization, there will be corrections [...] This should have observable consequences!' (Thiemann [unpublished]). Non-standard empirical consequences are of course a virtue in a physical theory. Furthermore it is worth noting that Thiemann's approach in the paper in question is connected to a specific research programme in terms of Phantom k-essence cosmology (Aguirregabiria et al. [2004]) and thus the modifications in question could be independently motivated and in principle tested via the relevant modified matter or gravity theory. There are thus good methodological reasons to pursue more radical approach to identifying cosmic time.

Can we plot a plausible path towards a more conservative approach that preserves both the distinct notions of partial and complete observable and the clock hypothesis? Recall once more that on the original Rovelli's definition a partial observable is a physical quantity associated with measuring procedure leading to a number. This definition fits well with the way the Hubble parameter features in our cosmological observational practice, even if its measurement is indirect. The problem was that proper time along the Hubble flow is not associated with a direct measuring procedure in any straightforward sense that involves a clock. Notwithstanding the clock hypothesis, there seems to be a tension between our intuitive notion of measurement of time (by a clock) and the role played by cosmic proper time in our scientific theories. It is worth considering at this juncture, however, that the intuitive notion of measurement has itself been disputed in the context of a practice orientated account of scientific measurement. This has lead to a reorientation of the philosophy of scientific measurement towards a model-based account of measurement. Furthermore, perhaps the most detailed and powerful example of the conceptual heavy lifting that a model-based account of measurement can do is in context of the measurement of time.

Let us briefly consider key ideas from the path-breaking work of Tal ([2016]) on the measurement of time via atomic clocks (but see Thébault [2021a]). Following this account, we recognize that the time 'measured' by atomic clocks does not correspond to a simple procedure of reading a number from a device. Rather, 'coordinated universal time' (UTC) is based upon a standardization procedure involving multiple atomic clocks distributed throughout the globe and systematic modelling at various stages. Caesium plays a particularly important role in modern time-keeping since it is transitions of an idealized caesium atom that are the basis for the definition of the second. However, as emphasized by Tal, this does not mean that one can simply read seconds from real caesium atoms. The caesium atom that defines the second is an idealized construct, at rest at zero degrees Kelvin and with no coupling to any external fields. Actual atomic clocks are built to approximately realize the ideal caesium clock, with known sources of difference minimized and modelled. However, the 'primary' standards caesium clocks typically only operates for a few weeks at a time in order to calibrate 'secondary standards'. The secondary standards are a different class of atomic clocks that are less accurate but can be run continuously for a number of years. The secondary standard clocks also must be modelled. In particular, the 'readings' of the clocks are subject to quantitive adjustments relating to the known sources of difference between their ideal physical operation and their actual physical realization. This allows the time that they read to be a close approximation to that read by their idealized counterpart. The crucial point is that UTC is not 'read' by either primary or secondary standards. Rather it is a product of a further abstraction based upon the readings of the different participant clocks throughout the world. Furthermore, not only are different clocks weighted differently in UTC, since some clocks are more noisy, but since the clocks are at different physical locations on the earth, one must also take account of their differing proper times, as determined by the relevant differences in gravitational field and four-acceleration (Tal [2016], p. 302). One also requires a procedure for synchronization that introduces an element of conventionality. However, despite including conventional elements, the synchronization of the clock network towards UTC is partially anchored in underlying regularities in nature that are required for the explanation of successful stabilization of a synchronization standards in 'metrological practice' (Tal [2016], sec. 3.1).[25]

The general implication for a model-based account of measurement for the definition of a partial observable is thus to substantially liberalize the sense of 'procedure' in Rovelli's definition. In particular, on a model-based account of measurement, the procedure involved may include not just indirect measurement but various tiers of modelling, calibration, and aggregation. Furthermore, and more importantly for our discussion, comparison between the status of UTC and cosmic time throws into relief our failure in the latter case to find an actual physical system that plays the role of the clock. There are no actual physical systems that measure UTC either. Consider, then, that we can 'measure' cosmic time via various cosmological phenomena. For example, we can use the analysis of the power spectrum of temperature fluctuations of the CMB (TT power spectrum) measured by the Planck satellite. In particular, the CMB power spectrum represents the power distribution of the temperature anisotropies as a function of angular scale. The different angular scales correspond to the scales of the baryonic acoustic oscillations (BAOs), which are pressure waves generated by interactions between radiation (photons) and matter (baryons) in the primordial plasma before decoupling era. Anisotropies reflect primordial density differences, which led to the formation of galaxies and other structures (Kolb and Turner [1994], chap. 9). The power spectrum contains peaks and valleys at different scales and their position and amplitude are sensitive to cosmological parameters. For example, a higher density of matter leads to higher peaks that are closer together, and a flat universe tends to have a different distribution of peaks than a curved universe. The Hubble constant, H_0, also affects the scale of the fluctuations and the position of the

[25] For specific technical details regarding the relativity of synchronization in the context of UTC and the GPS system, see the discussion in (Ashby [2003]).

peaks.

By reconstructing the power spectrum it is possible to determine cosmological time. In particular, cosmological time can be determined by integrating the equations of the expansion of the universe:

$$\tau(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}, \tag{4}$$

where $H(z)$ is obtained from cosmological parameters Ω_I, H_0, which are determined by the experimental power spectrum. To be precise, $\tau(z)$ here does not correspond to the proper time of a real observer comoving with the Hubble flow, since in observational practice, we know the Universe is not perfectly homogeneous and isotropic: not even on cosmologically large scales. The FLRW model is an idealization to describe the average behaviour of the Universe. The Hubble parameter $H(z)$ in the formula depends on parameters such as Ω_{CDM} (dark matter density), which are in part determined using perturbative methods, which take into account small inhomogeneities. Therefore, in measuring the parameter $H(z)$, contributions of inhomogeneity (such as galaxy clusters, voids) and anisotropies are taken into account. Nonetheless, the value of $H(z)$ used in the theoretical calculation of $\tau(z)$ refers to the 'average' behaviour of the Universe, described by a homogeneous and isotropic model. We mean that the formula, $\tau(z)$, calculating cosmic time is based on a model that idealizes the Universe as perfectly homogeneous and isotropic, using an 'average value' of $H(z)$. This means that the cosmic time $\tau(z)$ is a global average, which does not reflect local fluctuations or deviations from isotropy, but rather provides an approximated estimate. One could say that, experimentally, the contributions from inhomogeneities and anisotropies of the actual Universe are 'averaged out to zero', in the sense that these contributions are present in the data but, when calculating the Hubble parameter $H(z)$, local fluctuations are essentially 'smoothed out' and do not significantly influence the overall result. Consequently, experimental measurements approximate the theoretical ideal cosmic time, which refers to an ideally homogeneous and isotropic Universe, analogously to what happens with primary and secondary clocks for measuring UTC. It is the case that $\tau(z)$ is used to make predictions in cosmology and it is a partial observable, *pace* (Thiemann [unpublished]). We thus have that in a more liberalized sense of 'measurement procedure', it is the case that we can treat proper time along the Hubble flow as a partial observable in the context of cosmology. This has not involved simply stipulating proper time as partial observable via the clock hypothesis. However, it also has not involved the abandonment of the clock hypothesis altogether. Rather, the clock hypothesis reemerges within cosmology as something like a 'coordinative definition' in sense of Reichenbach ([1928]). That is, the clock hypothesis allows for the coordination of a concept (an ideal clock measuring the Hubble flow temporal parametrization) with an empirical phenomenon (cosmic time). This broadly logical empiricist understanding accords with other discussions of the clock hypothesis in the recent philosophical literature (Adlam et al. [unpublished]) and makes sense of the physically non-trivial but partially definitional role (but see Fletcher [2024]). We recover the Friedmann equations and cosmic time while keeping both the clock hypothesis and the partial–complete observable distinction, albeit each in modified form. The conservative option for finding cosmic time is thus a live possibility.

6. Summary and Outlook

The complete and partial observables programme is a response to the problem of time that seeks to preserve the Dirac criterion for observables. Recent work on reference frames allows us to disambiguate the definition of a partial observable and better understand what it means for a physical variable to play the role of a clock in the context of a complete observable. Thiemann's approach to the interpretation of the Friedmann equations as dynamical equations leads to their de-parametrization via a phantom field through the Brown and Kuchař ([1995]) mechanism. An alternative approach is based on the idea of comic time as a proper time parameter along the Hubble flow. In this context, we can re-consider the observability and measurability of the Hubble parameter and its status as a complete observable whose dynamics is described by the unmodified Friedman equations. The problem is then that there is no physical system that can be understood to even approximately measure proper time along the Hubble flow. This leads to the introduction of a more liberalized sense of 'measurement procedure' in the context of cosmic time drawing upon the model-based account of measurement. Provided one is willing to reinterpret our criteria of observability in light of the clock hypothesis and the model-based account of measurement, one can preserve the Friedmann equations and find time in cosmology.

The problem of time is often presented as arcane question in the foundations of quantum gravity. If the loss of time within our physical theories were only an issue at the Planck scale then it would hardly seem an urgent one. We hope to have given good reasons to dispel such complacency. In particular, we have show how aspects of the problem occur even in the familiar context of classical cosmology and even at scales at which we already have a wealth of observable evidence, such as those relevant to the age of the universe. How we approach the problem can have empirical, physical consequences. Moreover, on our account, the cosmic problem of time is closely connected to both important methodological questions relating to idealization and the nature of measurement and to familiar foundational topics such as the interpretation of the clock hypothesis and the nature of gauge degrees of freedom. Given this, we suggest the search for cosmic time warrants more attention from both philosophers of science and scientists alike.

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