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DOCTORAL THESIS

The Role of Reference Frames In The Foundations of General Relativity

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Declaration of Authorship

I, Nicola BAMONTI, declare that this thesis titled, "The Role of Reference Frames In The Foundations of General Relativity" and the work presented in it are my own. I confirm that:

- This work was done wholly while in candidature for a Doctoral degree at both Universities.
- Where I have consulted the published work of others, this is always clearly attributed.
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"We seek the truth and will endure the consequences"

The Harbinger — John Wick: Chapter 4

Abstract

This thesis offers a systematic investigation of reference frames in General Relativity, with a particular emphasis on distinguishing them from coordinate systems. A novel classification of reference frames is proposed, grounded in their dynamical coupling to the gravitational field. Building on this classification, the dissertation revisits Earman's symmetry principles, demonstrating that relaxing the assumption of dynamical coupling gives rise to new violations of the SP1 principle. The notions of influence and correlation between fields are analysed, enabling a principled distinction between physical and dynamical fields. The thesis also addresses the concept of observability in General Relativity, criticising existing notions in the relevant literature and advancing a new interpretation of Einstein's point-coincidence argument, with implications for the ontology of spacetime.

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Snoop Dogg

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List of Abbreviations

GR	General Relativity
SR	Special Relativity
EFEs	Einstein Field Equations
URF	Uncoupled Reference Frame
IRF	Idealised Reference Frame
ARF	Auxiliary Reference Frame
CRF	Coupled Reference Frame
DRF	Dynamical Reference Frame
RRF	Real Reference Frame
KPM	Kinematically Possible Model
DPM	Dinamically Possible Model
BPM	Boundary Possible Model
GDS	Generalised Dynamical Symmetry
AR Principle	Action-Reaction Principle

CMB	Cosmic Microwave Background
DI	Diffeomorphisms-Invariance
DET	Deterministic evolution
GI	Gauge-Invariance

List of Symbols

Indices

$(a, b, c, \dots)$	Abstract spacetime indices
$(\mu, \nu, \rho, \dots)$	Coordinate-based spacetime indices
$(i, j, k, \dots)$	Spatial indices (e.g. $i = 1, 2, 3$)
$(I, J, K, \dots)$	Reference frame indices
$(A, B, C, \dots)$	Internal Minkowski indices (tetrads)

Spacetime & Geometry

$\mathcal{M}$	Smooth spacetime manifold
g_{ab}	Metric tensor (abstract index notation)
η_{ab}	Minkowski metric
$R^a_{\ bcd}$	Riemann curvature tensor
G_{ab}	Einstein tensor ($G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R$)
T^{ab}	Stress-energy tensor
∇_a	Covariant derivative
$\Box_g$	d'Alembertian operator ($g^{ab} \nabla_a \nabla_b$)

Symmetry Groups

Diff($\mathcal{M}$)	Diffeomorphism group of $\mathcal{M}$
Poin($\mathcal{M}$)	Poincaré group on $\mathcal{M}$

Fields & Reference Frames

$\phi^{(I)}$	Scalar fields defining reference frames ($I = 1, 2, 3, 4$)
$g_{IJ}(\phi)$	Relational metric in reference frame coordinates
$e_a^{(A)}$	Tetrad fields (orthonormal frames)
(Θ, Ψ)	Generic fields

Lagrangians & Dynamics

$\mathcal{L}$	Lagrangian density
S	Action functional
D_a	Covariant gauge derivative (e.g., $D_a\phi = \nabla_a\phi + ieA_a\phi$)
F_{ab}	Electromagnetic field tensor

Mathematical Operators

$[\bullet]^*$	Pullback
d^*	Pullback under diffeomorphism d (e.g. $[d^*g]_{ab}$)
$\det(\bullet)$	Determinant (e.g., $\det(\partial\phi^{(I)}/\partial x^\mu)$)
α_C^t	Gauge flow generated by constraint C

Miscellanea

$F_{[f,T]}(s;x)$	Complete observable constructed from partial observables f and T
$\mathbf{r}^{S}_{\Theta,\Psi}$	Symmetric direct influence between fields Θ and Ψ
$\mathbf{r}^{NS}_{\Theta,\Psi}$	Non-symmetric direct influence
$\tilde{\mathbf{r}}^{S}_{\Theta,\Psi}$	Symmetric indirect influence
$\tilde{\mathbf{r}}^{NS}_{\Theta,\Psi}$	Non-symmetric indirect influence

Chapter 1

Introduction

General Relativity (GR) has long provided a compelling framework for understanding the nature of spacetime and gravity and the dynamics of both the universe as a whole and its constituents.

A central role in this framework is played by *reference frames*, which are essential for defining and interpreting relevant results of the theory. However, despite their invaluable significance, as this thesis contends, the precise definition and role of reference frames are frequently obscured by ambiguities that have persisted since the inception of GR. This thesis aims to address some of these ambiguities through a rigorous analysis of reference frames in the context of GR, thus highlighting the power of the reference frame machinery to address some of the deeper conceptual problems of the theory.

One such problems is the interpretation of the diffeomorphic symmetry of the theory. Diffeomorphisms in GR are often understood as *gauge symmetries*, which are transformations that leave the physical content of a theory unchanged and somewhat obscure the relationship between 'meaningless' mathematical objects and objects describing real physical quantities. By defining and clarifying the dynamically relevant role of reference frames – as opposed to that of coordinates, which are mere mathematical labellings without any dynamical role in the theory — my work seeks to provide an understanding of how gauge symmetries operate in GR, how they should be interpreted and ultimately how they relate to the construction of the so-called local observables (Chapter 2). *Local observables* are quantities that describe the physical properties of a system at specific spatiotemporal locations. For example, in GR one of such local observables could be the value of the Ricci curvature scalar 'here and now'. However, the *background independence* of GR does not allow the definition of a 'here and now' that is *a priori* assigned and external to the dynamics of the theory. GR is the dynamic theory *of* spacetime, not a dynamical theory *in* spacetime. In particular, the challenge of defining local observables arises because the theory is invariant under a particular group of gauge symmetries: the *diffeomorphisms*, which are transformations that 'reassign spacetime points'.[1] Consequently, the value of Ricci curvature scalar at a point is not a gauge-invariant quantity and, according to Dirac (1964)'s criterion, cannot be qualified as a local observable.

[1] We arguably have to just become more refined in what we mean by 'at a point'. This will be one of the main focuses of the thesis.

At the heart of the thesis is a relational framework, which treats reference frames not as abstract mathematical constructs like coordinates, but as physical systems *on the same footing* of other physical systems. In particular, the use of reference frames contributes to define a specific notion of locality, which here will be referred to as *relational locality*: fields localise *relative to each other*. Thus, saying 'here and now' requires defining the relative values of two sets of fields, one of which plays the role of a reference frame, allowing the notion of 'here and now' to be defined in a *physical manner*. This approach aligns with the growing recognition in the literature that local observable quantities in GR must be understood relationally, especially in light of the theory's physical content being invariant under the action of diffeomorphisms: a property which stems directly from the theory's background-independence (a much more complex property to define; see Read, 2023).

Another key theme of the thesis is the relationship between *spacetime symmetries*, which describe the *geometric* structure of a spatiotemporal theory, and *dynamical symmetries*, which pertain to the *dynamical* structure of the theory. By analysing the dynamical (un)coupling properties of reference frames, I will revisit and refine the well-established Earman (1992)'s *symmetry principles* (**SP**) concerning the spatiotemporal (spacetime and dynamical) symmetries which govern any spatiotemporal theory (Chapter 3). In particular, I will show how my nuanced understanding of coupled and uncoupled reference frames can reconcile mismatches between the symmetries of spacetime and those of the dynamics. Remarkably, my results will extend Earman's reflection on this possible mismatch in an entirely original and innovative way.

All the results which will be derived in the first two chapters hinge on a critical distinction: the ability to differentiate between coupled and uncoupled reference frames. This distinction is foundational and extends to all fields, not only to those acting like reference frames. In particular, only when reference fields are coupled can we meaningfully define local relational observables. Moreover, it is under these conditions—when fields are coupled—that dynamical symmetries and spacetime symmetries align, as outlined by Earman's **SP** principles.

But what exactly does it mean for fields to be *coupled*? And how do we differentiate between *dynamical* fields and *physical* fields?

While in the first chapter the notion of coupling is conceptualised in terms of influence between the reference frame and the gravitational field, in the second chapter the notion of coupling will be framed as dynamical dependence, or correlation. This duality clarifies a key distinction: while physical fields (e.g., the GR metric) influence other fields and are in turn influenced by them, dynamical fields are not identified by their 'mutual causal efficacy', but by the structure of the solution space that characterises them.

Building on this groundwork, the thesis (Chapter 4) aims to shed light on these issues standing at the crossroads of two key debates: the analysis of the action-reaction principle (as discussed in Brown and Lehmkuhl, 2013) and the ongoing debate between the geometric and dynamical views of spacetime (explored in Brown, 2005), which probe the ontological status of spacetime structures.

By exploring these connections, my research provides a clearer framework for addressing foundational questions and advancing our understanding of the interplay between fields, symmetries, and observables.

The concept of an *observable* serves as a unifying thread throughout this thesis. From the beginning, observables are framed through Dirac's lens as gauge-invariant quantities. However, this definition is enriched by Rovelli's influential work, which introduces a critical dichotomy: *partial observables*, which are measurable but gauge-variant, and *complete observables*, which are both measurable and gauge-invariant, thereby enabling deterministic predictions (Rovelli, 2002b).

Building on this foundation, the thesis (Chapter 5) addresses two pivotal questions: In what sense are complete observables inherently *relational*? How can partial observables—despite their gauge-variance—be meaningfully *measured*?

Central to addressing these questions is the thesis's novel analysis of reference frames. This analysis not only refines Rovelli's definitions but also counters critiques by Thiemann (2006) and Adlam (2024a), who argue against the viability of partial observables. Crucially, the argument hinges on disentangling two frequently conflated concepts: *relationalism* (quantities defined relative to reference systems) and *gauge-invariance* (quantities invariant under symmetry transformations). While all gauge-invariant observables are relational, the converse does not hold—a distinction often overlooked yet essential for reconciling empirical practice with theoretical formalism.

This refined framework enables a reappraisal of Einstein's 'point-coincidence argument'. By distinguishing operational measurements—modelled as relational *coincidences* (partial observables)—from theoretical predictions encoded as gauge-invariant *events* (complete observables), the thesis challenges prevailing assumptions about spacetime ontology. The result is a coherent account of how empirical data, though inherently approximate and gauge-variant, align with the deterministic structure of GR.

The general overview just presented shows that the study of reference frames is of central importance in GR, and perhaps beyond as I will argue in the Conclusion of the thesis (§6).

The motivation for this work is twofold. First, it aims to address the conceptual underpinnings of reference frames in GR, establishing a new classification that distinguishes reference frames based on their dynamical role in the theory, especially offering a structured framework for analysing their role in gravitational dynamics and their interplay with coordinate systems. Second, by framing reference frames in this way, the thesis provides a powerful machinery for addressing longstanding discussions about the nature of gauge symmetries, the relational nature of measurement outcomes and the construction of local observables to which associate predictions of the theory, as well as the distinction between physical and dynamical, coupled and uncoupled fields.

In sum, this thesis provides a comprehensive investigation into the role of reference frames in the foundations of GR, offering solutions to foundational problems in the theory while laying a valuable groundwork for future research into both classical and quantum realms of gravity, ensuring that the relational structure of the theory remains a central theme.

Structure of the thesis

- The first chapter (§2), titled *Distinction Between Reference Frames and Coordinate Systems in General Relativity*, lays the groundwork for the thesis. It begins by defining what is a reference frame in GR and distinguishing it from the concept of a coordinate system (§2.1). While coordinate systems are abstract mathematical constructs used to label spacetime points, reference frames are *instantiated* by physical systems and interact with the dynamical variables of the theory. The chapter introduces a novel classification of reference frames based on their level of coupling to the gravitational field. This classification divides reference frames into two main categories: uncoupled reference frames (**URFs**) and coupled reference frames (**CRFs**). Each category is further divided into subtypes: idealised (**IRFs**), auxiliary (**ARFs**), dynamical (**DRFs**), and real reference frames (**RRFs**), providing a systematic framework for analysing their roles (§ 2.2). The chapter demonstrates how this classification addresses the long-standing issues in GR regarding the definition of local observables and the interpretation of diffeomorphism symmetry not as mere mathematical redundancies (§2.3).

- In the second chapter (§ 3), *Symmetry Principles: Earman Reconsidered*, the focus shifts to *spatiotemporal symmetries* in GR. Building on the classification introduced in Chapter 2, this chapter revisits Earman (1992)'s foundational work on symmetry principles, known as *SP principles* (§3.1). In particular, in light of the possibility of considering (un)coupled reference frames, this chapter offers a refined formulation of **SP1** principle, named **SP1***. The chapter also reinterprets spacetime and dynamical symmetries within this context, offering a generalisation of the definition of dynamical symmetries, which will be named **GDS** (§3.2).

- Chapter 4, *Dynamical (Un)coupling: Influence or Correlation?*, analyses the concept of '*coupling*', distinguishing between '*influence*', which is typical of interaction terms in the Lagrangian formalism, and '*correlation*' between fields, which is related to the preservation of solutionhood by dynamical symmetries. The classification of influence as direct or indirect, symmetric or non-symmetric is explored in depth (§4.4), highlighting the role of the action-reaction principle and the differences to Lehmkuhl (2011)'s approach (§4.1; §4.2). The concept of correlation between fields is then introduced as a broader concept than influence, showing how it can also exist without influence, while the vice versa is not allowed. The physics of the cosmic microwave background provides a concrete example of the relationship between correlation and influence. (§4.5). Furthermore, physical fields and dynamical fields are distinguished (§4.3). This distinction serves to clarify the nature of the metric in special relativity, demonstrating that it does not fall within the categories of physical or dynamical field. Consequently, it cannot exert a causal influence on other fields, thereby supporting Brown (2005)'s so-called *dynamical view* (§4.2.2).

- The fourth chapter (§5), *Observability and Measurability: Rovelli Reconsidered*, revisits Rovelli (2002b)'s framework for relational observables, offering a nuanced reconsideration of observability and measurability in GR, maintaining the relational spirit. The traditional definition of partial observables is the subject of considerable criticism. Firstly, it is showed that partial observables must be dynamically coupled to each other to form complete observables (§5.2). Secondly, in order to be associated with measurement results, it is argued that a partial observable must be defined relationally in terms of uncoupled reference frames and not in terms of abstract coordinates (§5.3). Finally, the *point-coincidence argument* is reformulated, emphasising the relational nature of coincidences, rather than their gauge-invariance (§5.4). This operational interpretation of Einstein's argument suggests a distinction between the measurability of coincidences and the predictability of *events*.

- The thesis concludes with a synthesis of the findings and their broader implications for the foundations of GR and quantum gravity, as well as for the philosophical reflection on space, time and reality.

Chapter 2

Distinction Between Reference Frames and Coordinate Systems in General Relativity

The terms *reference frame* and *coordinate system* are frequently used interchangeably in the context of General Relativity (GR), resulting in considerable ambiguity. Within both the physics and philosophy literature, there has been a persistent tendency to conflate the concepts of a 'reference frame' and a 'coordinate system'. As highlighted by Norton (1989, 1993), this confusion can be traced, in part, to Einstein himself, who did not always draw a clear distinction between the two.

To appreciate the significance of this distinction, it is necessary to outline two key challenges in GR when formalised in terms of coordinates: (i) it becomes impossible to define local observables that are invariant under gauge transformations (the active diffeomoprhisms); (ii) gauge symmetries are seen as mathematical redundancies or, as Earman (2004) terms it, "descriptive fluff". However, since GR (like all field theories known to date) exhibits such a gauge redundancy in its formalism, an interesting question is whether it is possible to give a physical interpretation to this mathematical fact. Although there are some works that raise doubts about this (see Belot (2017) and references therein), in this context, I consider the gauge group of GR to be composed of active diffeomorphisms (Pooley, 2017).

This chapter will emphasise the importance of distinguishing between reference frames and coordinate systems—a distinction that has profound implications for the interpretation of diffeomorphism invariance and the identification of the so-called local, gauge-invariant observables. In particular, I demonstrate that issues (i) and (ii) can be addressed and solved when quantities are localised using reference frames rather than coordinates or manifold points.[1]

Let me now introduce what is a gauge-invariant observable and how it is usually defined. In physics, gauge transformations refer to specific types of transformations that result in descriptions of physical states that are inherently redundant. By altering the gauge, one can modify the mathematical representation of a physical system without affecting its

[1]Importantly, my approach is not limited to any specific coordinate-based representation.

observable properties, thus enabling different yet physically equivalent descriptions. In Hamiltonian formulations of physics, gauge freedom appears as the action of constraints, often called first-class constraints. Dirac (1950, 1958b, 1964) emphasised that only gauge-invariant quantities — those unchanged by gauge transformations — qualify as true observables, that is, quantities encapsulating the physical content of the theory.[2] In the vacuum solutions of GR, the first-class constraints relate to 3-diffeomorphisms within an hypersurface of the spacetime foliation, as well as transformations that modify the normal directions to the foliation surfaces, commonly referred to as '*refoliations*', or temporal diffeomorphisms (Gryb and Thébault (2016)).[3]

A local observable is typically understood as a gauge-invariant quantity defined *point by point* in spacetime. However, this approach runs into a problem: GR's gauge group effectively 'shuffles' these points, making such local objects gauge-dependent and therefore not true observables in Dirac's sense. As a result, any mathematical object defined *locally* in terms of the spacetime's points is inherently non-invariant under these symmetries.[4]

[2]This idea also traces back to Bergmann (1956). See Pitts (2019) and references therein for discussion.

[3]As shown by Lee and Wald (1990), there is a clear relationship between the Hamiltonian symmetries in a 3+1 decomposition and the full four-dimensional diffeomorphisms in spacetimes governed by Einstein's equations.

[4]Some early attempts are the well-known work of Komar (1958).

The situation is slightly better when matter is taken into account. The idea is as follows: instead of using 'naked' (*haecceitistic* in technical terms, so naked of all qualities) points of spacetime, we can use the values of material fields to localise other quantities in a relational fashion. Within this framework, Rovelli (2002b) addressed the issue of defining local gauge-invariant observables distinguishing between '*partial observables*', which can be measured but are not gauge-invariant, and '*complete observables*', constructed relating partial observables in a gauge-independent manner and which corresponds to Dirac observables. The so-called *relational observable* constructed by coupling two partial observables, one of which is a (typically material) reference frame, turns out to be a 'well-defined' local, Dirac observable for the theory. (See Chapter 5 for a detailed construction of complete observables). This approach suggests that locality in GR is best understood relationally, as interactions between fields rather than as associations with fixed spacetime structures (Rovelli, 2004; Westman and Sonego, 2009).

With this framework in mind, I will now categorise material reference frames in GR into four distinct types (see Table 2.1):

- **Idealised Reference Frames (IRFs)**: Both their dynamical equations and stress-energy contributions to the Einstein Field Equations (EFEs) are omitted.
- **Auxiliary Reference Frames (ARFs)**: They have dynamical equations written in an *auxiliary metric* which is treated as dynamically

isolated by the main metric. Their stress-energy contributions to the EFEs are omitted.

- **Dynamical Reference Frames (DRFs)**: Their stress-energy content is omitted, but their dynamical equations are taken into account.
- **Real Reference Frames (RRFs)**: Both their stress-energy contributions to the EFEs and their dynamics are taken into account.

	IRF	**ARFs**	**DRF**	**RRF**
Dynamics	no	yes, but uncoupled	yes	yes
Backreaction	no	no	no	yes

TABLE 2.1: Four possible classes of reference frames in GR.

It is worth saying that throughout the whole thesis, I will focus primarily on **IRFs, ARFs** and **DRFs**, leaving **RRFs** for future work. Although I believe that using **RRFs** is the more realistic and correct way of modelling a reference frames, it requires solving the non-linear EFEs in their entirety. While this is possible for simple models, it restricts the freedom in the choice of reference fields drastically (see Section 2.3).

As will be clear when discussing each class more in detail, my categorisation can reveal an interesting fact: it is generally agreed that GR is formally treated in terms of variables called coordinates. Well, I argue that what are called coordinates are actually **IRFs**. In fact, as I will clarify in § 2.2.1, **IRFs** and coordinate systems are often confused, as **IRFs** are practically equivalent to coordinate systems, though *not conceptually*.

Closely related to this confusion, another frequent source of confusion involves the terms 'idealisation' and 'approximation'. According to Norton (2012), an idealisation involves replacing the system under study with a simpler, often fictional one, while an approximation provides an inexact but direct description of the system. In our context, reference frames are structures composing GR models, usually given by tuples $\langle \mathcal{M}, g_{ab}, \phi \rangle$, where $\mathcal{M}$ represents the Manifold, g_{ab} the metric field written in abstract index notation (Penrose and Rindler (1987)) and ϕ any matter content that can play the role of the reference frame. **IRFs** (as well as **ARFs** and **DRFs**) are supposed to be derived from successive approximations to such structure ϕ modelling the target system playing the role of the reference frame (I will better focus on models in GR in Chapter 3). In contrast, coordinates are idealised constructs with no direct physical instantiation. Therefore, they do not appear as structures composing possible models.

To sum up, this chapter seeks to clarify the concept of 'reference frame' within the framework of GR by refining and building upon existing definitions found in the literature. Actually, reference frames turn out to be fundamental in physics in general. Experiments and theoretical analyses invariably rely on reference frames, *even if implicitly*. As noted by Anderson (1967, 128), measurements are essentially comparisons between physical systems. Similarly, Landau and Lifshitz (1987, 1) emphasise that natural processes require a reference frame for their description.

The central question addressed here is: what constitutes a reference frame, especially in GR? This is particularly relevant as recent research into quantum reference frames continues to gain interest (see e.g. Giacomini et al. (2019) and references therein). All known physical systems are fundamentally quantum, and a deeper understanding of classical reference frames in GR may shed light on their quantum counterparts.

To conclude, I emphasise that my intention is not to critique the approximation methodologies prevalent in physics. Rather, I underscore the utility of these approximations while advocating for awareness of their inherent limitations, as neglecting these can lead to significant issues, as exemplified by points (i) and (ii) outlined earlier.

This chapter is organized in the following manner: In Section 2.1, I undertake a review of the standard definitions of reference frames as found across various literature. Section 2.2 presents a comprehensive taxonomy of reference frames within the context of GR, accompanied by specific examples. I emphasise the importance of reference frames in tackling the issue of creating local gauge-invariant observables and in understanding the physical implications of diffeomorphism gauge symmetries in GR.

2.1 The realm of reference frames

This section aims to contextualise my characterisation of possible reference frames in GR within the available literature on the foundations of GR. As a cautionary note, I point out that I often alternate between saying that a reference frame *is* a physical system and that a reference frame *represents* a physical system. This distinction is conceptually important, and I believe the latter phrasing is more accurate. However, this nuance does not undermine the arguments presented herein.[5]

In Rovelli (1991), a reference frame is understood as a set of parameters that represent a material system, such as discrete bodies or a matter field. These parameters allow to define spatiotemporal position relationally. Rovelli also points out that reference frames in GR are '*dynamically uncoupled*' to the target system, like the metric field, *only if approximations are made*. My classification on possible reference frames stems from Rovelli's work, but it carefully examines the relationship between a 'material reference frame' and the metric field in GR, expanding on Rovelli's preliminary work, which in fact did not focus on this issue.

I will further elaborate in Chapter 4 on the concept of '*dynamically (un)coupled fields*'.[6] For the moment, it is sufficient to define this nomenclature in the following way: consider any two fields, if one field *influences* the dynamics of the other (the viceversa not necessarily has to apply), they are said

[5]My thanks to Erik Curiel for his insightful suggestion on this matter.
[6]The impatient reader can pause reading that chapter and go for a peek.

to be dynamically coupled. Conversely, the dynamical uncoupling is a stronger condition: I define two fields to be uncoupled if the dynamics of each does not depend on the other. It is evident that the only way to uncouple a field *from a metric field* in a spatiotemporal theory is to disregard its dynamical equations, since in *any* spatiotemporal theory the dynamics is always written relative to a metric.

The dominant interpretation, particularly in the philosophical literature led by John Earman and John Norton, considers a reference frame as a smooth, timelike 4-velocity $U^a = \frac{dx^a}{d\tau}$ (where τ is the proper time of a *comoving observer*).[7] The 4-velocity is tangent to the trajectories of a material system, with which an equivalence class of coordinate systems (x^0, x^1, x^2, x^3) is locally adapted (as per Bradley (2021); Jacobs (2024)).

This approach, being prevalent, will be explored more thoroughly in what follows. The definitions by Earman and Norton are noteworthy:

> In this context a reference frame is defined by a smooth, timelike vector field V. [...]Alternatively, a frame can, at least locally,

[7]See Earman and Friedman (1973); Earman (1974); Earman and Glymour (1978); Norton (1985, 1989, 1993). A comoving observer is defined as the observer for which the spatial coordinates are constants. Thus, such observer is *moving with* the flow of 4-velocity vector field. Is the converse true? If an observer has constant spatial coordinates, are they necessarily comoving? Not always. For example, in Minkowski space, an inertial observer can have constant spatial coordinates in their own frame, but that's trivial.

be construed as an equivalence class of coordinate systems. The coordinate system $\{x^i\}$,$i = 1, 2, 3, 4$, is said to be adapted to the frame F if the trajectories of the vector field which defines F have the form $x^a = \text{const}, a = 1, 2, 3$. If $\{x^i\}$ is adapted to F, then so is $\{x'^i\}$ where $x'^a = x'^a(x^b)$, $x'^4 = x'^4(x^b, x^4)$; such a transformation is called an internal coordinate transformation. F may be identified with a maximal class of internally related class of coordinate systems. (Earman, 1974, 270)

[...] it is now customary to represent the intuitive notion of a physical frame of reference as a congruence of time-like curves. Each curve represents the world line of a reference point of the frame. [...] A coordinate system $\{x^i\}$ $(i = 1, 2, 3, 4)$ is said to be 'adapted' to a given frame of reference just in case the curves of constant x^1, x^2 and x^3 are the curves of the frame. These three coordinates are 'spatial' coordinates and the x^4 coordinate a 'time' coordinate. (Norton, 1985, 209)

Thus a frame of reference is introduced in standard practice as a congruence of timelike curves defined on the manifold (with metric). The frame, if smooth, assigns a velocity, its tangent vector, to every event in the manifold. (Norton, 1989, 1242)

It is evident within this formalism that a coordinate-based narrative persists, utilising what is known as an 'adapted' coordinate system. To sharpen this observation, consider that Rovelli too employs a material system following geodesics as a reference frame. Nevertheless, he opts not to use the 4-velocity of the system in question as the reference frame, but rather the associated physical degrees of freedom.

To give a famous example to remove any doubt about the difference between the two approaches, let's consider the so-called Brown and Kuchař (1995) dust. It consists of a freely-falling (thus, following a geodesic according to the *geodesic principle* (Tamir, 2012)) pressureless material fluid, having a specific 4-velocity. However, the dust is described by eight scalar fields, four of which (T, Z^i) represent the spatiotemporal degrees of freedom to be used as the reference frame representing the dust fluid. Thus, each point of $\mathcal{M}$ is labelled by the set (T, Z^i). In particular, the Z^i are constant along the geodesics of the dust and the T measures the proper time parametrising the geodesics of the dust.

I claim that the standard characterisation of reference frames as a 'maximal class of adapted coordinate systems' may blur distinctions between coordinates and reference frames. For this reason, I will avoid such characterisation, in favour of that of Rovelli-Brown-Kuchar. In fact, as also noted by Earman and Glymour (1978):

> Of course, a reference frame can be represented by a maximal class of adapted coordinate systems. [...] *However, such a representation may obscure crucial distinctions* [between reference frames and coordinate systems]. (My emphasis).

It is particularly in the recent quantum reference frame literature that reference frames are linked to physical systems' degrees of freedom, which, at the quantum level, are inherently quantum themselves Castro-Ruiz et al. (2020). In this context, the distinction between reference frames and coordinates is more pressing, since not considering reference frames as physical degrees of freedom, but as mere coordinates, has obvious consequences as one misses their quantum nature. However, many studies on quantum reference frames assume that the material systems are non-backreacting (see de la Hamette et al. (2023) and references therein; see also Geng (2024)).

Another point of view widely used in the literature involves *tetrads*, or 'orthonormal frames' (Wald (1984); Duerr (2021)). Tetrads, denoted by $e_a^{(A)}$, consist of four smooth 4-vectors that satisfy the orthonormality condition $e_a^{(A)} e_b^{(B)} g^{ab} = \eta^{AB}$, with $\eta^{AB} = \mathrm{diag}(-1, 1, 1, 1)$. The indices A and B represent the so-called internal Minkowski coordinates, while a signifies the coordinate-free notation. A tetrad serves as a *local* orthonormal basis for the tangent space, simulating a local Minkowski spacetime associated to each point of the manifold. The use of tetrads allows a simplified, local expression of geometric objects. For instance, the metric g_{ab} in terms of tetrads is $g_{ab} = e_a^{(A)} e_b^{(B)} \eta_{AB}$. In particular, tetrads facilitate defining local *directions* — one temporal and three spatial — which constitute the so-called '*tetrad frame*.'

As I showed, reference frames has an extensive physical and philosophical literature. Throughout my thesis, I will formally define a reference frame as a set of four degrees of freedom describing a physical system and that form a local diffeomorphism $U \to \mathbb{R}^4$, for some $U \subseteq \mathcal{M}$, which *uniquely* assigns four numbers to each point in the manifold.

In this way, a physical quantity represented by a tensor on the manifold, such as a metric field g_{ab}, can be parametrised by the chosen reference frame. For example, as we will see in 2.3 we can use as a reference frame a set of four linearly independent scalar fields $\{\phi^{(I)}\}_{I=1,\dots,4}$ satisfying some dynamical equations (like Klein-Gordon equations). The use of an index in parenthesis emphasises that this is just a list, and in principle not the components of a vector field. In this way, we can define the quantity

$$g_{IJ}(\phi) := \left[(\phi^{(I)})^{-1}\right]^* g_{ab}, \tag{2.1}$$

which is the *relational observable* we introduced at the beginning of this chapter.[8]

Within this *relational framework*, where reference frames are regarded as physical degrees of freedom equivalent to other physical systems, lies the capability of reference systems to address the specified issues (i) and (ii). This capability is generally absent in other non-relational methodologies discussed above in the literature on reference frames within the foundations of classical GR.

[8] With the symbol $[\bullet]^*$ we denote the pullback of the metric field via the scalars.

To conclude this section and before moving on to my proposed classification of possible reference frames in GR, I want to quote some relevant passages that support my thesis according to which the distinction between reference frames and coordinates is central to addressing the issue of the meaning of gauge symmetries, which I identified in problem (ii).

In the context of the discussion regarding how physicists count physical states (Belot, 2017, 955-956) states that:

> in physicist's practice [. . .] they can't be bothered to make the distinction between global states and subsystem states [. . .] They say that a system of n particles has $3n$ degrees of freedom because it takes three numbers to specify the location of each particle relative to a chosen set of coordinates [. . .] The ambient system provides a reference frame, and each way of specifying the coordinates of a particle relative to this frame corresponds to a possible way that a particle of the subsystem could be located [. . .] When it comes to possible configurations of the universe as a whole [. . .] it just takes just $3n - 6$ numbers to fix the configuration of an n particle system.

In these passages, in my view Belot brings to light an analogy between subsystems and reference frames. Or rather: given a dynamical system, one can identify at least two subsystems, one of which constitutes a reference frame and the other represents the dynamical degrees of freedom of interest for the theory (e.g. in GR the metric, or in a Newtonian theory of Belot's example the degrees of freedom of the system of n particles). Thus, the carelessness in the distinction between global dynamical system (subsystem+reference frame *in interaction*) and isolated system (n particles in a coordinate system) echoes the carelessness in the distinction between reference frames and coordinate systems. The 6 fewer (as redundant to describe the physics of the subsystem of n particles) degrees of freedom required to characterise the physical state of the (sub)system of $3n$ particles corresponds to a redundancy fixed by the choice of a reference frame: the environment. I consider this case illustrative in discussing the relational approach to the problem of giving meaning to gauge redundancies. (Ça va sans dire that by this I do not in any way intend that *true* gauge symmetries in the technical meaning of the term exist in the specific Newtonian case examined. What matters is that the 6 degrees of freedom associated with the translation and rotation of the system can be considered *redundant* if the interest is primarily in describing the internal relative configurations between the particles).

An analogous argument can be found in Wallace (2022a), where he introduces the so-called 'cosmological assumption': the primary way to understand a physical theory is to suppose that it models the *entire universe*. According to the rationale of my work, to formalise physics in terms of coordinates is to claim (idealising) that the theory under consideration describes the entire universe. Working with reference frames instead means accepting the possibility that theories should be interpreted as modelling a *subsystem* (the dynamical system of interest) in relation to another subsystem (the reference frame). Wallace continues stressing that in the case of a Newtonian theory of n particles

> the environments of those systems have been removed to infinity is an idealization, not something to be understood in physical terms. Some authors] take for granted the physical theories should in the first instance be understood as modelling the whole universe. [. . .] This is fairly explicit in Belot's, and Greaves and Wallace's, papers. (*ibid*, 240-241)

In addition, in a sequel paper Wallace (2022b) states that " Isolated systems [. . .][are] idealized descriptions of larger systems ". In my jargon, the idealisation lies in not considering reference frames but only the dynamical target system of interest expressed in terms of coordinates.

So, I believe that the following quote by Wallace:

> [. . .] any model can be interpreted in the first instance as modelling a dynamically isolated subsystem under certain idealizations about its environment and where, if we want to remove those idealizations, we can embed the model in a model of a larger system within the same theory[. . .] (*ibid*, 242)

clearly express the interplay between the use of reference frames and coordinates. In my terms, *removing the idealisation* means first of all realising that the use of an environment isolated from the subsystem actually coincides with using an **IRF**. This is the first conceptual step to be taken to be ready to de-idealise the system. Now, since we already know that an **IRF** is actually an *approximated* (sub)system representing the environment which would be a naturally interacting system. At this point, one can finally lighten the approximations made and include the environment's dynamical content, thus addressing problem (ii) of finding a physical reason to gauge symmetries.

After all this discussion, it is now worth resuming the distinction between reference frames and coordinates in GR. To do that, let me schematise this distinction in two coarse-grained definition:

Reference Frame: a set of dynamically coupled and *instantiated* parameters[9]

Coordinate System: a set of non-dynamical and *uninstantiated* mathematical labellings[10]

[9]Recall, again, that the dynamical coupling is here understood *with respect to the dynamical system of interest* that we want to write in terms of the chosen reference frame. In GR, this is the metric field.

[10]It is straightforward that the logical relation between the two properties is the following:

$$\text{Dynamically coupled} \rightarrow \text{Instantiated.}$$

2.2 Reference frames in GR: a dynamical classification

In previous discussions, I provided an overview of various definitions of reference frames in spatiotemporal contexts, along with the distinctions between coordinates and reference frames. In this section, I introduce a novel methodology for classifying reference frames in the context of GR, organising them into two primary classes, each containing two sub-classes: the class of 'uncoupled reference frames' (**URFs**), composed by **IRFs** and **ARFs** and the class of 'coupled reference frames' (**CRFs**), composed by **DRFs** and **RRFs**.

Class	Type 1	Type 2
URFs	**IRFs**	**ARFs**
CRFs	**DRFs**	**RRFs**

TABLE 2.2: Classification of reference frames: the general overview.

The reason is simple: any object whose dynamics is coupled with the dynamics of the system under analysis has dynamics, by definition. Any object that has dynamics is a physical object, or at least a physically possible object. Therefore, its degrees of freedom are *instantiated*.

As per definition proposed at the end of the previous section, at the fundamental level, a reference frame can be described as a configuration of variables *instantiated* by a physical system. Crucially, one needs at least four independent scalar quantities that can be experimentally measured. Since most elementary physical fields—whether fermionic or bosonic—are not scalar by nature, these scalar quantities might also reflect 'collective' characteristics of matter. For example, in the context of FLRW cosmology, the entropy of the cosmological fluid can serve as a temporal reference (see works by Schutz (1970, 1971); Cianfrani et al. (2009); Campolongo and Montani (2020)). Alternatively, the scalars could derive from fundamental fields. Importantly, while the definition of a reference frames—requiring four independent parameters for space and time—remains broad, additional constraints are needed to refine these parameters. Just to give an example, if a Klein-Gordon scalar field is selected to play the role of the timelike variable (say $\phi^{(1)}$) it needs to satisfy some properties such as a homogeneity condition $\nabla^i\nabla_i\phi^{(1)}(x^\mu) = 0$, where $i = 1, 2, 3$ are spatial indices in some coordinates $\{x^\mu\}$. We could also assume a 'monotonicity condition' connected with some assumptions on its potential (when it is considered).

My classification of reference frames in GR is grounded in a systematic division of a physical system into two distinct subsystems: the target dynamical system (which in this context is the gravitational field) and the reference frame. This classification is based on the degree of dynamical approximation applied to the model of the target physical system. Specifically, the extent to which the dynamics of the reference frame is coupled with that of the gravitational field determines the nature of the reference frame.

2.2.1 Uncoupled Reference Frames

Idealised Reference Frames (IRFs)

Idealised Reference Frames (**IRFs**) are characterised by the omission of *any* dynamical interaction between the physical system that constitutes the reference frame and gravity. Specifically, this implies making two assumptions (see also Rovelli (1991)):

1. The stress-energy tensor contribution of the material reference frame is disregarded in the EFEs.
2. The equations that describe the dynamics of the material reference frame are not taken into account.

This type of reference frame bears resemblance to what (Rovelli, 2004, 64) terms as 'undetermined physical *coordinates*.' According to Rovelli, the explanation for this terminology is as follows:

> We obtain a system of equation for the gravitational field and other matter, expressed in terms of coordinates X^{μ} that are interpreted as the spacetime location of reference objects whose dynamics we *have chosen* to ignore. This set of equation is

underdetermined: same initial conditions can evolve into different solutions. However, the interpretation of such underdetermination is simply that we have chosen to neglect part of the equations of motion.

However, I opt not to use this terminology to avoid potential confusion between the concepts of reference frames and coordinates. Additionally, I find it more suitable to discuss about equations of motion in terms of indeterminism, rather than underdetermination—a term often associated with choices that are not inherently tied to dynamical aspects.

In fact, point (2) introduces *indeterminism* in the dynamics of the metric field when it is expressed through the degrees of freedom associated with the **IRF**. To be precise, using **IRFs**, *similar to using coordinates* in GR, lead to an *apparent*, not pernicious, indeterminism, which is a consequence of unaccounted gauge freedom, which allows for multiple solutions from the same initial data. These distinct solutions, sharing identical initial conditions, are connected by gauge transformations.[11]

To further illustrate the nature of indeterminism when using **IRFs**, let's consider a Lorentzian metric field g_{ab} that satisfies the EFEs, alongside four scalar fields $\{\phi^{(I)}\}$ each satisfying a Klein-Gordon equation and defining the **IRF**. NB: the fact that the dynamical equations of the **IRF** are *ignored*

[11]This kind of 'indeterminism' within GR is the core content of the so-called 'Hole Argument' Earman and Norton (1987); Pooley and Read (2021).

in the general dynamics of the system does not mean that the scalar fields that make up the **IRF** do not satisfy any dynamic equations *intrinsically*. Stating otherwise, for **IRFs** their dynamical equations are not included in the dynamical models of the theory.[12] The reason why **IRFs** seem to lead to indeterminism is that both $(g_{ab}, \phi^{(I)})$ and $(d^{*}g_{ab}, \phi^{(I)})$, for any diffeomorphism $d \in \mathrm{Diff}(M)$, are valid solutions for the dynamics. This is because **IRFs** are *dynamically uncoupled* to the metric field, causing a persistent gauge redundancy in how the metric can be represented. Consequently, this redundancy results in the apparent indeterminism.

Importantly, there is a (subtle) difference between **IRFs** and coordinates: an **IRF** can be understood as an '*instantiated*' coordinate system that allows for the assignment of a physical referent, though this referent's dynamical role is approximated away.[13] This is conceptually distinct from a coordinate system, which does not bear any physical instantiation. Additionally, while coordinate systems are *inherently* uncoupled from the fields of the theory, *not obeying any equations of motion at all*, **IRFs** achieve a similar uncoupling through an approximation method. However, there's a clear overlap between the use of **IRFs** and coordinates. Both are dynamically inert parameters. The overlap in the use of **IRFs** and coordinates in GR

[12]These are referred to as DPMs: dynamically possible models. See § 3.1 for an overview of models in spatiotemporal theories.

[13]We adhere to the distinction between approximations and idealisations as described in Norton (2012).

may be responsible for some of the confusion between the concepts, especially regarding the main issues (i) and (ii) we want to analyse in this chapter.

Auxiliary Reference Frames (ARFs)

Unlike **IRFs**, **ARFs** (*Auxiliary Reference Frames*) are governed by equations of motion that dictate their dynamics. However, these equations of motion are uncoupled from the dynamics of any system formulated with respect to the **ARF** itself. To illustrate this, let's revisit the example of four scalar fields obeying a Klein-Gordon-type equation. These fields can serve as a reference frame for a Lorentzian metric g_{ab}.

Now, imagine that these Klein-Gordon scalar fields evolve *exclusively* with respect to an 'auxiliary' Lorentzian metric h_{ab}, which is dynamically uncoupled from the 'primary' metric g_{ab}—the metric whose dynamics we are interested in. Importantly, also g_{ab} is not influenced by the scalar fields. In this setup, the reference frame is entirely uncoupled from the dynamics of g_{ab}, as the evolution of the scalar fields is determined solely by the auxiliary metric h_{ab} via the d'Alembertian $\Box_h \phi = 0$.

The reason for this uncoupling is straightforward: the dynamics of the scalar fields depend only on their initial conditions, which are tied to the auxiliary metric h_{ab} and not to g_{ab}. The auxiliary nature of h_{ab} ensures that the two metrics remain completely uncoupled. As a result, there is a clear dynamical independence between the scalar fields and the metric g_{ab}.

Like **IRFs**, **ARFs** are defined as a category of reference fields over the spacetime manifold. Consequently, they transform covariantly under active diffeomorphisms—a property known as *equivariance* (Weatherall, 2018).

However, the key distinction between the two subclasses lies in their dynamical behaviour: while **IRFs** are treated as purely kinematical entities, **ARFs** exhibit *explicit* dynamics governed by their own equations of motion. Crucially, these dynamics remain independent of the relevant dynamical system of the theory—in this case, the metric g_{ab}.

2.2.2 Coupled Reference Frames

Dynamical Reference Frames (DRFs)

Let's only apply the first approximation mentioned earlier — specifically, approximation 1 — we obtain a *Dynamical Reference Frame* (**DRF**). In this case, the equations of motion for matter, written using the metric under consideration, become available allowing us to establish a deterministic dynamical system.

How? The equations of motion for the material reference frame can be interpreted as setting gauge-fixing conditions. This idea aligns with Rovelli (2014)'s argument (also elaborated in Gomes (2024b, §2.3)) that gauge freedom is not a mere formal redundancy, but rather points to the inherently relational nature of physical quantities. This interpretation is also consistent with (Henneaux and Teitelboim, 1994, 3) description of a gauge theory as a theory

> [...] in which the dynamical variables are specified with respect to a 'reference frame'.

The attentive reader will immediately notice that the use of **DRFs** solves 'problem (ii)' about wanting to give physical meaning to gauge redundancies.

In the following, I would like to provide a useful conceptual tool for understanding how a **DRF** can be used: the *deparameterisation* of the dynamics of a system (see Henneaux et al. (1990),Brown and Kuchař (1995), Thiemann (2006), and Tambornino (2012) for details on deparametrisation procedure in physics).

To illustrate this with a simplified analogy, consider a parametrised Newtonian system in one dimension, represented by canonical variables $\{q(t), p(t)\}$. Though not a rigorous example of a **DRF** (which strictly speaking is defined only in general-relativistic settings), this serves as a helpful comparison.

By employing the so-called *parameterisation procedure* the configuration space $C = \{q(t)\}$ extends to $C_{\text{ext}} = \{q(\tau), t(\tau)\}$, effectively treating time t — which acts as an 'external' clock —- as a dynamical variable like q, with both now depending on a gauge parameter τ. In particular, they satisfy the following dynamical equations: $\frac{dt}{d\tau} = N(\tau)$ and $\frac{dq}{d\tau} = N(\tau)\frac{p(\tau)}{m}$, where $N(\tau)$ is a Lagrange multiplier (the equivalent of the *lapse function* in GR).

It turns out that the 'extended system' exhibits a reparametrisation symmetry $\tau \to \tau'(\tau)$, where different choices for the Lagrange multiplier $N(\tau)$ correspond to various parametrisations of the configuration space variables $\{q(\tau), t(\tau)\}$. This is a gauge redundancy—distinct mathematical descriptions correspond to identical physical states.[14]

This redundancy is governed by the constraint $C = p_t + \frac{p^2}{2m}$, which defines the constraint surface in phase space (the subspace of physically allowed states) and generates gauge orbits (trajectories on the surface representing the same physical evolution). By setting the *gauge condition* $N = 1$, the variables $t(\tau)$ increases linearly with respect to τ. Despite this, the dynamics *still* retains some redundancy, as the variables $(t(\tau), q(\tau))$ are not invariant under reparametrisations $\tau \to \tau'(\tau)$. In fact, the system's dynamical evolution relies on the arbitrary parameter τ, rendering it physically meaningless.

To fix this redundancy, one can impose what is known as the 'canonical

[14]This redundancy in the dynamics is akin to the diffeomorphism redundancy in GR. While gauge symmetries are traditionally associated with relativistic systems, here the redundancy arises artificially from the parametrised formulation, not Newtonian mechanics itself, which intrinsically treats time as absolute.

gauge condition' $t(\tau) \equiv t_0$, which eliminates any remaining formal redundancy. This condition geometrically specifies a slice that intersects the gauge orbits on the constraint surface *exactly once*.

In this way, physical quantities can be understood as *relational* gauge-invariant objects, such as the coincidence of q with t, meaning the value of q when $t = t_0$. Explicitly:

$$q(\tau)|_{t(\tau)=t_0} = q(\tau) + \frac{p}{m}[t_0 - t(\tau)]. \tag{2.2}$$

An alternative, but equivalent, strategy to fix the gauge redundancy of the parametrised system is to choose t as the 'temporal reference frame' or 'relational clock' by inverting the relation $t(\tau) = \tau$, giving $\tau(t) = t$. This is possible because the equations $\frac{dt}{d\tau} = 1$ can be straightforwardly solved for this system, and in this particular case, the function $t(\tau)$ is globally invertible.[15]

By inserting $\tau(t)$ into the gauge-dependent variable $q(\tau)$, we obtain a gauge-invariant, *relational* observable $q(\tau(t))$, defined for any specific value of t. This process is known as *deparameterisation*: one goes back to the formalism of the original, non-parameterised case. However, now the role of t as a *dynamical entity on the same footing with q* is clarified, providing a useful analogy to (the time component of) a **DRF**. The relational observable

[15] In general, these two conditions are not always met. The Hamiltonian of the system can be arbitrarily complicated and the equations not easily invertible even locally (see (Dittrich, 2006)).

$q(t)$ captures the gauge-invariant relational change of q with respect to the dynamical variable t, in fact the dynamics of $q(t)$ is fully deterministic and we have eliminated any gauge redundancy from the formalism.

In summary, the *deparameterisation* process enables certain dynamical variables to act as reference frames, freeing the formalism from explicit gauge dependence, which is present for the same theory when written in terms of gauge parameters, which actually correspond to coordinates. This procedure becomes particularly relevant in the context of GR, which *inherently* adopts a parameterised framework:

> The already [parametrised] system "per excellence" is the gravitational field in general relativity. (Henneaux and Teitelboim, 1994, p.102)[16]

Deparametrisation, therefore, consists of replacing the use of coordinates with the use of reference frames, understood as dynamical variables. Remarkably, this procedure is entirely equivalent to gauge-fixing, the result being the same: the possibility of constructing local gauge-invariant observables. This helps us to understand that problems (i) and (ii) are closely related: the use of reference frames gives a possible relational interpretation to gauge-fixing procedures and allows the construction of observables.

[16]This quote mirrors the claim, traced back to Kuchař (1990), that *the mark* of GR is its *lacking* a *non*-general covariant formulation.

The Klein-Gordon Test Scalars A *test* system provides a significant example of a **DRF**. Particularly in the physics literature, the term 'test' is associated with material systems whose backreaction on the spacetime structure is negligible.

I will consider the usual set of four real, massless, Klein-Gordon scalar fields in curved spacetime as a toy model of four *test scalar fields*. The equations of motion for these fields, expressed in abstract index notation, are:

$$\Box\phi^{(I)} = \nabla^a\nabla_a\phi^{(I)} = 0. \tag{2.3}$$

In this context, the metric field satisfies EFEs in which the stress-energy tensor corresponding to the scalar fields is disregarded. These scalar fields collectively define a *local* reference frame, which can be interpreted as a clock and three spatial rods, enabling the description of local quantities.[17]

This case illustrates the equivalence between employing a **DRF** and imposing a gauge-fixing condition. Specifically, the equations (2.3) governing the Klein-Gordon fields are equivalent to those applied when *De-Donder gauge-fixing* is fixed for some coordinates ϕ. This clearly supports a *relational interpretation of gauge-fixing*.

[17]Usually, with the word 'clock' we evoke a specific human invention for timekeeping. However, in principle any physical quantity can be considered a 'clock' as far as its physical evolution is associated with 'time elapsed'.

Furthermore, the use of the set of Klein-Gordon fields $\{\phi^{(I)}\}$ as a local reference frame, yields gauge-invariant relational observables $g_{IJ}(\phi)$ already introduced in equation (2.1). Contrary to the case of **URFs**, in this case no gauge redundancy appears, as dynamical solutions $(g_{ab}, \phi^{(I)})$ are *unique* given some initial conditions. In fact, transformations $([d^*g]_{ab}, \phi^{(I)})$, where $d \in \text{Diff}(\mathcal{M})$, do not satisfy the equations, so they are not allowed.

To make this crucial point clear, suppose we have some initial data for the Klein-Gordon equation $\Box_g \phi = 0$, with respect to a given metric g_{ab}. Given a generic diffeomorphism d acting on the metric, this will *not* produce a solution, since the d'Alembertian $\Box_{d^*g}$ will have different Christoffel symbols appearing, so $\Box_{d^*g}\phi \neq 0$ for the same chosen ϕ and the same initial data. Consequently, the observables $g_{IJ}(\phi)$ are *uniquely* defined.

Why is $g_{IJ}(\phi)$ a *local* quantity? This relational construction of observables emphasises that the 'physical metric' $g_{IJ}(\phi)$ is not localised on points of $\mathcal{M}$, but rather on the space $\mathbb{R}^4$, defined by the ordered real values of the scalar fields. In fact, recall that the scalar fields $\{\phi^I\}$ can be understood as a set of diffeomorphisms mapping $g_{ab} \in \mathcal{M}$ to $g_{IJ} \in \mathbb{R}^4$. By removing explicit reference to points in $\mathcal{M}$, this approach provides a well-defined formulation of local gauge-invariant observables in GR where physical objects localise *relative to each other* rather than to the manifold, encapsulating the concept of *relational localisation* (Rovelli, 2023).

Accordingly, a point $p \in \mathcal{M}$ is designated by the specific values of the scalar fields: the identity of point p is defined by the four scalar fields taking certain values and it is not the place *where* the four scalar fields take certain values. This way of defining a spatiotemporal points aligns with Einstein's original *operational* interpretation of coordinates, as discussed in Norton (1989).[18]

Finally, note that to ensure that they can be used as reference frames, each scalar field $\phi^{(I)}$ must be at least locally invertible, requiring a non-zero determinant of the Jacobian $\partial\phi^{(I)}(x)/\partial x^{\mu}$ within some open subset $U \subset M$. In this respect, an analogy with tetrads arises: if we express a tetrad in terms of scalar fields as $e^{(I)}_{\mu} = \partial\phi^{(I)}/\partial x^{\mu}$, this underscores the interpretation of scalar fields as internal parameters (analogous to internal Minkowski coordinates) associated with spacetime points. In such-defined '*physicalised tetrad frame*', the metric becomes:

$$g^{IJ}(\phi) = (\partial_{\mu}\phi^{(I)})(\partial_{\nu}\phi^{(J)})g^{\mu\nu}. \tag{2.4}$$

So, the condition $(\partial\phi^{(I)}(x)/\partial x^{\mu}) \neq 0$ highlights the non-degeneracy of tetrads. Furthermore, in non-orthonormal cases, the 'tetradric metric' g_{IJ} is not Minkowskian and this is in fact the general case when using scalar fields as reference frames.

[18] The term *operational* here does not imply any adherence to operationalism (Bridgman, 1927). An operationally defined quantity is one whose *definition* does not require that the exact measurement procedure be specified, but only that its definition be given through *some* measurement procedure. For more discussion of operationalism see Chang (2021); Fankhauser and Dürr (2021).

The Global Positioning System (GPS) The Global Positioning System (GPS) can be understood through a straightforward physical model with a high level of realism. The idea is to consider a system where GR is coupled with four test bodies, labelled as '*satellites*', each equipped with a clock that tracks the proper time along its timelike geodesic, originating from an initial meeting point O. These satellites broadcast their local proper times, which I will name ϕ, via radio signals. Imagine being at a point P, equipped with a device that records these four signals, displaying the respective times. Those times will be your spatiotemporal GPS frame parameters. Geometrically, as defined in Rovelli (2002a), they constitute the proper timelike distances between the four intersection points with the past lightcone of P and the starting point O. See Fig. 2.1.

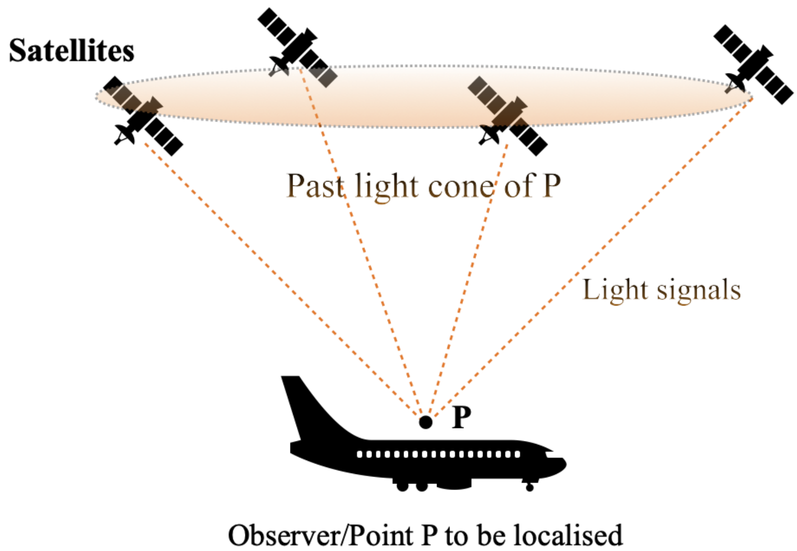

FIGURE 2.1: Pictorial representation of a GPS reference frame. To give an idea of the real usefulness of such a frame of reference, I decided to illustrate the case of an aeroplane in flight. Contrary to standard illustrations, I have drawn the past light cone of the aeroplane pointing upwards, because the satellites that broadcast the signals orbit at a higher altitude than the altitude of the aeroplane.

The GPS coordinates provide a tangible, realistic example of a **DRF** and the four distinct proper times $\phi^{(I)}_{,I=1,2,3,4}$ are the four physical parameters that define the **DRF**. This case emphasises the value of my systematic classification, serving as a coherent conceptual framework for organising reference frames described in relevant literature.

Let me analyse some fundamental aspects of the construction just presented.

The meeting point O plays a crucial part in the construction of the GPS frame, as each satellite starts 'counting' its proper time from the 'meeting event' with the other satellites, allowing the signals to be *synchronised* relative to a shared origin. This synchronisation is essential for accurately defining the times $\phi^{(I)}$ broadcasted by the satellites and elapsed from the 'starting point' O. This setup enables the definition of the spatiotemporal location of P through these measurements.

Notice that this model presupposes the existence of 'clocks' carried by the four satellites, tracking proper time as dictated by the Clock Hypothesis Malament (2012). Additionally, receivers at P are needed to log the satellites' time data. These clocks and receivers operate externally from the dynamics, leaving the metric unaffected. For the purposes of system construction, it is equivalent to consider the satellites themselves as clocks/emitters. There remains also the need to disregard the receiver placed at P in the schema.

Why GPS is so powerful as a reference frame? Rovelli's insight is that GPS coordinates avoid the typical non-locality (in the special-relativist causal meaning of the term) present when one uses coordinates. As an example of *non-local coordinates*, Rovelli considers a scenario where coordinates in the solar system are defined using cosmological time t_c and spatial distances, like the distances from the Sun (x_S), Earth (x_E), and Jupiter (x_J). In these coordinates, the metric tensor $g_{\mu\nu}(t_c, x_S, x_E, x_J)$ would depend on the spatial positions of these celestial bodies. Now, suppose that Jupiter is swept away by a comet at a certain cosmological time. This event would cause an *instantaneous* change in the spatial coordinate x_J. Consequently, the value of the metric tensor $g_{\mu\nu}(t_c, x_S, x_E, x_J)$ would change *instantaneously*. This instantaneous change due to a distant event introduces what I dub *causal non-locality*, since the metric at one location is being influenced by changes far away, without any intervening local interaction. In contrast, in the case of the GPS frame, the localisation of point P depends on signals from satellites broadcasting *from the past* of P. Since the signal broadcast from a satellite to P depends only on the past history of the satellite, the evolution of the observable metric $g_{IJ}(\phi)$ in these GPS coordinates is *causally local*. In essence, while the coordinates depend on distant satellites, the information they carry is causally related to the past, ensuring *causally local* evolution.

Light Clocks Let me give what I believe is another example of a **DRF**, which appears in relativistic literature under the name of 'light clocks'. The idea of light clocks is a longstanding one, dating back to Einstein's work on special relativity. For recent work on such clocks, see e.g. (Fletcher, 2013). The simplest form of a light clock consists of a ray of light bouncing between two perfectly reflecting parallel mirrors, separated by a distance of d. In summary, the clock is called 'light' because its construction is based on the use of light (or photons), which travels along null geodesics, as opposed to, say, mechanisms that might use massive particles (which would travel along timelike geodesics). The term 'light' therefore refers to the nature of the component (the light/photons) used as the basic element to 'mark' time through its bounces.

Henceforth, when the reader has to deal with 'light clocks' in GR, (s)he will know that such clocks have all the characteristics and advantages of the **DRFs** discussed in this chapter. The same applies to GPS coordinates, test fluids, test particles and so on.

Fletcher (2013) states:

> [showing that the clock hypothesis holds] for most real clocks is, and must be, enormously complicated. Thus some previous works [. . .] have required the use of extensive approximations regarding the dynamics, making the study of an idealised light clock attractive [. . .] One can represent a light clock's dynamics in a relativistic spacetime without appealing to Einstein's field equations.

I want to highlight some points about this quote. The first is that Fletcher does not seem to be attentive to the distinction between idealisation and approximation, or at least does not adopt Norton (2012)'s distinction. In my jargon, which adheres to Norton's one, light clocks are *approximated* clocks.

In this regard, the sentence "One can represent a light clock's dynamics in a relativistic spacetime without appealing to Einstein's field equations" should be interpreted to mean that in order to represent the dynamics of such clocks, it is not necessary to take their stress-energy tensor into account, which is in fact neglected by EFEs.

Finally, the validity of the clock hypothesis for light clocks supports my claim that **DRFs** can be used as good clocks. It also highlights how it is *very common* in experimental practice to approximate systems and neglect their backreaction. This is why it becomes absolutely relevant to characterise the theoretical properties of these approximate systems when used as spatiotemporal reference frames. A task that I set out to pursue in this chapter and in general throughout my thesis work.

2.3 Discussion

The classification I introduced holds particular significance, as it provides a well-defined conceptual framework to analyse foundational challenges in GR that can be addressed using reference frames understood as material systems coupled with gravity. This framework is particularly useful for situating the two challenges (i) and (ii) discussed above and for elucidating the consequences of employing different possible types of reference frames.

Let me summarise and rephrase the two main challenges I discussed:

(i) When **URFs** or coordinate systems are used, it becomes impossible to construct *local* observables that are invariant under gauge transformations.[19]

(ii) In the case of **URFs** or coordinates, the gauge freedom in GR appears as a purely mathematical redundancy.

However, by shifting the focus to **CRFs**—thus relaxing certain approximations and considering dynamical reference frames (**DRFs**) or real reference frames (**RRFs**)—the issues associated with (i) and (ii) are naturally solved.

(i-solved). As illustrated by examples provided in § 2.2.2, by employing **CRFs** it is possible to define local quantities that are invariant under gauge transformations, which in GR we consider to be diffeomorphisms. This approach involves adopting a notion of relational localisation between fields. By doing so, one can define gauge-invariant observables, develop a deterministic framework for their dynamics, and provide an unambiguous method for identifying spacetime points across equivalent models.

(ii-solved). Regarding the interpretation of gauge freedom within GR, employing **CRFs** allows gauge-fixing conditions to be interpreted as dynamical relations between the chosen reference frame and the target system. In this perspective, gauge freedom indicates that the dynamics of the reference frame being used have been omitted. As highlighted in Rovelli (2014), gauge invariance is more than just a redundancy in the

[19] Of course, it is always possible to define non-local gauge-invariant observables as integral quantities.

mathematical formalism; it reflects the relational character of fundamental observables in physics. Gauge symmetry is pervasive and it is not merely an artifact of mathematics but a profound expression of the relational structure of the physical world. Selecting a specific gauge can be realised *physically* by coupling our system with a reference frame.

My classification also aids in resolving semantic ambiguities, promoting clearer communication about the use of reference frames across the various contexts of classical and quantum gravity. One recent example is the use of my classification in Kabel et al. (2024). Moreover, it sheds light on the interplay between reference frames and coordinate systems within GR.

Finally, examining **URFs** and **DRFs** as possible categories of reference frames reveals an intriguing insight. Suppose the material reference frame is the *only* matter contribution in the total system. Then, ignoring its stress-energy tensor leads to vacuum EFEs. Thus, one might interpret the vacuum sector of GR as an *approximated sector* describing *observable* metrics that rely on **URFs** or **DRFs**. This implies that *exact* vacuum solutions could be absent in nature, with only approximated matter solutions *mimicking* vacuum behaviour being viable. Further discussion on this hypothesis, however, lies beyond the scope of this work and will deserve further study in the future.

To conclude my discussion, I want to emphasise that a scenario in which only assumption (2) is applied yields the same conclusion concerning indeterminism obtained with **URFs**. In such a situation, the material reference frame affects spacetime curvature and must be considered in the resolution of the EFEs, *yet* its dynamic behaviour is overlooked.

This is *evidently* an approximation, as differential Bianchi identities imply $\nabla_a G^{ab} = 0$. Consequently, the validity of the EFEs leads directly to $\nabla_a T^{ab} = 0$. All physicists know that the Euler-Lagrange equations for matter in GR are essentially equivalent to enforcing $\nabla_a T^{ab} = 0$. In this fact lies the non-linearity of EFEs and the fact that the equations of motion are not independent of the field equations. Thus, not incorporating the material equations of motion, despite a non-zero stress-energy content T^{ab}, clearly represents an approximation.

I decided to not study this case since, as far as I know, no examples of such an approximation can be found in theoretical practice in GR.

2.4 Conclusion

This chapter introduced a detailed classification of reference frames in GR, distinguishing them by their relevance in gravitational dynamics.

Specifically, reference frames were categorised as 'uncoupled' (**URFs**) and 'coupled')**CRFs**) to gravity. To the first class belong '*idealised reference frames*' (**IRFs**) whose physical properties play no role in the dynamics, and '*auxuliary reference frames*' which have a dynamics defined with respect to an 'auxiliary' metric field h_{ab}, dynamically uncoupled to the 'main' metric field g_{ab}. To the second class belong '*dynamical reference frame*s' (**DRFs**) which have associated a specific set of dynamical equations defined with respect to the main metric, and '*real reference frames*' (**RRFs**) whose stress-energy tensor is taken into consideration in the EFEs.

I have not dealt with **RRFs** in this chapter, but I give a brief characterisation of them here, leaving their analysis for future work. Obviously, all the properties of **DRFs** are shared by **RRFs**, which are only analytically more difficult to deal with. Examples of **RRFs** include pressureless dust fields (Brown and Kuchař, 1995) and massless scalar fields (Rovelli and Smolin, 1994). It is also possible to propose a sub-classification of **RRFs**. I define a $\mathbf{RRF}_{\text{dep}}$ as a **RRF** that permits the theory to be deparametrised, so that complete observables can be *analytically* described (see Tambornino (2012) for a review). As a matter of fact, in some special cases approximations can be made to the Hamiltonian of the material field used as the **RRF**, thereby implementing a deparametrisation procedure.

My novel categorisation provided a robust theoretical framework for addressing key challenges in GR, particularly: (i) interpreting diffeomorphism symmetry beyond its interpretation as a mere mathematical redundancy, and (ii) the issue of defining local gauge-invariant observables. My analysis placed particular emphasis on the role of **DRFs** in this context, extending and complementing existing literature.

A significant point addressed is the need for conceptual precision to avoid misinterpretations. Reference frames are often approximated as **IRFs** and erroneously conflated with coordinate systems. While an **IRF** can *functionally* resemble a coordinate system, conflating the two is a fundamental error. In GR, coordinates are mathematical abstractions defined by *uninstantiated* variables, whereas **IRFs** are representations of structures appearing in the models of the theory and are *instantiated* by physical systems.

My classification may have relevant implications for the study of quantum reference frames and discussions on the nature of vacuum solutions in GR. A critical question arises as to whether vacuum solutions can be reconceptualised as approximated matter solutions, where the stress-energy tensor is disregarded. Furthermore, the apparent effectiveness of treating reference frames as mere coordinates in experimental contexts warrants exploration (just think about the use of Schwarzschild coordinate in black hole physics).

I believe a further branch of research to be explored in the field of reference frames are non-material reference frames, such as those based solely on gravitational degrees of freedom. A well-known example are the so-called Komar-Kretschmann scalars: four scalar functions constructed from the Riemann tensor (Giovanelli, 2024).

In conclusion, as I will also show in the rest of the thesis, the classification introduced in this chapter holds potential for significantly advancing the conceptual foundations of Einsteinian gravity.

Chapter 3

Symmetry Principles: Earman reconsidered

3.1 Earman SP Principles: Spacetime and Dynamical Symmetries

This chapter does not intend to offer a thorough historical and philosophical account of the concept of *symmetry* in physics. While such an exploration would undoubtedly be fascinating, it lies beyond the scope of my discussion. For a comprehensive review of the concept of symmetry, the reader is directed to Brading and Castellani (2003) and related references.

Instead, the primary focus here is on Earman's SP principles (Earman, 1992). Specifically, acknowledging the possibility of **URFs** motivates a redefinition of *dynamical symmetries* and a revision of Earman's **SP1** principle. This argument highlights the importance of exploring and critically analysing the role of reference frames in theoretical physics and foundations of GR, which is the main focus of my whole thesis.

To achieve my goals, Earman's **SP** principles are framed using the concepts of internal and external parameters. *External parameters* represent the independent variables, typically corresponding to the points on the smooth manifold $\mathcal{M}$. *Internal parameters*, on the other hand, are functions F (dependent variables) that depend on the independent variables. The general-relativistic models I consider are described as tuples $\langle \mathcal{M}, F_n \rangle$, where n acts as an index enumerating these functions.

I have already introduced the terminology of 'models' of a theory in the previous chapter, defining them as n+1-uples akin to $\langle \mathcal{M}, F_n \rangle$. It is now appropriate to fully characterise the possible models of a spatiotemporal theory, particularly focusing on the case of GR.

For any given theory, the broadest associated class of models is identified as the class of *Kinematically Possible Models* (KPMs). These models are defined as tuples comprising specific geometric objects, usually tensor fields on a manifold. For instance, in the case of GR, KPMs correspond to *triples* of the form $\langle \mathcal{M}, g_{ab}, \Psi \rangle$, where Ψ represents here a placeholder encompassing the matter fields content and $\mathcal{M}$ is a four-dimensional, paracompact, Hausdorff, connected, differential manifold Malament (2012). The defining characteristic of KPMs is that the functions F_n — which, in the given example, correspond to g_{ab} and Ψ — and their domains are only subject to algebraic constraints, which notoriously do not encapsulate the physical evolution of the system.

Within this class, a subclass of models that satisfy the theory's equations of motion are distinguished as *Dynamically Possible Models* (DPMs).

While the concepts of KPMs and DPMs are well-documented in the literature (see, for example the recent Pooley, 2022), a further class of models can be introduced. Following Read (2023), this third class is the class of *Boundary Possible Models* (BPMs). BPMs constitute a subset of KPMs; in particular, these are the KPMs of a theory whose F_n adhere to specific *boundary conditions* and do not have to satisfy the EFEs. For instance, in GR, one often focuses on those KPMs where the metric tensor g_{ab} satisfies the boundary condition of asymptotic flatness (see Wald, 1984, ch.11).

In Figure 3.1 I present a useful graphical representation to visualise the relationships between KPMs, DPMs, and BPMs in the case of vacuum GR.

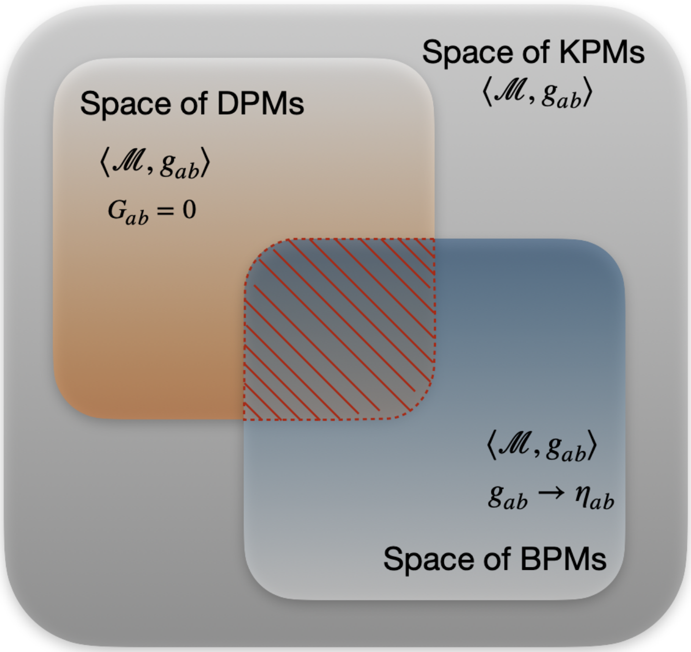

FIGURE 3.1: The space of **KPMs** is the most extended. The **DPMs** and **BPMs** form subspaces. In particular, the space of **BPMs** is partially overlapping with the subspace of **DPMs**: some **BPMs** may also be **DPMs** (see the dashed region). An example of **BPMs** that are not **DPMs** is for models with asymptotically flat metrics, irrespective of the satisfaction of EFEs (other different boundary conditions are of course possible).). An example of a **BPM** that is also a **DPM** is for the asymptotically flat Schwarzschild solution.

When discussing the symmetries inherent in spatiotemporal theories, it is critical to differentiate between 'background' and 'dynamical structures'.

While this division could be elucidated using Anderson (1967, ch.4)'s framework of *fixed*, *absolute* and *dynamical* fields, such details are unnecessary here. Broadly, Anderson divides *fixed* background structures, which stay *identically* the same across all KPMs and do not satisfy any dynamical equation, from dynamical structures, which satisfy some equations of motion. Furthermore, he also distinguishes a *dynamical field*, which *varies* (*up to isomorphisms*) from DPM to DPM, from an *absolute field*: i.e. a geometric object which satisfies some equations, but it is *the same* (*up to isomorphisms*) in every DPM.

However, here I follow another strategy, in order to avoid complications on the definition of the terms 'absolute, fixed, dynamical'.[1]

In GR, the background structure can be conventionally identified with the smooth manifold $\mathcal{M}$, or more specifically, according to the so-called '*chart-nominalist approach*', with the smooth structure defined by the manifold's maximal atlas of compatible charts (see Lang (1999). See also Wallace (2019) for a comprehensive discussion, contrasting chart-nominalism with Kobayashi and Nomizu (1963)'s *intrinsic* approach).

Talking of a 'background structure' within GR can be risky or confusing, since GR is understood as the background-independent theory *par excellence* (see Giulini, 2007; Belot, 2011; Pooley, 2017; Read, 2023). It is

[1]See Pooley (2022, §8.7) and Read (2023, ch.3) for a discussion. See also the next chapter for my characterisation of the term 'dynamical field' which is in line with Anderson's.

important, therefore, to emphasise that I do not intend to identify $\mathcal{M}$ as a background spacetime. Actually, different perspectives exist on how spacetime should be defined: it may be (i) the manifold $\mathcal{M}$[2]; (ii) the pair $(\mathcal{M}, g_{ab})$, where g_{ab} is the dynamical metric field representing the gravitational field[3]; (iii) the gravitational field g_{ab} alone; (iv) finally, I also will present my preferred option in §5.4, giving a physical, that is gauge-invariant, definition of spacetime in terms of *events*. In particular, the case (iv) shares with the case (iii) the idea that $\mathcal{M}$ is to be treated only as a *mathematical construct* without ontological status, as also argued in Rovelli and Gaul (2000); Rovelli and Vidotto (2015) and, famously, in Einstein et al. (2015).

Using the formal apparatus I have just presented in broad outline, I am now ready to define the difference between 'spacetime symmetries' and 'dynamical symmetries'.

Spacetime symmetries are defined as transformations that preserve the set of independent variables, or *external parameters*, that identify the background structure. In GR, these transformations correspond to the automorphisms

[2]I believe that this perspective is the most questionable, in light of the critiques to the so-called *naive* or *haecceitistic substantivalism* (Earman and Norton, 1987)

[3]This is the proposal adopted by the *sophisticated* wing of substantivalism, also known as *anti-haecceitistic substantivalism* (Pooley and Read, 2021)

of the manifold $\mathcal{M}$, forming the group $G_S \equiv \text{Diff}(\mathcal{M})$. These are *active diffeomorphisms*, acting on points.[4]

Dynamical symmetries, on the other hand, are transformations that act on DPMs and preserve *solutionhood*. *This means that solutions are mapped to solutions, and non-solutions to non-solutions.* These symmetries are assumed to form a group G_D, specific for each dynamical theory. In the context of GR, the EFEs select as $G_D \equiv \text{Diff}(\mathcal{M})$.

It is easy to see that, in the presented case of GR, the spacetime symmetry group and the dynamical symmetry group coincide. Is this a coincidence? The answer comes from Earman (1992)'s well-known work. Earman introduced two principles, *valid in all space-time theories*, relating these two types of symmetries:

(SP1) Every dynamical symmetry is a spacetime symmetry.

(SP2) Every spacetime symmetry is a dynamical symmetry.

Together, these principles imply that dynamical symmetries are exclusively those induced by automorphisms of $\mathcal{M}$. Consequently, in GR it is expected that these principles are always satisfied. However, as will be demonstrated in §3.2, certain assumptions characterising GR, when relaxed, can lead to violations of one of these principles: the **SP1**.

[4] There is of course a much more restricted sense in which a spacetime can have a symmetry—as captured by Killing fields (see below).

Before presenting my argument on how it is possible to break the **SP1** principle in an entirely new way compared to what is found in the literature, I would like to comment briefly on the definition of dynamical symmetries given above.

Firstly, dynamical symmetries — often referred to as 'symmetries' *simpliciter* — should be distinguished based on whether they pertain to equations or specific solutions.

For instance, in GR, while diffeomorphisms preserve *solutionhood* , they do not necessarily leave individual DPMs *unchanged*. For a solution g_{ab} of the EFEs, any diffeomorphism d preserves its being a solution, but usually results in $[d^{*}g]_{ab} \neq g_{ab}$. The subgroup of Diff($\mathcal{M}$) associated with the symmetries *of specific solutions* g_{ab} is named the Killing group, which is generated by Killing vector fields (Wald, 1984). Under a Killing transformation we have $[d^{*}g]_{ab} = g_{ab}$. That is: Killing transformations are *automorphisms*.

For generic metrics, this group is trivial or vanishing. Spacetimes with Killing symmetries are essential in simplifying EFEs and studying conserved quantities. Some famous examples of spacetimes with Killing symmetries include: the Schwarzschild metric which has four Killing vectors: one for time translation and the three rotational Killing vectors which together form the $SO(3)$ group of spherical symmetry; and the cosmological FLRW metric which has 6 spatial Killing vectors (3 translations + 3 rotations), forming the $SO(3) \times \mathbb{R}^3$ group.

Further clarification is needed as to what is meant by 'preserving solutionhood'. Pooley (2022, 121) suggests that a dynamical symmetry "preserves *the form* of the dynamical equations." However, this is misleading. Merely preserving *the form* is insufficient for a transformation to qualify as a dynamical symmetry. For example, in Special Relativity, the generally covariant Klein-Gordon equation $\eta^{ab}\nabla_a\nabla_b\phi = 0$ preserves its form under *any* $d \in \mathrm{Diff}(\mathcal{M})$, but solutionhood is preserved *only* for diffeomorphisms belonging to the Poincaré subgroup of Diff($\mathcal{M}$): $G_D \equiv \mathrm{Poin}(\mathcal{M}) \subset Diff(\mathcal{M})$. Explicitly, $\langle \mathcal{M}, [d^*\eta]_{ab}, d^*\phi \rangle$ is a DPM only for those d such that $[d^*\eta]_{ab} = \eta_{ab}$, which constitute the 10 Killing vectors group preserving Minkowski spacetime $(\mathcal{M}, \eta_{ab})$.[5]

As a further clarification of the concept of dynamical symmetry, I would like to point out that the very definition of dynamical symmetry is far from straightforward. This is also explicitly stressed in Belot (2013) (see also Fletcher (2019)).

I have defined above dynamical symmetries in terms of DPMs and solutionhood. An alternative definition frames symmetries as infinitesimal transformations generated by the flow of a vector field defined in the space of models (a manifold in the case of field theories), leaving the *action functional invariant* (modulo boundary terms). In details, (Gomes, 2023,

[5]Notice that in Special Relativity *the pair* $(\mathcal{M}, \eta_{ab})$ is defined as the background structure of the base set of independent variables. Dynamical objects are defined on such a background. So, careful, $\langle \mathcal{M}, \eta_{ab} \rangle$ is *not* a model of a special relativistic theory!

8) introduces the definition of symmetries as follows: given the action functional S on the space of models Φ, a vector field χ on Φ generates an *infinitesimal S-symmetry*, iff:

(a) χ respects the structure of Φ (e.g. its flow is continuous, smooth, symplectic, etc.);

(b) χ is definable without fixed parameters from Φ, i.e. all models enter as free variables in the argument of χ; and

(c) χ preserves the values of S: for any model m, $\chi[S](m) = 0$.

When an *infinitesimal S-symmetry* can be integrated for parameter time t, we have finite symmetries generated by the flow of $\chi : \Theta^t_\chi : \Phi \to \Phi$, such that (omitting χ and t): $S(\Theta(m)) = S(m)$ with Θ denotes the symmetry transformation $\Theta : \Phi \to \Phi$.

While advantageous in some contexts, such as quantum mechanics and the Hamiltonian formalism, this definition is not unproblematic.

For instance, diffeomorphisms — while dynamical symmetries of GR — do *not* leave the so-called *'gamma-gamma action'* $S_{\Gamma\Gamma} = \int d^4x \sqrt{-g} g^{\mu\nu} (\Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\nu\sigma} - \Gamma^\rho_{\mu\nu}\Gamma^\sigma_{\rho\sigma})$ invariant. Notably, from the variational principle, this action, introduced by Einstein (1916b) (but see also Dirac, 1958a) returns the EFEs like the well-known Einstein-Hilbert action $S_{EH} = \int d^4x \sqrt{-g} R$, with R the Ricci scalar. In fact, $S_{\Gamma\Gamma}$ contains the first derivatives of the metric, from which derive naturally the Einstein equations containing second derivatives of the metric (see Cianfrani (2014) for the demonstration). However, Christoffel symbols are *pseudo-tensors* and therefore non-covariant objects. It follows that $S_{\Gamma\Gamma}$ is not diffeomorphisms-invariant and condition (c) above is not met.

Another debate on dynamical symmetries, which I will only mention as it is not the focus of this chapter (for an in-depth discussion on this topic see Belot (2017)), is whether or not (isomorphisms induced by) dynamical symmetries acting on models generate new physical possibilities. In particular, Belot (2017) underscores a discrepancy between the (isomorphisms induced by) diffeomorphisms, which represent dynamical symmetries in GR, and gauge symmetries, understood as transformations that *do not* generate new physical possibilities. To clarify this distinction, he differentiates between:

> **Gauge symmetries**: [dynamical] symmetries that relate solutions that cannot be taken to represent distinct situations.

and

> **Physical symmmetries**: [dynamical] symmetries that relate solutions that can be taken to represent distinct situations.

Specifically, Belot mainly focuses on the context of asymptotically flat, vacuum spacetimes at spatial infinity. For a given diffeomorphism $d \in$

$Diff(\mathcal{M})$, two isomorphic models $\langle \mathcal{M}, g_{ab} \rangle$ and $\langle \mathcal{M}, [d^*g]_{ab} \rangle$ must represent flat spacetime within *the same asymptotic region*. This requirement necessitates a consistent notion of *identity* for the base manifold $\mathcal{M}$. Consequently, diffeomorphisms that preserve flatness but are *not the identity in the asymptotic region* correspond to different physical possibilities, meaning they cannot be classified as gauge symmetries. Only asymptotically trivial diffeomorphisms qualify as such (see also Ashtekar et al., 1991).

Therefore, diffeomorphisms that act non-trivially (i.e. do not reduce to the identity) at the *asymptotic* boundary, also referred to as *large diffeomorphisms*, can lead to distinct physical configurations.

By contrast, diffeomorphisms that act non-trivially only within the bulk while reducing to the identity at the asymptotic boundary are considered gauge transformations, also known as *small diffeomorphisms*, and do not produce physically distinguishable states.

In summary, large diffeomorphisms change the observable relation between the bulk and the boundary (creating distinct physical configurations), whereas small diffeomorphisms, by reducing to the identity at the boundary, leave that relation—and thus the physical state—unchanged.[6]

In the remainder of my work, as I did in Chapter 2, I will not follow

[6]Here is a practical example, which I find useful for its ease of depiction: the famous thought-experiment of Galileo's ship. Imagine a ship moving smoothly at a constant velocity over calm waters. Inside the ship, there is a closed cabin where an observer performs various experiments. Galileo argued that if the ship is moving uniformly (without acceleration), the observer inside will not be able to tell whether the ship is at rest or in motion. Imagine that the ship and the shore are two *subsystems*, representing the bulk and the boundary respectively. A small diffeomorphism could consist of a (Galilean) boost in velocity *inside* the boat, which does not change the position of the boat relative to the shore: this is the experiment Galileo refers to. A large diffeomorphism, on the other hand, also moves the boat so that its position relative to the shore changes. The whole system of ship and sea thus registers the difference between two such situations, namely in the different relative velocity between the ship and the shore. In this case, the velocity boost have a *direct empirical significance* (Gomes, 2022). An interesting third case emerges from these examples: if we perform a transformation that displaces both the ship and the shore by the same amount, there is no change in the relative position of the ship with respect to the shore. In this case, even if the transformation is 'large' in the sense of being non-trivial at the boundary (the shore), the *compensating transformation* makes it effectively gauge, meaning it does not change physical observables, since physical observables often depend on relative positions and the relation between bulk and boundary remains the same.

this distinction and consider active diffeomorphisms as the gauge transformations of GR, which therefore do not generate new physical, gauge-invariant possibilities.

3.2 Generalised Dynamical Symmetries and SP1* Principle

Let me introduce two generic fields, $\Theta(p)$ and $\Psi(p)$, defined on the manifold $\mathcal{M}$. The specific nature of these fields is not critical for the current discussion. The only requirement is that they be tensor fields.

In what follows, I propose a revised notion of dynamical symmetries that challenges Earman's **SP1** principle and generalises the notions introduced in the literature and revised in §3.1. As already stressed in §3.1, Earman's approach distinguishes between background elements, A, and dynamical fields, F. In the general-relativistic framework, the fields Θ and Ψ fall into the category of dynamical entities, while the background structure A consists solely of the smooth structure of $\mathcal{M}$.

Specifically, using this distinction, Earman defines a dynamical symmetry — which will be referred to as *standard dynamical symmetry* — as follows:

> Consider a model $m = \langle \mathcal{M}, A_1, A_2, \cdots, F_1, F_2, \cdots \rangle$ and let d be a diffeomorphism that maps $\mathcal{M}$ onto $\mathcal{M}$. Define $M_d = \langle \mathcal{M}, A_1, A_2, \cdots, d^*F_1, d^*F_2, \cdots \rangle$. Now d will be said to be a dynamical symmetry just in case for any $m \in \mathbb{M}_T$, it is also the case

that $M_d \in \mathbb{M}_T$ [*here* $\mathbb{M}_T$ *represents the set of all DPMs*] (Earman, 1992, 45).[7]

In the notation adopted in §3.1, a **standard dynamical symmetry** corresponds to a transformation $d \in G_D \subseteq \text{Diff}(\mathcal{M})$ that satisfies: $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM if and only if $\langle \mathcal{M}, d^*\Theta, d^*\Psi \rangle$ is also a DPM.[8]

Thus, a standard dynamical symmetry acts *uniformly* across all dynamical fields via a single diffeomorphism $d \in G_D$. Consequently, configurations of DPMs such as $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ or $\langle \mathcal{M}, \Theta, d^*\Psi \rangle$ do not classify d as a dynamical symmetry.

This limitation amounts to the underlying assumption that *all* dynamical fields are *dynamically coupled*, an assumption that my work aims to relax.

It is crucial to note that, unlike Chapter 2, the term *coupling* here pertains to DPMs and dynamical symmetries, which differs from its interpretation as influence. The subsequent Chapter 4 will provide a precise analysis of the interrelation between these two distinct definitions of coupling.

For the time being, I will keep the generic wording 'coupling' to indicate that two fields are dynamically coupled if and only if a standard dynamical symmetry *must* act *uniformly* across all dynamical fields via a single diffeomorphism $d \in G_D$ in order to preserve solutionhood. That is, two

[7] I have conformed his nomenclature with mine to avoid confusion.

[8] Note that this definition is the basis for that of 'Diff-Invariance' *of a theory*, as discussed in Pooley (2017), not to be confused with the diffeomorphism-invariance *of a quantity*, see Chapter 5.

fields are dynamically coupled if and only if, if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, neither $\langle \mathcal{M}, d^{*}\Theta, \Psi \rangle$ nor $\langle \mathcal{M}, \Theta, d^{*}\Psi \rangle$ are, $\forall d \in G_D$.

Having realised the role of the dynamical coupling assumption for the definition of standard dynamical symmetry, it is possible to reformulate the definition of spacetime symmetries and dynamical symmetries in a generalised way, so as to make these definitions more flexible to the abandonment of the dynamical coupling assumption. The motivation behind the generalisation of dynamical symmetries is two-fold:

The first is to shed light on some implicit assumptions made in the definition of standard dynamical symmetries and, hence, of the **SP1** principle that makes use of the notion of dynamical symmetry explicitly.

The second reason, which is perhaps more physical in nature, concerns the exploration of the consequences of introducing dynamically uncoupled fields into the theory, which can serve as reference frames, as argued in Chapter 2.

In detail, the definitions of spacetime and dynamical symmetries in §3.1 are reinterpreted and formalised as follows: (i) **Generalised Spacetime symmetries** form a group $G_S \subseteq \mathrm{Diff}(\mathcal{M})$ acting as: $\langle \mathcal{M}, \Theta, \Psi \rangle \to \langle \mathcal{M}, d^{*}\Theta, d^{*}\Psi \rangle$, for all kinematically possible models (KPMs); (ii) **Generalised Dynamical symmetries** are transformations $d, f \in G_D \subseteq \mathrm{Diff}(\mathcal{M}) \times \mathrm{Diff}(\mathcal{M})$ satisfying: $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM if and only if $\langle \mathcal{M}, d^{*}\Theta, f^{*}\Psi \rangle$ is a DPM.

As can easily be seen, the definition of spacetime symmetry has not changed, but that of dynamical symmetry acquires a new flexibility. In fact, my generalised definition of dynamical symmetries, denoted as **(GDS)**, still preserves the solution space for Θ and Ψ, while ensuring the smooth structure of $\mathcal{M}$ remains intact, but unlike the Earman (1992)'s standard definition, it allows G_D to extend to $\text{Diff}(\mathcal{M}) \times \text{Diff}(\mathcal{M})$, permitting *different* transformations on different dynamical fields, once their dynamically coupling is abandoned.

In cases where $d = f \in G_D = \text{Diff}(\mathcal{M})$, the generalised definition of dynamical symmetry aligns with the standard definition.[9]

In Figure 3.2, there is a graphical representation of the different action of a standard and generalised dynamical symmetry on a DPM. For simplicity, but without loss of generality, I adopt the case where a generalised dynamical symmetry acts as a *single* transformation *independently* on either one or the other field. Explicitly, instead of $\langle \mathcal{M}, d^*\Theta, f^*\Psi \rangle$, I consider $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ with $d \in G_D = \text{Diff}(\mathcal{M}) \times \text{Diff}(\mathcal{M})$.

[9]This shows that Pooley (2017)'s Diff-Invariance of a theory presupposes dynamical coupling between fields (see fn.8). However, by relaxing the dynamical coupling assumption, this generalisation enables *distinct* diffeomorphisms to act separately on each dynamical field.

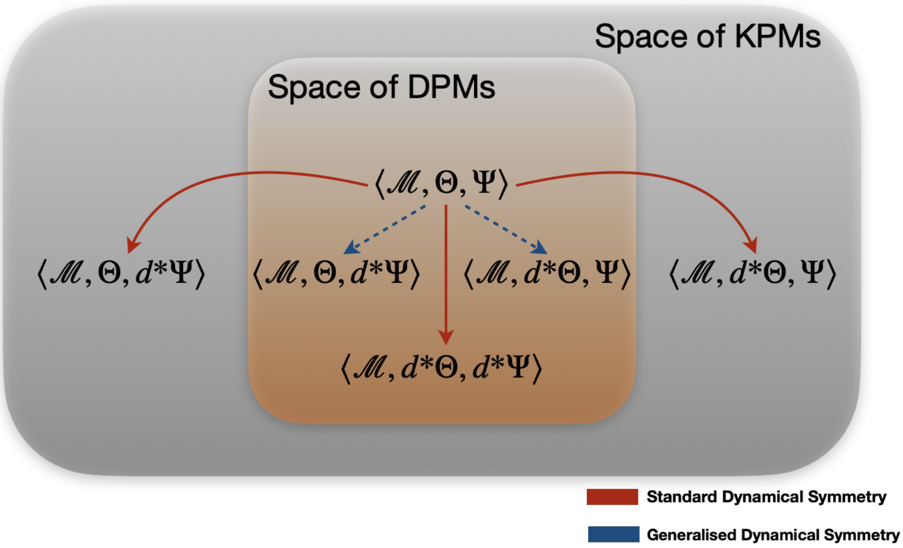

FIGURE 3.2: Action of a standard dynamical symmetry (in red arrows) and a generalised dynamical symmetry (in blue, dotted arrows) on a DPM $\langle \mathcal{M}, \Theta, \Psi \rangle$.

We now have the necessary theoretical tools to underscore an important result of my work: the contravention of the **SP1** principle in an entirely new way that, to my knowledge, has never been highlighted in the literature.

It should be noted, as Earman himself acknowledges, that the violation of the **SP1** principle is not unprecedented in the literature. Nonetheless, I will present a novel breach that avoids the limitations of previously documented cases concerning Newtonian vs. Galilean space-time, which I will review below.

Before presenting the result, I want to elaborate more on the status of the

SP1 principle. The **SP1** principle's validity depends on constraining the G_D group to a subgroup of Diff($\mathcal{M}$). As previously argued, this restriction on the G_D group, and consequently the **SP1** principle, rely on a tacit *necessary* assumption: all dynamical fields of the theory are dynamically coupled.

Within the framework of GR, this assumption is a *consequence* of one of the core tenets of the theory—the universality of gravitational interaction, which serves as a 'common cause' (a concept reminiscent of Reichenbach (1956) famous argument). Abandoning the assumption of dynamical coupling leads to interesting outcomes, such as the violation of **SP1** and the deviation from the universality of gravitational coupling, which are *sufficient but not necessary* for dynamical coupling to subsist.

In the next Chapter 4, I will analyse in more details the sufficiency and necessity relationship between dynamical coupling, as defined in this chapter, and the universality of gravity, which is here conceived in terms of a common cause, hence in terms of the *influence* of gravity on the dynamics of other fields, via the EFEs. Here, only a concise rationale is provided to substantiate the assertion that discarding dynamical coupling results in a departure from the universality of gravity. As previously stated, if two fields are *not* dynamically coupled, then if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, then, e.g., $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ will also be a DPM, for any d being a generalised dynamical symmetry. However, if $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ is a DPM, then it can be deduced that a change in Θ does not affect Ψ, hence Θ does not (dynamically) influence

Ψ. Conversely, if two fields *are* dynamically coupled, this provides no indication of whether or not they influence each other; it merely indicates that their dynamics are interrelated. Therefore, if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, then only $\langle \mathcal{M}, d^*\Theta, d^*\Psi \rangle$ can be, with d a dynamical symmetry.

From this discussion, what emerges is that **SP1** *implies* dynamical coupling between all fields in the theory (but not *vice versa*), and that **SP1** (which only makes use of the concept of dynamical symmetry) is in no way related to the notion of influence between fields and the universality of gravity. Schematically:

$$\textbf{SP1} \longrightarrow \text{Dynamical Coupling between } (\Theta, \Psi), (\Theta, g_{ab}), (\Psi, g_{ab})$$

$$\uparrow$$

$$\text{Universality of gravity}$$

My use in this chapter of 'dynamical coupling' in terms of DPMs, dynamical symmetries, and preservation of solutionhood, allows me to point out that in *every* spatiotemporal theory fields are dynamically coupled to each other through the common metric field.[10] The only way to uncouple a field *from a metric field* in a generic spatiotemporal theory is to disregard its dynamical equations, since in *any* spatiotemporal theory the dynamics is always written relative to a metric. This is consistent with the fact that **SP1** applies to *every* spatiotemporal theory and not only in GR, where the

[10]Keep this observation in mind. It will be useful in the next chapter when I talk about *indirect correlation* in §.4.5.

notion of *universality* of the gravitational metric, in terms of a common *influencing* cause, applies.

For instance, in Special Relativity, if $\langle \mathcal{M}, \eta_{ab}; \Theta, \Psi \rangle$[11] constitutes a DPM, then transformations like $(\eta_{ab} \to [\Lambda^* \eta]_{ab}, \Theta \to \Lambda^* \Theta, \Psi)$ fail to preserve this status under $\Lambda \in G_D = \text{Poin}(\mathbb{R}^4)$. The same holds for Galilean transformations in Newtonian physics. Breaking the coupling between fields can be achieved by neglecting the dynamics of at least one of them (if it helps, think about what it is done in the case of **IRFs** or **ARFs**).

It is also critical to clarify that **SP1** requires all fields in KPMs to be dynamically coupled, not only the dynamical fields. This ensures the inclusion of fields such as the Minkowski metric η_{ab} (in the next chapter, I will state why η_{ab} cannot be defined as a dynamical field. For now it is sufficient to say that it is not associated to some dynamical equations).

Now, back to the main result of the discussion: the violation of **SP1** in the case of uncoupled fields. I showed that in case of dynamical coupling, preserving solutionhood mandates that a dynamical symmetry act *uniformly* on all fields rather than independently on any subset.

This changes when the condition of dynamical coupling is removed: if all fields are not dynamically coupled, then: if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, then both

[11] I decide to adopt a personal notation in which I divide background structures from dynamical ones by means of a semicolon. Recall footnote 5.

$\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ and $\langle \mathcal{M}, \Theta, d^*\Psi \rangle$ *are* DPMs, $\forall d \in G_D$. Thus, the *independent* action of a dynamical symmetry is allowed in this case.

In terms of my definition of **GDS**, two fields are dynamically uncoupled if and only if, if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, both $\langle \mathcal{M}, d^*\Theta, f^*\Psi \rangle$ and $\langle \mathcal{M}, f^*\Theta, d^*\Psi \rangle$ are, $\forall d \neq f \in G_D$.

At this point, just to keep the discussion as simple as possible and also coherent with the usual examples of matter field, consider the fields Ψ and Θ as two sets of four dynamically uncoupled scalar fields, ϕ_1 and ϕ_2, mapping $\mathcal{M}$ to $\mathbb{R}^4$.

Both models $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$ and $\langle \mathcal{M}, d^*\phi_1, f^*\phi_2 \rangle$, where $d \neq f \in G_D = \text{Diff}(\mathcal{M}) \times \text{Diff}(\mathcal{M})$, *are* valid DPMs. Dynamical symmetries can *independently* affect ϕ_1 or ϕ_2, still preserving solutionhood.

This independence breaks Earman's **SP1** principle. In fact, models like $\langle \mathcal{M}, d^*\phi_1, d^*\phi_2 \rangle$ are related to $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$ by spacetime symmetries. However, this is not true for models such as $\langle \mathcal{M}, d^*\phi_1, f^*\phi_2 \rangle$, as spacetime symmetries require that a *single* diffeomorphism acts *uniformly on all fields*. On the other hand, $\langle \mathcal{M}, d^*\phi_1, f^*\phi_2 \rangle$ *is* related to $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$ by a generalised dynamical symmetry (**GDS**). This discrepancy demonstrates that **SP1** is violated: dynamical symmetries in theories with *uncoupled fields* do not simply mirror the automorphisms of $\mathcal{M}$, which are spacetime symmetries.

To address this violation, **SP1** can be reformulated as:

(SP1)* : every dynamical symmetry is a spacetime symmetry, *only if fields are dynamically coupled*. See Figure 3.3

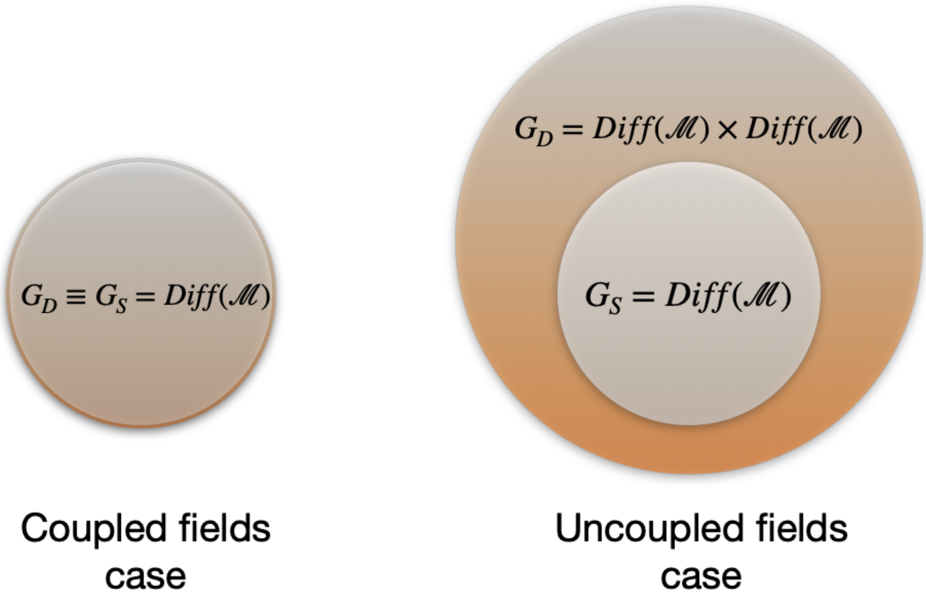

FIGURE 3.3: Mismatch between the spacetime symmetry group G_S and the dynamical symmetry group G_D of GR when fields are uncoupled.

In the literature, however, it is known that **SP1** can be hacked to some extent. So where is the novelty? To defend the **SP1** principle against possible violations, (Earman, 1992, p46) sustains that:

> The theory that fails **(SP1)** is thus using more space-time structure than is needed to support the laws, and slicing away this superfluous structure serves to restore **(SP1)**.

A pertinent question is whether the violation identified in my discussion aligns with such cases, where one introduces unnecessary structures. As will be demonstrated, this is not the case, as in the scenario presented here, no such superfluous structure exists, making Earman's argument to salvage **SP1** inapplicable.

A standard counterexample of **SP1** is found in Newtonian mechanics, which assumes a standard of absolute rest that the dynamical symmetries (forming the Galilean boosts group) fail to preserve. To resolve this issue, Newtonian mechanics is reformulated into Neo-Newtonian or Galilean spacetime, removing the absolute rest structure and thereby recovering **SP1**. See Figure 3.4.

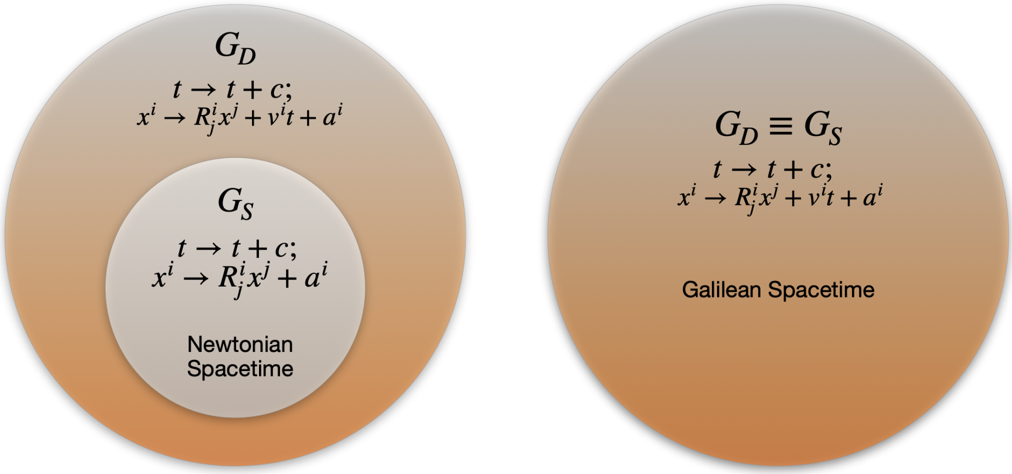

FIGURE 3.4: In accordance with normal usage, I coordinatise spacetimes with some coordinates (t, x^i). In the figure on the left, we find the case in which Newtonian spacetime has spatial and temporal translations as spacetime symmetries. The dynamical symmetries, on the other hand, are the Galilean Boosts. It can be seen that the G_S group is smaller than the G_D group. Hence there is a mismatch between dynamical and spacetime symmetries, resulting in the violation of **SP1**. The figure on the right is obtained by extending the G_S group, which therefore coincides with the G_D group of the Galilean Boosts. In this way, the spacetime under consideration is referred to as Galilean spacetime. The symbols used represent: t the time; c a constant; x^i the coordinates of a point, R^i_j is a 3×3 orthogonal rotation matrix; v^i the velocity; a^i a translation vector in space.

In contrast, in the case proposed by me, the manifold $\mathcal{M}$ does not contain unnecessary structures to remove. Adopting the so-called *Kleinian approach* and defining $\mathcal{M}$ as the 'G_S-structured space', structured by its spacetime symmetry group G_S, $Diff(\mathcal{M})$ is *the largest*, non-trivial possible G_S-group.

The only larger group would be that of continuous transformation and the less-structured space would be a topological manifold. However, even this generalisation would not save **SP1**, because the dynamics in GR require a *differentiable manifold* and not a mere topological manifold. Therefore, the $Diff(\mathcal{M})$-structured space $\mathcal{M}$ does not have any redundant structure!

To clarify, it's worth noting that if the group is the largest possible, the induced structure is the *smallest* possible (see Wallace, 2019). In Table 3.1, an example of the (inverse) relationship is shown between the dimension of the spacetime symmetry group and the amount of structure of spacetime.

G_S**-structured space**	G_S **Group**	**Spacetime Symmetries**
Vector Space	$GL(4, \mathbb{R})$	$x^\mu \rightarrow M^\mu_\nu x^\nu$
Inner Product Space	$O(4)$	$x^\mu \rightarrow O^\mu_\nu x^\nu$, with $O^T O = I$
Affine Space	$GL(4, \mathbb{R}) \ltimes \mathbb{R}^4$	$x^\mu \rightarrow M^\mu_\nu x^\nu + a^\mu$
Metric Space	$\text{Isom}(\bullet, s)$	Transformations f that preserve distances s $s(f(p), f(q)) = s(p, q)$
Metric Manifold	$\text{Isom}(\mathcal{M}, g)$	Transformations d preserving the metric $g_{\mu\nu}$ such that $[d^* g]_{\mu\nu} = g_{\mu\nu}$.
Differentiable Manifold	$\text{Diff}(\mathcal{M})$	Diffeomorphisms (smooth and with smooth inverse)
Topological Manifold	$\text{Homeo}(\mathcal{M})$	Homeomorphisms (continuous and with continuous inverse)
↓ **Larger** G_S **group**	↓ **Less structured space**	

TABLE 3.1: Inverse relationship between the size of the symmetry group and the amount of structure of spacetime. The larger the symmetry group, the less structured the space. Here M^μ_ν is a 4×4 invertible matrix; O^μ_ν is a 4×4 orthogonal matrix; $GL(4, \mathbb{R})$ is the group of invertible matrices 4×4 with real coefficients; $\ltimes$ is the semi-direct product symbol, which means that the two groups' operations are correlated.

In conclusion, I would like to make some remarks about the other principle by Earman: **SP2**. In particular, it's easy to note that **SP2** principle remains valid even under the assumption of uncoupling between fields. Recall that **SP2** establish that every spacetime symmetry is a dynamical symmetry.

For any given DPM $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$, where ϕ_1 and ϕ_2 are uncoupled, a spacetime symmetry-related configuration will read as $\langle \mathcal{M}, d^*\phi_1, d^*\phi_2 \rangle, d \in G_S$. However, it is the case that a DPM as $\langle \mathcal{M}, d^*\phi_1, d^*\phi_2 \rangle$ classified d as a dynamical symmetry.

3.3 Conclusion

This chapter critically re-examined Earman's symmetry principles (**SP1** and **SP2**) within the framework of GR, focusing on the interplay between spacetime and dynamical symmetries.

My analysis has revealed that the standard identification of dynamical symmetries with spacetime symmetries (embodied in the **SP1** principle) depends crucially on the assumption that all dynamical fields are dynamically coupled.

Dynamical coupling, in this context, means that a dynamical symmetry, in order to preserve solutionhood, cannot act independently on the fields composing a DPM. In GR this coupling is naturally provided by the universality of the gravitational interaction; every field is influenced by the metric, and vice versa.

By relaxing this assumption, as detailed in §3.2 , I introduced a more generalised notion of dynamical symmetry, labelled (**GDS**). This refined notion allows the *independent* actions of different elements of the generalised dynamical symmetry group on uncoupled fields, while still preserving solutionhood.

For example, in a DPM $\langle \mathcal{M}, \Theta, \Psi \rangle$, independent dynamical symmetry transformations d and f may act on Θ and Ψ, respectively (with $d \neq f$), thereby broadening the traditional definition which requires the *uniform* action of a *single* group element on all dynamical fields.

This generalisation leads directly to a violation of the original **SP1** principle, while preserving **SP2**. When dynamical fields are not coupled, a transformation may preserve solutionhood in one field while acting differently on another. In response, we have proposed a revised principle (**SP1***) stating that every dynamical symmetry is a spacetime symmetry only under the condition of dynamical coupling among fields.

The observed violation of **SP1** underscored the contingent nature of symmetry principles, which depend on the underlying assumptions about field coupling.

Importantly, this failure cannot be remedied by eliminating superfluous spacetime structure, as suggested in other contexts by Earman. Indeed, this violation does not stem from an unnecessary addition of spatiotemporal structure to the theory's dynamics, rather, it reflects a genuine dynamical assumption in the treatment of relation between fields.

The results presented in this chapter, which drew inspiration from the possibility of **URFs**, suggest that careful attention must be paid to the underlying, often implicit assumptions about field coupling. This insight has significant implications for the interpretation of reference frames in gravitational theory, especially given their increasing role in quantum contexts. In fact, as noted in Chapter 2, these contexts frequently employ uncoupled fields as (quantum) reference frames.

In summary, this chapter contributes to the ongoing debate about symmetry in physics by challenging long-held assumptions and offering a refined conceptual framework that explicitly accounts for the relaxation of dynamical coupling. The introduction of **GDS** and the consequent formulation of **SP1*** demonstrate that relaxing the coupling requirement naturally leads to a mismatch between the dynamical symmetry group and the group of spacetime automorphisms and leads to a richer, more flexible symmetry structure.

Moreover, this chapter highlighted the importance and fertility of conducting a thorough examination of reference frames. Such an analysis could shed light on specific elements of the fundamental principles underlying spacetime theories, which are frequently assumed without detailed scrutiny.

Chapter 4

Dynamical (un)coupling: influence or correlation?

In this chapter, I will refine and clarify the broad concept of *dynamical coupling* by distinguishing it into two main notions: *influence* and *correlation*. Additionally, I will examine the differences between symmetrical and non-symmetrical forms of influence, as well as the direct or indirect forms of both influence and correlation.

I think this work needs to be done, since the term *coupling* is inherently ambiguous. Indeed, it is interpreted in different ways in different types of literature.

In theoretical physics, this term typically refers to (symmetrical) *influence* between fields, as manifested in interaction terms in the total Lagrangian of the theory. This usage of the term coupling emphasises that fields interact *directly* through specific interaction terms. Thus, the notion of *influence* pertains to cases where fields affect one another.

If we use *coupling* in the sense of *influence* (see § 4.4), then only in GR it is the case that the coupling between *all* fields occurs. In GR, this 'universal coupling' exemplifies what I call *indirect influence*. This kind of coupling is mediated by the unavoidable *direct influence* of the gravitational field, encoded in the metric. This distinction between direct and indirect influence provides a clearer framework for understanding the nature of coupling in physical and philosophical contexts.

In Chapter 2, I introduced the concept of *dynamical coupling*, which referred to the (not necessarily symmetrical) influence between the reference frame and the gravitational field.

However, as I have shown in Chapter 3, in the context of philosophical analysis, the term *coupling* often carries a more specific meaning related to solutionhood and dynamical symmetries. This interpretation centers on what I refer to as *correlation*, as distinguished from *influence*. Under this interpretation of the term coupling, I will show in § 4.5 that in *any* spatiotemporal theory, *every* field is coupled to every other field through the common metric of the theory. This is a direct consequence of Earman's SP principles, which we have seen in Chapter 3 always apply, except in the case of approximations.[1] In short, all fields are connected (I will

[1] In fact, if fields in the DPMs are *uncorrelated*, then **SP1** fails. The logical *vice versa* is that the validity of **SP1** implies that fields in the DPMs are *correlated*.

say *indirectly correlated*) by the shared spatiotemporal structure. Furthermore, they are also *(directly) correlated* with the metric characterising the spatiotemporal theory.

This chapter aims to disentangle the various possible meanings to be attributed to the concept 'coupling', also in light of the implications between correlation and influence, once they are distinguished, that is: correlation implies influence, but not *vice versa*. In particular, I will argue that if two fields mutually influence each other, or even if only one influences the other, then they will also be correlated. But the converse is not true: we can have two correlated fields, without one influencing the other. The only way to break the correlation is to neglect the dynamics of the fields.

To complete this work, I will also provide a definition of *dynamical fields* and *physical fields*, distinguishing them in light of the recent literature on the action-reaction principle (§ 4.3).

4.1 Lehmkuhl (2011)'s proposal

To the best of my knowledge, such an attempt to distinguish between different notions of the term 'coupling' can only be found in Lehmkuhl (2011, sec.4.3). Here, I will elaborate on what distinguishes my proposal from Lehmkuhl's. My approach will be fully revealed in the course of the chapter, clarifying its innovative contributions.

Lehmkuhl, offering examples within the Lagrangian formalism, distinguishes between *direct coupling*, *indirect coupling*, and *interaction*, which is always considered direct.

According to his definition, *direct coupling* occurs when two fields appear as factors of the same Lagrangian (density) *product term* (*ibid.*, 467).

For instance, consider the interaction term between a complex scalar field and the electromagnetic field in a given *background metric* g_{ab}

$$\mathcal{L}_{\text{int}} = e^2\Big[A_a A^a \phi\phi^* + \frac{1}{e}A^a J_a\Big], \tag{4.1}$$

where e is the electric charge playing the role of the coupling constant and $J^a = ie[-\phi\nabla^a\phi^* + \phi^*\nabla^a\phi]$ is the conserved current. Here, the scalar field ϕ and the electromagnetic field A_a are considered *directly* coupled, resulting in coupled differential equations derived from the Euler-Lagrange equations.

In contrast, *indirect coupling* arises when two fields couple via an 'intermediate field'. If ϕ *directly couples* to the metric g_{ab} and g_{ab} *directly couples* to A_a, then the scalar field ϕ and A_a are *indirectly coupled*. An example for this case is the total Lagrangian (density):

$$\mathcal{L}_{tot} = \mathcal{L}_\phi + \mathcal{L}_A, \tag{4.2}$$

where $\mathcal{L}_\phi = \Big[g^{ab}(\nabla_a\phi)(\nabla_b\phi^*) - m^2\phi\phi^*)\Big]$ describes the scalar field and $\mathcal{L}_A =$

$-\frac{1}{4}g^{ac}g^{bd}F^{ab}F_{cd}$ represents the electromagnetic field. As can be easily seen, each Lagrangian represents a product term between the metric and the respective field, so ϕ and A_a are deemed *directly coupled* with g_{ab}, but *indirectly coupled* to each other.

Importantly, Lehmkuhl notices that coupling does not imply interaction.

Interaction requires both fields to be dynamical (*ibid.*, 469) and capable of influencing each other, whereas *coupling* can occur even if one field is non-dynamical. For instance, in the examples made above, the background metric g_{ab} is non-dynamical but *directly couples* to matter fields, since "they are factors of the same product term in the Lagrangian" (*ibid.*, 467). No interactions between the metric and matter fields are considered.

As noted by Lehmkuhl, interaction implies direct coupling. This is why the fields in the interaction term Lagrangian (4.1) are directly coupled. So, an interaction Lagrangian term will also be a product term, but the *vice versa* does not hold.

In which sense is the background metric g_{ab} in the Lagrangian (4.2) considered as non-dynamical? Lehmkuhl defines as a sufficient condition for defining a field as dynamical the fact that Hamilton's principle holds for variation over that field (*ibid.*, 469). I will show in Section 4.3 that such condition is not sufficient and encounters counterexamples that must be carefully addressed.

To conclude the presentation of Lehmkuhl's proposal, notice that in Lehmkuhl's terminology direct coupling with the metric is inevitable in *any* spatiotemporal theory. Consequently, indirect coupling between 'non-metric fields' is also inevitable, since they are necessarily directly coupled with the metric.

By contrast, in GR, the metric g_{ab} is dynamical, enabling true interaction with other fields. So, e.g., the term $\mathcal{L}_A = -\frac{1}{4} g^{ac} g^{bd} F^{ab} F_{cd}$ of equation (4.2) above, now represent an *interaction term* in the general-relativistic case and not just a *direct coupling* term (which was called a *product term* above). This also highlights that in GR not only coupling, but also the *interaction* between any field and the metric, is inevitable.

The forthcoming sections will elucidate that the notion of coupling as described by Lehmkuhl (2011) can be analogised to my notion of correlation (Section 4.5), whereas his concept of interaction aligns with my idea of influence, in particular *symmetrical* influence (Section 4.4).

However, this introduction already sets the stage for a glance at the differences between my conceptual framework and the one advanced by Lehmkuhl (2011).

The difference that immediately jumps out is that I divide the term coupling into influence and correlation, whereas Lehmkuhl uses coupling as a specific term and distinguishes it from interaction (corresponding to my notion of influence).

The other relevant difference from my approach is that I apply the distinction between 'direct' and 'indirect' not only to correlation (the equivalent of Lehmkuhl's coupling), but also to the concept of influence (corresponding to Lehmkuhl's interaction). Linking directness and indirectness to influence will allow me to discriminate cases whether the influence relation is symmetrical (that is, *mutual*) or not: an argument absent in Lehmkuhl's analysis (see Figure 4.1 below). See Table 4.1 for a summary of the differences between my approach and Lehmkuhl's.

Approach	**Concept**		**Properties**
Bamonti (this thesis)	Coupling	- Influence	- Direct, indirect, symmetrical, non-symmetrical
		- Correlation	- Direct, indirect, symmetrical
Lehmkuhl (2011)	- Interaction		- Direct, symmetrical
	- Coupling		- Direct, indirect

TABLE 4.1: The concept of coupling: comparison between Bamonti (this thesis) and Lehmkuhl (2011).

Notice that correlation *cannot* accommodate this differentiation because it is *inherently symmetrical*. Here is a simple explanation of this fact: the category of correlation is the same as that of the relation between two *relata* and I find at least questionable to state that only one of two *relata* is related (non-symmetrically) to the other.[2]

Conversely, influence aligns more closely with the notion of *action* of one entity upon another. To further clarify this point, alongside the notion of 'action', I will discuss below the action-reaction principle as detailed by Brown and Lehmkuhl (2013).

Furthermore, to explore the concept of influence versus correlation in more detail, I will also extend the discussion to analyse how this distinction applies to GR and pre-GR spatiotemporal theories. To this end, the introduction of fundamental principles of the so-called *dynamical approach* to Special Relativity (SR), championed by Brown (2005) will be helpful, since this approach describes a specific way of how to interpret the spatiotemporal structure of SR, encoded by the metric field.

All this discussions will shed light on why, contrary to GR, in the case of a background metric g_{ab}, I said that the Lagrangian $\mathcal{L}_A = -\frac{1}{4} g^{ac} g^{bd} F^{ab} F_{cd}$ does *not* identify an interaction term, and why the non-dynamical metric cannot *influence* fields, by acting on them.

[2]It is important to clarify that by 'correlation' I mean dynamical dependence in the sense relevant to solutionhood, not epistemic or probabilistic correlation as used in statistics or information theory. In those contexts, correlation can indeed be asymmetric — for instance, in causal Bayesian networks or mutual information relations conditioned on prior variables. My claim of symmetry applies specifically to dynamical correlation between fields within a spacetime theory.

4.2 Brown's Dynamical View and the Action–Reaction Principle

4.2.1 The Action-Reaction Principle

Newton's articulation of the action–reaction (AR) principle in the *Principia Mathematica* (Newton, 1687) is often taken as a foundational element of classical mechanics. According to this principle, every action between two entities results in a reciprocal reaction. This concept, as Brown and Lehmkuhl (2013) observed, extends beyond mechanics and reveals deep structural insights into physical theories in general.

In GR, for example, the AR principle manifests in the mutual interaction between the gravitational metric and matter fields. Unlike in Newtonian mechanics or SR, where spacetime remains causally inert, GR demonstrates a dynamical reciprocity: matter influences spacetime curvature, and spacetime curvature governs the motion of matter. This is one of the core of Einstein's theory, which is often summed up in the famous mantra: ''matter tells spacetime how to curve, and curved spacetime tells matter how to move'' (Misner et al., 2017, 5).

Einstein himself lauded GR as a triumph of the AR principle. The Einstein field equations encapsulate this principle by coupling the dynamics of spacetime to the distribution of stress-energy tensors of matter. Thus, unlike the spacetime structure of SR or Newtonian mechanics in GR spacetime is rendered a fully active participant in physical processes. This led

Einstein to believe that only GR was able to fulfill the demands of AR principle in a robust and unprecedented manner.

Nevertheless, Brown and Lehmkuhl (2013) acknowledged conceptual complexities in this belief. While GR satisfies AR, the broader applicability of the principle—particularly in theories where spacetime retains a fixed spatiotemporal structure—remains contentious. In sum:

> *What is debatable is not the claim that GR is consistent with the action–reaction principle, but the assertion that older theories involving absolute spacetime structures are not.* (Brown and Lehmkuhl, 2013, 2)

This clarifies that the AR principle does not require that all fields *both* act and react. For its validity it suffices that *if* a field acts, it is also acted upon. Violations occur only if a field acts without being acted upon, or vice versa.

It is true that GR's adherence to this principle highlights its conceptual advancement over theories where spacetime '*acts unilaterally*' upon matter without being influenced in return. However, this does *not* imply that if a field does *neither act nor reacts* the AR principle is not *still* respected.

This paves the way for the validity of AR principle also in SR, if interpreted in a specific way. In particular, adopting the so-called dynamical approach, asserting that the spacetime in theories like SR or Newtonian mechanics is a causally inert structure, the metric *neither acts nor reacts*, so the AR principle *still* holds.

To conclude this short paragraph it is worth noting that throughout the literature on AR principle 'to be acted upon' is often called 'reacting', so that a body satisfying the AR principle is said to 'act and react'. I claim this is an abuse of language. Re-action is a 'reflex action' by a body on which something acts. The crucial point of AR principle is *to act and to be acted upon*, which however is the 'passive' part of the principle. So, strictly speaking, a body satisfying the AR principle should be said to 'act and being acted upon'. Nonetheless, in the following I will also succumb to the temptation to abuse language.[3]

4.2.2 The Dynamical and Geometrical Approaches in Spacetime Theories

Although quickly, I have previously mentioned that there is a specific approach introduced by Brown (2005) to interpret spatiotemporal structure in SR (I will only consider Special Relativity, but the discussion is analogous to Newton physics), which is called the *dynamical approach*.

Actually, I have to make the reader aware that there are *two* main approaches.

The first approach — which is known as the *geometrical approach*, or the *causal-explanatory approach* (Samaroo, 2018)— interprets Minkowskian

[3]"Pleasure's a sin, and sometimes sin's a pleasure." (Byron, 1819).

spacetime as a causally active, but not reactive structure.[4] Spacetime is treated as an autonomous background that dictates causal order and determines the trajectories of free particles and light rays, analogous to how tracks guide a train[5]. For instance, Minkowski spacetime in SR imposes constraints on particle motion, emphasising the explanatory primacy of geometric structures. However, critics argue that this approach renders spacetime a "container" divorced from the dynamics of physical laws.

The other (heterodox) approach — the already mentioned *dynamical approach* pionereed by (Brown, 2005) (see also Brown and Pooley, 2006; Brown and Read, 2021) —- interprets Minkowski spacetime as a causally inert structure and reframes spacetime as a *codification* of the symmetries inherent in physical laws.[6] This approach characterises SR through its dynamical symmetry group, de-emphasising its geometrical underpinnings and challenging the traditional explanatory hierarchy by prioritising dynamics over geometry. In fact, Minkowski spacetime, in this context, is not an independent causal agent, but a manifestation of the Poincaré symmetry inherent in the laws governing matter and fields.

[4]Thus, according to this approach, Minkowski metric contravenes the AR principle.

[5]The geometrical approach is widely used in the physical and philosophical literature, so much so that it can be called the *orthodox approach*. For example, see Friedman (1983) and Maudlin (2015).

[6]Actually, I believe that the seeds of this approach are already found in the Einstein-Schlick correspondence of 1917 (see Brown and Lehmkuhl, 2013, 13). According to Schlick:

I believe it is useful to frame the debate between the dynamical and geometrical approaches in SR in 'deductive-nomological terms' as follows:

- **Dynamical Approach:** Geometry (*explanandum*) is Minkowskian *because* the laws of physics are Lorentz-covariant (*explanans*).
- **Geometrical Approach:** The laws of physics are Lorentz-covariant (*explanandum*) *because* geometry is Minkowskian (*explanans*).

In sum: while the geometrical approach underscores spacetime's role as an explanatory structure for the dynamics, the dynamical view situates spacetime as a derivative of the symmetries of physical laws.

In GR, the dynamical approach gains further traction. The metric field's chrono-metric significance arises from its interaction with matter fields, as governed by Einstein's equations. As discussed by Read et al. (2018), the 'principle of minimal coupling' underscores this interplay, ensuring that the fundamental equations governing matter fields remain locally

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> One does not have to understand Newtonian theory as taking Galilean space — after all, an unobservable thing — for the cause of the centrifugal forces. Instead, one might take the talk of absolute space a mere restatement of the bare fact that these forces exist. [...] One does not need to regard absolute rotation as the cause of the ellipsoidal shape [of S2]. Instead one can say: the former is defined by the latter.

invariant under Poincaré transformations.[7] The local Poincaré invariance is treated as an empirical fact—what Read et al. refer to as a '*brute fact*'. The consequence of this symmetry is profound: it dictates the local geometric structure of spacetime in GR. A particularly interesting implication of the dynamical approach is also how it reinterprets the geodesic principle.[8] Traditionally, the geodesic motion is viewed as a geometric property of spacetime. However, the dynamical approach suggests an alternative viewpoint: the geodesic motion of test particles emerges as a consequence of the Poincaré invariance of the laws governing matter dynamics. That is, the same symmetry principles that govern non-gravitational interactions also ensure that freely falling particles move along geodesics, thus providing a dynamical explanation for a principle often treated as purely geometric.

4.3 Physical fields or Dynamical fields?

This section begins *in medias res* with my definition of:

[7]The idea behind the minimal coupling is simple: when moving from SR to GR, replace the Minkowski metric η_{ab} with the general metric g_{ab} and ordinary derivatives ∂_a with covariant derivatives ∇_a, which take into account the curvature of spacetime.

[8]The geodesic principle states that the world-lines of force-free test particles are constrained to lie on geodesics of the connection (Brown and Pooley, 2006; Tamir, 2012; Weatherall, 2016).

Physical Field. A field is physical if and only if it is dynamical and respects the AR principle, *by both acting and reacting*.

This is the definition of a physical field that I adopt in this thesis and will now defend.

According to my definition, the constituent elements of a *physical field* are: compliance with the AR principle and being dynamical. This is a normative proposal: the justification lies not in fitting current usage but in structuring the distinctions needed for our analysis of coupling and observability. It is not just by definition, but because physical fields leave empirical traces and participate in energetic exchange, that they must influence and be influenced.

In §4.2.1, I already introduced the AR principle. What remains is to characterise what I mean by a *dynamical field*.

Specifically, I will present a case study in SR that challenges the conventional definition of a dynamical field as fields governed by Hamilton's principle (a view also critiqued by Read (2023)). While Lehmkuhl (2011) adopts this framework, my analysis will demonstrate its limitations through a concrete counterexample, showing why this criterion fails to fully capture what constitutes a dynamical field.

To do so, following Pooley (2017), I introduce **SR3**: a (real and massless) Klein-Gordon scalar theory in SR, whose dynamical equations

$$\begin{cases} \Box_\eta \phi = 0 \\ R^a_{\ bcd} = 0 \end{cases} \tag{4.3}$$

are derivable from the Hamilton's principle.[9] In fact, one can define the so-called Rosen (1966)-Sorkin (2002) Lagrangian (density) $\mathcal{L}_{R-S} = \Theta^{abcd} R_{abcd}$, in which the Lagrange multiplier Θ^{abcd} is introduced in addition to the fields η_{ab} and ϕ. Varying with respect to Θ^{abcd}, via Hamilton's principle, one recovers the dynamical equations $R^a_{\ bcd} = 0$ for the metric η_{ab}.

[9]Note that differently from Pooley (2017), I use the symbol η_{ab} to denote the Minkowskian metric and not g_{ab}. I use this choice to clearly divide the case of flat geometry from curved geometry. Also, note that with the abstract metric η_{ab} we do not necessarily refer to *the* Minkowski metric $\eta_{\mu\nu}$, which is specifically expressed in inertial coordinates. What I mean is that using η_{ab} also includes the case of a *curvilinear, not curved* metric. The difference lies in the fact that only in the case of a curved metric is the associated Riemann tensor non-zero. In the curvilinear case, some Christoffel symbols are non-zero, but the Riemann *is* zero. The Minkowski metric $\eta_{\mu\nu}$ represents the special case where *all* Christoffel symbols vanish.

However, I argue that this is not a sufficient criterion to define η_{ab} as a dynamical field (nor, consequently, as a physical field).[10]

Rovelli (2004) calls η_{ab} a ''fake dynamical field, since it is constrained to a single solution up to gauges [...]. Having no physical degrees of freedom, η_{ab} is physically a fixed background field, in spite of the trick of declaring

[10]A stricter, but equally insufficient criterion to demonstrate the 'dynamicity' of η_{ab} can be extrapolated from the discussion in Read (2023) regarding a Klein-Gordon scalar theory in a *parametrised* version of special relativity (named **SR4** in Pooley (2017). Quoting Read (2023, 25):

> To construct the parametrised theory [$\cdots$], one treats the four coordinate fields x^μ of this formulation as themselves dependent variables ('clock fields'), writes them as functions of arbitrary coordinates, $X^{(I)} = X^{(I)}(x^\mu)$, and re-expresses the Lagrangian in terms of these new variables.

Thus, we might demand as a stricter criterion of dynamism that, to define a field as dynamical, it *must* be subject to Hamilton's principle. In the parametrised case, it is indeed true that the clock fields $X^{(I)}$ *are* subject to Hamilton's principle applied to the parametrised action. However, variation of this action with respect to $X^{(I)}$ leads to equations that are *automatically* satisfied, so "it does not much matter [for the dynamics] whether $X^{(I)}$ are varied or not" (Brian Pitts, 2006, 9). Analogously, the new parametrised metric need *not* be varied to obtain its equation of motion $R^{\mu}_{\nu\rho\sigma} = 0$, so it does not satisfy this new stricter dynamical criterion. In fact, the parametrised metric field is *defined* in terms of the clock fields $X^{(I)}$ as $\eta_{\mu\nu}(x^\mu) := \eta_{IJ}(X^{(I)})\partial_\mu X^{(I)}\partial_\nu X^{(J)}$, so the DPMs of the theory are tuples $\langle \mathcal{M}, X^{(I)}; \phi \rangle$ since the clocks $X^{(I)}$ replace the metric field of the original non-parametrised theory, having as DPMs $\langle \mathcal{M}, \eta_{ab}; \phi \rangle$ (see Read, 2023, 26).

it a variable and then constraining the variable to a single solution.''

I fully agree with the underlying message behind this quote. I believe that the crucial point is that the solution space of equation $R^a_{bcd} = 0$ is *trivial* in the sense that different solutions are *all* related by dynamical symmetries that are *non-trivial* automorphisms of the background spatiotemporal structure, namely the transformations of the Poincaré group. I claim that *this* is what precludes η_{ab} from being a dynamical field, since *the absence* of non-trivial automorphisms for a spacetime structure is a necessary condition for it to be considered dynamical (see Pooley, 2017, 112).

Put differently, the equation $R^a_{bcd} = 0$ defines a solution space corresponding *exclusively* to a single *geometry*: the flat (or Riemann-flat) geometry. Here, the term 'geometry' refers to *the equivalence class of isometric metrics*. In the case of flat geometry, the isometric metrics are the those connected by Poincaré-transformations, since by 'isometry' I mean the isomorphism induced by the automorphism of the spatiotemporal structure.[11]

Note that this is not the case for EFEs $G_{ab} = 0$, where different solutions

[11] To be precise, in the case of particular topological assumptions, the isometric group of the Riemann-flat metrics can be a Poincaré *subgroup*. For example, if a periodic topology is introduced, such as identifying a spatial coordinate ($x \sim x + L$), a cylindrical topology is obtained. This periodicity constraint restricts the isometries to only those transformations within the Poincaré group that preserve the introduced periodicity, thereby forming a proper subgroup.

are *not all* related by dynamical symmetries that are *non-trivial* automorphisms of the background spatiotemporal structure identified with the differentiable manifold, namely the diffeomorphisms (see Chapter 3).

This is the case only *within* a given geometry, but different solutions of the EFEs can represent *different geometries*. For example, FLRW and Schwarzschild metrics are two solutions of the EFEs, but they are not connectable by a diffeomorphism. Only within each FLRW (resp. Schwarzschild) geometry we have a whole class of diff-related FLRW (resp. Schwarzschild) metrics.

Let me stress that even if the metric tensor in GR has *only* trivial automorphisms, in compliance with the necessary condition to be defined as a dynamical field, it is easy to give examples of particular solutions of EFEs *having* non-trivial automorphisms, understood as flows of Killing vector fields. Consider, for example, the aforementioned static Schwarzschild solution and the non-static FLRW solution (see Chapter 3 for an overview of their automorphism group).

The point is that one should not confuse GR, *qua* the dynamical theory *of* the metric tensor, with the dynamical theory of a field in a *given* curved metric solution of the EFEs.

I can now finally present my definition of:

Dynamical Field. A dynamical field is a field whose solution space is non-trivial.[12]

My analysis of the meaning of the terms 'physical' and 'dynamical' leads to the conclusion that the metric in SR, as well as any fixed Riemann-curved background metric, are not dynamical fields. Consequently, they cannot even be regarded as physical fields in the sense of being entities that both *influence* and *are influenced by* other fields — that is, they do not *act* and *are acted upon* in a reciprocal manner.

This aligns with the dynamical interpretation of SR presented by Brown (2005) in §4.2.2, which argues that the metric in SR should not be understood as an independent entity. Instead, it serves as a mathematical structure that encodes certain properties of the matter fields, reflecting their behaviour rather than existing as a dynamical participant in the physical system.

It is important to emphasise that a dynamical field does not have to satisfy the AR principle. Thus, it can act and not be acted upon; or be acted upon

[12]Note that my definition agrees with (Anderson, 1967, ch.4)'s definition of dynamical field as a field that changes from DPM to DPM, *up to isomorphism*. Notice that Anderson supports the view that a dynamical field has equations of motion derivable from a variational principle. He states: "we will assume that the equations of motion for the dynamical objects of a theory follow from a variational principle and that those for the absolute elements do not" (*ibid.*, 89). As I have shown above, this is, at most, a necessary, but not a sufficient condition.

and not act.[13]

The utility of differentiating between physical and dynamical fields lies also in the support it gives to the differentiation of the concepts of influence and correlation. In particular, *influence* is restricted to interactions involving *at least* dynamical fields (specifically, for symmetrical influence, *only* between physical fields). Conversely, *correlation* remains applicable even when one of the involved fields is non-dynamical.

The distinction between physical and dynamical fields also suggests that within the class of **CRFs**, **DRFs** are dynamical fields (hence the name), while **RRFs** are physical fields. Thus, another way to refer to **RRFs** could be *physical reference frames*. As they say, *de gustibus*.

[13]In the geometrical view, SR have such unsatisfactory element of a metric field that acts but does not react. The dissatisfaction with the existence of such a field in a physical theory comes from Einstein himself. Adopting what we now call a geometrical view, in Einstein (1966) he lamented that the metric of SR was a kind of 'new aether', or a 'merely factitious cause' (Einstein, 1916a). His Machian *empiriocriticist* spirit at the time contrasted with the impossibility of empirically testing the presence of the Minkowskian spacetime, thus prompting him to rethink spacetime structure in general-relativistic terms. As I argued, one can get rid of this 'factitious cause' also if the metric is conceived as *encoding* properties of the fully interacting matter fields. This is what is done in the dynamical view, where the metric becomes a 'glorious non-entity' (Brown and Pooley, 2006).

Before moving on to my precise characterisation of influence and correlation in the next two sections, I want to dwell on the case of GR. I have already argued above that the metric in GR is always a dynamical field. Now I want to stress that, to be more precise, the metric in GR is *always* a physical field. Let me consider some possible objections to this statement, to which I will answer.

The first is trivial and is based on the already mentioned and often underemphasised misunderstanding between a field theory on curved spacetime and GR coupled with a matter field. GR is the dynamical theory *of* the physical metric. One could argue that if I use a non-backreacting field on the metric (as in the case of **DRFs** in Chapter 2), then it would mean that the metric acts and is not acted upon, so it is not a physical field. However, this reasoning is wrong. Saying that a field does not backreact on the metric does not necessarily mean that the metric is a given solution of the EFEs, constituting a curved background. Quite the opposite: considering a non-backreacting test fields means that these fields will follow the geodesics of a metric that is, however, a *dynamical unknown* of the vacuum EFEs, that is the unknown variable to be found by solving the Einstein's dynamical field equations $G_{ab} = 0$.

Having clarified this point, which might seem trivial for the expert reader, let me now focus on the condition that the metric acts and, according to the AR principle, reacts in order to qualify as a physical field. Consider the case of vacuum GR. *In what sense does the metric act and react if there* *are no other fields in the theory on which to act*? Here, the *self-influence* of the metric field— which is due to the non-linear nature of the EFEs — comes into play. In the case of vacuum GR, the metric *acts on itself and reacts to its own action*.

Formally, this central aspect of GR can be justified as follows: vacuum EFEs can be rewritten in terms of Christoffel symbols as

$$R_{ab} \equiv \partial_c \Gamma^c_{ab} - \partial_b \Gamma^c_{ac} - \Gamma^c_{bd}\Gamma^d_{ac} + \Gamma^c_{cd}\Gamma^d_{ab} = 0. \tag{4.4}$$

This field equation can be divided into two parts: the first two terms, the derivatives of the Christoffel symbols, are linear in the second derivatives of the metric tensor and suggest a propagation phenomenon;[14] the last two terms, however, are non-linear and can be interpreted as the self-interaction part.

Proceeding in a non-rigorous, but physically intuitive manner, one can separate the propagative terms grouping them into a single term $\mathcal{G}_{ab} := (\partial_c \Gamma^c_{ab} - \partial_b \Gamma^c_{ac})$, from the non-linear, self-interaction part, moving the non-linear terms to the second member, which thus behave as a source term that can be termed as $\mathcal{T}_{ab} := \Gamma^c_{bd}\Gamma^d_{ac} - \Gamma^c_{cd}\Gamma^d_{ab})$. In such a way, vacuum EFEs in eq. (4.4) read as:

$$\mathcal{G}_{ab} = \mathcal{T}_{ab}. \tag{4.5}$$

[14]In the case of weak gravitational field, the vacuum EFEs represent the propagation of the gravitational field through a wave equation: the gravitational waves (Abbott et al. (2016)).

Written in this form, vacuum EFEs indicate that the gravitational field *always* has a part of self-action *and* backreaction, even in a vacuum where there is no matter.

This also confirms that gravity, if understood as spacetime curvature, exists even in the absence of matter, as the gravitational field itself always generates a 'gravitational energy-momentum source' capable of curving spacetime and propagating as a wave.[15]

[15]Historically, this realisation was a blow to Einstein's Machian view of GR, according to which the gravitational field was conditioned and determined solely by matter (see Einstein (1918)). It was de Sitter who in 1918 found a solution of the EFEs in vacuum — the de Sitter solution — that joined the already known vacuum Minkowski solution, whose existence was one of the reasons why Einstein (1917) modified the EFEs by adding a cosmological constant term (other reasons concerned his desire for a static and spatially closed Universe). Thus, Einstein was forced to admit the metric field as a physical player in its own right according to GR, akin to the electromagnetic field in Maxwell electrodynamics. See Janssen (2014) for more on the 'fight' between Einstein and de Sitter.

4.4 Coupling as Influence

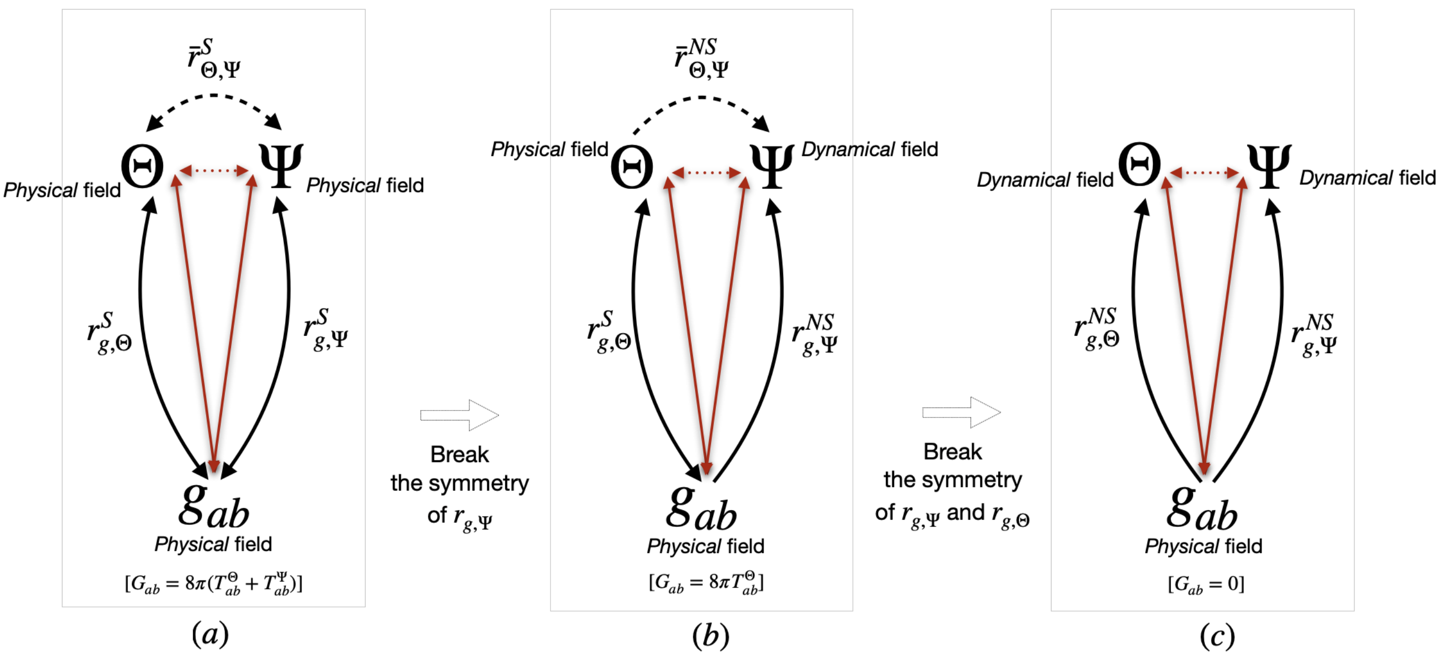

FIGURE 4.1: Direct *influence* is represented by solid, curved black arrows; indirect *influence* by dashed, curved black arrows.
Direct *correlation* is represented by straight, solid red arrows; indirect *correlation* by straight, dotted red arrows.
Note that Θ and Ψ are *directly correlated* to the physical, general-relativistic metric g_{ab}. This correlation is always guaranteed by the influence relation.

In this section I proceed to characterise the concept of *influence* between fields, where influence is understood here as a *binary* relationship between two fields. Of course, this is not intended to be a necessary restriction: it is also possible to speak of *n* fields influencing each other, but here I will study the influence relation as subsisting between pairs.

For that purpose, in the following, I will consider GR coupled with two generic *physical* fields Θ and Ψ, whose models are tuples $\langle \mathcal{M}, g_{ab}, \Theta, \Psi \rangle$. Importantly, I will take Θ and Ψ as *non-interacting* physical fields, in the sense that the total Lagrangian of the system does not contain an Lagrangian interaction term, or more physically, the two fields transform under distinct gauge groups (e.g. one field transforms under $SU(2)$ and the other under $SU(3)$, so they cannot interact via gauge bosons of a single group).

Influence between fields can occur either when both fields are physical, or also when one is a physical field and the other is a dynamical field. What will vary is the type of influence: in scenarios involving only physical fields, the influence is termed symmetrical, in compliance with the AR principle. Conversely, in the second case, influence is non-symmetrical.

In contrast, in the case where two fields are both dynamical, there can be no influence relationship between them.

Beyond this classification, I introduce a further distinction between direct and indirect influence. The key difference is that, in the case of indirect influence, the fields exert their influence through their respective direct interactions with a *common intermediary* field.

Below, I will discuss these four cases of influence in detail.

Case (I): Direct and Symmetrical Influence. The presence of a Lagrangian *interaction term* — not to be confused with a product term (see §4.1) — between two fields denotes their *direct* and *symmetrical* influence.

I define such a relation as $\mathbf{r}^S_{[\cdot,\cdot]}$, where the 'S' stays for 'symmetric'. For example, $\mathbf{r}^S_{\Theta,\Psi}(\equiv \mathbf{r}^S_{\Psi,\Theta})$ codifies the statement: 'the field Θ influences the dynamics of the field Ψ, *and vice versa*'. Their coupled dynamics is expressed through the interaction Lagrangian term $\mathcal{L}_{int_{(\Theta,\Psi)}}$.

I will now reformulate the examples made in §4.1 in terms of influence.

Consider a complex, Klein-Gordon scalar field ϕ and an electromagnetic field A_a, whose dynamics is described in a background metric field g_{ab}. The fields (ϕ, A_a) are a special case of generic fields (Θ, Ψ).[16] The total Lagrangian of the physical system will be:

$$\begin{aligned}\mathcal{L}_{tot} &= \mathcal{L}_\phi + \mathcal{L}_A + \mathcal{L}_{int_{(\phi,A_a)}} \\ &= (\nabla_a\phi\nabla^a\phi^* - m^2\phi\phi^*) - \frac{1}{4}g^{ac}g^{bd}F^{ab}F_{cd} + e^2\Big[A_aA^a\phi\phi^* + \frac{1}{e}A^aJ_a\Big].\end{aligned} \tag{4.6}$$

[16]The reader will forgive me if I relax for a moment the assumption of no direct influence between the two fields. However, I need to do so in this paragraph, because my aim is precisely to explain the direct influence.

The presence of the interaction Lagrangian (which is the already introduced term in 4.1) suggests that the fields influence each other *directly and symmetrically*. So, the influence relation characterising the considered system is: $\mathbf{r}^S_{\phi,A_a}$.

Now, consider the case of GR coupled with a complex, Klein-Gordon scalar field ϕ and an electromagnetic field A_a. Thus, we have now three physical fields: (g_{ab}, ϕ, A_a). The total Lagrangian of the physical system will be a single interaction Lagrangian, without free field terms:

$$\mathcal{L}_{int} = (g^{ab} D_a \phi D_b \phi^* - m^2 \phi \phi^*) - \frac{1}{4} g^{ab} g^{cd} F_{ac} F_{bd} - R, \tag{4.7}$$

where $D_a \phi = \nabla_a \phi + ieA_a \phi$ is the so-called covariant gauge derivative. This is the general-relativstic extension of the Lagrangian 4.2, *plus* the interaction term between (A_a, ϕ) encoded by D_a.

In GR, via the EFEs, all fields *directly* influence and are influenced by the metric. This results in the loss of free Lagrangian terms, because gravity *sticks* through the covariant derivative D_a. So, the influence relations characterising the system are: $\mathbf{r}^S_{\phi,A_a}$, $\mathbf{r}^S_{g,\phi}$ and $\mathbf{r}^S_{g,A_a}$.

Let me add a final consideration: not only in GR, but in *every theory*, physical fields by definition act and, in order to comply with the AR principle, are acted upon. The peculiarity of GR lies in the fact that *any* field mutually interacts with the physical field of the theory: i.e.

the gravitational field. Even the gravitational field itself.[17]

In contrast, e.g. in electromagnetism, *only* electrically charged fields interact with the electromagnetic field. The same applies to other known field theories (QCD, Electroweak Theory) for their respective charges.

Therefore, GR truly universalises gravity, making it an interaction that influence every form of energy and is influenced by every form of energy, *modulo approximations*. The next case will clarify the role of these approximations.

Case (I.1): Direct and Non-Symmetrical Influence. Now that I have reviewed the cases handled by Lehmkuhl, for the sake of generality, let me return to the more general case of GR coupled to two generic physical fields (Θ, Ψ). In particular, as already stressed at the beginning of this discussion, I choose Θ and Ψ such that they do not *directly* influence each other. This means that in the total Lagrangian of the system, there is no interaction term $\mathcal{L}_{int_{(\Theta,\Psi)}}$ between the two fields.

To recap what I have established so far: *any* physical field has a *direct* and *symmetrical* influence relation with gravity. This is due not

[17]Note that in Newtonian gravity only massive bodies interact with gravity. In particular: (i) only the mass density sources the field via the Poisson equation, (ii) the gravitational field itself carries no energy that can feed back into its own source (no self-coupling).

only to the universality of gravity, but also to the non-linearity of EFEs, whereby *any* field backreacts on the gravitational field via its stress-energy content.

However, inspired by the possibility of **DRFs**, we can make some *approximations* and neglect the backreaction of the material fields in the EFEs. In this way, we *break the symmetry* of the direct influence relation between gravity and the material fields.

For example, let's break the symmetry of the direct influence relation between (g_{ab}, Ψ). As a consequence, their influence relation is now described by: $\mathbf{r}^{NS}_{g,\Psi} \neq \mathbf{r}^{NS}_{\Psi,g}$, where 'NS' stays for 'non-symmetric'. In fact, $\mathbf{r}^{NS}_{\Psi,g} = \emptyset$, that is: '$\Psi$ does not influence the metric field g_{ab}'.

This move results in demoting Ψ to a *dynamical* field: is acted upon by gravity, but does not act on gravity, thus violating the AR principle. See figure 4.1(b).

As an aside, and to elaborate further on the notion of a physical field, I would like to point out that according to the approximation introduced, Ψ would be defined as a dynamical field, even though it mutually interacted with Θ (and I will show in **case (II.1)** below that this is not the case). For a dynamical field to be defined as physical, it must act and react with all fields *with which it can interact 'by constitution'*.

This means that a physical field need not act and react with all fields, *simpliciter*. For example, an electrically neutral field will not interact with the electromagnetic field, but this does not prevent it from being defined as a physical field. Conversely, an electrically charged field whose electromagnetic interaction is neglected is not definable as a physical field.

Case (II): Indirect and Symmetrical Influence. 'Indirect and symmetrical' influence can be understood as the 'mutual influence of physical fields, *mediated* by their direct and symmetrical influence with a common field'.

Therefore, we do not need an interaction term between two fields in the total Lagrangian to affirm that two fields indirectly influence each other. We define such a relation as a symmetrical relation $\bar{\mathbf{r}}^S_{[\cdot,\cdot]}$.

As I mentioned earlier, in GR, through EFEs, any field has a *direct* and *symmetrical* influence relation with gravity. This implies that in GR *all fields indirectly influence each other*.

In terms of the already considered general-relativistic system, this means that between Θ and Ψ there will *always* be an indirect influence relationship described by $\bar{\mathbf{r}}^S_{\Theta,\Psi}(\equiv \bar{\mathbf{r}}^S_{\Psi,\Theta})$. In Figure 4.1(a) we see the *symmetrical* and *direct* influence relations $\mathbf{r}^S_{g,\Theta}$ and $\mathbf{r}^S_{g,\Psi}$ between (Θ, Ψ) and g_{ab}. The symmetrical influence is encoded by the presence of the stress-energy tensors of both fields in the EFEs. We also see the induced *indirect* and *symmetrical* influence relation $\bar{\mathbf{r}}^S_{\Theta,\Psi}$ between Θ

and Ψ.

Importantly, this is *not* the case, for example, for a field theory in a given background metric (including the flat, Minkowski metric). According to my definition, the background metric is not even a dynamical field, thus the *influence* relation between such metric and other fields cannot be defined. As a consequence, also the *indirect influence* between fields cannot be defined.

To this regard, for the sake of clarity, I stress that my notion of indirect influence does *not* coincide with the indirect coupling introduced by Lehmkuhl, which characterised Lagrangian 4.2. In fact, in Lagrangian 4.2, the metric g_{ab} was considered to be a background metric, so the reference theory is not a general-relativistic one. In the case just presented, however, I consider as the reference theory GR coupled to Θ and Ψ fields (which are generalisations of the A_a and ϕ fields in 4.2).

The analogue of Lehmkuhl's indirect coupling will be introduced in Section 4.5.

Case (II.1): Indirect and Non-Symmetrical influence. As already mentioned in case (I.1) above, the fact that all fields influence and are influenced by gravity is valid *only modulo approximations*. Consider again the same approximated case of (i.a), where we neglect the backreaction of Ψ on the gravitational physical field g_{ab}. It is the case that (Θ, Ψ)

will still *indirectly* influence each other, but *not symmetrically*.[18]

Simply put, since Θ directly influences g_{ab} (and vice versa) and g_{ab} directly influences Ψ (but not vice versa), then Θ will *indirectly* influence Ψ, *but not vicev ersa*. Formally:

$$\mathbf{r}^{S}_{\Theta,g} \circ \mathbf{r}^{NS}_{g,\Psi} := \bar{\mathbf{r}}^{NS}_{\Theta,\Psi}. \tag{4.8}$$

The operator $\circ$ indicates the composition of relations and it is usually used for the composition of relations between sets. So, for this aim, the fields g_{ab}, Θ, Ψ can each be represented as *singleton sets*, i.e. sets with only one element. That is: $g_{ab} := \{g_{ab}\}$, $\Theta := \{\Theta\}$ and $\Psi := \{\Psi\}$.

Given two binary relations $\mathbf{r}^{S}_{\Theta,g}$ and $\mathbf{r}^{NS}_{g,\Psi}$ defined on sets $\{g_{ab}\}, \{\Theta\}, \{\Psi\}$, their composition is a new relation that connects (elements of) $\{\Theta\}$ with (elements of) $\{\Psi\}$, *via* the (elements of) $\{g_{ab}\}$. Formally: given $\mathbf{r}^{S}_{\Theta,g} \subseteq \{\Theta\} \times \{g_{ab}\}$ and $\mathbf{r}^{NS}_{g,\Psi} \subseteq \{g_{ab}\} \times \{\Psi\}$, then their composition is a new relation defined as:

$$\mathbf{r}^{S}_{\Theta,g} \circ \mathbf{r}^{NS}_{g,\Psi} = \{(\Theta, \Psi) \in \{\Theta\} \times \{\Psi\} \mid \exists g_{ab} \in \{g_{ab}\} \text{ such that } (\Theta, g_{ab}) \in \mathbf{r}^{S}_{\Theta,g} \text{ and } (g_{ab}, \Psi) \in \mathbf{r}^{NS}_{g,\Psi}\}, \tag{4.9}$$

whose interpretation is: 'there is a relation between Θ and Ψ via some *intermediate* element g_{ab}'. It is the case that $\circ$ retains the usual properties of *associativity*, *presence of the neutral element* and *non-commutativity*.

[18] We already know that they do not influence directly, by assumption.

It can be noted that the composition of a symmetrical and a non-symmetrical relation produces a non-symmetrical relation. So, by using the *transitivity* of the direct dynamical influence relation $\mathbf{r}_{[\bullet,\bullet]}$ and the *non-symmetry* of the direct relation $\mathbf{r}^{NS}_{g,\Psi}$, we obtain that $\bar{\mathbf{r}}^{NS}_{\Theta,\Psi}$ is a *non-symmetric and indirect* relation.[19]

To sum up: the breaking of the symmetry of the *direct* dynamical influence relation $\mathbf{r}_{g,\Psi}$ induces the breaking of the symmetry of the *indirect* influence between Θ and Ψ (see Figure 4.1(b)).

To conclude, at first sight one could say that after the approximation of neglecting the backreaction of Ψ on the gravitational physical field g_{ab}, thus demoting Ψ to a dynamical field, also Θ loses the property of being a physical field. In fact, Θ acts (indirectly) on Ψ, but is not acted upon by Ψ.

This fact allows me to make a further clarification on the notion of physical field. It is the absence of reaction by Ψ on g_{ab} that does not allow Θ to be acted upon (indirectly) by Ψ. Thus, it is *only* Ψ that is approximated from physical field to dynamical field. Θ remains a

[19] I do not have a formal argument to prove the *transitivity* of the direct dynamical influence. I rely, therefore, on the intuitive argument that: if a whatever element A influences the behaviour of B and B in turn influences the behaviour of C, it is reasonable to infer that A plays a role in the behaviour of C, using B as a 'conduit'.

physical field in the sense that the 'lack of being acted upon' by Ψ is Ψ's 'responsibility', not Θ's!

In the next section, I will address the concept of dynamical coupling understood as a correlation between fields, which corresponds to the concept of 'coupling' (simpliciter) adopted in Lehmkuhl's work. In particular, I will show how, even in the case where it is not possible to define an influence relationship between fields, this does not preclude the possibility of defining a correlation between fields. Influence is sufficient, but not necessary for the presence of correlation. Although this point is emphasised in Lehmkuhl's work, he only introduces direct coupling via an interaction term (as the Lagrangian 4.1), precisely by exploiting the sufficiency of the interaction for coupling. In contrast, I will show a case where there is no influence relationship, but there is still a correlation.

This will allow me to state that for *any* spacetime theory there is *always* correlation between fields. This is also true in the case of theories with a non-dynamical background metric, as is the case of SR.

4.5 Coupling as Correlation

The general-relativistic example of the three physical fields (g_{ab}, Θ, Ψ) proposed in the last Section has a further potential: that of shedding light on the nature of the correlation between fields.

Before showing, as promised, a case where the correlation between fields can be analysed without the presence of *any* influence in the total system, there is a further approximated case to consider, in which we break the symmetry of *both* the direct dynamical influence relations $\mathbf{r}^S_{g,\Psi}$ and $\mathbf{r}^S_{g,\Theta}$. In this way, both Θ and Ψ are demoted to *dynamical* fields. This manoeuvre breaks the indirect influence between Θ and Ψ. That is: $\bar{\mathbf{r}}^S_{\Theta,\Psi} = \bar{\mathbf{r}}^S_{\Psi,\Theta} = \emptyset$. Their dynamics is still influenced by the common metric, but the dynamics of one no longer indirectly influence the other. In Figure 4.1(c) we see that since Θ and Ψ are influenced by, but do not influence g_{ab} (EFEs do not include their stress-energy tensors), then the two fields lose their *indirect* influence. *Still* they remain *indirectly correlated*, via their direct correlation with g_{ab}.

However, it can be showed that Θ and Ψ are not yet 'dynamically independent' of each other, *even though there is no influence relationship between them*. They still remain dynamically coupled in a specific sense that I name *correlation* and that I will now investigate

Let me formalise the central notion:

Correlation (direct or indirect). Two fields Θ and Ψ are *correlated iff*, if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, then $\langle \mathcal{M}, \mathrm{g}^*_D\Theta, \Psi \rangle$ or $\langle \mathcal{M}, \Theta, \mathrm{g}^*_D\Psi \rangle$ is not, for $\mathrm{g}^*_D\Theta \neq \Theta$ (resp. $\mathrm{g}^*_D\Psi \neq \Psi$).

Here, $\mathrm{g}_D \in G_D$ represents the *dynamical symmetry transformation* acting on Θ and Ψ in the relevant theory. I showed in Chapter 3 that in order for

$\langle \mathcal{M}, g_D^* \Theta, \Psi \rangle$ or $\langle \mathcal{M}, \Theta, g_D^* \Psi \rangle$ to be DPMs, the dynamical symmetry group must be *extended* to $G_D \times G_D$, so that the dynamical symmetries can act *independently* on the fields.

This concept of correlation admits further subdivision, mirroring the structure introduced in §4.4. I distinguish two types:

Direct Correlation. A field Θ (resp. Ψ) is directly correlated with another field (typically a metric) when its equations of motion are explicitly written with respect to that field. This is the standard case for any field propagating in a fixed background η_{ab}.

The case study I offered above of the scalar Klein-Gordon scalar field is also valid here. Assuming both Θ and Ψ being two massless, Klein-Gordon scalars, they are *directly correlated* with η_{ab} via their equations of motion: $\Box_\eta \Theta = 0, \Box_\eta \Psi = 0$. The very existence of their equations of motion indicates that any transformation applied to η_{ab} necessitates a corresponding transformation of Θ and Ψ *in an identical manner* to yield possible solutions. I define this kind of correlation between (Θ, η_{ab}) and (Ψ, η_{ab}) a *direct correlation* (corresponding to Lehmkuhl's direct coupling).

The meaning of the term '*direct*' lies in the fact that the equations of motion of Θ (resp. Ψ) are *always* written involving a metric, explicitly or implicitly.[20]

It should be emphasised that the aforementioned Brown (2005)'s dynamical view on SR does not veto the possibility to speak of direct correlations of fields with the common Minkowski metrics which appears in the equations. It is only 'forbidden' to speak in terms of *influence* of metric on fields' dynamics, since the metric is an abstraction derived from the fact that all fields obey the same type of dynamical symmetries, i.e. the Poincaré symmetries.

Indirect Correlation. Two fields Θ and Ψ, are indirectly correlated if each is directly correlated with a third field (typically the metric), and this shared dependence induces constraints on their mutual configuration, even in the absence of direct influence. In the equations of motion of each field the other does not appear (hence indirectness), however, the two fields are indirectly correlated by virtue of *sharing the same metric* with respect to which their respective equations are expressed.

Importantly, while influence implies correlation, the converse does not hold. In fact, correlation may persist even when all influence — direct or

[20]By 'implicitly' I mean that in Minkowksian coordinates (t, x, y, z), the Klein Gordon equation for the scalar Θ (resp. Ψ) can be written as:

$$-\frac{\partial^2\phi}{\partial t^2} + \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0,$$

where the metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ does not appear explicitly.

indirect — has ceased. This marks a key point of departure from Lehmkuhl (2011) treatment of coupling: whereas his analysis equates coupling with terms in the Lagrangian, mine recognises that dynamical dependence can remain even when causal efficacy is absent.

Using this terminology, I can now clarify the structure exhibited in Figure 4.1(c). Since both fields Θ and Ψ are *directly* influenced by the same *physical* metric g_{ab}, they must also be *directly* correlated with it. This follows from the general point established above: direct influence is sufficient for direct correlation. Moreover, if two fields are each directly correlated with the same third field — in this case, the metric — they are thereby indirectly correlated with each other.

Indirect influence (whether symmetrical or non-symmetrical), as illustrated in Figure 4.1(a) and Figure 4.1(b), likewise entails *indirect* correlation.

In all such cases, correlation tracks the structural dependence imposed by shared dynamical background.

This structure illustrates a point made earlier: while influence may be non-symmetric, correlation is necessarily symmetric. The case shown in Figure4.1(c) serves to make this vivid. Although all influence relations have been removed, the fields Θ and Ψ remain interdependent: any transformation of one requires a corresponding transformation of the other to preserve dynamical admissibility.

This framework also highlights that influence, while sufficient for correlation, is not necessary. In Figure 4.1(c), although all influence relations have been eliminated — neither Θ nor Ψ influences the metric, and they do not influence one another — Θ and Ψ remain indirectly correlated.

it can be showed that the two fields Θ and Ψ exhibit *indirect correlation*. This is underscored by the fact that, to preserve solutionhood, we cannot act with a dynamical symmetry (a diffeomorphism $d \in \text{Diff}(\mathcal{M})$) *independently* on Θ and Ψ. The reason is structural: their equations of motion still depend on a common field, the *physical metric* g_{ab}. Any attempt to apply a dynamical symmetry (for example, a diffeomorphism $d \in \text{Diff}(\mathcal{M})$) independently to Θ and Ψ would violate the requirement of joint solutionhood. Their correlation persists, even in the absence of mutual influence.

To make this more explicit, imagine the case where Θ is a massless scalar field and satisfies a Klein-Gordon equation $\Box_g \Theta = 0$, which are formulated in terms of g_{ab}. This equation constrains the dynamically allowed solutions of Θ. If we apply a transformation to such solutions, we must expect that for the new configuration to reamin dynamically admissible, the 'cause' that constrains the field to assume such solutions — i.e. the physical metric g_{ab} —- must undergo the *same* transformation.

So, if we change the field $\Theta \rightarrow d^*\Theta$, we want the metric to change according to *the same* transformation $g_{ab} \rightarrow [d^*g]_{ab}$. But, since the metric directly influences Ψ, also Ψ must change according to *the same* transformation $\Psi \rightarrow d^*\Psi$. Therefore, the transformation $g_{ab} \rightarrow d^*g_{ab}$ requires that $\Psi \rightarrow$

$d^*\Psi$ in order to preserve its own solutionhood. In short, a transformation of Θ necessitates a matching transformation of Ψ, mediated via their shared dependence on g_{ab}.

This argument concludes that Θ is *indirectly correlated* to Ψ due to their being *influenced by the same field*, acting as a 'common cause' (a concept reminiscent of Reichenbach (1956) famous argument). Thus, even in the absence of any direct or indirect influence between Θ and Ψ, the structural role of the metric enforces a dynamical constraint on their relative configurations. Their correlation is not causal but structural — a shared consequence of their dependence on a common, shared cause (understood as a source of influence).

Let me now offer a physical example of indirect correlation arising from a common cause: the Cosmic Microwave Background (CMB). For empirical confirmation of this possibility, one need only look up at the sky.

The CMB is the faint, uniform radiation left over from the Big Bang, representing the Universe's earliest light, emitted about 380,000 years after its formation when atoms first formed, making the Universe transparent.[21] It permeates the entire universe and has approximately the same temperature at every observed point in the sky, no matter how far apart they are, about 2.725*K*. Inhomogeneities in temperature are of the order of one part in ten thousand. See Fig. 4.2.

This experimental fact underlies the so-called '*horizon problem*': how do distant regions of the universe, which according to FLRW dynamics could never exchange information with each other and which *currently* are not in causal contact, share the same temperature today? This problem can be translated into an everyday example:

> Now, if you invite two people to a potluck dinner, and they both bring potato salad, you can dismiss that as coincidence, even if they had 10^5 different dishes to chose from. However,

[21]The epoch of CMB formation is sometimes called the *recombination epoch*, which refers to the combination of protons and electrons in the primordial plasma and the formation of the first atoms. Strictly speaking, however, the CMB formed in an epoch after that of recombination, called the *decoupling epoch*, which is the name given to the last time the CMB photons interacted with matter. The recombination and decoupling events are distinct because it is necessary to wait for the mean free path of the photons to become larger than the *Hubble length*, which sets the scale for the propagation of causal signals. Only then are the photons free to travel throughout the Universe. Intuitively, it takes time for enough atoms to recombine to allow photons to propagate without encountering too many protons or free electrons and be scattered (Serjeant, 2010).

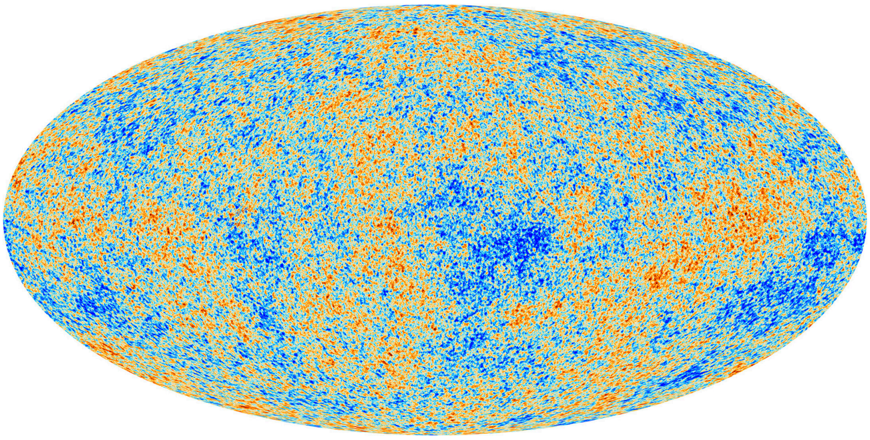

FIGURE 4.2: Anisotropies in the Cosmic Microwave Background (CMB) as observed by Planck. The CMB provides a snapshot of the oldest light in the Universe, emitted when it was just 380,000 years old. The temperature fluctuations reveal regions of varying density, which seeded the formation of cosmic structures. While distant points exhibit statistical correlations, they do not causally influence each other. CREDIT: ESA and the Planck Collaboration.

if you invite 40,000 people to a potluck dinner, and they all bring potato salad, it starts to dawn on you that they must have been in contact with each other: "*Psst...let's all bring potato salad. Pass it on*." Similarly, it starts to dawn on you that the different patches of the last scattering surface, in order to be so nearly equal in temperature, must have been in contact with each other: "*Psst...let's all be at* $T = 2.725K$ *when the universe is*

13.5 *gigayears old. Pass it on*." (Ryden, 2017, 189)

For our purposes, the key point is this: we currently observe correlations in temperature between regions of the CMB that are not in causal contact. That is, we observe *current* correlation *without current influence*.

The most widely proposed explanation for this fact is offered by the theory of *cosmic inflation*. According to this paradigm, FLRW dynamics must be supplemented with a phase of extremely rapid expansion of the universe that occurred between 10^{-36} and 10^{-34} seconds after the Big Bang. This inflationary phase exponentially increased a causally connected region, 'stretching it' to encompass the entire observable universe, which corresponds to the CMB we now observe (Montani et al., 2011).[22]

As a result, regions of the universe that now appear causally disconnected were once in thermal contact and thus influenced one another. Influence occurred during the inflationary phase, imprinting a common thermal history onto all such regions. That influence has long since ceased — but the correlation it produced endures. This perfectly exemplifies the principle established earlier: *influence implies correlation*, but *correlation does not require ongoing influence*.

It is important to remark again that this is not a case of correlation without any influence whatsoever. It is important to note that this is *not* a case

[22]To date, inflation remains a promising, yet speculative theory. Therefore, it would be better to call it a paradigm rather than a theory (Koberinski and Smeenk, 2024).

of correlation arising *without* any influence whatsoever. Rather, the correlation we observe today is a persistent trace of past mutual influence during the inflationary era. What makes the CMB an illustrative example of 'correlation without influence' is that at present, the correlated regions are causally disconnected—yet the correlation remains, preserved from an earlier epoch when interaction was possible. This temporal separation between cause and continued dependence exemplifies how correlation can survive the breakdown of direct influence.

So far, we have examined systems in which some form of influence is always present — whether direct or indirect, symmetric or asymmetric — between at least two fields.

I now turn to a final, more radical possibility: *fields may be correlated even in the complete absence of influence anywhere in the system as a whole*. This means that no physical metric that influences two fields is required for them to be correlated. It is sufficient that the fields' equations of motion are written with respect to a shared, *non-dynamical* background. This situation is illustrated in Figure 4.3.

For example, as discussed already in §3.2, in SR, where the Minkowski metric η_{ab} is *not even dynamical*, if $\langle \mathcal{M}, \eta_{ab}; \Theta, \Psi \rangle$ constitutes a DPM, then transformations like $(\eta_{ab} \to [\Lambda^* \eta]_{ab}, \Theta \to \Lambda^* \Theta, \Psi)$ fail to preserve this status under the dynamical symmetry transformation $g_D = \Lambda \in G_D = \text{Poin}(\mathbb{R}^4)$. This reveals that (Θ, Ψ) are *indirectly correlated* due to their being *directly correlated* with the same field — the Minkowski metric. No 'common

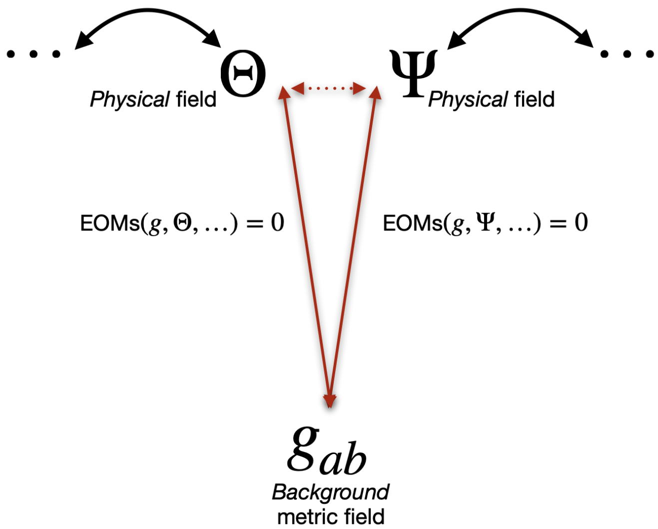

FIGURE 4.3: As in Figure 4.1, direct *correlation* is represented by straight and solid red arrows; indirect *correlation* by straight and dotted red arrows.

cause', understood in terms of influence, is needed.

This yields the central structural insight:

> Correlation between fields is preserved so long as their equations of motion share a background — even in the absence of causal influence.

To conclude, let me consider also the limiting case in which there is *neither influence nor correlation between fields*. This would be represented by a diagram in which the fields appear 'without any connecting arrows': they are dynamically uncoupled.

But such a situation is highly exceptional. In any realistic spacetime theory — general-relativistic or otherwise — the equations of motion of physical fields are expressed with respect to a shared background metric, whether fixed or dynamical. As shown in §3.2, *direct correlation* between a field and the metric is lost only if we disregard the dynamical equations altogether. Once direct correlation with the 'common metric' is lost, so too is any indirect correlation between fields *via* that metric. Thus, correlation is a structural feature of how physical theories represent the spatiotemporal dynamics governing fields, rather than a mere byproduct of direct interactions.

4.6 Conclusion

In this chapter, I developed a refined framework for understanding the ambiguous and multifaceted use of the term *dynamical coupling* in both theoretical physics and philosophical contexts. I extended and deepened Lehmkuhl (2011)'s work by proposing a systematic methodology for a fine-grained analysis of the notions of *influence* and *correlation*, as well as their direct, indirect, symmetrical and non-symmetrical manifestations.

To this purpose, in Section 4.3, I established a clear distinction between physical and dynamical fields. A field is classified as physical *iff* it is dynamical and adheres to the action-reaction principle by both exerting and experiencing influence. By contrast, a field is considered only dynamical *iff* its solution space is *non-trivial*, that is if and only if the field is a dynamical solution that is not linked to *all* other dynamical solutions via dynamical symmetries which are *non-trivial* automorphisms of the spatiotemporal structure. This approach rectifies limitations in earlier definitions of dynamical fields, which were often restricted to their dynamics being derived from Hamilton's principle.

By reinterpreting coupling in terms of *influence* in Section 4.4, I demonstrated how fields influence each other, either directly or indirectly *via* a 'common cause'. I also introduced the possibility of both symmetrical and non-symmetrical influence. In a general-relativistic theory, since the metric field universally influences *all* fields, which in turn influence the metric field by reaction, influence is both direct and symmetrical, exemplifying the action-reaction principle. However, by employing carefully constructed approximations inspired by approximated reference frames, such as neglecting the backreaction of certain fields (see Chapter 2), I showed how symmetry in these interactions can be broken. This led to new insights into the concept of non-symmetrical and indirect influence between fields, mediated by the shared gravitational field, acting as a 'common cause'.

In Section 4.5, I shifted the focus to *correlation*, which I demonstrated to be a broader concept than influence. While influence necessarily implies correlation, the converse does not hold, as fields may be indirectly correlated through a common metric background. Importantly, in support of the discussion of Chapter 3, I showed that correlation between fields is a concept valid also for theories possessing non-dynamical metrics, like Special Relativity, in alignment with the dynamical theory of spacetime. In particular, all fields directly couple with the metric of the relevant spatiotemporal theory. Finally, introducing the Cosmic Microwave Background, I presented a real example in nature showing how correlation can exist even in the absence of influence.

All these distinctions between direct, indirect, symmetrical, non-symmetrical influence and correlation were clarified with graphical representations that highlighted the structural relationships between fields under varying coupling scenarios.

The findings of this chapter establish a robust analytical framework for addressing the complexities of dynamical coupling. I have elucidated the interplay between influence and correlation, providing a more precise conceptual toolkit for analysing field dynamics and relationships. These insights not only deepen our understanding of coupling in GR and other spatiotemporal theories, but also open new avenues for exploring connections between this theme, the action-reaction principle, and the ongoing debate between geometrical and dynamical approaches to spacetime, which were carefully reviewed in §4.1, §4.2.

Chapter 5

Observability and Measurability: Rovelli Reconsidered

5.1 The Construction of Partial and Complete Observables

When articulating a physical theory through mathematics, a major concern is to identify the mathematical elements within the formalism that encapsulate the measurable aspects of the theory. Notably, Bergmann — inspired by Einstein's pioneering ideas —- connected this pragmatic concern with the search for observables (Giovanelli, 2024). He emphasised the following point:

> The equations of mathematical physics cease being mere mathematics to become honest physics only when one is able (a) to point to spatial quantities and expressions in the formalism

> and designate them as 'observable' and (b) to prescribe operational procedures by which such quantities may, in fact, be measured (observed), either by laboratory experiments or by astronomical measurements.[1] (Bergmann, 1957)

As explained in Chapter 2, the term observable not only has the operational meaning of a measurable quantity, but is also used to characterise quantities that can be *predicted* by theory. Observables in this sense are often called *Dirac* observables. Notably, the distinction between a theory's general variables and its Dirac observables becomes particularly crucial in systems with inherent mathematical redundancies, such as those featuring gauge freedom.

In Chapter 2 I already mentioned the proposal of Rovelli (2002b) to solve this challenge, which consists of distinguishing between two types of observables in GR: gauge-variant *partial* observables and gauge-invariant *complete* observables, corresponding to Dirac observables.

In particular, Rovelli argued that the physically significant content of GR lies not only in fully gauge-invariant quantities, but also in the relationships among gauge-dependent fields. This perspective emphasises the relational nature of observable quantities in GR, focusing on the dynamical correlations between variables, rather than their evolution against an abstract, unobservable manifold of points.

[1] However, Bergmann changed his view at various points. See Pitts (2022) for discussion.

Specifically, in Rovelli (2002b) a partial observable is defined as a coordinate-dependent physical quantity *associated with a measurement procedure*, while a complete observable is characterised by its theoretical predictability and gauge-invariant status, including the possibility of defining its probability distribution in quantum contexts.[2] Furthermore, in Rovelli's framework, *independent* partial observables are often associated with objects that have spatio-temporal localisation (it is to these quantities that I have assigned the role of frame of reference), while *dependent* partial observables are associated with field values defined with respect to those 'locators'. However, in GR, this distinction may become blurred because the theory does not inherently prioritise one set of variables as independent parameters. Instead, variables evolve relative to each other, and in general (and most common) cases—that is *non-deparameterisable scenarios*—this interdependence cannot be resolved into a simple function of one variable in terms of another. As Rovelli correctly observed:

The key difference between general relativistic physics and

[2]As also emphasised in Chapter 2 with respect to Einstein's operational way of understanding coordinates, Rovelli in his original definition stresses that partial observables should not be understood in the strict operationalist sense à la Bridgman. In particular, he notes ''The operational tone of the [partial observable] definition does not imply any adherence to operationalism here (Bridgman, 1927): the reference to measuring procedures is just instrumental for clarifying a distinction'' (*ibid*., 2). Thus, Rovelli in defining a partial observable, does not necessitate detailing the measurement process; rather, he is satisfied with merely its existence.

> pre-GR physics is the fact that in general relativistic physics the distinction between dependent and independent partial observables is lost. (Rovelli, 2002b, 3)

This conceptual shift implies that independent and dependent partial observables must be treated on *equal footing* within GR.

The mathematical formalism underpinning Rovelli's relational framework on observability was significantly elaborated by Dittrich (2006, 2007), who clarified the construction and implications of partial and complete observables in theories that are constrained with *infinitely* many constraints, such as field theories like GR. For comprehensive discussions, see Thiemann (2007) and Tambornino (2012). Extensions of this approach, including insights from Gambini and Porto (2001); Gambini et al. (2009) and Bojowald et al. (2011a,b), have broadened its applicability to quantum contexts. For an overview of these debates, see Anderson et al. (2014) and Anderson (2017). A philosophical analysis exploring the implications of partial and complete observables is provided by Rickles (2007, 161-171).

To apply the machinery of partial and complete observables, for each Hamiltonian constraint one selects one phase space function — corresponding to a partial observable — to play the role of an '*internal clock*'. Then, one defines expressions for both clock and non-clock variables in terms of the parameter defined along the Lie flow of the vector field generated by the constraint associated with the internal clock. Next, the clock variable's flow equation is inverted and substituted into the flow equations of a non-clock variable, producing a *parameter-free* algebraic description of their correlation, calculated at a particular value of the clock variable. The resulting quantity constitutes a *complete observable*.

For completeness, I will give below the relevant passages of such construction, though omitting the whole formal construction. For the rigorous treatment, see Dittrich (2007, 1895-6).

Consider a finite-dimensional mechanical system with a single Hamiltonian constraint $C(x)$ to which we associate the clock phase function $T(x)$. Consider a function $f(x)$ as the non-clock variable. Here, $x \in \Gamma = \mathbb{R}^{\mathbb{N}} \times \mathbb{R}^{N}$ denotes a point of the $2N-$dimensional phase space Γ.

In the following, I explain the standard way in which a first-class constraint in a Hamiltonian (or symplectic) system 'generates' a one-parameter family of canonical transformations, i.e. a gauge flow $\alpha^t_C(x)$ with t the gauge parameter of the flow, and how any phase-space function (a 'partial observable') evolves under that flow.

A first-class constraint $C(x)$ defines a Hamiltonian vector field $X_C(\varphi) = \{\varphi, C\}$, for any phase space function φ. Here $\{\cdot,\cdot\}$ is the Poisson bracket. Integrating the vector field one obtains a one-parameter family of diffeomorphisms (canonical transformations) $\alpha^t_C : x \to \alpha^t_C(x)$. Once we have the gauge flow $\alpha^t_C(x)$ on each points $x \in \Gamma$, it automatically induces an action on phase space functions by *pull-back* of the function φ along the gauge flow: $\left[(\alpha^t_C)^{-1}\right]^* \varphi\ (x) = \varphi\big(\alpha^t_C(x)\big) = \alpha^t_C(f)(x)$. Following this procedure, one constructs parametrised flow expressions for the evolution of the partial observables $f(x)$ and $T(x)$ under the gauge flow, obtaining:

$$\begin{cases} f(t;x) := f(\alpha_C^t(x)) = \alpha_C^t(f)(x) \\ T(t;x) := T(\alpha_C^t(x)) = \alpha_C^t(T)(x). \end{cases} \tag{5.1}$$

Geometrically, α_C^t drags the point x along the gauge orbit generated by C by 'amount' t. Then, one reads off the values of f and T at the dragged-point. A complete observable is defined as a function:[3]

$$F_{[f,T]}(s;x) := \alpha_C^t(f)(x)\Big|_{\alpha_C^t(T)(x)=s} \tag{5.2}$$

and expresses the value that the partial observable $f(t;x)$ assumes if the partial observable $T(t;x)$ assumes the value $T \equiv s \in \mathbb{R}$.[4]

Notably (see Dittrich, 2007, §3), in order to implement such a construction the clock variable $T(t;x)$ has to be at least *locally invertible*. This example also clarifies what was already presented in §2.2.2 within the Lagrangian

[3]To be rigorous, it is a whole family of complete observables parametrised by t.

[4]As a concrete example one can consider a free parametrised particle in 1D, where the phase space variables (q, t) represent the two partial observables. For explicit calculations see Henneaux and Teitelboim (1994, ch.4). For a qualitative example, one can think of f as the position of a pendulum and T as the time measured by a clock. The complete observable (4.2) represents the position of the pendulum at a specific time (see Rovelli, 2002b).

framework, where I introduced the technique of deparametrisation by inverting the clock variable.[5]

Notice that this complete observable F obtained with the Rovelli-Dittrich methodology is a Dirac observable, since for any specification of s we have $\{C, F(s)\} = 0$.[6] This result is summarised in **Theorem 3.1** in Dittrich (2007), which reads as follows (in my notation):

Theorem 5.1.1 (Theorem 3.1) *Let f, T be two phase space functions and $x \in \Gamma$ a phase space point, fulfilling the condition: $\alpha_C^t(f)(x) = \alpha_C^\tau(f)(x)$ for all values $t, \tau \in \mathbb{R}$ for which $\alpha_C^t(T)(x) = \alpha_C^\tau(T)(x)$. Then $F_{[f,T]}(s; x)$ is invariant under the flow generated by C [that is: is gauge-invariant].*

In particular, the condition that the function $f(t; x) = f(\tau; x)$ for all $t, \tau \in \mathbb{R}$ for which $T(t; x) = T(\tau; x)$ (where τ is a gauge parameter) is a *sufficient* condition for $T(t; x)$ to be *locally* invertible.

[5]It is evident that the term 'parameter' here refers to the variable t that parametrises the gauge flow generated by the Hamiltonian constraint C.

[6]Committing a slight abuse of notation, but gaining in understanding in relation to how partial observables are used as reference frames, we can also redefine the complete observable as $f(T)|_{T=s} : \Gamma \to \mathbb{R}$ and $\{C, f(T)|_{T=s}\} = 0$. This isn't strictly precise in this gauge-theoretic formalism, since a complete observable is a function constructed by flowing f along the gauge orbits parametrised by t until $T = s$.

Although the formal development of Dittrich-Rovelli's observability proposal is widely recognised, consensus on the differentiation between partial and complete observables, as well as their corresponding interpretations, remains elusive.

As noted above, while Rovelli interpreted partial observables as "physical quantities associated with a measurement procedure leading to a number" (*ibid.*, 2), Thiemann (2007) argued that "a measurable quantity is always a complete observable, even pointers of a clock are observables and not partial observables. Now complete observables are defined with respect to non-measurable quantities...which we will simply call non-observables" (*ibid.*, 78), in effect rejecting the distinction between partial and complete observables. In the same vein, Adlam (2024a) has recently claimed that ''[. . .] we must somehow measure the complete relational observable all together.'' (*ibid.*, 8). From the 'Thiemann-Adlam perspective', therefore, when we say 'observable', we always mean 'Dirac observable'; there are no observables that are measurable but not predictable.

This chapter will delve into the significance of partial observables in shaping the notion of measurement and provide a critical analysis and intepretation of their formal definition. Furthermore, it aims to elucidate the connection between partial observables and complete observables, the latter being interpreted as correlations among partial observables.

In particular, in §5.2 I will describe a degenerate case, in which if we want to use **URFs** as partial observables, we reintroduce a kind of indeterminism in the theory, thus precluding the possibility of defining complete gauge-invariant observables via a correlation of partial observables. This case can be seen as exploiting a *lacuna* in the definition of partial and complete observables.

Section 5.3 will focus on the notion of measurement. I will argue how, *qua* measurable quantities, partial observables should be formally defined as relational quantities and not as functions of some coordinates. This will also allow me to disentangle the (often identified) notions of relationalism and gauge-invariance.

Finally, in §5.4, I will propose a purely operational interpretation of the so-called 'point-coincidence argument' (see Stachel, 1989; Giovanelli, 2021), introduced by Einstein (1916b).

5.2 Observability Reconsidered

The conventional approach to defining partial observables usually involves associating them with a measurement outcome, thereby making them 'observables' in the standard, theoretically deflated sense.

However, existing literature rarely emphasise that partial observables *need* to be *dynamically coupled* in order for their relation to form a complete observable. Rovelli, in his examination of the significance of coordinates in GR, identifies what I categorise as **IRFs**, as partial observables. He says:

> The coordinates are partial observables in (i) physical coordinates, interpreted as positions relative to entities whose equations of motion are considered, and (ii) physical coordinates, seen as positions relative to entities *whose equations of motion are disregarded* [my emphasis]. (Rovelli, 2004, 63)

In this section, I contend, however, that the assertion (ii) is inaccurate. For a set of quantities to be *bona-fide* partial observables, they must be dynamically coupled. Only in this way their coupling form a complete observable. In Adlam (2024a)'s terms: ''partial observables are simply how complete observables look 'from the inside,' so to speak.''

This description of partial observables emphasises that it is not enough to have any two measurable quantities, implement the inversion procedure, and construct some relationship between them to obtain a complete observable as a result. The quantities we take as partial observables must already be dynamically coupled. So, in a certain sense (the same as Adlam), the procedure is *top-down*: we obtain two partial observables because we 'unpack' a complete observable. This also clarifies the common misconception that there is no distinction between measurable quantities and partial observables and therefore partial observables are *by definition* measurable quantities: while a partial observable is always a measurable quantity, the reverse is not true.

Before proceeding, I would like to point out immediately that whenever I mention the term dynamically coupled in this chapter, I will do so in the sense of *correlation* §4.5.

From the above, it is clear that the characterisation of partial observables goes hand in hand with that of complete observables: *bona-fide* partial observables return *bona-fide* complete observables (and *vice versa*).

Let me start the analysis using again the now familiar case where we have a metric field g_{ab} satisfying EFEs, along with four scalar fields $\{\phi^{(I)}\}$, each following distinct Klein-Gordon equations with distinct initial conditions. These scalar fields are used as a reference frame, so each of them provides a local mapping $U \to \mathbb{R}^4$ for some region $U \subset \mathcal{M}$. If the manifold $\mathcal{M}$ is diffeomorphic to $\{R\}^4$, we can take $U = \mathcal{M}$. In this case, global invertibility,

$$\det\left(\frac{\partial \phi^I}{\partial x^\mu}\right) \neq 0 \quad \text{everywhere on } \mathcal{M}$$

ensures that each spacetime point is uniquely labeled by the values of the scalar fields. Although this assumption is not generic—frame patches in realistic situations typically cover only part of the manifold—it greatly simplifies the construction of complete observables. Without invertibility, the fields could, for instance, be constant everywhere, mapping all of $\mathcal{M}$ to a single point in $\mathbb{R}^4$ and obstructing the definition of the complete observable $g_{IJ}(\phi)$, here termed a *gauge-invariant, relational observable*.

In this section, it will become clear how the specification of distinction between the terms 'gauge-invariant' and 'relational' is crucial. In fact, as I will argue below, it does not immediately follow from their definition that a *relational* observable is a complete, *gauge-invariant* observable.

For a quantity to qualify as a *bona-fide* complete observable, it must meet the following two conditions (they are also sufficient conditions):

(DI) It must remain invariant under active diffeomorphisms, which 'move manifold points around', making it a *relational*, *diffeomorphism-invariant* (**DI**) quantity.

(DET) Its evolution over time must be deterministic.

These criteria define what I call the '*Gauge-Invariance*' **(GI)** property:

$$\textbf{(GI)} \leftrightarrow [\textbf{(DI)} \wedge \textbf{(DET)}], \tag{5.3}$$

where $\wedge$ denotes the logical conjunction, indicating that **(GI)** is true only when *both* **(DI)** and **(DET)** are satisfied.

Depending on whether the set $\{\phi^{(I)}\}$ constitutes an **URF** or a **CRF**, one can determine if $g_{IJ}(\phi)$ is a **GI** observable or not. This assessment can be guided by examining initial conditions and determinism in the equations of motion. [7]

In the context of **URFs**, *any* $\phi^{(I)}$ (with its Klein-Gordon dynamics disregarded) is compatible with *all* metrics, including those that are *isometrically*

[7] **(DI)** is inherently satisfied for $g_{IJ}(\phi)$ with *any* frame field.

equivalent, implying that both $\langle \mathcal{M}, g_{ab}, \phi^{(I)} \rangle$ and $\langle \mathcal{M}, [d^* g]_{ab}, \phi^{(I)} \rangle$ represent DPMs for *any* $d \in \text{Diff}(\mathcal{M})$.[8]

It's crucial to highlight that the term '*isometry*' here is used to signify the induced *isomorphism* on fields through the pullback $[\bullet]^*$ of the diffeomorphisms.[9] This way of using the term isometry contrasts with its other possible meaning of *automorphism*: a transformation that leaves the metric invariant. For example, in many physical contexts (e.g. in Landsman (2021, 71), but also Wald, 1984), an isometry strictly refers to a diffeomorphism d such that $(d^* g)_{ab} = g_{ab}$, effectively equating isometries with flows of Killing fields (Landsman, 2021, 57). This distinction reflects two possible different understandings of the term isometry. In the first case one means *iso-(geo)metry*: 'same geometry', where a geometry is an equivalence class of diff-related metrics. In the second case *iso-metry*: 'same metric'. Finally, as can be inferred from Weatherall (2018) and Cudek (2024), the word 'isometry' actually has a third meaning, in addition to the two I have considered. Isometry can also be a(n isomoprhism induced by a) diffeomorphism $d : \mathcal{M} \to \mathcal{M}'$ that preserves the metric in the sense of sending $g_{ab} \in \mathcal{M} \to g'_{ab} \in \mathcal{M}'$, such that $g'_{ab} = (d^* g)_{ab}$. Thus it differs from both the standard automorphism (iso-metry) and the diffeomorphism-induced isomorphism which sends $g_{ab} \in \mathcal{M} \to (d^* g)_{ab} \in \mathcal{M}$ (iso-geometry).

[8]The same goes for $\langle \mathcal{M}, g_{ab}, d^* \phi^{(I)} \rangle$. Notice that d in such a case is what I defined in §3.2 as a *generalised dynamical symmetry*.

[9]I owe this nomenclature to Henrique Gomes.

I now set aside this etymological digression and return to the main point.

The reason why it is not sufficient to say that $g_{IJ}(\phi)$ is relationally constructed for it to be a complete observable is that both $\left[(\phi^{(I)})^{-1}\right]^* g_{ab}$ and $\left[(\phi^{(I)})^{-1}\right]^* [d^*g]_{ab}$ are quantities thus constructed, hence they are **DI**. Given some initial data, we have *no reason* to prefer one or the other, so the condition for determinism **(DET)** is not satisfied, leading to the same level of redundancy that existed before selecting a reference frame, having still an action of $Diff(\mathcal{M})$ in the chamber. Consequently, if the set $\{\phi^{(I)}\}$ compose an **URF**, they cannot be considered as partial observables, as they cannot form a *unique*, **GI** complete observable, which is **DET**, according to relation (5.3).

In the scenario involving **CRFs**, as already argued in §2.2.2, if the couple $(g_{ab}, \phi^{(I)})$ is dynamically allowed, then typically $([d^*g]_{ab}, \phi^{(I)})$ will *not* be for a generic $d \in \text{Diff}(\mathcal{M})$. Consequently, the observables $g_{IJ}(\phi)$ are *uniquely* defined. In this situation, *given a choice of* $\phi^{(I)}$, a particular choice of g_{ab} (instead of any of its isomorphic alternatives) alongside initial conditions *uniquely* determines $g_{IJ}(\phi)$. Hence, $\phi^{(I)}$ and g_{ab} qualify as valid partial observables since $\left[\phi^{(I)})^{-1}\right]^* g_{ab}$ is relational (**DI**) and only the *diagonal* subgroup $\text{Diag}(\text{Diff}(\mathcal{M}))$ preserves the solutionhood.[10] Thus, for any given initial data, $\phi^{(I)}$ effectively fixes the gauge for g_{IJ}, ensuring the determinism

[10]Notice that d is what I defined in §3.2 as a *standard dynamical symmetry*.

criterion **(DET)** is also met.[11]

The arguments just presented not only shed light on the definition of both partial and complete observability, but are also capable of providing innovative connections between between relationalism, gauge-invariance, and determinism — as expressed in the relation (5.3) —- that to the best of my knowledge have never been explicitly presented in the literature.

For completeness, I summarise all the logical relationships between these three concepts in the following table.

[11] With the aim of clarifying the discussion the reader may find in Dittrich (2007, 1914), I point out that complete observables and gauge-fixed observables *are* equivalent, as selecting a reference frame can be interpreted as making a gauge choice (in Gomes (2024a)'s terminology this consists in choosing a section of the principal fiber bundle of models of the theory). At the very least, it is always the case that a gauge-fixed observable qualifies as a complete observable, while only the reverse is less immediately obvious.

	Relations
(L1)	Relational $\equiv$ **(DI)**
(L2)	**(DI)** $\nleftrightarrow$ **(DET)**
(L3)	**(GI)** $\rightarrow$ **(DET)**
	(GI) $\nleftarrow$ **(DET)**
(L4)	**(GI)** $\rightarrow$ **(DI)**
	(GI) $\nleftarrow$ **(DI)**

TABLE 5.1: Relations between the concepts of **(DI)**, **(GI)**, **(DET)**.

Before concluding, note that I have only argued one direction of the implication—namely, that diffeomorphism invariance **(DI)** of a quantity does not imply deterministic evolution **(DET)** for that quantity.

Let me now provide a simple example illustrating why the converse does not hold: that is, why deterministic evolution does not imply diffeomorphism invariance. I will consider the case of *time-independent diffeomorphisms*. Specifically, I will explain why the deterministic evolution (**DET**) of a quantity—such as the metric—does not entail invariance of that quantity under spatial diffeomorphisms (i.e. **DI**).

Consider the 3 + 1 ADM Hamiltonian formulation of GR, where $h_{ij}(t, x)$ is a Riemannian 3-metric induced on Σ_t by a generic solution of the EFEs. In any field theory whose equations of motion are second-order in time—including Einstein's equations—specifying only the 'initial position' (in this case, the 3-metric $h_{ij}(0, x)$) is insufficient for a unique evolution. One also needs the 'initial velocity', roughly the time derivative $\dot{h}_{ij}(0, x)$, which in the ADM formalism is encoded in the *extrinsic curvature* K_{ij}. Geometrically, K_{ij} measures how a spatial slice Σ_t is embedded in the full spacetime.

Given initial data $(h_{ij}(0, x), K_{ij}(0, x))$, **DET** simply means that there exists a *unique* future evolution $h_{ij}(t, x)$ consistent with this initial data.

Now, consider any time-*independent* spatial diffeomorphism f such that $h_{ij}(t, x) \mapsto h'_{ij}(t, x) := h_{ij}(t, f(x))$. The fields h and h' will generally correspond to different initial data—$(h_{ij}(0, x), K_{ij}(0, x))$ and $(h'_{ij}(0, x), K'_{ij}(0, x))$ respectively—defined on *the same* initial slice Σ_0. Both h and h' evolve *uniquely* from its own initial data (**DET** is satisfied), but h' is not the same field as h, hence **DI** is not satisfied.

By contrast, time-*dependent* diffeomorphisms introduce genuine gauge freedom into the evolution, rendering the evolution of the field $h_{ij}(t, x)$ non-unique for some given initial data. Such diffeomorphisms can be constructed to act *trivially* at $t = 0$, yet their generators appear in the canonical Hamiltonian, multiplied by arbitrary functions of time—the lapse function and the shift vector, also known as Lagrange multipliers. Because these multipliers are freely specifiable as functions of time, the evolution of the diff-variant variables $h_{ij}(t, x)$ becomes non-unique: different choices of the multipliers 'push' $h_{ij}(t, x)$ along the same gauge orbit in different ways. This non-uniqueness is the hallmark of gauge redundancy and the source of apparent gauge indeterminism and the failure of **DET**.

Such failure does not arise from time-*independent* diffeomorphisms— since such transformations cannot be constructed to act trivially at $t = 0$, unless the diffeomorphism is the identity. Wallace (2002) refers to these as *global (in time) symmetries* because they act uniformly across the entire history: if they leave the initial data unchanged, they must preserve the entire solution, thereby introducing no 'gauge-indeterminism'. As Wallace notes, such symmetries merely relabel points on each slice and do not generate any new gauge freedom in the time evolution. Consequently, they do not violate **DET** and always map each hypersurface onto itself.

To summarise in one sentence: only time-*dependent* diffeomorphisms give rise to the familiar gauge indeterminism in the evolution of $h_{ij}(t, x)$. Purely spatial (time-*independent*) relabellings leave the metric's evolution uniquely determined once the initial data are fixed.

5.3 Measurability reconsidered

A partial observable, as defined by Rovelli (2002b), represents a measurable and gauge-variant quantity expressed in a specific coordinate system

$\{x^\mu\} \in \mathbb{R}^4$. For coherence with earlier discussions, an example could be the metric field $g_{\mu\nu}(x^\rho)$. However, coordinates are often treated as mere mathematical, extra-empirical constructs, devoid of physical realisation. This raises the question: how can partial observables be empirically significant?

Empirical comparisons between theory and experiment inherently require a *physically instantiated* coordinate system. Since empirical data are often understood as relational,[12] I propose to refine the definition of a partial observable as a relational quantity $g_{IJ}(\phi)$, where $\{\phi^{(I)}\}$ comprises four scalar degrees of freedom that may constitute an **IRF**. Thus, measurement outcomes are formalised by *relational partial observables* which are no longer tied to *uninstantiated* coordinates but to a(n idealised) reference frame $\{\phi^{(I)}\}$.

Agreed: **IRFs** serve as convenient approximations of a real measurement. Strictly speaking, only complete observables—relational quantities expressed relative to a **CRF**—are empirically accessible. Real physical measurements concern dynamically coupled quantities, given the universality of gravitational interaction (§4). While **IRFs** are useful for constructing

[12]The relational nature of empirical measurements is supported e.g. by the "Unobservability Thesis," articulated as follows:

The Unobservability Thesis: Given a family of models of a system which are related by a symmetry transformation, it is impossible to determine empirically which model in fact represents the system. (Wallace, 2022c, 327-8) relational partial observables modelling approximated measurement outcomes, **CRFs** should rigorously be used to represent real measurements.

Notably, Thiemann (2007) sustained the view that all measurable quantities are (gauge-invariant) complete observables, arguing: ''a measurable quantity is always a complete observable, even pointers of a clock are observables and not partial observables. Now complete observables are defined with respect to non-measurable quantities...which we will simply call non-observables'' (*ibid*., 78).[13] Recently, Adlam (2024a) supported Thiemann's stance, arguing: ''we surely cannot measure complete relational observables by measuring two partial observables separately and then comparing their values; we must somehow measure the complete relational observable all together [. . .] So a natural way to think about Rovelli's proposal is to say that an observer is able to observe partial observables relativized to her body or location, because she is thereby measuring a complete relational variable pertaining to the relation between her and the entity she is measuring.'' (*ibid*., 8-9).

Nevertheless, in contrast to the 'Thiemann-Adlam approach,' I uphold Rovelli's perspective that partial observables are indeed measurable. However, this assertion relies on the redefining of partial observables as relational gauge-variant quantities framed within an **IRF**.

[13] Pitts (2022) attributes the paternity of the notion of 'observable' as a quantity related to physical measurements mainly to Peter Bergmann.

Reconciling (i) the position that we *do* measure relational partial observables and (ii) their definition in section 5.2 as 'inseparable, coupled components' of a complete observable requires careful deliberation.[14]

To support this argument, I argue that while actual measurements inherently involve complete observables, theoretical models of these (local) outcomes are often approximated (Elgin, 2017; Frigg and Nguyen, 2021) and frequently employ relational, non-gauge-invariant quantities. These are partial observables.[15]

Consider, for example, the (indirect and model-based (Tal, 2013, 2016) measurement of $g_{IJ}(\phi)$. In modelling such measurements, the reference frame ϕ is often approximated as an **IRF**, allowing the interpretation of $g_{IJ}(\phi)$ as a relational partial observable. However, this approximation must later be refined, reinterpreting ϕ as a **CRF**. This refinement restores a

[14]The relationship between partial observables and their gauge-invariant relation forming a complete observable must be understood *holistically*: the resulting complete observable cannot be decomposed into separate elements.

[15]Actually, in this thesis I am using the distinction between approximations and idealisations brought up in Norton (2012). These meanings take on a different role in Frigg (2022) where it would be more correct to speak of *idealised models*. The substantive point does not change: one models target systems by intentionally 'distorting' some of their properties in order to give them an easier representation to study.

faithful depiction of reality,[16] aligning the measured $g_{IJ}(\phi)$ with a complete observable and theoretical predictions.

This two-step process—initial approximate theoretical modelling by approximating reference frames to **URFs**, followed by theoretical de-approximation of these reference frames in order to return to interpreting them as **CRFs**—reconciles the apparent tension between (i) and (ii). This shows that although the *actually* measured $g_{ab}(\phi)$ is strictly speaking a complete observable since g_{ab} and ϕ are dynamically coupled, being ϕ a **CRF**, $g_{ab}(\phi)$ can be also interpreted as a relational partial observable when modelling the measurement results.

This interplay between actual measurement, theoretical modelling of measurements and theoretical interpretation establishes consistency between the inevitable dynamical coupling between fields and the distinction between relational partial observables and gauge-invariant complete observables.[17]

[16]To be precise, one always has a description to a certain approximation. A useful mantra to keep in mind (which I heard during a talk by Erik Curiel and uttered by John Norton) is:“there is more in mathematics than in the world, and viceversa!”

[17]To be extremely clear, I am portraying physical practice as consisting of three stages: 1) actual measurements as interactions between coupled physical systems: data collection. 2) theoretical modelling of these measurements in which approximations occur. 3) comparison between collected data and predictions, in which the approximations of step 2 must be recognised and eliminated.

In the next section, I will use this theoretical apparatus to draw a lesson from what has become the cornerstone of the relationalist view in GR, distinguishing it from pre-GR theories: the 'point-coincidence argument'. In particular, I will highlight how the moral of this argument is *purely operational* (see fn. 2), with no implications for observability in the Dirac sense of the term.

5.4 Point-Coincidence Argument: An Operational Perspective

This section delves into two possible interpretations of the famous 'point-coincidence argument' (Einstein, 1916b; Stachel, 1989). The argument can be summarised in the often-quoted remark that:

> If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings [coincidences] of two or more of these points. (Einstein, 1916b)

Through an analysis of Einstein quotes often used in the literature on the subject, I will challenge the common (tacit) assumption that a "*coincidence*" is represented by a "*gauge-invariant*" quantity in the full sense of **GI** that I have described above. Instead, I propose an alternative interpretation, emphasising the relational aspect of measurement in physics and the difference between the concepts of **(DI)** and **(GI)** clarified in §5.2.

It is widely accepted that, as noted by Anderson (1967):

> All measurements are comparisons between different physical systems. (*ibid.*, 128)

Similar views are expressed by Landau and Lifshitz (1987, 1), who stress that every physical measurement requires a reference frame (see also Rovelli, 1991, 298).

Einstein's original formulation of the point-coincidence argument can be reconstructed according to this operational framework. There is a widespread agreement that Einstein argued that the true physical content of a theory resides in the spacetime coincidences of material points. To support this thesis, the following quotations are often given (see Giovanelli, 2021):

> All our spacetime *verifications* invariably amount to a determination of spacetime coincidences [my italics]. (Einstein, 1916b)

and:

> Physical *experiences* [are] always assessments of point coincidences [my italics]. (Einstein, 1919)

On closer inspection, it is evident that Einstein names *verifications* and *experience*, two concepts that do not directly relate to theoretical *predictions*.

My argument builds on the following arguments: (a) only complete, gauge-invariant observables are associated with Dirac observables and hence with predictions. Partial observables, on the other hand, are only associated with measurements; (b) as discussed above in §5.3, physical *measurements* inherently rely on *relational* quantities; (c) relational quantities do not necessarily guarantee deterministic evolution, which is essential for defining complete observables, but not for partial observables.

My conclusion, which I will explore below, is that the point-coincidence argument concerns relational partial observables.

To support my thesis, I compare my position with that of three authors: Rovelli, Westman and Sonego, and Giovanelli.

Rovelli. In line with the 'received-view on coincidences', Rovelli interprets the point-coincidence argument not only associating observability with measurability, but he also further connects this to Dirac's notion of observability. He thereby equates relationalism with gauge-invariance, supporting the identification of coincidences with *complete observables*. For instance, he asserts:

> The GPS observables [which are complete ones] are [...] *precisely* Einstein's "spacetime coincidences." (Rovelli, 2004)

In contrast, I maintain a stricter interpretation of coincidences, focusing purely on measurability. My position is consistent to the decoupling of the notions of relationalism (**DI**) and gauge-invariance (**GI**) to a degree not addressed in Rovelli's work. However, in agreement with Rovelli, I affirm the connection between (approximated) empirical *verification* and relationalism. What I dispute is the assumption that relationalism necessarily implies gauge-invariance. This implication presupposes that all measurable quantities are dynamically coupled — a defensible but often dropped assumption in the context of reference frames. Thus, GPS observables, which are Dirac observables, are *not precisely* spacetime coincidences in the sense that coincidences are by definition **GI** quantities. It takes less to be a spatial coincidence than to be a Dirac observable: it is sufficient to be **DI**.

Westman and Sonego. The conflation of Einsteinian point-coincidences with gauge-invariant quantities and not merely with (relational) measurements is also found in Westman and Sonego (2009). I structure my criticism to the authors' position into three main critiques.

First: Events.

The notion of 'event' can be taken as a primitive notion that corresponds to some kind of 'happening' — for example, a collision of particles. Westman and Sonego (2009) state that:

> [...] the space of point-coincidences $\mathcal{E}$ contains all local gauge-independent data and represents physical events. (*ibid.*, 28)
>
> It is obvious that $\mathcal{E}$ is invariant under diffeomorphisms of $\mathcal{M}$, *so it contains all the local gauge-independent data*, and its elements represent physical events [my italics]. (*ibid.*, 26)

Based on my distinction between relationalism (**DI**) and gauge-invariance (**GI**), I propose instead to distinguish the notion of event from that of coincidence. While a coincidence is **DI**, but it is *not* observable in Dirac's sense, any *event* occurring at a spacetime point is uniquely defined through the choice of a reference frame and, being **GI**, is represented by Dirac observables.

Consequently, I interpret the space $\mathcal{E}$ of coincidences as the space of all relational measurements, and not as ''the space of all gauge-independent data''.

Second: Defining Point-Coincidences.

> [$\cdots$] in order to identify uniquely a point-coincidence [$\cdots$] locally the values of *four* different quantities are enough [...] [my italics]. (*ibid.*, 25)

I defined a spatiotemporal coincidence as a *relational* quantity, such as $g_{IJ}(\phi)$, where ϕ is (at least) an **IRF**.

Thus, according to my definition a spatiotemporal coincidence is at least a '*five-party relationship*': i.e. a relational (not necessarily gauge-invariant) expression of some quantity, e.g., the Ricci scalar of a metric, at a certain point descriptively designated by the values of the four *linearly independent* scalar components of a reference frame.

This implies that the choice of a reference frame, consisting of four *independent* scalars, does not inherently constitute a realisation of Einstein's point-coincidences, nor is it a realisation of an event which, *according to my definition*, is a deterministic coincidence, so to speak.

My stance is in line with the work of Einstein's assistant, Bergmann, who brought the language of coincidences to the fore again in the 1960's , which are intended as quantities construed from *at least five scalars* (Bergmann and Komar, 1962, 315-316).[18]

Third Point: Ontology.

Once I have clarified my position on what a coincidence is and on the distinction between relational (**DI**) coincidence and gauge-invariant (**GI**) event, I also disagree with the authors' conclusion on the ontology of spacetime.

Westman and Sonego (2009, sec.6) identify spacetime with the space $\mathcal{E}$ of point-coincidences, understanding it as the space of **GI** data. I agree with the conclusion, but not with the way they got there.

I agree that spacetime is the space of **GI** data, but I understand this as the *space of events*, that is as the space of all the "five-party relation" of the kind $g_{IJ}(\phi)$ defined by *measurable and predictable* configurations of the physical and geometrical fields. Thus, spacetime is not a container for the configurations of fields and particles.

[18]However, Bergmann's view differs from mine and agrees with that of Rovelli and Westman and Sonego in defining a *coincidence* as a gauge-invariant quantity and thus assimilating it to the notion of *event*.

This conclusion accords with the famous remarks by Einstein et al. (2015) (see also Einstein's contribution in Jammer, 1954):

> There is no such thing as an empty space, i.e., a space without field. Space-time does not claim existence on its own, but only as a structural property of the field.

In particular, it makes no sense to think of a region of spacetime, understood as the space of events, where there are no fields at all. That region would simply not exist at all.

The resulting notion of space-time is that of an *emergent* structure defined in terms of what is observable in the Dirac's sense. Such observables, or *happenings*, or *events*, define the "when and the where" of happenings themselves: there are no happenings at a pre-existing when and where.

Importantly, this characterisation of spacetime in terms of events does not contradict the standard *mathematical* formulation of physical fields on a manifold $\mathcal{M}$. What is contradicted is only the assumption of identifying *physical* spacetime with some geometric structure $\langle \mathcal{M} \rangle$ or $\langle \mathcal{M}, g_{ab} \rangle$ which are not definable as **GI** structures. If we want to give a physical, i.e. **GI**, characterisation of spacetime, my (relational) proposal of a 'space of events' has its natural place.

Giovanelli. Giovanelli (2021) owes much to the work of Westman and Sonego. In his historical reconstruction of the point-coincidence argument (which I will not present here), he interprets the mantra that encapsulates the point-coincidence argument: “nothing but coincidences are *observable* in GR (*ibid*., 4)”, assigning the concept of observability the sense of Dirac-observability.

For example, on page 44 he argues that:

> Physicists used to think that the goal of a physical theory was to *predict* the successive positions x, y, z, of, say, a material point with respect to a suitably chosen reference frame at time t measured by clocks placed at fixed positions on such scaffolding. At closer inspection, however, what they were actually able to *predict* was only the “the encounters [Begegnungen] of this [material] point with particular points” of a particular physical system that we have chosen as a “reference-body” [. . .] [my italics, where I highlight the use of the term *predict* associated with the notion of coincidence ([Begegnungen])]

In conclusion, I want to make it clear that the disagreement between this position and mine is not based on the fact that what the theory predicts are relations between fields. This is always true, since for the relation 5.3 **GI** requires **DI**.

What I criticise is that the theory’s predictions are made to coincide with Einstein’s spacetime coincidences. This is not necessary: theory predicts events, spacetime coincidences are instead what we *model* as measurements outcomes, where, as argued in §5.3, approximations are often used.

5.5 Summary

In this chapter, building on the classification of possible reference frames in GR as outlined in Chapter 2, I proposed two main distinct critiques of the concept of partial observables.

The first criticism concerns the conditions under which two quantities can be combined in a relational way to obtain a complete observable and thus qualify as partial observables. Identifying a *lacuna* in the literature, I showed that this is achievable only when the two quantities exhibit a dynamical coupling, such that one of them can serve as a **CRF**. Consequently, a complete observable is understood as a **(DI)** quantity characterised by deterministic evolution **(DET)**, while a partial observable is a **(DI)** quantity only. The attributes of **(DI)** and **(DET)** together define what was termed gauge-invariance **(GI)** (§5.2).

The second critique shifts focus to the empirical side and pertains to the very formal definition of partial observables. Since coordinates in GR lack any physical instantiation, I argued that partial observables must be defined *relationally*, as quantities localised using an **URF**. This analysis clarifies the distinction between relationalism — here referred to as **(DI)** —, and gauge-invariance **(GI)**, two concepts that are frequently conflated in the literature (§5.3).

These considerations led to an operational reinterpretation of Einstein's famous point-coincidence argument. Contrary to the standard view held by, e.g., Rovelli, Westman&Songego and Giovanelli, I argue that the observability of spacetime coincidences lies in their relational nature, rather than their status as gauge-invariant quantities. Relational coincidences, although not fully gauge-invariant in the Dirac sense, capture the essence of model-based measurement outcomes that often rely on approximations to the reference frames used to locate these measurements. This also suggested the distinction between the concepts of coincidence and event. *Events* are represented by complete observables and embody the deterministic, empirically testable aspects of a theory, while *coincidences* are the relational outcomes measured in experimental practice, which arise from approximations using **URFs**. This distinction allowed me to define spacetime as the set of *events*: i.e. as as the set of 'where and when' defined by the *happenings*, represented by complete observables (§5.4).

Chapter 6

Conclusion Of The Thesis

The conclusions of my work can only be in favour of recognising the fundamental role that reference frames have been shown to play in understanding GR. Reference frames are not just mathematical objects that can be conveniently used to describe physics, but they are a central element in the description of spatiotemporal structure, highlighting its intrinsically relational nature.

By systematically disentangling reference frames and coordinate systems, the work laid a solid foundation for addressing questions central not only to the foundations of GR, but also to physics in general:

- Why reference frames and not coordinate systems?
- Why do gauge symmetries permeate all known fundamental physics? Is this just a 'joke' of mathematics?
- What are the observable quantities of a theory, and how are they defined locally? What does observable mean?

- What does it mean that a quantity is measurable and what does it mean that it is predictable? Are there hidden assumptions in the modelling of measurement results?
- What notion of locality is available in GR, as background-independent and diff-invariant theory? And what is the space-time that defines the here and now in which measurements and predictions occur?
- Is the relationship between spacetime symmetries and dynamic symmetries based on any physical assumptions?
- What is the nature of the coupling between fields, which underlies the relational perspective for the dynamics of a theory?
- What does it mean to claim that the metric in GR is a physical field, while in special relativity it is not? And what is a dynamical field instead?

Throughout the chapters, I have demonstrated the power of reference frames in providing a conceptual framework to answer all these questions:

- Physics aims to give a mathematical representation of the world. In such a representation, the role of localisation is played by physical objects internal to the dynamic theory of reference and not by mathematical objects external to the physical world that is to be represented.

- Gauge symmetries are an expression of the relational nature of the dynamics of any physical system in which the coupled dynamical equations serve as gauge-fixing conditions.
- Observables, as invariant quantities under gauge transformations, are objects that can be measured and predicted by theory. They are locally well-defined once reference frames are used as 'spacetime locators', giving rise to relational observables that are gauge-invariant when the dynamics of the reference frames are coupled to that of the observable. Observing, therefore, can mean both measuring and predicting.
- When speaking of observing, in the sense of measuring, the role of reference frames clearly emerges, as only relational objects are objects of a local measurement, consistent with the fact that phenomenology cannot concern non-physical and non-instantiated entities. While it is true that in the act of measurement reference frames and observables have a coupled dynamic, in the modelling of measurement results this coupling is usually neglected.
- The notion of locality proper to local observables is thus inherently relational. The here and now is not given a priori, but dynamically established by the interaction between physical objects. It is their interaction that defines the here and now of the interaction event itself.

- The role of dynamical coupling between reference frame and observable is also central to the relationship between spacetime symmetries and dynamical symmetries, which constitute Earman's **SP1** principle. The acknowledgement of this centrality highlights a necessary assumption, often hidden and never made explicit, for the **SP1** principle: that all objects in the theory are dynamically coupled.
- Coupling can be understood as influence or correlation. *Influence* refers to a dynamical relationship in which one field acts on the dynamics of another (and vice versa), and is often mediated by Lagrangian interaction terms. *Correlation*, on the other hand, occurs when fields are constrained in such a way that a dynamical symmetry transformation acting on one field requires a corresponding transformation on the other fields to preserve the status of a physically possible model, even in the absence of direct dynamical influence. An example is the correlation between fields in Special Relativity due to the non- dynamical Minkowski metric.
- A *dynamical field* is a field whose space of solutions is non-trivial, i.e. its equations of motion admit solutions that are not all bound by non-trivial isomorphisms induced by symmetry transformations of the space-time structure. In GR the metric is considered a *physical field* because it is both dynamical and a field that interacts with all other fields (and itself) and is subject to their backreaction, thus satisfying the action-reaction principle. This does not apply to the Minkowski

metric in special relativity whose solution space consists of a single flat (Riemann-flat) geometry.

Beneath, a schematic synthesis of each chapter's primary contributions is provided, accompanied by a summary of their results and methodology.

Detailed Summary

Chapter 2: Distinction Between Reference Frames and Coordinate Systems in General Relativity

Section 2.1 reviewed how reference frames in GR are best understood not as mere coordinate labels but as physical systems endowed with four measurable degrees of freedom that map manifold points to $\mathbb{R}^4$. It contrasted the traditional view of Earman&Norton—where a reference frame is often taken as a congruence of timelike curves with an adapted coordinate system—with a more nuanced perspective that emphasises the dynamical coupling (or uncoupling) between the reference frame and the metric field. This relational approach underlined that while coordinates are abstract idealisations, true reference frames are instantiated by material systems, a distinction crucial for addressing gauge redundancies and constructing local observables in GR.

Section 2.2 introduced a novel systematic classification of reference frames in GR by dividing them into two main categories: uncoupled and coupled reference frames. Uncoupled reference frames (**URFs**) (§2.2.1) are subdivided into Idealised Reference Frames (**IRFs**), which neglect both the stress-energy contributions and the dynamics of the frame, and Auxiliary Reference Frames (**ARFs**), which, although governed by their own dynamics via an auxiliary metric, remain independent of the gravitational field's dynamics. In contrast, coupled reference frames (**CRFs**) (§2.2.2) include Dynamical Reference Frames (**DRFs**), where the frame's dynamics are intertwined with the gravitational field—effectively serving as a gauge-fixing mechanism—and Real Reference Frames (**RRFs**), which fully incorporate both dynamical behaviour and backreaction.

Section 2.3 discussed the implications of the proposed classification of reference frames for resolving the identified issues of the definition of local observables and the interpretation of diffeomorphism symmetry. Local observables are well-defined once one adopts a a relational localisation of physical quantities. Gauge redundancies are reinterpreted as overlooked dynamical equations of coupled reference frames. The section underlined that a proper treatment of reference frames not only clarifies the dynamical role of matter and geometry but also provides a more robust foundation for understanding gravitational dynamics in a gauge-invariant manner.

Chapter 3: Symmetry Principles: Earman reconsidered

Section 3.1 offered a general review on symmetries and examined Earman's **SP** principles, which establish a connection between spacetime and dynamical symmetries. Spacetime symmetries are defined as transformations preserving the background structure, while dynamical symmetries preserve the solutionhood of equations. These two symmetry groups coincide, forming the basis of Earman's principles: (**SP1**) every dynamical symmetry is a spacetime symmetry, and (**SP2**) every spacetime symmetry is a dynamical symmetry. The discussion also introduced different classes of models and set the stage for later arguments that propose breaking **SP1** in a novel way relaxing certain assumptions, particularly about field coupling.

Section 3.2 introduced a generalised notion of dynamical symmetries (**GDS**) that challenges Earman's **SP1** principle. The key idea is that the traditional definition of dynamical symmetries assumes all dynamical fields are coupled, meaning a single diffeomorphism must act uniformly on all fields. By relaxing this assumption, the chapter allowed for transformations that *independently* affect different fields while still preserving solutionhood, leading to a violation of **SP1**. This motivates the revised principle **SP1***, which states that every dynamical symmetry is a spacetime symmetry *only if fields are dynamically coupled*. Unlike previous cases where **SP1** violations stemmed from unnecessary structures in the theory, the presented argument shows that even within the standard framework of GR, **SP1** can be broken without modifying spacetime structure.

Chapter 4: Dynamical (Un)coupling:Influence or Correlation?

Section 4.1 discussed Lehmkuhl (2011)'s proposal on coupling in the Lagrangian formalism, distinguishing between direct coupling, indirect coupling, and interaction. *Direct coupling* occurs when two fields appear as factors in the same Lagrangian term, whereas *indirect coupling* occurs when fields couple via an intermediate field. *Interaction*, on the other hand, requires both fields to be dynamical and capable of influencing each other. The section paved the way for the redefinition of the term coupling coupling by distinguishing between *influence* (direct or indirect, symmetrical or non-symmetrical) and *correlation* (direct or indirect, symmetrical). Influence refers to the *action* of one field on another, while correlation describes the constrained relationship between fields that is expressed through the *diagonal* action of dynamical symmetries as the only possible action that preserves solutionhood.

Section 4.2 explored Brown's Dynamical View and the Action–Reaction (AR) principle. The AR principle (§4.2.1) states that every action between entities results in a reciprocal reaction. In GR, this manifests as a mutual interaction between the gravitational metric and matter fields, as dictated by the EFEs. This reciprocal nature of spacetime contrasts with Special Relativity and Newtonian mechanics, where the metric remains causally inert. While GR is often viewed as uniquely fulfilling the AR principle, this section clarified that the principle does not require every field to both act and react; rather, a field must be acted upon *if* it acts. This allows for an alternative interpretation of Special Relativity where the spacetime metric, being causally inert, neither acts nor reacts and still satisfies the AR principle. The section also contrasted two perspectives on spacetime (§4.2.2): the geometrical approach, which treats Minkowski spacetime as a causally active structure, and the dynamical approach, which interprets spacetime as encoding the symmetries of physical laws rather than being an independent entity. The latter approach was presented as more consistent for the division between physical and dynamical fields offered in the chapter.

Section 4.3 introduced the distinction between physical and dynamical fields. A *physical field* is defined as one that is both dynamical and respects the Action-Reaction (AR) principle by both acting and reacting. In contrast, a *dynamical field* is any field whose solution space is non-trivial, meaning its solutions are not all related by dynamical symmetries that are automorphisms of the underlying spacetime structure. The section critiqued the traditional view that a field is dynamical if its equations follow from Hamilton's principle, showing that this criterion alone is insufficient. This analysis reinforced the argument that the metric in SR is not a physical field, supporting the dynamical view of spacetime in contrast to the geometrical perspective.

Section 4.4 conducted a deep analysis of the influence relation between fields. Influence was defined as a binary relation between fields, occurring either symmetrically (when both fields act on each other) or non-symmetrically (when one acts on the other without reciprocation). The section further distinguished between direct influence, where fields interact explicitly through terms in the Lagrangian, and indirect influence, where they interact through a common intermediary cause, such as the gravitational metric in GR. The section presented a framework in which all physical fields directly influence and are influenced by gravity, ensuring mutual interaction. However, breaking this symmetry—such as neglecting backreaction effects—can lead to cases where one field indirectly influences another without reciprocal action, redefining it as merely dynamical rather than physical. This refined classification provided a more precise conceptual basis for understanding field interactions in GR and other spacetime theories.

Section 4.5 highlighted that correlation is fundamentally associated with the dynamical symmetries of the theory and equations of motion. While influence involves one field acting upon another, correlation refers to a relationship between fields that persists even without direct or indirect influence. The section defined direct correlation as cases where two fields appear in each other's equations of motion, while indirect correlation occurs when they have the same metric in their equations of motion. In both cases, for solutions to be mapped in solutions, dynamical symmetries must apply to all fields uniformly. A key argument proposed is that direct correlation with the metric field exists in *all* theories of spacetime, including those with a fixed background metric such as Special Relativity, as their dynamics are expressed with respect to the common Minkowski metric, although without being influenced by it. This analysis reinforced that correlation is a broader concept than influence, as influence always implies correlation, but correlation does not necessarily imply influence.

Methodologically, the whole chapter employed diagrammatic representations and specific case studies to elucidate the complex interplay between fields in terms of influence and correlation.

Chapter 5: Observability and Measurability Reconsidered

Section 5.1 introduced in more detail the notion of the observable, which is used throughout the rest of the thesis. In particular, the two meanings of observable as measurable and observable as predictable were distinguished. It also explored the technical construction of partial and complete observables in GR, following the framework of Rovelli and Dittrich. The section set out the mathematical formalism required to construct complete observables, and it also examined different interpretations of observables, highlighting Thiemann's critique of the partial/complete distinction and Adlam's recent arguments that measurable quantities must correspond to complete observables. The discussion underscored the conceptual and mathematical challenges in defining observables in constrained systems like GR.

Section 5.2 explored the conditions under which partial observables form complete observables, challenging the idea that any measurable quantity qualifies as a partial observable. Clarifying what is often not made explicit in the literature on partial observables, it was argued that partial observables must be dynamically coupled to form complete observables. The section distinguished between diffeomorphism-invariance (**DI**) and gauge-invariance (**GI**), arguing that **GI** requires both **DI** and deterministic evolution (**DET**). The analysis provided a novel framework connecting relationalism, gauge-invariance, and determinism, advocating that only coupled reference frames (**CRFs**) can generate valid complete observables.

Section 5.3 focused on the notion of measurement. It questioned the empirical significance of partial observables, when defined as gauge-variant quantities within a coordinate system, arguing that partial observables, as measurable quantities, must be defined as relational quantities in terms of uncoupled reference frames (**URFs**). The section engaged with Thiemann and Adlam's position that all measurable quantities must be complete observables but upheld Rovelli's view that partial observables remain meaningful if framed relationally. Finally a dual-stage procedure for obtaining measurements was proposed: initially, measurement results are represented using an approximate model with relational partial observables described within an **IRF**. In the subsequent stage, these approximations concerning the reference frame need adjustment, redefining it as a **CRF**, and thereby allowing the characterization of the measurement outcome as a complete observable, which represents the actual entities measured.

Section 5.4 offered a new interpretation of Einstein's point coincidence argument, challenging the conventional view that spacetime coincidences must be gauge-invariant, instead proposing that they should be understood relationally in terms of partial observables. It critically assessed Rovelli, Westman&Sonego, and Giovanelli, highlighting that their interpretations conflate gauge-invariance with measurability which only requires relationality. The difference between coincidence as a relational quantity and event as a gauge-invariant quantity was then highlighted, proposing an ontological view of space-time as the set of 'where and when' emerging from the happenings of gauge-invariant events, represented by complete observables.

Broader Implications

I firmly believe that the role of reference systems is not limited to the field of classical GR. There are a number of other research fields where reference systems have the potential to make a difference. In the following I review some of them, without any claim to completeness in the list of areas that would benefit from recognising the use of reference systems and analysing their (un)coupling with observable quantities of interest.

Quantum World. The results obtained in this thesis may have implications that extend beyond classical GR providing a conceptual toolkit for addressing challenges within quantum reference frames. For instance, the insights gained into the relational nature of observables can inform approaches to quantum spacetime, where gauge-invariance and relational localisation remain central concerns. Thus, the ramifications of this research may extend beyond the classical foundations of GR, potentially indicating a novel paradigm for comprehending quantum reference frames that may prove fundamental in the context of quantum gravity.

Gravitational Frames. In my work, I have focused exclusively on material reference frames, providing a dynamic classification. However, a further step in my research will be to study gravitational reference frames, such as Kretschmann-Komar scalars (Kretschmann, 1918; Komar, 1958). These reference frames are fundamental in the context of vacuum GR for the study of vacuum solutions.

Hole Argument. The uses of reference frames are like chocolates. Just when you think they are finished, another one surprises you. Thanks to the crucial role of CRFs in defining local gauge-invariant observables, their use has all the makings of a solution to the well-known problem of indeterminism in GR foundations: the hole argument (Earman and Norton, 1987; Weatherall, 2018; Pooley and Read, 2021). However, several questions remain to be addressed in future work: Is the choice of reference frame arbitrary? If so, on what basis do we choose one reference frame or another? What role does the use of URFs play in the indeterminism typical of the hole argument? How do these questions translate into the quantum realm, with respect to the so-called quantum hole argument (Adlam et al., 2022; Kabel et al., 2024)?

Metaphysics. The 'radically' relational approach of the reference frame machinery implies that the description of physical systems is dependent on the reference frame adopted, suggesting fundamental limits to objectivity. A natural continuation of my thesis will be to investigate the ontological implications of such a framework from both classical and quantum viewpoints. This includes analysing concepts such as perspectivalism and the interconnection between frames (Adlam and Rovelli, 2023; Adlam, 2024b).

By providing a systematic framework that reconciles theoretical constructs with physical practice, this thesis laid the foundation for the future research in (quantum) General Relativity.

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