

Averages of b -hadron, c -hadron, and τ -lepton properties as of summer 2014

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Abstract

This article reports world averages of measurements of b -hadron, c -hadron, and τ -lepton properties obtained by the Heavy Flavor Averaging Group (HFAG) using results available through summer 2014. For the averaging, common input parameters used in the various analyses are adjusted (rescaled) to common values, and known correlations are taken into account. The averages include branching fractions, lifetimes, neutral meson mixing parameters, CP violation parameters, parameters of semileptonic decays and CKM matrix elements.

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1 Introduction

Flavor dynamics is an important element in understanding the nature of particle physics. The accurate knowledge of properties of heavy flavor hadrons, especially b hadrons, plays an essential role for determining the elements of the Cabibbo-Kobayashi-Maskawa (CKM) weak-mixing matrix [1, 2]. The operation of the Belle and BABAR e^+e^- B factory experiments led to a large increase in the size of available B -meson, D -hadron and τ -lepton samples, enabling dramatic improvement in the accuracies of related measurements. The CDF and D0 experiments at the Fermilab Tevatron have also provided important results in heavy flavour physics, most notably in the B_s^0 sector. Run I of the CERN Large Hadron Collider delivered high luminosity, enabling the collection of even higher statistics samples of b and c hadrons, and thus a further leap in precision in many areas, at the ATLAS, CMS, and (especially) LHCb experiments.

The Heavy Flavor Averaging Group (HFAG) was formed in 2002 to continue the activities of the LEP Heavy Flavor Steering group [3]. This group was responsible for calculating averages of measurements of b -flavor related quantities. HFAG has evolved since its inception and currently consists of seven subgroups:

- the “ B Lifetime and Oscillations” subgroup provides averages for b -hadron lifetimes, b -hadron fractions in $\Upsilon(4S)$ decay and pp or $p\bar{p}$ collisions, and various parameters governing B^0 - $\bar{B}^0$ and B_s - $\bar{B}_s^0$ mixing;
- the “Unitarity Triangle Parameters” subgroup provides averages for time-dependent CP asymmetry parameters and studies of $B \rightarrow DK$ decays, and resulting determinations of the angles of the CKM unitarity triangle;
- the “Semileptonic B Decays” subgroup provides averages for inclusive and exclusive B -decay branching fractions, and subsequent determinations of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$;
- the “ B to Charm Decays” subgroup provides averages of branching fractions for B decays to final states involving open charm or charmonium mesons;
- the “Rare Decays” subgroup provides averages of branching fractions and CP asymmetries for charmless, radiative, leptonic, and baryonic B -meson and b -baryon decays;
- the “Charm Physics” subgroup provides averages of numerous quantities in the charm sector, including branching fractions properties of charm baryons and of excited D^{**} and D_{sJ} mesons, averages of D^0 - $\bar{D}^0$ mixing and CP and T violation parameters, and an average value for the D_s decay constant f_{D_s} .
- the “Tau Physics” subgroup provides documentation and averages for a selection of τ lepton quantities that most profit from the adoption of the HFAG prescriptions. In particular, the τ lepton branching fractions, uncertainties and correlations are obtained from a global fit of the experimental results, and this information is further elaborated to compute several lepton universality tests and the CKM matrix element $|V_{us}|$. The τ lepton-flavor-violating decays are documented and, starting with this edition, combinations of such upper limits are also computed.

The “Lifetime and Oscillations” and “Semileptonic” subgroups were formed from the merger of four LEP working groups. The “Unitary Triangle,” “ B to Charm Decays,” and “Rare Decays” subgroups were formed to provide averages for new results obtained from the B factory experiments (and now also from the Fermilab Tevatron and CERN LHC experiments). The “Charm” and “Tau” subgroups were formed more recently in response to the wealth of new data concerning D and τ physics. Subgroups typically include representatives from Belle, *BABAR* and LHCb, plus, when relevant, CLEO, CDF and D0.

This article is an update of the last HFAG preprint, which used results available at least through early 2012 [4]. Here we report world averages using results available at least through summer 2014. In some cases results made available in the latter part of 2014 have been included.¹ In general, we use all publicly available results that have written documentation. These include preliminary results presented at conferences or workshops. However, we do not use preliminary results that remain unpublished for an extended period of time, or for which no publication is planned. Close contacts have been established between representatives from the experiments and members of subgroups that perform averaging to ensure that the data are prepared in a form suitable for combinations.

In the case of obtaining a world average for which $\chi^2/\text{dof} > 1$, where dof is the number of degrees of freedom in the average calculation, we do not usually scale the resulting error, as is presently done by the Particle Data Group [5]. Rather, we examine the systematics of each measurement to better understand them. Unless we find possible systematic discrepancies between the measurements, we do not apply any additional correction to the calculated error. We provide the confidence level of the fit as an indicator for the consistency of the measurements included in the average. In case some special treatment was necessary to calculate an average, or if an approximation used in an average calculation might not be sufficiently accurate (*e.g.*, assuming Gaussian errors when the likelihood function indicates non-Gaussian behavior), we include a warning message.

Chapter 2 describes the methodology used for calculating averages. In the averaging procedure, common input parameters used in the various analyses are adjusted (rescaled) to common values, and, where possible, known correlations are taken into account. Chapters 3–9 present world average values from each of the subgroups listed above. A brief summary of the averages presented is given in Chapter 10. A complete listing of the averages and plots, including updates since this document was prepared, are also available on the HFAG web site:

<http://www.slac.stanford.edu/xorg/hfag>

¹* Particularly important new results have been included whenever possible. The precise cut-off date for including results in the averages varies between subgroups.

2 Methodology

The general averaging problem that HFAG faces is to combine information provided by different measurements of the same parameter to obtain our best estimate of the parameter's value and uncertainty. The methodology described here focuses on the problems of combining measurements performed with different systematic assumptions and with potentially-correlated systematic uncertainties. Our methodology relies on the close involvement of the people performing the measurements in the averaging process.

Consider two hypothetical measurements of a parameter x , which might be summarized as

$$\begin{aligned} x &= x_1 \pm \delta x_1 \pm \Delta x_{1,1} \pm \Delta x_{2,1} \dots \\ x &= x_2 \pm \delta x_2 \pm \Delta x_{1,2} \pm \Delta x_{2,2} \dots, \end{aligned}$$

where the δx_k are statistical uncertainties, and the $\Delta x_{i,k}$ are contributions to the systematic uncertainty. One popular approach is to combine statistical and systematic uncertainties in quadrature

$$\begin{aligned} x &= x_1 \pm (\delta x_1 \oplus \Delta x_{1,1} \oplus \Delta x_{2,1} \oplus \dots) \\ x &= x_2 \pm (\delta x_2 \oplus \Delta x_{1,2} \oplus \Delta x_{2,2} \oplus \dots) \end{aligned}$$

and then perform a weighted average of x_1 and x_2 , using their combined uncertainties, as if they were independent. This approach suffers from two potential problems that we attempt to address. First, the values of the x_k may have been obtained using different systematic assumptions. For example, different values of the B^0 lifetime may have been assumed in separate measurements of the oscillation frequency Δm_d . The second potential problem is that some contributions of the systematic uncertainty may be correlated between experiments. For example, separate measurements of Δm_d may both depend on an assumed Monte-Carlo branching fraction used to model a common background.

The problems mentioned above are related since, ideally, any quantity y_i that x_k depends on has a corresponding contribution $\Delta x_{i,k}$ to the systematic error which reflects the uncertainty Δy_i on y_i itself. We assume that this is the case and use the values of y_i and Δy_i assumed by each measurement explicitly in our averaging (we refer to these values as $y_{i,k}$ and $\Delta y_{i,k}$ below). Furthermore, since we do not lump all the systematics together, we require that each measurement used in an average have a consistent definition of the various contributions to the systematic uncertainty. Different analyses often use different decompositions of their systematic uncertainties, so achieving consistent definitions for any potentially correlated contributions requires close coordination between HFAG and the experiments. In some cases, a group of systematic uncertainties must be combined to obtain a coarser description that is consistent between measurements. Systematic uncertainties that are uncorrelated with any other sources of uncertainty appearing in an average are lumped together with the statistical error, so that the only systematic uncertainties treated explicitly are those that are correlated with at least one other measurement via a consistently-defined external parameter y_i . When asymmetric statistical or systematic uncertainties are quoted, we symmetrize them since our combination method implicitly assumes parabolic likelihoods for each measurement.

The fact that a measurement of x is sensitive to the value of y_i indicates that, in principle, the data used to measure x could equally-well be used for a simultaneous measurement of x and y_i , as illustrated by the large contour in Fig. 1(a) for a hypothetical measurement. However,

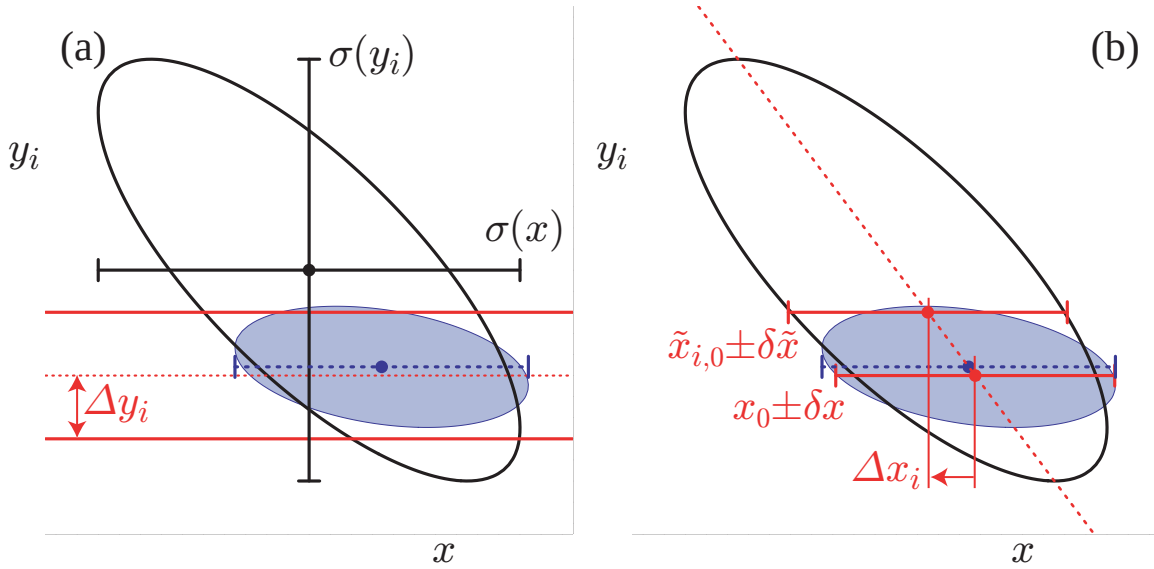


Figure 1: The left-hand plot (a) compares the 68% confidence-level contours of a hypothetical measurement's unconstrained (large ellipse) and constrained (filled ellipse) likelihoods, using the Gaussian constraint on y_i represented by the horizontal band. The solid error bars represent the statistical uncertainties $\sigma(x)$ and $\sigma(y_i)$ of the unconstrained likelihood. The dashed error bar shows the statistical error on x from a constrained simultaneous fit to x and y_i . The right-hand plot (b) illustrates the method described in the text of performing fits to x with y_i fixed at different values. The dashed diagonal line between these fit results has the slope $\rho(x, y_i)\sigma(y_i)/\sigma(x)$ in the limit of a parabolic unconstrained likelihood. The result of the constrained simultaneous fit from (a) is shown as a dashed error bar on x .

we often have an external constraint Δy_i on the value of y_i (represented by the horizontal band in Fig. 1(a)) that is more precise than the constraint $\sigma(y_i)$ from our data alone. Ideally, in such cases we would perform a simultaneous fit to x and y_i , including the external constraint, obtaining the filled (x, y) contour and corresponding dashed one-dimensional estimate of x shown in Fig. 1(a). Throughout, we assume that the external constraint Δy_i on y_i is Gaussian.

In practice, the added technical complexity of a constrained fit with extra free parameters is not justified by the small increase in sensitivity, as long as the external constraints Δy_i are sufficiently precise when compared with the sensitivities $\sigma(y_i)$ to each y_i of the data alone. Instead, the usual procedure adopted by the experiments is to perform a baseline fit with all y_i fixed to nominal values $y_{i,0}$, obtaining $x = x_0 \pm \delta x$. This baseline fit neglects the uncertainty due to Δy_i , but this error can be mostly recovered by repeating the fit separately for each external parameter y_i with its value fixed at $y_i = y_{i,0} + \Delta y_i$ to obtain $x = \tilde{x}_{i,0} \pm \delta \tilde{x}$, as illustrated in Fig. 1(b). The absolute shift, $|\tilde{x}_{i,0} - x_0|$, in the central value of x is what the experiments usually quote as their systematic uncertainty Δx_i on x due to the unknown value of y_i . Our procedure requires that we know not only the magnitude of this shift but also its sign. In the limit that the unconstrained data is represented by a parabolic likelihood, the signed shift is given by

$$\Delta x_i = \rho(x, y_i) \frac{\sigma(x)}{\sigma(y_i)} \Delta y_i, \quad (1)$$

where $\sigma(x)$ and $\rho(x, y_i)$ are the statistical uncertainty on x and the correlation between x and y_i in the unconstrained data. While our procedure is not equivalent to the constrained fit with extra parameters, it yields (in the limit of a parabolic unconstrained likelihood) a central value x_0 that agrees to $\mathcal{O}(\Delta y_i / \sigma(y_i))^2$ and an uncertainty $\delta x \oplus \Delta x_i$ that agrees to $\mathcal{O}(\Delta y_i / \sigma(y_i))^4$.

In order to combine two or more measurements that share systematics due to the same external parameters y_i , we would ideally perform a constrained simultaneous fit of all data samples to obtain values of x and each y_i , being careful to only apply the constraint on each y_i once. This is not practical since we generally do not have sufficient information to reconstruct the unconstrained likelihoods corresponding to each measurement. Instead, we perform the two-step approximate procedure described below.

Figs. 2(a,b) illustrate two statistically-independent measurements, $x_1 \pm (\delta x_1 \oplus \Delta x_{i,1})$ and $x_2 \pm (\delta x_2 \oplus \Delta x_{i,2})$, of the same hypothetical quantity x (for simplicity, we only show the contribution of a single correlated systematic due to an external parameter y_i). As our knowledge of the external parameters y_i evolves, it is natural that the different measurements of x will assume different nominal values and ranges for each y_i . The first step of our procedure is to adjust the values of each measurement to reflect the current best knowledge of the values y'_i and ranges $\Delta y'_i$ of the external parameters y_i , as illustrated in Figs. 2(c,b). We adjust the central values x_k and correlated systematic uncertainties $\Delta x_{i,k}$ linearly for each measurement (indexed by k) and each external parameter (indexed by i):

$$x'_k = x_k + \sum_i \frac{\Delta x_{i,k}}{\Delta y_{i,k}} (y'_i - y_{i,k}) \quad (2)$$

$$\Delta x'_{i,k} = \Delta x_{i,k} \cdot \frac{\Delta y'_i}{\Delta y_{i,k}}. \quad (3)$$

This procedure is exact in the limit that the unconstrained likelihoods of each measurement is parabolic.

The second step of our procedure is to combine the adjusted measurements, $x'_k \pm (\delta x_k \oplus \Delta x'_{k,1} \oplus \Delta x'_{k,2} \oplus \dots)$ using the chi-square

$$\chi^2_{\text{comb}}(x, y_1, y_2, \dots) \equiv \sum_k \frac{1}{\delta x_k^2} \left[x'_k - \left(x + \sum_i (y_i - y'_i) \frac{\Delta x'_{i,k}}{\Delta y'_i} \right) \right]^2 + \sum_i \left(\frac{y_i - y'_i}{\Delta y'_i} \right)^2, \quad (4)$$

and then minimize this χ^2 to obtain the best values of x and y_i and their uncertainties, as illustrated in Fig. 3. Although this method determines new values for the y_i , we do not report them since the $\Delta x_{i,k}$ reported by each experiment are generally not intended for this purpose (for example, they may represent a conservative upper limit rather than a true reflection of a 68% confidence level).

For comparison, the exact method we would perform if we had the unconstrained likelihoods $\mathcal{L}_k(x, y_1, y_2, \dots)$ available for each measurement is to minimize the simultaneous constrained likelihood

$$\mathcal{L}_{\text{comb}}(x, y_1, y_2, \dots) \equiv \prod_k \mathcal{L}_k(x, y_1, y_2, \dots) \prod_i \mathcal{L}_i(y_i), \quad (5)$$

with an independent Gaussian external constraint on each y_i

$$\mathcal{L}_i(y_i) \equiv \exp \left[-\frac{1}{2} \left(\frac{y_i - y'_i}{\Delta y'_i} \right)^2 \right]. \quad (6)$$

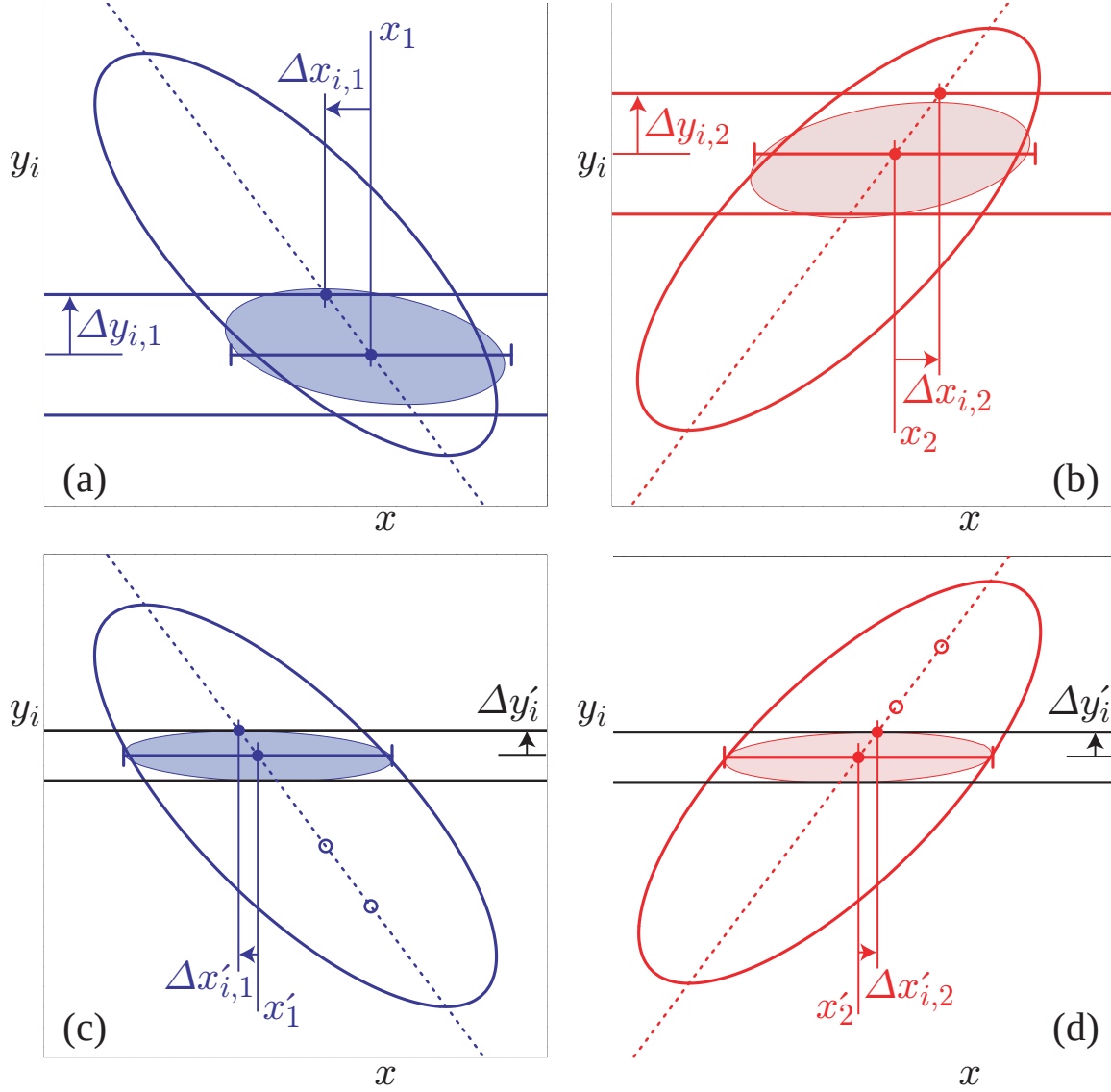


Figure 2: The upper plots (a) and (b) show examples of two individual measurements to be combined. The large ellipses represent their unconstrained likelihoods, and the filled ellipses represent their constrained likelihoods. Horizontal bands indicate the different assumptions about the value and uncertainty of y_i used by each measurement. The error bars show the results of the approximate method described in the text for obtaining x by performing fits with y_i fixed to different values. The lower plots (c) and (d) illustrate the adjustments to accommodate updated and consistent knowledge of y_i as described in the text. Open circles mark the central values of the unadjusted fits to x with y fixed; these determine the dashed line used to obtain the adjusted values.

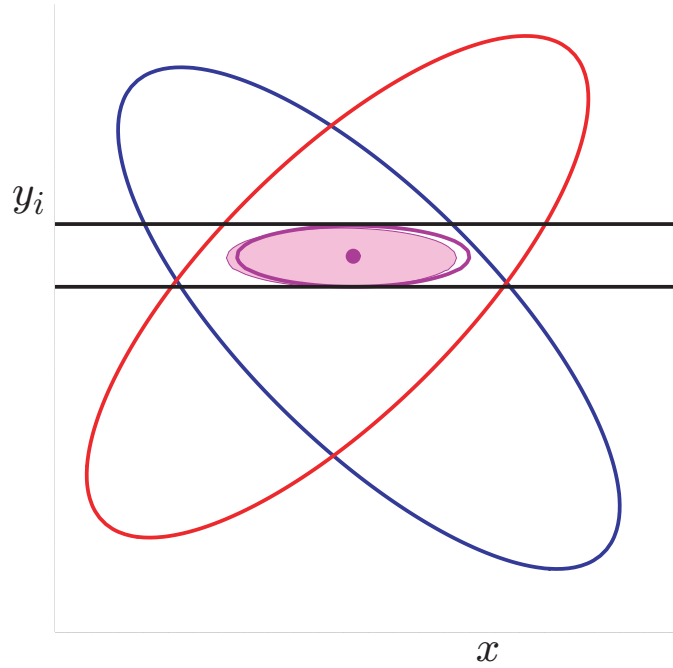


Figure 3: An illustration of the combination of two hypothetical measurements of x using the method described in the text. The ellipses represent the unconstrained likelihoods of each measurement, and the horizontal band represents the latest knowledge about y_i that is used to adjust the individual measurements. The filled small ellipse shows the result of the exact method using $\mathcal{L}_{\text{comb}}$, and the hollow small ellipse and dot show the result of the approximate method using χ^2_{comb} .

The results of this exact method are illustrated by the filled ellipses in Figs. 3(a,b) and agree with our method in the limit that each $\mathcal{L}_k$ is parabolic and that each $\Delta y'_i \ll \sigma(y_i)$. In the case of a non-parabolic unconstrained likelihood, experiments would have to provide a description of $\mathcal{L}_k$ itself to allow an improved combination. In the case of $\sigma(y_i) \simeq \Delta y'_i$, experiments are advised to perform a simultaneous measurement of both x and y so that their data will improve the world knowledge about y .

The algorithm described above is used as a default in the averages reported in the following sections. For some cases, somewhat simplified or more complex algorithms are used and noted in the corresponding sections. Some examples for extensions of the standard method for extracting averages are given here. These include the case where measurement errors depend on the measured value, *i.e.* are relative errors, unknown correlation coefficients and the breakdown of error sources.

For measurements with Gaussian errors, the usual estimator for the average of a set of measurements is obtained by minimizing the following χ^2 :

$$\chi^2(t) = \sum_i^N \frac{(y_i - t)^2}{\sigma_i^2}, \quad (7)$$

where y_i is the measured value for input i and σ_i^2 is the variance of the distribution from which y_i was drawn. The value $\hat{t}$ of t at minimum χ^2 is our estimator for the average. (This discussion is given for independent measurements for the sake of simplicity; the generalization to correlated measurements is straightforward, and has been used when averaging results.) The true σ_i are unknown but typically the error as assigned by the experiment σ_i^{raw} is used as an estimator for it. Caution is advised, however, in the case where σ_i^{raw} depends on the value measured for y_i . Examples of this include an uncertainty in any multiplicative factor (like an acceptance) that enters the determination of y_i , *i.e.* the $\sqrt{N}$ dependence of Poisson statistics, where $y_i \propto N$ and $\sigma_i \propto \sqrt{N}$. Failing to account for this type of dependence when averaging leads to a biased average. Biases in the average can be avoided (or at least reduced) by minimizing the following χ^2 :

$$\chi^2(t) = \sum_i^N \frac{(y_i - t)^2}{\sigma_i^2(\hat{t})}. \quad (8)$$

In the above $\sigma_i(\hat{t})$ is the uncertainty assigned to input i that includes the assumed dependence of the stated error on the value measured. As an example, consider a pure acceptance error, for which $\sigma_i(\hat{t}) = (\hat{t}/y_i) \times \sigma_i^{\text{raw}}$. It is easily verified that solving Eq. 8 leads to the correct behavior, namely

$$\hat{t} = \frac{\sum_i^N y_i^3 / (\sigma_i^{\text{raw}})^2}{\sum_i^N y_i^2 / (\sigma_i^{\text{raw}})^2},$$

i.e. weighting by the inverse square of the fractional uncertainty, $\sigma_i^{\text{raw}}/y_i$. It is sometimes difficult to assess the dependence of σ_i^{raw} on $\hat{t}$ from the errors quoted by experiments.

Another issue that needs careful treatment is the question of correlation among different measurements, *e.g.* due to using the same theory for calculating acceptances. A common practice is to set the correlation coefficient to unity to indicate full correlation. However, this is not a “conservative” thing to do, and can in fact lead to a significantly underestimated uncertainty on the average. In the absence of better information, the most conservative choice

of correlation coefficient between two measurements i and j is the one that maximizes the uncertainty on $\hat{t}$ due to that pair of measurements:

$$\sigma_{\hat{t}(i,j)}^2 = \frac{\sigma_i^2 \sigma_j^2 (1 - \rho_{ij}^2)}{\sigma_i^2 + \sigma_j^2 - 2 \rho_{ij} \sigma_i \sigma_j}, \quad (9)$$

namely

$$\rho_{ij} = \min \left(\frac{\sigma_i}{\sigma_j}, \frac{\sigma_j}{\sigma_i} \right), \quad (10)$$

which corresponds to setting $\sigma_{\hat{t}(i,j)}^2 = \min(\sigma_i^2, \sigma_j^2)$. Setting $\rho_{ij} = 1$ when $\sigma_i \neq \sigma_j$ can lead to a significant underestimate of the uncertainty on $\hat{t}$, as can be seen from Eq. 9.

Finally, we carefully consider the various sources of error contributing to the overall uncertainty of an average. The overall covariance matrix is constructed from a number of individual sources, *e.g.* $\mathbf{V} = \mathbf{V}_{\text{stat}} + \mathbf{V}_{\text{sys}} + \mathbf{V}_{\text{th}}$. The variance on the average $\hat{t}$ can be written

$$\sigma_{\hat{t}}^2 = \frac{\sum_{i,j} (\mathbf{V}^{-1} [\mathbf{V}_{\text{stat}} + \mathbf{V}_{\text{sys}} + \mathbf{V}_{\text{th}}] \mathbf{V}^{-1})_{ij}}{\left(\sum_{i,j} V_{ij}^{-1} \right)^2} = \sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2 + \sigma_{\text{th}}^2. \quad (11)$$

Written in this form, one can readily determine the contribution of each source of uncertainty to the overall uncertainty on the average. This breakdown of the uncertainties is used in the following sections.

Following the prescription described above, the central values and errors are rescaled to a common set of input parameters in the averaging procedures according to the dependency on any of these input parameters. We try to use the most up-to-date values for these common inputs and the same values among the HFAG subgroups. For the parameters whose averages are produced by HFAG, we use the values in the current update cycle. For other external parameters, we use the most recent PDG values available (usually Ref. [5]). The parameters and values used are listed in each subgroup section.

3 b -hadron production fractions, lifetimes and mixing parameters

Quantities such as b -hadron production fractions, b -hadron lifetimes, and neutral B -meson oscillation frequencies have been studied in the nineties at LEP and SLC (e^+e^- colliders at $\sqrt{s} = m_Z$) as well as at the first version of the Tevatron ($p\bar{p}$ collider at $\sqrt{s} = 1.8$ TeV). Since then precise measurements of the B^0 and B^+ mesons have also been performed at the asymmetric B factories, KEKB and PEP-II (e^+e^- colliders at $\sqrt{s} = m_{\Upsilon(4S)}$) while measurements related to the other b hadrons, in particular B_s^0 , B_c^+ and Λ_b^0 , have been performed at the upgraded Tevatron ($\sqrt{s} = 1.96$ TeV) and are continuing at the LHC (pp collider at $\sqrt{s} = 7$ and 8 TeV). In most cases, these basic quantities, although interesting by themselves, became necessary ingredients for the more complicated and refined analyses at the asymmetric B factories, the Tevatron and the LHC, in particular the time-dependent CP asymmetry measurements. It is therefore important that the best experimental values of these quantities continue to be kept up-to-date and improved.

In several cases, the averages presented in this chapter are needed and used as input for the results given in the subsequent chapters. Within this chapter, some averages need the knowledge of other averages in a circular way. This coupling, which appears through the b -hadron fractions whenever inclusive or semi-exclusive measurements have to be considered, has been reduced drastically in the past several years with increasingly precise exclusive measurements becoming available and dominating practically all averages.

In addition to b -hadron fractions, lifetimes and mixing parameters, this chapter also deals with the CP -violating phase $\phi_s^{c\bar{c}s} \simeq -2\beta_s$, which is the phase difference between the B_s^0 mixing amplitude and the $b \rightarrow c\bar{c}s$ decay amplitude, as well as the parameters of CP violation in the B mixing amplitudes. The angle β , which is the equivalent of β_s for the B^0 system, is discussed in Chapter 4.

3.1 b -hadron production fractions

We consider here the relative fractions of the different b -hadron species found in an unbiased sample of weakly decaying b hadrons produced under some specific conditions. The knowledge of these fractions is useful to characterize the signal composition in inclusive b -hadron analyses, to predict the background composition in exclusive analyses, or to convert (relative) observed rates into (relative) branching fraction measurements. Many B -physics analyses need these fractions as input. We distinguish here the following three conditions: $\Upsilon(4S)$ decays, $\Upsilon(5S)$ decays, and high-energy collisions (including Z^0 decays).

3.1.1 b -hadron production fractions in $\Upsilon(4S)$ decays

Only pairs of the two lightest (charged and neutral) B mesons can be produced in $\Upsilon(4S)$ decays, and it is enough to determine the following branching fractions:

$$f^{+-} = \Gamma(\Upsilon(4S) \rightarrow B^+ B^-) / \Gamma_{\text{tot}}(\Upsilon(4S)), \quad (12)$$

$$f^{00} = \Gamma(\Upsilon(4S) \rightarrow B^0 \bar{B}^0) / \Gamma_{\text{tot}}(\Upsilon(4S)). \quad (13)$$

Table 1: Published measurements of the B^+/B^0 production ratio in $\Upsilon(4S)$ decays, together with their average (see text). Systematic uncertainties due to the imperfect knowledge of $\tau(B^+)/\tau(B^0)$ are included. The latest *BABAR* result [6] supersedes the earlier *BABAR* measurements [7, 8].

Experiment and year	Ref.	Decay modes or method	Published value of $R^{+-/00} = f^{+-}/f^{00}$	Assumed value of $\tau(B^+)/\tau(B^0)$
CLEO, 2001	[9]	$J/\psi K^{(*)}$	$1.04 \pm 0.07 \pm 0.04$	1.066 ± 0.024
<i>BABAR</i> , 2002	[7]	$(c\bar{c})K^{(*)}$	$1.10 \pm 0.06 \pm 0.05$	1.062 ± 0.029
CLEO, 2002	[10]	$D^*\ell\nu$	$1.058 \pm 0.084 \pm 0.136$	1.074 ± 0.028
Belle, 2003	[11]	dilepton events	$1.01 \pm 0.03 \pm 0.09$	1.083 ± 0.017
<i>BABAR</i> , 2004	[8]	$J/\psi K$	$1.006 \pm 0.036 \pm 0.031$	1.083 ± 0.017
<i>BABAR</i> , 2005	[6]	$(c\bar{c})K^{(*)}$	$1.06 \pm 0.02 \pm 0.03$	1.086 ± 0.017
Average			1.059 ± 0.027 (tot)	1.076 ± 0.004

In practice, most analyses measure their ratio

$$R^{+-/00} = f^{+-}/f^{00} = \Gamma(\Upsilon(4S) \rightarrow B^+ B^-) / \Gamma(\Upsilon(4S) \rightarrow B^0 \bar{B}^0), \quad (14)$$

which is easier to access experimentally. Since an inclusive (but separate) reconstruction of B^+ and B^0 is difficult, specific exclusive decay modes, $B^+ \rightarrow x^+$ and $B^0 \rightarrow x^0$, are usually considered to perform a measurement of $R^{+-/00}$, whenever they can be related by isospin symmetry (for example $B^+ \rightarrow J/\psi K^+$ and $B^0 \rightarrow J/\psi K^0$). Under the assumption that $\Gamma(B^+ \rightarrow x^+) = \Gamma(B^0 \rightarrow x^0)$, *i.e.* that isospin invariance holds in these B decays, the ratio of the number of reconstructed $B^+ \rightarrow x^+$ and $B^0 \rightarrow x^0$ mesons, after correcting for efficiency, is proportional to

$$\frac{f^{+-} \mathcal{B}(B^+ \rightarrow x^+)}{f^{00} \mathcal{B}(B^0 \rightarrow x^0)} = \frac{f^{+-} \Gamma(B^+ \rightarrow x^+) \tau(B^+)}{f^{00} \Gamma(B^0 \rightarrow x^0) \tau(B^0)} = \frac{f^{+-}}{f^{00}} \frac{\tau(B^+)}{\tau(B^0)}, \quad (15)$$

where $\tau(B^+)$ and $\tau(B^0)$ are the B^+ and B^0 lifetimes respectively. Hence the primary quantity measured in these analyses is $R^{+-/00} \tau(B^+)/\tau(B^0)$, and the extraction of $R^{+-/00}$ with this method therefore requires the knowledge of the $\tau(B^+)/\tau(B^0)$ lifetime ratio.

The published measurements of $R^{+-/00}$ are listed in Table 1 together with the corresponding assumed values of $\tau(B^+)/\tau(B^0)$. All measurements are based on the above-mentioned method, except the one from Belle, which is a by-product of the B^0 mixing frequency analysis using dilepton events (but note that it also assumes isospin invariance, namely $\Gamma(B^+ \rightarrow \ell^+ X) = \Gamma(B^0 \rightarrow \ell^+ X)$). The latter is therefore treated in a slightly different manner in the following procedure used to combine these measurements:

- each published value of $R^{+-/00}$ from CLEO and *BABAR* is first converted back to the original measurement of $R^{+-/00} \tau(B^+)/\tau(B^0)$, using the value of the lifetime ratio assumed in the corresponding analysis;
- a simple weighted average of these original measurements of $R^{+-/00} \tau(B^+)/\tau(B^0)$ from CLEO and *BABAR* (which do not depend on the assumed value of the lifetime ratio) is then computed, assuming no statistical or systematic correlations between them;

- the weighted average of $R^{+-/00} \tau(B^+)/\tau(B^0)$ is converted into a value of $R^{+-/00}$, using the latest average of the lifetime ratios, $\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004$ (see Sec. 3.2.3);
- the Belle measurement of $R^{+-/00}$ is adjusted to the current values of $\tau(B^0) = 1.520 \pm 0.004$ ps and $\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004$ (see Sec. 3.2.3), using the quoted systematic uncertainties due to these parameters;
- the combined value of $R^{+-/00}$ from CLEO and BABAR is averaged with the adjusted value of $R^{+-/00}$ from Belle, assuming a 100% correlation of the systematic uncertainty due to the limited knowledge on $\tau(B^+)/\tau(B^0)$; no other correlation is considered.

The resulting global average,

$$R^{+-/00} = \frac{f^{+-}}{f^{00}} = 1.059 \pm 0.027, \quad (16)$$

is consistent with equal production of charged and neutral B mesons, although only at the 2.2σ level.

On the other hand, the BABAR collaboration has performed a direct measurement of the f^{00} fraction using an original method, which neither relies on isospin symmetry nor requires the knowledge of $\tau(B^+)/\tau(B^0)$. Its analysis, based on a comparison between the number of events where a single $B^0 \rightarrow D^{*-}\ell^+\nu$ decay could be reconstructed and the number of events where two such decays could be reconstructed, yields [12]

$$f^{00} = 0.487 \pm 0.010 \text{ (stat)} \pm 0.008 \text{ (syst)}. \quad (17)$$

The two results of Eqs. (16) and (17) are of very different natures and completely independent of each other. Their product is equal to $f^{+-} = 0.516 \pm 0.019$, while another combination of them gives $f^{+-} + f^{00} = 1.003 \pm 0.029$, compatible with unity. Assuming² $f^{+-} + f^{00} = 1$, also consistent with CLEO's observation that the fraction of $\Upsilon(4S)$ decays to $B\bar{B}$ pairs is larger than 0.96 at 95% CL [14], the results of Eqs. (16) and (17) can be averaged (first converting Eq. (16) into a value of $f^{00} = 1/(R^{+-/00} + 1)$) to yield the following more precise estimates:

$$f^{00} = 0.486 \pm 0.006, \quad f^{+-} = 1 - f^{00} = 0.514 \pm 0.006, \quad \frac{f^{+-}}{f^{00}} = 1.058 \pm 0.024. \quad (18)$$

The latter ratio differs from one by 2.4σ .

3.1.2 b -hadron production fractions in $\Upsilon(5S)$ decays

Hadronic events produced in e^+e^- collisions at the $\Upsilon(5S)$ (also known as $\Upsilon(10860)$) energy can be classified into three categories: light-quark (u, d, s, c) continuum events, $b\bar{b}$ continuum events, and $\Upsilon(5S)$ events. The latter two cannot be distinguished and will be called $b\bar{b}$ events in the following. These $b\bar{b}$ events, which also include $b\bar{b}\gamma$ events because of possible initial-state radiation, can hadronize in different final states. We define $f_{u,d}^{\Upsilon(5S)}$ as the fraction of $b\bar{b}$

²A few non- $B\bar{B}$ decay modes of the $\Upsilon(4S)$ ($\Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$, $\Upsilon(1S)\eta$) have been observed with branching fractions of the order of 10^{-4} [13], corresponding to a partial width several times larger than that in the e^+e^- channel. However, this can still be neglected and the assumption $f^{+-} + f^{00} = 1$ remains valid in the present context of the determination of f^{+-} and f^{00} .

Table 2: Published measurements of $f_s^{\Upsilon(5S)}$. All values have been obtained assuming $f_{\mathcal{B}}^{\Upsilon(5S)} = 0$. They are quoted as in the original publications, except for the most recent measurement which is quoted as $1 - f_{u,d}^{\Upsilon(5S)}$, with $f_{u,d}^{\Upsilon(5S)}$ from Ref. [15]. The last line gives our average of $f_s^{\Upsilon(5S)}$ assuming $f_{\mathcal{B}}^{\Upsilon(5S)} = 0$.

Experiment, year, dataset	Decay mode or method	Value of $f_s^{\Upsilon(5S)}$
CLEO, 2006, 0.42 fb^{-1} [16]	$\Upsilon(5S) \rightarrow D_s X$	$0.168 \pm 0.026^{+0.067}_{-0.034}$
	$\Upsilon(5S) \rightarrow \phi X$	$0.246 \pm 0.029^{+0.110}_{-0.053}$
	$\Upsilon(5S) \rightarrow B\bar{B}X$	$0.411 \pm 0.100 \pm 0.092$
	CLEO average of above 3	$0.21^{+0.06}_{-0.03}$
Belle, 2006, 1.86 fb^{-1} [17]	$\Upsilon(5S) \rightarrow D_s X$	$0.179 \pm 0.014 \pm 0.041$
	$\Upsilon(5S) \rightarrow D^0 X$	$0.181 \pm 0.036 \pm 0.075$
	Belle average of above 2	$0.180 \pm 0.013 \pm 0.032$
Belle, 2010, 23.6 fb^{-1} [15]	$\Upsilon(5S) \rightarrow B\bar{B}X$	$0.263 \pm 0.032 \pm 0.051$
Average of all above after adjustments to inputs of Table 3		0.215 ± 0.031

Table 3: External inputs on which the $f_s^{\Upsilon(5S)}$ averages are based.

Branching fraction	Value	Explanation and reference
$\mathcal{B}(B \rightarrow D_s X) \times \mathcal{B}(D_s \rightarrow \phi\pi)$	0.00374 ± 0.00014	derived from [18]
$\mathcal{B}(B_s^0 \rightarrow D_s X)$	0.92 ± 0.11	model-dependent estimate [19]
$\mathcal{B}(D_s \rightarrow \phi\pi)$	0.045 ± 0.004	[18]
$\mathcal{B}(B \rightarrow D^0 X) \times \mathcal{B}(D^0 \rightarrow K\pi)$	0.0243 ± 0.0011	derived from [18]
$\mathcal{B}(B_s^0 \rightarrow D^0 X)$	0.08 ± 0.07	model-dependent estimate [17, 19]
$\mathcal{B}(D^0 \rightarrow K\pi)$	0.0387 ± 0.0005	[18]
$\mathcal{B}(B \rightarrow \phi X)$	0.0343 ± 0.0012	world average [16, 18]
$\mathcal{B}(B_s^0 \rightarrow \phi X)$	0.161 ± 0.024	model-dependent estimate [16]

events with a pair of non-strange bottom mesons ($B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}$, $B^*\bar{B}^*$, $B\bar{B}\pi$, $B\bar{B}^*\pi$, $B^*\bar{B}\pi$, $B^*\bar{B}^*\pi$, and $B\bar{B}\pi\pi$ final states, where B denotes a B^0 or B^+ meson and $\bar{B}$ denotes a $\bar{B}^0$ or B^- meson), $f_s^{\Upsilon(5S)}$ as the fraction of $b\bar{b}$ events with a pair of strange bottom mesons ($B_s^0\bar{B}_s^0$, $B_s^0\bar{B}_s^{*0}$, $B_s^{*0}\bar{B}_s^0$, and $B_s^{*0}\bar{B}_s^{*0}$ final states), and $f_{\mathcal{B}}^{\Upsilon(5S)}$ as the fraction of $b\bar{b}$ events without any bottom meson in the final state. Note that the excited bottom-meson states decay via $B^* \rightarrow B\gamma$ and $B_s^{*0} \rightarrow B_s^0\gamma$. These fractions satisfy

$$f_{u,d}^{\Upsilon(5S)} + f_s^{\Upsilon(5S)} + f_{\mathcal{B}}^{\Upsilon(5S)} = 1. \quad (19)$$

The CLEO and Belle collaborations have published measurements of several inclusive $\Upsilon(5S)$ branching fractions, $\mathcal{B}(\Upsilon(5S) \rightarrow D_s X)$, $\mathcal{B}(\Upsilon(5S) \rightarrow \phi X)$ and $\mathcal{B}(\Upsilon(5S) \rightarrow D^0 X)$, from which they extracted the model-dependent estimates of $f_s^{\Upsilon(5S)}$ reported in Table 2.³ This extraction

³ It was realized just before finalizing this document that more recent results from Belle [20], $f_s^{\Upsilon(5S)} =$

was performed under the implicit assumption $f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0$, using the relation

$$\frac{1}{2}\mathcal{B}(\mathcal{Y}(5S) \rightarrow D_s X) = f_s^{\mathcal{Y}(5S)} \times \mathcal{B}(B_s^0 \rightarrow D_s X) + \left(1 - f_s^{\mathcal{Y}(5S)} - f_{\mathcal{B}}^{\mathcal{Y}(5S)}\right) \times \mathcal{B}(B \rightarrow D_s X), \quad (20)$$

and similar relations for $\mathcal{B}(\mathcal{Y}(5S) \rightarrow D^0 X)$ and $\mathcal{B}(\mathcal{Y}(5S) \rightarrow \phi X)$. In Table 2 we list also the values of $f_s^{\mathcal{Y}(5S)}$ derived from measurements of $f_{u,d}^{\mathcal{Y}(5S)} = \mathcal{B}(\mathcal{Y}(5S) \rightarrow B\bar{B}X)$ [15, 16], as well as our average value of $f_s^{\mathcal{Y}(5S)}$, all obtained under the assumption $f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0$.

However, the assumption $f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0$ is known to be invalid since the observation of the following final states in e^+e^- collisions at the $\mathcal{Y}(5S)$ energy: $\mathcal{Y}(1S)\pi^+\pi^-$, $\mathcal{Y}(2S)\pi^+\pi^-$, $\mathcal{Y}(3S)\pi^+\pi^-$ and $\mathcal{Y}(1S)K^+K^-$ [21, 22], $h_b(1P)\pi^+\pi^-$ and $h_b(2P)\pi^+\pi^-$ [23], and more recently $\mathcal{Y}(1S)\pi^0\pi^0$, $\mathcal{Y}(2S)\pi^0\pi^0$ and $\mathcal{Y}(3S)\pi^0\pi^0$ [24]. The sum of the measurements of the corresponding visible cross-sections, adding also the contributions of the unmeasured $\mathcal{Y}(1S)K^0\bar{K}^0$, $h_b(1P)\pi^0\pi^0$ and $h_b(2P)\pi^0\pi^0$ final states assuming isospin conservation, amounts to

$$\sigma^{\text{vis}}(e^+e^- \rightarrow (b\bar{b})hh) = 13.2 \pm 1.4 \text{ pb}, \quad \text{for } (b\bar{b}) = \mathcal{Y}(1S, 2S, 3S), h_b(1P, 2P) \text{ and } hh = \pi\pi, KK.$$

We divide this by the $b\bar{b}$ production cross section, $\sigma(e^+e^- \rightarrow b\bar{b}X) = 337 \pm 15 \text{ pb}$, obtained as the average of the CLEO [19] and Belle [20]⁴ measurements, to obtain

$$\mathcal{B}(\mathcal{Y}(5S) \rightarrow (b\bar{b})hh) = 0.039 \pm 0.004, \quad \text{for } (b\bar{b}) = \mathcal{Y}(1S, 2S, 3S), h_b(1P, 2P) \text{ and } hh = \pi\pi, KK,$$

which is to be considered as a lower bound for $f_{\mathcal{B}}^{\mathcal{Y}(5S)}$.

Following the method described in Ref. [25], we perform a χ^2 fit of the original measurements of the $\mathcal{Y}(5S)$ branching fractions of Refs. [15–17], using the inputs of Table 3, the relations of Eqs. (19) and (20) and the one-sided Gaussian constraint $f_{\mathcal{B}}^{\mathcal{Y}(5S)} \geq \mathcal{B}(\mathcal{Y}(5S) \rightarrow (b\bar{b})hh)$, to simultaneously extract $f_{u,d}^{\mathcal{Y}(5S)}$, $f_s^{\mathcal{Y}(5S)}$ and $f_{\mathcal{B}}^{\mathcal{Y}(5S)}$. Taking all known correlations into account, the best fit values are

$$f_{u,d}^{\mathcal{Y}(5S)} = 0.761_{-0.042}^{+0.027}, \quad (21)$$

$$f_s^{\mathcal{Y}(5S)} = 0.200_{-0.031}^{+0.030}, \quad (22)$$

$$f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0.039_{-0.004}^{+0.050}, \quad (23)$$

where the strongly asymmetric uncertainty on $f_{\mathcal{B}}^{\mathcal{Y}(5S)}$ is due to the one-sided constraint from the observed $(b\bar{b})hh$ decays. These results, together with their correlation, imply

$$f_s^{\mathcal{Y}(5S)} / f_{u,d}^{\mathcal{Y}(5S)} = 0.263_{-0.044}^{+0.052}, \quad (24)$$

in fair agreement with the results of a *BABAR* analysis [26], performed as a function of centre-of-mass energy.⁵

The production of B_s^0 mesons at the $\mathcal{Y}(5S)$ is observed to be dominated by the $B_s^{*0}\bar{B}_s^{*0}$ channel, with $\sigma(e^+e^- \rightarrow B_s^{*0}\bar{B}_s^{*0}) / \sigma(e^+e^- \rightarrow B_s^{(*)0}\bar{B}_s^{(*)0}) = (87.0 \pm 1.7)\%$ [27, 28]. The proportions of the various production channels for non-strange B mesons have also been measured [15].

0.172 $\pm$ 0.030, have been overlooked. These results are not included in Table 2 nor in the averages presented here.

⁴ The results of Ref. [20] supersede the $\sigma(e^+e^- \rightarrow b\bar{b}X)$ and $f_s^{\mathcal{Y}(5S)}$ results of Ref. [17].

⁵ This has not been included in the average, since no numerical value is given for $f_s^{\mathcal{Y}(5S)} / f_{u,d}^{\mathcal{Y}(5S)}$ in Ref. [26].

3.1.3 b -hadron production fractions at high energy

At high energy, all species of weakly decaying b hadrons may be produced, either directly or in strong and electromagnetic decays of excited b hadrons. It is often assumed that the fractions of these different species are the same in unbiased samples of high- p_T b jets originating from Z^0 decays, from $p\bar{p}$ collisions at the Tevatron, or from pp collisions at the LHC. This hypothesis is plausible under the condition that the square of the momentum transfer to the produced b quarks, Q^2 , is large compared with the square of the hadronization energy scale, $Q^2 \gg \Lambda_{\text{QCD}}^2$. On the other hand, there is no strong argument that the fractions at different machines should be strictly equal, so this assumption should be checked experimentally. Although the available data is not sufficient at this time to perform a definitive check, it is expected that more refined analyses of the Tevatron Run II data and new analyses from LHC experiments may improve this situation and allow one to confirm or disprove this assumption with reasonable confidence. Meanwhile, the attitude adopted here is that these fractions are assumed to be equal at all high-energy colliders until demonstrated otherwise by experiment. Both CDF and LHCb report a p_T dependence for Λ_b^0 production relative to B^+ and B^0 ; the number of Λ_b^0 baryons observed at low p_T is enhanced with respect to that seen at LEP's higher p_T . Therefore we present three sets of complete averages: one set including only measurements performed at LEP, a second set including only measurements performed at the Tevatron, a third set including measurements performed at LEP, Tevatron and LHCb. The LHCb production fractions results by themselves are still incomplete, lacking measurements of the production of other weakly decaying heavy-flavour baryons, Ξ_b and Ω_b , and a measurement of $\bar{\chi}$ giving an extra constraint between f_d and f_s .

Contrary to what happens in the charm sector where the fractions of D^+ and D^0 are different, the relative amount of B^+ and B^0 is not affected by the electromagnetic decays of excited B^{*+} and B^{*0} states and strong decays of excited B^{**+} and B^{**0} states. Decays of the type $B_s^{**0} \rightarrow B^{(*)}K$ also contribute to the B^+ and B^0 rates, but with the same magnitude if mass effects can be neglected. We therefore assume equal production of B^+ and B^0 mesons. We also neglect the production of weakly decaying states made of several heavy quarks (like B_c^+ and doubly heavy baryons) which is known to be very small. Hence, for the purpose of determining the b -hadron fractions, we use the constraints

$$f_u = f_d \quad \text{and} \quad f_u + f_d + f_s + f_{\text{baryon}} = 1, \quad (25)$$

where f_u , f_d , f_s and f_{baryon} are the unbiased fractions of B^+ , B^0 , B_s^0 and b baryons, respectively.

We note that there are many measurements of the production cross-sections of different species of b hadrons. In principle these could be included in a global fit to determine the production fractions. We do not perform such a fit at the current time, and instead average only the measured production fractions.

The LEP experiments have measured $f_s \times \mathcal{B}(B_s^0 \rightarrow D_s^- \ell^+ \nu_\ell X)$ [29], $\mathcal{B}(b \rightarrow \Lambda_b^0) \times \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell X)$ [30, 31] and $\mathcal{B}(b \rightarrow \Xi_b^-) \times \mathcal{B}(\Xi_b^- \rightarrow \Xi^- \ell^- \bar{\nu}_\ell X)$ [32, 33]⁶ from partially reconstructed final states including a lepton, f_{baryon} from protons identified in b events [35], and the production rate of charged b hadrons [36]. Ratios of b -hadron fractions have been measured at CDF using lepton+charm final states [37–39]⁷, double semileptonic decays with $K^* \mu \mu$ and $\phi \mu \mu$ final

⁶ The DELPHI result of Ref. [33] is considered to supersede an older one [34].

⁷ CDF updated their measurement of $f_{\Lambda_b^0}/f_d$ [37] to account for a measured p_T dependence between exclusively reconstructed Λ_b^0 and B^0 [39].

states [40], and fully reconstructed $B_s^0 \rightarrow J/\psi \phi$ decays [41]. Measurements of the production of other heavy flavour baryons at the Tevatron are included in the determination of f_{baryon} [42–44]⁸ using the constraint

$$\begin{aligned} f_{\text{baryon}} &= f_{\Lambda_b^0} + f_{\Xi_b^0} + f_{\Xi_b^-} + f_{\Omega_b^-} \\ &= f_{\Lambda_b^0} \left(1 + 2 \frac{f_{\Xi_b^-}}{f_{\Lambda_b^0}} + \frac{f_{\Omega_b^-}}{f_{\Lambda_b^0}} \right), \end{aligned} \quad (26)$$

where isospin invariance is assumed in the production of Ξ_b^0 and Ξ_b^- . Other b baryons are expected to decay strongly or electromagnetically to those baryons listed. For the production measurements, both CDF and D0 reconstruct their b baryons exclusively to final states which include a J/ψ and a hyperon ($\Lambda_b^0 \rightarrow J/\psi \Lambda$, $\Xi_b^- \rightarrow J/\psi \Xi^-$ and $\Omega_b^- \rightarrow J/\psi \Omega^-$). We assume that the partial decay width of a b baryon to a J/ψ and the corresponding hyperon is equal to the partial width of any other b baryon to a J/ψ and the corresponding hyperon. LHCb has also measured ratios of b -hadron fractions in charm+lepton final states [45] and in fully reconstructed hadronic two-body decays $B^0 \rightarrow D^- \pi^+$, $B_s^0 \rightarrow D_s^- \pi^+$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$ [46, 47].⁹

Both CDF and LHCb observe that the ratio $f_{\Lambda_b^0}/f_d$ depends on the p_T of the charm+lepton system [39, 45].¹⁰ CDF chose to correct an older result to account for the p_T dependence. In a second result, CDF binned their data in p_T of the charm+electron system [38]. The more recent LHCb measurement using hadronic decays [47] obtains the scale for $R_{\Lambda_b^0} = f_{\Lambda_b^0}/f_d$ from their previous charm + lepton data [45]. The LHCb measurement using hadronic data also bins the same data in pseudorapidity (η) and sees a linear dependence of $R_{\Lambda_b^0}$. Since η is not entirely independent of p_T it is impossible to tell at this time whether this dependence is just an artifact of the p_T dependence. Figure 4 shows the ratio $R_{\Lambda_b^0}$ as a function of p_T for the b hadron, as measured by LHCb. LHCb fits their scaled hadronic data to obtain

$$R_{\Lambda_b^0} = (0.151 \pm 0.030) + \exp \left\{ -(0.57 \pm 0.11) - (0.095 \pm 0.016)[\text{GeV}/c]^{-1} \times p_T \right\}. \quad (27)$$

A value of $R_{\Lambda_b^0}$ is also calculated for LEP and placed at the approximate p_T for the charm+lepton system, but this value does not participate in any fit.¹¹ Because the two LHCb results for $R_{\Lambda_b^0}$ are not independent, we use only their semileptonic data for the averages. Note that the p_T dependence of $R_{\Lambda_b^0}$ combined with the constraint from Eq. (25) implies a compensating p_T dependence in one or more of the production fractions, f_u , f_d , or f_s .

Both CDF and LHCb have investigated the p_T dependence of f_s/f_d using fully reconstructed B_s^0 and B^0 decays. The CDF analysis reconstructed decays that include a J/ψ in the final state [41] and reports no significant p_T dependence on the ratio. However, their result is dominated by an 18% scale uncertainty from preliminary measurements of the branching ratios of the $\mathcal{B}(B_s^0 \rightarrow J/\psi \phi)$ and $\mathcal{B}(B^0 \rightarrow J/\psi K^*(892))$. LHCb reported 3σ evidence that the ratio

⁸ D0 reports $f_{\Omega_b^-}/f_{\Xi_b^-}$. We use the CDF+D0 average of $f_{\Xi_b^-}/f_{\Lambda_b^0}$ to obtain $f_{\Omega_b^-}/f_{\Lambda_b^0}$ and then combine it with the CDF result.

⁹ The results of Ref. [46] supersede those of Ref. [48].

¹⁰ CDF compares the p_T distribution of fully reconstructed $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$ with $\bar{B}^0 \rightarrow D^+ \pi^-$, which gives $f_{\Lambda_b^0}/f_d$ up to a scale factor. LHCb compares the p_T in the charm+lepton system between Λ_b^0 and B^0 and B^+ , giving $R_{\Lambda_b^0}/2 = f_{\Lambda_b^0}/(f_u + f_d) = f_{\Lambda_b^0}/2f_d$.

¹¹ The CDF semileptonic data would require significant corrections to obtain the p_T of the b hadron and be included on the same plot with the LHCb data. We do not have these corrections at this time.

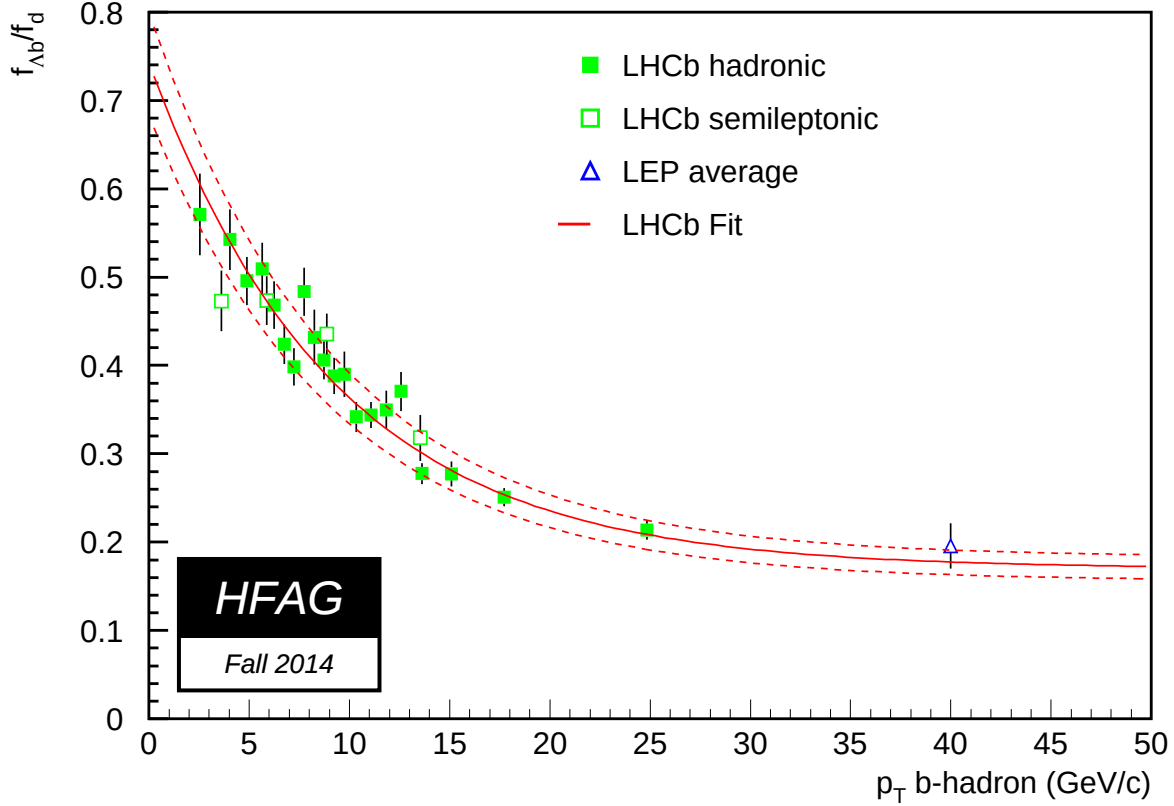


Figure 4: Ratio of production fractions $f_{\Lambda_b^0}/f_d$ as a function of p_T of the b hadron from LHCb data for b hadrons decaying semileptonically [45] and fully reconstructed in hadronic decays [47]. The curve represents a fit to the LHCb hadronic data [47]. The computed LEP ratio is included at an approximate p_T in Z decays, but does not participate in any fit.

f_s/f_d decreases with p_T using fully reconstructed B_s^0 and B^0 decays and theoretical predictions for branching ratios [46]⁹. Figure 5 shows the ratio $R_s = f_s/f_u$ as a function of p_T measured by CDF and LHCb. Two fits are performed. The first fit, using a linear parameterization, yields $R_s = (0.2760 \pm 0.0068) - (0.00191 \pm 0.00059)[\text{GeV}/c]^{-1} \times p_T$. A second fit, using a simple exponential, yields $R_s = \exp \{(-1.293 \pm 0.028) - (0.0077 \pm 0.0025)[\text{GeV}/c]^{-1} \times p_T\}$. The two fits are nearly indistinguishable over the p_T range of the results, but the second gives a physical value for all p_T . R_s is also calculated for LEP and placed at the approximate p_T for the b hadron, though the LEP result doesn't participate in the fit. Our world average for R_s is also included in the figure for reference.

In order to combine or compare LHCb results with other experiments, the p_T -dependent $f_{\Lambda_b^0}/(f_u + f_d)$ is weighted by the p_T spectrum.¹² Table 4 compares the p_T -weighted LHCb data with comparable averages from CDF. The average CDF and LHCb data are in good agreement

¹² In practice the LHCb data are given in 14 bins in p_T and η with a full covariance matrix [45]. The weighted average is calculated as $D^T C^{-1} M / \sigma$, where $\sigma = D^T C^{-1} D$, M is a vector of measurements, C^{-1} is the inverse

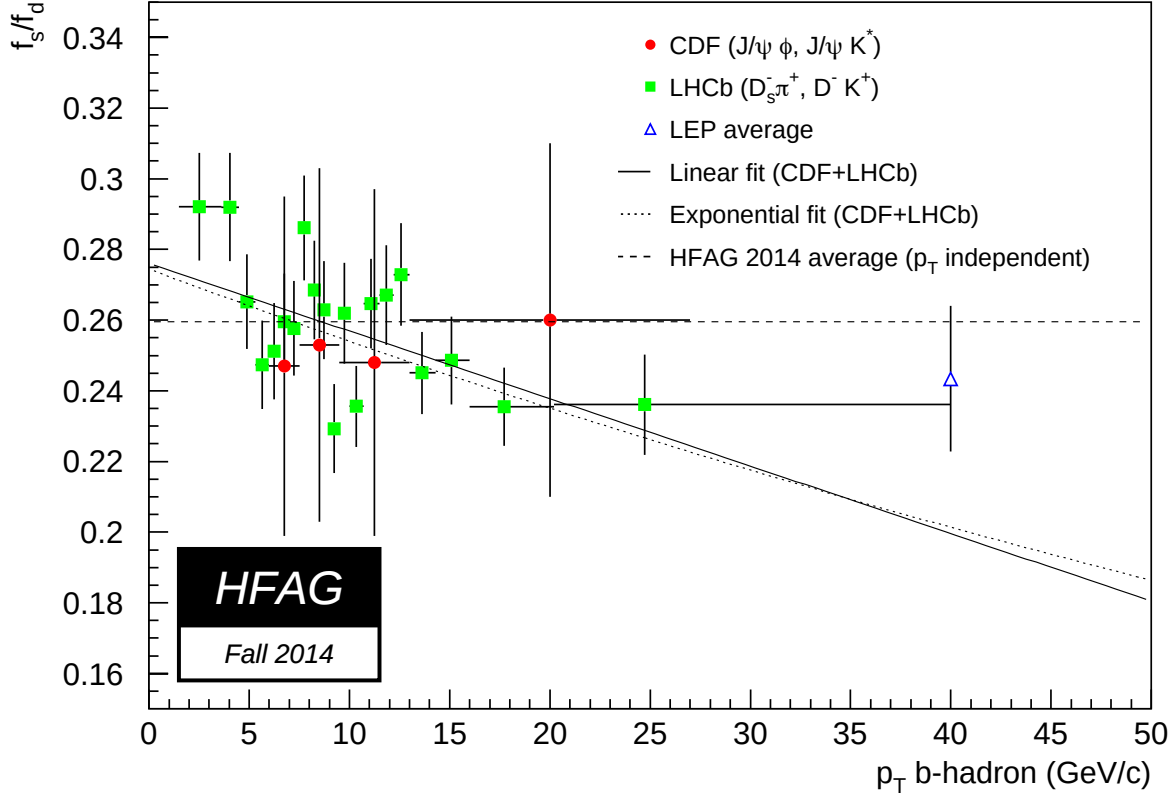


Figure 5: Ratio of production fractions f_s/f_d as a function of p_T of the reconstructed b hadrons for the CDF [41] and LHCb [46]⁹ data. Note the suppressed zero for the vertical axis. The curves represent fits to the data: a linear fit (solid), and an exponential fit described in the text (dotted). The p_T independent value average of R_s (dashed) is shown for comparison. The computed LEP ratio is included at an approximate p_T in Z decays, but does not participate in any fit.

despite the b hadrons being produced in different kinematic regimes.

All these published results have been combined following the procedure and assumptions described in Ref. [3], to yield $f_u = f_d = 0.406 \pm 0.005$, $f_s = 0.104 \pm 0.006$ and $f_{\text{baryon}} = 0.084 \pm 0.011$ under the constraints of Eq. (25). Repeating the combinations for LEP and the Tevatron, we obtain $f_u = f_d = 0.414 \pm 0.008$, $f_s = 0.088 \pm 0.013$ and $f_{\text{baryon}} = 0.083 \pm 0.011$ when using the LEP data only, and $f_u = f_d = 0.348 \pm 0.020$, $f_s = 0.097 \pm 0.012$ and $f_{\text{baryon}} = 0.207 \pm 0.046$ when using the Tevatron data only. As noted previously, the LHCb data are insufficient to determine a complete set of b -hadron production fractions. The world averages (LEP, Tevatron and LHCb) for the various fractions are presented here for comparison with previous averages. Significant differences exist between the LEP and Tevatron fractions, therefore use of the world

covariance matrix and D^T is the transpose of the design matrix (vector of 1's).

Table 4: Comparison of average production fraction ratios from CDF and LHCb. The kinematic regime of the charm+lepton system reconstructed in each experiment is also shown.

Quantity	CDF	LHCb
$f_s/(f_u + f_d)$	0.211 ± 0.054	0.131 ± 0.009
$f_{\Lambda_b^0}/(f_u + f_d)$	0.212 ± 0.058	0.223 ± 0.022
Average charm+lepton p_T	$\sim 13 \text{ GeV}/c$	$\sim 7 \text{ GeV}/c$
Pseudorapidity range	$-1 < \eta < 1$	$2 < \eta < 5$

averages should be taken with some care. For these combinations other external inputs are used, *e.g.* the branching ratios of B mesons to final states with a D , D^* or D^{**} in semileptonic decays, which are needed to evaluate the fraction of semileptonic B_s^0 decays with a D_s^- in the final state.

Time-integrated mixing analyses performed with lepton pairs from $b\bar{b}$ events produced at high-energy colliders measure the quantity

$$\bar{\chi} = f'_d \chi_d + f'_s \chi_s, \quad (28)$$

where f'_d and f'_s are the fractions of B^0 and B_s^0 hadrons in a sample of semileptonic b -hadron decays, and where χ_d and χ_s are the B^0 and B_s^0 time-integrated mixing probabilities. Assuming that all b hadrons have the same semileptonic decay width implies $f'_i = f_i R_i$, where $R_i = \tau_i/\tau_b$ is the ratio of the lifetime τ_i of species i to the average b -hadron lifetime $\tau_b = \sum_i f_i \tau_i$. Hence measurements of the mixing probabilities $\bar{\chi}$, χ_d and χ_s can be used to improve our knowledge of f_u , f_d , f_s and f_{baryon} . In practice, the above relations yield another determination of f_s obtained from f_{baryon} and mixing information,

$$f_s = \frac{1}{R_s} \frac{(1+r)\bar{\chi} - (1 - f_{\text{baryon}} R_{\text{baryon}})\chi_d}{(1+r)\chi_s - \chi_d}, \quad (29)$$

where $r = R_u/R_d = \tau(B^+)/\tau(B^0)$.

The published measurements of $\bar{\chi}$ performed by the LEP experiments have been combined by the LEP Electroweak Working Group to yield $\bar{\chi} = 0.1259 \pm 0.0042$ [49]. This can be compared with the Tevatron average, $\bar{\chi} = 0.127 \pm 0.008$, obtained from D0 [50] and CDF [51] measurements with Run II data.¹³ The two averages agree, showing no evidence that the production fractions of B^0 and B_s^0 mesons at the Z peak or at the Tevatron are different. We combine these two results in a simple weighted average, assuming no correlations, and obtain $\bar{\chi} = 0.1260 \pm 0.0037$.

Introducing the $\bar{\chi}$ average in Eq. (29), together with our world average $\chi_d = 0.1875 \pm 0.0017$ (see Eq. (57) of Sec. 3.3.1), the assumption $\chi_s = 1/2$ (justified by Eq. (66) in Sec. 3.3.2), the best knowledge of the lifetimes (see Sec. 3.2) and the estimate of f_{baryon} given above, yields $f_s = 0.109 \pm 0.010$ (or $f_s = 0.108 \pm 0.011$ using only LEP data, or $f_s = 0.110 \pm 0.019$ using only Tevatron data), an estimate dominated by the mixing information. Taking into account all known correlations (including that introduced by f_{baryon}), this result is then combined with

¹³ As explained in Ref. [51], a previous CDF analysis [52] performed with Run I data overlooked a background component, so the corresponding result is not included in the average.

Table 5: Time-integrated mixing probability $\bar{\chi}$ (defined in Eq. (28)), and fractions of the different b -hadron species in an unbiased sample of weakly decaying b hadrons, obtained from both direct and mixing measurements. The correlation coefficients between the fractions are also given. The last column includes measurements performed at LEP, Tevatron and LHCb.

Quantity		Z decays	Tevatron	LHCb [46] ⁹	all
Mixing probability $\bar{\chi}$		0.1259 ± 0.0042	0.127 ± 0.008		0.1260 ± 0.0037
B^+ or B^0 fraction	$f_u = f_d$	0.410 ± 0.007	0.350 ± 0.020		0.406 ± 0.005
B_s^0 fraction	f_s	0.100 ± 0.008	0.100 ± 0.010		0.105 ± 0.005
b -baryon fraction	f_{baryon}	0.080 ± 0.010	0.199 ± 0.044		0.083 ± 0.010
B_s^0/B^0 ratio	f_s/f_d	0.243 ± 0.023	0.286 ± 0.029	0.256 ± 0.020	0.259 ± 0.013
$\rho(f_s, f_u) = \rho(f_s, f_d)$		-0.646	$+0.255$		-0.350
$\rho(f_{\text{baryon}}, f_u) = \rho(f_{\text{baryon}}, f_d)$		-0.797	-0.976		-0.899
$\rho(f_{\text{baryon}}, f_s)$		$+0.053$	-0.459		-0.096

the set of fractions obtained from direct measurements (given above), to yield the improved estimates of Table 5, still under the constraints of Eq. (25). As can be seen, our knowledge on the mixing parameters substantially reduces the uncertainty on f_s . It should be noted that the results are correlated, as indicated in Table 5.

Although no recent measurements of the fractions have become available, the averages of Table 5 (and most notably the b -baryon fraction) have significantly improved in precision as compared to those given in our previous report [4]. This is mostly due to a new and precise model-independent measurement of the $\Lambda_c^+ \rightarrow pK^-\pi^+$ branching fraction from Belle [53], which has been used to adjust the fractions obtained from direct measurements.

3.2 b -hadron lifetimes

In the spectator model the decay of b -flavoured hadrons H_b is governed entirely by the flavour changing $b \rightarrow Wq$ transition ($q = c, u$). For this very reason, lifetimes of all b -flavoured hadrons are the same in the spectator approximation regardless of the (spectator) quark content of the H_b . In the early 1990's experiments became sophisticated enough to start seeing the differences of the lifetimes among various H_b species. The first theoretical calculations of the spectator quark effects on H_b lifetime emerged only few years earlier.

Currently, most such calculations are performed in the framework of the Heavy Quark Expansion, HQE. In the HQE, under certain assumptions (the most important of which is that of quark-hadron duality [54]), the decay rate of an H_b to an inclusive final state f is expressed as the sum of a series of expectation values of operators of increasing dimension, multiplied by the correspondingly higher powers of Λ_{QCD}/m_b :

$$\Gamma_{H_b \rightarrow f} = |\text{CKM}|^2 \sum_n c_n^{(f)} \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^n \langle H_b | O_n | H_b \rangle, \quad (30)$$

where $|\text{CKM}|^2$ is the relevant combination of the CKM matrix elements. The coefficients $c_n^{(f)}$ of this expansion, known as the Operator Product Expansion [55], can be calculated perturbatively. Hence, the HQE predicts $\Gamma_{H_b \rightarrow f}$ in the form of an expansion in both Λ_{QCD}/m_b and

$\alpha_s(m_b)$. The precision of current experiments makes it mandatory to go to the next-to-leading order in QCD, *i.e.* to include corrections of the order of $\alpha_s(m_b)$ to the $c_n^{(f)}$ terms. All non-perturbative physics is shifted into the expectation values $\langle H_b | O_n | H_b \rangle$ of operators O_n . These can be calculated using lattice QCD or QCD sum rules, or can be related to other observables via the HQE [56]. One may reasonably expect that powers of Λ_{QCD}/m_b provide enough suppression that only the first few terms of the sum in Eq. (30) matter.

Theoretical predictions are usually made for the ratios of the lifetimes (with $\tau(B^0)$ chosen as the common denominator) rather than for the individual lifetimes, for this allows several uncertainties to cancel. The precision of the current HQE calculations (see Refs. [57–59] for the latest updates) is in some instances already surpassed by the measurements, *e.g.* in the case of $\tau(B^+)/\tau(B^0)$. Also, HQE calculations are not assumption-free. More accurate predictions are a matter of progress in the evaluation of the non-perturbative hadronic matrix elements and verifying the assumptions that the calculations are based upon. However, the HQE, even in its present shape, draws a number of important conclusions, which are in agreement with experimental observations:

- The heavier the mass of the heavy quark, the smaller is the variation in the lifetimes among different hadrons containing this quark, which is to say that as $m_b \rightarrow \infty$ we retrieve the spectator picture in which the lifetimes of all H_b states are the same. This is well illustrated by the fact that lifetimes are rather similar in the b sector, while they differ by large factors in the c sector ($m_c < m_b$).
- The non-perturbative corrections arise only at the order of $\Lambda_{\text{QCD}}^2/m_b^2$, which translates into differences among H_b lifetimes of only a few percent.
- It is only the difference between meson and baryon lifetimes that appears at the $\Lambda_{\text{QCD}}^2/m_b^2$ level. The splitting of the meson lifetimes occurs at the $\Lambda_{\text{QCD}}^3/m_b^3$ level, yet it is enhanced by a phase space factor $16\pi^2$ with respect to the leading free b decay.

To ensure that certain sources of systematic uncertainty cancel, lifetime analyses are sometimes designed to measure ratios of lifetimes. However, because of the differences in decay topologies, abundance (or lack thereof) of decays of a certain kind, *etc.*, measurements of the individual lifetimes are also common. In the following section we review the most common types of lifetime measurements. This discussion is followed by the presentation of the averaging of the various lifetime measurements, each with a brief description of its particularities.

3.2.1 Lifetime measurements, uncertainties and correlations

In most cases, the lifetime of an H_b state is estimated from a flight distance measurement and a $\beta\gamma$ factor which is used to convert the geometrical distance into the proper decay time. Methods of accessing lifetime information can roughly be divided in the following five categories:

1. **Inclusive (flavour-blind) measurements.** These measurements are aimed at extracting the lifetime from a mixture of b -hadron decays, without distinguishing the decaying species. Often the knowledge of the mixture composition is limited, which makes these measurements experiment-specific. Also, these measurements have to rely on Monte Carlo simulation for estimating the $\beta\gamma$ factor, because the decaying hadrons are not fully reconstructed. On the bright side, these are usually the largest statistics b -hadron lifetime

measurements that are accessible to a given experiment, and can, therefore, serve as an important performance benchmark.

2. **Measurements in semileptonic decays of a specific H_b .** The W boson from $b \rightarrow Wc$ produces a $\ell\nu_\ell$ pair ($\ell = e, \mu$) in about 21% of the cases. The electron or muon from such decays provides a clean and efficient trigger signature. The c quark from the $b \rightarrow Wc$ transition and the other quark(s) making up the decaying H_b combine into a charm hadron, which is reconstructed in one or more exclusive decay channels. Knowing what this charmed hadron is allows one to separate, at least statistically, different H_b species. The advantage of these measurements is in statistics, which is usually superior to the case of exclusively reconstructed H_b decays. Some of the main disadvantages are related to the difficulty of estimating the lepton+charm sample composition and to the Monte Carlo reliance for the momentum (and hence $\beta\gamma$ factor) estimate.
3. **Measurements in exclusively reconstructed hadronic decays.** These have the advantage of complete reconstruction of the decaying H_b state, which allows one to infer the decaying species as well as to perform precise measurement of the $\beta\gamma$ factor. Both lead to generally smaller systematic uncertainties than in the above two categories. The downsides are smaller branching ratios and larger combinatorial backgrounds, especially in $H_b \rightarrow H_c\pi(\pi\pi)$ and multi-body H_c decays, or in a hadron collider environment with non-trivial underlying event. Decays of the type $H_b \rightarrow J/\psi H_s$ are relatively clean and easy to trigger, due to the $J/\psi \rightarrow \ell^+\ell^-$ signature, but their branching fraction is only about 1%.
4. **Measurements at asymmetric B factories.** In the $\Upsilon(4S) \rightarrow B\bar{B}$ decay, the B mesons (B^+ or B^0) are essentially at rest in the $\Upsilon(4S)$ frame. This makes direct lifetime measurements impossible in experiments at symmetric colliders producing $\Upsilon(4S)$ at rest. At asymmetric B factories the $\Upsilon(4S)$ meson is boosted resulting in B and $\bar{B}$ moving nearly parallel to each other with the same boost. The lifetime is inferred from the distance Δz separating the B and $\bar{B}$ decay vertices along the beam axis and from the $\Upsilon(4S)$ boost known from the beam energies. This boost is equal to $\beta\gamma \approx 0.55$ (0.43) in the BABAR (Belle) experiment, resulting in an average B decay length of approximately 250 (190) μm .

In order to determine the charge of the B mesons in each event, one of them is fully reconstructed in a semileptonic or hadronic decay mode. The other B is typically not fully reconstructed, only the position of its decay vertex is determined from the remaining tracks in the event. These measurements benefit from large statistics, but suffer from poor proper time resolution, comparable to the B lifetime itself. This resolution is dominated by the uncertainty on the decay vertices, which is typically 50 (100) μm for a fully (partially) reconstructed B meson. With very large future statistics, the resolution and purity could be improved (and hence the systematics reduced) by fully reconstructing both B mesons in the event.

5. **Direct measurement of lifetime ratios.** This method, initially applied in the measurement of $\tau(B^+)/\tau(B^0)$, is now also used for other b -hadron species at the LHC. The ratio of the lifetimes is extracted from the proper time dependence of the ratio of the observed yields of two different b -hadron species, both reconstructed in decay modes

with similar topologies. The advantage of this method is that subtle efficiency effects (partially) cancel in the ratio.

In some of the latest analyses, measurements of two (*e.g.* $\tau(B^+)$ and $\tau(B^+)/\tau(B^0)$) or three (*e.g.* $\tau(B^+)$, $\tau(B^+)/\tau(B^0)$, and Δm_d) quantities are combined. This introduces correlations among measurements. Another source of correlations among the measurements are the systematic effects, which could be common to an experiment or to an analysis technique across the experiments. When calculating the averages, such correlations are taken into account following the general procedure described in Ref. [60].

3.2.2 Inclusive b -hadron lifetimes

The inclusive b hadron lifetime is defined as $\tau_b = \sum_i f_i \tau_i$ where τ_i are the individual species lifetimes and f_i are the fractions of the various species present in an unbiased sample of weakly decaying b hadrons produced at a high-energy collider.¹⁴ This quantity is certainly less fundamental than the lifetimes of the individual species, the latter being much more useful in comparisons of the measurements with the theoretical predictions. Nonetheless, we perform the averaging of the inclusive lifetime measurements for completeness as well as for the reason that they might be of interest as “technical numbers.”

Table 6: Measurements of average b -hadron lifetimes.

Experiment	Method	Data set	τ_b (ps)	Ref.
ALEPH	Dipole	91	$1.511 \pm 0.022 \pm 0.078$	[61]
DELPHI	All track i.p. (2D)	91–92	$1.542 \pm 0.021 \pm 0.045$	[62] ^a
DELPHI	Sec. vtx	91–93	$1.582 \pm 0.011 \pm 0.027$	[63] ^a
DELPHI	Sec. vtx	94–95	$1.570 \pm 0.005 \pm 0.008$	[64]
L3	Sec. vtx + i.p.	91–94	$1.556 \pm 0.010 \pm 0.017$	[65] ^b
OPAL	Sec. vtx	91–94	$1.611 \pm 0.010 \pm 0.027$	[66]
SLD	Sec. vtx	93	$1.564 \pm 0.030 \pm 0.036$	[67]
Average set 1 (b vertex)			1.572 ± 0.009	
ALEPH	Lepton i.p. (3D)	91–93	$1.533 \pm 0.013 \pm 0.022$	[68]
L3	Lepton i.p. (2D)	91–94	$1.544 \pm 0.016 \pm 0.021$	[65] ^b
OPAL	Lepton i.p. (2D)	90–91	$1.523 \pm 0.034 \pm 0.038$	[69]
Average set 2 ($b \rightarrow \ell$)			1.537 ± 0.020	
CDF1	J/ψ vtx	92–95	$1.533 \pm 0.015^{+0.035}_{-0.031}$	[70]
ATLAS	J/ψ vtx	2010	$1.489 \pm 0.016 \pm 0.043$	[71] ^p
Average set 3 ($b \rightarrow J/\psi$)			1.516 ± 0.028	

^a The combined DELPHI result quoted in [63] is $1.575 \pm 0.010 \pm 0.026$ ps.

^b The combined L3 result quoted in [65] is $1.549 \pm 0.009 \pm 0.015$ ps.

^p Preliminary.

In practice, an unbiased measurement of the inclusive lifetime is difficult to achieve, because it would imply an efficiency which is guaranteed to be the same across species. So most of the

¹⁴In principle such a quantity could be slightly different in Z decays, at the Tevatron or at the LHC, in case the fractions of b -hadron species are not exactly the same; see the discussion in Sec. 3.1.3.

measurements are biased. In an attempt to group analyses which are expected to select the same mixture of b hadrons, the available results (given in Table 6) are divided into the following three sets:

1. measurements at LEP and SLD that accept any b -hadron decay, based on topological reconstruction (secondary vertex or track impact parameters);
2. measurements at LEP based on the identification of a lepton from a b decay; and
3. measurements at the Tevatron based on inclusive $H_b \rightarrow J/\psi X$ reconstruction, where the J/ψ is fully reconstructed.

The measurements of the first set are generally considered as estimates of τ_b , although the efficiency to reconstruct a secondary vertex most probably depends, in an analysis-specific way, on the number of tracks coming from the vertex, thereby depending on the type of the H_b . Even though these efficiency variations can in principle be accounted for using Monte Carlo simulations (which inevitably contain assumptions on branching fractions), the H_b mixture in that case can remain somewhat ill-defined and could be slightly different among analyses in this set.

On the contrary, the mixtures corresponding to the other two sets of measurements are better defined in the limit where the reconstruction and selection efficiency of a lepton or a J/ψ from an H_b does not depend on the decaying hadron type. These mixtures are given by the production fractions and the inclusive branching fractions for each H_b species to give a lepton or a J/ψ . In particular, under the assumption that all b hadrons have the same semileptonic decay width, the analyses of the second set should measure $\tau(b \rightarrow \ell) = (\sum_i f_i \tau_i^3) / (\sum_i f_i \tau_i^2)$ which is necessarily larger than τ_b if lifetime differences exist. Given the present knowledge on τ_i and f_i , $\tau(b \rightarrow \ell) - \tau_b$ is expected to be of the order of 0.003 ps. On the other hand, the third set measuring $\tau(b \rightarrow J/\psi)$ is expected to give an average smaller than τ_b because of the B_c^+ meson which has a significantly larger probability to decay to a J/ψ than other b -hadron species.

Measurements by SLC and LEP experiments are subject to a number of common systematic uncertainties, such as those due to (lack of knowledge of) b and c fragmentation, b and c decay models, $\mathcal{B}(B \rightarrow \ell)$, $\mathcal{B}(B \rightarrow c \rightarrow \ell)$, $\mathcal{B}(c \rightarrow \ell)$, τ_c , and H_b decay multiplicity. In the averaging, these systematic uncertainties are assumed to be 100% correlated. The averages for the sets defined above (also given in Table 6) are

$$\tau(b \text{ vertex}) = 1.572 \pm 0.009 \text{ ps}, \quad (31)$$

$$\tau(b \rightarrow \ell) = 1.537 \pm 0.020 \text{ ps}, \quad (32)$$

$$\tau(b \rightarrow J/\psi) = 1.516 \pm 0.028 \text{ ps}. \quad (33)$$

3.2.3 B^0 and B^+ lifetimes and their ratio

After a number of years of dominating these averages the LEP experiments yielded the scene to the asymmetric B factories and the Tevatron experiments. The B factories have been very successful in utilizing their potential – in only a few years of running, *BABAR* and, to a greater extent, *Belle*, have struck a balance between the statistical and the systematic uncertainties, with both being close to (or even better than) the impressive 1%. In the meanwhile, CDF and D0 have emerged as significant contributors to the field as the Tevatron Run II data flowed in.

Recently, the LHCb experiment reached a further step in precision, improving by a factor ~ 2 over the previous best measurement.

At present time we are in an interesting position of having three sets of measurements (from LEP/SLC, B factories and the Tevatron) that originate from different environments, obtained using substantially different techniques and are precise enough for incisive comparison.

Table 7: Measurements of the B^0 lifetime.

Experiment	Method	Data set	$\tau(B^0)$ (ps)	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.518 \pm 0.053 \pm 0.034$	[72]
ALEPH	Exclusive	91–94	$1.25^{+0.15}_{-0.13} \pm 0.05$	[73]
ALEPH	Partial rec. $\pi^+\pi^-$	91–94	$1.49^{+0.17+0.08}_{-0.15-0.06}$	[73]
DELPHI	$D^{(*)}\ell$	91–93	$1.61^{+0.14}_{-0.13} \pm 0.08$	[74]
DELPHI	Charge sec. vtx	91–93	$1.63 \pm 0.14 \pm 0.13$	[75]
DELPHI	Inclusive $D^*\ell$	91–93	$1.532 \pm 0.041 \pm 0.040$	[76]
DELPHI	Charge sec. vtx	94–95	$1.531 \pm 0.021 \pm 0.031$	[64]
L3	Charge sec. vtx	94–95	$1.52 \pm 0.06 \pm 0.04$	[77]
OPAL	$D^{(*)}\ell$	91–93	$1.53 \pm 0.12 \pm 0.08$	[78]
OPAL	Charge sec. vtx	93–95	$1.523 \pm 0.057 \pm 0.053$	[79]
OPAL	Inclusive $D^*\ell$	91–00	$1.541 \pm 0.028 \pm 0.023$	[80]
SLD	Charge sec. vtx ℓ	93–95	$1.56^{+0.14}_{-0.13} \pm 0.10$	[81] ^a
SLD	Charge sec. vtx	93–95	$1.66 \pm 0.08 \pm 0.08$	[81] ^a
CDF1	$D^{(*)}\ell$	92–95	$1.474 \pm 0.039^{+0.052}_{-0.051}$	[82]
CDF1	Excl. $J/\psi K^{*0}$	92–95	$1.497 \pm 0.073 \pm 0.032$	[83]
CDF2	Excl. $J/\psi K_S^0, J/\psi K^{*0}$	02–09	$1.507 \pm 0.010 \pm 0.008$	[84]
D0	Excl. $J/\psi K^{*0}$	03–07	$1.414 \pm 0.018 \pm 0.034$	[85]
D0	Excl. $J/\psi K_S^0$	02–11	$1.508 \pm 0.025 \pm 0.043$	[86]
D0	Inclusive $D^-\mu^+$	02–11	$1.534 \pm 0.019 \pm 0.021$	[87]
BABAR	Exclusive	99–00	$1.546 \pm 0.032 \pm 0.022$	[88]
BABAR	Inclusive $D^*\ell$	99–01	$1.529 \pm 0.012 \pm 0.029$	[89]
BABAR	Exclusive $D^*\ell$	99–02	$1.523^{+0.024}_{-0.023} \pm 0.022$	[90]
BABAR	Incl. $D^*\pi, D^*\rho$	99–01	$1.533 \pm 0.034 \pm 0.038$	[91]
BABAR	Inclusive $D^*\ell$	99–04	$1.504 \pm 0.013^{+0.018}_{-0.013}$	[92]
Belle	Exclusive	00–03	$1.534 \pm 0.008 \pm 0.010$	[93]
ATLAS	Excl. $J/\psi K^{*0}$	2010	$1.51 \pm 0.04 \pm 0.04$	[94] ^p
LHCb	Excl. $J/\psi K^{*0}$	2011	$1.524 \pm 0.006 \pm 0.004$	[95]
LHCb	Excl. $J/\psi K_S^0$	2011	$1.499 \pm 0.013 \pm 0.005$	[95]
LHCb	$K^+\pi^-$	2011	$1.524 \pm 0.011 \pm 0.004$	[96]
Average			1.520 ± 0.004	

^a The combined SLD result quoted in [81] is $1.64 \pm 0.08 \pm 0.08$ ps.

^p Preliminary.

The averaging of $\tau(B^+)$, $\tau(B^0)$ and $\tau(B^+)/\tau(B^0)$ measurements is summarized¹⁵ in Tables 7,

¹⁵We do not include the old unpublished measurements of Refs. [99, 100].

Table 8: Measurements of the B^+ lifetime.

Experiment	Method	Data set	$\tau(B^+)$ (ps)	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.648 \pm 0.049 \pm 0.035$	[72]
ALEPH	Exclusive	91–94	$1.58^{+0.21+0.04}_{-0.18-0.03}$	[73]
DELPHI	$D^{(*)}\ell$	91–93	$1.61 \pm 0.16 \pm 0.12$	[74] ^a
DELPHI	Charge sec. vtx	91–93	$1.72 \pm 0.08 \pm 0.06$	[75] ^a
DELPHI	Charge sec. vtx	94–95	$1.624 \pm 0.014 \pm 0.018$	[64]
L3	Charge sec. vtx	94–95	$1.66 \pm 0.06 \pm 0.03$	[77]
OPAL	$D^{(*)}\ell$	91–93	$1.52 \pm 0.14 \pm 0.09$	[78]
OPAL	Charge sec. vtx	93–95	$1.643 \pm 0.037 \pm 0.025$	[79]
SLD	Charge sec. vtx ℓ	93–95	$1.61^{+0.13}_{-0.12} \pm 0.07$	[81] ^b
SLD	Charge sec. vtx	93–95	$1.67 \pm 0.07 \pm 0.06$	[81] ^b
CDF1	$D^{(*)}\ell$	92–95	$1.637 \pm 0.058^{+0.045}_{-0.043}$	[82]
CDF1	Excl. $J/\psi K$	92–95	$1.636 \pm 0.058 \pm 0.025$	[83]
CDF2	Excl. $J/\psi K$	02–09	$1.639 \pm 0.009 \pm 0.009$	[84]
CDF2	Excl. $D^0\pi$	02–06	$1.663 \pm 0.023 \pm 0.015$	[97]
<i>BABAR</i>	Exclusive	99–00	$1.673 \pm 0.032 \pm 0.023$	[88]
Belle	Exclusive	00–03	$1.635 \pm 0.011 \pm 0.011$	[93]
LHCb	Excl. $J/\psi K$	2011	$1.637 \pm 0.004 \pm 0.003$	[95]
Average			1.638 ± 0.004	

^a The combined DELPHI result quoted in [75] is 1.70 ± 0.09 ps.

^b The combined SLD result quoted in [81] is $1.66 \pm 0.06 \pm 0.05$ ps.

Table 9: Measurements of the ratio $\tau(B^+)/\tau(B^0)$.

Experiment	Method	Data set	Ratio $\tau(B^+)/\tau(B^0)$	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.085 \pm 0.059 \pm 0.018$	[72]
ALEPH	Exclusive	91–94	$1.27^{+0.23+0.03}_{-0.19-0.02}$	[73]
DELPHI	$D^{(*)}\ell$	91–93	$1.00^{+0.17}_{-0.15} \pm 0.10$	[74]
DELPHI	Charge sec. vtx	91–93	$1.06^{+0.13}_{-0.11} \pm 0.10$	[75]
DELPHI	Charge sec. vtx	94–95	$1.060 \pm 0.021 \pm 0.024$	[64]
L3	Charge sec. vtx	94–95	$1.09 \pm 0.07 \pm 0.03$	[77]
OPAL	$D^{(*)}\ell$	91–93	$0.99 \pm 0.14^{+0.05}_{-0.04}$	[78]
OPAL	Charge sec. vtx	93–95	$1.079 \pm 0.064 \pm 0.041$	[79]
SLD	Charge sec. vtx ℓ	93–95	$1.03^{+0.16}_{-0.14} \pm 0.09$	[81] ^a
SLD	Charge sec. vtx	93–95	$1.01^{+0.09}_{-0.08} \pm 0.05$	[81] ^a
CDF1	$D^{(*)}\ell$	92–95	$1.110 \pm 0.056^{+0.033}_{-0.030}$	[82]
CDF1	Excl. $J/\psi K$	92–95	$1.093 \pm 0.066 \pm 0.028$	[83]
CDF2	Excl. $J/\psi K^{(*)}$	02–09	$1.088 \pm 0.009 \pm 0.004$	[84]
D0	$D^{*+}\mu D^0\mu$ ratio	02–04	$1.080 \pm 0.016 \pm 0.014$	[98]
BABAR	Exclusive	99–00	$1.082 \pm 0.026 \pm 0.012$	[88]
Belle	Exclusive	00–03	$1.066 \pm 0.008 \pm 0.008$	[93]
LHCb	Excl. $J/\psi K^{(*)}$	2011	$1.074 \pm 0.005 \pm 0.003$	[95]
Average			1.076 ± 0.004	

^a The combined SLD result quoted in [81] is $1.01 \pm 0.07 \pm 0.06$.

8, and 9. For $\tau(B^+)/\tau(B^0)$ we average only the measurements of this quantity provided by experiments rather than using all available knowledge, which would have included, for example, $\tau(B^+)$ and $\tau(B^0)$ measurements which did not contribute to any of the ratio measurements.

The following sources of correlated (within experiment/machine) systematic uncertainties have been considered:

- for SLC/LEP measurements – D^{**} branching ratio uncertainties [3], momentum estimation of b mesons from Z^0 decays (b -quark fragmentation parameter $\langle X_E \rangle = 0.702 \pm 0.008$ [3]), B_s^0 and b -baryon lifetimes (see Secs. 3.2.4 and 3.2.6), and b -hadron fractions at high energy (see Table 5);
- for BABAR measurements – alignment, z scale, PEP-II boost, sample composition (where applicable);
- for D0 and CDF Run II measurements – alignment (separately within each experiment).

The resultant averages are:

$$\tau(B^0) = 1.520 \pm 0.004 \text{ ps}, \quad (34)$$

$$\tau(B^+) = 1.638 \pm 0.004 \text{ ps}, \quad (35)$$

$$\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004. \quad (36)$$

3.2.4 B_s^0 lifetimes

Like neutral kaons, neutral B mesons contain short- and long-lived components, since the light (L) and heavy (H) eigenstates, B_L and B_H , differ not only in their masses, but also in their total decay widths, with a decay width difference defined as $\Delta\Gamma = \Gamma_L - \Gamma_H$. Neglecting CP violation in $B - \bar{B}$ mixing, which is expected to be very small [101, 102] (see also Sec. 3.3.3), the mass eigenstates are also CP eigenstates, with the light B_L state being CP -even and the heavy B_H state being CP -odd. While the decay width difference $\Delta\Gamma_d$ can be neglected in the B^0 system, the B_s^0 system exhibits a significant value of $\Delta\Gamma_s$: the sign of $\Delta\Gamma_s$ is known to be positive [103], *i.e.* the heavy eigenstate lives longer than the light eigenstate. Specific measurements of $\Delta\Gamma_s$ and $\Gamma_s = (\Gamma_L + \Gamma_H)/2$ are explained and averaged in Sec. 3.3.2, but the results for $1/\Gamma_L$, $1/\Gamma_H$ and the mean B_s^0 lifetime, defined as $\tau(B_s^0) = 1/\Gamma_s$, are also quoted at the end of this section.

Many B_s^0 lifetime analyses, in particular the early ones performed before the non-zero value of $\Delta\Gamma_s$ was firmly established, ignore $\Delta\Gamma_s$ and fit the proper time distribution of a sample of B_s^0 candidates reconstructed in a certain final state f with a model assuming a single exponential function for the signal. We denote such effective lifetime measurements [104] as $\tau_{\text{single}}(B_s^0 \rightarrow f)$; their true values may lie *a priori* anywhere between $1/\Gamma_L = 1/(\Gamma_s + \Delta\Gamma_s/2)$ and $1/\Gamma_H = 1/(\Gamma_s - \Delta\Gamma_s/2)$, depending on the proportion of B_L and B_H in the final state f . More recent determinations of effective lifetimes may be interpreted as measurements of the relative composition of B_L and B_H decaying to the final state f . Table 10 summarizes the effective lifetime measurements.

Averaging measurements of $\tau_{\text{single}}(B_s^0 \rightarrow f)$ over several final states f will yield a result corresponding to an ill-defined observable when the proportions of B_L and B_H differ. Therefore, the effective B_s^0 lifetime measurements are broken down into several categories and averaged separately.

Table 10: Measurements of the effective B_s^0 lifetimes obtained from single exponential fits.

Experiment	Final state f		Data set		$\tau_{\text{single}}(B_s^0 \rightarrow f)$ (ps)	Ref.
ALEPH	$D_s^- \ell^+$	flavour-specific	91–95		$1.54_{-0.13}^{+0.14} \pm 0.04$	[105]
CDF1	$D_s^- \ell^+$	flavour-specific	92–96		$1.36 \pm 0.09_{-0.05}^{+0.06}$	[106]
DELPHI	$D_s^- \ell^+$	flavour-specific	91–95		$1.42_{-0.13}^{+0.14} \pm 0.03$	[107]
OPAL	$D_s^- \ell^+$	flavour-specific	90–95		$1.50_{-0.15}^{+0.16} \pm 0.04$	[108]
D0	$D_s^- \mu^+ X$	flavour-specific	Run II	10.4 fb $^{-1}$	$1.479 \pm 0.010 \pm 0.021$	[87]
CDF2	$D_s^- \pi^+(X)$	flavour-specific	02–06	1.3 fb $^{-1}$	$1.518 \pm 0.041 \pm 0.027$	[109]
LHCb	$D_s^- D^+$	flavour-specific	11–12	3 fb $^{-1}$	$1.52 \pm 0.15 \pm 0.01$	[110]
LHCb	$D_s^- \pi^+$	flavour-specific	11	1 fb $^{-1}$	$1.535 \pm 0.015 \pm 0.014$	[111]
Average of above 8 flavour-specific lifetime measurements					1.511 ± 0.014	
LHCb	$\pi^+ K^-$	flavour-specific	11	1.0 fb $^{-1}$	$1.60 \pm 0.06 \pm 0.01$	[96]
ALEPH	$D_s h$	ill-defined	91–95		$1.47 \pm 0.14 \pm 0.08$	[112]
DELPHI	$D_s h$	ill-defined	91–95		$1.53_{-0.15}^{+0.16} \pm 0.07$	[113]
OPAL	D_s incl.	ill-defined	90–95		$1.72_{-0.19-0.17}^{+0.20+0.18}$	[114]
CDF1	$J/\psi \phi$	CP even+odd	92–95		$1.34_{-0.19}^{+0.23} \pm 0.05$	[70]
D0	$J/\psi \phi$	CP even+odd	02–04		$1.444_{-0.090}^{+0.098} \pm 0.02$	[115]
ATLAS	$J/\psi \phi$	CP even+odd	10	40 pb $^{-1}$	$1.41 \pm 0.08 \pm 0.05$	[94] ^p
LHCb	$J/\psi \phi$	CP even+odd	11	1 fb $^{-1}$	$1.480 \pm 0.011 \pm 0.005$	[95]
Average of above 4 $J/\psi \phi$ lifetime measurements					1.478 ± 0.012	
ALEPH	$D_s^{(*)+} D_s^{(*)-}$	mostly CP even	91–95		$1.27 \pm 0.33 \pm 0.08$	[116]
LHCb	$K^+ K^-$	CP -even	10	0.037 fb $^{-1}$	$1.440 \pm 0.096 \pm 0.009$	[117]
LHCb	$K^+ K^-$	CP -even	11	1.0 fb $^{-1}$	$1.407 \pm 0.016 \pm 0.007$	[96]
Average of above 2 $K^+ K^-$ lifetime measurements					1.408 ± 0.017	
LHCb	$D_s^+ D_s^-$	CP -even	11–12	3 fb $^{-1}$	$1.379 \pm 0.026 \pm 0.017$	[110]
Average of above 1 measurement of $1/\Gamma_L$					1.379 ± 0.031	
LHCb	$J/\psi K_S^0$	CP -odd	11	1.0 fb $^{-1}$	$1.75 \pm 0.12 \pm 0.07$	[118]
CDF2	$J/\psi f_0(980)$	CP -odd	02–08	3.8 fb $^{-1}$	$1.70_{-0.11}^{+0.12} \pm 0.03$	[119]
LHCb	$J/\psi \pi^+ \pi^-$	CP -odd	11	1.0 fb $^{-1}$	$1.652 \pm 0.024 \pm 0.024$	[120]
Average of above 2 measurements of $1/\Gamma_H$					1.656 ± 0.033	

^p Preliminary.

- **Decays to a flavour-specific final state** without CP violation in the decay amplitude, such as $B_s^0 \rightarrow D_s^- \ell^+ \nu$ or $B_s^0 \rightarrow D_s^- \pi^+$, have equal fractions of B_L and B_H at time zero.¹⁶ If the resulting superposition of two exponential distributions is fitted with a single exponential function, one obtains a measure of the so-called *flavour-specific lifetime* [121]:

$$\tau_{\text{single}}(B_s^0 \rightarrow \text{flavour specific}) = \frac{1}{\Gamma_s} \frac{1 + \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}{1 - \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}. \quad (37)$$

The average of all flavour-specific B_s^0 lifetime measurements¹⁷ is

$$\tau_{\text{single}}(B_s^0 \rightarrow \text{flavour specific}) = 1.511 \pm 0.014 \text{ ps}. \quad (38)$$

This average does not include an effective lifetime measurement of $B_s^0 \rightarrow \pi^+ K^-$ decays [96].

- **$B_s^0 \rightarrow D_s^\mp X$ decays** include flavour-specific decays but also decays with an unknown mixture of light and heavy components. Measurements performed with such inclusive states are no longer used in averages.
- **$B_s^0 \rightarrow J/\psi \phi$ decays** contain a well-measured mixture of CP -even and CP -odd states. There are no known correlations between the existing $B_s^0 \rightarrow J/\psi \phi$ effective lifetime measurements; these are combined into the average¹⁸ $\tau_{\text{single}}(B_s^0 \rightarrow J/\psi \phi) = 1.478 \pm 0.012 \text{ ps}$. A caveat is that different experimental acceptances may lead to different admixtures of the CP -even and CP -odd states, and simple fits to a single exponential may result in inherently different values of $\tau_{\text{single}}(B_s^0 \rightarrow J/\psi \phi)$. Analyses that separate the CP -even and CP -odd components in this decay through a full angular study, outlined in Sec. 3.3.2, provide directly precise measurements of $1/\Gamma_s$ and $\Delta\Gamma_s$ (see Table 22).
- **Decays to CP eigenstates** have also been measured, in the CP -even modes $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$ by ALEPH [116], $B_s^0 \rightarrow K^+ K^-$ by LHCb [96, 117]¹⁹ and $B_s^0 \rightarrow D_s^+ D_s^-$ by LHCb [110], as well as in the CP -odd modes $B_s^0 \rightarrow J/\psi f_0(980)$ by CDF [119], $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ by LHCb [120] and $B_s^0 \rightarrow J/\psi K_S^0$ by LHCb [118]. If these decays are dominated by a single weak phase and if CP violation can be neglected, then $\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-even}) = 1/\Gamma_L$ and $\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-odd}) = 1/\Gamma_H$ (see Eqs. (61) and (62) for approximate relations in presence of CP violation in the mixing). However, not all these modes can be considered as pure CP eigenstates: a small CP -odd component is most probably present in $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$ decays. Furthermore the decays $B_s^0 \rightarrow K^+ K^-$ and $B_s^0 \rightarrow J/\psi K_S^0$ may suffer from CP violation due to interfering tree and loop amplitudes. The averages for the effective lifetimes obtained for decays to pure CP -even ($D_s^+ D_s^-$) and CP -odd ($J/\psi f_0(980)$, $J/\psi \pi^+ \pi^-$) final states, where CP conservation can be assumed, are

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-even}) = 1.379 \pm 0.031 \text{ ps}, \quad (39)$$

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-odd}) = 1.656 \pm 0.033 \text{ ps}. \quad (40)$$

¹⁶The assumption that such decays are flavour-specific is valid to an excellent approximation in the SM. However, there are few experimental tests of it.

¹⁷An old unpublished measurement [122] is not included.

¹⁸An old unpublished measurement [123] is not included.

¹⁹An old unpublished measurement of the $B_s^0 \rightarrow K^+ K^-$ effective lifetime by CDF [124] is no longer considered.

Table 11: Measurements of the B_c^+ lifetime.

Experiment	Method	Data set		$\tau(B_c^+)$ (ps)	Ref.
CDF1	$J/\psi \ell$	92–95	0.11 fb $^{-1}$	$0.46_{-0.16}^{+0.18} \pm 0.03$	[125]
CDF2	$J/\psi e$	02–04	0.36 fb $^{-1}$	$0.463_{-0.065}^{+0.073} \pm 0.036$	[127]
D0	$J/\psi \mu$	02–06	1.3 fb $^{-1}$	$0.448_{-0.036}^{+0.038} \pm 0.032$	[128]
CDF2	$J/\psi \pi$		6.7 fb $^{-1}$	$0.452 \pm 0.048 \pm 0.027$	[129]
LHCb	$J/\psi \mu$	12	2 fb $^{-1}$	$0.509 \pm 0.008 \pm 0.012$	[130]
LHCb	$J/\psi \pi$	11–12	3 fb $^{-1}$	$0.5134 \pm 0.0110 \pm 0.0057$	[131]
Average				0.507 ± 0.009	

As described in Sec. 3.3.2, the effective lifetime averages of Eqs. (38), (39), and (40) are used as ingredients to improve the determination of $1/\Gamma_s$ and $\Delta\Gamma_s$ obtained from the full angular analyses of $B_s^0 \rightarrow J/\psi \phi$ and $B_s^0 \rightarrow J/\psi K^+ K^-$ decays. The resulting world averages for the B_s^0 lifetimes are

$$\tau(B_{sL}) = \frac{1}{\Gamma_L} = \frac{1}{\Gamma_s + \Delta\Gamma_s/2} = 1.422 \pm 0.006 \text{ ps}, \quad (41)$$

$$\tau(B_{sH}) = \frac{1}{\Gamma_H} = \frac{1}{\Gamma_s - \Delta\Gamma_s/2} = 1.607 \pm 0.010 \text{ ps}, \quad (42)$$

$$\tau(B_s^0) = \frac{1}{\Gamma_s} = \frac{2}{\Gamma_L + \Gamma_H} = 1.509 \pm 0.004 \text{ ps}. \quad (43)$$

3.2.5 B_c^+ lifetime

Early measurements of the B_c^+ meson lifetime, from CDF [125–127] and D0 [128], use the semileptonic decay mode $B_c^+ \rightarrow J/\psi \ell^+ \nu$ and are based on a simultaneous fit to the mass and lifetime using the vertex formed with the leptons from the decay of the J/ψ and the third lepton. Correction factors to estimate the boost due to the missing neutrino are used. Correlated systematic errors include the impact of the uncertainty of the B_c^+ p_T spectrum on the correction factors, the level of feed-down from $\psi(2S)$ decays, Monte Carlo modeling of the decay model varying from phase space to the ISGW model, and mass variations. With more statistics, CDF2 was able to perform the first B_c^+ lifetime based on fully reconstructed $B_c^+ \rightarrow J/\psi \pi^+$ decays [129], which does not suffer from a missing neutrino. Recent measurements from LHCb, both with $B_c^+ \rightarrow J/\psi \mu^+ \nu$ [130] and $B_c^+ \rightarrow J/\psi \pi^+$ [131] decays, achieve the highest level of precision.

All the measurements²⁰ are summarized in Table 11 and the world average, dominated by the LHCb measurements, is determined to be

$$\tau(B_c^+) = 0.507 \pm 0.009 \text{ ps}. \quad (44)$$

²⁰We do not list (nor include in the average) an unpublished result from CDF2 [126].

3.2.6 Λ_b^0 and b -baryon lifetimes

The first measurements of b -baryon lifetimes, performed at LEP, originate from two classes of partially reconstructed decays. In the first class, decays with an exclusively reconstructed Λ_c^+ baryon and a lepton of opposite charge are used. These products are more likely to occur in the decay of Λ_b^0 baryons. In the second class, more inclusive final states with a baryon (p , $\bar{p}$, Λ , or $\bar{\Lambda}$) and a lepton have been used, and these final states can generally arise from any b baryon. With the large b -hadron samples available at the Tevatron and the LHC, the most precise measurements of b baryons now come from fully reconstructed exclusive decays.

The following sources of correlated systematic uncertainties have been considered: experimental time resolution within a given experiment, b -quark fragmentation distribution into weakly decaying b baryons, Λ_b^0 polarization, decay model, and evaluation of the b -baryon purity in the selected event samples. In computing the averages the central values of the masses are scaled to $M(\Lambda_b^0) = 5620 \pm 2 \text{ MeV}/c^2$ [132] and $M(b\text{-baryon}) = 5670 \pm 100 \text{ MeV}/c^2$.

For the semi-inclusive lifetime measurements, the meaning of the decay model systematic uncertainties and the correlation of these uncertainties between measurements are not always clear. Uncertainties related to the decay model are dominated by assumptions on the fraction of n -body semileptonic decays. To be conservative, it is assumed that these are 100% correlated whenever given as an error. DELPHI varies the fraction of 4-body decays from 0.0 to 0.3. In computing the average, the DELPHI result is corrected to a value of 0.2 ± 0.2 for this fraction.

Furthermore, in computing the average, the semileptonic decay results from LEP are corrected for a polarization of $-0.45_{-0.17}^{+0.19}$ [3] and a Λ_b^0 fragmentation parameter $\langle X_E \rangle = 0.70 \pm 0.03$ [133].

Inputs to the averages are given in Table 12. For the Λ_b^0 lifetime average, we only include measurements obtained with inclusive $\Lambda_c^\pm \ell^\mp$, inclusive $\Lambda \ell^- \ell^+$, and fully exclusive final states. The CDF $\Lambda_b^0 \rightarrow J/\psi \Lambda$ lifetime result [140] is larger than the world average computed excluding this result by 2.5σ . It is nonetheless combined with the rest without adjustment of input errors. The world average Λ_b^0 lifetime is then

$$\tau(\Lambda_b^0) = 1.467 \pm 0.010 \text{ ps} . \quad (45)$$

It turns out that the average obtained using only measurements performed with semileptonic Λ_b^0 decays ($1.245_{-0.069}^{+0.071} \text{ ps}$) is significantly different from the one using only measurements performed with exclusively reconstructed Λ_b^0 decays ($1.471 \pm 0.010 \text{ ps}$). The latter is much more precise (and less prone to systematic uncertainties) than the former. This discrepancy can only be attributed to a systematic experimental effect or to a statistical fluctuation.

For the strange b baryons, we no longer include measurements based on $\Xi^\mp \ell^\mp$ [32–34] final states which consist of a mixture of Ξ_b^0 and Ξ_b^- baryons. Instead we only average results obtained with fully exclusive modes, and obtain

$$\tau(\Xi_b^-) = 1.559 \pm 0.037 \text{ ps} , \quad (46)$$

$$\tau(\Xi_b^0) = 1.465 \pm 0.031 \text{ ps} , \quad (47)$$

$$\tau(\Omega_b^-) = 1.57_{-0.20}^{+0.23} \text{ ps} . \quad (48)$$

3.2.7 Summary and comparison with theoretical predictions

Averages of lifetimes of specific b -hadron species are collected in Table 13. As described in

Table 12: Measurements of the b -baryon lifetimes.

Experiment	Method	Data set	Lifetime (ps)	Ref.
ALEPH	$\Lambda\ell$	91–95	$1.20 \pm 0.08 \pm 0.06$	[31]
DELPHI	$\Lambda\ell\pi$ vtx	91–94	$1.16 \pm 0.20 \pm 0.08$	[134] ^b
DELPHI	$\Lambda\mu$ i.p.	91–94	$1.10^{+0.19}_{-0.17} \pm 0.09$	[135] ^b
DELPHI	$p\ell$	91–94	$1.19 \pm 0.14 \pm 0.07$	[134] ^b
OPAL	$\Lambda\ell$ i.p.	90–94	$1.21^{+0.15}_{-0.13} \pm 0.10$	[136] ^c
OPAL	$\Lambda\ell$ vtx	90–94	$1.15 \pm 0.12 \pm 0.06$	[136] ^c
ALEPH	$\Lambda_c^+\ell$	91–95	$1.18^{+0.13}_{-0.12} \pm 0.03$	[31] ^a
ALEPH	$\Lambda\ell^-\ell^+$	91–95	$1.30^{+0.26}_{-0.21} \pm 0.04$	[31] ^a
DELPHI	$\Lambda_c^+\ell$	91–94	$1.11^{+0.19}_{-0.18} \pm 0.05$	[134] ^b
OPAL	$\Lambda_c^+\ell, \Lambda\ell^-\ell^+$	90–95	$1.29^{+0.24}_{-0.22} \pm 0.06$	[108]
CDF1	$\Lambda_c^+\ell$	91–95	$1.32 \pm 0.15 \pm 0.07$	[137]
D0	$\Lambda_c^+\mu$	02–06	$1.290^{+0.119+0.087}_{-0.110-0.091}$	[138]
Average of above 6 (semileptonic Λ_b^0 decays)			$1.245^{+0.071}_{-0.069}$	
CDF2	$\Lambda_c^+\pi$	02–06	$1.401 \pm 0.046 \pm 0.035$	[139]
CDF2	$J/\psi\Lambda$	02–11	$1.565 \pm 0.035 \pm 0.020$	[140]
D0	$J/\psi\Lambda$	02–11	$1.303 \pm 0.075 \pm 0.035$	[86]
ATLAS	$J/\psi\Lambda$	2011	$1.449 \pm 0.036 \pm 0.017$	[141]
CMS	$J/\psi\Lambda$	2011	$1.503 \pm 0.052 \pm 0.031$	[142]
LHCb	$J/\psi\Lambda$	2011	$1.415 \pm 0.027 \pm 0.006$	[95]
LHCb	$J/\psi pK$	11–12	$1.479 \pm 0.009 \pm 0.010$	[143]
Average of above 7 (fully reconstructed Λ_b^0 decays)			1.471 ± 0.010	
Average of above 13: Λ_b^0 lifetime =			1.467 ± 0.010	
ALEPH	$\Xi\ell$	90–95	$1.35^{+0.37+0.15}_{-0.28-0.17}$	[32]
DELPHI	$\Xi\ell$	91–93	$1.5^{+0.7}_{-0.4} \pm 0.3$	[34] ^d
DELPHI	$\Xi\ell$	92–95	$1.45^{+0.55}_{-0.43} \pm 0.13$	[33] ^d
CDF2	$J/\psi\Xi^-$	02–11	$1.32 \pm 0.14 \pm 0.02$	[140]
LHCb	$J/\psi\Xi^-$	11–12	$1.55^{+0.10}_{-0.09} \pm 0.03$	[144]
LHCb	$\Xi_c^0\pi^-$	11–12	$1.599 \pm 0.041 \pm 0.022$	[145]
Average of above 3: Ξ_b^- lifetime =			1.559 ± 0.037	
LHCb	$\Xi_c^+\pi^-$	11–12	$1.477 \pm 0.026 \pm 0.019$	[146]
Average of above 1: Ξ_b^0 lifetime =			1.465 ± 0.031	
CDF2	$J/\psi\Omega^-$	02–11	$1.66^{+0.53}_{-0.40} \pm 0.02$	[140]
LHCb	$J/\psi\Omega^-$	11–12	$1.54^{+0.26}_{-0.21} \pm 0.05$	[144]
Average of above 2: Ω_b^- lifetime =			$1.57^{+0.23}_{-0.20}$	

^a The combined ALEPH result quoted in [31] is 1.21 ± 0.11 ps.

^b The combined DELPHI result quoted in [134] is $1.14 \pm 0.08 \pm 0.04$ ps.

^c The combined OPAL result quoted in [136] is $1.16 \pm 0.11 \pm 0.06$ ps.

^d The combined DELPHI result quoted in [33] is $1.48^{+0.40}_{-0.31} \pm 0.12$ ps.

the introduction to Sec. 3.2, the HQE can be employed to explain the hierarchy of $\tau(B_c^+) \ll$

Table 13: Summary of the lifetime averages for the different b -hadron species.

b -hadron species	Measured lifetime
B^+	1.638 ± 0.004 ps
B^0	1.520 ± 0.004 ps
B_s^0 $1/\Gamma_s =$	1.509 ± 0.004 ps
B_{sL} $1/\Gamma_L =$	1.422 ± 0.006 ps
B_{sH} $1/\Gamma_H =$	1.607 ± 0.010 ps
B_c^+	0.507 ± 0.009 ps
Λ_b^0	1.467 ± 0.010 ps
Ξ_b^-	1.559 ± 0.037 ps
Ξ_b^0	1.465 ± 0.031 ps
Ω_b^-	$1.57^{+0.23}_{-0.20}$ ps

Table 14: Measured ratios of b -hadron lifetimes relative to the B^0 lifetime and ranges predicted by theory [58, 59].

Lifetime ratio	Measured value	Predicted range
$\tau(B^+)/\tau(B^0)$	1.076 ± 0.004	$1.04 - 1.08$
$\tau(B_s^0)/\tau(B^0)$	0.993 ± 0.004	$0.99 - 1.01$
$\tau(\Lambda_b^0)/\tau(B^0)$	0.965 ± 0.007	$0.86 - 0.95$

$\tau(\Lambda_b^0) < \tau(B_s^0) \approx \tau(B^0) < \tau(B^+)$, and used to predict the ratios between lifetimes. Typical predictions are compared to the measured lifetime ratios in Table 14. The prediction of the ratio between the B^+ and B^0 lifetimes, 1.06 ± 0.02 [58], is in good agreement with experiment.

The total widths of the B_s^0 and B^0 mesons are expected to be very close and differ by at most 1% [59, 147]. This prediction is consistent with the experimental ratio $\tau(B_s^0)/\tau(B^0) = \Gamma_d/\Gamma_s$, which is smaller than 1 by $(0.7 \pm 0.4)\%$.

The ratio $\tau(\Lambda_b^0)/\tau(B^0)$ has particularly been the source of theoretical scrutiny since earlier calculations using the HQE [55, 148] predicted a value larger than 0.90, almost 2σ above the world average at the time. Many predictions cluster around a most likely central value of 0.94 [149]. More recent calculations of this ratio that include higher-order effects predict a lower ratio between the Λ_b^0 and B^0 lifetimes [58, 59] and reduce this difference. References [58, 59] present probability density functions of their predictions with a variation of theoretical inputs, and the indicated ranges in Table 14 are the RMS of the distributions from the most probable values, and for $\tau(\Lambda_b^0)/\tau(B^0)$, also encompass the earlier theoretical predictions [55, 148, 149]. Note that in contrast to the B mesons, complete NLO QCD corrections and fully reliable lattice determinations of the matrix elements for Λ_b^0 are not yet available. As already mentioned, the CDF measurement of the Λ_b^0 lifetime in the exclusive decay mode $J/\psi \Lambda$ [140] is significantly higher than the world average before inclusion, with a ratio to the $\tau(B^0)$ world average of $\tau(\Lambda_b^0)/\tau(B^0) = 1.012 \pm 0.031$, resulting in continued interest in lifetimes of b baryons.

The lifetimes of the most abundant b -hadron species are now all known to sub-percent precision. Neglecting the contributions of the rarer species (B_c^+ meson and b baryons other

than the Λ_b^0), one can compute the average b -hadron lifetime from the individual lifetimes and production fractions as

$$\tau_b = \frac{f_d \tau(B^0)^2 + f_u \tau(B^+)^2 + 0.5 f_s \tau(B_{sH})^2 + 0.5 f_s \tau(B_{sL})^2 + f_{\text{baryon}} \tau(\Lambda_b^0)^2}{f_d \tau(B^0) + f_u \tau(B^+) + 0.5 f_s \tau(B_{sH}) + 0.5 f_s \tau(B_{sL}) + f_{\text{baryon}} \tau(\Lambda_b^0)}. \quad (49)$$

Using the lifetimes of Table 13 and the fractions in Z decays of Table 5, taking into account the correlations between the fractions (Table 5) as well as the correlation between $\tau(B_{sH})$ and $\tau(B_{sL})$ (-0.464), one obtains

$$\tau_b(Z) = 1.567 \pm 0.003 \text{ ps}. \quad (50)$$

This is in very good agreement with (and three times more precise than) the average of Eq. (31) for the inclusive measurements performed at LEP.

3.3 Neutral B -meson mixing

The $B^0 - \bar{B}^0$ and $B_s^0 - \bar{B}_s^0$ systems both exhibit the phenomenon of particle-antiparticle mixing. For each of them, there are two mass eigenstates which are linear combinations of the two flavour states, B and $\bar{B}$. The heaviest (lightest) of these mass states is denoted B_H (B_L), with mass m_H (m_L) and total decay width Γ_H (Γ_L). We define

$$\Delta m = m_H - m_L, \quad x = \Delta m / \Gamma, \quad (51)$$

$$\Delta \Gamma = \Gamma_L - \Gamma_H, \quad y = \Delta \Gamma / (2\Gamma), \quad (52)$$

where $\Gamma = (\Gamma_H + \Gamma_L)/2 = 1/\bar{\tau}(B)$ is the average decay width. Δm is positive by definition, and $\Delta \Gamma$ is expected to be positive within the Standard Model.²¹

There are four different time-dependent probabilities describing the case of a neutral B meson produced as a flavour state and decaying without CP violation to a flavour-specific final state. If CPT is conserved (which will be assumed throughout), they can be written as

$$\begin{cases} \mathcal{P}(B \rightarrow B) &= \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{\Delta \Gamma}{2} t\right) + \cos(\Delta m t) \right] \\ \mathcal{P}(B \rightarrow \bar{B}) &= \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{\Delta \Gamma}{2} t\right) - \cos(\Delta m t) \right] \left| \frac{q}{p} \right|^2 \\ \mathcal{P}(\bar{B} \rightarrow B) &= \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{\Delta \Gamma}{2} t\right) - \cos(\Delta m t) \right] \left| \frac{p}{q} \right|^2 \\ \mathcal{P}(\bar{B} \rightarrow \bar{B}) &= \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{\Delta \Gamma}{2} t\right) + \cos(\Delta m t) \right] \end{cases}, \quad (53)$$

where t is the proper time of the system (*i.e.* the time interval between the production and the decay in the rest frame of the B meson). At the B factories, only the proper-time difference Δt between the decays of the two neutral B mesons from the $\Upsilon(4S)$ can be determined, but, because the two B mesons evolve coherently (keeping opposite flavours as long as neither of them has decayed), the above formulae remain valid if t is replaced with Δt and the production flavour is replaced by the flavour at the time of the decay of the accompanying B meson in a flavour-specific state. As can be seen in the above expressions, the mixing probabilities depend on three mixing observables: Δm , $\Delta \Gamma$, and $|q/p|^2$, which signals CP violation in the mixing if $|q/p|^2 \neq 1$.

²¹ For reasons of symmetry in Eqs. (51) and (52), $\Delta \Gamma$ is sometimes defined with the opposite sign. The definition adopted here, *i.e.* Eq. (52), is the one used by most experimentalists and many phenomenologists in B physics.

In the next sections we review in turn the experimental knowledge on the B^0 decay-width and mass differences, the B_s^0 decay-width and mass differences, CP violation in B^0 and B_s^0 mixing, and mixing-induced CP violation in B_s^0 decays.

3.3.1 B^0 mixing parameters $\Delta\Gamma_d$ and Δm_d

A large number of time-dependent $B^0\text{--}\overline{B}^0$ oscillation analyses have been performed since almost 20 years by the ALEPH, DELPHI, L3, OPAL, CDF, D0, *BABAR*, Belle and LHCb collaborations. The corresponding measurements of Δm_d are summarized in Table 15, where only the most recent results are listed (*i.e.* measurements superseded by more recent ones are omitted²²). Although a variety of different techniques have been used, the individual Δm_d results obtained at different colliders have remarkably similar precision. The systematic uncertainties are comparable to the statistical uncertainties; they are often dominated by sample composition, mistag probability, or b -hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the averages of the b -hadron fractions and lifetimes given in this report (see Secs. 3.1 and 3.2). Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fractions, lifetimes, branching ratios of b hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, flavour tagging, background description). Combining all published measurements listed in Table 15 and accounting for all identified correlations as described in Ref. [3] yields $\Delta m_d = 0.510 \pm 0.003 \pm 0.002 \text{ ps}^{-1}$.

On the other hand, ARGUS and CLEO have published measurements of the time-integrated mixing probability χ_d [170–172], which average to $\chi_d = 0.182 \pm 0.015$. Following Ref. [172], the decay width difference $\Delta\Gamma_d$ could in principle be extracted from the measured value of $\Gamma_d = 1/\tau(B^0)$ and the above averages for Δm_d and χ_d (provided that $\Delta\Gamma_d$ has a negligible impact on the Δm_d and $\tau(B^0)$ analyses that have assumed $\Delta\Gamma_d = 0$), using the relation

$$\chi_d = \frac{x_d^2 + y_d^2}{2(x_d^2 + 1)} \quad \text{with} \quad x_d = \frac{\Delta m_d}{\Gamma_d} \quad \text{and} \quad y_d = \frac{\Delta\Gamma_d}{2\Gamma_d}. \quad (54)$$

However, direct time-dependent studies provide much stronger constraints: $|\Delta\Gamma_d|/\Gamma_d < 18\%$ at 95% CL from DELPHI [152], $-6.8\% < \text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d < 8.4\%$ at 90% CL from *BABAR* [173], and $\text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d = (1.7 \pm 1.8 \pm 1.1)\%$ [174] from Belle, where $\lambda_{CP} = (q/p)_d(\overline{A}_{CP}/A_{CP})$ is defined for a CP -even final state (the sensitivity to the overall sign of $\text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d$ comes from the use of B^0 decays to CP final states). In addition, the D0 collaboration has recently extracted a value of $\Delta\Gamma_d/\Gamma_d = (0.50 \pm 1.38)\%$ [175] from their measurements of the same-sign dimuon charge asymmetry, under the interpretation that the observed asymmetries are due to CP violation in neutral B -meson mixing and interference. More recently LHCb has obtained $\Delta\Gamma_d/\Gamma_d = (-0.044 \pm 0.025 \pm 0.011)\%$ [95] by comparing measurements of the $B^0 \rightarrow J/\psi K^{*0}$ and $B^0 \rightarrow J/\psi K_S^0$ decays, following the method of Ref. [176]. Assuming $\text{Re}\lambda_{CP} > 0$, as expected from the global fits of the Unitarity Triangle within the Standard Model [177], a combination of these five results (after adjusting the DELPHI and *BABAR* results to $1/\Gamma_d = \tau(B^0) = 1.520 \pm 0.004 \text{ ps}$) yields

$$\Delta\Gamma_d/\Gamma_d = 0.001 \pm 0.010. \quad (55)$$

²² Two old unpublished CDF2 measurements [168, 169] are also omitted from our averages, Table 15 and Fig. 6.

Table 15: Time-dependent measurements included in the Δm_d average. The results obtained from multi-dimensional fits involving also the B^0 (and B^+) lifetimes as free parameter(s) [90, 92, 93] have been converted into one-dimensional measurements of Δm_d . All the measurements have then been adjusted to a common set of physics parameters before being combined.

Experiment and Ref.	Method		Δm_d in ps^{-1}	Δm_d in ps^{-1}
	rec.	tag	before adjustment	after adjustment
ALEPH [150]	ℓ	Q_{jet}	$0.404 \pm 0.045 \pm 0.027$	
ALEPH [150]	ℓ	ℓ	$0.452 \pm 0.039 \pm 0.044$	
ALEPH [150]	above two combined		$0.422 \pm 0.032 \pm 0.026$	$0.440 \pm 0.032 \begin{smallmatrix} +0.020 \\ -0.019 \end{smallmatrix}$
ALEPH [150]	D^*	ℓ, Q_{jet}	$0.482 \pm 0.044 \pm 0.024$	$0.482 \pm 0.044 \pm 0.024$
DELPHI [151]	ℓ	Q_{jet}	$0.493 \pm 0.042 \pm 0.027$	$0.497 \pm 0.042 \pm 0.024$
DELPHI [151]	$\pi^* \ell$	Q_{jet}	$0.499 \pm 0.053 \pm 0.015$	$0.500 \pm 0.053 \pm 0.015$
DELPHI [151]	ℓ	ℓ	$0.480 \pm 0.040 \pm 0.051$	$0.496 \pm 0.040 \begin{smallmatrix} +0.042 \\ -0.040 \end{smallmatrix}$
DELPHI [151]	D^*	Q_{jet}	$0.523 \pm 0.072 \pm 0.043$	$0.518 \pm 0.072 \pm 0.043$
DELPHI [152]	vtx	comb	$0.531 \pm 0.025 \pm 0.007$	$0.525 \pm 0.025 \pm 0.006$
L3 [153]	ℓ	ℓ	$0.458 \pm 0.046 \pm 0.032$	$0.467 \pm 0.046 \pm 0.028$
L3 [153]	ℓ	Q_{jet}	$0.427 \pm 0.044 \pm 0.044$	$0.438 \pm 0.044 \pm 0.042$
L3 [153]	ℓ	$\ell(\text{IP})$	$0.462 \pm 0.063 \pm 0.053$	$0.470 \pm 0.063 \pm 0.044$
OPAL [154]	ℓ	ℓ	$0.430 \pm 0.043 \begin{smallmatrix} +0.028 \\ -0.030 \end{smallmatrix}$	$0.466 \pm 0.043 \begin{smallmatrix} +0.017 \\ -0.016 \end{smallmatrix}$
OPAL [155]	ℓ	Q_{jet}	$0.444 \pm 0.029 \begin{smallmatrix} +0.020 \\ -0.017 \end{smallmatrix}$	$0.481 \pm 0.029 \pm 0.013$
OPAL [156]	$D^* \ell$	Q_{jet}	$0.539 \pm 0.060 \pm 0.024$	$0.544 \pm 0.060 \pm 0.023$
OPAL [156]	D^*	ℓ	$0.567 \pm 0.089 \begin{smallmatrix} +0.029 \\ -0.023 \end{smallmatrix}$	$0.572 \pm 0.089 \begin{smallmatrix} +0.028 \\ -0.022 \end{smallmatrix}$
OPAL [80]	$\pi^* \ell$	Q_{jet}	$0.497 \pm 0.024 \pm 0.025$	$0.496 \pm 0.024 \pm 0.025$
CDF1 [157]	$D \ell$	SST	$0.471 \begin{smallmatrix} +0.078 \\ -0.068 \end{smallmatrix} \begin{smallmatrix} +0.033 \\ -0.034 \end{smallmatrix}$	$0.470 \begin{smallmatrix} +0.078 \\ -0.068 \end{smallmatrix} \begin{smallmatrix} +0.033 \\ -0.034 \end{smallmatrix}$
CDF1 [158]	μ	μ	$0.503 \pm 0.064 \pm 0.071$	$0.513 \pm 0.064 \begin{smallmatrix} +0.070 \\ -0.069 \end{smallmatrix}$
CDF1 [159]	ℓ	ℓ, Q_{jet}	$0.500 \pm 0.052 \pm 0.043$	$0.544 \pm 0.052 \pm 0.036$
CDF1 [160]	$D^* \ell$	ℓ	$0.516 \pm 0.099 \begin{smallmatrix} +0.029 \\ -0.035 \end{smallmatrix}$	$0.523 \pm 0.099 \begin{smallmatrix} +0.028 \\ -0.035 \end{smallmatrix}$
D0 [161]	$D^{(*)} \mu$	OST	$0.506 \pm 0.020 \pm 0.016$	$0.506 \pm 0.020 \pm 0.016$
BABAR [162]	B^0	ℓ, K, NN	$0.516 \pm 0.016 \pm 0.010$	$0.521 \pm 0.016 \pm 0.008$
BABAR [163]	ℓ	ℓ	$0.493 \pm 0.012 \pm 0.009$	$0.487 \pm 0.012 \pm 0.006$
BABAR [92]	$D^* \ell \nu(\text{part})$	ℓ	$0.511 \pm 0.007 \pm 0.007$	$0.513 \pm 0.007 \pm 0.007$
BABAR [90]	$D^* \ell \nu$	ℓ, K, NN	$0.492 \pm 0.018 \pm 0.014$	$0.493 \pm 0.018 \pm 0.013$
Belle [164]	$D^* \pi(\text{part})$	ℓ	$0.509 \pm 0.017 \pm 0.020$	$0.513 \pm 0.017 \pm 0.019$
Belle [11]	ℓ	ℓ	$0.503 \pm 0.008 \pm 0.010$	$0.506 \pm 0.008 \pm 0.008$
Belle [93]	$B^0, D^* \ell \nu$	comb	$0.511 \pm 0.005 \pm 0.006$	$0.513 \pm 0.005 \pm 0.006$
LHCb [165]	B^0	OST	$0.499 \pm 0.032 \pm 0.003$	$0.499 \pm 0.032 \pm 0.003$
LHCb [166]	B^0	OST, SST	$0.516 \pm 0.005 \pm 0.003$	$0.516 \pm 0.005 \pm 0.003$
LHCb [167]	$D \mu$	OST, SST	$0.503 \pm 0.011 \pm 0.013$	$0.503 \pm 0.011 \pm 0.013$
World average (all above measurements included):				$0.510 \pm 0.003 \pm 0.002$
– ALEPH, DELPHI, L3, OPAL and CDF1 only:				$0.496 \pm 0.010 \pm 0.009$
– BABAR and Belle only:				$0.509 \pm 0.003 \pm 0.003$
– LHCb only:				$0.514 \pm 0.005 \pm 0.003$

Table 16: Simultaneous measurements of Δm_d and $\tau(B^0)$, and their average. The Belle analysis also measures $\tau(B^+)$ at the same time, but it is converted here into a two-dimensional measurement of Δm_d and $\tau(B^0)$, for an assumed value of $\tau(B^+)$. The first quoted error on each measurement is statistical and the second is systematic; in the case of adjusted measurements, the latter includes a contribution obtained from the variation of $\tau(B^+)$ or $\tau(B^+)/\tau(B^0)$ in the indicated range. Units are ps^{-1} for Δm_d and ps for lifetimes. The three different values of $\rho(\Delta m_d, \tau(B^0))$ correspond to the statistical, systematic and total correlation coefficients between the adjusted measurements of Δm_d and $\tau(B^0)$.

Exp. & Ref.	Measured Δm_d	Measured $\tau(B^0)$	Measured $\tau(B^+)$	Assumed $\tau(B^+)$
<i>BABAR</i> [90]	$0.492 \pm 0.018 \pm 0.013$	$1.523 \pm 0.024 \pm 0.022$	—	$(1.083 \pm 0.017)\tau(B^0)$
<i>BABAR</i> [92]	$0.511 \pm 0.007 \begin{smallmatrix} +0.007 \\ -0.006 \end{smallmatrix}$	$1.504 \pm 0.013 \begin{smallmatrix} +0.018 \\ -0.013 \end{smallmatrix}$	—	1.671 ± 0.018
Belle [93]	$0.511 \pm 0.005 \pm 0.006$	$1.534 \pm 0.008 \pm 0.010$	$1.635 \pm 0.011 \pm 0.011$	—
	Adjusted Δm_d	Adjusted $\tau(B^0)$	$\rho(\Delta m_d, B^0)$	Assumed $\tau(B^+)$
<i>BABAR</i> [90]	$0.492 \pm 0.018 \pm 0.013$	$1.523 \pm 0.024 \pm 0.022$	$-0.22 \begin{smallmatrix} +0.71 \\ +0.16 \end{smallmatrix}$	$(1.076 \pm 0.004)\tau(B^0)$
<i>BABAR</i> [92]	$0.512 \pm 0.007 \pm 0.007$	$1.506 \pm 0.013 \pm 0.018$	$+0.01 \begin{smallmatrix} -0.85 \\ -0.48 \end{smallmatrix}$	1.638 ± 0.004
Belle [93]	$0.511 \pm 0.005 \pm 0.006$	$1.535 \pm 0.008 \pm 0.011$	$-0.27 \begin{smallmatrix} -0.14 \\ -0.19 \end{smallmatrix}$	1.638 ± 0.004
Average	$0.509 \pm 0.004 \pm 0.004$	$1.527 \pm 0.006 \pm 0.008$	$-0.19 \begin{smallmatrix} -0.25 \\ -0.23 \end{smallmatrix}$	1.638 ± 0.004

Assuming $\Delta\Gamma_d = 0$ and using $1/\Gamma_d = \tau(B^0) = 1.520 \pm 0.004$ ps, the Δm_d and χ_d results are combined through Eq. (54) to yield the world average

$$\Delta m_d = 0.510 \pm 0.003 \text{ ps}^{-1}, \quad (56)$$

or, equivalently,

$$x_d = 0.775 \pm 0.006 \quad \text{and} \quad \chi_d = 0.1875 \pm 0.0017. \quad (57)$$

Figure 6 compares the Δm_d values obtained by the different experiments.

The B^0 mixing averages given in Eqs. (56) and (57) and the b -hadron fractions of Table 5 have been obtained in a fully consistent way, taking into account the fact that the fractions are computed using the χ_d value of Eq. (57) and that many individual measurements of Δm_d at high energy depend on the assumed values for the b -hadron fractions. Furthermore, this set of averages is consistent with the lifetime averages of Sec. 3.2.

It should be noted that the most recent (and precise) analyses at the asymmetric B factories measure Δm_d as a result of a multi-dimensional fit. Two *BABAR* analyses [90, 92], based on fully and partially reconstructed $B^0 \rightarrow D^* \ell \nu$ decays respectively, extract simultaneously Δm_d and $\tau(B^0)$ while the latest Belle analysis [93], based on fully reconstructed hadronic B^0 decays and $B^0 \rightarrow D^* \ell \nu$ decays, extracts simultaneously Δm_d , $\tau(B^0)$ and $\tau(B^+)$. The measurements of Δm_d and $\tau(B^0)$ of these three analyses are displayed in Table 16 and in Fig. 7. Their two-dimensional average, taking into account all statistical and systematic correlations, and expressed at $\tau(B^+) = 1.638 \pm 0.004$ ps, is

$$\left. \begin{aligned} \Delta m_d &= 0.509 \pm 0.006 \text{ ps}^{-1} \\ \tau(B^0) &= 1.527 \pm 0.010 \text{ ps} \end{aligned} \right\} \text{ with a total correlation of } -0.23. \quad (58)$$

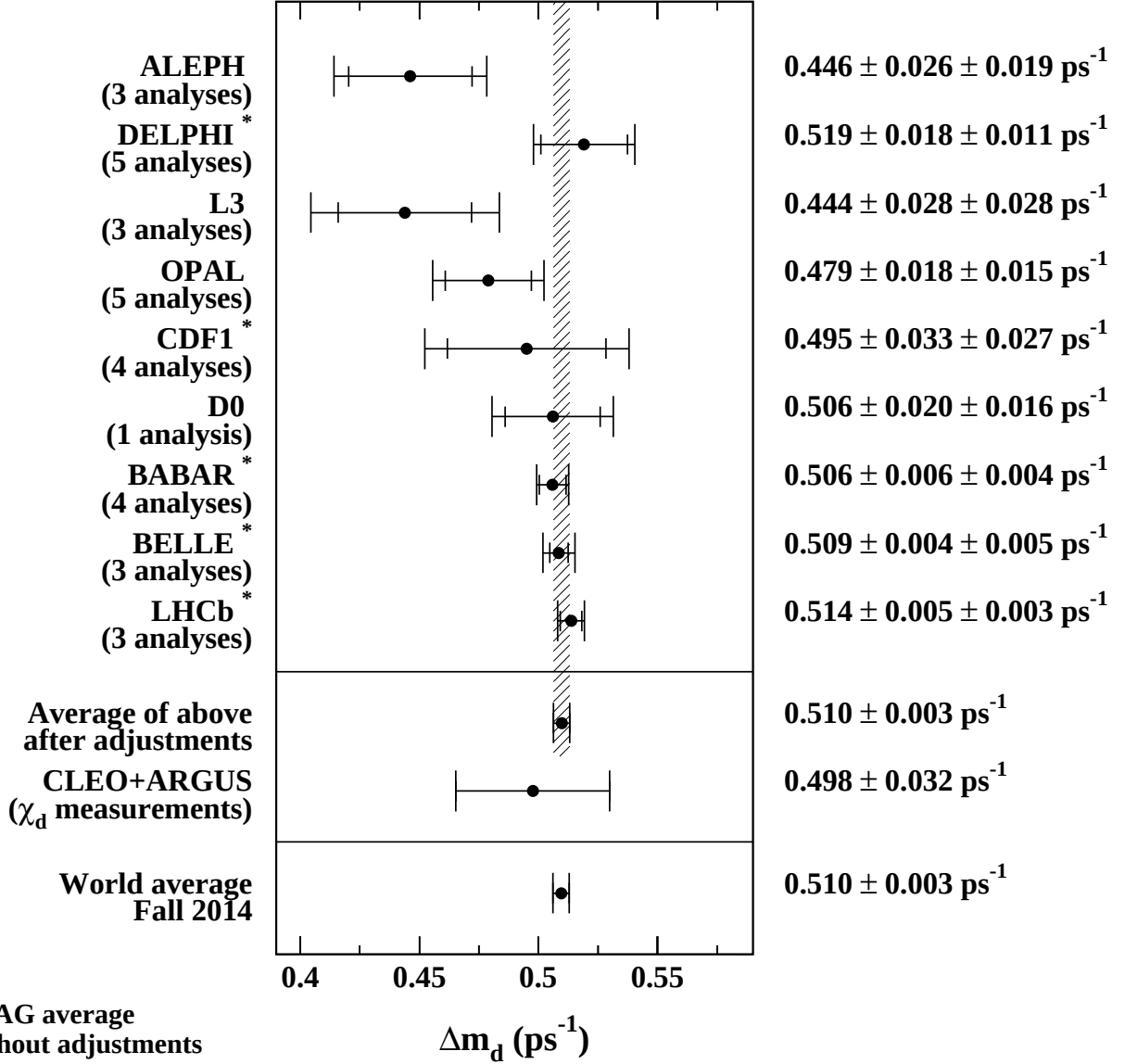


Figure 6: The $B^0\text{--}\bar{B}^0$ oscillation frequency Δm_d as measured by the different experiments. The averages quoted for ALEPH, L3 and OPAL are taken from the original publications, while the ones for DELPHI, CDF, *BABAR*, Belle and LHCb have been computed from the individual results listed in Table 15 without performing any adjustments. The time-integrated measurements of χ_d from the symmetric B factory experiments ARGUS and CLEO have been converted to a Δm_d value using $\tau(B^0) = 1.520 \pm 0.004$ ps. The two global averages have been obtained after adjustments of all the individual Δm_d results of Table 15 (see text).

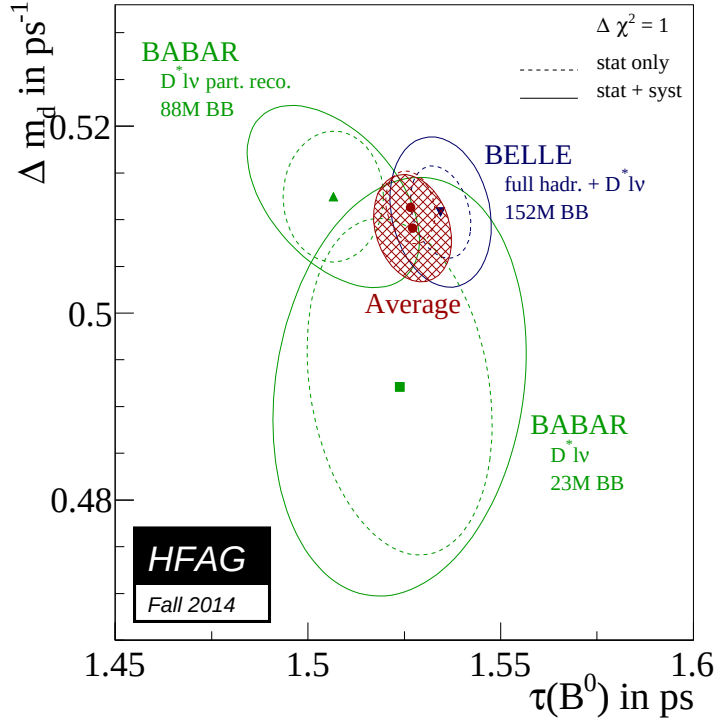


Figure 7: Simultaneous measurements of Δm_d and $\tau(B^0)$ [90, 92, 93], after adjustment to a common set of parameters (see text). Statistical and total uncertainties are represented as dashed and solid contours respectively. The average of the three measurements is indicated by a hatched ellipse.

3.3.2 B_s^0 mixing parameters $\Delta\Gamma_s$ and Δm_s

Definitions and an introduction to $\Delta\Gamma_s$ have been given in Sec. 3.2.4. Neglecting CP violation, the mass eigenstates are also CP eigenstates, with the short-lived state being CP -even and the long-lived state being CP -odd.

The best sensitivity to $\Delta\Gamma_s$ is currently achieved by the recent time-dependent measurements of the $B_s^0 \rightarrow J/\psi \phi$ (or more generally $B_s^0 \rightarrow J/\psi K^+ K^-$) decay rates performed at CDF [178], D0 [179], ATLAS [180], CMS [181, 182] and LHCb [183], where the CP -even and CP -odd amplitudes are statistically separated through a full angular analysis (see last two columns of Table 22). With the exception of the first CMS analysis [181], these studies use both untagged and tagged B_s^0 candidates and are optimized for the measurement of the CP -violating phase $\phi_s^{c\bar{c}s}$, defined later in Sec. 3.3.4. The LHCb collaboration analyzed the $B_s^0 \rightarrow J/\psi K^+ K^-$ decay, considering that the $K^+ K^-$ system can be in a P -wave or S -wave state, and measured the dependence of the strong phase difference between the P -wave and S -wave amplitudes as a function of the $K^+ K^-$ invariant mass [103]. This allowed, for the first time, the unambiguous determination of the sign of $\Delta\Gamma_s$, which was found to be positive at the 4.7σ level. The following averages present only the $\Delta\Gamma_s > 0$ solutions.

The available results [178–183] are shown in Table 17. They are combined, taking into account, in each analysis, the correlation between $\Delta\Gamma_s$ and Γ_s . The results, displayed as the red contours labelled “ $B_s^0 \rightarrow J/\psi KK$ measurements” in the plots of Fig. 8, are given in the first column of numbers of Table 18.

Table 17: Measurements of $\Delta\Gamma_s$ and Γ_s using $B_s^0 \rightarrow J/\psi\phi$ and $B_s^0 \rightarrow J/\psi K^+K^-$ decays. Only the solution with $\Delta\Gamma_s > 0$ is shown, since the two-fold ambiguity has been resolved in Ref. [103]. The first error is due to statistics, the second one to systematics. The last line gives our average.

Exp.	Mode	Dataset	$\Delta\Gamma_s$ (ps ⁻¹)	Γ_s (ps ⁻¹)	Ref.
CDF	$J/\psi\phi$	9.6 fb ⁻¹	$0.068 \pm 0.026 \pm 0.009$	$0.654 \pm 0.008 \pm 0.004$	[178]
D0	$J/\psi\phi$	8.0 fb ⁻¹	$0.163^{+0.065}_{-0.064}$	$0.693^{+0.018}_{-0.017}$	[179]
ATLAS	$J/\psi\phi$	4.9 fb ⁻¹	$0.053 \pm 0.021 \pm 0.010$	$0.677 \pm 0.007 \pm 0.004$	[180]
CMS	$J/\psi\phi$	5.0 fb ⁻¹	$0.048 \pm 0.024 \pm 0.003$	$0.655 \pm 0.008 \pm 0.003$	[181] ^p
CMS	$J/\psi\phi$	20 fb ⁻¹	$0.096 \pm 0.014 \pm 0.007$	$0.670 \pm 0.004 \pm 0.005$	[182] ^p
LHCb	$J/\psi K^+K^-$	3.0 fb ⁻¹	$0.0805 \pm 0.0091 \pm 0.0033$	$0.6603 \pm 0.0027 \pm 0.0015$	[183]
All combined			$+0.077 \pm 0.007$	0.6629 ± 0.0020	

^p Preliminary.

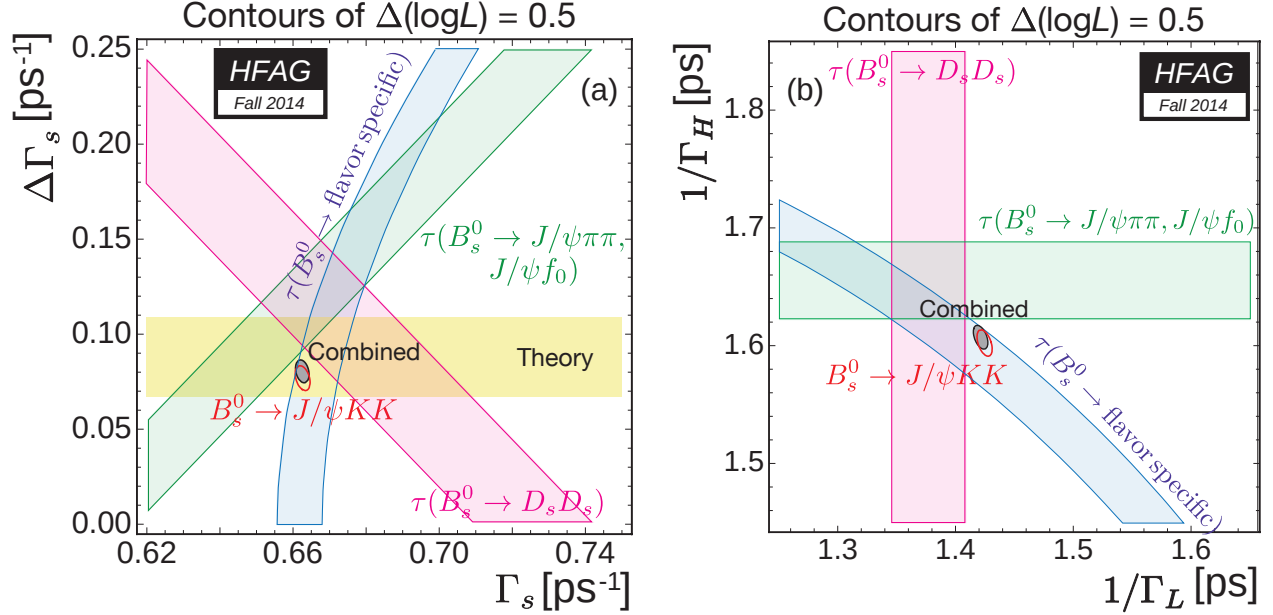


Figure 8: Contours of $\Delta \ln L = 0.5$ (39% CL for the enclosed 2D regions, 68% CL for the bands) shown in the $(\Gamma_s, \Delta\Gamma_s)$ plane on the left and in the $(1/\Gamma_L, 1/\Gamma_H)$ plane on the right. The average of all the $B_s^0 \rightarrow J/\psi\phi$ and $B_s^0 \rightarrow J/\psi K^+K^-$ results is shown as the red contour, and the constraints given by the effective lifetime measurements of B_s^0 to flavour-specific, pure CP -odd and pure CP -even final states are shown as the blue, green and purple bands, respectively. The average taking all constraints into account is shown as the gray-filled contour. The yellow band is a theory prediction $\Delta\Gamma_s = 0.087 \pm 0.021$ ps⁻¹ [101] that assumes no new physics in B_s^0 mixing.

Table 18: Averages of $\Delta\Gamma_s$, Γ_s and related quantities, obtained from $B_s^0 \rightarrow J/\psi\phi$ and $B_s^0 \rightarrow J/\psi K^+K^-$ alone (first column), adding the constraints from the effective lifetimes measured in pure CP modes $B_s^0 \rightarrow D_s^+D_s^-$ and $B_s^0 \rightarrow J/\psi f_0(980), J/\psi\pi^+\pi^-$ (second column), and adding the constraint from the effective lifetime measured in flavour-specific modes $B_s^0 \rightarrow D_s^-\ell^+\nu X$, $D_s^-\pi^+$, $D_s^-D^+$ (third column, recommended world averages).

	$B_s^0 \rightarrow J/\psi K^+K^-$ modes only (see Table 17)	$B_s^0 \rightarrow J/\psi K^+K^-$ modes + pure CP modes	$B_s^0 \rightarrow J/\psi K^+K^-$ modes + pure CP modes + flavour-specific modes
Γ_s	$0.6629 \pm 0.0020 \text{ ps}^{-1}$	$0.6623 \pm 0.0020 \text{ ps}^{-1}$	$0.6628 \pm 0.0019 \text{ ps}^{-1}$
$1/\Gamma_s$	$1.509 \pm 0.005 \text{ ps}$	$1.510 \pm 0.004 \text{ ps}$	$1.509 \pm 0.004 \text{ ps}$
$1/\Gamma_L$	$1.426 \pm 0.006 \text{ ps}$	$1.422 \pm 0.006 \text{ ps}$	$1.422 \pm 0.006 \text{ ps}$
$1/\Gamma_H$	$1.602 \pm 0.011 \text{ ps}$	$1.609 \pm 0.011 \text{ ps}$	$1.607 \pm 0.010 \text{ ps}$
$\Delta\Gamma_s$	$+0.077 \pm 0.007 \text{ ps}^{-1}$	$+0.082 \pm 0.006 \text{ ps}^{-1}$	$+0.081 \pm 0.006 \text{ ps}^{-1}$
$\Delta\Gamma_s/\Gamma_s$	$+0.116 \pm 0.010$	$+0.123 \pm 0.009$	$+0.122 \pm 0.009$
$\rho(\Gamma_s, \Delta\Gamma_s)$	-0.353	-0.323	-0.271

An alternative approach, which is directly sensitive to first order in $\Delta\Gamma_s/\Gamma_s$, is to determine the effective lifetime of untagged B_s^0 candidates decaying to pure CP eigenstates; we use here measurements with $B_s^0 \rightarrow D_s^+D_s^-$ [110], $B_s^0 \rightarrow J/\psi f_0(980)$ [119] and $B_s^0 \rightarrow J/\psi\pi^+\pi^-$ [120] decays. The precise extraction of $1/\Gamma_s$ and $\Delta\Gamma_s$ from such measurements, discussed in detail in Ref. [104], requires additional information in the form of theoretical assumptions or external inputs on weak phases and hadronic parameters. If f designates a final state in which both B_s^0 and $\bar{B}_s^0$ can decay, the ratio of the effective B_s^0 lifetime decaying to f relative to the mean B_s^0 lifetime is [104]²³

$$\frac{\tau_{\text{single}}(B_s^0 \rightarrow f)}{\tau(B_s^0)} = \frac{1}{1 - y_s^2} \left[\frac{1 - 2A_f^{\Delta\Gamma} y_s + y_s^2}{1 - A_f^{\Delta\Gamma} y_s} \right], \quad (59)$$

where

$$A_f^{\Delta\Gamma} = -\frac{2\text{Re}(\lambda_f)}{1 + |\lambda_f|^2}. \quad (60)$$

To include the measurements of the effective $B_s^0 \rightarrow D_s^+D_s^-$ (CP -even), $B_s^0 \rightarrow J/\psi f_0(980)$ (CP -odd) and $B_s^0 \rightarrow J/\psi\pi^+\pi^-$ (CP -odd) lifetimes as constraints in the $\Delta\Gamma_s$ fit,²⁴ we neglect sub-leading penguin contributions and possible direct CP violation. Explicitly, in Eq. (60), we set $A_{CP\text{-even}}^{\Delta\Gamma} = \cos\phi_s^{c\bar{c}s}$ and $A_{CP\text{-odd}}^{\Delta\Gamma} = -\cos\phi_s^{c\bar{c}s}$. Given the small value of $\phi_s^{c\bar{c}s}$, we have, to first order in y_s :

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-even}) \approx \frac{1}{\Gamma_L} \left(1 + \frac{(\phi_s^{c\bar{c}s})^2 y_s}{2} \right), \quad (61)$$

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-odd}) \approx \frac{1}{\Gamma_H} \left(1 - \frac{(\phi_s^{c\bar{c}s})^2 y_s}{2} \right). \quad (62)$$

²³ The definition of $A_f^{\Delta\Gamma}$ given in Eq. (60) has the sign opposite to that given in Ref. [104].

²⁴ The effective lifetimes measured in $B_s^0 \rightarrow K^+K^-$ (mostly CP -even) and $B_s^0 \rightarrow J/\psi K_S^0$ (mostly CP -odd) are not used because we can not quantify the penguin contributions in those modes.

The numerical inputs are taken from Eqs. (39) and (40) and the resulting averages, combined with the $B_s^0 \rightarrow J/\psi K^+ K^-$ information, are indicated in the second column of numbers of Table 18. These averages assume $\phi_s^{\bar{c}c s} = 0$, which is compatible with the $\phi_s^{\bar{c}c s}$ average presented in Sec. 3.3.4.

Information on $\Delta\Gamma_s$ can also be obtained from the study of the proper time distribution of untagged samples of flavour-specific B_s^0 decays [121], where the flavour (*i.e.* B_s^0 or $\bar{B}_s^0$) at the time of decay can be determined by the decay products. In such decays, *e.g.* semileptonic B_s^0 decays, there is an equal mix of the heavy and light mass eigenstates at time zero. The proper time distribution is then a superposition of two exponential functions with decay constants $\Gamma_{L,H} = \Gamma_s \pm \Delta\Gamma_s/2$. This provides sensitivity to both $1/\Gamma_s$ and $(\Delta\Gamma_s/\Gamma_s)^2$. Ignoring $\Delta\Gamma_s$ and fitting for a single exponential leads to an estimate of Γ_s with a relative bias proportional to $(\Delta\Gamma_s/\Gamma_s)^2$, as shown in Eq. (37). Including the constraint from the world-average flavour-specific B_s^0 lifetime, given in Eq. (38), leads to the results shown in the last column of Table 18. These world averages are displayed as the gray contours labelled “Combined” in the plots of Fig. 8. They correspond to the lifetime averages $1/\Gamma_s = 1.509 \pm 0.004$ ps, $1/\Gamma_L = 1.422 \pm 0.006$ ps, $1/\Gamma_H = 1.607 \pm 0.010$ ps, and to the decay-width difference

$$\Delta\Gamma_s = +0.081 \pm 0.006 \text{ ps}^{-1} \quad \text{and} \quad \Delta\Gamma_s/\Gamma_s = +0.122 \pm 0.009, \quad (63)$$

which is in good agreement with the Standard Model prediction $\Delta\Gamma_s = 0.087 \pm 0.021 \text{ ps}^{-1}$ [101].

Independent estimates of $\Delta\Gamma_s/\Gamma_s$ obtained from measurements of the $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$ branching fraction [116, 184–186]²⁵ have not been used, since they are based on the questionable [101] assumption that these decays account for all CP -even final states. The results of early lifetime analyses attempting to measure $\Delta\Gamma_s/\Gamma_s$ [70, 77, 107, 113] have not been used either.

The strength of B_s^0 mixing is known to be large since more than 20 years. Indeed the time-integrated measurements of $\bar{\chi}$ (see Sec. 3.1.3), when compared to our knowledge of χ_d and the b -hadron fractions, indicated that χ_s should be close to its maximal possible value of $1/2$. Many searches of the time dependence of this mixing were performed by ALEPH [188], CDF (Run I) [189], DELPHI [107, 113, 152, 190], OPAL [191, 192] and SLD [193–195], but did not have enough statistical power and proper time resolution to resolve the small period of the B_s^0 oscillations.

B_s^0 oscillations have been observed for the first time in 2006 by the CDF collaboration [196], based on samples of flavour-tagged hadronic and semileptonic B_s^0 decays (in flavour-specific final states), partially or fully reconstructed in 1 fb^{-1} of data collected during Tevatron’s Run II. This was shortly followed by independent evidence obtained by the D0 collaboration with 2.4 fb^{-1} of data [197]. More recently the LHCb collaboration obtained the most precise results using fully reconstructed $B_s^0 \rightarrow D_s^- \pi^+$ and $B_s^0 \rightarrow D_s^- \pi^+ \pi^- \pi^+$ decays at the LHC [198, 199]. LHCb has also observed B_s^0 oscillations with $B_s^0 \rightarrow J/\psi K^+ K^-$ decays [183] and with semileptonic $B_s^0 \rightarrow D_s^- \mu^+ X$ decays [167]. The measurements of Δm_s are summarized in Table 19.

A simple average of the CDF and LHCb results²⁶, taking into account the correlated systematic uncertainties between the three LHCb measurements, yields

$$\Delta m_s = 17.757 \pm 0.020 \pm 0.007 \text{ ps}^{-1} = 17.757 \pm 0.021 \text{ ps}^{-1} \quad (64)$$

²⁵ The result of Ref. [185] supersedes that of Ref. [187].

²⁶ We do not include the old unpublished D0 [197] result in the average.

Table 19: Measurements of Δm_s .

Experiment	Method	Data set	Δm_s (ps ⁻¹)	Ref.
CDF2	$D_s^{(*)-} \ell^+ \nu$, $D_s^{(*)-} \pi^+$, $D_s^- \rho^+$	1 fb ⁻¹	17.77 ± 0.10 ± 0.07	[196]
D0	$D_s^- \ell^+ X$, $D_s^- \pi^+ X$	2.4 fb ⁻¹	18.53 ± 0.93 ± 0.30	[197] ^u
LHCb	$D_s^- \pi^+$, $D_s^- \pi^+ \pi^- \pi^+$	2010 0.034 fb ⁻¹	17.63 ± 0.11 ± 0.02	[198]
LHCb	$D_s^- \mu^+ X$	2011 1.0 fb ⁻¹	17.93 ± 0.22 ± 0.15	[167]
LHCb	$D_s^- \pi^+$	2011 1.0 fb ⁻¹	17.768 ± 0.023 ± 0.006	[199]
LHCb	$J/\psi K^+ K^-$	2011–2012 3.0 fb ⁻¹	17.711 ^{+0.055} _{-0.057} ± 0.011	[183]
Average of CDF and LHCb measurements			17.757 ± 0.020 ± 0.007	

^u Unpublished.

and is illustrated in Figure 9. Multiplying this result with the mean B_s^0 lifetime of Eq. (43), $1/\Gamma_s = 1.509 \pm 0.004$ ps, yields

$$x_s = \frac{\Delta m_s}{\Gamma_s} = 26.79 \pm 0.08. \quad (65)$$

With $2y_s = \Delta\Gamma_s/\Gamma_s = +0.122 \pm 0.009$ (see Eq. (63)) and under the assumption of no CP violation in B_s^0 mixing, this corresponds to

$$\chi_s = \frac{x_s^2 + y_s^2}{2(x_s^2 + 1)} = 0.499307 \pm 0.000004. \quad (66)$$

The ratio of the B^0 and B_s^0 oscillation frequencies, obtained from Eqs. (56) and (64),

$$\frac{\Delta m_d}{\Delta m_s} = 0.02870 \pm 0.00020, \quad (67)$$

can be used to extract the following ratio of CKM matrix elements,

$$\left| \frac{V_{td}}{V_{ts}} \right| = \xi \sqrt{\frac{\Delta m_d}{\Delta m_s} \frac{m(B_s^0)}{m(B^0)}} = 0.2166 \pm 0.0007 \pm 0.0108, \quad (68)$$

where the first quoted error is from experimental uncertainties (with the masses $m(B_s^0)$ and $m(B^0)$ taken from Ref. [18]), and where the second quoted error is from theoretical uncertainties in the estimation of the SU(3) flavour-symmetry breaking factor $\xi = 1.268 \pm 0.063$, obtained from unquenched lattice QCD calculations [200]. Note that Eq. (68) assumes that Δm_s and Δm_d only receive Standard Model contributions. An alternative approach would be to take V_{td}/V_{ts} from global fits to predict $\Delta m_d/\Delta m_s$, and then compare the prediction with the measurement of Eq. (67) to set limits on new physics effects.

3.3.3 CP violation in B^0 and B_s^0 mixing

Evidence for CP violation in B^0 mixing has been searched for, both with flavour-specific and inclusive B^0 decays, in samples where the initial flavour state is tagged. In the case of semileptonic (or other flavour-specific) decays, where the final state tag is also available, the following

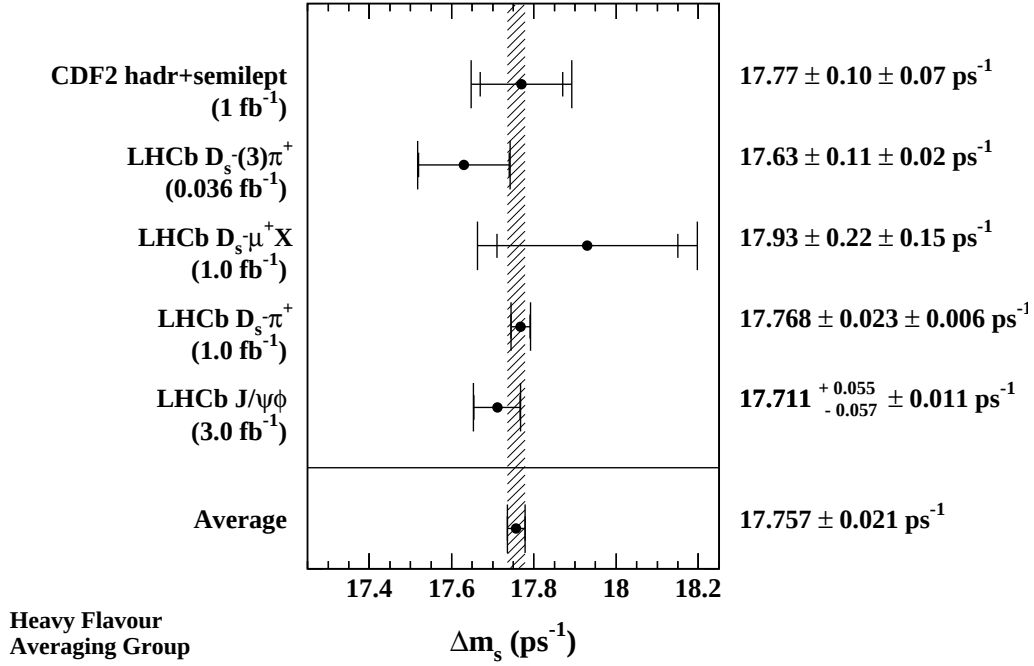


Figure 9: Published measurements of Δm_s , together with their average.

asymmetry

$$\mathcal{A}_{\text{SL}}^d = \frac{N(\bar{B}^0(t) \rightarrow \ell^+ \nu_\ell X) - N(B^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)}{N(\bar{B}^0(t) \rightarrow \ell^+ \nu_\ell X) + N(B^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)} = \frac{|p/q|_d^2 - |q/p|_d^2}{|p/q|_d^2 + |q/p|_d^2} \quad (69)$$

has been measured, either in time-integrated analyses at CLEO [172,201], *BABAR* [202], CDF [203]²⁷ and D0 [175,205], or in time-dependent analyses at OPAL [155], ALEPH [206], *BABAR* [173, 207,208] and Belle [209]. Note that the asymmetry of time-dependent decay rates in Eq. (69) is time-independent. In the inclusive case, also investigated and published at ALEPH [206] and OPAL [79], no final state tag is used, and the asymmetry [210]

$$\frac{N(B^0(t) \rightarrow \text{all}) - N(\bar{B}^0(t) \rightarrow \text{all})}{N(B^0(t) \rightarrow \text{all}) + N(\bar{B}^0(t) \rightarrow \text{all})} \simeq \mathcal{A}_{\text{SL}}^d \left[\frac{\Delta m_d}{2\Gamma_d} \sin(\Delta m_d t) - \sin^2 \left(\frac{\Delta m_d t}{2} \right) \right] \quad (70)$$

must be measured as a function of the proper time to extract information on CP violation.

On the other hand, LHCb has studied the time-dependence of the charge asymmetry of $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X$ decays without tagging the initial state [211], which would be equal to

$$\frac{N(D^{(*)-} \mu^+ \nu_\mu X) - N(D^{(*)+} \mu^- \bar{\nu}_\mu X)}{N(D^{(*)-} \mu^+ \nu_\mu X) + N(D^{(*)+} \mu^- \bar{\nu}_\mu X)} = \mathcal{A}_{\text{SL}}^d [1 - \cos(\Delta m_d t)] \quad (71)$$

in absence of detection and production asymmetries.

Table 20 summarizes the different measurements: in all cases asymmetries compatible with zero have been found, with a precision limited by the available statistics.

²⁷ We do not include the unpublished measurement of Ref. [204] in our average.

Table 20: Measurements²⁸ of CP violation in B^0 mixing and their average in terms of both $\mathcal{A}_{\text{SL}}^d$ and $|q/p|_d$. The individual results are listed as quoted in the original publications, or converted³² to an $\mathcal{A}_{\text{SL}}^d$ value. When two errors are quoted, the first one is statistical and the second one systematic. The ALEPH and OPAL results assume no CP violation in B_s^0 mixing.

Exp. & Ref.	Method	Measured $\mathcal{A}_{\text{SL}}^d$	Measured $ q/p _d$
CLEO [172]	partial hadronic rec.	+0.017 ±0.070 ±0.014	
CLEO [201]	dileptons	+0.013 ±0.050 ±0.005	
CLEO [201]	average of above two	+0.014 ±0.041 ±0.006	
BABAR [173]	full hadronic rec.		1.029 ±0.013 ±0.011
BABAR [207]	part. rec. $D^* X \ell \nu$	+0.0006 ±0.0017 ^{+0.0038} _{-0.0032}	0.99971 ±0.00084 ±0.00175
BABAR [202]	dileptons	-0.0039 ±0.0035 ±0.0019	
Belle [209]	dileptons	-0.0011 ±0.0079 ±0.0085	1.0005 ±0.0040 ±0.0043
Average of above 6 B factory results		-0.0019 ± 0.0027 (tot)	1.0009 ± 0.0013 (tot)
D0 [205]	$B^0 \rightarrow D^{(*)-} \mu + X$	+0.0068 ±0.0045 ±0.0014	
LHCb [211]	$B^0 \rightarrow D^{(*)-} \mu + X$	-0.0002 ±0.0019 ±0.0030	
Average of above 8 pure B^0 results		+0.0001 ± 0.0020 (tot)	1.0000 ± 0.0010 (tot)
D0 [175]	dimuons	-0.0062 ± 0.0043 (tot)	
Average of above 9 direct measurements		-0.0010 ± 0.0018 (tot)	1.0005 ± 0.0009 (tot)
OPAL [155]	leptons	+0.008 ±0.028 ±0.012	
OPAL [79]	inclusive (Eq. (70))	+0.005 ±0.055 ±0.013	
ALEPH [206]	leptons	-0.037 ±0.032 ±0.007	
ALEPH [206]	inclusive (Eq. (70))	+0.016 ±0.034 ±0.009	
ALEPH [206]	average of above two	-0.013 ± 0.026 (tot)	
Average of above 14 results		-0.0010 ± 0.0018 (tot)	1.0005 ± 0.0009 (tot)
Best fit value from 2D combination of $\mathcal{A}_{\text{SL}}^d$ and $\mathcal{A}_{\text{SL}}^s$ results (see Eq. (75))		-0.0015 ± 0.0017 (tot)	1.0007 ± 0.0009 (tot)

A simple average of all measurements performed at B factories [172, 173, 201, 202, 207, 209] yields $\mathcal{A}_{\text{SL}}^d = -0.0019 \pm 0.0027$; adding also the D0 [205] and LHCb [211] measurements obtained with reconstructed semileptonic B^0 decays yields

$$\mathcal{A}_{\text{SL}}^d = +0.0001 \pm 0.0020 \quad \Longleftrightarrow \quad |q/p|_d = 1.0000 \pm 0.0010, \quad (72)$$

where the relation between $\mathcal{A}_{\text{SL}}^d$ and $|q/p|_d$ is given in Eq. (69). As discussed in more detail later in this section, the latest dimuon D0 analysis [175] separates the B^0 and B_s^0 contributions by exploiting the dependence on the muon impact parameter cut; combining the $\mathcal{A}_{\text{SL}}^d$ result quoted by D0 with the above B^0 average of Eq. (72) yields $\mathcal{A}_{\text{SL}}^d = -0.0010 \pm 0.0018$.

All the other B^0 analyses performed at high energy, either at LEP or at the Tevatron, did not separate the contributions from the B^0 and B_s^0 mesons. Under the assumption of no CP violation in B_s^0 mixing, a number of these analyses [50, 79, 155, 206] quote a measurement of $\mathcal{A}_{\text{SL}}^d$ or $|q/p|_d$ for the B^0 meson. Including also these results²⁸ in the previous average under the assumption $\mathcal{A}_{\text{SL}}^s = 0$ leads to $\mathcal{A}_{\text{SL}}^d = -0.0010 \pm 0.0018$ (*i.e.* no change). The latter

²⁸ A low-statistics result published by CDF using the Run I data [203] and an unpublished result by CDF using Run II data [204] are not included in our averages, nor in Table 20.

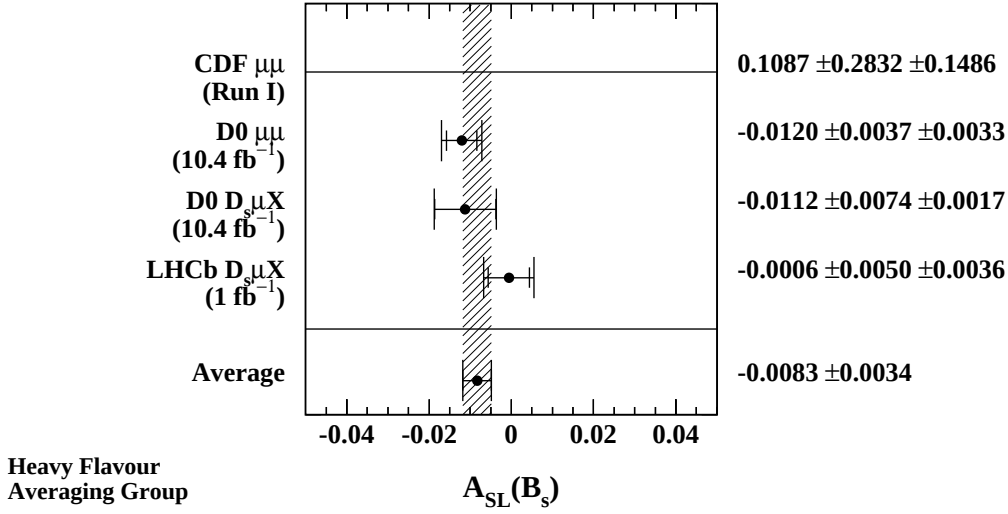


Figure 10: Measurements of $\mathcal{A}_{SL}^s$, derived from CDF [203],²⁹ D0 [175, 212] and LHCb [213] analyses, adjusted to the pure B^0 average of $\mathcal{A}_{SL}^d$. The combined value of $\mathcal{A}_{SL}^s$ is also shown.

assumption makes sense within the Standard Model, since $\mathcal{A}_{SL}^s$ is predicted to be much smaller than $\mathcal{A}_{SL}^d$ [101], but may not be suitable in the presence of new physics.

The following constraints on a combination of $\mathcal{A}_{SL}^d$ and $\mathcal{A}_{SL}^s$ (or equivalently $|q/p|_d$ and $|q/p|_s$) have been obtained by the Tevatron experiments, using inclusive semileptonic decays of b hadrons:

$$\frac{1}{4} (f'_d \chi_d \mathcal{A}_{SL}^d + f'_s \chi_s \mathcal{A}_{SL}^s) = +0.0015 \pm 0.0038(\text{stat}) \pm 0.0020(\text{syst}) \quad \text{CDF1 [203]}, \quad (73)$$

$$\mathcal{A}_{SL}^b = \frac{f'_d \chi_d \mathcal{A}_{SL}^d + f'_s \chi_s \mathcal{A}_{SL}^s}{f'_d \chi_d + f'_s \chi_s} = -0.00496 \pm 0.00153(\text{stat}) \pm 0.00072(\text{syst}) \quad \text{D0 [175]}, \quad (74)$$

where the fractions f' have been defined in Eq. (28). While the imprecise CDF1 result is compatible with no CP violation²⁹, the D0 result of Eq. (74), obtained by measuring the charge asymmetry of like-sign dimuons, differs by 2.8 standard deviations from the Standard Model expectation of $\mathcal{A}_{SL}^b(\text{SM}) = (-2.3 \pm 0.4) \times 10^{-4}$ [101, 175]. With a more sophisticated analysis in bins of the muon impact parameters, D0 conclude that the overall deviation of the their measurements from the SM is at the level of 3.6σ .

Using the average $\mathcal{A}_{SL}^d = +0.0001 \pm 0.0020$ of Eq. (72), obtained from pure B^0 measurements, the two results of Eqs. (73) and (74) are turned³⁰ into the measurements of $\mathcal{A}_{SL}^s$ displayed in the top part of Fig. 10. Taking into account the uncertainties on the b -hadron fractions and mixing parameters, the value derived from the D0 analysis does not show evidence of CP violation in the B_s^0 system. In addition, the third and fourth lines of Fig. 10 show direct

²⁹ A measurement from CDF2, $\mathcal{A}_{SL}^b = +0.0080 \pm 0.0090(\text{stat}) \pm 0.0068(\text{syst})$ [204], more precise than the D0 measurement, is also compatible with no CP violation, but since it is unpublished since 2007 we no longer include it in our averages, nor in Fig. 10.

³⁰ For simplicity, we set $f'_q = f_q$.

Table 21: Direct measurements of CP violation in B_s^0 and B^0 mixing, together with their two-dimensional average. Only total errors are quoted.

Exp. & Ref.	Method	Measured $\mathcal{A}_{\text{SL}}^s$	Measured $\mathcal{A}_{\text{SL}}^d$	$\rho(\mathcal{A}_{\text{SL}}^s, \mathcal{A}_{\text{SL}}^d)$
D0 [212]	$B_s^0 \rightarrow D_s \mu X$	-0.0112 ± 0.0076		
LHCb [213]	$B_s^0 \rightarrow D_s \mu X$	-0.0006 ± 0.0062		
Average of above B_s^0 results		-0.0048 ± 0.0048		
Average of B^0 results (Eq. (72))			$+0.0001 \pm 0.0020$	
D0 [175]	dimuons	-0.0082 ± 0.0099	-0.0062 ± 0.0043	-0.61
Average of all above		-0.0075 ± 0.0041	-0.0015 ± 0.0017	-0.158

determination of $\mathcal{A}_{\text{SL}}^s$ obtained by D0 [212] and LHCb [213] by measuring the time-integrated charge asymmetry of untagged $B_s^0 \rightarrow D_s \mu X$ decays. The four results of Fig. 10 are combined to yield $\mathcal{A}_{\text{SL}}^s = -0.0083 \pm 0.0027(\text{stat}) \pm 0.0021(\text{syst}) = -0.0083 \pm 0.0034$ or, equivalently through Eq. (69), $|q/p|_s = 1.0042 \pm 0.0014(\text{stat}) \pm 0.0010(\text{syst}) = 1.0042 \pm 0.0017$. The quoted systematic errors include experimental systematics as well as the correlated dependence on external parameters.

As mentioned above, the D0 like-sign dimuon analysis investigates the dependence of the charge asymmetry as a function of the muon impact parameters. Interpreting the observed asymmetries in terms of CP violation in B -meson mixing and interference, and using the mixing parameters and the world b -hadron fractions of Ref. [4], the D0 collaboration extracts [175] values for $\mathcal{A}_{\text{SL}}^d$ and $\mathcal{A}_{\text{SL}}^s$ and their correlation coefficient³¹, as shown in Table 21. However, the individual contributions to the total quoted errors from this analysis and from the external inputs are not given, so the adjustment of these results to different or more recent values of the external inputs cannot (easily) be done. Using a two-dimensional fit, these values are combined with the pure B^0 average of Eq. (72) and with the results from the $B_s^0 \rightarrow D_s \mu X$ analyses [212, 213], assumed to be independent and also shown in Table 21. The result, shown graphically in Fig. 11, is

$$\mathcal{A}_{\text{SL}}^d = -0.0015 \pm 0.0017 \iff |q/p|_d = 1.0007 \pm 0.0009, \quad (75)$$

$$\mathcal{A}_{\text{SL}}^s = -0.0075 \pm 0.0041 \iff |q/p|_s = 1.0038 \pm 0.0021, \quad (76)$$

$$\rho(\mathcal{A}_{\text{SL}}^d, \mathcal{A}_{\text{SL}}^s) = -0.158. \quad (77)$$

The average of Fig. 10 ignores the impact parameter study of D0. The average of Eq. (76) ignores the CDF1 result, which has a very large uncertainty anyway. We choose the results of Eqs. (75), (76), and (77) as our final averages,³² since they incorporate better the available published data.

The above averages show no evidence of CP violation in B^0 and B_s^0 mixing. They deviate by 1.5σ from the very small predictions of the Standard Model, $\mathcal{A}_{\text{SL}}^{d, \text{SM}} = -(4.1 \pm 0.6) \times 10^{-4}$ and $\mathcal{A}_{\text{SL}}^{s, \text{SM}} = +(1.9 \pm 0.3) \times 10^{-5}$ [101]. Given the current size of the experimental uncertainties,

³¹ They also extract at the same time a value for $\Delta\Gamma_d/\Gamma_d$ (see Sec. 3.3.1).

³² Early analyses and (perhaps hence) the PDG use the complex parameter $\epsilon_B = (p - q)/(p + q)$; if CP violation in the mixing is small, $\mathcal{A}_{\text{SL}}^d \cong 4\text{Re}(\epsilon_B)/(1 + |\epsilon_B|^2)$ and the averages of Eqs. (72) and (75) correspond to $\text{Re}(\epsilon_B)/(1 + |\epsilon_B|^2) = +0.0000 \pm 0.0005$ and -0.0004 ± 0.0004 , respectively.

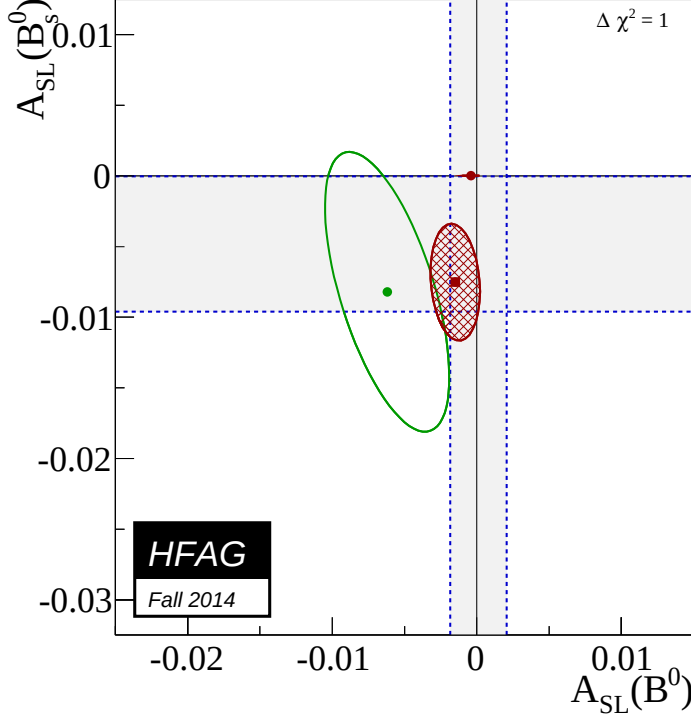


Figure 11: Direct measurements of $\mathcal{A}_{\text{SL}}^s$ and $\mathcal{A}_{\text{SL}}^d$ listed in Table 21 (B^0 average as the vertical band, B_s^0 average as the horizontal band, D0 dimuon result as the green ellipse), together with their two-dimensional average (red hatched ellipse). The red point close to (0,0) is the Standard Model prediction of Ref. [101] with error bars multiplied by 10. The prediction and the experimental average deviate from each other by 1.5σ .

there is still significant room for a possible new physics contribution, especially in the B_s^0 system. In this respect, the deviation of the D0 dimuon asymmetry [175] from expectation has generated a lot of excitement, however recent results from D0 and LHCb have not yet settled the issue, and more experimental data (especially from LHCb) is awaited eagerly.

At the more fundamental level, CP violation in B_s^0 mixing³³ is caused by the weak phase difference

$$\phi_{12} = \arg[-M_{12}/\Gamma_{12}], \quad (78)$$

where M_{12} and Γ_{12} are the off-diagonal elements of the mass and decay matrices of the $B_s^0 - \bar{B}_s^0$ system. This is related to the observed decay-width difference through the relation

$$\Delta\Gamma_s = 2|\Gamma_{12}|\cos\phi_{12} + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right), \quad (79)$$

where quadratic (or higher-order) terms in the small quantity $|\Gamma_{12}/M_{12}| \sim \mathcal{O}(m_b^2/m_t^2)$ can be neglected. The SM prediction for this phase is tiny [101],

$$\phi_{12}^{\text{SM}} = 0.0038 \pm 0.0010; \quad (80)$$

³³ Of course, a similar formalism exists for the B^0 system; for simplicity we omit here the subscript s for ϕ_{12} , M_{12} and Γ_{12} .

however, new physics in B_s^0 mixing could change this observed phase to

$$\phi_{12} = \phi_{12}^{\text{SM}} + \phi_{12}^{\text{NP}}. \quad (81)$$

The B_s^0 semileptonic asymmetry can be expressed as [214]

$$\mathcal{A}_{\text{SL}}^s = \text{Im} \left(\frac{\Gamma_{12}}{M_{12}} \right) + \mathcal{O} \left(\left| \frac{\Gamma_{12}}{M_{12}} \right|^2 \right) = \frac{\Delta\Gamma_s}{\Delta m_s} \tan \phi_{12} + \mathcal{O} \left(\left| \frac{\Gamma_{12}}{M_{12}} \right|^2 \right). \quad (82)$$

Using this relation, the current knowledge of $\mathcal{A}_{\text{SL}}^s$, $\Delta\Gamma_s$ and Δm_s , given in Eqs. (76), (63), and (64) respectively, yield an experimental determination of ϕ_{12} ,

$$\tan \phi_{12} = \mathcal{A}_{\text{SL}}^s \frac{\Delta m_s}{\Delta\Gamma_s} = -1.7 \pm 0.9, \quad (83)$$

which represents only a very weak constraint at present.

3.3.4 Mixing-induced CP violation in B_s^0 decays

CP violation induced by $B_s^0 - \bar{B}_s^0$ mixing has been a field of very active study and fast experimental progress in the past couple of years. The main observable is the CP -violating phase $\phi_s^{c\bar{c}s}$, defined as the weak phase difference between the $B_s^0 - \bar{B}_s^0$ mixing amplitude and the $b \rightarrow c\bar{c}s$ decay amplitude.

The golden mode for such studies is $B_s^0 \rightarrow J/\psi \phi$, followed by $J/\psi \rightarrow \mu^+ \mu^-$ and $\phi \rightarrow K^+ K^-$, for which a full angular analysis of the decay products is performed to separate statistically the CP -even and CP -odd contributions in the final state. As already mentioned in Sec. 3.3.2, CDF [178], D0 [179], ATLAS [180], CMS [182] and LHCb [183] have used both untagged and tagged $B_s^0 \rightarrow J/\psi \phi$ (and $B_s^0 \rightarrow J/\psi K^+ K^-$) events for the measurement of $\phi_s^{c\bar{c}s}$. LHCb [215] has used $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ events, analyzed with a full amplitude model including several $\pi^+ \pi^-$ resonances (*e.g.* $f_0(980)$), although the $J/\psi \pi^+ \pi^-$ final state had already been shown to be almost CP pure with a CP -odd fraction larger than 0.977 at 95% CL [216]. In addition, LHCb has used the $B_s^0 \rightarrow D_s^+ D_s^-$ channel [217] to measure $\phi_s^{c\bar{c}s}$.

All CDF, D0, ATLAS and CMS analyses provide two mirror solutions related by the transformation $(\Delta\Gamma_s, \phi_s^{c\bar{c}s}) \rightarrow (-\Delta\Gamma_s, \pi - \phi_s^{c\bar{c}s})$. However, the LHCb analysis of $B_s^0 \rightarrow J/\psi K^+ K^-$ resolves this ambiguity and rules out the solution with negative $\Delta\Gamma_s$ [103], a result in agreement with the Standard Model expectation. Therefore, in what follows, we only consider the solution with $\Delta\Gamma_s > 0$.

We perform a combination of the CDF [178], D0 [179], ATLAS [180], CMS [182] and LHCb [183, 215] results summarized in Table 22. This is done by adding the two-dimensional log profile-likelihood scans of $\Delta\Gamma_s$ and $\phi_s^{c\bar{c}s}$ from the four $B_s^0 \rightarrow J/\psi \phi$ ($B_s^0 \rightarrow J/\psi K^+ K^-$) analyses and a one-dimensional log profile-likelihood of $\phi_s^{c\bar{c}s}$ from the $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ and $B_s^0 \rightarrow D_s^+ D_s^-$ analyses; the combined likelihood is then maximized with respect to $\Delta\Gamma_s$ and $\phi_s^{c\bar{c}s}$.

In the $B_s^0 \rightarrow J/\psi \phi$ and $B_s^0 \rightarrow J/\psi K^+ K^-$ analyses, $\phi_s^{c\bar{c}s}$ and $\Delta\Gamma_s$ come from a simultaneous fit that determines also the B_s^0 lifetime, the polarisation amplitudes and strong phases. While the correlation between $\phi_s^{c\bar{c}s}$ and all other parameters is small, the correlations between $\Delta\Gamma_s$ and the polarisation amplitudes are sizeable. However, since the various experiments use different conventions for the amplitudes and phases, a full combination including all correlations is not performed. Instead, our average only takes into account the correlation between $\phi_s^{c\bar{c}s}$ and $\Delta\Gamma_s$.

Table 22: Direct experimental measurements of $\phi_s^{c\bar{c}s}$, $\Delta\Gamma_s$ and Γ_s using $B_s^0 \rightarrow J/\psi \phi$, $J/\psi K^+ K^-$, $J/\psi \pi^+ \pi^-$ and $D_s^+ D_s^-$ decays. Only the solution with $\Delta\Gamma_s > 0$ is shown, since the two-fold ambiguity has been resolved in Ref. [103]. The first error is due to statistics, the second one to systematics. The last line gives our average.

Exp.	Mode	Dataset	$\phi_s^{c\bar{c}s}$	$\Delta\Gamma_s$ (ps ⁻¹)	Ref.
CDF	$J/\psi \phi$	9.6 fb ⁻¹	$[-0.60, 0.12]$, 68% CL	$0.068 \pm 0.026 \pm 0.009$	[178]
D0	$J/\psi \phi$	8.0 fb ⁻¹	$-0.55^{+0.38}_{-0.36}$	$0.163^{+0.065}_{-0.064}$	[179]
ATLAS	$J/\psi \phi$	4.9 fb ⁻¹	$+0.12 \pm 0.25 \pm 0.05$	$0.053 \pm 0.021 \pm 0.010$	[180]
CMS	$J/\psi \phi$	20 fb ⁻¹	$-0.03 \pm 0.11 \pm 0.03$	$0.096 \pm 0.014 \pm 0.007$	[182] ^p
LHCb	$J/\psi K^+ K^-$	3.0 fb ⁻¹	$-0.058 \pm 0.049 \pm 0.006$	$0.0805 \pm 0.0091 \pm 0.0033$	[183]
LHCb	$J/\psi \pi^+ \pi^-$	3.0 fb ⁻¹	$+0.070 \pm 0.068 \pm 0.008$	—	[215]
LHCb	$J/\psi h^+ h^-$	3.0 fb ⁻¹	$-0.010 \pm 0.039(\text{tot})$	—	[183] ^a
LHCb	$D_s^+ D_s^-$	3.0 fb ⁻¹	$+0.02 \pm 0.17 \pm 0.02$	—	[217]
All combined			-0.015 ± 0.035	$+0.081 \pm 0.007$	

^a LHCb combination of $J/\psi K^+ K^-$ [183] and $J/\psi \pi^+ \pi^-$ [215].

^p Preliminary.

In the recent LHCb $B_s^0 \rightarrow J/\psi K^+ K^-$ analysis [183], the $\phi_s^{c\bar{c}s}$ values are measured for the first time for each polarization of the final state. Since those values are compatible within each other, we still use the unique value of $\phi_s^{c\bar{c}s}$ for our world average, corresponding to the one measured by the other-than-LHCb analyses. In the same analysis, the statistical correlation coefficient between $\phi_s^{c\bar{c}s}$ and $|\lambda|$ (which signals CP violation in the decay if different from unity) is measured to be very small (-0.02). We neglect this correlation in our average. Furthermore, the statistical correlation coefficient between $\phi_s^{c\bar{c}s}$ and $\Delta\Gamma_s$ is measured to be small (-0.08). When averaging LHCb results of $B_s^0 \rightarrow J/\psi K^+ K^-$, $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ and $B_s^0 \rightarrow D_s^+ D_s^-$, we neglect this correlation coefficient (putting it to zero). Given the increasing experimental precision, we have also stopped using the two-dimensional $\Delta\Gamma_s - \phi_s^{c\bar{c}s}$ histograms provided by the CDF and D0 collaborations: we are now approximating those with two-dimensional Gaussian likelihoods.

We obtain the individual and combined contours shown in Fig. 12. Maximizing the likelihood, we find, as summarized in Table 22:

$$\Delta\Gamma_s = +0.081 \pm 0.007 \text{ ps}^{-1}, \quad (84)$$

$$\phi_s^{c\bar{c}s} = -0.015 \pm 0.035. \quad (85)$$

The above $\Delta\Gamma_s$ average is consistent, but highly correlated with the average of Eq. (63). Our final recommended average for $\Delta\Gamma_s$ is the one of Eq. (63), which includes all available information on $\Delta\Gamma_s$.

In the Standard Model and ignoring sub-leading penguin contributions, $\phi_s^{c\bar{c}s}$ is expected to be equal to $-2\beta_s$, where $\beta_s = \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$ is a phase analogous to the angle β of the usual CKM unitarity triangle (aside from a sign change). An indirect determination via global fits to experimental data gives [177]

$$(\phi_s^{c\bar{c}s})^{\text{SM}} = -2\beta_s = -0.0363^{+0.0012}_{-0.0014}. \quad (86)$$

The average value of $\phi_s^{c\bar{c}s}$ from Eq. (85) is consistent with this Standard Model expectation.

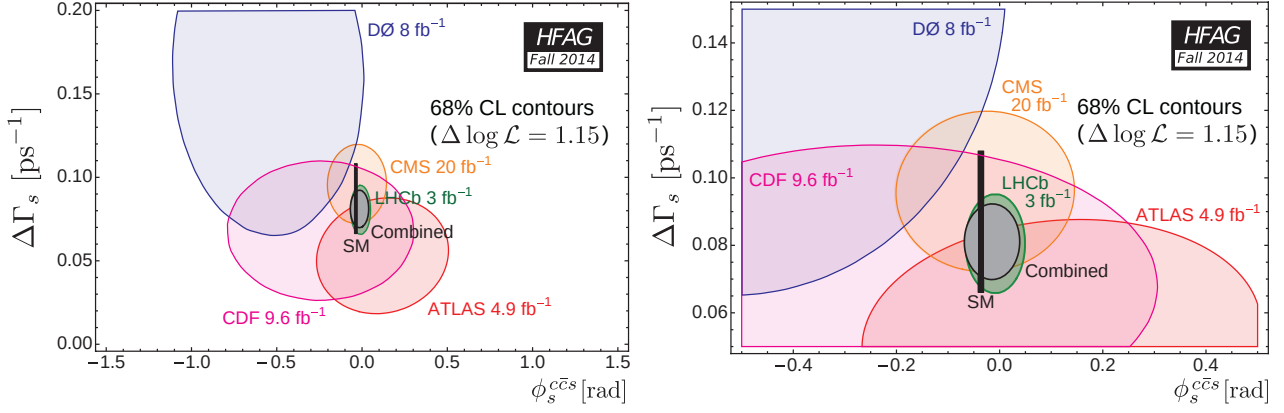


Figure 12: Left: 68% CL regions in B_s^0 width difference $\Delta\Gamma_s$ and weak phase $\phi_s^{c\bar{c}s}$ obtained from individual and combined CDF [178], DØ [179], ATLAS [180], CMS [182] and LHCb [183, 215] likelihoods of $B_s^0 \rightarrow J/\psi \phi$, $B_s^0 \rightarrow J/\psi K^+ K^-$ and $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$. The expectation within the Standard Model [101, 177] is shown as the black rectangle. Right: same, but zoomed on the region of interest.

New physics could contribute to $\phi_s^{c\bar{c}s}$. Assuming that new physics only enters in M_{12} (rather than in Γ_{12}), one can write [101]

$$\phi_s^{c\bar{c}s} = -2\beta_s + \phi_{12}^{\text{NP}}, \quad (87)$$

where the new physics phase ϕ_{12}^{NP} is the same as that appearing in Eq. (81). In this case

$$\phi_{12} = \phi_{12}^{\text{SM}} + 2\beta_s + \phi_s^{c\bar{c}s} = 0.025 \pm 0.035, \quad (88)$$

where the numerical estimation was performed with the values of Eqs. (80), (86), and (85). This can serve as a reference value to which the measurement of Eq. (83) can be compared.

4 Measurements related to Unitarity Triangle angles

The charge of the “ $CP(t)$ and Unitarity Triangle angles” group is to provide averages of measurements from time-dependent asymmetry analyses, and other quantities that are related to the angles of the Unitarity Triangle (UT). In cases where considerable theoretical input is required to extract the fundamental quantities, no attempt is made to do so at this stage. However, straightforward interpretations of the averages are given, where possible.

In Sec. 4.1 a brief introduction to the relevant phenomenology is given. In Sec. 4.2 an attempt is made to clarify the various different notations in use. In Sec. 4.3 the common inputs to which experimental results are rescaled in the averaging procedure are listed. We also briefly introduce the treatment of experimental errors. In the remainder of this section, the experimental results and their averages are given, divided into subsections based on the underlying quark-level decays.

4.1 Introduction

The Standard Model Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix V must be unitary. A 3×3 unitary matrix has four free parameters,³⁴ and these are conventionally written by the product of three (complex) rotation matrices [218], where the rotations are characterised by the Euler angles θ_{12} , θ_{13} and θ_{23} , which are the mixing angles between the generations, and one overall phase δ ,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (89)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ for $i < j = 1, 2, 3$.

Following the observation of a hierarchy between the different matrix elements, the Wolfenstein parametrisation [219] is an expansion of V in terms of the four real parameters λ (the expansion parameter), A , ρ and η . Defining to all orders in λ [220]

$$\begin{aligned} s_{12} &\equiv \lambda, \\ s_{23} &\equiv A\lambda^2, \\ s_{13}e^{-i\delta} &\equiv A\lambda^3(\rho - i\eta), \end{aligned} \quad (90)$$

and inserting these into the representation of Eq. (89), unitarity of the CKM matrix is achieved to all orders. A Taylor expansion of V leads to the familiar approximation

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (91)$$

At order λ^5 , the obtained CKM matrix in this extended Wolfenstein parametrisation is:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6). \quad (92)$$

³⁴ In the general case there are nine free parameters, but five of these are absorbed into unobservable quark phases.

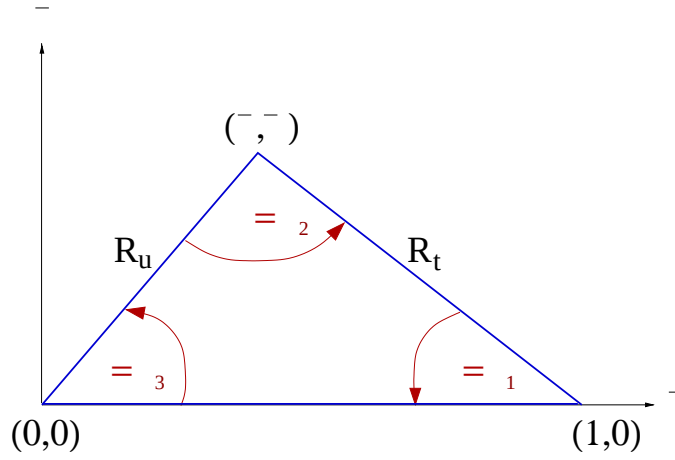


Figure 13: The Unitarity Triangle.

The non-zero imaginary part of the CKM matrix, which is the origin of CP violation in the Standard Model, is encapsulated in a non-zero value of η .

The unitarity relation $V^\dagger V = 1$ results in a total of nine expressions, that can be written as $\sum_{i=u,c,t} V_{ij}^* V_{ik} = \delta_{jk}$, where δ_{jk} is the Kronecker symbol. Of the off-diagonal expressions ($j \neq k$), three can be transformed into the other three leaving six relations, in which three complex numbers sum to zero, which therefore can be expressed as triangles in the complex plane. More details about unitarity triangles can be found in Refs. [221–226].

One of these relations,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \quad (93)$$

is of particular importance to the B system, being specifically related to flavour changing neutral current $b \rightarrow d$ transitions. The three terms in Eq. (93) are of the same order ($\mathcal{O}(\lambda^3)$), and this relation is commonly known as the Unitarity Triangle. For presentational purposes, it is convenient to rescale the triangle by $(V_{cd}V_{cb}^*)^{-1}$, as shown in Fig. 13.

Two popular naming conventions for the UT angles exist in the literature:

$$\alpha \equiv \phi_2 = \arg \left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right], \quad \beta \equiv \phi_1 = \arg \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right], \quad \gamma \equiv \phi_3 = \arg \left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]. \quad (94)$$

In this document the (α, β, γ) set is used.³⁵ The sides R_u and R_t of the Unitarity Triangle (the third side being normalised to unity) are given by

$$R_u = \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}, \quad R_t = \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}. \quad (95)$$

where $\bar{\rho}$ and $\bar{\eta}$ define the apex of the Unitarity Triangle [220]

$$\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \equiv 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = \frac{\sqrt{1 - \lambda^2}(\rho + i\eta)}{\sqrt{1 - A^2\lambda^4} + \sqrt{1 - \lambda^2}A^2\lambda^4(\rho + i\eta)}. \quad (96)$$

³⁵ The relevant unitarity triangle for the B_s^0 system is obtained by replacing $d \leftrightarrow s$ in Eq. (93). Definitions of the set of angles $(\alpha_s, \beta_s, \gamma_s)$ can be obtained using equivalent relations to those of Eq. (94), for example $\beta_s = \arg [-(V_{cs}V_{cb}^*)/(V_{ts}V_{tb}^*)]$. This definition gives a value of β_s that is negative in the Standard Model, so that the sign is often flipped in the literature.

The exact relation between (ρ, η) and $(\bar{\rho}, \bar{\eta})$ is

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\bar{\rho} + i\bar{\eta})}{\sqrt{1 - \lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}. \quad (97)$$

By expanding in powers of λ , several useful approximate expressions can be obtained, including

$$\bar{\rho} = \rho(1 - \frac{1}{2}\lambda^2) + \mathcal{O}(\lambda^4), \quad \bar{\eta} = \eta(1 - \frac{1}{2}\lambda^2) + \mathcal{O}(\lambda^4), \quad V_{td} = A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) + \mathcal{O}(\lambda^6). \quad (98)$$

4.2 Notations

Several different notations for CP violation parameters are commonly used. This section reviews those found in the experimental literature, in the hope of reducing the potential for confusion, and to define the frame that is used for the averages.

In some cases, when B mesons decay into multibody final states via broad resonances (ρ , K^* , *etc.*), the experimental analyses ignore the effects of interference between the overlapping structures. This is referred to as the quasi-two-body (Q2B) approximation in the following.

4.2.1 CP asymmetries

The CP asymmetry is defined as the difference between the rate involving a b quark and that involving a $\bar{b}$ quark, divided by the sum. For example, the partial rate (or charge) asymmetry for a charged B decay would be given as

$$\mathcal{A}_f \equiv \frac{\Gamma(B^- \rightarrow f) - \Gamma(B^+ \rightarrow \bar{f})}{\Gamma(B^- \rightarrow f) + \Gamma(B^+ \rightarrow \bar{f})}. \quad (99)$$

4.2.2 Time-dependent CP asymmetries in decays to CP eigenstates

If the amplitudes for B^0 and $\bar{B}^0$ to decay to a final state f , which is a CP eigenstate with eigenvalue η_f , are given by A_f and $\bar{A}_f$, respectively, then the decay distributions for neutral B mesons, with known flavour at time $\Delta t = 0$, are given by

$$\Gamma_{\bar{B}^0 \rightarrow f}(\Delta t) = \frac{e^{-|\Delta t|/\tau(B^0)}}{4\tau(B^0)} \left[1 + \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin(\Delta m \Delta t) - \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta m \Delta t) \right], \quad (100)$$

$$\Gamma_{B^0 \rightarrow f}(\Delta t) = \frac{e^{-|\Delta t|/\tau(B^0)}}{4\tau(B^0)} \left[1 - \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin(\Delta m \Delta t) + \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta m \Delta t) \right]. \quad (101)$$

Here $\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$ contains terms related to B^0 - $\bar{B}^0$ mixing and to the decay amplitude (the eigenstates of the effective Hamiltonian in the $B^0\bar{B}^0$ system are $|B_{\pm}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$). This formulation assumes CPT invariance, and neglects possible lifetime differences (between the eigenstates of the effective Hamiltonian; see Sec. 3.3 where the mass difference Δm is also defined) in the neutral B meson system. The case where non-zero lifetime differences are taken into account is discussed in Sec. 4.2.3. Note that the notation and normalisation used here is that which is relevant for the e^+e^- B factory experiments. At hadron collider experiments, the flavour tagging is done at production ($\Delta t = t = 0$), and therefore t is usually used in place

of Δt . Moreover, since negative values of t are not allowed, the normalisation is such that $\int_0^{+\infty} (\Gamma_{\bar{B}^0 \rightarrow f}(t) + \Gamma_{B^0 \rightarrow f}(t)) dt = 1$, rather than $\int_{-\infty}^{+\infty} (\Gamma_{\bar{B}^0 \rightarrow f}(\Delta t) + \Gamma_{B^0 \rightarrow f}(\Delta t)) d(\Delta t) = 1$, as in Eqs. (100) and (101).

The time-dependent CP asymmetry, again defined as the difference between the rate involving a b quark and that involving a $\bar{b}$ quark, is then given by

$$\mathcal{A}_f(\Delta t) \equiv \frac{\Gamma_{\bar{B}^0 \rightarrow f}(\Delta t) - \Gamma_{B^0 \rightarrow f}(\Delta t)}{\Gamma_{\bar{B}^0 \rightarrow f}(\Delta t) + \Gamma_{B^0 \rightarrow f}(\Delta t)} = \frac{2 \operatorname{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin(\Delta m \Delta t) - \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta m \Delta t). \quad (102)$$

While the coefficient of the $\sin(\Delta m \Delta t)$ term in Eq. (102) is everywhere³⁶ denoted S_f :

$$S_f \equiv \frac{2 \operatorname{Im}(\lambda_f)}{1 + |\lambda_f|^2}, \quad (103)$$

different notations are in use for the coefficient of the $\cos(\Delta m \Delta t)$ term:

$$C_f \equiv -A_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}. \quad (104)$$

The C notation is used by the *BABAR* collaboration (see *e.g.* Ref. [227]), and also in this document. The A notation is used by the Belle collaboration (see *e.g.* Ref. [228]).

Neglecting effects due to CP violation in mixing (by taking $|q/p| = 1$), if the decay amplitude contains terms with a single weak (*i.e.* CP violating) phase then $|\lambda_f| = 1$ and one finds $S_f = -\eta_f \sin(\phi_{\text{mix}} + \phi_{\text{dec}})$, $C_f = 0$, where $\phi_{\text{mix}} = \arg(q/p)$ and $\phi_{\text{dec}} = \arg(\bar{A}_f/A_f)$. Note that the B^0 – $\bar{B}^0$ mixing phase $\phi_{\text{mix}} \approx 2\beta$ in the Standard Model (in the usual phase convention) [229,230].

If amplitudes with different weak phases contribute to the decay, no clean interpretation of S_f is possible without further input. If the decay amplitudes have in addition different CP conserving strong phases, then $|\lambda_f| \neq 1$ and additional input is required for interpretation. The coefficient of the cosine term becomes non-zero, indicating CP violation in decay.

Due to the fact that $\sin(\Delta m \Delta t)$ and $\cos(\Delta m \Delta t)$ are respectively odd and even functions of Δt , only small correlations (that can be induced by backgrounds, for example) between S_f and C_f are expected at an e^+e^- B factory experiment, where the range of Δt is $-\infty < \Delta t < +\infty$. The situation is different for measurements at hadron collider experiments, where the range of the time variable is $0 < \Delta t < +\infty$, so that more sizable correlations can be expected. We include the correlations in the averages where available.

Frequently, we are interested in combining measurements governed by similar or identical short-distance physics, but with different final states (*e.g.*, $B^0 \rightarrow J/\psi K_S^0$ and $B^0 \rightarrow J/\psi K_L^0$). In this case, we remove the dependence on the CP eigenvalue of the final state by quoting $-\eta S_f$. In cases where the final state is not a CP eigenstate but has an effective CP content (see below), the reported $-\eta S$ is corrected by the effective CP .

4.2.3 Time-dependent distributions with non-zero decay width difference

A complete analysis of the time-dependent decay rates of neutral B mesons must also take into account the lifetime difference between the eigenstates of the effective Hamiltonian, denoted by $\Delta\Gamma$. This is particularly important in the B_s system, since a non-negligible value of $\Delta\Gamma_s$ has

³⁶ Occasionally one also finds Eq. (102) written as $\mathcal{A}_f(\Delta t) = \mathcal{A}_f^{\text{mix}} \sin(\Delta m \Delta t) + \mathcal{A}_f^{\text{dir}} \cos(\Delta m \Delta t)$, or similar.

been established (see Sec. 3.3 for the latest experimental constraints). Neglecting CP violation in mixing, the relevant replacements for Eqs. (100) and (101) are [231]

$$\Gamma_{\bar{B}_s^0 \rightarrow f}(\Delta t) = \mathcal{N} \frac{e^{-|\Delta t|/\tau(B_s^0)}}{4\tau(B_s^0)} \left[\cosh\left(\frac{\Delta\Gamma\Delta t}{2}\right) + \frac{2\text{Im}(\lambda_f)}{1+|\lambda_f|^2} \sin(\Delta m\Delta t) - \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \cos(\Delta m\Delta t) - \frac{2\text{Re}(\lambda_f)}{1+|\lambda_f|^2} \sinh\left(\frac{\Delta\Gamma\Delta t}{2}\right) \right], \quad (105)$$

and

$$\Gamma_{B_s^0 \rightarrow f}(\Delta t) = \mathcal{N} \frac{e^{-|\Delta t|/\tau(B_s^0)}}{4\tau(B_s^0)} \left[\cosh\left(\frac{\Delta\Gamma\Delta t}{2}\right) - \frac{2\text{Im}(\lambda_f)}{1+|\lambda_f|^2} \sin(\Delta m\Delta t) + \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \cos(\Delta m\Delta t) - \frac{2\text{Re}(\lambda_f)}{1+|\lambda_f|^2} \sinh\left(\frac{\Delta\Gamma\Delta t}{2}\right) \right]. \quad (106)$$

To be consistent with our earlier notation,³⁷ we write here the coefficient of the sinh term as

$$A_f^{\Delta\Gamma} = -\frac{2\text{Re}(\lambda_f)}{1+|\lambda_f|^2}. \quad (107)$$

A time-dependent analysis of CP asymmetries in flavour-tagged B_s decays to a CP eigenstate f can thus obtain the parameters S_f , C_f and $A_f^{\Delta\Gamma}$. Note that, by definition,

$$(S_f)^2 + (C_f)^2 + (A_f^{\Delta\Gamma})^2 = 1, \quad (108)$$

and this constraint can be imposed or not in the fits. Since these parameters have sensitivity to both $\text{Im}(\lambda_f)$ and $\text{Re}(\lambda_f)$, alternative choices of parametrisation, including those directly involving CP violating phases (such as β_s), are possible. These can also be adopted for vector-vector final states.

The *untagged* time-dependent decay rate is given by

$$\Gamma_{\bar{B}_s^0 \rightarrow f}(\Delta t) + \Gamma_{B_s^0 \rightarrow f}(\Delta t) = \mathcal{N} \frac{e^{-|\Delta t|/\tau(B_s^0)}}{2\tau(B_s^0)} \left[\cosh\left(\frac{\Delta\Gamma\Delta t}{2}\right) - \frac{2\text{Re}(\lambda_f)}{1+|\lambda_f|^2} \sinh\left(\frac{\Delta\Gamma\Delta t}{2}\right) \right]. \quad (109)$$

With the requirement $\int_{-\infty}^{+\infty} \left\{ \Gamma_{\bar{B}_s^0 \rightarrow f}(\Delta t) + \Gamma_{B_s^0 \rightarrow f}(\Delta t) \right\} d(\Delta t) = 1$, the normalisation factor $\mathcal{N}$ is fixed to $1 - (\frac{\Delta\Gamma}{2\Gamma})^2$. Note that an untagged time-dependent analysis can probe λ_f , through $\text{Re}(\lambda_f)$, when $\Delta\Gamma \neq 0$. This is equivalent to determining the “*effective lifetime*” [104], as discussed in Sec. 3.2.4. The tagged analysis is, of course, more sensitive.

Other expressions can be similarly modified to take into account non-zero lifetime differences. Note that when the final state contains a mixture of CP -even and CP -odd states (as, for example, for vector-vector or multibody self-conjugate states), that $\text{Re}(\lambda_f)$ contains terms proportional to both the sine and cosine of the weak phase difference, albeit with rather different sensitivities.

4.2.4 Time-dependent CP asymmetries in decays to vector-vector final states

Consider B decays to states consisting of two spin-1 particles, such as $J/\psi K^{*0} (\rightarrow K_s^0 \pi^0)$, $J/\psi \phi$, $D^{*+} D^{*-}$ and $\rho^+ \rho^-$, which are eigenstates of charge conjugation but not of parity.³⁸ In

³⁷ As ever, alternative and conflicting notations appear in the literature. One popular alternative notation for this parameter is $\mathcal{A}_{\Delta\Gamma}$. Particular care must be taken over the signs.

³⁸ This is not true of all vector-vector final states, *e.g.*, $D^{*\pm} \rho^\mp$ is clearly not an eigenstate of charge conjugation.

fact, for such a system, there are three possible final states; in the helicity basis these can be written h_{-1}, h_0, h_{+1} . The h_0 state is an eigenstate of parity, and hence of CP ; however, CP transforms $h_{+1} \leftrightarrow h_{-1}$ (up to an unobservable phase). In the transversity basis, these states are transformed into $h_{\parallel} = (h_{+1} + h_{-1})/2$ and $h_{\perp} = (h_{+1} - h_{-1})/2$. In this basis all three states are CP eigenstates, and $h_{\perp}$ has the opposite CP to the others.

The amplitudes to these states are usually given by $A_{0,\perp,\parallel}$ (here we use normalisation such that $|A_0|^2 + |A_{\perp}|^2 + |A_{\parallel}|^2 = 1$). Then the effective CP of the vector-vector state is known if $|A_{\perp}|^2$ is measured. An alternative strategy is to measure just the longitudinally polarised component, $|A_0|^2$ (sometimes denoted by f_{long}), which allows a limit to be set on the effective CP since $|A_{\perp}|^2 \leq |A_{\perp}|^2 + |A_{\parallel}|^2 = 1 - |A_0|^2$. The most complete treatment for neutral B decays to vector-vector final states is time-dependent angular analysis (also known as time-dependent transversity analysis). In such an analysis, the interference between the CP -even and CP -odd states provides additional sensitivity to the weak and strong phases involved.

In most analyses of time-dependent CP asymmetries in decays to vector-vector final states carried out to date, an assumption has been made that each helicity (or transversity) amplitude has the same weak phase. This is a good approximation for decays that are dominated by amplitudes with a single weak phase, such as $B^0 \rightarrow J/\psi K^{*0}$, and is a reasonable approximation in any mode for which only very limited statistics are available. However, for modes that have contributions from amplitudes with different weak phases, the relative size of these contributions can be different for each helicity (or transversity) amplitude, and therefore the time-dependent CP asymmetry parameters can also differ. The most generic analysis, suitable for modes with sufficient statistics, would allow for this effect; an intermediate analysis can allow different parameters for the CP -even and CP -odd components. Such an analysis has been carried out by *BABAR* for the decay $B^0 \rightarrow D^{*+} D^{*-}$ [232]. The independent treatment of each helicity (or transversity) amplitude, as in the latest result on $B_s^0 \rightarrow J/\psi \phi$ (discussed in Sec. 3), becomes increasingly important for high precision measurements.

4.2.5 Time-dependent asymmetries: self-conjugate multiparticle final states

Amplitudes for neutral B decays into self-conjugate multiparticle final states such as $\pi^+ \pi^- \pi^0$, $K^+ K^- K_S^0$, $\pi^+ \pi^- K_S^0$, $J/\psi \pi^+ \pi^-$ or $D \pi^0$ with $D \rightarrow K_S^0 \pi^+ \pi^-$ may be written in terms of CP -even and CP -odd amplitudes. As above, the interference between these terms provides additional sensitivity to the weak and strong phases involved in the decay, and the time-dependence depends on both the sine and cosine of the weak phase difference. In order to perform unbinned maximum likelihood fits, and thereby extract as much information as possible from the distributions, it is necessary to select a model for the multiparticle decay, and therefore the results acquire some model dependence (binned, model independent methods are also possible, though are not as statistically powerful). The number of observables depends on the final state (and on the model used); the key feature is that as long as there are regions where both CP -even and CP -odd amplitudes contribute, the interference terms will be sensitive to the cosine of the weak phase difference. Therefore, these measurements allow distinction between multiple solutions for, *e.g.*, the four values of β from the measurement of $\sin(2\beta)$.

We now consider the various notations which have been used in experimental studies of time-dependent asymmetries in decays to self-conjugate multiparticle final states.

$B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_s^0\pi^+\pi^-$

The states $D\pi^0$, $D^*\pi^0$, $D\eta$, $D^*\eta$, $D\omega$ are collectively denoted $D^{(*)}h^0$. When the D decay model is fixed, fits to the time-dependent decay distributions can be performed to extract the weak phase difference. However, it is experimentally advantageous to use the sine and cosine of this phase as fit parameters, since these behave as essentially independent parameters, with low correlations and (potentially) rather different uncertainties. A parameter representing CP violation in the B decay can also be floated. For consistency with other analyses, this could be chosen to be C_f , but could equally well be $|\lambda_f|$, or other possibilities.

Belle performed an analysis of these channels with $\sin(2\phi_1)$ and $\cos(2\phi_1)$ as free parameters [233]. *BABAR* has performed an analysis floating also $|\lambda_f|$ [234] (and, of course, replacing ϕ_1 with β).

$B^0 \rightarrow D^{*+}D^{*-}K_s^0$

The hadronic structure of the $B^0 \rightarrow D^{*+}D^{*-}K_s^0$ decay is not sufficiently well understood to perform a full time-dependent Dalitz plot analysis. Instead, following Ref. [235], *BABAR* [236] and Belle [237] divide the Dalitz plane in two: $m(D^{*+}K_s^0)^2 > m(D^{*-}K_s^0)^2$ ($\eta_y = +1$) and $m(D^{*+}K_s^0)^2 < m(D^{*-}K_s^0)^2$ ($\eta_y = -1$); and then fit to a decay time distribution with asymmetry given by

$$\mathcal{A}_f(\Delta t) = \eta_y \frac{J_c}{J_0} \cos(\Delta m \Delta t) - \left[\frac{2J_{s1}}{J_0} \sin(2\beta) + \eta_y \frac{2J_{s2}}{J_0} \cos(2\beta) \right] \sin(\Delta m \Delta t). \quad (110)$$

The measured values are $\frac{J_c}{J_0}$, $\frac{2J_{s1}}{J_0} \sin(2\beta)$ and $\frac{2J_{s2}}{J_0} \cos(2\beta)$, where the parameters J_0 , J_c , J_{s1} and J_{s2} are the integrals over the half Dalitz plane $m(D^{*+}K_s^0)^2 < m(D^{*-}K_s^0)^2$ of the functions $|a|^2 + |\bar{a}|^2$, $|a|^2 - |\bar{a}|^2$, $\text{Re}(\bar{a}a^*)$ and $\text{Im}(\bar{a}a^*)$ respectively, where a and $\bar{a}$ are the decay amplitudes of $B^0 \rightarrow D^{*+}D^{*-}K_s^0$ and $\bar{B}^0 \rightarrow D^{*+}D^{*-}K_s^0$ respectively. The parameter J_{s2} (and hence J_{s2}/J_0) is predicted to be positive; with this assumption it is possible to determine the sign of $\cos(2\beta)$.

$B^0 \rightarrow K^+K^-K^0$

Studies of $B^0 \rightarrow K^+K^-K^0$ [238–240] and of the related decay $B^+ \rightarrow K^+K^-K^+$ [240–242], show that the decay is dominated by a large nonresonant contribution with significant components from the intermediate K^+K^- resonances $\phi(1020)$, $f_0(980)$, and other higher resonances,³⁹ as well a contribution from χ_{c0} .

The full time-dependent Dalitz plot analysis allows the complex amplitudes of each contributing term to be determined from data, including CP violation effects (*i.e.* allowing the complex amplitude for the B^0 decay to be independent from that for $\bar{B}^0$ decay), although one amplitude must be fixed to give a reference point. There are several choices for parametrisation of the complex amplitudes (*e.g.* real and imaginary part, or magnitude and phase). Similarly, there are various approaches to include CP violation effects. Note that positive definite parameters such as magnitudes are disfavoured in certain circumstances (they inevitably lead to biases for small values). In order to compare results between analyses, it is useful for each experiment to present results in terms of the parameters that can be measured in a Q2B analysis (such as $\mathcal{A}_f$, S_f , C_f , $\sin(2\beta^{\text{eff}})$, $\cos(2\beta^{\text{eff}})$, *etc.*)

³⁹ The broad structure that peaks near $m(K^+K^-) \sim 1550$ MeV/ c^2 and was denoted $X_0(1550)$ is now believed to originate from interference effects.

In the *BABAR* analysis of $B^0 \rightarrow K^+ K^- K^0$ [240], the complex amplitude for each resonant contribution is written as

$$A_f = c_f(1 + b_f)e^{i(\phi_f + \delta_f)}, \quad \bar{A}_f = c_f(1 - b_f)e^{i(\phi_f - \delta_f)}, \quad (111)$$

where b_f and δ_f introduce CP violation in the magnitude and phase respectively. Belle [239] use the same parametrisation but with a different notation for the parameters.⁴⁰ [The weak phase in $B^0 - \bar{B}^0$ mixing (2β) also appears in the full formula for the time-dependent decay distribution.] The Q2B parameter of CP violation in decay is directly related to b_f

$$\mathcal{A}_f = \frac{-2b_f}{1 + b_f^2} \approx C_f, \quad (112)$$

and the mixing-induced CP violation parameter can be used to obtain $\sin(2\beta^{\text{eff}})$

$$-\eta_f S_f \approx \frac{1 - b_f^2}{1 + b_f^2} \sin(2\beta_f^{\text{eff}}), \quad (113)$$

where the approximations are exact in the case that $|q/p| = 1$.

Both *BABAR* [240] and Belle [239] present results for c_f and ϕ_f , for each resonant contribution, and in addition present results for $\mathcal{A}_f$ and β_f^{eff} for $\phi(1020)K^0$, $f_0(980)K^0$ and for the remainder of the contributions to the $K^+ K^- K^0$ Dalitz plot combined.⁴¹ The models used to describe the resonant structure of the Dalitz plot differ, however. Both analyses suffer from multiple solutions, from which we select only one for averaging.

$B^0 \rightarrow \pi^+ \pi^- K_s^0$

Studies of $B^0 \rightarrow \pi^+ \pi^- K_s^0$ [243, 244] and of the related decay $B^+ \rightarrow \pi^+ \pi^- K^+$ [241, 245–247] show that the decay is dominated by components from intermediate resonances in the $K\pi$ ($K^*(892)$, $K_0^*(1430)$) and $\pi\pi$ ($\rho(770)$, $f_0(980)$, $f_2(1270)$) spectra, together with a poorly understood scalar structure that peaks near $m(\pi\pi) \sim 1300$ MeV/ c^2 and is denoted $f_X(1300)$ (that could be identified as either the $f_0(1370)$ or $f_0(1500)$), and a large nonresonant component. There is also a contribution from the χ_{c0} state.

The full time-dependent Dalitz plot analysis allows the complex amplitudes of each contributing term to be determined from data, including CP violation effects. In the *BABAR* analysis [243], the magnitude and phase of each component (for both B^0 and $\bar{B}^0$ decays) are measured relative to $B^0 \rightarrow f_0(980)K_s^0$, using the following parametrisation

$$A_f = |A_f| e^{i \arg(A_f)}, \quad \bar{A}_f = |\bar{A}_f| e^{i \arg(\bar{A}_f)}. \quad (114)$$

In the Belle analysis [244], the $B^0 \rightarrow K^{*+} \pi^-$ amplitude is chosen as the reference, and the amplitudes are parametrised as

$$A_f = a_f(1 + c_f)e^{i(b_f + d_f)}, \quad \bar{A}_f = a_f(1 - c_f)e^{i(b_f - d_f)}. \quad (115)$$

In both cases, the results are translated into Q2B parameters such as $2\beta_f^{\text{eff}}$, S_f , C_f for each CP eigenstate f , and parameters of CP violation in decay for each flavour-specific state. Relative phase differences between resonant terms are also extracted.

⁴⁰ $(c, b, \phi, \delta) \leftrightarrow (a, c, b, d)$.

⁴¹ *BABAR* also present results for the Q2B parameter S_f for these channels.

$$B^0 \rightarrow \pi^+ \pi^- \pi^0$$

The $B^0 \rightarrow \pi^+ \pi^- \pi^0$ decay is dominated by intermediate ρ resonances. Though it is possible, as above, to determine directly the complex amplitudes for each component, an alternative approach [248, 249] has been used by both *BABAR* [250, 251] and *Belle* [252, 253]. The amplitudes for B^0 and $\bar{B}^0$ to $\pi^+ \pi^- \pi^0$ are written

$$A_{3\pi} = f_+ A_+ + f_- A_- + f_0 A_0, \quad \bar{A}_{3\pi} = f_+ \bar{A}_+ + f_- \bar{A}_- + f_0 \bar{A}_0 \quad (116)$$

respectively. The symbols A_+ , A_- and A_0 represent the complex decay amplitudes for $B^0 \rightarrow \rho^+ \pi^-$, $B^0 \rightarrow \rho^- \pi^+$ and $B^0 \rightarrow \rho^0 \pi^0$ while $\bar{A}_+$, $\bar{A}_-$ and $\bar{A}_0$ represent those for $\bar{B}^0 \rightarrow \rho^+ \pi^-$, $\bar{B}^0 \rightarrow \rho^- \pi^+$ and $\bar{B}^0 \rightarrow \rho^0 \pi^0$ respectively. The terms f_+ , f_- and f_0 incorporate kinematic and dynamical factors and depend on the Dalitz plot coordinates. The full time-dependent decay distribution can then be written in terms of 27 free parameters, one for each coefficient of the form factor bilinears, as listed in Table 23. These parameters are sometimes referred to as “the U s and I s”, and can be expressed in terms of A_+ , A_- , A_0 , $\bar{A}_+$, $\bar{A}_-$ and $\bar{A}_0$. If the full set of parameters is determined, together with their correlations, other parameters, such as weak and strong phases, parameters of CP violation in decay, *etc.*, can be subsequently extracted. Note that one of the parameters (typically U_+^+) is often fixed to unity to provide a reference point; this does not affect the analysis.

4.2.6 Time-dependent CP asymmetries in decays to non- CP eigenstates

Consider a non- CP eigenstate f , and its conjugate $\bar{f}$. For neutral B decays to these final states, there are four amplitudes to consider: those for B^0 to decay to f and $\bar{f}$ (A_f and $A_{\bar{f}}$, respectively), and the equivalents for $\bar{B}^0$ ($\bar{A}_f$ and $\bar{A}_{\bar{f}}$). If CP is conserved in the decay, then $A_f = \bar{A}_{\bar{f}}$ and $A_{\bar{f}} = \bar{A}_f$.

The time-dependent decay distributions can be written in many different ways. Here, we follow Sec. 4.2.2 and define $\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$ and $\lambda_{\bar{f}} = \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}$. The time-dependent CP asymmetries that are sensitive to mixing-induced CP violation effects then follow Eq. (102):

$$\mathcal{A}_f(\Delta t) \equiv \frac{\Gamma_{\bar{B}^0 \rightarrow f}(\Delta t) - \Gamma_{B^0 \rightarrow f}(\Delta t)}{\Gamma_{\bar{B}^0 \rightarrow f}(\Delta t) + \Gamma_{B^0 \rightarrow f}(\Delta t)} = S_f \sin(\Delta m \Delta t) - C_f \cos(\Delta m \Delta t), \quad (117)$$

$$\mathcal{A}_{\bar{f}}(\Delta t) \equiv \frac{\Gamma_{\bar{B}^0 \rightarrow \bar{f}}(\Delta t) - \Gamma_{B^0 \rightarrow \bar{f}}(\Delta t)}{\Gamma_{\bar{B}^0 \rightarrow \bar{f}}(\Delta t) + \Gamma_{B^0 \rightarrow \bar{f}}(\Delta t)} = S_{\bar{f}} \sin(\Delta m \Delta t) - C_{\bar{f}} \cos(\Delta m \Delta t), \quad (118)$$

with the definitions of the parameters C_f , S_f , $C_{\bar{f}}$ and $S_{\bar{f}}$, following Eqs. (103) and (104).

The time-dependent decay rates are given by

$$\Gamma_{\bar{B}^0 \rightarrow f}(\Delta t) = \frac{e^{-|\Delta t|/\tau(B^0)}}{8\tau(B^0)} (1 + \langle \mathcal{A}_{f\bar{f}} \rangle) \{1 + S_f \sin(\Delta m \Delta t) - C_f \cos(\Delta m \Delta t)\}, \quad (119)$$

$$\Gamma_{B^0 \rightarrow f}(\Delta t) = \frac{e^{-|\Delta t|/\tau(B^0)}}{8\tau(B^0)} (1 + \langle \mathcal{A}_{f\bar{f}} \rangle) \{1 - S_f \sin(\Delta m \Delta t) + C_f \cos(\Delta m \Delta t)\}, \quad (120)$$

$$\Gamma_{\bar{B}^0 \rightarrow \bar{f}}(\Delta t) = \frac{e^{-|\Delta t|/\tau(B^0)}}{8\tau(B^0)} (1 - \langle \mathcal{A}_{f\bar{f}} \rangle) \{1 + S_{\bar{f}} \sin(\Delta m \Delta t) - C_{\bar{f}} \cos(\Delta m \Delta t)\}, \quad (121)$$

$$\Gamma_{B^0 \rightarrow \bar{f}}(\Delta t) = \frac{e^{-|\Delta t|/\tau(B^0)}}{8\tau(B^0)} (1 - \langle \mathcal{A}_{f\bar{f}} \rangle) \{1 - S_{\bar{f}} \sin(\Delta m \Delta t) + C_{\bar{f}} \cos(\Delta m \Delta t)\}, \quad (122)$$

Table 23: Definitions of the U and I coefficients. Modified from Ref. [250].

Parameter	Description
U_+^+	Coefficient of $ f_+ ^2$
U_0^+	Coefficient of $ f_0 ^2$
U_-^+	Coefficient of $ f_- ^2$
U_0^-	Coefficient of $ f_0 ^2 \cos(\Delta m \Delta t)$
U_-^-	Coefficient of $ f_- ^2 \cos(\Delta m \Delta t)$
U_+^-	Coefficient of $ f_+ ^2 \cos(\Delta m \Delta t)$
I_0	Coefficient of $ f_0 ^2 \sin(\Delta m \Delta t)$
I_-	Coefficient of $ f_- ^2 \sin(\Delta m \Delta t)$
I_+	Coefficient of $ f_+ ^2 \sin(\Delta m \Delta t)$
$U_{+-}^{+, \text{Im}}$	Coefficient of $\text{Im}[f_+ f_-^*]$
$U_{+-}^{+, \text{Re}}$	Coefficient of $\text{Re}[f_+ f_-^*]$
$U_{+-}^{-, \text{Im}}$	Coefficient of $\text{Im}[f_+ f_-^*] \cos(\Delta m \Delta t)$
$U_{+-}^{-, \text{Re}}$	Coefficient of $\text{Re}[f_+ f_-^*] \cos(\Delta m \Delta t)$
I_{+-}^{Im}	Coefficient of $\text{Im}[f_+ f_-^*] \sin(\Delta m \Delta t)$
I_{+-}^{Re}	Coefficient of $\text{Re}[f_+ f_-^*] \sin(\Delta m \Delta t)$
$U_{+0}^{+, \text{Im}}$	Coefficient of $\text{Im}[f_+ f_0^*]$
$U_{+0}^{+, \text{Re}}$	Coefficient of $\text{Re}[f_+ f_0^*]$
$U_{+0}^{-, \text{Im}}$	Coefficient of $\text{Im}[f_+ f_0^*] \cos(\Delta m \Delta t)$
$U_{+0}^{-, \text{Re}}$	Coefficient of $\text{Re}[f_+ f_0^*] \cos(\Delta m \Delta t)$
I_{+0}^{Im}	Coefficient of $\text{Im}[f_+ f_0^*] \sin(\Delta m \Delta t)$
I_{+0}^{Re}	Coefficient of $\text{Re}[f_+ f_0^*] \sin(\Delta m \Delta t)$
$U_{-0}^{+, \text{Im}}$	Coefficient of $\text{Im}[f_- f_0^*]$
$U_{-0}^{+, \text{Re}}$	Coefficient of $\text{Re}[f_- f_0^*]$
$U_{-0}^{-, \text{Im}}$	Coefficient of $\text{Im}[f_- f_0^*] \cos(\Delta m \Delta t)$
$U_{-0}^{-, \text{Re}}$	Coefficient of $\text{Re}[f_- f_0^*] \cos(\Delta m \Delta t)$
I_{-0}^{Im}	Coefficient of $\text{Im}[f_- f_0^*] \sin(\Delta m \Delta t)$
I_{-0}^{Re}	Coefficient of $\text{Re}[f_- f_0^*] \sin(\Delta m \Delta t)$

where the time-independent parameter $\langle \mathcal{A}_{f\bar{f}} \rangle$ represents an overall asymmetry in the production of the f and $\bar{f}$ final states,⁴²

$$\langle \mathcal{A}_{f\bar{f}} \rangle = \frac{\left(|A_f|^2 + |\bar{A}_f|^2 \right) - \left(|A_{\bar{f}}|^2 + |\bar{A}_{\bar{f}}|^2 \right)}{\left(|A_f|^2 + |\bar{A}_f|^2 \right) + \left(|A_{\bar{f}}|^2 + |\bar{A}_{\bar{f}}|^2 \right)}. \quad (123)$$

Assuming $|q/p| = 1$, the parameters C_f and $C_{\bar{f}}$ can also be written in terms of the decay

⁴² This parameter is often denoted $\mathcal{A}_f$ (or $\mathcal{A}_{CP}$), but here we avoid this notation to prevent confusion with the time-dependent CP asymmetry.

amplitudes as follows:

$$C_f = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2} \quad \text{and} \quad C_{\bar{f}} = \frac{|A_{\bar{f}}|^2 - |\bar{A}_{\bar{f}}|^2}{|A_{\bar{f}}|^2 + |\bar{A}_{\bar{f}}|^2}, \quad (124)$$

giving asymmetries in the decay amplitudes of B^0 and $\bar{B}^0$ to the final states f and $\bar{f}$ respectively. In this notation, the conditions for absence of CP violation in decay are $\langle \mathcal{A}_{f\bar{f}} \rangle = 0$ and $C_f = -C_{\bar{f}}$. Note that C_f and $C_{\bar{f}}$ are typically non-zero; *e.g.*, for a flavour-specific final state, $\bar{A}_f = A_{\bar{f}} = 0$ ($A_f = \bar{A}_{\bar{f}} = 0$), they take the values $C_f = -C_{\bar{f}} = 1$ ($C_f = -C_{\bar{f}} = -1$).

The coefficients of the sine terms contain information about the weak phase. In the case that each decay amplitude contains only a single weak phase (*i.e.*, no CP violation in decay), these terms can be written

$$S_f = \frac{-2|A_f||\bar{A}_f|\sin(\phi_{\text{mix}} + \phi_{\text{dec}} - \delta_f)}{|A_f|^2 + |\bar{A}_f|^2} \quad \text{and} \quad S_{\bar{f}} = \frac{-2|A_{\bar{f}}||\bar{A}_{\bar{f}}|\sin(\phi_{\text{mix}} + \phi_{\text{dec}} + \delta_f)}{|A_{\bar{f}}|^2 + |\bar{A}_{\bar{f}}|^2}, \quad (125)$$

where δ_f is the strong phase difference between the decay amplitudes. If there is no CP violation, the condition $S_f = -S_{\bar{f}}$ holds. If decay amplitudes with different weak and strong phases contribute, no clean interpretation of S_f and $S_{\bar{f}}$ is possible.

Since two of the CP invariance conditions are $C_f = -C_{\bar{f}}$ and $S_f = -S_{\bar{f}}$, there is motivation for a rotation of the parameters:

$$S_{f\bar{f}} = \frac{S_f + S_{\bar{f}}}{2}, \quad \Delta S_{f\bar{f}} = \frac{S_f - S_{\bar{f}}}{2}, \quad C_{f\bar{f}} = \frac{C_f + C_{\bar{f}}}{2}, \quad \Delta C_{f\bar{f}} = \frac{C_f - C_{\bar{f}}}{2}. \quad (126)$$

With these parameters, the CP invariance conditions become $S_{f\bar{f}} = 0$ and $C_{f\bar{f}} = 0$. The parameter $\Delta C_{f\bar{f}}$ gives a measure of the “flavour-specificity” of the decay: $\Delta C_{f\bar{f}} = \pm 1$ corresponds to a completely flavour-specific decay, in which no interference between decays with and without mixing can occur, while $\Delta C_{f\bar{f}} = 0$ results in maximum sensitivity to mixing-induced CP violation. The parameter $\Delta S_{f\bar{f}}$ is related to the strong phase difference between the decay amplitudes of B^0 to f and to $\bar{f}$. We note that the observables of Eq. (126) exhibit experimental correlations (typically of $\sim 20\%$, depending on the tagging purity, and other effects) between $S_{f\bar{f}}$ and $\Delta S_{f\bar{f}}$, and between $C_{f\bar{f}}$ and $\Delta C_{f\bar{f}}$. On the other hand, the final state specific observables of Eqs. (119)–(122) tend to have low correlations.

Alternatively, if we recall that the CP invariance conditions at the decay amplitude level are $A_f = \bar{A}_{\bar{f}}$ and $A_{\bar{f}} = \bar{A}_f$, we are led to consider the parameters [254]

$$\mathcal{A}_{f\bar{f}} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2} \quad \text{and} \quad \mathcal{A}_{\bar{f}f} = \frac{|\bar{A}_f|^2 - |A_{\bar{f}}|^2}{|\bar{A}_f|^2 + |A_{\bar{f}}|^2}. \quad (127)$$

These are sometimes considered more physically intuitive parameters since they characterise CP violation in decay in decays with particular topologies. For example, in the case of $B^0 \rightarrow \rho^\pm \pi^\mp$ (choosing $f = \rho^+ \pi^-$ and $\bar{f} = \rho^- \pi^+$), $\mathcal{A}_{f\bar{f}}$ (also denoted $\mathcal{A}_{\rho\pi}^{+-}$) parametrises CP violation in decays in which the produced ρ meson does not contain the spectator quark, while $\mathcal{A}_{\bar{f}f}$ (also denoted $\mathcal{A}_{\rho\pi}^{-+}$) parametrises CP violation in decays in which it does. Note that we have again followed the sign convention that the asymmetry is the difference between the rate involving

a b quark and that involving a $\bar{b}$ quark, *cf.* Eq. (99). Of course, these parameters are not independent of the other sets of parameters given above, and can be written

$$\mathcal{A}_{f\bar{f}} = -\frac{\langle\mathcal{A}_{f\bar{f}}\rangle + C_{f\bar{f}} + \langle\mathcal{A}_{f\bar{f}}\rangle\Delta C_{f\bar{f}}}{1 + \Delta C_{f\bar{f}} + \langle\mathcal{A}_{f\bar{f}}\rangle C_{f\bar{f}}} \quad \text{and} \quad \mathcal{A}_{\bar{f}f} = \frac{-\langle\mathcal{A}_{f\bar{f}}\rangle + C_{f\bar{f}} + \langle\mathcal{A}_{f\bar{f}}\rangle\Delta C_{f\bar{f}}}{-1 + \Delta C_{f\bar{f}} + \langle\mathcal{A}_{f\bar{f}}\rangle C_{f\bar{f}}}. \quad (128)$$

They usually exhibit strong correlations.

We now consider the various notations which have been used in experimental studies of time-dependent CP asymmetries in decays to non- CP eigenstates.

$B^0 \rightarrow D^{*\pm} D^\mp$

The $(\langle\mathcal{A}_{f\bar{f}}\rangle, C_f, S_f, C_{\bar{f}}, S_{\bar{f}})$ set of parameters was used in early publications by both *BABAR* [255] and *Belle* [256] (albeit with slightly different notations) in the $D^{*\pm} D^\mp$ system ($f = D^{*+} D^-$, $\bar{f} = D^{*-} D^+$). In their most recent paper on this topic *Belle* [257] instead used the parametrisation $(A_{D^*D}, S_{D^*D}, \Delta S_{D^*D}, C_{D^*D}, \Delta C_{D^*D})$, while *BABAR* [232] give results in both sets of parameters. We therefore use the $(A_{D^*D}, S_{D^*D}, \Delta S_{D^*D}, C_{D^*D}, \Delta C_{D^*D})$ set.

$B^0 \rightarrow \rho^\pm \pi^\mp$

In the $\rho^\pm \pi^\mp$ system, the $(\langle\mathcal{A}_{f\bar{f}}\rangle, C_{f\bar{f}}, S_{f\bar{f}}, \Delta C_{f\bar{f}}, \Delta S_{f\bar{f}})$ set of parameters has been used originally by *BABAR* [258] and *Belle* [259], in the Q2B approximation; the exact names⁴³ used in this case are $(\mathcal{A}_{CP}^{\rho\pi}, C_{\rho\pi}, S_{\rho\pi}, \Delta C_{\rho\pi}, \Delta S_{\rho\pi})$, and these names are also used in this document.

Since $\rho^\pm \pi^\mp$ is reconstructed in the final state $\pi^+ \pi^- \pi^0$, the interference between the ρ resonances can provide additional information about the phases (see Sec. 4.2.5). Both *BABAR* [250] and *Belle* [252, 253] have performed time-dependent Dalitz plot analyses, from which the weak phase α is directly extracted. In such an analysis, the measured Q2B parameters are also naturally corrected for interference effects.

$B^0 \rightarrow D^\mp \pi^\pm, D^{*\mp} \pi^\pm, D^\mp \rho^\pm$

Time-dependent CP analyses have also been performed for the final states $D^\mp \pi^\pm$, $D^{*\mp} \pi^\pm$ and $D^\mp \rho^\pm$. In these theoretically clean cases, no penguin contributions are possible, so there is no CP violation in decay. Furthermore, due to the smallness of the ratio of the magnitudes of the suppressed ($b \rightarrow u$) and favoured ($b \rightarrow c$) amplitudes (denoted R_f), to a very good approximation, $C_f = -C_{\bar{f}} = 1$ (using $f = D^{(*)-} h^+$, $\bar{f} = D^{(*)+} h^-$, $h = \pi, \rho$), and the coefficients of the sine terms are given by

$$S_f = -2R_f \sin(\phi_{\text{mix}} + \phi_{\text{dec}} - \delta_f) \quad \text{and} \quad S_{\bar{f}} = -2R_f \sin(\phi_{\text{mix}} + \phi_{\text{dec}} + \delta_f). \quad (129)$$

Thus weak phase information can be cleanly obtained from measurements of S_f and $S_{\bar{f}}$, although external information on at least one of R_f or δ_f is necessary. (Note that $\phi_{\text{mix}} + \phi_{\text{dec}} = 2\beta + \gamma \equiv 2\phi_1 + \phi_3$ for all the decay modes in question, while R_f and δ_f depend on the decay mode.)

Again, different notations have been used in the literature. *BABAR* [260, 261] defines the time-dependent probability function by

$$f^\pm(\eta, \Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \mp S_\zeta \sin(\Delta m \Delta t) \mp \eta C_\zeta \cos(\Delta m \Delta t)], \quad (130)$$

⁴³ *BABAR* has used the notations $A_{CP}^{\rho\pi}$ [258] and $\mathcal{A}_{\rho\pi}$ [250] in place of $\mathcal{A}_{CP}^{\rho\pi}$.

Table 24: Conversion between the various notations used for CP violation parameters in the $D^\pm\pi^\mp$, $D^{*\pm}\pi^\mp$ and $D^\pm\rho^\mp$ systems. The b_i terms used by *BABAR* have been neglected. Recall that $(\alpha, \beta, \gamma) = (\phi_2, \phi_1, \phi_3)$.

	<i>BABAR</i>	Belle partial rec.	Belle full rec.
$S_{D^+\pi^-}$	$-S_- = -(a + c_i)$	N/A	$2R_{D\pi} \sin(2\phi_1 + \phi_3 + \delta_{D\pi})$
$S_{D^-\pi^+}$	$-S_+ = -(a - c_i)$	N/A	$2R_{D\pi} \sin(2\phi_1 + \phi_3 - \delta_{D\pi})$
$S_{D^{*+}\pi^-}$	$-S_- = -(a + c_i)$	S^+	$-2R_{D^*\pi} \sin(2\phi_1 + \phi_3 + \delta_{D^*\pi})$
$S_{D^{*-}\pi^+}$	$-S_+ = -(a - c_i)$	S^-	$-2R_{D^*\pi} \sin(2\phi_1 + \phi_3 - \delta_{D^*\pi})$
$S_{D^+\rho^-}$	$-S_- = -(a + c_i)$	N/A	N/A
$S_{D^-\rho^+}$	$-S_+ = -(a - c_i)$	N/A	N/A

where the upper (lower) sign corresponds to the tagging meson being a B^0 ($\bar{B}^0$). Note here that a tagging B^0 ($\bar{B}^0$) corresponds to $-S_\zeta$ ($+S_\zeta$). The parameters η and ζ take the values $+1$ and -1 when the final state is, *e.g.*, $D^-\pi^+$ ($D^+\pi^-$). However, in the fit, the substitutions $C_\zeta = 1$ and $S_\zeta = a \mp \eta b_i - \eta c_i$ are made.⁴⁴ (Note that, neglecting b terms, $S_+ = a - c$ and $S_- = a + c$, so that $a = (S_+ + S_-)/2$, $c = (S_- - S_+)/2$, in analogy to the parameters of Eq. (126).) These are motivated by the possibility of CP violation on the tag side [262], which is absent for semileptonic B decays (mostly lepton tags). The parameter a is not affected by tag side CP violation. The parameter b only depends on tag side CP violation parameters and is not directly useful for determining UT angles. A clean interpretation of the c parameter is only possible for lepton-tagged events, so the *BABAR* measurements report c measured with those events only.

The parameters used by Belle in the analysis using partially reconstructed B decays [263], are similar to the S_ζ parameters defined above. However, in the Belle convention, a tagging B^0 corresponds to a $+$ sign in front of the sine coefficient; furthermore the correspondence between the super/subscript and the final state is opposite, so that $S_\pm$ (*BABAR*) = $-S^\mp$ (Belle). In this analysis, only lepton tags are used, so there is no effect from tag side CP violation. In the Belle analysis using fully reconstructed B decays [264], this effect is measured and taken into account using $D^*\ell\nu$ decays; in neither Belle analysis are the a , b and c parameters used. In the latter case, the measured parameters are $2R_{D^{(*)}\pi} \sin(2\phi_1 + \phi_3 \pm \delta_{D^{(*)}\pi})$; the definition is such that $S^\pm$ (Belle) = $-2R_{D^*\pi} \sin(2\phi_1 + \phi_3 \pm \delta_{D^*\pi})$. However, the definition includes an angular momentum factor $(-1)^L$ [265], and so for the results in the $D\pi$ system, there is an additional factor of -1 in the conversion.

Explicitly, the conversion then reads as given in Table 24, where we have neglected the b_i terms used by *BABAR* (which are zero in the absence of tag side CP violation). For the averages in this document, we use the a and c parameters, and give the explicit translations used in Table 25. It is to be fervently hoped that the experiments will converge on a common notation in future.

$$B_s^0 \rightarrow D_s^\mp K^\pm$$

The phenomenology of $B_s^0 \rightarrow D_s^\mp K^\pm$ decays is similar to that for $B^0 \rightarrow D^\mp \pi^\pm$, with some

⁴⁴ The subscript i denotes tagging category.

Table 25: Translations used to convert the parameters measured by Belle to the parameters used for averaging in this document. The angular momentum factor L is -1 for $D^*\pi$ and $+1$ for $D\pi$. Recall that $(\alpha, \beta, \gamma) = (\phi_2, \phi_1, \phi_3)$.

	$D^*\pi$ partial rec.	$D^{(*)}\pi$ full rec.
a	$-(S^+ + S^-)$	$\frac{1}{2}(-1)^{L+1} (2R_{D^{(*)}\pi} \sin(2\phi_1 + \phi_3 + \delta_{D^{(*)}\pi}) + 2R_{D^{(*)}\pi} \sin(2\phi_1 + \phi_3 - \delta_{D^{(*)}\pi}))$
c	$-(S^+ - S^-)$	$\frac{1}{2}(-1)^{L+1} (2R_{D^{(*)}\pi} \sin(2\phi_1 + \phi_3 + \delta_{D^{(*)}\pi}) - 2R_{D^{(*)}\pi} \sin(2\phi_1 + \phi_3 - \delta_{D^{(*)}\pi}))$

important caveats. The larger size of the ratio R of the magnitudes of the suppressed and favoured amplitudes allows it to be determined from the data, as the deviation of C_f and $C_{\bar{f}}$ from unity (in magnitude) can be observed. Moreover, the non-zero value of $\Delta\Gamma_s$ allows the determination of additional terms, $A_f^{\Delta\Gamma}$ and $A_{\bar{f}}^{\Delta\Gamma}$ (see Sec. 4.2.3), that break ambiguities in the solutions for $\phi_{\text{mix}} + \phi_{\text{dec}}$, which for $B_s^0 \rightarrow D_s^\mp K^\pm$ decays is equal to $\gamma - 2\beta_s$.

LHCb [266] have performed such an analysis with $B_s^0 \rightarrow D_s^\mp K^\pm$ decays. The absence of CP violation in decay is assumed, and the parameters that are determined from the fit are labelled $C, A^{\Delta\Gamma}, \bar{A}^{\Delta\Gamma}, S, \bar{S}$. These are trivially related to the definitions used in this Section.

Time-dependent asymmetries in radiative B decays

As a special case of decays to non- CP eigenstates, let us consider radiative B decays. Here, the emitted photon has a distinct helicity, which is in principle observable, but in practise is not usually measured. Thus the measured time-dependent decay rates are given by [267, 268]

$$\begin{aligned} \Gamma_{\bar{B}^0 \rightarrow X\gamma}(\Delta t) &= \Gamma_{\bar{B}^0 \rightarrow X\gamma_L}(\Delta t) + \Gamma_{\bar{B}^0 \rightarrow X\gamma_R}(\Delta t) \\ &= \frac{e^{-|\Delta t|/\tau(B^0)}}{4\tau(B^0)} \{1 + (S_L + S_R) \sin(\Delta m \Delta t) - (C_L + C_R) \cos(\Delta m \Delta t)\}, \end{aligned} \quad (131)$$

$$\begin{aligned} \Gamma_{B^0 \rightarrow X\gamma}(\Delta t) &= \Gamma_{B^0 \rightarrow X\gamma_L}(\Delta t) + \Gamma_{B^0 \rightarrow X\gamma_R}(\Delta t) \\ &= \frac{e^{-|\Delta t|/\tau(B^0)}}{4\tau(B^0)} \{1 - (S_L + S_R) \sin(\Delta m \Delta t) + (C_L + C_R) \cos(\Delta m \Delta t)\}, \end{aligned} \quad (132)$$

where in place of the subscripts f and $\bar{f}$ we have used L and R to indicate the photon helicity. In order for interference between decays with and without B^0 - $\bar{B}^0$ mixing to occur, the X system must not be flavour-specific, *e.g.*, in case of $B^0 \rightarrow K^{*0}\gamma$, the final state must be $K_s^0\pi^0\gamma$. The sign of the sine term depends on the C eigenvalue of the X system. At leading order, the photons from $b \rightarrow q\gamma$ ($\bar{b} \rightarrow \bar{q}\gamma$) are predominantly left (right) polarised, with corrections of order of m_q/m_b , thus interference effects are suppressed. Higher order effects can lead to corrections of order Λ_{QCD}/m_b [269, 270], though explicit calculations indicate such corrections are small for exclusive final states [271, 272]. The predicted smallness of the S terms in the Standard Model results in sensitivity to new physics contributions.

The formalism discussed above is valid from any radiative decay to a final state where the hadronic system is an eigenstate of C . In addition to $K_s^0\pi^0\gamma$, experiments have presented results using B^0 decays to $K_s^0\eta\gamma$, $K_s^0\rho\gamma$ and $K_s^0\phi\gamma$. For the case of the $K_s^0\rho\gamma$ final state, particular care is needed, as due to the non-negligible width of the ρ^0 meson, decays selected as $B^0 \rightarrow K_s^0\rho^0\gamma$ can include a significant contribution from $K^{*\pm}\pi^\mp\gamma$ decays, which are flavour-specific and do

not have the same oscillation phenomenology. It is therefore necessary to correct the fitted asymmetry parameter for a “dilution factor”.

4.2.7 Asymmetries in $B \rightarrow D^{(*)}K^{(*)}$ decays

CP asymmetries in $B \rightarrow D^{(*)}K^{(*)}$ decays are sensitive to γ . The neutral $D^{(*)}$ meson produced is an admixture of $D^{(*)0}$ (produced by a $b \rightarrow c$ transition) and $\bar{D}^{(*)0}$ (produced by a colour-suppressed $b \rightarrow u$ transition) states. If the final state is chosen so that both $D^{(*)0}$ and $\bar{D}^{(*)0}$ can contribute, the two amplitudes interfere, and the resulting observables are sensitive to γ , the relative weak phase between the two B decay amplitudes [273]. Various methods have been proposed to exploit this interference, including those where the neutral D meson is reconstructed as a CP eigenstate (GLW) [274, 275], in a suppressed final state (ADS) [276, 277], or in a self-conjugate three-body final state, such as $K_S^0\pi^+\pi^-$ (Dalitz) [278, 279]. It should be emphasised that while each method differs in the choice of D decay, they are all sensitive to the same parameters of the B decay, and can be considered as variations of the same technique.

Consider the case of $B^\mp \rightarrow DK^\mp$, with D decaying to a final state f , which is accessible to both D^0 and $\bar{D}^0$. We can write the decay rates for B^- and B^+ ($\Gamma_\mp$), the charge averaged rate ($\Gamma = (\Gamma_- + \Gamma_+)/2$) and the charge asymmetry ($\mathcal{A} = (\Gamma_- - \Gamma_+)/(\Gamma_- + \Gamma_+)$, see Eq. (99)) as

$$\Gamma_\mp \propto r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D \mp \gamma), \quad (133)$$

$$\Gamma \propto r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma), \quad (134)$$

$$\mathcal{A} = \frac{2r_B r_D \sin(\delta_B + \delta_D) \sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)}, \quad (135)$$

where the ratio of B decay amplitudes⁴⁵ is usually defined to be less than one,

$$r_B = \frac{|A(B^- \rightarrow \bar{D}^0 K^-)|}{|A(B^- \rightarrow D^0 K^-)|}, \quad (136)$$

and the ratio of D decay amplitudes is correspondingly defined by

$$r_D = \frac{|A(D^0 \rightarrow f)|}{|A(\bar{D}^0 \rightarrow f)|}. \quad (137)$$

The strong phase differences between the B and D decay amplitudes are given by δ_B and δ_D , respectively. The values of r_D and δ_D depend on the final state f : for the GLW analysis, $r_D = 1$ and δ_D is trivial (either zero or π), in the Dalitz plot analysis r_D and δ_D vary across the Dalitz plot, and depend on the D decay model used, for the ADS analysis, the values of r_D and δ_D are not trivial.

Note that, for given values of r_B and r_D , the maximum size of $\mathcal{A}$ (at $\sin(\delta_B + \delta_D) = 1$) is $2r_B r_D \sin(\gamma) / (r_B^2 + r_D^2)$. Thus even for D decay modes with small r_D , large asymmetries, and hence sensitivity to γ , may occur for B decay modes with similar values of r_B . For this reason, the ADS analysis of the decay $B^\mp \rightarrow D\pi^\mp$ is also of interest.

⁴⁵ Note that here we use the notation r_B to denote the ratio of B decay amplitudes, whereas in Sec. 4.2.6 we used, *e.g.*, $R_{D\pi}$, for a rather similar quantity. The reason is that here we need to be concerned also with D decay amplitudes, and so it is convenient to use the subscript to denote the decaying particle. Hopefully, using r in place of R will reduce the potential for confusion.

In the GLW analysis, the measured quantities are the partial rate asymmetry and the charge averaged rate, which are measured both for CP -even and CP -odd D decays. The latter is defined as

$$R_{CP} = \frac{2\Gamma(B^- \rightarrow D_{CP}K^-)}{\Gamma(B^- \rightarrow D^0K^-)}. \quad (138)$$

It is experimentally convenient to measure R_{CP} using a double ratio,

$$R_{CP} = \frac{\Gamma(B^- \rightarrow D_{CP}K^-) / \Gamma(B^- \rightarrow D^0K^-)}{\Gamma(B^- \rightarrow D_{CP}\pi^-) / \Gamma(B^- \rightarrow D^0\pi^-)} \quad (139)$$

that is normalised both to the rate for the favoured $D^0 \rightarrow K^-\pi^+$ decay, and to the equivalent quantities for $B^- \rightarrow D\pi^-$ decays (charge conjugate processes are implicitly included in Eq. (138) and (139)). In this way the constant of proportionality drops out of Eq. (134). Eq. (139) is exact in the limit that the contribution of the $b \rightarrow u$ decay amplitude to $B^- \rightarrow D\pi^-$ vanishes and when the flavour-specific rates $\Gamma(B^- \rightarrow D^0h^-)$ ($h = \pi, K$) are determined using appropriately flavour-specific D decays. In reality, the decay $D \rightarrow K\pi$ is invariably used, leading to a small source of systematic uncertainty. The CP asymmetry is defined as

$$A_{CP} = \frac{\Gamma(B^- \rightarrow D_{CP}K^-) - \Gamma(B^+ \rightarrow D_{CP}K^+)}{\Gamma(B^- \rightarrow D_{CP}K^-) + \Gamma(B^+ \rightarrow D_{CP}K^+)}. \quad (140)$$

For the ADS analysis, using a suppressed $D \rightarrow f$ decay, the measured quantities are again the partial rate asymmetry, and the charge averaged rate. In this case it is sufficient to measure the rate in a single ratio (normalised to the favoured $D \rightarrow \bar{f}$ decay) since detection systematics cancel naturally; the observed quantity is then

$$R_{\text{ADS}} = \frac{\Gamma(B^- \rightarrow [f]_D K^-) + \Gamma(B^+ \rightarrow [\bar{f}]_D K^+)}{\Gamma(B^- \rightarrow [\bar{f}]_D K^-) + \Gamma(B^+ \rightarrow [f]_D K^+)}, \quad (141)$$

where the inclusion of charge conjugate modes has been made explicit. The CP asymmetry is defined as

$$A_{\text{ADS}} = \frac{\Gamma(B^- \rightarrow [f]_D K^-) - \Gamma(B^+ \rightarrow [f]_D K^+)}{\Gamma(B^- \rightarrow [f]_D K^-) + \Gamma(B^+ \rightarrow [f]_D K^+)}. \quad (142)$$

Since the uncertainty of A_{ADS} depends on the central value of R_{ADS} , for some statistical treatments it is preferable to use an alternative pair of parameters [280])

$$R_- = \frac{\Gamma(B^- \rightarrow [f]_D K^-)}{\Gamma(B^- \rightarrow [\bar{f}]_D K^-)} \quad R_+ = \frac{\Gamma(B^+ \rightarrow [\bar{f}]_D K^+)}{\Gamma(B^+ \rightarrow [f]_D K^+)}, \quad (143)$$

where there is no inclusion of charge conjugated processes. We use the $(R_{\text{ADS}}, A_{\text{ADS}})$ set in our compilation.

In the ADS analysis, there are an additional two unknowns (r_D and δ_D) compared to the GLW case. However, the value of r_D can be measured using decays of D mesons of known flavour, and δ_D can be measured from interference effects in decays of quantum-correlated $D\bar{D}$ pairs produced at the $\psi(3770)$ resonance. More generally, one needs access to two different linear admixtures of D^0 and $\bar{D}^0$ states in order to determine the relative phase: one such sample can be flavour tagged D mesons which are available in abundant quantities in many

experiments; the other can be CP -tagged D mesons from $\psi(3770)$ decays or could be mixed D mesons, or could for that matter be the combination of D^0 and $\bar{D}^0$ that is found in $B \rightarrow DK$ decays. In fact, the most precise information on both r_D and δ_D currently comes from global fits on charm mixing parameters, as discussed in Sec. 8.1.

The relation of $\mathcal{A}_{\text{ADS}}$ to the underlying parameters given in Eq. (135) and Table 26 is exact for a two-body D decay. For multibody decays, a similar formalism can be used with the introduction of a coherence factor [281]. This is most appropriate for doubly-Cabibbo-suppressed decays to non-self-conjugate final states, but can also be modified for use with singly-Cabibbo-suppressed decays [282]. For multibody self-conjugate final states, such as $K_S^0 \pi^+ \pi^-$, a Dalitz plot analysis (discussed below) is often more appropriate.

Additional coherence factors enter the expressions when the B decay is to a multibody final state. In particular, experiments have studied $B^+ \rightarrow DK^*(892)^+$, $B^0 \rightarrow DK^*(892)^0$ and $B^+ \rightarrow DK^+ \pi^+ \pi^-$ decays. The non-negligible width of the $K^*(892)$ resonance implies that contributions from other $B \rightarrow DK\pi$ decays can pass the selection requirements. Their effect on the Q2B analysis can be accounted for with a coherence factor [283]. An alternative approach, not yet pursued by experiments, but in certain cases potentially more advantageous [284, 285], is Dalitz plot analysis of the full $B \rightarrow DK\pi$ phase space.

In the Dalitz plot analysis of D decays to multibody self-conjugate final states, once a model is assumed for the D decay, which gives the values of r_D and δ_D across the Dalitz plot, it is possible to perform a simultaneous fit to the B^+ and B^- samples and directly extract γ , r_B and δ_B . However, the uncertainties on the phases depend approximately inversely on r_B . Furthermore, r_B is positive definite (and small), and therefore tends to be overestimated, which leads to an underestimation of the uncertainty on γ that must be corrected statistically. An alternative approach is to extract from the data the ‘‘Cartesian’’ variables

$$(x_{\pm}, y_{\pm}) = (\text{Re}(r_B e^{i(\delta_B \pm \gamma)}), \text{Im}(r_B e^{i(\delta_B \pm \gamma)})) = (r_B \cos(\delta_B \pm \gamma), r_B \sin(\delta_B \pm \gamma)). \quad (144)$$

These variables are approximately statistically uncorrelated and almost Gaussian. The pairs of variables $(x_{\pm}, y_{\pm})$ can be extracted from independent fits of the $B^{\pm}$ data samples. Use of these variables makes the combination of results much simpler.

The assumption of a model for the D decay can, however, lead to a non-negligible, and hard to quantify, source of uncertainty. To obviate this, it is possible to use instead a model-independent approach, in which the Dalitz plot (or, more generally, the phase-space) is binned [278, 286, 287]. In this case, hadronic parameters describing the average strong phase difference in each bin between the suppressed and favoured decay amplitudes enter the equations. These parameters can be determined from interference effects in decays of quantum-correlated $D\bar{D}$ pairs produced at the $\psi(3770)$ resonance.

However, if the Dalitz plot is effectively dominated by one CP state, there will be additional sensitivity to γ in the numbers of events in the $B^{\pm}$ data samples. This can be taken into account in various ways. One possibility is to extract GLW-like variables in addition to the $(x_{\pm}, y_{\pm})$ parameters. An alternative approach proceeds by defining $z_{\pm} = x_{\pm} + iy_{\pm}$ and $x_0 = -\int \text{Re}[f(s_1, s_2)f^*(s_2, s_1)] ds_1 ds_2$, where s_1, s_2 are the coordinates of invariant mass squared that define the Dalitz plot and f is the complex amplitude for D decay as a function of the

Dalitz plot coordinates.⁴⁶ The fitted parameters $(\rho^\pm, \theta^\pm)$ are then defined by

$$\rho^\pm e^{i\theta^\pm} = z_\pm - x_0. \quad (145)$$

Note that the yields of $B^\pm$ decays are proportional to $1 + (\rho^\pm)^2 - (x_0)^2$. This choice of variables has been used by *BABAR* in the analysis of $B^\mp \rightarrow DK^\mp$ with $D \rightarrow \pi^+\pi^-\pi^0$ [288]; for this D decay, $x_0 = 0.850$. More recently, it has been noted that $D \rightarrow \pi^+\pi^-\pi^0$ can be used in a GLW-like analysis [289].

The relations between the measured quantities and the underlying parameters are summarised in Table 26. Note carefully that the hadronic factors r_B and δ_B are different, in general, for each B decay mode.

Table 26: Summary of relations between measured and physical parameters in GLW, ADS and Dalitz analyses of $B \rightarrow D^{(*)}K^{(*)}$.

GLW analysis	
$R_{CP\pm}$	$1 + r_B^2 \pm 2r_B \cos(\delta_B) \cos(\gamma)$
$A_{CP\pm}$	$\pm 2r_B \sin(\delta_B) \sin(\gamma) / R_{CP\pm}$
ADS analysis	
R_{ADS}	$r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)$
A_{ADS}	$2r_B r_D \sin(\delta_B + \delta_D) \sin(\gamma) / R_{\text{ADS}}$
Dalitz analysis ($D \rightarrow K_S^0 \pi^+ \pi^-$)	
$x_\pm$	$r_B \cos(\delta_B \pm \gamma)$
$y_\pm$	$r_B \sin(\delta_B \pm \gamma)$
Dalitz analysis ($D \rightarrow \pi^+ \pi^- \pi^0$)	
$\rho^\pm$	$ z_\pm - x_0 $
$\theta^\pm$	$\tan^{-1}(\text{Im}(z_\pm) / (\text{Re}(z_\pm) - x_0))$

4.3 Common inputs and error treatment

The common inputs used for rescaling are listed in Table 27. The B^0 lifetime ($\tau(B^0)$), mixing parameter (Δm_d) and relative width difference ($\Delta \Gamma_d / \Gamma_d$) averages are provided by the HFAG Lifetimes and Oscillations subgroup (Sec. 3). The fraction of the perpendicularly polarised component ($|A_\perp|^2$) in $B \rightarrow J/\psi K^*(892)$ decays, which determines the CP composition in these decays, is averaged from results by *BABAR* [290], Belle [291], CDF [292], D0 [85] and LHCb [293]. See also HFAG B to Charm Decay Parameters subgroup (Sec. 6).

At present, we only rescale to a common set of input parameters for modes with reasonably small statistical errors ($b \rightarrow c\bar{c}s$ transitions). Correlated systematic errors are taken into account in these modes as well. For all other modes, the effect of such a procedure is currently negligible.

As explained in Sec. 1, we do not apply a rescaling factor on the error of an average that has $\chi^2/\text{dof} > 1$ (unlike the procedure currently used by the PDG [5]). We provide a confidence level

⁴⁶ The x_0 parameter is closely related to the c_i parameters of the model dependent Dalitz plot analysis [278, 286, 287], and the coherence factor of inclusive ADS-type analyses [281], integrated over the entire Dalitz plot.

Table 27: Common inputs used in calculating the averages.

$\tau(B^0)$ (ps)	1.519 ± 0.005
Δm_d (ps ⁻¹)	0.510 ± 0.003
$\Delta\Gamma_d/\Gamma_d$	0.001 ± 0.010
$ A_\perp ^2(J/\psi K^*)$	0.209 ± 0.006

of the fit so that one can know the consistency of the measurements included in the average, and attach comments in case some care needs to be taken in the interpretation. Note that, in general, results obtained from data samples with low statistics will exhibit some non-Gaussian behaviour. We average measurements with asymmetric errors using the PDG [5] prescription. In cases where several measurements are correlated (*e.g.* S_f and C_f in measurements of time-dependent CP violation in B decays to a particular CP eigenstate) we take these into account in the averaging procedure if the uncertainties are sufficiently Gaussian. For measurements where one error is given, it represents the total error, where statistical and systematic uncertainties have been added in quadrature. If two errors are given, the first is statistical and the second systematic. If more than two errors are given, the origin of the additional uncertainty will be explained in the text.

4.4 Time-dependent asymmetries in $b \rightarrow c\bar{c}s$ transitions

4.4.1 Time-dependent CP asymmetries in $b \rightarrow c\bar{c}s$ decays to CP eigenstates

In the Standard Model, the time-dependent parameters for $b \rightarrow c\bar{c}s$ transitions are predicted to be: $S_{b \rightarrow c\bar{c}s} = -\eta \sin(2\beta)$, $C_{b \rightarrow c\bar{c}s} = 0$ to very good accuracy. The averages for $-\eta S_{b \rightarrow c\bar{c}s}$ and $C_{b \rightarrow c\bar{c}s}$ are provided in Table 28. The averages for $-\eta S_{b \rightarrow c\bar{c}s}$ are shown in Fig. 14.

Both *BABAR* and Belle have used the $\eta = -1$ modes $J/\psi K_S^0$, $\psi(2S)K_S^0$, $\chi_{c1}K_S^0$ and $\eta_c K_S^0$, as well as $J/\psi K_L^0$, which has $\eta = +1$ and $J/\psi K^{*0}(892)$, which is found to have η close to $+1$ based on the measurement of $|A_\perp|$ (see Sec. 4.3). The most recent Belle result does not use $\eta_c K_S^0$ or $J/\psi K^{*0}(892)$ decays.⁴⁷ ALEPH, OPAL, CDF and LHCb have used only the $J/\psi K_S^0$ final state. *BABAR* has also determined the CP violation parameters of the $B^0 \rightarrow \chi_{c0} K_S^0$ decay from the time-dependent Dalitz plot analysis of $B^0 \rightarrow \pi^+\pi^-K_S^0$ (see Sec. 4.6.2). In addition, Belle has performed a measurement with data accumulated at the $\Upsilon(5S)$ resonance, using the $J/\psi K_S^0$ final state – this involves a different flavour tagging method compared to the measurements performed with data accumulated at the $\Upsilon(4S)$ resonance. A breakdown of results in each charmonium-kaon final state is given in Table 29.

It should be noted that, while the uncertainty in the average for $-\eta S_{b \rightarrow c\bar{c}s}$ is still limited by statistics, the uncertainty for $C_{b \rightarrow c\bar{c}s}$ is close to being dominated by systematics. This occurs due to the possible effect of tag side interference on the $C_{b \rightarrow c\bar{c}s}$ measurement, an effect which is correlated between different $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ experiments. Understanding of this effect may continue to improve in future, allowing the uncertainty to reduce.

⁴⁷ Previous analyses from Belle did include these channels [93], but it is not possible to obtain separate results for those modes from the published information.

Table 28: $S_{b \rightarrow c\bar{c}s}$ and $C_{b \rightarrow c\bar{c}s}$.

Experiment	Sample size		$-\eta S_{b \rightarrow c\bar{c}s}$	$C_{b \rightarrow c\bar{c}s}$
<i>BABAR</i>	[294]	$N(B\bar{B}) = 465\text{M}$	$0.687 \pm 0.028 \pm 0.012$	$0.024 \pm 0.020 \pm 0.016$
<i>BABAR</i> $\chi_{c0} K_S^0$	[243]	$N(B\bar{B}) = 383\text{M}$	$0.69 \pm 0.52 \pm 0.04 \pm 0.07$	$-0.29^{+0.53}_{-0.44} \pm 0.03 \pm 0.05$
<i>BABAR</i> $J/\psi K_S^0$ (*)	[295]	$N(B\bar{B}) = 88\text{M}$	$1.56 \pm 0.42 \pm 0.21$	–
<i>Belle</i>	[296]	$N(B\bar{B}) = 772\text{M}$	$0.667 \pm 0.023 \pm 0.012$	$-0.006 \pm 0.016 \pm 0.012$
<i>B</i> factory average			0.679 ± 0.020	0.005 ± 0.017
Confidence level			0.28	0.47
ALEPH	[297]	–	$0.84^{+0.82}_{-1.04} \pm 0.16$	–
OPAL	[298]	–	$3.2^{+1.8}_{-2.0} \pm 0.5$	–
CDF	[299]	–	$0.79^{+0.41}_{-0.44}$	–
LHCb	[300]	1.0 fb^{-1}	$0.73 \pm 0.07 \pm 0.04$	$0.03 \pm 0.09 \pm 0.01$
<i>Belle</i> $\Upsilon(5S)$	[301]	121 fb^{-1}	$0.57 \pm 0.58 \pm 0.06$	–
Average			0.682 ± 0.019	0.005 ± 0.017

* This result uses “*hadronic and previously unused muonic decays of the J/ψ* ”. We neglect a small possible correlation of this result with the main *BABAR* result [294] that could be caused by reprocessing of the data.

From the average for $-\eta S_{b \rightarrow c\bar{c}s}$ above, we obtain the following solutions for β (in $[0, \pi]$):

$$\beta = (21.5^{+0.8}_{-0.7})^\circ \quad \text{or} \quad \beta = (68.5^{+0.7}_{-0.8})^\circ \quad (146)$$

In radians, these values are $\beta = (0.375 \pm 0.013)$, $\beta = (1.196 \pm 0.013)$.

This result gives a precise constraint on the $(\bar{\rho}, \bar{\eta})$ plane, as shown in Fig. 14. The measurement is in remarkable agreement with other constraints from CP conserving quantities, and with CP violation in the kaon system, in the form of the parameter ϵ_K . Such comparisons have been performed by various phenomenological groups, such as CKMfitter [254] and UTFit [302].

4.4.2 Time-dependent transversity analysis of $B^0 \rightarrow J/\psi K^{*0}$

B meson decays to the vector-vector final state $J/\psi K^{*0}$ are also mediated by the $b \rightarrow c\bar{c}s$ transition. When a final state that is not flavour-specific ($K^{*0} \rightarrow K_S^0 \pi^0$) is used, a time-dependent transversity analysis can be performed allowing sensitivity to both $\sin(2\beta)$ and $\cos(2\beta)$ [303]. Such analyses have been performed by both B factory experiments. In principle, the strong phases between the transversity amplitudes are not uniquely determined by such an analysis, leading to a discrete ambiguity in the sign of $\cos(2\beta)$. The *BABAR* collaboration resolves this ambiguity using the known variation [304] of the P-wave phase (fast) relative to the S-wave phase (slow) with the invariant mass of the $K\pi$ system in the vicinity of the $K^*(892)$ resonance. The result is in agreement with the prediction from s quark helicity conservation, and corresponds to Solution II defined by Suzuki [305]. We use this phase convention for the averages given in Table 30 and Fig. 15.

At present the results are dominated by large and non-Gaussian statistical errors, and exhibit significant correlations. We perform uncorrelated averages, the interpretation of which has to be done with the greatest care. Nonetheless, it is clear that $\cos(2\beta) > 0$ is preferred

Table 29: Breakdown of B factory results on $S_{b \rightarrow c\bar{c}s}$ and $C_{b \rightarrow c\bar{c}s}$.

Mode		$N(B\bar{B})$	$-\eta S_{b \rightarrow c\bar{c}s}$	$C_{b \rightarrow c\bar{c}s}$
<i>BABAR</i>				
$J/\psi K_S^0$	[294]	465M	$0.657 \pm 0.036 \pm 0.012$	$0.026 \pm 0.025 \pm 0.016$
$J/\psi K_L^0$	[294]	465M	$0.694 \pm 0.061 \pm 0.031$	$-0.033 \pm 0.050 \pm 0.027$
$J/\psi K^0$	[294]	465M	$0.666 \pm 0.031 \pm 0.013$	$0.016 \pm 0.023 \pm 0.018$
$\psi(2S)K_S^0$	[294]	465M	$0.897 \pm 0.100 \pm 0.036$	$0.089 \pm 0.076 \pm 0.020$
$\chi_{c1}K_S^0$	[294]	465M	$0.614 \pm 0.160 \pm 0.040$	$0.129 \pm 0.109 \pm 0.025$
$\eta_c K_S^0$	[294]	465M	$0.925 \pm 0.160 \pm 0.057$	$0.080 \pm 0.124 \pm 0.029$
$J/\psi K^{*0}(892)$	[294]	465M	$0.601 \pm 0.239 \pm 0.087$	$0.025 \pm 0.083 \pm 0.054$
All	[294]	465M	$0.687 \pm 0.028 \pm 0.012$	$0.024 \pm 0.020 \pm 0.016$
<i>Belle</i>				
$J/\psi K_S^0$	[296]	772M	$0.670 \pm 0.029 \pm 0.013$	$0.015 \pm 0.021^{+0.023}_{-0.045}$
$J/\psi K_L^0$	[296]	772M	$0.642 \pm 0.047 \pm 0.021$	$-0.019 \pm 0.026^{+0.041}_{-0.017}$
$\psi(2S)K_S^0$	[296]	772M	$0.738 \pm 0.079 \pm 0.036$	$-0.104 \pm 0.055^{+0.027}_{-0.047}$
$\chi_{c1}K_S^0$	[296]	772M	$0.640 \pm 0.117 \pm 0.040$	$0.017 \pm 0.083^{+0.026}_{-0.046}$
All	[296]	772M	$0.667 \pm 0.023 \pm 0.012$	$-0.006 \pm 0.016 \pm 0.012$
Averages				
$J/\psi K_S^0$			0.665 ± 0.024	0.024 ± 0.026
$J/\psi K_L^0$			0.663 ± 0.041	-0.023 ± 0.030
$\psi(2S)K_S^0$			0.807 ± 0.067	-0.009 ± 0.055
$\chi_{c1}K_S^0$			0.632 ± 0.099	0.066 ± 0.074

 Table 30: Averages from $B^0 \rightarrow J/\psi K^{*0}$ transversity analyses.

Experiment		$N(B\bar{B})$	$\sin 2\beta$	$\cos 2\beta$	Correlation
<i>BABAR</i>	[306]	88M	$-0.10 \pm 0.57 \pm 0.14$	$3.32^{+0.76}_{-0.96} \pm 0.27$	-0.37
<i>Belle</i>	[291]	275M	$0.24 \pm 0.31 \pm 0.05$	$0.56 \pm 0.79 \pm 0.11$	0.22
Average			0.16 ± 0.28	1.64 ± 0.62	uncorrelated averages
Confidence level			$0.61 (0.5\sigma)$	$0.03 (2.2\sigma)$	

by the experimental data in $J/\psi K^{*0}$ (for example, *BABAR* [306] find a confidence level for $\cos(2\beta) > 0$ of 89%).

4.4.3 Time-dependent CP asymmetries in $B^0 \rightarrow D^{*+}D^{*-}K_S^0$ decays

Both *BABAR* [236] and *Belle* [237] have performed time-dependent analyses of the $B^0 \rightarrow D^{*+}D^{*-}K_S^0$ decay, to obtain information on the sign of $\cos(2\beta)$. More information can be found in Sec. 4.2.5. The results are shown in Table 31, and Fig. 16.

From the above result and the assumption that $J_{s2} > 0$, *BABAR* infer that $\cos(2\beta) > 0$ at the 94% confidence level [236].

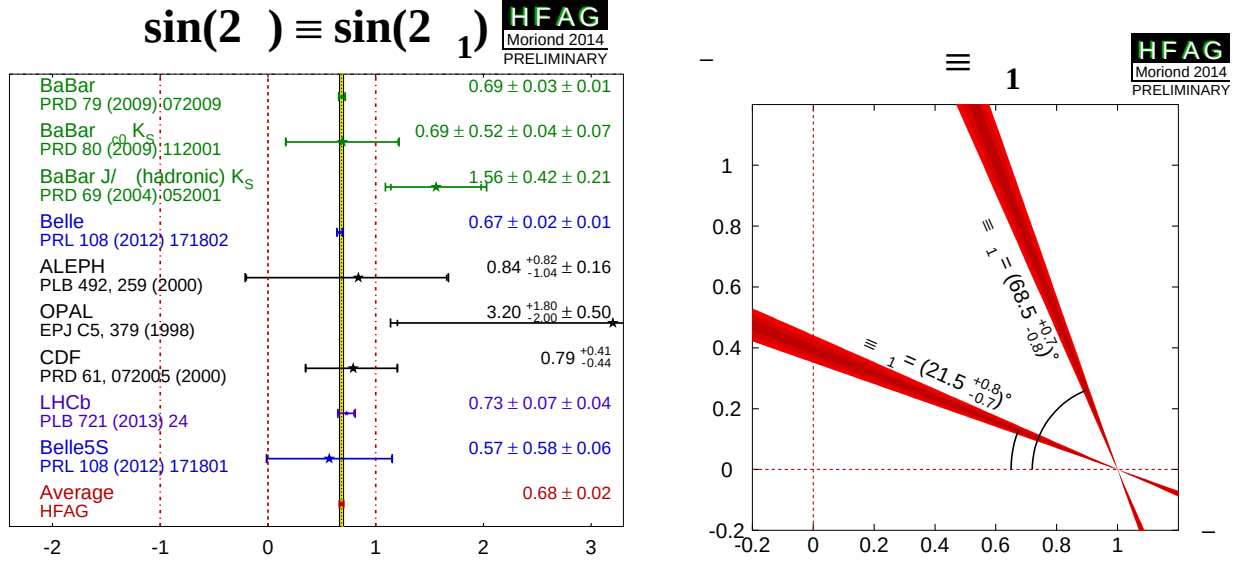


Figure 14: (Left) Average of measurements of $S_{b \to c \bar{c} s}$. (Right) Constraints on the $(\bar{\rho}, \bar{\eta})$ plane, obtained from the average of $-\eta S_{b \to c \bar{c} s}$ and Eq. (146).

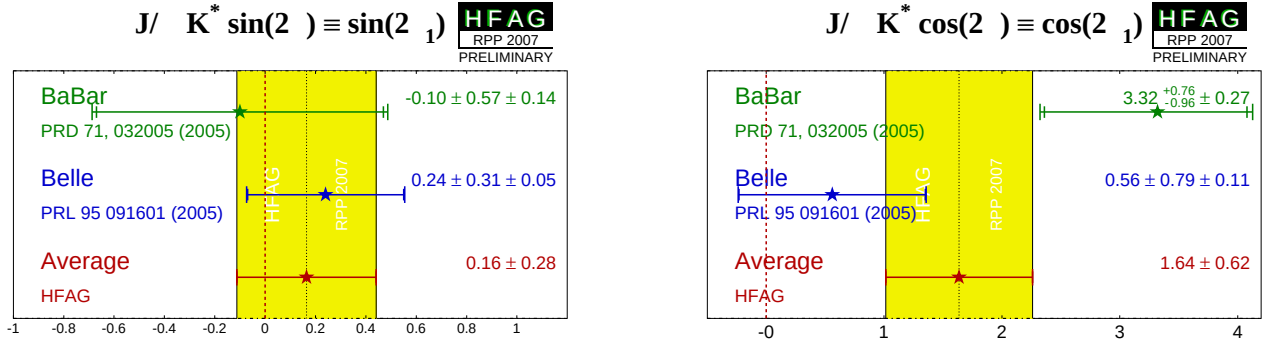


Figure 15: Averages of (left) $\sin(2\beta) \equiv \sin(2\phi_1)$ and (right) $\cos(2\beta) \equiv \cos(2\phi_1)$ from time-dependent analyses of $B^0 \to J/\psi K^{*0}$ decays.

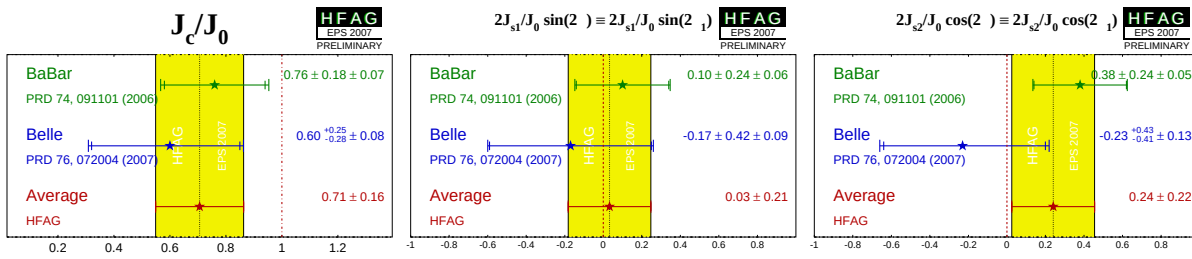


Figure 16: Averages of (left) (J_c/J_0) , (middle) $(2J_{s1}/J_0) \sin(2\beta)$ and (right) $(2J_{s2}/J_0) \cos(2\beta)$ from time-dependent analyses of $B^0 \to D^{*+} D^{*-} K_S^0$ decays.

Table 31: Results from time-dependent analysis of $B^0 \rightarrow D^{*+} D^{*-} K_s^0$.

Experiment	$N(B\bar{B})$	$\frac{J_c}{J_0}$	$\frac{2J_{s1}}{J_0} \sin(2\beta)$	$\frac{2J_{s2}}{J_0} \cos(2\beta)$
BABAR [236]	230M	$0.76 \pm 0.18 \pm 0.07$	$0.10 \pm 0.24 \pm 0.06$	$0.38 \pm 0.24 \pm 0.05$
Belle [237]	449M	$0.60^{+0.25}_{-0.28} \pm 0.08$	$-0.17 \pm 0.42 \pm 0.09$	$-0.23^{+0.43}_{-0.41} \pm 0.13$
Average		0.71 ± 0.16	0.03 ± 0.21	0.24 ± 0.22
Confidence level		$0.63 (0.5\sigma)$	$0.59 (0.5\sigma)$	$0.23 (1.2\sigma)$

4.4.4 Time-dependent analysis of B_s^0 decays through the $b \rightarrow c\bar{c}s$ transition

As described in Sec. 4.2.3, time-dependent analysis of decays such as $B_s^0 \rightarrow J/\psi\phi$ probes the CP violating phase of $B_s^0\text{--}\bar{B}_s^0$ oscillations, ϕ_s . Within the Standard Model, this parameter is predicted to be small.⁴⁸ The combination of results on $B_s^0 \rightarrow J/\psi\phi$ decays, including also results from $B_s^0 \rightarrow J/\psi\pi^+\pi^-$ and $B_s^0 \rightarrow D_s^+D_s^-$ decays, is performed by the HFAG Lifetimes and Oscillations group, see Sec. 3.

4.5 Time-dependent CP asymmetries in colour-suppressed $b \rightarrow c\bar{u}d$ transitions

4.5.1 Time-dependent CP asymmetries: $b \rightarrow c\bar{u}d$ decays to CP eigenstates

Decays of B mesons to final states such as $D\pi^0$ are governed by $b \rightarrow c\bar{u}d$ transitions. If the final state is a CP eigenstate, *e.g.* $D_{CP}\pi^0$, the usual time-dependence formulae are recovered, with the sine coefficient sensitive to $\sin(2\beta)$. Since there is no penguin contribution to these decays, there is even less associated theoretical uncertainty than for $b \rightarrow c\bar{c}s$ decays such as $B \rightarrow J/\psi K_s^0$. Such measurements therefore allow to test the Standard Model prediction that the CP violation parameters in $b \rightarrow c\bar{u}d$ transitions are the same as those in $b \rightarrow c\bar{c}s$ [307].

Note that there is an additional contribution from CKM suppressed $b \rightarrow u\bar{c}d$ decays. The effect of this contribution is small, and can be taken into account in the analysis [308, 309].

Results of such an analysis are available from BABAR [310]. The decays $B^0 \rightarrow D\pi^0$, $B^0 \rightarrow D\eta$, $B^0 \rightarrow D\omega$, $B^0 \rightarrow D^*\pi^0$ and $B^0 \rightarrow D^*\eta$ are used. In the latter two modes, the daughter decay $D^* \rightarrow D\pi^0$ is used. The CP -even D decay to K^+K^- is used for all decay modes, with the CP -odd D decay to $K_s^0\omega$ also used in $B^0 \rightarrow D^{(*)}\pi^0$ and the additional CP -odd D decay to $K_s^0\pi^0$ also used in $B^0 \rightarrow D\omega$. Results are presented separately for CP -even and CP -odd $D^{(*)}$ decays (denoted $D_+^{(*)}h^0$ and $D_-^{(*)}h^0$ respectively), and for both combined, taking into account the different CP factors (denoted $D_{CP}^{(*)}h^0$). The results are summarised in Table 32.

4.5.2 Time-dependent Dalitz plot analyses of $b \rightarrow c\bar{u}d$ decays

When multibody D decays, such as $D \rightarrow K_s^0\pi^+\pi^-$ are used, a time-dependent analysis of the Dalitz plot of the neutral D decay allows for a direct determination of the weak phase: 2β .

⁴⁸ We make the approximation $\phi_s = -2\beta_s$, where $\phi_s \equiv \arg[-M_{12}/\Gamma_{12}]$ and $2\beta_s \equiv 2\arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$ (see Sec. 4.1). This is a reasonable approximation since, although the equality does not hold in the Standard Model [101], both are much smaller than the current experimental resolution, whereas new physics contributions add a phase ϕ_{NP} to ϕ_s and subtract the same phase from $2\beta_s$, such that the approximation remains valid.

Table 32: Results from analyses of $B^0 \rightarrow D^{(*)}h^0$, $D \rightarrow CP$ eigenstates decays.

Experiment		$N(B\bar{B})$	S_{CP}	C_{CP}	Correlation
			$D_+^{(*)}h^0$		
<i>BABAR</i>	[310]	383M	$-0.65 \pm 0.26 \pm 0.06$	$-0.33 \pm 0.19 \pm 0.04$	0.04
			$D_-^{(*)}h^0$		
<i>BABAR</i>	[310]	383M	$-0.46 \pm 0.46 \pm 0.13$	$-0.03 \pm 0.28 \pm 0.07$	-0.14
			$D_{CP}^{(*)}h^0$		
<i>BABAR</i>	[310]	383M	$-0.56 \pm 0.23 \pm 0.05$	$-0.23 \pm 0.16 \pm 0.04$	-0.02

(Equivalently, both $\sin(2\beta)$ and $\cos(2\beta)$ can be measured.) This information can be used to resolve the ambiguity in the measurement of 2β from $\sin(2\beta)$ [311].

Results of such analyses are available from both Belle [233] and *BABAR* [234]. The decays $B \rightarrow D\pi^0$, $B \rightarrow D\eta$, $B \rightarrow D\omega$, $B \rightarrow D^*\pi^0$ and $B \rightarrow D^*\eta$ are used. (This collection of states is denoted by $D^{(*)}h^0$.) The daughter decays are $D^* \rightarrow D\pi^0$ and $D \rightarrow K_s^0\pi^+\pi^-$. The results are shown in Table 33, and Fig. 17. Note that *BABAR* quote uncertainties due to the D decay model separately from other systematic errors as a third source of uncertainty, while Belle do not.

Table 33: Averages from $B^0 \rightarrow D^{(*)}h^0$, $D \rightarrow K_s\pi^+\pi^-$ analyses.

Experiment		$N(B\bar{B})$	$\sin 2\beta$	$\cos 2\beta$	$ \lambda $
<i>BABAR</i>	[234]	383M	$0.29 \pm 0.34 \pm 0.03 \pm 0.05$	$0.42 \pm 0.49 \pm 0.09 \pm 0.13$	$1.01 \pm 0.08 \pm 0.02$
Belle	[233]	386M	$0.78 \pm 0.44 \pm 0.22$	$1.87^{+0.40+0.22}_{-0.53-0.32}$	—
Average			0.45 ± 0.28	1.01 ± 0.40	1.01 ± 0.08
Confidence level			$0.59 (0.5\sigma)$	$0.12 (1.6\sigma)$	—

Again, it is clear that the data prefer $\cos(2\beta) > 0$. Indeed, Belle [233] determine the sign of $\cos(2\phi_1)$ to be positive at 98.3% confidence level, while *BABAR* [234] favour the solution of β with $\cos(2\beta) > 0$ at 87% confidence level. Note, however, that the Belle measurement has strongly non-Gaussian behaviour. Therefore, we perform uncorrelated averages, from which any interpretation has to be done with the greatest care.

4.6 Time-dependent CP asymmetries in charmless $b \rightarrow q\bar{q}s$ transitions

The flavour changing neutral current $b \rightarrow s$ penguin can be mediated by any up-type quark in the loop, and hence the amplitude can be written as

$$\begin{aligned}
A_{b \rightarrow s} &= F_u V_{ub} V_{us}^* + F_c V_{cb} V_{cs}^* + F_t V_{tb} V_{ts}^* \\
&= (F_u - F_c) V_{ub} V_{us}^* + (F_t - F_c) V_{tb} V_{ts}^* \\
&= \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2)
\end{aligned} \tag{147}$$

using the unitarity of the CKM matrix. Therefore, in the Standard Model, this amplitude is dominated by $V_{tb} V_{ts}^*$, and to within a few degrees ($\delta\beta \lesssim 2^\circ$ for $\beta \approx 20^\circ$) the time-dependent

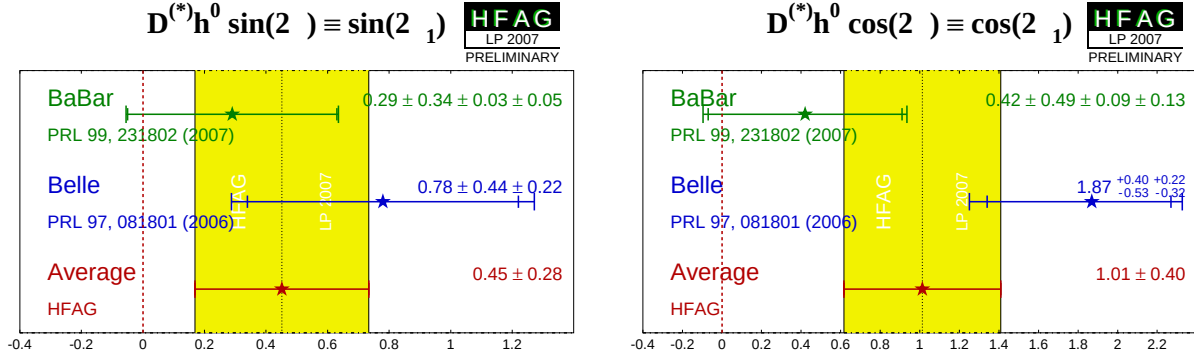


Figure 17: Averages of (left) $\sin(2\beta)$ and (right) $\cos(2\beta)$ measured in colour-suppressed $b \rightarrow c\bar{u}d$ transitions.

parameters can be written⁴⁹ $S_{b \rightarrow q\bar{q}s} \approx -\eta \sin(2\beta)$, $C_{b \rightarrow q\bar{q}s} \approx 0$, assuming $b \rightarrow s$ penguin contributions only ($q = u, d, s$).

Due to the suppression of the Standard Model amplitude, contributions of additional diagrams from physics beyond the Standard Model, with heavy virtual particles in the penguin loops, may have observable effects. In general, these contributions will affect the values of $S_{b \rightarrow q\bar{q}s}$ and $C_{b \rightarrow q\bar{q}s}$. A discrepancy between the values of $S_{b \rightarrow c\bar{c}s}$ and $S_{b \rightarrow q\bar{q}s}$ can therefore provide a clean indication of new physics [307, 312–314].

However, there is an additional consideration to take into account. The above argument assumes that only the $b \rightarrow s$ penguin contributes to the $b \rightarrow q\bar{q}s$ transition. For $q = s$ this is a good assumption, which neglects only rescattering effects. However, for $q = u$ there is a colour-suppressed $b \rightarrow u$ tree diagram (of order $\mathcal{O}(\lambda^4)$), which has a different weak (and possibly strong) phase. In the case $q = d$, any light neutral meson that is formed from $d\bar{d}$ also has a $u\bar{u}$ component, and so again there is “tree pollution”. The B^0 decays to $\pi^0 K_s^0$, $\rho^0 K_s^0$ and ωK_s^0 belong to this category. The mesons ϕ , f_0 and η' are expected to have predominant $s\bar{s}$ parts, which reduces the relative size of the possible tree pollution. If the inclusive decay $B^0 \rightarrow K^+ K^- K^0$ (excluding ϕK^0) is dominated by a nonresonant three-body transition, an OZI-rule suppressed tree-level diagram can occur through insertion of an $s\bar{s}$ pair. The corresponding penguin-type transition proceeds via insertion of a $u\bar{u}$ pair, which is expected to be favoured over the $s\bar{s}$ insertion by fragmentation models. Neglecting rescattering, the final state $K^0 \bar{K}^0 K^0$ (reconstructed as $K_s^0 K_s^0 K_s^0$) has no tree pollution [315]. Various estimates, using different theoretical approaches, of the values of $\Delta S = S_{b \rightarrow q\bar{q}s} - S_{b \rightarrow c\bar{c}s}$ exist in the literature [316–329]. In general, there is agreement that the modes ϕK^0 , $\eta' K^0$ and $K^0 \bar{K}^0 K^0$ are the cleanest, with values of $|\Delta S|$ at or below the few percent level (ΔS is usually positive).

⁴⁹ The presence of a small ($\mathcal{O}(\lambda^2)$) weak phase in the dominant amplitude of the s penguin decays introduces a phase shift given by $S_{b \rightarrow q\bar{q}s} = -\eta \sin(2\beta) \cdot (1 + \Delta)$. Using the CKMfitter results for the Wolfenstein parameters [254], one finds: $\Delta \simeq 0.033$, which corresponds to a shift of 2β of +2.1 degrees. Nonperturbative contributions can alter this result.

4.6.1 Time-dependent CP asymmetries: $b \rightarrow q\bar{q}s$ decays to CP eigenstates

The averages for $-\eta S_{b \rightarrow q\bar{q}s}$ and $C_{b \rightarrow q\bar{q}s}$ can be found in Table 34, and are shown in Figs. 18, 19 and 20. Results from both *BABAR* and Belle are averaged for the modes $\eta' K^0$ (K^0 indicates that both K_S^0 and K_L^0 are used) $K_S^0 K_S^0 K_S^0$, $\pi^0 K_S^0$ and ωK_S^0 .⁵⁰ Results on ϕK_S^0 and $K^+ K^- K_S^0$ (implicitly excluding ϕK_L^0 and $f_0 K_S^0$) are taken from time-dependent Dalitz plot analyses of $K^+ K^- K_S^0$; results on $\rho^0 K_S^0$, $f_2 K_S^0$, $f_X K_S^0$ and $\pi^+ \pi^- K_S^0$ nonresonant are taken from time-dependent Dalitz plot analyses of $\pi^+ \pi^- K_S^0$ (see subsection 4.6.2). The results on $f_0 K_S^0$ are from combinations of both Dalitz plot analyses. *BABAR* has also presented results with the final states $\pi^0 \pi^0 K_S^0$,⁵¹ and $\phi K_S^0 \pi^0$.

Of these final states, ϕK_S^0 , $\eta' K_S^0$, $\pi^0 K_S^0$, $\rho^0 K_S^0$, ωK_S^0 and $f_0 K_L^0$ have CP eigenvalue $\eta = -1$, while ϕK_L^0 , $\eta' K_L^0$, $K_S^0 K_S^0 K_S^0$, $f_0 K_S^0$, $f_2 K_S^0$, $f_X K_S^0$,⁵² $\pi^0 \pi^0 K_S^0$ and $\pi^+ \pi^- K_S^0$ nonresonant have $\eta = +1$. The final state $K^+ K^- K_S^0$ (with ϕK_S^0 and $f_0 K_S^0$ implicitly excluded) is not a CP eigenstate, but the CP -content can be absorbed in the amplitude analysis to allow the determination of a single effective S parameter. (In earlier analyses of the $K^+ K^- K^0$ final state, its CP composition was determined using an isospin argument [332] and a moments analysis [333].)

The final state $\phi K_S^0 \pi^0$ is also not a CP eigenstate but its CP -composition can be determined from an angular analysis. Since the parameters are common to the $B^0 \rightarrow \phi K_S^0 \pi^0$ and $B^0 \rightarrow \phi K^+ \pi^-$ decays (because only $K\pi$ resonances contribute), *BABAR* perform a simultaneous analysis of the two final states [340] (see Sec. 4.6.3).

It must be noted that Q2B parameters extracted from Dalitz plot analyses are constrained to lie within the physical boundary ($S_{CP}^2 + C_{CP}^2 < 1$) and consequently the obtained errors are highly non-Gaussian when the central value is close to the boundary. This is particularly evident in the *BABAR* results for $B^0 \rightarrow f_0 K^0$ with $f_0 \rightarrow \pi^+ \pi^-$ [243]. These results must be treated with extreme caution.

As explained above, each of the modes listed in Table 34 has potentially different subleading contributions within the Standard Model, and thus each may have a different value of $-\eta S_{b \rightarrow q\bar{q}s}$. Therefore, there is no strong motivation to make a combined average over the different modes. We refer to such an average as a “naïve s -penguin average.” It is naïve not only because of the theoretical uncertainties are neglected, but also since possible correlations of systematic effects between different modes are not included. In spite of these caveats there remains interest in the value of this quantity and therefore it is given here: $\langle -\eta S_{b \rightarrow q\bar{q}s} \rangle = 0.655 \pm 0.032$, with confidence level 0.77 (0.3σ). This value is in agreement with the average $-\eta S_{b \rightarrow c\bar{c}s}$ given in Sec. 4.4.1. (The average for $C_{b \rightarrow q\bar{q}s}$ is $\langle C_{b \rightarrow q\bar{q}s} \rangle = -0.006 \pm 0.026$ with confidence level 0.53 (0.6σ).) We emphasise again that we do not advocate the use of these averages.

From Table 34 it may be noted that the averages for $-\eta S_{b \rightarrow q\bar{q}s}$ in ϕK_S^0 , $\eta' K^0$, $f_0 K_S^0$ and $K^+ K^- K_S^0$ are all now more than 5σ away from zero, so that CP violation in these modes can be considered well established. There is no evidence (above 2σ) for CP violation in any $b \rightarrow q\bar{q}s$ decay.

⁵⁰ Belle [330] include the $\pi^0 K_L^0$ final state together with $\pi^0 K_S^0$ in order to improve the constraint on the parameter of CP violation in decay; these events cannot be used for time-dependent analysis.

⁵¹ We do not include a preliminary result from Belle [331], which remains unpublished after more than two years.

⁵² The f_X is assumed to be spin even.

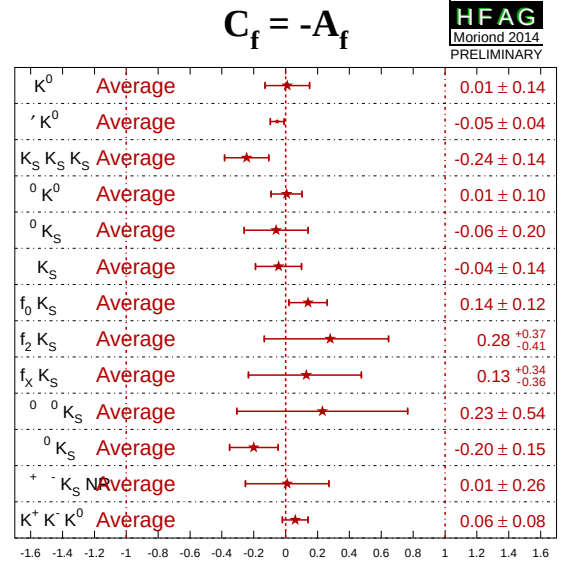
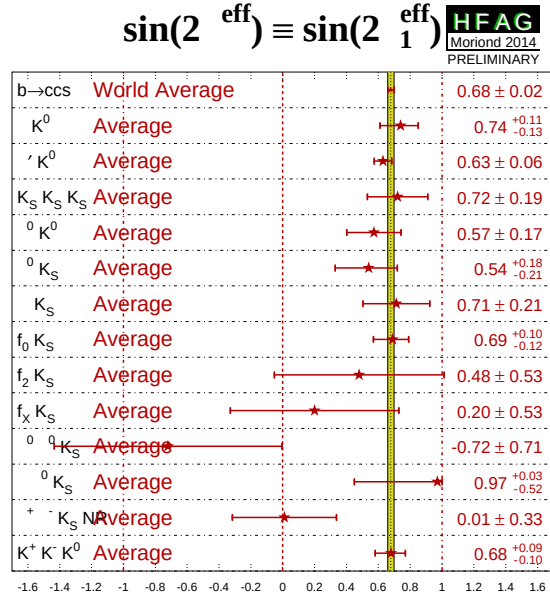
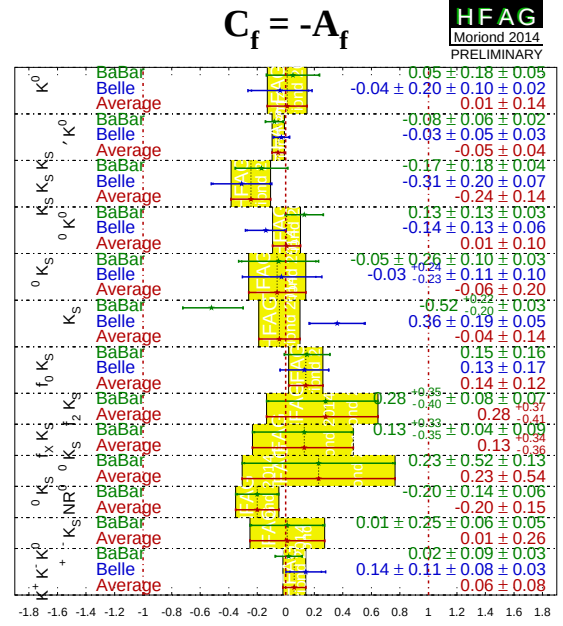
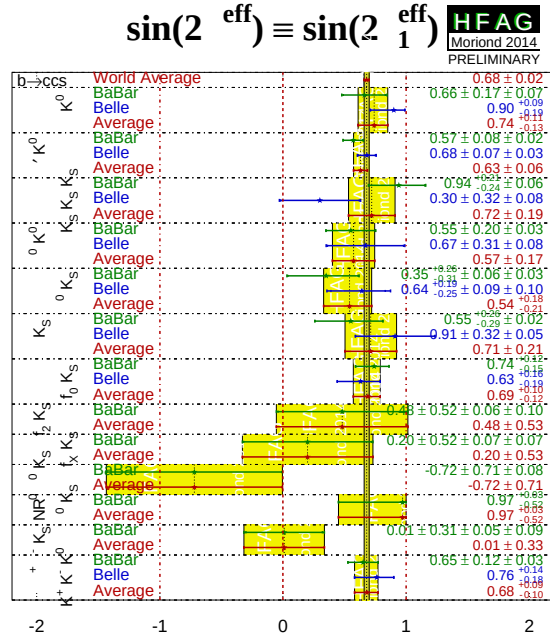


Figure 18: (Top) Averages of (left) $-\eta S_{b \rightarrow q\bar{q}s}$ and (right) $C_{b \rightarrow q\bar{q}s}$. The $-\eta S_{b \rightarrow q\bar{q}s}$ figure compares the results to the world average for $-\eta S_{b \rightarrow c\bar{c}s}$ (see Sec. 4.4.1). (Bottom) Same, but only averages for each mode are shown. More figures are available from the HFAG web pages.

Table 34: Averages of $-\eta S_{b \rightarrow q\bar{q}s}$ and $C_{b \rightarrow q\bar{q}s}$. Where a third source of uncertainty is given, it is due to model uncertainties arising in Dalitz plot analyses.

Experiment		$N(B\bar{B})$	$-\eta S_{b \rightarrow q\bar{q}s}$	$C_{b \rightarrow q\bar{q}s}$	Correlation
ϕK^0					
<i>BABAR</i>	[240]	470M	$0.66 \pm 0.17 \pm 0.07$	$0.05 \pm 0.18 \pm 0.05$	–
<i>Belle</i>	[239]	657M	$0.90^{+0.09}_{-0.19}$	$-0.04 \pm 0.20 \pm 0.10 \pm 0.02$	–
Average			$0.74^{+0.11}_{-0.13}$	0.01 ± 0.14	uncorrelated averages
$\eta' K^0$					
<i>BABAR</i>	[334]	467M	$0.57 \pm 0.08 \pm 0.02$	$-0.08 \pm 0.06 \pm 0.02$	0.03
<i>Belle</i>	[335]	772M	$0.68 \pm 0.07 \pm 0.03$	$-0.03 \pm 0.05 \pm 0.03$	0.03
Average			0.63 ± 0.06	-0.05 ± 0.04	0.02
Confidence level			0.53 (0.6 σ)		
$K_s^0 K_s^0 K_s^0$					
<i>BABAR</i>	[336]	468M	$0.94^{+0.21}_{-0.24} \pm 0.06$	$-0.17 \pm 0.18 \pm 0.04$	0.16
<i>Belle</i>	[337]	535M	$0.30 \pm 0.32 \pm 0.08$	$-0.31 \pm 0.20 \pm 0.07$	–
Average			0.72 ± 0.19	-0.24 ± 0.14	0.09
Confidence level			0.26 (1.1 σ)		
$\pi^0 K^0$					
<i>BABAR</i>	[334]	467M	$0.55 \pm 0.20 \pm 0.03$	$0.13 \pm 0.13 \pm 0.03$	0.06
<i>Belle</i>	[330]	657M	$0.67 \pm 0.31 \pm 0.08$	$-0.14 \pm 0.13 \pm 0.06$	–0.04
Average			0.57 ± 0.17	0.01 ± 0.10	0.02
Confidence level			0.37 (0.9 σ)		
$\rho^0 K_s^0$					
<i>BABAR</i>	[243]	383M	$0.35^{+0.26}_{-0.31} \pm 0.06 \pm 0.03$	$-0.05 \pm 0.26 \pm 0.10 \pm 0.03$	–
<i>Belle</i>	[244]	657M	$0.64^{+0.19}_{-0.25} \pm 0.09 \pm 0.10$	$-0.03^{+0.24}_{-0.23} \pm 0.11 \pm 0.10$	–
Average			$0.54^{+0.18}_{-0.21}$	-0.06 ± 0.20	uncorrelated averages
ωK_s^0					
<i>BABAR</i>	[334]	467M	$0.55^{+0.26}_{-0.29} \pm 0.02$	$-0.52^{+0.22}_{-0.20} \pm 0.03$	0.03
<i>Belle</i>	[338]	772M	$0.91 \pm 0.32 \pm 0.05$	$0.36 \pm 0.19 \pm 0.05$	–0.00
Average			0.71 ± 0.21	-0.04 ± 0.14	0.01
Confidence level			0.007 (2.7 σ)		
$f_0 K^0$					
<i>BABAR</i>	[240, 243]	–	$0.74^{+0.12}_{-0.15}$	0.15 ± 0.16	–
<i>Belle</i>	[239, 244]	–	$0.63^{+0.16}_{-0.19}$	0.13 ± 0.17	–
Average			$0.69^{+0.10}_{-0.12}$	0.14 ± 0.12	uncorrelated averages
$f_2 K_s^0$					
<i>BABAR</i>	[243]	383M	$0.48 \pm 0.52 \pm 0.06 \pm 0.10$	$0.28^{+0.35}_{-0.40} \pm 0.08 \pm 0.07$	–
$f_X K_s^0$					
<i>BABAR</i>	[243]	383M	$0.20 \pm 0.52 \pm 0.07 \pm 0.07$	$0.13^{+0.33}_{-0.35} \pm 0.04 \pm 0.09$	–

4.6.2 Time-dependent Dalitz plot analyses: $B^0 \rightarrow K^+ K^- K^0$ and $B^0 \rightarrow \pi^+ \pi^- K_s^0$

As mentioned in Sec. 4.2.5 and above, both BABAR and Belle have performed time-dependent Dalitz plot analysis of $B^0 \rightarrow K^+ K^- K^0$ and $B^0 \rightarrow \pi^+ \pi^- K_s^0$ decays. The results are summarised in Tables 36 and 37. Averages for the $B^0 \rightarrow f_0 K_s^0$ decay, which contributes to both Dalitz plots, are shown in Fig. 21. Results are presented in terms of the effective weak phase (from mixing and decay) difference β^{eff} and the parameter of CP violation in decay $\mathcal{A}$ ($\mathcal{A} = -C$) for each of the resonant contributions. Note that Dalitz plot analyses, including all those included in these averages, often suffer from ambiguous solutions – we quote the results corresponding to those presented as solution 1 in all cases. Results on flavour specific amplitudes that may contribute

Table 35: Averages of $-\eta S_{b \rightarrow q\bar{q}s}$ and $C_{b \rightarrow q\bar{q}s}$ (continued). Where a third source of uncertainty is given, it is due to model uncertainties arising in Dalitz plot analyses.

Experiment	$N(B\bar{B})$	$-\eta S_{b \rightarrow q\bar{q}s}$	$C_{b \rightarrow q\bar{q}s}$	Correlation
$\pi^0 \pi^0 K_s^0$				
BABAR [339]	227M	$-0.72 \pm 0.71 \pm 0.08$	$0.23 \pm 0.52 \pm 0.13$	-0.02
$\phi K_s^0 \pi^0$				
BABAR [340]	465M	$0.97^{+0.03}_{-0.52}$	$-0.20 \pm 0.14 \pm 0.06$	—
$\pi^+ \pi^- K_s^0$ nonresonant				
BABAR [243]	383M	$0.01 \pm 0.31 \pm 0.05 \pm 0.09$	$0.01 \pm 0.25 \pm 0.06 \pm 0.05$	—
$K^+ K^- K^0$				
BABAR [240]	470M	$0.65 \pm 0.12 \pm 0.03$	$0.02 \pm 0.09 \pm 0.03$	—
Belle [239]	657M	$0.76^{+0.14}_{-0.18}$	$0.14 \pm 0.11 \pm 0.08 \pm 0.03$	—
Average		$0.68^{+0.09}_{-0.10}$	0.06 ± 0.08	uncorrelated averages

to these Dalitz plots (such as $K^{*+} \pi^-$) are averaged by the HFAG Rare Decays subgroup (Sec. 7).

For the $B^0 \rightarrow K^+ K^- K^0$ decay, both BABAR and Belle measure the CP violation parameters for the ϕK^0 , $f_0 K^0$ and “other $K^+ K^- K^0$ ” amplitudes, where the latter includes all remaining resonant and nonresonant contributions to the charmless three-body decay. For the $B^0 \rightarrow \pi^+ \pi^- K_s^0$ decay, BABAR report CP violation parameters for all of the CP eigenstate components in the Dalitz plot model ($\rho^0 K_s^0$, $f_0 K_s^0$, $f_2 K_s^0$, $f_X K_s^0$ and nonresonant decays⁵³), while Belle report the CP violation parameters for only the $\rho^0 K_s^0$ and $f_0 K_s^0$ amplitudes, although the used Dalitz plot model is rather similar.

4.6.3 Time-dependent analyses of $B^0 \rightarrow \phi K_s^0 \pi^0$

The final state in the decay $B^0 \rightarrow \phi K_s^0 \pi^0$ is a mixture of CP -even and CP -odd amplitudes. However, since only ϕK^{*0} resonant states contribute (in particular, $\phi K^{*0}(892)$, $\phi K_0^{*0}(1430)$ and $\phi K_2^{*0}(1430)$ are seen), the composition can be determined from the analysis of $B \rightarrow \phi K^+ \pi^-$, assuming only that the ratio of branching fractions $\mathcal{B}(K^{*0} \rightarrow K_s^0 \pi^0)/\mathcal{B}(K^{*0} \rightarrow K^+ \pi^-)$ is the same for each excited kaon state.

BABAR [340] have performed a simultaneous analysis of $B^0 \rightarrow \phi K_s^0 \pi^0$ and $B^0 \rightarrow \phi K^+ \pi^-$ that is time-dependent for the former mode and time-integrated for the latter. Such an analysis allows, in principle, all parameters of the $B^0 \rightarrow \phi K^{*0}$ system to be determined, including mixing-induced CP violation effects. The latter is determined to be $\Delta\phi_{00} = 0.28 \pm 0.42 \pm 0.04$, where $\Delta\phi_{00}$ is half the weak phase difference between B^0 and $\bar{B}^0$ decays to $\phi K_0^{*0}(1430)$. As discussed above, this can also be presented in terms of the Q2B parameter $\sin(2\beta_{00}^{\text{eff}}) = \sin(2\beta + 2\Delta\phi_{00}) = 0.97^{+0.03}_{-0.52}$. The highly asymmetric uncertainty arises due to the conversion from the phase to the sine of the phase, and the proximity of the physical boundary.

Similar $\sin(2\beta^{\text{eff}})$ parameters can be defined for each of the helicity amplitudes for both $\phi K^{*0}(892)$ and $\phi K_2^{*0}(1430)$. However, the relative phases between these decays are constrained due to the nature of the simultaneous analysis of $B^0 \rightarrow \phi K_s^0 \pi^0$ and $B^0 \rightarrow \phi K^+ \pi^-$, and therefore

⁵³ The f_X resonance included in the model is a poorly understood excess in the $m(\pi^+ \pi^-)$ distribution, that may originate from interference between other states in this region and nonresonant amplitudes.

Table 36: Results from time-dependent Dalitz plot analyses of the $B^0 \rightarrow K^+ K^- K^0$ decay. Correlations (not shown) are taken into account in the average.

Experiment	$N(BB)$	ϕK_s^0		$f_0 K_s^0$		$K^+ K^- K_s^0$	
		$\beta^{\text{eff}} (^\circ)$	$\mathcal{A}$	$\beta^{\text{eff}} (^\circ)$	$\mathcal{A}$	$\beta^{\text{eff}} (^\circ)$	$\mathcal{A}$
BaBar [240]	470M	$21 \pm 6 \pm 2$	$-0.05 \pm 0.18 \pm 0.05$	$18 \pm 6 \pm 4$	$-0.28 \pm 0.24 \pm 0.09$	$20.3 \pm 4.3 \pm 1.2$	$-0.02 \pm 0.09 \pm 0.03$
Belle [239]	657M	$32.2 \pm 9.0 \pm 2.6 \pm 1.4$	$0.04 \pm 0.20 \pm 0.10 \pm 0.02$	$31.3 \pm 9.0 \pm 3.4 \pm 4.0$	$-0.30 \pm 0.29 \pm 0.11 \pm 0.09$	$24.9 \pm 6.4 \pm 2.1 \pm 2.5$	$-0.14 \pm 0.11 \pm 0.08 \pm 0.03$
Average		24 ± 5	-0.01 ± 0.14	22 ± 6	-0.29 ± 0.20	21.6 ± 3.7	-0.06 ± 0.08
Confidence level					$0.93 (0.1\sigma)$		

Table 37: Results from time-dependent Dalitz plot analysis of the $B^0 \rightarrow \pi^+ \pi^- K_s^0$ decay. Correlations (not shown) are taken into account in the average.

Experiment	$N(B\bar{B})$	β^{eff}	$\rho^0 K_s^0$	$\mathcal{A}$	β^{eff}	$f_0 K_s^0$	$\mathcal{A}$
BABAR	[243]	383M	$(10.2 \pm 8.9 \pm 3.0 \pm 1.9)^\circ$	$0.05 \pm 0.26 \pm 0.10 \pm 0.03$	$(36.0 \pm 9.8 \pm 2.1 \pm 2.1)^\circ$	$-0.08 \pm 0.19 \pm 0.03 \pm 0.04$	
Belle	[244]	657M	$(20.0^{+8.6}_{-8.5} \pm 3.2 \pm 3.5)^\circ$	$0.03^{+0.23}_{-0.24} \pm 0.11 \pm 0.10$	$(12.7^{+6.9}_{-6.5} \pm 2.8 \pm 3.3)^\circ$	$-0.06 \pm 0.17 \pm 0.07 \pm 0.09$	
Average			16.4 ± 6.8	0.06 ± 0.20	20.6 ± 6.2	-0.07 ± 0.14	
Confidence level					$0.39 \text{ (} 0.9\sigma \text{)}$		
Experiment	$N(B\bar{B})$	β^{eff}	$f_2 K_s^0$	$\mathcal{A}$	β^{eff}	$f_X K_s^0$	$\mathcal{A}$
BABAR	[243]	383M	$(14.9 \pm 17.9 \pm 3.1 \pm 5.2)^\circ$	$-0.28^{+0.40}_{-0.35} \pm 0.08 \pm 0.07$	$(5.8 \pm 15.2 \pm 2.2 \pm 2.3)^\circ$	$-0.13^{+0.35}_{-0.33} \pm 0.04 \pm 0.09$	
Experiment	$N(B\bar{B})$	$B^0 \rightarrow \pi^+ \pi^- K_s^0$ nonresonant β^{eff}			β^{eff}	$\chi_{c0} K_s^0$	$\mathcal{A}$
BABAR	[243]	383M	$(0.4 \pm 8.8 \pm 1.9 \pm 3.8)^\circ$	$-0.01 \pm 0.25 \pm 0.06 \pm 0.05$	$(23.2 \pm 22.4 \pm 2.3 \pm 4.2)^\circ$	$0.29^{+0.44}_{-0.53} \pm 0.03 \pm 0.05$	

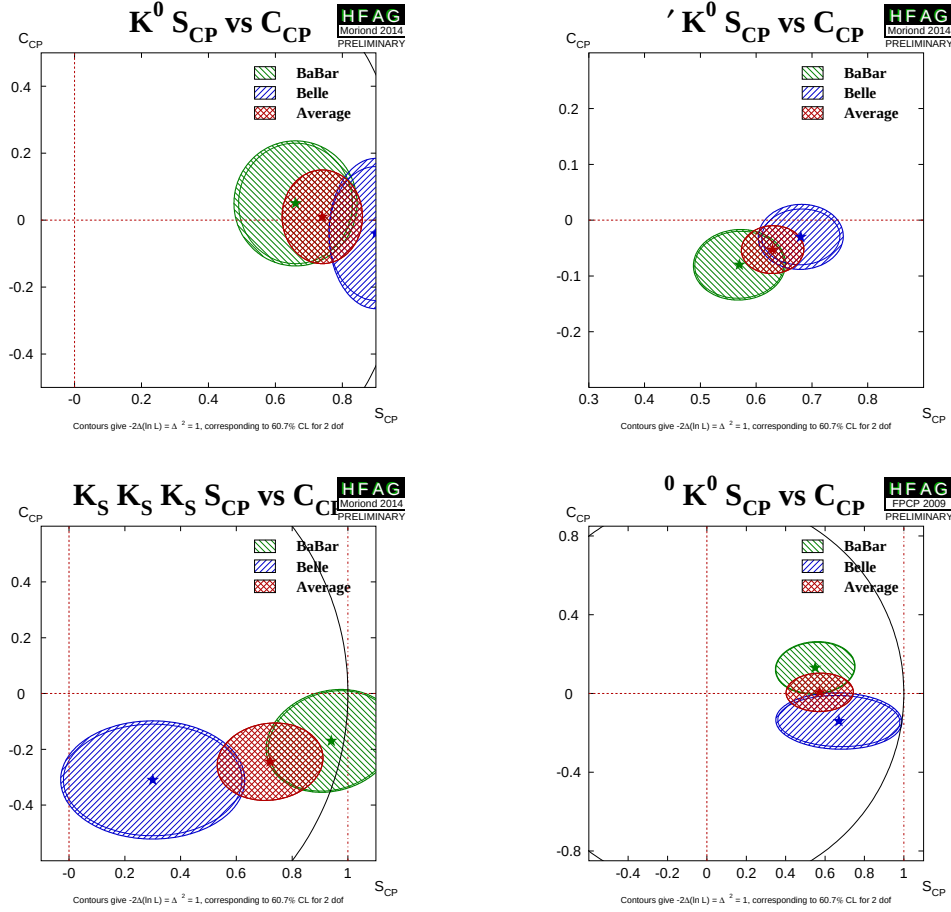


Figure 19: Averages of four $b \rightarrow q\bar{q}s$ dominated channels, for which correlated averages are performed, in the S_{CP} vs. C_{CP} plane, where S_{CP} has been corrected by the CP eigenvalue to give $\sin(2\beta^{\text{eff}})$. (Top left) $B^0 \rightarrow \phi K^0$, (top right) $B^0 \rightarrow \eta' K^0$, (bottom left) $B^0 \rightarrow K_S^0 K_S^0 K_S^0$, (bottom right) $B^0 \rightarrow \pi^0 K_S^0$. More figures are available from the HFAG web pages.

these measurements are highly correlated. Instead of quoting all these results, *BABAR* provide an illustration of their measurements with the following differences:

$$\sin(2\beta - 2\Delta\delta_{01}) - \sin(2\beta) = -0.42^{+0.26}_{-0.34} \quad (148)$$

$$\sin(2\beta - 2\Delta\phi_{\parallel 1}) - \sin(2\beta) = -0.32^{+0.22}_{-0.30} \quad (149)$$

$$\sin(2\beta - 2\Delta\phi_{\perp 1}) - \sin(2\beta) = -0.30^{+0.23}_{-0.32} \quad (150)$$

$$\sin(2\beta - 2\Delta\phi_{\perp 1}) - \sin(2\beta - 2\Delta\phi_{\parallel 1}) = 0.02 \pm 0.23 \quad (151)$$

$$\sin(2\beta - 2\Delta\delta_{02}) - \sin(2\beta) = -0.10^{+0.18}_{-0.29} \quad (152)$$

where the first subscript indicates the helicity amplitude and the second indicates the spin of the kaon resonance. For the complete definitions of the $\Delta\delta$ and $\Delta\phi$ parameters, please refer to the *BABAR* paper [340].

Parameters of CP violation in decay for each of the contributing helicity amplitudes can also be measured. Again, these are determined from a simultaneous fit of $B^0 \rightarrow \phi K_S^0 \pi^0$ and $B^0 \rightarrow \phi K^+ \pi^-$, with the precision being dominated by the statistics of the latter mode. Direct CP violation measurements are tabulated by HFAG - Rare Decays (Sec. 7).

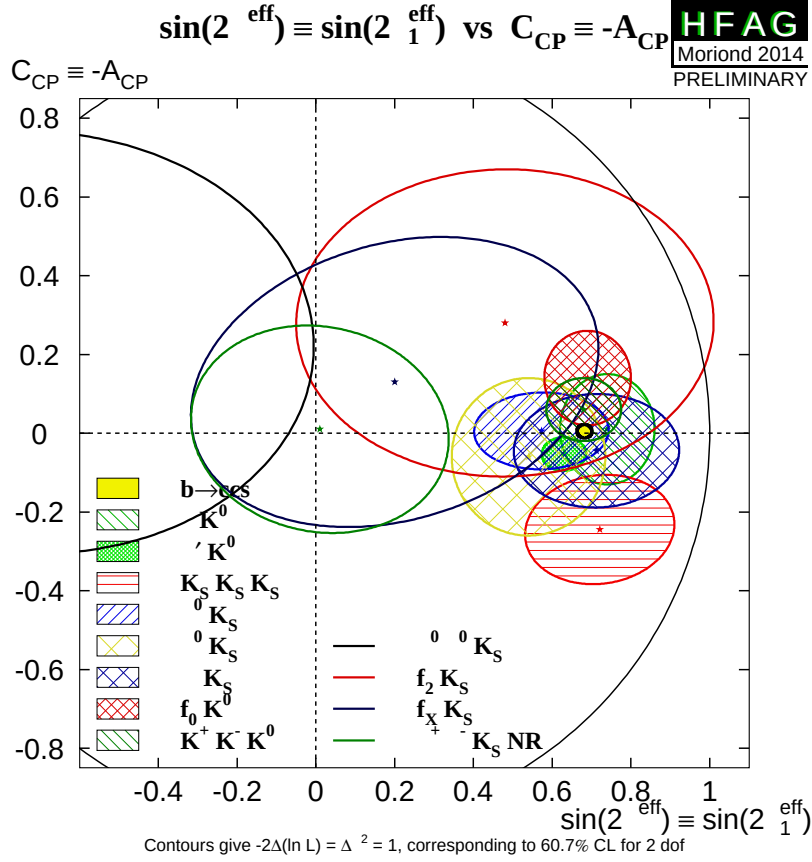


Figure 20: Compilation of constraints in the $-\eta S_{b \rightarrow q\bar{q}s}$ vs. $C_{b \rightarrow q\bar{q}s}$ plane.

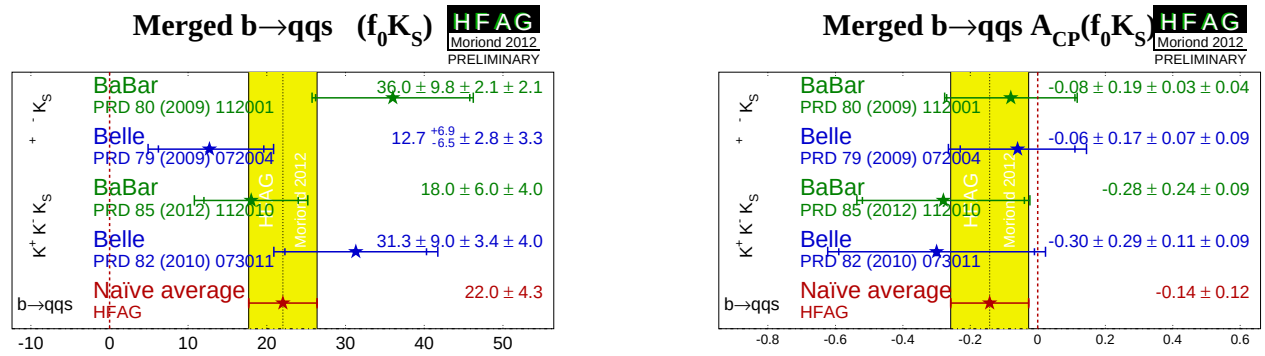


Figure 21: Averages of (left) $\beta^{\text{eff}} \equiv \phi_1^{\text{eff}}$ and (right) A_{CP} for the $B^0 \rightarrow f_0 K_S^0$ decay including measurements from Dalitz plot analyses of both $B^0 \rightarrow K^+ K^- K_S^0$ and $B^0 \rightarrow \pi^+ \pi^- K_S^0$.

4.6.4 Time-dependent CP asymmetries in $B_s^0 \rightarrow K^+ K^-$

The decay $B_s^0 \rightarrow K^+ K^-$ involves a $b \rightarrow u\bar{u}s$ transition, and hence has both penguin and tree contributions. Both mixing-induced and CP violation in decay effects may arise, and additional input is needed to disentangle the contributions and determine γ and β_s^{eff} . For example, the observables in $B^0 \rightarrow \pi^+ \pi^-$ can be related using U-spin, as proposed by Fleischer [341].

The observables are $A_{\text{mix}} = S_{CP}$, $A_{\text{dir}} = -C_{CP}$, and $A_{\Delta\Gamma}$. They can all be treated as free parameters, but are physically constrained to satisfy $A_{\text{mix}}^2 + A_{\text{dir}}^2 + A_{\Delta\Gamma}^2 = 1$. Note that the untagged decay distribution, from which an “effective lifetime” can be measured, retains sensitivity to $A_{\Delta\Gamma}$. Averages of effective lifetimes are performed by the HFAG Lifetimes and Oscillations group, see Sec. 3.

The observables in $B_s^0 \rightarrow K^+ K^-$ have been measured by LHCb, who impose the constraint mentioned above to eliminate $A_{\Delta\Gamma}$.

Table 38: Results from time-dependent analysis of the $B_s^0 \rightarrow K^+ K^-$ decay.

Experiment	Sample size	A_{mix}	A_{dir}	Correlation
LHCb [342]	1.0 fb^{-1}	$0.30 \pm 0.12 \pm 0.04$	$0.14 \pm 0.11 \pm 0.03$	0.02

4.6.5 Time-dependent CP asymmetries in $B_s^0 \rightarrow \phi\phi$

The decay $B_s^0 \rightarrow \phi\phi$ involves a $b \rightarrow s\bar{s}s$ transition, and hence is a “pure penguin” mode (in the limit that the ϕ meson is treated as a pure $s\bar{s}$ state). Since the mixing phase and the decay phase are expected to cancel in the Standard Model, the prediction for the phase from the interference of mixing and decay is predicted to be $\phi_s(\phi\phi) = 0$ with low uncertainty [343]. Due to the vector-vector nature of the final state, angular analysis is needed to separate the CP -even and CP -odd contributions. Such an analysis also makes it possible to fit directly for $\phi_s(\phi\phi)$.

A constraint on $\phi_s(\phi\phi)$ has been obtained by LHCb using 3.0 fb^{-1} of data [344]. The result is $\phi_s(\phi\phi) = -0.17 \pm 0.15 \pm 0.03 \text{ rad}$ where the first uncertainty is statistical and the second is systematic.

4.7 Time-dependent CP asymmetries in $b \rightarrow c\bar{c}d$ transitions

The transition $b \rightarrow c\bar{c}d$ can occur via either a $b \rightarrow c$ tree or a $b \rightarrow d$ penguin amplitude. Similarly to Eq. (147), the amplitude for the $b \rightarrow d$ penguin can be written

$$\begin{aligned}
A_{b \rightarrow d} &= F_u V_{ub} V_{ud}^* + F_c V_{cb} V_{cd}^* + F_t V_{tb} V_{td}^* \\
&= (F_u - F_c) V_{ub} V_{ud}^* + (F_t - F_c) V_{tb} V_{td}^* \\
&= \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3).
\end{aligned} \tag{153}$$

From this it can be seen that the $b \rightarrow d$ penguin amplitude contains terms with different weak phases at the same order of CKM suppression.

In the above, we have followed Eq. (147) by eliminating the F_c term using unitarity. However, we could equally well write

$$\begin{aligned} A_{b \rightarrow d} &= (F_u - F_t)V_{ub}V_{ud}^* + (F_c - F_t)V_{cb}V_{cd}^*, \\ &= (F_c - F_u)V_{cb}V_{cd}^* + (F_t - F_u)V_{tb}V_{td}^*. \end{aligned} \quad (154)$$

Since the $b \rightarrow c\bar{c}d$ tree amplitude has the weak phase of $V_{cb}V_{cd}^*$, either of the above expressions allow the penguin to be decomposed into parts with weak phases the same and different to the tree amplitude (the relative weak phase can be chosen to be either β or γ). However, if the tree amplitude dominates, there is little sensitivity to any phase other than that from B^0 - $\bar{B}^0$ mixing.

The $b \rightarrow c\bar{c}d$ transitions can be investigated with studies of various different final states. Results are available from both *BABAR* and *Belle* using the final states $J/\psi\pi^0$, D^+D^- , $D^{*+}D^{*-}$ and $D^{*\pm}D^\mp$, the averages of these results are given in Tables 39 and 40. The results using the CP eigenstate ($\eta = +1$) modes $J/\psi\pi^0$ and D^+D^- are shown in Fig. 22 and Fig. 23 respectively, with two-dimensional constraints shown in Fig. 24.

The vector-vector mode $D^{*+}D^{*-}$ is found to be dominated by the CP -even longitudinally polarised component; *BABAR* measures a CP -odd fraction of $0.158 \pm 0.028 \pm 0.006$ [232] while *Belle* measures a CP -odd fraction of $0.125 \pm 0.043 \pm 0.023$ [345]. These values, listed as $R_\perp$, are included in the averages which ensures that the correlations are taken into account.⁵⁴ *BABAR* has also performed an additional fit in which the CP -even and CP -odd components are allowed to have different CP violation parameters S and C . These results are included in Table 40. Results using $D^{*+}D^{*-}$ are shown in Fig. 25.

As discussed in Sec. 4.2.6, the most recent papers on the non- CP eigenstate mode $D^{*\pm}D^\mp$ use the $(A, S, \Delta S, C, \Delta C)$ set of parameters, and we therefore perform the averages with this choice.

Table 39: Averages for the $b \rightarrow c\bar{c}d$ modes, $B^0 \rightarrow J/\psi\pi^0$ and D^+D^- .

Experiment		$N(B\bar{B})$	S_{CP}	C_{CP}	Correlation
$J/\psi\pi^0$					
$BABAR$	[346]	466M	$-1.23 \pm 0.21 \pm 0.04$	$-0.20 \pm 0.19 \pm 0.03$	0.20
Belle	[347]	535M	$-0.65 \pm 0.21 \pm 0.05$	$-0.08 \pm 0.16 \pm 0.05$	-0.10
Average			-0.93 ± 0.15	-0.10 ± 0.13	0.04
Confidence level			0.15 (1.4 σ)		
D^+D^-					
$BABAR$	[232]	467M	$-0.65 \pm 0.36 \pm 0.05$	$-0.07 \pm 0.23 \pm 0.03$	-0.01
Belle	[257]	772M	$-1.06^{+0.21}_{-0.14} \pm 0.08$	$-0.43 \pm 0.16 \pm 0.05$	-0.12
Average			-0.98 ± 0.17	-0.31 ± 0.14	-0.08
Confidence level			0.26 (1.1 σ)		

In the absence of the penguin contribution (tree dominance), the time-dependent parameters would be given by $S_{b \rightarrow c\bar{c}d} = -\eta \sin(2\beta)$, $C_{b \rightarrow c\bar{c}d} = 0$, $S_{+-} = \sin(2\beta + \delta)$, $S_{-+} = \sin(2\beta - \delta)$,

⁵⁴ Note that the *BABAR* value given in Table 40 differs from that given above, since that in the table is not corrected for efficiency.

Table 40: Averages for the $b \rightarrow \bar{c}\bar{d}$ modes, $D^{*+}D^{*-}$ and $D^{*\pm}D^\mp$.

Experiment	$N(B\bar{B})$	S_{CP}	C_{CP}	$R_\perp$
$D^{*+}D^{*-}$				
BABAR	[232]	467M	$-0.70 \pm 0.16 \pm 0.03$	$0.05 \pm 0.09 \pm 0.02$
BABAR part. rec.	[348]	471M	$-0.49 \pm 0.18 \pm 0.07 \pm 0.04$	$0.15 \pm 0.09 \pm 0.04$
Belle	[349]	772M	$-0.79 \pm 0.13 \pm 0.03$	$-0.15 \pm 0.08 \pm 0.02$
Average			-0.71 ± 0.09	-0.01 ± 0.05
Confidence level			$0.72 \text{ (0.4}\sigma\text{)}$	0.15 ± 0.02

Experiment	$N(B\bar{B})$	S_{CP+}	C_{CP+}	S_{CP-}	C_{CP-}	$R_\perp$
$D^{*+}D^{*-}$						
BABAR	[232]	467M	$-0.76 \pm 0.16 \pm 0.04$	$0.02 \pm 0.12 \pm 0.02$	$-1.81 \pm 0.71 \pm 0.16$	$0.41 \pm 0.50 \pm 0.08$
						0.15 ± 0.03

Experiment	$N(B\bar{B})$	S	C	ΔS	ΔC	$\mathcal{A}$
$D^{*\pm}D^\mp$						
BABAR	[232]	467M	$-0.68 \pm 0.15 \pm 0.04$	$0.04 \pm 0.12 \pm 0.03$	$0.05 \pm 0.15 \pm 0.02$	$0.04 \pm 0.12 \pm 0.03$
Belle	[257]	772M	$-0.78 \pm 0.15 \pm 0.05$	$-0.01 \pm 0.11 \pm 0.04$	$-0.13 \pm 0.15 \pm 0.04$	$0.12 \pm 0.11 \pm 0.03$
Average			-0.73 ± 0.11	0.01 ± 0.09	-0.04 ± 0.11	0.08 ± 0.08
Confidence level			$0.65 \text{ (0.5}\sigma\text{)}$	$0.77 \text{ (0.3}\sigma\text{)}$	$0.41 \text{ (0.8}\sigma\text{)}$	$0.63 \text{ (0.5}\sigma\text{)}$
						$0.01 \pm 0.05 \pm 0.01$
						$0.06 \pm 0.05 \pm 0.02$
						0.03 ± 0.04
						$0.48 \text{ (0.7}\sigma\text{)}$

$C_{+-} = -C_{-+}$ and $\mathcal{A} = 0$, where δ is the strong phase difference between the $D^{*+}D^-$ and $D^{*-}D^+$ decay amplitudes. In the presence of the penguin contribution, there is no clean interpretation in terms of CKM parameters, however direct CP violation may be observed as any of $C_{b \rightarrow c\bar{c}d} \neq 0$, $C_{+-} \neq -C_{-+}$ or $A_{+-} \neq 0$.

The averages for the $b \rightarrow c\bar{c}d$ modes are shown in Figs. 26 and 27. Results are consistent with tree dominance, and with the Standard Model, though the Belle results in $B^0 \rightarrow D^+D^-$ [350] show an indication of CP violation in decay, and hence a non-zero penguin contribution. The average of $S_{b \rightarrow c\bar{c}d}$ in both $J/\psi\pi^0$ and $D^{*+}D^{*-}$ final states is more than 5σ from zero, corresponding to observations of CP violation in these decay channels. That in the D^+D^- final state is more than 3σ from zero; however, due to the large uncertainty and possible non-Gaussian effects, any strong conclusion should be deferred.

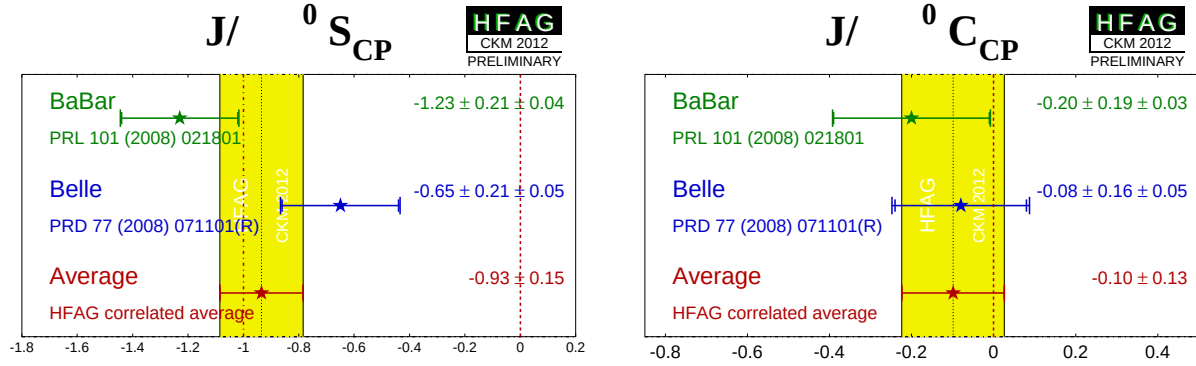


Figure 22: Averages of (left) $S_{b \rightarrow c\bar{c}d}$ and (right) $C_{b \rightarrow c\bar{c}d}$ for the mode $B^0 \rightarrow J/\psi\pi^0$.

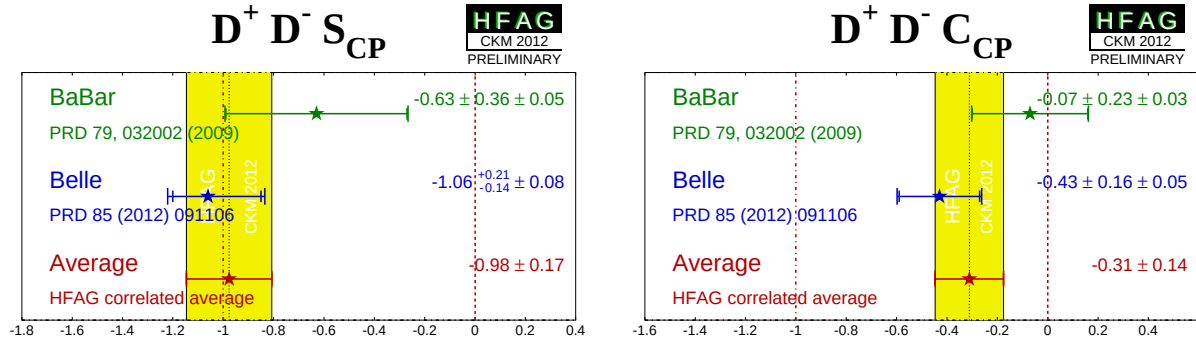


Figure 23: Averages of (left) $S_{b \rightarrow c\bar{c}d}$ and (right) $C_{b \rightarrow c\bar{c}d}$ for the mode $B^0 \rightarrow D^+D^-$.

4.8 Time-dependent CP asymmetries in $b \rightarrow q\bar{q}d$ transitions

Decays such as $B^0 \rightarrow K_s^0 K_s^0$ are pure $b \rightarrow q\bar{q}d$ penguin transitions. As shown in Eq. (153), this diagram has different contributing weak phases, and therefore the observables are sensitive to the difference (which can be chosen to be either β or γ). Note that if the contribution with the

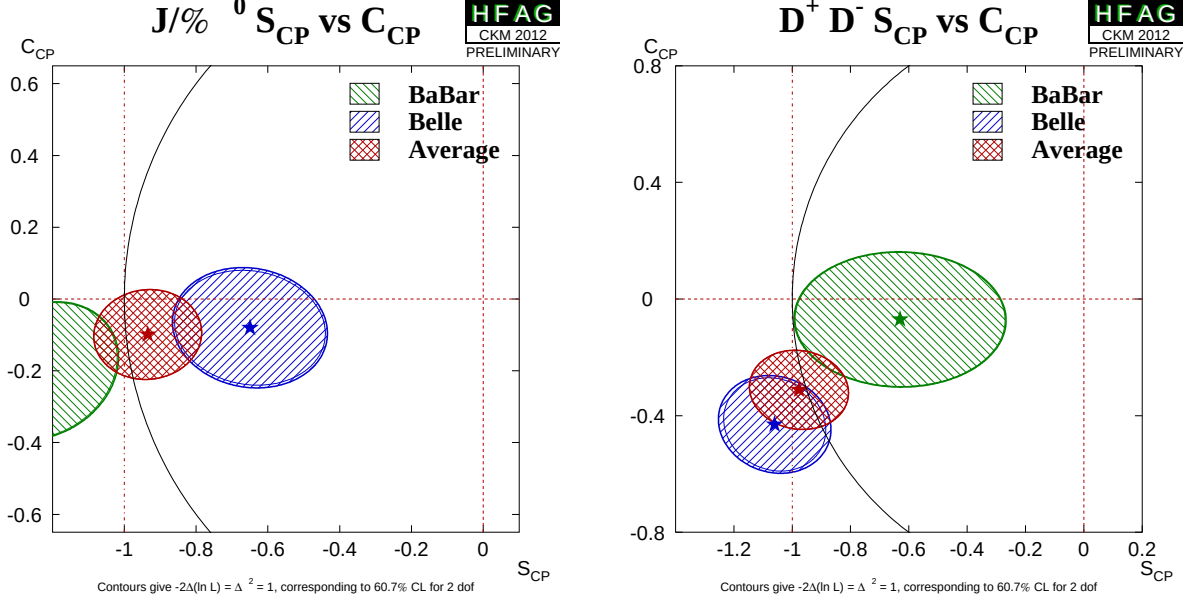


Figure 24: Averages of two $b \rightarrow c\bar{c}d$ dominated channels, for which correlated averages are performed, in the S_{CP} vs. C_{CP} plane. (Left) $B^0 \rightarrow J/\psi\pi^0$ and (right) $B^0 \rightarrow D^+D^-$.

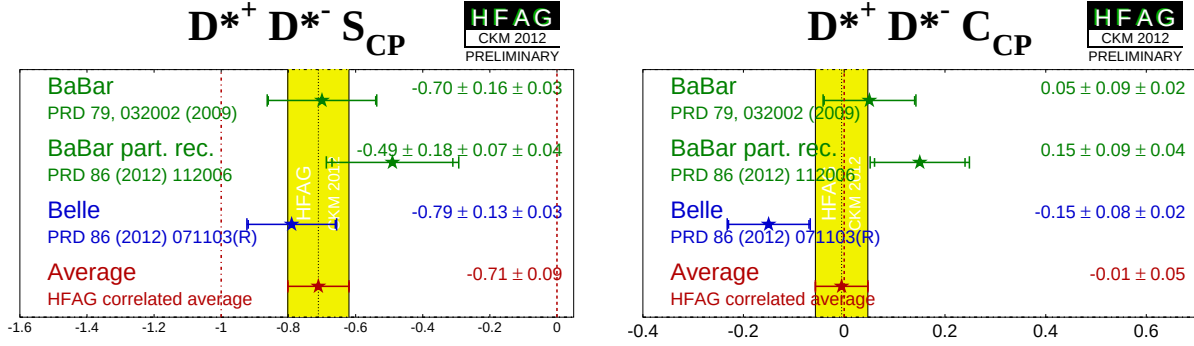


Figure 25: Averages of (left) $S_{b \rightarrow c\bar{c}d}$ and (right) $C_{b \rightarrow c\bar{c}d}$ for the mode $B^0 \rightarrow D^{*+}D^{*-}$.

top quark in the loop dominates, the weak phase from the decay amplitudes should cancel that from mixing, so that no CP violation (neither mixing-induced nor in decay) occurs. Non-zero contributions from loops with intermediate up and charm quarks can result in both types of effect (as usual, a strong phase difference is required for CP violation in decay to occur).

Both *BABAR* [351] and Belle [352] have performed time-dependent analyses of $B^0 \rightarrow K_S^0 K_S^0$. The results are shown in Table 41 and Fig. 28.

4.9 Time-dependent asymmetries in $b \rightarrow s\gamma$ transitions

The radiative decays $b \rightarrow s\gamma$ produce photons which are highly polarised in the Standard Model. The decays $B^0 \rightarrow F\gamma$ and $\bar{B}^0 \rightarrow F\gamma$ produce photons with opposite helicities, and since the polarisation is, in principle, observable, these final states cannot interfere. The finite

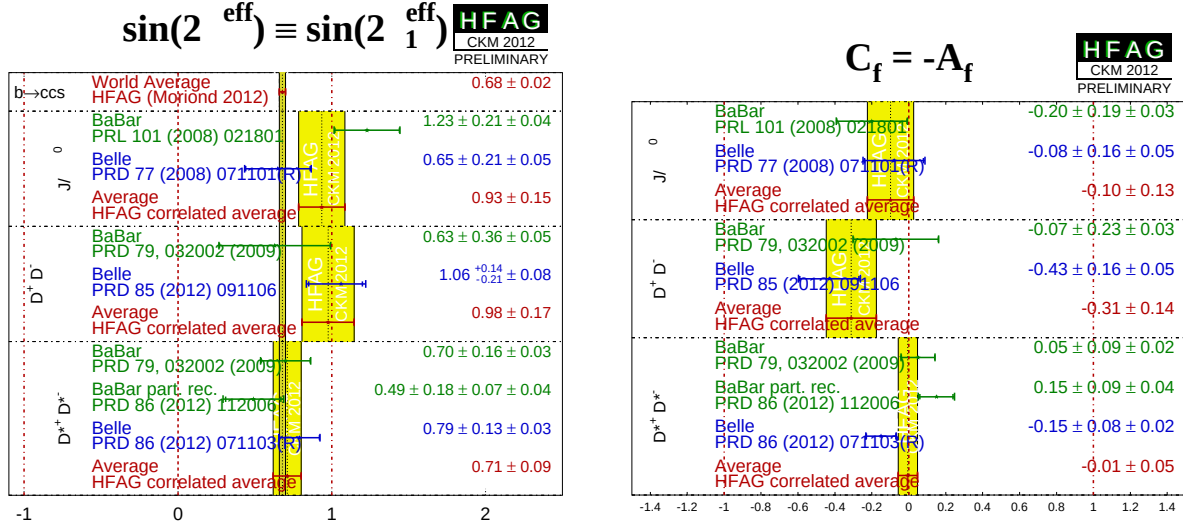


Figure 26: Averages of (left) $-\eta S_{b \rightarrow c\bar{c}d}$ and (right) $C_{b \rightarrow c\bar{c}d}$. The $-\eta S_{b \rightarrow q\bar{q}s}$ figure compares the results to the world average for $-\eta S_{b \rightarrow c\bar{c}s}$ (see Sec. 4.4.1).

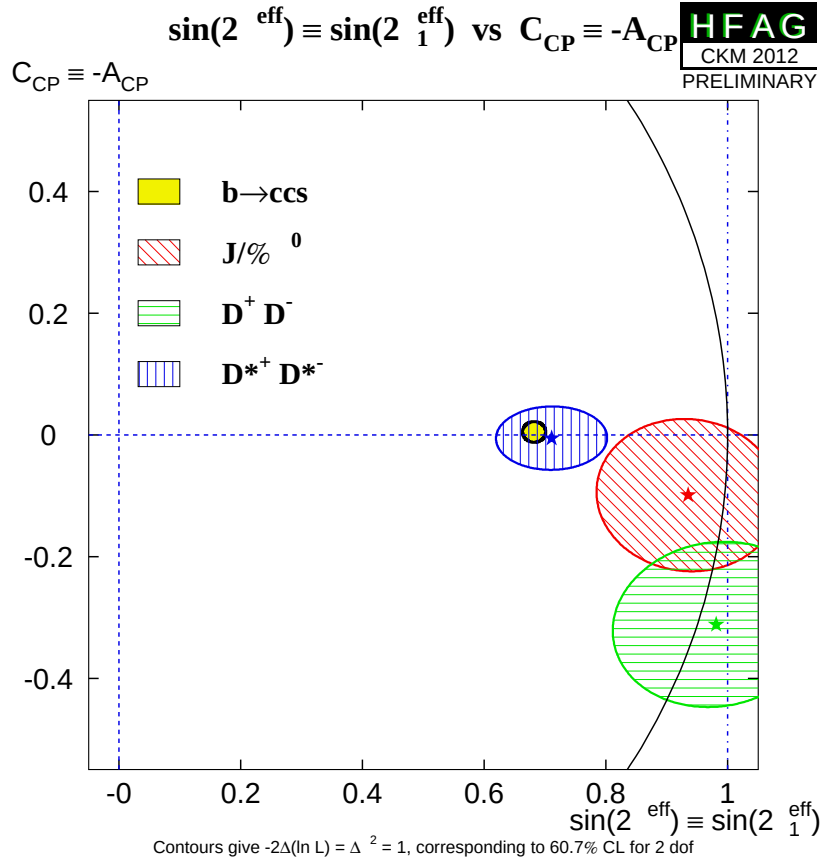
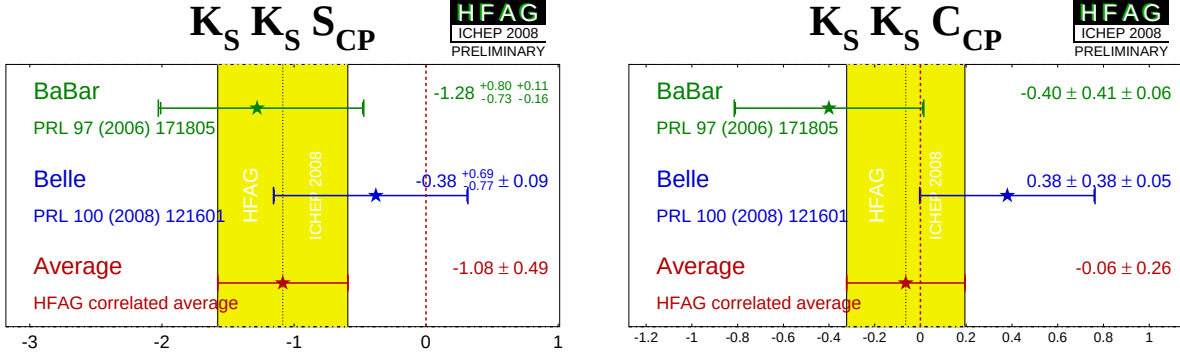


Figure 27: Compilation of constraints in the $-\eta S_{b \rightarrow c\bar{c}d}$ vs. $C_{b \rightarrow c\bar{c}d}$ plane.

Table 41: Results for $B^0 \rightarrow K_s^0 K_s^0$.

Experiment		$N(B\bar{B})$	S_{CP}	C_{CP}	Correlation
BABAR	[351]	350M	$-1.28^{+0.80+0.11}_{-0.73-0.16}$	$-0.40 \pm 0.41 \pm 0.06$	-0.32
Belle	[352]	657M	$-0.38^{+0.69}_{-0.77} \pm 0.09$	$0.38 \pm 0.38 \pm 0.05$	0.48
Average			-1.08 ± 0.49	-0.06 ± 0.26	0.14
Confidence level			$0.29 (1.1\sigma)$		


 Figure 28: Averages of (left) $S_{b \rightarrow q\bar{q}d}$ and (right) $C_{b \rightarrow q\bar{q}d}$ for the mode $B^0 \rightarrow K_s^0 K_s^0$.

mass of the s quark introduces small corrections to the limit of maximum polarisation, but any large mixing induced CP violation would be a signal for new physics. Since a single weak phase dominates the $b \rightarrow s\gamma$ transition in the Standard Model, the cosine term is also expected to be small.

Atwood *et al.* [268] have shown that an inclusive analysis with respect to $K_s^0 \pi^0 \gamma$ can be performed, since the properties of the decay amplitudes are independent of the angular momentum of the $K_s^0 \pi^0$ system. However, if non-dipole operators contribute significantly to the amplitudes, then the Standard Model mixing-induced CP violation could be larger than the naïve expectation $S \simeq -2(m_s/m_b) \sin(2\beta)$ [269, 270]. In this case, the CP parameters may vary over the $K_s^0 \pi^0 \gamma$ Dalitz plot, for example as a function of the $K_s^0 \pi^0$ invariant mass. Explicit calculations indicate such corrections are small for exclusive final states [271, 272].

With the above in mind, we quote two averages: one for $K^*(892)$ candidates only, and the other one for the inclusive $K_s^0 \pi^0 \gamma$ decay (including the $K^*(892)$). If the Standard Model dipole operator is dominant, both should give the same quantities (the latter naturally with smaller statistical error). If not, care needs to be taken in interpretation of the inclusive parameters, while the results on the $K^*(892)$ resonance remain relatively clean. Results from BABAR and Belle are used for both averages; both experiments use the invariant mass range $0.60 \text{ GeV}/c^2 < M_{K_s^0 \pi^0} < 1.80 \text{ GeV}/c^2$ in the inclusive analysis.

In addition to the $K_s^0 \pi^0 \gamma$ decay, both BABAR and Belle have presented results using $K_s^0 \eta \gamma$ and $K_s^0 \rho \gamma$, and Belle has in addition presented results using $K_s^0 \phi \gamma$. For the $K_s^0 \rho \gamma$ case, due to the non-negligible width of the ρ^0 meson, decays selected as $B^0 \rightarrow K_s^0 \rho^0 \gamma$ can include a significant contribution from $K^{*\pm} \pi^\mp \gamma$ decays, which are flavour-specific and do not have the same oscillation phenomenology. Both BABAR and Belle measure S_{eff} for all B decay candidates

with the ρ^0 selection being $0.6 < m(\pi^+\pi^-) < 0.9$ GeV/ c^2 , obtaining $0.14 \pm 0.25^{+0.04}_{-0.03}$ (*BABAR*) and $0.09 \pm 0.27^{+0.04}_{-0.07}$ (*Belle*). These values are then corrected for a “dilution factor”, that is evaluated with different methods in the two experiments: *BABAR* [353] obtain $0.549^{+0.096}_{-0.094}$ while *Belle* [354] obtain $0.83^{+0.19}_{-0.03}$. Until the discrepancy between these values is understood, the average of the results should be treated with caution.

Table 42: Averages for $b \rightarrow s\gamma$ modes.

Experiment		$N(B\bar{B})$	$S_{CP}(b \rightarrow s\gamma)$	$C_{CP}(b \rightarrow s\gamma)$	Correlation
$K^*(892)\gamma$					
<i>BABAR</i>	[355]	467M	$-0.03 \pm 0.29 \pm 0.03$	$-0.14 \pm 0.16 \pm 0.03$	0.05
<i>Belle</i>	[356]	535M	$-0.32^{+0.36}_{-0.33} \pm 0.05$	$0.20 \pm 0.24 \pm 0.05$	0.08
Average			-0.16 ± 0.22	-0.04 ± 0.14	0.06
Confidence level			0.40 (0.9 σ)		
$K_s^0\pi^0\gamma$ (including $K^*(892)\gamma$)					
<i>BABAR</i>	[355]	467M	$-0.17 \pm 0.26 \pm 0.03$	$-0.19 \pm 0.14 \pm 0.03$	0.04
<i>Belle</i>	[356]	535M	$-0.10 \pm 0.31 \pm 0.07$	$0.20 \pm 0.20 \pm 0.06$	0.08
Average			-0.15 ± 0.20	-0.07 ± 0.12	0.05
Confidence level			0.30 (1.0 σ)		
$K_s^0\eta\gamma$					
<i>BABAR</i>	[357]	465M	$-0.18^{+0.49}_{-0.46} \pm 0.12$	$-0.32^{+0.40}_{-0.39} \pm 0.07$	-0.17
<i>Belle</i>	[358]	772M	$-1.32 \pm 0.77 \pm 0.36$	$0.48 \pm 0.41 \pm 0.07$	-0.14
Average			-0.49 ± 0.42	0.06 ± 0.29	-0.15
Confidence level			0.24 (1.2 σ)		
$K_s^0\rho^0\gamma$					
<i>BABAR</i>	[359]	471M	$0.25 \pm 0.46^{+0.08}_{-0.06}$	$-0.39 \pm 0.20 \pm 0.05$	-0.09
<i>Belle</i>	[354]	657M	$0.11 \pm 0.33^{+0.05}_{-0.09}$	$-0.05 \pm 0.18 \pm 0.06$	0.04
Average			0.14 ± 0.27	-0.20 ± 0.14	-0.01
Confidence level			0.47 (0.7 σ)		
$K_s^0\phi\gamma$					
<i>Belle</i>	[360]	772M	$0.74^{+0.72}_{-1.05}^{+0.10}_{-0.24}$	$-0.35 \pm 0.58^{+0.10}_{-0.23}$	—

The results are shown in Table 42, and in Figs. 29 and 30. No significant CP violation results are seen; the results are consistent with the Standard Model and with other measurements in the $b \rightarrow s\gamma$ system (see Sec. 7).

4.10 Time-dependent asymmetries in $b \rightarrow d\gamma$ transitions

The formalism for the radiative decays $b \rightarrow d\gamma$ is much the same as that for $b \rightarrow s\gamma$ discussed above. Assuming dominance of the top quark in the loop, the weak phase in decay should cancel with that from mixing, so that the mixing-induced CP violation parameter S_{CP} should be very small. Corrections due to the finite light quark mass are smaller compared to $b \rightarrow s\gamma$, since $m_d < m_s$, and although QCD corrections may still play a role, they cannot significantly affect the prediction $S_{b \rightarrow d\gamma} \simeq 0$. Large CP violation effects could, however, be seen through a non-zero value of $C_{b \rightarrow d\gamma}$, since the top loop is not the only contribution.

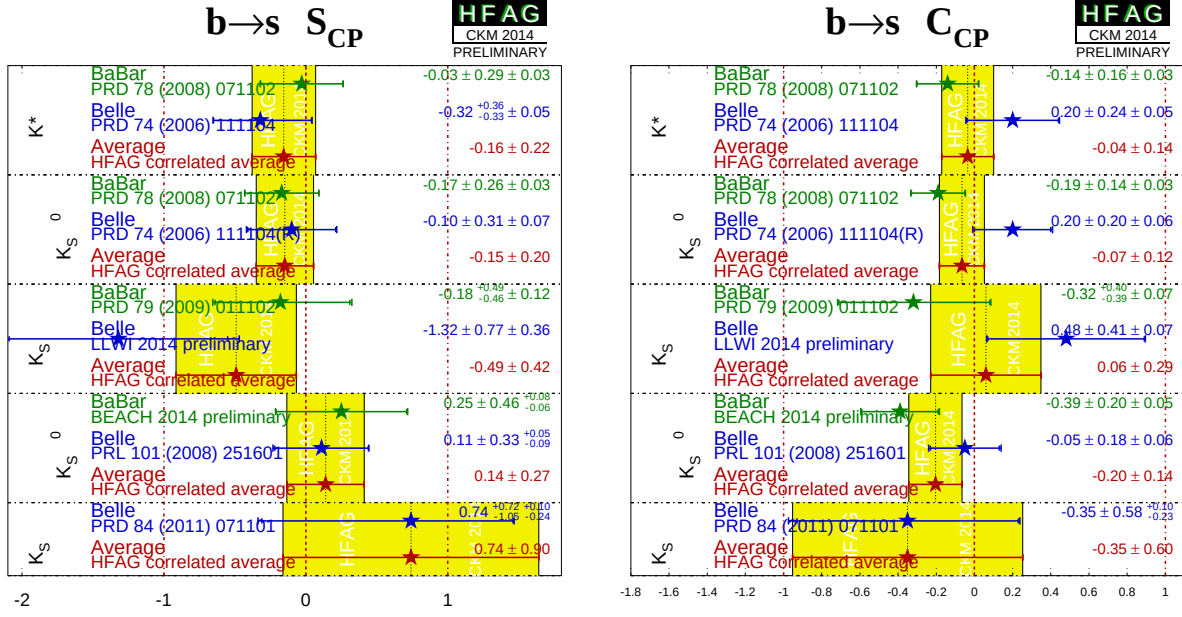


Figure 29: Averages of (left) $S_{b \rightarrow s \gamma}$ and (right) $C_{b \rightarrow s \gamma}$. Recall that the data for $K^* \gamma$ is a subset of that for $K_S^0 \pi^0 \gamma$.

Results using the mode $B^0 \rightarrow \rho^0 \gamma$ are available from Belle and are shown in Table 43.

Table 43: Averages for $B^0 \rightarrow \rho^0 \gamma$.

Experiment	$N(B\bar{B})$	S_{CP}	C_{CP}	Correlation
Belle [361]	657M	$-0.83 \pm 0.65 \pm 0.18$	$0.44 \pm 0.49 \pm 0.14$	-0.08

4.11 Time-dependent CP asymmetries in $b \rightarrow u\bar{u}d$ transitions

The $b \rightarrow u\bar{u}d$ transition can be mediated by either a $b \rightarrow u$ tree amplitude or a $b \rightarrow d$ penguin amplitude. These transitions can be investigated using the time dependence of B^0 decays to final states containing light mesons. Results are available from both *BABAR* and Belle for the CP eigenstate ($\eta = +1$) $\pi^+ \pi^-$ final state and for the vector-vector final state $\rho^+ \rho^-$, which is found to be dominated by the CP -even longitudinally polarised component (*BABAR* measure $f_{\text{long}} = 0.992 \pm 0.024^{+0.026}_{-0.013}$ [362] while Belle measure $f_{\text{long}} = 0.941^{+0.034}_{-0.040} \pm 0.030$ [363]). *BABAR* has also performed a time-dependent analysis of the vector-vector final state $\rho^0 \rho^0$ [364], in which they measure $f_{\text{long}} = 0.70 \pm 0.14 \pm 0.05$; Belle measure a smaller branching fraction than *BABAR* for $B^0 \rightarrow \rho^0 \rho^0$ [365] with corresponding signal yields too small to perform time-dependent or angular analyses. *BABAR* has furthermore performed a time-dependent analysis of the $B^0 \rightarrow a_1^\pm \pi^\mp$ decay [366]; further experimental input for the extraction of α from this channel is reported in a later publication [367].

Results, and averages, of time-dependent CP violation parameters in $b \rightarrow u\bar{u}d$ transitions

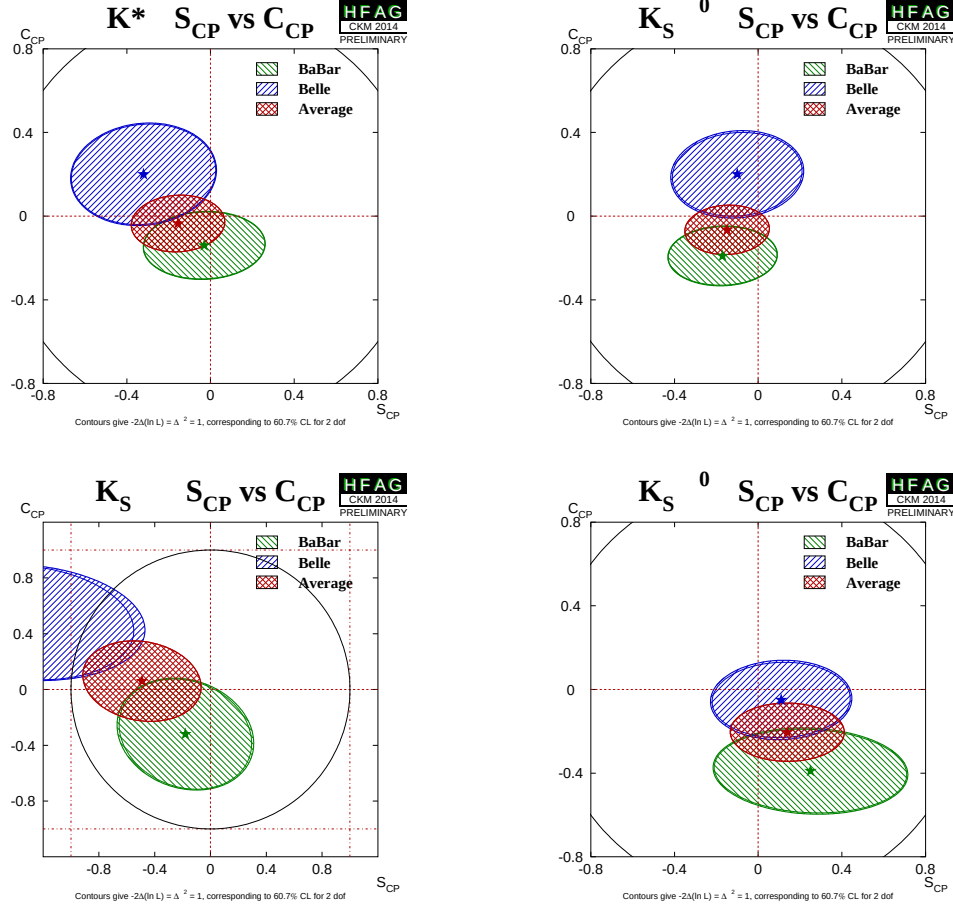


Figure 30: Averages of four $b \rightarrow s\gamma$ dominated channels, for which correlated averages are performed, in the S_{CP} vs. C_{CP} plane. (Top left) $B^0 \rightarrow K^*\gamma$, (top right) $B^0 \rightarrow K_S^0\pi^0\gamma$ (including $K^*\gamma$), (bottom left) $B^0 \rightarrow K_S^0\eta\gamma$, (bottom right).

are listed in Table 44. The averages for $\pi^+\pi^-$ are shown in Fig. 31, and those for $\rho^+\rho^-$ are shown in Fig. 32, with the averages in the S_{CP} vs. C_{CP} plane shown in Fig. 33 and averages of CP violation parameters in $B^0 \rightarrow a_1^\pm\pi^\mp$ decay shown in Fig. 34.

If the penguin contribution is negligible, the time-dependent parameters for $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow \rho^+\rho^-$ are given by $S_{b \rightarrow u\bar{u}d} = \eta \sin(2\alpha)$ and $C_{b \rightarrow u\bar{u}d} = 0$. In the presence of the penguin contribution, CP violation in decay may arise, and there is no straightforward interpretation of $S_{b \rightarrow u\bar{u}d}$ and $C_{b \rightarrow u\bar{u}d}$. An isospin analysis [372] can be used to disentangle the contributions and extract α .

For the non- CP eigenstate $\rho^\pm\pi^\mp$, both *BABAR* [250] and *Belle* [252, 253] have performed time-dependent Dalitz plot (DP) analyses of the $\pi^+\pi^-\pi^0$ final state [248]; such analyses allow direct measurements of the phases. Both experiments have measured the U and I parameters discussed in Sec. 4.2.5 and defined in Table 23. We have performed a full correlated average of these parameters, the results of which are summarised in Fig. 35.

Both experiments have also extracted the Q2B parameters. We have performed a full correlated average of these parameters, which is equivalent to determining the values from the

Table 44: Averages for $b \rightarrow u\bar{u}d$ modes.

Experiment	Sample size	S_{CP}	C_{CP}	Correlation		
		$\pi^+\pi^-$				
BABAR	[368]	467M	$-0.68 \pm 0.10 \pm 0.03$	$-0.25 \pm 0.08 \pm 0.02$		
Belle	[369]	772M	$-0.64 \pm 0.08 \pm 0.03$	$-0.33 \pm 0.06 \pm 0.03$		
LHCb	[342]	1.0 fb ⁻¹	$-0.71 \pm 0.13 \pm 0.02$	$-0.38 \pm 0.15 \pm 0.02$		
Average		-0.66 ± 0.06	-0.31 ± 0.05	0.00		
Confidence level		0.92 (0.1σ)				
		$\rho^+\rho^-$				
BABAR	[362]	$N(B\bar{B}) = 387\text{M}$	$-0.17 \pm 0.20^{+0.05}_{-0.06}$	$0.01 \pm 0.15 \pm 0.06$		
Belle	[370]	$N(B\bar{B}) = 535\text{M}$	$0.19 \pm 0.30 \pm 0.07$	$-0.16 \pm 0.21 \pm 0.07$		
Average			-0.05 ± 0.17	-0.06 ± 0.13		
Confidence level		0.50 (0.7σ)				
		$\rho^0\rho^0$				
BABAR	[364]	$N(B\bar{B}) = 465\text{M}$	$0.3 \pm 0.7 \pm 0.2$	$0.2 \pm 0.8 \pm 0.3$		
				-0.04		
Experiment	$N(B\bar{B})$	$A_{CP}^{a_1\pi}$	$C_{a_1\pi}$	$S_{a_1\pi}$	$\Delta C_{a_1\pi}$	$\Delta S_{a_1\pi}$
			$a_1^\pm\pi^\mp$			
BABAR	[366]	384M	$-0.07 \pm 0.07 \pm 0.02$	$-0.10 \pm 0.15 \pm 0.09$	$0.37 \pm 0.21 \pm 0.07$	$0.26 \pm 0.15 \pm 0.07$
Belle	[371]	772M	$-0.06 \pm 0.05 \pm 0.07$	$-0.01 \pm 0.11 \pm 0.09$	$-0.51 \pm 0.14 \pm 0.08$	$0.54 \pm 0.11 \pm 0.07$
Average			-0.06 ± 0.06	-0.05 ± 0.11	-0.20 ± 0.13	$-0.09 \pm 0.14 \pm 0.06$
Confidence level					0.43 ± 0.10	-0.10 ± 0.12
				0.03 (2.1σ)		
Experiment	$N(B\bar{B})$	$\mathcal{A}_{a_1\pi}^{+-}$	$\mathcal{A}_{a_1\pi}^{+-}$	Correlation		
BABAR	[366]	384M	$0.07 \pm 0.21 \pm 0.15$	$0.15 \pm 0.15 \pm 0.07$		
Belle	[371]	772M	$-0.04 \pm 0.26 \pm 0.19$	$0.07 \pm 0.08 \pm 0.10$		
Average			0.02 ± 0.20	0.10 ± 0.10		
Confidence level				0.92 (0.1σ)		

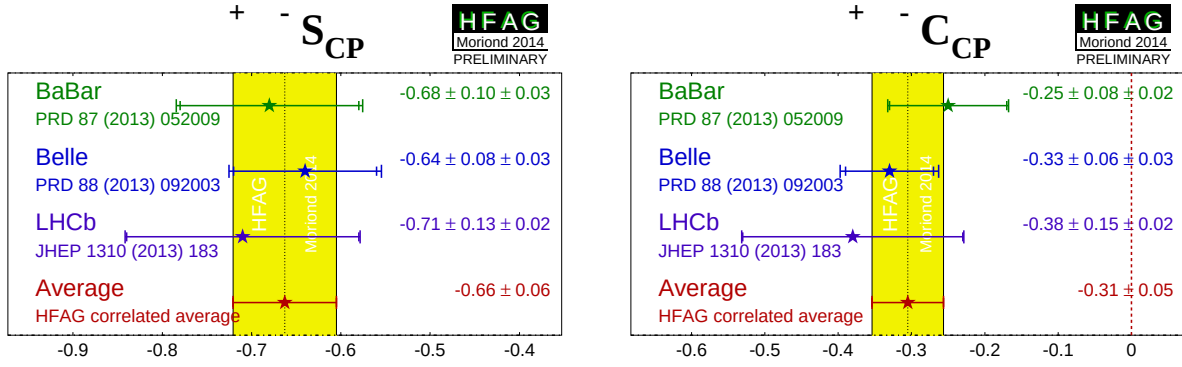


Figure 31: Averages of (left) $S_{b \rightarrow u\bar{u}d}$ and (right) $C_{b \rightarrow u\bar{u}d}$ for the mode $B^0 \rightarrow \pi^+ \pi^-$.

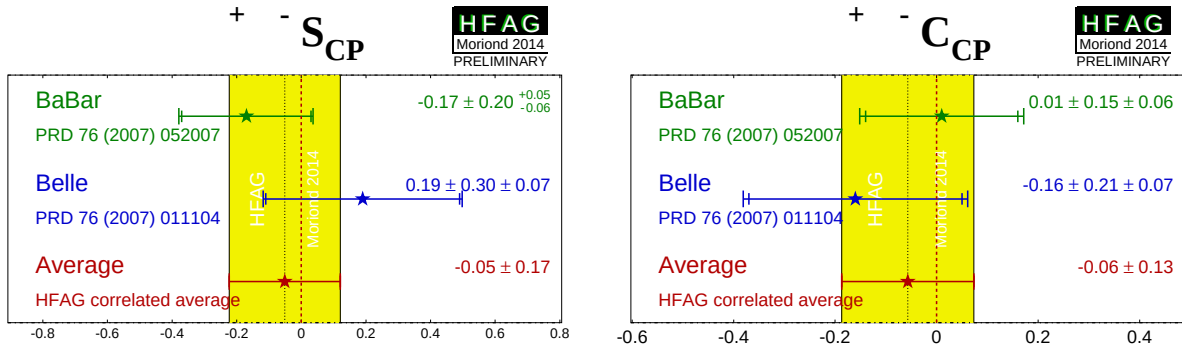


Figure 32: Averages of (left) $S_{b \rightarrow u\bar{u}d}$ and (right) $C_{b \rightarrow u\bar{u}d}$ for the mode $B^0 \rightarrow \rho^+ \rho^-$.

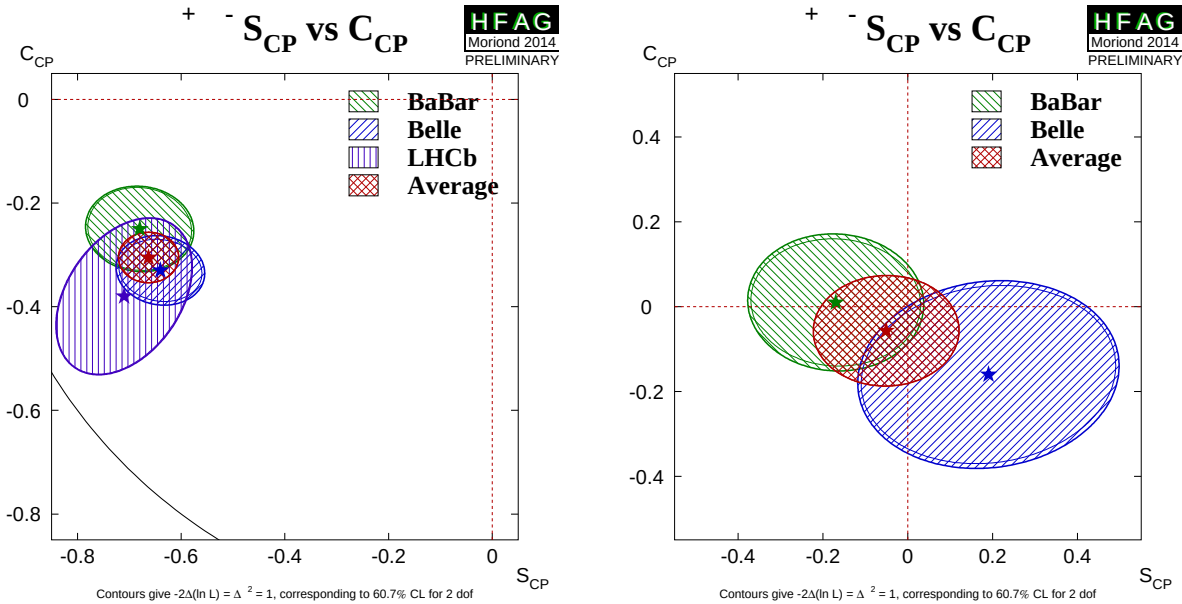


Figure 33: Averages of $b \rightarrow u\bar{u}d$ dominated channels, for which correlated averages are performed, in the S_{CP} vs. C_{CP} plane. (Left) $B^0 \rightarrow \pi^+ \pi^-$ and (right) $B^0 \rightarrow \rho^+ \rho^-$.

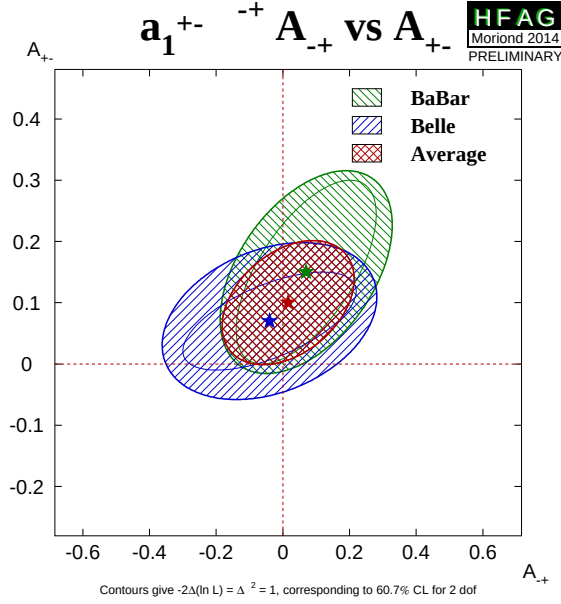


Figure 34: Averages of CP violation parameters in $B^0 \rightarrow a_1^\pm \pi^\mp$ in $\mathcal{A}_{a_1\pi}^{+-}$ vs. $\mathcal{A}_{a_1\pi}^{+-}$ space.

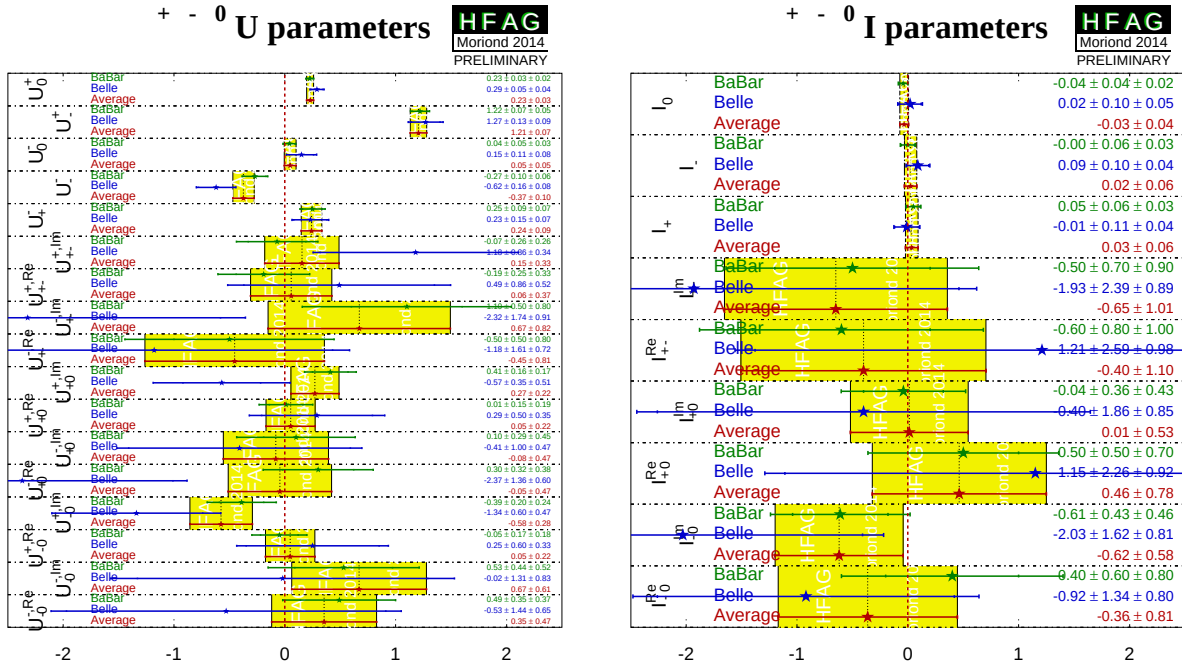


Figure 35: Summary of the U and I parameters measured in the time-dependent $B^0 \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot analysis.

averaged U and I parameters. The results are shown in Table. 45.⁵⁵ Averages of the $B^0 \rightarrow \rho^0 \pi^0$ Q2B parameters are shown in Figs. 36 and 37.

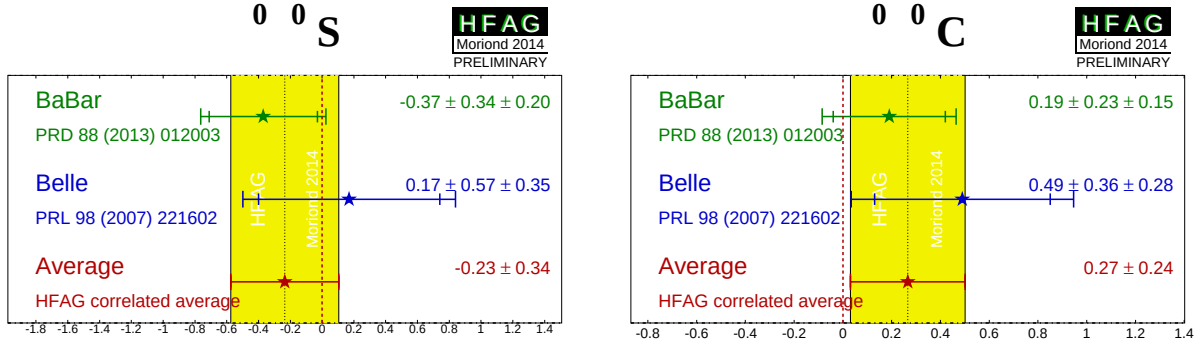


Figure 36: Averages of (left) $S_{b \rightarrow u\bar{u}d}$ and (right) $C_{b \rightarrow u\bar{u}d}$ for the mode $B^0 \rightarrow \rho^0 \pi^0$.

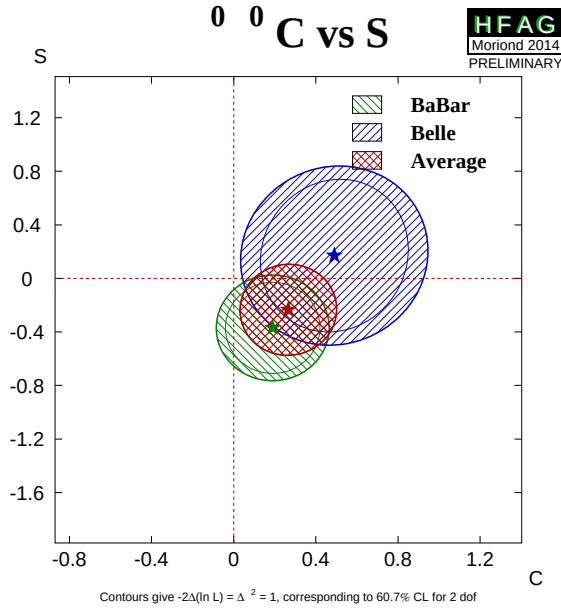


Figure 37: Averages of $b \rightarrow u\bar{u}d$ dominated channels, for the mode $B^0 \rightarrow \rho^0 \pi^0$ in the S_{CP} vs. C_{CP} plane.

With the notation described in Sec. 4.2 (Eq. (126)), the time-dependent parameters for the Q2B $B^0 \rightarrow \rho^\pm \pi^\mp$ analysis are, neglecting penguin contributions, given by

$$S_{\rho\pi} = \sqrt{1 - \left(\frac{\Delta C}{2}\right)^2} \sin(2\alpha) \cos(\delta), \quad \Delta S_{\rho\pi} = \sqrt{1 - \left(\frac{\Delta C}{2}\right)^2} \cos(2\alpha) \sin(\delta) \quad (155)$$

⁵⁵ The $B^0 \rightarrow \rho^\pm \pi^\mp$ Q2B parameters are equivalent to the parameters used for $B^0 \rightarrow a_1^\pm \pi^\mp$ decays, reported in Table 44. For the $B^0 \rightarrow a_1^\pm \pi^\mp$ case there has not yet been a full amplitude analysis of $B^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ and therefore only the Q2B parameters are available.

Table 45: Averages of quasi-two-body parameters extracted from time-dependent Dalitz plot analysis of $B^0 \rightarrow \pi^+ \pi^- \pi^0$.

Experiment	$N(B\bar{B})$	$\mathcal{A}_{CP}^{\rho\pi}$	$C_{\rho\pi}$	$S_{\rho\pi}$	$\Delta C_{\rho\pi}$	$\Delta S_{\rho\pi}$
BABAR	[251]	$-0.10 \pm 0.03 \pm 0.02$	$0.02 \pm 0.06 \pm 0.04$	$0.05 \pm 0.08 \pm 0.03$	$0.23 \pm 0.06 \pm 0.05$	$0.05 \pm 0.08 \pm 0.04$
Belle	[252, 253]	$-0.12 \pm 0.05 \pm 0.04$	$-0.13 \pm 0.09 \pm 0.05$	$0.06 \pm 0.13 \pm 0.05$	$0.36 \pm 0.10 \pm 0.05$	$-0.08 \pm 0.13 \pm 0.05$
Average		-0.11 ± 0.03	-0.03 ± 0.06	0.06 ± 0.07	0.27 ± 0.06	0.01 ± 0.08
Confidence level				$0.63 \text{ (} 0.5\sigma \text{)}$		

Experiment	$N(B\bar{B})$	$\mathcal{A}_{\rho\pi}^{+-}$	$\mathcal{A}_{\rho\pi}^{+-}$	Correlation
BABAR	[251]	$-0.12 \pm 0.08^{+0.04}_{-0.05}$	$0.09^{+0.05}_{-0.06} \pm 0.04$	0.55
Belle	[252, 253]	$0.08 \pm 0.16 \pm 0.11$	$0.21 \pm 0.08 \pm 0.04$	0.47
Average		-0.08 ± 0.08	0.13 ± 0.05	0.37
Confidence level			$0.47 \text{ (} 0.7\sigma \text{)}$	

Experiment	$N(B\bar{B})$	$C_{\rho^0\pi^0}$	$S_{\rho^0\pi^0}$	Correlation
BABAR	[251]	$0.19 \pm 0.23 \pm 0.15$	$-0.37 \pm 0.34 \pm 0.20$	0.00
Belle	[252, 253]	$0.49 \pm 0.36 \pm 0.28$	$0.17 \pm 0.57 \pm 0.35$	0.08
Average		0.27 ± 0.24	-0.23 ± 0.34	0.02
Confidence level			$0.68 \text{ (} 0.4\sigma \text{)}$	

and $C_{\rho\pi} = \mathcal{A}_{CP}^{\rho\pi} = 0$, where $\delta = \arg(A_{-+}A_{+-}^*)$ is the strong phase difference between the $\rho^-\pi^+$ and $\rho^+\pi^-$ decay amplitudes. In the presence of the penguin contribution, there is no straightforward interpretation of the Q2B observables in the $B^0 \rightarrow \rho^\pm\pi^\mp$ system in terms of CKM parameters. However, CP violation in decay may arise, resulting in either or both of $C_{\rho\pi} \neq 0$ and $\mathcal{A}_{CP}^{\rho\pi} \neq 0$. Equivalently, CP violation in decay may be seen by either of the decay-type-specific observables $\mathcal{A}_{\rho\pi}^{+-}$ and $\mathcal{A}_{\rho\pi}^{-+}$, defined in Eq. (127), deviating from zero. Results and averages for these parameters are also given in Table 45. Averages of CP violation parameters in $B^0 \rightarrow \rho^\pm\pi^\mp$ decays are shown in Fig. 38, both in $\mathcal{A}_{CP}^{\rho\pi}$ vs. $C_{\rho\pi}$ space and in $\mathcal{A}_{\rho\pi}^{-+}$ vs. $\mathcal{A}_{\rho\pi}^{+-}$ space.

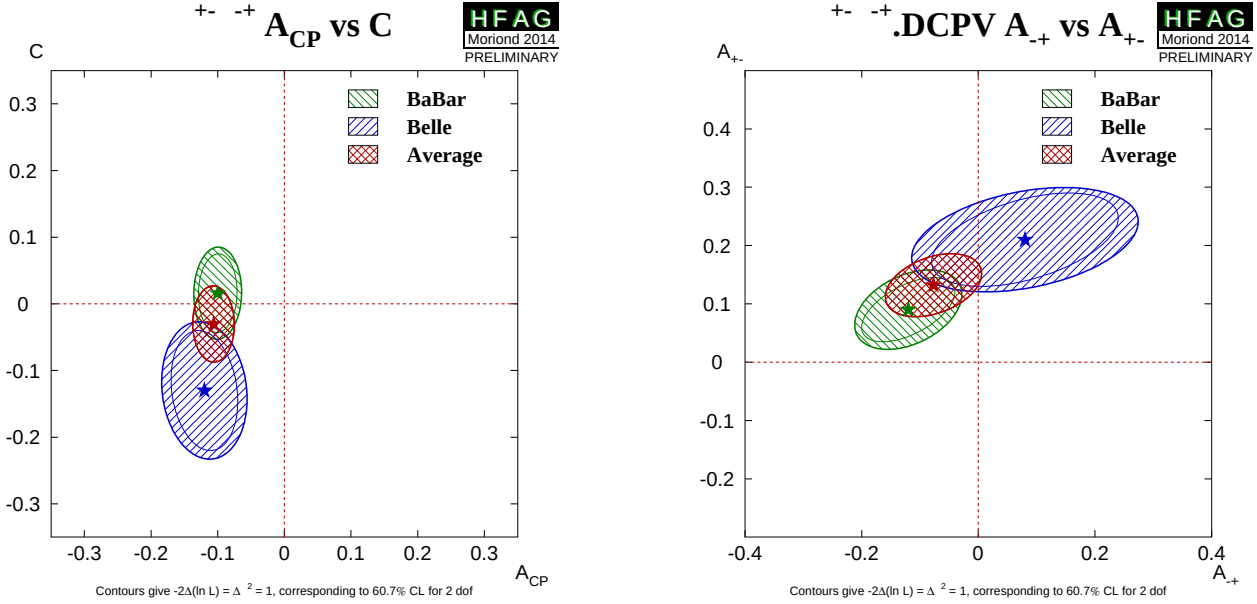


Figure 38: CP violation in $B^0 \rightarrow \rho^\pm\pi^\mp$ decays. (Left) $\mathcal{A}_{CP}^{\rho\pi}$ vs. $C_{\rho\pi}$ space, (right) $\mathcal{A}_{\rho\pi}^{-+}$ vs. $\mathcal{A}_{\rho\pi}^{+-}$ space.

The averages for $S_{b \rightarrow u\bar{u}d}$ and $C_{b \rightarrow u\bar{u}d}$ in $B^0 \rightarrow \pi^+\pi^-$ are both more than 5σ away from zero, suggesting that both mixing-induced and CP violation in decay are well-established in this channel. The discrepancy between results from *BABAR* and Belle that used to exist in this channel (see, for example, Ref. [373]) is no longer apparent, and the results from LHCb are also fully consistent with other measurements. Some difference is, however, seen between the *BABAR* and Belle measurements in the $a_1^\pm\pi^\mp$ system. The confidence level of the five-dimensional average is 0.03, which corresponds to a 2.1σ discrepancy. As seen in Table 44, this discrepancy is primarily in the values of $S_{a_1\pi}$, and is not evident in the $\mathcal{A}_{a_1\pi}^{-+}$ vs. $\mathcal{A}_{a_1\pi}^{+-}$ projection shown in Fig. 34. Since there is no evidence of systematic problems in either analysis, we do not rescale the errors of the averages.

In $B^0 \rightarrow \rho^\pm\pi^\mp$ decays, both experiments see an indication of CP violation in the $\mathcal{A}_{CP}^{\rho\pi}$ parameter (as seen in Fig. 38). The average is more than 3σ from zero, providing evidence of direct CP violation in this channel. In $B^0 \rightarrow \rho^+\rho^-$ decays there is no evidence for CP violation, either mixing-induced or in decay. The absence of evidence of penguin contributions in this mode leads to strong constraints on $\alpha \equiv \phi_2$.

4.11.1 Constraints on $\alpha \equiv \phi_2$

The precision of the measured CP violation parameters in $b \rightarrow u\bar{u}d$ transitions allows constraints to be set on the UT angle $\alpha \equiv \phi_2$. Constraints have been obtained with various methods:

- Both *BABAR* [368] and Belle [369] have performed isospin analyses in the $\pi\pi$ system. Belle exclude $23.8^\circ < \phi_2 < 66.8^\circ$ at the 68% C.L. while *BABAR* give a confidence level interpretation for α , and constrain $\alpha \in [71^\circ, 109^\circ]$ at the 68% C.L. considering only the solution consistent with the Standard Model. Values in the range $[23^\circ, 67^\circ]$ at the 90% C.L. are excluded. In both cases, only solutions in 0° – 180° are considered.
- Both experiments have also performed isospin analyses in the $\rho\rho$ system. The most recent result from *BABAR* is given in an update of the measurements of the $B^+ \rightarrow \rho^+\rho^0$ decay [374], and sets the constraint $\alpha = (92.4^{+6.0}_{-6.5})^\circ$. The most recent result from Belle is given in an update of the search for the $B^0 \rightarrow \rho^0\rho^0$ decay and sets the constraint $\phi_2 = (91.7 \pm 14.9)^\circ$ [365].
- The time-dependent Dalitz plot analysis of the $B^0 \rightarrow \pi^+\pi^-\pi^0$ decay allows a determination of α without input from any other channels. *BABAR* [251] present a scan, but not an interval, for α , since their studies indicate that the scan is not statistically robust and cannot be interpreted as 1-C.L. Belle [252, 253] have obtained a constraint on α using additional information from the SU(2) partners of $B \rightarrow \rho\pi$, which can be used to constrain α via an isospin pentagon relation [375]. With this analysis, Belle obtain the constraint $\phi_2 = (83^{+12}_{-23})^\circ$ (where the errors correspond to 1σ , *i.e.* 68.3% confidence level).
- The results from *BABAR* on $B^0 \rightarrow a_1^\pm\pi^\mp$ [366] can be combined with results from modes related by isospin (a_1K and $K_1\pi$) [376] leading to the constraint $\alpha = (79 \pm 7 \pm 11)^\circ$ [367].
- The CKMfitter [254] and UTFit [302] groups use the measurements from Belle and *BABAR* given above with other branching fractions and CP asymmetries in $B \rightarrow \pi\pi$, $\rho\pi$ and $\rho\rho$ modes, to perform isospin analyses for each system, and to make combined constraints on α .
- The *BABAR* and Belle collaborations have combined their results on $B \rightarrow \pi\pi$, $\pi\pi\pi^0$ and $\rho\rho$ to obtain [377]

$$\alpha \equiv \phi_2 = (88 \pm 5)^\circ. \quad (156)$$

The above solution is that consistent with the Standard Model (an ambiguous solution shifted by 180° exists). The strongest constraint currently comes from the $B \rightarrow \rho\rho$ system. The inclusion of results from $B^0 \rightarrow a_1^\pm\pi^\mp$ does not significantly affect the average.

Note that methods based on isospin symmetry make extensive use of measurements of branching fractions and CP asymmetries, as averaged by the HFAG Rare Decays subgroup (Sec. 7). Note also that each method suffers from discrete ambiguities in the solutions. The model assumption in the $B^0 \rightarrow \pi^+\pi^-\pi^0$ analysis helps resolve some of the multiple solutions, and results in a single preferred value for α in $[0, \pi]$. All the above measurements correspond to the choice that is in agreement with the global CKM fit.

At present we make no attempt to provide an HFAG average for $\alpha \equiv \phi_2$. More details on procedures to calculate a best fit value for α can be found in Refs. [254, 302].

4.12 Time-dependent CP asymmetries in $b \rightarrow c\bar{u}d/u\bar{c}d$ transitions

Non- CP eigenstates such as $D^\mp\pi^\pm$, $D^{*\mp}\pi^\pm$ and $D^\mp\rho^\pm$ can be produced in decays of B^0 mesons either via Cabibbo favoured ($b \rightarrow c$) or doubly Cabibbo suppressed ($b \rightarrow u$) tree amplitudes. Since no penguin contribution is possible, these modes are theoretically clean. The ratio of the magnitudes of the suppressed and favoured amplitudes, R , is sufficiently small (predicted to be about 0.02), that terms of $\mathcal{O}(R^2)$ can be neglected, and the sine terms give sensitivity to the combination of UT angles $2\beta + \gamma$.

As described in Sec. 4.2.6, the averages are given in terms of parameters a and c . CP violation would appear as $a \neq 0$. Results are available from both *BABAR* and Belle in the modes $D^\mp\pi^\pm$ and $D^{*\mp}\pi^\pm$; for the latter mode both experiments have used both full and partial reconstruction techniques. Results are also available from *BABAR* using $D^\mp\rho^\pm$. These results, and their averages, are listed in Table 46, and are shown in Fig. 39. The constraints in c vs. a space for the $D\pi$ and $D^*\pi$ modes are shown in Fig. 40. It is notable that the average value of a from $D^*\pi$ is more than 3σ from zero, providing evidence of CP violation in this channel.

Table 46: Averages for $b \rightarrow c\bar{u}d/u\bar{c}d$ modes.

Experiment		$N(B\bar{B})$	a	c
			$D^\mp\pi^\pm$	
<i>BABAR</i> (full rec.)	[260]	232M	$-0.010 \pm 0.023 \pm 0.007$	$-0.033 \pm 0.042 \pm 0.012$
Belle (full rec.)	[264]	386M	$-0.050 \pm 0.021 \pm 0.012$	$-0.019 \pm 0.021 \pm 0.012$
Average			-0.030 ± 0.017	-0.022 ± 0.021
Confidence level			0.24 (1.2 σ)	0.78 (0.3 σ)
			$D^{*\mp}\pi^\pm$	
<i>BABAR</i> (full rec.)	[260]	232M	$-0.040 \pm 0.023 \pm 0.010$	$0.049 \pm 0.042 \pm 0.015$
<i>BABAR</i> (partial rec.)	[261]	232M	$-0.034 \pm 0.014 \pm 0.009$	$-0.019 \pm 0.022 \pm 0.013$
Belle (full rec.)	[264]	386M	$-0.039 \pm 0.020 \pm 0.013$	$-0.011 \pm 0.020 \pm 0.013$
Belle (partial rec.)	[263]	657M	$-0.046 \pm 0.013 \pm 0.015$	$-0.015 \pm 0.013 \pm 0.015$
Average			-0.039 ± 0.010	-0.010 ± 0.013
Confidence level			0.97 (0.03 σ)	0.59 (0.6 σ)
			$D^\mp\rho^\pm$	
<i>BABAR</i> (full rec.)	[260]	232M	$-0.024 \pm 0.031 \pm 0.009$	$-0.098 \pm 0.055 \pm 0.018$

For each mode, $D\pi$, $D^*\pi$ and $D\rho$, there are two measurements (a and c , or S^+ and S^-) which depend on three unknowns (R , δ and $2\beta + \gamma$), of which two are different for each decay mode. Therefore, there is not enough information to solve directly for $2\beta + \gamma$. However, for each choice of R and $2\beta + \gamma$, one can find the value of δ that allows a and c to be closest to their measured values, and calculate the distance in terms of numbers of standard deviations. (We currently neglect experimental correlations in this analysis.) These values of $N(\sigma)_{\min}$ can then be plotted as a function of R and $2\beta + \gamma$ (and can trivially be converted to confidence levels). These plots are given for the $D\pi$ and $D^*\pi$ modes in Fig. 40; the uncertainties in the $D\rho$ mode are currently too large to give any meaningful constraint.

The constraints can be tightened if one is willing to use theoretical input on the values of R and/or δ . One popular choice is the use of SU(3) symmetry to obtain R by relating

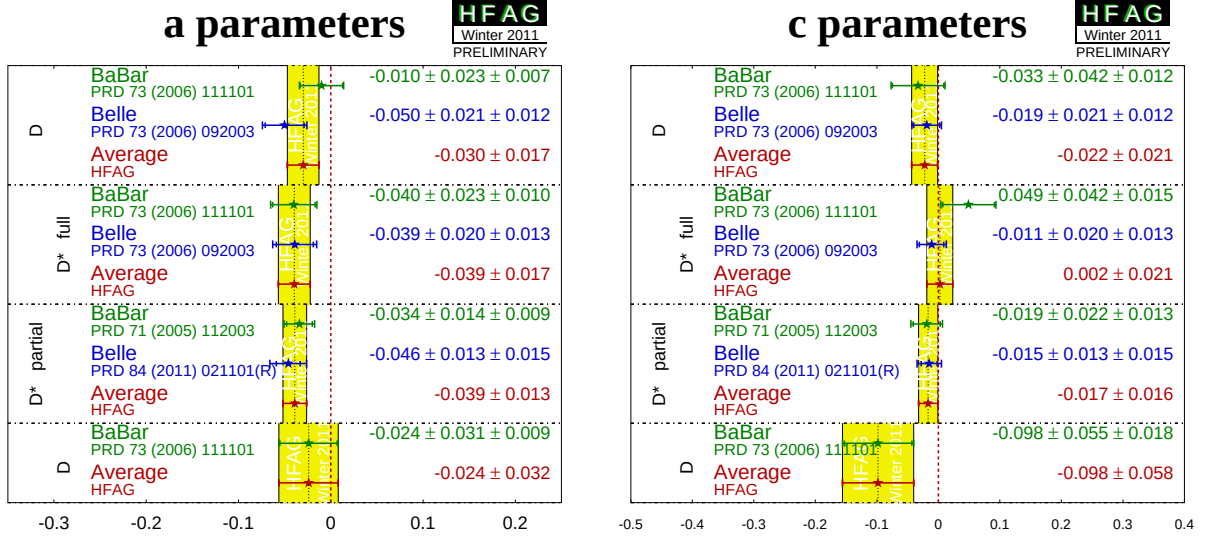


Figure 39: Averages for $b \rightarrow c\bar{u}d/u\bar{c}d$ modes.

the suppressed decay mode to B decays involving D_s mesons. More details can be found in Refs. [254, 302].

4.13 Time-dependent CP asymmetries in $b \rightarrow c\bar{u}s/u\bar{c}s$ transitions

4.13.1 Time-dependent CP asymmetries in $B^0 \rightarrow D^\mp K_s^0 \pi^\pm$

Time-dependent analyses of transitions such as $B^0 \rightarrow D^\mp K_s^0 \pi^\pm$ can be used to probe $\sin(2\beta + \gamma)$ in a similar way to that discussed above (Sec. 4.12). Since the final state contains three particles, a Dalitz plot analysis is necessary to maximise the sensitivity. BABAR [378] have carried out such an analysis. They obtain $2\beta + \gamma = (83 \pm 53 \pm 20)^\circ$ (with an ambiguity $2\beta + \gamma \leftrightarrow 2\beta + \gamma + \pi$) assuming the ratio of the $b \rightarrow u$ and $b \rightarrow c$ amplitude to be constant across the Dalitz plot at 0.3.

4.13.2 Time-dependent CP asymmetries in $B_s^0 \rightarrow D_s^\mp K^\pm$

Time-dependent analysis of $B_s^0 \rightarrow D_s^\mp K^\pm$ decays can be used to determine $\gamma - 2\beta_s$ [265, 379, 380]. Compared to the situation for $B^0 \rightarrow D^{(*)\mp} \pi^\pm$ discussed above (Sec. 4.12), the larger value of the ratio R of the magnitudes of the suppressed and favoured amplitudes allows it to be determined from the data. Moreover, the non-zero value of $\Delta\Gamma_s$ allows the determination of additional terms, labelled $A^{\Delta\Gamma}$ and $\bar{A}^{\Delta\Gamma}$, that break ambiguities in the solutions for $\gamma - 2\beta_s$.

LHCb [266] have measured the time-dependent CP violation parameters in $B_s^0 \rightarrow D_s^\mp K^\pm$ decays, using 1.0 fb^{-1} of data. The results are given in Table 47. From these results, and a constraint on $2\beta_s$ from independent LHCb measurements, LHCb determine $\gamma = (115^{+28}_{-43})^\circ$, $\delta_{D_s K} = (3^{+19}_{-20})^\circ$ and $R_{D_s K} = 0.53^{+0.17}_{-0.16}$.

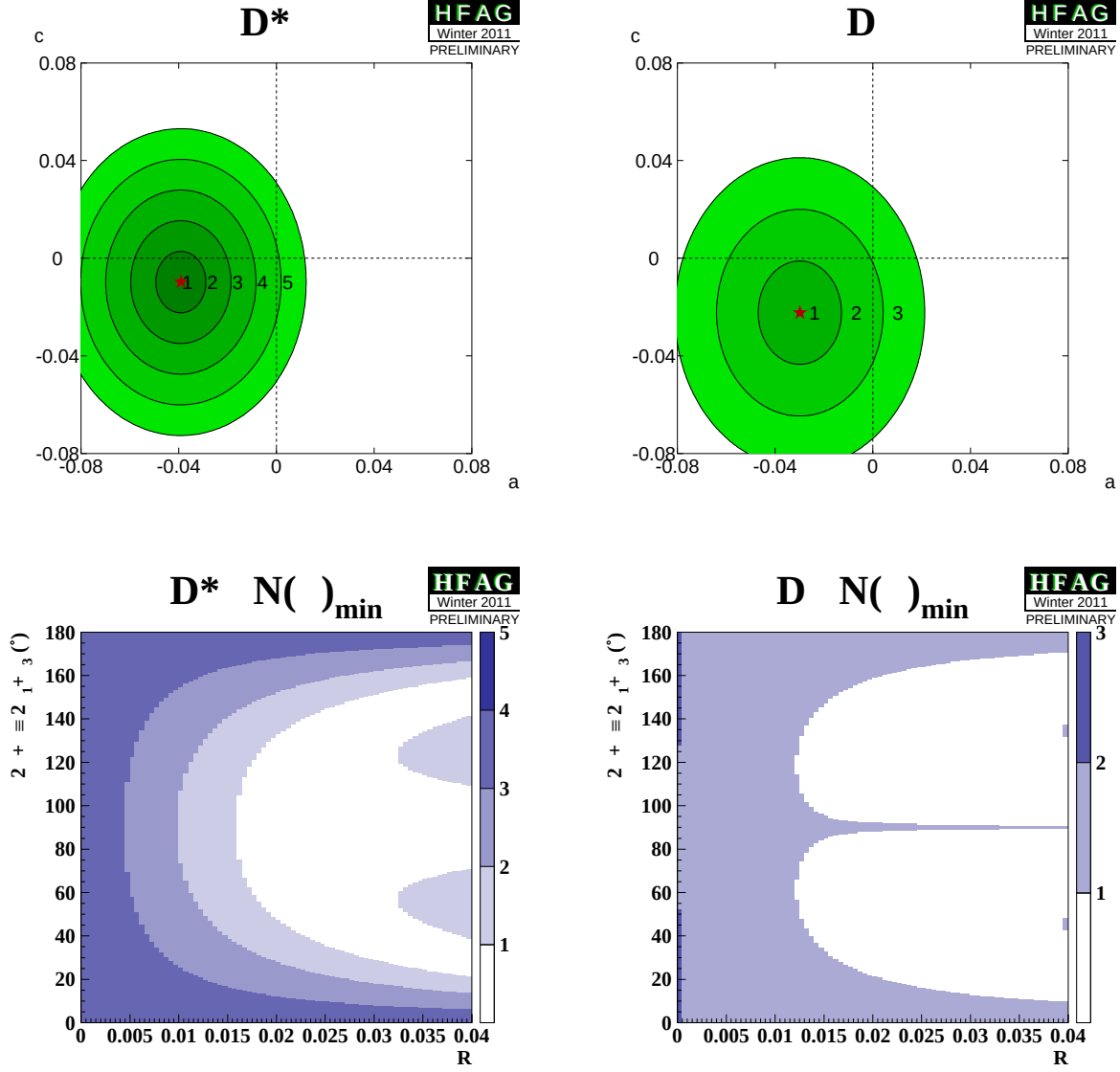


Figure 40: Results from $b \rightarrow \bar{c}u d / \bar{u}c d$ modes. (Top) Constraints in c vs. a space. (Bottom) Constraints in $2\beta + \gamma$ vs. R space. (Left) $D^*\pi$ and (right) $D\pi$ modes.

Table 47: Results for $B_s^0 \rightarrow D_s^\mp K^\pm$.

Experiment	Sample size	C	$A^{\Delta\Gamma}$	$\bar{A}^{\Delta\Gamma}$	S	$\bar{S}$
LHCb [266]	1 fb^{-1}	$0.53 \pm 0.25 \pm 0.04$	$0.37 \pm 0.42 \pm 0.20$	$0.20 \pm 0.41 \pm 0.20$	$-1.09 \pm 0.33 \pm 0.08$	$-0.36 \pm 0.34 \pm 0.08$

4.14 Rates and asymmetries in $B^\mp \rightarrow D^{(*)}K^{(*)\mp}$ decays

As explained in Sec. 4.2.7, rates and asymmetries in $B^\mp \rightarrow D^{(*)}K^{(*)\mp}$ decays are sensitive to γ , and have negligible theoretical uncertainty [381]. Various methods using different $D^{(*)}$ final states have been used.

4.14.1 D decays to CP eigenstates

Results are available from both *BABAR* and *Belle* on GLW analyses in the decay modes $B^\pm \rightarrow DK^\mp$, $B^\pm \rightarrow D^*K^\mp$ and $B^\pm \rightarrow DK^{*\mp}$. Both experiments use the CP -even D decay final states K^+K^- and $\pi^+\pi^-$ in all three modes; both experiments generally use the CP -odd decay modes $K_s^0\pi^0$, $K_s^0\omega$ and $K_s^0\phi$, though care is taken to avoid statistical overlap with the $K_s^0K^+K^-$ sample used for Dalitz plot analyses (see Sec. 4.14.3), and asymmetric systematic errors are assigned due to CP -even pollution under the $K_s^0\omega$ and $K_s^0\phi$ signals. Both experiments use both the $D^* \rightarrow D\pi^0$ decay, which gives $CP(D^*) = CP(D)$, and the $D^* \rightarrow D\gamma$ decays, which gives $CP(D^*) = -CP(D)$. In addition, results from CDF and LHCb are available in the decay mode $B^\pm \rightarrow DK^\pm$, and from LHCb in the decay mode $B^\pm \rightarrow DK^\pm\pi^+\pi^-$, for CP -even final states (K^+K^- and $\pi^+\pi^-$) only. The results and averages are given in Table 48 and shown in Fig. 41.

Table 48: Averages from GLW analyses of $b \rightarrow c\bar{u}s/u\bar{c}s$ modes.

Experiment	Sample size	A_{CP+}	A_{CP-}	R_{CP+}	R_{CP-}
$D_{CP}K^-$					
<i>BABAR</i> [382]	$N(B\bar{B}) = 467\text{M}$	$0.25 \pm 0.06 \pm 0.02$	$-0.09 \pm 0.07 \pm 0.02$	$1.18 \pm 0.09 \pm 0.05$	$1.07 \pm 0.08 \pm 0.04$
<i>Belle</i> [383]	$N(B\bar{B}) = 772\text{M}$	$0.29 \pm 0.06 \pm 0.02$	$-0.12 \pm 0.06 \pm 0.01$	$1.03 \pm 0.07 \pm 0.03$	$1.13 \pm 0.09 \pm 0.05$
CDF [384]	1 fb^{-1}	$0.39 \pm 0.17 \pm 0.04$	—	$1.30 \pm 0.24 \pm 0.12$	—
LHCb [385]	1 fb^{-1}	$0.14 \pm 0.03 \pm 0.01$	—	$1.01 \pm 0.04 \pm 0.01$	—
Average		0.19 ± 0.03	-0.11 ± 0.05	1.03 ± 0.03	1.10 ± 0.07
Confidence level		$0.09 (1.7\sigma)$	$0.75 (0.3\sigma)$	$0.33 (1.0\sigma)$	$0.66 (0.4\sigma)$
$D_{CP}^*K^-$					
<i>BABAR</i> [386]	$N(B\bar{B}) = 383\text{M}$	$-0.11 \pm 0.09 \pm 0.01$	$0.06 \pm 0.10 \pm 0.02$	$1.31 \pm 0.13 \pm 0.03$	$1.09 \pm 0.12 \pm 0.04$
<i>Belle</i> [387]	772M	$-0.14 \pm 0.10 \pm 0.01$	$0.22 \pm 0.11 \pm 0.01$	$1.19 \pm 0.13 \pm 0.03$	$1.03 \pm 0.13 \pm 0.03$
Average		-0.12 ± 0.07	0.13 ± 0.07	1.25 ± 0.09	1.06 ± 0.09
Confidence level		$0.82 (0.2\sigma)$	$0.29 (1.1\sigma)$	$0.52 (0.6\sigma)$	$0.74 (0.3\sigma)$
$D_{CP}K^{*-}$					
<i>BABAR</i> [388]	$N(B\bar{B}) = 379\text{M}$	$0.09 \pm 0.13 \pm 0.06$	$-0.23 \pm 0.21 \pm 0.07$	$2.17 \pm 0.35 \pm 0.09$	$1.03 \pm 0.27 \pm 0.13$
$D_{CP}K^-\pi^+\pi^-$					
LHCb [389]	1 fb^{-1}	$-0.14 \pm 0.10 \pm 0.01$		$0.95 \pm 0.11 \pm 0.02$	

LHCb have performed a GLW analysis using the $B^0 \rightarrow DK^{*0}$ decay with $D \rightarrow K^+K^-$ and $D \rightarrow \pi^+\pi^-$ channels [390]. The results are presented separately to allow for possible CP violation effects in the charm decays, which are, however, known to be small. The results are shown in Table 49 where an average is also given.

Table 49: Results from GLW analysis of $B^0 \rightarrow DK^{*0}$.

Experiment	Sample size	A_{CP+}	R_{CP+}
LHCb ($D \rightarrow K^+K^-$)	[390] 3 fb^{-1}	$-0.20 \pm 0.15 \pm 0.02$	$1.05^{+0.17}_{-0.15} \pm 0.04$
LHCb ($D \rightarrow \pi^+\pi^-$)	[390] 3 fb^{-1}	$-0.09 \pm 0.22 \pm 0.02$	$1.21^{+0.28}_{-0.25} \pm 0.05$
Average		-0.16 ± 0.12	1.10 ± 0.14

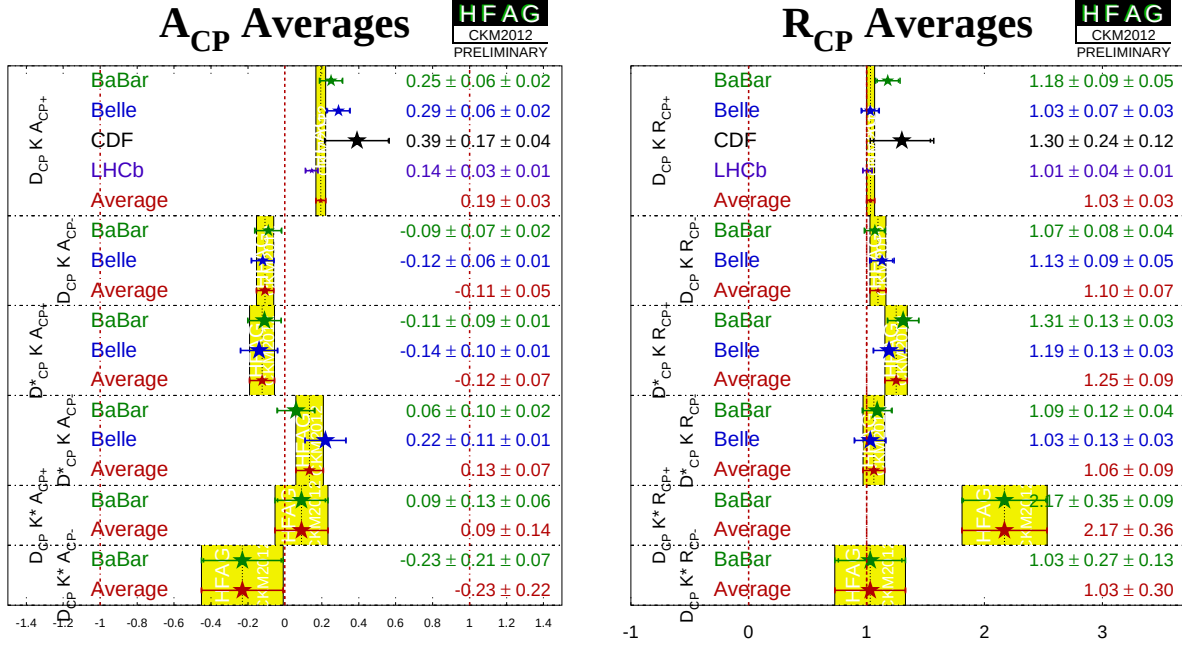


Figure 41: Averages of A_{CP} and R_{CP} from GLW analyses.

4.14.2 D decays to suppressed final states

For ADS analyses, all of *BABAR*, Belle, CDF and LHCb have studied the modes $B^\mp \rightarrow DK^\mp$ and $B^\mp \rightarrow D\pi^\mp$. *BABAR* and Belle have also analysed the $B^\mp \rightarrow D^*K^\mp$ mode. There is an effective shift of π in the strong phase difference between the cases that the D^* is reconstructed as $D\pi^0$ and $D\gamma$ [280], therefore these modes are studied separately. *BABAR* has also studied the $B^\mp \rightarrow DK^{*\mp}$ mode, where $K^{*\mp}$ is reconstructed as $K_s^0\pi^\mp$. In all cases the suppressed decay $D \rightarrow K^+\pi^-\pi^0$ has been used. *BABAR* and Belle also have results using $B^\mp \rightarrow DK^\mp$ with $D \rightarrow K^+\pi^-\pi^0$, while LHCb have results using $B^\mp \rightarrow DK^\mp$ with $D \rightarrow K^+\pi^-\pi^+\pi^-$. The results and averages are given in Table 50 and shown in Fig. 42.

Similar phenomenology as for $B \rightarrow DK$ decays holds for $B \rightarrow D\pi$ decays, though in this case the interference is between $b \rightarrow c\bar{u}d$ and $b \rightarrow u\bar{c}d$ transitions, and the ratio of suppressed to favoured amplitudes is expected to be much smaller, $\mathcal{O}(1\%)$. For most D meson final states this implies that the interference effect is too small to be of interest, but in the case of ADS analysis it is possible that effects due to γ may be observable. Accordingly, the experiments now measure the corresponding observables in the $D\pi$ final states. The results and averages are given in Table 51 and shown in Fig. 43.

BABAR [397], Belle [398] and LHCb [390] have also presented results from a similar analysis method with self-tagging neutral B decays: $B^0 \rightarrow DK^{*0}$ with $D \rightarrow K^-\pi^+$ (all), $D \rightarrow K^-\pi^+\pi^0$ and $D \rightarrow K^-\pi^+\pi^+\pi^-$ (*BABAR* only) (all with $K^{*0} \rightarrow K^+\pi^-$). Effects due to the natural width of the K^{*0} are handled using the parametrisation suggested by Gronau [283].

The following 95% C.L. limits are set by *BABAR*:

$$R_{\text{ADS}}(K\pi) < 0.244 \quad R_{\text{ADS}}(K\pi\pi^0) < 0.181 \quad R_{\text{ADS}}(K\pi\pi\pi) < 0.391, \quad (157)$$

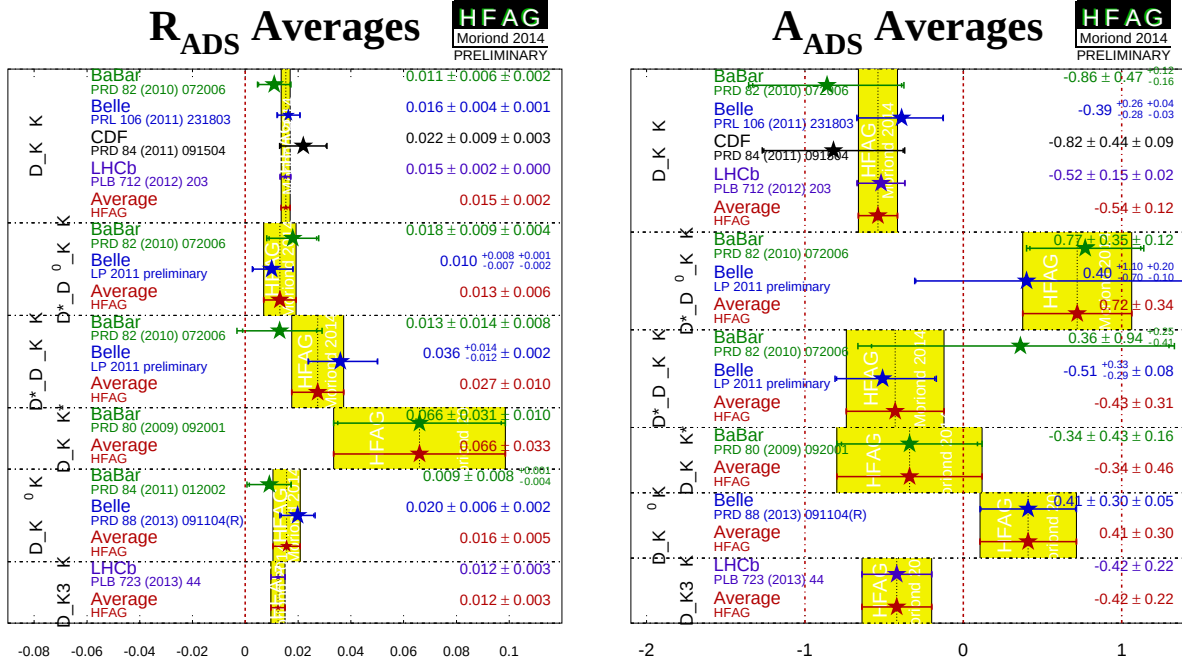


Figure 42: Averages of R_{ADS} and A_{ADS} for $B \rightarrow D^{(*)}K^{(*)}$ decays.

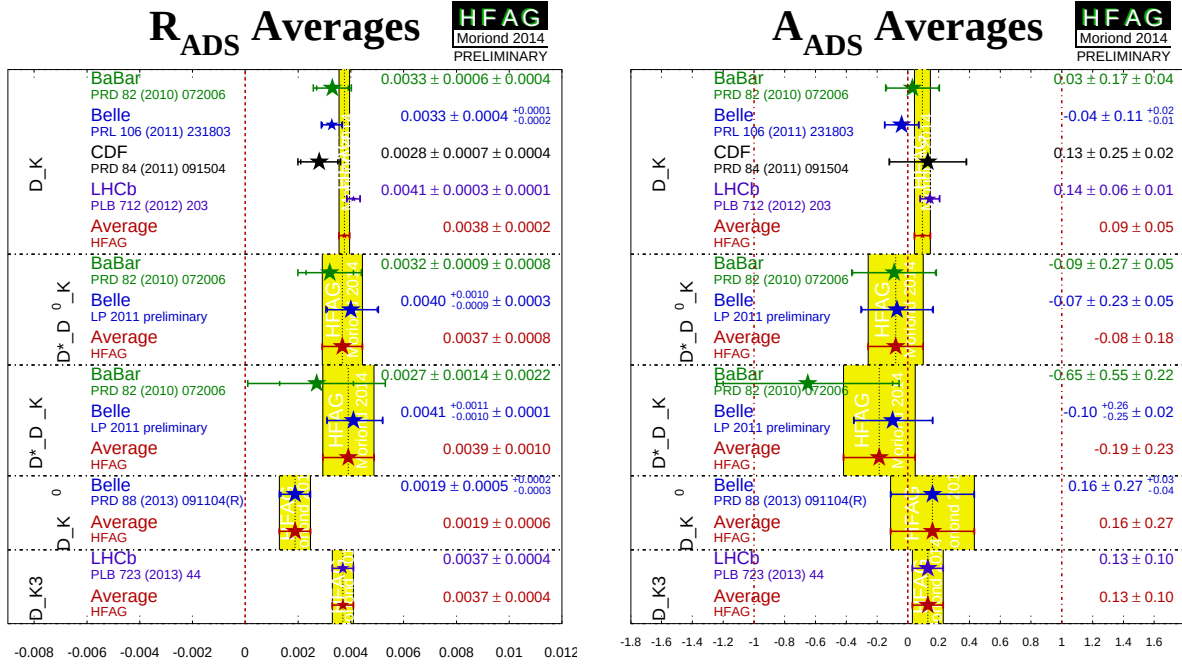


Figure 43: Averages of R_{ADS} and A_{ADS} for $B \rightarrow D^{(*)}\pi$ decays.

Table 50: Averages from ADS analyses of $b \rightarrow c\bar{u}s/u\bar{c}s$ modes.

Experiment	Sample size	A_{ADS}	R_{ADS}	
$DK^-, D \rightarrow K^+\pi^-$				
<i>BABAR</i>	[391]	$N(B\bar{B}) = 467\text{M}$	$-0.86 \pm 0.47^{+0.12}_{-0.16}$	$0.011 \pm 0.006 \pm 0.002$
<i>Belle</i>	[392]	$N(B\bar{B}) = 772\text{M}$	$-0.39^{+0.26+0.04}_{-0.28-0.03}$	$0.0163^{+0.0044+0.0007}_{-0.0041-0.0013}$
CDF	[393]	7 fb^{-1}	$-0.82 \pm 0.44 \pm 0.09$	$0.0220 \pm 0.0086 \pm 0.0026$
LHCb	[385]	1 fb^{-1}	$-0.52 \pm 0.15 \pm 0.02$	$0.0152 \pm 0.0020 \pm 0.0004$
Average		-0.54 ± 0.12	0.0153 ± 0.0017	
Confidence level		$0.77 (0.3\sigma)$	$0.78 (0.3\sigma)$	
Experiment	$N(B\bar{B})$	A_{ADS}	R_{ADS}	
$D^*K^-, D^* \rightarrow D\pi^0, D \rightarrow K^+\pi^-$				
<i>BABAR</i>	[391]	467M	$0.77 \pm 0.35 \pm 0.12$	$0.018 \pm 0.009 \pm 0.004$
<i>Belle</i>	[383]	772M	$0.4^{+1.1+0.2}_{-0.7-0.1}$	$0.010^{+0.008+0.001}_{-0.007-0.002}$
Average		0.72 ± 0.34	0.013 ± 0.006	
Confidence level		$0.71 (0.4\sigma)$	$0.52 (0.6\sigma)$	
$D^*K^-, D^* \rightarrow D\gamma, D \rightarrow K^+\pi^-$				
<i>BABAR</i>	[391]	467M	$0.36 \pm 0.94^{+0.25}_{-0.41}$	$0.013 \pm 0.014 \pm 0.008$
<i>Belle</i>	[383]	772M	$-0.51^{+0.33}_{-0.29} \pm 0.08$	$0.036^{+0.014}_{-0.012} \pm 0.002$
Average		-0.43 ± 0.31	0.027 ± 0.010	
Confidence level		$0.42 (0.8\sigma)$	$0.26 (1.1\sigma)$	
$DK^{*-}, D \rightarrow K^+\pi^-, K^{*-} \rightarrow K^0_s\pi^-$				
<i>BABAR</i>	[388]	379M	$-0.34 \pm 0.43 \pm 0.16$	$0.066 \pm 0.031 \pm 0.010$
$DK^-, D \rightarrow K^+\pi^-\pi^0$				
<i>BABAR</i>	[394]	474M	–	$0.0091^{+0.0082+0.0014}_{-0.0076-0.0037}$
<i>Belle</i>	[395]	772M	$0.41 \pm 0.30 \pm 0.05$	$0.0198 \pm 0.0062 \pm 0.0024$
Average		–	0.0156 ± 0.0052	
Confidence level			$0.32 (1.0\sigma)$	
$DK^-, D \rightarrow K^+\pi^-\pi^+\pi^-$				
LHCb	[396]	1 fb^{-1}	-0.42 ± 0.22	0.0124 ± 0.0027

while Belle obtain

$$R_{\text{ADS}}(K\pi) < 0.16. \quad (158)$$

The results from LHCb, which are presented in terms of the parameters R_+ and R_- instead of R_{ADS} and A_{ADS} , are shown in Table 52.

Combining the results and using additional input from CLEOc [399,400] a limit on the ratio between the $b \rightarrow u$ and $b \rightarrow c$ amplitudes of $r_s \in [0.07, 0.41]$ at 95% C.L. limit is set by *BABAR*. Belle set a limit of $r_B(DK^{*0}) < 0.4$ at 95% C.L. LHCb take input from Sec. 8 and obtain $r_B(DK^{*0}) = 0.240^{+0.055}_{-0.048}$ (different from zero with 2.7σ significance).

Table 51: Averages from ADS analyses of $b \rightarrow c\bar{u}d/u\bar{c}d$ modes.

Experiment	Sample size	A_{ADS}	R_{ADS}
$D\pi^-, D \rightarrow K^+\pi^-$			
BABAR [391]	$N(B\bar{B}) = 467\text{M}$	$0.03 \pm 0.17 \pm 0.04$	$0.0033 \pm 0.0006 \pm 0.0004$
Belle [392]	$N(B\bar{B}) = 772\text{M}$	$-0.04 \pm 0.11^{+0.02}_{-0.01}$	$0.00328^{+0.00038+0.00012}_{-0.00036-0.00018}$
CDF [393]	7 fb^{-1}	$0.13 \pm 0.25 \pm 0.02$	$0.0028 \pm 0.0007 \pm 0.0004$
LHCb [385]	1 fb^{-1}	$0.143 \pm 0.062 \pm 0.011$	$0.00410 \pm 0.00025 \pm 0.00005$
Average		0.09 ± 0.05	0.00375 ± 0.00020
Confidence level		$0.53 (0.6\sigma)$	$0.17 (1.4\sigma)$
$D^*\pi^-, D^* \rightarrow D\pi^0, D \rightarrow K^+\pi^-$			
BABAR [391]	467M	$-0.09 \pm 0.27 \pm 0.05$	$0.0032 \pm 0.0009 \pm 0.0008$
Belle [383]	772M	$-0.07 \pm 0.23 \pm 0.05$	$0.0040^{+0.0010}_{-0.0009} \pm 0.0003$
Average		-0.08 ± 0.18	0.0037 ± 0.0008
Confidence level		$0.96 (0.1\sigma)$	$0.61 (0.5\sigma)$
$D^*\pi^-, D^* \rightarrow D\gamma, D \rightarrow K^+\pi^-$			
BABAR [391]	467M	$-0.65 \pm 0.55 \pm 0.22$	$0.0027 \pm 0.0014 \pm 0.0022$
Belle [383]	772M	$-0.10^{+0.26}_{-0.25} \pm 0.02$	$0.0041^{+0.0011}_{-0.0010} \pm 0.0001$
Average		-0.19 ± 0.23	0.0039 ± 0.0010
Confidence level		$0.39 (0.9\sigma)$	$0.62 (0.5\sigma)$
$DK^-, D \rightarrow K^+\pi^-\pi^0$			
Belle [395]	772M	$0.16 \pm 0.27^{+0.03}_{-0.04}$	$0.00189 \pm 0.00054^{+0.00022}_{-0.00025}$
$DK^-, D \rightarrow K^+\pi^-\pi^+\pi^-$			
LHCb [396]	1 fb^{-1}	0.13 ± 0.10	0.0037 ± 0.0004

Table 52: Results from ADS analysis of $B^0 \rightarrow DK^{*0}, D \rightarrow K^+\pi^-$.

Experiment	Sample size	R_+	R_-
LHCb [390]	3 fb^{-1}	$0.06 \pm 0.03 \pm 0.01$	$0.06 \pm 0.03 \pm 0.01$

4.14.3 D decays to multiparticle self-conjugate final states (model-dependent analysis)

For the model-dependent Dalitz plot analysis, both BABAR [401] and Belle [402,403] have studied the modes $B^\mp \rightarrow DK^\mp$, $B^\mp \rightarrow D^*K^\mp$ and $B^\mp \rightarrow DK^{*\mp}$. For $B^\mp \rightarrow D^*K^\mp$, both experiments have used both D^* decay modes, $D^* \rightarrow D\pi^0$ and $D^* \rightarrow D\gamma$, taking the effective shift in the strong phase difference into account. In all cases the decay $D \rightarrow K_s^0\pi^+\pi^-$ has been used. BABAR also used the decay $D \rightarrow K_s^0K^+K^-$. LHCb [404] has also studied $B^\mp \rightarrow DK^\mp$ decays with $D \rightarrow K_s^0\pi^+\pi^-$. BABAR has also performed an analysis of $B^\mp \rightarrow DK^\mp$ with $D \rightarrow \pi^+\pi^-\pi^0$ [288]. Results and averages are given in Table 53, and shown in Figs. 44 and 45. The third error on each measurement is due to D decay model uncertainty.

The parameters measured in the analyses are explained in Sec. 4.2.7. Both BABAR and Belle have measured the ‘‘Cartesian’’ ($x_\pm, y_\pm$) variables, defined in Sec. 4.2.7, and perform frequentist

statistical procedures, to convert these into measurements of γ , r_B and δ_B . In the $B^\mp \rightarrow DK^\mp$ with $D \rightarrow \pi^+\pi^-\pi^0$ analysis, the parameters $(\rho^\pm, \theta^\pm)$ are used instead.

Both experiments reconstruct $K^{*\mp}$ as $K_s^0\pi^\mp$, but the treatment of possible nonresonant $K_s^0\pi^\mp$ differs: Belle assign an additional model uncertainty, while *BABAR* use a parametrisation suggested by Gronau [283]. The parameters r_B and δ_B are replaced with effective parameters κr_s and δ_s ; no attempt is made to extract the true hadronic parameters of the $B^\mp \rightarrow DK^{*\mp}$ decay.

We perform averages using the following procedure, which is based on a set of reasonable, though imperfect, assumptions.

- It is assumed that effects due to the different D decay models used by the two experiments are negligible. Therefore, we do not rescale the results to a common model.
- It is further assumed that the model uncertainty is 100% correlated between experiments, and therefore this source of error is not used in the averaging procedure. (This approximation is significantly less valid now that the *BABAR* results include $D \rightarrow K_s^0 K^+ K^-$ decays in addition to $D \rightarrow K_s^0 \pi^+ \pi^-$.)
- We include in the average the effect of correlations within each experiment's set of measurements.
- At present it is unclear how to assign an average model uncertainty. We have not attempted to do so. Our average includes only statistical and systematic errors. An unknown amount of model uncertainty should be added to the final error.
- We follow the suggestion of Gronau [283] in making the DK^* averages. Explicitly, we assume that the selection of $K^{*\pm} \rightarrow K_s^0 \pi^\pm$ is the same in both experiments (so that κ , r_s and δ_s are the same), and drop the additional source of model uncertainty assigned by Belle due to possible nonresonant decays.
- We do not consider common systematic errors, other than the D decay model.

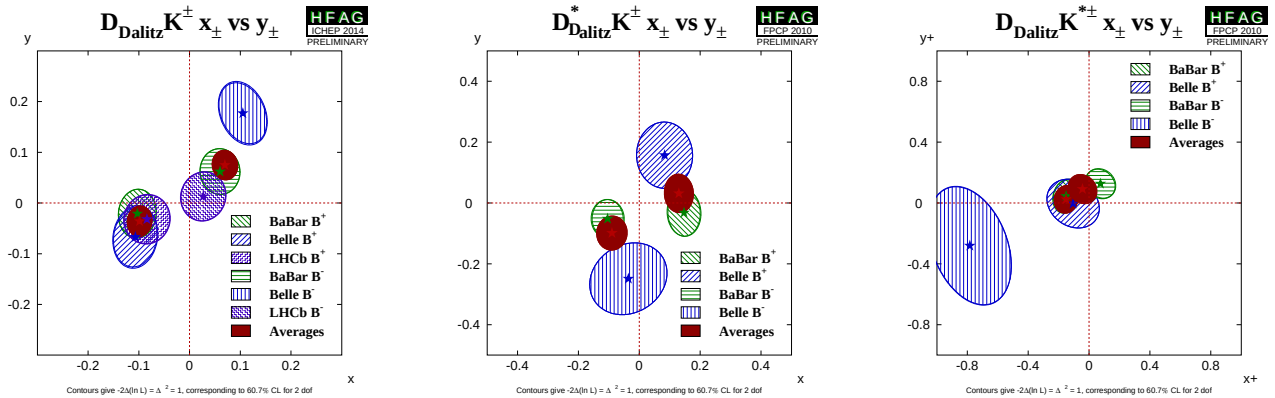


Figure 44: Contours in the $(x_\pm, y_\pm)$ from model-dependent analysis of $B^\mp \rightarrow D^{(*)}K^{(*)\pm}$, $D \rightarrow K_s^0 h^+ h^-$ ($h = \pi, K$). (Left) $B^\mp \rightarrow DK^\mp$, (middle) $B^\mp \rightarrow D^*K^\mp$, (right) $B^\mp \rightarrow DK^{*\mp}$. Note that the uncertainties assigned to the averages given in these plots do not include model errors.

Table 53: Averages from Dalitz plot analyses of $b \rightarrow \bar{c}us/\bar{u}cs$ modes. Note that the uncertainties assigned to the averages do not include model errors.

Experiment	$N(BB)$	x_+	y_+	x_-	y_-
$DK^-, D \rightarrow K_s^0 \pi^+ \pi^-$					
BABAR	[405]	468M	$-0.103 \pm 0.037 \pm 0.006 \pm 0.007$	$-0.021 \pm 0.048 \pm 0.004 \pm 0.009$	$0.060 \pm 0.039 \pm 0.007 \pm 0.006$
Belle	[402]	657M	$-0.107 \pm 0.043 \pm 0.011 \pm 0.055$	$-0.067 \pm 0.059 \pm 0.018 \pm 0.063$	$0.105 \pm 0.047 \pm 0.011 \pm 0.064$
LHCb	[404]	1 fb $^{-1}$	$-0.084 \pm 0.045 \pm 0.009 \pm 0.005$	$-0.032 \pm 0.048^{+0.010}_{-0.009} \pm 0.008$	$0.027 \pm 0.044^{+0.010}_{-0.008} \pm 0.001$
Average			-0.098 ± 0.024	-0.036 ± 0.030	0.070 ± 0.025
Confidence level				0.52 (0.7 σ)	0.075 $\pm$ 0.029
$D^* K^-, D^* \rightarrow D\pi^0$ or $D\gamma, D \rightarrow K_s^0 \pi^+ \pi^-$					
BABAR	[405]	468M	$0.147 \pm 0.053 \pm 0.017 \pm 0.003$	$-0.032 \pm 0.077 \pm 0.008 \pm 0.006$	$-0.104 \pm 0.051 \pm 0.019 \pm 0.002$
Belle	[402]	657M	$0.083 \pm 0.092 \pm 0.081$	$0.157 \pm 0.109 \pm 0.063$	$-0.036 \pm 0.127 \pm 0.090$
Average			0.130 ± 0.048	0.031 ± 0.063	-0.090 ± 0.050
Confidence level				0.29 (1.1 σ)	-0.099 ± 0.056
$DK^{*-}, D \rightarrow K_s^0 \pi^+ \pi^-$					
BABAR	[405]	468M	$-0.151 \pm 0.083 \pm 0.029 \pm 0.006$	$0.045 \pm 0.106 \pm 0.036 \pm 0.008$	$0.075 \pm 0.096 \pm 0.029 \pm 0.007$
Belle	[403]	386M	$-0.105^{+0.177}_{-0.167} \pm 0.006 \pm 0.088$	$-0.004^{+0.164}_{-0.156} \pm 0.013 \pm 0.095$	$-0.784^{+0.249}_{-0.295} \pm 0.029 \pm 0.097$
Average			-0.152 ± 0.077	0.024 ± 0.091	-0.043 ± 0.094
Confidence level				0.011 (2.5 σ)	0.127 $\pm$ 0.095 $\pm$ 0.027 $\pm$ 0.006 $-0.281^{+0.440}_{-0.335} \pm 0.046 \pm 0.086$ 0.091 $\pm$ 0.096
θ^+, θ^-					
Experiment	$N(BB)$	ρ^+	θ^+	ρ^-	θ^-
BABAR	[288]	324M	$0.75 \pm 0.11 \pm 0.04$	$147 \pm 23 \pm 1$	$0.72 \pm 0.11 \pm 0.04$
					173 $\pm$ 42 $\pm$ 2

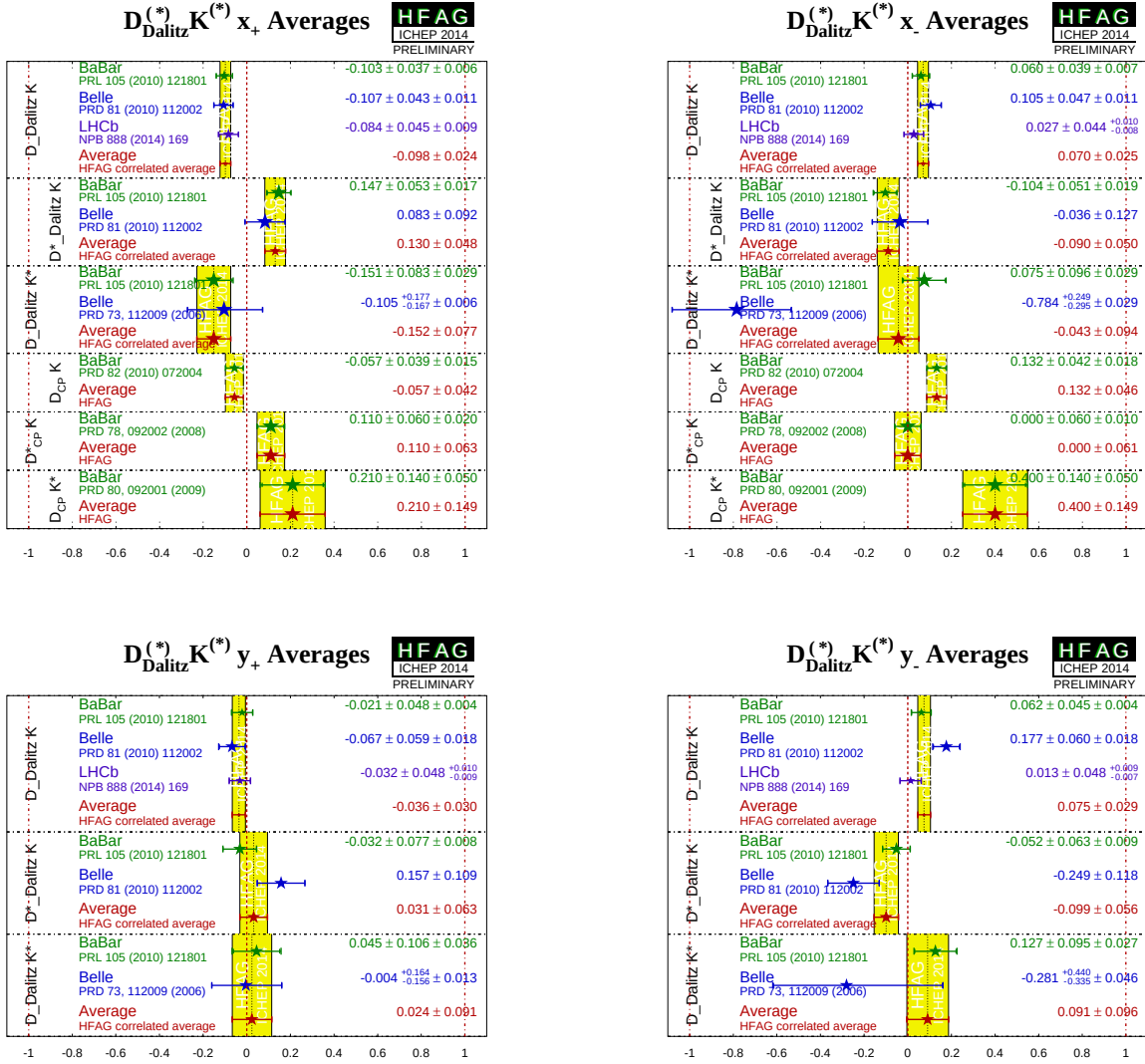


Figure 45: Averages of $(x_{\pm}, y_{\pm})$ from model-dependent analyses of $B^{\pm} \rightarrow D^{(*)}K^{(*)\pm}$ with $D \rightarrow K_S^0 h^+ h^-$ ($h = \pi, K$). (Top left) x_+ , (top right) x_- , (bottom left) y_+ , (bottom right) y_- . The top plots include constraints on $x_{\pm}$ obtained from GLW analyses (see Sec. 4.14.1). Note that the uncertainties assigned to the averages given in these plots do not include model errors.

Constraints on γ

The measurements of $(x_{\pm}, y_{\pm})$ can be used to obtain constraints on γ , as well as the hadronic parameters r_B and δ_B . BABAR [405], Belle [402, 403] and LHCb [404] have all done so using a frequentist procedure (there are some differences in the details of the techniques used).

- BABAR obtain $\gamma = (68^{+15}_{-14} \pm 4 \pm 3)^{\circ}$ from $DK^{\pm}$, $D^*K^{\pm}$ and $DK^{*\pm}$.
- Belle obtain $\phi_3 = (78^{+11}_{-12} \pm 4 \pm 9)^{\circ}$ from $DK^{\pm}$ and $D^*K^{\pm}$.
- LHCb obtain $\gamma = (84^{+49}_{-42})^{\circ}$ from $DK^{\pm}$ using 1 fb^{-1} of data (a more precise result using 3 fb^{-1} and the model-independent method is reported below).
- The experiments also obtain values for the hadronic parameters as detailed in Table 54.

Table 54: Summary of constraints on hadronic parameters in $B^\pm \rightarrow D^{(*)}K^{(*)\pm}$ decays. Note the alternative parametrisation of the hadronic parameters used by *BABAR* in the $DK^{*\pm}$ mode.

	r_B	δ_B
In $DK^\pm$		
<i>BABAR</i>	$0.096 \pm 0.029 \pm 0.005 \pm 0.004$	$(119^{+19}_{-20} \pm 3 \pm 3)^\circ$
Belle	$0.160^{+0.040}_{-0.038} \pm 0.011^{+0.05}_{-0.010}$	$(138^{+13}_{-16} \pm 4 \pm 23)^\circ$
LHCb	0.06 ± 0.04	$(115^{+41}_{-51})^\circ$
In $D^*K^\pm$		
<i>BABAR</i>	$0.133^{+0.042}_{-0.039} \pm 0.014 \pm 0.003$	$(-82 \pm 21 \pm 5 \pm 3)^\circ$
Belle	$0.196^{+0.072}_{-0.069} \pm 0.012^{+0.062}_{-0.012}$	$(342^{+19}_{-21} \pm 3 \pm 23)^\circ$
In $DK^{*\pm}$		
<i>BABAR</i>	$\kappa r_S = 0.149^{+0.066}_{-0.062} \pm 0.026 \pm 0.006$	$\delta_S = (111 \pm 32 \pm 11 \pm 3)^\circ$
Belle	$0.56^{+0.22}_{-0.16} \pm 0.04 \pm 0.08$	$(243^{+20}_{-23} \pm 3 \pm 50)^\circ$

- The CKMfitter [254] and UTFit [302] groups use the measurements from Belle and *BABAR* given above to make combined constraints on γ .
- In the *BABAR* analysis of $B^\mp \rightarrow DK^\mp$ with $D \rightarrow \pi^+\pi^-\pi^0$ [288], a constraint of $-30^\circ < \gamma < 76^\circ$ is obtained at the 68% confidence level.

At present we make no attempt to provide an HFAG average for γ , nor indeed for the hadronic parameters. More details on procedures to calculate a best fit value for γ can be found in Refs. [254, 302].

BABAR [406] have also performed a similar Dalitz plot analysis to that described above using the self-tagging neutral B decay $B^0 \rightarrow DK^{*0}$ (with $K^{*0} \rightarrow K^+\pi^-$). Effects due to the natural width of the K^{*0} are handled using the parametrisation suggested by Gronau [283].

BABAR extract the three-dimensional likelihood for the parameters (γ, δ_S, r_S) and, combining with a separately measured PDF for r_S (using a Bayesian technique), obtain bounds on each of the three parameters.

$$\gamma = (162 \pm 56)^\circ \quad \delta_S = (62 \pm 57)^\circ \quad r_S < 0.55, \quad (159)$$

where the limit on r_S is at 95% probability. Note that there is an ambiguity in the solutions $(\gamma, \delta_S \leftrightarrow \gamma + \pi, \delta_S + \pi)$.

4.14.4 D decays to multiparticle self-conjugate final states (model-independent analysis)

A model-independent approach to the analysis of $B^- \rightarrow D^{(*)}K^-$ with multibody D decays was proposed by Giri, Grossman, Soffer and Zupan [278], and further developed by Bondar and Poluektov [286, 287]. The method relies on information on the average strong phase difference between D^0 and $\bar{D}^0$ decays in bins of Dalitz plot position that can be obtained from quantum-correlated $\psi(3770) \rightarrow D^0\bar{D}^0$ events. This information is measured in the form of parameters c_i and s_i that are the amplitude weighted averages of the cosine and sine of the strong phase difference in a Dalitz plot bin labelled by i , respectively. These quantities have been obtained

for $D \rightarrow K_s^0 \pi^+ \pi^-$ (and $D \rightarrow K_s^0 K^+ K^-$) by CLEOc [407,408]. (Preliminary results from BESIII are also available.)

Belle [409] and LHCb [410,411] have used the model-independent Dalitz plot analysis approach to study the mode $B^\mp \rightarrow DK^\mp$ with $D \rightarrow K_s^0 \pi^+ \pi^-$. LHCb have also included the $D \rightarrow K_s^0 K^+ K^-$ decay. The variables $(x_\pm, y_\pm)$, defined in Sec. 4.2.7, are determined from the data. Note that due to the strong statistical and systematic correlations with the model-dependent results given in Sec. 4.14.3, these results cannot be combined.

The results and averages are shown in Table 55, and shown in Figs. 46. The results have three sets of errors, which are statistical, systematic, and uncertainty coming from the knowledge of c_i and s_i respectively. To perform the average, we remove the last uncertainty, which should be 100% correlated between the measurements. Since the size of the uncertainty from c_i and s_i is found to depend on the size of the $B \rightarrow DK$ data sample, we assign the LHCb uncertainties (which are mostly the smaller of the Belle and LHCb values) to the averaged result. This procedure should be conservative.

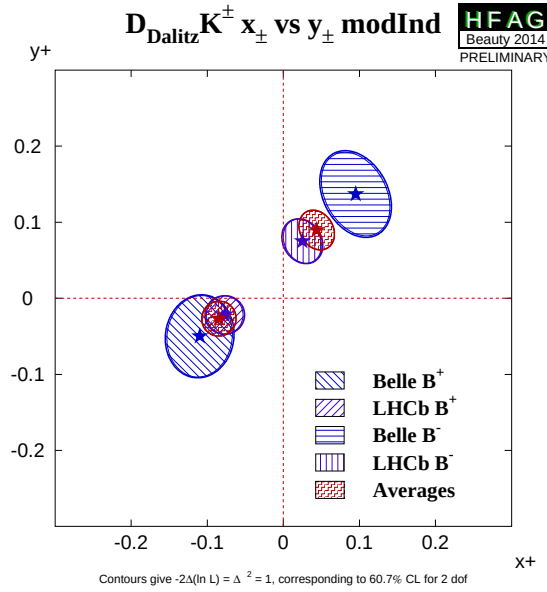


Figure 46: Contours in the $(x_\pm, y_\pm)$ plane from model-independent analysis of $B^\mp \rightarrow DK^\mp$, $D \rightarrow K_s^0 h^+ h^-$ ($h = \pi, K$). Note that the uncertainties assigned to the averages given in these plots do not include model errors.

Constraints on γ

The measurements of $(x_\pm, y_\pm)$ can be used to obtain constraints on γ , as well as the hadronic parameters r_B and δ_B . Belle [409] and LHCb [410,411] have done so using a frequentist procedure (there are some differences in the details of the techniques used).

- Belle obtain $\phi_3 = (77.3^{+15.1}_{-14.9} \pm 4.1 \pm 4.3)^\circ$.
- LHCb obtain $\gamma = (62^{+15}_{-14})^\circ$.
- The experiments also obtain values for the hadronic parameters as detailed in Table 57.

Table 55: Averages from model-independent Dalitz plot analyses of $b \rightarrow c\bar{u}s/u\bar{c}s$ modes.

Experiment	Sample size	x_+	y_+	x_-	y_-
$DK^-, D \rightarrow K_s^0 \pi^+ \pi^-$					
Belle [409]	772M	$-0.110 \pm 0.043 \pm 0.014 \pm 0.007$	$-0.050^{+0.052}_{-0.055} \pm 0.011 \pm 0.007$	$0.095 \pm 0.045 \pm 0.014 \pm 0.010$	$0.137^{+0.053}_{-0.057} \pm 0.015 \pm 0.023$
LHCb [411]	3 fb ⁻¹	$-0.077 \pm 0.024 \pm 0.010 \pm 0.004$	$-0.022 \pm 0.025 \pm 0.004 \pm 0.010$	$0.025 \pm 0.025 \pm 0.010 \pm 0.005$	$0.075 \pm 0.029 \pm 0.005 \pm 0.014$
Average		$-0.085 \pm 0.023 \pm 0.04$	$-0.027 \pm 0.023 \pm 0.010$	$0.044 \pm 0.023 \pm 0.005$	$0.090 \pm 0.026 \pm 0.014$
Confidence level			0.39 (0.9 σ)		

Table 56: Results from model-independent Dalitz plot analysis of $B^- \rightarrow DK^-$, $D \rightarrow K_s^0 K^\pm \pi^\mp$.

Experiment	Sample size	R_{SS}	R_{OS}	$A_{SS,D\pi}$ $D \rightarrow K_s^0 K^\pm \pi^\mp$ (whole Dalitz plot)	$A_{OS,D\pi}$	$A_{OS,D\pi}$
LHCb [412]	3 fb^{-1}	$0.092 \pm 0.009 \pm 0.004$	$0.066 \pm 0.009 \pm 0.002$	$0.040 \pm 0.091 \pm 0.018$	$0.233 \pm 0.129 \pm 0.024$	$-0.025 \pm 0.024 \pm 0.010$
LHCb [412]	3 fb^{-1}	$0.084 \pm 0.011 \pm 0.003$	$0.056 \pm 0.013 \pm 0.002$	$0.026 \pm 0.109 \pm 0.029$	$0.336 \pm 0.208 \pm 0.026$	$-0.012 \pm 0.028 \pm 0.010$
				$D \rightarrow K^*(892)^\pm \pi^\mp$		$-0.052 \pm 0.029 \pm 0.017$
						$-0.054 \pm 0.043 \pm 0.017$

Table 57: Summary of constraints on hadronic parameters from model-independent analyses of $B^\pm \rightarrow DK^\pm$, $D \rightarrow K_s^0 h^+ h^-$ ($h = \pi, K$) decays.

	r_B	δ_B
Belle	$0.145 \pm 0.030 \pm 0.010 \pm 0.011$	$(129.9 \pm 15.0 \pm 3.8 \pm 4.7)^\circ$
LHCb	$0.080^{+0.019}_{-0.021}$	$(134^{+14}_{-15})^\circ$

At present we make no attempt to provide an HFAG average for γ , nor indeed for the hadronic parameters. More details on procedures to calculate a best fit value for γ can be found in Refs. [254, 302].

4.14.5 D decays to multiparticle non-self-conjugate final states (model-independent analysis)

Following the original suggestion of Grossman, Ligeti and Soffer [282], decays of D mesons to $K_s^0 K^\pm \pi^\mp$ can be used in a similar approach to that discussed above to determine $\gamma \equiv \phi_3$. Since these decays are less abundant, the event samples available to date have not been sufficient for a fine binning of the Dalitz plots, but the analysis can be performed using only an overall coherence factor and related strong phase difference for the decay. These quantities have been determined by CLEOc [413] both for the full Dalitz plots and in a restricted region $\pm 100 \text{ MeV}/c^2$ around the peak of the $K^*(892)^\pm$ resonance.

LHCb [412] have reported results of an analysis of $B^- \rightarrow DK^-$ and $B^- \rightarrow D\pi^-$ decays with $D \rightarrow K_s^0 K^\pm \pi^\mp$. The decays with different final states of the D meson are distinguished by the charge of the kaon from the decay of the D meson relative to the charge of the B meson, and are labelled “same sign” (SS) and “opposite sign” (OS). Six observables potentially sensitive to $\gamma \equiv \phi_3$ are measured: two ratios of rates for DK and $D\pi$ decays (one each for SS and OS) and four asymmetries (for DK & $D\pi$, SS & OS). This is done both for the full Dalitz plot and for the $K^*(892)^\pm$ -dominated region (with the same boundaries as used by CLEOc). Note that there is a significant overlap of events between the two samples. The results, shown in Table 56 do not yet have sufficient precision to set significant constraints on $\gamma \equiv \phi_3$.

4.14.6 Combinations of results on rates and asymmetries in $B^\mp \rightarrow D^{(*)} K^{(*)\mp}$ decays to obtain constraints on $\gamma \equiv \phi_3$

BABAR, Belle and LHCb have all presented constraints on $\gamma \equiv \phi_3$ from combinations of their results on $B^- \rightarrow DK^-$ and related processes. All use a frequentist procedure (there are some differences in the details of the techniques used).

- *BABAR* [414] use results from DK , D^*K and DK^* modes with GLW, ADS and GGSZ analyses, to obtain $\gamma = (69^{+17}_{-16})^\circ$.
- Belle [387] use results from DK and D^*K modes with GLW, ADS and GGSZ analyses, to obtain $\phi_3 = (68^{+15}_{-14})^\circ$.
- LHCb [415] use results from the DK mode with GLW, ADS (both $K\pi$ and $K3\pi$), GGSZ ($K_s^0 h^+ h^-$) and GLS ($K_s^0 K^\pm \pi^\mp$) analyses, as well as DK^{*0} with GLW and ADS analyses and $B_s^0 \rightarrow D_s^\mp K^\pm$ decays. LHCb have in addition obtained a constraint (not quoted here)

Table 58: Summary of constraints on hadronic parameters obtained from global combinations of results in $B^\pm \rightarrow D^{(*)}K^{(*)\pm}$ decays.

	r_B	δ_B
<i>BABAR</i>	$0.092^{+0.013}_{-0.012}$	$(105^{+16}_{-17})^\circ$
<i>Belle</i>	$0.112^{+0.014}_{-0.015}$	$(116^{+18}_{-21})^\circ$
<i>LHCb</i>	$0.0914^{+0.0083}_{-0.0088}$	$(127^{+10}_{-12})^\circ$

including results from $B \rightarrow D\pi$. The LHCb combination takes into account subleading effects due to charm mixing and CP violation [416]. The result is $\gamma = (73^{+9}_{-10})^\circ$.

- All the combinations use inputs from CLEOc (and/or from the HFAG - Charm Physics global fits on charm mixing parameters, see Sec. 8.1) to constrain the hadronic parameters in the charm system.
- Constraints are also obtained on the hadronic parameters involved in the decays. A summary of these is given in Table 58.
- The CKMfitter [254] and UTfit [302] groups perform similar combinations of all available results to make combined constraints on γ .

5 Semileptonic B decays

This section contains our averages for semileptonic B meson decays, *i.e.* decays of the type $B \rightarrow X \ell \nu_\ell$, where X is a hadronic system, ℓ a charged lepton and ν_ℓ its corresponding neutrino. Unless otherwise stated, ℓ stands for an electron *or* a muon, lepton universality is assumed, and both charge conjugate states are included. Some averages assume isospin symmetry and this will be explicitly mentioned at every instance.

The averages are organized by the flavour changing transition (the CKM-favoured $b \rightarrow c$ transition and the CKM-suppressed $b \rightarrow u$ transition) and by the experimental definition of the hadronic X system. Measurements which are sensitive to only a specific hadronic state ($X = D, D^*, \pi, \dots$) are called *exclusive* while analyses that measure all hadronic states within a given region of phase space are *inclusive*. The principal reason why semileptonic B decays are studied in experiments is the determination of the CKM matrix element magnitudes $|V_{cb}|$ and $|V_{ub}|$. The averages in the different subsections thus focus very much on these two fundamental parameters of the Standard Model. In the last subsection, we discuss semileptonic B decays with a τ -lepton. These decays are relevant to the search for physics beyond the Standard Model, *e.g.* in the context of the type II Two-Higgs-Doublet-Model (2HDM).

The technique for obtaining the averages follows the general HFAG procedure (Sec. 2) unless otherwise stated. More information on the averages, in particular the common input parameters is available on the HFAG semileptonic webpage:

<http://www.slac.stanford.edu/xorg/hfag/semi/pdg14/>

5.1 Exclusive CKM-favoured decays

This section is organized as follows: First, we present averages for the decays $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$ and $\bar{B} \rightarrow D \ell^- \bar{\nu}_\ell$. In addition to the branching fractions, the CKM element $|V_{cb}|$ is extracted. We then provide averages for the inclusive branching fractions $\mathcal{B}(\bar{B} \rightarrow D^{(*)} \pi \ell^- \bar{\nu}_\ell)$ and for B semileptonic decays into orbitally-excited P -wave charm mesons (D^{**}). As the D^{**} branching fraction is poorly known, we report the averages for the products $\mathcal{B}(B^- \rightarrow D^{**} (D^{(*)} \pi) \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D^{**} \rightarrow D^{(*)} \pi)$.

5.1.1 $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$

The kinematics of the decay $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$ are described by the form factor $\eta_{EW} \mathcal{F}(w)$, where η_{EW} is a known electro-weak correction factor and w is the product of the B and D^* meson 4-velocities, $w = v_B \cdot v_{D^*}$. Experiments measure the differential decay width as a function of w and determine the form factor $\eta_{EW} \mathcal{F}(w)$ in the parameterization of Caprini, Lellouch and Neubert (CLN), which describes the shape and normalization in terms of four quantities: the normalization $\eta_{EW} \mathcal{F}(1)$, the slope ρ^2 , and the amplitude ratios $R_1(1)$ and $R_2(1)$ [417]. Our main average and the determination of $|V_{cb}|$ are based on this parameterization.

We use the measurements of these form factor parameters shown in Table 59 and rescale them to the latest values of the input parameters (mainly branching fractions of charmed mesons) [418]. Most of the measurements in Table 59 are based exclusively on the decay $\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$. Some measurements [419, 420] are sensitive also to $B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell$ and one measurement [421] is based exclusively on the decay $B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell$. Our analysis thus assumes isospin symmetry.

Table 59: Measurements of the Caprini, Lellouch and Neubert (CLN) [417] form factor parameters in $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$ before and after rescaling. Most analyses (except [422,423]) measure only $\eta_{\text{EW}} \mathcal{F}(1)|V_{cb}|$, and ρ^2 , so only these two parameters are shown here. The average is the result of a 4-dimensional fit to the rescaled measurements of $\eta_{\text{EW}} \mathcal{F}(1)|V_{cb}|$, ρ^2 , $R_1(1)$ and $R_2(1)$ – see the text for more details. The χ^2 value of the combination is 30.0 for 23 degrees of freedom (CL=15.0%).

Experiment	$\eta_{\text{EW}} \mathcal{F}(1) V_{cb} [10^{-3}]$ (rescaled) $\eta_{\text{EW}} \mathcal{F}(1) V_{cb} [10^{-3}]$ (published)	ρ^2 (rescaled) ρ^2 (published)
ALEPH [424]	$31.23 \pm 1.80_{\text{stat}} \pm 1.30_{\text{syst}}$ $31.9 \pm 1.8_{\text{stat}} \pm 1.9_{\text{syst}}$	$0.493 \pm 0.228_{\text{stat}} \pm 0.144_{\text{syst}}$ $0.37 \pm 0.26_{\text{stat}} \pm 0.14_{\text{syst}}$
CLEO [419]	$39.94 \pm 1.23_{\text{stat}} \pm 1.62_{\text{syst}}$ $43.1 \pm 1.3_{\text{stat}} \pm 1.8_{\text{syst}}$	$1.367 \pm 0.085_{\text{stat}} \pm 0.086_{\text{syst}}$ $1.61 \pm 0.09_{\text{stat}} \pm 0.21_{\text{syst}}$
OPAL excl [425]	$36.50 \pm 1.60_{\text{stat}} \pm 1.49_{\text{syst}}$ $36.8 \pm 1.6_{\text{stat}} \pm 2.0_{\text{syst}}$	$1.234 \pm 0.212_{\text{stat}} \pm 0.145_{\text{syst}}$ $1.31 \pm 0.21_{\text{stat}} \pm 0.16_{\text{syst}}$
OPAL partial reco [425]	$37.14 \pm 1.19_{\text{stat}} \pm 2.36_{\text{syst}}$ $37.5 \pm 1.2_{\text{stat}} \pm 2.5_{\text{syst}}$	$1.152 \pm 0.145_{\text{stat}} \pm 0.294_{\text{syst}}$ $1.12 \pm 0.14_{\text{stat}} \pm 0.29_{\text{syst}}$
DELPHI partial reco [426]	$35.32 \pm 1.40_{\text{stat}} \pm 2.33_{\text{syst}}$ $35.5 \pm 1.4_{\text{stat}}^{+2.3}_{-2.4_{\text{syst}}}$	$1.174 \pm 0.126_{\text{stat}} \pm 0.377_{\text{syst}}$ $1.34 \pm 0.14_{\text{stat}}^{+0.24}_{-0.22_{\text{syst}}}$
DELPHI excl [427]	$36.10 \pm 1.70_{\text{stat}} \pm 1.97_{\text{syst}}$ $39.2 \pm 1.8_{\text{stat}} \pm 2.3_{\text{syst}}$	$1.081 \pm 0.142_{\text{stat}} \pm 0.152_{\text{syst}}$ $1.32 \pm 0.15_{\text{stat}} \pm 0.33_{\text{syst}}$
Belle [422]	$34.60 \pm 0.17_{\text{stat}} \pm 1.02_{\text{syst}}$ $34.6 \pm 0.2_{\text{stat}} \pm 1.0_{\text{syst}}$	$1.212 \pm 0.034_{\text{stat}} \pm 0.009_{\text{syst}}$ $1.214 \pm 0.034_{\text{stat}} \pm 0.009_{\text{syst}}$
<i>BABAR</i> excl [423]	$33.94 \pm 0.30_{\text{stat}} \pm 0.99_{\text{syst}}$ $34.7 \pm 0.3_{\text{stat}} \pm 1.1_{\text{syst}}$	$1.185 \pm 0.048_{\text{stat}} \pm 0.029_{\text{syst}}$ $1.18 \pm 0.05_{\text{stat}} \pm 0.03_{\text{syst}}$
<i>BABAR</i> D^{*0} [421]	$35.22 \pm 0.59_{\text{stat}} \pm 1.33_{\text{syst}}$ $35.9 \pm 0.6_{\text{stat}} \pm 1.4_{\text{syst}}$	$1.128 \pm 0.058_{\text{stat}} \pm 0.055_{\text{syst}}$ $1.16 \pm 0.06_{\text{stat}} \pm 0.08_{\text{syst}}$
<i>BABAR</i> global fit [420]	$35.76 \pm 0.20_{\text{stat}} \pm 1.10_{\text{syst}}$ $35.7 \pm 0.2_{\text{stat}} \pm 1.2_{\text{syst}}$	$1.193 \pm 0.020_{\text{stat}} \pm 0.061_{\text{syst}}$ $1.21 \pm 0.02_{\text{stat}} \pm 0.07_{\text{syst}}$
Average	$35.81 \pm 0.11_{\text{stat}} \pm 0.44_{\text{syst}}$	$1.207 \pm 0.015_{\text{stat}} \pm 0.021_{\text{syst}}$

In the next step, we perform a four-dimensional fit of the parameters $\eta_{\text{EW}} \mathcal{F}(1)|V_{cb}|$, ρ^2 , $R_1(1)$ and $R_2(1)$ using the rescaled measurements and taking into account correlated statistical and systematic uncertainties. Only two measurements constrain all four parameters [422,423], the remaining measurements determine only the normalization $\eta_{\text{EW}} \mathcal{F}(1)|V_{cb}|$ and the slope ρ^2 . The result of the fit is

$$\eta_{\text{EW}} \mathcal{F}(1)|V_{cb}| = (35.81 \pm 0.45) \times 10^{-3} , \quad (160)$$

$$\rho^2 = 1.207 \pm 0.026 , \quad (161)$$

$$R_1(1) = 1.406 \pm 0.033 , \quad (162)$$

$$R_2(1) = 0.853 \pm 0.020 , \quad (163)$$

and the correlation coefficients are

$$\rho_{\eta_{EW}\mathcal{F}(1)|V_{cb}|,\rho^2} = 0.323 , \quad (164)$$

$$\rho_{\eta_{EW}\mathcal{F}(1)|V_{cb}|,R_1(1)} = -0.108 , \quad (165)$$

$$\rho_{\eta_{EW}\mathcal{F}(1)|V_{cb}|,R_2(1)} = -0.063 , \quad (166)$$

$$\rho_{\rho^2,R_1(1)} = 0.568 , \quad (167)$$

$$\rho_{\rho^2,R_2(1)} = -0.809 , \quad (168)$$

$$\rho_{R_1(1),R_2(1)} = -0.758 . \quad (169)$$

The uncertainties and correlations quoted here include both statistical and systematic contributions. The χ^2 of the fit is 30.0 for 23 degrees of freedom, which corresponds to a confidence level of 15.0%. An illustration of this fit result is given in Fig. 47.

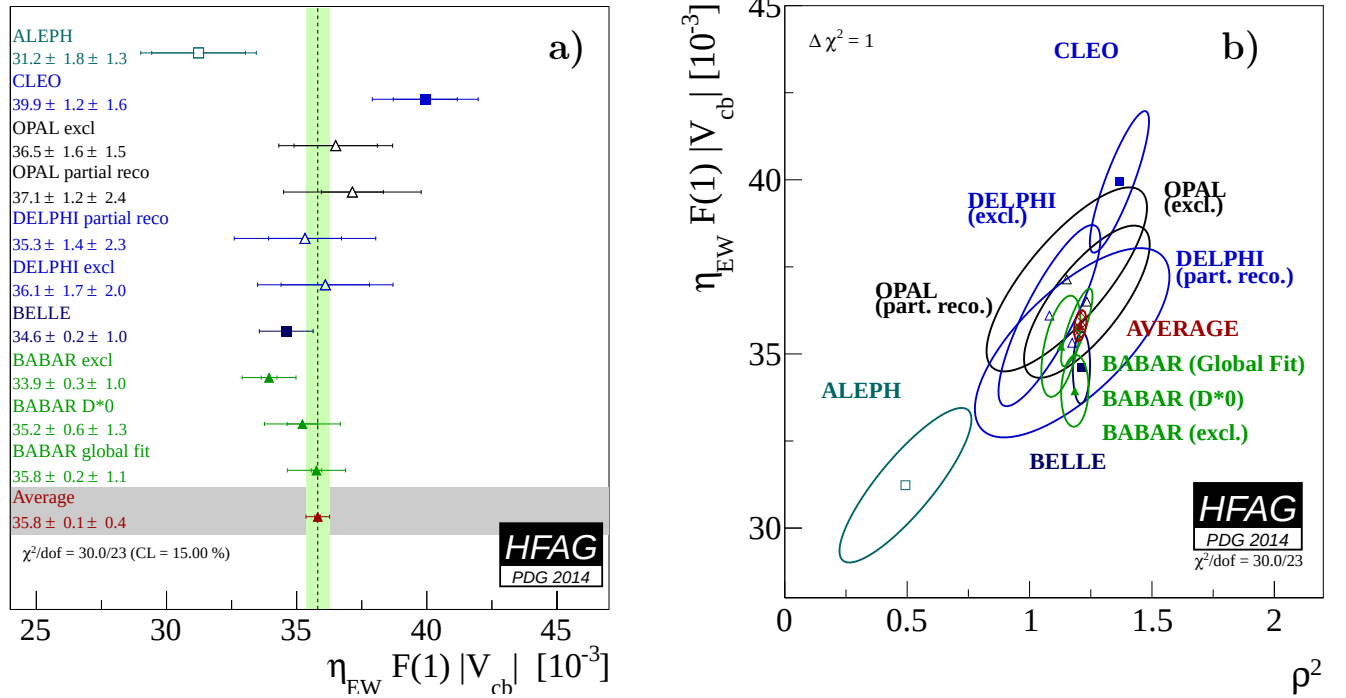


Figure 47: (a) Illustration of the $\eta_{EW}\mathcal{F}(1)|V_{cb}|$ average. (b) Illustration of the $\eta_{EW}\mathcal{F}(1)|V_{cb}|$ vs. ρ^2 average. The error ellipses correspond to $\Delta\chi^2 = 1$ (CL=39%).

Using the latest update from the FNAL/MILC group [428], the form factor normalization $\eta_{EW}\mathcal{F}(1)$ is

$$\eta_{EW}\mathcal{F}(1) = 0.920 \pm 0.014 , \quad (170)$$

which results in the following determination of $|V_{cb}|$ from Eq. 160,

$$|V_{cb}| = (38.94 \pm 0.49_{\text{exp}} \pm 0.58_{\text{th}}) \times 10^{-3} , \quad (171)$$

where the first uncertainty is experimental and the second error is theoretical (lattice QCD calculation and electro-weak correction).

From each rescaled measurement in Table 59, we calculate the $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$ form factor $\eta_{\text{EW}} \mathcal{F}(w)$ and, by numerical integration, the branching ratio of the decay $\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$. For measurements which do not determine the parameters $R_1(1)$ and $R_2(1)$ we assume the average values Eqs. 162 and 163. The results are quoted in Table 60. The branching ratio found for the average values of $\eta_{\text{EW}} \mathcal{F}(1)|V_{cb}|$, ρ^2 , $R_1(1)$ and $R_2(1)$ is

$$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}) = (4.93 \pm 0.11)\% . \quad (172)$$

To test isospin symmetry, we have performed a simple 1-dimensional average of measurements sensitive to the decay $B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell$ only, which is shown in Table 61. Fig. 48 illustrates our two averages of $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$.

Table 60: $\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}$ branching fractions calculated from the rescaled CLN parameters in Table 59. For Ref. [421] the published value of $\mathcal{B}(B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell)$ has been rescaled by the factor $\tau(B^0)/\tau(B^+)$ for comparison to the other measurements.

Experiment	$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu})$ [%] (calculated)	$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu})$ [%] (published)
ALEPH [424]	$5.35 \pm 0.25_{\text{stat}} \pm 0.31_{\text{syst}}$	$5.53 \pm 0.26_{\text{stat}} \pm 0.52_{\text{syst}}$
CLEO [419]	$5.62 \pm 0.18_{\text{stat}} \pm 0.26_{\text{syst}}$	$6.09 \pm 0.19_{\text{stat}} \pm 0.40_{\text{syst}}$
OPAL excl [425]	$5.05 \pm 0.19_{\text{stat}} \pm 0.42_{\text{syst}}$	$5.11 \pm 0.19_{\text{stat}} \pm 0.49_{\text{syst}}$
OPAL partial reco [425]	$5.46 \pm 0.25_{\text{stat}} \pm 0.52_{\text{syst}}$	$5.92 \pm 0.27_{\text{stat}} \pm 0.68_{\text{syst}}$
DELPHI partial reco [426]	$4.88 \pm 0.13_{\text{stat}} \pm 0.72_{\text{syst}}$	$4.70 \pm 0.13_{\text{stat}}^{+0.36}_{-0.31} \text{ syst}$
DELPHI excl [427]	$5.35 \pm 0.20_{\text{stat}} \pm 0.37_{\text{syst}}$	$5.90 \pm 0.22_{\text{stat}} \pm 0.50_{\text{syst}}$
Belle [422]	$4.56 \pm 0.03_{\text{stat}} \pm 0.26_{\text{syst}}$	$4.58 \pm 0.03_{\text{stat}} \pm 0.26_{\text{syst}}$
BABAR excl [423]	$4.54 \pm 0.04_{\text{stat}} \pm 0.25_{\text{syst}}$	$4.69 \pm 0.04_{\text{stat}} \pm 0.34_{\text{syst}}$
BABAR D^{*0} [421]	$4.97 \pm 0.07_{\text{stat}} \pm 0.34_{\text{syst}}$	$5.15 \pm 0.07_{\text{stat}} \pm 0.38_{\text{syst}}$
BABAR global fit [420]	$4.95 \pm 0.02_{\text{stat}} \pm 0.20_{\text{syst}}$	$5.00 \pm 0.02_{\text{stat}} \pm 0.19_{\text{syst}}$
Average	$4.93 \pm 0.01_{\text{stat}} \pm 0.11_{\text{syst}}$	$\chi^2/\text{dof} = 30.0/23$ (CL=15.0%)

Table 61: Average of the $B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell$ branching fraction measurements. This fit uses only measurements of $B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell$.

Experiment	$\mathcal{B}(B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell)$ [%] (rescaled)	$\mathcal{B}(B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell)$ [%] (published)
CLEO [419]	$6.60 \pm 0.20_{\text{stat}} \pm 0.39_{\text{syst}}$	$6.50 \pm 0.20_{\text{stat}} \pm 0.43_{\text{syst}}$
BABAR tagged [429]	$5.74 \pm 0.15_{\text{stat}} \pm 0.30_{\text{syst}}$	$5.83 \pm 0.15_{\text{stat}} \pm 0.30_{\text{syst}}$
BABAR [421]	$5.36 \pm 0.08_{\text{stat}} \pm 0.40_{\text{syst}}$	$5.56 \pm 0.08_{\text{stat}} \pm 0.41_{\text{syst}}$
BABAR [420]	$5.42 \pm 0.02_{\text{stat}} \pm 0.21_{\text{syst}}$	$5.40 \pm 0.02_{\text{stat}} \pm 0.21_{\text{syst}}$
Average	$5.69 \pm 0.02_{\text{stat}} \pm 0.19_{\text{syst}}$	$\chi^2/\text{dof} = 9.0/3$ (CL=2.94%)

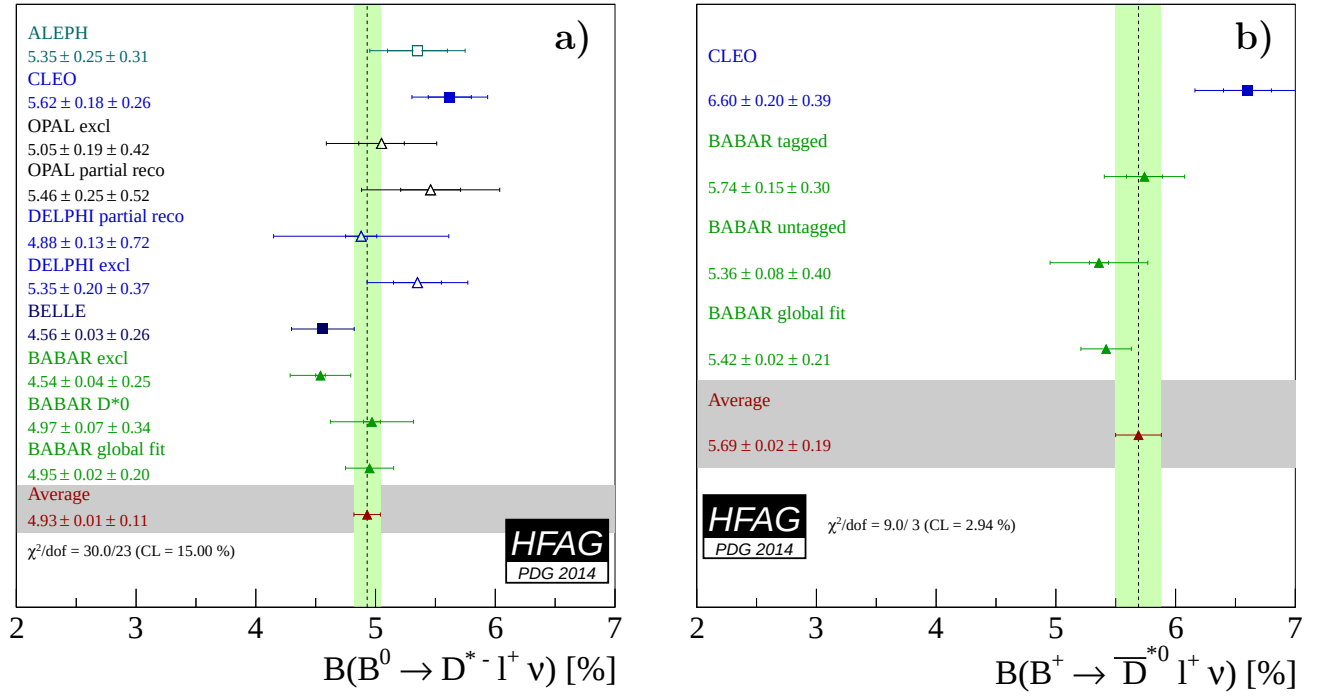


Figure 48: (a) Average branching fractions of exclusive semileptonic B decays $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$: (a) $\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}$ (Table 60) and (b) $B^- \rightarrow D^{*0} \ell^- \bar{\nu}_\ell$ (Table 61).

5.1.2 $\bar{B} \rightarrow D \ell^- \bar{\nu}_\ell$

The relevant form factor for the decay $\bar{B} \rightarrow D \ell^- \bar{\nu}_\ell$ is $\eta_{\text{EW}} \mathcal{G}(w)$, which in CLN [417] is described by only two parameters: the normalization $\eta_{\text{EW}} \mathcal{G}(1) |V_{cb}|$ and the slope ρ^2 .

Experiments measure the differential decay width as a function of w and determine these two form factor parameters. We use the analyses shown in Table 62 and correct them to match the latest values of the input parameters [418]. These measurements are sensitive to both isospin states ($\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}$ and $B^- \rightarrow D^0 \ell^- \bar{\nu}_\ell$). So, isospin symmetry is assumed in the analysis.

The form factor parameters are extracted by a two-dimensional fit to the rescaled measurements of $\eta_{\text{EW}} \mathcal{G}(1) |V_{cb}|$ and ρ^2 taking into account correlated statistical and systematic uncertainties. The result of the fit reads

$$\eta_{\text{EW}} \mathcal{G}(1) |V_{cb}| = (42.65 \pm 1.53) \times 10^{-3}, \quad (173)$$

$$\rho^2 = 1.185 \pm 0.054, \quad (174)$$

with a correlation of

$$\rho_{\eta_{\text{EW}} \mathcal{G}(1) |V_{cb}|, \rho^2} = 0.824. \quad (175)$$

The uncertainties and the correlation coefficient include both statistical and systematic contributions. The χ^2 of the fit is 0.5 for 8 degrees of freedom, which corresponds to a confidence level of 100.0%. An illustration of this fit result is given in Fig. 49.

The most recent lattice QCD result obtained for the form factor normalization $\eta_{\text{EW}} \mathcal{G}(1)$ is [433]

$$\eta_{\text{EW}} \mathcal{G}(1) = 1.081 \pm 0.024, \quad (176)$$

Table 62: Measurements of the Caprini, Lellouch and Neubert (CLN) [417] form factor parameters in $\bar{B} \rightarrow D\ell^-\bar{\nu}_\ell$ before and after rescaling. The average is the result of a 2-dimensional fit to the rescaled measurements of $\eta_{EW}\mathcal{G}(1)|V_{cb}|$ and ρ^2 – see the text for more details. The χ^2 value of the combination is 0.5 for 8 degrees of freedom (CL=100.0%).

Experiment	$\mathcal{G}(1) V_{cb} $ [10^{-3}] (rescaled) $\mathcal{G}(1) V_{cb} $ [10^{-3}] (published)	ρ^2 (rescaled) ρ^2 (published)
ALEPH [424]	$38.66 \pm 11.80_{\text{stat}} \pm 5.19_{\text{syst}}$ $31.1 \pm 9.9_{\text{stat}} \pm 8.6_{\text{syst}}$	$0.942 \pm 0.980_{\text{stat}} \pm 0.272_{\text{syst}}$ $0.70 \pm 0.98_{\text{stat}} \pm 0.50_{\text{syst}}$
CLEO [430]	$44.88 \pm 5.96_{\text{stat}} \pm 3.25_{\text{syst}}$ $44.8 \pm 6.1_{\text{stat}} \pm 3.7_{\text{syst}}$	$1.270 \pm 0.220_{\text{stat}} \pm 0.119_{\text{syst}}$ $1.30 \pm 0.27_{\text{stat}} \pm 0.14_{\text{syst}}$
Belle [431]	$40.96 \pm 4.39_{\text{stat}} \pm 5.03_{\text{syst}}$ $41.1 \pm 4.4_{\text{stat}} \pm 5.1_{\text{syst}}$	$1.120 \pm 0.190_{\text{stat}} \pm 0.111_{\text{syst}}$ $1.12 \pm 0.22_{\text{stat}} \pm 0.14_{\text{syst}}$
BABAR global fit [420]	$43.25 \pm 0.80_{\text{stat}} \pm 2.07_{\text{syst}}$ $43.1 \pm 0.8_{\text{stat}} \pm 2.3_{\text{syst}}$	$1.201 \pm 0.040_{\text{stat}} \pm 0.057_{\text{syst}}$ $1.20 \pm 0.04_{\text{stat}} \pm 0.07_{\text{syst}}$
BABAR tagged [432]	$42.54 \pm 1.88_{\text{stat}} \pm 1.02_{\text{syst}}$ $42.3 \pm 1.9_{\text{stat}} \pm 1.0_{\text{syst}}$	$1.177 \pm 0.088_{\text{stat}} \pm 0.057_{\text{syst}}$ $1.20 \pm 0.09_{\text{stat}} \pm 0.04_{\text{syst}}$
Average	$42.65 \pm 0.72_{\text{stat}} \pm 1.35_{\text{syst}}$	$1.185 \pm 0.035_{\text{stat}} \pm 0.041_{\text{syst}}$

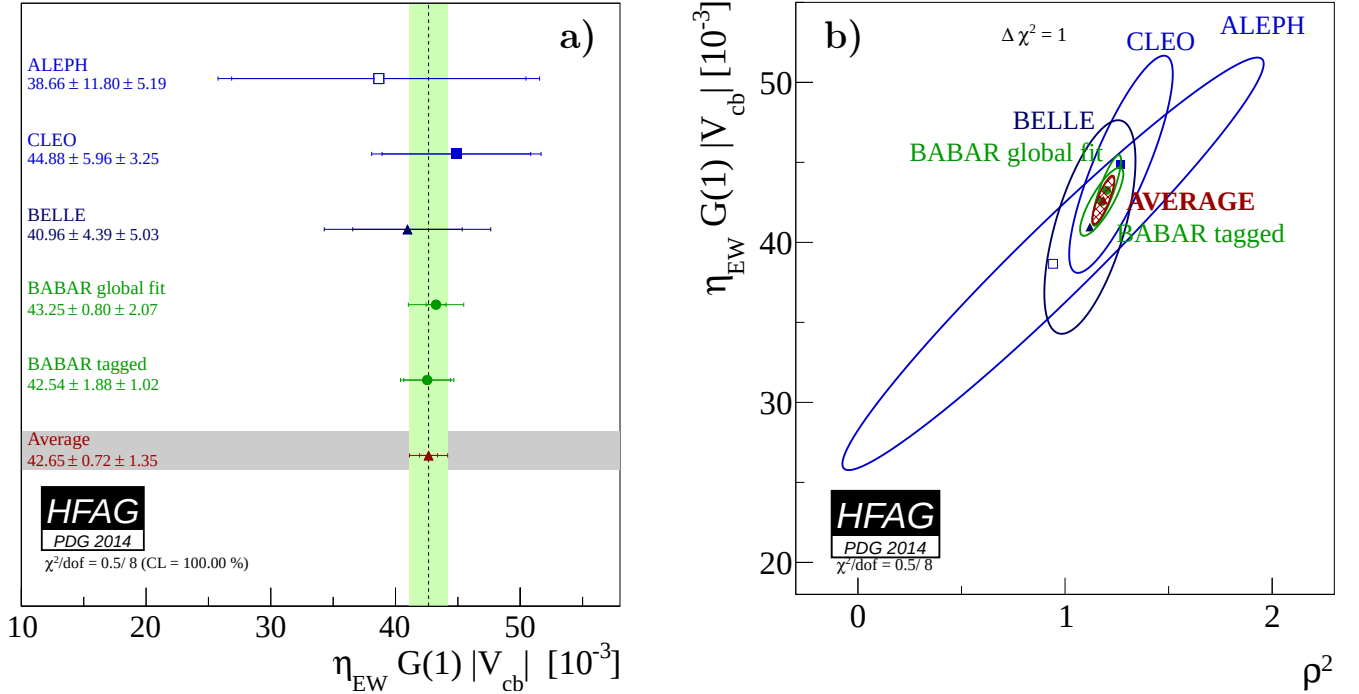


Figure 49: (a) Illustration of the $\eta_{EW}\mathcal{G}(1)|V_{cb}|$ average. (b) Illustration of the $\eta_{EW}\mathcal{G}(1)|V_{cb}|$ vs. ρ^2 average. The error ellipses correspond to $\Delta\chi^2 = 1$ (CL=39%).

which can be used to turn Eq. 173 into a determination of $|V_{cb}|$,

$$|V_{cb}| = (39.45 \pm 1.42_{\text{exp}} \pm 0.88_{\text{th}}) \times 10^{-3} , \quad (177)$$

where the first error is experimental and the second theoretical. This number is in excellent agreement with $|V_{cb}|$ obtained from decays $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$ (Eq. 171).

From each rescaled measurement in Table 62, we have calculated the $\bar{B} \rightarrow D \ell^- \bar{\nu}_\ell$ form factor $\mathcal{G}(w)$ and, by numerical integration, the branching ratio of the decay $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}$. The results are quoted in Table 63 and illustrated in Fig. 50. The branching ratio found for the average values of $\eta_{\text{EW}} \mathcal{G}(1) |V_{cb}|$ and ρ^2 is

$$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}) = (2.13 \pm 0.09)\% . \quad (178)$$

Table 63: $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell$ branching fractions calculated from the rescaled CLN parameters in Table 62.

Experiment	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell) [\%]$ (calculated)	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}_\ell) [\%]$ (published)
ALEPH [424]	$2.15 \pm 0.18_{\text{stat}} \pm 0.45_{\text{syst}}$	$2.35 \pm 0.20_{\text{stat}} \pm 0.44_{\text{syst}}$
CLEO [430]	$2.19 \pm 0.16_{\text{stat}} \pm 0.35_{\text{syst}}$	$2.20 \pm 0.16_{\text{stat}} \pm 0.19_{\text{syst}}$
Belle [431]	$2.08 \pm 0.12_{\text{stat}} \pm 0.52_{\text{syst}}$	$2.13 \pm 0.12_{\text{stat}} \pm 0.39_{\text{syst}}$
BABAR global fit [420]	$2.16 \pm 0.03_{\text{stat}} \pm 0.13_{\text{syst}}$	$2.34 \pm 0.03_{\text{stat}} \pm 0.13_{\text{syst}}$
BABAR tagged [432]	$2.14 \pm 0.11_{\text{stat}} \pm 0.08_{\text{syst}}$	$2.23 \pm 0.11_{\text{stat}} \pm 0.11_{\text{syst}}$
Average	$2.13 \pm 0.03_{\text{stat}} \pm 0.09_{\text{syst}}$	$\chi^2/\text{dof} = 0.5/8$ (CL=100.0%)

We have also performed simple 1-dimensional averages of measurements of $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}$ and $B^- \rightarrow D^0 \ell^- \bar{\nu}_\ell$. These fits are shown Tables 64 and 65.

Table 64: Average of $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}$ branching fraction measurements. This fit uses only measurements of the neutral mode.

Experiment	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}) [\%]$ (rescaled)	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}) [\%]$ (published)
ALEPH [424]	$2.28 \pm 0.18_{\text{stat}} \pm 0.35_{\text{syst}}$	$2.35 \pm 0.20_{\text{stat}} \pm 0.44_{\text{syst}}$
CLEO [430]	$2.13 \pm 0.13_{\text{stat}} \pm 0.15_{\text{syst}}$	$2.20 \pm 0.16_{\text{stat}} \pm 0.19_{\text{syst}}$
Belle [431]	$2.11 \pm 0.12_{\text{stat}} \pm 0.39_{\text{syst}}$	$2.13 \pm 0.12_{\text{stat}} \pm 0.39_{\text{syst}}$
BABAR [429]	$2.22 \pm 0.11_{\text{stat}} \pm 0.12_{\text{syst}}$	$2.21 \pm 0.11_{\text{stat}} \pm 0.12_{\text{syst}}$
Average	$2.19 \pm 0.06_{\text{stat}} \pm 0.10_{\text{syst}}$	$\chi^2/\text{dof} = 0.2/3$ (CL=97.3%)

5.1.3 $\bar{B} \rightarrow D^{(*)} \pi \ell^- \bar{\nu}_\ell$

The average inclusive branching fractions for $\bar{B} \rightarrow D^* \pi \ell^- \bar{\nu}_\ell$ decays, where no constraint is applied to the hadronic $D^{(*)} \pi$ system, are determined by the combination of the results provided

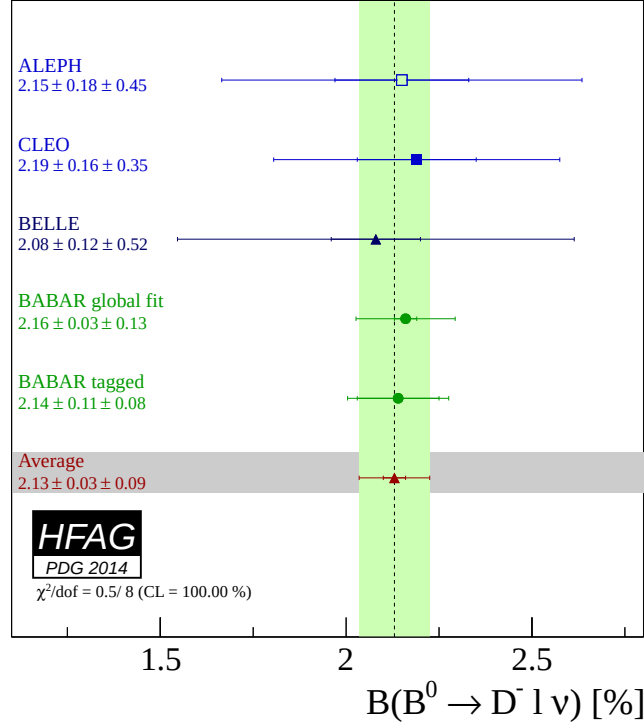


Figure 50: Illustration of Table 63.

Table 65: Average of $B^- \rightarrow D^0 \ell^- \bar{\nu}_\ell$ branching fraction measurements. This fit uses only measurements of the charged mode.

Experiment	$\mathcal{B}(B^- \rightarrow D^0 \ell^- \bar{\nu}_\ell)$ [%] (rescaled)	$\mathcal{B}(B^- \rightarrow D^0 \ell^- \bar{\nu}_\ell)$ [%] (published)
CLEO [430]	$2.20 \pm 0.13_{\text{stat}} \pm 0.17_{\text{syst}}$	$2.32 \pm 0.17_{\text{stat}} \pm 0.20_{\text{syst}}$
BABAR [429]	$2.29 \pm 0.09_{\text{stat}} \pm 0.09_{\text{syst}}$	$2.33 \pm 0.09_{\text{stat}} \pm 0.09_{\text{syst}}$
Average	$2.27 \pm 0.07_{\text{stat}} \pm 0.08_{\text{syst}}$	$\chi^2/\text{dof} = 0.1/1$ (CL=72.1%)

in Table 66 for $\bar{B}^0 \rightarrow D^0 \pi^+ \ell^- \bar{\nu}_\ell$, $\bar{B}^0 \rightarrow D^{*0} \pi^+ \ell^- \bar{\nu}_\ell$, $B^- \rightarrow D^+ \pi^- \ell^- \bar{\nu}_\ell$, and $B^- \rightarrow D^{*+} \pi^- \ell^- \bar{\nu}_\ell$. The measurements included in the average are scaled to a consistent set of input parameters and their errors [418]. For both the BABAR and Belle results, the B semileptonic signal yields are extracted from a fit to the missing mass squared in a sample of fully reconstructed $B\bar{B}$ events. Figure 51 illustrates the measurements and the resulting average.

5.1.4 $\bar{B} \rightarrow D^{**} \ell^- \bar{\nu}_\ell$

The D^{**} mesons contain one charm quark and one light quark with relative angular momentum $L = 1$. According to Heavy Quark Symmetry (HQS) [435], they form one doublet of states with angular momentum $j \equiv s_q + L = 3/2$ [$D_1(2420), D_2^*(2460)$] and another doublet with $j = 1/2$ [$D_0^*(2400), D_1'(2430)$], where s_q is the light quark spin. Parity and angular momentum conservation constrain the decays allowed for each state. The D_1 and D_2^* states decay through

Table 66: Average of the branching fraction $B \rightarrow D^{(*)}\pi^-\ell^-\bar{\nu}_\ell$ and individual results.

Experiment	$\mathcal{B}(B^- \rightarrow D^+\pi^-\ell^-\bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(B^- \rightarrow D^+\pi^-\ell^-\bar{\nu}_\ell)[\%]$ (published)
Belle [434]	$0.42 \pm 0.04_{\text{stat}} \pm 0.05_{\text{syst}}$	$0.40 \pm 0.04_{\text{stat}} \pm 0.06_{\text{syst}}$
BABAR [429]	$0.41 \pm 0.06_{\text{stat}} \pm 0.03_{\text{syst}}$	$0.42 \pm 0.06_{\text{stat}} \pm 0.03_{\text{syst}}$
Average	0.42 ± 0.05	$\chi^2/\text{dof} = 0.01$ (CL=90%)
Experiment	$\mathcal{B}(B^- \rightarrow D^{*+}\pi^-\ell^-\bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(B^- \rightarrow D^{*+}\pi^-\ell^-\bar{\nu}_\ell)[\%]$ (published)
Belle [434]	$0.68 \pm 0.08_{\text{stat}} \pm 0.07_{\text{syst}}$	$0.64 \pm 0.08_{\text{stat}} \pm 0.09_{\text{syst}}$
BABAR [429]	$0.57 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}$	$0.59 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}$
Average	0.60 ± 0.06	$\chi^2/\text{dof} = 0.5$ (CL=52%)
Experiment	$\mathcal{B}(\bar{B}^0 \rightarrow D^0\pi^+\ell^-\bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(\bar{B}^0 \rightarrow D^0\pi^+\ell^-\bar{\nu}_\ell)[\%]$ (published)
Belle [434]	$0.43 \pm 0.07_{\text{stat}} \pm 0.05_{\text{syst}}$	$0.42 \pm 0.07_{\text{stat}} \pm 0.06_{\text{syst}}$
BABAR [429]	$0.41 \pm 0.08_{\text{stat}} \pm 0.03_{\text{syst}}$	$0.43 \pm 0.08_{\text{stat}} \pm 0.03_{\text{syst}}$
Average	0.42 ± 0.06	$\chi^2/\text{dof} = 0.04$ (CL=85%)
Experiment	$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}\pi^+\ell^-\bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}\pi^+\ell^-\bar{\nu}_\ell)[\%]$ (published)
Belle [434]	$0.58 \pm 0.21_{\text{stat}} \pm 0.07_{\text{syst}}$	$0.56 \pm 0.21_{\text{stat}} \pm 0.08_{\text{syst}}$
BABAR [429]	$0.46 \pm 0.08_{\text{stat}} \pm 0.04_{\text{syst}}$	$0.48 \pm 0.08_{\text{stat}} \pm 0.04_{\text{syst}}$
Average	0.48 ± 0.08	$\chi^2/\text{dof} = 0.25$ (CL=62%)

a D-wave to $D^*\pi$ and $D^{(*)}\pi$, respectively, and have small decay widths, while the D_0^* and D_1' states decay through an S-wave to $D\pi$ and $D^*\pi$ and are very broad. For the narrow states, the average are determined by the combination of the results provided in Table 67 and 68 for $\mathcal{B}(B^- \rightarrow D_1^0\ell^-\bar{\nu}_\ell) \times \mathcal{B}(D_1^0 \rightarrow D^{*+}\pi^-)$ and $\mathcal{B}(B^- \rightarrow D_2^0\ell^-\bar{\nu}_\ell) \times \mathcal{B}(D_2^0 \rightarrow D^{*+}\pi^-)$. For the broad states, the average are determined by the combination of the results provided in Table 69 and 70 for $\mathcal{B}(B^- \rightarrow D_1^0\ell^-\bar{\nu}_\ell) \times \mathcal{B}(D_1^0 \rightarrow D^{*+}\pi^-)$ and $\mathcal{B}(B^- \rightarrow D_0^{*0}\ell^-\bar{\nu}_\ell) \times \mathcal{B}(D_0^{*0} \rightarrow D^+\pi^-)$. The measurements included in the average are scaled to a consistent set of input parameters and their errors [418].

For both the B-factory and the LEP and Tevatron results, the B semileptonic signal yields are extracted from a fit to the invariant mass distribution of the $D^{(*)+}\pi^-$ system. Apart for the CLEO, Belle and BABAR results, the other measurements are for the $\bar{B} \rightarrow D^{**}(D^*\pi^-)X\ell^-\bar{\nu}_\ell$ final state and we assume that no particles are left in the X system. The BABAR tagged measurement [436] measures $\bar{B} \rightarrow D_2^*(D\pi)X\ell^-\bar{\nu}_\ell$ and it has been translated in a result on $D_2^* \rightarrow D^*\pi$ decay mode, assuming $\mathcal{B}(D_2^* \rightarrow D\pi)/\mathcal{B}(D_2^* \rightarrow D^*\pi) = 1.54 \pm 0.15$ [5]. Figure 52 and 53 illustrate the measurements and the resulting average.

5.2 Inclusive CKM-favored decays

5.2.1 Global analysis of $\bar{B} \rightarrow X_c\ell^-\bar{\nu}_\ell$

The semileptonic width $\Gamma(\bar{B} \rightarrow X_c\ell^-\bar{\nu}_\ell)$ has been calculated in the framework of the Operator Product Expansion [55]. The result is a double-expansion in Λ_{QCD}/m_b and α_s , which depends on a number of non-perturbative parameters. These parameters give information on the dynamics of the b -quark inside the B hadron and can be measured using other observables in $\bar{B} \rightarrow X_c\ell^-\bar{\nu}_\ell$

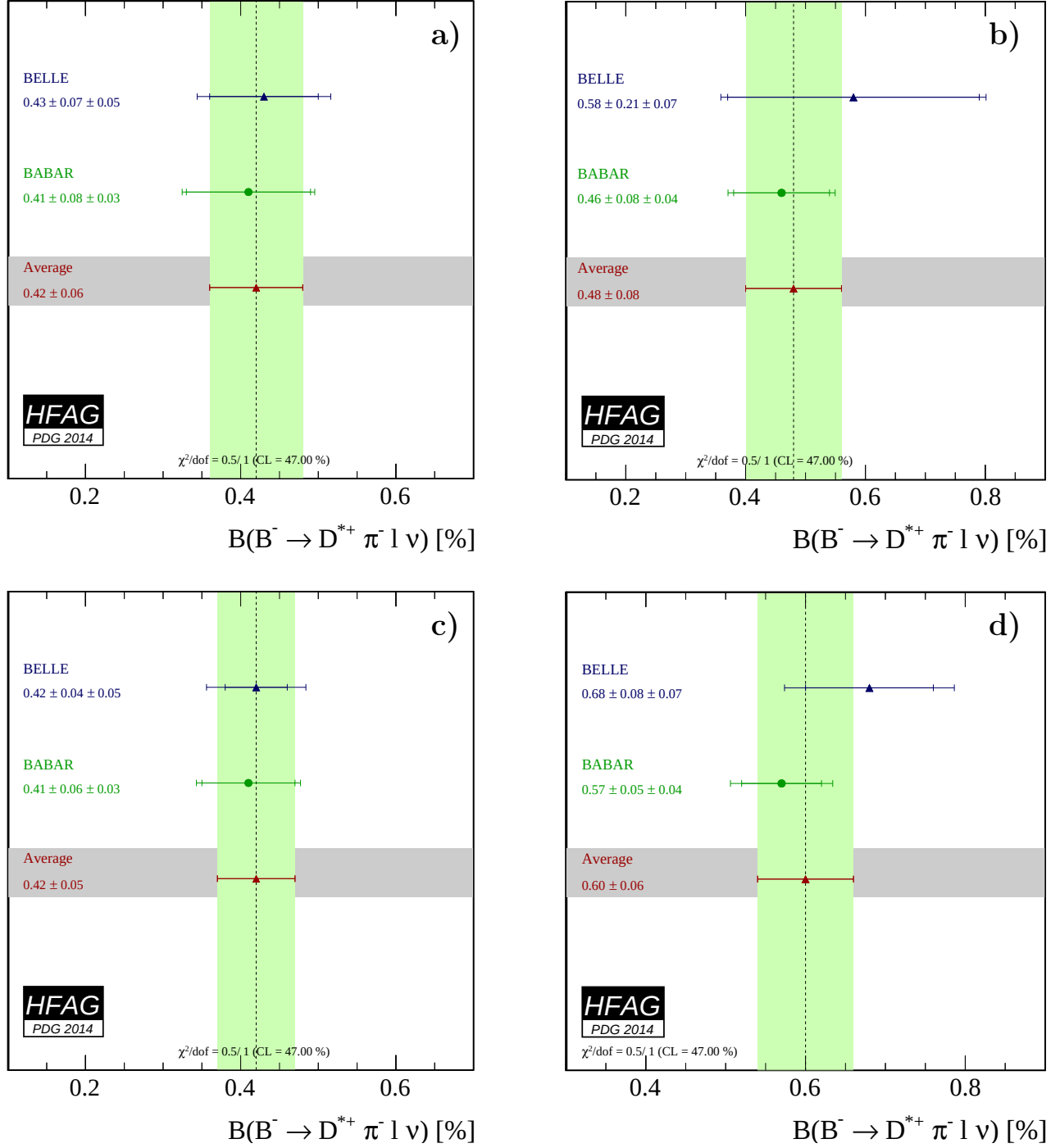


Figure 51: Average branching fraction of exclusive semileptonic B decays (a) $\bar{B}^0 \rightarrow D^0 \pi^+ \ell^- \bar{\nu}_\ell$, (b) $\bar{B}^0 \rightarrow D^{*0} \pi^+ \ell^- \bar{\nu}_\ell$, (c) $B^- \rightarrow D^+ \pi^- \ell^- \bar{\nu}_\ell$, and (d) $B^- \rightarrow D^{*+} \pi^- \ell^- \bar{\nu}_\ell$. The corresponding individual results are also shown.

decays, such as the moments of the lepton energy and the hadronic mass spectrum.

Two independent sets of theoretical expressions, named after the definition of the b -quark

Table 67: Average of the branching fraction $\mathcal{B}(B^- \rightarrow D_1^0 \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_1^0 \rightarrow D^{*+} \pi^-)$ and individual results. The ALEPH, OPAL and D0 measurements are for the $D_1(D^* \pi)X$ final state and we assum that no particles are left in the X system.

Experiment	$\mathcal{B}(B^- \rightarrow D_1^0(D^{*+} \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(B^- \rightarrow D_1^0(D^{*+} \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (published)
ALEPH [437]	$0.440 \pm 0.098_{\text{stat}} \pm 0.068_{\text{syst}}$	$0.47 \pm 0.10_{\text{stat}} \pm 0.07_{\text{syst}}$
OPAL [438]	$0.578 \pm 0.210_{\text{stat}} \pm 0.101_{\text{syst}}$	$0.70 \pm 0.21_{\text{stat}} \pm 0.10_{\text{syst}}$
CLEO [439]	$0.354 \pm 0.085_{\text{stat}} \pm 0.056_{\text{syst}}$	$0.373 \pm 0.085_{\text{stat}} \pm 0.057_{\text{syst}}$
D0 [440]	$0.215 \pm 0.018_{\text{stat}} \pm 0.035_{\text{syst}}$	$0.219 \pm 0.018_{\text{stat}} \pm 0.035_{\text{syst}}$
Belle Tagged B^- [434]	$0.443 \pm 0.070_{\text{stat}} \pm 0.059_{\text{syst}}$	$0.42 \pm 0.07_{\text{stat}} \pm 0.07_{\text{syst}}$
Belle Tagged B^0 [434]	$0.612 \pm 0.200_{\text{stat}} \pm 0.077_{\text{syst}}$	$0.42 \pm 0.07_{\text{stat}} \pm 0.07_{\text{syst}}$
BABAR Tagged [436]	$0.278 \pm 0.030_{\text{stat}} \pm 0.028_{\text{syst}}$	$0.29 \pm 0.03_{\text{stat}} \pm 0.03_{\text{syst}}$
BABAR Untagged B^- [441]	$0.295 \pm 0.017_{\text{stat}} \pm 0.016_{\text{syst}}$	$0.30 \pm 0.02_{\text{stat}} \pm 0.02_{\text{syst}}$
BABAR Untagged B^0 [441]	$0.299 \pm 0.026_{\text{stat}} \pm 0.027_{\text{syst}}$	$0.30 \pm 0.02_{\text{stat}} \pm 0.02_{\text{syst}}$
Average	$0.285 \pm 0.011 \pm 0.014$	$\chi^2/\text{dof} = 13.0/8$ (CL=11.1%)

Table 68: Average of the branching fraction $\mathcal{B}(B^- \rightarrow D_2^0 \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_2^0 \rightarrow D^{*+} \pi^-)$ and individual results. The D0 measurement is for the $D_2^*(D^* \pi)X$ final state and we assume that no particles are left in the X system. The BABAR tagged measurement has been translated in a result on $D_2^* \rightarrow D^* \pi$ decay mode, assuming $\mathcal{B}(D_2^* \rightarrow D \pi)/\mathcal{B}(D_2^* \rightarrow D^* \pi) = 1.54 \pm 0.15$ [5].

Experiment	$\mathcal{B}(B^- \rightarrow D_2^0(D^{*+} \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(B^- \rightarrow D_2^0(D^{*+} \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (published)
CLEO [439]	$0.056 \pm 0.066_{\text{stat}} \pm 0.011_{\text{syst}}$	$0.059 \pm 0.066_{\text{stat}} \pm 0.011_{\text{syst}}$
D0 [440]	$0.087 \pm 0.018_{\text{stat}} \pm 0.020_{\text{syst}}$	$0.088 \pm 0.018_{\text{stat}} \pm 0.020_{\text{syst}}$
Belle [434]	$0.190 \pm 0.060_{\text{stat}} \pm 0.025_{\text{syst}}$	$0.18 \pm 0.06_{\text{stat}} \pm 0.03_{\text{syst}}$
BABAR tagged [436]	$0.076 \pm 0.013_{\text{stat}} \pm 0.009_{\text{syst}}$	$0.078 \pm 0.013_{\text{stat}} \pm 0.010_{\text{syst}}$
BABAR untagged B^- [441]	$0.090 \pm 0.009_{\text{stat}} \pm 0.007_{\text{syst}}$	$0.087 \pm 0.013_{\text{stat}} \pm 0.007_{\text{syst}}$
BABAR untagged B^0 [441]	$0.067 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}}$	$0.087 \pm 0.013_{\text{stat}} \pm 0.007_{\text{syst}}$
Average	$0.078 \pm 0.007 \pm 0.004$	$\chi^2/\text{dof} = 5.6/5$ (CL=34.7%)

mass used, are available for this kind of analysis: the kinetic [443–445] and 1S scheme expressions [446]. The non-perturbative parameters in the kinetic scheme are: the quark masses m_b and m_c , μ_π^2 and μ_G^2 at $O(1/m_b^2)$, and ρ_D^3 and ρ_{LS}^3 at $O(1/m_b^3)$. In the 1S scheme, the parameters are: m_b , λ_1 at $O(1/m_b^2)$, and ρ_1 , τ_1 , τ_2 and τ_3 at $O(1/m_b^3)$. Note that due to the different definitions, the results for the quark masses cannot be compared directly between the two schemes.

Our analysis uses all available measurements of moments in $\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell$, excluding only points with too high correlation to avoid numerical issues. The list of included measurements is given in Table 71. The only external input is the average lifetime τ_B of neutral and charged B mesons, taken to be (1.579 ± 0.005) ps (Sec. 3).

Both in the kinetic and 1S schemes, the moments in $\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell$ are not sufficient to

Table 69: Average of the branching fraction $\mathcal{B}(B^- \rightarrow D_1'^0 \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_1'^0 \rightarrow D^{*+} \pi^-)$ and individual results. The DELPHI measurement is for the final state $D_1'(D^* \pi) X$ and we assume that no particles are left in the X system.

Experiment	$\mathcal{B}(B^- \rightarrow D_1'^0(D^{*+} \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(B^- \rightarrow D_1'^0(D^{*+} \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (published)
DELPHI [442]	$0.74 \pm 0.17_{\text{stat}} \pm 0.18_{\text{syst}}$	$0.83 \pm 0.17_{\text{stat}} \pm 0.18_{\text{syst}}$
Belle [434]	$-0.03 \pm 0.06_{\text{stat}} \pm 0.07_{\text{syst}}$	$-0.03 \pm 0.06_{\text{stat}} \pm 0.07_{\text{syst}}$
BABAR [436]	$0.26 \pm 0.04_{\text{stat}} \pm 0.04_{\text{syst}}$	$0.27 \pm 0.04_{\text{stat}} \pm 0.05_{\text{syst}}$
Average	$0.13 \pm 0.03 \pm 0.02$	$\chi^2/\text{dof} = 18./2$ (CL=0.0001%)

Table 70: Average of the branching fraction $\mathcal{B}(B^- \rightarrow D_0^{*0} \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_0^{*0} \rightarrow D^+ \pi^-)$ and individual results.

Experiment	$\mathcal{B}(B^- \rightarrow D_0^{*0}(D^+ \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (rescaled)	$\mathcal{B}(B^- \rightarrow D_0^{*0}(D^+ \pi^-) \ell^- \bar{\nu}_\ell)[\%]$ (published)
Belle Tagged B^- [434]	$0.25 \pm 0.04_{\text{stat}} \pm 0.06_{\text{syst}}$	$0.24 \pm 0.04_{\text{stat}} \pm 0.06_{\text{syst}}$
Belle Tagged B^0 [434]	$0.23 \pm 0.08_{\text{stat}} \pm 0.06_{\text{syst}}$	$0.24 \pm 0.04_{\text{stat}} \pm 0.06_{\text{syst}}$
BABAR Tagged [436]	$0.31 \pm 0.04_{\text{stat}} \pm 0.05_{\text{syst}}$	$0.26 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}$
Average	$0.29 \pm 0.03 \pm 0.04$	$\chi^2/\text{dof} = 0.61/2$ (CL=73.6%)

determine the b -quark mass precisely. In the kinetic scheme analysis we constrain the c -quark mass (defined in the $\overline{\text{MS}}$ scheme) to the value of Ref. [453],

$$m_c^{\overline{\text{MS}}}(3 \text{ GeV}) = (0.986 \pm 0.013) \text{ GeV} . \quad (179)$$

In the 1S scheme analysis, the b -quark mass is constrained with measurements of the photon energy moments in $B \rightarrow X_s \gamma$ [454–457].

5.2.2 Analysis in the kinetic scheme

The fit relies on the calculations of the spectral moments in $\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell$ decays described in Ref. [445] and closely follows the procedure of Ref. [458]. The analysis determines $|V_{cb}|$ and the 6 non-perturbative parameters mentioned above.

The result in terms of the main parameters is

$$|V_{cb}| = (42.46 \pm 0.88) \times 10^{-3} , \quad (180)$$

$$m_b^{\text{kin}} = 4.541 \pm 0.023 \text{ GeV} , \quad (181)$$

$$\mu_\pi^2 = 0.414 \pm 0.078 \text{ GeV}^2 , \quad (182)$$

with a χ^2 of 14.6 for $50 - 7$ degrees of freedom. The detailed result and the matrix of the correlation coefficients is given in Table 72. The fit to the lepton energy and hadronic mass moments is shown in Figs. 54 and 55, respectively.

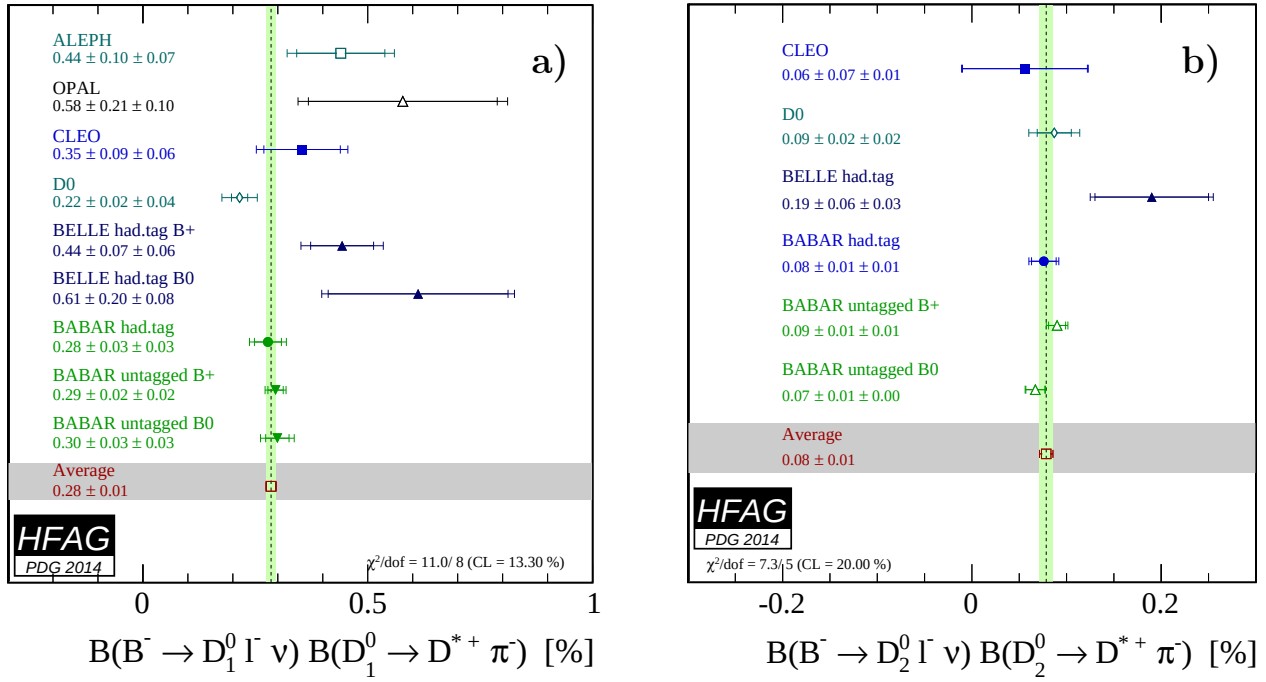


Figure 52: Average of the product of branching fraction (a) $\mathcal{B}(B^- \rightarrow D_1^0 \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_1^0 \rightarrow D^{*+} \pi^-)$ and (b) $\mathcal{B}(B^- \rightarrow D_2^0 \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_2^0 \rightarrow D^{*+} \pi^-)$. The corresponding individual results are also shown.

The inclusive $\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell$ branching fraction determined by this analysis is

$$\mathcal{B}(\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell) = (10.65 \pm 0.16)\% . \quad (183)$$

Correcting for charmless semileptonic decays (Sec. 5.4), $\mathcal{B}(\bar{B} \rightarrow X_u \ell^- \bar{\nu}_\ell) = (2.14 \pm 0.31) \times 10^{-3}$, we obtain the semileptonic branching fraction,

$$\mathcal{B}(\bar{B} \rightarrow X \ell^- \bar{\nu}_\ell) = (10.86 \pm 0.16)\% . \quad (184)$$

5.2.3 Analysis in the 1S scheme

The fit relies on the calculations of the spectral moments described in Ref. [446]. The theoretical uncertainties are estimated as explained in Ref. [459]. Only trivial theory correlations, *i.e.* between the same moment at the same threshold are included in the analysis. The fit determines $|V_{cb}|$ and the 6 non-perturbative parameters mentioned above.

The result of the fit using the $B \rightarrow X_s \gamma$ constraint is

$$|V_{cb}| = (41.98 \pm 0.45) \times 10^{-3} , \quad (185)$$

$$m_b^{1S} = 4.691 \pm 0.037 \text{ GeV} , \quad (186)$$

$$\lambda_1 = -0.362 \pm 0.067 \text{ GeV}^2 , \quad (187)$$

with a χ^2 of 23.0 for 66 – 7 degrees of freedom. The detailed result of the fit is given in Table 73.

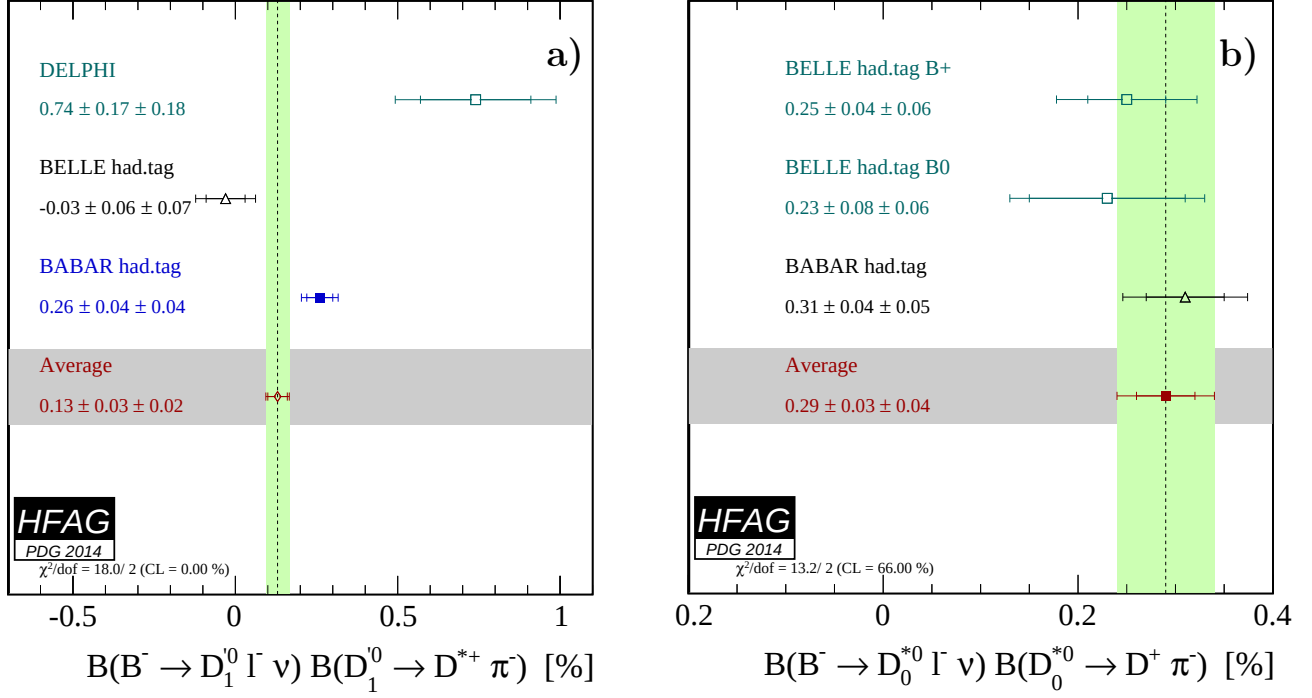


Figure 53: Average of the product of branching fraction (a) $\mathcal{B}(B^- \rightarrow D_1^0 \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_1^0 \rightarrow D^{*+} \pi^-)$ and (b) $\mathcal{B}(B^- \rightarrow D_0^{*0} \ell^- \bar{\nu}_\ell) \times \mathcal{B}(D_0^{*0} \rightarrow D^+ \pi^-)$. The corresponding individual results are also shown.

5.3 Exclusive CKM-suppressed decays

In this section, we list results on exclusive charmless semileptonic branching fractions and determinations of $|V_{ub}|$ based on $\bar{B} \rightarrow \pi \ell \bar{\nu}$ decays. The measurements are based on two different event selections: tagged events, in which case the second B meson in the event is fully reconstructed in either a hadronic decay (“had. tag”) or in a CKM-favored semileptonic decay (“sl. tag”); and untagged events, in which case the momentum of the undetected neutrino is inferred from measurements of the total momentum sum of the detected particles and the knowledge of the initial state. We also present averages for $\bar{B} \rightarrow \rho \ell \bar{\nu}$, $\bar{B} \rightarrow \omega \ell \bar{\nu}$, $\bar{B} \rightarrow \eta \ell \bar{\nu}$ and $\bar{B} \rightarrow \eta' \ell \bar{\nu}$.

The results for the full and partial branching fractions for $\bar{B} \rightarrow \pi \ell \bar{\nu}$ are given in Table 74 and shown in Figure 56.

When averaging these results, systematic uncertainties due to external inputs, *e.g.* form factor shapes and background estimates from the modeling of $\bar{B} \rightarrow X_c \ell \bar{\nu}$ and $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays, are treated as fully correlated (in the sense of Eq. 10 in Sec. 2). Uncertainties due to experimental reconstruction effects are treated as fully correlated among measurements from a given experiment.

The determination of $|V_{ub}|$ from $\bar{B} \rightarrow \pi \ell \bar{\nu}$ decays is shown in Table 75, and uses our averages for the partial branching fractions given in Table 74, combined with various form factor calculations. Two theoretical approaches are used: unquenched lattice QCD (LQCD) and QCD light-cone sum rules (LCSR). The calculations make predictions for different regions of q^2 , where q^2 is the four-momentum transferred to the lepton-neutrino system squared, $q^2 = (p_\ell + p_\nu)^2$. Lattice calculations of the form factors are limited to small hadron momenta, *i.e.* large q^2 , while calculations based on light-cone sum rules are restricted to small q^2 .

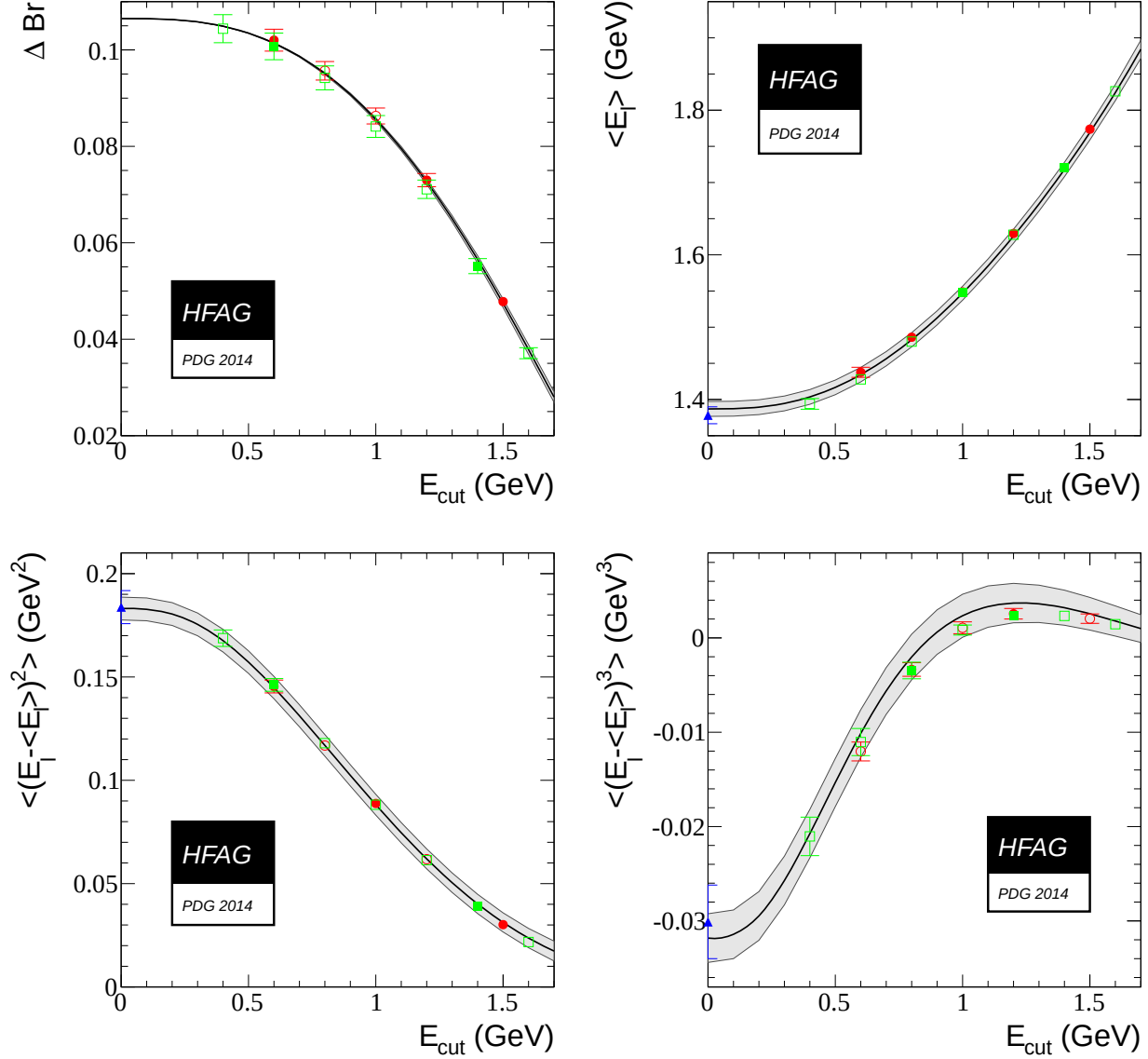


Figure 54: Fit to the partial semileptonic branching ratios and to the lepton energy moments in the kinetic mass scheme. In all plots, the grey band is the theory prediction with total theory error. *BABAR* data are shown by circles, *Belle* by squares and other experiments (*DELPHI*, *CDF*, *CLEO*) by triangles. Filled symbols mean that the point was used in the fit. Open symbols are measurements that were not used in the fit.

Table 71: Experimental inputs used in the global analysis of $\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell$. n is the order of the moment, c is the threshold value of the lepton momentum in GeV. In total, there are 23 measurements from *BABAR*, 15 measurements from Belle and 12 from other experiments.

Experiment	Hadron moments $\langle M_X^n \rangle$	Lepton moments $\langle E_\ell^n \rangle$
<i>BABAR</i>	$n = 2, c = 0.9, 1.1, 1.3, 1.5$ $n = 4, c = 0.8, 1.0, 1.2, 1.4$ $n = 6, c = 0.9, 1.3$ [447]	$n = 0, c = 0.6, 1.2, 1.5$ $n = 1, c = 0.6, 0.8, 1.0, 1.2, 1.5$ $n = 2, c = 0.6, 1.0, 1.5$ $n = 3, c = 0.8, 1.2$ [447, 448]
Belle	$n = 2, c = 0.7, 1.1, 1.3, 1.5$ $n = 4, c = 0.7, 0.9, 1.3$ [449]	$n = 0, c = 0.6, 1.4$ $n = 1, c = 1.0, 1.4$ $n = 2, c = 0.6, 1.4$ $n = 3, c = 0.8, 1.2$ [450]
CDF	$n = 2, c = 0.7$ $n = 4, c = 0.7$ [451]	
CLEO	$n = 2, c = 1.0, 1.5$ $n = 4, c = 1.0, 1.5$ [452]	
DELPHI	$n = 2, c = 0.0$ $n = 4, c = 0.0$ $n = 6, c = 0.0$ [442]	$n = 1, c = 0.0$ $n = 2, c = 0.0$ $n = 3, c = 0.0$ [442]

Table 72: Fit result in the kinetic scheme, using a precise c -quark mass constraint. The error matrix of the fit contains experimental and theoretical contributions. In the lower part of the table, the correlation matrix of the parameters is given.

	$ V_{cb} $ [10^{-3}]	m_b^{kin} [GeV]	m_c^{MS} [GeV]	μ_π^2 [GeV ²]	ρ_D^3 [GeV ³]	μ_G^2 [GeV ²]	ρ_{LS}^3 [GeV ³]
value	42.46	4.541	0.987	0.414	0.154	0.340	-0.147
error	0.88	0.023	0.013	0.078	0.045	0.066	0.098
$ V_{cb} $	1.000	-0.466	-0.049	0.344	0.161	-0.190	0.019
m_b^{kin}		1.000	0.506	-0.113	0.219	0.487	-0.156
m_c^{MS}			1.000	-0.018	0.020	0.008	-0.002
μ_π^2				1.000	0.610	0.001	0.058
ρ_D^3					1.000	-0.038	-0.126
μ_G^2						1.000	-0.014
ρ_{LS}^3							1.000

An alternative method to determine $|V_{ub}|$ from $\bar{B} \rightarrow \pi \ell \bar{\nu}$ decays that makes use of the measurement over the full q^2 range is based on a simultaneous fit of a $B \rightarrow \pi$ form factor parameterization to data and theory predictions. We choose the BCL (Bourrely, Caprini, Lellouch) parameterization [472] up to order z^2 . There are 3+1 fit parameters: the three coefficients of the BCL power series (b_0, b_1, b_2) and $|V_{ub}|$, which is determined from the relative normalization between data and theory predictions. As the shape of the q^2 spectrum is deter-

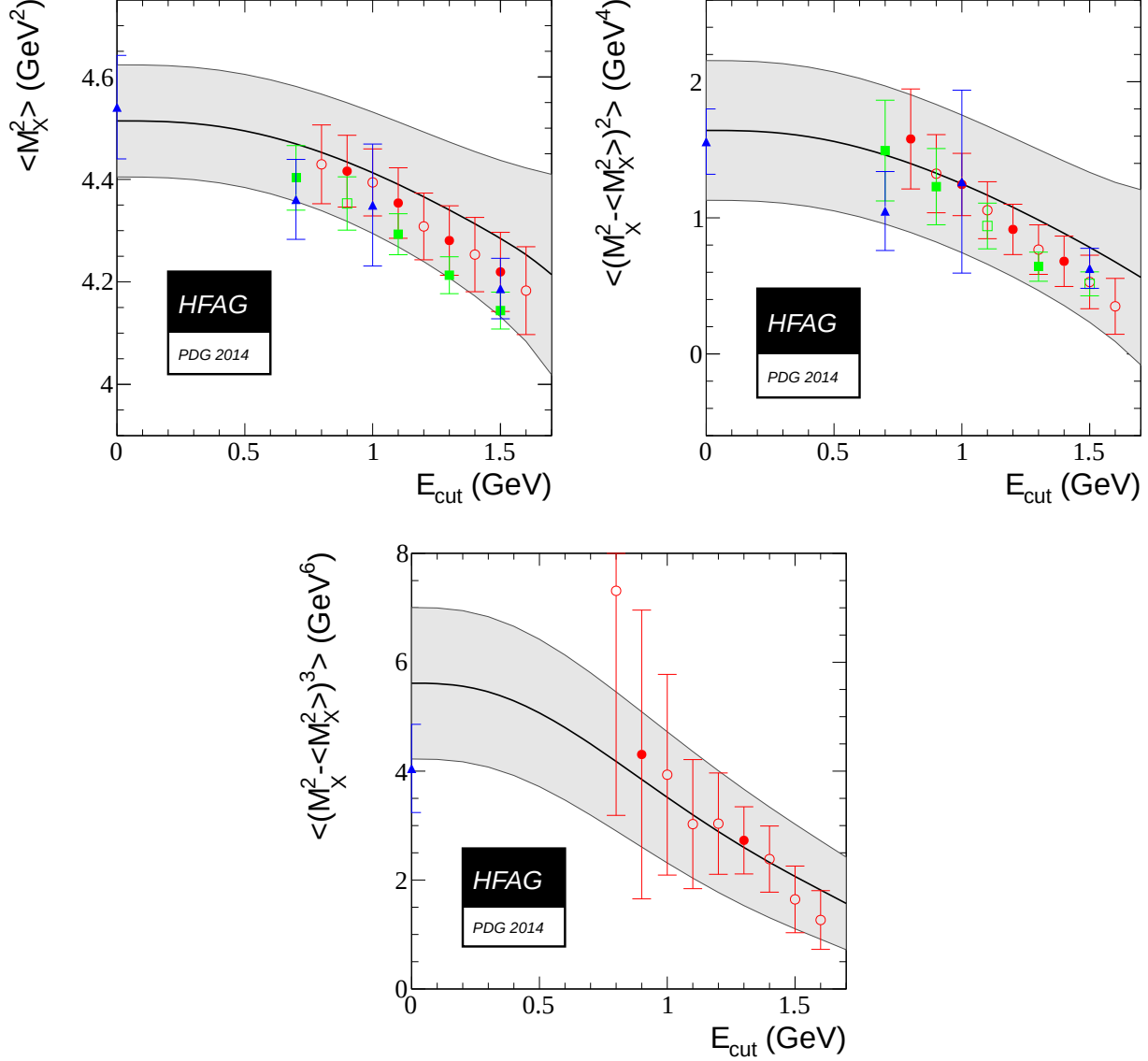


Figure 55: Same as Fig. 54 for the fit to the hadronic mass moments in the kinetic mass scheme.

mined by only two parameters (b_1 and b_2), we quote the ratios b_1/b_0 and b_2/b_0 as results for the shape determined in the fit.

The result of the simultaneous fit to the four most precise measurements from *BABAR* and Belle (*BABAR* untagged 6 q^2 bins, *BABAR* untagged 12 q^2 bins, Belle untagged, Belle had. tag) and the FNAL/MILC LQCD calculations is shown in Figure 57 (a). The fit probability is 0.053 ($\chi^2/dof = 60.2/44$) and we obtain the following values:

$$|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3}, \quad (188)$$

$$b_1/b_0 = -1.02 \pm 0.18, \quad (189)$$

$$b_2/b_0 = -1.20 \pm 0.57. \quad (190)$$

The fit results correspond to a value of the product $f_+(0)|V_{ub}|$ of $(0.923 \pm 0.024) \times 10^{-3}$. The

Table 73: Fit result in the 1S scheme, using $B \rightarrow X_s \gamma$ moments as a constraint. In the lower part of the table, the correlation matrix of the parameters is given.

	m_b^{1S} [GeV]	λ_1 [GeV ²]	ρ_1 [GeV ³]	τ_1 [GeV ³]	τ_2 [GeV ³]	τ_3 [GeV ³]	$ V_{cb} $ [10^{-3}]
value	4.691	-0.362	0.043	0.161	-0.017	0.213	41.98
error	0.037	0.067	0.048	0.122	0.062	0.102	0.45
m_b^{1S}	1.000	0.434	0.213	-0.058	-0.629	-0.019	-0.215
λ_1		1.000	-0.467	-0.602	-0.239	-0.547	-0.403
ρ_1			1.000	0.129	-0.624	0.494	0.286
τ_1				1.000	0.062	-0.148	0.194
τ_2					1.000	-0.009	-0.145
τ_3						1.000	0.376
$ V_{cb} $							1.000

correlation matrix of the fit parameters is:

	b_0	b_1	b_2	$ V_{ub} $
b_0	1.00	-0.54	0.15	-0.29
b_1		1.00	-0.89	0.24
b_2			1.00	-0.14
$ V_{ub} $				1.00

The simultaneous fit has also been performed using the most recent result for $f_+(0)$ from LCSR [473] ($f_+(0) = 0.261^{+0.020}_{-0.023}$), *i.e.* one point at $q^2 = 0$ is used as form factor normalization instead of the LQCD points at high q^2 . This fit has a probability of 0.029 ($\chi^2/dof = 59.8/41$) and yields consistent results (Fig. 57 (b)):

$$|V_{ub}| = (3.53 \pm 0.29) \times 10^{-3}, \quad (191)$$

$$b_1/b_0 = -0.99 \pm 0.20, \quad (192)$$

$$b_2/b_0 = -1.28 \pm 0.61, \quad (193)$$

$$f_+(0)|V_{ub}| = (0.922 \pm 0.024) \times 10^{-3}. \quad (194)$$

The branching fractions for $\bar{B} \rightarrow \rho \ell \bar{\nu}$ decays is computed based on the measurements in Table 76 and is shown in Figure 58. The determination of $|V_{ub}|$ from these other channels looks less promising than for $\bar{B} \rightarrow \pi \ell \bar{\nu}$ and at the moment it is not extracted.

We also report the branching fraction average for $\bar{B} \rightarrow \omega \ell \bar{\nu}$, $\bar{B} \rightarrow \eta \ell \bar{\nu}$ and $\bar{B} \rightarrow \eta' \ell \bar{\nu}$. The measurements for $\bar{B} \rightarrow \omega \ell \bar{\nu}$ are reported in Table 77 and shown in Figure 58, while the ones for $\bar{B} \rightarrow \eta \ell \bar{\nu}$ and $\bar{B} \rightarrow \eta' \ell \bar{\nu}$ are reported in Table 78 and 79, and are shown in Figure 59.

5.4 Inclusive CKM-suppressed decays

The large background from $B \rightarrow X_c \ell^+ \nu_\ell$ decays is the chief experimental limitation in determinations of $|V_{ub}|$. Cuts designed to reject this background limit the acceptance for $B \rightarrow X_u \ell^+ \nu_\ell$

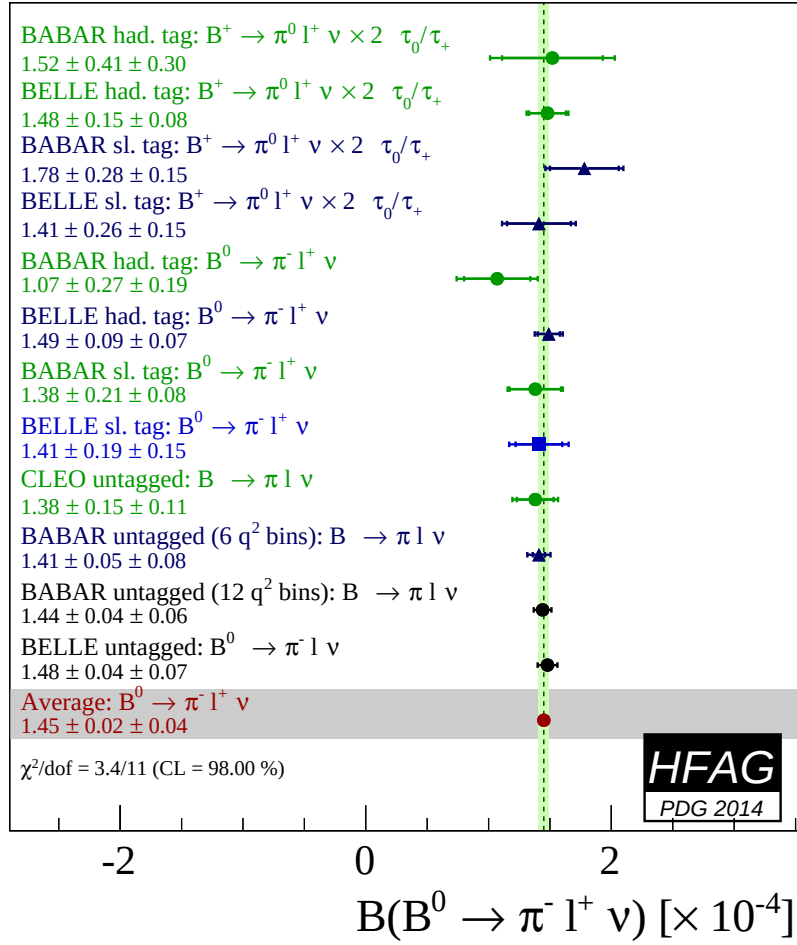


Figure 56: Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \pi \ell \bar{\nu})$ and their average. Measured branching fractions for $B^+ \rightarrow \pi^0 l^+ \nu$ have been multiplied by $2 \times \tau_{B^0}/\tau_{B^+}$ in accordance with isospin symmetry. The labels “had. tag”, “sl. tag” and “untagged” refer to the constraint used for the second B meson in the event. The results from the untagged measurements by CLEO and BABAR are based on a combined analysis of $B^0 \rightarrow \pi^- l^+ \nu$ and $B^+ \rightarrow \pi^0 l^+ \nu$ decays using isospin relations.

decays. The calculation of partial rates for these restricted acceptances is more complicated and requires substantial theoretical machinery. In this update, we use several theoretical calculations to extract $|V_{ub}|$. We do not advocate the use of one method over another. The authors for the different calculations have provided codes to compute the partial rates in limited regions of phase space covered by the measurements. Latest results by Belle [480] and BABAR [481] explore bigger and bigger portions of phase space, with a consequent reduction of the theoretical uncertainties.

For the averages we performed, the systematic errors associated with the modeling of $B \rightarrow X_c \ell^+ \nu_\ell$ and $B \rightarrow X_u \ell^+ \nu_\ell$ decays and the theoretical uncertainties are taken as fully correlated among all measurements. Reconstruction-related uncertainties are taken as fully correlated within a given experiment. We use all results published by BABAR in Ref. [481], since the statistical correlations are given. To make use of the theoretical calculations of Ref. [482], we restrict the kinematic range in M_X and q^2 , thereby reducing the size of the data sample

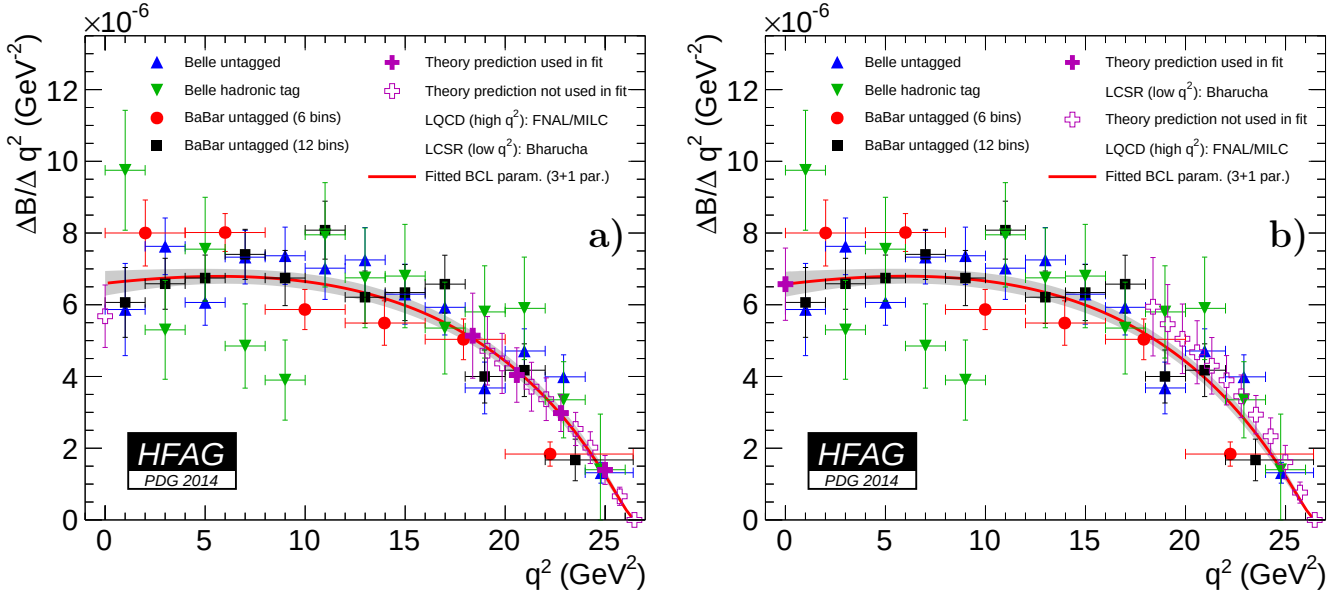


Figure 57: Simultaneous fit of the BCL parameterization with 3+1 parameters to *BABAR* and Belle $\bar{B} \rightarrow \pi \ell \bar{\nu}$ data and the theory prediction from (a) FNAL/MILC LQCD calculations [471] (yielding $|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3}$) and (b) LCSR [473] at $q^2 = 0$ (yielding $|V_{ub}| = (3.53 \pm 0.29) \times 10^{-3}$).

significantly, but also the theoretical uncertainty, as stated by the authors [482]. The dependence of the quoted error on the measured value for each source of error is taken into account in the calculation of the averages. Measurements of partial branching fractions for $B \rightarrow X_u \ell^+ \nu_\ell$ transitions from $\mathcal{T}(4S)$ decays, together with the corresponding accepted region, are given in Table 80. The signal yields for all the measurements shown in Table 80 are not rescaled to common input values of the B meson lifetime (see Sec. 3) and the semileptonic width [5].

It has been first suggested by Neubert [483] and later detailed by Leibovich, Low, and Rothstein (LLR) [484] and Lange, Neubert and Paz (LNP) [485], that the uncertainty of the leading shape functions can be eliminated by comparing inclusive rates for $B \rightarrow X_u \ell^+ \nu_\ell$ decays with the inclusive photon spectrum in $B \rightarrow X_s \gamma$, based on the assumption that the shape functions for transitions to light quarks, u or s , are the same to first order. However, shape function uncertainties are only eliminated at the leading order and they still enter via the signal models used for the determination of efficiency. For completeness, we provide a comparison of the results using calculations with reduced dependence on the shape function, as just introduced, with our averages using different theoretical approaches. Results are presented by *BABAR* in Ref. [486] using the LLR prescription. In another work (Ref. [487]), $|V_{ub}|$ was extracted from the endpoint spectrum of $B \rightarrow X_u \ell^+ \nu_\ell$ from *BABAR* [488], using several theoretical approaches with reduced dependence on the shape function. In both cases, the photon energy spectrum in the rest frame of the B -meson by *BABAR* [454] has been used.

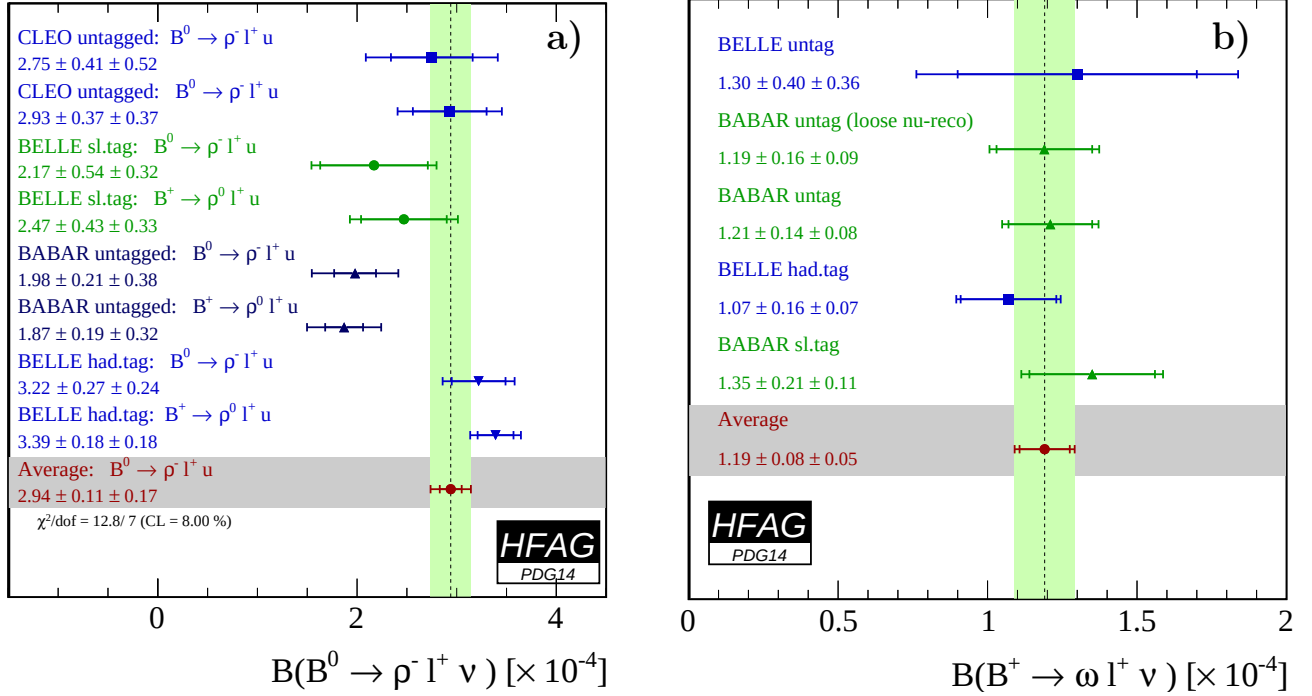


Figure 58: (a) Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \rho^- l^+ \nu)$ and their average. Measurements of $B^+ \rightarrow \rho^0 l^+ \nu$ branching fractions have been multiplied by $2\tau_{B^0}/\tau_{B^+}$ in accordance with isospin symmetry. (b) Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \omega l^+ \nu)$ and their average.

5.4.1 BLNP

Bosch, Lange, Neubert and Paz (BLNP) [495–498] provide theoretical expressions for the triple differential decay rate for $B \rightarrow X_u \ell^+ \nu_\ell$ events, incorporating all known contributions, whilst smoothly interpolating between the “shape-function region” of large hadronic energy and small invariant mass, and the “OPE region” in which all hadronic kinematical variables scale with the b -quark mass. BLNP assign uncertainties to the b -quark mass which enters through the leading shape function, to sub-leading shape function forms, to possible weak annihilation contribution, and to matching scales. The BLNP calculation uses the shape function renormalization scheme; the heavy quark parameters determined from the global fit in the kinetic scheme, described in 5.2.2, were therefore translated into the shape function scheme by using a prescription by Neubert [499, 500]. The resulting parameters are $m_b(SF) = (4.569 \pm 0.023 \pm 0.018)$ GeV, $\mu_\pi^2(SF) = (0.145 \pm 0.089_{-0.040}^{+0.020})$ GeV², where the second uncertainty is due to the scheme translation. The extracted values of $|V_{ub}|$ for each measurement along with their average are given in Table 81 and illustrated in Figure 60(a). The total uncertainty is $^{+5.8\%}_{-5.9\%}$ and is due to: statistics ($^{+2.0\%}_{-2.1\%}$), detector ($^{+1.7\%}_{-1.8\%}$), $B \rightarrow X_c \ell^+ \nu_\ell$ model ($^{+1.2\%}_{-1.2\%}$), $B \rightarrow X_u \ell^+ \nu_\ell$ model ($^{+1.9\%}_{-1.8\%}$), heavy quark parameters ($^{+2.6\%}_{-2.7\%}$), SF functional form ($^{+0.1\%}_{-0.3\%}$), sub-leading shape functions ($^{+0.6\%}_{-0.6\%}$), BLNP theory: matching scales μ, μ_i, μ_h ($^{+3.8\%}_{-3.8\%}$), and weak annihilation ($^{+0.0\%}_{-1.4\%}$). The error on the HQE parameters (b -quark mass and μ_π^2) is the source of the largest uncertainty, while the uncertainty assigned for the matching scales is a close second. The uncertainty due to weak

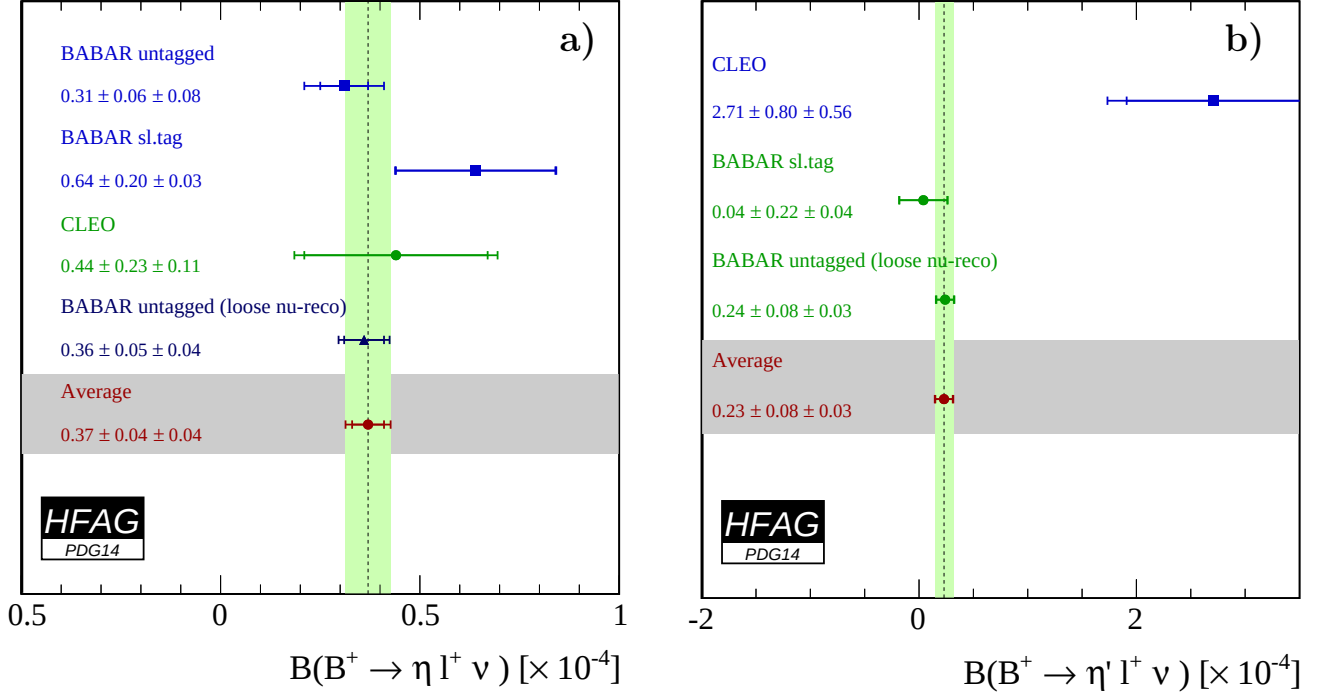


Figure 59: (a) Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \eta \ell \bar{\nu})$ and their average. (b) Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \eta' \ell \bar{\nu})$ and their average.

annihilation has been assumed to be asymmetric, *i.e.* it only tends to decrease $|V_{ub}|$.

5.4.2 DGE

J.R. Andersen and E. Gardi (Dressed Gluon Exponentiation, DGE) [503] provide a framework where the on-shell b -quark calculation, converted into hadronic variables, is directly used as an approximation to the meson decay spectrum without the use of a leading-power non-perturbative function (or, in other words, a shape function). The on-shell mass of the b -quark within the B -meson (m_b) is required as input. The DGE calculation uses the $\overline{MS}$ renormalization scheme; the heavy quark parameters determined from the global fit in the kinetic scheme, described in 5.2.2, were therefore translated into the $\overline{MS}$ scheme by using a calculation by Gardi, giving $m_b(\overline{MS}) = (4.177 \pm 0.043)$ GeV. The extracted values of $|V_{ub}|$ for each measurement along with their average are given in Table 81 and illustrated in Figure 60(b). The total error is $^{+4.8\%}_{-5.0\%}$, whose breakdown is: statistics ($^{+1.8\%}_{-1.8\%}$), detector ($^{+1.7\%}_{-1.7\%}$), $B \rightarrow X_c \ell^+ \nu_\ell$ model ($^{+1.3\%}_{-1.3\%}$), $B \rightarrow X_u \ell^+ \nu_\ell$ model ($^{+2.1\%}_{-1.9\%}$), strong coupling α_s ($^{+0.5\%}_{-0.5\%}$), m_b ($^{+3.2\%}_{-3.0\%}$), weak annihilation ($^{+0.0\%}_{-1.9\%}$), DGE theory: matching scales ($^{+0.5\%}_{-0.3\%}$). The largest contribution to the total error is due to the effect of the uncertainty on m_b . The uncertainty due to weak annihilation has been assumed to be asymmetric, *i.e.* it only tends to decrease $|V_{ub}|$.

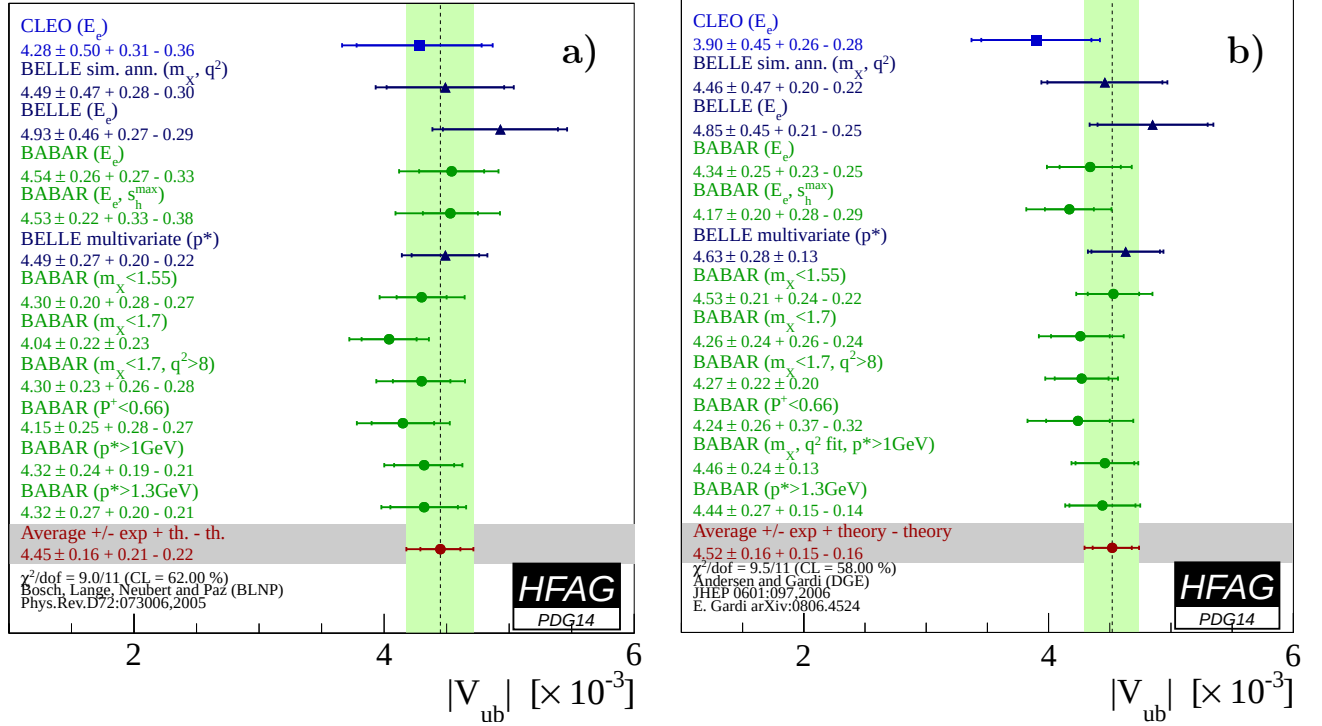


Figure 60: Measurements of $|V_{ub}|$ from inclusive semileptonic decays and their average based on the BLNP (a) and DGE (b) prescription. The labels indicate the distributions and selections used to define the signal regions in the different analyses, where E_e is the electron energy in the B rest frame, p^* the lepton momentum in the B frame and m_X is the invariant mass of the hadronic system. The light-cone momentum P_+ is defined in the B rest frame as $P_+ = E_X - |\vec{p}_X|$.

5.4.3 GGOU

Gambino, Giordano, Ossola and Uraltsev (GGOU) [504] compute the triple differential decay rates of $B \rightarrow X_u \ell^+ \nu_\ell$, including all perturbative and non-perturbative effects through $O(\alpha_s^2 \beta_0)$ and $O(1/m_b^3)$. The Fermi motion is parameterized in terms of a single light-cone function for each structure function and for any value of q^2 , accounting for all subleading effects. The calculations are performed in the kinetic scheme, a framework characterized by a Wilsonian treatment with a hard cutoff $\mu \sim 1$ GeV. GGOU have not included calculations for the “ (E_e, s_h^{max}) ” analysis. The heavy quark parameters determined from the global fit in the kinetic scheme, described in 5.2.2, are used as inputs: $m_b(kin) = (4.541 \pm 0.023)$ GeV, $\mu_\pi^2(kin) = (0.414 \pm 0.078)$ GeV². The extracted values of $|V_{ub}|$ for each measurement along with their average are given in Table 81 and illustrated in Figure 61(a). The total error is $^{+4.3\%}_{-4.8\%}$ whose breakdown is: statistics ($^{+1.9\%}_{-1.9\%}$), detector ($^{+1.7\%}_{-1.7\%}$), $B \rightarrow X_c \ell^+ \nu_\ell$ model ($^{+1.3\%}_{-1.3\%}$), $B \rightarrow X_u \ell^+ \nu_\ell$ model ($^{+1.9\%}_{-1.9\%}$), α_s , m_b and other non-perturbative parameters ($^{+1.6\%}_{-1.6\%}$), higher order perturbative and non-perturbative corrections ($^{+1.5\%}_{-1.5\%}$), modelling of the q^2 tail ($^{+1.4\%}_{-1.4\%}$), weak annihilations matrix element ($^{+0.0\%}_{-2.0\%}$), functional form of the distribution functions ($^{+0.2\%}_{-0.2\%}$). The leading uncertainties on $|V_{ub}|$ are both from theory, and are due to perturbative and non-perturbative parameters and the modelling of the q^2 tail. The uncertainty due to weak annihilation has been assumed to be asymmetric, *i.e.* it only tends to decrease $|V_{ub}|$.

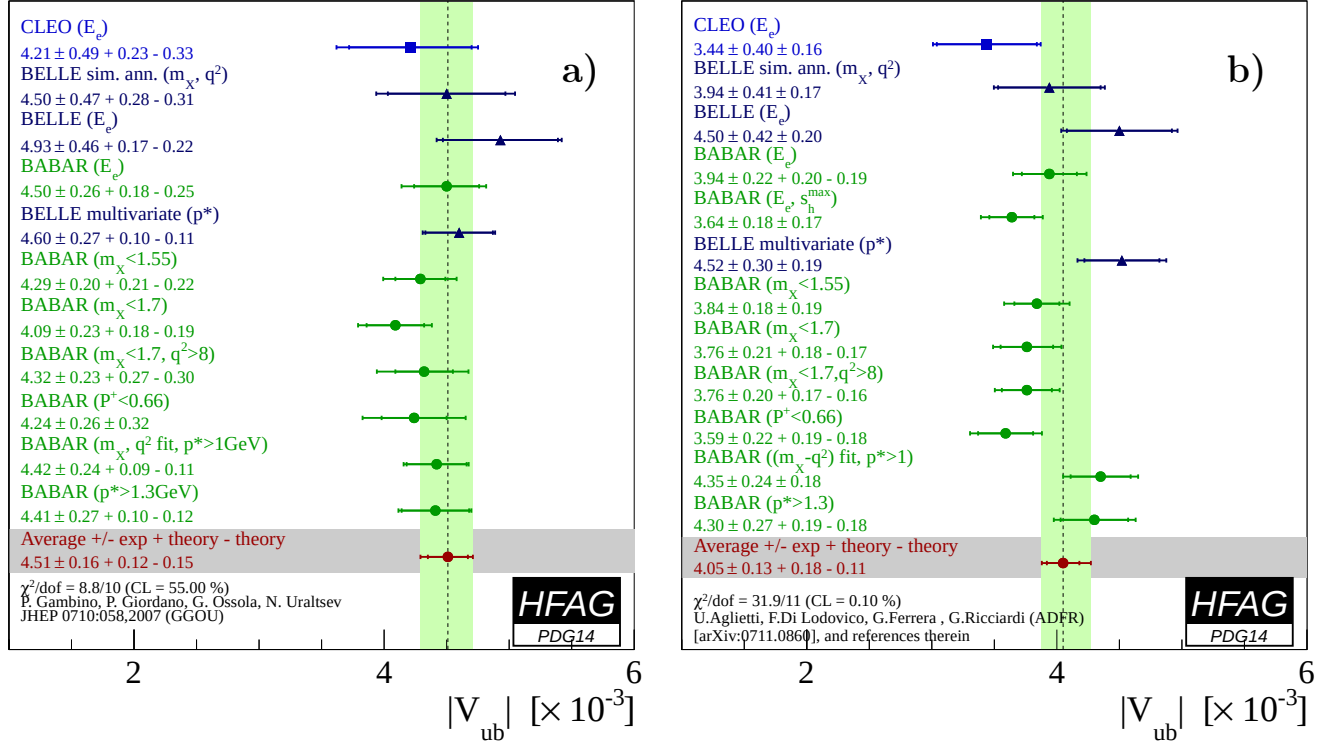


Figure 61: Measurements of $|V_{ub}|$ from inclusive semileptonic decays and their average based on the GGOU (a) and ADFR (b) prescription. The labels indicate the distributions and selections used to define the signal regions in the different analyses, where E_e is the electron energy in the B rest frame, p^* the lepton momentum in the B frame and m_X is the invariant mass of the hadronic system. The light-cone momentum P_+ is defined in the B rest frame as $P_+ = E_X - |\vec{p}_X|$.

5.4.4 ADFR

Aglietti, Di Lodovico, Ferrera and Ricciardi (ADFR) [505] use an approach to extract $|V_{ub}|$, which makes use of the ratio of the $B \rightarrow X_c \ell^+ \nu_\ell$ and $B \rightarrow X_u \ell^+ \nu_\ell$ widths. The normalized triple differential decay rate for $B \rightarrow X_u \ell^+ \nu_\ell$ [506–509] is calculated with a model based on (i) soft-gluon resummation to next-to-next-leading order and (ii) an effective QCD coupling without Landau pole. This coupling is constructed by means of an extrapolation to low energy of the high-energy behaviour of the standard coupling. More technically, an analyticity principle is used. The lower cut on the electron energy for the endpoint analyses is 2.3 GeV [506]. The ADFR calculation uses the $\overline{MS}$ renormalization scheme; the heavy quark parameters determined from the global fit in the kinetic scheme, described in 5.2.2, were therefore translated into the $\overline{MS}$ scheme by using a calculation by Gardi, giving $m_b(\overline{MS}) = (4.177 \pm 0.043)$ GeV. The extracted values of $|V_{ub}|$ for each measurement along with their average are given in Table 81 and illustrated in Figure 61(b). The total error is $^{+5.4\%}_{-5.3\%}$ whose breakdown is: statistics ($^{+1.8\%}_{-1.8\%}$), detector ($^{+1.8\%}_{-2.0\%}$), $B \rightarrow X_c \ell^+ \nu_\ell$ model ($^{+1.4\%}_{-1.4\%}$), $B \rightarrow X_u \ell^+ \nu_\ell$ model ($^{+1.3\%}_{-1.3\%}$), α_s ($^{+1.1\%}_{-1.0\%}$), $|V_{cb}|$ ($^{+2.0\%}_{-2.0\%}$), m_b ($^{+0.7\%}_{-0.7\%}$), m_c ($^{+0.4\%}_{-0.7\%}$), semileptonic branching fraction ($^{+0.8\%}_{-0.7\%}$), theory model ($^{+3.5\%}_{-3.5\%}$). The leading uncertainty, from theory, is due to the theory model.

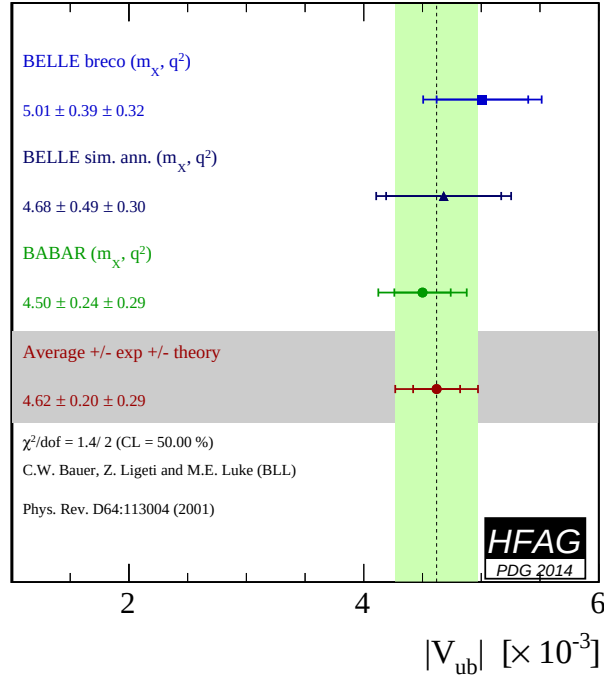


Figure 62: Measurements of $|V_{ub}|$ from inclusive semileptonic decays and their average in the BLL prescription. “(M_X, q^2)” indicates the analysis type.

5.4.5 BLL

Bauer, Ligeti, and Luke (BLL) [482] give a HQET-based prescription that advocates combined cuts on the dilepton invariant mass, q^2 , and hadronic mass, m_X , to minimise the overall uncertainty on $|V_{ub}|$. In their reckoning a cut on m_X only, although most efficient at preserving phase space ($\sim 80\%$), makes the calculation of the partial rate untenable due to uncalculable corrections to the b -quark distribution function or shape function. These corrections are suppressed if events in the low q^2 region are removed. The cut combination used in measurements is $M_x < 1.7 \text{ GeV}/c^2$ and $q^2 > 8 \text{ GeV}^2/c^2$. The extracted values of $|V_{ub}|$ for each measurement along with their average are given in Table 81 and illustrated in Figure 62. The total error is $^{+7.7\%}_{-7.7\%}$ whose breakdown is: statistics ($^{+3.3\%}_{-3.3\%}$), detector ($^{+3.0\%}_{-3.0\%}$), $B \rightarrow X_c \ell^+ \nu_\ell$ model ($^{+1.6\%}_{-1.6\%}$), $B \rightarrow X_u \ell^+ \nu_\ell$ model ($^{+1.1\%}_{-1.1\%}$), spectral fraction (m_b) ($^{+3.0\%}_{-3.0\%}$), perturbative : strong coupling α_s ($^{+3.0\%}_{-3.0\%}$), residual shape function ($^{+2.5\%}_{-2.5\%}$), third order terms in the OPE ($^{+4.0\%}_{-4.0\%}$). The leading uncertainties, both from theory, are due to residual shape function effects and third order terms in the OPE expansion. The leading experimental uncertainty is due to statistics.

5.4.6 Summary

A summary of the averages presented in several different frameworks and results by V.B. Golubev, V.G. Luth and Yu.I. Skovpen [487], based on prescriptions by LLR [484] and LNP [485] to reduce the leading shape function uncertainties are presented in Table 82. A value judgement based on a direct comparison should be avoided at the moment, experimental and theoretical uncertainties play out differently between the schemes and the theoretical assumptions for the theory calculations are different.

5.5 Summary of the $B \rightarrow D^{(*)}\tau\nu_\tau$ decays

The leptonic and semileptonic decays with τ in the final state are probes of physics beyond the SM. In the SM these decays proceed via the W emission diagrams. In models with extended Higgs sectors, such as the Two Higgs Doublet Models (2HDM) or the MSSM, charged Higgs can contribute to the decay amplitude at the tree level. These further contributions can affect the branching fraction. Compared to $B^+ \rightarrow \tau\nu_\tau$, the $B \rightarrow D^{(*)}\tau\nu_\tau$ decay has advantages: the branching fraction is relatively high, because it is not Cabibbo-suppressed, and because it is a three-body decay, many observables beside the branching fraction can be studied, such as the D^* polarisation, or the q^2 distribution (see Ref. [510] and reference therein for recent calculations).

The $B^0 \rightarrow D^{*+}\tau\nu_\tau$ decay was first observed by Belle [511] performing an inclusive reconstruction of the B_{tag} candidates using all the particles that remain after the selection of the B_{sig} decay products. Since then, both BABAR and Belle have published improved measurements and have found evidence for the $B \rightarrow D\tau\nu_\tau$ decays [512–514]. The most powerful way to study these decays is the full hadronic B_{tag} technique widely used by both BABAR and Belle.

Using the full dataset and an improved B_{tag} selection, BABAR measured [515] the ratios:

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu_\ell)} = 0.440 \pm 0.058 \pm 0.042 \quad (195)$$

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \rightarrow D^*\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^*\ell\nu_\ell)} = 0.332 \pm 0.024 \pm 0.018 \quad (196)$$

where $\ell = e, \mu$ and the B^0 and B^+ are combined in a isospin-constrained fit. The ratios $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are independent of $|V_{cb}|$ and to a large extent of the parametrizations of the form factors, so the SM predictions for these ratios are quite precise, $\mathcal{R}(D) = 0.297 \pm 0.017$ and $\mathcal{R}(D^*) = 0.252 \pm 0.003$ (results obtained in Ref. [515, 516] updating the calculations in Ref. [517, 518] with the recent $B \rightarrow D^{(*)}$ measurements from the B-Factories). The BABAR result exceed SM predictions, in both D and D^* channels, by 2.0σ and 2.7σ respectively. The combined result disagree with the SM by 3.4σ . BABAR also interpreted these measurements in terms of the 2HDM type-II and found that their results are not compatible with this model for any value of $\tan\beta$ and m_H . The result obtained by BABAR and Belle on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ are reported in Tab.83. Before a published result by Belle using the new B_{tag} reconstruction, we do not attempt to average the existing results.

Table 74: Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \pi \ell \bar{\nu})$. The errors quoted correspond to statistical and systematic uncertainties, respectively. Measured branching fractions for $B^+ \rightarrow \pi^0 \ell^+ \nu$ have been multiplied by $2 \times \tau_{B^0}/\tau_{B^+}$ in accordance with isospin symmetry. The labels “had. tag” and “sl. tag” refer to the type of B tag used in the measurement, and “untagged” refers to an untagged measurement.

	$\mathcal{B}[10^{-4}]$	$\mathcal{B}(q^2 < 12 \text{ GeV}^2/c^2)[10^{-4}]$	$\mathcal{B}(q^2 < 16 \text{ GeV}^2/c^2)[10^{-4}]$	$\mathcal{B}(q^2 > 16 \text{ GeV}^2/c^2)[10^{-4}]$
CLEO untagged π^+, π^0 [460]	$1.38 \pm 0.15 \pm 0.11$	$0.69 \pm 0.12 \pm 0.07$	$0.97 \pm 0.13 \pm 0.09$	$0.41 \pm 0.08 \pm 0.04$
BABAR untagged π^+, π^0 [461]	$1.41 \pm 0.05 \pm 0.08$	$0.88 \pm 0.04 \pm 0.05$	$1.10 \pm 0.04 \pm 0.06$	$0.32 \pm 0.02 \pm 0.03$
BABAR untagged π^+, π^0 [462]	$1.44 \pm 0.04 \pm 0.06$	$0.83 \pm 0.03 \pm 0.04$	$1.08 \pm 0.03 \pm 0.05$	$0.40 \pm 0.02 \pm 0.02$
Belle untagged π^+ [463]	$1.48 \pm 0.04 \pm 0.07$	$0.82 \pm 0.03 \pm 0.04$	$1.09 \pm 0.03 \pm 0.05$	$0.40 \pm 0.02 \pm 0.02$
Belle sl. tag π^+ [464]	$1.41 \pm 0.19 \pm 0.15$	$0.80 \pm 0.14 \pm 0.08$	$1.04 \pm 0.16 \pm 0.11$	$0.37 \pm 0.10 \pm 0.04$
Belle sl. tag π^0 [464]	$1.41 \pm 0.26 \pm 0.15$	$0.71 \pm 0.17 \pm 0.08$	$1.05 \pm 0.22 \pm 0.12$	$0.37 \pm 0.15 \pm 0.04$
BABAR sl. tag π^+ [465]	$1.38 \pm 0.21 \pm 0.08$	$0.77 \pm 0.14 \pm 0.05$	$0.92 \pm 0.16 \pm 0.05$	$0.46 \pm 0.13 \pm 0.03$
BABAR sl. tag π^0 [465]	$1.78 \pm 0.28 \pm 0.15$	$1.07 \pm 0.20 \pm 0.09$	$1.35 \pm 0.22 \pm 0.11$	$0.44 \pm 0.17 \pm 0.06$
BABAR had. tag π^+ [466]	$1.07 \pm 0.27 \pm 0.19$	$0.26 \pm 0.15 \pm 0.04$	$0.42 \pm 0.18 \pm 0.06$	$0.65 \pm 0.20 \pm 0.13$
BABAR had. tag π^0 [466]	$1.52 \pm 0.41 \pm 0.30$	$0.67 \pm 0.30 \pm 0.12$	$1.04 \pm 0.35 \pm 0.18$	$0.48 \pm 0.22 \pm 0.12$
Belle had. tag π^+ [467]	$1.49 \pm 0.09 \pm 0.07$	$0.81 \pm 0.06 \pm 0.04$	$1.06 \pm 0.07 \pm 0.05$	$0.45 \pm 0.05 \pm 0.02$
Belle had. tag π^0 [467]	$1.48 \pm 0.15 \pm 0.08$	$0.78 \pm 0.11 \pm 0.04$	$1.09 \pm 0.12 \pm 0.06$	$0.36 \pm 0.07 \pm 0.02$
Average	$1.45 \pm 0.02 \pm 0.04$	$0.81 \pm 0.02 \pm 0.02$	$1.06 \pm 0.02 \pm 0.03$	$0.38 \pm 0.01 \pm 0.01$

Table 75: Determinations of $|V_{ub}|$ based on the average partial $\bar{B} \rightarrow \pi \ell \bar{\nu}$ decay branching fractions stated in Table 74. The q^2 ranges for the partial branching fractions corresponding to the validity ranges of the form factor calculations are indicated. The first uncertainty is experimental and the second is from theory.

Method	q^2 range [GeV ² /c ²]	$ V_{ub} [10^{-3}]$
Khodjamirian et al. (LCSR) [468]	0 – 12	$3.41 \pm 0.06^{+0.37}_{-0.32}$
Ball & Zwicky (LCSR) [469]	0 – 16	$3.58 \pm 0.06^{+0.59}_{-0.40}$
HPQCD (LQCD) [470]	16 – 26.4	$3.52 \pm 0.08^{+0.61}_{-0.40}$
FNAL/MILC (LQCD) [471]	16 – 26.4	$3.36 \pm 0.08^{+0.37}_{-0.31}$

Table 76: Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \rho \ell \bar{\nu})$. The errors quoted correspond to statistical and systematic uncertainties, respectively.

$\mathcal{B}[10^{-4}]$	
CLEO ρ^+ [474]	$2.75 \pm 0.41 \pm 0.52$
CLEO ρ^+ [460]	$2.93 \pm 0.37 \pm 0.37$
Belle ρ^+ [467]	$3.22 \pm 0.27 \pm 0.24$
Belle ρ^0 [467]	$3.39 \pm 0.18 \pm 0.18$
Belle ρ^+ [464]	$2.17 \pm 0.54 \pm 0.32$
Belle ρ^0 [464]	$2.47 \pm 0.43 \pm 0.33$
BABAR ρ^+ [461]	$1.98 \pm 0.21 \pm 0.38$
BABAR ρ^0 [461]	$1.87 \pm 0.19 \pm 0.32$
Average	$2.94 \pm 0.11 \pm 0.17$

Table 77: Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \omega \ell \bar{\nu})$. The errors quoted correspond to statistical and systematic uncertainties, respectively.

$\mathcal{B}[10^{-4}]$	
Belle ω [475]	$1.30 \pm 0.40 \pm 0.36$
BABAR ω [462]	$1.19 \pm 0.16 \pm 0.09$
BABAR ω [476]	$1.21 \pm 0.14 \pm 0.08$
Belle ω [467]	$1.07 \pm 0.16 \pm 0.07$
BABAR ω [477]	$1.35 \pm 0.21 \pm 0.11$
Average	$1.19 \pm 0.08 \pm 0.06$

Table 78: Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \eta \ell \bar{\nu})$. The errors quoted correspond to statistical and systematic uncertainties, respectively.

$\mathcal{B}[10^{-4}]$	
CLEO η [478]	$0.44 \pm 0.23 \pm 0.11$
BABAR η [479]	$0.31 \pm 0.06 \pm 0.08$
BABAR η [465]	$0.64 \pm 0.20 \pm 0.03$
BABAR η [462]	$0.36 \pm 0.05 \pm 0.04$
Average	$0.37 \pm 0.04 \pm 0.04$

Table 79: Summary of exclusive determinations of $\mathcal{B}(\bar{B} \rightarrow \eta' \ell \bar{\nu})$. The errors quoted correspond to statistical and systematic uncertainties, respectively.

$\mathcal{B}[10^{-4}]$	
CLEO η' [478]	$2.71 \pm 0.80 \pm 0.56$
BABAR η' [465]	$0.04 \pm 0.22 \pm 0.04$
BABAR η' [462]	$0.24 \pm 0.08 \pm 0.03$
Average	$0.23 \pm 0.08 \pm 0.03$

Table 80: Summary of inclusive determinations of partial branching fractions for $B \rightarrow X_u \ell^+ \nu_\ell$ decays. The errors quoted on $\Delta\mathcal{B}$ correspond to statistical and systematic uncertainties. The $s_h^{\max}$ variable is described in Refs. [489, 490].

Measurement	Accepted region	$\Delta\mathcal{B}[10^{-4}]$	Notes
CLEO [491]	$E_e > 2.1 \text{ GeV}$	$3.3 \pm 0.2 \pm 0.7$	used only in BLL average
BABAR [490]	$E_e > 2.0 \text{ GeV}, s_h^{\max} < 3.5 \text{ GeV}^2$	$4.0 \pm 0.2 \pm 0.3$	
BABAR [488]	$E_e > 2.0 \text{ GeV}$	$5.7 \pm 0.4 \pm 0.5$	
Belle [492]	$E_e > 1.9 \text{ GeV}$	$8.5 \pm 0.4 \pm 1.5$	
BABAR [481]	$M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^2$	$6.9 \pm 0.6 \pm 0.4$	
Belle [493]	$M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^2$	$7.4 \pm 0.9 \pm 1.3$	
Belle [494]	$M_X < 1.7 \text{ GeV}/c^2, q^2 > 8 \text{ GeV}^2/c^2$	$8.5 \pm 0.9 \pm 1.0$	
BABAR [481]	$P_+ < 0.66 \text{ GeV}$	$9.9 \pm 0.9 \pm 0.8$	
BABAR [481]	$M_X < 1.7 \text{ GeV}/c^2$	$11.6 \pm 1.0 \pm 0.8$	
BABAR [481]	$M_X < 1.55 \text{ GeV}/c^2$	$10.9 \pm 0.8 \pm 0.6$	
Belle [480]	$p_\ell^* > 1 \text{ GeV}/c$	$19.6 \pm 1.7 \pm 1.6$	
BABAR [481]	$(M_X, q^2) \text{ fit}, p_\ell^* > 1 \text{ GeV}/c$	$18.2 \pm 1.3 \pm 1.5$	
BABAR [481]	$p_\ell^* > 1.3 \text{ GeV}/c$	$15.5 \pm 1.3 \pm 1.4$	

Table 81: Summary of input parameters used by the different theory calculations, corresponding inclusive determinations of $|V_{ub}|$ and their average. The errors quoted on $|V_{ub}|$ correspond to experimental and theoretical uncertainties, respectively.

	BLNP	DGE	GGOU	ADFR	BLL
Input parameters					
scheme	SF	$\overline{MS}$	kinetic	$\overline{MS}$	1S
Ref.	[499, 500]	Ref. [501]	see Sec. 5.2.2	Ref. [501]	Ref. [502]
m_b (GeV)	4.569 ± 0.025	4.177 ± 0.043	4.541 ± 0.023	4.177 ± 0.043	4.704 ± 0.029
μ_π^2 (GeV ²)	$0.145^{+0.091}_{-0.097}$	-	0.414 ± 0.078	-	-
Ref.	$ V_{ub} $ values				
E_e [491]	$4.28 \pm 0.50^{+0.31}_{-0.36}$	$3.90 \pm 0.45^{+0.26}_{-0.28}$	$4.21 \pm 0.49^{+0.23}_{-0.33}$	$3.44 \pm 0.40^{+0.16}_{-0.16}$	-
M_X, q^2 [493]	$4.49 \pm 0.47^{+0.28}_{-0.30}$	$4.46 \pm 0.47^{+0.20}_{-0.22}$	$4.50 \pm 0.47^{+0.28}_{-0.31}$	$3.94 \pm 0.41^{+0.17}_{-0.17}$	$4.68 \pm 0.49^{+0.30}_{-0.30}$
E_e [492]	$4.93 \pm 0.46^{+0.27}_{-0.29}$	$4.85 \pm 0.45^{+0.21}_{-0.25}$	$4.93 \pm 0.46^{+0.17}_{-0.22}$	$4.50 \pm 0.42^{+0.20}_{-0.20}$	-
E_e [488]	$4.54 \pm 0.26^{+0.27}_{-0.33}$	$4.34 \pm 0.25^{+0.23}_{-0.25}$	$4.50 \pm 0.26^{+0.18}_{-0.25}$	$3.94 \pm 0.22^{+0.20}_{-0.19}$	-
$E_e, s_h^{\max}$ [490]	$4.53 \pm 0.22^{+0.33}_{-0.38}$	$4.17 \pm 0.20^{+0.28}_{-0.29}$	-	$3.64 \pm 0.18^{+0.17}_{-0.17}$	-
p_ℓ^* [480]	$4.49 \pm 0.27^{+0.20}_{-0.22}$	$4.63 \pm 0.28^{+0.13}_{-0.13}$	$4.60 \pm 0.27^{+0.10}_{-0.11}$	$4.52 \pm 0.30^{+0.19}_{-0.19}$	-
M_X [481]	$4.30 \pm 0.20^{+0.28}_{-0.27}$	$4.53 \pm 0.21^{+0.24}_{-0.22}$	$4.29 \pm 0.20^{+0.21}_{-0.22}$	$3.84 \pm 0.18^{+0.19}_{-0.19}$	-
M_X [481]	$4.04 \pm 0.22^{+0.23}_{-0.23}$	$4.26 \pm 0.24^{+0.26}_{-0.24}$	$4.09 \pm 0.23^{+0.18}_{-0.19}$	$3.76 \pm 0.21^{+0.18}_{-0.17}$	-
M_X, q^2 [481]	$4.30 \pm 0.23^{+0.26}_{-0.28}$	$4.27 \pm 0.22^{+0.20}_{-0.20}$	$4.32 \pm 0.23^{+0.27}_{-0.30}$	$3.76 \pm 0.20^{+0.17}_{-0.16}$	$4.50 \pm 0.24^{+0.29}_{-0.29}$
P_+ [481]	$4.15 \pm 0.25^{+0.28}_{-0.27}$	$4.24 \pm 0.26^{+0.37}_{-0.32}$	$4.24 \pm 0.26^{+0.32}_{-0.32}$	$3.59 \pm 0.22^{+0.19}_{-0.18}$	-
$p_\ell^*, (M_X, q^2)$ fit [481]	$4.32 \pm 0.24^{+0.19}_{-0.21}$	$4.46 \pm 0.24^{+0.13}_{-0.13}$	$4.42 \pm 0.24^{+0.09}_{-0.11}$	$4.35 \pm 0.24^{+0.18}_{-0.18}$	-
p_ℓ^* [481]	$4.32 \pm 0.27^{+0.20}_{-0.21}$	$4.44 \pm 0.27^{+0.15}_{-0.14}$	$4.41 \pm 0.27^{+0.10}_{-0.12}$	$4.30 \pm 0.27^{+0.19}_{-0.18}$	-
M_X, q^2 [494]	-	-	-	-	$5.01 \pm 0.39^{+0.32}_{-0.32}$
Average	$4.45 \pm 0.16^{+0.21}_{-0.22}$	$4.52 \pm 0.16^{+0.15}_{-0.16}$	$4.51 \pm 0.16^{+0.12}_{-0.15}$	$4.05 \pm 0.13^{+0.18}_{-0.11}$	$4.62 \pm 0.20^{+0.29}_{-0.29}$

Table 82: Summary of inclusive determinations of $|V_{ub}|$. The errors quoted on $|V_{ub}|$ correspond to experimental and theoretical uncertainties, except for the last two measurements where the errors are due to the *BABAR* endpoint analysis, the *BABAR* $b \rightarrow s\gamma$ analysis [486], the theoretical errors and V_{ts} for the last averages.

Framework	$ V_{ub} [10^{-3}]$
BLNP	$4.45 \pm 0.15^{+0.20}_{-0.21}$
DGE	$4.52 \pm 0.16^{+0.15}_{-0.16}$
GGOU	$4.51 \pm 0.16^{+0.12}_{-0.15}$
ADFR	$4.05 \pm 0.13^{+0.18}_{-0.11}$
BLL (m_X/q^2 only)	$4.62 \pm 0.20 \pm 0.29$
LLR (<i>BABAR</i>) [486]	$4.43 \pm 0.45 \pm 0.29$
LLR (<i>BABAR</i>) [487]	$4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28$
LNP (<i>BABAR</i>) [487]	$4.40 \pm 0.30 \pm 0.41 \pm 0.23$

Table 83: Summary of the results on $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. The errors quoted correspond to statistical and systematic uncertainties, respectively.

	Belle [513]	<i>BABAR</i> [515]
$\mathcal{R}(D^0)$	$0.70^{+0.19}_{-0.18} \text{ } ^{+0.11}_{-0.09}$	$0.99 \pm 0.19 \pm 0.13$
$\mathcal{R}(D^{*0})$	$0.47^{+0.11}_{-0.10} \text{ } ^{+0.06}_{-0.07}$	$1.71 \pm 0.17 \pm 0.13$
$\mathcal{R}(D^+)$	$0.48^{+0.22}_{-0.19} \text{ } ^{+0.06}_{-0.05}$	$1.01 \pm 0.18 \pm 0.12$
$\mathcal{R}(D^{*+})$	$0.48^{+0.14}_{-0.12} \text{ } ^{+0.06}_{-0.04}$	$1.74 \pm 0.19 \pm 0.12$

6 b -hadron decays to charmed hadrons

Ground state B mesons and b -baryons dominantly decay to particles containing a charm quark via the $b \rightarrow c$ quark transition. Therefore these decays are sensitive to the $|V_{cb}|$ CKM matrix element. Usually semileptonic modes are used for $|V_{cb}|$ measurements which are discussed in Section 5. Some B meson decays to open charmed hadrons are fundamental decays for the measurements of CP -violation phases like $\phi_s^{c\bar{c}s}$ (Section 3), $\beta = \phi_1$ and $\gamma = \phi_3$ (Section 4).

The fact that decays to charmed hadrons are the dominant b -hadron decays makes them a very important part of the experimental programme. Understanding the rate of charm production in b -hadron decays is crucial to validate the HQE that underpins much of the theoretical framework for b physics (see, for example, Ref. [519] for a recent review). Moreover, such decays are often used as the normalization mode for measurements of rarer decays. In addition, they are the dominant background in many analyses. To accurately model the background with simulated data it is essential to have a precise knowledge of the contributing decay modes. In particular, with the expected increase in the data samples at LHCb and Belle II, the enhanced statistical sensitivity has to be matched by low systematic uncertainties due to knowledge of the dominant b -hadron decay modes. For multibody decays, knowledge of the distribution of decays across the phase-space (*e.g.* the Dalitz plot density for three-body decays or the polarization amplitudes for vector-vector final states) is required in addition to the total branching fraction.

The large yields of b -hadron decays to multibody final states containing charm makes them ideal to study the spectroscopy of both open charm and charmonia (or charmonia-like) mesons. In particular, they have been used to both discover and measure the properties of exotic particles such as the $X(3872)$ [520,521] and $Z(4430)$ [522,523] states. The large yields available similarly make $b \rightarrow c$ decays very useful to study baryon-antibaryon pair production.

In addition to the dominant b -hadron decays to final states containing charmed hadrons, there are several decays in this category that are expected to be highly suppressed in the Standard Model. These are of interest to probe particular decay topologies (*e.g.* the $B^- \rightarrow D_s^- \phi$ decay, which is dominated by the so-called annihilation diagram) and thereby constrain effects in other hadronic decays or to search for new physics. There are also other $b \rightarrow c$ decays, such as $\bar{B}^0 \rightarrow D_s^- \pi^+$, that are mediated by the W emission involving the $|V_{ub}|$ CKM matrix element. Finally, some $b \rightarrow c$ decays involving lepton flavour or number violation are completely forbidden in the Standard Model, and therefore provide highly sensitive null tests.

In this section, we give an exhaustive list of measured branching ratios of decay modes to charmed hadrons. Compared to the previous version of B to charm results the averaging procedure was updated to follow the methodology described in Section 2. Where available, correlations between measurements are taken into account. We provide averages of the polarization amplitudes of B meson decays to vector-vector states, but we do not currently provide detailed averages of quantities obtained from Dalitz plot analyses, due to the complications arising from the dependence on the model used.

The results are presented in subsections organized by the decaying bottom hadron: $\bar{B}^0$ (Sec. 6.1), B^- (Sec. 6.2), $\bar{B}^0/B^-$ admixture (Sec. 6.3), $\bar{B}_s^0$ (Sec. 6.4), B_c^- (Sec. 6.5), b baryons (Sec. 6.6). For each subsection the measurements are arranged, considering the final state, into the following groups: a single charmed meson, two charmed mesons, a charmonium state, a charm baryon, or other states, like for example the $X(3872)$ meson. The individual measurements and averages are shown as numerical values in tables followed by a graphical represen-

tation of the averages. The symbol $\mathcal{B}$ is used for branching ratios, f for production fractions (see Section 3), and σ for cross sections. The decay amplitudes for longitudinal, parallel, and perpendicular transverse polarization in pseudoscalar to vector-vector decays are denoted $\mathcal{A}_0$, $\mathcal{A}_\parallel$, and $\mathcal{A}_\perp$, respectively, and the definitions $\delta_\parallel = \arg(\mathcal{A}_\parallel/\mathcal{A}_0)$ and $\delta_\perp = \arg(\mathcal{A}_\perp/\mathcal{A}_0)$ are used for their relative phases. The inclusion of charge conjugate modes is always implied.

Following the approach used by the PDG [5], for decays that involve neutral kaons we mainly quote results in terms of final states including either a K^0 or $\bar{K}^0$ meson (instead of a K_s^0 or K_L^0). In some cases where the decay is not flavour-specific and the final state is not self-conjugate, the inclusion of the conjugate final state is implied – in fact, the flavour of the neutral kaon is never determined experimentally, and so the specification as K^0 or $\bar{K}^0$ simply follows the quark model expectation for the dominant decay. An exception occurs for some B_s^0 decays, specifically those to CP eigenstates, where the width difference between the mass eigenstates (see Sec. 3) means that the measured branching fraction, integrated over decay time, is specific to the studied final state [524]. Therefore it is appropriate to quote the branching fraction for, *e.g.*, $\bar{B}_s^0 \rightarrow J/\psi K_s^0$ instead of $\bar{B}_s^0 \rightarrow J/\psi \bar{K}^0$.

6.1 Decays of $\bar{B}^0$ mesons

Measurements of $\bar{B}^0$ decays to charmed hadrons are summarized in Sections 6.1.1 to 6.1.5.

6.1.1 Decays to a single open charm meson

Averages of $\bar{B}^0$ decays to a single open charm meson are shown in Tables 84–93 and Figs. 63–72.

Table 84: Decays to a $D^{(*)}$ meson and one or more pions [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \pi^0)$	BABAR [525]: $0.269 \pm 0.009 \pm 0.013$ Belle [526]: $0.225 \pm 0.014 \pm 0.035$	0.262 ± 0.015
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \pi^0)$	BABAR [525]: $0.305 \pm 0.014 \pm 0.028$ Belle [526]: $0.139 \pm 0.018 \pm 0.026$	0.223 ± 0.022
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$	BABAR [527]: $3.03 \pm 0.23 \pm 0.23$ BABAR [528]: $2.55 \pm 0.05 \pm 0.16$	2.65 ± 0.15
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \pi^-)$	BABAR [527]: $2.99 \pm 0.23 \pm 0.24$ BABAR [528]: $2.79 \pm 0.08 \pm 0.17$ Belle [529]: $2.30 \pm 0.06 \pm 0.19$	2.62 ± 0.13
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \pi^+ \pi^-)$	Belle [529]: $1.07 \pm 0.06 \pm 0.10$ Belle [530]: $0.80 \pm 0.06 \pm 0.15$	0.98 ± 0.09
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \pi^+ \pi^-)$	Belle [529]: $1.09 \pm 0.08 \pm 0.16$ Belle [530]: $0.62 \pm 0.12 \pm 0.18$	0.90 ± 0.14
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \pi^- \pi^+ \pi^-)$	Belle [531]: $6.81 \pm 0.23 \pm 0.72$	6.81 ± 0.76
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \pi^- \pi^+ \pi^- \pi^+)$	Belle [531]: $2.60 \pm 0.47 \pm 0.37$	2.60 ± 0.60
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \pi^- \pi^+ \pi^- \pi^+ \pi^-)$	Belle [531]: $4.72 \pm 0.59 \pm 0.71$	4.72 ± 0.92
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \omega(782) \pi^-)$	BABAR [532]: $2.88 \pm 0.21 \pm 0.31$	2.88 ± 0.38

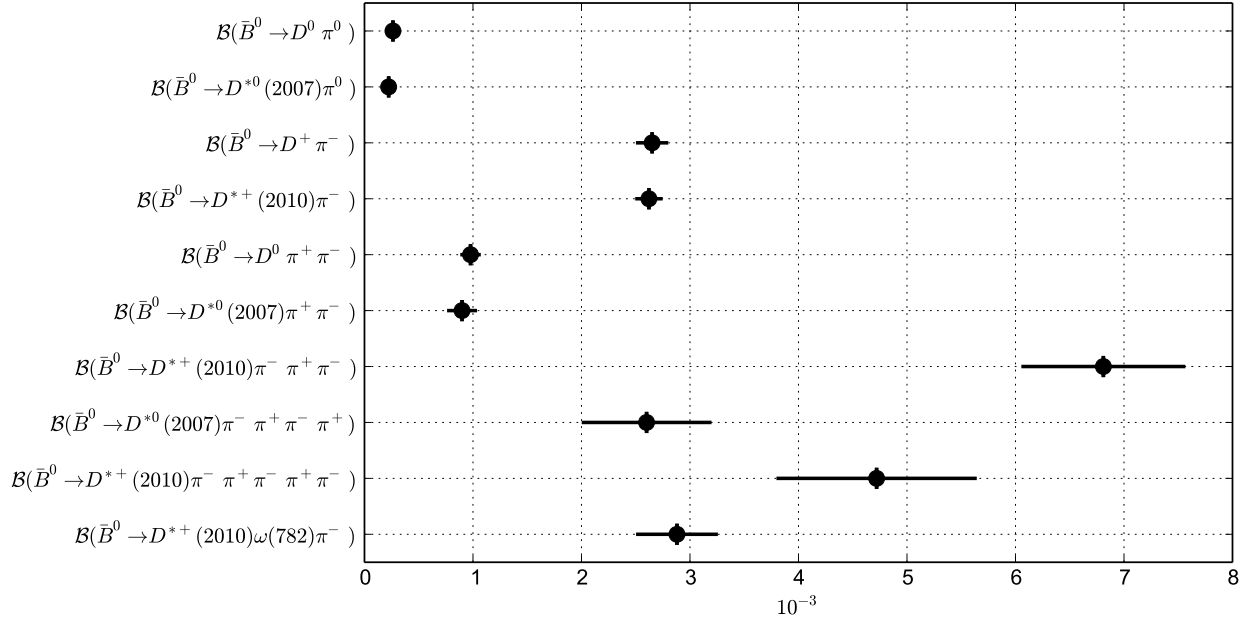


Figure 63: Summary of the averages from Table 84.

Table 85: Decays to a $D^{(*)0}$ meson and a light meson [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \rho^0(770))$	Belle [529]: $2.91 \pm 0.28^{+0.34}_{-0.63}$ Belle [530]: $2.9 \pm 1.0 \pm 0.4$	2.91 ± 0.51
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \rho^0(770))$	Belle [529]: $3.73 \pm 0.87^{+0.49}_{-0.47}$ Belle [530]: < 5.1	$3.73^{+1.00}_{-0.99}$
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \eta)$	BABAR [525]: $2.53 \pm 0.09 \pm 0.11$ Belle [526]: $1.77 \pm 0.16 \pm 0.21$	2.36 ± 0.13
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \eta)$	BABAR [525]: $2.69 \pm 0.14 \pm 0.23$ Belle [526]: $1.40 \pm 0.28 \pm 0.26$	2.26 ± 0.22
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \eta'(958))$	BABAR [525]: $1.48 \pm 0.13 \pm 0.07$ Belle [533]: $1.14 \pm 0.20^{+0.10}_{-0.13}$	1.38 ± 0.12
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \eta'(958))$	BABAR [525]: $1.48 \pm 0.22 \pm 0.13$ Belle [533]: $1.21 \pm 0.34 \pm 0.22$	1.40 ± 0.22
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \omega(782))$	BABAR [525]: $2.57 \pm 0.11 \pm 0.14$ Belle [526]: $2.37 \pm 0.23 \pm 0.28$	2.53 ± 0.16
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \omega(782))$	BABAR [525]: $4.55 \pm 0.24 \pm 0.39$ Belle [526]: $2.29 \pm 0.39 \pm 0.40$	3.64 ± 0.35
$\mathcal{B}(\bar{B}^0 \rightarrow f_2(1270) D^0)$	Belle [529]: $1.95 \pm 0.34^{+0.50}_{-0.38}$	$1.95^{+0.60}_{-0.51}$
$\mathcal{B}(\bar{B}^0 \rightarrow f_2(1270) D^{*0}(2007))$	Belle [529]: $1.86 \pm 0.65^{+1.00}_{-0.79}$	$1.86^{+1.19}_{-1.03}$

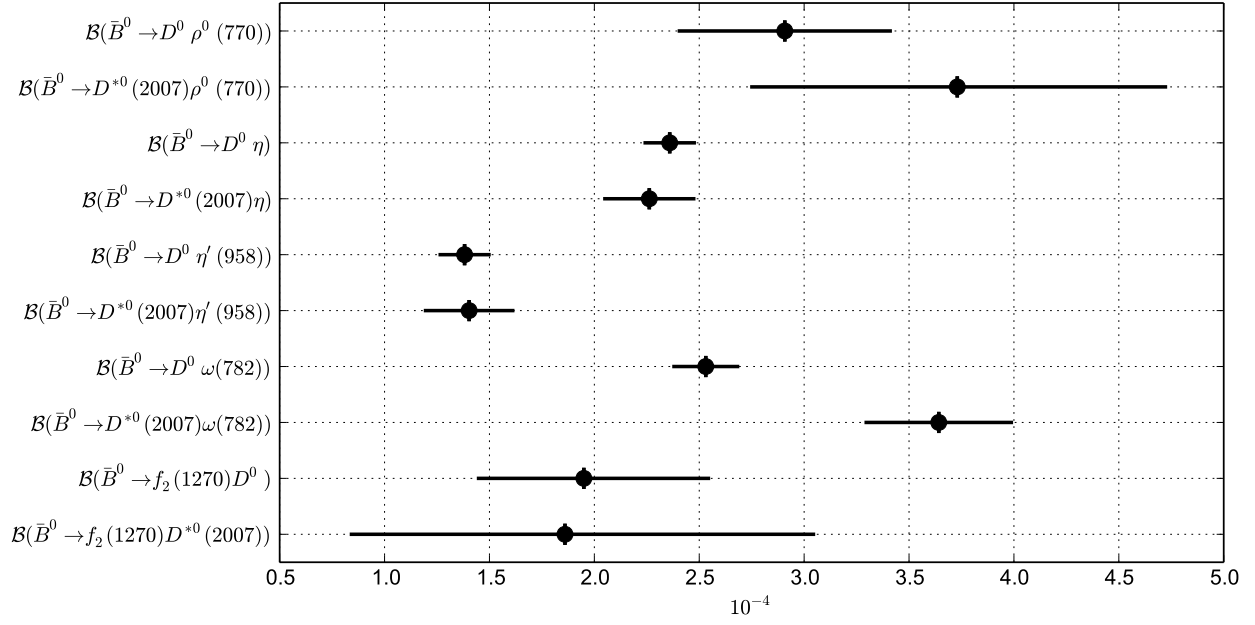


Figure 64: Summary of the averages from Table 85.

Table 86: Decays to a $D^{(*)+}$ meson and one or more kaons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^-)$	LHCb [46]: $0.22 \pm 0.00 \pm 0.01$ LHCb [48]: $0.201 \pm 0.018 \pm 0.014$ Belle [534]: $0.204 \pm 0.045 \pm 0.034$	0.215 ± 0.011
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} (2010) K^-)$	Belle [534]: $0.204 \pm 0.041 \pm 0.023$	0.204 ± 0.047
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^{*-} (892))$	BABAR [535]: $0.46 \pm 0.06 \pm 0.05$	0.46 ± 0.08
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} (2010) K^{*-} (892))$	BABAR [535]: $0.32 \pm 0.06 \pm 0.03$	0.32 ± 0.07
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^0 \pi^-)$	BABAR [535]: $0.49 \pm 0.07 \pm 0.05$	0.49 ± 0.09
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} (2010) K^0 \pi^-)$	BABAR [535]: $0.30 \pm 0.07 \pm 0.03$	0.30 ± 0.08
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^- K^0)$	Belle [536]: < 0.31	< 0.31
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} (2010) K^- K^0)$	Belle [536]: < 0.47	< 0.47
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^- K^{*0} (892))$	Belle [536]: $0.88 \pm 0.11 \pm 0.15$	0.88 ± 0.19
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} (2010) K^- K^{*0} (892))$	Belle [536]: $1.29 \pm 0.22 \pm 0.25$	1.29 ± 0.33

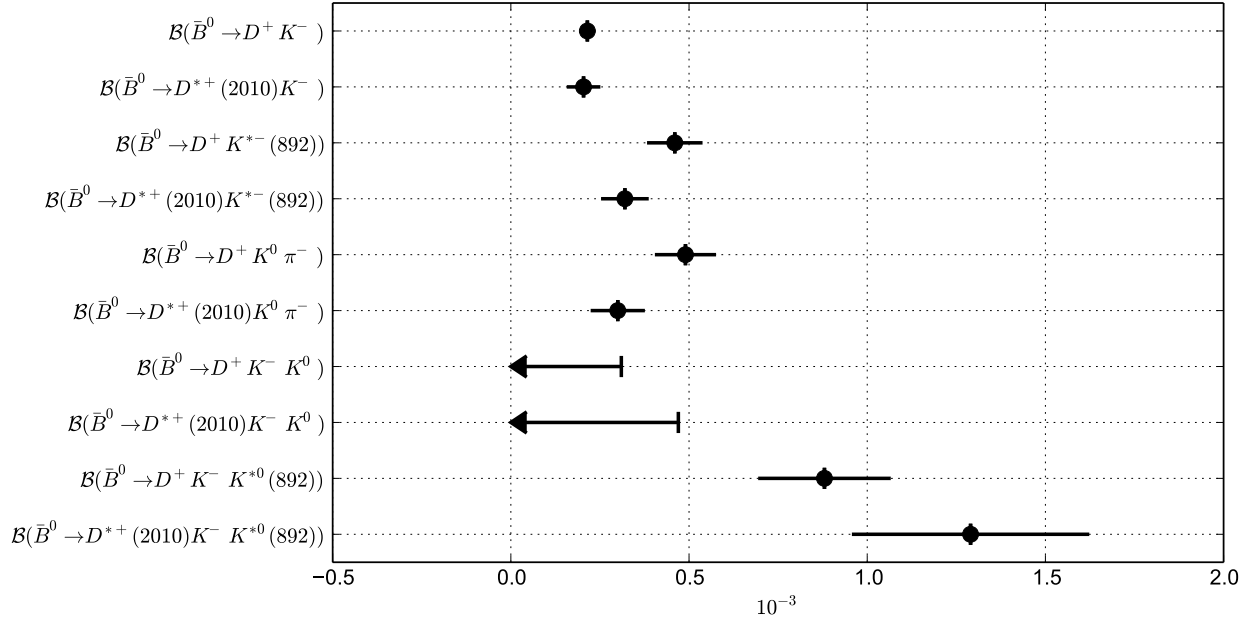


Figure 65: Summary of the averages from Table 86.

Table 87: Decays to a $D^{(*)0}$ meson and a kaon [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{K}^0)$	BABAR [537]: $0.53 \pm 0.07 \pm 0.03$ Belle [538]: $0.50^{+0.13}_{-0.12} \pm 0.06$	0.52 ± 0.07
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \bar{K}^0)$	BABAR [537]: $0.36 \pm 0.12 \pm 0.03$ Belle [538]: < 0.66	0.36 ± 0.12
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 K^- \pi^+)$	BABAR [539]: $0.88 \pm 0.15 \pm 0.09$	0.88 ± 0.17
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{K}^{*0}(892))$	BABAR [537]: $0.40 \pm 0.07 \pm 0.03$ Belle [538]: $0.48^{+0.11}_{-0.10} \pm 0.05$	0.42 ± 0.06
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{K}^{*0}(892)) \times \mathcal{B}(\bar{K}^{*0}(892) \rightarrow K^- \pi^+)$	BABAR [539]: $0.38 \pm 0.06 \pm 0.04$	0.38 ± 0.07
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \bar{K}^{*0}(892))$	Belle [538]: < 0.69	< 0.69
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^{*0}(2007) \bar{K}^{*0}(892))$	Belle [538]: < 0.40	< 0.40
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^0 K^- \pi^+)$	BABAR [539]: < 0.19	< 0.19
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^{*0}(892))$	BABAR [537]: $0.00 \pm 0.05 \pm 0.03$ Belle [538]: < 0.18	0.00 ± 0.06

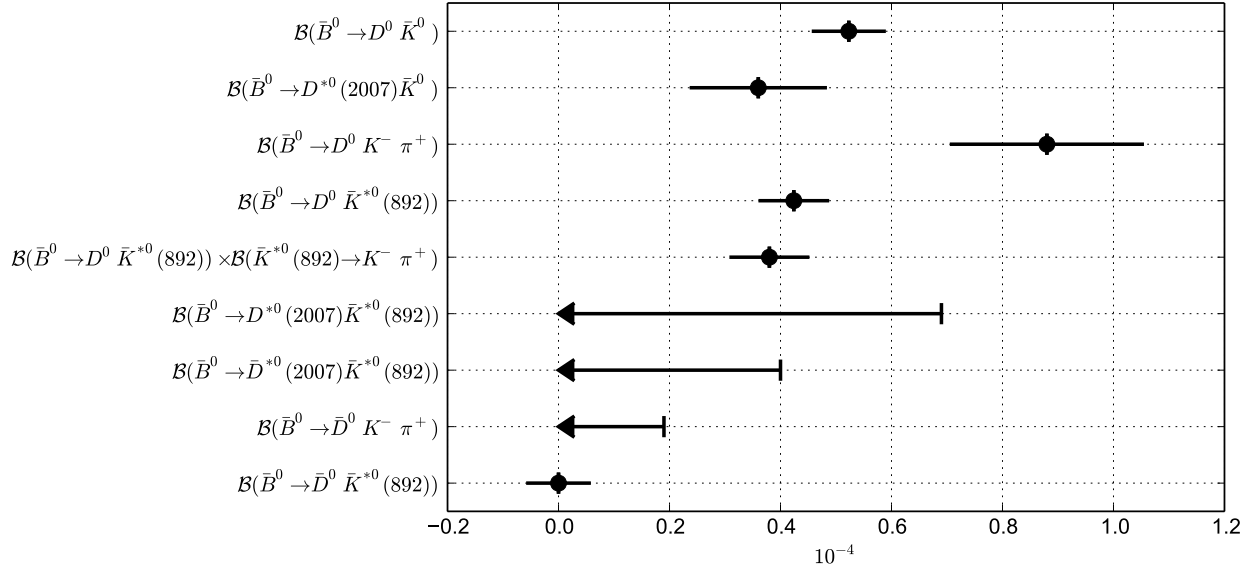


Figure 66: Summary of the averages from Table 87.

Table 88: Decays to a $D_s^{(*)}$ meson [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- \pi^+)$	BABAR [540]: $0.25 \pm 0.04 \pm 0.02$ Belle [541]: $0.199 \pm 0.026 \pm 0.018$	0.216 ± 0.026
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} \pi^+)$	BABAR [540]: $0.26^{+0.05}_{-0.04} \pm 0.02$ Belle [542]: $0.175 \pm 0.034 \pm 0.020$	0.207 ± 0.032
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- \rho^+(770))$	BABAR [540]: $0.11^{+0.09}_{-0.08} \pm 0.03$	$0.11^{+0.09}_{-0.09}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} \rho^+(770))$	BABAR [540]: $0.41^{+0.13}_{-0.12} \pm 0.04$	$0.41^{+0.14}_{-0.13}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- a_0^+(980))$	BABAR [543]: $0.06^{+0.14}_{-0.11} \pm 0.01$	$0.06^{+0.14}_{-0.11}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} a_0^+(980))$	BABAR [543]: $0.14^{+0.21}_{-0.16} \pm 0.03$	$0.14^{+0.21}_{-0.16}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- a_2^+(1320))$	BABAR [543]: $0.64^{+1.04}_{-0.57} \pm 0.15$	$0.64^{+1.05}_{-0.59}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} a_2^+(1320))$	BABAR [543]: < 2.0	< 2.0
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^+ K^-)$	BABAR [540]: $0.29 \pm 0.04 \pm 0.02$ Belle [541]: $0.191 \pm 0.024 \pm 0.017$	0.221 ± 0.025
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*+} K^-)$	BABAR [540]: $0.24 \pm 0.04 \pm 0.02$ Belle [542]: $0.202 \pm 0.033 \pm 0.022$	0.219 ± 0.031
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^+ K^{*-}(892))$	BABAR [540]: $0.35^{+0.10}_{-0.09} \pm 0.04$	$0.35^{+0.11}_{-0.10}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*+} K^{*-}(892))$	BABAR [540]: $0.32^{+0.14}_{-0.12} \pm 0.04$	$0.32^{+0.15}_{-0.13}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^+ K_S^0 \pi^-)$	BABAR [544]: $0.55 \pm 0.13 \pm 0.10$	0.55 ± 0.17
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*+} K^0 \pi^-)$	BABAR [544]: < 0.55	< 0.55

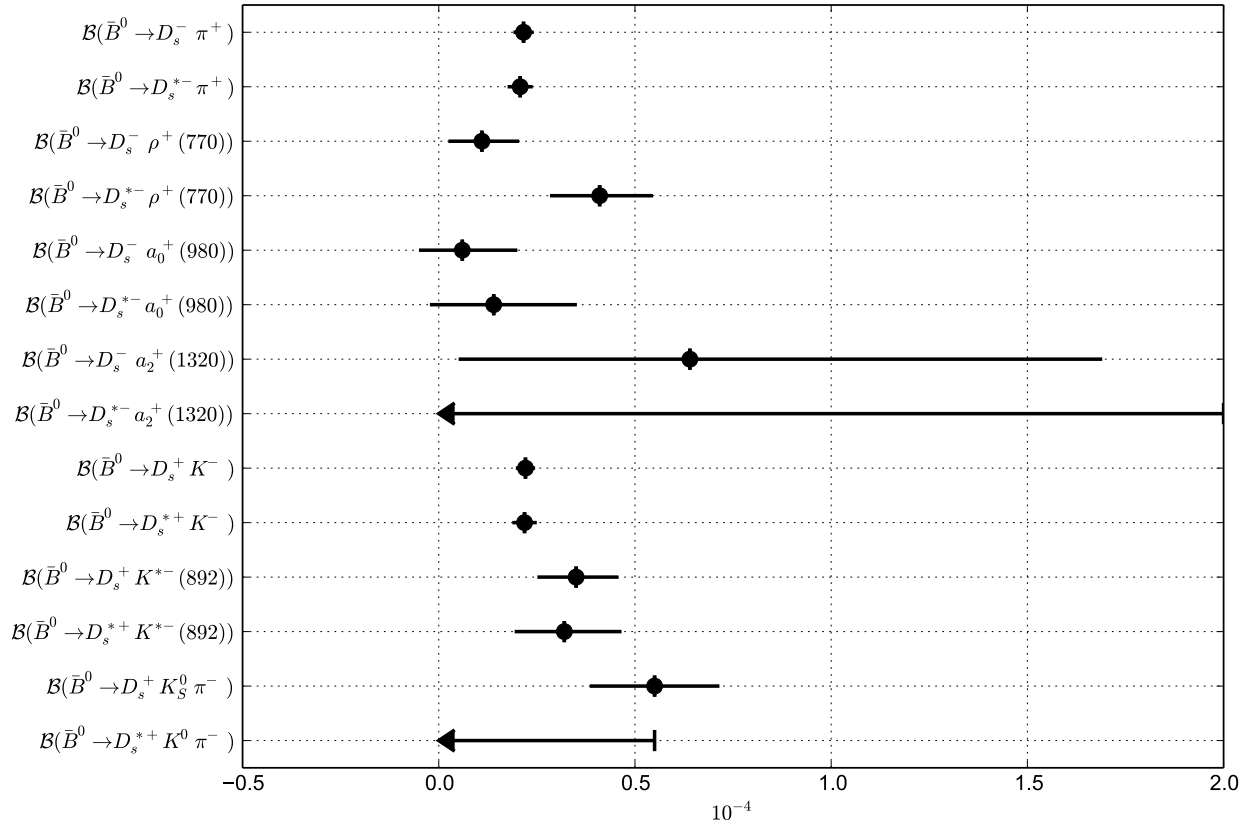


Figure 67: Summary of the averages from Table 88.

Table 89: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \rho^0(770))/\mathcal{B}(\bar{B}^0 \rightarrow D^0 \omega(782))$	Belle [530]: 1.6 ± 0.8	1.6 ± 0.8
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^0 K^- \pi^+)/\mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^0 \pi^- \pi^+)$	LHCb [545]: $0.106 \pm 0.007 \pm 0.008$	0.106 ± 0.011
$\mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^0 K^- K^+)/\mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^0 \pi^- \pi^+)$	LHCb [546]: $0.056 \pm 0.110 \pm 0.007$	0.056 ± 0.110
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^+ \pi^- \pi^-)/\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$	LHCb [547]: $2.38 \pm 0.11 \pm 0.21$	2.38 ± 0.24
$\frac{\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^-)}{\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)}$	LHCb [46]: $0.0822 \pm 0.0011 \pm 0.0025$ Belle [534]: $0.068 \pm 0.015 \pm 0.007$	0.0818 ± 0.0027
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^- \pi^+ \pi^-)/\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^+ \pi^- \pi^-)$	LHCb [548]: $0.059 \pm 0.011 \pm 0.005$	0.059 ± 0.012
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \pi^-)/\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$	BABAR [527]: $0.99 \pm 0.11 \pm 0.08$	0.99 ± 0.14
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) K^-)/\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \pi^-)$	BABAR [539]: $0.0776 \pm 0.0034 \pm 0.0029$ Belle [534]: $0.074 \pm 0.015 \pm 0.006$	0.0773 ± 0.0043
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \pi^-)/\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$	BABAR [527]: $0.77 \pm 0.22 \pm 0.29$	0.77 ± 0.36
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^+ K^- \pi^+ \pi^-)/\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ K^- \pi^+ \pi^-)$	LHCb [549]: $0.54 \pm 0.07 \pm 0.07$	0.54 ± 0.10

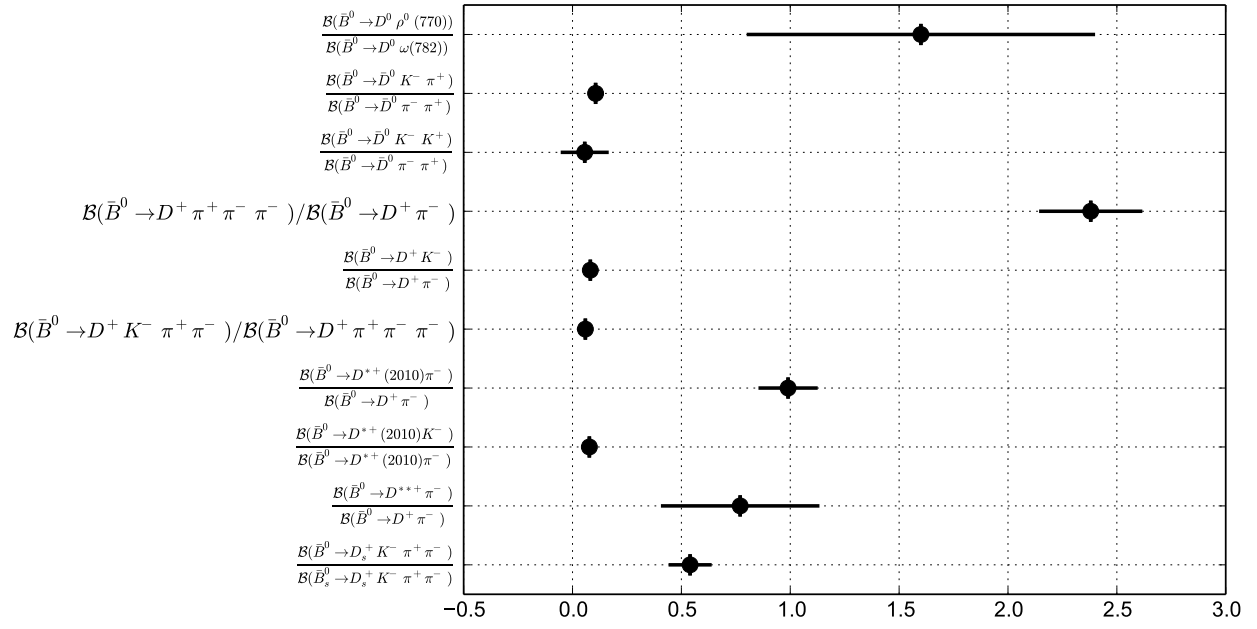


Figure 68: Summary of the averages from Table 89.

Table 90: Absolute product decay rates to excited D mesons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+(2420)\pi^-) \times \mathcal{B}(D_1^+(2420) \rightarrow D^{*0}(2007)\pi^+)$	Belle [529]: $0.368 \pm 0.060^{+0.096}_{-0.050}$	$0.368^{+0.113}_{-0.078}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+(2420)\pi^-) \times \mathcal{B}(D_1^+(2420) \rightarrow D^+\pi^-\pi^+)$	Belle [550]: $0.089 \pm 0.015^{+0.017}_{-0.031}$	$0.089^{+0.023}_{-0.034}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+(2420)\pi^-) \times \mathcal{B}(D_1^+(2420) \rightarrow D^{*+}(2010)\pi^-\pi^+)$	Belle [550]: < 0.033	< 0.033
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+}(2460)\pi^-) \times \mathcal{B}(D_2^{*+}(2460) \rightarrow D^0\pi^+)$	Belle [529]: $0.308 \pm 0.033^{+0.017}_{-0.009}$	$0.308^{+0.037}_{-0.034}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+}(2460)\pi^-) \times \mathcal{B}(D_2^{*+}(2460) \rightarrow D^{*0}(2007)\pi^+)$	Belle [529]: $0.245 \pm 0.042^{+0.052}_{-0.048}$	$0.245^{+0.067}_{-0.064}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+}(2460)\pi^-) \times \mathcal{B}(D_2^{*+}(2460) \rightarrow D^{*+}(2010)\pi^-\pi^+)$	Belle [550]: < 0.024	< 0.024
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+}(2460)K^-) \times \mathcal{B}(D_2^{*+}(2460) \rightarrow D^0\pi^+)$	BABAR [539]: $0.0183 \pm 0.0040 \pm 0.0031$	0.0183 ± 0.0051
$\mathcal{B}(\bar{B}^0 \rightarrow D_0^{*+}\pi^-) \times \mathcal{B}(D_0^{*+} \rightarrow D^0\pi^+)$	Belle [529]: < 0.12	< 0.12
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+(H)\pi^-) \times \mathcal{B}(D_1^+(H) \rightarrow D^{*0}(2007)\pi^+)$	Belle [529]: < 0.07	< 0.07
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^0(H)\omega(782)) \times \mathcal{B}(D_1^0(H) \rightarrow D^{*+}(2010)\pi^-)$	BABAR [532]: $0.41 \pm 0.12 \pm 0.11$	0.41 ± 0.16
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^-(2460)\pi^+) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^-\gamma)$	Belle [551]: < 0.0040	< 0.0040
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^+(2460)K^-) \times \mathcal{B}(D_{sJ}^+(2460) \rightarrow D_s^+\gamma)$	Belle [552]: $0.0053 \pm 0.0020^{+0.0016}_{-0.0015}$	$0.0053^{+0.0026}_{-0.0025}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^*(2317)^-\pi^+) \times \mathcal{B}(D_{sJ}^*(2317)^- \rightarrow D_s^-\pi^0)$	Belle [551]: < 0.025	< 0.025
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^*(2317)^+K^-) \times \mathcal{B}(D_{sJ}^*(2317)^+ \rightarrow D_s^+\pi^0)$	Belle [552]: $0.044 \pm 0.008 \pm 0.013$	0.044 ± 0.015

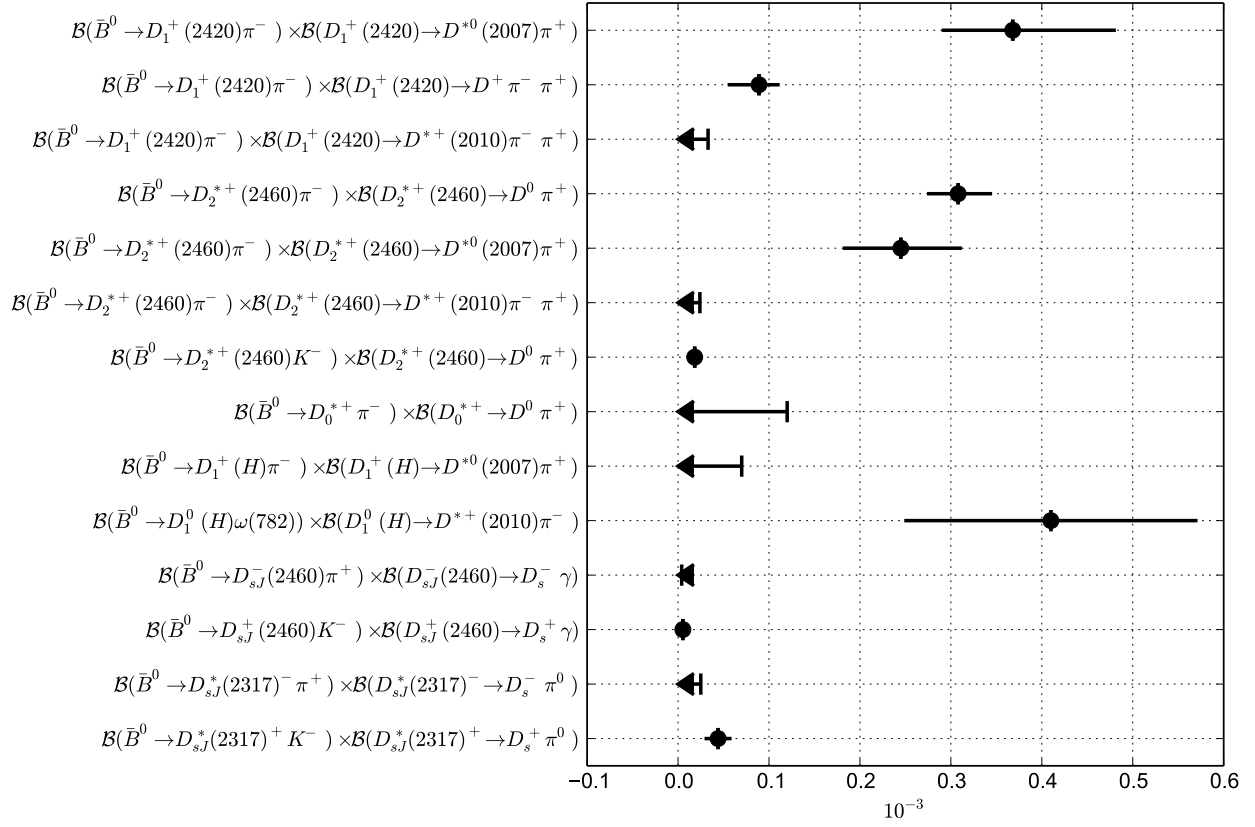


Figure 69: Summary of the averages from Table 90.

Table 91: Absolute and relative decay rates to excited D mesons [10^{-2}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \pi^-)$	<i>BABAR</i> [527]: $0.234 \pm 0.065 \pm 0.088$	0.234 ± 0.109
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+ \pi^-) \times \mathcal{B}(D_1^+ \rightarrow D^+ \pi^+ \pi^-) / \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^+ \pi^- \pi^-)$	LHCb [547]: $2.1 \pm 0.5^{+0.3}_{-0.5}$	$2.1^{+0.6}_{-0.7}$

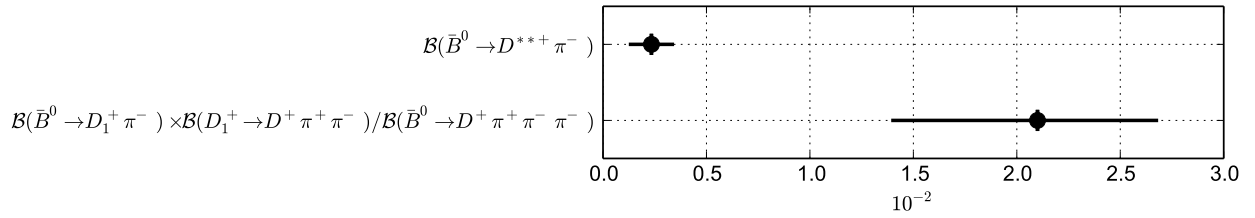


Figure 70: Summary of the averages from Table 91.

Table 92: Baryonic decays $[10^{-4}]$.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 p \bar{p})$	BABAR [553]: $1.02 \pm 0.04 \pm 0.06$ Belle [554]: $1.18 \pm 0.15 \pm 0.16$	1.04 ± 0.07
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) p \bar{p})$	BABAR [553]: $0.97 \pm 0.07 \pm 0.09$ Belle [554]: $1.20^{+0.33}_{-0.29} \pm 0.21$	0.99 ± 0.11
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ p \bar{p} \pi^-)$	BABAR [553]: $3.32 \pm 0.10 \pm 0.29$	3.32 ± 0.31
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) p \bar{p} \pi^-)$	BABAR [553]: $4.55 \pm 0.16 \pm 0.39$	4.55 ± 0.42
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 p \bar{p} \pi^- \pi^+)$	BABAR [553]: $2.99 \pm 0.21 \pm 0.45$	2.99 ± 0.50
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) p \bar{p} \pi^- \pi^+)$	BABAR [553]: $1.91 \pm 0.36 \pm 0.29$	1.91 ± 0.46
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^+ \Lambda \bar{p})$	Belle [555]: $0.29 \pm 0.07 \pm 0.06$	0.29 ± 0.09
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \Lambda^0 \bar{\Lambda}^0)$	BABAR [556]: $0.098^{+0.029}_{-0.026} \pm 0.019$ Belle [557]: $0.105^{+0.057}_{-0.044} \pm 0.014$	0.100 ± 0.028
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \Sigma^0 \bar{\Lambda} + \bar{B}^0 \rightarrow D^0 \Lambda \bar{\Sigma}^0)$	BABAR [556]: $0.15^{+0.09}_{-0.08} \pm 0.03$	$0.15^{+0.09}_{-0.09}$

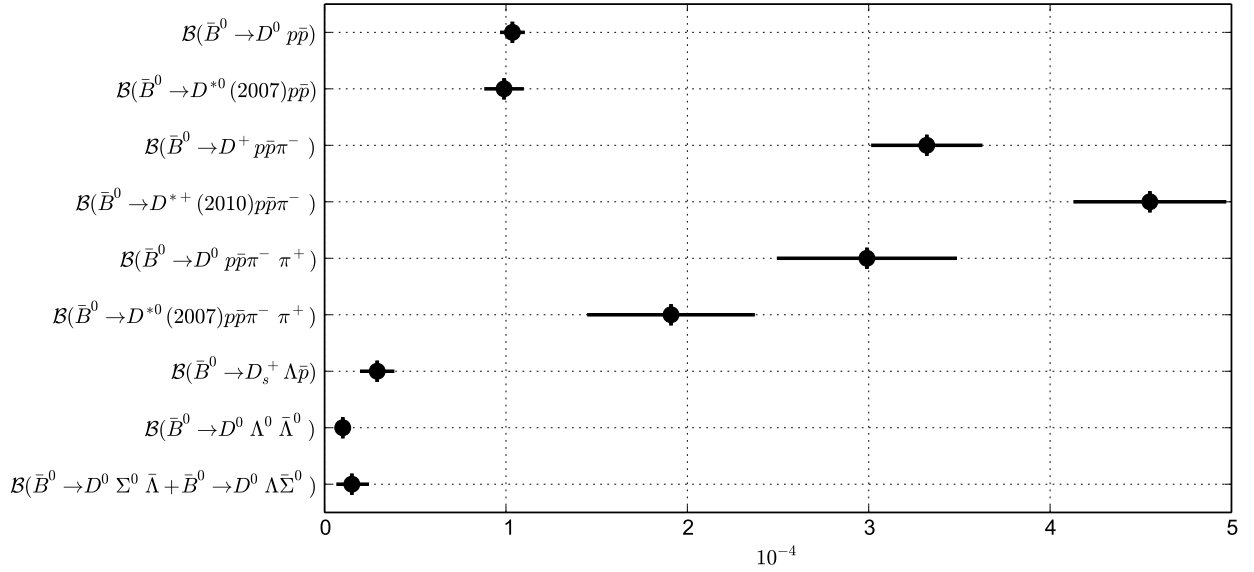


Figure 71: Summary of the averages from Table 92.

Table 93: Semileptonic decays.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \mu^- \bar{\nu}_\mu) / \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$	CDF [39]: $9.9 \pm 1.0 \pm 0.9$	9.9 ± 1.3
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \mu^- \bar{\nu}_\mu) / \mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \pi^-)$	CDF [39]: $16.5 \pm 2.3 \pm 1.1$	16.5 ± 2.6

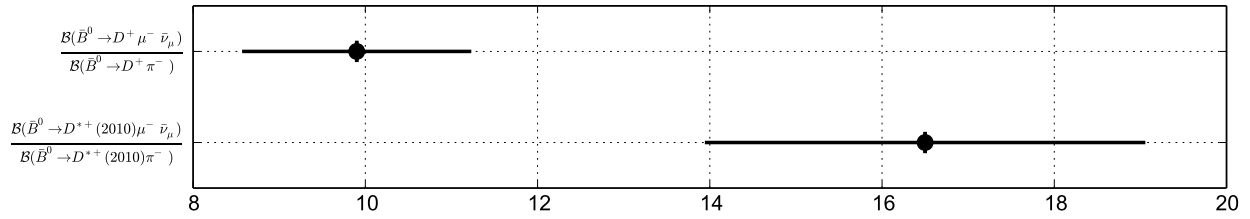


Figure 72: Summary of the averages from Table 93.

6.1.2 Decays to two open charm mesons

Averages of $\bar{B}^0$ decays to two open charm mesons are shown in Tables 94–100 and Figs. 73–79.

Table 94: Decays to $D^{(*)+} D^{(*)-}$ [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ D^-)$	BABAR [558]: $0.28 \pm 0.04 \pm 0.05$ Belle [257]: $0.212 \pm 0.016 \pm 0.018$	0.220 ± 0.023
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+)$	BABAR [558]: $0.57 \pm 0.07 \pm 0.07$ Belle [257]: $0.614 \pm 0.029 \pm 0.050$	0.603 ± 0.050
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*-} D^{*+}(2010))$	BABAR [558]: $0.81 \pm 0.06 \pm 0.10$ Belle [349]: $0.782 \pm 0.038 \pm 0.060$	0.790 ± 0.061
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{D}^0)$	BABAR [558]: < 0.06 Belle [559]: < 0.043	< 0.043
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{D}^{*0}(2007))$	BABAR [558]: < 0.29	< 0.29
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \bar{D}^{*0}(2007))$	BABAR [558]: < 0.09	< 0.09

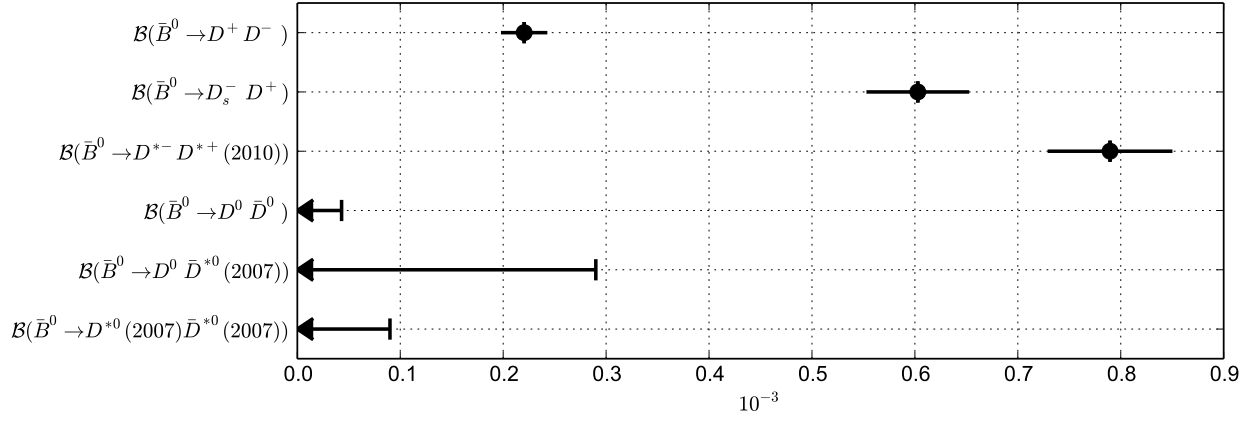


Figure 73: Summary of the averages from Table 94.

Table 95: Decays to two D mesons and a kaon [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ D^- \bar{K}^0)$	<i>BABAR</i> [560]: $0.75 \pm 0.12 \pm 0.12$	0.75 ± 0.17
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) D^- \bar{K}^0)$	<i>BABAR</i> [560]: $6.41 \pm 0.36 \pm 0.39$	6.41 ± 0.53
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*-}(2010) D^{*+}(2010) \bar{K}^0)$	<i>BABAR</i> [560]: $8.26 \pm 0.43 \pm 0.67$	8.26 ± 0.80
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) D^{*-}(2010) K_S^0)$	<i>BABAR</i> [236]: $4.4 \pm 0.4 \pm 0.7$ <i>Belle</i> [237]: $3.4 \pm 0.4 \pm 0.7$	3.9 ± 0.6
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \bar{D}^0 K^-)$	<i>BABAR</i> [560]: $1.07 \pm 0.07 \pm 0.09$	1.07 ± 0.11
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \bar{D}^0 K^-)$	<i>BABAR</i> [560]: $2.47 \pm 0.10 \pm 0.18$	2.47 ± 0.21
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \bar{D}^{*0}(2007) K^-)$	<i>BABAR</i> [560]: $3.46 \pm 0.18 \pm 0.37$	3.46 ± 0.41
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}(2010) \bar{D}^{*0}(2007) K^-)$	<i>BABAR</i> [560]: $10.6 \pm 0.3 \pm 0.9$	10.6 ± 0.9
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{D}^0 \bar{K}^0)$	<i>BABAR</i> [560]: $0.27 \pm 0.10 \pm 0.05$	0.27 ± 0.11
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{D}^{*0}(2007) \bar{K}^0)$	<i>BABAR</i> [560]: $1.08 \pm 0.32 \pm 0.36$	1.08 ± 0.48
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}(2007) \bar{D}^{*0}(2007) \bar{K}^0)$	<i>BABAR</i> [560]: $2.40 \pm 0.55 \pm 0.67$	2.40 ± 0.87
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{D}^0 \pi^0 \bar{K}^0)$	<i>Belle</i> [561]: $0.173 \pm 0.070^{+0.031}_{-0.053}$	$0.173^{+0.077}_{-0.088}$

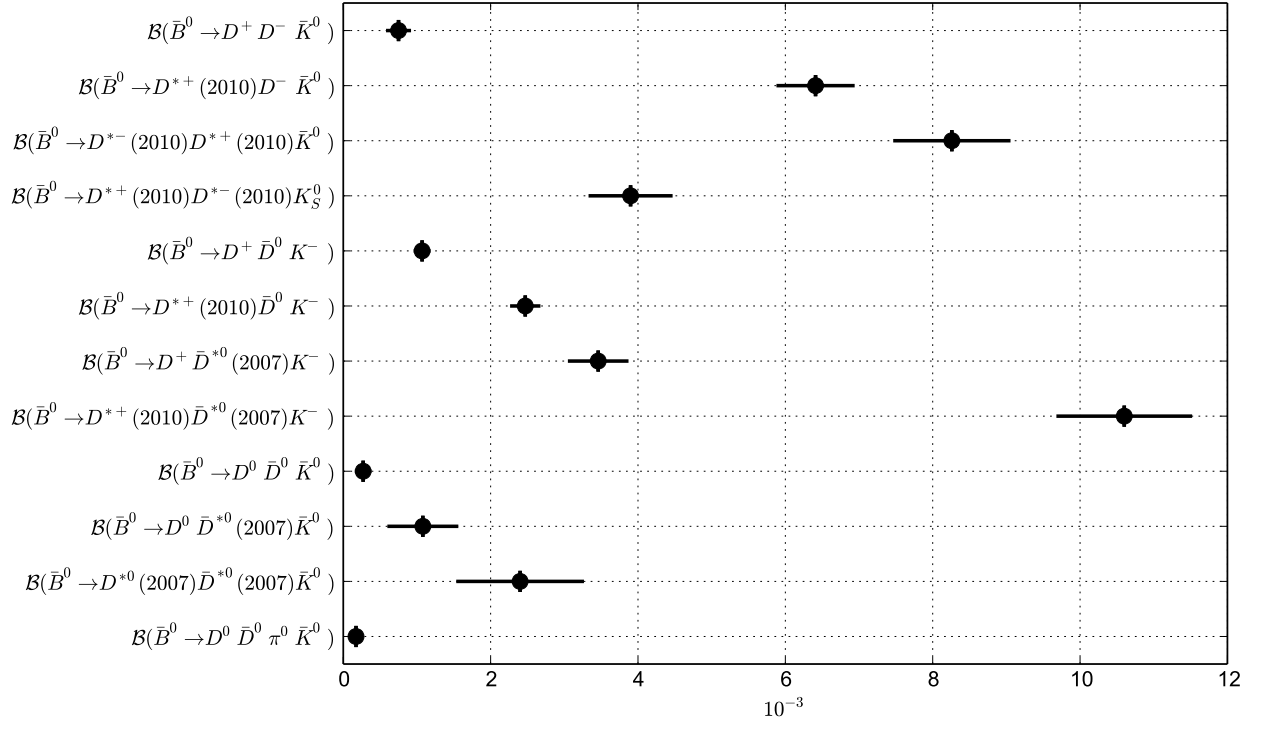


Figure 74: Summary of the averages from Table 95.

Table 96: Decays to $D_s^{(*)-} D^{(*)+}$ [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+)$	BABAR [562]: $9.0 \pm 1.8 \pm 1.4$ Belle [563]: $7.5 \pm 0.2 \pm 1.1$	7.8 ± 1.0
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^{*+}(2010))$	BABAR [564]: $10.3 \pm 1.4 \pm 2.9$ BABAR [562]: $5.7 \pm 1.6 \pm 0.9$	6.8 ± 1.6
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} D^{*+}(2010))$	BABAR [564]: $19.7 \pm 1.5 \pm 5.7$ BABAR [565]: $18.8 \pm 0.9 \pm 1.7$ BABAR [562]: $16.5 \pm 2.3 \pm 1.9$	18.1 ± 1.6
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} D^+)$	BABAR [562]: $6.7 \pm 2.0 \pm 1.1$	6.7 ± 2.3
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-) \times \mathcal{B}(\phi(1020) \rightarrow K^+ K^-)$	Belle [566]: $0.147 \pm 0.005 \pm 0.021$	0.147 ± 0.022
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $0.267 \pm 0.061 \pm 0.047$	0.267 ± 0.077
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^{*+}(2010)) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $0.511 \pm 0.094 \pm 0.072$	0.511 ± 0.118
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} D^+) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $0.414 \pm 0.119 \pm 0.094$	0.414 ± 0.152
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} D^{*+}(2010)) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $1.22 \pm 0.22 \pm 0.22$	1.22 ± 0.31

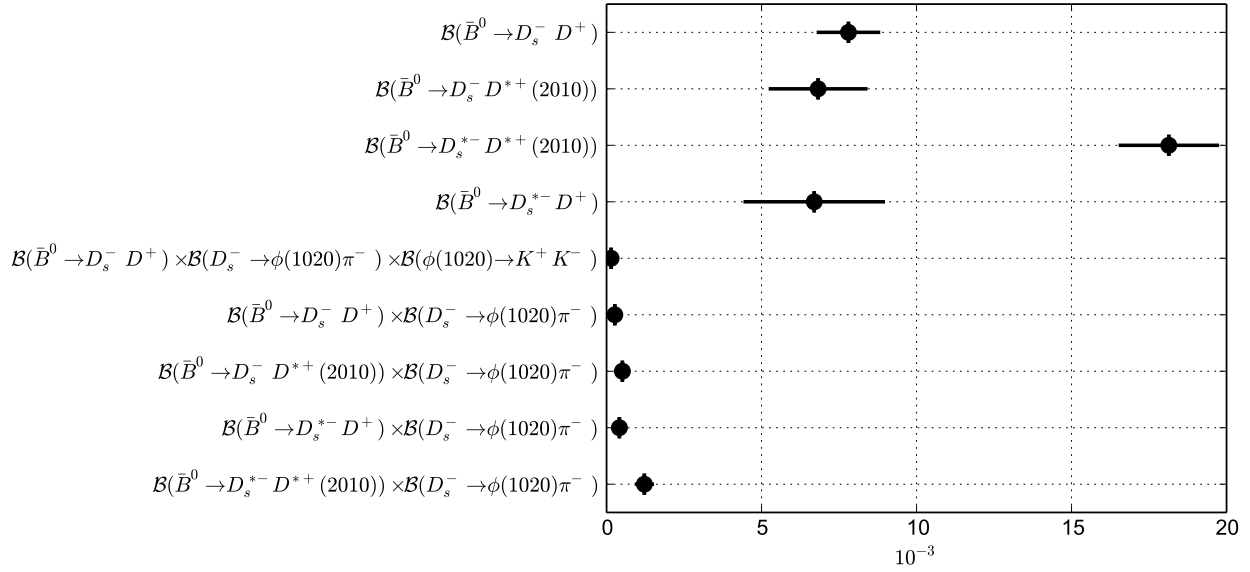


Figure 75: Summary of the averages from Table 96.

Table 97: Decays to $D_s^{(*)+}D_s^{(*)-}$ [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D_s^+)$	<i>BABAR</i> [567]: < 0.10 Belle [563]: < 0.036	< 0.036
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D_s^{*+})$	<i>BABAR</i> [567]: < 0.13	< 0.13
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*+} D_s^{*-})$	<i>BABAR</i> [567]: < 0.24	< 0.24

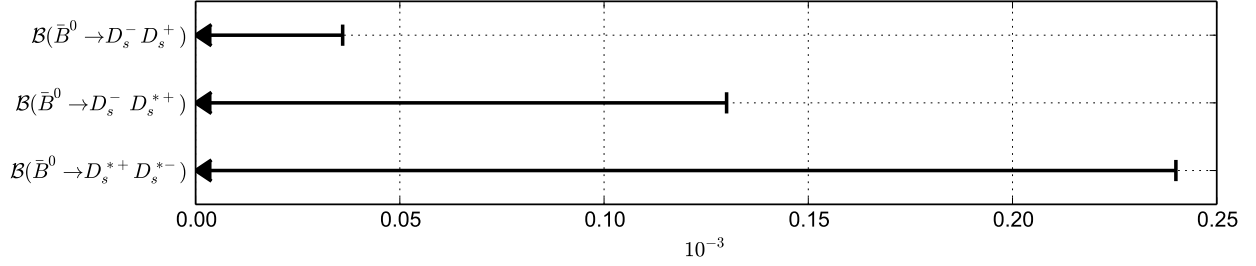


Figure 76: Summary of the averages from Table 97.

Table 98: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D^0 \bar{D}^0) / \mathcal{B}(\bar{B}^- \rightarrow D^0 \bar{D}_s^-)$	LHCb [568]: $0.0014 \pm 0.0006 \pm 0.0002$	0.0014 ± 0.0006
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+) / \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^+ \pi^- \pi^-)$	CDF [569]: $1.99 \pm 0.13 \pm 0.46$	1.99 ± 0.48
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^{*+}(2010)) / \mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+)$	CDF [569]: $1.5 \pm 0.4 \pm 0.1$	1.5 ± 0.4
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} D^+) / \mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+)$	CDF [569]: $0.9 \pm 0.2 \pm 0.1$	0.9 ± 0.2
$\mathcal{B}(\bar{B}^0 \rightarrow D_s^{*-} D^{*+}(2010)) / \mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+)$	CDF [569]: $2.6 \pm 0.5 \pm 0.2$	2.6 ± 0.5

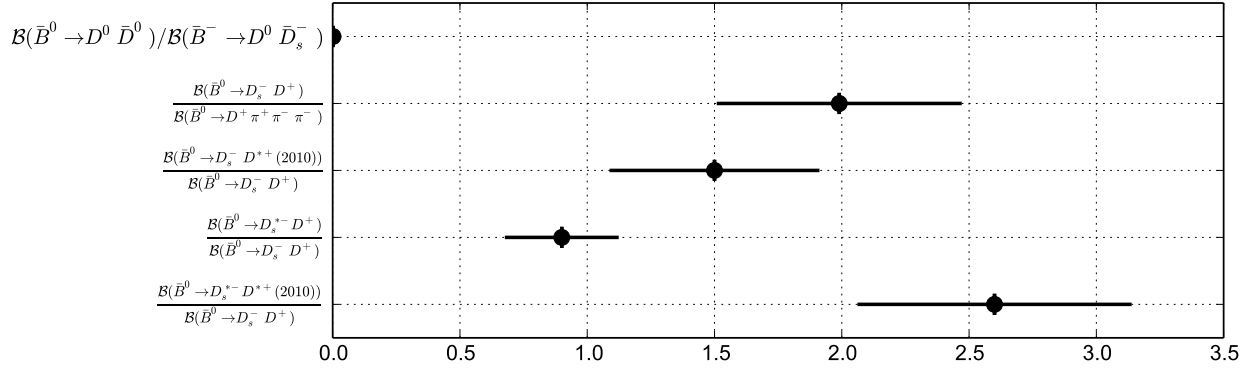


Figure 77: Summary of the averages from Table 98.

Table 99: Absolute decays rates to excited D_s mesons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^- (2460) D^+)$	<i>BABAR</i> [562]: $2.6 \pm 1.5 \pm 0.7$	2.6 ± 1.7
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^- (2460) D^{*+} (2010))$	<i>BABAR</i> [562]: $8.8 \pm 2.0 \pm 1.4$	8.8 ± 2.4
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^- (2536) D^{*+} (2010))$	<i>BABAR</i> [236]: $92 \pm 24 \pm 1$	92 ± 24

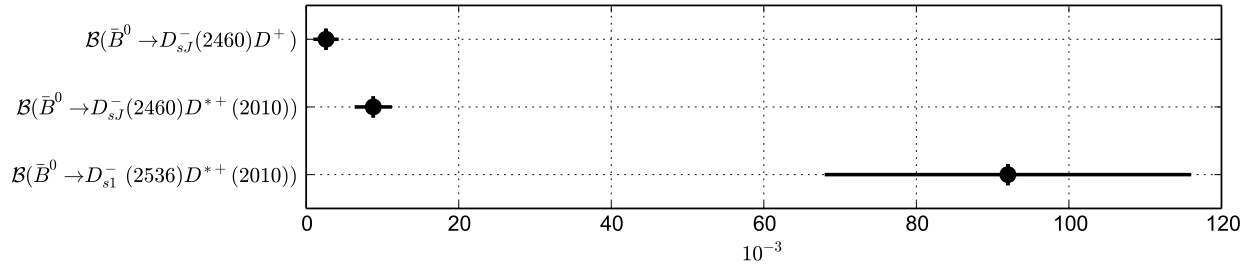


Figure 78: Summary of the averages from Table 99.

Table 100: Product decays rates to excited D_s mesons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^-(2536)D^+) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow D^{*-}(2010)\bar{K}^0)$	BABAR [570]: $0.261 \pm 0.103 \pm 0.031$	0.261 ± 0.108
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^-(2536)D^+) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow K^-\bar{D}^{*0}(2007))$	BABAR [570]: $0.171 \pm 0.048 \pm 0.032$	0.171 ± 0.058
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^-(2536)D^{*+}(2010)) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow D^{*-}(2010)\bar{K}^0)$	BABAR [570]: $0.500 \pm 0.151 \pm 0.067$	0.500 ± 0.165
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^-(2536)D^{*+}(2010)) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow \bar{D}^{*0}(2007)K^+)$	BABAR [570]: $0.332 \pm 0.088 \pm 0.066$	0.332 ± 0.110
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^+(2536)D^-) \times \mathcal{B}(D_{s1}^+(2536) \rightarrow D^{*0}(2007)K^+ + D^{*+}(2010)K^0)$	Belle [571]: $0.275 \pm 0.062 \pm 0.036$	0.275 ± 0.072
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^+(2536)D^{*-}(2010)) \times \mathcal{B}(D_{s1}^+(2536) \rightarrow D^{*0}(2007)K^+ + D^{*+}(2010)K^0)$	Belle [571]: $0.501 \pm 0.121 \pm 0.070$	0.501 ± 0.140
$\mathcal{B}(\bar{B}^0 \rightarrow D_{s1}^+(2536)D^{*-}(2010)) \times \mathcal{B}(D_{s1}^+(2536) \rightarrow D^{*+}(2010)K_S^0)$	Belle [237]: < 0.60	< 0.60
$\mathcal{B}(\bar{B}^0 \rightarrow D^+D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^-\pi^+\pi^-)$	Belle [572]: < 0.20	< 0.20
$\mathcal{B}(\bar{B}^0 \rightarrow D^+D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^-\pi^0)$	Belle [572]: < 0.36	< 0.36
$\mathcal{B}(\bar{B}^0 \rightarrow D^+D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^-\gamma)$	BABAR [573]: $0.8 \pm 0.2^{+0.3}_{-0.2}$ Belle [572]: $0.82^{+0.22}_{-0.19} \pm 0.25$	0.81 ± 0.23
$\mathcal{B}(\bar{B}^0 \rightarrow D^+D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^{*-}\pi^0)$	BABAR [573]: $2.8 \pm 0.8^{+1.1}_{-0.8}$ Belle [572]: $2.27^{+0.73}_{-0.62} \pm 0.68$	2.47 ± 0.76
$\mathcal{B}(\bar{B}^0 \rightarrow D^+D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^{*-}\gamma)$	Belle [572]: $0.13^{+0.20}_{-0.14} \pm 0.00$	$0.13^{+0.20}_{-0.14}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^-(2460)D^{*+}(2010)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^{*-}\pi^0)$	BABAR [573]: $5.5 \pm 1.2^{+2.1}_{-1.6}$	$5.5^{+2.5}_{-2.0}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^-(2460)D^{*+}(2010)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^-\gamma)$	BABAR [573]: $2.3 \pm 0.3^{+0.9}_{-0.6}$	$2.3^{+0.9}_{-0.7}$
$\mathcal{B}(\bar{B}^0 \rightarrow D^+D_{sJ}^*(2317)^-) \times \mathcal{B}(D_{sJ}^*(2317)^- \rightarrow D_s^-\pi^0)$	BABAR [573]: $1.8 \pm 0.4^{+0.7}_{-0.5}$ Belle [572]: $0.86^{+0.33}_{-0.26} \pm 0.26$	1.08 ± 0.35
$\mathcal{B}(\bar{B}^0 \rightarrow D^+D_{sJ}^*(2317)^-) \times \mathcal{B}(D_{sJ}^*(2317)^- \rightarrow D_s^{*-}\gamma)$	Belle [572]: $0.27^{+0.29}_{-0.22} \pm 0.00$	$0.27^{+0.29}_{-0.22}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_{sJ}^*(2317)^-D^{*+}(2010)) \times \mathcal{B}(D_{sJ}^*(2317)^- \rightarrow D_s^-\pi^0)$	BABAR [573]: $1.5 \pm 0.4^{+0.5}_{-0.4}$	$1.5^{+0.7}_{-0.5}$

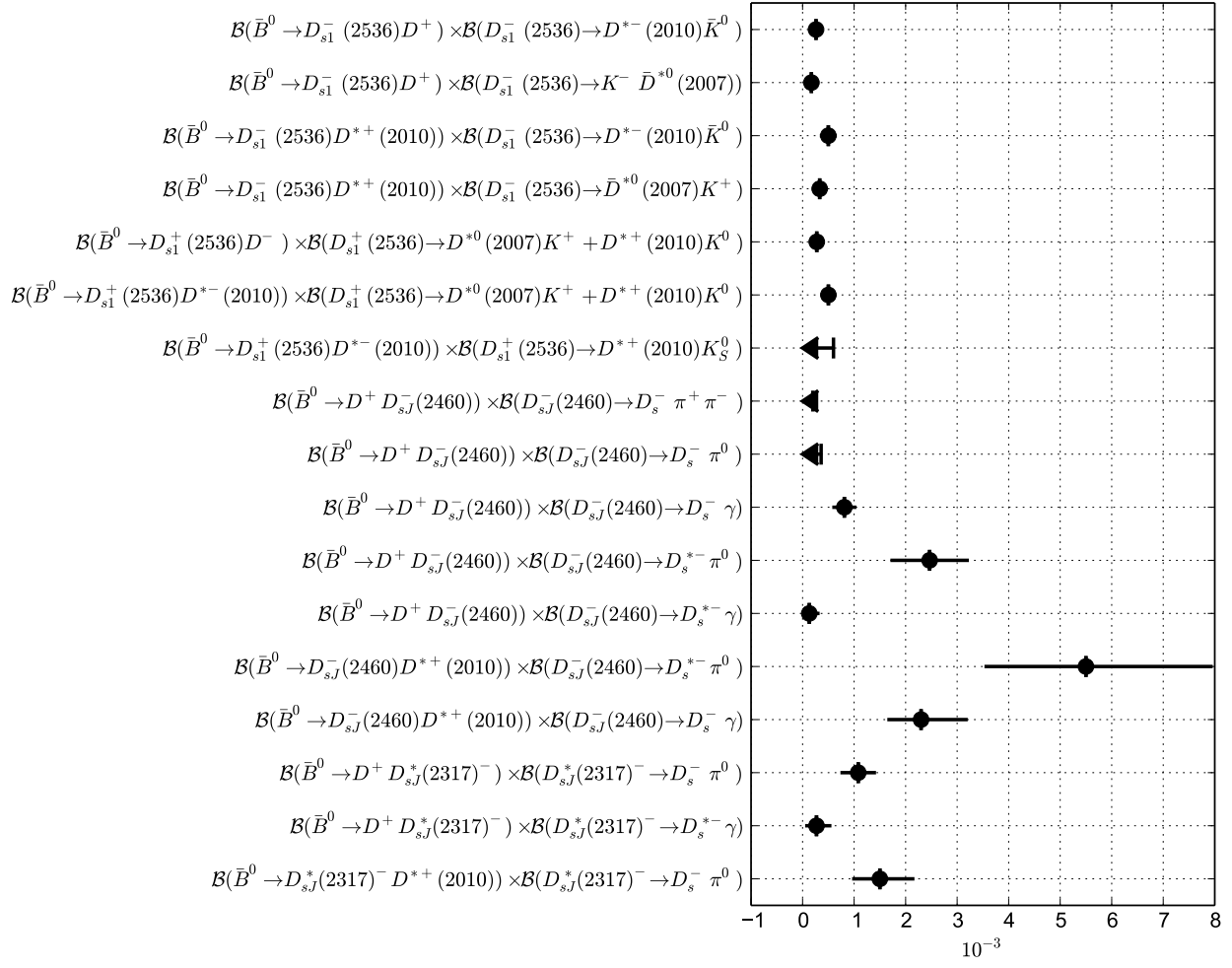


Figure 79: Summary of the averages from Table 100.

6.1.3 Decays to charmonium states

Averages of $\bar{B}^0$ decays to charmonium states are shown in Tables 101–106 and Figs. 80–85.

Table 101: Decays to J/ψ and one kaon [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^0)$	CDF [574]: $1.15 \pm 0.23 \pm 0.17$ BABAR [6]: $0.869 \pm 0.022 \pm 0.030$ Belle [575]: $0.79 \pm 0.04 \pm 0.09$	0.863 ± 0.035
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K^-\pi^+)$	Belle [576]: $1.15 \pm 0.01 \pm 0.05$	1.15 ± 0.05
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^{*0}(892))$	CDF [577]: $1.74 \pm 0.20 \pm 0.18$ BABAR [6]: $1.309 \pm 0.026 \pm 0.077$ Belle [576]: $1.19 \pm 0.01 \pm 0.08$	1.270 ± 0.056
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^0\pi^+\pi^-)$	CDF [578]: $1.03 \pm 0.33 \pm 0.15$	1.03 ± 0.36
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^0\rho^0(770))$	CDF [578]: $0.54 \pm 0.29 \pm 0.09$	0.54 ± 0.30
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K^{*-}(892)\pi^+)$	CDF [578]: $0.77 \pm 0.41 \pm 0.13$	0.77 ± 0.43
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\omega(782)\bar{K}^0)$	BABAR [579]: $0.23 \pm 0.03 \pm 0.03$	0.23 ± 0.04
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\phi(1020)\bar{K}^0)$	BABAR [580]: $0.102 \pm 0.038 \pm 0.010$	0.102 ± 0.039
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K^0\pi^-\pi^+)$	LHCb [581]: $0.430 \pm 0.030 \pm 0.037$	0.430 ± 0.047
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}_1^0(1270))$	Belle [582]: $1.30 \pm 0.34 \pm 0.31$	1.30 ± 0.46
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\eta K_S^0)$	BABAR [583]: $0.084 \pm 0.026 \pm 0.027$ Belle [584]: $0.0522 \pm 0.0078 \pm 0.0049$	0.0540 ± 0.0089
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^{*0}(892)\pi^+\pi^-)$	CDF [578]: $0.66 \pm 0.19 \pm 0.11$	0.66 ± 0.22

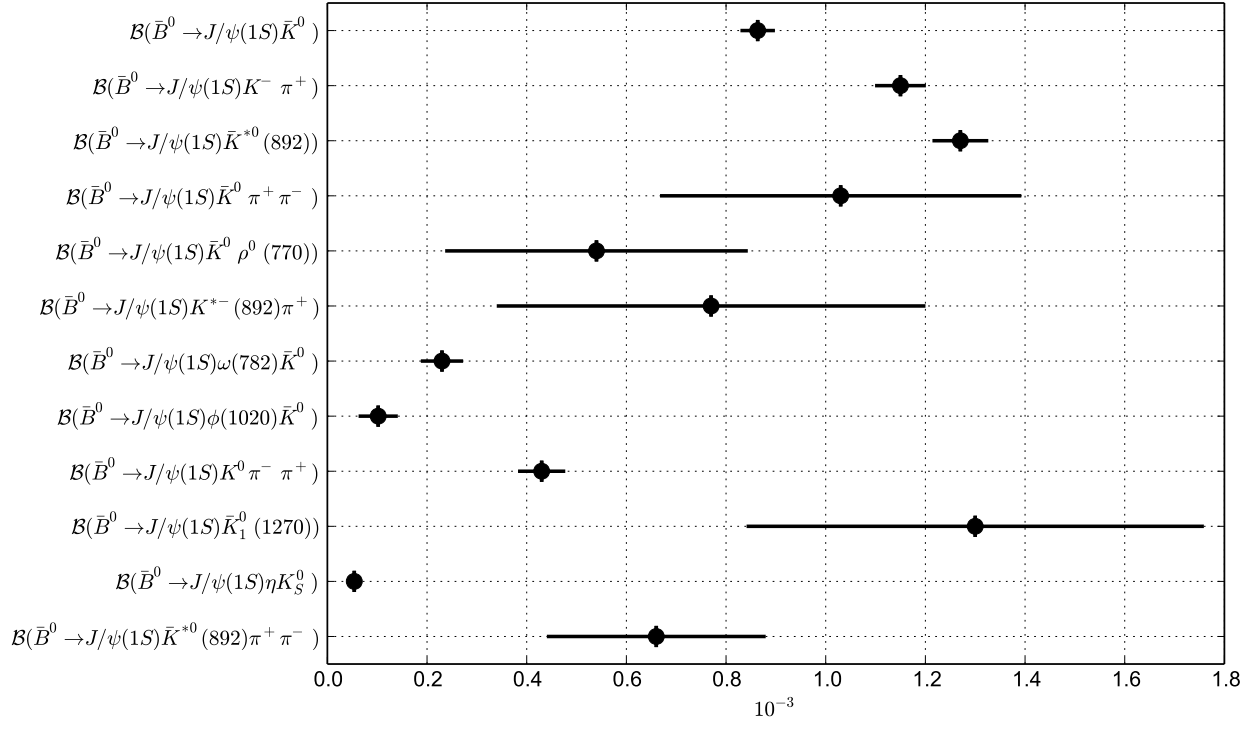


Figure 80: Summary of the averages from Table 101.

Table 102: Decays to charmonium other than J/ψ and one kaon [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)\bar{K}^0)$	BABAR [6]: $0.646 \pm 0.065 \pm 0.051$ Belle [575]: 0.67 ± 0.11	0.655 ± 0.066
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)\bar{K}^{*0}(892))$	CDF [577]: $0.90 \pm 0.22 \pm 0.09$ BABAR [6]: $0.649 \pm 0.059 \pm 0.097$ Belle [585]: $0.720 \pm 0.043 \pm 0.065$	0.711 ± 0.062
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)K^0)$	LHCb [581]: $0.47 \pm 0.07 \pm 0.07$	0.47 ± 0.10
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)K^0) \times \mathcal{B}(\psi(2S) \rightarrow \chi_{c1}(1P)\gamma)$	Belle [586]: $0.68 \pm 0.10 \pm 0.07$	0.68 ± 0.12
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)K^0) \times \mathcal{B}(\psi(2S) \rightarrow \chi_{c2}\gamma)$	Belle [586]: $0.47 \pm 0.16 \pm 0.08$	0.47 ± 0.18
$\mathcal{B}(\bar{B}^0 \rightarrow K^{*0}(892)\psi(2S))$	Belle [587]: $0.552^{+0.035}_{-0.032}{}^{+0.053}_{-0.058}$	$0.552^{+0.064}_{-0.066}$
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(3770)\bar{K}^0) \times \mathcal{B}(\psi(3770) \rightarrow D^0\bar{D}^0)$	BABAR [570]: < 0.123	< 0.123
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(3770)\bar{K}^0) \times \mathcal{B}(\psi(3770) \rightarrow D^+D^-)$	BABAR [570]: < 0.188	< 0.188
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c0}(1P)\bar{K}^0)$	BABAR [588]: < 1.24	< 1.24
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c0}(1P)\bar{K}^{*0}(892))$	BABAR [588]: < 0.77 BABAR [589]: $0.17 \pm 0.03 \pm 0.02$	0.17 ± 0.04
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c1}(1P)\bar{K}^0)$	BABAR [590]: $0.42 \pm 0.03 \pm 0.03$ Belle [591]: $0.378^{+0.017}_{-0.016} \pm 0.033$	0.396 ± 0.028
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c1}(1P)K^-\pi^+)$	BABAR [592]: $0.511 \pm 0.014 \pm 0.058$	0.511 ± 0.060
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c1}(1P)\bar{K}^{*0}(892))$	BABAR [590]: $0.25 \pm 0.02 \pm 0.02$ Belle [593]: $0.31 \pm 0.03 \pm 0.07$	0.26 ± 0.03
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c2}(1P)\bar{K}^0)$	BABAR [590]: $0.015 \pm 0.009 \pm 0.003$ Belle [591]: < 0.015	0.015 ± 0.009
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c2}(1P)\bar{K}^{*0}(892))$	BABAR [590]: $0.066 \pm 0.018 \pm 0.005$	0.066 ± 0.019
$\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^0)$	BABAR [594]: $1.14 \pm 0.15 \pm 0.34$ BABAR [595]: $0.64^{+0.22}_{-0.20}{}^{+0.28}_{-0.16}$ Belle [596]: $1.23 \pm 0.23^{+0.40}_{-0.41}$	0.85 ± 0.24
$\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^{*0}(892))$	BABAR [597]: $0.57 \pm 0.06 \pm 0.09$ BABAR [595]: $0.80^{+0.21}_{-0.19}{}^{+0.37}_{-0.23}$ Belle [596]: $1.62 \pm 0.32^{+0.55}_{-0.60}$	0.61 ± 0.10
$\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(2S)\bar{K}^{*0}(892))$	BABAR [597]: < 0.39	< 0.39
$\mathcal{B}(\bar{B}^0 \rightarrow h_c(1P)\bar{K}^{*0}(892)) \times \mathcal{B}(h_c(1P) \rightarrow \eta_c(1S)\gamma)$	BABAR [597]: < 0.22	< 0.22

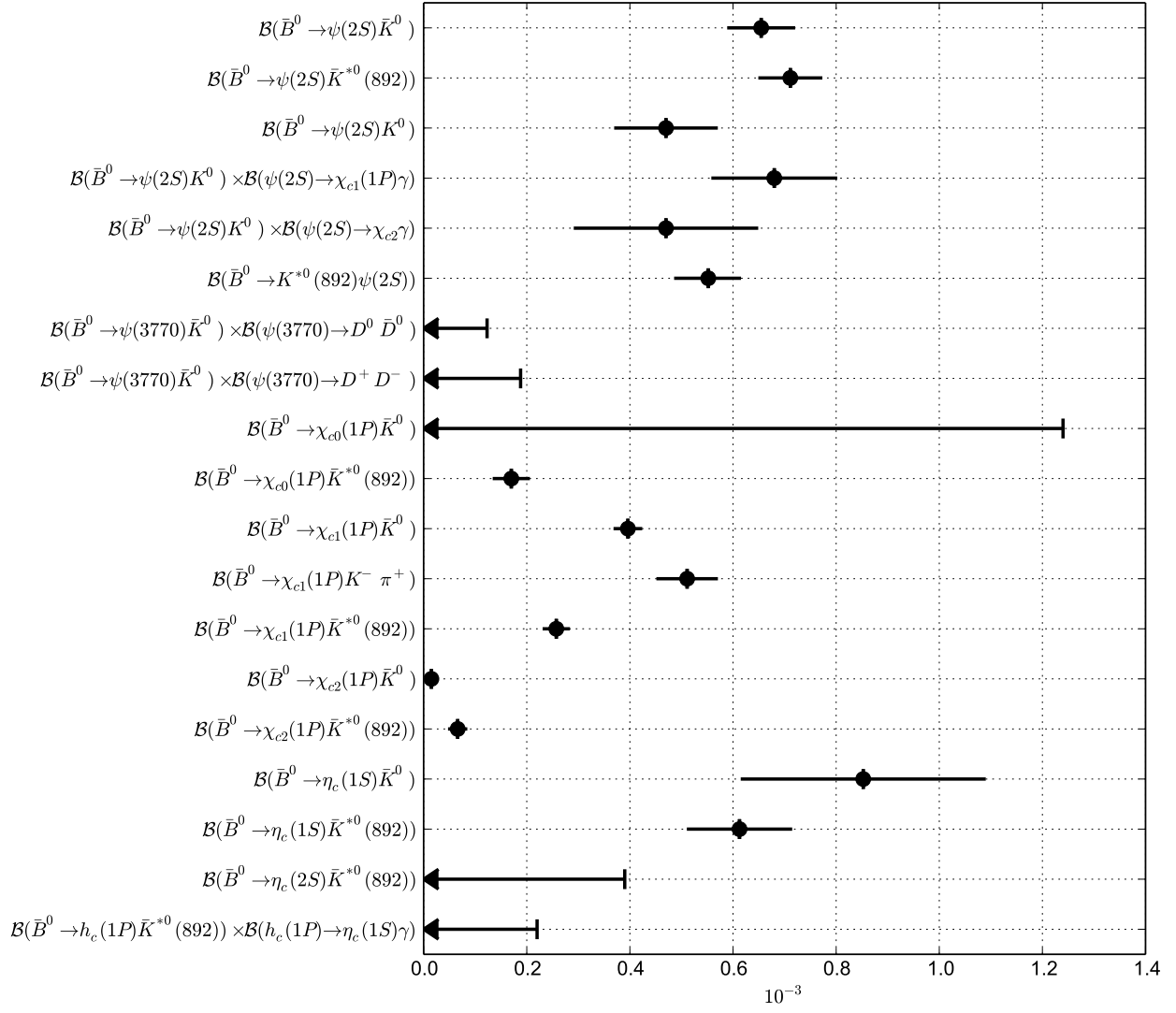


Figure 81: Summary of the averages from Table 102.

Table 103: Decays to charmonium and light mesons $[10^{-5}]$.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\pi^0)$	BABAR [346]: $1.69 \pm 0.14 \pm 0.07$ Belle [575]: $2.3 \pm 0.5 \pm 0.2$	1.74 ± 0.15
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\pi^+\pi^-)$	BABAR [598]: < 1.2 Belle [599]: < 1	< 1
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\rho^0(770))$	BABAR [598]: $2.7 \pm 0.3 \pm 0.2$ Belle [599]: $2.8 \pm 0.3 \pm 0.3$	2.7 ± 0.3
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\eta)$	BABAR [580]: < 2.7 Belle [600]: $1.23^{+0.18}_{-0.17} \pm 0.07$	$1.23^{+0.19}_{-0.18}$
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\eta'(958))$	BABAR [580]: < 6.3 Belle [600]: < 0.74	< 0.74
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)f_2(1270))$	BABAR [598]: < 0.46 Belle [599]: $0.98 \pm 0.39 \pm 0.20$	0.98 ± 0.44
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)a_0(980)) \times \mathcal{B}(a_0(980) \rightarrow K^+K^-)$	LHCb [601]: < 0.090	< 0.090
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)f_0(980)) \times \mathcal{B}(f_0(980) \rightarrow \pi^+\pi^-)$	LHCb [602]: < 0.11	< 0.11
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)f_1(1285))$	LHCb [603]: $0.837 \pm 0.195^{+0.079}_{-0.075}$	$0.837^{+0.210}_{-0.209}$
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)f_1(1285)) \times \mathcal{B}(f_1(1285) \rightarrow \pi^+\pi^-\pi^+\pi^-)$	LHCb [603]: $0.0921 \pm 0.0214 \pm 0.0064$	0.0921 ± 0.0223
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K^+K^-)$	LHCb [601]: $0.253 \pm 0.031 \pm 0.019$	0.253 ± 0.036
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\phi(1020))$	LHCb [601]: < 0.019 BABAR [580]: < 0.9 Belle [604]: < 0.094	< 0.019
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K^0K^\pm\pi^\mp)$	LHCb [581]: < 2.1	< 2.1
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K^0K^+K^-)$	LHCb [581]: $2.02 \pm 0.43 \pm 0.19$	2.02 ± 0.47
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c1}(1P)\pi^0)$	Belle [605]: $1.12 \pm 0.25 \pm 0.12$	1.12 ± 0.28

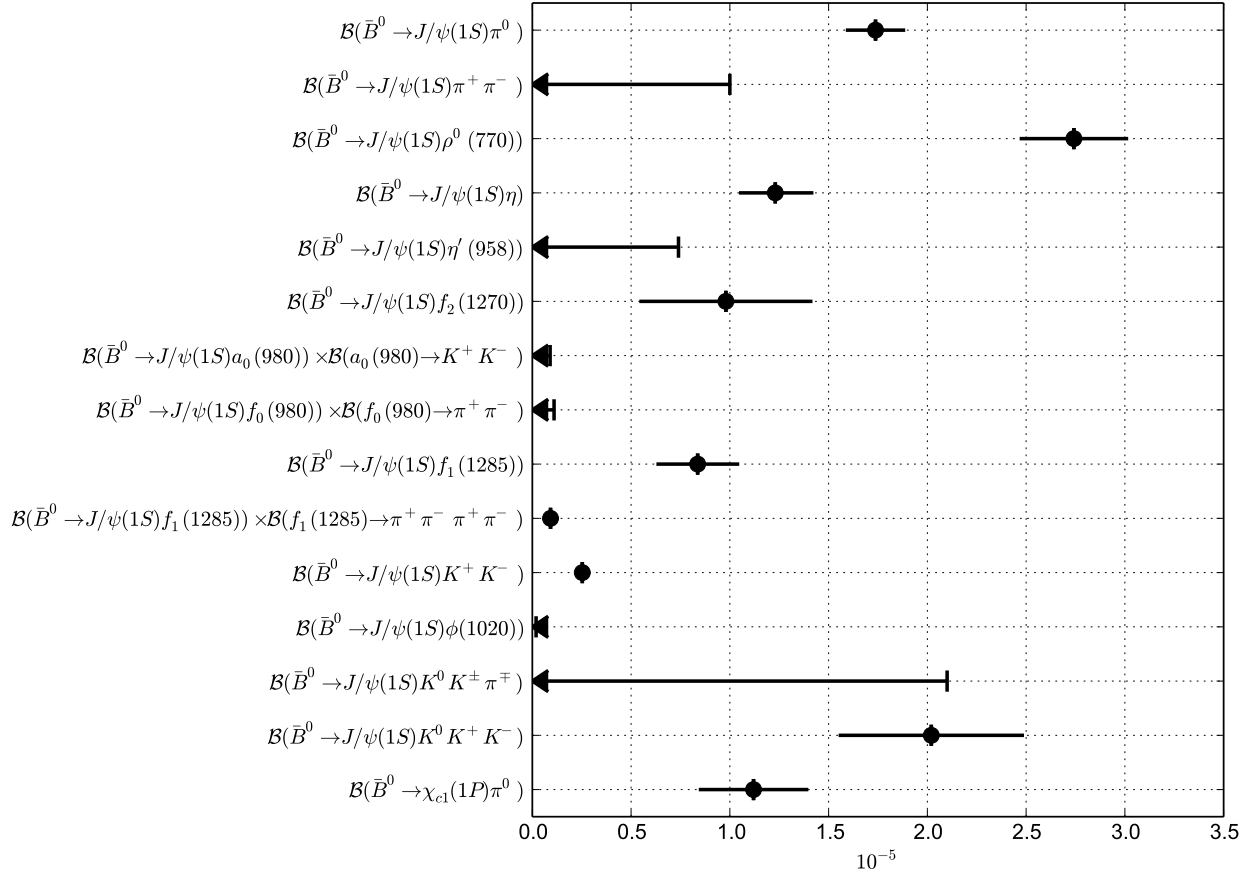


Figure 82: Summary of the averages from Table 103.

Table 104: Decays to J/ψ and photons, baryons, or heavy mesons [10^{-5}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\gamma)$	<i>BABAR</i> [606]: < 0.16	< 0.16
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{p}p)$	LHCb [607]: < 0.052	< 0.052
	<i>BABAR</i> [608]: < 0.19	
	Belle [609]: < 0.083	
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)D^0)$	<i>BABAR</i> [610]: < 1.3	< 1.3
	Belle [611]: < 2.0	

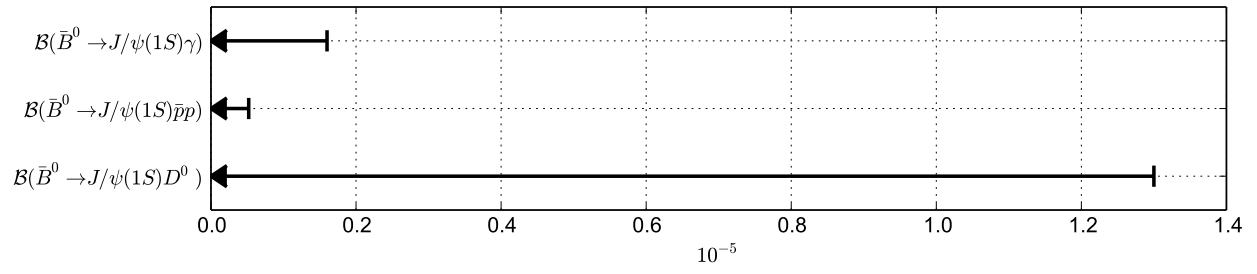


Figure 83: Summary of the averages from Table 104.

Table 105: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}_1^0(1270))/\mathcal{B}(B^- \rightarrow J/\psi(1S)K^-)$	Belle [582]: $1.30 \pm 0.34 \pm 0.28$	1.30 ± 0.44
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^{*0}(892))/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^0)$	CDF [612]: $1.39 \pm 0.36 \pm 0.10$ BABAR [6]: $1.51 \pm 0.05 \pm 0.08$	1.50 ± 0.09
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi\omega)/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi\rho)$	LHCb [613]: $0.89 \pm 0.19^{+0.07}_{-0.13}$	$0.89^{+0.20}_{-0.23}$
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi\omega\bar{K}^0)/\mathcal{B}(B^- \rightarrow J/\psi\omega K^-)$	BABAR [579]: $0.7 \pm 0.1 \pm 0.1$	0.7 ± 0.1
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K_S^0\pi^-\pi^+)/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K_S^0)$	LHCb [581]: $0.493 \pm 0.034 \pm 0.027$	0.493 ± 0.043
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K_S^0K^\pm\pi^\mp)/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\pi^+\pi^-)$	LHCb [581]: < 0.048	< 0.048
$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K_S^0K^+K^-)/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K_S^0\pi^+\pi^-)$	LHCb [581]: $0.047 \pm 0.010 \pm 0.004$	0.047 ± 0.011
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)K_S^0) \times (\psi(2S) \rightarrow J/\psi(1S)\pi^-\pi^+)/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K_S^0)$	LHCb [581]: $0.183 \pm 0.027 \pm 0.015$	0.183 ± 0.031
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)\bar{K}^{*0}(892))/\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)\bar{K}^0)$	BABAR [6]: $1.00 \pm 0.14 \pm 0.09$	1.00 ± 0.17
$\mathcal{B}(\bar{B}^0 \rightarrow \psi(2S)K(892)^{*0})/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K(892)^{*0})$	LHCb [614]: $0.476 \pm 0.014 \pm 0.016$	0.476 ± 0.021
$\mathcal{B}\bar{B}^0 \rightarrow \psi(2S)\pi^+\pi^-/\mathcal{B}\bar{B}^0 \rightarrow J/\psi\pi^+\pi^-$	LHCb [615]: $0.56 \pm 0.07 \pm 0.05$	0.56 ± 0.09
$\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^0)/\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-)$	BABAR [594]: $0.87 \pm 0.13 \pm 0.07$	0.87 ± 0.15
$\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^0)/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^0)$	BABAR [594]: $1.34 \pm 0.19 \pm 0.40$	1.34 ± 0.44
$\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^{*0}(892))/\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-)$	BABAR [616]: $0.67 \pm 0.09 \pm 0.07$	0.67 ± 0.11
$\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^{*0}(892))/\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^0)$	Belle [596]: $1.33 \pm 0.36^{+0.24}_{-0.33}$	$1.33^{+0.43}_{-0.49}$
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c1}(1P)K^-\pi^+)/\mathcal{B}(\bar{B}^0 \rightarrow J/\psi(1S)K^-\pi^+)$	BABAR [592]: $0.474 \pm 0.013 \pm 0.054$	0.474 ± 0.056
$\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c1}(1P)\bar{K}^{*0}(892))/\mathcal{B}(\bar{B}^0 \rightarrow \chi_{c1}(1P)\bar{K}^0)$	BABAR [6]: $0.72 \pm 0.11 \pm 0.12$	0.72 ± 0.16
$\mathcal{B}(\bar{B}^0 \rightarrow h_c(1P)\bar{K}^{*0}(892)) \times \mathcal{B}(h_c(1P) \rightarrow \eta_c(1S)\gamma)/\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-)$	BABAR [616]: < 0.26	< 0.26
$\mathcal{B}(\bar{B}^0 \rightarrow h_c(1P)\bar{K}^{*0}(892)) \times \mathcal{B}(h_c(1P) \rightarrow \eta_c(1S)\gamma)/\mathcal{B}(\bar{B}^0 \rightarrow \eta_c(1S)\bar{K}^{*0}(892))$	BABAR [616]: < 0.39	< 0.39

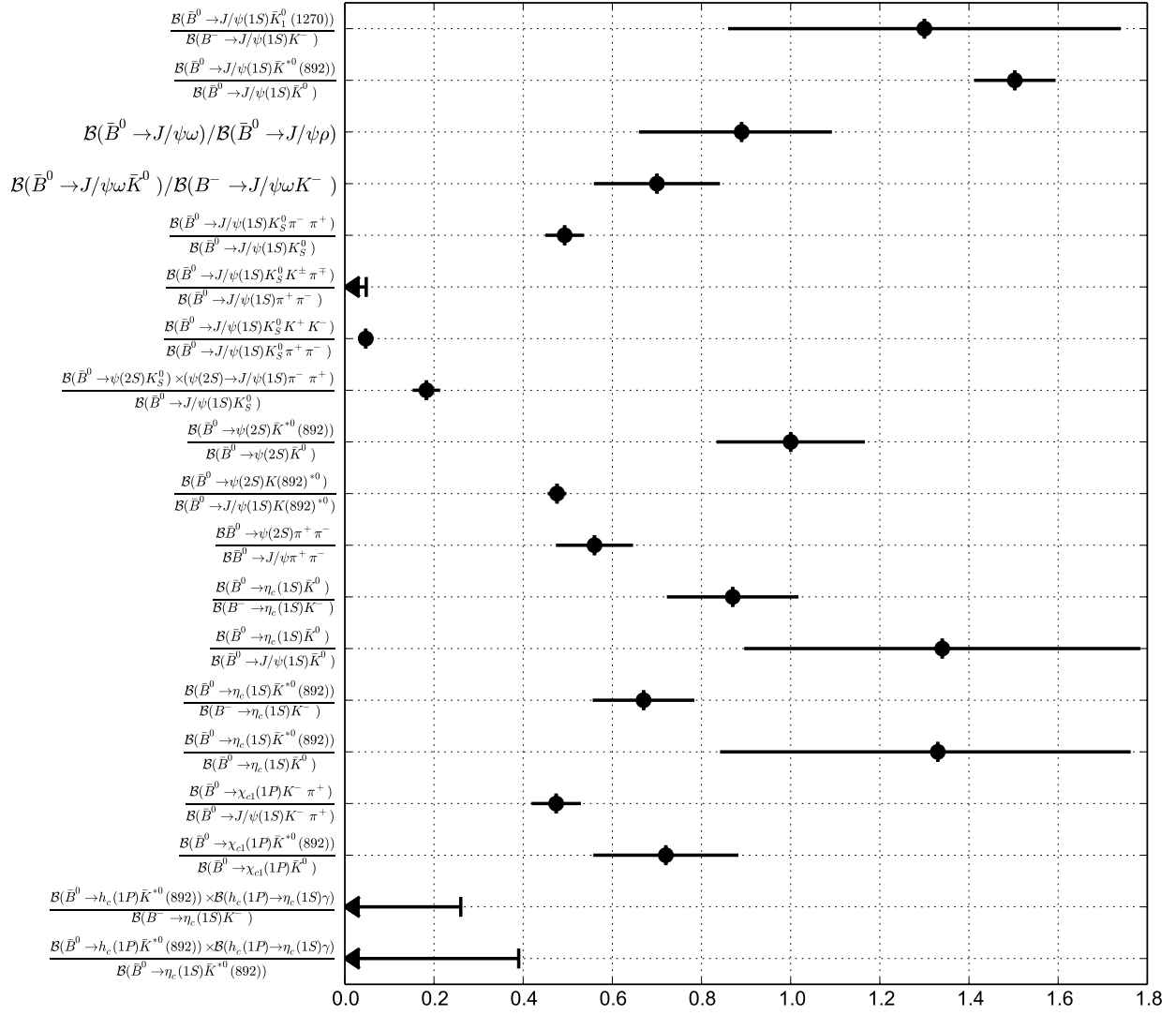


Figure 84: Summary of the averages from Table 105.

Table 106: Polarization fractions.

Parameter	Measurements	Average
$ \mathcal{A}_0 ^2(B^0 \rightarrow J/\psi(1S)\bar{K}^{*0}(892))/ \mathcal{A}_0 ^2(\bar{B}^0 \rightarrow J/\psi(1S)\bar{K}^{*0}(892))$	BABAR [617]: < 0.32	< 0.32
$ \mathcal{A}_0 ^2(\bar{B}^0 \rightarrow J/\psi(1S)K^{*0}(892))/ \mathcal{A}_0 ^2(B^0 \rightarrow J/\psi(1S)K^{*0}(892))$	BABAR [617]: < 0.26	< 0.26

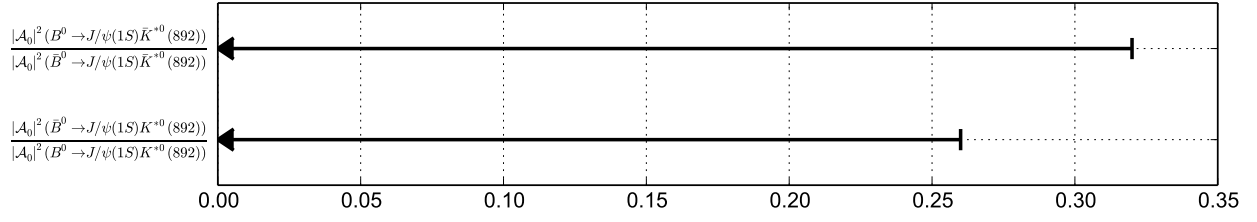


Figure 85: Summary of the averages from Table 106.

6.1.4 Decays to charm baryons

Averages of $\bar{B}^0$ decays to charm baryons are shown in Tables 107–108 and Figs. 86–87.

Table 107: Absolute decay rates to charm baryons [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p} \pi^0)$	<i>BABAR</i> [618]: $1.94 \pm 0.17 \pm 0.52$	1.94 ± 0.55
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p} \pi^+ \pi^-)$	<i>BABAR</i> [619]: $12.3 \pm 0.5 \pm 3.3$ Belle [620]: $11.0^{+1.2}_{-1.2} \pm 3.5$	11.9 ± 3.2
$\mathcal{B}(\bar{B}^0 \rightarrow \Sigma_c^{++} \bar{p} \pi^-)$	<i>BABAR</i> [619]: $2.13 \pm 0.10 \pm 0.56$ Belle [621]: $2.1 \pm 0.2 \pm 0.6$	2.12 ± 0.55
$\mathcal{B}(\bar{B}^0 \rightarrow \Sigma_c^{*++} \bar{p} \pi^-)$	<i>BABAR</i> [619]: $1.15 \pm 0.10 \pm 0.30$ Belle [621]: $1.2 \pm 0.1 \pm 0.4$	1.16 ± 0.32
$\mathcal{B}(\bar{B}^0 \rightarrow \Sigma_c^0 \bar{p} \pi^+)$	<i>BABAR</i> [619]: $0.91 \pm 0.07 \pm 0.24$ Belle [621]: $1.4 \pm 0.2 \pm 0.4$	0.77 ± 0.23
$\mathcal{B}(\bar{B}^0 \rightarrow \Sigma_c^{*0} \bar{p} \pi^+)$	<i>BABAR</i> [619]: $0.22 \pm 0.07 \pm 0.06$ Belle [621]: < 0.33	0.22 ± 0.09
$\mathcal{B}(\bar{B}^0 \rightarrow \Sigma_c^+ \bar{p}) \times \mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+)$	<i>BABAR</i> [618]: < 0.015	< 0.015
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p})$	<i>BABAR</i> [622]: $0.189 \pm 0.021 \pm 0.049$ Belle [623]: $0.219^{+0.056}_{-0.049} \pm 0.065$	0.190 ± 0.054
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p} \bar{K}^{*0}(892))$	<i>BABAR</i> [624]: $0.160 \pm 0.061 \pm 0.044$	0.160 ± 0.075
$\mathcal{B}(\bar{B}^0 \rightarrow \Sigma_c^{++} \bar{p} K^-)$	<i>BABAR</i> [624]: $0.111 \pm 0.030 \pm 0.030$	0.111 ± 0.043
$\mathcal{B}(\bar{B}^0 \rightarrow \Xi_c^+ \Lambda_c^-) \times \mathcal{B}(\Xi_c^+ \rightarrow \Xi^- \pi^+ \pi^+)$	<i>BABAR</i> [625]: $0.15 \pm 0.11 \pm 0.04$ Belle [626]: $0.93^{+0.37}_{-0.28} \pm 0.31$	0.17 ± 0.12
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \Lambda_c^-)$	Belle [627]: < 0.57	< 0.57
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{\Lambda} K^-)$	<i>BABAR</i> [628]: $0.38 \pm 0.08 \pm 0.10$	0.38 ± 0.13
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p} K^- \pi^+)$	<i>BABAR</i> [624]: $0.433 \pm 0.082 \pm 0.118$	0.433 ± 0.143
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \Lambda_c^- \bar{K}^0)$	<i>BABAR</i> [625]: $3.8 \pm 3.1 \pm 2.1$ Belle [629]: $7.9^{+2.9}_{-2.3} \pm 4.3$	4.8 ± 3.5
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p} \pi^+ \pi^-_{\text{non-}\Sigma_c})$	<i>BABAR</i> [619]: $7.9 \pm 0.4 \pm 2.0$	7.9 ± 2.1
$\mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p} p \bar{p}) \times \frac{\mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+)}{0.050}$	<i>BABAR</i> [630]: < 0.028	< 0.028

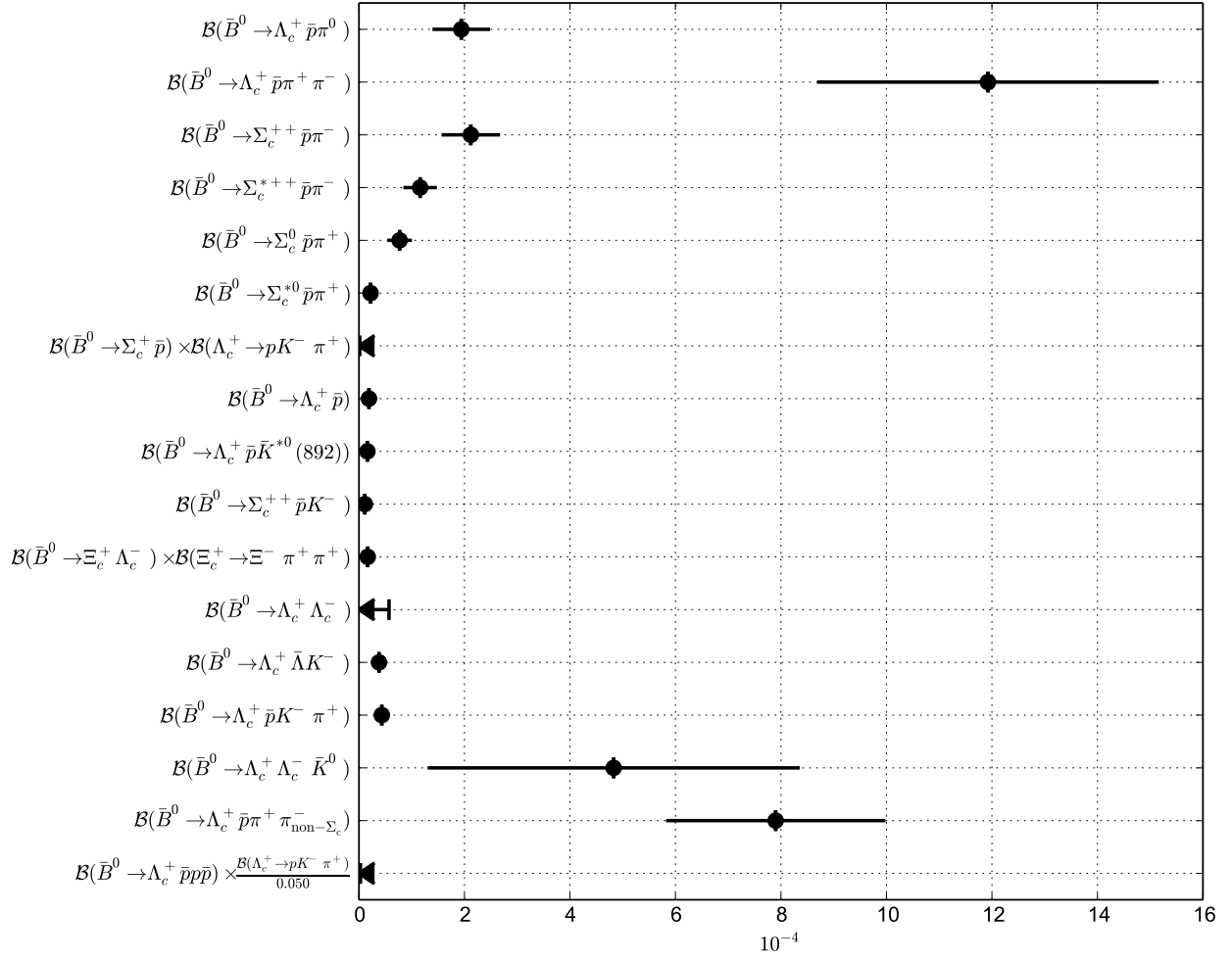


Figure 86: Summary of the averages from Table 107.

Table 108: Relative decay rates to charm baryons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow \Lambda_c^- \Lambda_c^+) / \mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^-)$	LHCb [631]: < 2	< 2

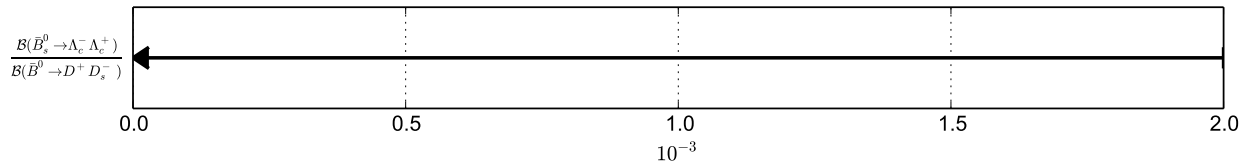


Figure 87: Summary of the averages from Table 108.

6.1.5 Decays to other (XYZ) states

Averages of $\bar{B}^0$ decays to other (XYZ) states are shown in Tables 109–113 and Figs. 88–92.

Table 109: Decays to $X(3872)$ [10^{-5}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^0) \times \mathcal{B}(X(3872) \rightarrow J/\psi(1S)\pi^+\pi^-)$	BABAR [632]: $0.35 \pm 0.19 \pm 0.04$	0.35 ± 0.19
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^0) \times \mathcal{B}(X(3872) \rightarrow J/\psi\omega)$	BABAR [579]: $0.6 \pm 0.3 \pm 0.1$	0.6 ± 0.3
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^0) \times \mathcal{B}(X(3872) \rightarrow J/\psi(1S)\gamma)$	BABAR [590]: $0.26 \pm 0.18 \pm 0.02$ Belle [591]: $0.24^{+0.13}_{-0.14} \pm 0.07$	0.25 ± 0.12
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^{*0}(892)) \times \mathcal{B}(X(3872) \rightarrow J/\psi(1S)\gamma)$	BABAR [590]: $0.07 \pm 0.14 \pm 0.01$	0.07 ± 0.14
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^0) \times \mathcal{B}(X(3872) \rightarrow \psi(2S)\gamma)$	BABAR [590]: $1.14 \pm 0.55 \pm 0.10$ Belle [591]: $0.662^{+0.130}_{-0.140} \pm 0.070$	0.695 ± 0.147
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^{*0}(892)) \times \mathcal{B}(X(3872) \rightarrow \psi(2S)\gamma)$	BABAR [590]: $-0.13 \pm 0.31 \pm 0.03$	-0.13 ± 0.31
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)K^0) \times \mathcal{B}(X(3872) \rightarrow \chi_{c1}(1P)\gamma)$	Belle [586]: < 0.96	< 0.96
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)K^0) \times \mathcal{B}(X(3872) \rightarrow \chi_{c2}\gamma)$	Belle [586]: < 1.22	< 1.22

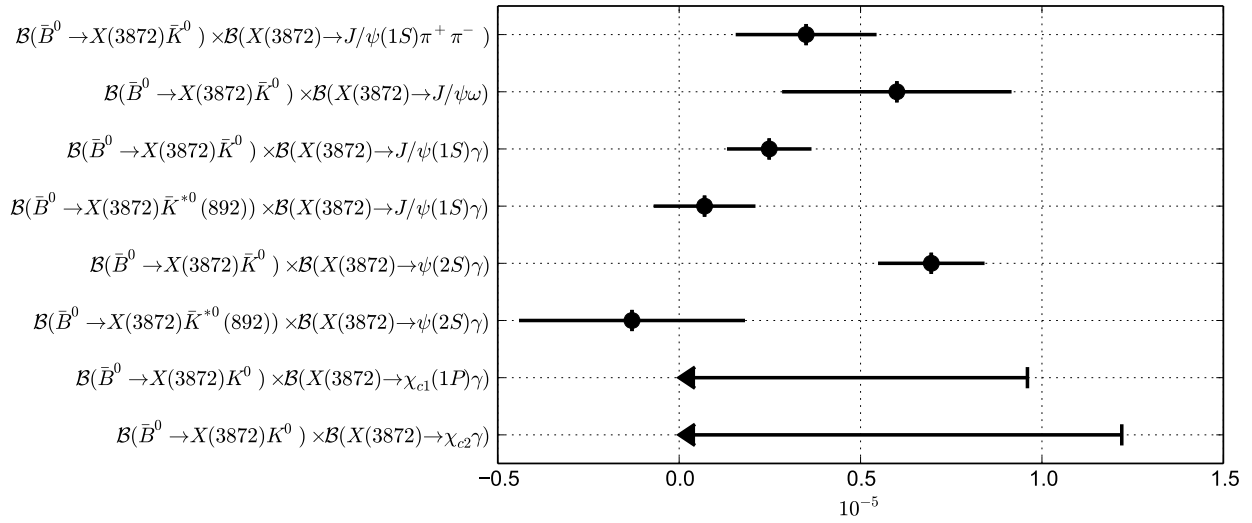


Figure 88: Summary of the averages from Table 109.

Table 110: Decays to $X(3872)$ with $X(3872) \rightarrow D\bar{D}$ [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^0) \times \mathcal{B}(X(3872) \rightarrow \bar{D}^{*0}(2007)D^0)$	$BABAR$ [570]: < 4.37	< 4.37

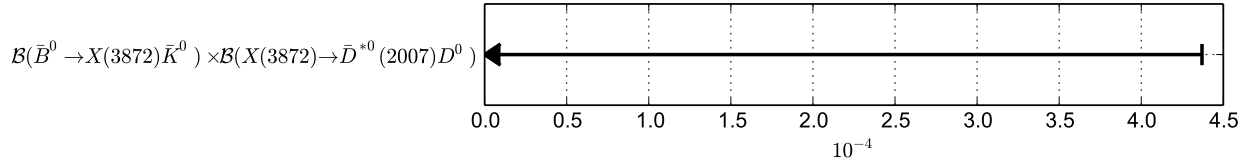


Figure 89: Summary of the averages from Table 110.

Table 111: Decays to neutral states other than $X(3872)$ [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow X(3823)K^0) \times \mathcal{B}(X(3823) \rightarrow \chi_{c1}(1P)\gamma)$	$Belle$ [586]: < 0.099	< 0.099
$\mathcal{B}(\bar{B}^0 \rightarrow X(3823)K^0) \times \mathcal{B}(X(3823) \rightarrow \chi_{c2}\gamma)$	$Belle$ [586]: < 0.228	< 0.228
$\mathcal{B}(\bar{B}^0 \rightarrow Y(3940)\bar{K}^0) \times \mathcal{B}(Y(3940) \rightarrow J/\psi(1S)\omega(782))$	$BABAR$ [579]: $0.21 \pm 0.09 \pm 0.03$	0.21 ± 0.09
$\mathcal{B}(\bar{B}^0 \rightarrow Z1(4050)K^-) \times \mathcal{B}(Z1(4050) \rightarrow \chi_{c1}(1P)\pi^+)$	$BABAR$ [592]: < 0.18 $Belle$ [633]: $0.30^{+0.15}_{-0.08} {}^{+0.37}_{-0.16}$	$0.30^{+0.40}_{-0.18}$
$\mathcal{B}(\bar{B}^0 \rightarrow Z2(4250)K^-) \times \mathcal{B}(Z2(4250) \rightarrow \chi_{c1}(1P)\pi^+)$	$BABAR$ [592]: < 0.47	< 0.47
$\mathcal{B}(\bar{B}^0 \rightarrow Z2(4250)K^-) \times \mathcal{B}(Z2(4250) \rightarrow \pi^+\psi(2S))$	$Belle$ [633]: $0.40^{+0.23}_{-0.09} {}^{+1.97}_{-0.05}$	$0.40^{+1.98}_{-0.10}$

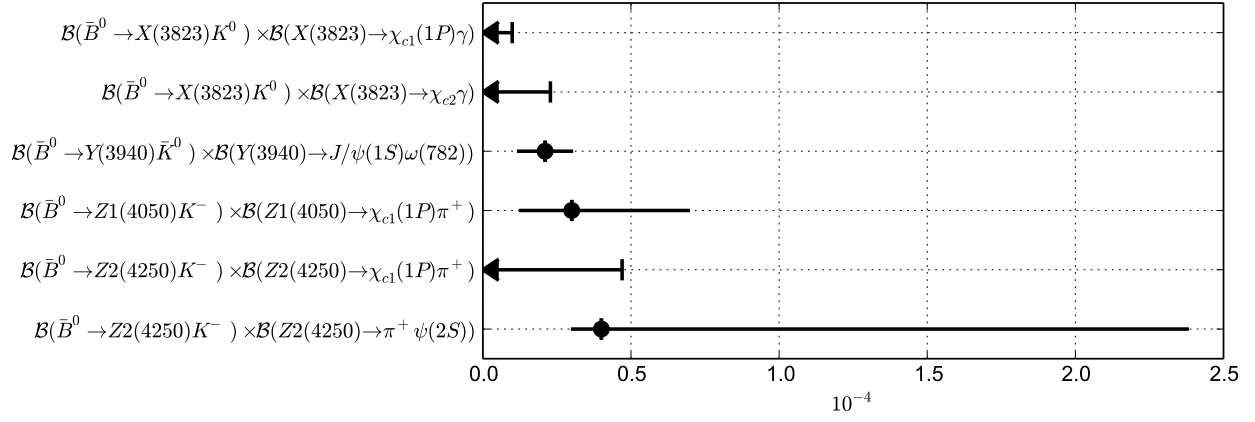


Figure 90: Summary of the averages from Table 111.

Table 112: Decays to charged states [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow X^+(3872)K^-)$	<i>BABAR</i> [634]: < 5.0	< 5.0
$\mathcal{B}(\bar{B}^0 \rightarrow X^+(3872)K^-) \times \mathcal{B}(X^+(3872) \rightarrow J/\psi(1S)\pi^+\pi^0)$	<i>BABAR</i> [635]: < 0.054	< 0.054
$\mathcal{B}(\bar{B}^0 \rightarrow Z(4430)^+K^-) \times \mathcal{B}(Z(4430)^+ \rightarrow J/\psi\pi^+)$	<i>BABAR</i> [636]: $-0.12 \pm 0.04 \pm 0.00$ <i>Belle</i> [576]: $0.054^{+0.040+0.011}_{-0.010-0.009}$	-0.011 ± 0.024
$\mathcal{B}(\bar{B}^0 \rightarrow Z(4430)^+K^-) \times \mathcal{B}(Z(4430)^+ \rightarrow \psi(2S)\pi^+)$	<i>BABAR</i> [636]: $0.19 \pm 0.08 \pm 0.00$ <i>Belle</i> [587]: $0.32^{+0.18+0.53}_{-0.09-0.16}$	0.19 ± 0.08
$\mathcal{B}(\bar{B}^0 \rightarrow Z_c(3900)^+K^-) \times \mathcal{B}(Z_c(3900)^+ \rightarrow J/\psi\pi^+)$	<i>Belle</i> [576]: < 0.009	< 0.009
$\mathcal{B}(\bar{B}^0 \rightarrow Z_c(4200)^+K^-) \times \mathcal{B}(Z_c(4200)^+ \rightarrow J/\psi\pi^+)$	<i>Belle</i> [576]: $0.22^{+0.07+0.11}_{-0.05-0.06}$	$0.22^{+0.13}_{-0.08}$

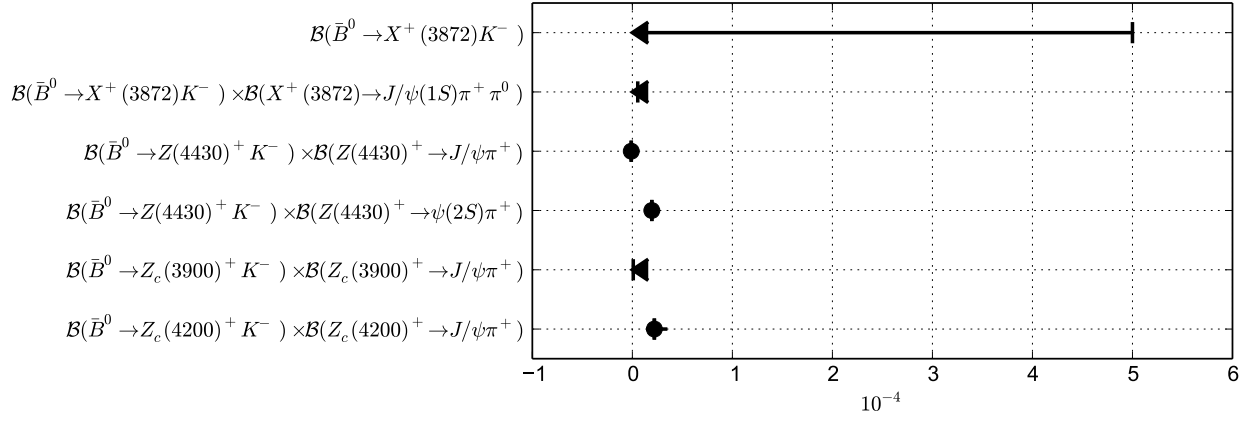


Figure 91: Summary of the averages from Table 112.

Table 113: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}^0 \rightarrow X(3872)\bar{K}^0)/\mathcal{B}(B^- \rightarrow X(3872)K^-)$	<i>BABAR</i> [579]: $1^{+1}_{-1}{}^{+0}_{-0}$ <i>BABAR</i> [632]: $0.41 \pm 0.24 \pm 0.05$	0.47 ± 0.23
$\mathcal{B}(\bar{B}^0 \rightarrow Y(3940)\bar{K}^0)/\mathcal{B}(B^- \rightarrow Y(3940)K^-)$	<i>BABAR</i> [579]: $0.7^{+0.4}_{-0.3} \pm 0.1$	$0.7^{+0.4}_{-0.3}$

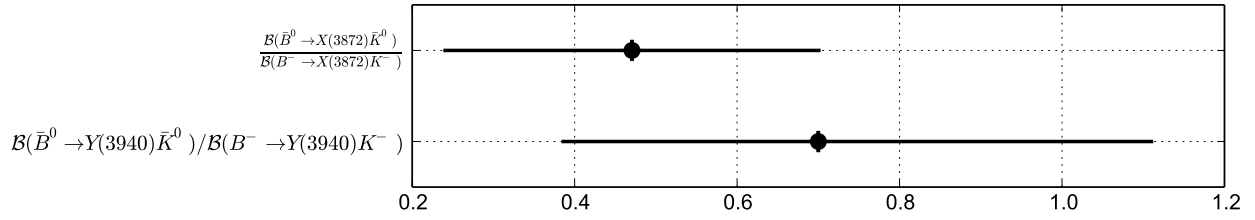


Figure 92: Summary of the averages from Table 113.

6.2 Decays of B^- mesons

Measurements of B^- decays to charmed hadrons are summarized in sections 6.2.1 to 6.2.5.

6.2.1 Decays to a single open charm meson

Averages of B^- decays to a single open charm meson are shown in Tables 114–124 and Figs. 93–103.

Table 114: Decays to a $D^{(*)}$ meson and one or more pions [10^{-2}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D_s^- \pi^0)$	Belle [637]: < 0.00036	< 0.00036
$\mathcal{B}(B^- \rightarrow D^0 \pi^-)$	BABAR [527]: $0.449 \pm 0.021 \pm 0.023$ BABAR [528]: $0.490 \pm 0.007 \pm 0.022$	0.475 ± 0.019
$\mathcal{B}(B^- \rightarrow D^{*0}(2007) \pi^-)$	BABAR [527]: $0.513 \pm 0.022 \pm 0.028$ BABAR [528]: $0.552 \pm 0.017 \pm 0.042$	0.528 ± 0.028
$\mathcal{B}(B^- \rightarrow D^+ \pi^- \pi^-)$	BABAR [638]: $0.108 \pm 0.003 \pm 0.005$ Belle [639]: $0.102 \pm 0.004 \pm 0.015$	0.107 ± 0.005
$\mathcal{B}(B^- \rightarrow D^{*+}(2010) \pi^- \pi^-)$	BABAR [640]: $0.122 \pm 0.005 \pm 0.018$ Belle [639]: $0.125 \pm 0.008 \pm 0.022$	0.123 ± 0.015
$\mathcal{B}(B^- \rightarrow D^{*0}(2007) \pi^- \pi^+ \pi^-)$	Belle [531]: $1.055 \pm 0.047 \pm 0.129$	1.055 ± 0.137
$\mathcal{B}(B^- \rightarrow D^{*+}(2010) \pi^- \pi^+ \pi^- \pi^-)$	Belle [531]: $0.256 \pm 0.026 \pm 0.033$	0.256 ± 0.042
$\mathcal{B}(B^- \rightarrow D^{*0}(2007) \pi^- \pi^+ \pi^- \pi^+ \pi^-)$	Belle [531]: $0.567 \pm 0.091 \pm 0.085$	0.567 ± 0.125

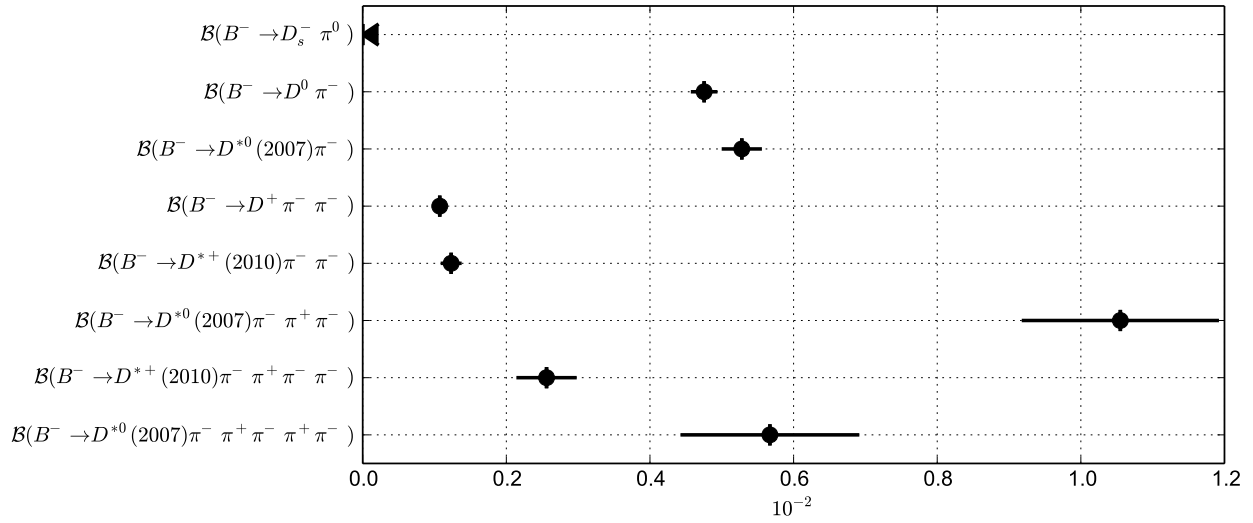


Figure 93: Summary of the averages from Table 114.

Table 115: Decays to a $D^{(*)0}$ meson and one or more kaons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^0 K^-)$	Belle [641]: $0.383 \pm 0.025 \pm 0.037$	0.383 ± 0.045
$\mathcal{B}(B^- \rightarrow D^{*0}(2007)K^-)$	Belle [534]: $0.359 \pm 0.087 \pm 0.051$	0.359 ± 0.101
$\mathcal{B}(B^- \rightarrow D^0 K^- K^0)$	Belle [536]: $0.55 \pm 0.14 \pm 0.08$	0.55 ± 0.16
$\mathcal{B}(B^- \rightarrow D^{*0}(2007)K^- K^0)$	Belle [536]: < 1.06	< 1.06
$\mathcal{B}(B^- \rightarrow D^0 K^- K^{*0}(892))$	Belle [536]: $0.75 \pm 0.13 \pm 0.11$	0.75 ± 0.17
$\mathcal{B}(B^- \rightarrow D^{*0}(2007)K^- K^{*0}(892))$	Belle [536]: $1.53 \pm 0.31 \pm 0.29$	1.53 ± 0.42
$\mathcal{B}(B^- \rightarrow D^0 K^{*-}(892))$	BABAR [642]: $0.529 \pm 0.030 \pm 0.034$	0.529 ± 0.045
$\mathcal{B}(B^- \rightarrow D^{*0}(2007)K^{*-}(892))$	BABAR [643]: $0.83 \pm 0.11 \pm 0.10$	0.83 ± 0.15

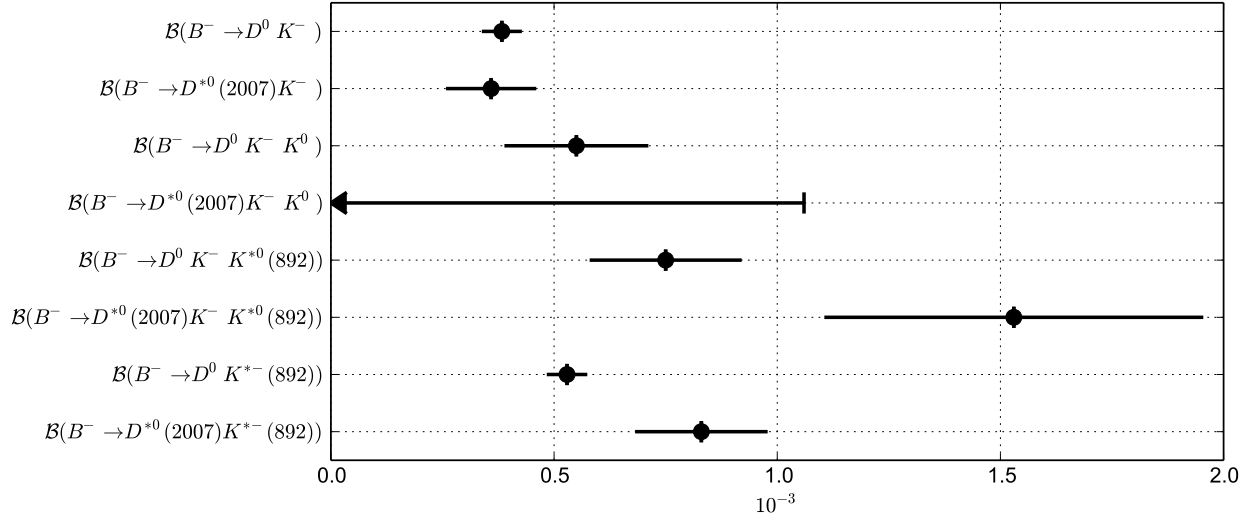


Figure 94: Summary of the averages from Table 115.

Table 116: Decays to a $D^{(*)-}$ meson and a neutral kaon [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^- \bar{K}^0)$	BABAR [644]: $-0.038^{+0.022}_{-0.018} {}^{+0.012}_{-0.016}$	$-0.038^{+0.025}_{-0.024}$
$\mathcal{B}(B^- \rightarrow D^- \bar{K}^{*0}(892))$	BABAR [644]: $-0.053^{+0.023}_{-0.020} {}^{+0.014}_{-0.018}$	$-0.053^{+0.027}_{-0.027}$
$\mathcal{B}(B^- \rightarrow D^{*-}(2010)\bar{K}^0)$	BABAR [645]: < 0.09	< 0.09

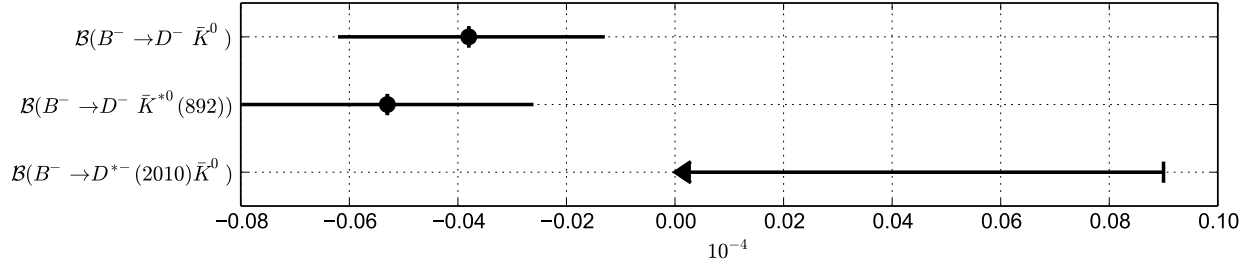


Figure 95: Summary of the averages from Table 116.

Table 117: Relative decay rates to D^0 mesons.

Parameter	Measurements	Average
$\frac{\mathcal{B}(B^- \rightarrow D^0 \pi^-)}{\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)}$	CDF [646]: $1.97 \pm 0.10 \pm 0.21$	1.97 ± 0.23
$\frac{\mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)}{\mathcal{B}(B^- \rightarrow D^0 \pi^-)}$	LHCb [547]: $1.27 \pm 0.06 \pm 0.11$	1.27 ± 0.13
$\frac{\mathcal{B}(B^- \rightarrow \bar{D}^0 K^-)}{\mathcal{B}(B^- \rightarrow D^0 K^-)}$	Belle [647]: < 0.19	< 0.19
$\frac{\mathcal{B}(B^- \rightarrow D^0 K^-)}{\mathcal{B}(B^- \rightarrow D^0 \pi^-)}$	LHCb [648]: $0.063 \pm 0.004 \pm 0.004$ CDF [649]: $0.065 \pm 0.007 \pm 0.004$ BABAR [650]: $0.0831 \pm 0.0035 \pm 0.0020$ Belle [641]: $0.077 \pm 0.005 \pm 0.006$ Belle [647]: $0.0677 \pm 0.0023 \pm 0.0030$	0.0723 ± 0.0023
$\mathcal{B}(B^- \rightarrow D^0 K^- \pi^+ \pi^-) / \mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)$	LHCb [548]: $0.094 \pm 0.013 \pm 0.009$	0.094 ± 0.016

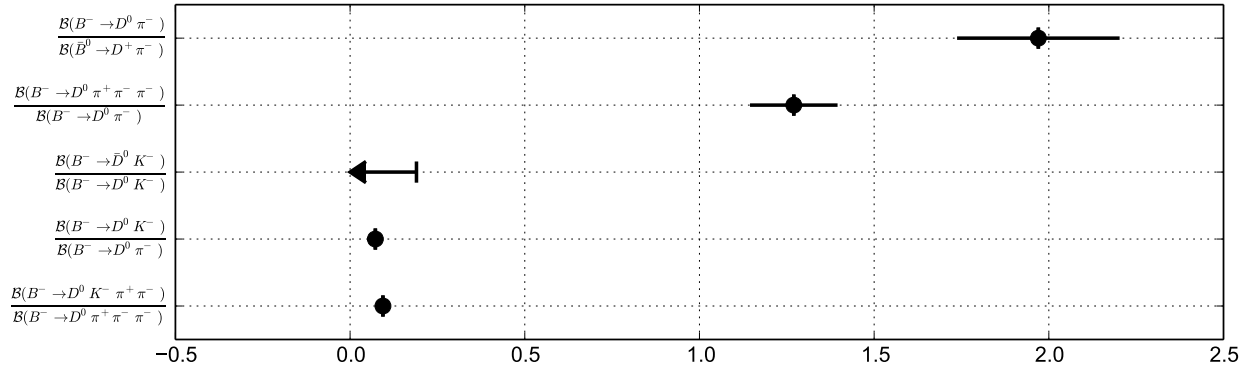


Figure 96: Summary of the averages from Table 117.

Table 118: Absolute decay rates to excited D mesons [10^{-2}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^{*0} \pi^-)$	<i>BABAR</i> [527]: $0.550 \pm 0.052 \pm 0.104$	0.550 ± 0.116

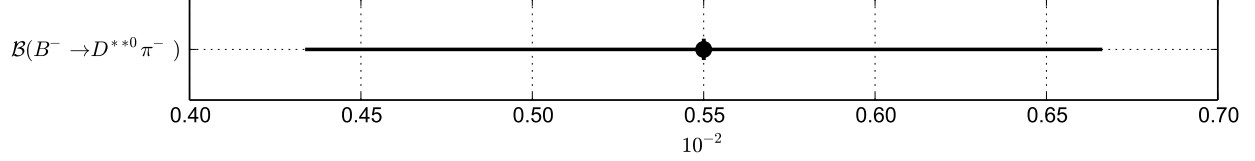


Figure 97: Summary of the averages from Table 118.

Table 119: Absolute product decay rates to excited D mesons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D_1^0(2420)\pi^-) \times \mathcal{B}(D_1^0(2420) \rightarrow D^{*+}(2010)\pi^-)$	<i>BABAR</i> [640]: $0.59 \pm 0.03 \pm 0.11$ Belle [639]: $0.68 \pm 0.07 \pm 0.13$	0.62 ± 0.09
$\mathcal{B}(B^- \rightarrow D_1^0(2420)\pi^-) \times \mathcal{B}(D_1^0(2420) \rightarrow D^0 \pi^- \pi^+)$	Belle [550]: $0.185 \pm 0.029^{+0.035}_{-0.058}$	$0.185^{+0.045}_{-0.065}$
$\mathcal{B}(B^- \rightarrow D_1^0(2420)\pi^-) \times \mathcal{B}(D_1^0(2420) \rightarrow D^{*0}(2007)\pi^- \pi^+)$	Belle [550]: < 0.006	< 0.006
$\mathcal{B}(B^- \rightarrow D_0^{*0}\pi^-) \times \mathcal{B}(D_0^{*0} \rightarrow D^+ \pi^-)$	<i>BABAR</i> [638]: $0.68 \pm 0.03 \pm 0.20$ Belle [639]: $0.61 \pm 0.06 \pm 0.18$	0.63 ± 0.19
$\mathcal{B}(B^- \rightarrow D_1^0(H)\pi^-) \times \mathcal{B}(D_1^0(H) \rightarrow D^{*+}(2010)\pi^-)$	Belle [639]: $0.50 \pm 0.04 \pm 0.11$	0.50 ± 0.11
$\mathcal{B}(B^- \rightarrow D_2^{*0}(2460)\pi^-) \times \mathcal{B}(D_2^{*0}(2460) \rightarrow D^{*+}(2010)\pi^-)$	<i>BABAR</i> [640]: $0.18 \pm 0.03 \pm 0.05$ Belle [639]: $0.18 \pm 0.03 \pm 0.04$	0.18 ± 0.04
$\mathcal{B}(B^- \rightarrow D_2^{*0}(2460)\pi^-) \times \mathcal{B}(D_2^{*0}(2460) \rightarrow D^+ \pi^-)$	<i>BABAR</i> [638]: $0.35 \pm 0.02 \pm 0.04$ Belle [639]: $0.34 \pm 0.03 \pm 0.07$	0.35 ± 0.05
$\mathcal{B}(B^- \rightarrow D_2^{*0}(2460)\pi^-) \times \mathcal{B}(D_2^{*0}(2460) \rightarrow D^{*0}(2007)\pi^- \pi^+)$	Belle [550]: < 0.022	< 0.022

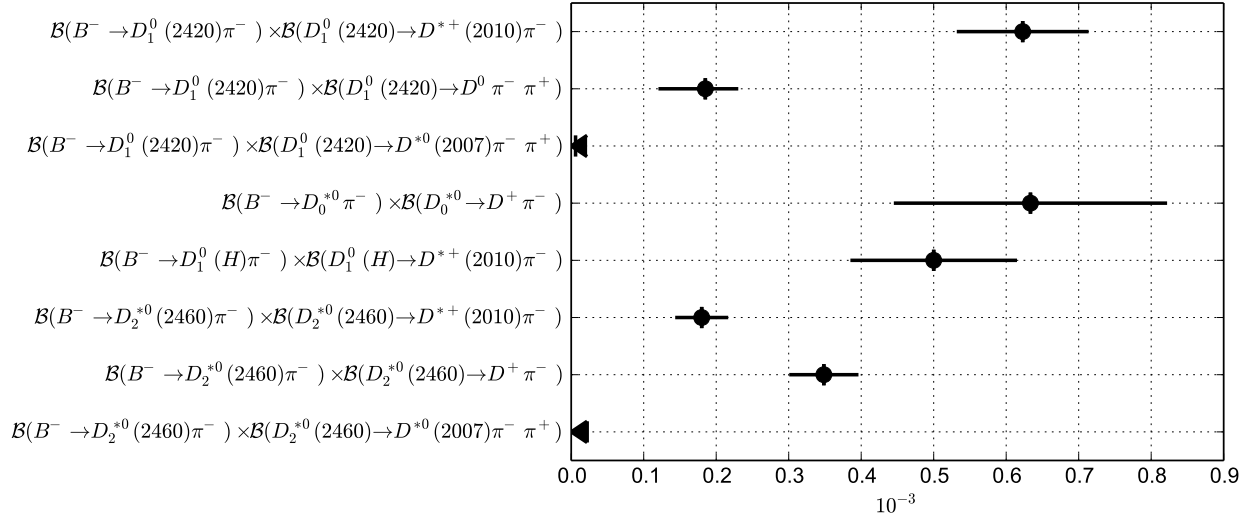


Figure 98: Summary of the averages from Table 119.

Table 120: Relative decay rates to excited D mesons.

Parameter	Measurements	Average
$\frac{\mathcal{B}(B^- \rightarrow D^{*0}(2007)\pi^-)}{\mathcal{B}(B^- \rightarrow D^0\pi^-)}$	BABAR [527]: $1.14 \pm 0.07 \pm 0.04$	1.14 ± 0.08
$\frac{\mathcal{B}(B^- \rightarrow D^{*0}\pi^-)}{\mathcal{B}(B^- \rightarrow D^0\pi^-)}$	BABAR [527]: $1.22 \pm 0.13 \pm 0.23$	1.22 ± 0.26
$\mathcal{B}(B^- \rightarrow D_2^{*0}(2460)\pi^-)/\mathcal{B}(B^- \rightarrow D_1^0(2420)\pi^-)$	BABAR [640]: $0.80 \pm 0.07 \pm 0.16$	0.80 ± 0.17
$\mathcal{B}(B^- \rightarrow D^{*0}(2007)K^-)/\mathcal{B}(B^- \rightarrow D^{*0}(2007)\pi^-)$	BABAR [651]: $0.0813 \pm 0.0040^{+0.0042}_{-0.0031}$ Belle [534]: $0.078 \pm 0.019 \pm 0.009$	0.0811 ± 0.0053

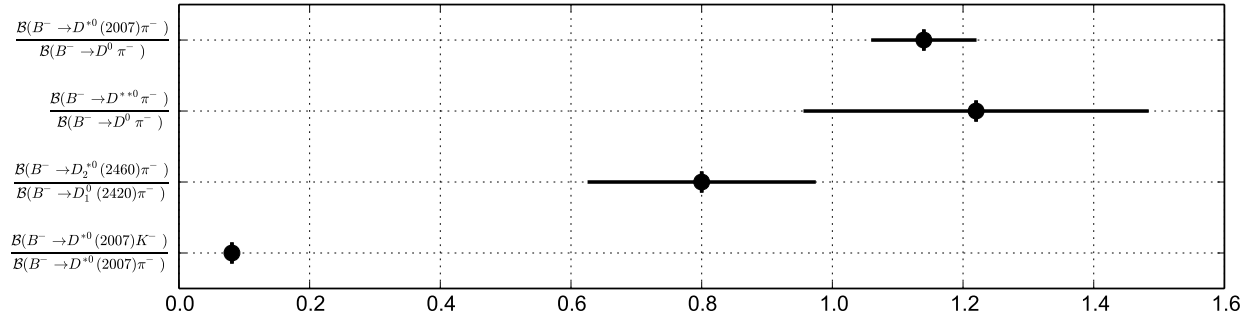


Figure 99: Summary of the averages from Table 120.

Table 121: Relative product decay rates to excited D mesons.

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D_1^0 \pi^-) \times \mathcal{B}(D_1^0 \rightarrow D^0 \pi^+ \pi^-) / \mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.040 \pm 0.007 \pm 0.005$	0.040 ± 0.009
$\mathcal{B}(B^- \rightarrow D_1^{*0} \pi^-) \times \mathcal{B}(D_1^{*0} \rightarrow D^{*+} \pi^-) / \mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.093 \pm 0.016 \pm 0.009$	0.093 ± 0.018
$\mathcal{B}(B^- \rightarrow D_1^{*0} \pi^-) \times \mathcal{B}(D_1^{*0} \rightarrow D^0 \pi^+ \pi^-) / \mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.103 \pm 0.015 \pm 0.009$	0.103 ± 0.017
$\mathcal{B}(B^- \rightarrow D_2^{*0} \pi^-) \times \mathcal{B}(D_2^{*0} \rightarrow D^{*+} \pi^-) / \mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.039 \pm 0.012 \pm 0.004$	0.039 ± 0.013
$\mathcal{B}(B^- \rightarrow D_2^{*0} \pi^-) \times \mathcal{B}(D_2^{*0} \rightarrow D^0 \pi^+ \pi^-) / \mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.040 \pm 0.010 \pm 0.004$	0.040 ± 0.011
$\mathcal{B}(B^- \rightarrow D_2^{*+} \pi^-) \times \mathcal{B}(D_2^{*+} \rightarrow D^0 \pi^- \pi^+) / \mathcal{B}(B^- \rightarrow D^0 \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.014 \pm 0.006 \pm 0.002$	0.014 ± 0.006

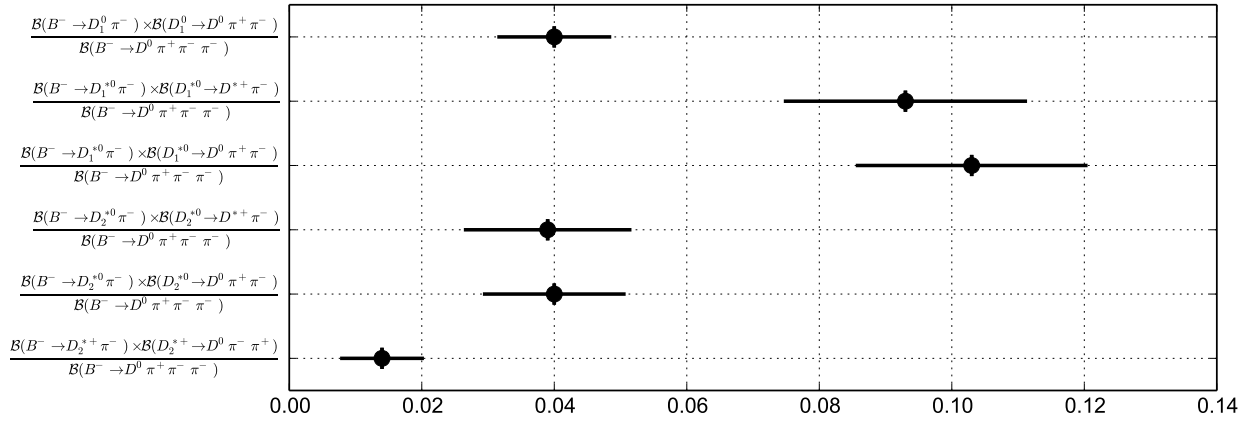


Figure 100: Summary of the averages from Table 121.

Table 122: Decays to $D_s^{(*)}$ mesons [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D_s^+ K^- \pi^-)$	BABAR [544]: $2.02 \pm 0.13 \pm 0.38$ Belle [652]: $1.94^{+0.09+0.26}_{-0.08-0.26}$	1.97 ± 0.23
$\mathcal{B}(B^- \rightarrow D_s^{*+} K^- \pi^-)$	BABAR [544]: $1.67 \pm 0.16 \pm 0.35$ Belle [652]: $1.47^{+0.15+0.23}_{-0.14-0.23}$	1.54 ± 0.22
$\mathcal{B}(B^- \rightarrow D_s^+ K^- K^-)$	BABAR [544]: $0.11 \pm 0.04 \pm 0.02$	0.11 ± 0.04
$\mathcal{B}(B^- \rightarrow D_s^{*+} K^- K^-)$	BABAR [544]: < 0.15	< 0.15
$\mathcal{B}(B^- \rightarrow D_s^- \pi^0)$	BABAR [653]: $0.15^{+0.05}_{-0.04} \pm 0.02$	$0.15^{+0.05}_{-0.05}$
$\mathcal{B}(B^- \rightarrow D_s^- \phi(1020))$	LHCb [654]: $0.0187^{+0.0125}_{-0.0073} \pm 0.0037$ BABAR [655]: < 0.019	$0.0187^{+0.0130}_{-0.0082}$
$\mathcal{B}(B^- \rightarrow D_s^{*-} \phi(1020))$	BABAR [655]: < 0.12	< 0.12

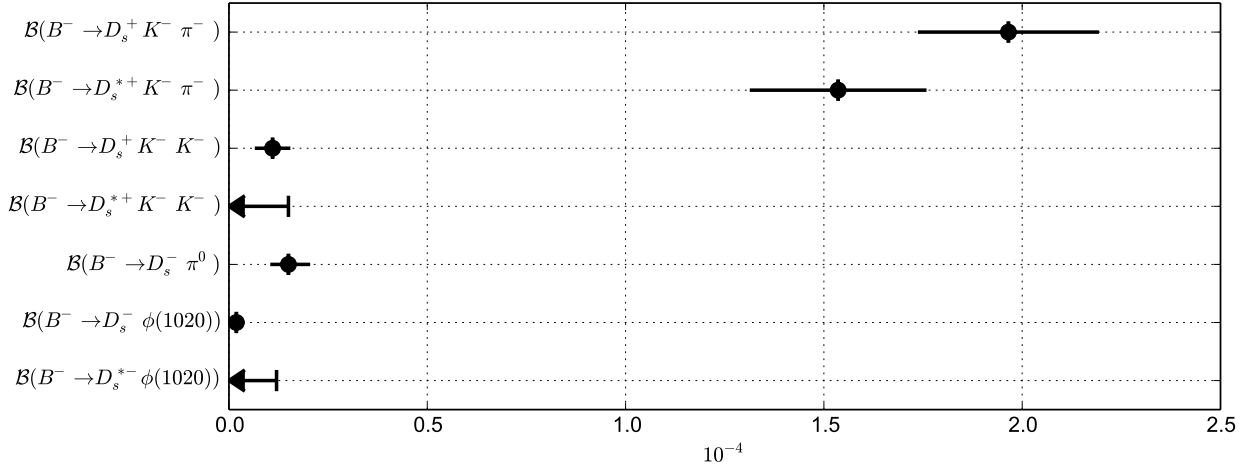


Figure 101: Summary of the averages from Table 122.

 Table 123: Baryonic decays [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^0 p \bar{p} \pi^-)$	BABAR [553]: $3.72 \pm 0.11 \pm 0.25$	3.72 ± 0.27
$\mathcal{B}(B^- \rightarrow D^{*0}(2007) p \bar{p} \pi^-)$	BABAR [553]: $3.73 \pm 0.17 \pm 0.27$	3.73 ± 0.32
$\mathcal{B}(B^- \rightarrow D^+ p \bar{p} \pi^- \pi^-)$	BABAR [553]: $1.66 \pm 0.13 \pm 0.27$	1.66 ± 0.30
$\mathcal{B}(B^- \rightarrow D^{*+}(2010) p \bar{p} \pi^- \pi^-)$	BABAR [553]: $1.86 \pm 0.16 \pm 0.19$	1.86 ± 0.25
$\mathcal{B}(B^- \rightarrow \bar{p} \Lambda D^0)$	Belle [656]: $0.143^{+0.028}_{-0.025} \pm 0.018$	$0.143^{+0.033}_{-0.031}$
$\mathcal{B}(B^- \rightarrow \bar{p} \Lambda D^{*0}(2007))$	Belle [656]: < 0.48	< 0.48
$\mathcal{B}(B^- \rightarrow D^- p \bar{p})$	Belle [554]: < 0.15	< 0.15
$\mathcal{B}(B^- \rightarrow D^{*-}(2010) p \bar{p})$	Belle [554]: < 0.15	< 0.15

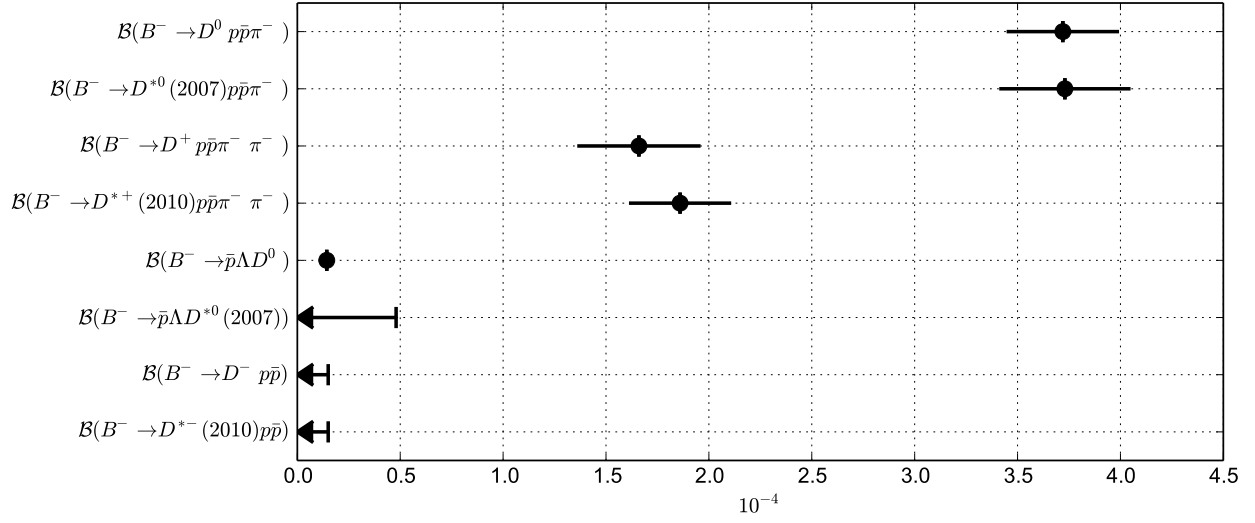


Figure 102: Summary of the averages from Table 123.

Table 124: Lepton number violating decays [10^{-6}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^- e^+ e^+)$	Belle [657]: < 2.6	< 2.6
$\mathcal{B}(B^- \rightarrow D^- e^+ \mu^+)$	Belle [657]: < 1.8	< 1.8
$\mathcal{B}(B^- \rightarrow D^- \mu^+ \mu^+)$	Belle [657]: < 1.0	< 1.0

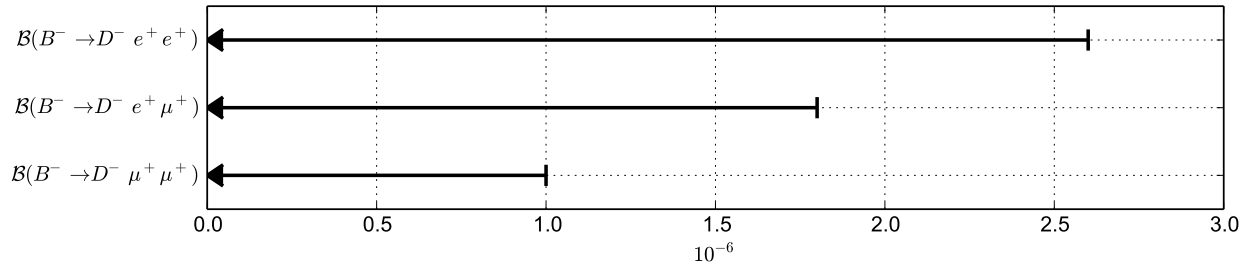


Figure 103: Summary of the averages from Table 124.

6.2.2 Decays to two open charm mesons

Averages of B^- decays to two open charm mesons are shown in Tables 125–130 and Figs. 104–109.

Table 125: Decays to $D^{(*)-}D^{(*)0}$ [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^- D^0)$	<i>BABAR</i> [558]: $0.38 \pm 0.06 \pm 0.05$ <i>Belle</i> [559]: $0.385 \pm 0.031 \pm 0.038$	0.384 ± 0.042
$\mathcal{B}(B^- \rightarrow D^{*0} D^-)$	<i>BABAR</i> [558]: $0.63 \pm 0.14 \pm 0.10$	0.63 ± 0.17
$\mathcal{B}(B^- \rightarrow D_s^- D^0)$	<i>BABAR</i> [558]: $0.36 \pm 0.05 \pm 0.04$ <i>Belle</i> [658]: $0.459 \pm 0.072 \pm 0.056$	0.385 ± 0.046
$\mathcal{B}(B^- \rightarrow D^{*0}(2007)D^{*-}(2010))$	<i>BABAR</i> [558]: $0.81 \pm 0.12 \pm 0.12$	0.81 ± 0.17

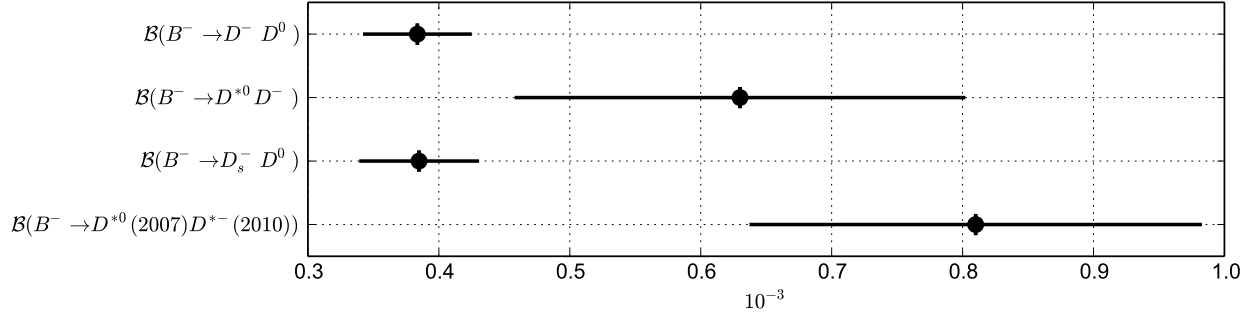


Figure 104: Summary of the averages from Table 125.

 Table 126: Decays to two D mesons and a kaon [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^0 \bar{D}^0 K^-)$	<i>BABAR</i> [560]: $1.31 \pm 0.07 \pm 0.12$ <i>Belle</i> [659]: $2.22 \pm 0.22^{+0.26}_{-0.24}$	1.44 ± 0.13
$\mathcal{B}(B^- \rightarrow D^{*0}(2007) \bar{D}^0 K^-)$	<i>BABAR</i> [560]: $2.26 \pm 0.16 \pm 0.17$	2.26 ± 0.23
$\mathcal{B}(B^- \rightarrow D^0 \bar{D}^{*0}(2007) K^-)$	<i>BABAR</i> [560]: $6.32 \pm 0.19 \pm 0.45$	6.32 ± 0.49
$\mathcal{B}(B^- \rightarrow \bar{D}^{*0}(2007) D^{*0}(2007) K^-)$	<i>BABAR</i> [560]: $11.23 \pm 0.36 \pm 1.26$	11.23 ± 1.31
$\mathcal{B}(B^- \rightarrow D^0 \bar{D}^0 \pi^0 K^-)$	<i>Belle</i> [561]: $0.107 \pm 0.031^{+0.019}_{-0.033}$	$0.107^{+0.036}_{-0.045}$
$\mathcal{B}(B^- \rightarrow D^+ D^- K^-)$	<i>BABAR</i> [560]: $0.22 \pm 0.05 \pm 0.05$ <i>Belle</i> [660]: < 0.90	0.22 ± 0.07
$\mathcal{B}(B^- \rightarrow D^{*+}(2010) D^- K^-)$	<i>BABAR</i> [560]: $0.60 \pm 0.10 \pm 0.08$	0.60 ± 0.13
$\mathcal{B}(B^- \rightarrow D^+ D^{*-}(2010) K^-)$	<i>BABAR</i> [560]: $0.63 \pm 0.09 \pm 0.06$	0.63 ± 0.11
$\mathcal{B}(B^- \rightarrow D^{*-}(2010) D^{*+}(2010) K^-)$	<i>BABAR</i> [560]: $1.32 \pm 0.13 \pm 0.12$	1.32 ± 0.18
$\mathcal{B}(B^- \rightarrow D^0 D^- \bar{K}^0)$	<i>BABAR</i> [560]: $1.55 \pm 0.17 \pm 0.13$	1.55 ± 0.21
$\mathcal{B}(B^- \rightarrow D^{*0}(2007) D^- \bar{K}^0)$	<i>BABAR</i> [560]: $2.06 \pm 0.38 \pm 0.30$	2.06 ± 0.48
$\mathcal{B}(B^- \rightarrow D^0 D^{*-}(2010) \bar{K}^0)$	<i>BABAR</i> [560]: $3.81 \pm 0.31 \pm 0.23$	3.81 ± 0.39
$\mathcal{B}(B^- \rightarrow D^{*0}(2007) D^{*-}(2010) \bar{K}^0)$	<i>BABAR</i> [560]: $9.17 \pm 0.83 \pm 0.90$	9.17 ± 1.22

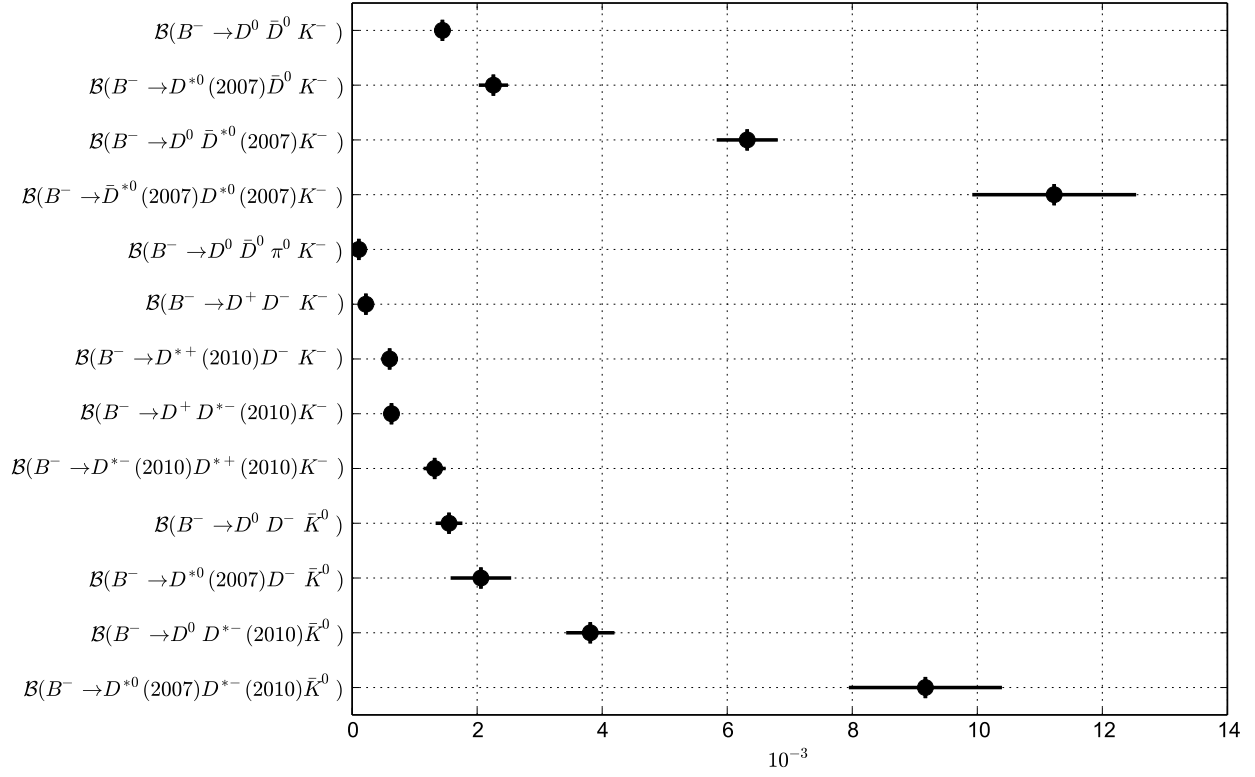


Figure 105: Summary of the averages from Table 126.

Table 127: Decays to $D_s^{(*)-} D^{(*)+}$ [10^{-2}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D_s^- D^0)$	BABAR [562]: $1.33 \pm 0.18 \pm 0.32$ Belle [661]: $0.852^{+0.039}_{-0.038} \pm 0.000$	0.857 ± 0.038
$\mathcal{B}(B^- \rightarrow D_s^- D^0) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $0.0400 \pm 0.0061 \pm 0.0061$	0.0400 ± 0.0086
$\mathcal{B}(B^- \rightarrow D_s^- D^{*0}(2007))$	BABAR [562]: $1.21 \pm 0.23 \pm 0.20$	1.21 ± 0.30
$\mathcal{B}(B^- \rightarrow D_s^- D^{*0}(2007)) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $0.0295 \pm 0.0065 \pm 0.0036$	0.0295 ± 0.0074
$\mathcal{B}(B^- \rightarrow D_s^{*-} D^0)$	BABAR [562]: $0.93 \pm 0.18 \pm 0.19$	0.93 ± 0.26
$\mathcal{B}(B^- \rightarrow D_s^{*-} D^0) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $0.0313 \pm 0.0119 \pm 0.0058$	0.0313 ± 0.0132
$\mathcal{B}(B^- \rightarrow D_s^{*-} D^{*0}(2007))$	BABAR [562]: $1.70 \pm 0.26 \pm 0.24$	1.70 ± 0.35
$\mathcal{B}(B^- \rightarrow D_s^{*-} D^{*0}(2007)) \times \mathcal{B}(D_s^- \rightarrow \phi(1020)\pi^-)$	BABAR [562]: $0.0857 \pm 0.0148 \pm 0.0112$	0.0857 ± 0.0186

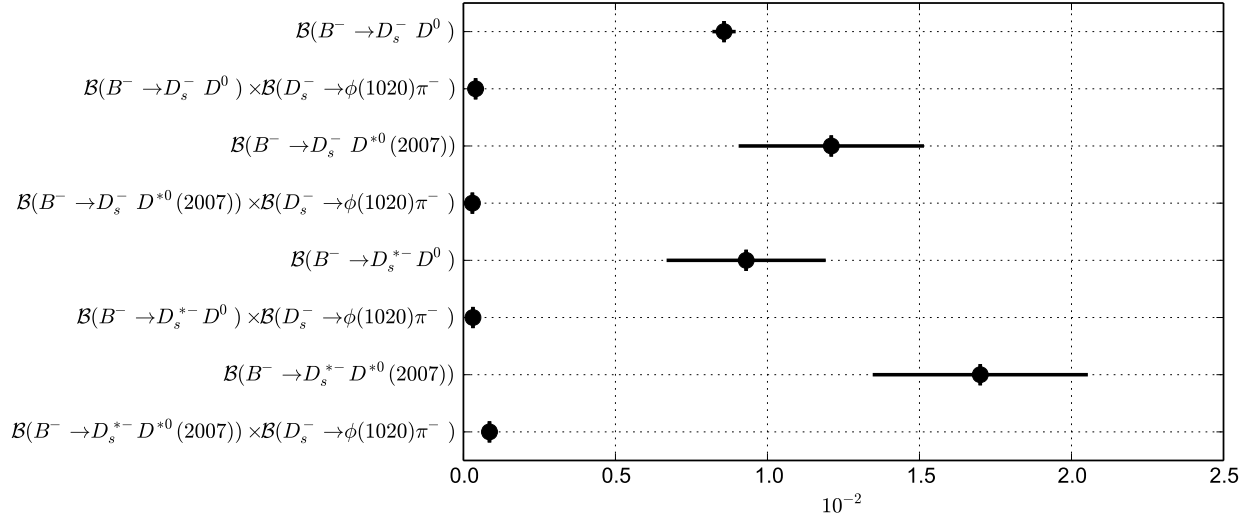


Figure 106: Summary of the averages from Table 127.

Table 128: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D_s^- D^0)/\mathcal{B}(\bar{B}^0 \rightarrow D_s^+ D^-)$	LHCb [568]: $1.22 \pm 0.02 \pm 0.07$	1.22 ± 0.07

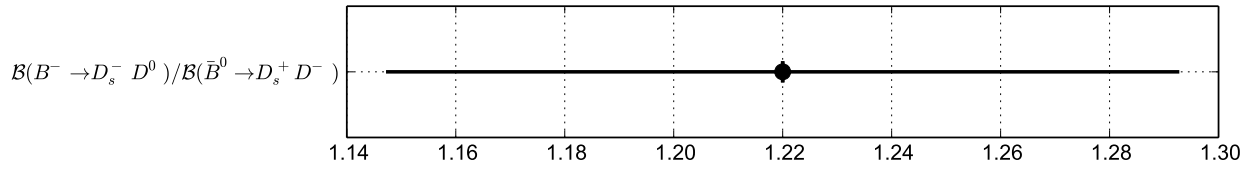


Figure 107: Summary of the averages from Table 128.

Table 129: Absolute decays rates to excited D_s mesons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D_{sJ}^-(2460) D^0)$	BABAR [562]: $4.3 \pm 1.6 \pm 1.3$	4.3 ± 2.1
$\mathcal{B}(B^- \rightarrow D_{sJ}^-(2460) D^{*0}(2007))$	BABAR [562]: $11.2 \pm 2.6 \pm 2.0$	11.2 ± 3.3

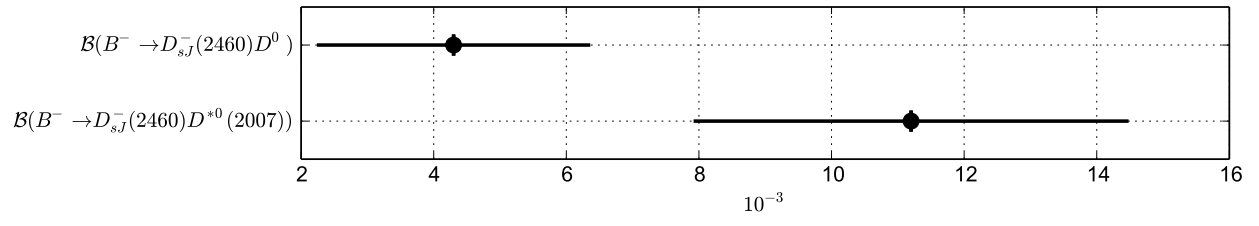


Figure 108: Summary of the averages from Table 129.

Table 130: Product decays rates to excited D_s mesons [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow D^0 D_{sJ}^*(2317)^- \times \mathcal{B}(D_{sJ}^*(2317)^- \rightarrow D_s^- \pi^0)$	BABAR [573]: $1.0 \pm 0.3^{+0.4}_{-0.2}$ Belle [572]: $0.81^{+0.30}_{-0.27} \pm 0.24$	0.89 ± 0.29
$\mathcal{B}(B^- \rightarrow D^0 D_{sJ}^*(2317)^- \times \mathcal{B}(D_{sJ}^*(2317)^- \rightarrow D_s^{*-} \gamma)$	Belle [572]: $0.25^{+0.21}_{-0.16} \pm 0.00$	$0.25^{+0.21}_{-0.16}$
$\mathcal{B}(B^- \rightarrow D_{sJ}^*(2317)^- D^{*0}(2007)) \times \mathcal{B}(D_{sJ}^*(2317)^- \rightarrow D_s^- \pi^0)$	BABAR [573]: $0.9 \pm 0.6^{+0.4}_{-0.3}$	$0.9^{+0.7}_{-0.7}$
$\mathcal{B}(B^- \rightarrow D^0 D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^- \gamma)$	BABAR [573]: $0.6 \pm 0.2^{+0.2}_{-0.1}$ Belle [572]: $0.56^{+0.16}_{-0.15} \pm 0.17$	0.58 ± 0.18
$\mathcal{B}(B^- \rightarrow D^0 D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^{*-} \gamma)$	Belle [572]: $0.31^{+0.27}_{-0.23} \pm 0.00$	$0.31^{+0.27}_{-0.23}$
$\mathcal{B}(B^- \rightarrow D^0 D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^- \pi^0)$	Belle [572]: < 0.27	< 0.27
$\mathcal{B}(B^- \rightarrow D^0 D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^{*-} \pi^0)$	BABAR [573]: $2.7 \pm 0.7^{+1.0}_{-0.8}$ Belle [572]: $1.19^{+0.61}_{-0.49} \pm 0.36$	1.56 ± 0.57
$\mathcal{B}(B^- \rightarrow D^0 D_{sJ}^-(2460)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^- \pi^+ \pi^-)$	Belle [572]: < 0.22	< 0.22
$\mathcal{B}(B^- \rightarrow D_{sJ}^-(2460) D^{*0}(2007)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^- \gamma)$	BABAR [573]: $1.4 \pm 0.4^{+0.6}_{-0.4}$	$1.4^{+0.7}_{-0.6}$
$\mathcal{B}(B^- \rightarrow D_{sJ}^-(2460) D^{*0}(2007)) \times \mathcal{B}(D_{sJ}^-(2460) \rightarrow D_s^{*-} \pi^0)$	BABAR [573]: $7.6 \pm 1.7^{+3.2}_{-2.4}$	$7.6^{+3.6}_{-2.9}$
$\mathcal{B}(B^+ \rightarrow D_{s1}^+(2536) \bar{D}^0) \times \mathcal{B}(D_{s1}^+(2536) \rightarrow D^{*0}(2007) K^+ + D^{*+}(2010) K^0)$	Belle [571]: $0.397 \pm 0.085 \pm 0.056$	0.397 ± 0.102
$\mathcal{B}(B^- \rightarrow D_{s1}^-(2536) D^0) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow \bar{D}^{*0}(2007) K^-)$	BABAR [570]: $0.216 \pm 0.052 \pm 0.045$	0.216 ± 0.069
$\mathcal{B}(B^- \rightarrow D_{s1}^-(2536) D^0) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow D^{*-}(2010) \bar{K}^0)$	BABAR [570]: $0.230 \pm 0.098 \pm 0.043$	0.230 ± 0.107
$\mathcal{B}(B^- \rightarrow D_{s1}^-(2536) \bar{D}^{*0}(2007)) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow \bar{D}^{*0}(2007) K^-)$	BABAR [570]: $0.546 \pm 0.117 \pm 0.104$	0.546 ± 0.157
$\mathcal{B}(B^- \rightarrow D_{s1}^-(2536) D^{*0}(2007)) \times \mathcal{B}(D_{s1}^-(2536) \rightarrow D^{*-}(2010) \bar{K}^0)$	BABAR [570]: < 1.069	< 1.069

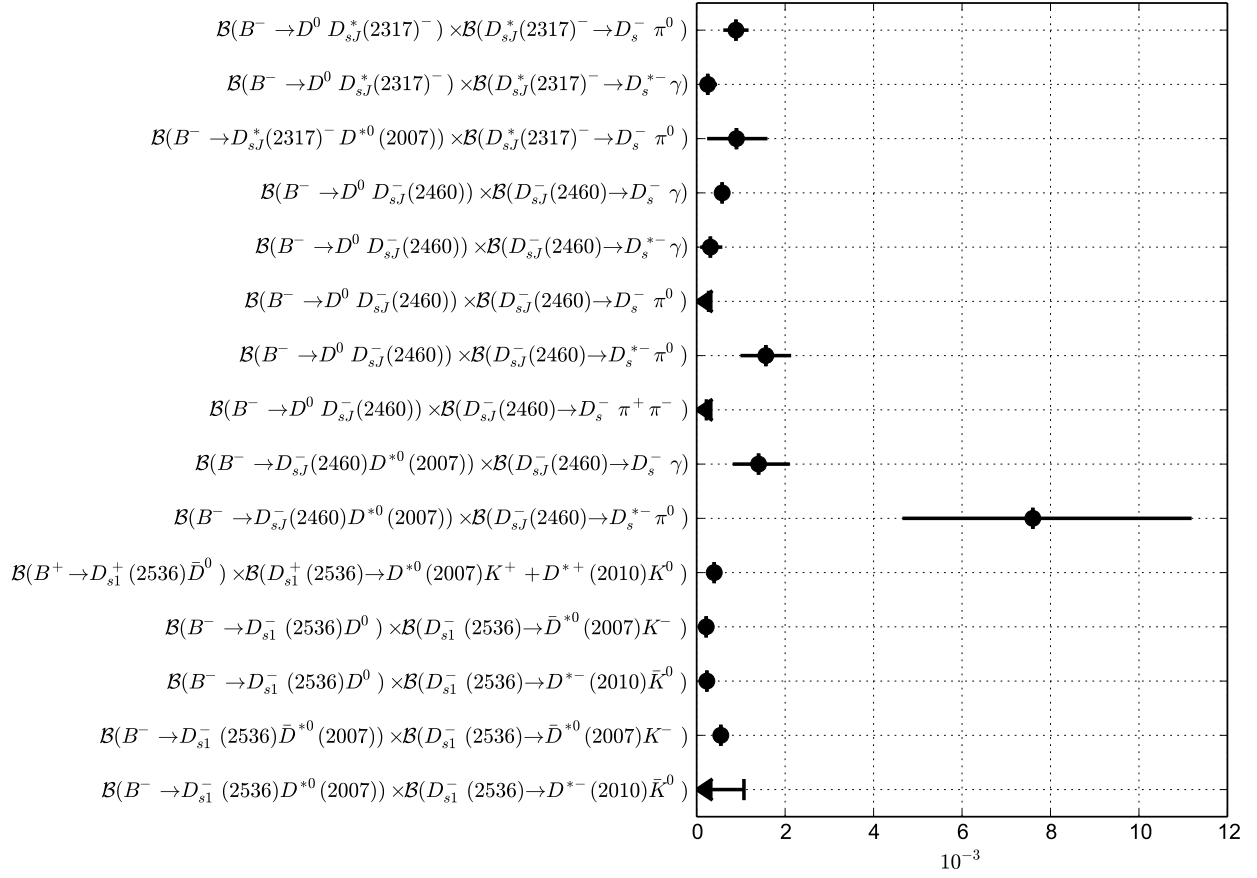


Figure 109: Summary of the averages from Table 130.

6.2.3 Decays to charmonium states

Averages of B^- decays to charmonium states are shown in Tables 131–136 and Figs. 110–115.

Table 131: Decays to J/ψ and one kaon [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow J/\psi(1S)K^-)$	BABAR [6]: $1.061 \pm 0.015 \pm 0.048$ BABAR [634]: $0.81 \pm 0.13 \pm 0.07$ Belle [575]: $1.01 \pm 0.02 \pm 0.07$	1.028 ± 0.040
$\mathcal{B}(B^- \rightarrow J/\psi(1S)K^{*-}(892))$	CDF [574]: $1.58 \pm 0.47 \pm 0.27$ BABAR [6]: $1.454 \pm 0.047 \pm 0.097$ Belle [662]: $1.28 \pm 0.07 \pm 0.14$	1.404 ± 0.089
$\mathcal{B}(B^- \rightarrow J/\psi(1S)K_1^-(1270))$	Belle [582]: $1.80 \pm 0.34 \pm 0.39$	1.80 ± 0.52
$\mathcal{B}(B^- \rightarrow J/\psi(1S)K^- \pi^+ \pi^-)$	CDF [663]: $0.69 \pm 0.18 \pm 0.12$ BABAR [664]: $1.16 \pm 0.07 \pm 0.09$ Belle [665]: $0.716 \pm 0.010 \pm 0.060$	0.807 ± 0.052
$\mathcal{B}(B^- \rightarrow J/\psi(1S)\eta K^-)$	BABAR [583]: $0.108 \pm 0.023 \pm 0.024$ Belle [584]: $0.127 \pm 0.011 \pm 0.011$	0.124 ± 0.014
$\mathcal{B}(B^- \rightarrow J/\psi(1S)\omega(782)K^-)$	BABAR [579]: $0.32 \pm 0.01^{+0.06}_{-0.03}$	$0.32^{+0.06}_{-0.03}$
$\mathcal{B}(B^- \rightarrow J/\psi(1S)\phi(1020)K^-)$	BABAR [580]: $0.044 \pm 0.014 \pm 0.005$	0.044 ± 0.015

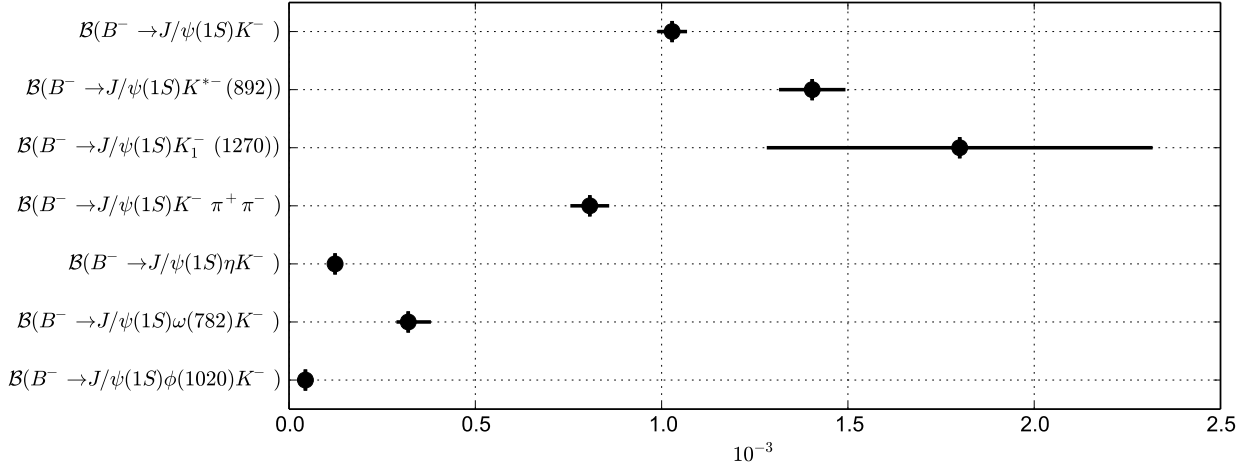


Figure 110: Summary of the averages from Table 131.

 Table 132: Decays to charmonium other than J/ψ and one kaon [10^{-3}]

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow \psi(2S)K^-)$	CDF [577]: $0.55 \pm 0.10 \pm 0.06$ BABAR [6]: $0.617 \pm 0.032 \pm 0.044$ BABAR [634]: $0.49 \pm 0.16 \pm 0.04$ Belle [575]: 0.69 ± 0.06	0.632 ± 0.037
$\mathcal{B}(B^- \rightarrow \psi(2S)K^-) \times \mathcal{B}(\psi(2S) \rightarrow \chi_{c1}(1P)\gamma)$	Belle [586]: $0.77 \pm 0.08 \pm 0.09$	0.77 ± 0.12

continued from previous page

$\mathcal{B}(B^- \rightarrow \psi(2S)K^-) \times \mathcal{B}(\psi(2S) \rightarrow \chi_{c2}\gamma)$	Belle [586]: $0.63 \pm 0.09 \pm 0.06$	0.63 ± 0.11
$\mathcal{B}(B^- \rightarrow \psi(2S)K^{*-}(892))$	BABAR [6]: $0.592 \pm 0.085 \pm 0.089$ Belle [585]: $0.813 \pm 0.077 \pm 0.089$	0.707 ± 0.085
$\mathcal{B}(B^- \rightarrow \psi(2S)K^- \pi^+ \pi^-)$	Belle [665]: $0.431 \pm 0.020 \pm 0.050$	0.431 ± 0.054
$\mathcal{B}(B^- \rightarrow \psi(3770)K^-)$	BABAR [634]: $0.35 \pm 0.25 \pm 0.03$ Belle [660]: $0.48 \pm 0.11 \pm 0.07$	0.45 ± 0.12
$\mathcal{B}(B^- \rightarrow \psi(3770)K^-) \times \mathcal{B}(\psi(3770) \rightarrow D^+ D^-)$	BABAR [570]: $0.084 \pm 0.032 \pm 0.021$	0.084 ± 0.038
$\mathcal{B}(B^- \rightarrow \psi(3770)K^-) \times \mathcal{B}(\psi(3770) \rightarrow D^0 \bar{D}^0)$	BABAR [570]: $0.141 \pm 0.030 \pm 0.022$	0.141 ± 0.037
$\mathcal{B}(B^- \rightarrow \chi_{c0}(1P)K^-)$	BABAR [634]: < 0.18 BABAR [242]: $0.184 \pm 0.032 \pm 0.031$ Belle [666]: $0.60^{+0.21}_{-0.18} \pm 0.11$	0.200 ± 0.044
$\mathcal{B}(B^- \rightarrow \chi_{c1}(1P)K^-)$	CDF [663]: $1.55 \pm 0.54 \pm 0.20$ BABAR [634]: $0.80 \pm 0.14 \pm 0.07$ BABAR [590]: $0.45 \pm 0.01 \pm 0.03$ Belle [591]: $0.494 \pm 0.011 \pm 0.033$	0.479 ± 0.023
$\mathcal{B}(B^- \rightarrow \chi_{c1} \bar{K}^0 \pi^-)$	BABAR [592]: $0.552 \pm 0.026 \pm 0.061$	0.552 ± 0.066
$\mathcal{B}(B^- \rightarrow \chi_{c2}(1P)K^-)$	BABAR [590]: $0.01 \pm 0.01 \pm 0.00$ Belle [591]: $0.0111^{+0.0036}_{-0.0034} \pm 0.0009$	0.0108 ± 0.0031
$\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-)$	BABAR [594]: $1.29 \pm 0.09 \pm 0.38$ BABAR [634]: 0.87 ± 0.15 Belle [596]: $1.25 \pm 0.14^{+0.39}_{-0.40}$	0.92 ± 0.14
$\mathcal{B}(B^- \rightarrow K^- \eta_c(1S)) \times \mathcal{B}(\eta_c(1S) \rightarrow K^0 K^+ \pi^+)$	Belle [667]: $0.000267 \pm 0.000014^{+0.000057}_{-0.000055}$	$0.000267^{+0.000059}_{-0.000057}$
$\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-) \times \mathcal{B}(\eta_c(1S) \rightarrow p \bar{p})$	BABAR [668]: $0.0018^{+0.0003}_{-0.0002} \pm 0.0002$ Belle [669]: $0.00142 \pm 0.00011^{+0.00016}_{-0.00020}$	0.00153 ± 0.00018
$\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-) \times \mathcal{B}(\eta_c(1S) \rightarrow \Lambda \Lambda)$	Belle [669]: $0.00095^{+0.00025}_{-0.00022}^{+0.00008}_{-0.00011}$	$0.00095^{+0.00026}_{-0.00025}$
$\mathcal{B}(B^- \rightarrow \eta_c(1S)K^{*-}(892))$	BABAR [595]: $1.21^{+0.43}_{-0.35}^{+0.64}_{-0.40}$	$1.21^{+0.77}_{-0.53}$
$\mathcal{B}(B^- \rightarrow \eta_c(2S)K^-)$	BABAR [634]: $0.34 \pm 0.18 \pm 0.03$	0.34 ± 0.18
$\mathcal{B}(B^- \rightarrow K^- \eta_c(2S)) \times \mathcal{B}(\eta_c(2S) \rightarrow K^0 K^- \pi^+)$	Belle [667]: $0.000034^{+0.000022}_{-0.000015}^{+0.000005}_{-0.000004}$	$0.000034^{+0.000023}_{-0.000016}$
$\mathcal{B}(B^- \rightarrow \chi_{c0}(1P)K^{*-}(892))$	BABAR [588]: < 2.86 BABAR [589]: $0.14 \pm 0.05 \pm 0.02$	0.14 ± 0.05
$\mathcal{B}(B^- \rightarrow \chi_{c1}(1P)K^{*-}(892))$	BABAR [590]: $0.26 \pm 0.05 \pm 0.04$ Belle [593]: $0.41 \pm 0.06 \pm 0.09$	0.30 ± 0.06
$\mathcal{B}(B^- \rightarrow \chi_{c2}(1P)K^{*-}(892))$	BABAR [590]: $0.011 \pm 0.043 \pm 0.055$	0.011 ± 0.070
$\mathcal{B}(B^- \rightarrow h_c(1P)K^-)$	Belle [670]: < 0.0038	< 0.0038
$\mathcal{B}(B^- \rightarrow h_c(1P)K^-) \times \mathcal{B}(h_c(1P) \rightarrow \eta_c(1S)\gamma)$	BABAR [597]: < 0.048	< 0.048
$\mathcal{B}(B^- \rightarrow h_c(1P)K^-) \times \mathcal{B}(h_c(1P) \rightarrow J/\psi(1S)\pi^+ \pi^-)$		

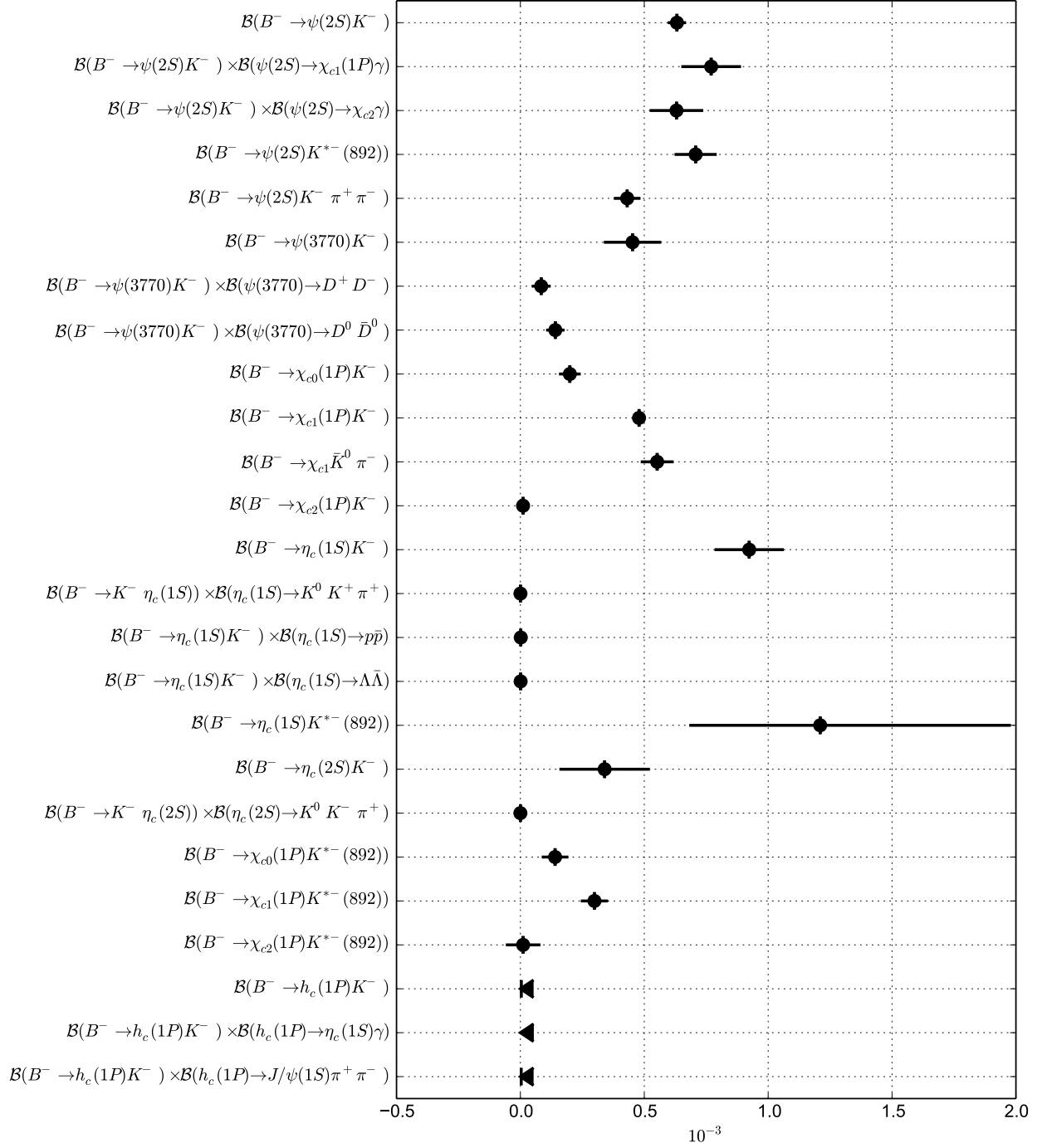


Figure 111: Summary of the averages from Table 132.

Table 133: Decays to charmonium and light mesons [10^{-5}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow J/\psi \pi^-)$	LHCb [671]: $3.88 \pm 0.11 \pm 0.15$ BABAR [672]: $5.37 \pm 0.45 \pm 0.24$ Belle [575]: $3.8 \pm 0.6 \pm 0.3$	4.04 ± 0.17
$\mathcal{B}(B^- \rightarrow \pi^- \pi^0 J/\psi(1S))$	BABAR [598]: < 0.73	< 0.73
$\mathcal{B}(B^- \rightarrow \rho^- (770) J/\psi(1S))$	BABAR [598]: $5 \pm 1 \pm 0$	5 ± 1
$\mathcal{B}(B^- \rightarrow \psi(2S) \pi^-)$	LHCb [671]: $2.52 \pm 0.26 \pm 0.15$	2.52 ± 0.30
$\mathcal{B}(B^- \rightarrow \chi_{c0}(1P) \pi^-)$	BABAR [673]: < 6.1	< 6.1
$\mathcal{B}(B^- \rightarrow \chi_{c1}(1P) \pi^-)$	Belle [674]: $2.2 \pm 0.4 \pm 0.3$	2.2 ± 0.5

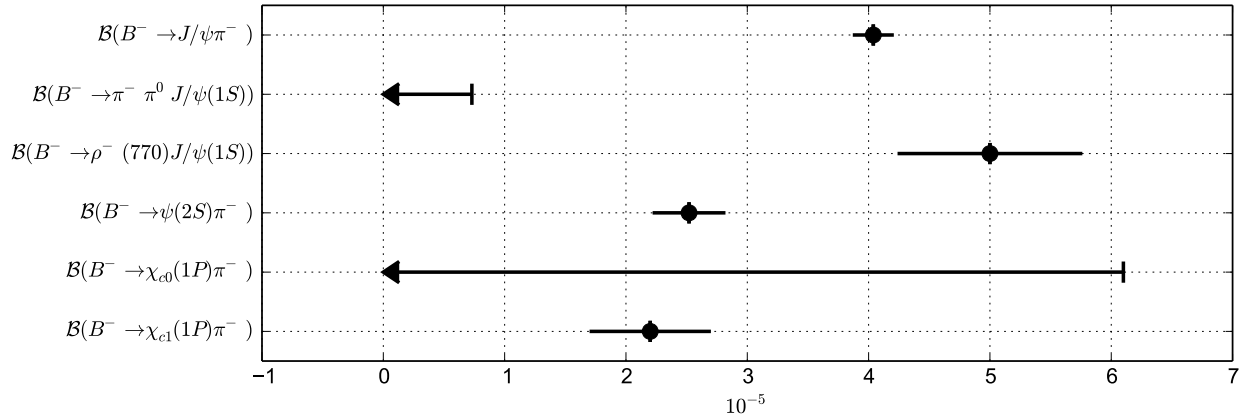


Figure 112: Summary of the averages from Table 133.

Table 134: Decays to J/ψ and a heavy mesons [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow J/\psi(1S) D^-)$	BABAR [610]: < 1.2	< 1.2
$\mathcal{B}(B^- \rightarrow J/\psi(1S) D^0 \pi^-)$	BABAR [664]: < 0.52 Belle [611]: < 0.25	< 0.25

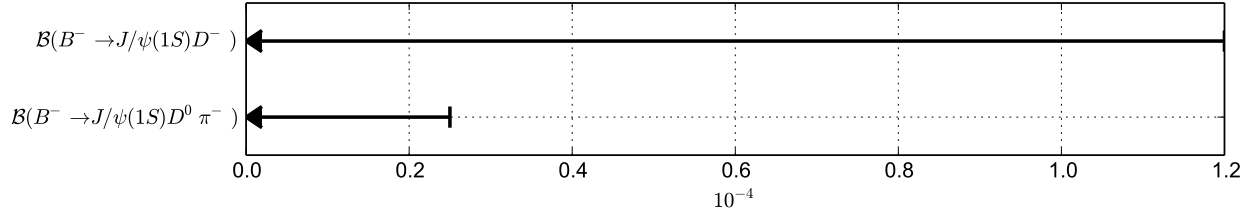


Figure 113: Summary of the averages from Table 134.

Table 135: Decays with baryons [10^{-5}].

Parameter	Measurements	Average
$\mathcal{B}(B^+ \rightarrow J/\psi(1S) p \bar{p} \pi^+)$	LHCb [607]: < 0.050	< 0.050
$\mathcal{B}(B^- \rightarrow J/\psi(1S) \Lambda \bar{p})$	BABAR [608]: $1.16^{+0.74}_{-0.53} {}^{+0.42}_{-0.18}$ Belle [609]: $1.16 \pm 0.28 {}^{+0.18}_{-0.23}$	1.16 ± 0.31
$\mathcal{B}(B^- \rightarrow J/\psi(1S) \Sigma^0 \bar{p})$	Belle [609]: < 1.1	< 1.1
$\mathcal{B}(B^- \rightarrow J/\psi(1S) K^-) \times \mathcal{B}(J/\psi(1S) \rightarrow \Lambda \bar{\Lambda})$	Belle [669]: $0.20^{+0.03}_{-0.03} \pm 0.03$	$0.20^{+0.05}_{-0.04}$
$\mathcal{B}(B^- \rightarrow J/\psi(1S) K^-) \times \mathcal{B}(J/\psi(1S) \rightarrow p \bar{p})$	BABAR [668]: $0.22 \pm 0.02 \pm 0.01$ Belle [669]: $0.221 \pm 0.013 \pm 0.010$	0.221 ± 0.013

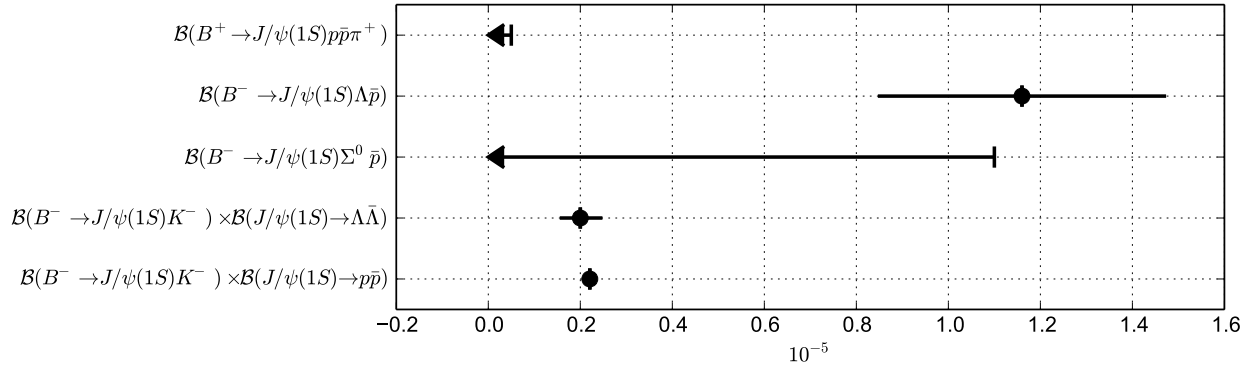


Figure 114: Summary of the averages from Table 135.

Table 136: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow J/\psi(1S)K^{*-}(892))/\mathcal{B}(B^- \rightarrow J/\psi(1S)K^-)$	CDF [612]: $1.92 \pm 0.60 \pm 0.17$ BABAR [6]: $1.37 \pm 0.05 \pm 0.08$	1.38 ± 0.09
$\mathcal{B}(B^- \rightarrow J/\psi(1S)K_1^-(1270))/\mathcal{B}(B^- \rightarrow J/\psi(1S)K^-)$	Belle [582]: $1.80 \pm 0.34 \pm 0.34$	1.80 ± 0.48
$\mathcal{B}(B^- \rightarrow J/\psi(1S)K_1^-(1400))/\mathcal{B}(B^- \rightarrow J/\psi(1S)K_1^-(1270))$	Belle [582]: < 0.30	< 0.30
$\frac{\mathcal{B}(B^- \rightarrow J/\psi(1S)\pi^-)}{\mathcal{B}(B^- \rightarrow J/\psi(1S)K^-)}$	LHCb [675]: $0.0394 \pm 0.0039 \pm 0.0017$ CDF [676]: $0.0486 \pm 0.0082 \pm 0.0015$ CDF [677]: $0.050^{+0.019}_{-0.017} \pm 0.001$ BABAR [672]: $0.0537 \pm 0.0045 \pm 0.0011$	0.0464 ± 0.0029
$\mathcal{B}(B^- \rightarrow \psi(2S)K^-)/\mathcal{B}(B^- \rightarrow J/\psi K^-)$	LHCb [614]: $0.594 \pm 0.006 \pm 0.022$ D0 [678]: $0.65 \pm 0.04 \pm 0.08$	0.598 ± 0.022
$\mathcal{B}(B^- \rightarrow \psi(2S)K^-) \times \mathcal{B}(\psi(2S) \rightarrow p\bar{p})/\mathcal{B}(B^- \rightarrow J/\psi K^-) \times \mathcal{B}(J/\psi \rightarrow p\bar{p})$	LHCb [679]: $0.080 \pm 0.012 \pm 0.009$	0.080 ± 0.015
$\mathcal{B}(B^- \rightarrow \psi(2S)K^{*-}(892))/\mathcal{B}(B^- \rightarrow \psi(2S)K^-)$	BABAR [6]: $0.96 \pm 0.15 \pm 0.09$	0.96 ± 0.17
$\frac{\mathcal{B}(B^- \rightarrow \chi_{c0}(1P)K^-)}{\mathcal{B}(B^- \rightarrow J/\psi(1S)K^-)}$	Belle [666]: $0.60^{+0.21}_{-0.18} \pm 0.09$	$0.60^{+0.23}_{-0.20}$
$\mathcal{B}(B^- \rightarrow \chi_{c1}(1P)K^{*-}(892))/\mathcal{B}(B^- \rightarrow \chi_{c1}(1P)K^-)$	BABAR [6]: $0.51 \pm 0.17 \pm 0.16$	0.51 ± 0.23
$\mathcal{B}(B^- \rightarrow \chi_{c1}(1P)\pi^-)/\mathcal{B}(B^- \rightarrow \chi_{c1}(1P)K^-)$	Belle [674]: $0.043 \pm 0.008 \pm 0.003$	0.043 ± 0.009
$\mathcal{B}(B^- \rightarrow \chi_{c1}\bar{K}^0\pi^-)/\mathcal{B}(B^- \rightarrow J/\psi\bar{K}^0\pi^-)$	BABAR [592]: $0.501 \pm 0.024 \pm 0.055$	0.501 ± 0.060
$\frac{\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-)}{\mathcal{B}(B^- \rightarrow J/\psi(1S)K^-)}$	BABAR [594]: $1.28 \pm 0.10 \pm 0.38$ BABAR [634]: $1.06 \pm 0.23 \pm 0.04$	1.12 ± 0.20
$\mathcal{B}(B^- \rightarrow \eta_c K^-) \times \mathcal{B}(\eta_c \rightarrow p\bar{p})/\mathcal{B}(B^- \rightarrow J/\psi K^-) \times \mathcal{B}(J/\psi \rightarrow p\bar{p})$	LHCb [679]: $0.578 \pm 0.035 \pm 0.027$	0.578 ± 0.044
$\mathcal{B}(B^- \rightarrow h_c(1P)K^-) \times \mathcal{B}(h_c(1P) \rightarrow \eta_c(1S)\gamma)/\mathcal{B}(B^- \rightarrow \eta_c(1S)K^-)$	BABAR [616]: < 0.058	< 0.058

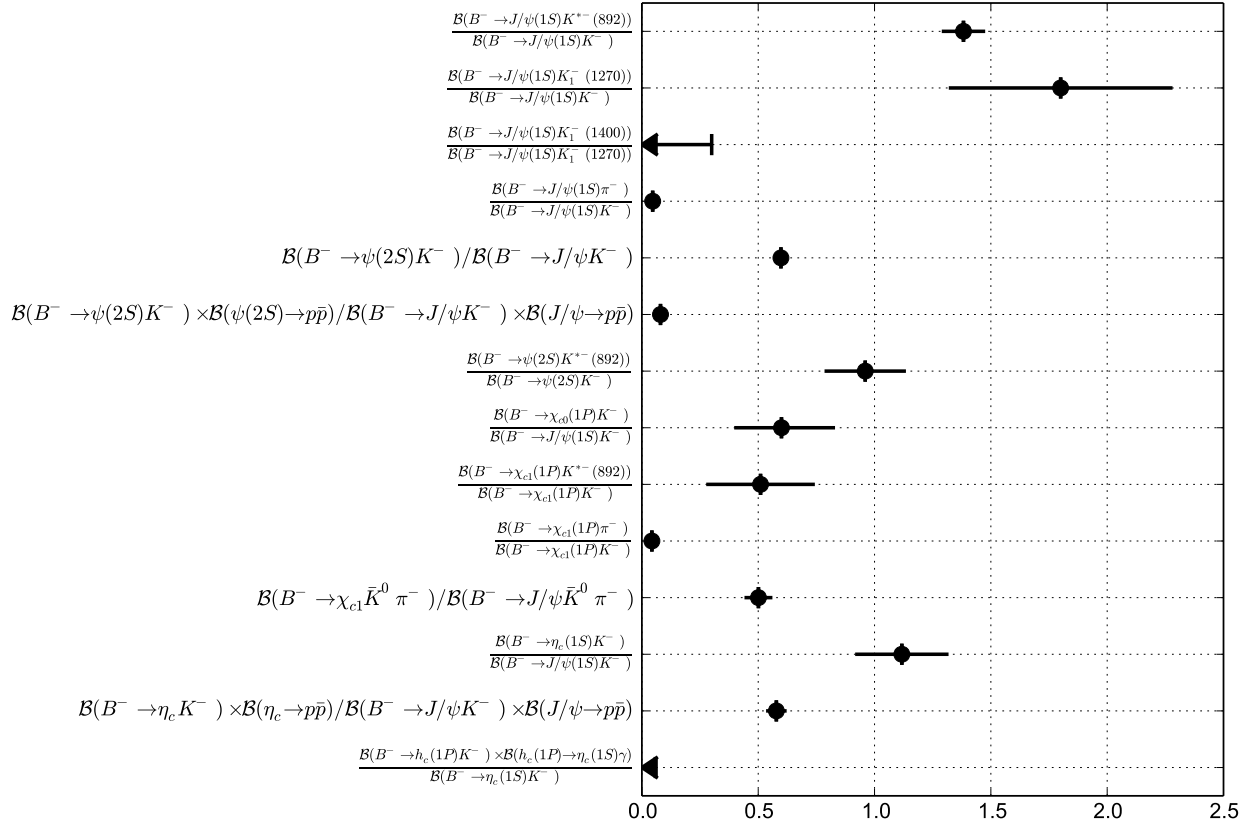


Figure 115: Summary of the averages from Table 136.

6.2.4 Decays to charm baryons

Averages of B^- decays to charm baryons are shown in Tables 137–138 and Figs. 116–117.

Table 137: Absolute (product) decay rates [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow \Lambda_c^+ \Lambda_c^- K^-)$	BABAR [625]: $1.14 \pm 0.15 \pm 0.62$ Belle [629]: $0.65^{+0.10}_{-0.09} \pm 0.36$	0.41 ± 0.32
$\mathcal{B}(B^- \rightarrow \Xi_c^0 \Lambda_c^-) \times \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$	BABAR [625]: $0.0208 \pm 0.0065 \pm 0.0061$ Belle [626]: $0.048^{+0.010}_{-0.009} \pm 0.016$	0.0221 ± 0.0089
$\mathcal{B}(B^- \rightarrow \Lambda_c^+ \bar{p} \pi^-)$	BABAR [622]: $0.338 \pm 0.012 \pm 0.089$ Belle [620]: $0.187^{+0.043}_{-0.040} \pm 0.056$	0.208 ± 0.069
$\mathcal{B}(B^- \rightarrow \Sigma_c^0 \bar{p})$	Belle [620]: $0.045^{+0.026}_{-0.019} \pm 0.014$	$0.045^{+0.029}_{-0.024}$
$\mathcal{B}(B^- \rightarrow \Sigma_c^{*0} \bar{p})$	Belle [620]: < 0.046	< 0.046
$\mathcal{B}(B^- \rightarrow \Sigma_c^{++} \bar{p} \pi^- \pi^-)$	BABAR [680]: $0.298 \pm 0.016 \pm 0.078$	0.298 ± 0.080

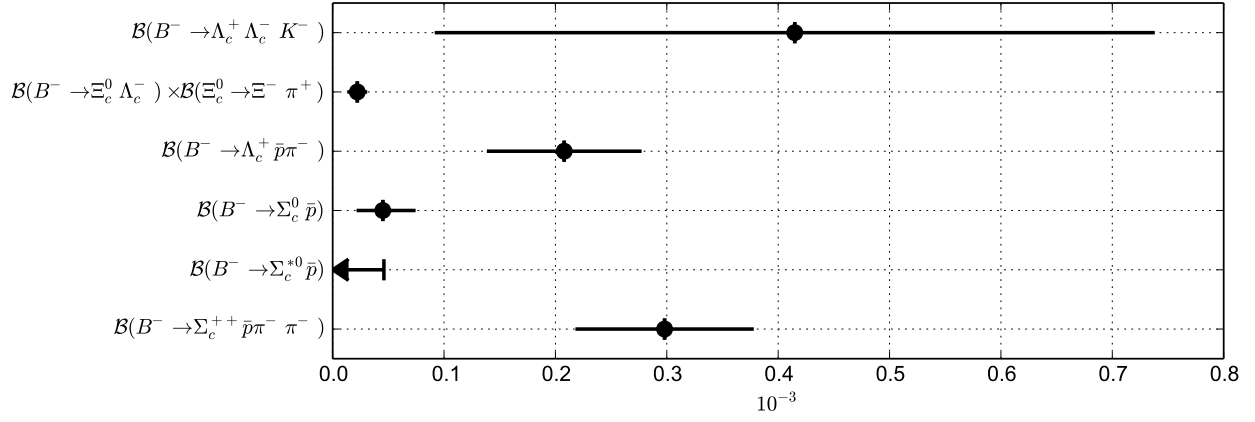


Figure 116: Summary of the averages from Table 137.

Table 138: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow \Lambda_c^+ \bar{p} \pi^-) / \mathcal{B}(\bar{B}^0 \rightarrow \Lambda_c^+ \bar{p})$	BABAR [622]: $15.4 \pm 1.8 \pm 0.3$	15.4 ± 1.8
$\mathcal{B}(B^- \rightarrow \Sigma_c^0(2455) \bar{p}) / \mathcal{B}(B^- \rightarrow \Lambda_c^+ \bar{p} \pi^-)$	BABAR [622]: $0.123 \pm 0.012 \pm 0.008$	0.123 ± 0.014
$\mathcal{B}(B^- \rightarrow \Sigma_c^0(2800) \bar{p}) / \mathcal{B}(B^- \rightarrow \Lambda_c^+ \bar{p} \pi^-)$	BABAR [622]: $0.117 \pm 0.023 \pm 0.024$	0.117 ± 0.033

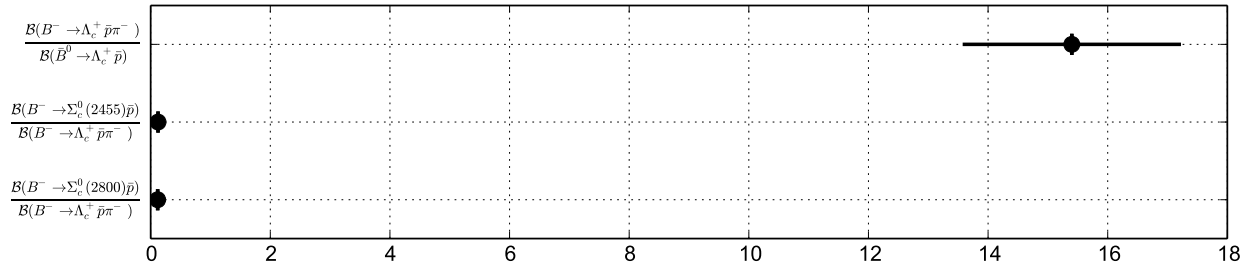


Figure 117: Summary of the averages from Table 138.

6.2.5 Decays to other (XYZ) states

Averages of B^- decays to other (XYZ) states are shown in Tables 139–143 and Figs. 118–122.

Table 139: Absolute decay rates [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow X(3872)K^-)$	<i>BABAR</i> [634]: < 3.2	< 3.2

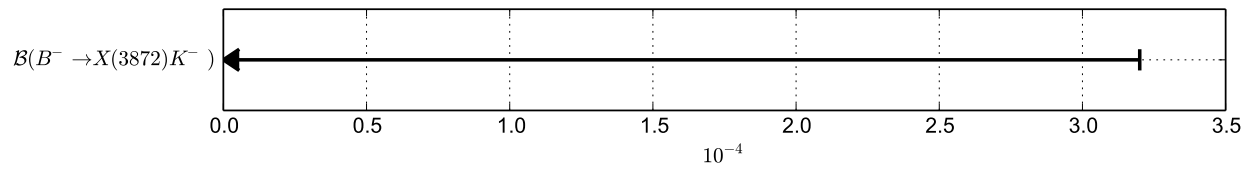


Figure 118: Summary of the averages from Table 139.

Table 140: Product decay rates to $X(3872)$ [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow \bar{D}^{*0}(2007)D^0)$	$BABAR$ [570]: $1.67 \pm 0.36 \pm 0.47$	1.67 ± 0.59
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow D^0 \bar{D}^0 \pi^0)$	$Belle$ [660]: < 0.6	< 0.6
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow D^0 \bar{D}^0)$	$Belle$ [660]: < 0.6	< 0.6
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow D^+ D^-)$	$Belle$ [660]: < 0.4	< 0.4
$\mathcal{B}(B^- \rightarrow K^- X(3872)) \times \mathcal{B}(X(3872) \rightarrow \pi^+ \pi^- J/\psi(1S))$	$BABAR$ [632]: $0.084 \pm 0.015 \pm 0.007$ $Belle$ [681]: $0.131 \pm 0.024 \pm 0.013$	0.097 ± 0.014
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow J/\psi(1S)\omega(782))$	$BABAR$ [579]: $0.06 \pm 0.02 \pm 0.01$	0.06 ± 0.02
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow J/\psi(1S)\eta)$	$BABAR$ [583]: < 0.077	< 0.077
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow J/\psi(1S)\gamma)$	$BABAR$ [590]: $0.028 \pm 0.008 \pm 0.001$ $Belle$ [591]: $0.0178^{+0.0048}_{-0.0044} \pm 0.0012$	0.0204 ± 0.0041
$\mathcal{B}(B^- \rightarrow X(3872)K^{*-}(892)) \times \mathcal{B}(X(3872) \rightarrow J/\psi(1S)\gamma)$	$BABAR$ [590]: $0.007 \pm 0.026 \pm 0.001$	0.007 ± 0.026
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow \psi(2S)\gamma)$	$BABAR$ [590]: $0.095 \pm 0.027 \pm 0.006$ $Belle$ [591]: < 0.0345	0.095 ± 0.028
$\mathcal{B}(B^- \rightarrow X(3872)K^{*-}(892)) \times \mathcal{B}(X(3872) \rightarrow \psi(2S)\gamma)$	$BABAR$ [590]: $0.064 \pm 0.098 \pm 0.096$	0.064 ± 0.137
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow \chi_{c1}(1P)\gamma)$	$Belle$ [586]: < 0.019	< 0.019
$\mathcal{B}(B^- \rightarrow X(3872)K^-) \times \mathcal{B}(X(3872) \rightarrow \chi_{c2}\gamma)$	$Belle$ [586]: < 0.067	< 0.067

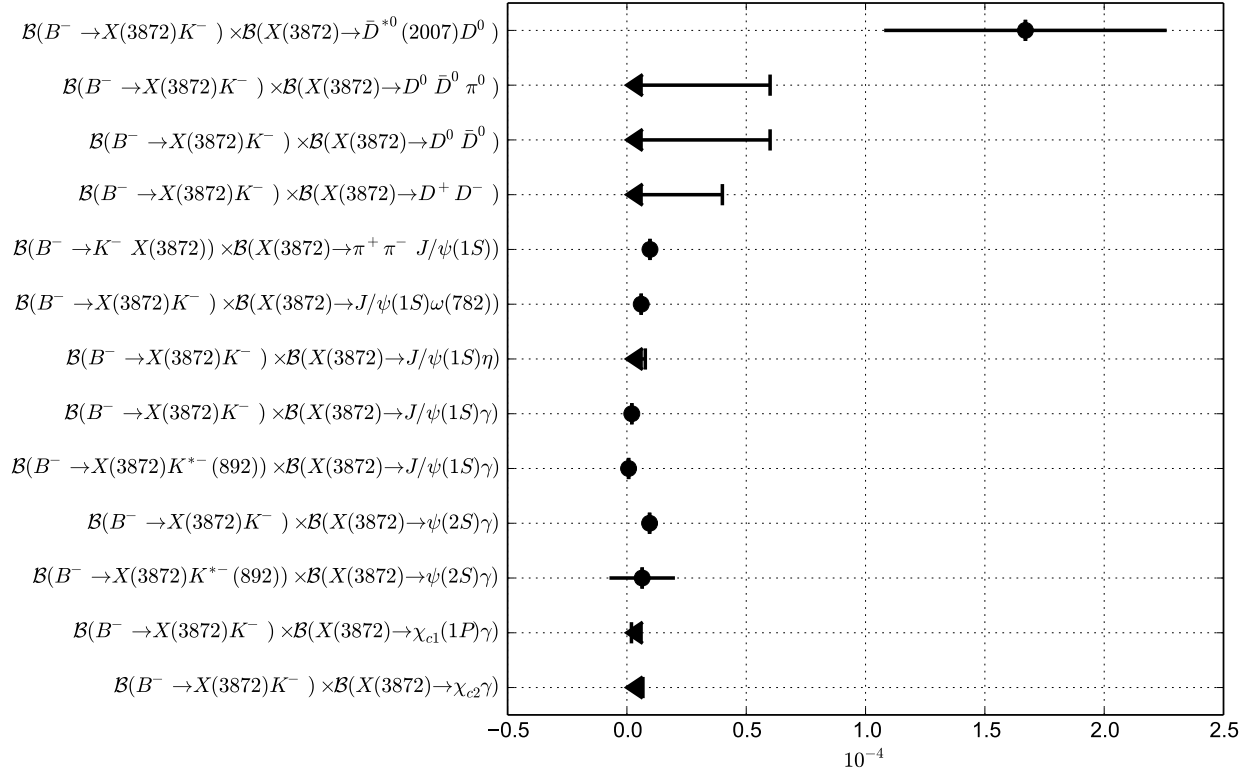


Figure 119: Summary of the averages from Table 140.

Table 141: Product decay rates to neutral states other than $X(3872)$ [10^{-5}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow X(3823)K^-) \times \mathcal{B}(X(3823) \rightarrow \chi_{c1}(1P)\gamma)$	Belle [586]: $0.97 \pm 0.28 \pm 0.11$	0.97 ± 0.30
$\mathcal{B}(B^- \rightarrow X(3823)K^-) \times \mathcal{B}(X(3823) \rightarrow \chi_{c2}\gamma)$	Belle [586]: < 0.36	< 0.36
$\mathcal{B}(B^- \rightarrow Y(3940)K^-) \times \mathcal{B}(Y(3940) \rightarrow J/\psi(1S)\gamma)$	BABAR [682]: < 1.4	< 1.4
$\mathcal{B}(B^- \rightarrow Y(3940)K^-) \times \mathcal{B}(Y(3940) \rightarrow J/\psi(1S)\omega(782))$	BABAR [579]: $3.0^{+0.7+0.5}_{-0.6-0.3}$	$3.0^{+0.9}_{-0.7}$
$\mathcal{B}(B^- \rightarrow Y(4260)K^-) \times \mathcal{B}(Y(4260) \rightarrow J/\psi(1S)\pi^+\pi^-)$	BABAR [683]: $2.0 \pm 0.7 \pm 0.2$	2.0 ± 0.7

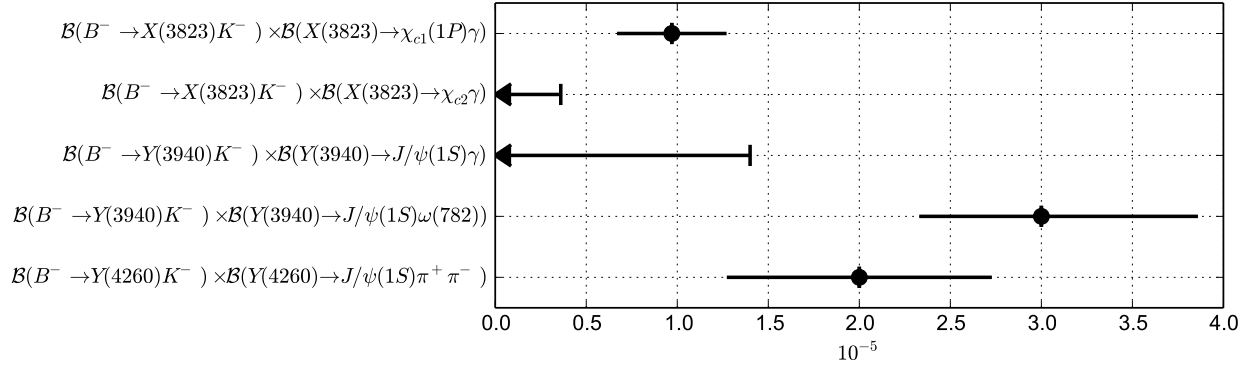


Figure 120: Summary of the averages from Table 141.

Table 142: Relative product decay rates to states with $s\bar{s}$ component.

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow X(4140)K^-) \times \mathcal{B}(X(4140) \rightarrow J/\psi\phi) / \mathcal{B}(B^- \rightarrow J/\psi\phi K^-)$	LHCb [684]: < 0.07	
	D0 [685]: $0.21 \pm 0.08 \pm 0.04$	0.162 ± 0.041
	CDF [686]: $0.149 \pm 0.039 \pm 0.024$	

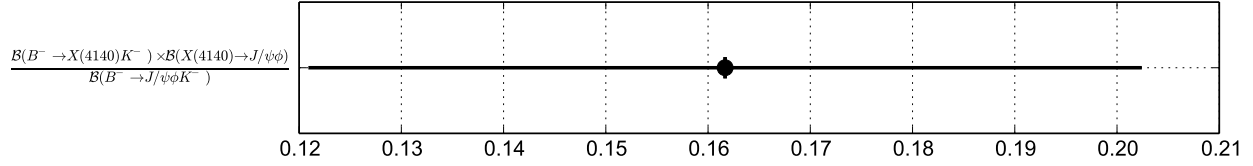


Figure 121: Summary of the averages from Table 142.

Table 143: Product decay rates to charged states [10^{-5}].

Parameter	Measurements	Average
$\mathcal{B}(B^- \rightarrow X^-(3872)\bar{K}^0) \times \mathcal{B}(X^-(3872) \rightarrow J/\psi(1S)\pi^-\pi^0)$	BABAR [635]: < 2.2	< 2.2
$\mathcal{B}(B^- \rightarrow Z^-(4430)\bar{K}^0) \times \mathcal{B}(Z^-(4430) \rightarrow J/\psi(1S)\pi^-)$	BABAR [636]: $-0.1 \pm 0.8 \pm 0.0$	-0.1 ± 0.8
$\mathcal{B}(B^- \rightarrow Z^-(4430)\bar{K}^0) \times \mathcal{B}(Z^-(4430) \rightarrow \psi(2S)\pi^-)$	BABAR [636]: $2.0 \pm 1.7 \pm 0.0$	2.0 ± 1.7

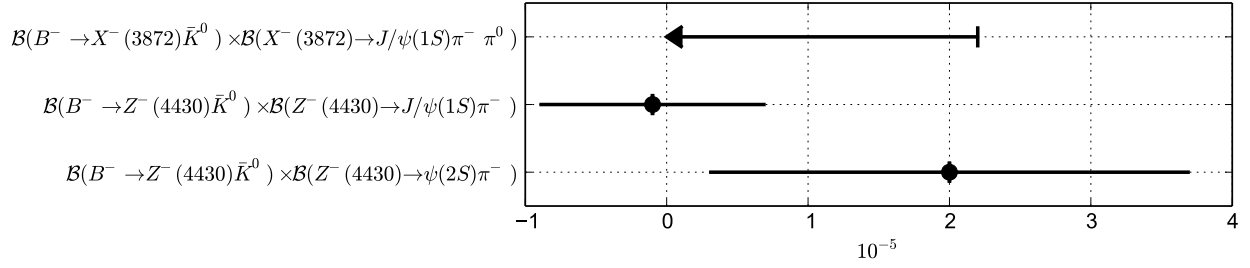


Figure 122: Summary of the averages from Table 143.

6.3 Decays of admixtures of $\bar{B}^0$ / B^- mesons

Measurements of $\bar{B}^0$ / B^- decays to charmed hadrons are summarized in sections 6.3.1 to 6.3.3.

6.3.1 Decays to two open charm mesons

Averages of $\bar{B}^0$ / B^- decays to two open charm mesons are shown in Table 144 and Fig. 123.

Table 144: B decays to double charm $[10^{-4}]$.

Parameter	Measurements	Average
$\mathcal{B}(B \rightarrow D^0 \bar{D}^0 \pi^0 K)$	Belle [561]: $1.27 \pm 0.31^{+0.22}_{-0.39}$	$1.27^{+0.38}_{-0.50}$

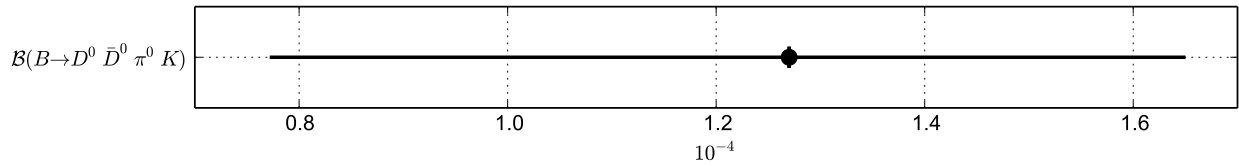


Figure 123: Summary of the averages from Table 144.

6.3.2 Decays to charmonium states

Averages of $\bar{B}^0$ / B^- decays to charmonium states are shown in Tables 145–149 and Figs. 124–128.

Table 145: Decay amplitudes for parallel transverse polarization.

Parameter	Measurements	Average
$ \mathcal{A}_{\parallel} ^2(B \rightarrow J/\psi(1S)K^*)$	<i>BABAR</i> [290]: $0.211 \pm 0.010 \pm 0.006$ Belle [291]: $0.231 \pm 0.012 \pm 0.008$	0.219 ± 0.009
$ \mathcal{A}_{\parallel} ^2(B \rightarrow \chi_{c1}(1P)K^*)$	<i>BABAR</i> [290]: $0.20 \pm 0.07 \pm 0.04$	0.20 ± 0.08
$ \mathcal{A}_{\parallel} ^2(B \rightarrow \psi(2S)K^*)$	<i>BABAR</i> [290]: $0.22 \pm 0.06 \pm 0.02$	0.22 ± 0.06

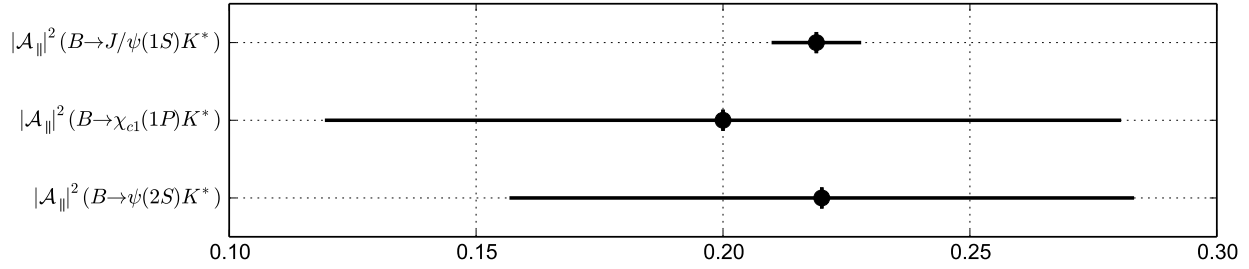


Figure 124: Summary of the averages from Table 145.

Table 146: Decay amplitudes for perpendicular transverse polarization.

Parameter	Measurements	Average
$ \mathcal{A}_{\perp} ^2(B \rightarrow J/\psi(1S)K^*)$	<i>BABAR</i> [290]: $0.233 \pm 0.010 \pm 0.005$ Belle [291]: $0.195 \pm 0.012 \pm 0.008$	0.219 ± 0.009
$ \mathcal{A}_{\perp} ^2(B \rightarrow \chi_{c1}(1P)K^*)$	<i>BABAR</i> [290]: $0.03 \pm 0.04 \pm 0.02$	0.03 ± 0.04
$ \mathcal{A}_{\perp} ^2(B \rightarrow \psi(2S)K^*)$	<i>BABAR</i> [290]: $0.30 \pm 0.06 \pm 0.02$	0.30 ± 0.06

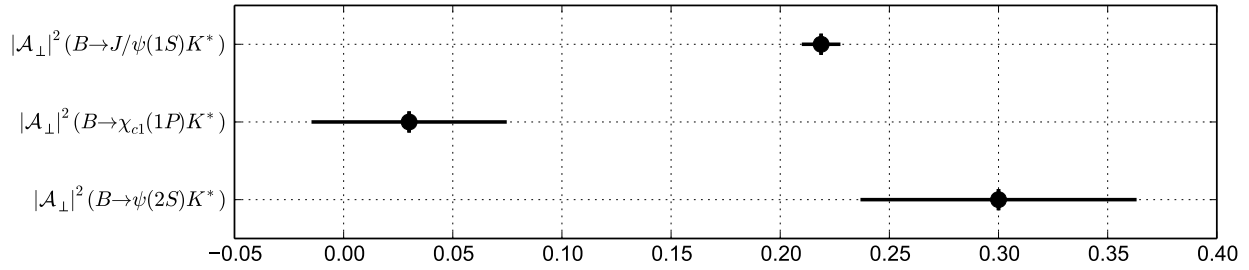


Figure 125: Summary of the averages from Table 146.

Table 147: Decay amplitudes for longitudinal polarization.

Parameter	Measurements	Average
$ \mathcal{A}_0 ^2(B \rightarrow J/\psi(1S)K^*)$	<i>BABAR</i> [290]: $0.556 \pm 0.009 \pm 0.010$ Belle [291]: $0.574 \pm 0.012 \pm 0.009$	0.564 ± 0.010
$ \mathcal{A}_0 ^2(B \rightarrow \chi_{c1}(1P)K^*)$	<i>BABAR</i> [290]: $0.77 \pm 0.07 \pm 0.04$	0.77 ± 0.08
$ \mathcal{A}_0 ^2(B \rightarrow \psi(2S)K^*)$	<i>BABAR</i> [290]: $0.48 \pm 0.05 \pm 0.02$	0.48 ± 0.05

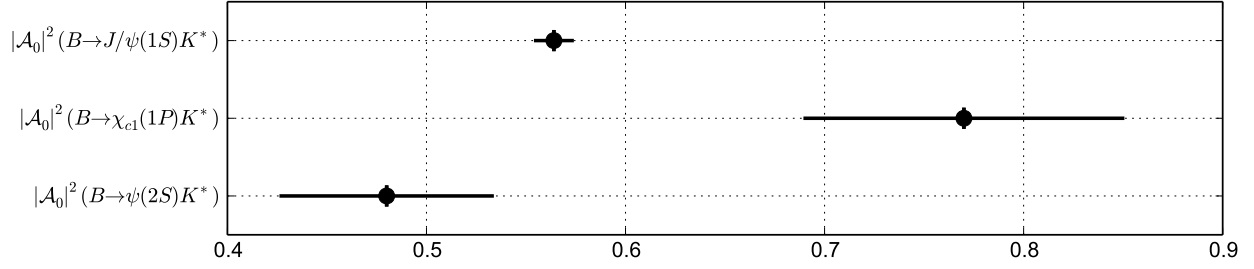


Figure 126: Summary of the averages from Table 147.

Table 148: Relative phases of parallel transverse polarization decay amplitudes.

Parameter	Measurements	Average
$\delta_{\parallel}(B \rightarrow J/\psi(1S)K^*)$	<i>BABAR</i> [290]: $-2.93 \pm 0.08 \pm 0.04$ Belle [291]: $-2.887 \pm 0.090 \pm 0.008$	-2.909 ± 0.064
$\delta_{\parallel}(B \rightarrow \chi_{c1}(1P)K^*)$	<i>BABAR</i> [290]: $0.0 \pm 0.3 \pm 0.1$	0.0 ± 0.3
$\delta_{\parallel}(B \rightarrow \psi(2S)K^*)$	<i>BABAR</i> [290]: $-2.8 \pm 0.4 \pm 0.1$	-2.8 ± 0.4

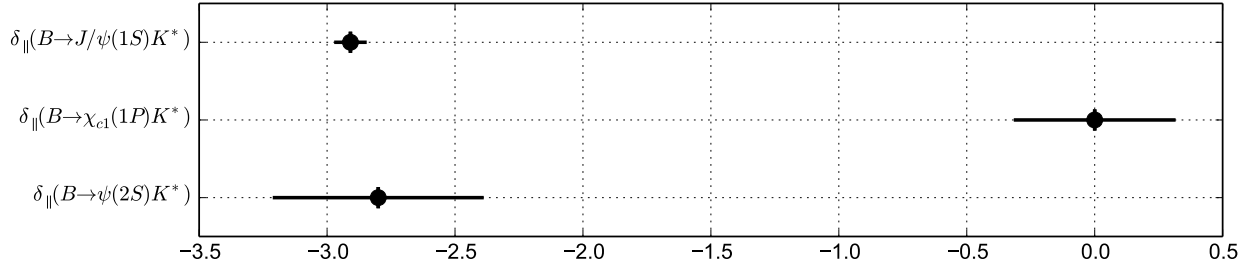


Figure 127: Summary of the averages from Table 148.

Table 149: Relative phases of perpendicular transverse polarization decay amplitudes.

Parameter	Measurements	Average
$\delta_{\perp}(B \rightarrow J/\psi(1S)K^*)$	<i>BABAR</i> [290]: $2.91 \pm 0.05 \pm 0.03$ Belle [291]: $2.938 \pm 0.064 \pm 0.010$	2.923 ± 0.043
$\delta_{\perp}(B \rightarrow \psi(2S)K^*)$	<i>BABAR</i> [290]: $2.8 \pm 0.3 \pm 0.1$	2.8 ± 0.3

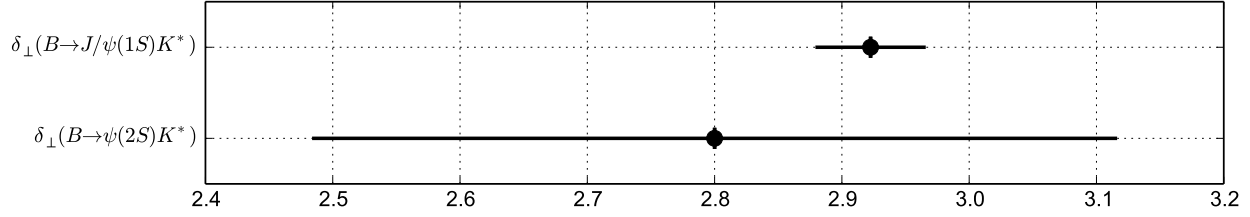


Figure 128: Summary of the averages from Table 149.

6.3.3 Decays to other (XYZ) states

Averages of $\bar{B}^0$ / B^- decays to other (XYZ) states are shown in Table 150 and Fig. 129.

Table 150: Absolute decay rates to X/Y states [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(B \rightarrow X(3872)K) \times \mathcal{B}(X(3872) \rightarrow D^{*0}(2007)\bar{D}^0)$	Belle [687]: $0.80 \pm 0.20 \pm 0.10$	0.80 ± 0.22
$\mathcal{B}(B \rightarrow Y(3940)K) \times \mathcal{B}(Y(3940) \rightarrow D^{*0}(2007)\bar{D}^0)$	Belle [687]: < 0.67	< 0.67
$\mathcal{B}(B \rightarrow KY(3940)) \times \mathcal{B}(Y(3940) \rightarrow \omega(782)J/\psi(1S))$	Belle [688]: $0.71 \pm 0.13 \pm 0.31$	0.71 ± 0.34

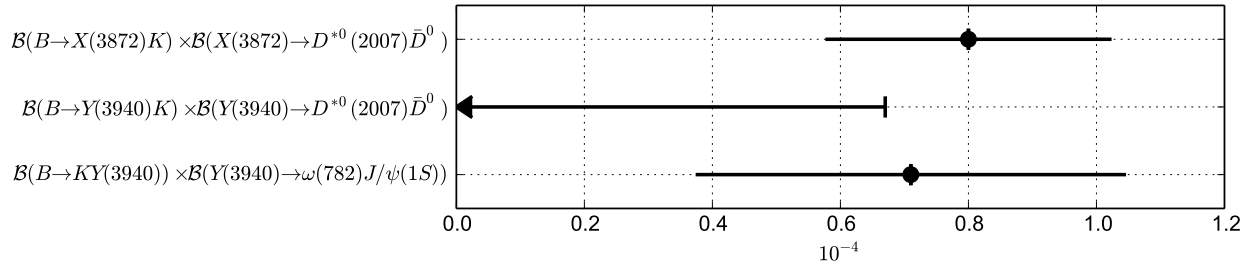


Figure 129: Summary of the averages from Table 150.

6.4 Decays of $\bar{B}_s^0$ mesons

Measurements of $\bar{B}_s^0$ decays to charmed hadrons are summarized in sections 6.4.1 to 6.4.4.

6.4.1 Decays to a single open charm meson

Averages of $\bar{B}_s^0$ decays to a single open charm meson are shown in Tables 151–154 and Figs. 130–133.

Table 151: Decays to a $D_s^{(*)}$ and a light meson [10^{-3}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^-)$	LHCb [689]: $2.95 \pm 0.05^{+0.25}_{-0.28}$ Belle [28]: $3.67^{+0.35+0.65}_{-0.33-0.65}$	3.03 ± 0.25
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{*+} \pi^-)$	Belle [690]: $2.4^{+0.5}_{-0.4} \pm 0.4$	$2.4^{+0.7}_{-0.6}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \rho^-(770))$	Belle [690]: $8.5^{+1.3}_{-1.2} \pm 1.7$	$8.5^{+2.1}_{-2.1}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{*+} \rho^-(770))$	Belle [690]: $11.8^{+2.2}_{-2.0} \pm 2.5$	$11.8^{+3.3}_{-3.2}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ K^-)$	LHCb [689]: $0.190 \pm 0.012^{+0.018}_{-0.019}$ Belle [28]: $0.24^{+0.12}_{-0.10} \pm 0.04$	0.192 ± 0.022

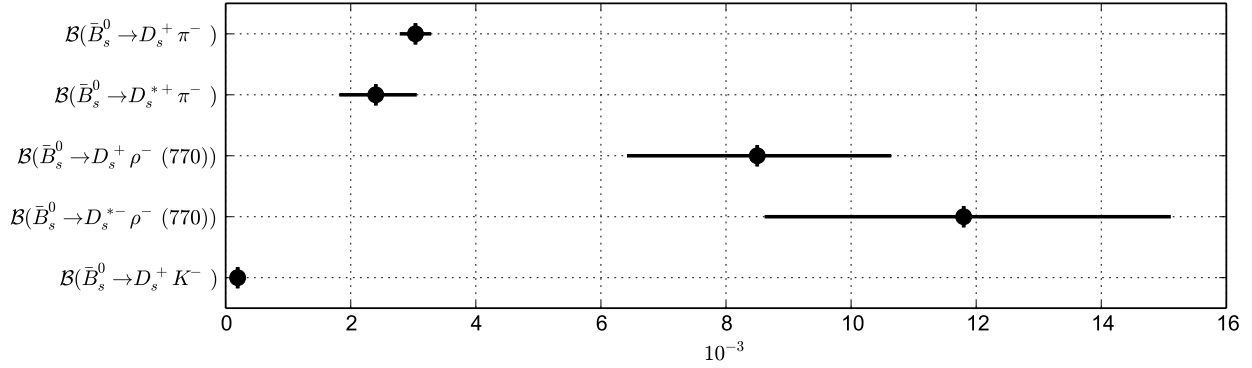


Figure 130: Summary of the averages from Table 151.

Table 152: Decays to a $D^{(*)}$ and a light meson [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^-)$	LHCb [691]: < 0.061	< 0.061
$\mathcal{B}(\bar{B}_s^0 \rightarrow D^0 \bar{K}^{*0})$	LHCb [692]: $4.72 \pm 1.07 \pm 0.96$	4.72 ± 1.44

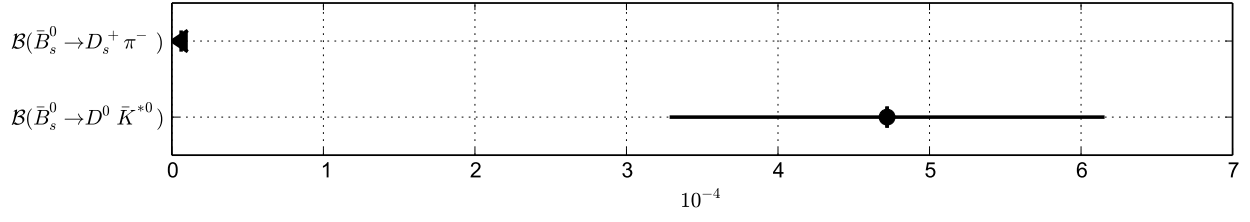


Figure 131: Summary of the averages from Table 152.

Table 153: Relative decay rates.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^-) / \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$	CDF [693]: $1.13 \pm 0.08 \pm 0.23$	1.13 ± 0.25
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^+ \pi^- \pi^-) / \mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^-)$	LHCb [547]: $2.01 \pm 0.37 \pm 0.20$	2.01 ± 0.42
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^+ \pi^- \pi^-) / \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^+ \pi^- \pi^-)$	CDF [693]: $1.05 \pm 0.10 \pm 0.22$	1.05 ± 0.24
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ K^-) / \mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ \pi^-)$	CDF [694]: $0.097 \pm 0.018 \pm 0.009$	0.097 ± 0.020
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ K^- \pi^+ \pi^-) / \mathcal{B}(\bar{B}^0 \rightarrow D_s^+ \pi^- \pi^+ \pi^-)$	LHCb [549]: $0.052 \pm 0.005 \pm 0.003$	0.052 ± 0.006
$\mathcal{B}(\bar{B}_s^0 \rightarrow D^0 K^{*0}) / \mathcal{B}(B^- \rightarrow D^0 \rho^0)$	LHCb [692]: $1.48 \pm 0.34 \pm 0.19$	1.48 ± 0.39
$\mathcal{B}(\bar{B}_s^0 \rightarrow D^0 K^{*0}) / \mathcal{B}(\bar{B}^0 \rightarrow D^0 K^{*0})$	LHCb [695]: $7.8 \pm 0.7 \pm 0.7$	7.8 ± 1.0
$\mathcal{B}(\bar{B}_s^0 \rightarrow D^0 \phi(1020)) / \mathcal{B}(\bar{B}_s^0 \rightarrow D^0 K^{*0})$	LHCb [695]: $0.069 \pm 0.013 \pm 0.007$	0.069 ± 0.015
$\mathcal{B}(\bar{B}_s^0 \rightarrow \bar{D}^0 K^- \pi^+) / \mathcal{B}(\bar{B}^0 \rightarrow \bar{D}^0 \pi^- \pi^+)$	LHCb [545]: $1.18 \pm 0.05 \pm 0.12$	1.18 ± 0.13
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_{s1}^+ \pi^-) \times (D_{s1}^+ \rightarrow D_s^+ \pi^- \pi^+) / \mathcal{B}(\bar{B}^0 \rightarrow D_s^+ \pi^- \pi^+ \pi^-)$	LHCb [549]: $0.0040 \pm 0.0010 \pm 0.0004$	0.0040 ± 0.0011

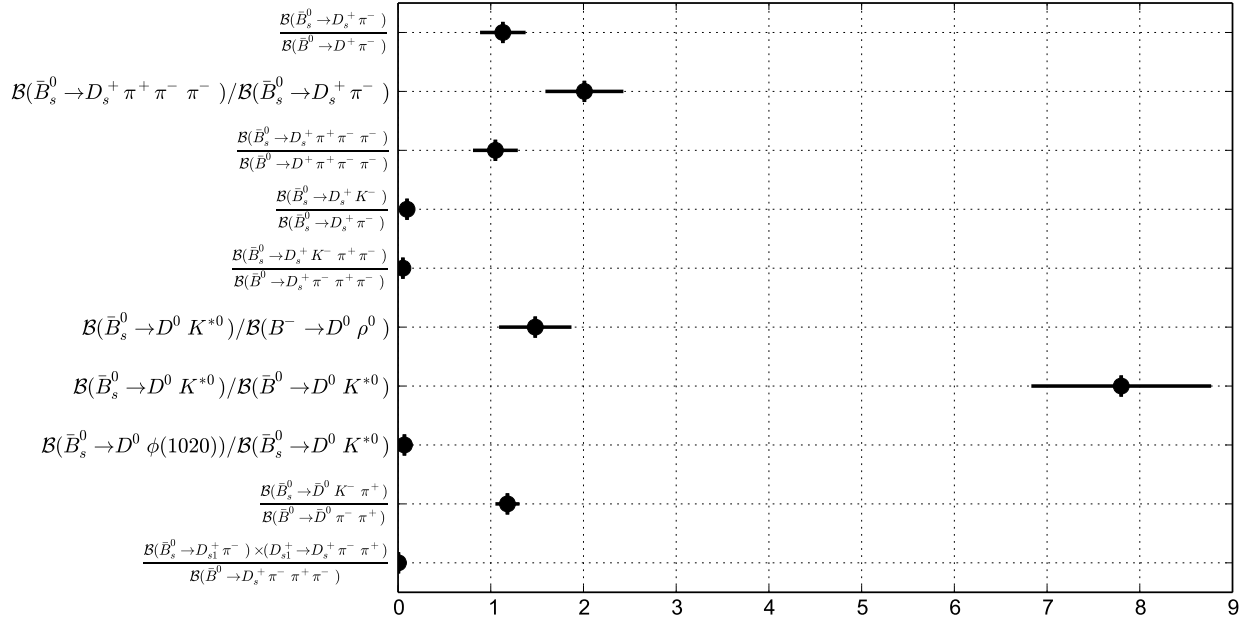


Figure 132: Summary of the averages from Table 153.

Table 154: Semileptonic decays $[10^{-2}]$.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_{s1}^+ \mu^- \bar{\nu}_\mu X) / \mathcal{B}(\bar{B}_s^0 \rightarrow \mu^- \bar{\nu}_\mu X)$	LHCb [696]: $5.4 \pm 1.2 \pm 0.5$	5.4 ± 1.3
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_{s2}^{*+} \mu^- \bar{\nu}_\mu X) / \mathcal{B}(\bar{B}_s^0 \rightarrow \mu^- \bar{\nu}_\mu X)$	LHCb [696]: $3.3 \pm 1.1 \pm 0.4$	3.3 ± 1.2

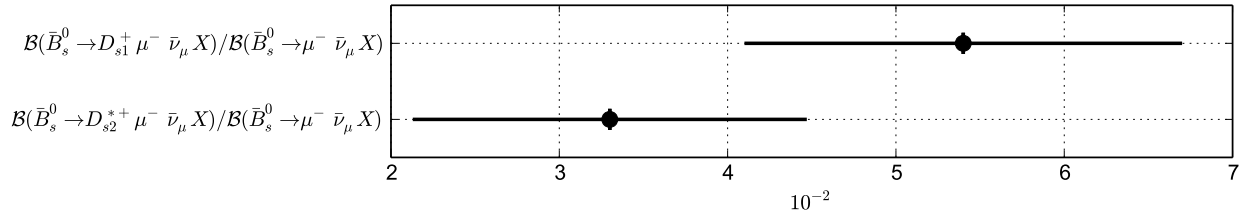


Figure 133: Summary of the averages from Table 154.

6.4.2 Decays to two open charm mesons

Averages of $\bar{B}_s^0$ decays to two open charm mesons are shown in Tables 155–156 and Figs. 134–135.

Table 155: Absolute branching fractions $[10^{-2}]$.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ D_s^-)$	CDF [697]: $0.49 \pm 0.06 \pm 0.09$ Belle [20]: $0.58^{+0.11}_{-0.09} \pm 0.13$	0.52 ± 0.09
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ D_s^{*-})$	CDF [697]: $1.13 \pm 0.12 \pm 0.21$ Belle [20]: $1.76^{+0.23}_{-0.22} \pm 0.40$	1.27 ± 0.21
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{*+} D_s^{*-})$	CDF [697]: $1.75 \pm 0.19 \pm 0.34$ Belle [20]: $1.98^{+0.33+0.51}_{-0.31-0.50}$	1.82 ± 0.32
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-})$	D0 [185]: $3.5 \pm 1.0 \pm 1.1$ CDF [697]: $3.38 \pm 0.25 \pm 0.64$	3.40 ± 0.62

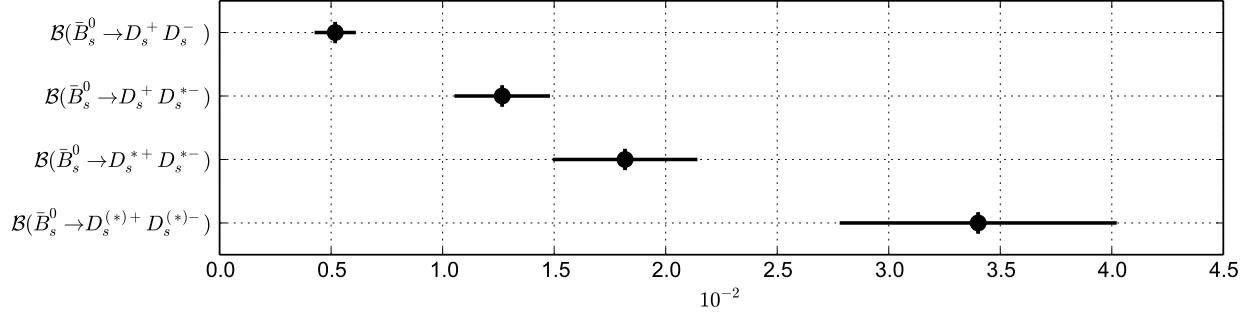


Figure 134: Summary of the averages from Table 155.

Table 156: Relative branching fractions.

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow D^- D^+)/\mathcal{B}(\bar{B}^0 \rightarrow D^- D^+)$	LHCb [568]: $1.08 \pm 0.20 \pm 0.10$	1.08 ± 0.22
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^- D_s^+)/\mathcal{B}(\bar{B}^0 \rightarrow D_s^- D^+)$	LHCb [568]: $0.56 \pm 0.03 \pm 0.04$	0.56 ± 0.05
$\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+ D^-)/\mathcal{B}(B^0 \rightarrow D_s^+ D^-)$	LHCb [568]: $0.050 \pm 0.008 \pm 0.004$	0.050 ± 0.009
$\mathcal{B}(\bar{B}_s^0 \rightarrow \bar{D}^0 D^0)/\mathcal{B}(B^- \rightarrow D^0 D_s^-)$	LHCb [568]: $0.019 \pm 0.003 \pm 0.003$	0.019 ± 0.004

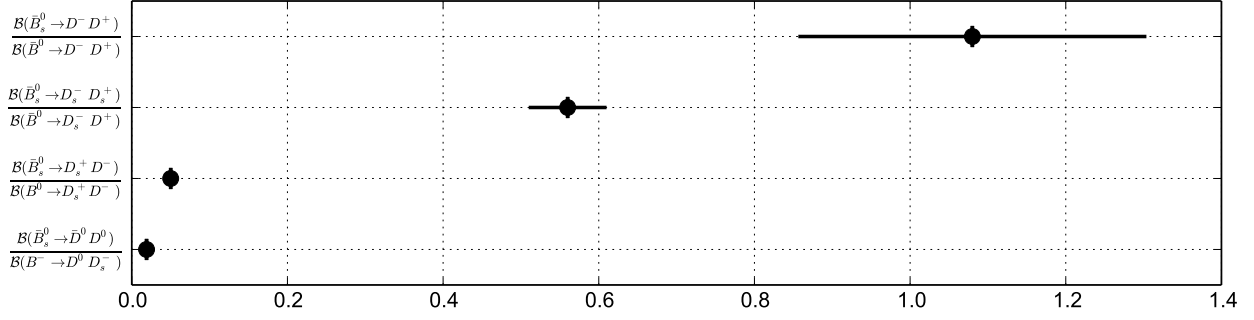


Figure 135: Summary of the averages from Table 156.

6.4.3 Decays to charmonium states

Averages of $\bar{B}_s^0$ decays to charmonium states are shown in Tables 157–160 and Figs. 136–139.

Table 157: Absolute decay rates I [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)\eta)$	Belle [698]: $5.10 \pm 0.50^{+1.17}_{-0.83}$	$5.10^{+1.27}_{-0.97}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)\eta')$	Belle [698]: $3.71 \pm 0.61^{+0.85}_{-0.60}$	$3.71^{+1.05}_{-0.85}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)\phi(1020))$	CDF [612]: $9.3 \pm 2.8 \pm 1.7$	9.3 ± 3.3
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)K^0 K^\pm \pi^\mp)$	LHCb [581]: $9.1 \pm 0.6 \pm 0.7$	9.1 ± 0.9
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)f_0(980)) \times \mathcal{B}(f_0(980) \rightarrow \pi^+ \pi^-)$	Belle [27]: $1.16^{+0.31}_{-0.19}{}^{+0.30}_{-0.25}$	$1.16^{+0.43}_{-0.32}$

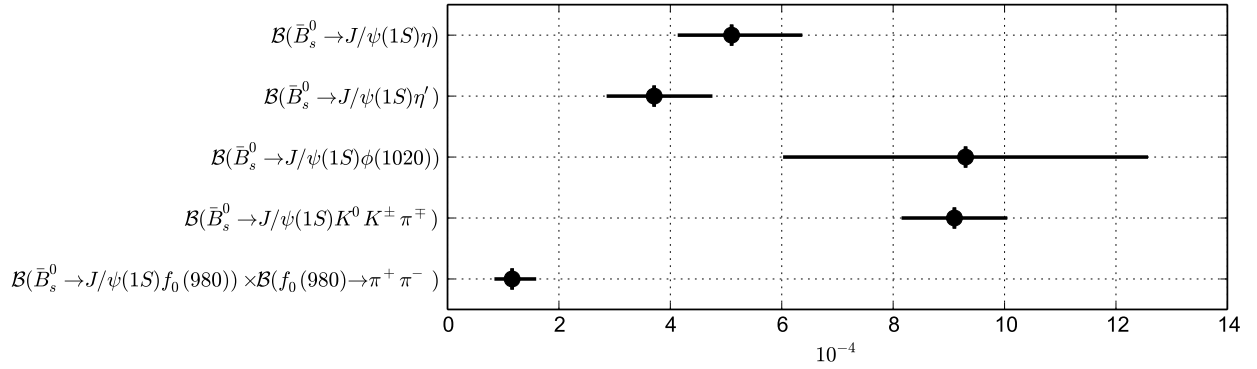


Figure 136: Summary of the averages from Table 157.

Table 158: Absolute decay rates II [10^{-5}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi \bar{K}^0)$	CDF [699]: $3.5 \pm 0.6 \pm 0.6$	3.5 ± 0.8
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi K^{*0})$	LHCb [700]: $3.5^{+1.1}_{-1.0} \pm 0.9$	$3.5^{+1.4}_{-1.3}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi \bar{K}^{*0})$	LHCb [701]: $4.4^{+0.5}_{-0.4} \pm 0.8$ CDF [699]: $8.3 \pm 1.2 \pm 3.6$	4.6 ± 0.9
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi K_S^0)$	LHCb [702]: $1.83 \pm 0.21 \pm 0.19$	1.83 ± 0.28
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)p\bar{p})$	LHCb [607]: < 0.48	< 0.48
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi K^+ \pi^-)$	LHCb [700]: $3.94^{+0.71}_{-0.62} \pm 0.66$	$3.94^{+0.97}_{-0.91}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)f_1(1285))$	LHCb [603]: $7.14 \pm 0.99^{+0.93}_{-1.00}$	$7.14^{+1.36}_{-1.41}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)K^0 \pi^+ \pi^-)$	LHCb [581]: < 4.4	< 4.4
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)K^0 K^+ K^-)$	LHCb [581]: < 1.2	< 1.2
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)f_0(1370)) \times \mathcal{B}(f_0(1370) \rightarrow \pi^+ \pi^-)$	Belle [27]: $3.4^{+1.1}_{-1.4}{}^{+0.9}_{-0.5}$	$3.4^{+1.4}_{-1.5}$
$\mathcal{B}(\bar{B}_s^0 \rightarrow J/\psi(1S)f_1(1285)) \times \mathcal{B}(f_1(1285) \rightarrow \pi^+ \pi^- \pi^+ \pi^-)$	LHCb [603]: $0.785 \pm 0.109^{+0.089}_{-0.101}$	$0.785^{+0.141}_{-0.149}$

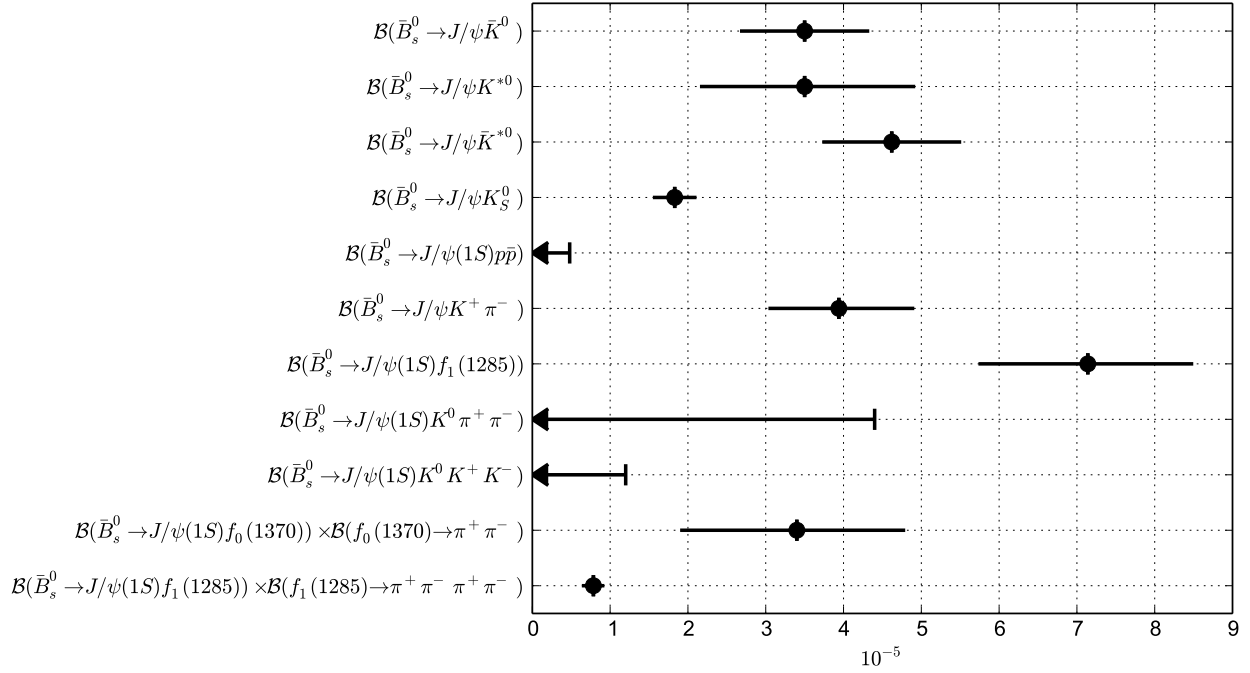


Figure 137: Summary of the averages from Table 158.

Table 159: Relative decay rates I.

Parameter	Measurements	Average
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi\eta)/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi\rho)$	LHCb [613]: $14.0 \pm 1.2^{+1.6}_{-1.8}$	$14.0^{+2.0}_{-2.2}$
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi\eta')/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi\rho)$	LHCb [613]: $12.7 \pm 1.1^{+1.1}_{-0.9}$	$12.7^{+1.6}_{-1.4}$
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi(1S)K_S^0 K^\pm \pi^\mp)/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi(1S)\pi^+\pi^-)$	LHCb [581]: $2.12 \pm 0.15 \pm 0.18$	2.12 ± 0.23

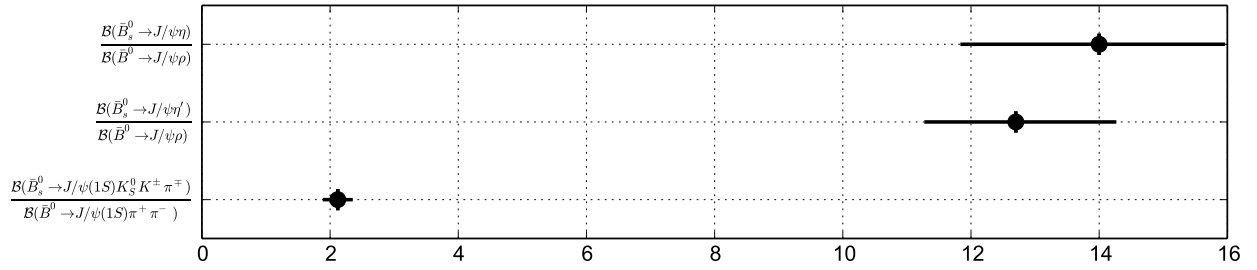


Figure 138: Summary of the averages from Table 159.

Table 160: Relative decay rates II.

Parameter	Measurements	Average
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi K_S^0)/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi K_S^0)$	LHCb [702]: $0.0420 \pm 0.0049 \pm 0.0040$	0.0420 ± 0.0063
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \eta)/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi \eta')$	Belle [698]: $0.73 \pm 0.14 \pm 0.02$	0.73 ± 0.14
$\frac{\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \eta')}{\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \eta)}$	LHCb [613]: $0.90 \pm 0.09^{+0.06}_{-0.02}$	$0.90^{+0.11}_{-0.09}$
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi f_2')/\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \phi)$	LHCb [703]: $0.264 \pm 0.027 \pm 0.024$ D0 [704]: $0.19 \pm 0.05 \pm 0.04$	0.246 ± 0.031
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \phi)/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi \overline{K}^{*0})$	CDF [705]: $0.89 \pm 0.01 \pm 0.13$	0.89 ± 0.13
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi f_0(980))/\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \phi)$	LHCb [706]: $0.252^{+0.046}_{-0.032}^{+0.027}_{-0.033}$ D0 [707]: $0.275 \pm 0.041 \pm 0.061$	0.259 ± 0.041
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \pi^+ \pi^-)/\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \phi)$	LHCb [706]: $0.162 \pm 0.022 \pm 0.016$	0.162 ± 0.027
$\mathcal{B}\overline{B}_s^0 \rightarrow \psi(2S)\pi^+\pi^-/\mathcal{B}\overline{B}_s^0 \rightarrow J/\psi\pi^+\pi^-$	LHCb [615]: $0.34 \pm 0.04 \pm 0.03$	0.34 ± 0.05
$\mathcal{B}(\overline{B}_s^0 \rightarrow \psi(2S)\phi(1020))/\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi(1S)\phi(1020))$	LHCb [614]: $0.489 \pm 0.026 \pm 0.024$ D0 [678]: $0.55 \pm 0.11 \pm 0.09$ CDF [708]: $0.52 \pm 0.13 \pm 0.07$	0.494 ± 0.034
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi(1S)K_S^0\pi^+\pi^-)/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi(1S)\pi^+\pi^-)$	LHCb [581]: < 0.10	< 0.10
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi(1S)K_S^0 K^+ K^-)/\mathcal{B}(\overline{B}^0 \rightarrow J/\psi(1S)\pi^+\pi^-)$	LHCb [581]: < 0.027	< 0.027
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi f_0(980)) \times \mathcal{B}(f_0(980) \rightarrow \pi^+\pi^-)/\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi \phi) \times \mathcal{B}(\phi \rightarrow K^+ K^-)$	CDF [119]: $0.257 \pm 0.020 \pm 0.014$	0.257 ± 0.024
$\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi f_0(500)) \times \mathcal{B}(f_0(500) \rightarrow \pi^+\pi^-)/\mathcal{B}(\overline{B}_s^0 \rightarrow J/\psi f_0(980)) \times \mathcal{B}(f_0(500) \rightarrow \pi^+\pi^-)$	LHCb [709]: < 0.034	< 0.034

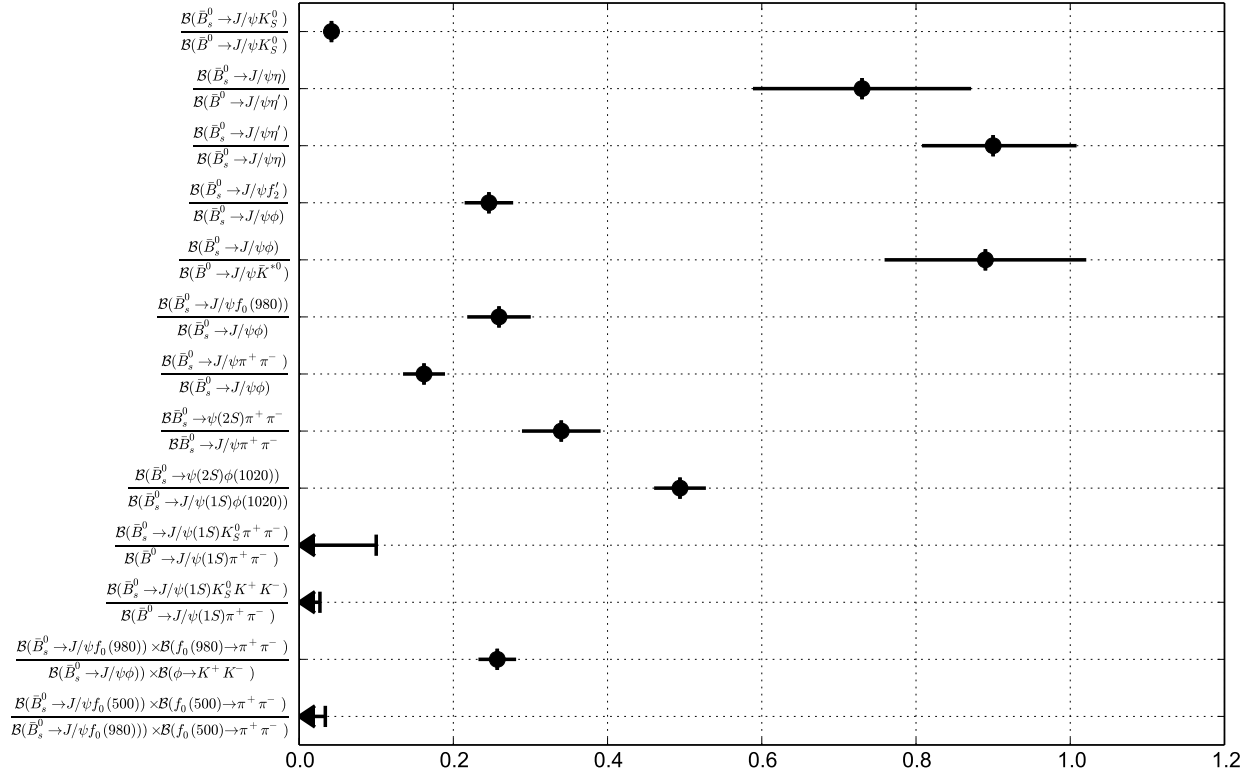


Figure 139: Summary of the averages from Table 160.

6.4.4 Decays to charm baryons

Averages of $\bar{B}_s^0$ decays to charm baryons are shown in Tables 161–162 and Figs. 140–141.

Table 161: Decays to one charm baryon [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\bar{B}_s^0 \rightarrow \Lambda_c^+ \pi^- \bar{\Lambda})$	Belle [710]: $3.6 \pm 1.1^{+1.2}_{-1.2}$	$3.6^{+1.6}_{-1.7}$

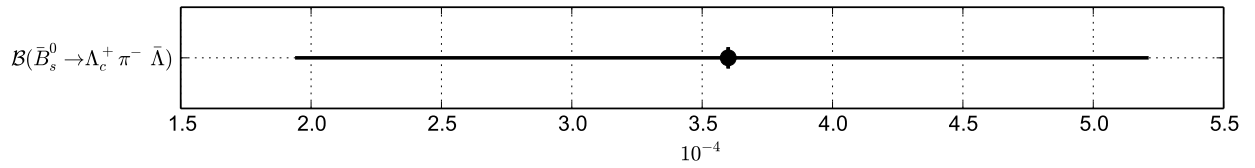


Figure 140: Summary of the averages from Table 161.

Table 162: Decays to two charm baryons.

Parameter	Measurements	Average
$\mathcal{B}(\overline{B}_s^0 \rightarrow \Lambda_c^- \Lambda_c^+)/\mathcal{B}(\overline{B}_s^0 \rightarrow D^- D_s^+)$	LHCb [631]: < 0.30	< 0.30



Figure 141: Summary of the averages from Table 162.

6.5 Decays of B_c^- mesons

Measurements of B_c^- decays to charmed hadrons are summarized in sections 6.5.1 to 6.5.2.

6.5.1 Decays to charmonium states

Averages of B_c^- decays to charmonium states are shown in Tables 163–164 and Figs. 142–143.

Table 163: Relative decay rates.

Parameter	Measurements	Average
$\frac{\mathcal{B}(B_c^- \rightarrow J/\psi D_s^-)}{\mathcal{B}(B_c^- \rightarrow J/\psi \pi^-)}$	LHCb [711]: $2.90 \pm 0.57 \pm 0.24$	2.90 ± 0.62
$\frac{\mathcal{B}(B_c^- \rightarrow J/\psi K^-)}{\mathcal{B}(B_c^- \rightarrow J/\psi \pi^-)}$	LHCb [712]: $0.069 \pm 0.019 \pm 0.005$	0.069 ± 0.020
$\mathcal{B}(B_c^- \rightarrow J/\psi K^- K^+ \pi^-)/\mathcal{B}(B_c^- \rightarrow J/\psi \pi^-)$	LHCb [713]: $0.53 \pm 0.10 \pm 0.05$	0.53 ± 0.11
$\mathcal{B}(B_c^- \rightarrow J/\psi \pi^+ \pi^- \pi^-)/\mathcal{B}(B_c^- \rightarrow J/\psi \pi^-)$	LHCb [714]: $3.0 \pm 0.6 \pm 0.4$ LHCb [715]: $2.41 \pm 0.30 \pm 0.33$ CMS [716]: $2.43 \pm 0.76^{+0.46}_{-0.44}$	2.55 ± 0.35
$\frac{\mathcal{B}(B_c^- \rightarrow \psi(2S) \pi^-)}{\mathcal{B}(B_c^- \rightarrow J/\psi \pi^-)}$	LHCb [717]: $0.25 \pm 0.07 \pm 0.02$	0.25 ± 0.07

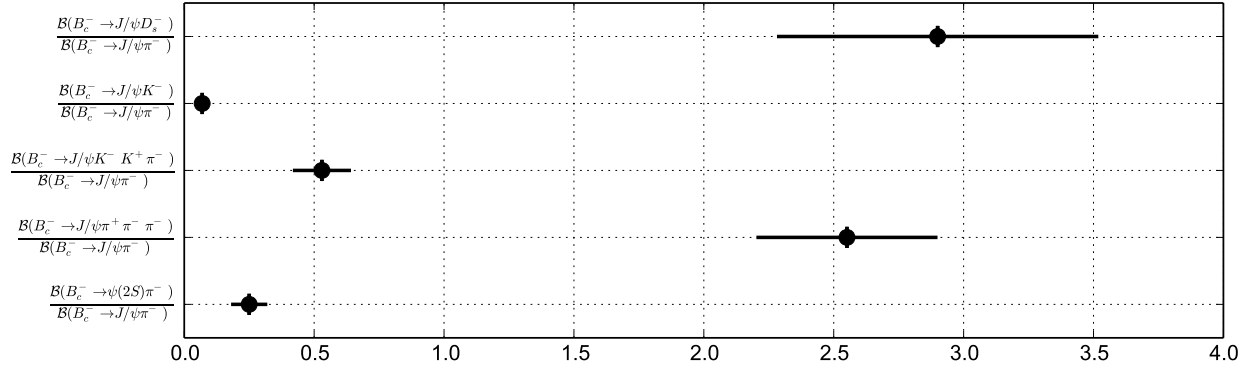


Figure 142: Summary of the averages from Table 163.

Table 164: Relative production times decay rates [10^{-3}].

Parameter	Measurements	Average
$\sigma(B_c^-) \times \mathcal{B}(B_c^- \rightarrow J/\psi \pi^-) / \sigma(B^-) \times \mathcal{B}(B^- \rightarrow J/\psi K^-)$	LHCb [718]: $22 \pm 8 \pm 2$ CMS [716]: $4.8 \pm 0.5^{+0.6}_{-0.5}$	4.9 ± 0.8

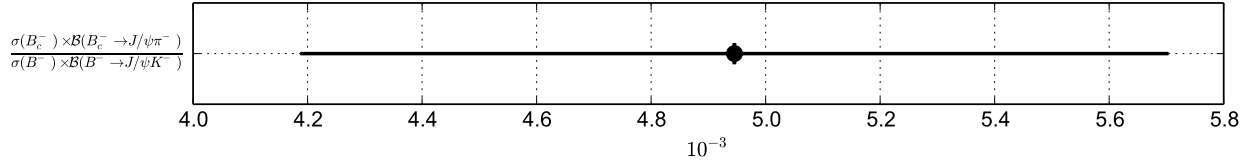


Figure 143: Summary of the averages from Table 164.

6.5.2 Decays to a B meson

Averages of B_c^- decays to a B meson are shown in Table 165 and Fig. 144.

Table 165: Decays to B_s^0 meson [10^{-3}].

Parameter	Measurements	Average
$\frac{\sigma(B_c^+)}{\sigma(B_s^0)} \times \mathcal{B}(B_c^+ \rightarrow B_s^0 \pi^+)$	LHCb [719]: $2.37 \pm 0.31^{+0.20}_{-0.17}$	$2.37^{+0.37}_{-0.35}$

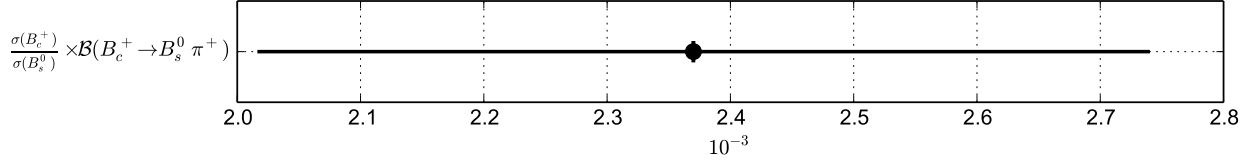


Figure 144: Summary of the averages from Table 165.

6.6 Decays of b baryons

Measurements of b baryons decays to charmed hadrons are summarized in sections 6.6.1 to 6.6.3.

6.6.1 Decays to a single open charm meson

Averages of b baryons decays to a single open charm meson are shown in Table 166 and Fig. 145.

Table 166: Relative decay rates to D^0 mesons.

Parameter	Measurements	Average
$\mathcal{B}(\Lambda_b^0 \rightarrow D^0 p K^-) / \mathcal{B}(\Lambda_b^+ \rightarrow D^0 p \pi^-)$	LHCb [720]: $0.073 \pm 0.008^{+0.005}_{-0.006}$	$0.073^{+0.009}_{-0.010}$
$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow D^0 p \pi^-) \times \mathcal{B}(D^0 \rightarrow K^+ \pi^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-) \times \mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+)}$	LHCb [720]: $0.0806 \pm 0.0023 \pm 0.0035$	0.0806 ± 0.0042
$\frac{f_{\Xi_b^0} \times \mathcal{B}(\Xi_b^0 \rightarrow D^0 p K^-)}{f_{\Lambda_b^0} \times \mathcal{B}(\Lambda_b^0 \rightarrow D^0 p K^-)}$	LHCb [720]: $0.44 \pm 0.09 \pm 0.06$	0.44 ± 0.11

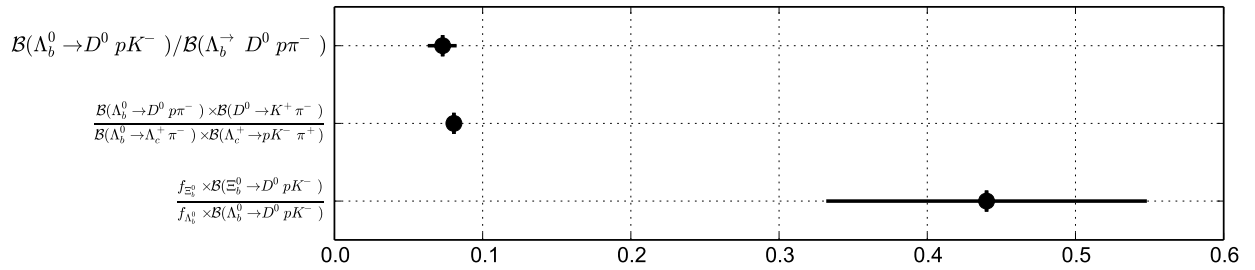


Figure 145: Summary of the averages from Table 166.

6.6.2 Decays to charmonium states

Averages of b baryons decays to charmonium states are shown in Tables 167–169 and Figs. 146–148.

Table 167: Λ_b^0 decays to charmonium [10^{-4}].

Parameter	Measurements	Average
$\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi(1S)\Lambda)$	CDF [721]: $4.7 \pm 2.1 \pm 1.9$	4.7 ± 2.8

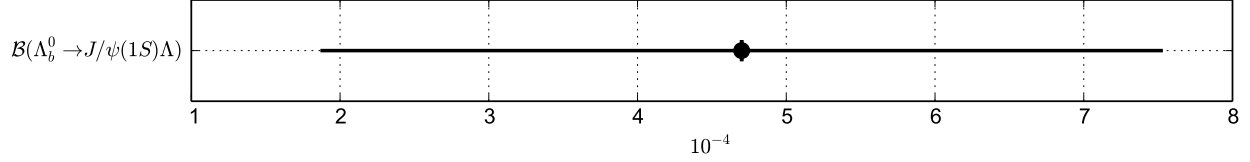


Figure 146: Summary of the averages from Table 167.

Table 168: f_b times Λ_b^0 decay to charmonium [10^{-5}].

Parameter	Measurements	Average
$f_b \times \mathcal{B}(\Lambda_b^0 \rightarrow J/\psi\Lambda)$	D0 [722]: $6.01 \pm 0.60 \pm 0.64$	6.01 ± 0.88

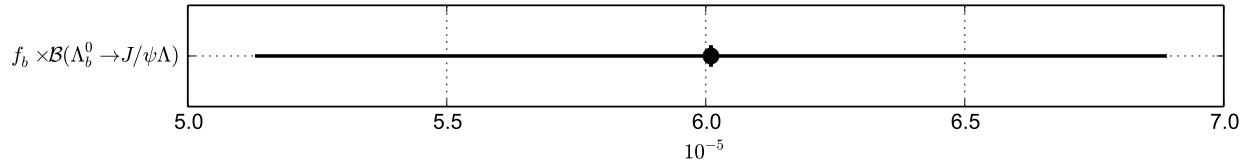


Figure 147: Summary of the averages from Table 168.

Table 169: Ξ_b^- and Ω_b^- decays to charmonium.

Parameter	Measurements	Average
$\sigma(\Xi_b^-) \times \mathcal{B}(\Xi_b^- \rightarrow J/\psi\Xi^-)/\sigma(\Lambda_b^0) \times \mathcal{B}(\Lambda_b^0 \rightarrow J/\psi\Lambda)$	CDF [44]: $0.167^{+0.037}_{-0.025} \pm 0.012$	$0.167^{+0.039}_{-0.028}$
$\sigma(\Omega_b^-) \times \mathcal{B}(\Omega_b^- \rightarrow J/\psi\Omega^-)/\sigma(\Lambda_b^0) \times \mathcal{B}(\Lambda_b^0 \rightarrow J/\psi\Lambda)$	CDF [44]: $0.045^{+0.017}_{-0.012} \pm 0.004$	$0.045^{+0.017}_{-0.013}$

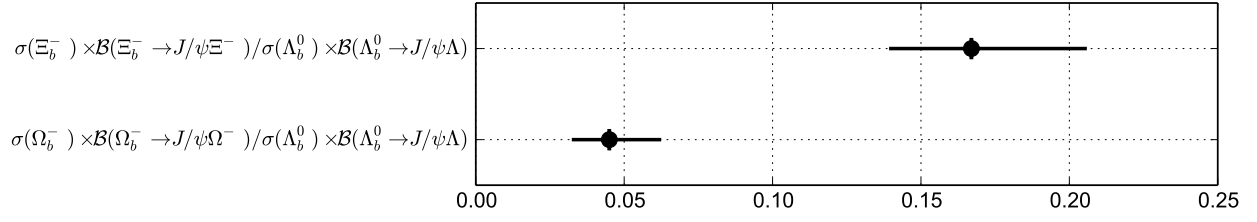


Figure 148: Summary of the averages from Table 169.

6.6.3 Decays to charm baryons

Averages of b baryons decays to charm baryons are shown in Tables 170–172 and Figs. 149–151.

Table 170: Absolute decay rates [10^{-2}].

Parameter	Measurements	Average
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)$	LHCb [47]: $0.430 \pm 0.003^{+0.036}_{-0.035}$	$0.430^{+0.036}_{-0.035}$
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \pi^-)$	CDF [723]: $2.68 \pm 0.29^{+1.15}_{-1.09}$	$2.68^{+1.19}_{-1.12}$

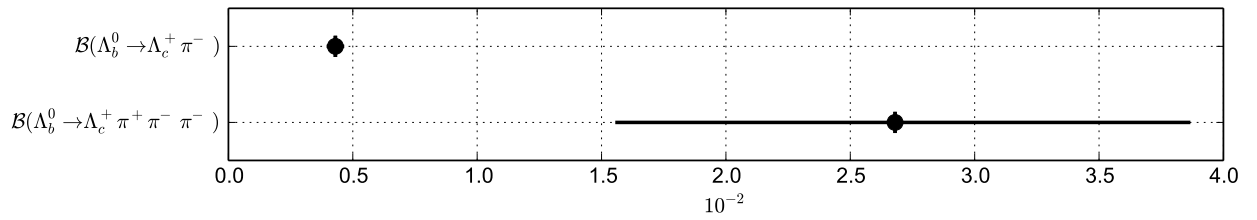


Figure 149: Summary of the averages from Table 170.

Table 171: Relative decay rates to Λ_c .

Parameter	Measurements	Average
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)$	CDF [39]: $16.6 \pm 3.0^{+2.8}_{-3.6}$	$16.6^{+4.1}_{-4.7}$
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-) / \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$	CDF [724]: $3.3 \pm 0.3 \pm 1.2$	3.3 ± 1.2
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \pi^-) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)$	LHCb [547]: $1.43 \pm 0.16 \pm 0.13$ CDF [723]: $3.04 \pm 0.33^{+0.70}_{-0.55}$	1.55 ± 0.20
$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)}$	LHCb [720]: $0.0731 \pm 0.0016 \pm 0.0016$	0.0731 ± 0.0023
$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ D^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-)}$	LHCb [631]: $0.042 \pm 0.003 \pm 0.003$	0.042 ± 0.004
$\frac{\mathcal{B}(\Xi_b^0 \rightarrow \Lambda_c^+ K^-) \times \mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+)}{\mathcal{B}(\Xi_b^0 \rightarrow D^0 p K^-) \times \mathcal{B}(D^0 \rightarrow K^+ \pi^-)}$	LHCb [720]: $0.57 \pm 0.22 \pm 0.21$	0.57 ± 0.30

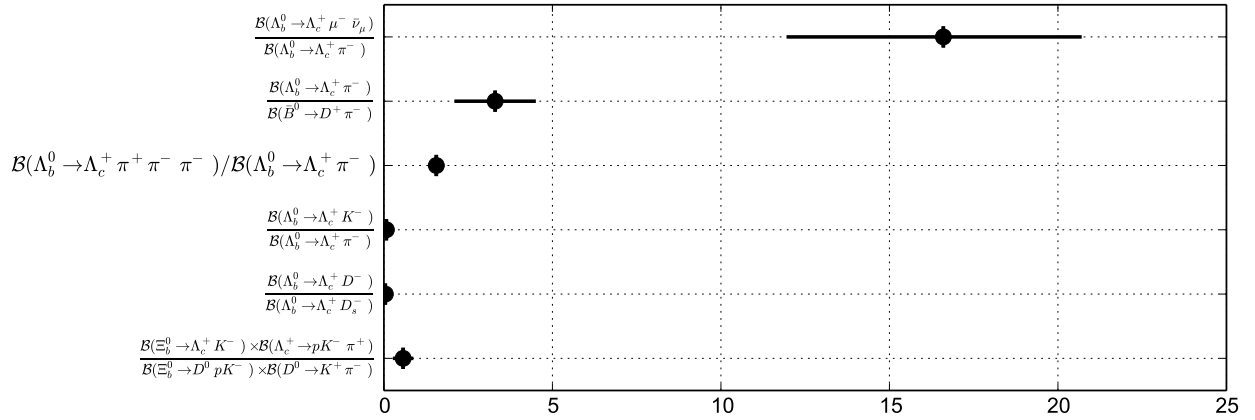


Figure 150: Summary of the averages from Table 171.

Table 172: Relative decay rates to excited or Σ_c states.

Parameter	Measurements	Average
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c(2595)^+ \mu^- \bar{\nu}_\mu) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu)$	CDF [39]: $0.126 \pm 0.033^{+0.047}_{-0.038}$	$0.126^{+0.057}_{-0.050}$
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c(2695)^+ \mu^- \bar{\nu}_\mu) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu)$	CDF [39]: $0.210 \pm 0.042^{+0.071}_{-0.050}$	$0.210^{+0.082}_{-0.065}$
$\mathcal{B}(\Lambda_b^0 \rightarrow \Sigma_c \pi \mu^- \bar{\nu}_\mu) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu)$	CDF [39]: $0.108 \pm 0.044^{+0.042}_{-0.036}$	$0.108^{+0.061}_{-0.057}$
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c(2595)^+ \pi^-) \times \mathcal{B}(\Lambda_c(2595)^+ \rightarrow \Lambda_c^+ \pi^+ \pi^-) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.044 \pm 0.017^{+0.006}_{-0.004}$	$0.044^{+0.018}_{-0.017}$
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c(2625)^+ \pi^-) \times \mathcal{B}(\Lambda_c(2625)^+ \rightarrow \Lambda_c^+ \pi^+ \pi^-) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.043 \pm 0.015 \pm 0.004$	0.043 ± 0.016
$\mathcal{B}(\Lambda_b^0 \rightarrow \Sigma_c^0 \pi^+ \pi^-) \times \mathcal{B}(\Sigma_c^0 \rightarrow \Lambda_c^+ \pi^-) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.074 \pm 0.024 \pm 0.012$	0.074 ± 0.027
$\mathcal{B}(\Lambda_b^0 \rightarrow \Sigma_c^{++} \pi^- \pi^-) \times \mathcal{B}(\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi^+) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \pi^-)$	LHCb [547]: $0.042 \pm 0.018 \pm 0.007$	0.042 ± 0.019

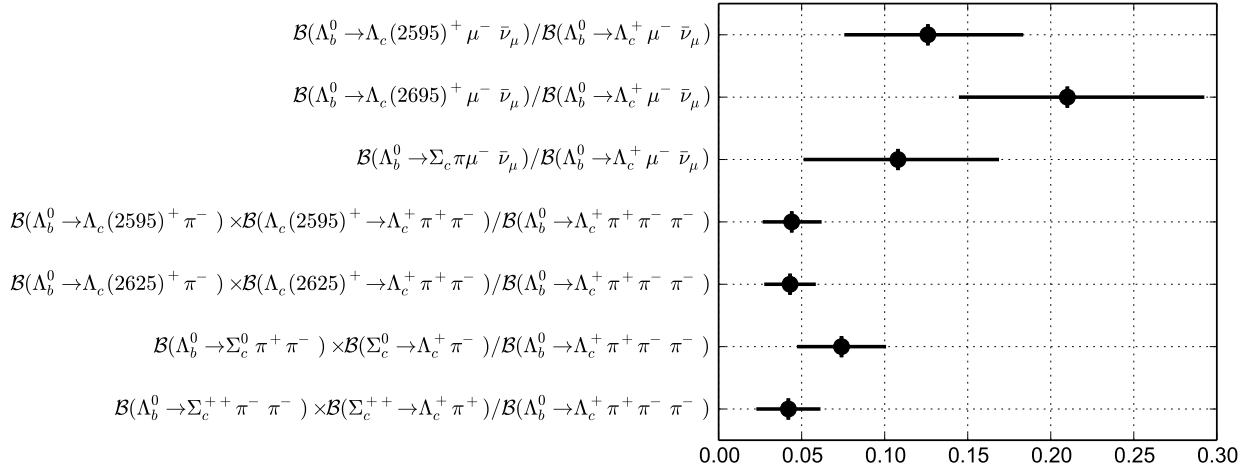


Figure 151: Summary of the averages from Table 172.

7 B decays to charmless final states

This section provides the branching fractions, polarization fractions, the partial rate asymmetries (A_{CP}) and other observables of charmless B decays. The order of entries in the Tables correspond to that in the PDG, and the quoted RPP numbers correspond to the PDG numbers of the corresponding branching fractions. The asymmetry is defined as $A_{CP} = \frac{N_{\bar{B}} - N_B}{N_{\bar{B}} + N_B}$, where $N_{\bar{B}}$ and N_B are respectively number of $\bar{B}^0/B^-$ and B^0/B^+ decaying into a specific final state. Four different B decay categories are considered: charmless mesonic, baryonic, radiative and leptonic. We also include measurements of B_s decays. Measurements supported with written documents are accepted in the averages; written documents include journal papers, conference contributed papers, preprints or conference proceedings. Results from A_{CP} measurements obtained from time-dependent analyses are listed and described in Sec. 4.

Most of the branching fractions from *BABAR* and *Belle* assume equal production of charged and neutral B pairs. The best measurements to date show that this is still a reasonable approximation (see Sec. 3). For branching fractions, we provide either averages or the most stringent 90% confidence level (CL) upper limits. If one or more experiments have measurements with $>4\sigma$ for a decay channel, all available central values for that channel are used in the averaging. We also give central values and errors for cases where the significance of the average value is at least 3σ , even if no single measurement is above 4σ . Since a few decay modes are sensitive to the contribution of new physics and the current experimental upper limits are not far from the Standard Model expectation, we provide the combined upper limits or averages in these cases. Their upper limits can be estimated assuming that the errors are Gaussian. For A_{CP} we provide averages in all cases.

Our averaging is performed by maximizing the likelihood, $\mathcal{L} = \prod_i \mathcal{P}_i(x)$, where $\mathcal{P}_i$ is the probability density function (PDF) of the i^{th} measurement, and x is the branching fraction or A_{CP} . The PDF is modeled by an asymmetric Gaussian function with the measured central value as its mean and the quadratic sum of the statistical and systematic errors as the standard deviations. The experimental uncertainties are considered to be uncorrelated with each other when the averaging is performed. No error scaling is applied when the fit χ^2 is greater than 1 since we believe that tends to overestimate the errors except in cases of extreme disagreement (we have no such cases).

At present, we have measurements of more than 500 decay modes, reported in hundreds of papers. Because the number of references is so large, we do not include them with the tables shown here but the full set of references is available from active GIF files at the “2014” link on the rare decays HFAG web page: <http://www.slac.stanford.edu/xorg/hfag/rare/index.html>. The largest improvement since the last report has come from the inclusion of a variety of new measurements from the LHC, especially LHCb. The measurements of B_s decays are particularly noteworthy.

Sections 7.1 and 7.2 provide compilations of branching fractions of B^0 and B^+ to mesonic and baryonic charmless final states, respectively, while Sec. 7.3 gives branching fractions of Λ_b decays. In Sec. 7.4 and 7.5 different observables of interest are detailed in addition to branching fractions: in the former for B_s^0 -meson charmless decays, and in the latter for leptonic and radiative B^0 and B^+ meson decays, including processes in which the photon yields a pair of charged or neutral leptons. This section also contains limits from searches for lepton-number-violating decays. Finally, Sec. 7.6 and 7.7 give CP asymmetries and results of polarization

measurements, respectively, in different b -hadron charmless decays.

7.1 Mesonic decays of B^0 and B^+ mesons

This section provides branching fractions of charmless mesonic decays: Tables 173 to 175 for B^+ and Tables 176 to 178 for B^0 mesons. The tables are separated according to the presence or absence of kaons in the final state. Finally, Table 179 details several relative branching fractions of B^0 decays.

Table 173: Branching fractions (BF) of charmless mesonic B^+ decays with kaons (part 1) in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	LHCb	New avg.
262	$K^0\pi^+$	23.7 ± 0.8	$23.9 \pm 1.1 \pm 1.0$	$23.97 \pm 0.53 \pm 0.71$	$18.8^{+3.7+2.1}_{-3.3-1.8}$			23.79 ± 0.75
263	$K^+\pi^0$	12.9 ± 0.5	$13.6 \pm 0.6 \pm 0.7$	$12.62 \pm 0.31 \pm 0.56$	$12.9^{+2.4+1.2}_{-2.2-1.1}$			$12.94^{+0.52}_{-0.51}$
264	$\eta'K^+$	70.6 ± 2.5	$71.5 \pm 1.3 \pm 3.2$	$69.2 \pm 2.2 \pm 3.7$	$80^{+10}_{-9} \pm 7$			71.1 ± 2.6
265	$\eta'K^{*+}$	$4.8^{+1.8}_{-1.6}$	$4.8^{+1.6}_{-1.4} \pm 0.8$	< 2.9	< 35			$4.8^{+1.8}_{-1.6}$
266	$\eta'K_0^*(1430)^+$	5.2 ± 2.1	$5.2 \pm 1.9 \pm 1.0$					5.2 ± 2.1
267	$\eta'K_2^*(1430)^+$	28 ± 5	$28.0^{+4.6}_{-4.3} \pm 2.6$					$28.0^{+5.3}_{-5.0}$
268	ηK^+	2.4 ± 0.4	$2.94^{+0.39}_{-0.34} \pm 0.21$	$2.12 \pm 0.23 \pm 0.11$	$2.2^{+2.8}_{-2.2}$			$2.36^{+0.22}_{-0.21}$
269	ηK^{*+}	19.3 ± 1.6	$18.9 \pm 1.8 \pm 1.3$	$19.3^{+2.0}_{-1.9} \pm 1.5$	$26.4^{+9.6}_{-8.2} \pm 3.3$			19.3 ± 1.6
270	$\eta K_0^*(1430)^+$	18 ± 4	$18.2 \pm 2.6 \pm 2.6$					18.2 ± 3.7
271	$\eta K_2^*(1430)^+$	9.1 ± 3.0	$9.1 \pm 2.7 \pm 1.4$					9.1 ± 3.0
272	$\eta(1295)K^+ \dagger$	$2.9^{+0.8}_{-0.7}$	$2.9^{+0.8}_{-0.7} \pm 0.2 \S$					$2.9^{+0.8}_{-0.7}$
274	$\eta(1405)K^+ \dagger$	< 1.2	< 1.2					< 1.2
275	$\eta(1475)K^+ \dagger$	$13.8^{+2.1}_{-1.8}$	$13.8^{+1.8+1.0}_{-1.7-0.6}$					$13.8^{+2.1}_{-1.8}$
276	$f_1(1285)K^+$	< 2.0	< 2.0					< 2.0
277	$f_1(1420)K^+ \dagger$	< 2.9	< 2.9					< 2.9
279	$\phi(1680)K^+ \dagger$	< 3.4	< 3.4					< 3.4
279	$\phi(1680)K^+ \dagger$	< 3.4	< 3.4					< 3.4
280	$f_0(1500)K^+$	3.7 ± 2.2	$3.7 \pm 2.2 \ddagger$					3.7 ± 2.2
281	ωK^+	6.7 ± 0.8	$6.3 \pm 0.5 \pm 0.3$	$8.1 \pm 0.6 \pm 0.6$	$3.2^{+2.4}_{-1.9} \pm 0.8$			6.7 ± 0.5
282	ωK^{*+}	< 7.4	< 7.4		< 87			< 7.4
283	$\omega(K\pi)_0^{*+}$	28 ± 4	$27.5^{+3.0}_{-2.6}$					$27.5^{+3.0}_{-2.6}$
284	$\omega K_0^*(1430)^+$	24 ± 5	$24.0 \pm 2.6 \pm 4.4$					24.0 ± 5.1
285	$\omega K_2^*(1430)^+$	21 ± 4	$21.5 \pm 3.6 \pm 2.4$					21.5 ± 4.3
286	$a_0(980)^+K^0 \dagger$	< 3.9	< 3.9					< 3.9
287	$a_0(980)^0K^+ \dagger$	< 2.5	< 2.5					< 2.5
288	$K^{*0}\pi^+$	10.1 ± 0.9	$10.8 \pm 0.6^{+1.2}_{-1.4}$	$9.7 \pm 0.6^{+0.8}_{-0.9}$	< 16			$10.1^{+0.8}_{-0.9}$
289	$K^{*+}\pi^0$	8.2 ± 1.9	$8.2 \pm 1.5 \pm 1.1$		$7.1^{+11.4}_{-7.1} \pm 1.0$			8.2 ± 1.8
290	$K^+\pi^+\pi^-$	51 ± 2.9	$54.4 \pm 1.1 \pm 4.6$	$48.8 \pm 1.1 \pm 3.6$				51.0 ± 3.0
291	$K^+\pi^+\pi^- (NR)$	$16.3^{+2.1}_{-1.5}$	$9.3 \pm 1.0^{+6.9}_{-1.7}$	$16.9 \pm 1.3^{+1.7}_{-1.6}$	< 28			16.3 ± 2.0
292	$\omega(782)K^+$	6 ± 9	$5.9^{+8.8+0.5}_{-9.0-0.4}$	$6.8 \pm 0.4 \pm 0.4$				6.8 ± 0.6
293	$f_0(980)K^+ \dagger$	$9.4^{+1.0}_{-1.2}$	$10.3 \pm 0.5^{+2.0}_{-1.4}$	$8.8 \pm 0.8^{+0.9}_{-1.8}$				$9.4^{+0.9}_{-1.0}$
294	$f_2(1270)^0K^+$	1.07 ± 0.27	$0.88^{+0.38+0.01}_{-0.33-0.03}$	$1.33 \pm 0.30^{+0.23}_{-0.34}$				1.07 ± 0.29
295	$f_0(1370)^0K^+ \dagger$	< 10.7	< 10.7					< 10.7
296	$\rho^0(1450)K^+$	< 11.7	< 11.7					< 11.7
297	$f_2'(1525)K^+$	< 3.4	< 3.4					< 3.4
298	ρ^0K^+	3.7 ± 0.5	$3.56 \pm 0.45^{+0.57}_{-0.46}$	$3.89 \pm 0.47^{+0.43}_{-0.41}$	< 17			$3.74^{+0.49}_{-0.45}$
299	$K_0^*(1430)^0\pi^+$	45^{+9}_{-7}	$32.0 \pm 1.2^{+10.8}_{-6.0}$	$51.6 \pm 1.7^{+7.0}_{-7.5}$				45.1 ± 6.3
300	$K_2^*(1430)^0\pi^+$	$5.6^{+2.2}_{-1.5}$	$5.6 \pm 1.2^{+1.8}_{-0.8}$	< 6.9				$5.6^{+2.2}_{-1.4}$
301	$K^*(1410)^0\pi^+$	< 45		< 45				< 45
302	$K^*(1680)^0\pi^+$	< 12	< 15	< 12				< 12
303	$K^+\pi^0\pi^0$	16.2 ± 1.9	$16.2 \pm 1.2 \pm 1.5$					16.2 ± 1.9
304	$f_0(980)K^+$	2.8 ± 0.8	$2.8 \pm 0.6 \pm 0.5$					2.8 ± 0.8
305	$K^-\pi^+\pi^+$	< 0.95	< 0.95	< 4.5				< 0.95
306	$K^-\pi^+\pi^+ (NR)$	< 56			< 56			< 56
307	$K_1(1270)^0\pi^+$	< 40	< 40					< 40
308	$K_1(1400)^0\pi^+$	< 39	< 39					< 39
309	$K^0\pi^+\pi^0$	< 66			< 66			< 66
310	ρ^+K^0	8.0 ± 1.5	$8.0^{+1.4}_{-1.3} \pm 0.6$		< 48			$8.0^{+1.5}_{-1.4}$
311	$K^{*+}\pi^+\pi^-$	75 ± 10	$75.3 \pm 6.0 \pm 8.1$					75.3 ± 10.1
312	$K^{*+}\rho^0$	4.6 ± 1.1	$4.6 \pm 1.0 \pm 0.4$		< 74			4.6 ± 1.1
313	$f_0(980)K^{*+} \dagger$	4.2 ± 0.7	$4.2 \pm 0.6 \pm 0.3$					4.2 ± 0.7

$\dagger$ Product BF - daughter BF taken to be 100%.

$\S$ Product BF - $\times \mathcal{B}(\eta(1295) \rightarrow \eta\pi\pi)$.

$\ddagger$ Average of results in $K_S^0 K^+ K^-$, $K_S^0 K_S^0 K^+$ [240] and $K^+\pi^+\pi^-$ [247]. Reference [247] includes an f_X resonance with parameters that are compatible with $f_0(1500)$.

Table 174: Branching fractions (BF) of charmless mesonic B^+ decays with kaons (part 2) in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	LHCb	New avg.
314	$a_1^+ K^0$	35 ± 7	$34.9 \pm 5.0 \pm 4.4$					34.9 ± 6.7
315	$b_1^+ K^0 \dagger$	9.6 ± 1.9	$9.6 \pm 1.7 \pm 0.9$					9.6 ± 1.9
317	$K_1(1400)^+ \rho^0$	< 780	$< 780 \diamond$					$< 780 \diamond$
318	$K_2(1430)^+ \rho^0$	< 1500	$< 1500 \diamond$					$< 1500 \diamond$
319	$b_1^0 K^+ \dagger$	9.1 ± 2.0	$9.1 \pm 1.7 \pm 1.0$					9.1 ± 2.0
320	$b_1^+ K^{*0} \dagger$	< 5.9	< 5.9					< 5.9
321	$b_1^0 K^{*+} \dagger$	< 6.7	< 6.7					< 6.7
322	$K^+ \bar{K}^0$	1.31 ± 0.17	$1.61 \pm 0.44 \pm 0.09$	$1.11 \pm 0.19 \pm 0.05$	< 3.3		$1.52 \pm 0.21 \pm 0.05$	1.32 ± 0.14
323	$\bar{K}^0 K^+ \pi^0$	< 24			< 24			< 24
324	$K^+ K_S K_S$	10.8 ± 0.6	$10.6 \pm 0.5 \pm 0.3$	$13.4 \pm 1.9 \pm 1.5$				10.8 ± 0.6
325	$f_0(980) K^+$	14.7 ± 3.3	$14.7 \pm 2.8 \pm 1.8$					14.7 ± 3.3
326	$f_0(1710) K^+$	$0.48^{+0.40}_{-0.26}$	$0.48^{+0.40}_{-0.24} \pm 0.11$					$0.48^{+0.41}_{-0.26}$
327	$K^+ K_S K_S(NR)$	20 ± 4	$19.8 \pm 3.7 \pm 2.5$					19.8 ± 4.5
328	$K_S K_S \pi^+$	< 0.51	< 0.51	< 3.2				< 0.51
329	$K^+ K^- \pi^+$	5.0 ± 0.7	$5.0 \pm 0.5 \pm 0.5$	< 13				5.0 ± 0.7
330	$K^+ K^- \pi^+(NR)$	< 75			< 75			< 75
331	$\bar{K}^{*0} K^+$	< 1.1	< 1.1		< 5.3			< 1.1
332	$\bar{K}_0^*(1430)^0 K^+$	< 2.2	< 2.2					< 2.2
333	$K^+ K^+ \pi^-$	< 0.16	< 0.16	< 2.4				< 0.16
334	$K^+ K^+ \pi^-(NR)$	< 87.9						< 87.9
335	$f_2'(1525) K^+$	1.8 ± 0.5	$1.8 \pm 0.5 \dagger$	< 8				1.8 ± 0.5
336	$f_J(2220) K^+$	< 1.2		< 1.2				< 1.2
337	$K^{*+} \pi^+ K^-$	< 11.8	< 11.8					< 11.8
338	$K^{*+} \bar{K}^{*0}$	1.2 ± 0.5	$1.2 \pm 0.5 \pm 0.1$		< 71			1.2 ± 0.5
339	$K^{*+} K^+ \pi^-$	< 6.1	< 6.1					< 6.1
340	$K^+ K^- K^+$	34.0 ± 1.4	$34.6 \pm 0.6 \pm 0.9$	$30.6 \pm 1.2 \pm 2.3$				34.0 ± 1.0
341	ϕK^+	$8.8^{+0.7}_{-0.6}$	$9.2 \pm 0.4^{+0.7}_{-0.5}$	$9.6 \pm 0.9^{+1.1}_{-0.8}$	$5.5^{+2.1}_{-1.8} \pm 0.6$	$7.6 \pm 1.3 \pm 0.6$		8.8 ± 0.5
342	$f_0(980) K^+$	9.4 ± 3.2	$9.4^{+1.6}_{-2.8}$					$9.4^{+1.6}_{-2.8}$
343	$a_2(1320) K^+ \dagger$	< 1.1		< 1.1				< 1.1
344	$X_0(1550) K^+$	4.3 ± 0.7	$4.3 \pm 0.60 \pm 0.30$					4.30 ± 0.67
345	$\phi(1680) K^+ \dagger$	< 0.8		< 0.8				< 0.8
346	$f_0(1710) K^+ \dagger$	1.1 ± 0.6	$1.12 \pm 0.25 \pm 0.50$					1.12 ± 0.56
347	$K^+ K^- K^+(NR)$	$23.8^{+2.8}_{-5.0}$	$22.8 \pm 2.7 \pm 7.6$	$24.0 \pm 1.5^{+2.6}_{-6.0}$				$23.8^{+2.9}_{-5.1}$
348	$K^{*+} K^+ K^-$	36 ± 5	$36.2 \pm 3.3 \pm 3.6$					36.2 ± 4.9
349	ϕK^{*+}	10.0 ± 2.0	$11.2 \pm 1.0 \pm 0.9$	$6.7^{+2.1+0.7}_{-1.9-1.0}$	$10.6^{+6.4+1.8}_{-4.9-1.6}$			10.0 ± 1.1
350	$\phi(K\pi)_0^{*+}$	8.3 ± 1.6	$8.3^{+1.4}_{-0.8}$					$8.3^{+1.4}_{-0.8}$
351	$\phi K_1(1270)^+$	6.1 ± 1.9	$6.1 \pm 1.6 \pm 1.1$					6.1 ± 1.9
352	$\phi K_1(1400)^+$	< 3.2	< 3.2					< 3.2
353	$\phi K^*(1410)^+$	< 4.3	< 4.3					< 4.3
354	$\phi K_0^*(1430)^+$	7.0 ± 1.6	$7.0 \pm 1.3 \pm 0.9$					7.0 ± 1.6
355	$\phi K_2^*(1430)^+$	8.4 ± 2.1	$8.4 \pm 1.8 \pm 1.0$					8.4 ± 2.1
356	$\phi K_2(1770)^+$	< 15	< 15					< 15
357	$\phi K_2(1820)^+$	< 16.3	< 16.3					< 16.3
358	$a_1^+ K^{*0}$	< 3.6	< 3.6					< 3.6
359	$\phi \phi K^+ \S$	5.0 ± 1.2	$5.6 \pm 0.5 \pm 0.3$	$2.6^{+1.1}_{-0.9} \pm 0.3$				5.0 ± 0.5
360	$\eta' \eta' K^+$	< 25	< 25					< 25
361	$K^+ \omega \phi$	< 1.9		< 1.9				< 1.9
362	$K^+ X(1812) \dagger$	< 0.32		< 0.32				< 0.32

$\dagger$ Product BF - daughter BF taken to be 100%.

$\S$ $M_{\phi\phi} < 2.85 \text{ GeV}/c^2$.

$\ddagger$ Average of results in $K^+ K^- K^+$ and $K_S^0 K_S^0 K^+$.

$\diamond$ Result from ARGUS. Cited in the *BABAR* column to avoid adding adding a column to the table.

Table 175: Branching fractions (BF) of charmless mesonic B^+ decays without kaons in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in red (blue) are new published (preliminary) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	LHCb	New avg.
379	$\pi^+\pi^0$	5.5 ± 0.4	$5.02 \pm 0.46 \pm 0.29$	$5.86 \pm 0.26 \pm 0.38$	$4.6^{+1.8+0.6}_{-1.6-0.7}$			$5.48^{+0.35}_{-0.34}$
380	$\pi^+\pi^+\pi^-$	15.2 ± 1.4	$15.2 \pm 0.6 \pm 1.3$					15.2 ± 1.4
381	$\rho^0\pi^+$	8.3 ± 1.2	$8.1 \pm 0.7^{+1.3}_{-1.6}$	$8.0^{+2.3}_{-2.0} \pm 0.7$	$10.4^{+3.3}_{-3.4} \pm 2.1$			$8.3^{+1.2}_{-1.3}$
382	$f_0(980)\pi^+ \dagger$	< 1.5	< 1.5					< 1.5
383	$f_2(1270)\pi^+$	$1.6^{+0.7}_{-0.4}$	$1.57 \pm 0.42^{+0.55}_{-0.25}$					$1.57^{+0.69}_{-0.49}$
384	$\rho(1450)^0\pi^+ \dagger$	$1.4^{+0.6}_{-0.9}$	$1.4 \pm 0.4^{+0.5}_{-0.8}$					$1.4^{+0.6}_{-0.9}$
385	$f_0(1370)\pi^+ \dagger$	< 4.0	< 4.0					< 4.0
386	$f_0(500)\pi^+ \dagger$	< 4.1	< 4.1					< 4.1
387	$\pi^+\pi^-\pi^+(NR)$	$5.3^{+1.5}_{-1.1}$	$5.3 \pm 0.7^{+1.3}_{-0.8}$					$5.3^{+1.5}_{-1.1}$
388	$\pi^+\pi^0\pi^0$	< 890	< 890					< 890
389	$\rho^+\pi^0$	10.9 ± 1.4	$10.2 \pm 1.4 \pm 0.9$	$13.2 \pm 2.3^{+1.4}_{-1.9}$	< 43			$10.9^{+1.4}_{-1.5}$
391	$\rho^+\rho^0$	24.0 ± 1.9	$23.7 \pm 1.4 \pm 1.4$	$31.7 \pm 7.1^{+3.8}_{-6.7}$				$24.0^{+1.9}_{-2.0}$
392	$f_0(980)\rho^+ \dagger$	< 2.0	< 2.0					< 2.0
393	$a_1^+\pi^0$	26 ± 7	$26.4 \pm 5.4 \pm 4.1$					26.4 ± 6.8
394	$a_1^0\pi^+$	20 ± 6	$20.4 \pm 4.7 \pm 3.4$					20.4 ± 5.8
395	$\omega\pi^+$	6.9 ± 0.5	$6.7 \pm 0.5 \pm 0.4$	$6.9 \pm 0.6 \pm 0.5$	$11.3^{+3.3}_{-2.9} \pm 1.4$			6.9 ± 0.5
396	$\omega\rho^+$	15.9 ± 2.1	$15.9 \pm 1.6 \pm 1.4$		< 61			15.9 ± 2.1
397	$\eta\pi^+$	4.02 ± 0.27	$4.00 \pm 0.40 \pm 0.24$	$4.07 \pm 0.26 \pm 0.21$	$1.2^{+2.8}_{-1.2}$			4.02 ± 0.27
398	$\eta\rho^+$	7.0 ± 2.9	$9.9 \pm 1.2 \pm 0.8$	$4.1^{+1.4}_{-1.3} \pm 0.4$	$4.8^{+5.2}_{-3.8}$			6.9 ± 1.0
399	$\eta'\pi^+$	2.7 ± 0.9	$3.5 \pm 0.6 \pm 0.2$	$1.8^{+0.7}_{-0.6} \pm 0.1$	$1.0^{+5.8}_{-1.0}$			$2.7^{+0.5}_{-0.4}$
400	$\eta'\rho^+$	9.7 ± 2.2	$9.7^{+1.9}_{-1.8} \pm 1.1$	< 5.8	< 33			$9.7^{+2.2}_{-2.1}$
401	$\phi\pi^+$	< 0.15	< 0.24	< 0.33	< 5		< 0.15	< 0.15
402	$\phi\rho^+$	< 3.0	< 3.0		< 16			< 3.0
403	$a_0(980)^0\pi^+ \dagger$	< 5.8	< 5.8					< 5.8
404	$a_0(980)^+\pi^0 \dagger$	< 1.4	< 1.4					< 1.4
405	$\pi^+\pi^+\pi^+\pi^-\pi^-$	< 860	< 860					< 860
406	$\rho^0 a_1(1260)^+$	< 620			< 620			< 620
407	$\rho^0 a_2(1320)^+$	< 720			< 720			< 720
408	$b_1^0\pi^+ \dagger$	6.7 ± 2.0	$6.7 \pm 1.7 \pm 1.0$					6.7 ± 2.0
409	$b_1^+\pi^0 \dagger$	< 3.3	< 3.3					< 3.3
410	$\pi^+\pi^+\pi^+\pi^-\pi^-\pi^0$	< 6300	< 6300					< 6300
411	$b_1^+\rho^0 \dagger$	< 5.2	< 5.2					< 5.2
413	$b_1^0\rho^+ \dagger$	< 3.3	< 3.3					< 3.3

$\dagger$ Product BF - daughter BF taken to be 100%.

Table 176: Branching fractions (BF) of charmless mesonic B^0 decays with kaons (part 1) in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	LHCb	New avg.
227	$K^+\pi^-$	19.6 ± 0.5	$19.1 \pm 0.6 \pm 0.6$	$20.0 \pm 0.34 \pm 0.60$	$18.0^{+2.3+1.2}_{-2.1-0.9}$			$19.57^{+0.53}_{-0.52}$
228	$K^0\pi^0$	9.9 ± 0.5	$10.1 \pm 0.6 \pm 0.4$	$9.68 \pm 0.46 \pm 0.50$	$12.8^{+4.0+1.7}_{-3.3-1.4}$			9.93 ± 0.49
229	$\eta'K^0$	66 ± 4	$68.5 \pm 2.2 \pm 3.1$	$58.9^{+3.6}_{-3.5} \pm 4.3$	$89^{+18}_{-16} \pm 9$			66.1 ± 3.1
230	$\eta'K^{*0}$	3.1 ± 0.9	$3.1^{+0.9}_{-0.8} \pm 0.3$	$2.6 \pm 0.7 \pm 0.2$	$7.8^{+7.7}_{-5.7}$			$2.8^{+0.6}_{-0.5}$
231	$\eta'K_0^*(1430)^0$	6.3 ± 1.6	$6.3 \pm 1.3 \pm 0.9$					6.3 ± 1.6
232	$\eta'K_2^*(1430)^0$	13.7 ± 3.2	$13.7^{+3.0}_{-1.9} \pm 1.2$					$13.7^{+3.2}_{-2.2}$
233	ηK^0	$1.23^{+0.27}_{-0.24}$	$1.15^{+0.43}_{-0.38} \pm 0.09$	$1.27^{+0.33}_{-0.29} \pm 0.08$	$0.0^{+3.0}_{-0.0}$			$1.23^{+0.27}_{-0.24}$
234	ηK^{*0}	15.9 ± 1.0	$16.5 \pm 1.1 \pm 0.8$	$15.2 \pm 1.2 \pm 1.0$	$13.8^{+5.5}_{-4.6} \pm 1.6$			15.9 ± 1.0
235	$\eta K_0^*(1430)^0$	11.0 ± 2.2	$11.0 \pm 1.6 \pm 1.5$					11.0 ± 2.2
236	$\eta K_2^*(1430)^0$	9.6 ± 2.1	$9.6 \pm 1.8 \pm 1.1$					9.6 ± 2.1
237	ωK^0	5.0 ± 0.6	$5.4 \pm 0.8 \pm 0.3$	$4.5 \pm 0.4 \pm 0.3$	$10.0^{+5.4}_{-4.2} \pm 1.4$			4.8 ± 0.4
238	$a_0(980)^0 K^0 \dagger$	< 7.8	< 7.8					< 7.8
239	$b_1^0 K^0 \dagger$	< 7.8	< 7.8					< 7.8
240	$a_0(980)^- K^+ \dagger$	< 1.9	< 1.9					< 1.9
241	$b_1^- K^+ \dagger$	7.4 ± 1.4	$7.4 \pm 1.0 \pm 1.0$					7.4 ± 1.4
242	$b_1^0 K^{*0} \dagger$	< 8.0	< 8.0					< 8.0
243	$b_1^- K^{*+} \dagger$	< 5.0	< 5.0					< 5.0
244	$a_0(1450)^- K^+ \dagger$	< 3.1	< 3.1					< 3.1
245	$K_S X^0(\text{Familon}) \dagger$	< 53			< 53			< 53
246	ωK^{*0}	2.0 ± 0.5	$2.2 \pm 0.6 \pm 0.2$	$1.8 \pm 0.7^{+0.3}_{-0.2}$	< 23			2.0 ± 0.5
247	ωK^{*0}	18.4 ± 2.5	$18.4^{+1.8}_{-1.7}$					$18.4^{+1.8}_{-1.7}$
248	$\omega K_0^*(1430)^0$	16.0 ± 3.4	$16.0 \pm 1.6 \pm 3.0$					16.0 ± 3.4
249	$\omega K_2^*(1430)^0$	10.1 ± 2.3	$10.1 \pm 2.0 \pm 1.1$					10.1 ± 2.3
250	$\omega K^+\pi^- (NR)^{(1)}$	5.1 ± 1.0		$5.1 \pm 0.7 \pm 0.7$				5.1 ± 1.0
251	$K^+\pi^-\pi^0$	37.8 ± 3.2	$38.5 \pm 1.0 \pm 3.9$	$36.6^{+4.2}_{-4.3} \pm 3.0$	< 40			37.8 ± 3.2
252	$\rho^- K^+$	7.0 ± 0.9	$6.6 \pm 0.5 \pm 0.8$	$15.1^{+3.4+2.4}_{-3.3-2.6}$	< 32			7.0 ± 0.9
253	$\rho(1450)^- K^+$	2.4 ± 1.2	$2.4 \pm 1.0 \pm 0.6$					2.4 ± 1.2
254	$\rho(1700)^- K^+$	0.6 ± 0.7	$0.6 \pm 0.6 \pm 0.4$					0.6 ± 0.7
255	$K^+\pi^-\pi^0(NR)$	2.8 ± 0.6	$2.8 \pm 0.5 \pm 0.4$	< 9.4				2.8 ± 0.6
256	$(K\pi)_0^{*+}\pi^-$	34 ± 5	$34.2 \pm 2.4 \pm 4.1$					34.2 ± 4.8
257	$(K\pi)_0^{*+}\pi^0$	8.5 ± 1.7	$8.6^{+1.1}_{-1.3}$					$8.6^{+1.1}_{-1.3}$
258	$K_2^*(1430)^0\pi^0$	< 4.0	< 4.0					< 4.0
259	$K^*(1680)^0\pi^0$	< 7.5	< 7.5					< 7.5
260	$K_x^{*0}\pi^0 (2)$	6.1 ± 1.6		$6.1^{+1.6+0.5}_{-1.5-0.6}$				$6.1^{+1.7}_{-1.6}$
261	$K^0\pi^+\pi^-$	65 ± 8	$50.2 \pm 1.5 \pm 1.8$	$47.5 \pm 2.4 \pm 3.7$	$50^{+10}_{-9} \pm 7$		$65.2^{+6.0}_{-5.1} \diamond$	51.8 ± 1.9
262	$K^0\pi^+\pi^- (NR)$	$14.7^{+4.0}_{-2.6}$	$11.1^{+2.5}_{-1.0} \pm 0.9$	$19.9 \pm 2.5^{+1.7}_{-2.0}$				14.7 ± 2.0
263	$\rho^0 K^0$	4.7 ± 0.6	$4.4 \pm 0.7 \pm 0.3$	$6.1 \pm 1.0^{+1.1}_{-1.2}$	< 39			4.7 ± 0.7
264	$K^{*+}\pi^-$	8.4 ± 0.8	$8.2 \pm 0.9^{(3)}$	$8.4 \pm 1.1^{+1.0}_{-0.9}$	$16^{+6}_{-5} \pm 2$			8.4 ± 0.8
265	$K_0^*(1430)^+\pi^-$	33 ± 7	$29.9^{+2.3}_{-1.7} \pm 3.6$	$49.7 \pm 3.8^{+6.8}_{-8.2}$				$33.5^{+3.9}_{-3.8}$
266	$K_x^{*+}\pi^- (2)$	5.1 ± 1.6		$5.1^{+1.5+0.6}_{-1.5-0.7}$				$5.1^{+1.6}_{-1.7}$
267	$K^*(1410)^+\pi^- \dagger$	< 3.8		< 3.8				< 3.8
268	$f_0(980)K^0 \dagger$	7.0 ± 0.9	$6.9 \pm 0.8 \pm 0.6$	$7.6 \pm 1.7^{+0.9}_{-1.3}$				7.0 ± 0.9
269	$f_2(1270)^0 K^0$	$2.7^{+1.3}_{-1.2}$	$2.7^{+1.0}_{-0.8} \pm 0.9$	$< 2.5 \dagger$				$2.7^{+1.3}_{-1.2}$
270	$f_x(1300)^0 K^0$	1.8 ± 0.7	$1.81^{+0.55}_{-0.45} \pm 0.48$					$1.81^{+0.73}_{-0.66}$

$\dagger$ Product BF - daughter BF taken to be 100%.

$\ddagger$ Relative BF converted to absolute BF.

⁽¹⁾ $0.755 < M(K\pi) < 1.250 \text{ GeV}/c^2$.

⁽²⁾ K_x^{*0} stands for the possible candidates for $K^*(1410)$, $K_0^*(1430)$, $K_2^*(1430)$.

⁽³⁾ Average of BABAR results from $B^0 \rightarrow K^+\pi^-\pi^0$ and $B^0 \rightarrow K^0\pi^+\pi^-$.

$\diamond$ Obtained from a fit to the ratios of BF's measured by LHCb (Ref. [725]) and to the averages of the BF's in their numerators, as measured by other experiments (RPP 292 and 298).

Table 177: Branching fractions (BF) of charmless mesonic B^0 decays with kaons (part 2) in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	LHCb	New avg.
271	$K^{*0}\pi^0$	3.3 ± 0.6	$3.3 \pm 0.5 \pm 0.4$	< 3.5	< 3.6			3.3 ± 0.6
272	$K_2^*(1430)^+\pi^-$	< 6	< 16.2	< 6.3				< 6.3
273	$K^*(1680)^+\pi^-$	< 10	< 25	< 10.1				< 10.1
275	$\rho^0 K^+\pi^-$	2.8 ± 0.7		$2.8 \pm 0.5 \pm 0.5^{(2)}$				2.8 ± 0.7
276	$f_0(980)K^+\pi^-$	$1.4^{+0.5}_{-0.6}$		$1.4 \pm 0.4^{+0.3}_{-0.4}{}^{(2)}$				$1.4^{+0.5}_{-0.6}$
277	$K^+\pi^-\pi^+\pi^-$	< 2.1		< 2.1				< 2.1
278	$K^{*0}\pi^+\pi^-$	55 ± 5	$54.5 \pm 2.9 \pm 4.3$					54.5 ± 5.2
279	$K^{*0}\rho^0$	3.9 ± 1.3	$5.1 \pm 0.6^{+0.6}_{-0.8}$	$2.1^{+0.8+0.9}_{-0.7-0.5}$	< 34			3.9 ± 0.8
280	$f_0(980)K^{*0}\dagger$	$3.9^{+2.1}_{-1.8}$	$5.7 \pm 0.6 \pm 0.4$	$1.4^{+0.6+0.6}_{-0.5-0.4}$				3.9 ± 0.5
—	$f_0(980)K_2^*(1430)^0\dagger$	New	$8.6 \pm 1.7 \pm 1.0$					8.6 ± 2.0
281	$K_1(1270)^+\pi^-$	< 30	17^{+6}_{-25}					17^{+6}_{-25}
282	$K_1(1400)^+\pi^-$	< 27	16^{+8}_{-24}					16^{+8}_{-24}
283	$a_1^- K^+$	16 ± 4	$16.3 \pm 2.9 \pm 2.3$					16.3 ± 3.7
284	$K^{*+}\rho^-$	10.3 ± 0.26	$10.3 \pm 2.3 \pm 1.3$					10.3 ± 2.6
285	$K_0(1430)^+\rho^-$	28 ± 12	$28 \pm 10 \pm 6$					28 ± 11
287	$K_2^*(1430)^0\rho^0$	27 ± 6	$27 \pm 4 \pm 4$					27 ± 5
288	$K_0^*(1430)^0 f_0(980)$	2.7 ± 0.9	$2.7 \pm 0.7 \pm 0.6$					2.7 ± 0.9
289	$K_2^*(1430)^0 f_0(980)$	8.6 ± 2.0	$8.6 \pm 1.7 \pm 1.0$					8.6 ± 2.0
290	K^+K^-	0.13 ± 0.05	< 0.5	$0.10 \pm 0.08 \pm 0.04$	< 0.8	$0.23 \pm 0.10 \pm 0.10\dagger$	$0.12^{+0.08}_{-0.07} \pm 0.01\dagger$	$0.13^{+0.06}_{-0.05}$
291	$K^0\bar{K}^0$	1.21 ± 0.16	$1.08 \pm 0.28 \pm 0.11$	$1.26 \pm 0.19 \pm 0.05$	< 3.3			1.21 ± 0.16
292	$K^0 K^-\pi^+$	7.3 ± 1.1	$6.4 \pm 1.0 \pm 0.6$	< 18	< 21		$6.64 \pm 0.99 \diamond$	6.54 ± 0.75
293	$K^{*0}\bar{K}^0$	< 1.9	< 1.9					< 1.9
—	$K^{*+}K^\pm$	New					$< 0.4\dagger$	$< 0.4\dagger$
294	$K^+K^-\pi^0$	2.2 ± 0.6		$2.17 \pm 0.60 \pm 0.24$	< 19			2.17 ± 0.65
295	$K_S K_S \pi^0$	< 0.9	< 0.9					< 0.9
296	$K_S K_S \eta$	< 1.0	< 1.0					< 1.0
297	$K_S K_S \eta'$	< 2.0	< 2.0					< 2.0
298	$K^+K^-K^0$	26.3 ± 1.5	$26.5 \pm 0.9 \pm 0.8$	$28.3 \pm 3.3 \pm 4.0$			$19.1 \pm 1.9 \diamond$	24.5 ± 1.0
299	ϕK^0	7.3 ± 0.7	$7.1 \pm 0.6^{+0.4}_{-0.3}$	$9.0^{+2.2}_{-1.8} \pm 0.7$	$5.4^{+3.7}_{-2.7} \pm 0.7$			$7.3^{+0.7}_{-0.6}$
300	$f_0(980)K^0\dagger$	$7.0^{+3.5}_{-3.0}$	$7.0^{+2.6}_{-1.8} \pm 2.4$					$7.0^{+3.5}_{-3.0}$
301	$f_0(1500)K^0\dagger$	13^{+7}_{-5}	$13.3^{+5.8}_{-4.4} \pm 3.2$					$13.3^{+6.6}_{-5.4}$
302	$f_2'(1525)K^0$	$0.3^{+0.5}_{-0.4}$	$0.29^{+0.27}_{-0.18} \pm 0.36$					$0.29^{+0.45}_{-0.40}$
303	$f_0(1710)K^0\dagger$	4.4 ± 0.9	$4.4 \pm 0.7 \pm 0.5$					4.4 ± 0.9
304	$K^0 K^+ K^-$ (NR)	33 ± 10	$33 \pm 5 \pm 9$					33 ± 10
305	$K_S K_S K_S$	$6.2^{+1.2}_{-1.1}$	$6.19 \pm 0.48 \pm 0.19$	$4.2^{+1.6}_{-1.3} \pm 0.8$				6.04 ± 0.50
306	$f_0(980)K_S\dagger$	2.7 ± 1.8	$2.7^{+1.3}_{-1.2} \pm 1.3\dagger$					2.7 ± 1.8
307	$f_0(1710)K_S\dagger$	$0.50^{+0.050}_{-0.026}$	$0.50^{+0.46}_{-0.24} \pm 0.11\dagger$					$0.50^{+0.47}_{-0.26}$
308	$f_0(2010)K_S\dagger$	0.5 ± 0.6	$0.54^{+1.1}_{-0.20} \pm 0.52\dagger$					0.54 ± 0.56
309	$K_S K_S K_S$ (NR)	13.3 ± 3.1	$13.3^{+2.3}_{-2.3} \pm 2.2$					$13.3^{+3.1}_{-3.2}$
310	$K_S K_S K_L$	< 16	$< 16^{(2)}$					$< 16^{(2)}$
311	$K^{*0}K^+K^-$	27.5 ± 2.6	$27.5 \pm 1.3 \pm 2.2$					27.5 ± 2.6
312	ϕK^{*0}	10.0 ± 0.5	$9.7 \pm 0.5 \pm 0.6$	$10.4 \pm 0.5 \pm 0.6$	$11.5^{+4.5+1.8}_{-3.7-1.7}$			$10.1^{+0.6}_{-0.5}$
313	$K^+\pi^-\pi^+K^-$	< 72		$< 72^{(3)}$				$< 72^{(3)}$
314	$K^{*0}\pi^+K^-$	4.5 ± 1.3	$4.6 \pm 1.1 \pm 0.8$	$< 13.9^{(3)}$				4.6 ± 1.4
315	$K^{*0}\bar{K}^{*0}$	0.8 ± 0.5	$1.28^{+0.35}_{-0.30} \pm 0.11$	$0.26^{+0.33+0.10}_{-0.29-0.08}$	< 22			0.81 ± 0.23
316	$K^+\pi^-K^+\pi^-(NR)$	< 6.0		$< 6.0^{(3)}$				$< 6.0^{(3)}$
317	$K^{*0}K^+\pi^-$	< 2.2	< 2.2	$< 7.6^{(3)}$				< 2.2
318	$K^{*0}K^{*0}$	< 0.2	< 0.41	< 0.2				< 0.2
319	$K^{*+}K^{*-}$	< 2.0	< 2.0		< 37			< 2.0
320	$K_1^*(1400)^0\phi$	< 5000	$< 5000^{(5)}$		< 141			$< 5000^{(5)}$
321	$(K\pi)_0^0\phi$	4.3 ± 0.4	$4.3 \pm 0.4 \pm 0.4$	$4.3 \pm 0.4 \pm 0.4$				4.3 ± 0.4
322	$(K\pi)_0^0\phi^{(4)}$	< 1.7	< 1.7					< 1.7
323	$K_0^*(1430)^0\pi^+K^-$	< 31.8		$< 31.8^{(3)}$				$< 31.8^{(3)}$
324	$K_0^*(1430)^0\bar{K}^0$	< 3.3		< 3.3				< 3.3
325	$K_0^*(1430)^0\bar{K}_0^*(1430)^0$	< 8.4		< 8.4				< 8.4
326	$\phi K_0^*(1430)^0$	3.9 ± 0.8	$3.9 \pm 0.5 \pm 0.6$	$4.3 \pm 0.4 \pm 0.4$				4.2 ± 0.5
327	$K_0^*(1430)^0 K^{*0}$	< 1.7		< 1.7				< 1.7
328	$K_0^*(1430)^0 K_0^*(1430)^0$	< 4.7		< 4.7				< 4.7
329	$\phi K^*(1680)^0$	< 3.5	< 3.5					< 3.5
330	$\phi K_2^*(1780)^0$	< 2.7	< 2.7					< 2.7
331	$\phi K_4^*(2045)^0$	< 15.3	< 15.3					< 15.3
332	$\rho^0 K_2^*(1430)^0$	< 1100	$< 1100^{(5)}$					$< 1100^{(5)}$
333	$\phi K_2^*(1430)^0$	6.8 ± 0.9	$7.5 \pm 0.9 \pm 0.5$	$5.5^{+0.9}_{-0.7} \pm 1.0$				6.8 ± 0.8
334	$\phi\phi K^0 \S$	4.5 ± 0.9	$4.5 \pm 0.8 \pm 0.3$					4.5 ± 0.9
335	$\eta'\eta'K^0$	< 31	< 31					< 31

$\dagger$ Product BF - daughter BF taken to be 100%; $\S$ $M_{\phi\phi} < 2.85 \text{ GeV}/c^2$; $\ddagger$ Relative BF converted to absolute BF; $^{(1)}$ $0.55 < M(\pi\pi) < 1.42 \text{ GeV}/c^2$; $^{(2)}$ $0.75 < M(K\pi) < 1.20 \text{ GeV}/c^2$; $^{(3)}$ $0.70 < M(K\pi) < 1.70 \text{ GeV}/c^2$; $^{(4)}$ $1.60 < M(K\pi) < 2.15 \text{ GeV}/c^2$; $^{(5)}$ Result from ARGUS; $\diamond$ Obtained from a fit to the ratios of BF's measured by LHCb (Ref. [725]) and to the averages of the BF's therein, as measured by other experiments (excluding the present line).

Table 178: Branching fractions (BF) of charmless mesonic B^0 decays without kaons in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	LHCb	New avg.
356	$\pi^+\pi^-$	5.15 ± 0.19	$5.5 \pm 0.4 \pm 0.3$	$5.04 \pm 0.21 \pm 0.18$	$4.5^{+1.4+0.5}_{-1.2-0.4}$	$5.02 \pm 0.33 \pm 0.35$ ‡	$5.08 \pm 0.17 \pm 0.37$ ‡	5.10 ± 0.19
357	$\pi^0\pi^0$	1.91 ± 0.22	$1.83 \pm 0.21 \pm 0.13$	$0.90 \pm 0.12 \pm 0.10$	< 4.4			1.17 ± 0.13
358	$\eta\pi^0$	< 1.5	< 1.5	< 2.5	< 2.9			< 1.5
359	$\eta\eta$	< 1.0	< 1.0	< 2.0	< 18			< 1.0
360	$\eta'\pi^0$	1.2 ± 0.6	$0.9 \pm 0.4 \pm 0.1$	$2.8 \pm 1.0 \pm 0.3$	$0.0^{+1.8}_{-0.0}$			1.2 ± 0.4
361	$\eta'\eta'$	< 1.7	< 1.7	< 6.5	< 47			< 1.7
362	$\eta'\eta$	< 1.2	< 1.2	< 4.5	< 27			< 1.2
363	$\eta'\rho^0$	< 1.3	< 2.8	< 1.3	< 12			< 1.3
364	$f_0(980)\eta'$ †	< 0.9	< 0.9					< 0.9
365	$\eta\rho^0$	< 1.5	< 1.5	< 1.9	< 10			< 1.5
366	$f_0(980)\eta$ †	< 0.4	< 0.4					< 0.4
367	$\omega\eta$	$0.94^{+0.40}_{-0.31}$	$0.94^{+0.35}_{-0.30} \pm 0.09$		< 12			$0.94^{+0.36}_{-0.31}$
368	$\omega\eta'$	$1.0^{+0.5}_{-0.4}$	$1.01^{+0.46}_{-0.38} \pm 0.09$	< 2.2	< 60			$1.01^{+0.47}_{-0.39}$
369	$\omega\rho^0$	< 1.6	< 1.6		< 11			< 1.6
370	$f_0(980)\omega$ †	< 1.5	< 1.5					< 1.5
371	$\omega\omega$	1.2 ± 0.4	$1.2 \pm 0.3^{+0.3}_{-0.2}$		< 19			1.2 ± 0.4
372	$\phi\pi^0$	< 0.15	< 0.28	< 0.15	< 5			< 0.15
373	$\phi\eta$	< 0.5	< 0.5		< 9			< 0.5
374	$\phi\eta'$	< 0.5	< 1.1	< 0.5	< 31			< 0.5
375	$\phi\rho^0$	< 0.33	< 0.33		< 13			< 0.33
376	$f_0(980)\phi$ †	< 0.38	< 0.38					< 0.38
377	$\omega\phi$	< 0.7	< 0.7		< 21			< 0.7
378	$\phi\phi$	< 0.2	< 0.2		< 12			< 0.2
379	$a_0^+(980)\pi^\pm$ †	< 3.1	< 3.1					< 3.1
379	$a_0^+(1450)\pi^\pm$	< 2.3	< 2.3					< 2.3
380	$a_0^+(1450)\pi^\pm$ †	< 2.3	< 2.3					< 2.3
382	$\rho^0\pi^0$	2.0 ± 0.5	$1.4 \pm 0.6 \pm 0.3$	$3.0 \pm 0.5 \pm 0.7$	$1.6^{+2.0}_{-1.4} \pm 0.8$			2.0 ± 0.5
383	$\rho^+\pi^\pm$	23.0 ± 2.3	$22.6 \pm 1.8 \pm 2.2$	$22.6 \pm 1.1 \pm 4.4$	$27.6^{+8.4}_{-7.4} \pm 4.2$			23.0 ± 2.3
384	$\pi^+\pi^-\pi^+\pi^-$	< 19.3	< 23.1	< 11.2				< 11.2
385	$\rho^0\pi^+\pi^-(NR)$	< 8.8	< 8.8	< 12				< 8.8
386	$\rho^0\rho^0$	0.73 ± 0.28	$0.92 \pm 0.32 \pm 0.14$	$1.02 \pm 0.30 \pm 0.15$	< 18			0.97 ± 0.24
387	$f_0(980)\pi^+\pi^-(NR)$ †	< 3.8		< 3.0				< 3.0
388	$f_0(980)\rho^0$ †	< 0.3	< 0.40	$0.78 \pm 0.22 \pm 0.11$				0.78 ± 0.25
389	$f_0(980)f_0(980)$ †	< 0.1	< 0.19	< 0.2				< 0.19
391	$a_1^+\pi^\pm$	26 ± 5	$33.2 \pm 3.8 \pm 3.0$	$22.2 \pm 2.0 \pm 2.8$				25.9 ± 2.8
392	$a_2^+\pi^\pm$	< 6.3		< 6.3				< 6.3
393	$\pi^+\pi^-\pi^0\pi^0$	< 3100	< 3100					< 3100
394	$\rho^+\rho^-$	24.2 ± 3.1	$25.5 \pm 2.1^{+3.6}_{-3.9}$	$22.8 \pm 3.8^{+2.3}_{-2.6}$				$24.2^{+3.1}_{-3.2}$
395	$a_1(1260)^0\pi^0$	< 1100	< 1100					< 1100
396	$\omega\pi^0$	< 0.5	< 0.5	< 2.0	< 5.5			< 0.5
397	$\pi^+\pi^+\pi^-\pi^-\pi^0$	< 9000	< 9000					< 9000
398	$a_1^+\rho^\mp$	< 61	< 61					< 61
399	$a_1^+\rho^0$	< 2400	< 2400					< 2400
400	$b_1^+\pi^\pm$ †	10.9 ± 1.5	$10.9 \pm 1.2 \pm 0.9$					10.9 ± 1.5
401	$b_1^0\pi^0$ †	< 1.9	< 1.9					< 1.9
402	$b_1^+\rho^\mp$ †	< 1.4	< 1.4					< 1.4
403	$b_1^0\rho^0$ †	< 3.4	< 3.4					< 3.4
404	$\pi^+\pi^+\pi^+\pi^-\pi^-\pi^-$	< 3000	< 3000					< 3000
405	$a_1^+a_1^+$	11.8 ± 2.6	11.8 ± 2.6					11.8 ± 2.6
406	$\pi^+\pi^+\pi^+\pi^-\pi^-\pi^-$	< 11000	< 11000					< 11000

† Product BF - daughter BF taken to be 100%.

‡ Relative BF converted to absolute BF.

Table 179: Relative branching fractions (BF) of charmless mesonic B^0 decays. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	CDF	LHCb	New avg.
273	$\mathcal{B}(B^0 \rightarrow K^+ K^-)/\mathcal{B}(B^0 \rightarrow K^+ \pi^-)$		$0.012 \pm 0.005 \pm 0.005$		0.012 ± 0.007
356	$\mathcal{B}(B^0 \rightarrow \pi^+ \pi^-)/\mathcal{B}(B^0 \rightarrow K^+ \pi^-)$	0.261 ± 0.010	$0.259 \pm 0.017 \pm 0.016$	$0.262 \pm 0.009 \pm 0.017$	0.261 ± 0.015
—	$\mathcal{B}(\Lambda B^0 \rightarrow K^{*\mp} K^\pm)/\mathcal{B}(\Lambda B^0 \rightarrow K^{*+} \pi^-)$	New		< 0.05	< 0.05

7.2 Baryonic decays of B^0 and B^+ mesons

This section provides branching fractions of charmless baryonic decays of B^0 and B^+ mesons in Tables 180 and 181, respectively. Relative branching fractions are given in Table 182.

Table 180: Branching fractions (BF) of charmless baryonic B^+ decays in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	LHCb	New Avg.
417	$p\bar{p}\pi^+$	1.62 ± 0.20	$1.69 \pm 0.29 \pm 0.26$ †	$1.60^{+0.22}_{-0.19} \pm 0.12$		$1.62^{+0.21}_{-0.20}$
417	$p\bar{p}\pi^+ \S$				$1.07 \pm 0.11 \pm 0.11$ ¶	1.07 ± 0.16
420	$p\bar{p}K^+$	5.9 ± 0.5	$6.7 \pm 0.5 \pm 0.4$ †	$5.54^{+0.27}_{-0.25} \pm 0.36$	$4.46 \pm 0.21 \pm 0.27$ ¶ ◊	5.14 ± 0.25
421	$\Theta^{++}\bar{p}^{(1)}$	< 0.091	< 0.09	< 0.091		< 0.09
422	$f_J(2221)K^+^{(2)}$	< 0.41		< 0.41		< 0.41
423	$p\bar{\Lambda}(1520)$	< 1.5	< 1.5		$0.315 \pm 0.048 \pm 0.027$ ¶	0.315 ± 0.055
425	$p\bar{p}K^{*+}$	$3.6^{+0.8}_{-0.7}$	$5.3 \pm 1.5 \pm 1.3$ †	$3.38^{+0.73}_{-0.60} \pm 0.39$ ‡		$3.64^{+0.79}_{-0.70}$
426	$f_J(2221)K^{*+ (2)}$	< 0.77	< 0.77			< 0.77
427	$p\bar{\Lambda}$	< 0.32		< 0.32		< 0.32
429	$p\bar{\Lambda}\pi^0$	$3.00^{+0.7}_{-0.6}$		$3.00^{+0.61}_{-0.53} \pm 0.33$		$3.00^{+0.69}_{-0.62}$
430	$p\bar{\Sigma}(1385)^0$	< 0.47		< 0.47		< 0.47
431	$\Delta^+\bar{\Lambda}$	< 0.82		< 0.82		< 0.82
433	$p\bar{\Lambda}\pi^+\pi^-$ (NR)	5.9 ± 1.1		$5.92^{+0.88}_{-0.84} \pm 0.69$		$5.92^{+1.12}_{-1.09}$
434	$p\bar{\Lambda}\rho^0$	4.8 ± 0.9		$4.78^{+0.67}_{-0.64} \pm 0.60$		$4.78^{+0.90}_{-0.88}$
435	$p\bar{\Lambda}f_2(1270)$	2.0 ± 0.8		$2.03^{+0.77}_{-0.72} \pm 0.27$		$2.03^{+0.82}_{-0.77}$
436	$\Lambda\bar{\Lambda}\pi^+$	< 0.94		< 0.94 §		< 0.94 §
437	$\Lambda\bar{\Lambda}K^+$	3.4 ± 0.6		$3.38^{+0.41}_{-0.36} \pm 0.41$ ‡		$3.38^{+0.58}_{-0.55}$
438	$\Lambda\bar{\Lambda}K^{*+}$	$2.2^{+1.2}_{-0.9}$		$2.19^{+1.13}_{-0.88} \pm 0.33$ §		$2.19^{+1.18}_{-0.94}$
439	$\bar{\Delta}^0 p$	< 1.38		< 1.38 §		< 1.38 §
440	$\Delta^{++}\bar{p}$	< 0.14		< 0.14 §		< 0.14 §

§ Di-baryon mass is less than $2.85\text{GeV}/c^2$.

† Charmonium decays to $p\bar{p}$ have been statistically subtracted.

¶ Relative BF converted to absolute BF.

◊ Includes contribution where $p\bar{p}$ is produced in charmonia decays.

⁽¹⁾ $\Theta(1540)^{++} \rightarrow K^+ p$ (pentaquark candidate).

⁽²⁾ Product BF — daughter BF taken to be 100%.

‡ The charmonium mass region has been vetoed.

Table 181: Branching fractions (BF) of charmless baryonic B^0 decays in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	LHCb	New Avg.
407	$p\bar{p}$	$0.015^{+0.007}_{-0.005}$	< 0.27	< 0.11	$0.0147^{+0.0062+0.0035}_{-0.0051-0.0014}$	$0.0150^{+0.0070}_{-0.0050}$
409	$p\bar{p}K^0$	2.66 ± 0.32	$3.0 \pm 0.5 \pm 0.3 \uparrow$	$2.51^{+0.35}_{-0.29} \pm 0.21 \ddagger$		$2.66^{+0.34}_{-0.32}$
410	$\Theta^+\bar{p}^{(1)}$	< 0.05	< 0.05	< 0.23		< 0.05
411	$f_J(2221)K^0^{(2)}$	< 0.45	< 0.45			< 0.45
412	$p\bar{p}K^{*0}$	$1.24^{+0.28}_{-0.25}$	$1.47 \pm 0.45 \pm 0.40 \uparrow$	$1.18^{+0.29}_{-0.25} \pm 0.11 \ddagger$		$1.24^{+0.28}_{-0.25}$
413	$f_J(2221)K^{*0(2)}$	< 0.15	< 0.15			< 0.15
414	$p\bar{\Lambda}\pi^-$	3.14 ± 0.29	$3.07 \pm 0.31 \pm 0.23$	$3.23^{+0.33}_{-0.29} \pm 0.29$		$3.14^{+0.29}_{-0.28}$
415	$p\bar{\Sigma}(1385)^-$	< 0.26		< 0.26		< 0.26
416	$\Delta^0\bar{\Lambda}$	< 0.93		< 0.93		< 0.93
417	$p\bar{\Lambda}K^-$	< 0.82		< 0.82		< 0.82
418	$p\bar{\Sigma}^0\pi^-$	< 3.8		< 3.8		< 3.8
419	$\bar{\Lambda}\Lambda$	< 0.32		< 0.32		< 0.32
420	$\bar{\Lambda}\Lambda K^0$	$4.8^{+1.0}_{-0.9}$		$4.76^{+0.84}_{-0.68} \pm 0.61 \ddagger$		$4.76^{+1.04}_{-0.91}$
421	$\Lambda\bar{\Lambda}K^{*0}$	$2.5^{+0.9}_{-0.8}$		$2.46^{+0.87}_{-0.72} \pm 0.34 \ddagger$		$2.46^{+0.93}_{-0.80}$

$\uparrow$ Charmonium decays to $p\bar{p}$ have been statistically subtracted.

$\ddagger$ The charmonium mass region has been vetoed.

⁽¹⁾ $\Theta(1540)^+ \rightarrow pK^0$ (pentaquark candidate).

⁽²⁾ Product BF — daughter BF taken to be 100%.

Table 182: Relative branching fractions (BF) of charmless baryonic B decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	LHCb	New Avg.
417	$\mathcal{B}(B^+ \rightarrow p\bar{p}\pi^+, m_{p\bar{p}} < 2.85\text{GeV}/c^2)/\mathcal{B}(B^+ \rightarrow J/\psi(\rightarrow p\bar{p})\pi^+)$		$12.0 \pm 1.2 \pm 0.3$	12.0 ± 1.2
420	$\mathcal{B}(B^+ \rightarrow p\bar{p}K^+)/\mathcal{B}(B^+ \rightarrow J/\psi(\rightarrow p\bar{p})K^+)$		$4.91 \pm 0.19 \pm 0.14 \uparrow$	4.91 ± 0.24
420	$\mathcal{B}(B^+ \rightarrow p\bar{p}K^+)/\mathcal{B}(B^+ \rightarrow J/\psi K^+)$	$0.0104 \pm 0.0005 \pm 0.0001$	$0.0104 \pm 0.0005 \pm 0.0001 \uparrow \S$	0.0104 ± 0.0005
423	$\mathcal{B}(B^+ \rightarrow \bar{\Lambda}(1520)(\rightarrow K^+\bar{p})p)/\mathcal{B}(B^+ \rightarrow J/\psi(\rightarrow p\bar{p})\pi^+)$		$0.033 \pm 0.005 \pm 0.007$	0.033 ± 0.009

$\uparrow$ Includes contribution where $p\bar{p}$ is produced in charmonia decays.

$\S$ Original experimental relative BF multiplied by the best values (PDG2014) of certain reference BF. The first error is experimental, the second is from reference BF.

7.3 Decays of b baryons

A compilation of branching fractions of Λ_b^0 baryon decays is given in Table 183. Table 184 provides the partial branching fractions of $\Lambda_b^0 \rightarrow \Lambda\mu^+\mu^-$ decays.

Table 183: Branching fractions (BF) of charmless Λ_b^0 decays in units of $\times 10^{-6}$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2012 Avg.	CDF	LHCb	New Avg.
19	$p\pi^-$	$3.5 \pm 0.8 \pm 0.6$	$3.5 \pm 0.8 \pm 0.6$		3.5 ± 1.0
20	pK^-	$5.5 \pm 1.0 \pm 1.0$	$5.5 \pm 1.0 \pm 1.0$		5.5 ± 1.4
21	$\Lambda\mu^+\mu^-$	$17.2 \pm 4.2 \pm 5.5$	$17.2 \pm 4.2 \pm 5.5$	$0.96 \pm 0.16 \pm 0.25$	0.99 ± 0.30

Table 184: Partial branching fractions (BF) of $\Lambda_b^0 \rightarrow \mu^+\mu^-$ decays in intervals of $q^2 = m_{\mu\mu}^2$ in units of $\times 10^{-6}$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	q^2 [(GeV/ c^2) ^{2}] †	PDG2012 Avg.	CDF	LHCb	New Avg.
21	$\Lambda\mu^+\mu^-$	< 2.0	$0.15 \pm 2.01 \pm 0.05$	$0.15 \pm 2.01 \pm 0.05$	$0.56 \pm 0.76 \pm 0.80$	0.41 ± 0.87
	$\Lambda\mu^+\mu^-$	$[2.0, 4.3]$	$1.8 \pm 1.7 \pm 0.6$	$1.8 \pm 1.7 \pm 0.6$	$0.71 \pm 0.60 \pm 0.10$	0.91 ± 0.55
	$\Lambda\mu^+\mu^-$	$[4.3, 8.68]$	$-0.2 \pm 1.6 \pm 0.1$	$-0.2 \pm 1.6 \pm 0.1$	$0.66 \pm 0.72 \pm 0.16$	0.40 ± 0.62
	$\Lambda\mu^+\mu^-$	$[10.09, 12.86]$	$3.0 \pm 1.5 \pm 1.0$	$3.0 \pm 1.5 \pm 1.0$	$1.55 \pm 0.58 \pm 0.55$	1.96 ± 0.68
	$\Lambda\mu^+\mu^-$	$[14.18, 16.00]$	$1.0 \pm 0.7 \pm 0.3$	$1.0 \pm 0.7 \pm 0.3$	$1.44 \pm 0.44 \pm 0.42$	1.19 ± 0.40
	$\Lambda\mu^+\mu^-$	> 16.00	$7.0 \pm 1.9 \pm 2.2$	$7.0 \pm 1.9 \pm 2.2$	$4.7 \pm 0.8 \pm 1.2$	5.5 ± 1.2

† see the original papers for the exact q^2 selection.

7.4 Decays of B_s^0 mesons

Tables 185 and 186 detail branching fractions and relative branching fractions of B_s^0 meson decays, respectively. Tables 187 to 191 give different observables for $B_s^0 \rightarrow \phi \mu^+ \mu^-$ decays in bins of the dimuon invariant mass.

Table 185: Branching fractions (BF) of charmless B_s^0 decays in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	Belle	CDF	D0	LHCb	CMS	ATLAS	New Avg.
45	$\pi^+ \pi^-$	0.76 ± 0.19	< 12	$0.60 \pm 0.17 \pm 0.04$ ‡		$0.98^{+0.23}_{-0.19} \pm 0.07$ ‡			0.76 ± 0.13
51	$\phi \phi$	19.1 ± 3.1		$19.1 \pm 2.6 \pm 1.6$ ‡					19.1 ± 3.1
52	$\pi^+ K^-$	5.5 ± 0.6	< 26	$5.3 \pm 0.9 \pm 0.3$ ‡		$5.6 \pm 0.6 \pm 0.3$ ‡			5.5 ± 0.5
53	$K^+ K^-$	24.9 ± 1.7	$38^{+10}_{-9} \pm 7$	$25.9 \pm 2.2 \pm 1.7$ ‡		$23.7 \pm 1.6 \pm 1.5$ ‡			24.8 ± 1.7
54	$K^0 \bar{K}^0$	< 66	< 66						< 66
55	$K^0 \pi^+ \pi^-$	19 ± 5				$19 \pm 5 \pm 2$ ‡			19 ± 5
56	$K^0 K^+ \pi^-$ ¶	97 ± 17				$97 \pm 12 \pm 12$ ‡			97 ± 16
57	$K^0 K^+ K^-$	< 4				< 4 ‡			< 4 ‡
—	$K^{*+} K^{\mp}$	New				$12.7 \pm 1.9 \pm 1.9$ ‡			12.7 ± 2.7
—	$K^{*-} \pi^+$	New				$3.3 \pm 1.1 \pm 0.5$ ‡			3.3 ± 1.2
59	$K^{*0} \bar{K}^{*0}$	$28.1 \pm 4.6 \pm 5.6$				$28.1 \pm 4.6 \pm 5.6$ ‡			28.1 ± 7.2
60	$\phi \bar{K}^0$	1.13 ± 0.3				$1.13 \pm 0.29 \pm 0.06$ ‡			1.13 ± 0.30
61	$p \bar{p}$	$0.028^{+0.022}_{-0.017}$				$0.0284^{+0.0203+0.0085}_{-0.0168-0.0018}$ ‡			$0.0280^{+0.0220}_{-0.0170}$
63	$\gamma \gamma$	< 8.7	< 8.7						< 8.7
64	$\phi \gamma$	36 ± 4	57^{+18+12}_{-15-11}			$35.1 \pm 3.5 \pm 1.2$ ‡			35.9 ± 3.6
65	$\mu^+ \mu^-$	0.0031 ± 0.0007		$0.013^{+0.009}_{-0.007}$ ‡	< 0.012 ‡	$0.0029^{+0.0011+0.0003}_{-0.0010-0.0001}$ ‡	$0.0030^{+0.0010}_{-0.0009}$ ‡	< 0.019 ‡	0.0031 ± 0.0007
65	$\mu^+ \mu^-$	CMS-LHCb comb.				$0.0028^{+0.0007}_{-0.0006}$ (blue)	$0.0028^{+0.0007}_{-0.0006}$ (blue)		
66	$e^+ e^-$	< 0.28		< 0.28					< 0.28
67	$e^+ \mu^{\mp}$	< 0.011		< 0.20		< 0.011 ‡			< 0.011 ‡
68	$\mu^+ \mu^- \mu^+ \mu^-$	< 0.012				< 0.012			< 0.012
70	$\phi \mu^+ \mu^-$	0.76 ± 0.15		$1.17 \pm 0.18 \pm 0.37$ ‡	< 3.2 ‡	$0.707^{+0.064}_{-0.059} \pm 0.073$ ‡			$0.731^{+0.095}_{-0.092}$

‡ Original experimental relative BF multiplied by the best values (PDG2014) of reference BF. The first error is experimental, the second is from reference BF.

† Relative BF converted to absolute BF.

¶ Sum of charge conjugate states.

Table 186: Relative branching fractions (BF) of charmless B_s^0 decays. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	CDF	LHCb	New Avg.
45	$f_s \mathcal{B}(B_s^0 \rightarrow \pi^+ \pi^-) / f_d \mathcal{B}(B^0 \rightarrow K^+ \pi^-)$		$0.008 \pm 0.002 \pm 0.001$		0.008 ± 0.002
45	$f_s \mathcal{B}(B_s^0 \rightarrow \pi^+ \pi^-) / f_d \mathcal{B}(B^0 \rightarrow \pi^+ \pi^-)$			$0.050^{+0.011}_{-0.009} \pm 0.004$	$0.050^{+0.012}_{-0.010}$
51	$\mathcal{B}(B_s^0 \rightarrow \phi \phi) / \mathcal{B}(B_s^0 \rightarrow J/\psi \phi)$		$0.0178 \pm 0.0014 \pm 0.0020$		0.0180 ± 0.0020
52	$f_s \mathcal{B}(B_s^0 \rightarrow K^+ \pi^-) / f_d \mathcal{B}(B_d^0 \rightarrow K^+ \pi^-)$		$0.071 \pm 0.010 \pm 0.007$	$0.074 \pm 0.006 \pm 0.006$	0.073 ± 0.007
53	$f_s \mathcal{B}(B_s^0 \rightarrow K^+ K^-) / f_d \mathcal{B}(B_d^0 \rightarrow K^+ \pi^-)$		$0.347 \pm 0.020 \pm 0.021$	$0.316 \pm 0.009 \pm 0.019$	0.327 ± 0.017
55	$f_s \mathcal{B}(B_s^0 \rightarrow K^0 \pi^+ \pi^-) / f_d \mathcal{B}(B^0 \rightarrow K^0 \pi^+ \pi^-)$			$0.29 \pm 0.06 \pm 0.04$	0.29 ± 0.07
56	$f_s \mathcal{B}(B_s^0 \rightarrow K^0 K^- \pi^+) / f_d \mathcal{B}(B^0 \rightarrow K^0 K^- \pi^+) \P$			$1.48 \pm 0.12 \pm 0.14$	1.48 ± 0.18
57	$f_s \mathcal{B}(B_s^0 \rightarrow K^0 K^+ K^-) / f_d \mathcal{B}(B^0 \rightarrow K^0 K^+ K^-)$			< 0.068	< 0.068
—	$\mathcal{B}(\emptyset B_s^0 \rightarrow K^{*-} K^+) / \mathcal{B}(\emptyset B^0 \rightarrow K^{*+} \pi^-)$	New		$1.49 \pm 0.22 \pm 0.18$	1.49 ± 0.28
—	$\mathcal{B}(\emptyset B_s^0 \rightarrow K^{*-} \pi^+) / \mathcal{B}(\emptyset B^0 \rightarrow K^{*+} \pi^-)$	New		$0.39 \pm 0.13 \pm 0.05$	0.39 ± 0.14
60	$\mathcal{B}(\emptyset B_s^0 \rightarrow \phi \bar{K}^{*0}) / \mathcal{B}(\emptyset B^0 \rightarrow \phi K^{*0})$			$0.113 \pm 0.024 \pm 0.016$	0.113 ± 0.029
64	$\mathcal{B}(\emptyset B_s^0 \rightarrow \phi \gamma) / \mathcal{B}(\emptyset B^0 \rightarrow K^{*0} \gamma)$			$0.81 \pm 0.04 \pm 0.07$	0.81 ± 0.08
70	$\mathcal{B}(B_s^0 \rightarrow \phi \mu^+ \mu^-) / \mathcal{B}(B_s^0 \rightarrow J/\psi \phi) \times 10^3$	0.71 ± 0.13	$0.90 \pm 0.14 \pm 0.07$	$0.674^{+0.061}_{-0.056} \pm 0.016$	$0.704^{+0.060}_{-0.056}$

$\P$ Sum of charge conjugate states in the numerator.

Table 187: Partial branching fractions (BF) of $B_s^0 \rightarrow \phi \mu^+ \mu^-$ decays in units of $\times 10^{-7}$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 [(GeV/ c^2) ^{2}]	PDG2014 Avg.	CDF	LHCb	New Avg.
$\phi \mu^+ \mu^-$	< 2.0 †	0.93 ± 0.21	$3.16 \pm 0.92 \pm 1.00$	$0.90^{+0.21}_{-0.19} \pm 0.10$	$0.96^{+0.23}_{-0.22}$
	[2.0, 4.3]	$0.55^{+0.18}_{-0.16}$	$0.27 \pm 0.41 \pm 0.09$	$0.53^{+0.18}_{-0.16} \pm 0.06$	$0.49^{+0.17}_{-0.16}$
	[4.3, 8.68]	1.40 ± 0.26	$0.64 \pm 0.68 \pm 0.20$	$1.38^{+0.25}_{-0.23} \pm 0.15$	$1.28^{+0.26}_{-0.25}$
	[10.09, 12.86]	1.22 ± 0.25	$2.25 \pm 0.69 \pm 0.71$	$1.20^{+0.23}_{-0.21} \pm 0.14$	$1.27^{+0.26}_{-0.25}$
	[14.18, 16.00]	0.80 ± 0.20	$1.11 \pm 0.42 \pm 0.35$	$0.76^{+0.19}_{-0.17} \pm 0.09$	$0.80^{+0.20}_{-0.18}$
	> 16.00 †	1.08 ± 0.24	$2.31 \pm 0.59 \pm 0.73$	$1.06^{+0.23}_{-0.21} \pm 0.12$	$1.14^{+0.25}_{-0.24}$
	[1.00, 6.00]	1.15 ± 0.25	$1.03 \pm 0.70 \pm 0.33$	$1.14^{+0.25}_{-0.23} \pm 0.13$	$1.13^{+0.26}_{-0.25}$

† See the original papers for the exact q^2 interval, which very slightly differs between experiments.

Table 188: Longitudinal polarization fraction (F_L) of $B_s^0 \rightarrow \phi \mu^+ \mu^-$ decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 [(GeV/ c^2) ^{2}]	PDG2014 Avg.	LHCb	New Avg.
$\phi \mu^+ \mu^-$	0.1 – 2.0	$0.37^{+0.19}_{-0.17} \pm 0.07$	$0.37^{+0.19}_{-0.17} \pm 0.07$	$0.37^{+0.20}_{-0.18}$
	[2.0, 4.3]	$0.53^{+0.25}_{-0.23} \pm 0.10$	$0.53^{+0.25}_{-0.23} \pm 0.10$	$0.53^{+0.27}_{-0.25}$
	[4.3, 8.68]	$0.81^{+0.11}_{-0.13} \pm 0.05$	$0.81^{+0.11}_{-0.13} \pm 0.05$	$0.81^{+0.12}_{-0.14}$
	[10.09, 12.86]	$0.33^{+0.14}_{-0.12} \pm 0.06$	$0.33^{+0.14}_{-0.12} \pm 0.06$	$0.33^{+0.15}_{-0.13}$
	[14.18, 16.00]	$0.34^{+0.18}_{-0.17} \pm 0.07$	$0.34^{+0.18}_{-0.17} \pm 0.07$	$0.34^{+0.19}_{-0.18}$
	16.00 – 19.00	$0.16^{+0.17}_{-0.10} \pm 0.07$	$0.16^{+0.17}_{-0.10} \pm 0.07$	$0.16^{+0.18}_{-0.12}$
	[1.00, 6.00]	$0.56^{+0.17}_{-0.16} \pm 0.09$	$0.56^{+0.17}_{-0.16} \pm 0.09$	$0.56^{+0.19}_{-0.18}$

Table 189: The parameter S_3 from the angular analysis of $B_s^0 \rightarrow \phi\mu^+\mu^-$ decays (see reference for definition). Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 $[(\text{GeV}/c^2)^2]$ †	PDG2014 Avg.	LHCb	New Avg.
$\phi\mu^+\mu^-$	0.1 – 2.0		$-0.11^{+0.28}_{-0.25} \pm 0.05$	$-0.11^{+0.28}_{-0.26}$
	[2.0, 4.3]		$-0.97^{+0.53}_{-0.03} \pm 0.17$	$-0.97^{+0.56}_{-0.17}$
	[4.3, 8.68]		$0.25^{+0.21}_{-0.24} \pm 0.05$	$0.25^{+0.22}_{-0.24}$
	[10.09, 12.86]		$0.24^{+0.27}_{-0.25} \pm 0.06$	$0.24^{+0.28}_{-0.26}$
	[14.18, 16.00]		$-0.03^{+0.29}_{-0.31} \pm 0.06$	$-0.03^{+0.30}_{-0.32}$
	16.00 – 19.00		$0.19^{+0.30}_{-0.31} \pm 0.05$	$0.19^{+0.30}_{-0.31}$
	[1.00, 6.00]		$-0.21^{+0.24}_{-0.22} \pm 0.08$	$-0.21^{+0.25}_{-0.23}$

Table 190: The parameter A_6 from the angular analysis of $B_s^0 \rightarrow \phi\mu^+\mu^-$ decays (see reference for definition). Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 $[(\text{GeV}/c^2)^2]$	PDG2014 Avg.	LHCb	New Avg.
$\phi\mu^+\mu^-$	0.1 – 2.0		$0.04^{+0.27}_{-0.32} \pm 0.12$	$0.04^{+0.29}_{-0.34}$
	[2.0, 4.3]		$0.47^{+0.39}_{-0.42} \pm 0.14$	$0.47^{+0.41}_{-0.44}$
	[4.3, 8.68]		$-0.02^{+0.20}_{-0.21} \pm 0.10$	$-0.02^{+0.22}_{-0.23}$
	[10.09, 12.86]		$-0.06^{+0.20}_{-0.20} \pm 0.08$	-0.06 ± 0.21
	[14.18, 16.00]		$-0.06^{+0.30}_{-0.30} \pm 0.08$	-0.06 ± 0.31
	16.00 – 19.00		$0.26^{+0.22}_{-0.24} \pm 0.08$	$0.26^{+0.23}_{-0.25}$
	[1.00, 6.00]		$0.20^{+0.29}_{-0.27} \pm 0.07$	$0.20^{+0.30}_{-0.28}$

Table 191: The parameter A_9 from the angular analysis of $B_s^0 \rightarrow \phi\mu^+\mu^-$ decays (see reference for definition). Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 $[(\text{GeV}/c^2)^2]$	PDG2014 Avg.	LHCb	New Avg.
$\phi\mu^+\mu^-$	0.1 – 2.0		$-0.16^{+0.30}_{-0.27} \pm 0.09$	$-0.16^{+0.31}_{-0.28}$
	[2.0, 4.3]		$-0.40^{+0.52}_{-0.35} \pm 0.11$	$-0.40^{+0.53}_{-0.37}$
	[4.3, 8.68]		$-0.13^{+0.27}_{-0.26} \pm 0.10$	$-0.13^{+0.29}_{-0.28}$
	[10.09, 12.86]		$0.29^{+0.25}_{-0.26} \pm 0.10$	$0.29^{+0.27}_{-0.28}$
	[14.18, 16.00]		$0.24^{+0.36}_{-0.35} \pm 0.12$	$0.24^{+0.38}_{-0.37}$
	16.00 – 19.00		$0.27^{+0.31}_{-0.28} \pm 0.11$	$0.27^{+0.33}_{-0.30}$
	[1.00, 6.00]		$-0.30^{+0.30}_{-0.29} \pm 0.11$	$-0.30^{+0.32}_{-0.31}$

7.5 Radiative and leptonic decays of B^0 and B^+ mesons

This section gives different observables for leptonic and radiative B^0 and B^+ meson decays, including processes in which the photon yields a pair of charged or neutral leptons. Tables 192, 193 and 194 provide compilations of branching fractions of B^+ , B^0 , and $B^\pm/B^0$ admixture, respectively. Table 195 contains branching fractions of leptonic and radiative-leptonic B^+ and B^0 decays. It is followed by Tabs 196 and 197, which give relative branching fractions of B^+ decays and a compilations of inclusive decays, respectively. The next two tables detail isospin asymmetries: Table 198 contains overall measurements, and 199 measurements in bins of dimuon invariant mass, q^2 , bins for $B \rightarrow K^{(*)}\ell^+\ell^-$ decays. Tables 200 and 201 detail measurements of the forward-backward asymmetry and the fraction of longitudinal polarization, respectively, in $B \rightarrow K^{(*)}\ell^+\ell^-$ decays in bins of q^2 . Finally, Table 202 quotes the LHCb measurement of photon polarization via the Up-Down asymmetry in $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ decays.

Table 192: Branching fractions (BF) of charmless semileptonic and radiative B^+ decays in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABar	Belle	CLEO	CDF	LHCb	New Avg.
363	$K^{*+}\gamma$	42.1 ± 1.8	$42.2 \pm 1.4 \pm 1.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$			42.1 ± 1.8
364	$K_1^+(1270)\gamma$	43 ± 13	$44.0^{+6.0+5.8}_{-4.0-5.9} \dagger$	$43 \pm 9 \pm 9$				$43.7^{+7.0}_{-6.2}$
365	$K^+\eta\gamma$	7.9 ± 0.9	$7.7 \pm 1.0 \pm 0.4$	$8.4^{+1.5}_{-1.2} \pm 0.9$				7.9 ± 0.9
366	$K^+\eta'\gamma$	$2.9^{+1.0}_{-0.9}$	$1.9^{+1.5}_{-1.2} \pm 0.1$	$3.6 \pm 1.2 \pm 0.4$				$2.9^{+1.0}_{-0.9}$
367	$K^+\phi\gamma$	2.7 ± 0.4	$3.5 \pm 0.6 \pm 0.4$	$2.48 \pm 0.30 \pm 0.24$				2.71 ± 0.34
368	$K^+\pi^-\pi^+\gamma$	27.6 ± 2.2	$27.2 \pm 1.0^{+1.1}_{-1.3} \dagger$	$25.0 \pm 1.8 \pm 2.2 \ddagger$				26.7 ± 1.4
369	$K^{*0}\pi^+\gamma \S$	20^{+7}_{-6}	$26.0^{+1.4}_{-1.3} \pm 1.8 \dagger$	$20^{+7}_{-6} \pm 2$				$25.5^{+2.2}_{-2.1}$
370	$K^+\rho^0\gamma \S$	< 20	$9.2^{+0.8}_{-0.7} \pm 1.3 \dagger$	< 20				9.2 ± 1.5
—	$(K\pi)_0^0\pi^+\gamma$	New	$11.3 \pm 1.5^{+2.0}_{-2.6} \dagger$					$11.3^{+2.5}_{-3.0}$
371	$K^+\pi^-\pi^+\gamma(\text{N.R.}) \S$	< 9.2	$10.8^{+1.4+1.9}_{-1.5-2.5} \dagger$	< 9.2				$10.8^{+2.4}_{-2.5}$
—	$K_0^*(1430)\pi^+\gamma$	New	$0.82 \pm 0.11^{+0.17}_{-0.21} \dagger$					$0.82^{+0.20}_{-0.24}$
372	$K^0\pi^+\pi^0\gamma$	46 ± 5	$45.6 \pm 4.2 \pm 3.1 \dagger$					45.6 ± 5.2
373	$K_1^+(1400)\gamma$	< 15	$9.7^{+4.6+3.2}_{-2.9-1.9} \dagger$	< 15				$9.7^{+5.6}_{-3.5}$
—	$K^{*+}(1410)\gamma$	New	$23.8^{+5.2+6.4}_{-4.6-2.8} \dagger$					$23.8^{+8.2}_{-5.4}$
374	$K_S^+(1430)^+\gamma$	14 ± 4	$10.4^{+8.7+6.3}_{-7.0-9.9} \dagger$					$10.4^{+10.7}_{-12.1}$
375	$K^{*+}(1680)\gamma$	< 1900	$72^{+7+16}_{-6-14} \dagger$					72^{+17}_{-15}
376	$K_S^+(1780)^+\gamma$	< 39		< 39				< 39
378	$\rho^+\gamma$	0.98 ± 0.25	$1.20^{+0.42}_{-0.37} \pm 0.20$	$0.87^{+0.29+0.09}_{-0.27-0.11}$	< 13			$0.98^{+0.25}_{-0.24}$
428	$p\bar{A}\gamma$	$2.4^{+0.5}_{-0.4}$		$2.45^{+0.44}_{-0.38} \pm 0.22$				$2.45^{+0.49}_{-0.44}$
432	$p\bar{\Sigma}^0\gamma$	< 4.6		< 4.6				< 4.6
467	$\pi^+\ell^+\ell^-$	< 0.049	< 0.066	< 0.049				< 0.049
468	$\pi^+e^+e^-$	< 0.080	< 0.125	< 0.080				< 0.080
469	$\pi^+\mu^+\mu^-$	< 0.055	< 0.055	< 0.069			$0.023 \pm 0.006 \pm 0.001 \P^{(1)}$	0.023 ± 0.006
470	$\pi^+\nu\bar{\nu}$	< 98	< 100	< 98				< 98
471	$K^+\ell^+\ell^-$	0.451 ± 0.023	$0.48 \pm 0.09 \pm 0.02$	$0.53^{+0.06}_{-0.05} \pm 0.03$			(4)	0.51 ± 0.05
472	$K^+e^+e^-$	0.55 ± 0.07	$0.51^{+0.12}_{-0.11} \pm 0.02$	$0.57^{+0.09}_{-0.08} \pm 0.03$	< 2.4			0.55 ± 0.07
473	$K^+\mu^+\mu^-$	0.449 ± 0.023	$0.41^{+0.16}_{-0.15} \pm 0.02$	$0.53 \pm 0.08^{+0.07}_{-0.03}$	< 3.68	$0.45 \pm 0.03 \pm 0.02$	$0.429 \pm 0.007 \pm 0.021$	0.439 ± 0.018
476	$K^+\nu\bar{\nu}$	< 16	< 16	< 55	< 240			< 16
477	$\rho^+\nu\bar{\nu}$	< 213		< 213				< 213
478	$K^{*+}\ell^+\ell^-$	1.29 ± 0.21	$1.40^{+0.40}_{-0.37} \pm 0.09$	$1.24^{+0.23}_{-0.21} \pm 0.13$				$1.29^{+0.22}_{-0.21}$
479	$K^{*+}e^+e^-$	$1.55^{+0.40}_{-0.31}$	$1.38^{+0.47}_{-0.42} \pm 0.08$	$1.73^{+0.50}_{-0.42} \pm 0.20$				$1.55^{+0.35}_{-0.32}$
480	$K^{*+}\mu^+\mu^-$	1.12 ± 0.15	$1.46^{+0.79}_{-0.75} \pm 0.12$	$1.11^{+0.32}_{-0.27} \pm 0.10$		$0.89 \pm 0.25 \pm 0.09$	$0.924 \pm 0.093 \pm 0.067$	$0.949^{+0.099}_{-0.097}$
481	$K^{*+}\nu\bar{\nu}$	< 40	< 64	< 40				< 40
—	$K^+\pi^+\pi^-\mu^+\mu^-$	New					$0.436^{+0.029}_{-0.027} \pm 0.028 \P^{(2)}$	$0.436^{+0.040}_{-0.039}$
—	$K^+\phi\mu^+\mu^-$	New					$0.082^{+0.019+0.029}_{-0.017-0.027} \P$	$0.082^{+0.035}_{-0.032}$
484	$\pi^+e^\pm\mu^\mp$	< 0.17	< 0.17					< 0.17
485	$\pi^+e^+\tau^-$	< 74	< 74					< 74
486	$\pi^+e^-\tau^+$	< 20	< 20					< 20
487	$\pi^+e^\pm\tau^\mp$	< 75	< 75					< 75
488	$\pi^+\mu^+\tau^-$	< 62	< 62					< 62
489	$\pi^+\mu^-\tau^+$	< 45	< 45					< 45
490	$\pi^+\mu^\pm\tau^\mp$	< 72	< 72					< 72
491	$K^+e^+\mu^-$	< 0.091	< 0.091					< 0.091
492	$K^+e^-\mu^+$	< 0.13	< 0.13					< 0.13
493	$K^+e^\pm\mu^\mp$	< 0.091	< 0.091					< 0.091
494	$K^+e^+\tau^-$	< 43	< 43					< 43
495	$K^+e^-\tau^+$	< 15	< 15					< 15
496	$K^+e^\pm\tau^\mp$	< 30	< 30					< 30
497	$K^+\mu^+\tau^-$	< 45	< 45					< 45
498	$K^+\mu^-\tau^+$	< 28	< 28					< 28
499	$K^+\mu^\pm\tau^\mp$	< 48	< 48					< 48
500	$K^{*+}e^+\mu^-$	< 1.3	< 1.3					< 1.3
501	$K^{*+}e^-\mu^+$	< 0.99	< 0.99					< 0.99
502	$K^{*+}e^\pm\mu^\mp$	< 1.4	< 1.4					< 1.4
503	$\pi^-e^+\mu^+$	< 0.023	< 0.023		< 1.6			< 0.023
504	$\pi^-\mu^+\mu^+$	< 0.013	< 0.107		< 1.4		$< 0.004^{(3)}$	$< 0.004^{(3)}$
505	$\pi^-e^+\mu^+$	< 0.15	< 0.15		< 1.3			< 0.15
506	$\rho^-e^+\mu^+$	< 0.17	< 0.17		< 2.6			< 0.17
507	$\rho^-\mu^+\mu^+$	< 0.42	< 0.42		< 5.0			< 0.42
508	$\rho^-e^+\mu^+$	< 0.47	< 0.47		< 3.3			< 0.47
509	$K^-e^+e^+$	< 0.03	< 0.03		< 1.0			< 0.03
510	$K^-\mu^+\mu^+$	< 0.041	< 0.067		< 1.8		< 0.041	< 0.041
511	$K^-e^+\mu^+$	< 0.16	< 0.16		< 2.0			< 0.16
512	$K^{*-}e^+e^+$	< 0.40	< 0.40		< 2.8			< 0.40
513	$K^{*-}\mu^+\mu^+$	< 0.59	< 0.59		< 8.3			< 0.59
514	$K^{*-}e^+\mu^+$	< 0.30	< 0.30		< 4.4			< 0.30

$\dagger M_{K\pi\pi} < 1.8 \text{ GeV}/c^2$; $\ddagger 1.0 < M_{K\pi\pi} < 2.0 \text{ GeV}/c^2$; $\S M_{K\pi\pi} < 2.4 \text{ GeV}/c^2$.

$\P$ Relative BF converted to absolute BF.

(1) PDG2014 cites only the measurement: $\mathcal{B}(\pi^+\mu^+\mu^-)/\mathcal{B}(K^+\mu^+\mu^-) = 0.053 \pm 0.014 \pm 0.01$.

(2) Differential BF in bins of $m(\mu\mu)$ is also available.

(3) At 95% CL.

(4) PDG considers here the BF measured in $B^+ \rightarrow K^+\mu^+\mu^-$.

Table 193: Branching fractions (BF) of charmless semileptonic and radiative B^0 decays in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	LHCb	New Avg.
336	$K^0\eta\gamma$	7.6 ± 1.8	$7.1^{+2.1}_{-2.0} \pm 0.4$	$8.7^{+3.1+1.9}_{-2.7-1.6}$				$7.6^{+1.8}_{-1.7}$
337	$K^0\eta'\gamma$	< 6.4	< 6.6	< 6.4				< 6.4
338	$K^0\phi\gamma$	2.7 ± 0.7	< 2.7	$2.74 \pm 0.60 \pm 0.32$				2.74 ± 0.68
339	$K^+\pi^-\gamma$ §	4.6 ± 1.4		$4.6^{+1.3+0.5}_{-1.2-0.7}$				4.6 ± 1.4
340	$K^{*0}\gamma$	43.3 ± 1.5	$44.7 \pm 1.0 \pm 1.6$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$			43.3 ± 1.5
341	$K^*(1410)^0\gamma$	< 130		< 130				< 130
342	$K^+\pi^-\gamma$ (N.R.) §	< 2.6		< 2.6				< 2.6
344	$K^0\pi^+\pi^-\gamma$	19.5 ± 2.2	$18.5 \pm 2.1 \pm 1.2$ †	$24 \pm 4 \pm 3$ ‡				19.5 ± 2.2
345	$K^+\pi^-\pi^0\gamma$	41 ± 4	$40.7 \pm 2.2 \pm 3.1$ †					40.7 ± 3.8
346	$K_1^0(1270)\gamma$	< 58		< 58				< 58
347	$K_1^0(1400)\gamma$	< 12		< 12				< 12
348	$K_2^*(1430)^0\gamma$	12.4 ± 2.4	$12.2 \pm 2.5 \pm 1.0$	$13 \pm 5 \pm 1$				12.4 ± 2.4
350	$K_3^*(1780)^0\gamma$	< 83		< 83				< 83
352	$\rho^0\gamma$	0.86 ± 0.15	$0.97^{+0.24}_{-0.22} \pm 0.06$	$0.78^{+0.17+0.09}_{-0.16-0.10}$	< 17			$0.86^{+0.15}_{-0.14}$
354	$\omega\gamma$	$0.44^{+0.18}_{-0.16}$	$0.50^{+0.27}_{-0.23} \pm 0.09$	$0.40^{+0.19}_{-0.17} \pm 0.13$	< 9.2			$0.44^{+0.18}_{-0.16}$
355	$\phi\gamma$	< 0.85	< 0.85		< 3.3			< 0.85
—	$p\bar{A}\pi^-\gamma$	New		< 0.65				< 0.65
465	$\pi^0\ell^+\ell^-$	< 0.053	< 0.053	< 0.154				< 0.053
466	$\pi^0e^+e^-$	< 0.084	< 0.084	< 0.227				< 0.084
467	$\pi^0\mu^+\mu^-$	< 0.069	< 0.069	< 0.184				< 0.069
468	$\eta\ell^+\ell^-$	< 0.064	< 0.064					< 0.064
469	ηe^+e^-	< 0.108	< 0.108					< 0.108
470	$\eta\mu^+\mu^-$	< 0.112	< 0.112					< 0.112
471	$\pi^0\nu\bar{\nu}$	< 69		< 69				< 69
472	$K^0\ell^+\ell^-$	$0.31^{+0.08}_{-0.07}$	$0.21^{+0.15}_{-0.13} \pm 0.02$	$0.34^{+0.09}_{-0.08} \pm 0.02$				$0.31^{+0.08}_{-0.07}$
473	$K^0e^+e^-$	$0.16^{+0.10}_{-0.08}$	$0.08^{+0.15}_{-0.12} \pm 0.01$	$0.20^{+0.14}_{-0.10} \pm 0.01$	< 8.45			$0.16^{+0.10}_{-0.08}$
474	$K^0\mu^+\mu^-$	0.34 ± 0.05	$0.49^{+0.29}_{-0.25} \pm 0.03$	$0.44^{+0.13}_{-0.10} \pm 0.03$	< 6.64	$0.33 \pm 0.08 \pm 0.03$	$0.327 \pm 0.034 \pm 0.017$	$0.341^{+0.033}_{-0.032}$
475	$K^0\nu\bar{\nu}$	< 49	< 49	< 194				< 49
476	$\rho^0\nu\bar{\nu}$	< 208		< 208				< 208
477	$K^{*0}\ell^+\ell^-$	$0.99^{+0.12}_{-0.11}$	$1.03^{+0.22}_{-0.21} \pm 0.07$	$0.97^{+0.13}_{-0.11} \pm 0.07$				$0.99^{+0.13}_{-0.11}$
478	$K^{*0}e^+e^-$	$1.03^{+0.19}_{-0.17}$	$0.86^{+0.26}_{-0.24} \pm 0.05$	$1.18^{+0.27}_{-0.22} \pm 0.09$				$1.03^{+0.19}_{-0.17}$
479	$K^{*0}\mu^+\mu^-$	1.05 ± 0.10	$1.35^{+0.40}_{-0.37} \pm 0.10$	$1.06^{+0.19}_{-0.14} \pm 0.07$		$1.14 \pm 0.09 \pm 0.06$		$1.13^{+0.10}_{-0.09}$
480	$K^{*0}\nu\bar{\nu}$	< 55	< 120	< 55				< 55
481	$\phi\nu\bar{\nu}$	< 127		< 127				< 127
483	$\pi^0e^\pm\mu^\mp$	< 0.14	< 0.14					< 0.14
484	$K^0e^\pm\mu^\mp$	< 0.27	< 0.27					< 0.27
485	$K^{*0}e^\pm\mu^\mp$	< 0.53	< 0.53					< 0.53

† $M_{K\pi\pi} < 1.8 \text{ GeV}/c^2$.

‡ $1.0 < M_{K\pi\pi} < 2.0 \text{ GeV}/c^2$; § $1.25 \text{ GeV}/c^2 < M_{K\pi} < 1.6 \text{ GeV}/c^2$.

Table 194: Branching fractions (BF) of charmless semileptonic and radiative decays of $B^\pm/B^0$ admixture in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	CDF	New Avg.
66	$K\eta\gamma$	$8.5^{+1.8}_{-1.6}$		$8.5^{+1.3}_{-1.2} \pm 0.9$			$8.5^{+1.6}_{-1.5}$
68	$K_2^*(1430)\gamma$	17^{+6}_{-5}			$17 \pm 6 \pm 1$		17 ± 6
70	$K_3^*(1780)\gamma$	< 37		< 2.8 §			< 2.8 §
77	$s\gamma$	360 ± 23	$300 \pm 14 \pm 20$	$345 \pm 15 \pm 40$	$321 \pm 43^{+32}_{-29}$		$343 \pm 21 \pm 7$
78	$d\gamma$	9.2 ± 3.0	$9.2 \pm 2.0 \pm 2.3$				9.2 ± 3.0
84	$\rho\gamma$	1.39 ± 0.25	$1.73^{+0.34}_{-0.32} \pm 0.17$	$1.21^{+0.24}_{-0.22} \pm 0.12$	< 14		$1.39^{+0.22}_{-0.21}$
85	$\rho/\omega\gamma$	1.30 ± 0.23	$1.63^{+0.30}_{-0.28} \pm 0.16$	$1.14 \pm 0.20^{+0.10}_{-0.12}$	< 14		$1.30^{+0.18}_{-0.19}$
119	se^+e^- ‡	4.7 ± 1.3	$7.69^{+0.82+0.71}_{-0.77-0.69}$	$4.56 \pm 1.15^{+0.33}_{-0.40}$	< 57		6.44 ± 0.76
120	$s\mu^+\mu^-$ ‡	4.3 ± 1.2	$4.41^{+1.31+0.63}_{-1.17-0.50}$	$1.91 \pm 1.02^{+0.16}_{-0.18}$	< 58		2.90 ± 0.80
121	$s\ell^+\ell^-$ ‡	4.5 ± 1.0	$6.73^{+0.70+0.60}_{-0.64-0.56}$	$3.33 \pm 0.80^{+0.19}_{-0.24}$	< 42		4.97 ± 0.59
122	$\pi\ell^+\ell^-$	< 0.059	< 0.059	< 0.062			< 0.059
123	πe^+e^-	< 0.110	< 0.110				< 0.110
124	$\pi\mu^+\mu^-$	< 0.050	< 0.050				< 0.050
125	Ke^+e^-	0.44 ± 0.06	$0.39^{+0.09}_{-0.08} \pm 0.02$	$0.48^{+0.08}_{-0.07} \pm 0.03$			0.44 ± 0.06
126	$K^*e^+e^-$	1.19 ± 0.20	$0.99^{+0.23}_{-0.21} \pm 0.06$	$1.39^{+0.23}_{-0.20} \pm 0.12$			$1.19^{+0.17}_{-0.16}$
127	$K\mu^+\mu^-$	0.44 ± 0.04	$0.41^{+0.13}_{-0.12} \pm 0.02$	$0.50 \pm 0.06 \pm 0.03$		$4.2 \pm 0.4 \pm 0.2$	0.55 ± 0.06
128	$K^*\mu^+\mu^-$	1.06 ± 0.09	$1.35^{+0.35}_{-0.33} \pm 0.10$	$1.10^{+0.16}_{-0.14} \pm 0.08$		$10.1 \pm 1.0 \pm 0.5$	1.33 ± 0.16
129	$K\ell^+\ell^-$	0.48 ± 0.04	$0.47 \pm 0.06 \pm 0.02$	$0.48^{+0.05}_{-0.04} \pm 0.03$	< 1.7		0.48 ± 0.04
130	$K^*\ell^+\ell^-$	1.05 ± 0.10	$1.02^{+0.14}_{-0.13} \pm 0.05$	$1.07^{+0.11}_{-0.10} \pm 0.09$	< 3.3		1.05 ± 0.10
131	$K\nu\bar{\nu}$	< 17	< 17				< 17
132	$K^*\nu\bar{\nu}$	< 76	< 76				< 76
134	$\pi e^\pm\mu^\mp$	< 0.092	< 0.092		< 1.6		< 0.092
135	$\rho e^\pm\mu^\mp$	< 3.2			< 3.2		< 3.2
136	$Ke^\pm\mu^\mp$	< 0.038	< 0.038		< 1.6		< 0.038
137	$K^*e^\pm\mu^\mp$	< 0.51	< 0.51		< 6.2		< 0.51
—	$s\gamma$ with baryons	—			< 38 †		< 38 †

† $E_\gamma > 2.0$ GeV.

‡ Belle: $M(\ell^+\ell^-) > 0.2$ GeV/ c^2 , BABAR: $M^2(\ell^+\ell^-) > 0.1$ GeV $^2/c^4$.

§ Product BF ($\times \mathcal{B}(K_3^* \rightarrow K\eta)$). PDG gives the BF assuming $\mathcal{B}(K_3^* \rightarrow K\eta) = 11^{+5}_{-4}$.

Table 195: Branching fractions (BF) of leptonic and radiative-leptonic B^+ and B^0 decays in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CDF	LHCb	CMS	New Avg.
29	$e^+\nu$	< 0.98	< 1.9	< 0.98 †				< 0.98 †
30	$\mu^+\nu$	< 1.0	< 1.0	< 1.7 †				< 1.0
31	$\tau^+\nu$	114 ± 27	179 ± 48 ‡	96 ± 26 ‡				114 ± 22
32	$\ell^+\nu\ell\gamma$	< 15.6	< 15.6					< 15.6
33	$e^+\nu_e\gamma$	< 17	< 17					< 17
34	$\mu^+\nu_\mu\gamma$	< 24	< 24					< 24
457	$\gamma\gamma$	< 0.32	< 0.32	< 0.62				< 0.32
458	e^+e^-	< 0.083	< 0.113	< 0.19	< 0.083			< 0.083
459	$e^+e^-\gamma$	< 0.12	< 0.12					< 0.12
460	$\mu^+\mu^-$	< 0.00063	< 0.052	< 0.16	< 0.0038	< 0.00063	< 0.00092	< 0.00063
460	$\mu^+\mu^-$	CMS-LHCb comb.				$0.00039^{+0.00016}_{-0.00014}$	$0.00039^{+0.00016}_{-0.00014}$	
461	$\mu^+\mu^-\gamma$	< 0.16	< 0.16					< 0.16
462	$\mu^+\mu^-\mu^+\mu^-$	< 0.0053				< 0.0053		< 0.0053
464	$\tau^+\tau^-$	< 4100	< 4100					< 4100
482	$e^\pm\mu^\mp$	< 0.0028	< 0.092	< 0.17	< 0.064	< 0.0028		< 0.0028
488	$e^\pm\tau^\mp$	< 28	< 28					< 28
489	$\mu^\pm\tau^\mp$	< 22	< 22					< 22
490	$\nu\bar{\nu}$	< 24	< 24	< 130				< 24
491	$\nu\bar{\nu}\gamma$	< 17	< 17					< 17

† More recent results exist, with hadronic tagging (Ref. [726]). It does not improve the limits (< 3.4 and < 2.7 for $e^+\nu$ and $\mu^+\nu$, respectively).

‡ The authors average their results with earlier results from BABAR [727] and Belle [728].

Table 196: Relative branching fractions (BF) of semileptonic and radiative B^+ decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2012 Avg.	CDF	DØ	LHCb	New avg.
–	$10^4 \times \mathcal{B}(B^+ \rightarrow K^+\pi^+\pi^-\mu^+\mu^-)/\mathcal{B}(B^+ \rightarrow \psi(2S)K^+)$	New			$6.95^{+0.46}_{-0.43} \pm 0.34$	$6.95^{+0.57}_{-0.55}$
–	$10^4 \times \mathcal{B}(B^+ \rightarrow K^+\phi\mu^+\mu^-)/\mathcal{B}(B^+ \rightarrow \psi(2S)K^+)$	New			$1.58^{+0.36+0.19}_{-0.32-0.07}$	$1.58^{+0.41}_{-0.33}$
469	$\mathcal{B}(\pi^+\mu^+\mu^-)/\mathcal{B}(K^+\mu^+\mu^-)$	$0.053 \pm 0.014 \pm 0.01$			$0.053 \pm 0.014 \pm 0.01$	$0.053 \pm 0.014 \pm 0.01$

Table 197: Branching fractions (BF) of $B \rightarrow \bar{b} \rightarrow \bar{q}$ gluon decays in units of $\times 10^{-6}$. Upper limits are at 90% CL. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CLEO	New Avg.
80	ηX	260^{+50}_{-80}		$261 \pm 30^{+44}_{-74}$ §	< 440	261^{+53}_{-79}
81	$\eta' X$	420 ± 90	$390 \pm 80 \pm 90$ †		$460 \pm 110 \pm 60$ †	423 ± 86
82	$K^+ X$	< 187	< 187 ‡			< 187 ‡
83	$K^0 X$	195^{+71}_{-67}	$195^{+51}_{-45} \pm 50$ ‡			195^{+71}_{-67}
94	$\pi^+ X$	370 ± 80	$372^{+50}_{-47} \pm 59$ ¶			372^{+77}_{-75}

§ $0.4 < m_X < 2.6\text{GeV}/c$; † $2.0 < p^*(\eta') < 2.7\text{GeV}/c$;

‡ $m_X < 1.69\text{GeV}/c$; ¶ $m_X < 1.71\text{GeV}/c$.

Table 198: Isospin asymmetry in radiative and semileptonic B meson decays. The notations are those adopted by the PDG. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Parameter	PDG2014 Avg.	<i>BABAR</i>	Belle	LHCb	New Avg.
$\Delta_0-(X_s\gamma)$	-0.01 ± 0.06	-0.01 ± 0.06 §			-0.01 ± 0.06
$\Delta_{0+}(K^*\gamma)$	0.052 ± 0.026	$0.066 \pm 0.021 \pm 0.022$	$0.012 \pm 0.044 \pm 0.026$		0.012 ± 0.051
$\Delta_{\rho\gamma}$	-0.46 ± 0.17	$-0.43^{+0.25}_{-0.22} \pm 0.10$	$-0.48^{+0.21+0.08}_{-0.19-0.09}$		$-0.48^{+0.23}_{-0.21}$
$\Delta_0-(K\ell\ell)^\dagger$	-0.37 ± 0.13	$-0.58^{+0.29}_{-0.37} \pm 0.02$	$-0.31^{+0.17}_{-0.14} \pm 0.08$	$-0.35^{+0.23}_{-0.27}$	-0.32 ± 0.14
$\Delta_0-(K^*\ell\ell)^\dagger$	-0.22 ± 0.10	$-0.25^{+0.20}_{-0.17} \pm 0.03$	$-0.29 \pm 0.16 \pm 0.09$	-0.15 ± 0.16	-0.21 ± 0.12
$\Delta_0-(K^{(*)}\ell\ell)^\dagger$	-0.45 ± 0.17	$-0.64^{+0.15}_{-0.14} \pm 0.03$	$-0.30^{+0.12}_{-0.11} \pm 0.08$		-0.30 ± 0.14

† See the references for precise $q^2 = m_{\ell\ell}^2$ region. In all measurements $m_{\ell\ell} < m_{J/\psi}$.

§ Average of two independent measurements from *BABAR*.

Table 199: Isospin asymmetry in $K^{(*)}\ell^+\ell^-$ modes in bins of $q^2 = m_{\ell\ell}^2$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 [(GeV/c ²) ²] †	PDG2014 Avg.	<i>BABAR</i>	Belle	CDF ‡	LHCb ‡	New Avg.
$K\ell^+\ell^-$	< 2.0		$-0.51^{+0.49}_{-0.95}$	$-0.33^{+0.34}_{-0.26}$	0.19 ± 0.34 ± 0.05	$-0.55^{+0.40}_{-0.56}$	$-0.24^{+0.18}_{-0.19}$
	[2.0, 4.3]		$-0.73^{+0.48}_{-0.55}$	$-0.47^{+0.50}_{-0.39}$	-0.07 ± 0.34 ± 0.07	$-0.76^{+0.45}_{-0.79}$	$-0.42^{+0.20}_{-0.22}$
	[4.3, 8.68]		$-0.32^{+0.27}_{-0.30}$	$-0.19^{+0.26}_{-0.22}$	-0.20 ± 0.26 ± 0.08	$0.00^{+0.14}_{-0.15}$	-0.11 ± 0.11
	[10.09, 12.86]		$-0.05^{+0.25}_{-0.29}$	$-0.29^{+0.37}_{-0.29}$	-0.27 ± 0.37 ± 0.08	$-0.15^{+0.19}_{-0.22}$	$-0.16^{+0.14}_{-0.15}$
	[14.18, 16.00]		$0.05^{+0.32}_{-0.43}$	$-0.40^{+0.61}_{-0.69}$	0.04 ± 0.23 ± 0.05	-0.40 ± 0.22	$-0.17^{+0.14}_{-0.15}$
	> 16.00		$-0.93^{+0.83}_{-4.99}$	$0.11^{+0.25}_{-0.22}$	-0.29 ± 0.28 ± 0.06	$-0.52^{+0.18}_{-0.22}$	$-0.28^{+0.12}_{-0.13}$
	[1.00, 6.00]		$-0.41^{+0.25}_{-0.01}$	$-0.41^{+0.26}_{-0.21}$	-0.06 ± 0.24 ± 0.07	$-0.35^{+0.23}_{-0.27}$	-0.30 ± 0.12
	$K\ell^+\ell^-$ §					-0.37 ± 0.18 ± 0.02	$-0.37^{+0.18}_{-0.21}$
	0.1 – 2.0					-0.15 ± 0.13 ± 0.02	$-0.15^{+0.13}_{-0.15}$
	2.0 – 4.0					-0.10 ± 0.13 ± 0.02	$-0.10^{+0.13}_{-0.16}$
$K^*\ell^+\ell^-$	4.0 – 6.0					0.09 ± 0.10 ± 0.02	$0.09^{+0.10}_{-0.11}$
	6.0 – 8.0					-0.16 ± 0.15 ± 0.03	$-0.16^{+0.15}_{-0.18}$
	11.0 – 12.5					-0.04 ± 0.11 ± 0.02	$-0.04^{+0.11}_{-0.13}$
	15.0 – 17.0					-0.12 ± 0.10 ± 0.02	$-0.12^{+0.10}_{-0.11}$
	17.0 – 22.0					-0.10 ± 0.08 ± 0.02	$-0.10^{+0.08}_{-0.09}$
	1.1 – 6.0					-0.09 ± 0.08 ± 0.02	-0.09 ± 0.08
	15.0 – 22.0					-0.09 ± 0.08 ± 0.02	-0.09 ± 0.08
	$K^*\ell^+\ell^-$						
	< 2.0		$-0.17^{+0.29}_{-0.24}$	$-0.67^{+0.19}_{-0.17}$	0.15 ± 0.32 ± 0.06	$0.05^{+0.27}_{-0.21}$	$-0.25^{+0.12}_{-0.11}$
	[2.0, 4.3]		$-0.06^{+0.56}_{-0.36}$	$1.45^{+1.04}_{-1.15}$	0.00 ± 0.39 ± 0.07	$-0.27^{+0.29}_{-0.18}$	$-0.12^{+0.23}_{-0.17}$
$K^*\ell^+\ell^-$ §	[4.3, 8.68]		$0.03^{+0.43}_{-0.32}$	$-0.34^{+0.32}_{-0.30}$	0.29 ± 0.41 ± 0.13	$-0.06^{+0.19}_{-0.14}$	$-0.06^{+0.14}_{-0.11}$
	[10.09, 12.86]		$-0.48^{+0.23}_{-0.19}$	$0.00^{+0.22}_{-0.23}$	0.43 ± 0.35 ± 0.10	$-0.16^{+0.17}_{-0.16}$	-0.14 ± 0.11
	[14.18, 16.00]		$0.24^{+0.61}_{-0.39}$	$0.16^{+0.31}_{-0.36}$	0.17 ± 0.29 ± 0.07	$0.02^{+0.23}_{-0.21}$	$0.11^{+0.15}_{-0.14}$
	> 16.00		$1.07^{+4.28}_{-1.01}$	$-0.02^{+0.22}_{-0.23}$	-0.23 ± 0.23 ± 0.06	$0.02^{+0.21}_{-0.20}$	-0.05 ± 0.13
	[1.00, 6.00]		$-0.20^{+0.30}_{-0.23}$	$0.33^{+0.38}_{-0.44}$	-0.26 ± 0.21 ± 0.07	-0.15 ± 0.16	$-0.16^{+0.12}_{-0.11}$
	0.1 – 2.0					0.11 ± 0.12 ± 0.02	$0.11^{+0.12}_{-0.11}$
	2.0 – 4.0					-0.20 ± 0.15 ± 0.03	$-0.20^{+0.15}_{-0.12}$
	4.0 – 6.0					0.23 ± 0.21 ± 0.02	$0.23^{+0.21}_{-0.18}$
	6.0 – 8.0					0.19 ± 0.17 ± 0.02	$0.19^{+0.17}_{-0.15}$
	11.0 – 12.5					-0.25 ± 0.09 ± 0.03	$-0.25^{+0.10}_{-0.09}$
$K^*\ell^+\ell^-$ §	15.0 – 17.0					-0.10 ± 0.10 ± 0.03	-0.10 ± 0.10
	17.0 – 19.0					0.51 ± 0.29 ± 0.02	$0.51^{+0.29}_{-0.24}$
	1.1 – 6.0					0.00 ± 0.10 ± 0.02	$0.00^{+0.12}_{-0.10}$
	15.0 – 19.0					0.06 ± 0.10 ± 0.02	$0.06^{+0.10}_{-0.09}$

† See the papers for the exact $q^2 = M^2(\mu^+\mu^-)$ selection.

‡ Muon mode only ($\ell = \mu$).

§ Results in two different sets of q^2 bins are available.

Table 200: Forward-backward asymmetry (A_{FB}) in $K^{(*)}\ell^+\ell^-$ modes in bins of $q^2 = m_{\ell\ell}^2$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 [(GeV/ c^2) ^{2}] †	PDG2014 Avg.	Belle	CDF ‡	LHCb ‡	CMS ‡	ATLAS ‡	New Avg.
$K\ell^+\ell^-$	< 2.0	$0.00^{+0.06}_{-0.05}$	$0.06^{+0.32}_{-0.35} \pm 0.02$	$-0.19^{+0.37}_{-0.45} \pm 0.09$	$0.00^{+0.06+0.03}_{-0.05-0.01}$			$-0.00^{+0.06}_{-0.05}$
	[2.0, 4.3]	$0.09^{+0.10}_{-0.09}$	$-0.43^{+0.38}_{-0.42} \pm 0.09$	$0.32^{+0.17}_{-0.18} \pm 0.10$	$0.07^{+0.08+0.02}_{-0.03-0.01}$			$0.09^{+0.08}_{-0.07}$
	[4.3, 8.68]	$-0.04^{+0.04}_{-0.05}$	$-0.20^{+0.12}_{-0.14} \pm 0.03$	$0.08^{+0.08}_{-0.09} \pm 0.01$	$-0.02^{+0.03}_{-0.05} \pm 0.03$			$-0.02^{+0.04}_{-0.05}$
	[10.09, 12.86]	-0.05 ± 0.06	$-0.21^{+0.11}_{-0.17} \pm 0.06$	$-0.04^{+0.12}_{-0.08} \pm 0.03$	$-0.03 \pm 0.07 \pm 0.01$			-0.05 ± 0.06
	[14.18, 16.00]	$-0.02^{+0.07}_{-0.05}$	$0.04^{+0.15}_{-0.16} \pm 0.05$	$-0.07^{+0.08}_{-0.08} \pm 0.01$	$-0.01^{+0.12}_{-0.06} \pm 0.01$			$-0.03^{+0.06}_{-0.04}$
	[16.0, 18.0]	$-0.09^{+0.07}_{-0.09}$			$-0.09^{+0.07+0.02}_{-0.09-0.01}$			$-0.09^{+0.07}_{-0.09}$
	[18.0, 22.0]	0.02 ± 0.11			$0.02 \pm 0.11 \pm 0.01$			0.02 ± 0.11
	> 16.00	$0.04^{+0.09}_{-0.09}$	$0.02^{+0.11}_{-0.13} \pm 0.02$	$0.05^{+0.18}_{-0.10} \pm 0.05$				$0.03^{+0.09}_{-0.05}$
	[1.00, 6.00]	$0.034^{+0.040}_{-0.029}$	$-0.04^{+0.16}_{-0.16} \pm 0.05$	$0.13^{+0.11}_{-0.10} \pm 0.02$	$0.02^{+0.05+0.02}_{-0.03-0.01}$			$0.03^{+0.05}_{-0.03}$
	< 2.0	-0.01 ± 0.14	$0.47^{+0.26}_{-0.32} \pm 0.03$	$0.05^{+0.28}_{-0.27} \pm 0.10$	$-0.02 \pm 0.12 \pm 0.01$			0.04 ± 0.11
$K^*\ell^+\ell^-$	[1.0, 2.0]	$0.45^{+0.26}_{-0.30}$				$-0.29^{+0.37}_{-0.00} \pm 0.18$		$-0.29^{+0.41}_{-0.18}$
	[2.0, 4.3]	-0.15 ± 0.07	$0.11^{+0.31}_{-0.36} \pm 0.07$	$-0.11^{+0.34}_{-0.41} \pm 0.16$	$-0.20 \pm 0.08 \pm 0.01$	$-0.07 \pm 0.20 \pm 0.02$	$0.22 \pm 0.28 \pm 0.14$	-0.15 ± 0.07
	[4.3, 8.68]	$0.13^{+0.05}_{-0.06}$	$0.45^{+0.15}_{-0.21} \pm 0.15$	$0.09^{+0.14}_{-0.14} \pm 0.04$	$0.16^{+0.06}_{-0.06} \pm 0.01$	$-0.01 \pm 0.11 \pm 0.03$	$0.24 \pm 0.13 \pm 0.01$	0.15 ± 0.04
	[10.09, 12.86]	0.34 ± 0.05	$0.43^{+0.18}_{-0.20} \pm 0.03$	$0.44^{+0.12}_{-0.13} \pm 0.08$	$0.28^{+0.07}_{-0.06} \pm 0.02$	$0.40 \pm 0.08 \pm 0.05$	$0.09 \pm 0.09 \pm 0.03$	$0.29^{+0.05}_{-0.04}$
	[14.18, 16.00]	$0.47^{+0.07}_{-0.06}$	$0.70^{+0.16}_{-0.12} \pm 0.10$	$0.53^{+0.09}_{-0.09} \pm 0.07$	$0.51^{+0.07}_{-0.05} \pm 0.02$	$0.29 \pm 0.09 \pm 0.05$	$0.48 \pm 0.19 \pm 0.05$	0.48 ± 0.04
	[16.0, 19.0]	0.40 ± 0.06	$0.66^{+0.12}_{-0.16} \pm 0.04$	$0.35^{+0.19}_{-0.19} \pm 0.06$	$0.30 \pm 0.08^{+0.01}_{-0.02}$	$0.41 \pm 0.05 \pm 0.03$	$0.16 \pm 0.10 \pm 0.03$	0.36 ± 0.04
	[1.00, 6.00]	-0.12 ± 0.07	$0.26^{+0.27}_{-0.30} \pm 0.07$	$0.19^{+0.17}_{-0.21} \pm 0.05$	$-0.17 \pm 0.06 \pm 0.01$	$-0.07 \pm 0.12 \pm 0.01$	$0.07 \pm 0.20 \pm 0.07$	-0.11 ± 0.05

† see the original papers for the exact q^2 selection.

‡ muon mode only ($\ell = \mu$).

Table 201: Fraction of the Longitudinal Polarization (F_L) in $K^{(*)}\ell^+\ell^-$ modes in bins of $q^2 = m_{\ell\ell}^2$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Mode	q^2 [(GeV/ c^2) ²]	†	PDG2014 Avg.	Belle	CDF ‡	LHCb ‡	CMS ‡	ATLAS ‡	New Avg.
$K^*\ell^+\ell^-$	< 2.0	[1.0, 2.0]	$0.34^{+0.08}_{-0.07}$	$0.29^{+0.21}_{-0.18} \pm 0.02$	$0.25^{+0.14}_{-0.13} \pm 0.04$	$0.37^{+0.11}_{-0.09}$			$0.33^{+0.08}_{-0.07}$
			$0.60^{+0.00}_{-0.28} \pm 0.19$				$0.60^{+0.00}_{-0.28} \pm 0.19$		$0.60^{+0.19}_{-0.34}$
	[2.0, 4.3]	[4.3, 8.68]	0.69 ± 0.08	$0.71 \pm 0.24 \pm 0.05$	$0.71^{+0.15}_{-0.17} \pm 0.07$	$0.74^{+0.10}_{-0.09}$	$0.65 \pm 0.17 \pm 0.03$	$0.26 \pm 0.18 \pm 0.06$	0.66 ± 0.07
			0.64 ± 0.06	$0.64^{+0.23}_{-0.24} \pm 0.07$	$0.72^{+0.12}_{-0.13} \pm 0.05$	$0.57 \pm 0.07 \pm 0.03$	$0.81^{+0.13}_{-0.12} \pm 0.05$	$0.37 \pm 0.11 \pm 0.02$	0.59 ± 0.05
	[10.09, 12.86]	[14.18, 16.00]	0.43 ± 0.06	$0.17^{+0.15}_{-0.15} \pm 0.03$	$0.38^{+0.11}_{-0.11} \pm 0.04$	$0.48^{+0.08}_{-0.09} \pm 0.03$	$0.45^{+0.10}_{-0.11} \pm 0.04$	$0.50 \pm 0.09 \pm 0.04$	0.44 ± 0.05
			0.35 ± 0.08	$-0.15^{+0.27}_{-0.23} \pm 0.07$	$0.40^{+0.11}_{-0.11} \pm 0.04$	$0.33^{+0.08}_{-0.07} \pm 0.02$	$0.53 \pm 0.12 \pm 0.03$	$0.28 \pm 0.16 \pm 0.03$	$0.36^{+0.06}_{-0.05}$
	[16.0, 19.0]	[1.00, 6.00]	0.37 ± 0.06	$0.12^{+0.15}_{-0.13} \pm 0.02$	$0.19^{+0.12}_{-0.11} \pm 0.07$	$0.38^{+0.09}_{-0.08} \pm 0.03$	$0.44 \pm 0.07 \pm 0.03$	$0.35 \pm 0.08 \pm 0.02$	0.35 ± 0.04
			0.66 ± 0.06	$0.67 \pm 0.23 \pm 0.05$	$0.76^{+0.12}_{-0.14} \pm 0.07$	$0.65^{+0.08}_{-0.07} \pm 0.03$	$0.68 \pm 0.10 \pm 0.02$	$0.18 \pm 0.15 \pm 0.03$	0.62 ± 0.05

† see the original paper for the exact q^2 selection.

‡ muon mode only ($\ell = \mu$).

Table 202: Up-down Asymmetry in $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ decays in bins of $m_{K^+ \pi^- \pi^+}$. Values in red (blue) are new published (preliminary) results since PDG2014.

Mode	$m_{K^+ \pi^- \pi^+}$ [(GeV/ c^2)]	PDG2014 Avg.	LHCb	New Avg.
$K^+ \pi^- \pi^+ \gamma$	1.1-1.3		6.9 ± 1.7	6.9 ± 1.7
	1.3-1.4		4.9 ± 2.0	4.9 ± 2.0
	1.4-1.6		5.6 ± 1.8	5.6 ± 1.8
	1.6-1.9		-4.5 ± 1.9	-4.5 ± 1.9
	1.1-1.3		-1.1 ± 1.7	-1.1 ± 1.7
	1.3-1.4		7.2 ± 2.0	7.2 ± 2.0
	1.4-1.6		6.4 ± 1.8	6.4 ± 1.8
	1.6-1.9		-3.9 ± 1.9	-3.9 ± 1.9

7.6 Charge asymmetries in b -hadron decays

This section contains, in Tables 203 to 207 compilations of CP asymmetries in decays of different b -hadrons: B^+ , B^0 mesons, $B^\pm/B^0$ admixtures, B_s^0 mesons and finally Λ_b^0 baryons. Measurements of time-dependent CP asymmetries are not listed here but are discussed in Sec. 4. In addition, we do not attempt to include the model-independent studies of CP violation across the phase space of the three body decays $B^+ \rightarrow K^+K^-K^+$, $K^+\pi^-\pi^+$, $K^+K^-\pi^+$ and $\pi^+\pi^-\pi^+$, where large effects have been observed by LHCb [729–731]. The results, in terms of CP asymmetries of quasi-two-body final states, of model-dependent analyses, and of the CP asymmetries for the inclusive three-body final states, are however included.

Table 203: CP asymmetries of charmless hadronic B^+ decays. Values in red (blue) are new published (preliminary) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BaBAR	Belle	CDF	LHCb	New Avg.
262	$K^0\pi^+$	-0.017 ± 0.016	$-0.029 \pm 0.039 \pm 0.010$	$-0.011 \pm 0.021 \pm 0.006$		$-0.022 \pm 0.025 \pm 0.010$	-0.017 ± 0.016
263	$K^+\pi^0$	0.037 ± 0.021	$0.030 \pm 0.039 \pm 0.010$	$0.043 \pm 0.024 \pm 0.002$			0.040 ± 0.021
264	$\eta'K^+$	0.013 ± 0.017	$0.008^{+0.017}_{-0.018} \pm 0.009$	$0.028 \pm 0.028 \pm 0.021$			0.013 ± 0.017
265	$\eta'K^{*+}$	-0.26 ± 0.27	$-0.26 \pm 0.27 \pm 0.02$				-0.26 ± 0.27
266	$\eta'K_0^*(1430)^+$	0.06 ± 0.20	$0.06 \pm 0.20 \pm 0.02$				0.06 ± 0.20
267	$\eta'K_2^*(1430)^+$	0.15 ± 0.13	$0.15 \pm 0.13 \pm 0.02$				0.15 ± 0.13
268	ηK^+	-0.37 ± 0.08	$-0.36 \pm 0.11 \pm 0.03$	$-0.38 \pm 0.11 \pm 0.01$			-0.37 ± 0.08
269	ηK^{*+}	0.02 ± 0.06	$0.01 \pm 0.08 \pm 0.02$	$0.03 \pm 0.10 \pm 0.01$			0.02 ± 0.06
270	$\eta K_0^*(1430)^+$	$0.05 \pm 0.13 \pm 0.02$	$0.05 \pm 0.13 \pm 0.02$				0.05 ± 0.13
271	$\eta K_2^*(1430)^+$	$-0.45 \pm 0.30 \pm 0.02$	$-0.45 \pm 0.30 \pm 0.02$				-0.45 ± 0.30
281	ωK^+	0.02 ± 0.05	$-0.01 \pm 0.07 \pm 0.01$	$-0.03 \pm 0.04 \pm 0.01$			-0.02 ± 0.04
282	ωK^{*+}	0.29 ± 0.35	$0.29 \pm 0.35 \pm 0.02$				0.29 ± 0.35
284	$\omega K_0^*(1430)^+$	-0.10 ± 0.09	$-0.10 \pm 0.09 \pm 0.02$				-0.10 ± 0.09
285	$\omega K_2^*(1430)^+$	0.14 ± 0.15	$0.14 \pm 0.15 \pm 0.02$				0.14 ± 0.15
288	$K^{*0}\pi^+$	-0.04 ± 0.09	$0.032 \pm 0.052^{+0.016}_{-0.013}$	$-0.149 \pm 0.064 \pm 0.022$			-0.038 ± 0.042
289	$K^{*+}\pi^0$	-0.06 ± 0.24	$-0.06 \pm 0.24 \pm 0.04$				-0.06 ± 0.24
290	$K^+\pi^+\pi^-$	0.033 ± 0.010	$0.028 \pm 0.020 \pm 0.023$	$0.049 \pm 0.026 \pm 0.020$		$0.025 \pm 0.004 \pm 0.008$	0.027 ± 0.008
293	$f_0(980)K^+$	-0.08 ± 0.09	$-0.106 \pm 0.050^{+0.036}_{-0.015}$	$-0.077 \pm 0.065^{+0.046}_{-0.026}$			$-0.095^{+0.049}_{-0.042}$
294	$f_2(1270)K^+$	$-0.68^{+0.19}_{-0.017}$	$-0.85 \pm 0.22^{+0.26}_{-0.13}$	$-0.59 \pm 0.22 \pm 0.04$			$-0.68^{+0.20}_{-0.18}$
295	$f_0(1370)K^+$	$0.28^{+0.30}_{-0.29}$	$0.28 \pm 0.26^{+0.15}_{-0.14}$				$0.28^{+0.30}_{-0.29}$
298	$\rho^0 K^+$	0.37 ± 0.10	$0.44 \pm 0.10^{+0.06}_{-0.14}$	$0.30 \pm 0.11^{+0.11}_{-0.05}$			0.37 ± 0.11
299	$K_0^*(1430)^0\pi^+$	0.055 ± 0.033	$0.032 \pm 0.035^{+0.034}_{-0.028}$	$0.076 \pm 0.038^{+0.028}_{-0.022}$			$0.055^{+0.034}_{-0.032}$
300	$K_2^*(1430)^0\pi^+$	$0.05^{+0.29}_{-0.24}$	$0.05 \pm 0.23^{+0.18}_{-0.08}$				$0.05^{+0.29}_{-0.24}$
303	$K^+\pi^0\pi^0$	-0.06 ± 0.07	$-0.06 \pm 0.06 \pm 0.04$				-0.06 ± 0.07
310	$\rho^+ K^0$	-0.12 ± 0.17	$-0.12 \pm 0.17 \pm 0.02$				-0.12 ± 0.17
311	$K^{*+}\pi^+\pi^-$	0.07 ± 0.08	$0.07 \pm 0.07 \pm 0.04$				0.07 ± 0.08
312	$K^{*+}\rho^0$	0.31 ± 0.13	$0.31 \pm 0.13 \pm 0.03$				0.31 ± 0.13
313	$f_0(980)K^{*+}$	-0.15 ± 0.12	$-0.15 \pm 0.12 \pm 0.03$				-0.15 ± 0.12
314	$a_1^+ K^0$	0.12 ± 0.11	$0.12 \pm 0.11 \pm 0.02$				0.12 ± 0.11
315	$b_1^+ K^0$	-0.03 ± 0.15	$-0.03 \pm 0.15 \pm 0.02$				-0.03 ± 0.15
312	$K^{*0}\rho^+$	-0.01 ± 0.16	$-0.01 \pm 0.16 \pm 0.02$				-0.01 ± 0.16
319	$b_1^0 K^+$	-0.46 ± 0.20	$-0.46 \pm 0.20 \pm 0.02$				-0.46 ± 0.20
322	$K^+ \bar{K}^0$	0.04 ± 0.14	$0.10 \pm 0.26 \pm 0.03$	$0.014 \pm 0.168 \pm 0.002$		$-0.21 \pm 0.14 \pm 0.01$	-0.087 ± 0.100
324	$K^+ K_S K_S$	$0.04^{+0.04}_{-0.05}$	$0.04^{+0.04}_{-0.05} \pm 0.02$				$0.04^{+0.04}_{-0.05}$
329	$K^+ K^- \pi^+$	-0.12 ± 0.05	$0.00 \pm 0.10 \pm 0.03$			$-0.123 \pm 0.017 \pm 0.014$	-0.118 ± 0.022
340	$K^+ K^- K^+$	-0.036 ± 0.012	$-0.017^{+0.019}_{-0.014} \pm 0.014$			$-0.036 \pm 0.004 \pm 0.007$	-0.033 ± 0.007
341	ϕK^+	0.04 ± 0.04	$0.128 \pm 0.044 \pm 0.013$	$0.01 \pm 0.12 \pm 0.05$	$-0.07 \pm 0.17^{+0.03}_{-0.02}$	$0.022 \pm 0.021 \pm 0.009$	0.041 ± 0.020
348	$K^{*+} K^+ K^-$	0.11 ± 0.09	$0.11 \pm 0.08 \pm 0.03$				0.11 ± 0.09
349	ϕK^{*+}	-0.01 ± 0.08	$0.00 \pm 0.09 \pm 0.04$	$-0.02 \pm 0.14 \pm 0.03$			-0.01 ± 0.08
351	$\phi K_1(1270)^+$	0.15 ± 0.20	$0.15 \pm 0.19 \pm 0.05$				0.15 ± 0.20
354	$\phi K_0^*(1430)^+$	0.04 ± 0.15	$0.04 \pm 0.15 \pm 0.04$				0.04 ± 0.15
355	$\phi K_2^*(1430)^+$	-0.23 ± 0.20	$-0.23 \pm 0.19 \pm 0.06$				-0.23 ± 0.20
359	$\phi \phi K^+$	-0.10 ± 0.08	-0.10 ± 0.08				-0.10 ± 0.08
363	$K^{*+}\gamma$	0.18 ± 0.29	$0.18 \pm 0.28 \pm 0.07$				0.18 ± 0.29
365	$K^+\eta\gamma$	-0.12 ± 0.07	$-0.09 \pm 0.10 \pm 0.01$	$-0.16 \pm 0.09 \pm 0.06$			-0.12 ± 0.07
367	$K^+\phi\gamma$	-0.13 ± 0.11	$-0.26 \pm 0.14 \pm 0.05$	$-0.03 \pm 0.11 \pm 0.08$			-0.13 ± 0.10
378	$\rho^+\gamma$	-0.11 ± 0.33		$-0.11 \pm 0.32 \pm 0.09$			-0.11 ± 0.33
379	$\pi^+\pi^0$	0.03 ± 0.04	$0.03 \pm 0.08 \pm 0.01$	$0.025 \pm 0.043 \pm 0.007$			0.026 ± 0.039
380	$\pi^+\pi^-\pi^+$	0.105 ± 0.029	$0.032 \pm 0.044^{+0.040}_{-0.037}$			$0.058 \pm 0.008 \pm 0.011$	0.057 ± 0.014
381	$\rho^0\pi^+$	$0.18^{+0.09}_{-0.17}$	$0.18 \pm 0.07^{+0.05}_{-0.15}$				$0.18^{+0.09}_{-0.17}$
383	$f_2(1270)\pi^+$	$0.41^{+0.31}_{-0.29}$	$0.41 \pm 0.25^{+0.18}_{-0.15}$				$0.41^{+0.31}_{-0.29}$
384	$\rho(1450)^0\pi^+$	$-0.06^{+0.36}_{-0.42}$	$-0.06 \pm 0.28^{+0.23}_{-0.32}$				$-0.06^{+0.36}_{-0.42}$
385	$f_0(1370)\pi^+$	0.72 ± 0.22	$0.72 \pm 0.15 \pm 0.16$				0.72 ± 0.22
387	$\pi^+\pi^-\pi^+(NR)$	$-0.14^{+0.23}_{-0.16}$	$-0.14 \pm 0.14^{+0.18}_{-0.08}$				$-0.14^{+0.23}_{-0.16}$
389	$\rho^+\pi^0$	0.02 ± 0.11	$-0.01 \pm 0.13 \pm 0.02$	$0.06 \pm 0.17^{+0.04}_{-0.05}$			0.02 ± 0.11
391	$\rho^+\rho^0$	-0.05 ± 0.05	$-0.054 \pm 0.055 \pm 0.010$	$0.00 \pm 0.22 \pm 0.03$			-0.051 ± 0.054
397	$\eta\pi^+$	-0.14 ± 0.07	$-0.03 \pm 0.09 \pm 0.03$	$-0.19 \pm 0.06 \pm 0.01$			-0.14 ± 0.05
398	$\eta\rho^+$	0.11 ± 0.11	$0.13 \pm 0.11 \pm 0.02$	$-0.04^{+0.34}_{-0.32} \pm 0.01$			0.11 ± 0.11
399	$\eta'\pi^+$	0.06 ± 0.16	$0.03 \pm 0.17 \pm 0.02$	$0.20^{+0.37}_{-0.36} \pm 0.04$			0.06 ± 0.15
400	$\eta'\rho^+$	0.26 ± 0.17	$0.26 \pm 0.17 \pm 0.02$				0.26 ± 0.17
401	$\omega\pi^+$	$-0.04 \pm 0.06^{(1)}$	$-0.02 \pm 0.08 \pm 0.01$	$-0.02 \pm 0.09 \pm 0.01$			-0.02 ± 0.06
402	$\omega\rho^+$	-0.20 ± 0.09	$-0.20 \pm 0.09 \pm 0.02$				-0.20 ± 0.09
408	$b_1^0\pi^+$	0.05 ± 0.16	$0.05 \pm 0.16 \pm 0.02$				0.05 ± 0.16
417	$p\bar{p}\pi^+$	0.00 ± 0.04	$0.04 \pm 0.07 \pm 0.04$	$-0.17 \pm 0.10 \pm 0.02$			-0.04 ± 0.06
420	$p\bar{p}K^+$	-0.08 ± 0.04	$-0.16 \pm 0.08 \pm 0.04$	$-0.02 \pm 0.05 \pm 0.02$		$-0.047 \pm 0.036 \pm 0.007$	-0.051 ± 0.029
425	$p\bar{p}K^{*+}$	0.21 ± 0.16	$0.32 \pm 0.13 \pm 0.05$	$-0.01 \pm 0.19 \pm 0.02$			0.21 ± 0.11
428	$p\bar{A}\gamma$	0.17 ± 0.17		$0.17 \pm 0.16 \pm 0.05$			0.17 ± 0.17
429	$p\bar{A}\pi^0$	0.01 ± 0.17		$0.01 \pm 0.17 \pm 0.04$			0.01 ± 0.17
471	$K^+\ell\ell$	-0.02 ± 0.08	$-0.03 \pm 0.14 \pm 0.01$	$0.04 \pm 0.10 \pm 0.02$			0.02 ± 0.08
472	$K^+e^+e^-$	0.14 ± 0.14		$0.14 \pm 0.14 \pm 0.03$			0.14 ± 0.14
473	$K^+\mu^+\mu^-$	-0.003 ± 0.033		$-0.05 \pm 0.13 \pm 0.03$		$0.000 \pm 0.033 \pm 0.009$	-0.003 ± 0.033
478	$K^{*+}\ell\ell$	-0.09 ± 0.14	$0.01^{+0.26}_{-0.24} \pm 0.02$	$-0.13^{+0.17}_{-0.16} \pm 0.01$			$-0.09^{+0.14}_{-0.13}$
479	$K^{*+}e^+e^-$	$-0.14^{+0.23}_{-0.22}$		$-0.14^{+0.23}_{-0.22} \pm 0.02$			$-0.14^{+0.23}_{-0.22}$
480	$K^{*+}\mu^+\mu^-$	-0.12 ± 0.24		$-0.12 \pm 0.24 \pm 0.02$			-0.12 ± 0.24

† PDG takes the value from the BABAR amplitude analysis of $B^+ \rightarrow K^+ K^- K^+$ decays, while our numbers are from amplitude analyses of $B^+ \rightarrow K^+ \pi^- \pi^+$; ‡ PDG swaps the BELLE results corresponding to $A_{CP}(p\bar{p}\pi^+)$ and $A_{CP}(p\bar{p}K^+)$; § PDG uses also a previous result from BABAR ([732]); ⁽¹⁾ PDG uses also a result from CLEO.

Table 204: CP asymmetries of charmless hadronic B^0 decays. Values in red (blue) are new published (preliminary) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	BABAR	Belle	CDF	LHCb	New Avg.
227	$K^+\pi^-$	$-0.082 \pm 0.006^{(1)}$	$-0.107 \pm 0.016^{+0.006}_{-0.004}$	$-0.069 \pm 0.014 \pm 0.007$	$-0.083 \pm 0.013 \pm 0.004$	$-0.080 \pm 0.007 \pm 0.003$	-0.082 ± 0.006
230	$\eta' K^{*0}$	0.02 ± 0.23	$0.02 \pm 0.23 \pm 0.02$	$-0.22 \pm 0.29 \pm 0.07$			-0.07 ± 0.18
231	$\eta' K_0^*(1430)^0$	-0.19 ± 0.17	$-0.19 \pm 0.17 \pm 0.02$				-0.19 ± 0.17
232	$\eta' K_2^*(1430)^0$	0.14 ± 0.18	$0.14 \pm 0.18 \pm 0.02$				0.14 ± 0.18
234	ηK^{*0}	0.19 ± 0.05	$0.21 \pm 0.06 \pm 0.02$	$0.17 \pm 0.08 \pm 0.01$			0.19 ± 0.05
235	$\eta K_0^*(1430)^0$	0.06 ± 0.13	$0.06 \pm 0.13 \pm 0.02$				0.06 ± 0.13
236	$\eta K_2^*(1430)^0$	-0.07 ± 0.19	$-0.07 \pm 0.19 \pm 0.02$				-0.07 ± 0.19
241	$b_1^- K^+$	-0.07 ± 0.12	$-0.07 \pm 0.12 \pm 0.02$				-0.07 ± 0.12
246	ωK^{*0}	0.45 ± 0.25	$0.45 \pm 0.25 \pm 0.02$				0.45 ± 0.25
248	$\omega K_0^*(1430)^0$	-0.07 ± 0.09	$-0.07 \pm 0.09 \pm 0.02$				-0.07 ± 0.09
249	$\omega K_2^*(1430)^0$	-0.37 ± 0.17	$-0.37 \pm 0.17 \pm 0.02$				-0.37 ± 0.17
251	$K^+\pi^-\pi^0$	0.00 ± 0.06	$-0.030^{+0.045}_{-0.051} \pm 0.055$	$0.07 \pm 0.11 \pm 0.01$			$0.000^{+0.059}_{-0.061}$
252	$\rho^- K^+$	0.20 ± 0.11	$0.20 \pm 0.09 \pm 0.08$	$0.22^{+0.22+0.06}_{-0.23-0.02}$			0.20 ± 0.11
253	$\rho(1450)^- K^+$	-0.10 ± 0.33	$-0.10 \pm 0.32 \pm 0.09$				-0.10 ± 0.33
254	$\rho(1700)^- K^+$	-0.36 ± 0.61	$-0.36 \pm 0.57 \pm 0.23$				-0.36 ± 0.61
255	$K^+\pi^-\pi^0(NR)$	0.10 ± 0.18	$0.10 \pm 0.16 \pm 0.08$				0.10 ± 0.18
257	$K_0^*(1430)^0 \pi^0$	-0.15 ± 0.11	$-0.15 \pm 0.10 \pm 0.04$				-0.15 ± 0.11
261	$K^0 \pi^+ \pi^-$	-0.01 ± 0.05	$-0.01 \pm 0.05 \pm 0.01$				-0.01 ± 0.05
264	$K^{*+} \pi^-$	$-0.22 \pm 0.06^{(1)}$	$-0.24 \pm 0.07 \pm 0.02^{(2)}$	$-0.21 \pm 0.11 \pm 0.07$			-0.23 ± 0.06
265	$K_0^*(1430)^+ \pi^-$	0.09 ± 0.07	$0.09 \pm 0.07 \pm 0.03$				0.09 ± 0.08
271	$K^{*0} \pi^0$	-0.15 ± 0.13	$-0.15 \pm 0.12 \pm 0.04$				-0.15 ± 0.13
278	$K^{*0} \pi^+ \pi^-$	0.07 ± 0.05	$0.07 \pm 0.04 \pm 0.03$				0.07 ± 0.05
279	$K^{*0} \rho^0$	-0.06 ± 0.09	$-0.06 \pm 0.09 \pm 0.02$				-0.06 ± 0.09
280	$f_0(980) K^{*0}$	0.07 ± 0.10	$0.07 \pm 0.10 \pm 0.02$				0.07 ± 0.10
283	$a_1^- K^+$	-0.16 ± 0.12	$-0.16 \pm 0.12 \pm 0.01$				-0.16 ± 0.12
284	$K^{*+} \rho^-$	0.21 ± 0.15	$0.21 \pm 0.15 \pm 0.02$				0.21 ± 0.15
311	$K^{*0} K^+ K^-$	0.01 ± 0.05	$0.01 \pm 0.05 \pm 0.02$				0.01 ± 0.05
312	ϕK^{*0}	0.00 ± 0.04	$0.01 \pm 0.06 \pm 0.03$	$-0.007 \pm 0.048 \pm 0.021$		$-0.015 \pm 0.032 \pm 0.10^\dagger$	-0.003 ± 0.038
314	$K^{*0} \pi^+ K^-$	0.22 ± 0.39	$0.22 \pm 0.33 \pm 0.20$				0.22 ± 0.39
326	$\phi K_0^*(1430)^0$	0.12 ± 0.08	$0.20 \pm 0.14 \pm 0.06$	$0.093 \pm 0.094 \pm 0.017$			0.124 ± 0.081
333	$\phi K_2^*(1430)^0$	-0.11 ± 0.10	$-0.08 \pm 0.12 \pm 0.05$	$-0.155^{+0.152}_{-0.133} \pm 0.033$			$-0.113^{+0.102}_{-0.096}$
340	$K^{*0} \gamma$	-0.002 ± 0.015	$-0.016 \pm 0.022 \pm 0.007$			$0.008 \pm 0.017 \pm 0.009$	-0.002 ± 0.015
357	$\pi^0 \pi^0$	0.43 ± 0.14	$0.43 \pm 0.26 \pm 0.05$	$0.44^{+0.52}_{-0.55} \pm 0.17$			0.43 ± 0.24
391	$a_1^\mp \pi^\pm$	-0.07 ± 0.06	$-0.07 \pm 0.07 \pm 0.02$	$-0.06 \pm 0.05 \pm 0.07$			-0.07 ± 0.06
400	$b_1^\mp \pi^\pm$	-0.05 ± 0.10	$-0.05 \pm 0.10 \pm 0.02$				-0.05 ± 0.10
412	$p\bar{p} K^{*0}$	0.05 ± 0.12	$0.11 \pm 0.13 \pm 0.06$	$-0.08 \pm 0.20 \pm 0.02$			0.05 ± 0.12
414	$p\bar{A}\pi^-$	0.04 ± 0.07	$-0.10 \pm 0.10 \pm 0.02^{(3)}$	$-0.02 \pm 0.10 \pm 0.03$			-0.06 ± 0.07
477	$K^{*0} \ell \ell$	-0.05 ± 0.10	$0.02 \pm 0.20 \pm 0.02$	$-0.08 \pm 0.12 \pm 0.02$			-0.05 ± 0.10
478	$K^{*0} e^+ e^-$	-0.21 ± 0.19		$-0.21 \pm 0.19 \pm 0.02$			-0.21 ± 0.19
479	$K^{*0} \mu^+ \mu^-$	-0.07 ± 0.04		$0.00 \pm 0.15 \pm 0.03$		$-0.072 \pm 0.040 \pm 0.005$	-0.067 ± 0.039

Measurements of time-dependent CP asymmetries are listed in Sec. 4.

† Extracted from measured $\Delta A_{CP} = A_{CP}(\phi K^{*0}) - A_{CP}(J/\psi K^{*0}) = 0.015 \pm 0.032 \pm 0.005$.

$^{(1)}$ PDG uses also a result from CLEO.

$^{(2)}$ Average of BABAR results from $B^0 \rightarrow K^+ \pi^- \pi^0$ and $B^0 \rightarrow K^0 \pi^+ \pi^-$.

$^{(3)}$ PDG quotes the opposite asymmetry.

Table 205: CP asymmetries of charmless hadronic decays of $B^\pm/B^0$ admixture. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	<i>BABAR</i>	Belle	New Avg.
65	$K^*\gamma$	-0.003 ± 0.017 ‡	$-0.003 \pm 0.017 \pm 0.007$	$-0.015 \pm 0.044 \pm 0.012$	-0.005 ± 0.017
77	$s\gamma$	-0.008 ± 0.029	0.017 ± 0.019 ± 0.010 §	$0.002 \pm 0.050 \pm 0.030$	0.015 ± 0.020
80	$s\eta$	$-0.13^{+0.04}_{-0.05}$		$-0.13 \pm 0.04^{+0.02}_{-0.03}$	$-0.13^{+0.04}_{-0.05}$
86	π^+X	0.10 ± 0.17	$0.10 \pm 0.16 \pm 0.05$		0.10 ± 0.17
121	$s\ell\ell$	-0.22 ± 0.26	0.04 ± 0.11 ± 0.01		0.04 ± 0.11
126	$K^*e^+e^-$	-0.18 ± 0.15		$-0.18 \pm 0.15 \pm 0.01$	-0.18 ± 0.15
128	$K^*\mu^+\mu^-$	-0.03 ± 0.13		$-0.03 \pm 0.13 \pm 0.02$	-0.03 ± 0.13
129	$K\ell\ell$	New	-0.03 ± 0.14 ± 0.01		-0.03 ± 0.14
130	$K^*\ell\ell$	-0.04 ± 0.07	$0.03 \pm 0.13 \pm 0.01$ †	$-0.10 \pm 0.10 \pm 0.01$	-0.05 ± 0.08

§ *BABAR* also measures the difference in direct CP asymmetry for charged and neutral B mesons: $\Delta A_{CP} = +(5.0 \pm 3.9 \pm 1.5)\%$.

† Previous *BABAR* result is also included in the PDG Average.

‡ PDG include also a result from CLEO.

Table 206: CP asymmetries of charmless hadronic B_s^0 decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	Belle	CDF	LHCb	New Avg.
52	π^+K^-	0.28 ± 0.04		0.22 ± 0.07 ± 0.02	$0.27 \pm 0.04 \pm 0.01$	0.26 ± 0.04

Table 207: CP asymmetries of charmless hadronic Λ_b^0 baryons decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	CDF	LHCb	New Avg.
21	$p\pi^-$	0.03 ± 0.18	0.06 ± 0.07 ± 0.03		0.06 ± 0.08
22	pK^-	0.37 ± 0.17	-0.10 ± 0.08 ± 0.04		-0.10 ± 0.09
—	$\bar{K}^0 p\pi^-$	New		0.22 ± 0.13 ± 0.03	0.22 ± 0.13

7.7 Polarization measurements in b -hadron decays

In this section, compilations of polarization measurements in b -hadron decays are given. Tables 208 (209) detail measurements of the longitudinal fraction, f_L , in B^+ (B^0) decays, and Tables 210 (211) the results of the full angular analyses of B^+ (B^0) $\rightarrow \phi K^*$ decays. Table 212 gives results of the full angular analysis of $B^0 \rightarrow \phi K_2^{*0}(1430)$ decays. Tables 213 to 215 detail quantities of B_s^0 decays: f_L measurements, and observables from full angular analyses of decays to $\phi\phi$ and $\phi\bar{K}^{*0}$.

Table 208: Longitudinal polarization fraction f_L for B^+ decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	<i>BABAR</i>	<i>Belle</i>	New Avg.
282	ωK^{*+}	$0.41 \pm 0.18 \pm 0.05$	$0.41 \pm 0.18 \pm 0.05$		0.41 ± 0.19
285	$\omega K_2^{*+}(1430)^+$	$0.56 \pm 0.10 \pm 0.04$	$0.56 \pm 0.10 \pm 0.04$		0.56 ± 0.11
312	$K^{*+}\rho^0$	$0.78 \pm 0.12 \pm 0.03$	$0.78 \pm 0.12 \pm 0.03$		0.78 ± 0.12
316	$K^{*0}\rho^+$	0.48 ± 0.08	$0.52 \pm 0.10 \pm 0.04$	$0.43 \pm 0.11^{+0.05}_{-0.02}$	0.48 ± 0.08
338	$K^{*+}\bar{K}^{*0}$	$0.75^{+0.16}_{-0.26} \pm 0.03$	$0.75^{+0.16}_{-0.26} \pm 0.03$		$0.75^{+0.16}_{-0.26}$
349	ϕK^{*+}	0.50 ± 0.05	$0.49 \pm 0.05 \pm 0.03$	$0.52 \pm 0.08 \pm 0.03$	0.50 ± 0.05
351	$\phi K_1(1270)^+$	$0.46^{+0.12+0.06}_{-0.13-0.07}$	$0.46^{+0.12+0.06}_{-0.13-0.07}$		$0.46^{+0.13}_{-0.15}$
355	$\phi K_2^{*+}(1430)^+$	$0.80^{+0.09}_{-0.10} \pm 0.03$	$0.80^{+0.09}_{-0.10} \pm 0.03$		0.80 ± 0.10
391	$\rho^+\rho^0$	0.950 ± 0.016	$0.950 \pm 0.015 \pm 0.006$	$0.95 \pm 0.11 \pm 0.02$	0.950 ± 0.016
396	$\omega\rho^+$	$0.90 \pm 0.05 \pm 0.03$	$0.90 \pm 0.05 \pm 0.03$		0.90 ± 0.06

Table 209: Longitudinal polarization fraction f_L for B^0 decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	<i>BABAR</i>	<i>Belle</i>	<i>LHCb</i>	New Avg.
246	ωK^{*0}	0.69 ± 0.13	$0.72 \pm 0.14 \pm 0.02$	$0.56 \pm 0.29^{+0.18}_{-0.08}$		0.70 ± 0.13
249	$\omega K_2^{*0}(1430)^0$	$0.45 \pm 0.12 \pm 0.02$	$0.45 \pm 0.12 \pm 0.02$			0.45 ± 0.12
279	$\bar{K}^{*0}\rho^0$	$0.40 \pm 0.08 \pm 0.11$	$0.40 \pm 0.08 \pm 0.11$			0.40 ± 0.14
284	$K^{*+}\rho^-$	$0.38 \pm 0.13 \pm 0.03$	$0.38 \pm 0.13 \pm 0.03$			0.38 ± 0.13
312	ϕK^{*0}	0.497 ± 0.025	$0.494 \pm 0.034 \pm 0.013$	$0.499 \pm 0.030 \pm 0.018$	$0.497 \pm 0.019 \pm 0.015$	0.497 ± 0.017
315	$K^{*0}\bar{K}^{*0}$	$0.80^{+0.10}_{-0.12} \pm 0.06$	$0.80^{+0.10}_{-0.12} \pm 0.06$			$0.80^{+0.12}_{-0.13}$
333	$\phi K_2^{*0}(1430)^0$	$0.901^{+0.046}_{-0.058} \pm 0.037$	$0.901^{+0.046}_{-0.058} \pm 0.037$			$0.901^{+0.059}_{-0.069}$
386	$\rho^0\rho^0$	$0.75^{+0.11}_{-0.14} \pm 0.05$	$0.75^{+0.11}_{-0.14} \pm 0.05$	$0.21^{+0.18}_{-0.22} \pm 0.15$		0.59 ± 0.13
394	$\rho^+\rho^-$	$0.977^{+0.028}_{-0.024}$	$0.992 \pm 0.024^{+0.026}_{-0.013}$	$0.941^{+0.034}_{-0.040} \pm 0.030$		$0.978^{+0.025}_{-0.022}$
405	$a_1^+a_1^-$	$0.31 \pm 0.22 \pm 0.10$	$0.31 \pm 0.22 \pm 0.10$			0.31 ± 0.24

Table 210: Results of the full angular analyses of $B^+ \rightarrow \phi K^{*+}$. Values in red (blue) are new published (preliminary) results since PDG2014.

Parameter	PDG2014 Avg.	<i>BABAR</i>	Belle	New Avg.
$f_{\perp} = A_{\perp\perp}$	0.20 ± 0.05	$0.21 \pm 0.05 \pm 0.02$	$0.19 \pm 0.08 \pm 0.02$	0.20 ± 0.05
$\phi_{\parallel}$	2.34 ± 0.18	$2.47 \pm 0.20 \pm 0.07$	$2.10 \pm 0.28 \pm 0.04$	2.34 ± 0.17
$\phi_{\perp}$	2.58 ± 0.17	$2.69 \pm 0.20 \pm 0.03$	$2.31 \pm 0.30 \pm 0.07$	2.58 ± 0.17
δ_0	$3.07 \pm 0.18 \pm 0.06$	$3.07 \pm 0.18 \pm 0.06$		3.07 ± 0.19
A_{CP}^0	$0.17 \pm 0.11 \pm 0.02$	$0.17 \pm 0.11 \pm 0.02$		0.17 ± 0.11
$A_{CP}^{\perp}$	$0.22 \pm 0.24 \pm 0.08$	$0.22 \pm 0.24 \pm 0.08$		0.22 ± 0.25
$\Delta\phi_{\parallel}$	$0.07 \pm 0.20 \pm 0.05$	$0.07 \pm 0.20 \pm 0.05$		0.07 ± 0.21
$\Delta\phi_{\perp}$	$0.19 \pm 0.20 \pm 0.07$	$0.19 \pm 0.20 \pm 0.07$		0.19 ± 0.21
$\Delta\delta_0$	$0.20 \pm 0.18 \pm 0.03$	$0.20 \pm 0.18 \pm 0.03$		0.20 ± 0.18

Angles (ϕ , δ) are in radians. BF, f_L and A_{CP} are tabulated separately.

Table 211: Results of the full angular analyses of $B^0 \rightarrow \phi K^{*0}$. Values in red (blue) are new published (preliminary) results since PDG2014.

Parameter	PDG2014 Avg.	<i>BABAR</i>	Belle	LHCb	New Avg.
$f_{\perp} = A_{\perp\perp}$	0.228 ± 0.021	$0.212 \pm 0.032 \pm 0.013$	$0.238 \pm 0.026 \pm 0.008$	$0.221 \pm 0.016 \pm 0.013$	0.225 ± 0.015
$f_S(K\pi)$	New			$0.143 \pm 0.013 \pm 0.012$	0.143 ± 0.018
$f_S(KK)$	New			$0.122 \pm 0.013 \pm 0.008$	0.122 ± 0.015
$\phi_{\parallel}$	2.28 ± 0.08	$2.40 \pm 0.13 \pm 0.08$	$2.23 \pm 0.10 \pm 0.02$	$2.562 \pm 0.069 \pm 0.040$	2.430 ± 0.058
$\phi_{\perp}$	2.36 ± 0.09	$2.35 \pm 0.13 \pm 0.09$	$2.37 \pm 0.10 \pm 0.04$	$2.633 \pm 0.062 \pm 0.037$	2.527 ± 0.056
δ_0	2.88 ± 0.10	$2.82 \pm 0.15 \pm 0.09$	$2.91 \pm 0.10 \pm 0.08$		2.88 ± 0.10
$\phi_S(K\pi)^\dagger$	New			$2.222 \pm 0.063 \pm 0.081$	2.222 ± 0.103
$\phi_S(KK)^\dagger$	New			$2.481 \pm 0.072 \pm 0.048$	2.481 ± 0.087
A_{CP}^0	-0.01 ± 0.05	$0.01 \pm 0.07 \pm 0.02$	$-0.03 \pm 0.06 \pm 0.01$	$-0.003 \pm 0.038 \pm 0.005$	-0.007 ± 0.030
$A_{CP}^{\perp}$	-0.11 ± 0.09	$-0.04 \pm 0.15 \pm 0.06$	$-0.14 \pm 0.11 \pm 0.01$	$0.047 \pm 0.072 \pm 0.009$	-0.014 ± 0.057
$\mathcal{A}_{CP}^S(K\pi)$	New			$0.073 \pm 0.091 \pm 0.035$	0.073 ± 0.097
$\mathcal{A}_{CP}^S(KK)$	New			$-0.209 \pm 0.105 \pm 0.012$	-0.209 ± 0.106
$\Delta\phi_{\parallel}$	0.06 ± 0.11	$0.22 \pm 0.12 \pm 0.08$	$-0.02 \pm 0.10 \pm 0.01$	$0.045 \pm 0.068 \pm 0.015$	0.051 ± 0.053
$\Delta\phi_{\perp}$	0.10 ± 0.08	$0.21 \pm 0.13 \pm 0.08$	$0.05 \pm 0.10 \pm 0.02$	$0.062 \pm 0.062 \pm 0.006$	0.075 ± 0.050
$\Delta\delta_0$	0.13 ± 0.09	$0.27 \pm 0.14 \pm 0.08$	$0.08 \pm 0.10 \pm 0.01$		0.13 ± 0.08
$\Delta\phi_S(K\pi)^\dagger$	New			$0.062 \pm 0.062 \pm 0.022$	0.062 ± 0.066
$\Delta\phi_S(KK)^\dagger$	New			$0.022 \pm 0.072 \pm 0.004$	0.022 ± 0.072

Angles (ϕ , δ) are in radians. BF, f_L and A_{CP} are tabulated separately.

† Original LHCb notation adapted to match similar existing quantities.

Table 212: Results of the full angular analyses of $B^0 \rightarrow \phi K_2^{*0}(1430)$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Parameter	PDG2014 Avg.	<i>BABAR</i>	<i>Belle</i>	New Avg.
$f_{\perp} = A_{\perp\perp}$	$0.027^{+0.031}_{-0.025}$	$0.002^{+0.018}_{-0.002} \pm 0.031$	$0.056^{+0.050}_{-0.035} \pm 0.009$	$0.027^{+0.027}_{-0.024}$
$\phi_{\parallel}$	4.0 ± 0.4	$3.96 \pm 0.38 \pm 0.06$	$3.76 \pm 2.88 \pm 1.32$	3.96 ± 0.38
$\phi_{\perp}$	4.5 ± 0.4		$4.45^{+0.43}_{-0.38} \pm 0.13$	$4.45^{+0.45}_{-0.40}$
δ_0	3.46 ± 0.14	$3.41 \pm 0.13 \pm 0.13$	$3.53 \pm 0.11 \pm 0.19$	3.46 ± 0.14
A_{CP}^0	-0.03 ± 0.04	$-0.05 \pm 0.06 \pm 0.01$	$-0.016^{+0.066}_{-0.051} \pm 0.008$	$-0.032^{+0.043}_{-0.038}$
$A_{CP}^{\perp}$	$0.0^{+0.9}_{-0.7}$		$-0.01^{+0.85}_{-0.67} \pm 0.09$	$-0.01^{+0.85}_{-0.68}$
$\Delta\phi_{\parallel}$	-0.9 ± 0.4	$-1.00 \pm 0.38 \pm 0.09$	$-0.02 \pm 1.08 \pm 1.01$	-0.94 ± 0.38
$\Delta\phi_{\perp}$	-0.2 ± 0.4		$-0.19 \pm 0.42 \pm 0.11$	-0.19 ± 0.43
$\Delta\delta_0$	0.08 ± 0.09	$0.11 \pm 0.13 \pm 0.06$	$0.06 \pm 0.11 \pm 0.02$	0.08 ± 0.09

Angles (ϕ , δ) are in radians. BF, f_L and A_{CP} are tabulated separately.

Table 213: Longitudinal polarization fraction f_L for B_s^0 decays. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

RPP#	Mode	PDG2014 Avg.	CDF	LHCb	New Avg.
51	$\phi\phi$	0.361 ± 0.022	$0.348 \pm 0.041 \pm 0.021$	$0.365 \pm 0.022 \pm 0.012$	0.361 ± 0.022
59	$K^{*0}\bar{K}^{*0}$	0.31 ± 0.13		$0.31 \pm 0.12 \pm 0.04$	0.31 ± 0.13
60	$\phi\bar{K}^{*0}$	0.51 ± 0.17		$0.51 \pm 0.15 \pm 0.07$	0.51 ± 0.17

Table 214: Results of the full angular analyses of $B_s \rightarrow \phi\phi$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Parameter	PDG2014 Avg.	CDF	LHCb	New Avg.
$f_{\perp} = A_{\perp\perp}$	0.306 ± 0.030	$0.365 \pm 0.044 \pm 0.027$	$0.291 \pm 0.024 \pm 0.010$	0.306 ± 0.023
$\phi_{\parallel}$	2.59 ± 0.15	$2.71^{+0.31}_{-0.36} \pm 0.22$	$2.57 \pm 0.15 \pm 0.06$	2.59 ± 0.15

The parameter ϕ is in radians. BF, f_L and A_{CP} are tabulated separately.

Table 215: Results of the full angular analyses of $B_s \rightarrow \phi \bar{K}^{*0}$. Values in **red** (**blue**) are new **published** (**preliminary**) results since PDG2014.

Parameter	PDG2014 Avg.	LHCb	New Avg.
$f_{\perp} = A_{\perp\perp}$		$0.28 \pm 0.12 \pm 0.03$	0.28 ± 0.12
f_0		$0.51 \pm 0.15 \pm 0.07$	0.51 ± 0.17
$f_{\parallel}$	0.21 ± 0.11	$0.21 \pm 0.11 \pm 0.02$	0.21 ± 0.11
$\phi_{\parallel}^{\dagger}$	$1.75 \pm 0.53 \pm 0.29$	$1.75^{+0.59+0.38}_{-0.53-0.30}$	$1.75^{+0.70}_{-0.61}$

The parameter ϕ is in radians. BF, f_L and A_{CP} are tabulated separately.

$\dagger$ Converted from the measurement of $\cos(\phi_{\parallel})$. PDG takes the smallest resulting asymmetric error as parabolic.

8 D decays

8.1 D^0 - $\bar{D}^0$ mixing and CP violation

8.1.1 Introduction

In 2007 Belle [733] and *BABAR* [734] obtained the first evidence for D^0 - $\bar{D}^0$ mixing, which had been searched for for more than two decades. These results were later confirmed by CDF [735], and more recently by LHCb [736]. There are now numerous measurements of D^0 - $\bar{D}^0$ mixing with various levels of sensitivity. All the results are input into a global fit to determine world averages of mixing parameters, CP -violation (CPV) parameters, and strong phases.

Our notation is as follows. The mass eigenstates are denoted $D_1 = p|D^0\rangle - q|\bar{D}^0\rangle$ and $D_2 = p|D^0\rangle + q|\bar{D}^0\rangle$, where we use the convention $CP|D^0\rangle = -|\bar{D}^0\rangle$ and $CP|\bar{D}^0\rangle = -|D^0\rangle$. Thus in the absence of CP violation, D_1 is CP -even and D_2 is CP -odd. The weak phase ϕ is defined as $\text{Arg}(q/p)$. The mixing parameters are defined as $x \equiv (m_1 - m_2)/\Gamma$ and $y \equiv (\Gamma_1 - \Gamma_2)/(2\Gamma)$, where m_1, m_2 and Γ_1, Γ_2 are the masses and decay widths for the mass eigenstates, and $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$.

The global fit determines central values and errors for ten underlying parameters. These consist of the mixing parameters x and y ; indirect CPV parameters $|q/p|$ and ϕ ; the ratio of decay rates $R_D \equiv [\Gamma(D^0 \rightarrow K^+\pi^-) + \Gamma(\bar{D}^0 \rightarrow K^-\pi^+)]/[\Gamma(D^0 \rightarrow K^-\pi^+) + \Gamma(\bar{D}^0 \rightarrow K^+\pi^-)]$, direct CPV parameters A_K, A_π (see Table 219), and $A_D = (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$, where the $+$ ($-$) superscript corresponds to D^0 ($\bar{D}^0$) decays; the strong phase difference δ between $\bar{D}^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^-\pi^+$ amplitudes; and the strong phase difference $\delta_{K\pi\pi}$ between $\bar{D}^0 \rightarrow K^-\rho^+$ and $D^0 \rightarrow K^-\rho^+$ amplitudes.

The fit uses 45 observables taken from measurements of $D^0 \rightarrow K^+\ell^-\nu$, $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$, $D^0 \rightarrow K^+\pi^-$, $D^0 \rightarrow K^+\pi^-\pi^0$, $D^0 \rightarrow K_S^0\pi^+\pi^-$, and $D^0 \rightarrow K_S^0K^+K^-$ decays,⁵⁶ and from double-tagged branching fractions measured at the $\psi(3770)$ resonance. Correlations among observables are accounted for by using covariance matrices provided by the experimental collaborations. Errors are assumed to be Gaussian, and systematic errors among different experiments are assumed uncorrelated unless specific correlations have been identified. We have checked this method with a second method that adds together three-dimensional log-likelihood functions for x, y , and δ obtained from several analyses; this combination accounts for non-Gaussian errors. When both methods are applied to the same set of measurements, equivalent results are obtained.

Mixing in heavy flavor systems such as those of B^0 and B_s^0 is governed by a short-distance box diagram. In the D^0 system, however, this diagram is doubly-Cabibbo-suppressed relative to amplitudes dominating the decay width, and it is also GIM-suppressed. Thus the short-distance mixing rate is tiny, and D^0 - $\bar{D}^0$ mixing is expected to be dominated by long-distance processes. These are difficult to calculate reliably, and theoretical estimates for x and y range by up to three orders of magnitude [737–740].

Almost all methods besides that of the $\psi(3770) \rightarrow DD$ measurements [741] identify the flavor of the D^0 or $\bar{D}^0$ when produced by reconstructing the decay $D^{*+} \rightarrow D^0\pi^+$ or $D^{*-} \rightarrow \bar{D}^0\pi^-$. The charge of the pion, which has low momentum and is usually referred to as the “soft” pion (π_s), identifies the D flavor. For signal decays, $M_{D^*} - M_{D^0} - M_{\pi^+} \equiv Q \approx 6$ MeV, which is close to the threshold; thus analyses typically require that the reconstructed Q be

⁵⁶Charge-conjugate modes are implicitly included.

small to suppress backgrounds. A recent LHCb measurement [742] of the difference between time-integrated CP asymmetries $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$ identifies the flavor of the D^0 by partially reconstructing $\bar{B}^0 \rightarrow D^0 \mu^- X$ decays (and charge-conjugates); in this case the charge of the muon identifies the flavor of the D^0 .

For time-dependent measurements, the D^0 decay time is calculated as $(\vec{d} \cdot \vec{p})/p^2 \times M_{D^0}$, where $\vec{d}$ is the displacement vector between the D^* and D^0 decay vertices, and $\vec{p}$ is the D^0 momentum. The D^* vertex position is taken to be at the primary vertex for $\bar{p}p$ and pp experiments [735, 736], and at the intersection of the D^0 momentum vector with the beamspot profile for e^+e^- experiments.

8.1.2 Input observables

The global fit determines central values and errors for the underlying parameters using a χ^2 statistic. The fitted parameters are x , y , R_D , A_D , $|q/p|$, ϕ , δ , $\delta_{K\pi\pi}$, A_K and A_π . In the $D \rightarrow K^+\pi^-\pi^0$ Dalitz plot analysis that provides sensitivity to x and y , the $\bar{D}^0 \rightarrow K^+\pi^-\pi^0$ isobar phases are determined relative to that for $\mathcal{A}(\bar{D}^0 \rightarrow K^+\rho^-)$, and the $D^0 \rightarrow K^+\pi^-\pi^0$ isobar phases are determined relative to that for $\mathcal{A}(D^0 \rightarrow K^+\rho^-)$. As the $\bar{D}^0$ and D^0 Dalitz plots are fit independently, the phase difference $\delta_{K\pi\pi}$ between the two “normalizing amplitudes” cannot be determined from these fits.

All input measurements are listed in Tables 216-218. The observable $R_M = (x^2 + y^2)/2$ is calculated from $D^0 \rightarrow K^+\ell^-\nu$ decays [743–746] and is the world average (WA) value calculated by HFAG [747]. The inputs used for this average are plotted in Fig. 152. The observables $y_{CP} = (1/2)(|q/p| + |p/q|)y \cos \phi - (1/2)(|q/p| - |p/q|)x \sin \phi$ and $A_F = (1/2)(|q/p| - |p/q|)y \cos \phi - (1/2)(|q/p| + |p/q|)x \sin \phi$ are also HFAG WA values [747]; the inputs used for these averages are plotted in Figs. 153 and 154, respectively. The $D^0 \rightarrow K^+\pi^-$ observables used are from Belle [748, 749], BABAR [734], CDF [750], and LHCb [736]; earlier measurements have much less precision and are not used. The observables from $D^0 \rightarrow K_S^0 \pi^+\pi^-$ decays are measured in two ways: assuming CP conservation (D^0 and $\bar{D}^0$ decays are combined), and allowing for CP violation (D^0 and $\bar{D}^0$ decays are fitted separately). The no- CPV measurements are from Belle [751] and BABAR [752], but for the CPV -allowed case only Belle measurements [751] are available. The $D^0 \rightarrow K^+\pi^-\pi^0$ results are from BABAR [753], and the $\psi(3770) \rightarrow \bar{D}D$ results are from CLEOc [741].

The relationships between the observables and the fitted parameters are listed in Table 219. For each set of correlated observables we construct a difference vector $\vec{V}$ between measured values and those calculated from fitted parameters using the relations of Table 219; *e.g.* for $D^0 \rightarrow K_S^0 \pi^+\pi^-$ decays, $\vec{V} = (\Delta x, \Delta y, \Delta|q/p|, \Delta\phi)$. The contribution of a set of observables to the χ^2 is calculated as $\vec{V} \cdot (M^{-1}) \cdot \vec{V}^T$, where M^{-1} is the inverse of the covariance matrix for the measurement. Covariance matrices are constructed from the correlation coefficients among the measured observables. These coefficients (where applicable) are also listed in Tables 216-218.

8.1.3 Fit results

The global fit uses MINUIT with the MIGRAD minimizer, and all errors are obtained from MINOS [767]. Four separate fits are performed: (a) assuming CP conservation, *i.e.* fixing $A_D=0$, $A_K=0$, $A_\pi=0$, $\phi=0$, and $|q/p|=1$; (b) assuming no direct CPV in doubly Cabibbo-suppressed (DCS) decays and fitting for parameters $(x, y, |q/p|)$ or (x, y, ϕ) ; (c) assuming no

Table 216: Observables used in the global fit except those from $D^0 \rightarrow K^+ \pi^-$ and those used for measuring direct CPV . The $D^0 \rightarrow K^+ \pi^- \pi^0$ observables are $x'' \equiv x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$ and $y'' \equiv -x \sin \delta_{K\pi\pi} + y \cos \delta_{K\pi\pi}$.

Mode	Observable	Values	Correlation coefficients
$D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$,	y_{CP}	$(0.866 \pm 0.155)\%$	
ϕK_S^0 [747]	A_Γ	$(-0.014 \pm 0.052)\%$	
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ [751] (Belle: no CPV)	x	$(0.56 \pm 0.19^{+0.067}_{-0.127})\%$	+0.012
	y	$(0.30 \pm 0.15^{+0.050}_{-0.078})\%$	
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ [751] (Belle: no direct CPV)	$ q/p $	$0.90^{+0.16+0.078}_{-0.15-0.064}$	
	ϕ	$(-6 \pm 11^{+4.2}_{-5.0})$ degrees	
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ [751] (Belle: direct CPV allowed)	x	$(0.58 \pm 0.19^{+0.0734}_{-0.1177})\%$	$\begin{pmatrix} 1 & 0.054 & -0.074 & -0.031 \\ 0.054 & 1 & 0.034 & -0.019 \\ -0.074 & 0.034 & 1 & 0.044 \\ -0.031 & -0.019 & 0.044 & 1 \end{pmatrix}$
	y	$(0.27 \pm 0.16^{+0.0546}_{-0.0854})\%$	
	$ q/p $	$0.82^{+0.20+0.0807}_{-0.18-0.0645}$	
	ϕ	$(-13^{+12+4.15}_{-13-4.77})$ degrees	
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ [752] $K_S^0 K^+ K^-$ (BABAR: no CPV)	x	$(0.16 \pm 0.23 \pm 0.12 \pm 0.08)\%$	0.0615
	y	$(0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$	
$D^0 \rightarrow K^+ \ell^- \nu$ [747]	R_M	$(0.0130 \pm 0.0269)\%$	
$D^0 \rightarrow K^+ \pi^- \pi^0$ [753]	x''	$(2.61^{+0.57}_{-0.68} \pm 0.39)\%$	-0.75
	y''	$(-0.06^{+0.55}_{-0.64} \pm 0.34)\%$	
$\psi(3770) \rightarrow \bar{D} D$ [741] (CLEOc)	R_D	$(0.533 \pm 0.107 \pm 0.045)\%$	$\begin{pmatrix} 1 & 0 & 0 & -0.42 & 0.01 \\ & 1 & -0.73 & 0.39 & 0.02 \\ & & 1 & -0.53 & -0.03 \\ & & & 1 & 0.04 \\ & & & & 1 \end{pmatrix}$
	x^2	$(0.06 \pm 0.23 \pm 0.11)\%$	
	y	$(4.2 \pm 2.0 \pm 1.0)\%$	
	$\cos \delta$	$0.81^{+0.22+0.07}_{-0.18-0.05}$	
	$\sin \delta$	$-0.01 \pm 0.41 \pm 0.04$	

direct CPV in DCS decays and fitting for alternative parameters $x_{12} = 2|M_{12}|/\Gamma$, $y_{12} = \Gamma_{12}/\Gamma$, and $\phi_{12} = \text{Arg}(M_{12}/\Gamma_{12})$, where M_{12} and Γ_{12} are the off-diagonal elements of the D^0 - $\bar{D}^0$ mass and decay matrices, respectively; and (d) allowing full CPV , *i.e.* floating all parameters.

For the fits assuming no-direct- CPV in DCS decays, we set $A_D=0$. In addition, for fit (b) we impose the relation [768, 769] $\tan \phi = (1 - |q/p|^2)/(1 + |q/p|^2) \times (x/y)$, which reduces four independent parameters to three.⁵⁷ We impose this relationship in two ways. First we float parameters x , y , and ϕ and from these derive $|q/p|$; subsequently we repeat the fit floating x , y , and $|q/p|$ and from these derive ϕ . The central values returned by the two fits are identical, but the first fit yields MINOS errors for ϕ , while the second fit yields MINOS errors for $|q/p|$. For no-direct- CPV fit (c), we fit for the underlying parameters x_{12} , y_{12} , and ϕ_{12} , from which parameters x , y , $|q/p|$, and ϕ are derived.

All fit results are listed in Table 220. For the CPV -allowed fit, individual contributions to the χ^2 are listed in Table 221. The total χ^2 is 66.8 for $45 - 10 = 35$ degrees of freedom; this

⁵⁷One can also use Eq. (15) of Ref. [770] to reduce four parameters to three.

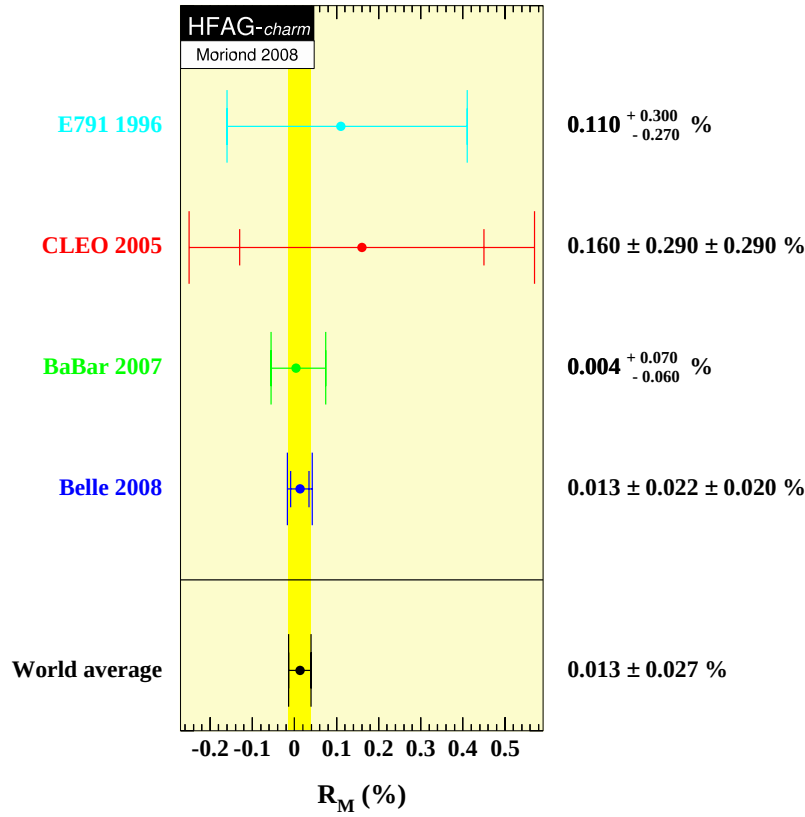


Figure 152: World average value of R_M from Ref. [747], as calculated from $D^0 \rightarrow K^+ \ell^- \nu$ measurements [743–746].

corresponds to a confidence level of 0.001, which is uncomfortably small.

Confidence contours in the two dimensions (x, y) or in $(|q/p|, \phi)$ are obtained by allowing, for any point in the two-dimensional plane, all other fitted parameters to take their preferred values. The resulting 1σ – 5σ contours are shown in Fig. 155 for the CP -conserving case, in Fig. 156 for the no-direct- CPV case, and in Fig. 157 for the CPV -allowed case. The contours are determined from the increase of the χ^2 above the minimum value. One observes that the (x, y) contours for the no- CPV fit are very similar to those for the CPV -allowed fit. In the latter fit, the χ^2 at the no-mixing point $(x, y) = (0, 0)$ is 421 units above the minimum value, which, for two degrees of freedom, corresponds to a confidence level $> 11.5\sigma$.⁵⁸ Thus, no mixing is excluded at this high level. In the $(|q/p|, \phi)$ plot, the point $(1, 0)$ is within the 1σ contour; thus the data is consistent with CP conservation.

One-dimensional confidence curves for individual parameters are obtained by allowing, for any value of the parameter, all other fitted parameters to take their preferred values. The resulting functions $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ ($\chi^2_{\min}$ is the minimum value) are shown in Fig. 158. The points where $\Delta\chi^2 = 3.84$ determine 95% C.L. intervals for the parameters. These intervals are listed in Table 220.

⁵⁸This is the limit of the CERNLIB PROB routine used for this calculation.

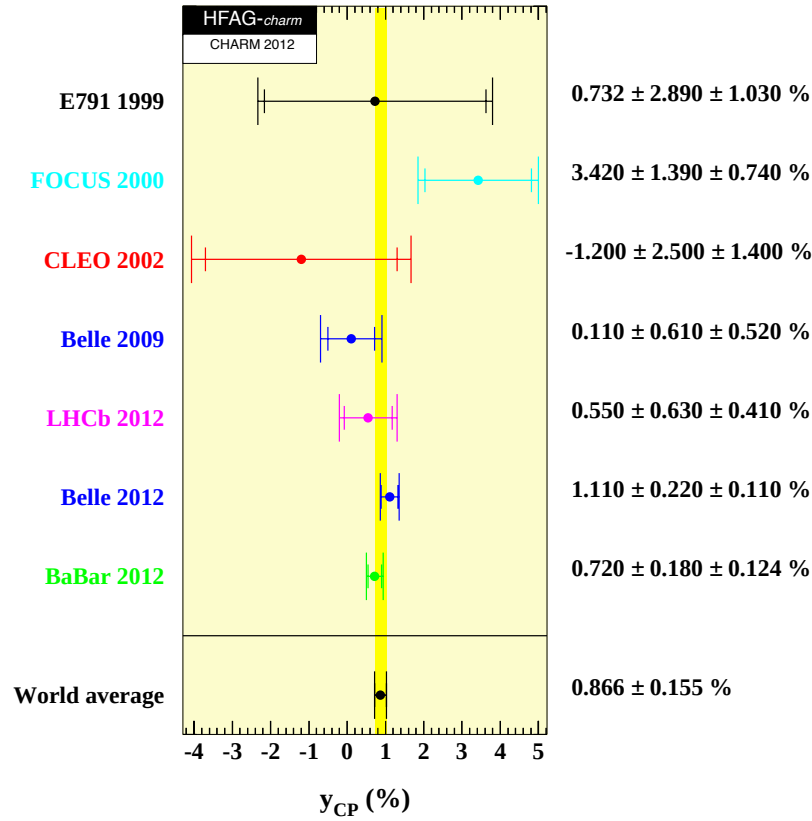


Figure 153: World average value of y_{CP} from Ref. [747], as calculated from $D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$ measurements [759–765].

8.1.4 Conclusions

From the fit results listed in Table 220 and shown in Figs. 157 and 158, we conclude that:

- the experimental data consistently indicate that D^0 mesons undergo mixing. The no-mixing point $x = y = 0$ is excluded at $> 11.5\sigma$. The parameter x differs from zero by 2.4σ , and y differs from zero by 9.4σ . This mixing is presumably dominated by long-distance processes, which are difficult to calculate. Thus, unless it turns out that $|x| \gg |y|$ [737] (which is not currently indicated), it will be difficult to identify new physics from (x, y) alone.
- Since y_{CP} is positive, the CP -even state is shorter-lived as in the $K^0-\bar{K}^0$ system. However, since x also appears to be positive, the CP -even state is heavier, unlike in the $K^0-\bar{K}^0$ system.
- The LHCb and CDF experiments measured time-integrated asymmetries that hint at *direct CPV* in D^0 decays (see Table 218). However, more statistics are needed to clarify this effect. There is no evidence for CPV arising from $D^0-\bar{D}^0$ mixing ($|q/p| \neq 1$) or from a phase difference between the mixing amplitude and a direct decay amplitude ($\phi \neq 0$).

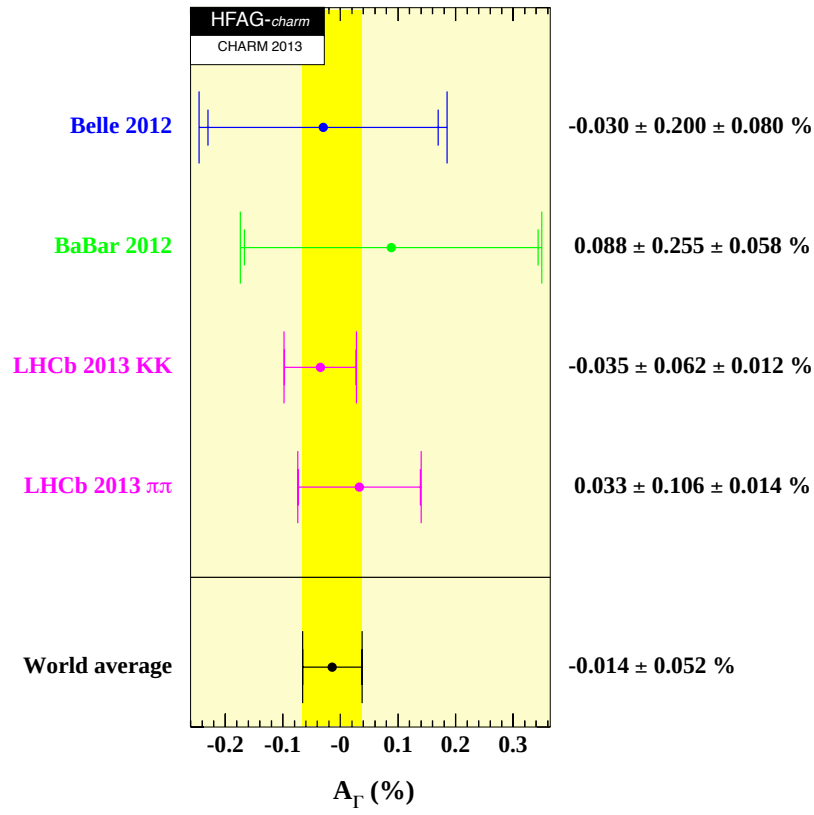


Figure 154: World average value of A_F from Ref. [747], as calculated from $D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$ measurements [763, 764, 766].

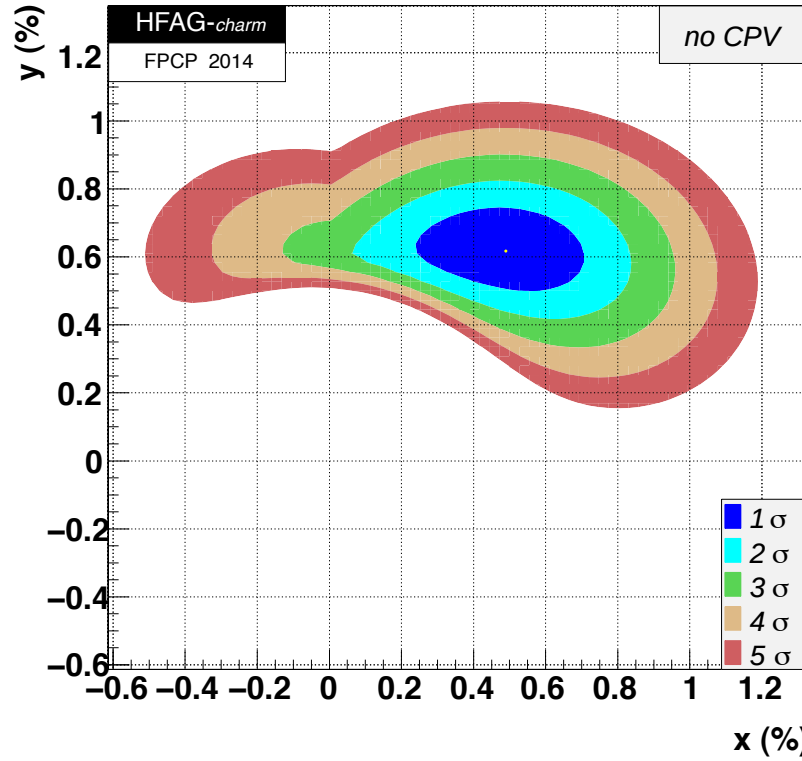


Figure 155: Two-dimensional contours for mixing parameters (x, y) , for no CPV .

Table 217: $D^0 \rightarrow K^+\pi^-$ observables used for the global fit ($x'^{2\pm}$ and $y'^{\pm}$ are defined in Ref. [734]).

Mode	Observable	Values	Correlation coefficients
$D^0 \rightarrow K^+\pi^-$ [734] (BABAR 384 fb $^{-1}$)	R_D	$(0.303 \pm 0.0189)\%$	$\begin{Bmatrix} 1 & 0.77 & -0.87 \\ 0.77 & 1 & -0.94 \\ -0.87 & -0.94 & 1 \end{Bmatrix}$
	x'^{2+}	$(-0.024 \pm 0.052)\%$	
	y'^+	$(0.98 \pm 0.78)\%$	
$\bar{D}^0 \rightarrow K^-\pi^+$ [734] (BABAR 384 fb $^{-1}$)	A_D	$(-2.1 \pm 5.4)\%$	same as above
	x'^{2-}	$(-0.020 \pm 0.050)\%$	
	y'^-	$(0.96 \pm 0.75)\%$	
$D^0 \rightarrow K^+\pi^-$ [749] (Belle 976 fb $^{-1}$ No CPV)	R_D	$(0.353 \pm 0.013)\%$	$\begin{Bmatrix} 1 & 0.737 & -0.865 \\ 0.737 & 1 & -0.948 \\ -0.865 & -0.948 & 1 \end{Bmatrix}$
	x'^2	$(0.009 \pm 0.022)\%$	
	y'	$(0.46 \pm 0.34)\%$	
$D^0 \rightarrow K^+\pi^-$ [748] (Belle 400 fb $^{-1}$ CPV -allowed)	R_D	$(0.364 \pm 0.018)\%$	$\begin{Bmatrix} 1 & 0.655 & -0.834 \\ 0.655 & 1 & -0.909 \\ -0.834 & -0.909 & 1 \end{Bmatrix}$
	x'^{2+}	$(0.032 \pm 0.037)\%$	
	y'^+	$(-0.12 \pm 0.58)\%$	
$\bar{D}^0 \rightarrow K^-\pi^+$ [748] (Belle 400 fb $^{-1}$ CPV -allowed)	A_D	$(2.3 \pm 4.7)\%$	same as above
	x'^{2-}	$(0.006 \pm 0.034)\%$	
	y'^-	$(0.20 \pm 0.54)\%$	
$D^0 \rightarrow K^+\pi^-$ [750] (CDF 9.6 fb $^{-1}$ No CPV)	R_D	$(0.351 \pm 0.035)\%$	$\begin{Bmatrix} 1 & 0.90 & -0.97 \\ 0.90 & 1 & -0.98 \\ -0.97 & -0.98 & 1 \end{Bmatrix}$
	x'^2	$(0.008 \pm 0.018)\%$	
	y'	$(0.43 \pm 0.43)\%$	
$D^0 \rightarrow K^+\pi^-$ [736] (LHCb 3.0 fb $^{-1}$ CPV -allowed)	R_D^+	$(0.3545 \pm 0.0095)\%$	$\begin{Bmatrix} 1 & 0.862 & -0.942 \\ 0.862 & 1 & -0.968 \\ -0.942 & -0.968 & 1 \end{Bmatrix}$
	x'^{2+}	$(0.0049 \pm 0.0070)\%$	
	y'^+	$(0.51 \pm 0.14)\%$	
$\bar{D}^0 \rightarrow K^-\pi^+$ [736] (LHCb 3.0 fb $^{-1}$ CPV -allowed)	R_D^+	$(0.3591 \pm 0.0094)\%$	$\begin{Bmatrix} 1 & 0.858 & -0.941 \\ 0.858 & 1 & -0.966 \\ -0.941 & -0.966 & 1 \end{Bmatrix}$
	x'^{2-}	$(0.0060 \pm 0.0068)\%$	
	y'^-	$(0.45 \pm 0.14)\%$	

Table 218: Measurements of time-integrated CP asymmetries. The observable $A_{CP}(f) \equiv [\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)]/[\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)]$, and $\Delta\langle t \rangle$ is the difference between mean lifetimes for $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ decays (due to different trigger/reconstruction efficiencies).

Mode	Observable	Values	$\Delta\langle t \rangle/\tau_D$
$D^0 \rightarrow h^+ h^-$ [754] (BABAR 386 fb $^{-1}$)	$A_{CP}(K^+ K^-)$ $A_{CP}(\pi^+ \pi^-)$	$(0.00 \pm 0.34 \pm 0.13)\%$ $(-0.24 \pm 0.52 \pm 0.22)\%$	0
$D^0 \rightarrow h^+ h^-$ [755] (Belle 976 fb $^{-1}$)	$A_{CP}(K^+ K^-)$ $A_{CP}(\pi^+ \pi^-)$	$(-0.32 \pm 0.21 \pm 0.09)\%$ $(0.55 \pm 0.36 \pm 0.09)\%$	0
$D^0 \rightarrow h^+ h^-$ [756, 757] (CDF 9.7 fb $^{-1}$)	$A_{CP}(K^+ K^-) - A_{CP}(\pi^+ \pi^-)$ $A_{CP}(K^+ K^-)$ $A_{CP}(\pi^+ \pi^-)$	$(-0.62 \pm 0.21 \pm 0.10)\%$ $(-0.32 \pm 0.21)\%$ $(0.31 \pm 0.22)\%$	0.27 ± 0.01
$D^0 \rightarrow h^+ h^-$ [758] (LHCb 1.0 fb $^{-1}$, $D^{*+} \rightarrow D^0 \pi^+$ tag)	$A_{CP}(K^+ K^-) - A_{CP}(\pi^+ \pi^-)$	$(-0.34 \pm 0.15 \pm 0.10)\%$	$0.1119 \pm 0.0013 \pm 0.0017$
$D^0 \rightarrow h^+ h^-$ [742] (LHCb 3 fb $^{-1}$, $B \rightarrow D^0 \mu^- X$ tag)	$A_{CP}(K^+ K^-) - A_{CP}(\pi^+ \pi^-)$	$(0.14 \pm 0.16 \pm 0.08)\%$	0.014 ± 0.004

Table 219: Left: decay modes used to determine fitted parameters x , y , δ , $\delta_{K\pi\pi}$, R_D , A_D , $|q/p|$, and ϕ . Middle: the observables measured for each decay mode. Right: the relationships between the observables measured and the fitted parameters. $\langle t \rangle$ is the mean lifetime for $D^0 \rightarrow K^+ K^-$ or $D^0 \rightarrow \pi^+ \pi^-$ decays.

Decay Mode	Observables	Relationship
$D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$	y_{CP} A_Γ	$2y_{CP} = (q/p + p/q) y \cos \phi -$ $(q/p - p/q) x \sin \phi$ $2A_\Gamma = (q/p - p/q) y \cos \phi -$ $(q/p + p/q) x \sin \phi$
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$	x y $ q/p $ ϕ	
$D^0 \rightarrow K^+ \ell^- \nu$	R_M	$R_M = (x^2 + y^2)/2$
$D^0 \rightarrow K^+ \pi^- \pi^0$ (Dalitz plot analysis)	x'' y''	$x'' = x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$ $y'' = y \cos \delta_{K\pi\pi} - x \sin \delta_{K\pi\pi}$
“Double-tagged” branching fractions measured in $\psi(3770) \rightarrow DD$ decays	R_M y R_D $\sqrt{R_D} \cos \delta$	$R_M = (x^2 + y^2)/2$
$D^0 \rightarrow K^+ \pi^-$	x'^2, y' x'^{2+}, x'^{2-} y'^+, y'^-	$x' = x \cos \delta + y \sin \delta$ $y' = y \cos \delta - x \sin \delta$ $A_M \equiv (q/p ^4 - 1)/(q/p ^4 + 1)$ $x'^{\pm} = [(1 \pm A_M)/(1 \mp A_M)]^{1/4} \times$ $(x' \cos \phi \pm y' \sin \phi)$ $y'^{\pm} = [(1 \pm A_M)/(1 \mp A_M)]^{1/4} \times$ $(y' \cos \phi \mp x' \sin \phi)$
$D^0 \rightarrow K^+ \pi^- / K^- \pi^+$ (time-integrated)	$\frac{\Gamma(D^0 \rightarrow K^+ \pi^-) + \Gamma(\bar{D}^0 \rightarrow K^- \pi^+)}{\Gamma(D^0 \rightarrow K^- \pi^+) + \Gamma(\bar{D}^0 \rightarrow K^+ \pi^-)}$ $\frac{\Gamma(D^0 \rightarrow K^+ \pi^-) - \Gamma(\bar{D}^0 \rightarrow K^- \pi^+)}{\Gamma(D^0 \rightarrow K^+ \pi^-) + \Gamma(\bar{D}^0 \rightarrow K^- \pi^+)}$	R_D A_D
$D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$ (time-integrated)	$\frac{\Gamma(D^0 \rightarrow K^+ K^-) - \Gamma(\bar{D}^0 \rightarrow K^+ K^-)}{\Gamma(D^0 \rightarrow K^+ K^-) + \Gamma(\bar{D}^0 \rightarrow K^+ K^-)}$ $\frac{\Gamma(D^0 \rightarrow \pi^+ \pi^-) - \Gamma(\bar{D}^0 \rightarrow \pi^+ \pi^-)}{\Gamma(D^0 \rightarrow \pi^+ \pi^-) + \Gamma(\bar{D}^0 \rightarrow \pi^+ \pi^-)}$	$A_K + \frac{\langle t \rangle}{\tau_D} \mathcal{A}_{CP}^{\text{indirect}} \quad (\mathcal{A}_{CP}^{\text{indirect}} \approx -A_\Gamma)$ $A_\pi + \frac{\langle t \rangle}{\tau_D} \mathcal{A}_{CP}^{\text{indirect}} \quad (\mathcal{A}_{CP}^{\text{indirect}} \approx -A_\Gamma)$

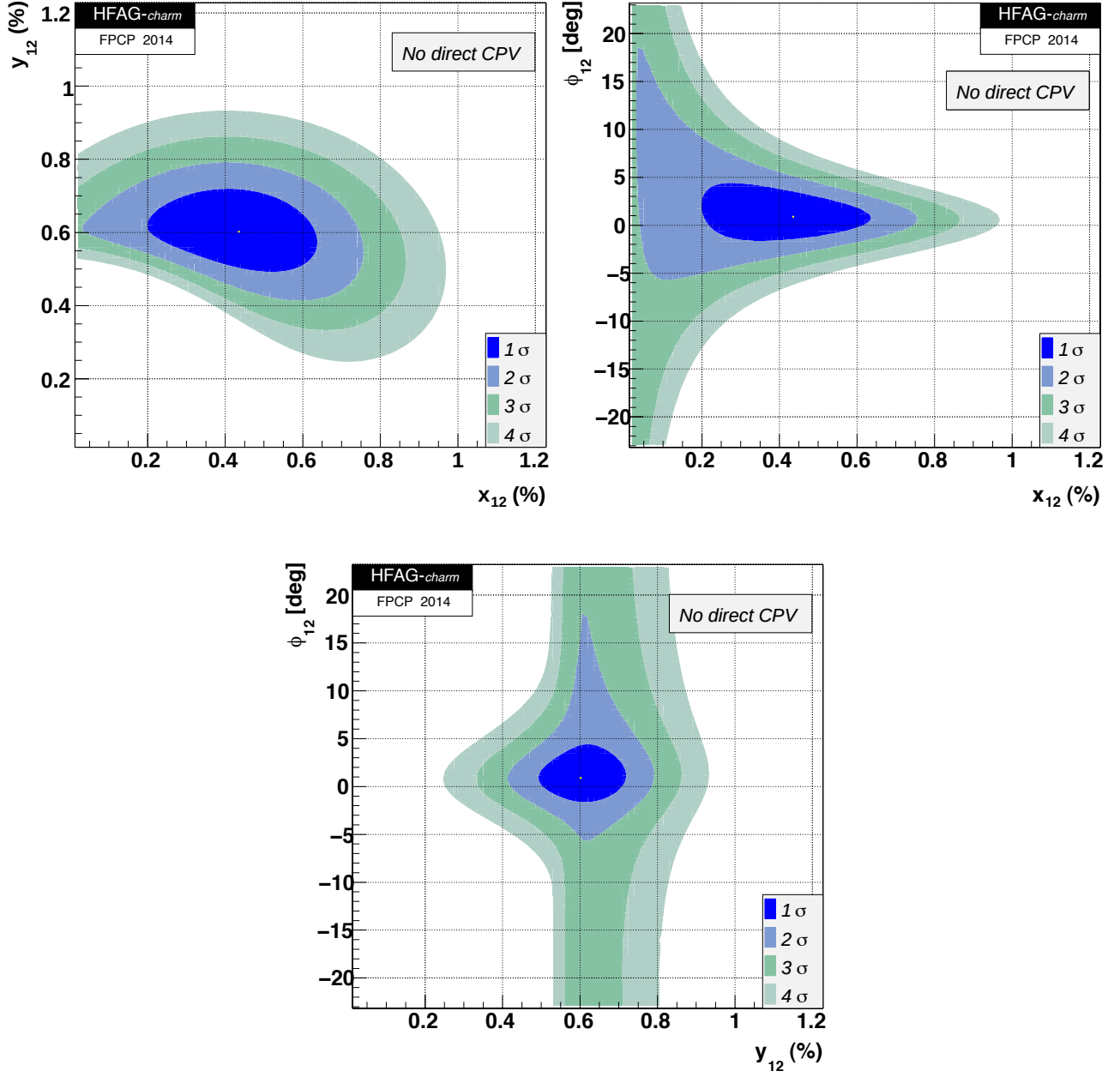


Figure 156: Two-dimensional contours for theoretical parameters (x_{12}, y_{12}) (top left), (x_{12}, ϕ_{12}) (top right), and (y_{12}, ϕ_{12}) (bottom), for no direct CPV .

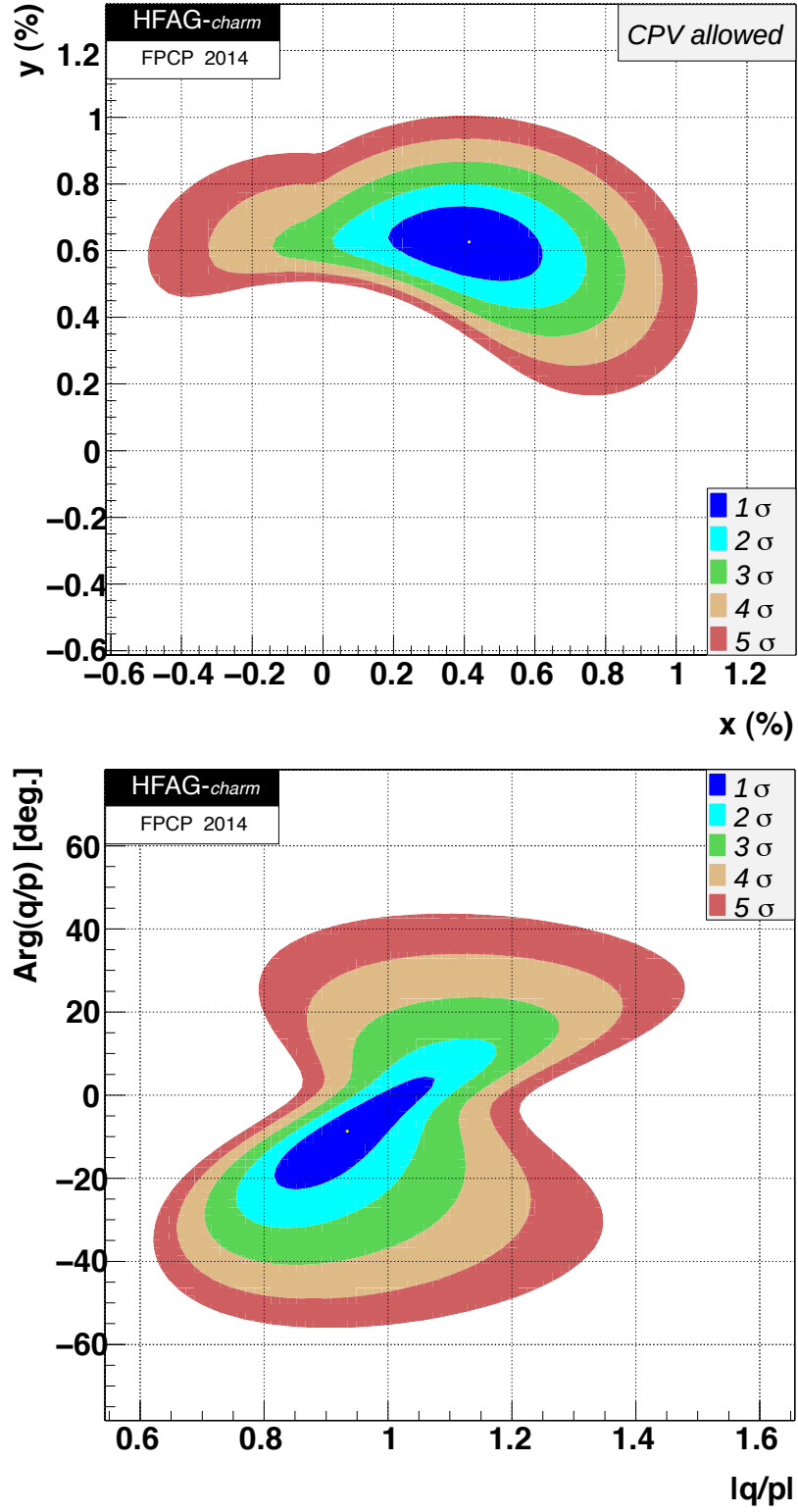


Figure 157: Two-dimensional contours for parameters (x, y) (top) and $(|q/p|, \phi)$ (bottom), allowing for CPV .

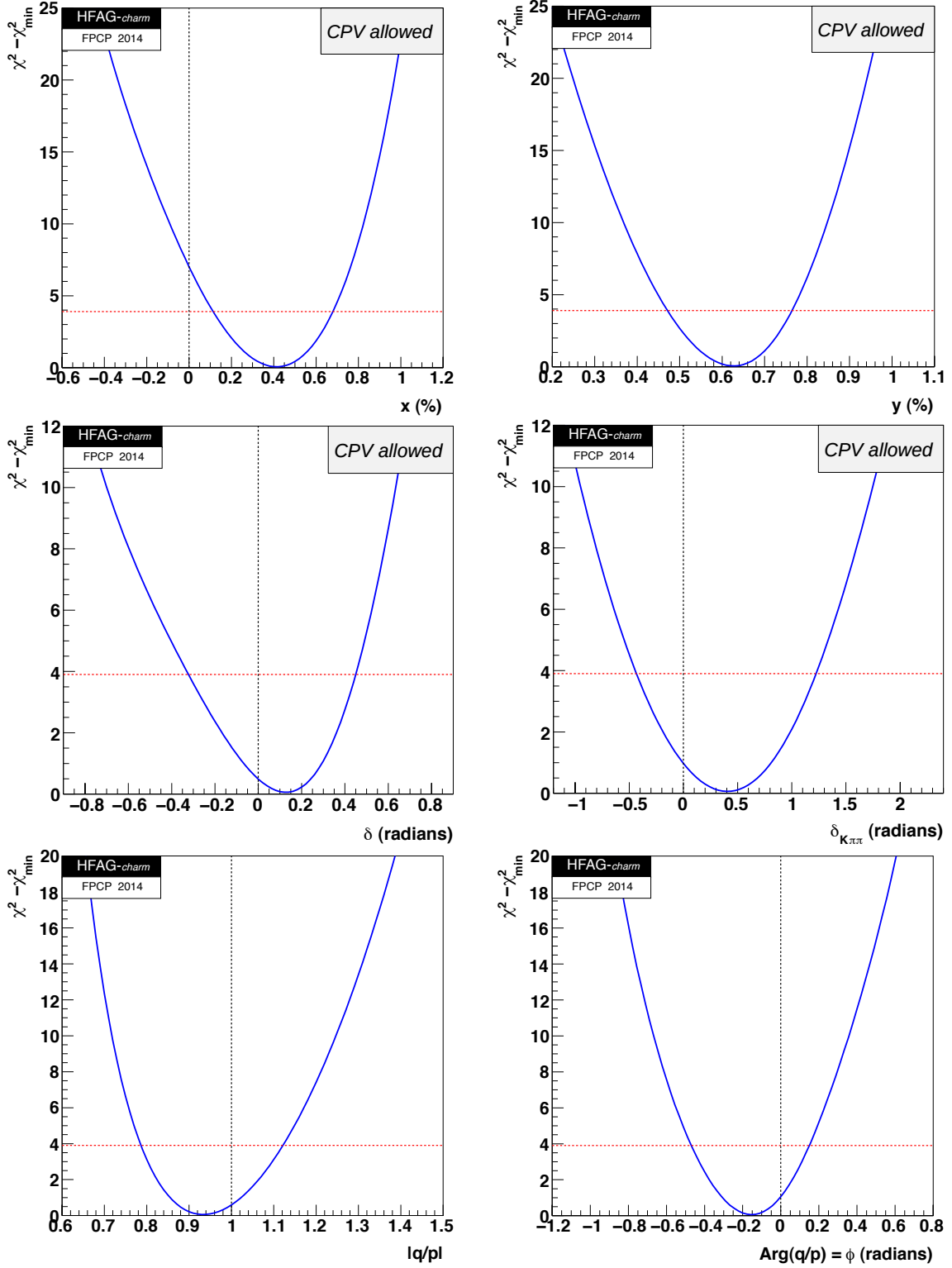


Figure 158: The function $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ for fitted parameters x , y , δ , $\delta_{K\pi\pi}$, $|q/p|$, and ϕ . The points where $\Delta\chi^2 = 3.84$ (denoted by dashed horizontal lines) determine 95% C.L. intervals.

Table 220: Results of the global fit for different assumptions concerning CPV .

Parameter	No CPV	No direct CPV in DCS decays	CPV -allowed	CPV -allowed 95% CL Interval
x (%)	$0.49^{+0.14}_{-0.15}$	$0.43^{+0.14}_{-0.15}$	$0.41^{+0.14}_{-0.15}$	[0.11, 0.68]
y (%)	0.62 ± 0.08	0.60 ± 0.07	$0.63^{+0.07}_{-0.08}$	[0.47, 0.76]
$\delta_{K\pi}$ ($^\circ$)	$7.8^{+9.6}_{-11.1}$	$4.6^{+10.3}_{-12.0}$	$7.3^{+9.8}_{-11.5}$	[-18.5, 25.8]
R_D (%)	0.350 ± 0.004	0.349 ± 0.004	0.349 ± 0.004	[0.342, 0.356]
A_D (%)	—	—	$-0.71^{+0.92}_{-0.95}$	[-2.6, 1.1]
$ q/p $	—	$1.007^{+0.015}_{-0.014}$	$0.93^{+0.09}_{-0.08}$	[0.79, 1.12]
ϕ ($^\circ$)	—	$-0.30^{+0.58}_{-0.60}$	$-8.7^{+8.7}_{-9.1}$	[-26.9, 8.6]
$\delta_{K\pi\pi}$ ($^\circ$)	$18.7^{+23.2}_{-23.7}$	$20.8^{+23.9}_{-24.3}$	$23.3^{+23.9}_{-24.4}$	[-24.8, 70.2]
A_π (%)	—	0.11 ± 0.14	0.14 ± 0.15	[-0.15, 0.42]
A_K (%)	—	-0.13 ± 0.13	$-0.11^{+0.14}_{-0.13}$	[-0.37, 0.15]
x_{12} (%)	—	$0.43^{+0.14}_{-0.15}$		[0.13, 0.69]
y_{12} (%)	—	0.60 ± 0.07		[0.45, 0.75]
ϕ_{12} ($^\circ$)	—	$0.9^{+1.9}_{-1.7}$		[-3.0, 6.1]

Table 221: Individual contributions to the χ^2 for the CPV -allowed fit.

Observable	χ^2	$\Sigma \chi^2$
y_{CP}	2.57	2.57
A_Γ	0.44	3.02
$x_{K^0\pi^+\pi^-}$ Belle	0.55	3.57
$y_{K^0\pi^+\pi^-}$ Belle	3.87	7.44
$ q/p _{K^0\pi^+\pi^-}$ Belle	0.28	7.72
$\phi_{K^0\pi^+\pi^-}$ Belle	0.16	7.88
$x_{K^0h^+h^-}$ BABAR	0.87	8.76
$y_{K^0h^+h^-}$ BABAR	0.03	8.79
$R_M(K^+\ell^-\nu)$	0.14	8.93
$x_{K^+\pi^-\pi^0}$ BABAR	7.15	16.08
$y_{K^+\pi^-\pi^0}$ BABAR	3.99	20.08
CLEOc		
$(x/y/R_D/\cos\delta/\sin\delta)$	10.08	30.16
$R_D^+/x'^{2+}/y'^+$ BABAR	11.66	41.82
$R_D^-/x'^{2-}/y'^-$ BABAR	6.02	47.83
$R_D^+/x'^{2+}/y'^+$ Belle	2.13	49.96
$R_D^-/x'^{2-}/y'^-$ Belle	3.18	53.14
$R_D/x'^2/y'$ CDF	1.15	54.29
$R_D^+/x'^{2+}/y'^+$ LHCb	1.11	55.50
$R_D^-/x'^{2-}/y'^-$ LHCb	1.27	56.67
$A_{KK}/A_{\pi\pi}$ BABAR	0.53	57.19
$A_{KK}/A_{\pi\pi}$ Belle	2.06	59.25
$A_{KK}/A_{\pi\pi}$ CDF	2.64	61.89
$A_{KK} - A_{\pi\pi}$ LHCb (D^* tag)	0.26	62.15
$A_{KK} - A_{\pi\pi}$ LHCb ($B^0 \rightarrow D^0 \mu X$ tag)	4.65	66.80

8.2 CP asymmetries

CP violation occurs if the decay rate for a particle differs from that of its CP -conjugate [771]. In general there are two classes of CP violation, termed *indirect* and *direct* [772]. Indirect CP violation refers to $\Delta C = 2$ processes and arises in D^0 decays due to D^0 - $\bar{D}^0$ mixing. It can occur as an asymmetry in the mixing itself, or it can result from interference between a decay amplitude arising via mixing and a non-mixed amplitude. Direct CP violation refers to $\Delta C = 1$ processes and can occur in both charged and neutral D decays. It results from interference between two different decay amplitudes (*e.g.* a penguin and tree amplitude) that have different weak (CKM) and strong phases⁵⁹. A difference in strong phases typically arises due to final-state interactions (FSI) [773]. A difference in weak phases arises from different CKM vertex couplings, as is often the case for spectator and penguin diagrams.

The CP asymmetry is defined as the difference between D and $\bar{D}$ partial widths divided by their sum:

$$A_{CP} = \frac{\Gamma(D) - \Gamma(\bar{D})}{\Gamma(D) + \Gamma(\bar{D})}. \quad (197)$$

However, to take into account differences in production rates between D and $\bar{D}$ (which would affect the number of respective decays observed), some experiments normalize to a Cabibbo-favored mode. In this case there is the additional benefit that most corrections due to inefficiencies cancel out, reducing systematic uncertainties. An implicit assumption is that there is no measurable CP violation in the Cabibbo-favored normalizing mode. The CP asymmetry is calculated as

$$A_{CP} = \frac{\eta(D) - \eta(\bar{D})}{\eta(D) + \eta(\bar{D})}, \quad (198)$$

where (considering, for example, $D^0 \rightarrow K^- K^+$)

$$\eta(D) = \frac{N(D^0 \rightarrow K^- K^+)}{N(D^0 \rightarrow K^- \pi^+)}, \quad (199)$$

$$\eta(\bar{D}) = \frac{N(\bar{D}^0 \rightarrow K^- K^+)}{N(\bar{D}^0 \rightarrow K^+ \pi^-)}. \quad (200)$$

In the case of D^+ and D_s^+ decays, A_{CP} measures direct CP violation; in the case of D^0 decays, A_{CP} measures direct and indirect CP violation combined. Values of A_{CP} for D^+ , D^0 and D_s^+ decays are listed in Tables 222, 223, 224, 225 and 226 respectively. In these tables we report asymmetries for the actual final state, *i.e.* resonant substructure is implicitly included by not considered separately.

Overall, CP asymmetry measurements have been carried out for 47 charm decay modes, and in several modes the sensitivity is below 5×10^{-3} . There is currently no evidence for CP violation in the charm sector, with one exception: the asymmetry observed for $D^+ \rightarrow K_s^0 \pi^+$.

⁵⁹The weak phase difference will have opposite signs for $D \rightarrow f$ and $\bar{D} \rightarrow \bar{f}$ decays, while the strong phase difference will have the same sign. As a result, squaring the total amplitudes to obtain the decay rates gives interference terms having opposite sign, *i.e.* non-identical decay rates.

However, the CP asymmetry observed is consistent with that expected due to CP violation in K^0 - $\bar{K}^0$ mixing [774], and thus it is not attributed to charm. Taken together, the limits obtained for CP asymmetries in the charm sector pose tight constraints on new physics models.

Table 222: CP asymmetries $A_{CP} = [\Gamma(D^+) - \Gamma(D^-)]/[\Gamma(D^+) + \Gamma(D^-)]$ for two-body $D^\pm$ decays.

Mode	Year	Collaboration	A_{CP}
$D^+ \rightarrow \mu^+ \nu$	2008	CLEOc [775]	$+0.08 \pm 0.08$
$D^+ \rightarrow \pi^+ \pi^0$	2010	CLEOc [776]	$+0.029 \pm 0.029 \pm 0.003$
$D^+ \rightarrow \pi^+ \eta$	2011	Belle [777]	$+0.0174 \pm 0.0113 \pm 0.0019$
	2010	CLEOc [776]	$-0.020 \pm 0.023 \pm 0.003$
		COMBOS average	$+0.010 \pm 0.010$
$D^+ \rightarrow \pi^+ \eta'$	2011	Belle [777]	$-0.0012 \pm 0.0112 \pm 0.0017$
	2010	CLEOc [776]	$-0.040 \pm 0.034 \pm 0.003$
		COMBOS average	-0.005 ± 0.011
$D^+ \rightarrow K^+ \pi^0$	2010	CLEOc [776]	$-0.035 \pm 0.107 \pm 0.009$
$D^+ \rightarrow K_s^0 \pi^+$	2014	CLEOc [778]	$-0.011 \pm 0.006 \pm 0.002$
	2012	Belle [779]	$-0.00363 \pm 0.00094 \pm 0.00067$
	2011	BABAR [780]	$-0.0044 \pm 0.0013 \pm 0.0010$
	2002	FOCUS [781]	$-0.016 \pm 0.015 \pm 0.009$
		COMBOS average	-0.0041 ± 0.0009
$D^+ \rightarrow K_s^0 K^+$	2014	LHCb [782]	$+0.0003 \pm 0.0017 \pm 0.0014$
	2013	BABAR [783]	$+0.0013 \pm 0.0036 \pm 0.0025$
	2013	Belle [784]	$-0.0025 \pm 0.0028 \pm 0.0014$
	2010	CLEOc [776]	$-0.002 \pm 0.015 \pm 0.009$
	2002	FOCUS [781]	$+0.071 \pm 0.061 \pm 0.012$
		COMBOS average	-0.0003 ± 0.0017

Table 223: CP asymmetries $A_{CP} = [\Gamma(D^+) - \Gamma(D^-)] / [\Gamma(D^+) + \Gamma(D^-)]$ for three- and four-body $D^\pm$ decays.

Mode	Year	Collaboration	A_{CP}
$D^+ \rightarrow \pi^+ \pi^- \pi^+$	2014	LHCb [785]	Dalitz plot analysis, no evidence for CP violation
	1997	E791 [786]	-0.017 ± 0.042 (stat.)
$D^+ \rightarrow K^- \pi^+ \pi^+$	2014	CLEOc [778]	$-0.003 \pm 0.002 \pm 0.004$
$D^+ \rightarrow K_s^0 \pi^+ \pi^0$	2014	CLEOc [778]	$-0.001 \pm 0.007 \pm 0.002$
$D^+ \rightarrow K^+ K^- \pi^+$	2014	CLEOc [778]	$-0.001 \pm 0.009 \pm 0.004$
	2013	BABAR [787]	$+0.0037 \pm 0.0030 \pm 0.0015$
	2008	CLEOc [788]	Dalitz plot analysis, no evidence for CP violation
	2000	FOCUS [789]	$+0.006 \pm 0.011 \pm 0.005$
	1997	E791 [786]	-0.014 ± 0.029 (stat.)
		COMBOS average	$+0.0032 \pm 0.0031$
$D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$	2014	CLEOc [778]	$-0.003 \pm 0.006 \pm 0.004$
$D^+ \rightarrow K_s^0 \pi^+ \pi^+ \pi^-$	2014	CLEOc [778]	$+0.000 \pm 0.012 \pm 0.003$
$D^+ \rightarrow K_s^0 K^+ \pi^+ \pi^-$	2005	FOCUS [790]	$-0.042 \pm 0.064 \pm 0.022$

Table 224: CP asymmetries $A_{CP} = [\Gamma(D^0) - \Gamma(\bar{D}^0)]/[\Gamma(D^0) + \Gamma(\bar{D}^0)]$ for two-body $D^0, \bar{D}^0$ decays.

Mode	Year	Collaboration	A_{CP}
$D^0 \rightarrow \pi^+ \pi^-$	2014	LHCb [742]	$-0.0020 \pm 0.0019 \pm 0.0010$
	2012	Belle [791]	$+0.0055 \pm 0.0036 \pm 0.0009$
	2012	CDF [792]	$+0.0022 \pm 0.0024 \pm 0.0011$
	2008	<i>BABAR</i> [754]	$-0.0024 \pm 0.0052 \pm 0.0022$
	2002	CLEO [761]	$+0.019 \pm 0.032 \pm 0.008$
	2000	FOCUS [789]	$+0.048 \pm 0.039 \pm 0.025$
	1998	E791 [793]	$-0.049 \pm 0.078 \pm 0.030$
		COMBOS average	$+0.0005 \pm 0.0015$
$D^0 \rightarrow \pi^0 \pi^0$	2014	Belle [794]	$-0.0003 \pm 0.0064 \pm 0.0010$
	2001	CLEO [795]	$+0.001 \pm 0.048$ (stat. and syst. combined)
		COMBOS average	-0.0003 ± 0.0064
$D^0 \rightarrow K_s^0 \pi^0$	2014	Belle [794]	$-0.0021 \pm 0.0016 \pm 0.0007$
	2001	CLEO [795]	$+0.001 \pm 0.013$ (stat. and syst. combined)
		COMBOS average	-0.0020 ± 0.0017
$D^0 \rightarrow K_s^0 \eta$	2011	Belle [796]	$+0.0054 \pm 0.0051 \pm 0.0016$
$D^0 \rightarrow K_s^0 \eta'$	2011	Belle [796]	$+0.0098 \pm 0.0067 \pm 0.0014$
$D^0 \rightarrow K_s^0 K_s^0$	2001	CLEO [795]	-0.23 ± 0.19 (stat. and syst. combined)
$D^0 \rightarrow K^+ K^-$	2014	LHCb [742]	$-0.0006 \pm 0.0015 \pm 0.0010$
	2012	Belle [791]	$-0.0032 \pm 0.0021 \pm 0.0009$
	2012	CDF [792]	$-0.0024 \pm 0.0022 \pm 0.0009$
	2008	<i>BABAR</i> [754]	$+0.0000 \pm 0.0034 \pm 0.0013$
	2002	CLEO [761]	$+0.000 \pm 0.022 \pm 0.008$
	2000	FOCUS [789]	$-0.001 \pm 0.022 \pm 0.015$
	1998	E791 [793]	$-0.010 \pm 0.049 \pm 0.012$
		COMBOS average	-0.0016 ± 0.0012

Table 225: CP asymmetries $A_{CP} = [\Gamma(D^0) - \Gamma(\bar{D}^0)]/[\Gamma(D^0) + \Gamma(\bar{D}^0)]$ for three- and four-body $D^0, \bar{D}^0$ decays.

Mode	Year	Collaboration	A_{CP}
$D^0 \rightarrow \pi^+ \pi^- \pi^0$	2008	<i>BABAR</i> [797]	$-0.0031 \pm 0.0041 \pm 0.0017$
	2008	Belle [798]	$+0.0043 \pm 0.0130$ (stat. and syst. combined)
	2005	CLEO [799]	$+0.01^{+0.09}_{-0.07} \pm 0.05$
		COMBOS average	-0.0023 ± 0.0042
$D^0 \rightarrow K^- \pi^+ \pi^0$	2014	CLEOc [778]	$+0.001 \pm 0.003 \pm 0.004$
	2001	CLEO [800]	-0.031 ± 0.086 (stat.)
		COMBOS average	$+0.0009 \pm 0.0050$
$D^0 \rightarrow K^+ \pi^- \pi^0$	2005	Belle [801]	-0.006 ± 0.053 (stat.)
	2001	CLEO [802]	$+0.09^{+0.25}_{-0.22}$ (stat.)
		COMBOS average	-0.0014 ± 0.0517
$D^0 \rightarrow K_s^0 \pi^+ \pi^-$	2012	CDF [803]	$-0.0005 \pm 0.0057 \pm 0.0054$
	2004	CLEO [804]	$-0.009 \pm 0.021^{+0.016}_{-0.057}$
		COMBOS average	-0.0008 ± 0.0077
$D^0 \rightarrow K^+ K^- \pi^0$	2008	<i>BABAR</i> [797]	$0.0100 \pm 0.0167 \pm 0.0025$
$D^0 \rightarrow \pi^- \pi^- \pi^+ \pi^+$	2013	LHCb [805]	Amplitude analysis, no evidence for CP violation
$D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$	2005	Belle [801]	-0.018 ± 0.044 (stat.)
$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$	2013	LHCb [805]	Amplitude analysis, no evidence for CP violation
	2005	FOCUS [790]	$-0.082 \pm 0.056 \pm 0.047$

Table 226: CP asymmetries $A_{CP} = [\Gamma(D_s^+) - \Gamma(D_s^-)]/[\Gamma(D_s^+) + \Gamma(D_s^-)]$ for $D_s^\pm$ decays.

Mode	Year	Collaboration	A_{CP}
$D_s^+ \rightarrow \mu^+ \nu$	2009	CLEOc [806]	$+0.048 \pm 0.061$
$D_s^+ \rightarrow \pi^+ \eta$	2013	CLEOc [807]	$+0.011 \pm 0.030 \pm 0.008$
$D_s^+ \rightarrow \pi^+ \eta'$	2013	CLEOc [807]	$-0.022 \pm 0.022 \pm 0.006$
$D_s^+ \rightarrow K_s^0 \pi^+$	2014	LHCb [782]	$+0.0038 \pm 0.0046 \pm 0.0017$
	2013	BABAR [783]	$+0.006 \pm 0.020 \pm 0.003$
	2010	Belle [808]	$+0.0545 \pm 0.0250 \pm 0.0033$
	2010	CLEOc [776]	$+0.163 \pm 0.073 \pm 0.003$
		COMBOS average	$+0.0063 \pm 0.0047$
$D_s^+ \rightarrow K_s^0 K^+$	2013	CLEOc [807]	$+0.026 \pm 0.015 \pm 0.006$
	2013	BABAR [783]	$-0.0005 \pm 0.0023 \pm 0.0024$
	2010	Belle [808]	$+0.0012 \pm 0.0036 \pm 0.0022$
		COMBOS average	$+0.0008 \pm 0.0026$
$D_s^+ \rightarrow K^+ \pi^0$	2010	CLEOc [776]	$+0.266 \pm 0.228 \pm 0.009$
$D_s^+ \rightarrow K^+ \eta$	2010	CLEOc [776]	$+0.093 \pm 0.152 \pm 0.009$
$D_s^+ \rightarrow K^+ \eta'$	2010	CLEOc [776]	$+0.060 \pm 0.189 \pm 0.009$
$D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$	2013	CLEOc [807]	$-0.007 \pm 0.030 \pm 0.006$
$D_s^+ \rightarrow \pi^+ \pi^0 \eta$	2013	CLEOc [807]	$-0.005 \pm 0.039 \pm 0.020$
$D_s^+ \rightarrow \pi^+ \pi^0 \eta'$	2013	CLEOc [807]	$-0.004 \pm 0.074 \pm 0.019$
$D_s^+ \rightarrow K_s^0 K^+ \pi^0$	2013	CLEOc [807]	$-0.016 \pm 0.060 \pm 0.011$
$D_s^+ \rightarrow K_s^0 K_s^0 \pi^+$	2013	CLEOc [807]	$+0.031 \pm 0.052 \pm 0.006$
$D_s^+ \rightarrow K^+ \pi^+ \pi^-$	2013	CLEOc [807]	$+0.045 \pm 0.048 \pm 0.006$
$D_s^+ \rightarrow K^+ K^- \pi^+$	2013	CLEOc [807]	$-0.005 \pm 0.008 \pm 0.004$
$D_s^+ \rightarrow K_s^0 K^- \pi^+ \pi^+$	2013	CLEOc [807]	$+0.041 \pm 0.027 \pm 0.009$
$D_s^+ \rightarrow K_s^0 K^+ \pi^+ \pi^-$	2013	CLEOc [807]	$-0.057 \pm 0.053 \pm 0.009$
$D_s^+ \rightarrow K^+ K^- \pi^+ \pi^0$	2013	CLEOc [807]	$+0.000 \pm 0.027 \pm 0.012$

8.3 T -violating asymmetries

T -violating asymmetries are measured using triple-product correlations and assuming the validity of the CPT theorem. Triple-product correlations of the form $\vec{a} \cdot (\vec{b} \times \vec{c})$, where a , b , and c are spins or momenta, are odd under time reversal (T). For example, for $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ decays, $C_T \equiv \vec{p}_{K^+} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})$ changes sign (*i.e.* is odd) under a T transformation. The corresponding quantity for $\bar{D}^0$ is $\bar{C}_T \equiv \vec{p}_{K^-} \cdot (\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})$. Defining

$$A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)} \quad (201)$$

for D^0 decays and

$$\bar{A}_T = \frac{\Gamma(-\bar{C}_T > 0) - \Gamma(-\bar{C}_T < 0)}{\Gamma(-\bar{C}_T > 0) + \Gamma(-\bar{C}_T < 0)} \quad (202)$$

for $\bar{D}^0$ decays, in the absence of strong phases either $A_T \neq 0$ or $\bar{A}_T \neq 0$ indicates T violation. In these expressions the Γ 's are partial widths. The asymmetry

$$A_{T \text{ viol}} \equiv \frac{A_T - \bar{A}_T}{2} \quad (203)$$

tests for T violation even with nonzero strong phases (see Refs. [809–813]). Values of $A_{T \text{ viol}}$ for some D^+ , D_s^+ , and D^0 decay modes are listed in Table 227.

T -violating asymmetry is a clean and alternative way to search for CP violation in the charm sector. Analogously to what is shown in Sec. 8.2 there is no evidence of CP violation.

Table 227: T -violating asymmetries $A_{T \text{ viol}} = (A_T - \bar{A}_T)/2$.

Mode	Year	Collaboration	$A_{T \text{ viol}}$
$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$	2014	LHCb [814]	$+0.0018 \pm 0.0029 \pm 0.0004$
	2010	BABAR [815]	$+0.0010 \pm 0.0051 \pm 0.0044$
	2005	FOCUS [790]	$+0.010 \pm 0.057 \pm 0.037$
		COMBOS average	$+0.0017 \pm 0.0027$
$D^+ \rightarrow K_s^0 K^+ \pi^+ \pi^-$	2011	BABAR [816]	$-0.0120 \pm 0.0100 \pm 0.0046$
	2005	FOCUS [790]	$+0.023 \pm 0.062 \pm 0.022$
		COMBOS average	-0.0110 ± 0.0109
$D_s^+ \rightarrow K_s^0 K^+ \pi^+ \pi^-$	2011	BABAR [816]	$-0.0136 \pm 0.0077 \pm 0.0034$
	2005	FOCUS [790]	$-0.036 \pm 0.067 \pm 0.023$
		COMBOS average	-0.0139 ± 0.0084

8.4 Interplay of direct and indirect CP violation

In decays of D^0 mesons, CP asymmetry measurements have contributions from both direct and indirect CP violation as discussed in Sec. 8.1. The contribution from indirect CP violation depends on the decay-time distribution of the data sample [769]. This section describes a combination of measurements that allows the extraction of the individual contributions of the two types of CP violation. At the same time, the level of agreement for a no- CP -violation hypothesis is tested. The observables are:

$$A_\Gamma \equiv \frac{\tau(\overline{D}^0 \rightarrow h^+ h^-) - \tau(D^0 \rightarrow h^+ h^-)}{\tau(\overline{D}^0 \rightarrow h^+ h^-) + \tau(D^0 \rightarrow h^+ h^-)}, \quad (204)$$

where $h^+ h^-$ can be $K^+ K^-$ or $\pi^+ \pi^-$, and

$$\Delta A_{CP} \equiv A_{CP}(K^+ K^-) - A_{CP}(\pi^+ \pi^-), \quad (205)$$

where A_{CP} are time-integrated CP asymmetries. The underlying theoretical parameters are:

$$\begin{aligned} a_{CP}^{\text{dir}} &\equiv \frac{|\mathcal{A}_{D^0 \rightarrow f}|^2 - |\mathcal{A}_{\overline{D}^0 \rightarrow f}|^2}{|\mathcal{A}_{D^0 \rightarrow f}|^2 + |\mathcal{A}_{\overline{D}^0 \rightarrow f}|^2}, \\ a_{CP}^{\text{ind}} &\equiv \frac{1}{2} \left[\left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) x \sin \phi - \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) y \cos \phi \right], \end{aligned} \quad (206)$$

where $\mathcal{A}_{D \rightarrow f}$ is the amplitude for $D \rightarrow f$ [817]. We use the following relations between the observables and the underlying parameters [818]:

$$\begin{aligned} A_\Gamma &= -a_{CP}^{\text{ind}} - a_{CP}^{\text{dir}} y_{CP}, \\ \Delta A_{CP} &= \Delta a_{CP}^{\text{dir}} \left(1 + y_{CP} \frac{\langle t \rangle}{\tau} \right) + a_{CP}^{\text{ind}} \frac{\Delta \langle t \rangle}{\tau} + \overline{a_{CP}^{\text{dir}}} y_{CP} \frac{\Delta \langle t \rangle}{\tau}, \end{aligned} \quad (207)$$

$$\approx \Delta a_{CP}^{\text{dir}} \left(1 + y_{CP} \frac{\langle t \rangle}{\tau} \right) + a_{CP}^{\text{ind}} \frac{\Delta \langle t \rangle}{\tau}. \quad (208)$$

The first relation constrains mostly indirect CP violation, and the direct CP violation contribution can differ for different final states. In the second relation, $\langle t \rangle / \tau$ denotes the mean decay time in units of the D^0 lifetime; ΔX denotes the difference in quantity X between $K^+ K^-$ and $\pi^+ \pi^-$ final states; and $\overline{X}$ denotes the average for quantity X . We neglect the last term in this relation as all three factors are $\mathcal{O}(10^{-2})$ or smaller, and thus this term is negligible with respect to the other two terms. Note that $\Delta \langle t \rangle / \tau \ll \langle t \rangle / \tau$, and it is expected that $|a_{CP}^{\text{dir}}| < |\Delta a_{CP}^{\text{dir}}|$ because $a_{CP}^{\text{dir}}(K^+ K^-)$ and $a_{CP}^{\text{dir}}(\pi^+ \pi^-)$ are expected to have opposite signs [817].

A χ^2 fit is performed in the plane $\Delta a_{CP}^{\text{dir}}$ vs. a_{CP}^{ind} . For the *BABAR* result the difference of the quoted values for $A_{CP}(K^+ K^-)$ and $A_{CP}(\pi^+ \pi^-)$ is calculated, adding all uncertainties in quadrature. This may overestimate the systematic uncertainty for the difference as it neglects correlated errors; however, the result is conservative and the effect is small as all measurements are statistically limited. For all measurements, statistical and systematic uncertainties are added in quadrature when calculating the χ^2 . We use the current world average value $y_{CP} = (0.866 \pm 0.155)\%$ (see Sec. 8.1) and the measurements listed in Table 228.

Table 228: Inputs to the fit for direct and indirect CP violation. The first uncertainty listed is statistical, and the second is systematic.

Year	Experiment	Results	$\Delta\langle t \rangle/\tau$	$\langle t \rangle/\tau$	Reference
2012	Belle prel.	$A_\Gamma = (-0.03 \pm 0.20 \pm 0.08)\%$	-	-	[763]
2012	<i>BABAR</i>	$A_\Gamma = (0.09 \pm 0.26 \pm 0.06)\%$	-	-	[764]
2013	LHCb	$A_\Gamma(KK) = (-0.035 \pm 0.062 \pm 0.012)\%$	-	-	[766]
		$A_\Gamma(\pi\pi) = (0.033 \pm 0.106 \pm 0.014)\%$	-	-	
2008	<i>BABAR</i>	$A_{CP}(KK) = (0.00 \pm 0.34 \pm 0.13)\%$			
		$A_{CP}(\pi\pi) = (-0.24 \pm 0.52 \pm 0.22)\%$	0.00	1.00	[754]
2012	Belle prel.	$\Delta A_{CP} = (-0.87 \pm 0.41 \pm 0.06)\%$	0.00	1.00	[819]
2012	CDF	$\Delta A_{CP} = (-0.62 \pm 0.21 \pm 0.10)\%$	0.25	2.58	[757]
2013	LHCb prel.	$\Delta A_{CP} = (-0.34 \pm 0.15 \pm 0.10)\%$	0.11	2.10	[820]
2014	LHCb	$\Delta A_{CP} = (0.14 \pm 0.16 \pm 0.08)\%$	0.01	1.07	[742]

In this fit, $A_\Gamma(KK)$ and $A_\Gamma(\pi\pi)$ are assumed to be identical. This assumption is supported by the most recent LHCb measurements [766]. A significant relative shift due to final-state dependent A_Γ values between ΔA_{CP} measurements with different mean decay times is excluded by these measurements.

The combination plot (see Fig. 159) shows the measurements listed in Table 228 for ΔA_{CP} and A_Γ , where the bands represent $\pm 1\sigma$ intervals. The point of no CP violation (0,0) is shown as a filled circle, and two-dimensional 68% CL, 95% CL, and 99.7% CL regions are plotted as ellipses. The best fit value is indicated by a cross showing the one-dimensional errors.

From the fit, the change in χ^2 from the minimum value for the no- CPV point (0,0) is 5.9, which corresponds to a CL of 5.1×10^{-2} for two degrees of freedom. Thus the data are consistent with the no- CP -violation hypothesis at 5.1% CL. This p -value corresponds to 2.0σ . The central values and $\pm 1\sigma$ errors for the individual parameters are

$$\begin{aligned}
 a_{CP}^{\text{ind}} &= (0.013 \pm 0.052)\% \\
 \Delta a_{CP}^{\text{dir}} &= (-0.253 \pm 0.104)\%.
 \end{aligned} \tag{209}$$

These values indicate that the present small deviation from no CP violation is primarily due to a difference between direct CP violation in the two final states, rather than due to common indirect CP violation.

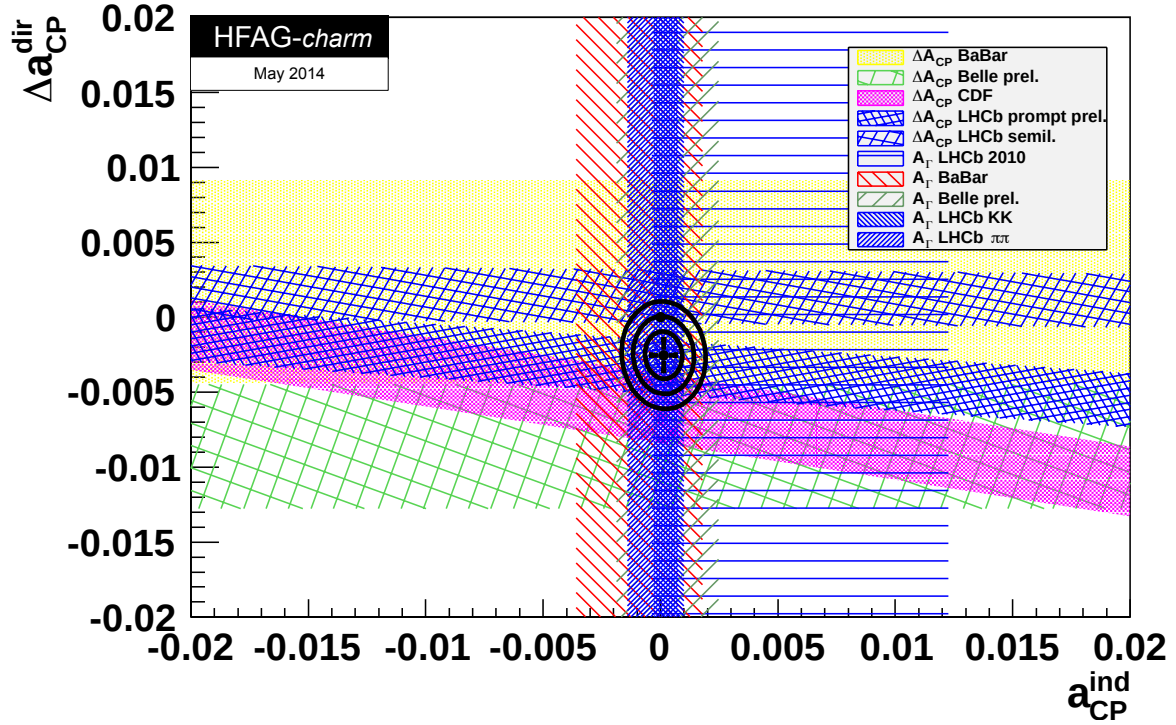


Figure 159: Plot of all data and the fit result. Individual measurements are plotted as bands showing their $\pm 1\sigma$ range. The no- CPV point $(0,0)$ is shown as a filled circle, and the best fit value is indicated by a cross showing the one-dimensional errors. Two-dimensional 68% CL, 95% CL, and 99.7% CL regions are plotted as ellipses.

8.5 Semileptonic decays

8.5.1 Introduction

Semileptonic decays of D mesons involve the interaction of a leptonic current with a hadronic current. The latter is nonperturbative and cannot be calculated from first principles; thus it is usually parameterized in terms of form factors. The transition matrix element is written

$$\mathcal{M} = -i \frac{G_F}{\sqrt{2}} V_{cq} L^\mu H_\mu, \quad (210)$$

where G_F is the Fermi constant and V_{cq} is a CKM matrix element. The leptonic current L^μ is evaluated directly from the lepton spinors and has a simple structure; this allows one to extract information about the form factors (in H_μ) from data on semileptonic decays [821]. Conversely, because there are no final-state interactions between the leptonic and hadronic systems, semileptonic decays for which the form factors can be calculated allow one to determine V_{cq} [2].

8.5.2 $D \rightarrow P \bar{\ell} \nu_\ell$ decays

When the final state hadron is a pseudoscalar, the hadronic current is given by

$$H_\mu = \langle P(p) | \bar{q} \gamma_\mu c | D(p') \rangle = f_+(q^2) \left[(p' + p)_\mu - \frac{m_D^2 - m_P^2}{q^2} q_\mu \right] + f_0(q^2) \frac{m_D^2 - m_P^2}{q^2} q_\mu, \quad (211)$$

where m_D and p' are the mass and four momentum of the parent D meson, m_P and p are those of the daughter meson, $f_+(q^2)$ and $f_0(q^2)$ are form factors, and $q = p' - p$. Kinematics require that $f_+(0) = f_0(0)$. The contraction $q_\mu L^\mu$ results in terms proportional to m_ℓ [822], and thus for $\ell = e$ the terms proportional to q_μ in Eq. (211) are negligible. For light leptons only the $f_+(q^2)$ form factor is relevant and the differential partial width is

$$\frac{d\Gamma(D \rightarrow P \bar{\ell} \nu_\ell)}{dq^2 d\cos\theta_\ell} = \frac{G_F^2 |V_{cq}|^2}{32\pi^3} p^{*3} |f_+(q^2)|^2 \sin^2\theta_\ell, \quad (212)$$

where p^* is the magnitude of the momentum of the final state hadron in the D rest frame, and θ_ℓ is the angle of the lepton in the $\ell\nu$ rest frame with respect to the direction of the pseudoscalar meson in the D rest frame.

8.5.3 Form factor parameterizations

The form factor is traditionally parameterized with an explicit pole and a sum of effective poles:

$$f_+(q^2) = \frac{f_+(0)}{(1 - \alpha)} \left[\left(\frac{1}{1 - q^2/m_{\text{pole}}^2} \right) + \sum_{k=1}^N \frac{\rho_k}{1 - q^2/(\gamma_k m_{\text{pole}}^2)} \right], \quad (213)$$

where ρ_k and γ_k are expansion parameters and α is such that provides the form factor normalization at $q^2 = 0$, $f_+(0)$. The parameter m_{pole} is the mass of the lowest-lying $c\bar{q}$ resonance with the appropriate quantum numbers; this is expected to provide the largest contribution to the form factor for the $c \rightarrow q$ transition. The sum over N gives the contribution of higher mass

states. For example, for $D \rightarrow \pi$ transitions the dominant resonance is expected to be $D^*(2010)$, and thus $m_{\text{pole}} = m_{D^*(2010)}$.

Simple pole

Equation (213) can be simplified by neglecting the sum over effective poles, leaving only the explicit vector meson pole. This approximation is referred to as “nearest pole dominance” or “vector-meson dominance.” The resulting parameterization is

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{\text{pole}}^2)}. \quad (214)$$

However, values of m_{pole} that give a good fit to the data do not agree with the expected vector meson masses [823]. To address this problem, the “modified pole” or Becirevic-Kaidalov (BK) parameterization [824] was introduced. $m_{\text{pole}}/\sqrt{\alpha_{\text{BK}}}$ is interpreted as the mass of an effective pole, higher than m_{pole} , thus it is expected that $\alpha_{\text{BK}} < 1$.

The parameterization takes the form

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{\text{pole}}^2)} \frac{1}{\left(1 - \alpha_{\text{BK}} \frac{q^2}{m_{\text{pole}}^2}\right)}. \quad (215)$$

These parameterizations have been used by several experiments to determine form factor parameters. Measured values of m_{pole} and α_{BK} are listed in Tables 229 and 230 for $D \rightarrow K\ell\nu_\ell$ and $D \rightarrow \pi\ell\nu_\ell$ decays, respectively.

z expansion

An alternative series expansion around some value $q^2 = t_0$ to parameterize $f_+(q^2)$ can be used [821, 825–827]. This parameterization is model independent and satisfies general QCD constraints, being suitable for fitting experimental data. The expansion is given in terms of a complex parameter z , which is the analytic continuation of q^2 into the complex plane:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (216)$$

where $t_\pm \equiv (m_D \pm m_P)^2$ and t_0 is the (arbitrary) q^2 value corresponding to $z = 0$. The physical region corresponds to $\pm|z|_{\text{max}} = \pm 0.051$ for $D \rightarrow K\ell\nu_\ell$ and $= \pm 0.17$ for $D \rightarrow \pi\ell\nu_\ell$, using $t_0 = t_+(1 - \sqrt{1 - t_-/t_+})$.

The form factor is expressed as

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2, t_0)]^k, \quad (217)$$

where the $P(q^2)$ factor accommodates sub-threshold resonances via

$$P(q^2) \equiv \begin{cases} 1 & (D \rightarrow \pi) \\ z(q^2, M_{D_s^*}^2) & (D \rightarrow K). \end{cases} \quad (218)$$

The “outer” function $\phi(t, t_0)$ can be any analytic function, but a preferred choice (see, *e.g.* Refs. [825, 826, 828]) obtained from the Operator Product Expansion (OPE) is

$$\phi(q^2, t_0) = \alpha \left(\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0} \right) \times \frac{t_+ - q^2}{(t_+ - t_0)^{1/4}} \frac{(\sqrt{t_+ - q^2} + \sqrt{t_+ - t_-})^{3/2}}{(\sqrt{t_+ - q^2} + \sqrt{t_+})^5}, \quad (219)$$

with $\alpha = \sqrt{\pi m_c^2/3}$. The OPE analysis provides a constraint upon the expansion coefficients, $\sum_{k=0}^N a_k^2 \leq 1$. These coefficients receive $1/M_D$ corrections, and thus the constraint is only approximate. However, the expansion is expected to converge rapidly since $|z| < 0.051$ (0.17) for $D \rightarrow K$ ($D \rightarrow \pi$) over the entire physical q^2 range, and Eq. (217) remains a useful parameterization. The main disadvantage as compared to phenomenological approaches is that there is no physical interpretation of the fitted coefficients a_K .

Three-pole formalism

A recent update of the vector pole dominance model has been developed for the $D \rightarrow \pi \ell \nu_\ell$ channel [829]. It uses information of the residues of the semileptonic form factor at its first two poles, the $D^*(2010)$ and $D^{*'}(2600)$ resonances. The form factor is expressed as an infinite sum of residues from $J^P = 1^-$ states with masses $m_{D_n^*}$:

$$f_+(q^2) = \sum_{n=0}^{\infty} \frac{\text{Res}_{q^2=m_{D_n^*}^2} f_+(q^2)}{m_{D_n^*}^2 - q^2}, \quad (220)$$

with the residues given by

$$\text{Res}_{q^2=m_{D_n^*}^2} f_+(q^2) = \frac{1}{2} m_{D_n^*} f_{D_n^*} g_{D_n^* D \pi}. \quad (221)$$

Values of the f_{D^*} and $f_{D^{*'}}$ decay constants have been obtained by Lattice QCD calculations, relative to f_D , with 2% and 28% precision, respectively [829]. The couplings to the $D\pi$ state, $g_{D^* D \pi}$ and $g_{D^{*' D \pi}$, are extracted from measurements of the $D^*(2010)$ and $D^{*'}(2600)$ widths by BABAR and LHCb experiments [830–832]. Thus the contribution from the first pole is known with a 3% accuracy. The contribution from the $D^{*'}(2600)$ is determined with poorer accuracy, $\sim 30\%$, mainly due to lattice uncertainties. A *superconvergence* condition is applied [833]:

$$\sum_{n=0}^{\infty} \text{Res}_{q^2=m_{D_n^*}^2} f_+(q^2) = 0, \quad (222)$$

protecting the form factor behavior at large q^2 . Within this model the first two poles are not sufficient to describe the data, and a third effective pole needs to be included.

One of the advantages of this phenomenological model is that it can be extrapolated outside the charm physical region, providing a method to extract the CKM matrix element V_{ub} using the ratio of the form factors of the $D \rightarrow \pi \ell \nu$ and $B \rightarrow \pi \ell \nu$ decay channels. It will be used once Lattice calculations provide the form factor ratio $f_{B\pi}^+(q^2)/f_{D\pi}^+(q^2)$ at the same pion energy.

8.5.4 Experimental techniques and results

Different techniques by several experiments have been used to measure D meson semileptonic decays with a pseudoscalar particle in the final state. The most recent results are provided by the *BABAR* and BES III collaborations. They have been presented at the ICHEP 2014 conference and are preliminary. Belle [834], *BABAR* [835] and CLEO-c [836, 837] collaborations have previously reported results. The Belle collaboration fully reconstructs the D events from the continuum under the $\Upsilon(4S)$ resonance, achieving a very good q^2 resolution ($\Delta q^2 = 15 \text{ MeV}^2$) and low background level, but having a low efficiency. Using 282 fb^{-1} , about 1300 and 115 signal semileptonic decays are isolated for each lepton flavor (e and μ), respectively. The *BABAR* experiment uses a partial reconstruction technique where the semileptonic decays are tagged through the $D^{*+} \rightarrow D^0 \pi^+$ decay. The D direction and neutrino energy is obtained using information from the rest of the event. With 75 fb^{-1} 74000 signal events in the $D^0 \rightarrow K^- e^+ \nu$ mode are obtained. This technique provides larger statistics but a higher background level and poorer q^2 resolution (Δq^2 ranges from 66 to 219 MeV^2). In this case the measurement of the branching fraction is obtained by normalizing to the $D^0 \rightarrow K^- \pi^+$ decay channel and will benefit from future improvements in the determination of this reference channel. The measurement of the Cabibbo suppressed mode has been recently obtained using the same technique and 350 fb^{-1} data. 5000 $D^0 \rightarrow \pi^- e^+ \nu$ signal events are reconstructed using this method [838]. The CLEO-c experiment uses two different methods to measure charm semileptonic decays. Tagged analyses [836] rely on the full reconstruction of $\Psi(3770) \rightarrow D\bar{D}$ events. One of the D mesons is reconstructed in a hadronic decay mode, the other in the semileptonic channel. The only missing particle is the neutrino so the q^2 resolution is very good and the background level very low. With the entire CLEO-c data sample, 818 pb^{-1} , 14123 and 1374 signal events are reconstructed for the $D^0 \rightarrow K^- e^+ \nu$ and $D^0 \rightarrow \pi^- e^+ \nu$ channels, and 8467 and 838 for the $D^+ \rightarrow \bar{K}^0 e^+ \nu$ and $D^+ \rightarrow \pi^0 e^+ \nu$ decays, respectively. Another technique without tagging the D meson in a hadronic mode (“untagged” in the following) has been also used by CLEO-c [837]. In this method, all missing energy and momentum in an event are associated with the neutrino four momentum, with the penalty of larger background as compared to the tagged method. Using the “tagged” method the BES III experiment has measured the $D^0 \rightarrow K^- e^+ \nu$ and $D^0 \rightarrow \pi^- e^+ \nu$ decay channels. With 2.9 fb^{-1} they fully reconstruct 70700 and 6300 signal events for each channel, respectively. These results are preliminary.

Previous measurements were also performed by CLEO III and FOCUS experiments. Events registered at the $\Upsilon(4S)$ energy corresponding to an integrated luminosity of 7 fb^{-1} were analyzed by CLEO III [839]. In the FOCUS fixed target photo-production experiment, D^0 semileptonic events were obtained from the decay of a D^{*+} , and the kaon or pion was reconstructed in the muon channel. Results of the hadronic form factor parameters by the different groups are given in Tables 229 and 230 for m_{pole} and α_{BK} .

The z -expansion formalism has been used by *BABAR* [835, 838], BES III [841] and CLEOc [836], [837]. Their fits use the first three terms of the expansion, and the results for the ratios $r_1 \equiv a_1/a_0$ and $r_2 \equiv a_2/a_0$ are listed in Tables 231 and 232. The CLEO III [839] and FOCUS [840] results listed are obtained by refitting their data using the full covariance matrix. The *BABAR* correlation coefficient listed is obtained by refitting their published branching fraction using their published covariance matrix. These measurements correspond to using the standard outer function $\phi(q^2, t_0)$ of Eq. (219) and $t_0 = t_+ \left(1 - \sqrt{1 - t_-/t_+}\right)$. This choice of t_0 constrains $|z|$ to vary between $\pm z_{max}$.

Table 229: Results for m_{pole} and α_{BK} from various experiments for $D^0 \rightarrow K^-\ell^+\nu$ and $D^+ \rightarrow K_S\ell^+\nu$ decays.

$D \rightarrow K\ell\nu_\ell$ Expt.	Ref.	m_{pole} (GeV/ c^2)	α_{BK}
CLEO III	[839]	$1.89 \pm 0.05^{+0.04}_{-0.03}$	$0.36 \pm 0.10^{+0.03}_{-0.07}$
FOCUS	[840]	$1.93 \pm 0.05 \pm 0.03$	$0.28 \pm 0.08 \pm 0.07$
Belle	[834]	$1.82 \pm 0.04 \pm 0.03$	$0.52 \pm 0.08 \pm 0.06$
<i>BABAR</i>	[835]	$1.889 \pm 0.012 \pm 0.015$	$0.366 \pm 0.023 \pm 0.029$
CLEO-c (tagged)	[836]	$1.93 \pm 0.02 \pm 0.01$	$0.30 \pm 0.03 \pm 0.01$
CLEO-c (untagged, D^0)	[837]	$1.97 \pm 0.03 \pm 0.01$	$0.21 \pm 0.05 \pm 0.03$
CLEO-c (untagged, D^+)	[837]	$1.96 \pm 0.04 \pm 0.02$	$0.22 \pm 0.08 \pm 0.03$
BESIII (preliminary)	[841]	$1.921 \pm 0.010 \pm 0.007$	$0.309 \pm 0.020 \pm 0.013$

Table 230: Results for m_{pole} and α_{BK} from various experiments for $D^0 \rightarrow \pi^-\ell^+\nu$ and $D^+ \rightarrow \pi^0\ell^+\nu$ decays.

$D \rightarrow \pi\ell\nu_\ell$ Expt.	Ref.	m_{pole} (GeV/ c^2)	α_{BK}
CLEO III	[839]	$1.86^{+0.10+0.07}_{-0.06-0.03}$	$0.37^{+0.20}_{-0.31} \pm 0.15$
FOCUS	[840]	$1.91^{+0.30}_{-0.15} \pm 0.07$	—
Belle	[834]	$1.97 \pm 0.08 \pm 0.04$	$0.10 \pm 0.21 \pm 0.10$
CLEO-c (tagged)	[836]	$1.91 \pm 0.02 \pm 0.01$	$0.21 \pm 0.07 \pm 0.02$
CLEO-c (untagged, D^0)	[837]	$1.87 \pm 0.03 \pm 0.01$	$0.37 \pm 0.08 \pm 0.03$
CLEO-c (untagged, D^+)	[837]	$1.97 \pm 0.07 \pm 0.02$	$0.14 \pm 0.16 \pm 0.04$
BES III (preliminary)	[841]	$1.911 \pm 0.012 \pm 0.004$	$0.279 \pm 0.035 \pm 0.011$
<i>BABAR</i> (preliminary)	[838]	$1.906 \pm 0.029 \pm 0.023$	$0.268 \pm 0.074 \pm 0.059$

The combined result for the $D \rightarrow K\ell\nu_\ell$ decay channel is obtained from a three-dimensional fit to *BABAR*, BES III (preliminary), CLEO III, CLEO-c and FOCUS data, taking the full correlations between $|V_{cq}|f_+(0)$, r_1 and r_2 into account. Data from each experiment is fitted with the z -expansion model and the combination is performed over the fitted results. The fit is constrained by the branching fraction measured at Belle [834]. The normalization of the form factor is fixed for the FOCUS data. The effect of radiative events has been taken into account slightly modifying the values from *BABAR* by correcting the numbers given in Tab. III of Ref. [835] by the shifts quoted in the last column of Tab. IV given in Ref. [835]. For this combination, the BES III (preliminary) result obtained with partial statistics is used [842]. Results of the combined fit are shown in Table 231, Table 235 and Figure 160. The $\chi^2/d.o.f$ of the combined fit is 12/13. The correlation matrix is given in Table 233.

The combined result for the $D \rightarrow \pi\ell\nu_\ell$ decay channel is obtained from a fit to *BABAR* (preliminary), Belle, BES III (preliminary), and CLEO-c data. The combination is performed in this case by fitting all the available measurements in bins of q^2 to the z -expansion model, instead of a combination fit to the individual fitted parameters in the z -expansion. Published

Table 231: Results for r_1 and r_2 from various experiments, for $D \rightarrow K\ell\nu_\ell$. The correlation coefficient listed is for the total uncertainties (statistical $\oplus$ systematic) on r_1 and r_2 .

The combined result does not include the new BES III result presented at the ICHEP2014 conference [841], but the previous one with partial statistics [842]. The fit is constrained by the branching fractions measured at Belle [834].

Expt. $D \rightarrow K\ell\nu_\ell$	mode	Ref.	r_1	r_2	ρ
CLEO III		[839]	$0.2^{+3.6}_{-3.0}$	-89^{+104}_{-120}	-0.99
FOCUS		[840]	-2.54 ± 0.75	7 ± 13	-0.97
<i>BABAR</i>		[835]	$-2.5 \pm 0.2 \pm 0.2$	$2.5 \pm 6.0 \pm 5.0$	-0.64
CLEO-c (tagged)	$D^0 \rightarrow K^-$	[836]	$-2.65 \pm 0.34 \pm 0.08$	$13 \pm 9 \pm 1$	-0.82
CLEO-c (tagged)	$D^+ \rightarrow \bar{K}^0$	[836]	$-1.66 \pm 0.44 \pm 0.10$	$-14 \pm 11 \pm 1$	-0.82
CLEO-c (untagged)	$D^0 \rightarrow K^-$	[837]	$-2.4 \pm 0.4 \pm 0.1$	$21 \pm 11 \pm 2$	-0.81
CLEO-c (untagged)	$D^+ \rightarrow \bar{K}^0$	[837]	$-2.8 \pm 6 \pm 2$	$32 \pm 18 \pm 4$	-0.84
BES III (0.9 fb $^{-1}$)		[842]	$-2.18 \pm 0.36 \pm 0.05$	$5 \pm 9 \pm 1$	
BES III (preliminary, 2.9 fb $^{-1}$)		[841]	$-2.33 \pm 0.16 \pm 0.08$	$3.4 \pm 3.9 \pm 2.4$	
Combined (preliminary)			-2.39 ± 0.17	6.2 ± 3.8	-0.82

Table 232: Results for r_1 and r_2 from various experiments, for $D \rightarrow \pi\ell\nu_\ell$. The correlation coefficient listed is for the total uncertainties (statistical $\oplus$ systematic) on r_1 and r_2 . The combined result includes preliminary results from *BABAR* and BES III presented at ICHEP 2014. The Belle data is refitted in the z -expansion formalism using published values of $f_+^{D\pi}(q^2) \times |V_{cd}|$, and removing the uncertainty on V_{cd} from the systematic error.

Expt. $D \rightarrow \pi\ell\nu_\ell$	mode	Ref.	r_1	r_2	ρ
CLEO-c (tagged)	$D^0 \rightarrow \pi^+$	[836]	$-2.80 \pm 0.49 \pm 0.04$	$6 \pm 3 \pm 0$	-0.94
CLEO-c (tagged)	$D^+ \rightarrow \pi^0$	[836]	$-1.37 \pm 0.88 \pm 0.24$	$-4 \pm 5 \pm 1$	-0.96
CLEO-c (untagged)	$D^0 \rightarrow \pi^+$	[837]	$-2.1 \pm 0.7 \pm 0.3$	$-1.2 \pm 4.8 \pm 1.7$	-0.96
CLEO-c (untagged)	$D^+ \rightarrow \pi^0$	[837]	$-0.2 \pm 1.5 \pm 0.4$	$-9.8 \pm 9.1 \pm 2.1$	-0.97
Belle		[834]	-1.84 ± 1.02	1.69 ± 6.5	-0.91
BES III (preliminary)		[841]	$-1.85 \pm 0.22 \pm 0.07$	$-1.4 \pm 1.5 \pm 0.5$	-0.93
<i>BABAR</i> (preliminary)		[838]	$-1.31 \pm 0.70 \pm 0.43$	$-4.2 \pm 4.0 \pm 1.9$	-0.97
Combined (preliminary)			-1.94 ± 0.19	-0.62 ± 1.19	-0.94

Table 233: Correlation matrix for the combined fit for the $D^0 \rightarrow K^- \ell^+ \nu_\ell$ channel

	$ V_{cs} f_+^K(0)$	r_1	r_2
$ V_{cs} f_+^K(0)$	1.000	-0.088	0.433
r_1	-0.088	1.000	-0.824
r_2	0.433	-0.824	1.000

values of $f_+^{D\pi}(q^2) \times |V_{cd}|$ Belle data [834] are modified by subtracting the uncertainty on V_{cd} from the systematic error. Since the experimental q^2 resolution is very high, measurements at different q^2 are assumed uncorrelated. Preliminary results obtained with the full BES III statistics [841] are included in the combination of the $D \rightarrow \pi \ell \nu_\ell$ decay channel. The new preliminary *BABAR* results are already corrected for radiation effects [838]. Results of the combined fit are shown in Table 232, Table 235 and Figure 160. The $\chi^2/d.o.f$ of the combined fit is 51/55. The correlation matrix is given in Table 234.

Results for the form factor normalization $f_+^K(0)|V_{cs}|$ and $f_+^\pi(0)|V_{cd}|$ for each individual measurement and for the combination is presented in Table 235, obtained using the z -expansion formalism. In the combination only the result for the $D \rightarrow \pi \ell \nu_\ell$ channel is obtained with the total BES III statistics [841], while the $D \rightarrow K \ell \nu_\ell$ channel includes results with partial statistics [842].

Assuming unitarity of the CKM matrix, the values of the CKM matrix elements entering in charm semileptonic decays are known from the V_{ud} , V_{td} and V_{cb} elements [5]:

$$\begin{aligned} V_{cs} &= 0.97343 \pm 0.00015 \\ V_{cd} &= 0.22521 \pm 0.00061. \end{aligned} \quad (223)$$

Using the combined values of $f_+^K(0)|V_{cs}|$ and $f_+^\pi(0)|V_{cd}|$ in Table 235, this leads to the form factor values:

$$\begin{aligned} f_+^K(0) &= 0.7479 \pm 0.0051 \\ f_+^\pi(0) &= 0.6327 \pm 0.0086, \end{aligned} \quad (224)$$

which are in agreement with present Lattice QCD computations [843]: $f_+^K(0) = 0.747 \pm 0.019$ and $f_+^\pi(0) = 0.666 \pm 0.029$. If on the contrary one assumes the form factor values from Lattice, one obtains for the CKM matrix elements using the combined results in Table 235:

$$\begin{aligned} V_{cs} &= 0.9746 \pm 0.0026 \\ V_{cd} &= 0.2140 \pm 0.0097, \end{aligned} \quad (225)$$

still compatible with unitarity of the CKM matrix.

Results of the three-pole model [829] to *BABAR* [838], Belle [834], BES III [841] and CLEOc [836], [837] $D \rightarrow \pi \ell \nu_\ell$ data are shown in Table 236. Fitted parameters are the first two residues $\gamma_0 = \text{Res}_{q^2=m_{D^*}^2} f_+(q^2)$ and $\gamma_1 = \text{Res}_{q^2=m_{D^{*'}}^2} f_+(q^2)$ (which are constrained using present measurements of masses and widths of the $D^*(2010)$ and $D^{*'}(2600)$ mesons, and lattice computations of decay constants, following [829]), and an effective mass, $m_{D_{eff}^{*''}}$, accounting for higher mass hadronic

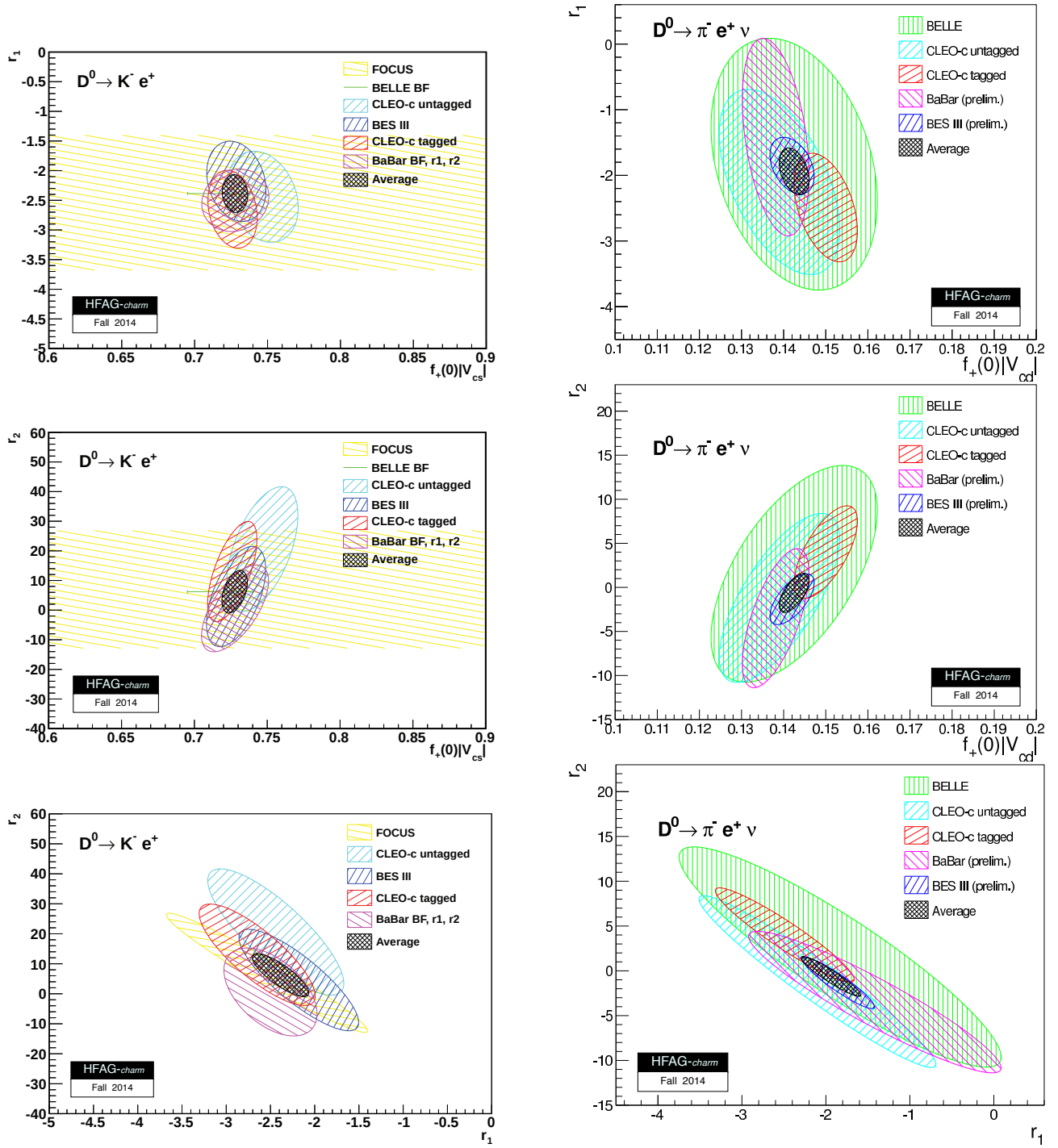


Figure 160: The $D^0 \rightarrow K^- e^+ \nu$ (left) and $D^0 \rightarrow \pi^- e^+ \nu$ (right) 68% C.L. error ellipses from the average fit of the 3-parameter z -expansion results.

Table 234: Correlation matrix for the combined fit for the $D^0 \rightarrow \pi^- \ell^+ \nu_\ell$ channel

	$ V_{cd} f_+^\pi(0)$	r_1	r_2
$ V_{cd} f_+^\pi(0)$	1.000	-0.379	0.634
r_1	-0.379	1.000	-0.936
r_2	0.634	-0.936	1.000

 Table 235: Results for the form factor normalization $f_+^K(0)|V_{cs}|$ and $f_+^\pi(0)|V_{cd}|$, obtained using the z -expansion formalism. Results of CLEO (2008) (untagged) shown in this table only refer to the D^0 channel. In the combination only the result for the $D \rightarrow \pi \ell \nu_\ell$ channel is obtained with the total BES III statistics [841], while the $D \rightarrow K \ell \nu_\ell$ channel includes results with partial statistics [842].

Experiment	Ref.	$f_+^K(0) V_{cs} $	$f_+^\pi(0) V_{cd} $
Belle (2006)	[834]	$0.692 \pm 0.007 \pm 0.022$	$0.140 \pm 0.004 \pm 0.007$
<i>BABAR</i> (2007), (preliminary 2014)	[835, 838]	$0.720 \pm 0.007 \pm 0.007$	$0.137 \pm 0.004 \pm 0.002$
CLEO-c (2008) (untagged)	[837]	$0.747 \pm 0.009 \pm 0.009$	$0.139 \pm 0.007 \pm 0.003$
CLEO-c (2009) (tagged)	[836]	$0.719 \pm 0.006 \pm 0.005$	$0.150 \pm 0.004 \pm 0.001$
BESIII (2014) (preliminary)	[841]	$0.720 \pm 0.004 \pm 0.004$	$0.142 \pm 0.002 \pm 0.001$
Combined fit (preliminary)		0.728 ± 0.005	0.1425 ± 0.0019

contributions. The V_{cd} value enters in the fit; the value used is that prescribed by unitarity in Eq. 223. The $\chi^2/d.o.f$ of the combined fit is 57.5/57.

The result for the effective mass $m_{D_{eff}^{*''}}$ is larger than the mass of the second radially excited state with $J^P = 1^-$ (~ 3.11 GeV), indicating that more contributions are needed to explain the form factor. Comparison of the combined fit with the individual data is shown in Figure 161.

8.5.5 $D \rightarrow V \bar{\ell} \nu_\ell$ decays

When the final state hadron is a vector meson, the decay can proceed through both vector and axial vector currents, and four form factors are needed. The hadronic current is $H_\mu = V_\mu + A_\mu$, where [822]

$$V_\mu = \langle V(p, \varepsilon) | \bar{q} \gamma_\mu c | D(p') \rangle = \frac{2V(q^2)}{m_D + m_V} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p'^\rho p^\sigma \quad (226)$$

$$\begin{aligned} A_\mu = \langle V(p, \varepsilon) | -\bar{q} \gamma_\mu \gamma_5 c | D(p') \rangle = & -i(m_D + m_V) A_1(q^2) \varepsilon_\mu^* \\ & + i \frac{A_2(q^2)}{m_D + m_V} (\varepsilon^* \cdot q) (p' + p)_\mu \\ & + i \frac{2m_V}{q^2} (A_3(q^2) - A_0(q^2)) [\varepsilon^* \cdot (p' + p)] q_\mu. \end{aligned} \quad (227)$$

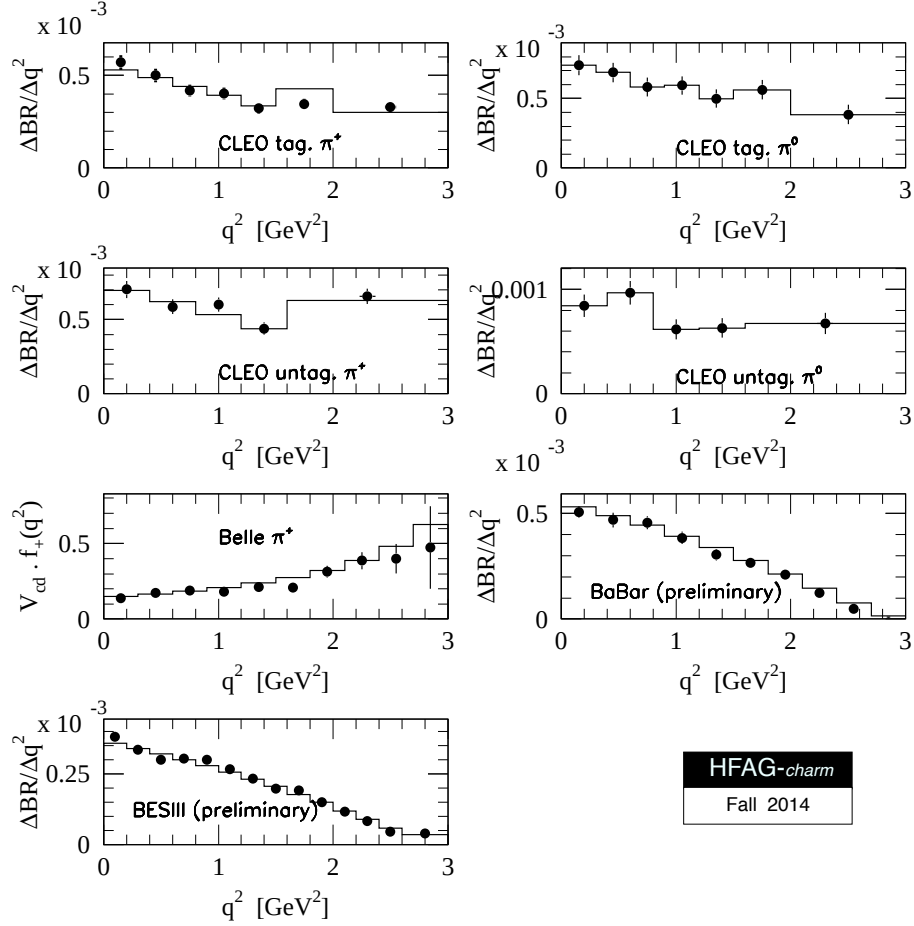


Figure 161: Result of the three-pole model fit [829] to *BABAR* [838], *Belle* [834], *BES III* [841] and *CLEOc* [836], [837] $D \rightarrow \pi \ell \nu_\ell$ data. Points are the measured data in q^2 bins and the black line correspond to the result of the combined fit.

Table 236: Results of the three-pole model to *BABAR* (preliminary), *Belle*, *BES III* (preliminary) and *CLEOc* (tagged and untagged) data. Fitted parameters are the first two residues γ_0 and γ_1 , which are constrained using present measurements of masses and widths of the D^* and $D^{*'}$ mesons, and lattice computations of decay constants, and the effective mass, $m_{D_{eff}^{*''}}$, accounting for higher mass hadronic contributions.

Parameter	Combined result ($D \rightarrow \pi \ell \nu_\ell$)
γ_0	$3.878 \pm 0.090 \text{ GeV}^2$
γ_1	$-1.18 \pm 0.30 \text{ GeV}^2$
$m_{D_{eff}^{*''}}$	$4.17 \pm 0.41 \text{ GeV}$

In this expression, m_V is the daughter meson mass and

$$A_3(q^2) = \frac{m_D + m_V}{2m_V} A_1(q^2) - \frac{m_D - m_V}{2m_V} A_2(q^2). \quad (228)$$

Kinematics require that $A_3(0) = A_0(0)$. Terms proportional to q_μ are only important for the case of τ leptons. Thus, only three form factors are relevant in these decays: $A_1(q^2)$, $A_2(q^2)$ and $V(q^2)$. The differential partial width is

$$\frac{d\Gamma(D \rightarrow V \bar{\ell} \nu_\ell)}{dq^2 d\cos\theta_\ell} = \frac{G_F^2 |V_{cq}|^2}{128\pi^3 m_D^2} p^* q^2 \times \left[\frac{(1 - \cos\theta_\ell)^2}{2} |H_-|^2 + \frac{(1 + \cos\theta_\ell)^2}{2} |H_+|^2 + \sin^2\theta_\ell |H_0|^2 \right], \quad (229)$$

where $H_\pm$ and H_0 are helicity amplitudes given by

$$H_\pm = \frac{1}{m_D + m_V} [(m_D + m_V)^2 A_1(q^2) \mp 2m_D p^* V(q^2)] \quad (230)$$

$$H_0 = \frac{1}{|q|} \frac{m_D^2}{2m_V(m_D + m_V)} \times \left[\left(1 - \frac{m_V^2 - q^2}{m_D^2} \right) (m_D + m_V)^2 A_1(q^2) - 4p^{*2} A_2(q^2) \right]. \quad (231)$$

p^* is the magnitude of the three-momentum of the V system, measured in the D rest frame, and θ_ℓ is defined in Figure 162 for the electron case (θ_e). The left-handed nature of the quark current manifests itself as $|H_-| > |H_+|$. The differential decay rate for $D \rightarrow V \ell \nu$ followed by the vector meson decaying into two pseudoscalars is

$$\begin{aligned} \frac{d\Gamma(D \rightarrow V \ell \nu, V \rightarrow P_1 P_2)}{dq^2 d\cos\theta_V d\cos\theta_\ell d\chi} &= \frac{3G_F^2 |V_{cq}|^2 p^*(q^2) q^2}{2048\pi^4 m_D^2} \mathcal{B}(V \rightarrow P_1 P_2) \times \\ &\left\{ (1 + \cos\theta_\ell)^2 \sin^2\theta_V |H_+(q^2)|^2 \right. \\ &+ (1 - \cos\theta_\ell)^2 \sin^2\theta_V |H_-(q^2)|^2 \\ &+ 4 \sin^2\theta_\ell \cos^2\theta_V |H_0(q^2)|^2 \\ &+ 4 \sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_+(q^2) H_0(q^2) \\ &- 4 \sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_-(q^2) H_0(q^2) \\ &\left. - 2 \sin^2\theta_\ell \sin^2\theta_V \cos 2\chi H_+(q^2) H_-(q^2) \right\}, \quad (232) \end{aligned}$$

where the angles θ_ℓ , θ_V , and χ are defined in Fig. 162.

Ratios between the values of the hadronic form factors expressed at $q^2 = 0$ are usually introduced:

$$r_V \equiv V(0)/A_1(0), \quad r_2 \equiv A_2(0)/A_1(0). \quad (233)$$

Table 237 lists measurements of r_V and r_2 from several experiments. Most of the measurements assume that the q^2 dependence of hadronic form factors is given by the simple pole ansatz. Some of these measurements do not consider a S-wave contribution and it is included in the measured values. The measurements are plotted in Fig. 163 which shows that they are all consistent.

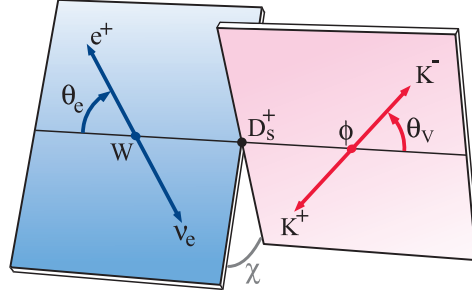


Figure 162: Decay angles θ_V , θ_ℓ and χ . Note that the angle χ between the decay planes is defined in the D -meson reference frame, whereas the angles θ_V and θ_ℓ are defined in the V meson and W reference frames, respectively.

Table 237: Results for r_V and r_2 from various experiments.

Experiment	Ref.	r_V	r_2
$D^+ \rightarrow \bar{K}^{*0} l^+ \nu$			
E691	[844]	$2.0 \pm 0.6 \pm 0.3$	$0.0 \pm 0.5 \pm 0.2$
E653	[845]	$2.00 \pm 0.33 \pm 0.16$	$0.82 \pm 0.22 \pm 0.11$
E687	[846]	$1.74 \pm 0.27 \pm 0.28$	$0.78 \pm 0.18 \pm 0.11$
E791 (e)	[847]	$1.90 \pm 0.11 \pm 0.09$	$0.71 \pm 0.08 \pm 0.09$
E791 (μ)	[848]	$1.84 \pm 0.11 \pm 0.09$	$0.75 \pm 0.08 \pm 0.09$
Beatrice	[849]	$1.45 \pm 0.23 \pm 0.07$	$1.00 \pm 0.15 \pm 0.03$
FOCUS	[850]	$1.504 \pm 0.057 \pm 0.039$	$0.875 \pm 0.049 \pm 0.064$
$D^0 \rightarrow \bar{K}^{*0} \pi^- \mu^+ \nu$			
FOCUS	[851]	$1.706 \pm 0.677 \pm 0.342$	$0.912 \pm 0.370 \pm 0.104$
BABAR	[852]	$1.493 \pm 0.014 \pm 0.021$	$0.775 \pm 0.011 \pm 0.011$
$D_s^+ \rightarrow \phi e^+ \nu$			
BABAR	[853]	$1.849 \pm 0.060 \pm 0.095$	$0.763 \pm 0.071 \pm 0.065$
$D^0, D^+ \rightarrow \rho e \nu$			
CLEO	[854]	$1.40 \pm 0.25 \pm 0.03$	$0.57 \pm 0.18 \pm 0.06$

8.5.6 S -wave component

In 2002 FOCUS reported [855] an asymmetry in the observed $\cos(\theta_V)$ distribution. This is interpreted as evidence for an S -wave component in the decay amplitude as follows. Since H_0 typically dominates over $H_\pm$, the distribution given by Eq. (232) is, after integration over χ , roughly proportional to $\cos^2 \theta_V$. Inclusion of a constant S -wave amplitude of the form $A e^{i\delta}$ leads to an interference term proportional to $|A H_0 \sin \theta_\ell \cos \theta_V|$; this term causes an asymmetry in $\cos(\theta_V)$. When FOCUS fit their data including this S -wave amplitude, they obtained $A = 0.330 \pm 0.022 \pm 0.015 \text{ GeV}^{-1}$ and $\delta = 0.68 \pm 0.07 \pm 0.05$ [850].

More recently, both BABAR [853] and CLEO-c [856] have also found evidence for an f_0 component in semileptonic D_s decays.

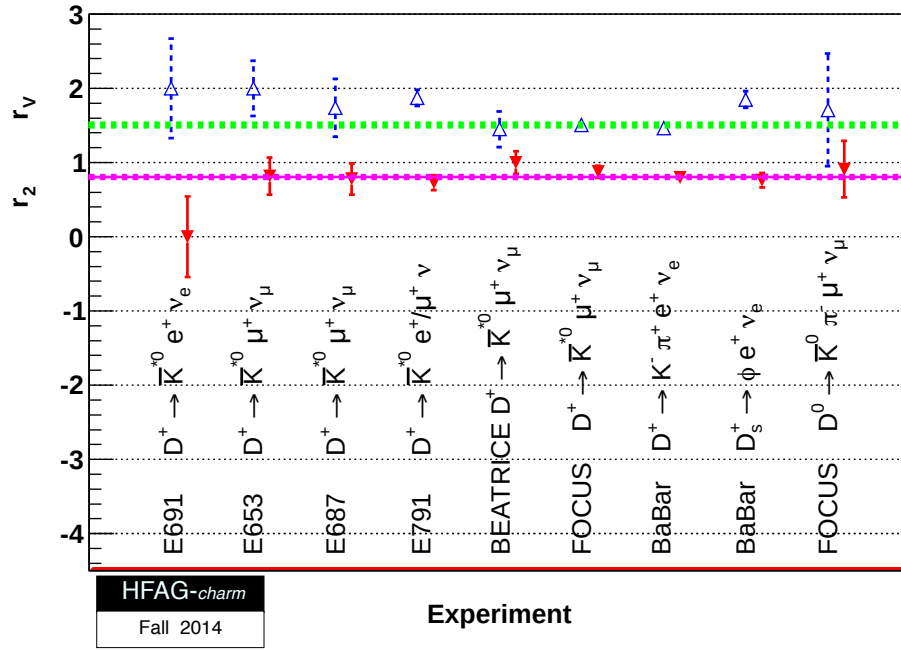


Figure 163: A comparison of r_2 and r_V values from various experiments. The first seven measurements are for $D^+ \rightarrow K^- \pi^+ l^+ \nu_l$ decays. Also shown as a line with $1\text{-}\sigma$ limits is the average of these. The last two points are D_s^+ decays and Cabibbo-suppressed D decays.

8.5.7 Model-independent form factor measurement

Subsequently the CLEO-c collaboration extracted the form factors $H_+(q^2)$, $H_-(q^2)$, and $H_0(q^2)$ in a model-independent fashion directly as functions of q^2 [857] and also determined the S -wave form factor $h_0(q^2)$ via the interference term, despite the fact that the $K\pi$ mass distribution appears dominated by the vector $K^*(892)$ state. Their results are shown in Figs. 164 and 165. Plots in Fig. 165 clearly show that $H_0(q^2)$ dominates over essentially the full range of q^2 , but especially at low q^2 . They also show that the transverse form factor $H_t(q^2)$, which can be related to $A_3(q^2)$, is small compared to Lattice Gauge Theory calculations and suggest that the form factor ratio $r_3 \equiv A_3(0)/A_1(0)$ is large and negative.

The product $H_0(q^2) \times h_0(q^2)$ is shown in Fig. 164 and clearly indicates the existence of $h_0(q^2)$, although it seems to fall faster with q^2 than $H_0(q^2)$. The other plots in this figure show that D - and F -wave versions of the S -wave $h_0(q^2)$ are not significant.

8.5.8 Detailed measurements of the $D^+ \rightarrow K^- \pi^+ e^+ \nu_e$ decay channel

BaBar [852] has selected a large sample of 244×10^3 signal events with a ratio $S/B \sim 2.3$ from an analyzed integrated luminosity of 347 fb^{-1} . With four particles emitted in the final state, the differential decay rate depends on five variables. In addition to the four variables defined in previous sections there is m^2 , the mass squared of the $K\pi$ system. Apart from this last variable, the reconstruction algorithm does not provide a high resolution on the other measured quantities and a multi-dimensional unfolding procedure is not used to correct for efficiency and resolution effects. However, these limitations still allow an essentially model independent measurement

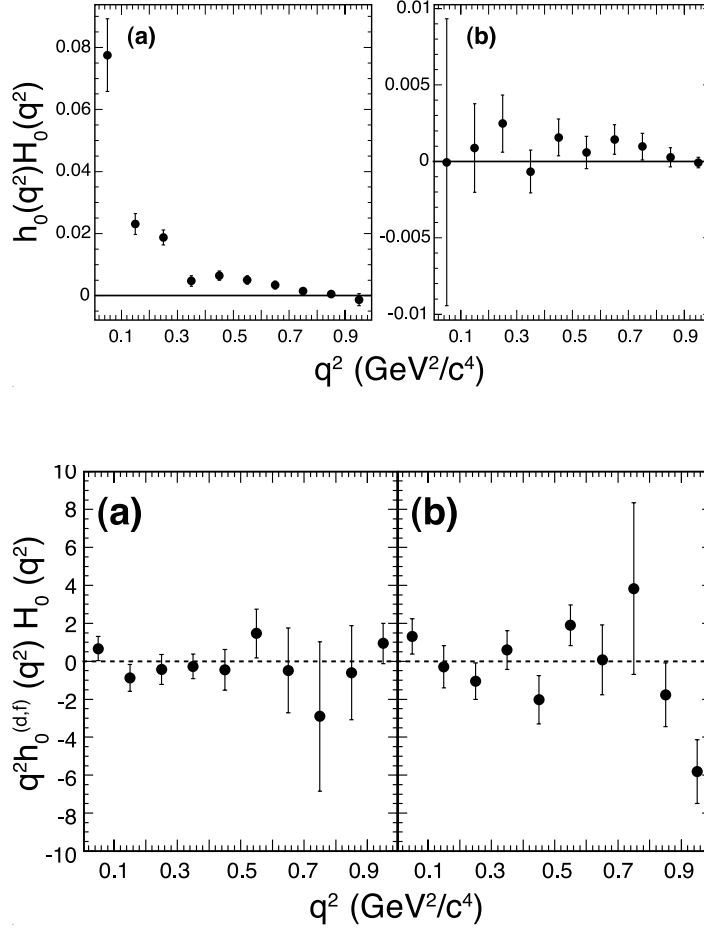


Figure 164: Model-independent form factors $h_0(q^2)$ measured by CLEO-c [857].

of the differential decay rate. This is because, apart from the q^2 and mass dependence of the form factors, angular distributions are fixed by kinematics. In addition, present accurate measurements of $D \rightarrow P\bar{\ell}\nu_\ell$ decays have shown that the q^2 dependence of the form factors can be well described by several models as long as the corresponding model parameter(s) are fitted from data. This is even more true in $D \rightarrow V\bar{\ell}\nu_\ell$ decays because the q^2 range is reduced. To analyze the $D^+ \rightarrow K^-\pi^+e^+\nu_e$ decay channel it is assumed that all form factors have a q^2 variation given by the simple pole model and the effective pole mass value, $m_A = (2.63 \pm 0.10 \pm 0.13) \text{ GeV}/c^2$, is fitted for the axial vector form factors. This value is compatible with expectations when comparing with the mass of $J^P = 1^+$ charm mesons. Data are not sensitive to the effective mass of the vector form factor for which $m_V = (2.1 \pm 0.1) \text{ GeV}/c^2$ is used, nor to the effective pole mass of the scalar component for which m_A is used. For the mass dependence of the form factors, a Breit-Wigner with a mass dependent width and a Blatt-Weisskopf damping factor is used. For the S-wave amplitude, considering what was measured in $D^+ \rightarrow K^-\pi^+\pi^+$ decays, a polynomial variation below the $\bar{K}_0^*(1430)$ and a Breit-Wigner distribution, above are assumed. For the polynomial part, a linear term is sufficient to fit data.

It is verified that the variation of the S-wave phase is compatible with expectations from elastic $K\pi$ scattering, according to the Watson theorem. At variance with elastic scattering, a negative relative sign between the S- and P-waves is measured; this is compatible with the

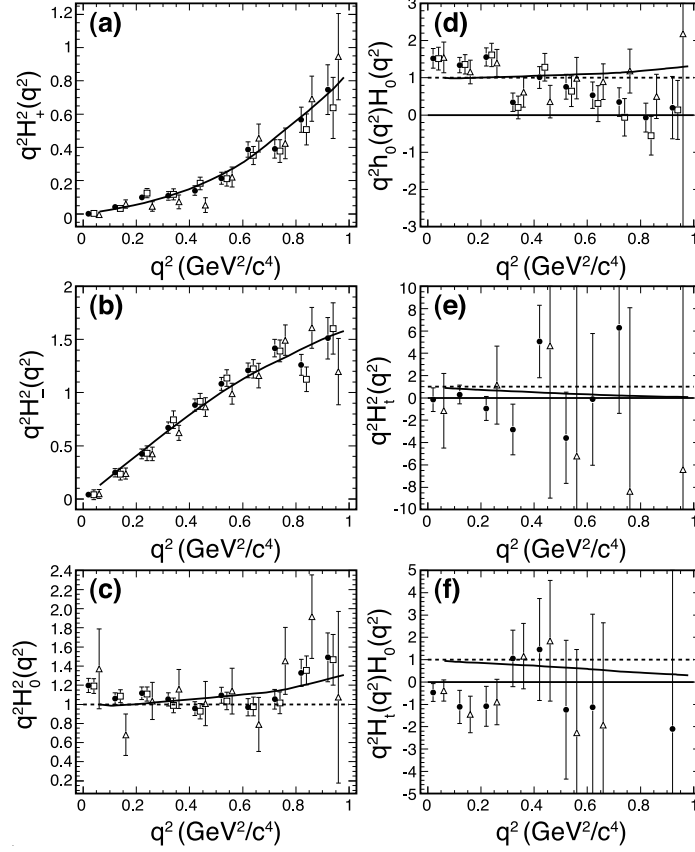


Figure 165: Model-independent form factors $H(q^2)$ measured by CLEO-c [857].

previous theorem. In Fig. 166, the measured S-wave phase is compared with the phase of the elastic, $I = 1/2$, $K\pi$ phase for different values of the $K\pi$ mass.

Contributions from other resonances decaying into $K^-\pi^+$ are considered. A small signal from the $\bar{K}^*(1410)$ is observed, compatible with expectations from τ decays and this component is included in the nominal fit. In total, 11 parameters are fitted in addition to the total number of signal events. They give a detailed description of the differential decay rate versus the 5 variables and corresponding matrices for statistical and systematic uncertainties are provided allowing to evaluate the compatibility of data with future theoretical expectations. Results of this analysis for the rates and few characteristics for S, P and D-waves are given in Table 238.

In Fig. 167, measured values from CLEO-c of the products $q^2 H_0^2(q^2)$ and $q^2 h_0(q^2) H_0(q^2)$ are compared with corresponding results from BABAR illustrating the difference in behavior of the scalar h_0 component and the helicity zero H_0 P-wave form factor. For this comparison, the plotted values from BABAR for the two distributions are fixed to 1 at $q^2 = 0$. The different behavior of $h_0(q^2)$ and $H_0(q^2)$ can be explained by they different dependence in the p^* variable.

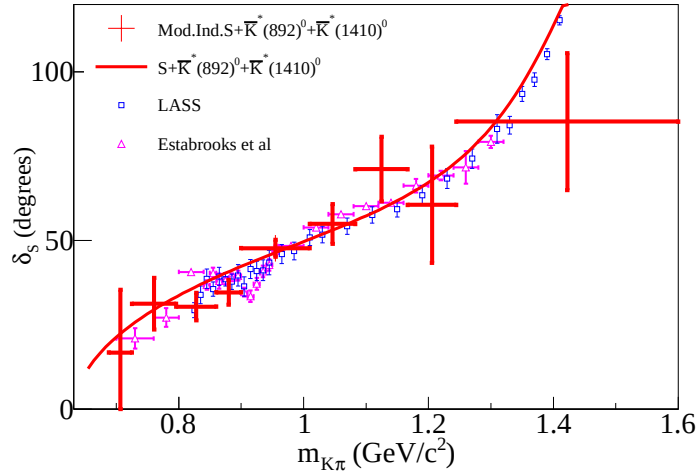


Figure 166: Points (full circles) give the *BABAR* S -wave phase variation assuming a signal containing S -wave, $\bar{K}^*(892)^0$ and $\bar{K}^*(1410)^0$ components [852]. Error bars include systematic uncertainties. The full line corresponds to a parameterized S -wave phase variation fitted on *BABAR* data. The phase variation measured in $K\pi$ scattering by Ref. [858] (triangles) and LASS [304] (squares), after correcting for $\delta^{3/2}$, are given.

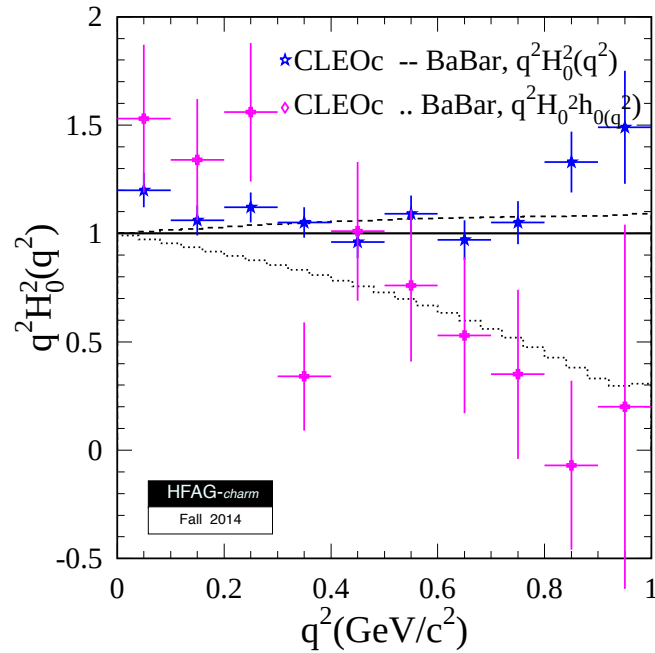


Figure 167: Comparison between CLEO-c measurements and *BABAR* results for the quantities $q^2 H_0^2(q^2)$ and $q^2 H_0(q^2) h_0(q^2)$.

Table 238: Detailed determination of the properties of the $D^+ \rightarrow K^- \pi^+ e^+ \nu_e$ decay channel from *BABAR* [852]. Values for $\mathcal{B}(D^+ \rightarrow \bar{K}^*(1410)^0 / \bar{K}_2^*(1430)^0 e^+ \nu_e)$ are corrected for their respective branching fractions into $K^- \pi^+$.

Measurement	<i>BABAR</i> result
$m_{K^*(892)^0} (MeV/c^2)$	$895.4 \pm 0.2 \pm 0.2$
$\Gamma_{K^*(892)^0}^0 (MeV/c^2)$	$46.5 \pm 0.3 \pm 0.2$
$r_{BW} (GeV/c)^{-1}$	$2.1 \pm 0.5 \pm 0.5$
r_V	$1.463 \pm 0.017 \pm 0.031$
r_2	$0.801 \pm 0.020 \pm 0.020$
$m_A (GeV/c^2)$	$2.63 \pm 0.10 \pm 0.13$
$\mathcal{B}(D^+ \rightarrow K^- \pi^+ e^+ \nu_e)(\%)$	$4.04 \pm 0.03 \pm 0.04 \pm 0.09$
$\mathcal{B}(D^+ \rightarrow K^- \pi^+ e^+ \nu_e)_{\bar{K}^{*0}}(\%)$	$3.80 \pm 0.04 \pm 0.05 \pm 0.09$
$\mathcal{B}(D^+ \rightarrow K^- \pi^+ e^+ \nu_e)_{S-wave}(\%)$	$0.234 \pm 0.007 \pm 0.007 \pm 0.005$
$\mathcal{B}(D^+ \rightarrow \bar{K}^*(1410)^0 e^+ \nu_e)(\%)$	$0.30 \pm 0.12 \pm 0.18 \pm 0.06$ (< 0.6 at 90% C.L.)
$\mathcal{B}(D^+ \rightarrow \bar{K}_2^*(1430)^0 e^+ \nu_e)(\%)$	$0.023 \pm 0.011 \pm 0.011 \pm 0.001$ (< 0.05 at 90% C.L.)

8.6 Leptonic decays

Purely leptonic decays of D^+ and D_s^+ mesons are among the simplest and theoretically cleanest probes of $c \rightarrow d$ and $c \rightarrow s$ quark flavor-changing transitions. The branching fraction of leptonic decays that proceed via the annihilation of the initial quark-antiquark pair ($c\bar{d}$ or $c\bar{s}$) into a virtual W^+ that finally materializes as an antilepton-neutrino pair ($\ell^+\nu_\ell$) is given in the Standard Model by

$$\mathcal{B}(D_q^+ \rightarrow \ell^+\nu_\ell) = \frac{G_F^2}{8\pi} \tau_{D_q} f_{D_q}^2 |V_{cq}|^2 m_{D_q} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_{D_q}^2}\right)^2. \quad (234)$$

Here, m_{D_q} is the D_q meson mass, τ_{D_q} is its lifetime, m_ℓ is the charged lepton mass, $|V_{cq}|$ is the magnitude of the relevant CKM matrix element, and G_F is the Fermi coupling constant. The parameter f_{D_q} is the D_q meson decay constant and is related to the wave-function overlap of the meson's constituent quark and anti-quark. Within the SM, the decay constants have been predicted using several methods, the most precise being the lattice gauge theory (LQCD) calculations. The Flavor Lattice Averaging Group [843] combines all LQCD calculations and provides averaged values for f_D and f_{D_s} (see Table 239) that are used within this section to extract the magnitudes of the V_{cd} and V_{cs} CKM matrix elements from experimentally measured branching fractions of $D^+ \rightarrow \ell^+\nu_\ell$ and $D_s^+ \rightarrow \ell^+\nu_\ell$ decays, respectively.

The leptonic decays of pseudoscalar mesons are suppressed by helicity conservation and their decay rates are thus proportional to the square of the charged lepton mass. Leptonic decays into electrons with $\mathcal{B} \lesssim 10^{-7}$ are not experimentally observable yet whereas decays to taus are favored over decays to muons. In particular, the ratio of the latter decays is equal to $R_{\tau/\mu}^{D_q} \equiv \mathcal{B}(D_q^+ \rightarrow \tau^+\nu_\tau)/\mathcal{B}(D_q^+ \rightarrow \mu^+\nu_\mu) = m_\tau^2/m_\mu^2 \cdot (1 - m_\tau^2/m_{D_q}^2)^2/(1 - m_\mu^2/m_{D_q}^2)^2 = 9.76 \pm 0.03$ in the case of D_s^+ decays and to 2.67 ± 0.01 in the case of D^+ decays based on the world average values of masses of the muon, tau and D_q meson given in Ref. [18]. Any deviation from this expectation could only be interpreted as violation of lepton universality in charged currents and would hence point to NP effects [859].

Averages presented within this subsection are weighted averages and correlations between measurements and dependencies on input parameters are taken into account.

Table 239: The LQCD average for D and D_s meson decay constants and their ratio from the Flavor Lattice Averaging Group [843].

Quantity	Value
f_D	$209.2 \pm 3.3 \text{ MeV}$
f_{D_s}	$248.6 \pm 2.7 \text{ MeV}$
f_{D_s}/f_D	1.187 ± 0.012

Table 240: Experimental results and world averages for $\mathcal{B}(D^+ \rightarrow \ell^+ \nu_\ell)$ and $f_D |V_{cd}|$. The first uncertainty is statistical and the second is experimental systematic. The third uncertainty in the case of $f_{D^+} |V_{cd}|$ is due to external inputs (dominated by the uncertainty of τ_D).

Mode	$\mathcal{B}$ (10^{-4})	$f_D V_{cd} $ (MeV)	Reference	
$\mu^+ \nu_\mu$	$3.82 \pm 0.32 \pm 0.09$	$46.4 \pm 1.9 \pm 0.5 \pm 0.2$	CLEO-c	[775]
	$3.71 \pm 0.19 \pm 0.06$	$45.7 \pm 1.2 \pm 0.4 \pm 0.2$	BESIII	[860]
$\mu^+ \nu_\mu$	$3.74 \pm 0.16 \pm 0.05$	$45.9 \pm 1.0 \pm 0.3 \pm 0.2$	Average	
$e^+ \nu_e$	< 0.088 at 90% C.L.		CLEO-c	[775]
$\tau^+ \nu_\tau$	< 12 at 90% C.L.		CLEO-c	[775]

8.6.1 $D^+ \rightarrow \ell^+ \nu_\ell$ decays and $|V_{cd}|$

We use measurements of the branching fraction $\mathcal{B}(D^+ \rightarrow \mu^+ \nu_\mu)$ from CLEO-c [775] and BESIII [860] to calculate its world average (WA) value. We obtain

$$\mathcal{B}^{\text{WA}}(D^+ \rightarrow \mu^+ \nu_\mu) = (3.74 \pm 0.17) \times 10^{-4}, \quad (235)$$

from which we determine the product of the decay constant and the CKM matrix element to be

$$f_D |V_{cd}| = (45.9 \pm 1.1) \text{ MeV}, \quad (236)$$

where the uncertainty includes the uncertainty on $\mathcal{B}^{\text{WA}}(D^+ \rightarrow \mu^+ \nu_\mu)$ and external inputs⁶⁰ needed to extract $f_D |V_{cd}|$ from the measured branching fraction using Eq. 234. Using the LQCD value for f_D from Table 239 we finally obtain the CKM matrix element V_{cd} to be

$$|V_{cd}| = 0.219 \pm 0.005(\text{exp.}) \pm 0.003(\text{LQCD}), \quad (237)$$

where the uncertainties are from the experiments and lattice calculations, respectively. All input values and the resulting world averages are summarized in Table 240 and plotted in Fig. 168.

The upper limit on the ratio of branching fractions is found to be $R_{\tau/\mu}^D < 3.2$ at 90% C.L., which is just slightly above the SM expected value, 2.67 ± 0.01 .

8.6.2 $D_s^+ \rightarrow \ell^+ \nu_\ell$ decays and $|V_{cs}|$

We use measurements of the absolute branching fraction $\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu_\mu)$ from CLEO-c [806], BABAR [861], and Belle [862] and obtain a WA value of

$$\mathcal{B}^{\text{WA}}(D_s^+ \rightarrow \mu^+ \nu_\mu) = (5.57 \pm 0.24) \times 10^{-3}. \quad (238)$$

The WA value for $\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu_\tau)$ is also calculated from CLEO-c, BABAR, and Belle measurements. CLEO-c made separate measurements for $\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau$ [863], $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ [806], and

⁶⁰These values (taken from the PDG [18]) are $m_\mu = (0.1056583715 \pm 0.0000000035) \text{ GeV}/c^2$, $m_D = (1.86962 \pm 0.00015) \text{ GeV}/c^2$ and $\tau_D = (1040 \pm 7) \times 10^{-15} \text{ s}$.

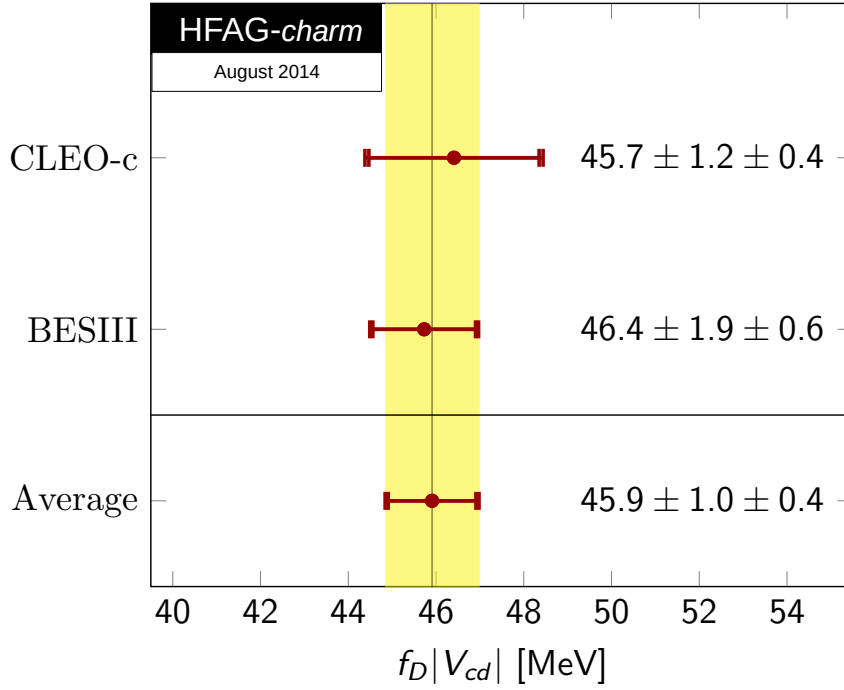


Figure 168: WA value for $f_D|V_{cd}|$. For each point, the first error listed is the statistical and the second error is the systematic error.

$\tau^+ \rightarrow \rho^+ \bar{\nu}_\tau$ [864]; *BABAR* made separate measurements for $\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau$ [861] and $\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau$; and Belle made separate measurements for $\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau$, $\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau$, and $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ [862]. Combining all of them we obtain the WA value of

$$\mathcal{B}^{\text{WA}}(D_s^+ \rightarrow \tau^+ \nu_\tau) = (5.55 \pm 0.24) \times 10^{-2}. \quad (239)$$

The ratio of branching fractions is found to be

$$R_{\tau/\mu}^{D_s} = 9.96 \pm 0.57, \quad (240)$$

and is consistent with the value expected in the SM, 9.76 ± 0.03 .

From the average values of branching fractions of muonic and tauonic decays we determine⁶¹ the product of D_s meson decay constant and the $|V_{cs}|$ CKM matrix element to be

$$f_{D_s}|V_{cs}| = (250.6 \pm 4.5) \text{ MeV}, \quad (241)$$

where the uncertainty is due to the uncertainties on $\mathcal{B}^{\text{WA}}(D_s^+ \rightarrow \mu^+ \nu_\mu)$ and $\mathcal{B}^{\text{WA}}(D_s^+ \rightarrow \tau^+ \nu_\tau)$ and the external inputs. All input values and the resulting world averages are summarized in Table 241 and plotted in Fig. 169. To obtain the averages given within this subsection and in Table 241 we have taken into account the correlations within each experiment⁶² for the uncertainties related to: normalization, tracking, particle identification, signal and background parameterizations, and peaking background contributions.

⁶¹ We use the following values (taken from PDG [18]) for external parameters entering Eq. 234: $m_\tau = (1.77682 \pm 0.00016) \text{ GeV}/c^2$, $m_{D_s} = (1.96850 \pm 0.00032) \text{ GeV}/c^2$ and $\tau_{D_s} = (500 \pm 7) \times 10^{-15} \text{ s}$.

⁶²In the case of *BABAR* we use the covariance matrix from the errata of Ref. [861].

Table 241: Experimental results and world averages for $\mathcal{B}(D_s^+ \rightarrow \ell^+ \nu_\ell)$ and $f_{D_s}|V_{cs}|$. The first uncertainty is statistical and the second is experimental systematic. The third uncertainty in the case of $f_{D_s}|V_{cs}|$ is due to external inputs (dominated by the uncertainty of τ_{D_s}). We have recalculated $\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu_\tau)$ quoted by CLEO-c and *BABAR* using the latest values for branching fractions of τ decays to electron, muon, or pion and neutrinos [18]. CLEO-c and *BABAR* include statistical uncertainty of number of D_s tags (denominator in the calculation of branching fraction) in the statistical uncertainty of measured $\mathcal{B}$. We subtract this uncertainty from the statistical one and add it to the systematic uncertainty.

Mode	$\mathcal{B} \text{ (} 10^{-2} \text{)}$	$f_{D_s} V_{cs} \text{ (MeV)}$	Reference	
$\mu^+\nu_\mu$	$0.565 \pm 0.044 \pm 0.020$	$250.8 \pm 9.8 \pm 4.4 \pm 1.8$	CLEO-c	[806]
	$0.602 \pm 0.037 \pm 0.032$	$258.9 \pm 8.0 \pm 6.9 \pm 1.8$	<i>BABAR</i>	[861]
	$0.531 \pm 0.028 \pm 0.020$	$243.1 \pm 6.4 \pm 4.6 \pm 1.7$	Belle	[862]
$\mu^+\nu_\mu$	$0.557 \pm 0.020 \pm 0.014$	$249.0 \pm 4.5 \pm 3.1 \pm 1.7$	Average	
$\tau^+(e^+)\nu_\tau$	$5.31 \pm 0.47 \pm 0.22$	$246.1 \pm 10.9 \pm 5.1 \pm 1.7$	CLEO-c	[864]
$\tau^+(\pi^+)\nu_\tau$	$6.46 \pm 0.80 \pm 0.23$	$271.4 \pm 16.8 \pm 4.8 \pm 1.9$	CLEO-c	[806]
$\tau^+(\rho^+)\nu_\tau$	$5.50 \pm 0.54 \pm 0.24$	$250.4 \pm 12.3 \pm 5.5 \pm 1.8$	CLEO-c	[863]
$\tau^+\nu_\tau$	$5.57 \pm 0.32 \pm 0.15$	$252.0 \pm 7.2 \pm 3.4 \pm 1.8$	CLEO-c	
$\tau^+(e^+)\nu_\tau$	$5.08 \pm 0.52 \pm 0.68$	$240.7 \pm 12.3 \pm 16.1 \pm 1.7$	<i>BABAR</i>	[861]
$\tau^+(\mu^+)\nu_\tau$	$4.90 \pm 0.46 \pm 0.54$	$236.4 \pm 11.1 \pm 13.0 \pm 1.7$		
$\tau^+\nu_\tau$	$4.95 \pm 0.36 \pm 0.58$	$237.6 \pm 8.6 \pm 13.8 \pm 1.7$	<i>BABAR</i>	
$\tau^+(e^+)\nu_\tau$	$5.37 \pm 0.33^{+0.35}_{-0.31}$	$247.4 \pm 7.6^{+8.1}_{-7.1} \pm 1.7$	Belle	[862]
$\tau^+(\mu^+)\nu_\tau$	$5.86 \pm 0.37^{+0.34}_{-0.59}$	$258.5 \pm 8.2^{+7.5}_{-13.0} \pm 1.8$		
$\tau^+(\pi^+)\nu_\tau$	$6.04 \pm 0.43^{+0.46}_{-0.40}$	$262.4 \pm 9.3^{+10.0}_{-8.7} \pm 1.8$		
$\tau^+\nu_\tau$	$5.70 \pm 0.21 \pm 0.31$	$254.9 \pm 4.7 \pm 6.9 \pm 1.8$	Belle	
$\tau^+\nu_\tau$	$5.55 \pm 0.18 \pm 0.17$	$251.5 \pm 4.1 \pm 3.9 \pm 1.8$	Average	
$\mu^+\nu_\mu$ $\tau^+\nu_\tau$		$250.6 \pm 3.1 \pm 2.8 \pm 1.8$	Average	
$e^+\nu_e$	< 0.0083 at 90% C.L.		Belle	[862]

Using the LQCD value for f_{D_s} from Table 239 we finally obtain the CKM matrix element V_{cs} to be

$$|V_{cs}| = 1.008 \pm 0.018(\text{exp.}) \pm 0.011(\text{LQCD}), \quad (242)$$

where the uncertainties are from the experiments and lattice calculations, respectively.

8.6.3 Comparison with other determinations of $|V_{cd}|$ and $|V_{cs}|$

Table 242 summarizes and Fig. 170 shows all determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$. As can be seen, the most precise direct determinations of these CKM matrix elements

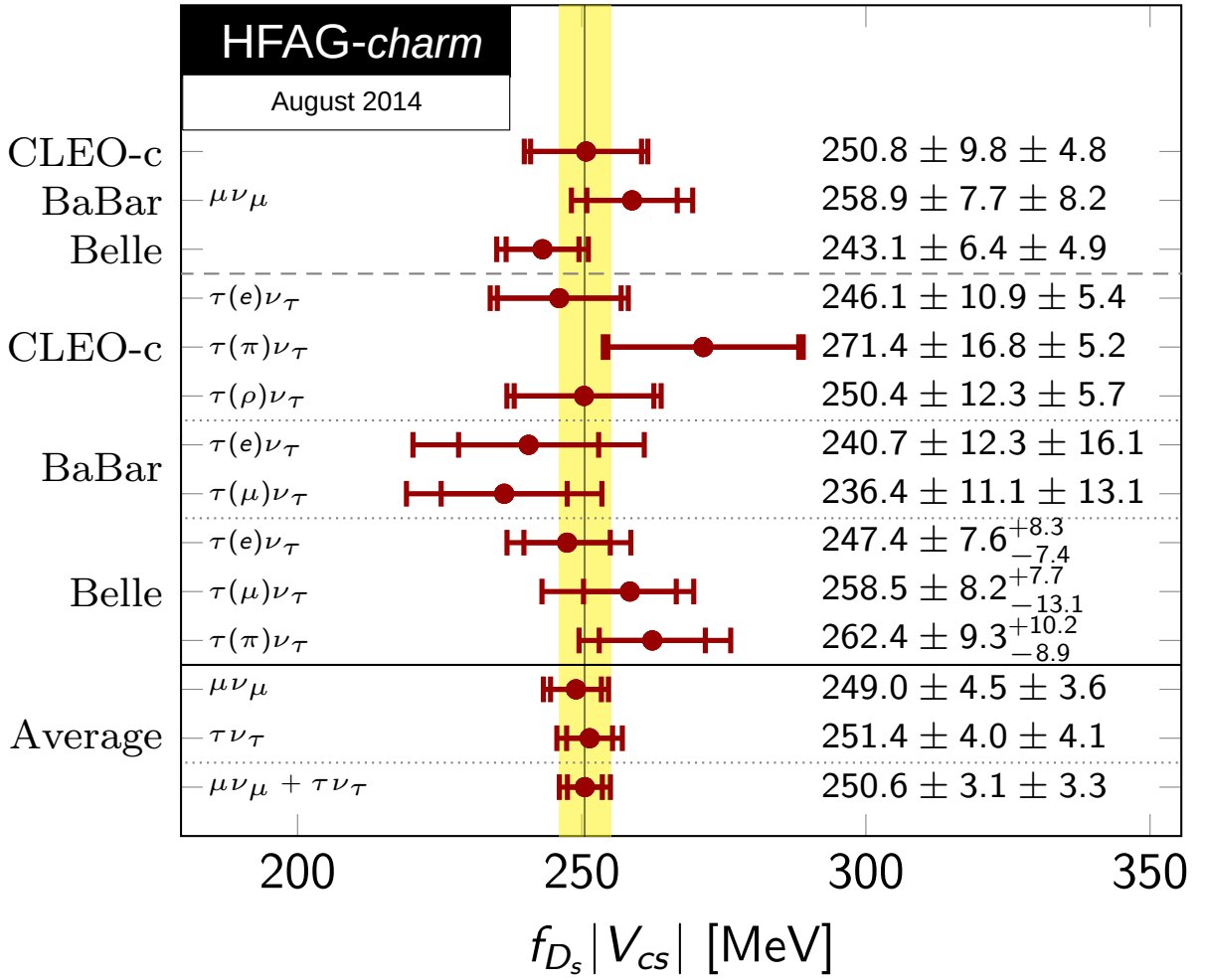


Figure 169: WA value for $f_{D_s}|V_{cs}|$. For each point, the first error listed is the statistical and the second error is the systematic error.

are those from leptonic and semileptonic $D_{(s)}$ decays. The values are in agreement within uncertainties with the values obtained from the global fit assuming CKM matrix unitarity.

8.6.4 Extraction of $D_{(s)}$ meson decay constants

Assuming unitarity of the CKM matrix, the values of the elements relevant in the case of (semi-)leptonic charm decays are known from the global fit of the CKM matrix, $|V_{cd}| = 0.22537^{+0.00068}_{-0.00035}$, and $|V_{cs}| = 0.973395^{+0.000095}_{-0.000176}$ [254]. These values can be used to extract the D and D_s meson decay constants from the experimentally measured products $f_D|V_{cd}|$ (Eq. 236) and $f_{D_s}|V_{cs}|$ (Eq. 241), respectively. This leads to the experimentally measured $D_{(s)}$ meson decay constants to be:

$$f_D^{\text{exp}} = (203.7 \pm 4.9) \text{ MeV}, \quad (243)$$

$$f_{D_s}^{\text{exp}} = (257.4 \pm 4.6) \text{ MeV}, \quad (244)$$

and the ratio of the constants is determined to be

$$f_{D_s}^{\text{exp}}/f_D^{\text{exp}} = 1.264 \pm 0.038. \quad (245)$$

Table 242: Average of the magnitudes of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ determined from the leptonic and semileptonic D and D_s decays. In the calculation of average values we assume 100% correlations in uncertainties due to LQCD. The values determined from neutrino scattering or W decays and indirect determination from the global fit to the CKM matrix are given for comparison as well.

Method	Reference	Value
$ V_{cd} $		
$D \rightarrow \ell \nu_\ell$	This section	$0.219 \pm 0.005(\text{exp.}) \pm 0.003(\text{LQCD})$
$D \rightarrow \pi \ell \nu_\ell$	Section 8.5	$0.214 \pm 0.003(\text{exp.}) \pm 0.009(\text{LQCD})$
$D \rightarrow \ell \nu_\ell$ $D \rightarrow \pi \ell \nu_\ell$	Average	0.219 ± 0.006
νN	PDG [18]	0.230 ± 0.011
Indirect	CKMFitter [254]	$0.22537^{+0.00068}_{-0.00035}$
$ V_{cs} $		
$D_s \rightarrow \ell \nu_\ell$	This section	$1.008 \pm 0.018(\text{exp.}) \pm 0.011(\text{LQCD})$
$D \rightarrow K \ell \nu_\ell$	Section 8.5	$0.975 \pm 0.007(\text{exp.}) \pm 0.025(\text{LQCD})$
$D_s \rightarrow \ell \nu_\ell$ $D \rightarrow K \ell \nu_\ell$	Average	0.998 ± 0.020
$W \rightarrow c \bar{s}$	PDG [18]	$0.94^{+0.32}_{-0.26} \pm 0.13$
Indirect	CKMFitter [254]	$0.973395^{+0.000095}_{-0.000176}$

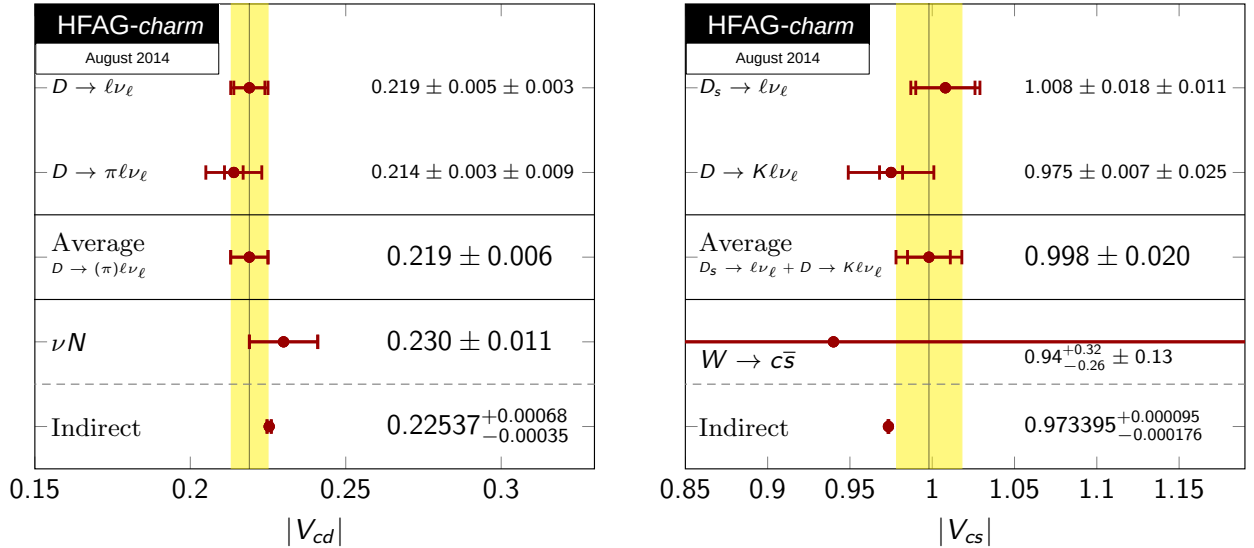


Figure 170: Comparison of magnitudes of the CKM matrix elements $|V_{cd}|$ (left) and $|V_{cs}|$ (right) determined from the (semi-)leptonic charm decays and from neutrino scattering data or W decays and indirect determination from the global fit assuming CKM unitarity [254].

The values are in agreement with the LQCD determinations given in Table 239 within the uncertainties. The largest discrepancy is in the determinations of the ratio of the decay constants where the agreement is only at the level of 1.9σ .

Table 243: Experimental results and world averages for branching fractions of $D_s^+ \rightarrow K^- K^+ \pi^+$, $D_s^+ \rightarrow \bar{K}^0 K^+$, and $D_s^+ \rightarrow \eta \pi^+$ decays. The first uncertainty is statistical and the second is experimental systematic. CLEO-c reports in Ref. [807] $\mathcal{B}(D_s^+ \rightarrow K_S^0 K^+)$. We include it in the average of $\mathcal{B}(D_s^+ \rightarrow \bar{K}^0 K^+)$ by using the relation $\mathcal{B}(D_s^+ \rightarrow \bar{K}^0 K^+) \equiv 2\mathcal{B}(D_s^+ \rightarrow K_S^0 K^+)$.

Mode	$\mathcal{B} (10^{-2})$	Reference	
$K^- K^+ \pi^+$	$5.78 \pm 0.20 \pm 0.30$	<i>BABAR</i>	[861]
	$5.55 \pm 0.14 \pm 0.13$	CLEO-c	[807]
	$5.06 \pm 0.15 \pm 0.21$	Belle	[862]
$K^- K^+ \pi^+$	$5.44 \pm 0.09 \pm 0.11$	Average	
$\bar{K}^0 K^+$	$3.04 \pm 0.10 \pm 0.06$	CLEO-c	[807]
	$2.95 \pm 0.11 \pm 0.09$	Belle	[862]
$\bar{K}^0 K^+$	$3.00 \pm 0.07 \pm 0.05$	Average	
$\eta \pi^+$	$1.67 \pm 0.08 \pm 0.06$	CLEO-c	[807]
	$1.82 \pm 0.14 \pm 0.07$	Belle	[862]
$\eta \pi^+$	$1.71 \pm 0.07 \pm 0.08$	Average	

8.7 Hadronic decays of D_s mesons

BABAR, CLEO-c and Belle collaborations have measured the absolute branching fractions of hadronic decays, $D_s^+ \rightarrow K^- K^+ \pi^+$, $D_s^+ \rightarrow \bar{K}^0 \pi^+$, and $D_s^+ \rightarrow \eta \pi^+$. The first two decay modes are the reference modes for the measurements of branching fractions of the D_s^+ decays to any other final state. Table 243 and Fig. 171 summarise the individual measurements and averaged values, which are found to be

$$\mathcal{B}^{\text{WA}}(D_s^+ \rightarrow K^- K^+ \pi^+) = (5.44 \pm 0.14)\%, \quad (246)$$

$$\mathcal{B}^{\text{WA}}(D_s^+ \rightarrow \bar{K}^0 \pi^+) = (3.00 \pm 0.09)\%, \quad (247)$$

$$\mathcal{B}^{\text{WA}}(D_s^+ \rightarrow \eta \pi^+) = (1.71 \pm 0.08)\%, \quad (248)$$

where the uncertainties are total uncertainties.

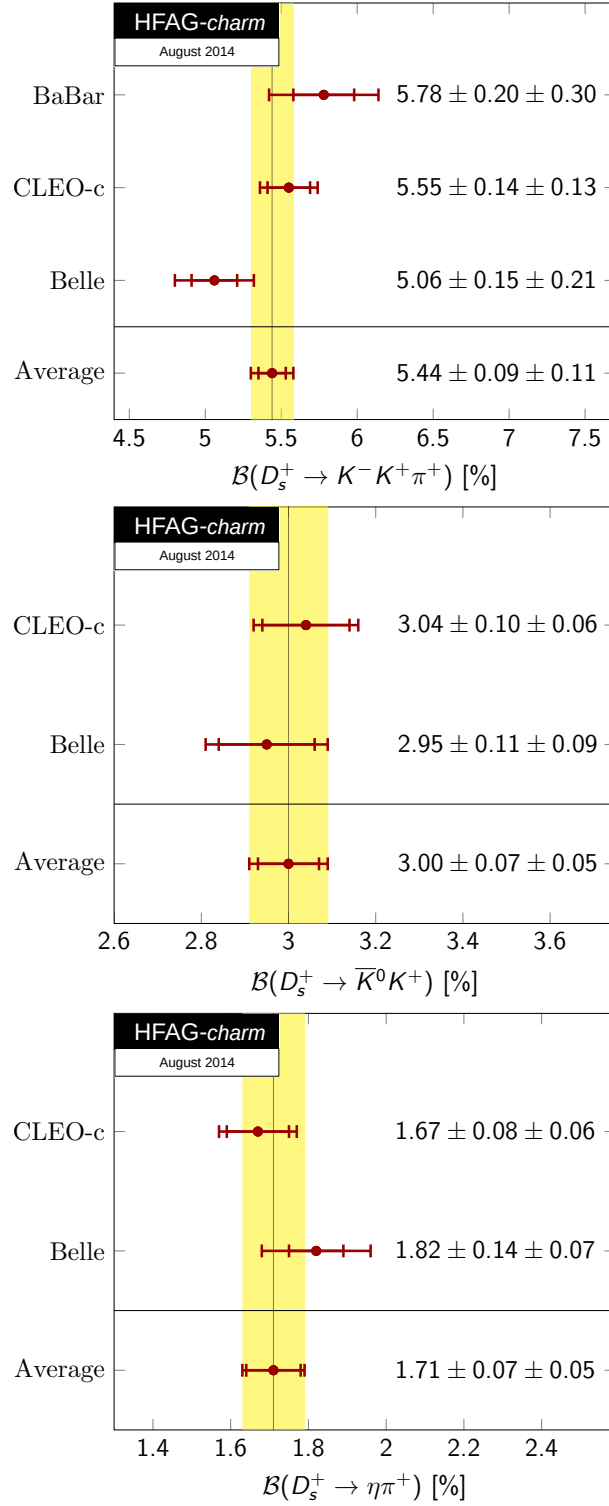


Figure 171: WA values for $\mathcal{B}(D_s^+ \rightarrow K^- K^+ \pi^+)$ (top), $\mathcal{B}(D_s^+ \rightarrow \bar{K}^0 \pi^+)$ (middle), $\mathcal{B}(D_s^+ \rightarrow \eta \pi^+)$ (bottom). For each point, the first error listed is the statistical and the second error is the systematic error.

8.8 Two-body hadronic D^0 decays and final state radiation

Measurements of the branching fractions for the decays $D^0 \rightarrow K^- \pi^+$, $D^0 \rightarrow \pi^+ \pi^-$, and $D^0 \rightarrow K^+ K^-$ have reached sufficient precision to allow averages with $\mathcal{O}(1\%)$ relative uncertainties. At these precisions, Final State Radiation (FSR) must be treated correctly and consistently across the input measurements for the accuracy of the averages to match the precision. The sensitivity of measurements to FSR arises because of a tail in the distribution of radiated energy that extends to the kinematic limit. The tail beyond $\sum E_\gamma \approx 30$ MeV causes typical selection variables like the hadronic invariant mass to shift outside the selection range dictated by experimental resolution, as shown in Fig. 172. While the differential rate for the tail is small, the integrated rate amounts to several percent of the total $h^+ h^- (n\gamma)$ rate because of the tail's extent. The tail therefore translates directly into a several percent loss in experimental efficiency.

All measurements that include an FSR correction have a correction based on the use of PHOTOS [865–868] within the experiment's Monte Carlo simulation. PHOTOS itself, however, has evolved, over the period spanning the set of measurements. In particular, the incorporation of interference between radiation off the two separate mesons has proceeded in stages: it was first available for particle–antiparticle pairs in version 2.00 (1993), extended to any two-body, all-charged, final states in version 2.02 (1999), and further extended to multi-body final states in version 2.15 (2005). The effects of interference are clearly visible, as shown in Figure 172, and cause a roughly 30% increase in the integrated rate into the high energy photon tail. To evaluate the FSR correction incorporated into a given measurement, we must therefore note whether any correction was made, the version of PHOTOS used in correction, and whether the interference terms in PHOTOS were turned on.

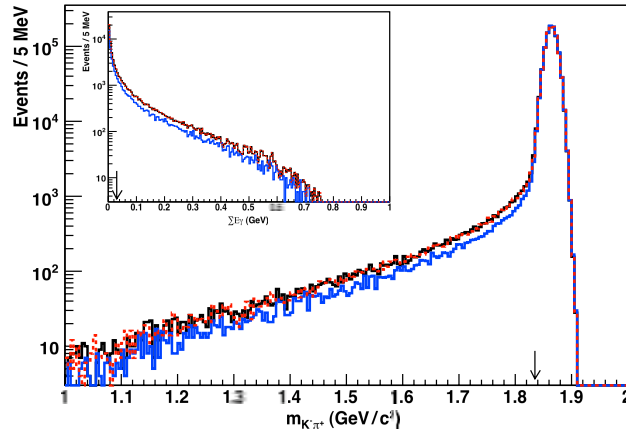


Figure 172: The $K\pi$ invariant mass distribution for $D^0 \rightarrow K^- \pi^+ (n\gamma)$ decays. The 3 curves correspond to three different configurations of PHOTOS for modeling FSR: version 2.02 without interference (blue/grey), version 2.02 with interference (red dashed) and version 2.15 with interference (black). The true invariant mass has been smeared with a typical experimental resolution of $10 \text{ MeV}/c^2$. Inset: The corresponding spectrum of total energy radiated per event. The arrow indicates the $\sum E_\gamma$ value that begins to shift kinematic quantities outside of the range typically accepted in a measurement.

8.8.1 Branching fraction corrections

Before averaging the measured branching fractions, the published results are updated, as necessary, to the FSR prediction of PHOTOS 2.15 with interference included. The correction will always shift a branching fraction to a higher value: with no FSR correction or with no interference term in the correction, the experimental efficiency determination will be biased high, and therefore the branching fraction will be biased low.

Most of the branching fraction analyses used the kinematic quantity sensitive to FSR in the candidate selection criteria. For the analyses at the $\psi(3770)$, this variable was ΔE , the difference between the candidate D^0 energy and the beam energy (*e.g.* $E_K + E_\pi - E_{\text{beam}}$ for $D^0 \rightarrow K^-\pi^+$). In the remainder of the analyses, the relevant quantity was the reconstructed hadronic two-body mass $m_{h^+h^-}$. To make the correction, we only need to evaluate the fraction of decays that FSR moves outside of the range accepted for the analysis.

The corrections were evaluated using an event generator (EvtGen [869]) that incorporates PHOTOS to simulate the portions of the decay process most relevant to the correction. We compared corrections determined both with and without smearing to account for experimental resolution. The differences were negligible, typically of $\mathcal{O}(1\%)$ of the correction itself. The immunity of the correction to resolution effects comes about because most of the long FSR-induced tail in, for example, the $m_{h^+h^-}$ distribution resides well away from the selection boundaries. The smearing from resolution, on the other hand, mainly affects the distribution of events right at the boundary.

For measurements incorporating an FSR correction that did not include interference, we update by assessing the FSR-induced efficiency loss for both the PHOTOS version and configuration used in the analysis and our nominal version 2.15 with interference. For measurements that published their sensitivity to FSR, our generator-level predictions for the original efficiency loss agreed to within a few percent of the correction. This agreement lends additional credence to the procedure.

Once the event loss from FSR in the most sensitive kinematic quantity is accounted for, the event loss from other quantities is very small. Analyses using D^* tags, for example, showed little sensitivity to FSR in the reconstructed $D^* - D^0$ mass difference: for example, in $m_{K^-\pi^+\pi^+} - m_{K^-\pi^+}$. Because the effect of FSR tends to cancel in the difference of the reconstructed masses, this difference showed a much smaller sensitivity than the two-body mass even before a two-body mass requirement. In the $\psi(3770)$ analyses, the beam-constrained mass distributions ($\sqrt{E_{\text{beam}}^2 - |\vec{p}_K + \vec{p}_\pi|^2}$) also show much smaller sensitivity than the two-body mass.

The FOCUS [870] analysis of the branching ratios $\mathcal{B}(D^0 \rightarrow \pi^+\pi^-)/\mathcal{B}(D^0 \rightarrow K^-\pi^+)$ and $\mathcal{B}(D^0 \rightarrow K^+K^-)/\mathcal{B}(D^0 \rightarrow K^-\pi^+)$ obtained yields using fits to the two-body mass distributions. FSR will both distort the low end of the signal mass peak, and will contribute a signal component to the low side tail used to estimate the background. The fitting procedure is not sensitive to signal events out in the FSR tail, which would be counted as part of the background.

A more complex toy Monte Carlo procedure was required to analyze the effect of FSR on the fitted yields, which were published with no FSR corrections applied. A detailed description of the procedure and results is available on the HFAG web site, and a brief summary is provided here. Determining the correction involved an iterative procedure in which samples of similar size to the FOCUS sample were generated and then fit using the FOCUS signal and background parameterizations. The MC parameterizations were tuned based on differences between the fits to the toy MC data and the FOCUS fits, and the procedure was repeated. These steps were

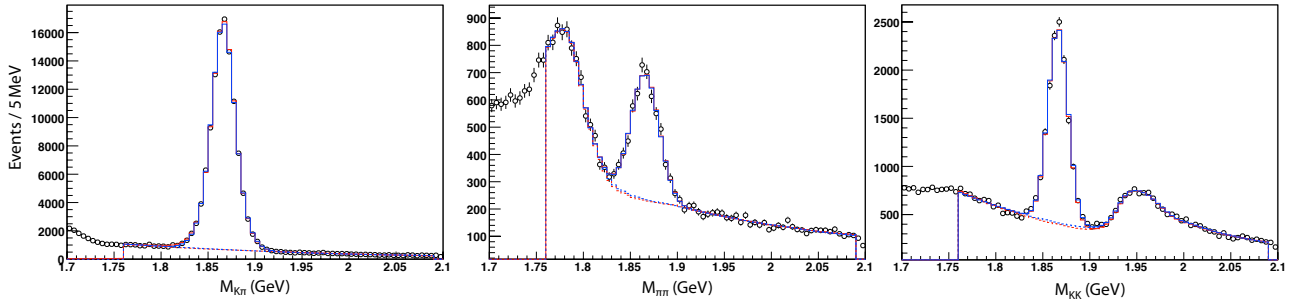


Figure 173: FOCUS data (dots), original fits (blue) and toy MC parameterization (red) for $D^0 \rightarrow K^- \pi^+$ (left), $D^0 \rightarrow \pi^+ \pi^-$ (center), and $D^0 \rightarrow \pi^+ \pi^-$ (right).

iterated until the fit parameters matched the original FOCUS parameters.

Table 244: The experimental measurements relating to $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$, $\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$, and $\mathcal{B}(D^0 \rightarrow K^+ K^-)$ after correcting to the common version and configuration of PHOTOS. The uncertainties are statistical and total systematic, with the FSR-related systematic estimated in this procedure shown in parentheses. Also listed are the percent shifts in the results from the correction, if any, applied here, as well as the original PHOTOS and interference configuration for each publication.

Experiment (acronym)	result (rescaled)	correction [%]	PHOTOS
$D^0 \rightarrow K^- \pi^+$			
CLEO-c 14 (CC14) [778]	$3.934 \pm 0.021 \pm 0.061(31)\%$	—	2.15/Yes
BABAR 07 (BB07) [871]	$4.035 \pm 0.037 \pm 0.074(24)\%$	0.69	2.02/No
CLEO II 98 (CL98) [872]	$3.920 \pm 0.154 \pm 0.168(32)\%$	2.80	none
ALEPH 97 (AL97) [873]	$3.930 \pm 0.091 \pm 0.125(32)\%$	0.79	2.0/No
ARGUS 94 (AR94) [874]	$3.490 \pm 0.123 \pm 0.288(24)\%$	2.33	none
CLEO II 93 (CL93) [875]	$3.960 \pm 0.080 \pm 0.171(15)\%$	0.38	2.0/No
ALEPH 91 (AL91) [876]	$3.730 \pm 0.351 \pm 0.455(34)\%$	3.12	none
$D^0 \rightarrow \pi^+ \pi^- / D^0 \rightarrow K^- \pi^+$			
CLEO-c 10 (CC10) [776]	$0.0370 \pm 0.0006 \pm 0.0009(02)$	—	2.15/Yes
CDF 05 (CD05) [877]	$0.03594 \pm 0.00054 \pm 0.00043(15)$	—	2.15/Yes
FOCUS 02 (FO02) [870]	$0.0364 \pm 0.0012 \pm 0.0006(02)$	3.10	none
$D^0 \rightarrow K^+ K^- / D^0 \rightarrow K^- \pi^+$			
CLEO-c 10 [776]	$0.1041 \pm 0.0011 \pm 0.0012(03)$	—	2.15/Yes
CDF 05 [877]	$0.0992 \pm 0.0011 \pm 0.0012(01)$	—	2.15/Yes
FOCUS 02 [870]	$0.0982 \pm 0.0014 \pm 0.0014(01)$	-1.12	none

The toy MC samples for the first iteration were based on the generator-level distribution of $m_{K^- \pi^+}$, $m_{\pi^+ \pi^-}$, and $m_{K^+ K^-}$, including the effects of FSR, smeared according to the original FOCUS resolution function, and on backgrounds generated using the parameterization from the final FOCUS fits. For each iteration, 400 to 1600 individual data-sized samples were generated and fit. The means of the parameters from these fits determined the corrections to the generator parameters for the following iteration. The ratio between the number of signal events generated and the final signal yield provides the required FSR correction in the final

iteration. Only a few iterations were required in each mode. Figure 173 shows the FOCUS data, the published FOCUS fits, and the final toy MC parameterizations. The toy MC provides an excellent description of the data.

The corrections obtained to the individual FOCUS yields were 1.0298 ± 0.0001 for $K^-\pi^+$, 1.062 ± 0.001 for $\pi^+\pi^-$, and 1.0183 ± 0.0003 for K^+K^- . These corrections tend to cancel in the branching ratios, leading to corrections of 1.031 to $\mathcal{B}(D^0 \rightarrow \pi^+\pi^-)/\mathcal{B}(D^0 \rightarrow K^-\pi^+)$, and 0.9888 for $\mathcal{B}(D^0 \rightarrow K^+K^-)/\mathcal{B}(D^0 \rightarrow K^-\pi^+)$.

Table 244 summarizes the corrected branching fractions. The published FSR-related modeling uncertainties have been replaced by with a new, common, estimate based on the assumption that the dominant uncertainty in the FSR corrections comes from the fact that the mesons are treated like structureless particles. No contributions from structure-dependent terms in the decay process (*e.g.* radiation off individual quarks) are included in PHOTOS. Internal studies done by various experiments have indicated that in $K\pi$ decays, the PHOTOS corrections agree with data at the 20-30% level. We therefore attribute a 25% uncertainty to the FSR prediction from potential structure-dependent contributions. For the other two modes, the only difference in structure is the final state valence quark content. While radiative corrections typically come in with a $1/M$ dependence, one would expect the additional contribution from the structure terms to come in on time scales shorter than the hadronization time scale. In this case, you might expect Λ_{QCD} to be the relevant scale, rather than the quark masses, and therefore that the amplitude is the same for the three modes. In treating the correlations among the measurements this is what we assume. We also assume that the PHOTOS amplitudes and any missing structure amplitudes are relatively real with constructive interference. The uncertainties largely cancel in the branching fraction ratios. For the final average branching fractions, the FSR uncertainty on $K\pi$ dominates. Note that because of the relative sizes of FSR in the different modes, the $\pi\pi/K\pi$ branching ratio uncertainty from FSR is positively correlated with that for the $K\pi$ branching fraction, while the $KK/K\pi$ branching ratio FSR uncertainty is negatively correlated.

The $\mathcal{B}(D^0 \rightarrow K^-\pi^+)$ measurement of reference [878], the $\mathcal{B}(D^0 \rightarrow \pi^+\pi^-)/\mathcal{B}(D^0 \rightarrow K^-\pi^+)$ measurements of references [793] and [761], and the $\mathcal{B}(D^0 \rightarrow K^+K^-)/\mathcal{B}(D^0 \rightarrow K^-\pi^+)$ measurement of reference [761] are excluded from the branching fraction averages presented here. These measurements appear not to have incorporated any FSR corrections, and insufficient information is available to determine the 2-3% corrections that would be required.

8.8.2 Average branching fractions

The average branching fractions for $D^0 \rightarrow K^-\pi^+$, $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ are obtained from a single χ^2 minimization procedure, in which the three branching fractions are floating parameters. The central values derive from a fit in which the full covariance matrix, accounting for all statistical, systematic (excluding FSR), and FSR measurement uncertainties, is used. Table 245 presents the correlation matrix for this nominal fit. We then obtain the three reported uncertainties on those central values as follows: The statistical uncertainties are obtained from a fit using only the statistical covariance matrix. The systematic uncertainties are obtained by subtracting (in quadrature) the statistical uncertainties from the uncertainties determined via a fit using a covariance matrix that accounts for both statistical and systematic measurement uncertainties. The FSR uncertainties are obtained by subtracting (in quadrature) the uncertainties determined via a fit using a covariance matrix that accounts for both statistical and

Table 245: The correlation matrix corresponding to the full covariance matrix. Subscripts h denote which of the $D^0 \rightarrow h^+ h^-$ decay results from a single experiment is represented in that row or column.

	CC14	BB07	CL98	AL97	AR94	CL93	AL91	FO02 $_{\pi}$	CD05 $_{\pi}$	CC10 $_{\pi}$	FO02 $_K$	CD05 $_K$	CC10 $_K$
CC14	1.000	0.139	0.057	0.084	0.031	0.033	0.023	0.070	0.103	0.068	-0.019	-0.032	-0.085
BB07	0.139	1.000	0.035	0.051	0.019	0.020	0.014	0.042	0.062	0.041	-0.012	-0.019	-0.051
CL98	0.057	0.035	1.000	0.021	0.008	0.298	0.006	0.017	0.026	0.017	-0.005	-0.008	-0.021
AL97	0.084	0.051	0.021	1.000	0.011	0.012	0.116	0.025	0.038	0.025	-0.007	-0.012	-0.031
AR94	0.031	0.019	0.008	0.011	1.000	0.004	0.003	0.009	0.014	0.009	-0.003	-0.004	-0.011
CL93	0.033	0.020	0.298	0.012	0.004	1.000	0.003	0.010	0.015	0.010	-0.003	-0.005	-0.012
AL91	0.023	0.014	0.006	0.116	0.003	0.003	1.000	0.007	0.010	0.007	-0.002	-0.003	-0.009
FO02 $_{\pi}$	0.070	0.042	0.017	0.025	0.009	0.010	0.007	1.000	0.031	0.021	-0.006	-0.010	-0.026
CD05 $_{\pi}$	0.103	0.062	0.026	0.038	0.014	0.015	0.010	0.031	1.000	0.031	-0.009	-0.014	-0.038
CC10 $_{\pi}$	0.068	0.041	0.017	0.025	0.009	0.010	0.007	0.021	0.031	1.000	-0.006	-0.010	-0.025
FO02 $_K$	-0.019	-0.012	-0.005	-0.007	-0.003	-0.003	-0.002	-0.006	-0.009	-0.006	1.000	0.003	0.007
CD05 $_K$	-0.032	-0.019	-0.008	-0.012	-0.004	-0.005	-0.003	-0.010	-0.014	-0.010	0.003	1.000	0.012
CC10 $_K$	-0.085	-0.051	-0.021	-0.031	-0.011	-0.012	-0.009	-0.026	-0.038	-0.025	0.007	0.012	1.000

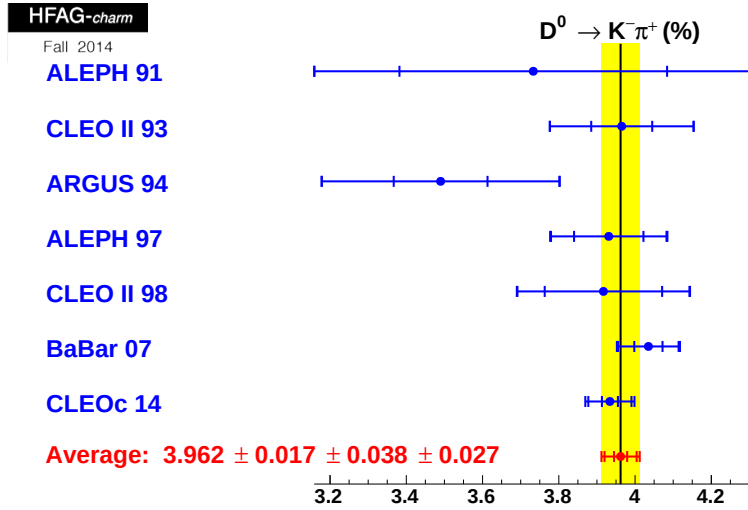


Figure 174: Comparison of measurements of $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$ (blue) with the average branching fraction obtained here (red, and yellow band).

systematic measurement uncertainties from the uncertainties determined via the fit using the full covariance matrix.

In forming the full covariance matrix, the FSR uncertainties are treated as fully correlated (or anti-correlated) as described above. For the covariance matrices involving systematic measurement uncertainties, ALEPH's systematic uncertainties in the θ_{D^*} parameter are treated as fully correlated between the ALEPH 97 and ALEPH 91 measurements. Similarly, the tracking efficiency uncertainties in the CLEO II 98 and the CLEO II 93 measurements are treated as fully correlated.

The averaging procedure results in a final χ^2 of 11.0 for 10 (13 – 3) degrees of freedom. The branching fractions obtained are

$$\begin{aligned}
 \mathcal{B}(D^0 \rightarrow K^- \pi^+) &= (3.962 \pm 0.017 \pm 0.038 \pm 0.027) \% \\
 \mathcal{B}(D^0 \rightarrow \pi^+ \pi^-) &= (0.144 \pm 0.002 \pm 0.002 \pm 0.002) \% \\
 \mathcal{B}(D^0 \rightarrow K^+ K^-) &= (0.399 \pm 0.003 \pm 0.005 \pm 0.002) \%.
 \end{aligned}$$

The uncertainties, estimated as described above, are statistical, systematic (excluding FSR), and FSR modeling. The correlation coefficients from the fit using the total uncertainties are

	$K^- \pi^+$	$\pi^+ \pi^-$	$K^+ K^-$
$K^- \pi^+$	1.00	0.71	0.76
$\pi^+ \pi^-$	0.71	1.00	0.53
$K^+ K^-$	0.76	0.53	1.00

As the χ^2 would suggest and Fig. 174 shows, the average value for $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$ and the input branching fractions agree very well. With the estimated uncertainty in the FSR modeling used here, the FSR uncertainty dominates the statistical uncertainty in the average, suggesting that experimental work in the near future should focus on verification of FSR with $\sum E_\gamma \gtrsim 100$ MeV. Note that the systematic uncertainty excluding FSR is still larger than the

Table 246: Evolution of the $D^0 \rightarrow K^- \pi^+$ branching fraction from a fit with no FSR corrections or correlations (similar to the average in the PDG 2014 update [5]) to the nominal fit presented here.

Modes fit	description	$\mathcal{B}(D^0 \rightarrow K^- \pi^+) (\%)$	$\chi^2 / (\text{d.o.f.})$
$K^- \pi^+$	PDG summer 2014 equivalent	$3.913 \pm 0.022 \pm 0.043$	6.0 / (8-1)
$K^- \pi^+$	drop Ref. [878]	$3.921 \pm 0.023 \pm 0.044$	4.8 / (7-1)
$K^- \pi^+$	use Ref. [778] instead of Ref. [879]	$3.938 \pm 0.017 \pm 0.042$	4.5 / (7-1)
$K^- \pi^+$	add FSR corrections	$3.955 \pm 0.017 \pm 0.038 \pm 0.018$	3.5 / (7-1)
$K^- \pi^+$	add FSR correlations	$3.956 \pm 0.017 \pm 0.038 \pm 0.027$	3.6 / (7-1)
all	—	$3.962 \pm 0.017 \pm 0.038 \pm 0.027$	11.0 / (13-3)

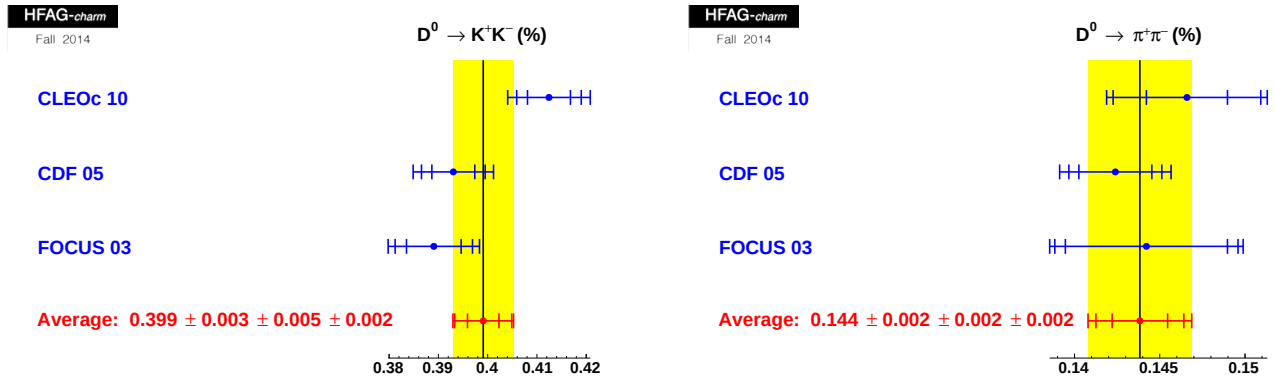


Figure 175: The $\mathcal{B}(D^0 \rightarrow K^+ K^-)$ (left) and $\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$ (right) values obtained by scaling the measured branching ratios with the $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$ branching fraction average obtained here. For the measurements (blue points), the error bars correspond to the statistical, systematic and $K\pi$ normalization uncertainties. The average obtained here (red point, yellow band) lists the statistical, systematics excluding FSR, and the FSR systematic.

FSR uncertainty; in the most precise measurements of these branching fractions, the largest systematic uncertainty is the uncertainty on the tracking efficiency. The $\mathcal{B}(D^0 \rightarrow K^+ K^-)$ and $\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$ measurements inferred from the branching ratio measurements also agree well (Fig. 175).

The $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$ average obtained here is approximately two statistical standard deviations higher than the 2014 PDG update average [5]. Table 246 shows the evolution from a fit similar to the PDG's (no FSR corrections or correlations, reference [878] included, uses reference [879] instead of reference [778] [the latter being a recent, superseding result]) to the average presented here. There are three main contributions to the difference. The branching fraction in reference [878] is low, and its exclusion shifts the result upwards. The dominant shifts ($+0.017\%$ each) are due to the FSR corrections, which as expected shift the result upwards, and the more precise result from reference [778].

8.9 Excited $D_{(s)}$ mesons

Excited charm meson states have received increased attention since the first observation of states that could not be accommodated by QCD predictions [880–883]. Tables 247 and 248 summarize recent measurements of the masses and widths of excited D and D_s mesons, respectively. If a preferred assignment of spin and parity was measured it is listed in the column J^P , where the label natural denotes $J^P = 0^-, 1^+, 2^-, \dots$ and unnatural $J^P = 0^+, 1^-, 2^+, \dots$. If possible an average mass and width was calculated, which is listed in the gray shaded row. The calculation of the averages assumes no correlation between individual measurements. A summary of the averaged masses and widths is shown in Figure 176.

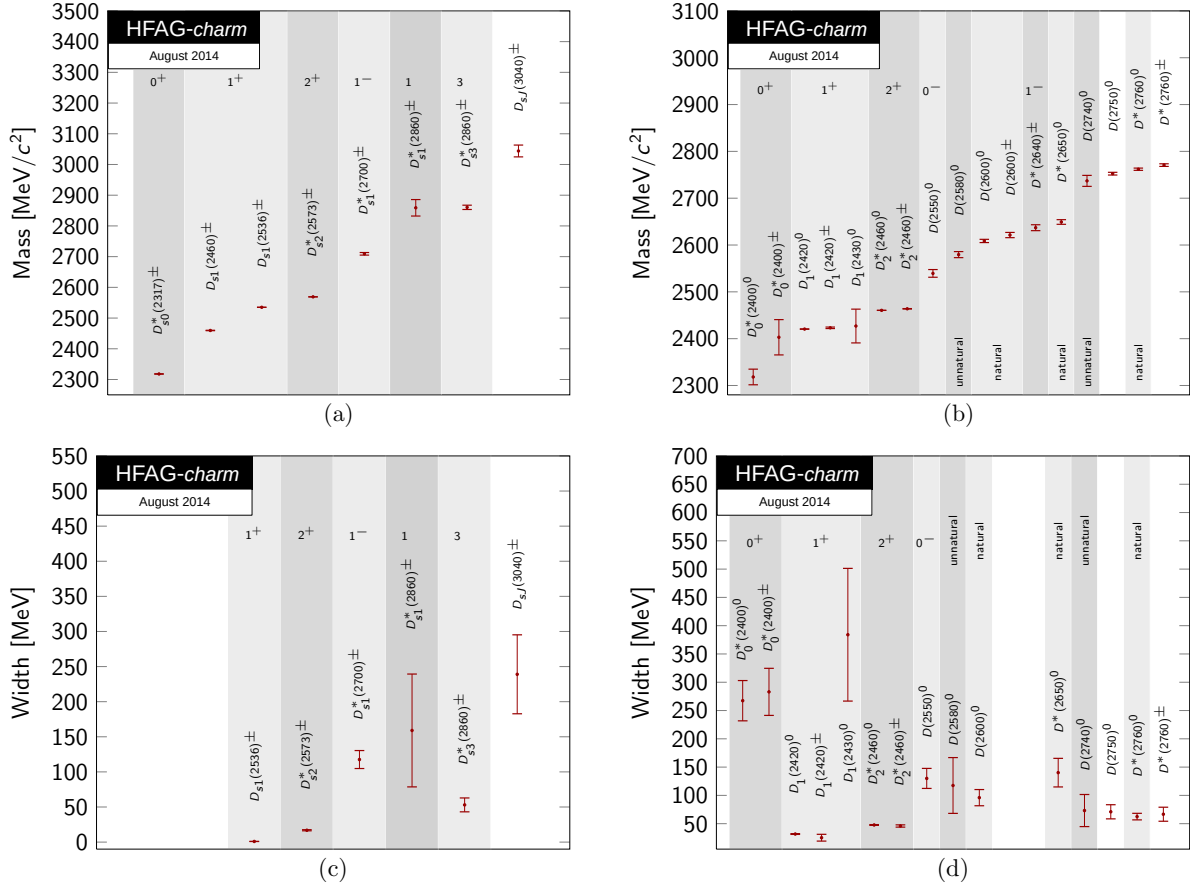


Figure 176: Averaged masses for D_s mesons are shown in subfigure (a) and for D mesons in subfigure (b). The average widths for D_s mesons are shown in subfigure (c) and for D mesons in subfigure (d). The vertical shaded regions distinguish between different spin parity states.

In the study of $B_s^0 \rightarrow \bar{D}^0 K^- \pi^+$ decays the LHCb collaboration searched for excited D_s mesons [884]. Previous measurements by *BABAR* [885] and LHCb [886] indicated the existence of a strange-charm $D_{sJ}^*(2860)^-$ meson. The new measurement of LHCb showed with 10σ significance that this state is comprised of two different particles, one of spin 1 and one of spin 3. This represents the first measurement of a heavy flavored spin-3 particle, and the first observation B mesons decays to spin 3 particles.

The masses and widths of narrow ($\Gamma < 50$ MeV) orbitally excited D mesons (denoted D^{**}), both neutral and charged, are well established. Measurements of broad states ($\Gamma \sim 200$ –

400 MeV) are less abundant, as identifying the signal is more challenging. There is a slight discrepancy between the $D_0^*(2400)^0$ masses measured by the Belle [639] and FOCUS [887] experiments. No data exist yet for the $D_1(2430)^\pm$ state. Dalitz plot analyses of $B \rightarrow D^{(*)}\pi\pi$ decays strongly favor the assignments 0^+ and 1^+ for the spin-parity quantum numbers of the $D_0^*(2400)^0/D_0^*(2400)^\pm$ and $D_1(2430)^0$ states, respectively. The measured masses and widths, as well as the J^P values, are in agreement with theoretical predictions based on potential models [435, 888–890].

Tables 249 and 250 summarize the branching fractions of B mesons decays to excited D and D_s states, respectively. It can be noted that the branching fractions for B mesons decaying to a narrow D^{**} state and a pion are similar for charged and neutral B initial states, the branching fractions to a broad D^{**} state and π^+ are much larger for B^+ than for B^0 . This may be due to the fact that color-suppressed amplitudes contribute only to the B^+ decay and not to the B^0 decay (for a theoretical discussion, see Ref. [891, 892]). Measurements of individual branching fractions of D mesons are difficult due to the unknown fragmentation of $c\bar{c} \rightarrow D^{**}$ or due to the unknown $B \rightarrow D^{**}X$ branching fractions.

The discoveries of the $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ have triggered increased interest in properties of, and searches for, excited D_s mesons (here generically denoted D_s^{**}). While the masses and widths of $D_{s1}(2536)^\pm$ and $D_{s2}(2573)^\pm$ states are in relatively good agreement with potential model predictions, the masses of $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ states are significantly lower than expected (see Ref. [893] for a discussion of $c\bar{s}$ models). Moreover, the mass splitting between these two states greatly exceeds that between the $D_{s1}(2536)^\pm$ and $D_{s2}(2573)^\pm$. These unexpected properties have led to interpretations of the $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ as exotic four-quark states [894, 895].

While there are few measurements of the J^P values of $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$, the available data favor 0^+ and 1^+ , respectively. A molecule-like (DK) interpretation of the $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ [894, 895] that can account for their low masses and isospin-breaking decay modes is tested by searching for charged and neutral isospin partners of these states; thus far such searches have yielded negative results. Therefore the subset of models that predict equal production rates for different charged states is excluded. The molecular picture can also be tested by measuring the rates for the radiative processes $D_{s0}^*(2317)^\pm/D_{s1}(2460)^\pm \rightarrow D_s^{(*)}\gamma$ and comparing to theoretical predictions. The predicted rates, however, are below the sensitivity of current experiments.

Another model successful in explaining the total widths and the $D_{s0}^*(2317)^\pm - D_{s1}(2460)^\pm$ mass splitting is based on the assumption that these states are chiral partners of the ground states D_s^+ and D_s^* [896]. While some measured branching fraction ratios agree with predicted values, further experimental tests with better sensitivity are needed to confirm or refute this scenario. A summary of the mass difference measurements is given in Table 251.

In addition to the $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ states, other excited D_s states may have been observed. SELEX has reported a $D_{sJ}(2632)^\pm$ candidate [897], but this has not been confirmed by other experiments. Recently, Belle, BABAR and LHCb have observed $D_{s1}(2700)^\pm$ which may be radial excitations of the $D_s^{*\pm}$. Equally the $D_{s1}(2860)^\pm$ measured by LHCb and $D_{sJ}(3040)^\pm$ measured by BABAR could be excitations of $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ or $D_{s1}(2536)^\pm$, respectively (for a theoretical discussion, see Ref [898]).

Table 252 summarizes measurements of the polarization amplitude A_D (sometimes also referred as helicity parameter), which describes the initial polarization of the D meson. In D^{**} meson decay the helicity distribution varies like $1 + A_D \cos^2 \theta_H$, where θ_H is the angle in

the D^* rest frame between the two pions emitted by decay $D^{**} \rightarrow D^*\pi$ and the $D^* \rightarrow D\pi$. The parameter is sensitive to possible S-wave contributions in the decay. In the case of an unpolarized D meson decay decaying purely via D-wave the polarization amplitude is predicted to give $A_D = 3$. Studies of the $D_1(2420)^0$ meson by the ZUES and *BABAR* collaborations suggest that there is an S-wave admixture in the decay, which is contrary to Heavy Quark Effective Theory calculations [899, 900].

Table 247: Recent measurements of mass and width for different excited D_s mesons. The column J^P list the most significant assignment of spin and parity. If possible an average mass or width is calculated.

Resonance	J^P	Decay mode	Mass [MeV/ c^2]	Width [MeV]	Measured by	Reference
$D_{s0}^*(2317)^\pm$	0^+	$D_s^+\pi^0$	$2319.6 \pm 0.2 \pm 1.4$		<i>BABAR</i>	[901]
		$D_s^+\pi^0$	$2317.3 \pm 0.4 \pm 0.8$		<i>BABAR</i>	[883]
			2318.0 ± 0.8		Our average	
$D_{s1}(2460)^\pm$	1^+	$D_s^+\pi^0\gamma, D_s^+\gamma, D_s^+\pi^+\pi^-$	$2460.1 \pm 0.2 \pm 0.8$		<i>BABAR</i>	[901]
		$D_s^+\pi^0\gamma$	$2458 \pm 1.0 \pm 1.0$		<i>BABAR</i>	[883]
			2459.6 ± 0.7		Our average	
$D_{s1}(2536)^\pm$	1^+	$D^{*+}K_S^0$	$2535.7 \pm 0.6 \pm 0.5$		DØ	[902]
		$D^{*+}K_S^0, D^{*0}K^+$	$2534.78 \pm 0.31 \pm 0.40$		<i>BABAR</i>	[570]
		$D_s^+\pi^+\pi^-$	$2534.6 \pm 0.3 \pm 0.7$		<i>BABAR</i>	[901]
		$D^{*+}K_S^0, D^{*0}K^+$	$2535.0 \pm 0.6 \pm 1.0$		E687	[903]
		$D^{*0}K^+$	$2535.3 \pm 0.2 \pm 0.5$		CLEO	[904]
		$D^{*+}K_S^0$	$2534.8 \pm 0.6 \pm 0.6$		CLEO	[904]
		$D^{*0}K^+$	$2535.2 \pm 0.5 \pm 1.5$		ARGUS	[905]
		$D^{*+}K_S^0$	$2535.6 \pm 0.7 \pm 0.4$		CLEO	[906]
		$D^{*+}K_S^0$	$2535.9 \pm 0.6 \pm 2.0$		ARGUS	[907]
		$D^{*+}K_S^0$		$0.92 \pm 0.03 \pm 0.04$	<i>BABAR</i>	[908]
			2535.10 ± 0.26	0.92 ± 0.05	Our average	
$D_{s2}^*(2573)^\pm$	2^+	D^0K^+	$2568.39 \pm 0.29 \pm 0.26$	$16.9 \pm 0.5 \pm 0.6$	LHCb	[909]
		$D^+K_S^0, D^0K^+$	$2569.4 \pm 1.6 \pm 0.5$	$12.1 \pm 4.5 \pm 1.6$	LHCb	[696]
		$D^+K_S^0, D^0K^+$	$2572.2 \pm 0.3 \pm 1.0$	$27.1 \pm 0.6 \pm 5.6$	<i>BABAR</i>	[910]
		D^0K^+	$2574.25 \pm 3.3 \pm 1.6$	$10.4 \pm 8.3 \pm 3.0$	ARGUS	[911]
		D^0K^+	$2573.2^{+1.7}_{-1.6} \pm 0.9$	$16^{+5}_{-4} \pm 3$	CLEO	[912]
			2569.08 ± 0.35	16.9 ± 0.8	Our average	
$D_{s1}^*(2700)^\pm$	1^-	$D^{*+}K_S^0, D^{*0}K^+$	$2709.2 \pm 1.9 \pm 4.5$	$115.8 \pm 7.3 \pm 12.1$	LHCb	[886]
		DK, D^*K	$2710 \pm 2^{+12}_{-7}$	$149 \pm 7^{+39}_{-52}$	<i>BABAR</i>	[885]
		D^0K^+	$2708 \pm 9^{+11}_{-10}$	$108 \pm 2^{+36}_{-31}$	Belle	[659]
			2709.2 ± 4.2	117.2 ± 12.5	Our average	
$D_{s1}^*(2860)^\pm$	1	D^0K^+	$2859 \pm 12 \pm 24$	$159 \pm 23 \pm 77$	LHCb	[884]
$D_{s3}^*(2860)^\pm$	3	D^0K^+	$2860.5 \pm 2.6 \pm 6.5$	$53 \pm 7 \pm 7$	LHCb	[884]
$D_{sJ}(3040)^\pm$		D^*K	$3044 \pm 8^{+30}_{-5}$	$239 \pm 35^{+46}_{-42}$	<i>BABAR</i>	[885]

Table 248: Recent measurements of mass and width for different excited D mesons. The column J^P list the most significant assignment of spin and parity. If possible an average mass or width is calculated.

Resonance	J^P	Decay mode	Mass [MeV/ c^2]	Width [MeV]	Measured by	Reference
$D_0^*(2400)^0$	0^+	$D^+\pi^-$	$2297 \pm 8 \pm 20$	$273 \pm 12 \pm 48$	<i>BABAR</i>	[638]
		$D^+\pi^-$	$2308 \pm 17 \pm 32$	$276 \pm 21 \pm 63$	Belle	[639]
		$D^+\pi^-$	$2407 \pm 21 \pm 35$	$240 \pm 55 \pm 59$	Focus	[887]
			2318.2 ± 16.9	267.4 ± 35.6	Our average	
$D_0^*(2400)^\pm$	0^+	$D^0\pi^+$	$2403 \pm 14 \pm 35$	$283 \pm 24 \pm 34$	Focus(m & Γ) + Belle(J^P) [887] + [913]	
$D_1(2420)^0$	1^+	$D^{*+}\pi^-$	$2419.6 \pm 0.1 \pm 0.7$	$35.2 \pm 0.4 \pm 0.9$	LHCb	[832]
		$D^{*+}\pi^-$	$2423.1 \pm 1.5^{+0.4}_{-1.0}$	$38.8 \pm 5^{+1.9}_{-5.4}$	Zeus	[914]
		$D^{*+}\pi^-$	$2420.1 \pm 0.1 \pm 0.8$	$31.4 \pm 0.5 \pm 1.3$	<i>BABAR</i>	[831]
		$D^{*+}\pi^-$		$20.0 \pm 1.7 \pm 1.3$	CDF	[915]
		$D^0\pi^+\pi^-$	$2426 \pm 3 \pm 1$	$24 \pm 7 \pm 8$	Belle	[550]
		$D^{*+}\pi^-$	$2421.4 \pm 1.5 \pm 0.9$	$23.7 \pm 2.7 \pm 4.0$	Belle	[639]
		$D^{*+}\pi^-$	$2421^{+1}_{-2} \pm 2$	20^{+6+3}_{-5-3}	CLEO	[916]
		$D^{*+}\pi^-$	$2422 \pm 2 \pm 2$	$15 \pm 8 \pm 4$	E687	[903]
		$D^{*+}\pi^-$	$2428 \pm 3 \pm 2$	23^{+8+10}_{-6-4}	CLEO	[906]
		$D^{*+}\pi^-$	$2414 \pm 2 \pm 5$	$13 \pm 6^{+10}_{-5}$	ARGUS	[917]
		$D^{*+}\pi^-$	$2428 \pm 8 \pm 5$	$58 \pm 14 \pm 10$	TPS	[918]
			2420.5 ± 0.5	31.7 ± 0.7	Our average	
$D_1(2420)^\pm$	1^+	$D^{*0}\pi^+$	$2421.9 \pm 4.7^{+3.4}_{-1.2}$		Zeus	[914]
		$D^+\pi^-\pi^+$	$2421 \pm 2 \pm 1$	$21 \pm 5 \pm 8$	Belle	[550]
		$D^{*0}\pi^+$	$2425 \pm 2 \pm 2$	$26^{+8}_{-7} \pm 4$	CLEO	[919]
		$D^{*0}\pi^+$	$2443 \pm 7 \pm 5$	$41 \pm 19 \pm 8$	TPS	[918]
			2423.2 ± 1.6	25.2 ± 6.0	Our average	
$D_1(2430)^0$	1^+	$D^{*+}\pi^-$	$2427 \pm 26 \pm 25$	$384^{+107}_{-75} \pm 74$	Belle	[639]
$D_2^*(2460)^0$	2^+	$D^{*+}\pi^-$	$2460.4 \pm 0.4 \pm 1.2$	$43.2 \pm 1.2 \pm 3.0$	LHCb	[832]
		$D^+\pi^-$	$2460.4 \pm 0.1 \pm 0.1$	$45.6 \pm 0.4 \pm 1.1$	LHCb	[832]
		$D^{*+}\pi^-, D^+\pi^-$	$2462.5 \pm 2.4^{+1.3}_{-1.1}$	$46.6 \pm 8.1^{+5.9}_{-3.8}$	Zeus	[914]
		$D^+\pi^-$	$2462.2 \pm 0.1 \pm 0.8$	$50.5 \pm 0.6 \pm 0.7$	<i>BABAR</i>	[831]
		$D^+\pi^-$	$2460.4 \pm 1.2 \pm 2.2$	$41.8 \pm 2.5 \pm 2.9$	<i>BABAR</i>	[638]
		$D^+\pi^-$		$49.2 \pm 2.3 \pm 1.3$	CDF	[915]
		$D^+\pi^-$	$2461.6 \pm 2.1 \pm 3.3$	$45.6 \pm 4.4 \pm 6.7$	Belle	[639]
		$D^+\pi^-$	$2464.5 \pm 1.1 \pm 1.9$	$38.7 \pm 5.3 \pm 2.9$	Focus	[887]
		$D^+\pi^-$	$2465 \pm 3 \pm 3$	$28^{+8}_{-7} \pm 6$	CLEO	[916]
		$D^+\pi^-$	$2453 \pm 3 \pm 2$	$25 \pm 10 \pm 5$	E687	[903]
		$D^{*+}\pi^-$	$2461 \pm 3 \pm 1$	$20^{+9}_{-12} \pm 10$	CLEO	[906]
		$D^+\pi^-$	$2455 \pm 3 \pm 5$	15^{+13+5}_{-10-10}	ARGUS	[920]
		$D^+\pi^-$	$2459 \pm 3 \pm 2$	$20 \pm 10 \pm 5$	TPS	[918]
			2460.47 ± 0.14	47.7 ± 0.7	Our average	
$D_2^*(2460)^\pm$	2^+	$D^0\pi^+$	$2463.1 \pm 0.2 \pm 0.6$	$48.6 \pm 1.3 \pm 1.9$	LHCb	[832]
		$D^{*0}\pi^+, D^0\pi^+$	$2460.6 \pm 4.4^{+3.6}_{-0.8}$		Zeus	[914]
		$D^0\pi^+$	$2465.4 \pm 0.2 \pm 1.1$		<i>BABAR</i>	[831]
		$D^0\pi^+$	$2465.7 \pm 1.8^{+1.4}_{-4.8}$	$49.7 \pm 3.8 \pm 6.4$	Belle	[913]
		$D^0\pi^+$	$2467.6 \pm 1.5 \pm 0.8$	$34.1 \pm 6.5 \pm 4.2$	Focus	[887]
		$D^0\pi^+$	$2463 \pm 3 \pm 3$	$27^{+11}_{-8} \pm 5$	CLEO	[919]
		$D^0\pi^+$	$2453 \pm 3 \pm 2$	$23 \pm 9 \pm 5$	E687	[903]
		$D^0\pi^+$	$2469 \pm 4 \pm 6$		ARGUS	[921]
			2463.8 ± 0.5	45.9 ± 2.0	Our average	
$D(2550)^0$	0^-	$D^{*+}\pi^-$	$2539.4 \pm 4.5 \pm 6.8$	$130 \pm 12 \pm 13$	<i>BABAR</i>	[831]
$D(2580)^0$	Unnatural	$D^{*+}\pi^-$	$2579.5 \pm 3.4 \pm 5.5$	$117.5 \pm 17.8 \pm 46.0$	LHCb	[832]
$D(2600)^0$	Natural	$D^+\pi^-$	$2608.7 \pm 2.4 \pm 2.5$	$93 \pm 6 \pm 13$	<i>BABAR</i>	[831]
$D(2600)^\pm$	Natural	$D^0\pi^+$	$2621.3 \pm 3.7 \pm 4.2$		<i>BABAR</i>	[831]
$D^*(2640)^\pm$	1^-	$D^{*+}\pi^+\pi^-$	$2637 \pm 2 \pm 6$		Delphi	[922]
$D^*(2650)^0$	Natural	$D^{*+}\pi^-$	$2649.2 \pm 3.5 \pm 3.5$	$140.2 \pm 17.1 \pm 18.6$	LHCb	[832]
$D(2740)^0$	Unnatural	$D^{*+}\pi^-$	$2737.0 \pm 3.5 \pm 11.2$	$73.2 \pm 13.4 \pm 25.0$	LHCb	[832]
$D(2750)^0$		$D^{*+}\pi^-$	$2752.4 \pm 1.7 \pm 2.7$	$71 \pm 6 \pm 11$	<i>BABAR</i>	[831]
$D^*(2760)^0$	Natural	$D^{*+}\pi^-$	$2761.1 \pm 5.1 \pm 6.5$	$74.4 \pm 3.4 \pm 37.0$	LHCb	[832]
		$D^+\pi^-$	$2760.1 \pm 1.1 \pm 3.7$	$74.4 \pm 3.4 \pm 19.1$	LHCb	[832]
		$D^+\pi^-$	$2763.3 \pm 2.3 \pm 2.3$	$60.9 \pm 5.1 \pm 3.6$	<i>BABAR</i>	[831]
			2761.9 ± 2.4	62.5 ± 5.9	Our average	
$D^*(2760)^\pm$		$D^0\pi^+$	$2771.7 \pm 1.7 \pm 3.8$	$66.7 \pm 6.6 \pm 10.5$	LHCb	[832]
		$D^0\pi^+$	$2769.7 \pm 3.8 \pm 1.5$		<i>BABAR</i>	[831]
			2770.7 ± 2.9	66.7 ± 12.4	Our average	

Table 249: Product of B meson branching fraction and (daughter) excited D meson branching fraction.

Resonance	Decay	$\mathcal{B}r[10^{-4}]$	Measured by	Reference
$D_0^*(2400)^0$	$B^- \rightarrow D_0^*(2400)^0(\rightarrow D^+\pi^+)\pi^-$	$6.1 \pm 0.6 \pm 1.8$	Belle	[639]
		$6.8 \pm 0.3 \pm 2.0$	BABAR	[638]
		6.4 ± 1.4	Our average	
$D_0^*(2400)^+$	$\bar{B}^0 \rightarrow D_0^*(2400)^+(\rightarrow D^0\pi^+)\pi^-$	$0.60 \pm 0.13 \pm 0.27$	Belle	[913]
$D_1(2420)^0$	$B^- \rightarrow D_1(2420)^0(\rightarrow D^{*+}\pi^-)\pi^-$	$6.8 \pm 0.7 \pm 1.3$	Belle	[639]
	$B^- \rightarrow D_1(2420)^0(\rightarrow D^0\pi^+\pi^-)\pi^-$	$1.85 \pm 0.29 \pm 0.27 \pm 0.41$	Belle	[550]
$D_1(2420)^+$	$\bar{B}^0 \rightarrow D_1(2420)^+(\rightarrow D^+\pi^-\pi^+)\pi^-$	$0.89 \pm 0.15 \pm 0.22$	Belle	[550]
$D_1(2430)^0$	$B^- \rightarrow D_1(2430)^0(\rightarrow D^{*+}\pi^-)\pi^-$	$5.0 \pm 0.4 \pm 1.08$	Belle	[639]
$D_2^*(2460)^0$	$B^- \rightarrow D_2^*(2460)^0(\rightarrow D^+\pi^-)\pi^-$	$3.4 \pm 0.3 \pm 0.7$	Belle	[639]
		$3.5 \pm 0.2 \pm 0.5$	BABAR	[638]
		3.5 ± 0.3	Our average	
	$B^- \rightarrow D_2^*(2460)^0(\rightarrow D^{*+}\pi^-)\pi^-$	$1.8 \pm 0.3 \pm 0.4$	Belle	[639]
$D_2^*(2460)^+$	$\bar{B}^0 \rightarrow D_2^*(2460)^+(\rightarrow D^0\pi^+)\pi^-$	$2.15 \pm 0.17 \pm 0.31$	Belle	[913]

Table 250: Product of B meson branching fraction and (daughter) excited D_s meson branching fraction.

Resonance	Decay	$\mathcal{B}r[10^{-4}]$	Measured by	Reference
$D_{s0}^*(2317)^+$	$B^0 \rightarrow D_{s0}^*(2317)^+(\rightarrow D_s^+\pi^0)D^-$	$8.6^{+3.3}_{-2.6} \pm 2.6$	Belle	[572]
		$18.0 \pm 4.0^{+6.7}_{-5.0}$	BABAR	[573]
		10.8 ± 3.4	Our average	
	$B^0 \rightarrow D_{s0}^*(2317)^+(\rightarrow D_s^+\pi^0)K^-$	$0.53^{+0.15}_{-0.13} \pm 0.16$	Belle	[551]
$D_{s1}(2460)^+$	$B^0 \rightarrow D_{s1}(2460)^+(\rightarrow D_s^{*+}\pi^0)D^-$	$22.7^{+7.3}_{-6.2} \pm 6.8$	Belle	[572]
		$28.0 \pm 8.0^{+11.2}_{-7.8}$	BABAR	[573]
		24.7 ± 7.6	Our average	
	$B^0 \rightarrow D_{s1}(2460)^+(\rightarrow D_s^{*+}\gamma)D^-$	$8.2^{+2.2}_{-1.9} \pm 2.5$	Belle	[572]
		$8.0 \pm 2.0^{+3.2}_{-2.3}$	BABAR	[573]
		8.1 ± 2.3	Our average	
	$D_{s1}(2460)^+ \rightarrow D_s^{*+}\pi^0$	$(56 \pm 13 \pm 9)\%$	BABAR	[562]
	$D_{s1}(2460)^+ \rightarrow D_s^{*+}\gamma$	$(16 \pm 4 \pm 3)\%$	BABAR	[562]
$D_{s1}(2536)^+$	$B^0 \rightarrow D_{s1}(2536)^+(\rightarrow D^{*0}K^+)D^-$	$1.71 \pm 0.48 \pm 0.32$	BABAR	[570]
	$B^0 \rightarrow D_{s1}(2536)^+(\rightarrow D^{*+}K^0)D^-$	$2.61 \pm 1.03 \pm 0.31$	BABAR	[570]
	$B^0 \rightarrow D_{s1}(2536)^+(\rightarrow D^{*0}K^+)D^{*-}$	$3.32 \pm 0.88 \pm 0.66$	BABAR	[570]
	$B^0 \rightarrow D_{s1}(2536)^+(\rightarrow D^{*+}K^0)D^{*-}$	$5.00 \pm 1.51 \pm 0.67$	BABAR	[570]
	$B^+ \rightarrow D_{s1}(2536)^+(\rightarrow D^{*0}K^+)\bar{D}^0$	$2.16 \pm 0.52 \pm 0.45$	BABAR	[570]
	$B^+ \rightarrow D_{s1}(2536)^+(\rightarrow D^{*+}K^0)\bar{D}^0$	$2.30 \pm 0.98 \pm 0.43$	BABAR	[570]
	$B^+ \rightarrow D_{s1}(2536)^+(\rightarrow D^{*0}K^+)\bar{D}^{*0}$	$5.46 \pm 1.17 \pm 1.04$	BABAR	[570]
	$B^+ \rightarrow D_{s1}(2536)^+(\rightarrow D^{*+}K^0)\bar{D}^{*0}$	$3.92 \pm 2.46 \pm 0.83$	BABAR	[570]
$D_{s1}(2700)^+$	$B^+ \rightarrow D_{s1}(2700)^+(\rightarrow D^0K^+)\bar{D}^0$	$11.3 \pm 2.2^{+1.4}_{-2.8}$	Belle	[659]

Table 251: Mass difference measurements for excited D mesons.

Resonance	Relative to	Δm [MeV/ c^2]	Measured by	Reference
$D_1^*(2420)^0$	D^{*+}	$410.2 \pm 2.1 \pm 0.9$	Zeus	[923]
		$411.7 \pm 0.7 \pm 0.4$	CDF	[915]
		411.5 ± 0.8	Our average	
$D_1(2420)^\pm$	$D_1^*(2420)^0$	$4_{-3}^{+2} \pm 3$	CLEO	[919]
$D_2^*(2460)^0$	D^+	$593.9 \pm 0.6 \pm 0.5$	CDF	[915]
	D^{*+}	$458.8 \pm 3.7_{-1.3}^{+1.2}$	Zeus	[923]
$D_2^*(2460)^\pm$	$D_2^*(2460)^0$	$3.1 \pm 1.9 \pm 0.9$	Focus	[887]
		$-2 \pm 4 \pm 4$	CLEO	[919]
		$14 \pm 5 \pm 8$	ARGUS	[921]
		3.0 ± 1.9	Our average	
$D_{s0}^*(2317)^\pm$	$D_s^\pm$	$348.7 \pm 0.5 \pm 0.7$	Belle	[882]
		$350.0 \pm 1.2 \pm 1.0$	CLEO	[881]
		$351.3 \pm 2.1 \pm 1.9$	Belle	[572]
		349.2 ± 0.7	Our average	
$D_{s1}(2460)^\pm$	$D_s^{*\pm}$	$344.1 \pm 1.3 \pm 1.1$	Belle	[882]
		$351.2 \pm 1.7 \pm 1.0$	CLEO	[881]
		$346.8 \pm 1.6 \pm 1.9$	Belle	[572]
	$D_s^\pm$	347.1 ± 1.1	Our average	
		$491.0 \pm 1.3 \pm 1.9$	Belle	[882]
$D_{s1}(2536)^\pm$	$D^*(2010)^\pm$	$491.4 \pm 0.9 \pm 1.5$	Belle	[882]
		491.3 ± 1.4	Our average	
		$524.83 \pm 0.01 \pm 0.04$	<i>BABAR</i>	[908]
	$D^*(2007)^0$	$525.30_{-0.41}^{+0.44} \pm 0.10$	Zeus	[923]
		$525.3 \pm 0.6 \pm 0.1$	ALEPH	[924]
$D_{s2}^*(2573)^\pm$	D^0	524.84 ± 0.04	Our average	
		$528.7 \pm 1.9 \pm 0.5$	ALEPH	[924]
$D_{s2}^*(2573)^\pm$	D^0	$704 \pm 3 \pm 1$	ALEPH	[924]

Table 252: Measurements of polarization amplitudes for excited D mesons.

Resonance	A_D	Measured by	Reference
$D_1(2420)^0$	$7.8^{+6.7+4.6}_{-2.7-1.8}$	ZEUS	[914]
	5.72 ± 0.25	<i>BABAR</i>	[831]
	$5.9^{+3.0+2.4}_{-1.7-1.0}$	ZEUS	[923]
	$3.8 \pm 0.6 \pm 0.8$	<i>BABAR</i>	[441]
	5.61 ± 0.24	Our average	
$D_1(2420)^\pm$	$3.8 \pm 0.6 \pm 0.8$	<i>BABAR</i>	[441]
$D_2^*(2460)^0$	-1.16 ± 0.35	ZEUS	[914]
$D(2750)^0$	-0.33 ± 0.28	<i>BABAR</i>	[831]

Table 253: Summary of excited Λ_c^+ baryons family.

Charmed Baryon Excited State	Mode	Mass (MeV/c ²)	Natural Width (MeV/c ²)	J^P
$\Lambda_c(2595)^+$	$\Lambda_c^+\pi^+\pi^-, \Sigma_c\pi$	2595.4 ± 0.6	$3.6_{-1.3}^{+2.0}$	$1/2^-$
$\Lambda_c(2625)^+$	$\Lambda_c^+\pi^+\pi^-, \Sigma_c\pi$	2628.1 ± 0.6	< 1.9	$3/2^-$
$\Lambda_c(2765)^+$	$\Lambda_c^+\pi^+\pi^-, \Sigma_c\pi$	2766.6 ± 2.4	50	??
$\Lambda_c(2880)^+$	$\Lambda_c^+\pi^+\pi^-, \Sigma_c\pi,$ $\Sigma_c(2520)\pi, D^0p$	2881.53 ± 0.35	5.8 ± 1.1	$5/2^+$ (experimental evidence)
$\Lambda_c(2940)^+$	$D^0p, \Sigma_c\pi$	$2939.3_{-1.5}^{+1.4}$	17_{-6}^{+8}	??

8.10 Charm baryons

In this section we summarize the present status of excited charm baryons, decaying strongly or electromagnetically. We list their masses (or the mass difference between the excited baryon and the corresponding ground state), natural widths, decay modes, and assigned quantum numbers. Table 253 summarizes the excited Λ_c^+ baryons. The first two states, $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$, are well-established. Based on the measured masses, it is believed they are orbitally excited Λ_c^+ baryons with total angular momentum of the light quarks $L = 1$. Thus their quantum numbers are assigned to be $J^P = (\frac{1}{2})^-$ and $J^P = (\frac{3}{2})^-$, respectively. Recently, their masses were precisely measured by CDF [925]: $M(\Lambda_c(2595)^+) = 2592.25 \pm 0.24 \pm 0.14$ MeV/c² and $M(\Lambda_c(2625)^+) = 2628.11 \pm 0.13 \pm 0.14$ MeV/c².

The next two states, $\Lambda_c(2765)^+$ and $\Lambda_c(2880)^+$, were discovered by CLEO [926] in the $\Lambda_c^+\pi^+\pi^-$ final state. CLEO found that $\Lambda_c(2880)^+$ decays also through the $\Sigma_c(2445)^{++/0}\pi^-/+$ mode. Later, BABAR [927] observed that this state has also a D^0p decay mode. It is the first example of an excited charm baryon decaying into a charm meson and a light baryon. (Excited charm baryons typically decay into charm baryons and light mesons.) In that analysis, BABAR observed for the first time an additional state, $\Lambda_c(2940)^+$, which decays into D^0p . Looking for the D^+p final state, BABAR found no signal; this implies that the $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are really Λ_c^+ excited states rather than Σ_c excitations. Belle reported the result of an angular analysis that favors 5/2 for the $\Lambda_c(2880)^+$ spin hypothesis. Moreover, the measured ratio of branching fractions $\mathcal{B}(\Lambda_c(2880)^+ \rightarrow \Sigma_c(2520)\pi^\pm)/\mathcal{B}(\Lambda_c(2880)^+ \rightarrow \Sigma_c(2455)\pi^\pm) = (0.225 \pm 0.062 \pm 0.025)$, combined with theoretical predictions based on HQS [435,928], favor even parity. The current open questions in the excited Λ_c^+ family are the determination of quantum numbers for almost all states, and the nature of the $\Lambda_c(2765)^+$ state, *i.e.* whether it is an excited Σ_c^+ or Λ_c^+ .

Table 254 summarizes the excited $\Sigma_c^{++,+,0}$ baryons. The triplet of $\Sigma_c(2520)^{++,+,0}$ baryons is well-established. Recently Belle [929] precisely measured the mass differences (see above for the definition) and widths of the double charged and neutral members of this triplet. The results are

$$\Delta M(\Sigma_c(2520)^{++}) = (231.99 \pm 0.10 \pm 0.02) \text{ MeV}/c^2 \quad (249)$$

$$\Gamma(\Sigma_c(2520)^{++}) = (14.77 \pm 0.25_{-0.30}^{+0.18}) \text{ MeV} \quad (250)$$

$$\Delta M(\Sigma_c(2520)^0) = (231.98 \pm 0.11 \pm 0.04) \text{ MeV}/c^2 \quad (251)$$

$$\Gamma(\Sigma_c(2520)^0) = (15.41 \pm 0.41_{-0.32}^{+0.20}) \text{ MeV} . \quad (252)$$

This short list of excited Σ_c baryons completes the triplet of $\Sigma_c(2800)$ states observed by Belle [930]. Based on the measured masses and theoretical predictions [931,932], these states are identified as members of the predicted Σ_{c2} $3/2^-$ triplet. From a study of resonant substructure in $B^- \rightarrow \Lambda_c^+ \bar{p} \pi^-$ decays, *BABAR* found a significant signal for $\Lambda_c^+ \pi^-$ with a mean value higher than that measured by Belle by about 3σ (Table 254). The decay widths measured by Belle and *BABAR* are consistent.

Table 254: Summary of excited $\Sigma_c^{++,+,0}$ baryons family.

Charmed Baryon Excited State	Mode	ΔM (MeV/c ²)	Natural Width (MeV/c ²)	J^P
$\Sigma_c(2520)^{++}$	$\Lambda_c^+ \pi^+$	$231.99 \pm 0.10 \pm 0.02$	$14.77 \pm 0.25^{+0.18}_{-0.30}$	$3/2^+$
$\Sigma_c(2520)^+$	$\Lambda_c^+ \pi^+$	231.0 ± 2.3	< 17 @ 90% CL	$3/2^+$
$\Sigma_c(2520)^0$	$\Lambda_c^+ \pi^+$	$231.98 \pm 0.11 \pm 0.04$	$15.41 \pm 0.41^{+0.20}_{-0.32}$	$3/2^+$
$\Sigma_c(2800)^{++}$	$\Lambda_c^+ \pi^+$	$514.5^{+3.4+2.8}_{-3.1-4.9}$	75^{+18+12}_{-13-11}	tentatively identified as members of the predicted Σ_{c2} $3/2^-$ isospin triplet
$\Sigma_c(2800)^+$	$\Lambda_c^+ \pi^0$	$505.4^{+5.8+12.4}_{-4.6-2.0}$	62^{+37+52}_{-23-38}	
$\Sigma_c(2800)^0$	$\Lambda_c^+ \pi^-$	$515.4^{+3.2+2.1}_{-3.1-6.0}$	61^{+18+22}_{-13-13}	
	$\Lambda_c^+ \pi^-$	$560 \pm 8 \pm 10$	86^{+33}_{-22}	

Table 255 summarizes the excited $\Xi_c^{+,0}$ and Ω_c^0 baryons. Recently, the list of excited Ξ_c baryons has increased, with several states having masses above 2900 MeV/c² and decaying into $\Lambda_c^+ K^-$, $\Lambda_c^+ K^{-/0} \pi^{+/-}$ and $\Sigma_c(2455/2520)K$. Some of these states ($\Xi_c(2980)^+$ and $\Xi_c(3080)^{+,0}$) are seen by both Belle [933,934] and *BABAR* [935] and are considered well-established. The $\Xi_c(2930)^0$ state decaying into $\Lambda_c^+ K^-$ is seen by *BABAR* [625] but still needs confirmation. The $\Xi_c(3055)^+$ found in the $\Sigma_c(2455)^{++} \pi^-$ final state by *BABAR* [935] was recently confirmed by Belle [934]. The width and mass measurements for the $\Xi_c(3055)^+$ listed in Table 255 are not averaged but quoted separately for *BABAR* and Belle. However, the $\Xi_c(3123)^+$ observed by *BABAR* [935] in the $\Sigma_c(2520)^{++} \pi^-$ final state has not been confirmed by Belle [934] with twice as much statistics. We note that Belle [934] recently performed the first significant measurement of the $\Xi_c(2645)^+$ width, as listed in Table 255.

The excited Ω_c^0 double charm baryon is seen by both *BABAR* [936] and Belle [937]; the mass differences $\delta M = M(\Omega_c^{*0}) - M(\Omega_c^0)$ measured by the experiments are in good agreement and are also consistent with most theoretical predictions [938–941].

Figure 177 shows the levels of excited charm baryons along with corresponding transitions between them, and also transitions to the ground states. In summary, we note that Belle and *BABAR* recently discovered that transitions between families are possible, *i.e.* between the Ξ_c and Λ_c^+ families of excited charm baryons. Also, highly excited Λ_c^+ baryons are found to decay into a charm meson and a proton.

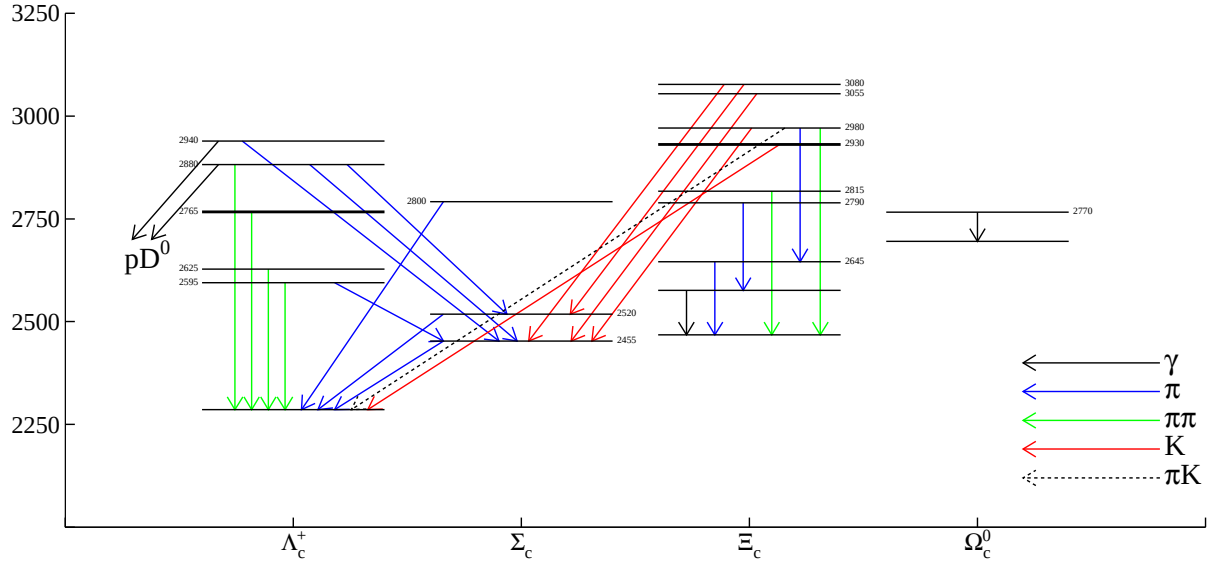


Figure 177: Level diagram for excited charm baryons.

Table 255: Summary of excited $\Xi_c^{+,0}$ and Ω_c^0 baryons families.

Charmed Baryon Excited State	Mode	Mass (MeV/c ²)	Natural Width (MeV/c ²)	J^P
$\Xi_c^{'+}$	$\Xi_c^+ \gamma$	2575.6 ± 3.1		$1/2^+$
$\Xi_c'^0$	$\Xi_c^0 \gamma$	2577.9 ± 2.9		$1/2^+$
$\Xi_c(2645)^+$	$\Xi_c^0 \pi^+$	$2645.9^{+0.6}_{-0.5}$	$2.6 \pm 0.2 \pm 0.4$	$3/2^+$
$\Xi_c(2645)^0$	$\Xi_c^+ \pi^-$	2645.9 ± 0.5	< 5.5	$3/2^+$
$\Xi_c(2790)^+$	$\Xi_c'^0 \pi^+$	2789.1 ± 3.2	< 15	$1/2^-$
$\Xi_c(2790)^0$	$\Xi_c'^+ \pi^-$	2791.8 ± 3.3	< 12	$1/2^-$
$\Xi_c(2815)^+$	$\Xi_c^+ \pi^+ \pi^-, \Xi_c(2645)^0 \pi^+$	2816.6 ± 0.9	< 3.5	$3/2^-$
$\Xi_c(2815)^0$	$\Xi_c^0 \pi^+ \pi^-, \Xi_c(2645)^+ \pi^-$	2819.6 ± 1.2	< 6.5	$3/2^-$
$\Xi_c(2930)^0$	$\Lambda_c^+ K^-$	2931.6 ± 6	36 ± 13	??
$\Xi_c(2980)^+$	$\Lambda_c^+ K^- \pi^+, \Sigma_c^{++} K^-, \Xi_c(2645)^0 \pi^+$	2971.4 ± 3.3	26 ± 7	??
$\Xi_c(2980)^0$	$\Xi_c(2645)^+ \pi^-$	2968.0 ± 2.6	20 ± 7	??
$\Xi_c(3055)^+$	$\Sigma_c^{++} K^-$	3054.2 ± 1.3 (BABAR) $3058.1 \pm 1.0 \pm 2.1$ (Belle)	17 ± 13 (BABAR) $9.7 \pm 3.4 \pm 3.3$ (Belle)	??
$\Xi_c(3080)^+$	$\Lambda_c^+ K^- \pi^+, \Sigma_c^{++} K^-, \Sigma_c(2520)^{++} K^-$	3077.0 ± 0.4	5.8 ± 1.0	??
$\Xi_c(3080)^0$	$\Lambda_c^+ K_S^0 \pi^-, \Sigma_c^0 K_S^0, \Sigma_c(2520)^0 K_S^0$	3079.9 ± 1.4	5.6 ± 2.2	??
$\Omega_c(2770)^0$	$\Omega_c^0 \gamma$	2765.9 ± 2.0	$70.7^{+0.8}_{-0.9}$	$3/2^+$

8.11 Rare and forbidden decays

This section provides a summary of rare and forbidden charm decays in tabular form. The decay modes can be categorized as flavor-changing neutral currents, lepton-flavor-violating, lepton-number-violating, and both baryon- and lepton-number-violating decays. Figures 178-180 plot the upper limits for D^0 , D^+ , D_s^+ , and Λ_c^+ decays. Tables 256-259 give the corresponding numerical results. Some theoretical predictions are given in Refs. [942–947].

In several cases the rare-decay final states have been observed with the di-lepton pair being the decay product of a hadronic resonance. For these measurements the quoted limits are those expected for the non-resonant di-lepton spectrum. For the extrapolation to the full spectrum a phase-space distribution of the non-resonant component has been assumed. This applies to the CLEO measurement of the decays $D_{(s)}^+ \rightarrow (K^+\pi^+)e^+e^-$ [948], to the D0 measurements of the decays $D_{(s)}^+ \rightarrow \pi^+\mu^+\mu^-$ [949], and to the *BABAR* measurements of the decays $D_{(s)}^+ \rightarrow (K^+\pi^+)e^+e^-$ and $D_{(s)}^+ \rightarrow (K^+\pi^+)\mu^+\mu^-$, where the contribution from $\phi \rightarrow l^+l^-$ ($l = e, \mu$) has been excluded. In the case of the LHCb measurements of the decays $D^0 \rightarrow \pi^+\pi^-\mu^+\mu^-$ [950] as well as the decays $D_{(s)}^+ \rightarrow \pi^+\mu^+\mu^-$ [951] the contributions from $\phi \rightarrow l^+l^-$ as well as from $\rho, \omega \rightarrow l^+l^-$ ($l = e, \mu$) have been excluded.

Table 256: Upper limits at 90% CL for D^0 decays.

Decay	Limit $\times 10^6$	Experiment	Reference
$\gamma\gamma$	26.0	CLEO II	[952]
	2.2	<i>BABAR</i> Preliminary	[953]
e^+e^-	220.0	CLEO	[954]
	170.0	ARGUS	[955]
	130.0	Mark3	[956]
	13.0	CLEO II	[957]
	8.19	E789	[958]
	6.2	E791	[959]
	1.2	<i>BABAR</i>	[960]
	0.079	Belle	[961]
$\mu^+\mu^-$	70.0	ARGUS	[955]
	44.0	E653	[962]
	34.0	CLEO II	[957]
	15.6	E789	[958]
	5.2	E791	[959]
	2.0	HERAb	[963]
	1.3	<i>BABAR</i>	[960]
	0.21	CDF	[964]
	0.14	Belle	[961]
	0.0062	LHCb	[965]
$\pi^0 e^+ e^-$	45.0	CLEO II	[957]
$\pi^0 \mu^+ \mu^-$	540.0	CLEO II	[957]
	180.0	E653	[962]
$\eta e^+ e^-$	110.0	CLEO II	[957]
$\eta \mu^+ \mu^-$	530.0	CLEO II	[957]

Table 256 – continued from previous page

Decay	Limit $\times 10^6$	Experiment	Reference
$\pi^+\pi^-\bar{e}^+e^-$	370.0	E791	[966]
ρe^+e^-	450.0	CLEO	[954]
	124.0	E791	[966]
	100.0	CLEO II	[957]
$\pi^+\pi^-\mu^+\mu^-$	30.0	E791	[966]
	0.55	LHCb	[950]
$\rho\mu^+\mu^-$	810.0	CLEO	[954]
	490.0	CLEO II	[957]
	230.0	E653	[962]
	22.0	E791	[966]
ωe^+e^-	180.0	CLEO II	[957]
$\omega\mu^+\mu^-$	830.0	CLEO II	[957]
$K^+K^-\bar{e}^+e^-$	315.0	E791	[966]
ϕe^+e^-	59.0	E791	[966]
	52.0	CLEO II	[957]
$K^+K^-\mu^+\mu^-$	33.0	E791	[966]
$\phi\mu^+\mu^-$	410.0	CLEO II	[957]
	31.0	E791	[966]
$\bar{K}^0 e^+e^-$	1700.0	Mark3	[967]
	110.0	CLEO II	[957]
$\bar{K}^0\mu^+\mu^-$	670.0	CLEO II	[957]
	260.0	E653	[962]
$K^-\pi^+e^+e^-$	385.0	E791	[966]
$\bar{K}^{*0}(892)e^+e^-$	140.0	CLEO II	[957]
	47.0	E791	[966]
$K^-\pi^+\mu^+\mu^-$	360.0	E791	[966]
$\bar{K}^{*0}(892)\mu^+\mu^-$	1180.0	CLEO II	[957]
	24.0	E791	[966]
$\pi^+\pi^-\pi^0\mu^+\mu^-$	810.0	E653	[962]
$\mu^\pm e^\mp$	270.0	CLEO	[954]
	120.0	Mark3	[968]
	100.0	ARGUS	[955]
	19.0	CLEO II	[957]
	17.2	E789	[958]
	8.1	E791	[959]
	0.81	BABAR	[960]
	0.26	Belle	[961]
$\pi^0 e^\pm \mu^\mp$	86.0	CLEO II	[957]
$\eta e^\pm \mu^\mp$	100.0	CLEO II	[957]
$\pi^+\pi^-\bar{e}^\pm \mu^\mp$	15.0	E791	[966]
$\rho e^\pm \mu^\mp$	66.0	E791	[966]
	49.0	CLEO II	[957]

Table 256 – continued from previous page

Decay	Limit $\times 10^6$	Experiment	Reference
$\omega e^\pm \mu^\mp$	120.0	CLEO II	[957]
$K^+ K^- e^\pm \mu^\mp$	180.0	E791	[966]
$\phi e^\pm \mu^\mp$	47.0	E791	[966]
	34.0	CLEO II	[957]
$\bar{K}^0 e^\pm \mu^\mp$	100.0	CLEO II	[957]
$K^- \pi^+ e^\pm \mu^\mp$	550.0	E791	[966]
$K^{*0}(892) e^\pm \mu^\mp$	100.0	CLEO II	[957]
	83.0	E791	[966]
$\pi^\mp \pi^\mp e^\pm e^\pm$	112.0	E791	[966]
$\pi^\mp \pi^\mp \mu^\pm \mu^\pm$	29.0	E791	[966]
$K^\mp \pi^\mp e^\pm e^\pm$	206.0	E791	[966]
$K^\mp \pi^\mp \mu^\pm \mu^\pm$	390.0	E791	[966]
$K^\mp K^\mp e^\pm e^\pm$	152.0	E791	[966]
$K^\mp K^\mp \mu^\pm \mu^\pm$	94.0	E791	[966]
$\pi^\mp \pi^\mp e^\pm \mu^\pm$	79.0	E791	[966]
$K^\mp \pi^\mp e^\pm \mu^\pm$	218.0	E791	[966]
$K^\mp K^\mp e^\pm \mu^\pm$	57.0	E791	[966]
pe^-	10.0	CLEO	[969]
$\bar{p}e^+$	11.0	CLEO	[969]

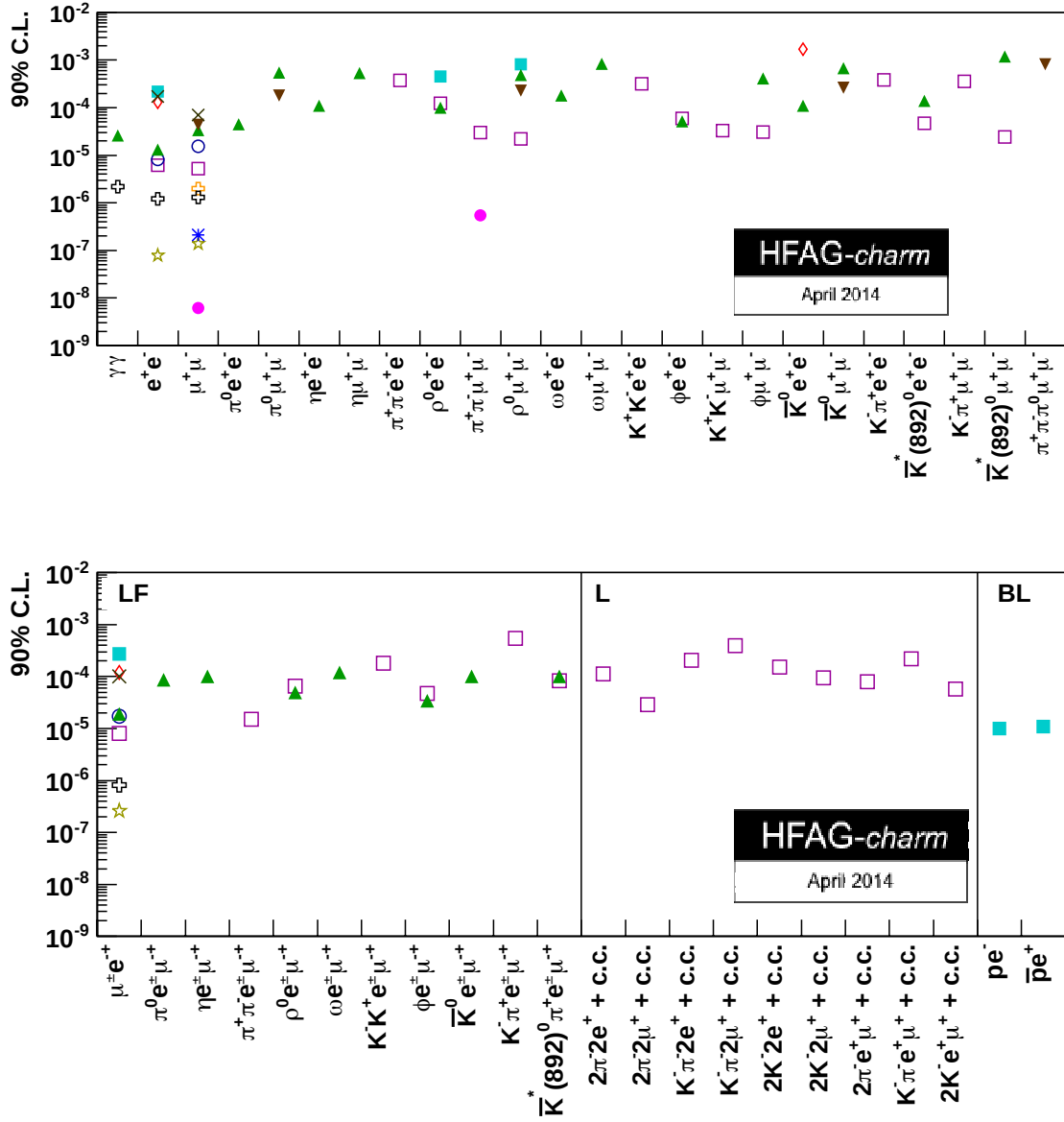


Figure 178: Upper limits at 90% CL for D^0 decays. The top plot shows flavor-changing neutral current decays, and the bottom plot shows lepton-flavor-changing (LF), lepton-number-changing (L), and both baryon- and lepton-number-changing (BL) decays. The legend is given in Fig. 180.

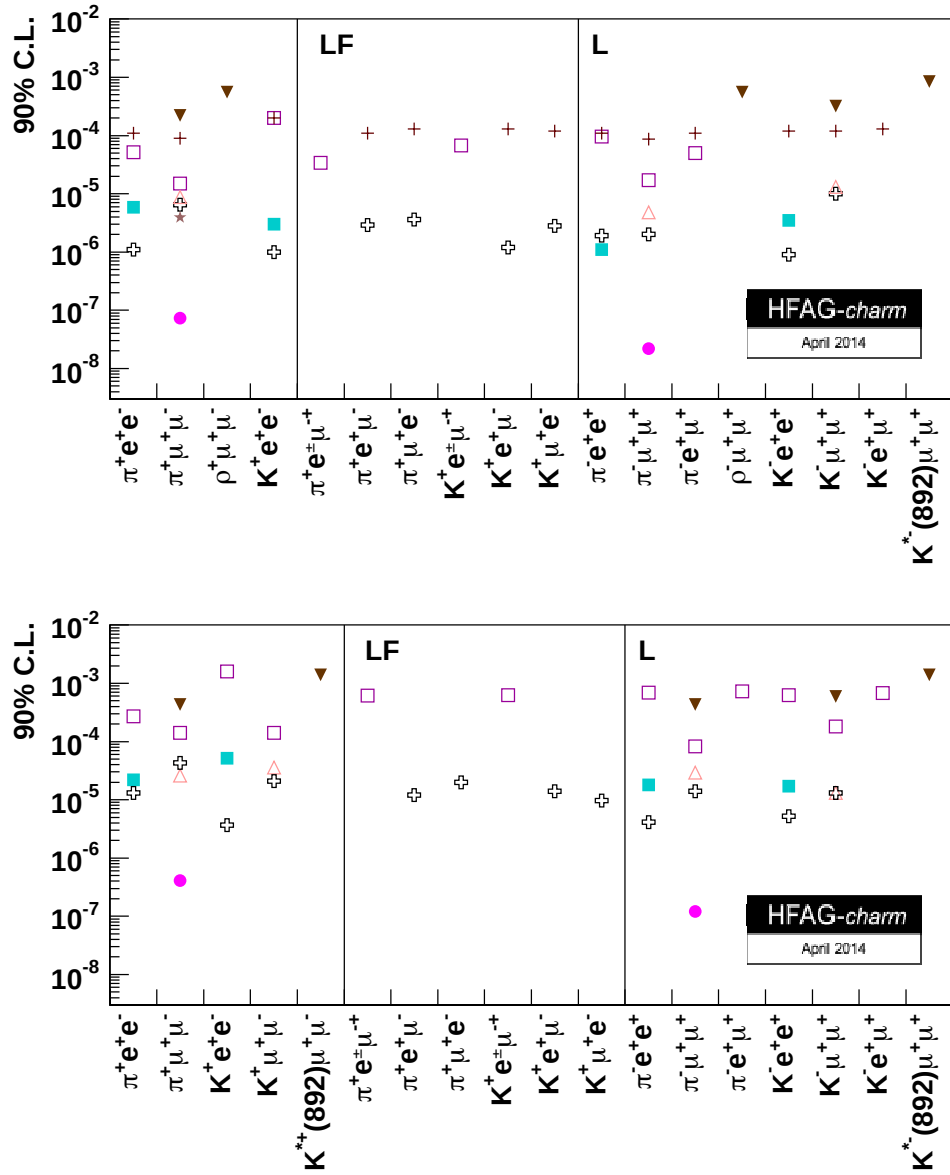


Figure 179: Upper limits at 90% CL for D^+ (top) and D_s^+ (bottom) decays. Each plot shows flavor-changing neutral current decays, lepton-flavor-changing decays (LF), and lepton-number-changing (L) decays. The legend is given in Fig. 180.

Table 257: Upper limits at 90% CL for D^+ decays.

Decay	Limit $\times 10^6$	Experiment	Reference
$\pi^+ e^+ e^-$	110.0	E687	[970]
	52.0	E791	[959]
	5.9	CLEO	[948]
	1.1	<i>BABAR</i>	[971]
$\pi^+ \mu^+ \mu^-$	220.0	E653	[962]
	89.0	E687	[970]

Table 257 – continued from previous page

Decay	Limit $\times 10^6$	Experiment	Reference
	15.0	E791	[959]
	8.8	Focus	[972]
	6.5	<i>BABAR</i>	[971]
	3.9	D0	[949]
	0.073	LHCb	[951]
$\rho^+\mu^+\mu^-$	560.0	E653	[962]
$K^+e^+e^-$	200.0	E687	[970]
	3.0	CLEO	[948]
	1.0	<i>BABAR</i>	[971]
$\pi^+e^\pm\mu^\mp$	34.0	E791	[959]
$\pi^+e^+\mu^-$	110.0	E687	[970]
	2.9	<i>BABAR</i>	[971]
$\pi^+\mu^+e^-$	130.0	E687	[970]
	3.6	<i>BABAR</i>	[971]
$K^+e^\pm\mu^\mp$	68.0	E791	[959]
$K^+e^+\mu^-$	130.0	E687	[970]
	1.2	<i>BABAR</i>	[971]
$K^+\mu^+e^-$	120.0	E687	[970]
	2.8	<i>BABAR</i>	[971]
$\pi^-e^+e^+$	110.0	E687	[970]
	96.0	E791	[959]
	1.9	<i>BABAR</i>	[971]
	1.1	CLEO	[948]
$\pi^-\mu^+\mu^+$	87.0	E687	[970]
	17.0	E791	[959]
	4.8	Focus	[972]
	2.0	<i>BABAR</i>	[971]
	0.022	LHCb	[951]
$\pi^-e^+\mu^+$	110.0	E687	[970]
	50.0	E791	[959]
$\rho^-\mu^+\mu^+$	560.0	E653	[962]
$K^-e^+e^+$	120.0	E687	[970]
	3.5	CLEO	[948]
	0.9	<i>BABAR</i>	[971]
$K^-\mu^+\mu^+$	320.0	E653	[962]
	120.0	E687	[970]
	13.0	Focus	[972]
	10.0	<i>BABAR</i>	[971]
$K^-e^+\mu^+$	130.0	E687	[970]
$K^{*-}(892)\mu^+\mu^+$	850.0	E653	[962]

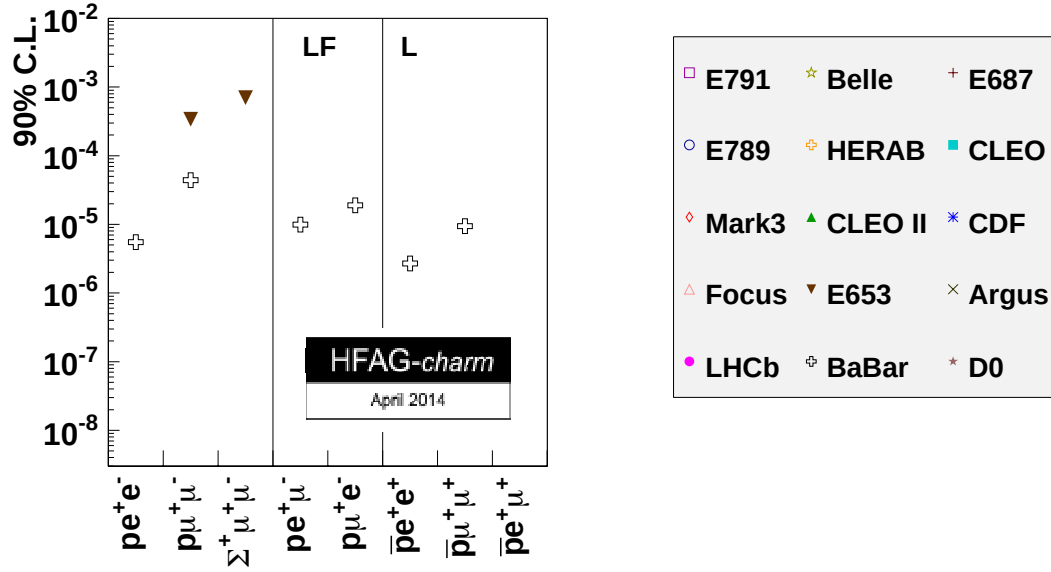


Figure 180: Upper limits at 90% CL for Λ_c^+ decays. Shown are flavor-changing neutral current decays, lepton-flavor-changing (LF) decays, and lepton-number-changing (L) decays.

Table 258: Upper limits at 90% CL for D_s^+ decays.

Decay	Limit $\times 10^6$	Experiment	Reference
$\pi^+ e^+ e^-$	270.0	E791	[959]
	22.0	CLEO	[948]
	13.0	BABAR	[971]
$\pi^+ \mu^+ \mu^-$	430.0	E653	[962]
	140.0	E791	[959]
	43.0	BABAR	[971]
	26.0	Focus	[972]
	0.41	LHCb	[951]
$K^+ e^+ e^-$	1600.0	E791	[959]
	52.0	CLEO	[948]
	3.7	BABAR	[971]
$K^+ \mu^+ \mu^-$	140.0	E791	[959]
	36.0	Focus	[972]
	21.0	BABAR	[971]
$K^{*+}(892)\mu^+\mu^-$	1400.0	E653	[962]
$\pi^+ e^\pm \mu^\mp$	610.0	E791	[959]
$\pi^+ e^+ \mu^-$	12.0	BABAR	[971]
$\pi^+ \mu^+ e^-$	20.0	BABAR	[971]
$K^+ e^\pm \mu^\mp$	630.0	E791	[959]
$K^+ e^+ \mu^-$	14.0	BABAR	[971]
$K^+ \mu^+ e^-$	9.7	BABAR	[971]

Table 258 – continued from previous page

Decay	Limit $\times 10^6$	Experiment	Reference
$\pi^- e^+ e^+$	690.0	E791	[959]
	18.0	CLEO	[948]
	4.1	<i>BABAR</i>	[971]
$\pi^- \mu^+ \mu^+$	430.0	E653	[962]
	82.0	E791	[959]
	29.0	Focus	[972]
	14.0	<i>BABAR</i>	[971]
	0.12	LHCb	[951]
$\pi^- e^+ \mu^+$	730.0	E791	[959]
$K^- e^+ e^+$	630.0	E791	[959]
	17.0	CLEO	[948]
	5.2	<i>BABAR</i>	[971]
$K^- \mu^+ \mu^+$	590.0	E653	[962]
	180.0	E791	[959]
	13.0	<i>BABAR</i>	[971]
$K^- e^+ \mu^+$	680.0	E791	[959]
$K^{*-}(892)\mu^+\mu^+$	1400.0	E653	[962]

Table 259: Upper limits at 90% CL for Λ_c^+ decays.

Decay	Limit $\times 10^6$	Experiment	Reference
$p e^+ e^-$	5.5	<i>BABAR</i>	[971]
$p \mu^+ \mu^-$	340.0	E653	[962]
	44.0	<i>BABAR</i>	[971]
$\sigma^+ \mu^+ \mu^-$	700.0	E653	[962]
$p e^+ \mu^-$	9.9	<i>BABAR</i>	[971]
$p \mu^+ e^-$	19.0	<i>BABAR</i>	[971]
$\bar{p} e^+ e^+$	2.7	<i>BABAR</i>	[971]
$\bar{p} \mu^+ \mu^+$	9.4	<i>BABAR</i>	[971]

9 Tau lepton properties

We present world averages of a selection of τ lepton quantities with the goal to provide the best up-to-date determinations of the tests of the universality of the charged weak interaction (Section 9.3) and of the Cabibbo-Kobayashi-Maskawa (CKM) matrix coefficient $|V_{us}|$ from τ decays (Section 9.5). We concentrate our effort in the averages that most benefit from the adoption of the HFAG methodology [373], namely a global fit of the τ branching fractions that best exploits the available experimental information. We also average the τ lifetime, in order to use the recent precise Belle result [973], which has not been used by PDG until the 2014 edition.

All published statistical correlations are used, and a selection of measurements, particularly the most precise and the most recent, were studied to take into account the significant systematic dependencies from external parameters and common systematic sources.

Finally, we report in Section 9.6 the most up-to-date limits on the lepton-flavour-violating τ branching fractions and we determine the combined upper limits 9.7 for the branching fractions that have multiple experimental results.

9.1 Branching fractions fit

The τ branching fractions provide a test for theory predictions based on the Standard Model (SM) EW and QCD interactions and can be further elaborated to test the EW charged-current universality for leptons, to determine the CKM matrix coefficient $|V_{us}|$ and the QCD coupling constant α_s at the τ mass. A global constrained fit of the available experimental measurements is used to obtain the averages of the τ branching fractions to a complete set of the observed final states, together with their uncertainties and statistical correlations: these data summarize the experimental information for further elaborations. The fit procedure is functionally equivalent to the one employed in the former HFAG reports [4, 373] and consists in a minimum χ^2 fit subject to linear and non-linear constraints.

The measurements listed in Table 266 have been used in a minimum χ^2 fit subject to the constraints that are listed either in the same table (where some fitted quantities and experimental measurements are expressed as ratios of fit quantities) or in Section 9.1.4. The fitted quantities and the measurements are labelled using the PDG Γ_n notation, where n is an integer number, which matches the PDG notation for $n < 800$. We use $n \geq 800$ to denote some additional branching fractions, as documented in the former HFAG report [4]. The PDG Γ_n notation does not maintain the same numbers across editions. We continue using the PDG 2010 [501] numbers with the aim to eventually switch to a different stable notation, probably based on PDG identifiers [974].

The fit output consists in 103 quantities, which correspond to either branching fractions or ratios of linear combinations of branching fractions. Although the fit treats all quantities in the same way, for the purpose of describing the results we divide the above quantities in a set of 47 “base nodes” that permit the definition of all the remaining ones as either a sum of “base nodes” (56 quantities, see Section 9.1.4) or as ratios of linear combinations of “base nodes” (the remaining quantities, see Table 266).

Furthermore we define (see Section 9.1.4) $\Gamma_{110} = B(\tau^- \rightarrow X_s^- \nu_\tau)$, the total branching fraction of the τ decays to final states with the strangeness quantum number equal to one, and Γ_{All} , the branching fraction of the τ into any measured final state, which is supposed to be equal

to 1 within the experimental uncertainty. We define the unitarity residual as $\Gamma_{998} = 1 - \Gamma_{\text{All}}$.

The fitted HFAG-Tau averages are reported in Table 266. The fit has $\chi^2/\text{d.o.f.} = 142.5/127$, corresponding to a confidence level $\text{CL} = 16.45\%$. We use a total of 174 measurements to fit the above mentioned 103 quantities. Although the unitarity constraint is not applied, the fit is statistically consistent with unitarity, and the unitarity residual is $\Gamma_{998} = 1 - \Gamma_{\text{All}} = (9.902 \pm 9.850) \cdot 10^{-4}$.

A scale factor of 5.44 (as in the two previous reports [4, 373]) has been applied to the published uncertainties of the two severely inconsistent measurements of $\Gamma_{96} = \tau \rightarrow KKK\nu$ by *BABAR* and Belle, following the same procedure as the PDG.

For several old results, for historical reasons, the table reports as statistical errors the sum in quadrature of the statistical and systematic errors and as systematic errors zero: this does not affect the fit results since the systematic errors are treated exactly like the statistical ones.

9.1.1 Changes with respect to the previous report

The following additions and changes have been done with respect to the previous HFAG report [4].

Published results from two *BABAR* papers and one Belle paper have been added. The following results have been used from the *BABAR* high multiplicity decay τ branching fractions paper [975]:

$$\begin{aligned}
\Gamma_{811} &= \pi^- 2\pi^0 \omega \nu_\tau \text{ (ex. } K^0) & (7.3 \pm 1.2 \pm 1.2) \cdot 10^{-5} \\
\Gamma_{812} &= 2\pi^- \pi^+ 3\pi^0 \nu_\tau \text{ (ex. } K^0, \eta, \omega, f_1) & (0.1 \pm 0.08 \pm 0.30) \cdot 10^{-4} \\
\Gamma_{821} &= 3\pi^- 2\pi^+ \nu_\tau \text{ (ex. } K^0, \omega, f_1) & (7.68 \pm 0.04 \pm 0.40) \cdot 10^{-4} \\
\Gamma_{822} &= K^- 2\pi^- 2\pi^+ \nu_\tau \text{ (ex. } K^0) & (0.6 \pm 0.5 \pm 1.1) \cdot 10^{-6} \\
\Gamma_{831} &= 2\pi^- \pi^+ \omega \nu_\tau \text{ (ex. } K^0) & (8.4 \pm 0.4 \pm 0.6) \cdot 10^{-5} \\
\Gamma_{832} &= 3\pi^- 2\pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0, \eta, \omega, f_1) & (0.36 \pm 0.03 \pm 0.09) \cdot 10^{-4} \\
\Gamma_{833} &= K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0) & (1.1 \pm 0.4 \pm 0.4) \cdot 10^{-6} \\
\Gamma_{910} &= 2\pi^- \pi^+ \eta \nu_\tau \text{ (} \eta \rightarrow 3\pi^0 \text{) (ex. } K^0) & (8.27 \pm 0.88 \pm 0.81) \cdot 10^{-5} \\
\Gamma_{911} &= \pi^- 2\pi^0 \eta \nu_\tau \text{ (} \eta \rightarrow \pi^+ \pi^- \pi^0 \text{) (ex. } K^0) & (4.57 \pm 0.77 \pm 0.50) \cdot 10^{-5} \\
\Gamma_{920} &= \pi^- f_1 \nu_\tau \text{ (} f_1 \rightarrow 2\pi^- 2\pi^+ \text{)} & (5.20 \pm 0.31 \pm 0.37) \cdot 10^{-5} \\
\Gamma_{930} &= 2\pi^- \pi^+ \eta \nu_\tau \text{ (} \eta \rightarrow \pi^+ \pi^- \pi^0 \text{) (ex. } K^0) & (5.39 \pm 0.27 \pm 0.41) \cdot 10^{-5} \\
\Gamma_{944} &= 2\pi^- \pi^+ \eta \nu_\tau \text{ (} \eta \rightarrow \gamma\gamma \text{) (ex. } K^0) & (8.26 \pm 0.35 \pm 0.51) \cdot 10^{-5}.
\end{aligned}$$

These results supersede the previous *BABAR* results $\Gamma_{136} = \tau^- \rightarrow \pi^- \pi^- \pi^+ \eta \nu_\tau$ (ex. K^0) [976] and $\Gamma_{103} = \tau^- \rightarrow 3h^- 2h^+ \nu_\tau$ (ex. K^0) [977]. The following results have been used from the *BABAR* paper on the τ branching fractions with two K_S [978]:

$$\begin{aligned}
\Gamma_{47} &= \pi^- K_S^0 K_S^0 \nu_\tau & (2.31 \pm 0.04 \pm 0.08) \cdot 10^{-4} \\
\Gamma_{50} &= \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau & (1.60 \pm 0.20 \pm 0.22) \cdot 10^{-5}.
\end{aligned}$$

The following results have been used from the Belle paper on the K_S final states [979]:

$$\begin{aligned}
\Gamma_{33} &= K_S^0 (\text{particles})^- \nu_\tau & (9.15 \pm 0.01 \pm 0.15) \cdot 10^{-3} \\
\Gamma_{35} &= \pi^- \bar{K}^0 \nu_\tau & (8.32 \pm 0.02 \pm 0.16) \cdot 10^{-3} \\
\Gamma_{37} &= K^- K^0 \nu_\tau & (14.8 \pm 0.14 \pm 0.54) \cdot 10^{-4} \\
\Gamma_{40} &= \pi^- \bar{K}^0 \pi^0 \nu_\tau & (3.86 \pm 0.04 \pm 0.14) \cdot 10^{-3}
\end{aligned}$$

$$\begin{aligned}
\Gamma_{42} &= K^- \pi^0 K^0 \nu_\tau & (14.96 \pm 0.20 \pm 0.74) \cdot 10^{-4} \\
\Gamma_{47} &= \pi^- K_S^0 K_S^0 \nu_\tau & (2.33 \pm 0.03 \pm 0.09) \cdot 10^{-4} \\
\Gamma_{50} &= \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau & (2.00 \pm 0.22 \pm 0.20) \cdot 10^{-5}.
\end{aligned}$$

These results supersede the preliminary ones [980] and the former published result on $\tau^- \rightarrow \pi^- \bar{K}^0 \nu_\tau$ [981]. In order to profit from the measurements of branching fractions with two K_S in the final state, we discard the inclusive ALEPH measurement on $\tau^- \rightarrow \pi^- K^0 \bar{K}^0 \pi^0 \nu_\tau$ [982] and we add the exclusive ALEPH measurement on $\tau^- \rightarrow \pi^- \pi^0 K_S^0 K_L^0 \nu_\tau$ [982].

The CLEO result on $\tau^- \rightarrow \pi^- \pi^- \pi^+ \eta \nu_\tau$ (ex. K^0) [983] has been discarded since it is very correlated with the branching fractions into six pions measured in the same paper, which are dominated by the η and ω resonances.

We added the $\tau^- \rightarrow K^- \eta \nu_\tau$ measurements by CLEO [984] and ALEPH [985] to complete the list of useful experimental inputs for this mode.

In order to best integrate the above new results in the global fit, the following constraints have been added:

$$\begin{aligned}
\Gamma_{33} &= \Gamma_{35} \cdot \Gamma_{<\bar{K}^0|K_S>} + \Gamma_{40} \cdot \Gamma_{<\bar{K}^0|K_S>} + \Gamma_{42} \cdot \Gamma_{<K^0|K_S>} + \Gamma_{47} + \Gamma_{48} + \Gamma_{50} + \Gamma_{51} + \Gamma_{37} \cdot \Gamma_{<K^0|K_S>} + \\
&\quad \Gamma_{132} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{\eta \rightarrow \text{neutral}}) + \Gamma_{44} \cdot \Gamma_{<\bar{K}^0|K_S>} + \Gamma_{801} \cdot \Gamma_{\phi \rightarrow K_S K_L} / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L}) \\
\Gamma_{49} &= \Gamma_{50} + \Gamma_{51} + \Gamma_{806} \\
\Gamma_{78} &= \Gamma_{810} + \Gamma_{50} \cdot 2 \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{132} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{\eta \rightarrow 3\pi^0}) \\
\Gamma_{103} &= \Gamma_{820} + \Gamma_{822} + \Gamma_{831} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} \\
\Gamma_{104} &= \Gamma_{830} + \Gamma_{833} \\
\Gamma_{806} &= \Gamma_{50} \cdot (\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>}) / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>}) \\
\Gamma_{810} &= \Gamma_{910} + \Gamma_{911} + \Gamma_{811} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{812} \\
\Gamma_{820} &= \Gamma_{920} + \Gamma_{821} \\
\Gamma_{830} &= \Gamma_{930} + \Gamma_{831} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{832} \\
\Gamma_{910} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
\Gamma_{930} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{944} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow \gamma \gamma} .
\end{aligned}$$

It was realised that the Γ_{44} fit quantity, which is exclusively determined by one single ALEPH result [982], does actually exclude the contribution from $K^0 \rightarrow \pi^0 \pi^0$, so we renamed it to $\Gamma_{44} = \pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0). The definition of the total branching fraction and of the inclusive branching fraction of the τ lepton into a strange final states have been updated as follows:

$$\begin{aligned}
\Gamma_{110} &= \Gamma_{10} + \Gamma_{16} + \Gamma_{23} + \Gamma_{28} + \Gamma_{35} + \Gamma_{40} + \Gamma_{128} + \Gamma_{802} + \Gamma_{803} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} + \Gamma_{44} + \\
&\quad \Gamma_{53} + \Gamma_{801} + \Gamma_{822} + \Gamma_{833} \\
\Gamma_{\text{All}} &= \Gamma_3 + \Gamma_5 + \Gamma_9 + \Gamma_{10} + \Gamma_{14} + \Gamma_{16} + \Gamma_{20} + \Gamma_{23} + \Gamma_{27} + \Gamma_{28} + \Gamma_{30} + \Gamma_{35} + \Gamma_{37} + \Gamma_{40} + \Gamma_{42} + \\
&\quad \Gamma_{47} + \Gamma_{48} + \Gamma_{804} + \Gamma_{62} + \Gamma_{70} + \Gamma_{77} + \Gamma_{78} + \Gamma_{93} + \Gamma_{94} + \Gamma_{104} + \Gamma_{126} + \Gamma_{128} + \Gamma_{802} + \Gamma_{803} + \\
&\quad \Gamma_{800} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} + \Gamma_{44} + \Gamma_{53} + \Gamma_{50} + \Gamma_{51} + \Gamma_{806} + \Gamma_{805} + \Gamma_{801} + \Gamma_{152} + \Gamma_{103} .
\end{aligned}$$

The total τ branching fraction Γ_{All} definition includes two modes that have overlapping final states, to a minor extent:

$$\begin{aligned}
\Gamma_{50} &= \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau \\
\Gamma_{132} &= \pi^- \bar{K}^0 \eta \nu_\tau .
\end{aligned}$$

The amount of overlap cannot be disentangled with the presently available measurements, however we consider it negligible since the involved branching fractions are small and their overlap is conceivably minor. An inaccurate constraint used in the two previous reports has been removed:

$$\Gamma_{136} = \Gamma_{104} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{78} \cdot \Gamma_{\eta \rightarrow 3\pi^0} .$$

The inaccurate constraint had negligible effect on the global fit in the previous HFAG reports, except for the involved specific branching ratios. In particular, the effects on the lepton universality tests and in the $|V_{us}|$ determination were negligible.

Finally, the constraint parameters (see Section 9.1.4) have been updated to the PDG 2013 results [18].

9.1.2 Branching ratio fit results and experimental inputs

Table 266 reports the τ branching ratio fit results and experimental inputs.

Table 266: HFAG Summer 2014 branching fractions fit results.

τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$\Gamma_3 = \mu^- \bar{\nu}_\mu \nu_\tau$	0.17391 ± 0.00040	HFAG Summer 2014 fit
$0.17319 \pm 0.00077 \pm 0.00000$	ALEPH	[986]
$0.17325 \pm 0.00122 \pm 0.00000$	DELPHI	[987]
$0.17342 \pm 0.00129 \pm 0.00000$	L3	[988]
$0.17340 \pm 0.00108 \pm 0.00000$	OPAL	[989]
$\frac{\Gamma_3}{\Gamma_5} = \frac{\mu^- \bar{\nu}_\mu \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	0.97610 ± 0.00278	HFAG Summer 2014 fit
$0.99700 \pm 0.05315 \pm 0.00000$	ARGUS	[990]
$0.97960 \pm 0.00390 \pm 0.00053$	BaBar	[991]
$0.97770 \pm 0.01074 \pm 0.00000$	CLEO	[992]
$\Gamma_5 = e^- \bar{\nu}_e \nu_\tau$	0.17817 ± 0.00041	HFAG Summer 2014 fit
$0.17837 \pm 0.00080 \pm 0.00000$	ALEPH	[986]
$0.17760 \pm 0.00180 \pm 0.00000$	CLEO	[992]
$0.17877 \pm 0.00155 \pm 0.00000$	DELPHI	[987]
$0.17806 \pm 0.00129 \pm 0.00000$	L3	[988]
$0.17810 \pm 0.00108 \pm 0.00000$	OPAL	[993]
$\Gamma_7 = h^- \geq 0K_L^0 \nu_\tau$	0.12026 ± 0.00054	HFAG Summer 2014 fit
$0.12400 \pm 0.00990 \pm 0.00000$	DELPHI	[994]
$0.12470 \pm 0.00502 \pm 0.00000$	L3	[995]
$0.12100 \pm 0.00860 \pm 0.00000$	OPAL	[996]
$\Gamma_8 = h^- \nu_\tau$	0.11509 ± 0.00054	HFAG Summer 2014 fit
$0.11524 \pm 0.00105 \pm 0.00000$	ALEPH	[986]

Table 266 – continued from previous page

τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$0.11520 \pm 0.00130 \pm 0.00000$	CLEO	[992]
$0.11571 \pm 0.00166 \pm 0.00000$	DELPHI	[997]
$0.11980 \pm 0.00206 \pm 0.00000$	OPAL	[998]
$\Gamma_9 = \pi^- \nu_\tau$	0.10813 ± 0.00053	HFAG Summer 2014 fit
$\frac{\Gamma_9}{\Gamma_5} = \frac{\pi^- \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	0.6069 ± 0.0032	HFAG Summer 2014 fit
$0.5945 \pm 0.0057 \pm 0.0025$	BaBar	[991]
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.6960 \pm 0.0287 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[999]
$(0.6600 \pm 0.1140 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1000]
$(0.8500 \pm 0.1800 \pm 0.0000) \cdot 10^{-2}$	DELPHI	[1001]
$(0.6580 \pm 0.0396 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1002]
$\frac{\Gamma_{10}}{\Gamma_5} = \frac{K^- \nu_\tau}{e^- \bar{\nu}_e \nu_\tau}$	$(3.903 \pm 0.054) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(3.882 \pm 0.063 \pm 0.017) \cdot 10^{-2}$	BaBar	[991]
$\Gamma_{13} = h^- \pi^0 \nu_\tau$	0.25936 ± 0.00090	HFAG Summer 2014 fit
$0.25924 \pm 0.00129 \pm 0.00000$	ALEPH	[986]
$0.25670 \pm 0.00010 \pm 0.00390$	Belle	[1003]
$0.25870 \pm 0.00437 \pm 0.00000$	CLEO	[1004]
$0.25740 \pm 0.00244 \pm 0.00000$	DELPHI	[997]
$0.25050 \pm 0.00610 \pm 0.00000$	L3	[995]
$0.25890 \pm 0.00336 \pm 0.00000$	OPAL	[998]
$\Gamma_{14} = \pi^- \pi^0 \nu_\tau$	0.25502 ± 0.00092	HFAG Summer 2014 fit
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4331 \pm 0.0149) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.4440 \pm 0.0354 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[999]
$(0.4160 \pm 0.0030 \pm 0.0180) \cdot 10^{-2}$	BaBar	[1005]
$(0.5100 \pm 0.1221 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1000]
$(0.4710 \pm 0.0633 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1006]
$\Gamma_{17} = h^- \geq 2\pi^0 \nu_\tau$	0.10804 ± 0.00095	HFAG Summer 2014 fit
$0.09910 \pm 0.00411 \pm 0.00000$	OPAL	[998]
$\Gamma_{19} = h^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(9.303 \pm 0.097) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(9.295 \pm 0.122 \pm 0.000) \cdot 10^{-2}$	ALEPH	[986]
$(9.498 \pm 0.422 \pm 0.000) \cdot 10^{-2}$	DELPHI	[997]
$(8.880 \pm 0.560 \pm 0.000) \cdot 10^{-2}$	L3	[995]
$\frac{\Gamma_{19}}{\Gamma_{13}} = \frac{h^- 2\pi^0 \nu_\tau \text{ (ex. } K^0\text{)}}{h^- \pi^0 \nu_\tau}$	0.3587 ± 0.0044	HFAG Summer 2014 fit
$0.3420 \pm 0.0171 \pm 0.0000$	CLEO	[1007]

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τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$\Gamma_{20} = \pi^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(9.240 \pm 0.100) \cdot 10^{-2}$	HFAG Summer 2014 fit
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0220) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.0560 \pm 0.0250 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[999]
$(0.0900 \pm 0.1044 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1000]
$\Gamma_{25} = h^- \geq 3\pi^0 \nu_\tau$ (ex. K^0)	$(1.233 \pm 0.065) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(1.403 \pm 0.310 \pm 0.000) \cdot 10^{-2}$	DELPHI	[997]
$\Gamma_{26} = h^- 3\pi^0 \nu_\tau$	$(1.157 \pm 0.072) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(1.082 \pm 0.093 \pm 0.000) \cdot 10^{-2}$	ALEPH	[986]
$(1.700 \pm 0.449 \pm 0.000) \cdot 10^{-2}$	L3	[995]
$\frac{\Gamma_{26}}{\Gamma_{13}} = \frac{h^- 3\pi^0 \nu_\tau}{h^- \pi^0 \nu_\tau}$	$(4.460 \pm 0.277) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(4.400 \pm 0.583 \pm 0.000) \cdot 10^{-2}$	CLEO	[1007]
$\Gamma_{27} = \pi^- 3\pi^0 \nu_\tau$ (ex. K^0)	$(1.030 \pm 0.075) \cdot 10^{-2}$	HFAG Summer 2014 fit
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(4.190 \pm 2.160) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(3.700 \pm 2.371 \pm 0.000) \cdot 10^{-4}$	ALEPH	[999]
$\Gamma_{29} = h^- 4\pi^0 \nu_\tau$ (ex. K^0)	$(0.1566 \pm 0.0391) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.1600 \pm 0.0707 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1007]
$\Gamma_{30} = h^- 4\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.1097 \pm 0.0391) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.1120 \pm 0.0509 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[986]
$\Gamma_{31} = K^- \geq 0\pi^0 \geq 0K^0 \geq 0\gamma \nu_\tau$	$(1.548 \pm 0.030) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(1.700 \pm 0.225 \pm 0.000) \cdot 10^{-2}$	CLEO	[1000]
$(1.540 \pm 0.240 \pm 0.000) \cdot 10^{-2}$	DELPHI	[1001]
$(1.528 \pm 0.056 \pm 0.000) \cdot 10^{-2}$	OPAL	[1002]
$\Gamma_{33} = K_S^0(\text{particles})^- \nu_\tau$	$(0.9019 \pm 0.0081) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.9700 \pm 0.0849 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[1008]
$(0.9150 \pm 0.0010 \pm 0.0150) \cdot 10^{-2}$	Belle	[979]
$(0.9700 \pm 0.1082 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1009]
$\Gamma_{34} = h^- \bar{K}^0 \nu_\tau$	$(0.9878 \pm 0.0119) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.8550 \pm 0.0814 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1010]
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8378 \pm 0.0123) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.9280 \pm 0.0564 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[999]
$(0.8400 \pm 0.0040 \pm 0.0230) \cdot 10^{-2}$	BaBar	[1011]
$(0.8320 \pm 0.0020 \pm 0.0160) \cdot 10^{-2}$	Belle	[979]
$(0.9500 \pm 0.1616 \pm 0.0000) \cdot 10^{-2}$	L3	[1012]
$(0.9330 \pm 0.0838 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1013]
$\Gamma_{37} = K^- K^0 \nu_\tau$	$(0.1500 \pm 0.0050) \cdot 10^{-2}$	HFAG Summer 2014 fit

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τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$(0.1580 \pm 0.0453 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[1008]
$(0.1620 \pm 0.0237 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[999]
$(0.1480 \pm 0.0014 \pm 0.0054) \cdot 10^{-2}$	Belle	[979]
$(0.1510 \pm 0.0304 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1010]
$\Gamma_{38} = K^- K^0 \geq 0\pi^0 \nu_\tau$	$(0.3029 \pm 0.0074) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.3300 \pm 0.0674 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1013]
$\Gamma_{39} = h^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.5209 \pm 0.0114) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.5620 \pm 0.0693 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1010]
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.3680 \pm 0.0103) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.2940 \pm 0.0818 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[1008]
$(0.3470 \pm 0.0646 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[999]
$(0.3420 \pm 0.0060 \pm 0.0150) \cdot 10^{-2}$	BaBar	[1014]
$(0.3860 \pm 0.0040 \pm 0.0140) \cdot 10^{-2}$	Belle	[979]
$(0.4100 \pm 0.1237 \pm 0.0000) \cdot 10^{-2}$	L3	[1012]
$\Gamma_{42} = K^- \pi^0 K^0 \nu_\tau$	$(0.1528 \pm 0.0070) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.1520 \pm 0.0789 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[1008]
$(0.1430 \pm 0.0291 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[999]
$(0.1496 \pm 0.0020 \pm 0.0074) \cdot 10^{-2}$	Belle	[979]
$(0.1450 \pm 0.0412 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1010]
$\Gamma_{43} = \pi^- \bar{K}^0 \geq 1\pi^0 \nu_\tau$	$(0.3805 \pm 0.0229) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.3240 \pm 0.0992 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1013]
$\Gamma_{44} = \pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	$(1.245 \pm 2.043) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(2.600 \pm 2.400 \pm 0.000) \cdot 10^{-4}$	ALEPH	[982]
$\Gamma_{46} = \pi^- K^0 \bar{K}^0 \nu_\tau$	$(0.1329 \pm 0.0110) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.1530 \pm 0.0340 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[1008]
$\Gamma_{47} = \pi^- K_S^0 K_S^0 \nu_\tau$	$(2.359 \pm 0.061) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(2.600 \pm 1.118 \pm 0.000) \cdot 10^{-4}$	ALEPH	[1008]
$(2.310 \pm 0.040 \pm 0.080) \cdot 10^{-4}$	BaBar	[978]
$(2.330 \pm 0.030 \pm 0.090) \cdot 10^{-4}$	Belle	[979]
$(2.300 \pm 0.583 \pm 0.000) \cdot 10^{-4}$	CLEO	[1010]
$\Gamma_{48} = \pi^- K_S^0 K_L^0 \nu_\tau$	$(0.0857 \pm 0.0104) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.1010 \pm 0.0264 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[1008]
$\Gamma_{49} = \pi^- K^0 \bar{K}^0 \pi^0 \nu_\tau$	$(2.896 \pm 1.051) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{50} = \pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$	$(1.845 \pm 0.206) \cdot 10^{-5}$	HFAG Summer 2014 fit
$(1.600 \pm 0.200 \pm 0.220) \cdot 10^{-5}$	BaBar	[978]

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τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$(2.000 \pm 0.220 \pm 0.200) \cdot 10^{-5}$	Belle	[979]
$\Gamma_{51} = \pi^- \pi^0 K_S^0 K_L^0 \nu_\tau$	$(2.527 \pm 1.047) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(3.100 \pm 1.100 \pm 0.500) \cdot 10^{-4}$	ALEPH	[1008]
$\Gamma_{53} = \bar{K}^0 h^- h^- h^+ \nu_\tau$	$(2.221 \pm 2.024) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(2.300 \pm 2.025 \pm 0.000) \cdot 10^{-4}$	ALEPH	[1008]
$\Gamma_{54} = h^- h^- h^+ \geq 0 \text{neutrals} \geq 0 K_L^0 \nu_\tau$	0.15201 ± 0.00059	HFAG Summer 2014 fit
$0.15000 \pm 0.00500 \pm 0.00000$	CELLO	[1015]
$0.14400 \pm 0.00671 \pm 0.00000$	L3	[1016]
$0.15100 \pm 0.01000 \pm 0.00000$	TPC	[1017]
$\Gamma_{55} = h^- h^- h^+ \geq 0 \text{neutrals} \nu_\tau$ (ex. K^0)	0.14573 ± 0.00056	HFAG Summer 2014 fit
$0.14556 \pm 0.00130 \pm 0.00000$	L3	[1018]
$0.14960 \pm 0.00238 \pm 0.00000$	OPAL	[1019]
$\Gamma_{57} = h^- h^- h^+ \nu_\tau$ (ex. K^0)	$(9.448 \pm 0.053) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(9.510 \pm 0.212 \pm 0.000) \cdot 10^{-2}$	CLEO	[1020]
$(9.317 \pm 0.122 \pm 0.000) \cdot 10^{-2}$	DELPHI	[997]
$\frac{\Gamma_{57}}{\Gamma_{55}} = \frac{h^- h^- h^+ \nu_\tau \text{ (ex. } K^0\text{)}}{h^- h^- h^+ \geq 0 \text{neutrals} \nu_\tau \text{ (ex. } K^0\text{)}}$	0.6483 ± 0.0029	HFAG Summer 2014 fit
$0.6600 \pm 0.0146 \pm 0.0000$	OPAL	[1019]
$\Gamma_{58} = h^- h^- h^+ \nu_\tau$ (ex. K^0, ω)	$(9.418 \pm 0.053) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(9.469 \pm 0.096 \pm 0.000) \cdot 10^{-2}$	ALEPH	[986]
$\Gamma_{60} = \pi^- \pi^- \pi^+ \nu_\tau$ (ex. K^0)	$(9.0097 \pm 0.0510) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(8.8337 \pm 0.0074 \pm 0.1267) \cdot 10^{-2}$	BaBar	[1021]
$(8.4200 \pm 0.0033 \pm 0.2588) \cdot 10^{-2}$	Belle	[1022]
$(9.1300 \pm 0.4627 \pm 0.0000) \cdot 10^{-2}$	CLEO3	[1023]
$\Gamma_{62} = \pi^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(8.980 \pm 0.051) \cdot 10^{-2}$	HFAG Summer 2014 fit
$\Gamma_{66} = h^- h^- h^+ \pi^0 \nu_\tau$ (ex. K^0)	$(4.603 \pm 0.051) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(4.734 \pm 0.077 \pm 0.000) \cdot 10^{-2}$	ALEPH	[986]
$(4.230 \pm 0.228 \pm 0.000) \cdot 10^{-2}$	CLEO	[1020]
$(4.545 \pm 0.148 \pm 0.000) \cdot 10^{-2}$	DELPHI	[997]
$\Gamma_{69} = \pi^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0)	$(4.516 \pm 0.052) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(4.190 \pm 0.233 \pm 0.000) \cdot 10^{-2}$	CLEO	[1024]
$\Gamma_{70} = \pi^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω)	$(2.767 \pm 0.071) \cdot 10^{-2}$	HFAG Summer 2014 fit
$\Gamma_{74} = h^- h^- h^+ \geq 2 \pi^0 \nu_\tau$ (ex. K^0)	$(0.5130 \pm 0.0311) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.5610 \pm 0.1168 \pm 0.0000) \cdot 10^{-2}$	DELPHI	[997]
$\Gamma_{76} = h^- h^- h^+ 2 \pi^0 \nu_\tau$ (ex. K^0)	$(0.4919 \pm 0.0310) \cdot 10^{-2}$	HFAG Summer 2014 fit

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τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$(0.4350 \pm 0.0461 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[986]
$\frac{\Gamma_{76}}{\Gamma_{54}} = \frac{h^- h^- h^+ 2\pi^0 \nu_\tau \text{ (ex. } K^0\text{)}}{h^- h^- h^+ \geq 0 \text{ neutrals} \geq 0 K_L^0 \nu_\tau}$	$(3.236 \pm 0.202) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(3.400 \pm 0.361 \pm 0.000) \cdot 10^{-2}$	CLEO	[1025]
$\Gamma_{77} = h^- h^- h^+ 2\pi^0 \nu_\tau \text{ (ex. } K^0, \omega, \eta\text{)}$	$(9.734 \pm 3.546) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{78} = h^- h^- h^+ 3\pi^0 \nu_\tau$	$(2.109 \pm 0.299) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(2.200 \pm 0.500 \pm 0.000) \cdot 10^{-4}$	CLEO	[983]
$\frac{\Gamma_{80}}{\Gamma_{60}} = \frac{K^- \pi^- h^+ \nu_\tau \text{ (ex. } K^0\text{)}}{\pi^- \pi^- \pi^+ \nu_\tau \text{ (ex. } K^0\text{)}}$	$(4.845 \pm 0.081) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(5.440 \pm 0.570 \pm 0.000) \cdot 10^{-2}$	CLEO	[1026]
$\frac{\Gamma_{81}}{\Gamma_{69}} = \frac{K^- \pi^- h^+ \pi^0 \nu_\tau \text{ (ex. } K^0\text{)}}{\pi^- \pi^- \pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0\text{)}}$	$(1.932 \pm 0.266) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(2.610 \pm 0.615 \pm 0.000) \cdot 10^{-2}$	CLEO	[1026]
$\Gamma_{82} = K^- \pi^- \pi^+ \geq 0 \text{ neutrals} \nu_\tau$	$(0.4796 \pm 0.0138) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.5800 \pm 0.1845 \pm 0.0000) \cdot 10^{-2}$	TPC	[1027]
$\Gamma_{85} = K^- \pi^- \pi^+ \nu_\tau \text{ (ex. } K^0\text{)}$	$(0.2929 \pm 0.0068) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.2140 \pm 0.0470 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[1028]
$(0.2726 \pm 0.0018 \pm 0.0092) \cdot 10^{-2}$	BaBar	[1021]
$(0.3300 \pm 0.0013 \pm 0.0166) \cdot 10^{-2}$	Belle	[1022]
$(0.3840 \pm 0.0405 \pm 0.0000) \cdot 10^{-2}$	CLEO3	[1023]
$(0.4150 \pm 0.0664 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1006]
$\Gamma_{88} = K^- \pi^- \pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0\text{)}$	$(8.113 \pm 1.168) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(6.100 \pm 4.295 \pm 0.000) \cdot 10^{-4}$	ALEPH	[1028]
$(7.400 \pm 1.360 \pm 0.000) \cdot 10^{-4}$	CLEO3	[1029]
$\Gamma_{92} = \pi^- K^- K^+ \geq 0 \text{ neutrals} \nu_\tau$	$(0.1497 \pm 0.0033) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.1590 \pm 0.0566 \pm 0.0000) \cdot 10^{-2}$	OPAL	[1030]
$(0.1500 \pm 0.0855 \pm 0.0000) \cdot 10^{-2}$	TPC	[1027]
$\Gamma_{93} = \pi^- K^- K^+ \nu_\tau$	$(0.14363 \pm 0.00274) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.16300 \pm 0.02702 \pm 0.00000) \cdot 10^{-2}$	ALEPH	[1028]
$(0.13461 \pm 0.00100 \pm 0.00364) \cdot 10^{-2}$	BaBar	[1021]
$(0.15500 \pm 0.00066 \pm 0.00556) \cdot 10^{-2}$	Belle	[1022]
$(0.15500 \pm 0.01082 \pm 0.00000) \cdot 10^{-2}$	CLEO3	[1023]
$\frac{\Gamma_{93}}{\Gamma_{60}} = \frac{\pi^- K^- K^+ \nu_\tau}{\pi^- \pi^- \pi^+ \nu_\tau \text{ (ex. } K^0\text{)}}$	$(1.594 \pm 0.030) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(1.600 \pm 0.335 \pm 0.000) \cdot 10^{-2}$	CLEO	[1026]
$\Gamma_{94} = \pi^- K^- K^+ \pi^0 \nu_\tau$	$(0.611 \pm 0.183) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(7.500 \pm 3.265 \pm 0.000) \cdot 10^{-4}$	ALEPH	[1028]

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τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$(0.550 \pm 0.184 \pm 0.000) \cdot 10^{-4}$	CLEO3	[1029]
$\frac{\Gamma_{94}}{\Gamma_{69}} = \frac{\pi^- K^- K^+ \pi^0 \nu_\tau}{\pi^- \pi^- \pi^+ \pi^0 \nu_\tau \text{ (ex. } K^0)}$	$(0.1353 \pm 0.0406) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.7900 \pm 0.4682 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1026]
$\Gamma_{96} = K^- K^- K^+ \nu_\tau$	$(2.156 \pm 0.800) \cdot 10^{-5}$	HFAG Summer 2014 fit
$(1.578 \pm 0.130 \pm 0.123) \cdot 10^{-5}$	BaBar	[1021]
$(3.290 \pm 0.169 \pm 0.196) \cdot 10^{-5}$	Belle	[1022]
$\Gamma_{102} = 3h^- 2h^+ \geq 0 \text{ neutrals } \nu_\tau \text{ (ex. } K^0)$	$(0.0986 \pm 0.0037) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.0970 \pm 0.0121 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1031]
$(0.1020 \pm 0.0290 \pm 0.0000) \cdot 10^{-2}$	HRS	[1032]
$(0.1700 \pm 0.0341 \pm 0.0000) \cdot 10^{-2}$	L3	[1018]
$\Gamma_{103} = 3h^- 2h^+ \nu_\tau \text{ (ex. } K^0)$	$(8.224 \pm 0.315) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(7.200 \pm 1.500 \pm 0.000) \cdot 10^{-4}$	ALEPH	[986]
$(6.400 \pm 2.508 \pm 0.000) \cdot 10^{-4}$	ARGUS	[1033]
$(7.700 \pm 1.030 \pm 0.000) \cdot 10^{-4}$	CLEO	[1031]
$(9.700 \pm 1.581 \pm 0.000) \cdot 10^{-4}$	DELPHI	[997]
$(5.100 \pm 2.000 \pm 0.000) \cdot 10^{-4}$	HRS	[1032]
$(9.100 \pm 1.523 \pm 0.000) \cdot 10^{-4}$	OPAL	[1034]
$\Gamma_{104} = 3h^- 2h^+ \pi^0 \nu_\tau \text{ (ex. } K^0)$	$(1.637 \pm 0.113) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(2.100 \pm 0.922 \pm 0.000) \cdot 10^{-4}$	ALEPH	[986]
$(1.700 \pm 0.283 \pm 0.000) \cdot 10^{-4}$	CLEO	[983]
$(1.600 \pm 1.342 \pm 0.000) \cdot 10^{-4}$	DELPHI	[997]
$(2.700 \pm 2.012 \pm 0.000) \cdot 10^{-4}$	OPAL	[1034]
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.882 \pm 0.047) \cdot 10^{-2}$	HFAG Summer 2014 fit
$\Gamma_{126} = \pi^- \pi^0 \eta \nu_\tau$	$(0.1387 \pm 0.0072) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.1800 \pm 0.0447 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[985]
$(0.1350 \pm 0.0030 \pm 0.0070) \cdot 10^{-2}$	Belle	[1035]
$(0.1700 \pm 0.0283 \pm 0.0000) \cdot 10^{-2}$	CLEO	[1036]
$\Gamma_{128} = K^- \eta \nu_\tau$	$(1.548 \pm 0.080) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(2.900^{+1.300}_{-1.200} \pm 0.700) \cdot 10^{-4}$	ALEPH	[985]
$(1.420 \pm 0.110 \pm 0.070) \cdot 10^{-4}$	BaBar	[1037]
$(1.580 \pm 0.050 \pm 0.090) \cdot 10^{-4}$	Belle	[1035]
$(2.600 \pm 0.500 \pm 0.500) \cdot 10^{-4}$	CLEO	[984]
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.483 \pm 0.116) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(0.460 \pm 0.110 \pm 0.040) \cdot 10^{-4}$	Belle	[1035]
$(1.770 \pm 0.904 \pm 0.000) \cdot 10^{-4}$	CLEO	[1038]

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τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$	$(0.934 \pm 0.149) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(0.880 \pm 0.140 \pm 0.060) \cdot 10^{-4}$	Belle	[1035]
$(2.200 \pm 0.734 \pm 0.000) \cdot 10^{-4}$	CLEO	[1038]
$\Gamma_{136} = \pi^- \pi^- \pi^+ \eta \nu_\tau$ (ex. K^0)	$(2.186 \pm 0.129) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{150} = h^- \omega \nu_\tau$	$(1.995 \pm 0.064) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(1.910 \pm 0.092 \pm 0.000) \cdot 10^{-2}$	ALEPH	[985]
$(1.600 \pm 0.491 \pm 0.000) \cdot 10^{-2}$	CLEO	[1039]
$\frac{\Gamma_{150}}{\Gamma_{66}} = \frac{h^- \omega \nu_\tau}{h^- h^- h^+ \pi^0 \nu_\tau}$ (ex. K^0)	0.4333 ± 0.0139	HFAG Summer 2014 fit
$0.4310 \pm 0.0330 \pm 0.0000$	ALEPH	[1040]
$0.4640 \pm 0.0233 \pm 0.0000$	CLEO	[1020]
$\Gamma_{151} = K^- \omega \nu_\tau$	$(4.100 \pm 0.922) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(4.100 \pm 0.922 \pm 0.000) \cdot 10^{-4}$	CLEO3	[1029]
$\Gamma_{152} = h^- \pi^0 \omega \nu_\tau$	$(0.4054 \pm 0.0418) \cdot 10^{-2}$	HFAG Summer 2014 fit
$(0.4300 \pm 0.0781 \pm 0.0000) \cdot 10^{-2}$	ALEPH	[985]
$\frac{\Gamma_{152}}{\Gamma_{76}} = \frac{h^- \omega \pi^0 \nu_\tau}{h^- h^- h^+ 2\pi^0 \nu_\tau}$ (ex. K^0)	0.8243 ± 0.0757	HFAG Summer 2014 fit
$0.8100 \pm 0.0848 \pm 0.0000$	CLEO	[1025]
$\Gamma_{800} = \pi^- \omega \nu_\tau$	$(1.954 \pm 0.065) \cdot 10^{-2}$	HFAG Summer 2014 fit
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \rightarrow KK)$	$(3.664 \pm 1.360) \cdot 10^{-5}$	HFAG Summer 2014 fit
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2922 \pm 0.0068) \cdot 10^{-2}$	HFAG Summer 2014 fit
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(4.101 \pm 1.429) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{804} = \pi^- K_L^0 K_L^0 \nu_\tau$	$(2.359 \pm 0.061) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{805} = a_1^- (\rightarrow \pi^- \gamma) \nu_\tau$	$(4.000 \pm 2.000) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(4.000 \pm 2.000 \pm 0.000) \cdot 10^{-4}$	ALEPH	[986]
$\Gamma_{806} = \pi^- \pi^0 K_L^0 K_L^0 \nu_\tau$	$(1.845 \pm 0.206) \cdot 10^{-5}$	HFAG Summer 2014 fit
$\Gamma_{810} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. K^0)	$(1.925 \pm 0.298) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{811} = \pi^- 2\pi^0 \omega \nu_\tau$ (ex. K^0)	$(7.110 \pm 1.586) \cdot 10^{-5}$	HFAG Summer 2014 fit
$(7.300 \pm 1.200 \pm 1.200) \cdot 10^{-5}$	BaBar	[975]
$\Gamma_{812} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)	$(1.336 \pm 2.682) \cdot 10^{-5}$	HFAG Summer 2014 fit
$(1.000 \pm 0.800 \pm 3.000) \cdot 10^{-5}$	BaBar	[975]
$\Gamma_{820} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. K^0, ω)	$(8.205 \pm 0.315) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{821} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. K^0, ω, f_1)	$(7.685 \pm 0.296) \cdot 10^{-4}$	HFAG Summer 2014 fit
$(7.680 \pm 0.040 \pm 0.400) \cdot 10^{-4}$	BaBar	[975]
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	$(0.596 \pm 1.208) \cdot 10^{-6}$	HFAG Summer 2014 fit
$(0.600 \pm 0.500 \pm 1.100) \cdot 10^{-6}$	BaBar	[975]

Table 266 – continued from previous page

τ lepton branching fraction	Fit value / Exp.	HFAG Fit / Ref.
$\Gamma_{830} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	$(1.626 \pm 0.113) \cdot 10^{-4}$	HFAG Summer 2014 fit
$\Gamma_{831} = 2\pi^- \pi^+ \omega \nu_\tau$ (ex. K^0) $(8.400 \pm 0.400 \pm 0.600) \cdot 10^{-5}$	$(8.370 \pm 0.624) \cdot 10^{-5}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{832} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1) $(3.600 \pm 0.300 \pm 0.900) \cdot 10^{-5}$	$(3.783 \pm 0.873) \cdot 10^{-5}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0) $(1.100 \pm 0.400 \pm 0.400) \cdot 10^{-6}$	$(1.108 \pm 0.566) \cdot 10^{-6}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{910} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \rightarrow 3\pi^0$) (ex. K^0) $(8.270 \pm 0.880 \pm 0.810) \cdot 10^{-5}$	$(7.144 \pm 0.423) \cdot 10^{-5}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{911} = \pi^- 2\pi^0 \eta \nu_\tau$ ($\eta \rightarrow \pi^+ \pi^- \pi^0$) (ex. K^0) $(4.570 \pm 0.770 \pm 0.500) \cdot 10^{-5}$	$(4.424 \pm 0.867) \cdot 10^{-5}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{920} = \pi^- f_1 \nu_\tau$ ($f_1 \rightarrow 2\pi^- 2\pi^+$) $(5.200 \pm 0.310 \pm 0.370) \cdot 10^{-5}$	$(5.202 \pm 0.444) \cdot 10^{-5}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{930} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \rightarrow \pi^+ \pi^- \pi^0$) (ex. K^0) $(5.390 \pm 0.270 \pm 0.410) \cdot 10^{-5}$	$(5.010 \pm 0.297) \cdot 10^{-5}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{944} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \rightarrow \gamma\gamma$) (ex. K^0) $(8.260 \pm 0.350 \pm 0.510) \cdot 10^{-5}$	$(8.615 \pm 0.510) \cdot 10^{-5}$ BaBar	HFAG Summer 2014 fit [975]
$\Gamma_{998} = 1 - \Gamma_{\text{All}}$	$(9.902 \pm 9.850) \cdot 10^{-4}$	HFAG Summer 2014 fit

9.1.3 Correlation between base nodes uncertainties

The following tables report the correlation coefficients between base nodes, in percent.

Table 267: Base nodes correlation coefficients in percent, section 1.

Γ_5	23													
Γ_9	7	5												
Γ_{10}	3	6	1											
Γ_{14}	-13	-14	-13	-3										
Γ_{16}	-0	-1	2	-1	-16									
Γ_{20}	-5	-5	-7	-1	-40	2								
Γ_{23}	0	0	-0	-2	2	-13	-22							
Γ_{27}	-4	-3	-8	-1	0	3	-36	6						
Γ_{28}	0	0	-0	-2	2	-13	5	-21	-29					
Γ_{30}	-5	-4	-11	-2	-9	-0	6	0	-42	0				
Γ_{35}	-0	-0	1	0	-0	2	-1	1	-0	1	-0			
Γ_{37}	0	0	-0	-0	0	-2	1	-2	1	-2	0	-29		
Γ_{40}	-0	-1	1	0	-0	2	-0	1	-2	1	-0	-10	-1	
	Γ_3	Γ_5	Γ_9	Γ_{10}	Γ_{14}	Γ_{16}	Γ_{20}	Γ_{23}	Γ_{27}	Γ_{28}	Γ_{30}	Γ_{35}	Γ_{37}	Γ_{40}

Table 268: Base nodes correlation coefficients in percent, section 2.

Γ_{42}	-0	0	-0	-0	1	-3	1	-5	0	-5	0	-3	-27	-17
Γ_{44}	-0	-0	0	-0	-0	-0	0	-1	0	-1	0	13	16	1
Γ_{47}	0	0	-0	0	-0	1	-0	1	-0	1	-0	-9	-9	-6
Γ_{48}	-0	-0	1	-0	0	-1	1	-3	0	-3	-0	40	51	12
Γ_{50}	0	0	-0	-0	0	-0	0	-0	0	-0	-0	-3	3	-1
Γ_{51}	-0	-0	0	-0	0	-0	0	-1	0	-1	-0	13	17	4
Γ_{53}	0	0	0	0	0	-0	0	0	0	0	0	-0	-0	-0
Γ_{62}	-3	-5	8	0	-4	5	-7	-1	-5	-1	-5	3	-1	2
Γ_{70}	-6	-6	-7	-1	-9	-1	-1	0	-1	0	3	-1	0	-1
Γ_{77}	-1	-0	-3	-1	-2	-0	-0	0	2	0	2	-0	0	-0
Γ_{93}	-1	-1	2	0	-1	2	-1	-0	-1	-0	-1	1	-0	1
Γ_{94}	-0	-0	-0	-0	-0	-0	-0	0	-0	0	0	-0	0	-0
Γ_{126}	0	0	0	0	0	0	-1	-0	0	-0	-2	0	-0	0
Γ_{128}	-0	-0	1	-0	-0	1	-0	-1	-0	-1	-0	1	-0	0
	Γ_3	Γ_5	Γ_9	Γ_{10}	Γ_{14}	Γ_{16}	Γ_{20}	Γ_{23}	Γ_{27}	Γ_{28}	Γ_{30}	Γ_{35}	Γ_{37}	Γ_{40}

Table 269: Base nodes correlation coefficients in percent, section 3.

Γ_{130}	0	0	0	0	0	0	-0	-0	0	-0	-0	0	-0	0
Γ_{132}	0	0	0	0	0	0	-0	-0	0	-0	-0	1	1	0
Γ_{136}	0	0	0	0	-0	0	-0	-0	-0	-0	-1	0	-0	0
Γ_{151}	0	0	0	-0	0	0	0	-0	0	-0	0	-0	-0	-0
Γ_{152}	-1	-0	-3	-1	-2	-0	-1	0	2	0	2	0	0	0
Γ_{800}	-2	-2	-2	-0	-3	-0	-0	0	-0	0	1	-0	0	-0
Γ_{801}	-0	-0	0	-0	-0	-0	0	-0	0	-0	-0	1	1	0
Γ_{802}	-1	-1	0	0	-1	-1	-2	0	-2	0	-1	-0	0	-0
Γ_{803}	-0	-0	-0	-0	-0	-0	-0	0	-0	0	0	-0	-0	-0
Γ_{805}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Γ_{811}	0	0	0	0	0	0	-0	-0	-0	-0	-0	0	-0	0
Γ_{812}	0	1	0	0	0	-0	0	0	-0	0	-0	-0	-0	-0
Γ_{821}	0	0	0	0	0	0	-1	-0	-0	-0	-1	0	-0	0
Γ_{822}	0	0	0	0	0	-0	-0	0	-0	0	-0	-0	0	-0
	Γ_3	Γ_5	Γ_9	Γ_{10}	Γ_{14}	Γ_{16}	Γ_{20}	Γ_{23}	Γ_{27}	Γ_{28}	Γ_{30}	Γ_{35}	Γ_{37}	Γ_{40}

Table 270: Base nodes correlation coefficients in percent, section 4.

Γ_{831}	-0	0	0	0	-0	0	-0	-0	-0	-0	-1	0	-0	0
Γ_{832}	-0	-0	-0	-0	-0	-0	0	0	0	0	-0	0	0	-0
Γ_{833}	-0	-0	-0	-0	-0	-0	0	0	0	0	-0	-0	0	-0
Γ_{920}	0	0	0	0	0	0	-0	-0	-0	-0	-0	0	-0	0
	Γ_3	Γ_5	Γ_9	Γ_{10}	Γ_{14}	Γ_{16}	Γ_{20}	Γ_{23}	Γ_{27}	Γ_{28}	Γ_{30}	Γ_{35}	Γ_{37}	Γ_{40}

Table 271: Base nodes correlation coefficients in percent, section 5.

Γ_{44}	7													
Γ_{47}	-4	17												
Γ_{48}	20	-96	48											
Γ_{50}	3	6	-12	20										
Γ_{51}	7	-32	17	-100	7									
Γ_{53}	-0	0	-0	-0	-0	-0								
Γ_{62}	-1	1	0	1	0	0	-0							
Γ_{70}	0	0	-0	-0	-0	-0	-0	-19						
Γ_{77}	0	-0	0	0	0	0	0	-1	-7					
Γ_{93}	-0	0	0	1	0	0	-0	14	-4	-0				
Γ_{94}	0	0	-0	-0	-0	-0	-0	-0	-2	-0	-0			
Γ_{126}	-0	0	1	0	0	0	-0	1	-0	-5	0	-0		
Γ_{128}	-0	0	0	0	0	0	-0	2	-0	-0	1	-0	4	
	Γ_{42}	Γ_{44}	Γ_{47}	Γ_{48}	Γ_{50}	Γ_{51}	Γ_{53}	Γ_{62}	Γ_{70}	Γ_{77}	Γ_{93}	Γ_{94}	Γ_{126}	Γ_{128}

Table 272: Base nodes correlation coefficients in percent, section 6.

Γ_{130}	-0	0	0	0	0	0	-0	0	-0	-1	0	-0	1	1
Γ_{132}	0	-1	1	-4	0	-1	-0	0	-0	-0	0	-0	2	1
Γ_{136}	-0	0	0	0	-0	0	-0	-0	-1	0	0	-0	0	0
Γ_{151}	-0	0	-0	0	-0	-0	-0	0	12	-0	0	0	-0	-0
Γ_{152}	0	-0	0	0	0	0	0	-1	-11	-64	-0	-0	-0	-0
Γ_{800}	0	0	-0	-0	-0	-0	-0	-8	-69	-2	-1	0	-0	-0
Γ_{801}	0	-1	1	-4	0	-1	-0	-1	-0	-0	1	-0	0	0
Γ_{802}	0	-0	-0	-0	-0	-0	-0	17	-6	-0	-0	-0	-0	-0
Γ_{803}	-0	-0	-0	-0	-0	-0	-0	-1	-19	-0	-0	-2	-0	-1
Γ_{805}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Γ_{811}	-0	-0	0	-0	-0	-0	-0	-0	-0	-0	-0	-0	0	0
Γ_{812}	-0	-0	0	-0	-1	-0	-0	-0	-1	-0	-0	-0	-0	-0
Γ_{821}	-0	0	0	0	0	0	0	0	-1	0	0	-0	0	0
Γ_{822}	0	-0	-0	-0	0	-0	0	-0	-0	0	-0	-0	-0	-0
	Γ_{42}	Γ_{44}	Γ_{47}	Γ_{48}	Γ_{50}	Γ_{51}	Γ_{53}	Γ_{62}	Γ_{70}	Γ_{77}	Γ_{93}	Γ_{94}	Γ_{126}	Γ_{128}

Table 273: Base nodes correlation coefficients in percent, section 7.

Γ_{831}	-0	0	0	0	-0	0	0	-0	-0	0	0	-0	0	0
Γ_{832}	-0	-0	0	-0	-0	-0	0	-0	-0	0	-0	-0	0	0
Γ_{833}	0	-0	-0	-0	0	-0	0	-0	0	0	-0	0	-0	-0
Γ_{920}	-0	0	0	0	0	0	0	0	-0	0	0	-0	0	0
	Γ_{42}	Γ_{44}	Γ_{47}	Γ_{48}	Γ_{50}	Γ_{51}	Γ_{53}	Γ_{62}	Γ_{70}	Γ_{77}	Γ_{93}	Γ_{94}	Γ_{126}	Γ_{128}

Table 274: Base nodes correlation coefficients in percent, section 8.

Γ_{132}	0													
Γ_{136}	0	-0												
Γ_{151}	-0	-0	-0											
Γ_{152}	-0	-0	0	-0										
Γ_{800}	-0	-0	-0	-14	-3									
Γ_{801}	0	-0	0	-0	-0	-0								
Γ_{802}	-0	-0	-0	-2	-0	-1	1							
Γ_{803}	-0	-0	-0	-58	-0	9	-0	1						
Γ_{805}	0	0	0	0	0	0	0	0	0					
Γ_{811}	0	-1	20	-0	-0	-0	-0	-0	-0	0				
Γ_{812}	-0	-2	-8	-0	-0	-0	-0	-0	-0	0	-16			
Γ_{821}	0	-0	47	-0	0	-0	0	-0	-0	0	8	-4		
Γ_{822}	-0	0	-1	-0	0	-0	-0	0	-0	0	-0	0	-1	
	Γ_{130}	Γ_{132}	Γ_{136}	Γ_{151}	Γ_{152}	Γ_{800}	Γ_{801}	Γ_{802}	Γ_{803}	Γ_{805}	Γ_{811}	Γ_{812}	Γ_{821}	Γ_{822}

Table 275: Base nodes correlation coefficients in percent, section 9.

Γ_{831}	0	-0	39	-0	0	-0	0	-0	-0	0	14	-4	39	-1
Γ_{832}	0	-0	3	-0	0	-0	-0	-0	-0	0	2	-0	3	-0
Γ_{833}	-0	0	-1	-0	0	0	-0	-0	0	0	-0	0	-1	0
Γ_{920}	0	-0	20	-0	0	-0	0	-0	-0	0	3	-2	35	-1
	Γ_{130}	Γ_{132}	Γ_{136}	Γ_{151}	Γ_{152}	Γ_{800}	Γ_{801}	Γ_{802}	Γ_{803}	Γ_{805}	Γ_{811}	Γ_{812}	Γ_{821}	Γ_{822}

Table 276: Base nodes correlation coefficients in percent, section 10.

Γ_{832}	-2		
Γ_{833}	-1	-1	
Γ_{920}	17	1	-0
	Γ_{831}	Γ_{832}	Γ_{833}

9.1.4 Equality constraints

We use equality constraints that relate a branching fraction to a sum of branching fractions. As mentioned above, the τ branching fractions are denoted with Γ_n labels. In the constraint relations we use the values of some non-tau branching fractions, denoted *e.g.* with the self-describing notation $\Gamma_{K_S \rightarrow \pi^0 \pi^0}$. We also use probabilities corresponding to modulus square amplitudes describing quantum mixtures of states such as K^0 , $\bar{K}^0$, K_S , K_L , denoted with *e.g.* $\Gamma_{<K^0|K_S>} = |\langle K^0|K_S \rangle|^2$. In the fit, all non-tau quantities are taken from the PDG 2013 [18] fits (when available) or averages, and are used without accounting for their uncertainties, which are however in general small with respect to the uncertainties on the τ branching fractions. The τ branching fractions are illustrated in Table 266. The equations in the following permit the computation of the values and uncertainties for branching fractions that are not listed in Table 266, once they are expressed as function of the quantities that are listed there. The following list does not include the (non-linear) constraints already introduced in Section 9.1, and illustrated in Table 266, where some measured branching fractions are expressed as ratios of “base” branching fractions.

$$\Gamma_7 = \Gamma_{35} \cdot \Gamma_{<\bar{K}^0|K_L>} + \Gamma_9 + \Gamma_{804} + \Gamma_{37} \cdot \Gamma_{<K^0|K_L>} + \Gamma_{10}$$

$$\Gamma_8 = \Gamma_9 + \Gamma_{10}$$

$$\Gamma_{13} = \Gamma_{14} + \Gamma_{16}$$

$$\begin{aligned} \Gamma_{17} = & \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{30} + \Gamma_{23} + \Gamma_{28} + \Gamma_{35} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\ & + \Gamma_{40} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{42} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{20} \\ & + \Gamma_{27} + \Gamma_{47} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{48} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{50} \cdot (\Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\ & + \Gamma_{51} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{37} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\ & + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \end{aligned}$$

$$\Gamma_{19} = \Gamma_{23} + \Gamma_{20}$$

$$\begin{aligned} \Gamma_{25} = & \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{30} + \Gamma_{28} + \Gamma_{27} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\ & + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \end{aligned}$$

$$\Gamma_{26} = \Gamma_{128} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{28} + \Gamma_{40} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{42} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) \\ + \Gamma_{27}$$

$$\Gamma_{29} = \Gamma_{30} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow 3\pi^0} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow 3\pi^0}$$

$$\Gamma_{31} = \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{neutral}} + \Gamma_{23} + \Gamma_{28} + \Gamma_{42} + \Gamma_{16} + \Gamma_{37} \\ + \Gamma_{10} + \Gamma_{801} \cdot (\Gamma_{\phi \rightarrow K_S K_L} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L})$$

$$\Gamma_{33} = \Gamma_{35} \cdot \Gamma_{<\bar{K}^0|K_S>} + \Gamma_{40} \cdot \Gamma_{<\bar{K}^0|K_S>} + \Gamma_{42} \cdot \Gamma_{<K^0|K_S>} + \Gamma_{47} \\ + \Gamma_{48} + \Gamma_{50} + \Gamma_{51} + \Gamma_{37} \cdot \Gamma_{<K^0|K_S>} + \Gamma_{132} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{\eta \rightarrow \text{neutral}}) \\ + \Gamma_{44} \cdot \Gamma_{<\bar{K}^0|K_S>} + \Gamma_{801} \cdot \Gamma_{\phi \rightarrow K_S K_L} / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L})$$

$$\Gamma_{34} = \Gamma_{35} + \Gamma_{37}$$

$$\Gamma_{38} = \Gamma_{42} + \Gamma_{37}$$

$$\Gamma_{39} = \Gamma_{40} + \Gamma_{42}$$

$$\Gamma_{43} = \Gamma_{40} + \Gamma_{44}$$

$$\Gamma_{46} = \Gamma_{48} + \Gamma_{47} + \Gamma_{804}$$

$$\Gamma_{49} = \Gamma_{50} + \Gamma_{51} + \Gamma_{806}$$

$$\Gamma_{54} = \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{152} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{35} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) \\ + \Gamma_{40} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) + \Gamma_{42} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) + \Gamma_{78} \\ + \Gamma_{47} \cdot (2 \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{77} + \Gamma_{48} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \\ + \Gamma_{50} \cdot (2 \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0}) + \Gamma_{51} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} + \Gamma_{94} \\ + \Gamma_{62} + \Gamma_{70} + \Gamma_{93} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{37} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) \\ + \Gamma_{802} + \Gamma_{803} + \Gamma_{800} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{151} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} \\ + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{132} \cdot (\Gamma_{<\bar{K}^0|K_L>} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\ + \Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{\eta \rightarrow 3\pi^0}) + \Gamma_{53} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{<\bar{K}^0|K_L>}) \\ + \Gamma_{801} \cdot (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L})$$

$$\Gamma_{55} = \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{152} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{78} + \Gamma_{77} \\ + \Gamma_{94} + \Gamma_{62} + \Gamma_{70} + \Gamma_{93} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{802} \\ + \Gamma_{803} + \Gamma_{800} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{151} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) \\ + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{801} \cdot \Gamma_{\phi \rightarrow K^+ K^-} / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L})$$

$$\Gamma_{57} = \Gamma_{62} + \Gamma_{93} + \Gamma_{802} + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} \\ + \Gamma_{801} \cdot \Gamma_{\phi \rightarrow K^+ K^-} / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L})$$

$$\Gamma_{58} = \Gamma_{62} + \Gamma_{93} + \Gamma_{802} + \Gamma_{801} \cdot \Gamma_{\phi \rightarrow K^+ K^-} / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L})$$

$$\Gamma_{60} = \Gamma_{62} + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-}$$

$$\Gamma_{66} = \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} + \Gamma_{94} + \Gamma_{70} + \Gamma_{803} \\ + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0}$$

$$\begin{aligned}
\Gamma_{69} &= \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} + \Gamma_{70} + \Gamma_{800} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{74} &= \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{78} + \Gamma_{77} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{76} &= \Gamma_{152} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{77} + \Gamma_{126} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{130} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{78} &= \Gamma_{810} + \Gamma_{50} \cdot 2 \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{K_S \rightarrow \pi^0 \pi^0} + \Gamma_{132} \cdot (\Gamma_{<\bar{K}^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-} \cdot \Gamma_{\eta \rightarrow 3\pi^0}) \\
\Gamma_{82} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \text{charged}} + \Gamma_{42} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) + \Gamma_{802} + \Gamma_{803} \\
&\quad + \Gamma_{151} \cdot (\Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{\omega \rightarrow \pi^+ \pi^-}) + \Gamma_{37} \cdot (\Gamma_{<K^0|K_S>} \cdot \Gamma_{K_S \rightarrow \pi^+ \pi^-}) \\
\Gamma_{85} &= \Gamma_{802} + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} \\
\Gamma_{88} &= \Gamma_{128} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{803} + \Gamma_{151} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{92} &= \Gamma_{94} + \Gamma_{93} \\
\Gamma_{96} &= \Gamma_{801} \cdot \Gamma_{\phi \rightarrow K^+ K^-} / (\Gamma_{\phi \rightarrow K^+ K^-} + \Gamma_{\phi \rightarrow K_S K_L}) \\
\Gamma_{102} &= \Gamma_{103} + \Gamma_{104} \\
\Gamma_{103} &= \Gamma_{820} + \Gamma_{822} + \Gamma_{831} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^-} \\
\Gamma_{104} &= \Gamma_{830} + \Gamma_{833} \\
\Gamma_{110} &= \Gamma_{10} + \Gamma_{16} + \Gamma_{23} + \Gamma_{28} + \Gamma_{35} + \Gamma_{40} + \Gamma_{128} \\
&\quad + \Gamma_{802} + \Gamma_{803} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} + \Gamma_{44} + \Gamma_{53} \\
&\quad + \Gamma_{801} + \Gamma_{822} + \Gamma_{833} \\
\Gamma_{150} &= \Gamma_{800} + \Gamma_{151} \\
\Gamma_{804} &= \Gamma_{47} \cdot (\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>}) / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>}) \\
\Gamma_{806} &= \Gamma_{50} \cdot (\Gamma_{<K^0|K_L>} \cdot \Gamma_{<\bar{K}^0|K_L>}) / (\Gamma_{<K^0|K_S>} \cdot \Gamma_{<\bar{K}^0|K_S>}) \\
\Gamma_{810} &= \Gamma_{910} + \Gamma_{911} + \Gamma_{811} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{812} \\
\Gamma_{820} &= \Gamma_{920} + \Gamma_{821} \\
\Gamma_{830} &= \Gamma_{930} + \Gamma_{831} \cdot \Gamma_{\omega \rightarrow \pi^+ \pi^- \pi^0} + \Gamma_{832} \\
\Gamma_{910} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow 3\pi^0} \\
\Gamma_{930} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow \pi^+ \pi^- \pi^0} \\
\Gamma_{944} &= \Gamma_{136} \cdot \Gamma_{\eta \rightarrow \gamma\gamma} \\
\Gamma_{\text{All}} &= \Gamma_3 + \Gamma_5 + \Gamma_9 + \Gamma_{10} + \Gamma_{14} + \Gamma_{16} + \Gamma_{20} \\
&\quad + \Gamma_{23} + \Gamma_{27} + \Gamma_{28} + \Gamma_{30} + \Gamma_{35} + \Gamma_{37} + \Gamma_{40} \\
&\quad + \Gamma_{42} + \Gamma_{47} + \Gamma_{48} + \Gamma_{804} + \Gamma_{62} + \Gamma_{70} + \Gamma_{77} \\
&\quad + \Gamma_{78} + \Gamma_{93} + \Gamma_{94} + \Gamma_{104} + \Gamma_{126} + \Gamma_{128} + \Gamma_{802} \\
&\quad + \Gamma_{803} + \Gamma_{800} + \Gamma_{151} + \Gamma_{130} + \Gamma_{132} + \Gamma_{44} + \Gamma_{53} \\
&\quad + \Gamma_{50} + \Gamma_{51} + \Gamma_{806} + \Gamma_{805} + \Gamma_{801} + \Gamma_{152} + \Gamma_{103}
\end{aligned}$$

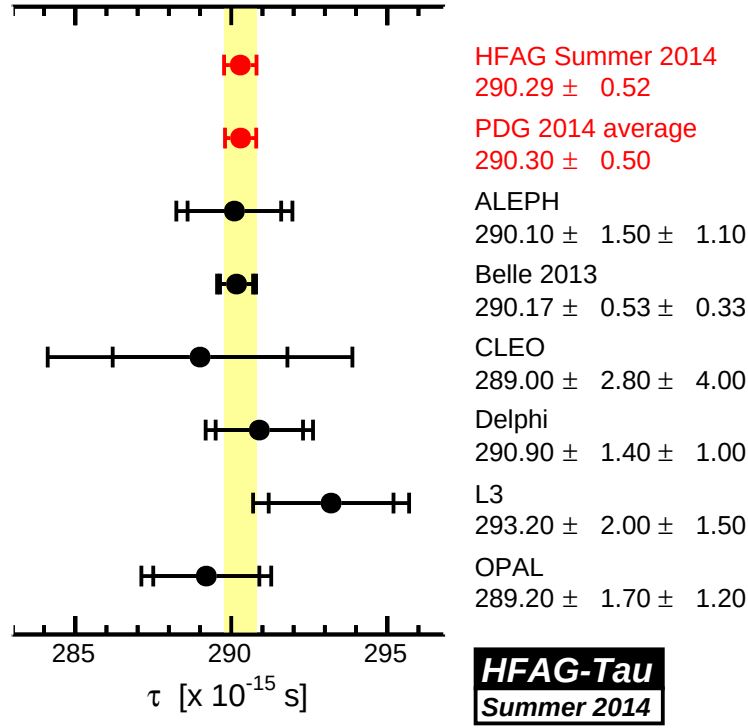


Figure 181: τ lifetime average.

9.2 τ lifetime average

When the work for this report started, the PDG did not yet include the 2013 precise Belle τ lifetime measurement [973]. In order to obtain the most precise and up-to-date lepton universality tests, we computed and used an updated τ lifetime average. In this case, the HFAG procedure does not improve the PDG average with additional information or technical treatment and we obtain the same result as PDG 2014 [5] (see Fig. 181) by doing a standard error-weighted average. With respect to PDG 2013, the uncertainty is now reduced by a factor two.

9.3 Tests of lepton universality

The lepton universality tests probe the Standard Model prediction that the charged weak current interaction has the same coupling for all leptons. The precision of such tests has been significantly improved by the addition of the recent Belle τ lifetime measurement [973], while improvements from the τ branching fraction fit are negligible. We compute the universality tests like in the previous report by using proper ratios of the partial widths of a heavier lepton L decaying to a lighter lepton ℓ [1041],

$$\Gamma(L \rightarrow \nu_L \ell \bar{\nu}_\ell (\gamma)) = \frac{B(L \rightarrow \nu_L \ell \bar{\nu}_\ell)}{\tau_L} = \frac{G_L G_\ell m_L^5}{192\pi^3} f\left(\frac{m_\ell^2}{m_L^2}\right) r_W^L r_\gamma^L,$$

where

$$G_\ell = \frac{g_\ell^2}{4\sqrt{2}M_W^2} , \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x ,$$

$$r_W^L = 1 + \frac{3}{5} \frac{m_L^2}{M_W^2} , \quad r_\gamma^L = 1 + \frac{\alpha(m_L)}{2\pi} \left(\frac{25}{4} - \pi^2 \right) .$$

We use $r_\gamma^\tau = 1 - 43.2 \cdot 10^{-4}$ and $r_\gamma^\mu = 1 - 42.4 \cdot 10^{-4}$ [1041] and M_W from PDG 2013 [18]. We use HFAG 2014 averages and PDG 2013 for the other quantities. Using pure leptonic processes we obtain

$$\left(\frac{g_\tau}{g_\mu} \right) = 1.0011 \pm 0.0015 , \quad \left(\frac{g_\tau}{g_e} \right) = 1.0029 \pm 0.0015 , \quad \left(\frac{g_\mu}{g_e} \right) = 1.0018 \pm 0.0014 .$$

Using semi-hadronic processes

$$\left(\frac{g_\tau}{g_\mu} \right)^2 = \frac{B(\tau \rightarrow h\nu_\tau)}{B(h \rightarrow \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1 + \delta_h) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2} \right)^2 ,$$

where $h = \pi$ or K and the radiative corrections are $\delta_\pi = (0.16 \pm 0.14)\%$ and $\delta_K = (0.90 \pm 0.22)\%$ [1042], we measure:

$$\left(\frac{g_\tau}{g_\mu} \right)_\pi = 0.9963 \pm 0.0027 , \quad \left(\frac{g_\tau}{g_\mu} \right)_K = 0.9858 \pm 0.0071 .$$

Similar tests could be performed with decays to electrons, however they are less precise because the hadron two body decays to electrons are helicity-suppressed. Averaging the three g_τ/g_μ ratios we obtain

$$\left(\frac{g_\tau}{g_\mu} \right)_{\tau+\pi+K} = 1.0001 \pm 0.0014 ,$$

accounting for statistical correlations. Table 277 reports the statistical correlation coefficients for the fitted coupling ratios:

Table 277: Universality coupling ratios correlation coefficients (%).

$\left(\frac{g_\tau}{g_e} \right)$	53			
$\left(\frac{g_\mu}{g_e} \right)$	-49	48		
$\left(\frac{g_\tau}{g_\mu} \right)_\pi$	24	26	2	
$\left(\frac{g_\tau}{g_\mu} \right)_K$	12	10	-2	5
	$\left(\frac{g_\tau}{g_\mu} \right)$	$\left(\frac{g_\tau}{g_e} \right)$	$\left(\frac{g_\mu}{g_e} \right)$	$\left(\frac{g_\tau}{g_\mu} \right)_\pi$

9.4 Universality improved $B(\tau \rightarrow e\nu\bar{\nu})$ and R_{had}

We compute two quantities that are used in this report and have been traditionally used for further elaborations and tests involving the τ branching fractions: the “universality improved” experimental determination of $B_e = B(\tau \rightarrow e\nu\bar{\nu})$, which relies on assuming that the Standard Model and lepton universality hold, and the ratio R_{had} between the total branching fraction of the τ to hadrons and the universality improved B_e , which is the same as the ratio of the two respective partial widths.

Following Ref. [1043], we obtain a more precise experimental determination of B_e using the τ branching fraction to muon and the τ lifetime. We average:

- the B_e fit value Γ_5 ,
- the B_e determination from the $B_\mu = B(\tau \rightarrow \mu\nu\bar{\nu})$ fit value Γ_3 assuming that $g_\mu/g_e = 1$ hence (see also Section 9.3) $B_e = B_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2)$,
- the B_e determination from the τ lifetime assuming that $g_\tau/g_\mu = 1$ hence $B_e = B(\mu \rightarrow e\bar{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2) \cdot (\delta_\gamma^\tau \delta_W^\tau)/(\delta_\gamma^\mu \delta_W^\mu)$ where $B(\mu \rightarrow e\bar{\nu}_e\nu_\mu) = 1$.

Accounting for statistical correlations, we obtain

$$B_e^{\text{uni}} = (17.814 \pm 0.023)\%.$$

The recent Belle τ lifetime measurement has brought a significant improvement. We use B_e^{uni} to obtain the ratio

$$R_{\text{had}} = \frac{\Gamma(\tau \rightarrow \text{hadrons})}{\Gamma(\tau \rightarrow e\nu\bar{\nu})} = \frac{\Gamma_{\text{hadrons}}}{B_e^{\text{uni}}} = 3.6315 \pm 0.0081,$$

where $\Gamma(\tau \rightarrow \text{hadrons})$ and $\Gamma(\tau \rightarrow e\nu\bar{\nu})$ indicate the partial widths and Γ_{hadrons} is the total branching fraction of the τ to hadrons, or the total branching fraction in any measured final state minus the leptonic branching fractions, *i.e.* with our notation $\Gamma_{\text{hadrons}} = \Gamma_{\text{All}} - \Gamma_3 - \Gamma_5 = (64.69 \pm 0.10)\%$ (see Section 9.1 and Table 266 for the definitions of Γ_{All} , Γ_3 , Γ_5). We underline that this report’s definition of Γ_{hadrons} corresponds to summing all τ hadronic decay modes, like in the previous report, rather than – as done elsewhere – subtracting the leptonic branching fractions from unity, *i.e.* $\Gamma_{\text{hadrons}} = 1 - \Gamma_3 - \Gamma_5$.

9.5 $|V_{us}|$ measurement

The CKM matrix element $|V_{us}|$ is most precisely determined from kaon decays [1044], and its precision is limited by the uncertainties of the lattice QCD estimates of $f_+^{K\pi}(0)$ and f_K/f_π . Using the τ branching fractions, it is possible to determine $|V_{us}|$ in an alternative way [1045] that does not depend on lattice QCD and has small theory uncertainties (see Section 9.5.1). Moreover, $|V_{us}|$ can be determined using the τ branching fractions similarly to the kaon case, using the same lattice QCD estimates, in order to check the overall experimental consistency.

We have updated the CKM coefficient $|V_{us}|$ determinations that we did in the previous report using the updated data from HFAG 2014 and PDG 2013.

9.5.1 Inclusive τ partial width to strange

The τ hadronic partial width is the sum of the τ partial widths to strange and to non-strange hadronic final states, $\Gamma_{\text{had}} = \Gamma_s + \Gamma_{\text{VA}}$. Dividing any partial width Γ_x by the electronic partial width, Γ_e , we obtain partial width ratios R_x (which are equal to the respective branching fraction ratios B_x/B_e) for which $R_{\text{had}} = R_s + R_{\text{VA}}$. In terms of such ratios, $|V_{us}|$ is measured as [1045]

$$|V_{us}|_{\tau s} = \sqrt{R_s / \left[\frac{R_{\text{VA}}}{|V_{ud}|^2} - \delta R_{\text{theory}} \right]},$$

where δR_{theory} can be determined in the context of low energy QCD theory, partly relying on experimental low energy scattering data. The literature reports several calculations [1045–1047]. In this report we use Ref. [1045], whose estimated uncertainty size is in between the two other ones. We use the information in that paper and the PDG 2013 value for the s -quark mass $m_s = 93.50 \pm 2.50$ MeV [18] to calculate $\delta R_{\text{theory}} = 0.239 \pm 0.030$.

We proceed following the same procedure of the 2012 HFAG report [4], using the universality improved $B_e^{\text{uni}} = (17.814 \pm 0.023)\%$ (see Section 9.4) to compute the R_x ratios, and using the sum of the τ branching fractions to strange and non-strange hadronic final states to compute R_s and R_{VA} , respectively.

Using the τ branching fraction fit results with their uncertainties and correlations (Section 9.1), we compute $B_s = (2.882 \pm 0.047)\%$ (see also Table 278) and $B_{\text{VA}} = B_{\text{hadrons}} - B_s = (61.81 \pm 0.10)\%$, where $B_{\text{hadrons}} = \Gamma_{\text{hadrons}}$ defined in section 9.4. PDG 2013 averages are used for non- τ quantities, including $|V_{ud}| = 0.97425 \pm 0.00022$, which comes from Ref. [1048] like for the previous HFAG report.

We obtain $|V_{us}|_{\tau s} = 0.2176 \pm 0.0021$, which is 3.4σ lower than the unitarity CKM prediction $|V_{us}|_{\text{uni}} = 0.22547 \pm 0.00095$, from $(|V_{us}|_{\text{uni}})^2 = 1 - |V_{ud}|^2$. The $|V_{us}|_{\tau s}$ uncertainty includes a systematic error contribution of 0.44% from the theory uncertainty on δR_{theory} . There is no significant change with respect to the previous HFAG report.

Kim Maltman has computed an alternative theoretical estimate based on Fixed Order Perturbation Theory and experimental inputs restricted to τ quantities [1049]. The result is $\delta R_{\text{theory}} = 0.254 \pm 0.038$. The uncertainty includes an additional contribution to account for the differences between using Fixed Order Perturbation Theory and Contour Improved Perturbation Theory. With this alternative value, we would obtain $|V_{us}|_{\tau s} = 0.2181 \pm 0.0022$, which would be 3.1σ lower than the unitarity CKM prediction.

9.5.2 $|V_{us}|$ from $B(\tau \rightarrow K\nu)/B(\tau \rightarrow \pi\nu)$ and from $B(\tau \rightarrow K\nu)$

We follow the same procedure of the HFAG 2012 report to compute $|V_{us}|$ from the ratio of branching fractions $B(\tau^- \rightarrow K^- \nu_\tau)/B(\tau^- \rightarrow \pi^- \nu_\tau) = (3.903 \pm 0.054) \cdot 10^{-2}$ from the equation

$$\frac{B(\tau^- \rightarrow K^- \nu_\tau)}{B(\tau^- \rightarrow \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2 (1 - m_K^2/m_\tau^2)^2 r_{\text{LD}}(\tau^- \rightarrow K^- \nu_\tau)}{f_\pi^2 |V_{ud}|^2 (1 - m_\pi^2/m_\tau^2)^2 r_{\text{LD}}(\tau^- \rightarrow \pi^- \nu_\tau)}.$$

We use $f_K/f_\pi = 1.1920 \pm 0.0050$ from the FLAG 2013 Lattice averages with $N_f = 2 + 1$ [1050]. We compute $|V_{us}|_{\tau K/\pi} = 0.2232 \pm 0.0019$, 1.0σ below the CKM unitarity prediction.

Table 278: HFAG Summer 2014 τ branching fractions to strange final states.

Branching fraction	HFAG Summer 2014 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4331 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0220) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0216) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8378 \pm 0.0123) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.3680 \pm 0.0103) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	$(0.0124 \pm 0.0204) \cdot 10^{-2}$
$\Gamma_{53} = \bar{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0155 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$	$(0.0093 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \rightarrow KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2922 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0410 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8817 \pm 0.0470) \cdot 10^{-2}$

We proceed like in 2012 also to determine $|V_{us}|$ from the branching fraction $B(\tau^- \rightarrow K^- \nu_\tau)$ using

$$B(\tau^- \rightarrow K^- \nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}.$$

We use $f_K = 156.3 \pm 0.9$ MeV from FLAG 2013 with $N_f = 2 + 1$ [1050]. We obtain $|V_{us}|_{\tau K} = 0.2212 \pm 0.0020$, which is 1.9σ below the CKM unitarity prediction. CODATA 2010 results [1051] and PDG 2013 have been used for the physics constants.

9.5.3 $|V_{us}|$ from τ summary

We summarize the $|V_{us}|$ results reporting the values, the discrepancy with respect to the $|V_{us}|$ determination from CKM unitarity, and an illustration of the measurement method:

$$\begin{aligned}
 |V_{us}|_{\text{uni}} &= 0.22547 \pm 0.00095 && \text{from } \sqrt{1 - |V_{ud}|^2} \quad (\text{CKM unitarity}), \\
 |V_{us}|_{\tau s} &= 0.2176 \pm 0.0021 && - 3.4\sigma \quad \text{from } \Gamma(\tau^- \rightarrow X_s^- \nu_\tau), \\
 |V_{us}|_{\tau K/\pi} &= 0.2232 \pm 0.0019 && - 1.0\sigma \quad \text{from } \Gamma(\tau^- \rightarrow K^- \nu_\tau) / \Gamma(\tau^- \rightarrow \pi^- \nu_\tau), \\
 |V_{us}|_{\tau K} &= 0.2212 \pm 0.0020 && - 1.9\sigma \quad \text{from } \Gamma(\tau^- \rightarrow K^- \nu_\tau).
 \end{aligned}$$

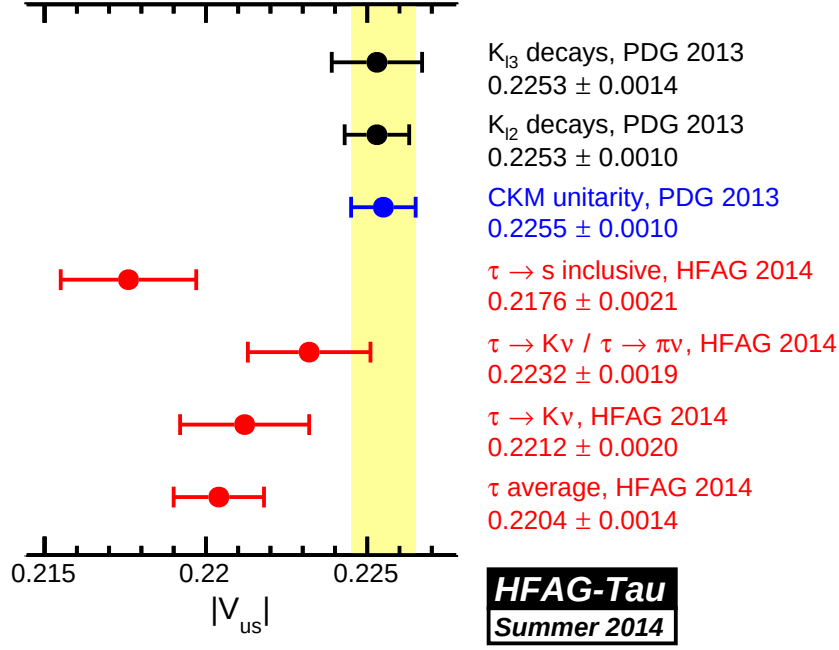


Figure 182: $|V_{us}|$ averages of this document compared with the FlaviaNet results [1044].

Averaging the three above $|V_{us}|$ determinations (taking into account all correlations due to the usage of the fitted τ branching fractions and the other mentioned inputs) we obtain:

$$|V_{us}|_{\tau} = 0.2204 \pm 0.0014 \quad -2.9\sigma \quad \text{average of 3 } |V_{us}| \text{ } \tau \text{ measurements.}$$

We could not find a published estimate of the correlation of the uncertainties on f_K and f_K/f_{π} , but even if we assume $\pm 100\%$ correlation, the uncertainty on $|V_{us}|_{\tau}$ does not change more than about $\pm 5\%$. Figure 182 summarizes the $|V_{us}|$ results.

9.6 Upper limits on τ LFV branching fractions

We report in Table 279 and Figure 183 the up-to-date upper limits on the τ LFV branching fractions in order to track and summarize the experimental results in this area, which are sensitive to physics beyond the Standard Model.

Table 279: Experimental upper limits on lepton flavor violating τ decays. The modes are grouped according to the particle content of their final states. Modes with lepton number violation are labeled with “(L)”, modes with baryon number violation are labeled with “(BNV)”.

Decay mode	Category	90% CL Limit	Exp.	Ref.
$\Gamma_{156} = e^- \gamma$	$l\gamma$	$< 12.0 \cdot 10^{-8}$	Belle	[1052]
		$< 3.3 \cdot 10^{-8}$	BABAR	[1053]
$\Gamma_{157} = \mu^- \gamma$		$< 4.5 \cdot 10^{-8}$	Belle	[1052]
		$< 4.4 \cdot 10^{-8}$	BABAR	[1053]
$\Gamma_{158} = e^- \pi^0$	lP^0	$< 2.2 \cdot 10^{-8}$	Belle	[1054]
		$< 13.0 \cdot 10^{-8}$	BABAR	[1055]
$\Gamma_{159} = \mu^- \pi^0$		$< 2.7 \cdot 10^{-8}$	Belle	[1054]
		$< 11.0 \cdot 10^{-8}$	BABAR	[1055]
$\Gamma_{162} = e^- \eta$		$< 4.4 \cdot 10^{-8}$	Belle	[1054]
		$< 16.0 \cdot 10^{-8}$	BABAR	[1055]
$\Gamma_{163} = \mu^- \eta$		$< 2.3 \cdot 10^{-8}$	Belle	[1054]
		$< 15.0 \cdot 10^{-8}$	BABAR	[1055]
$\Gamma_{172} = e^- \eta'(958)$		$< 3.6 \cdot 10^{-8}$	Belle	[1054]
		$< 24.0 \cdot 10^{-8}$	BABAR	[1055]
$\Gamma_{173} = \mu^- \eta'(958)$		$< 3.8 \cdot 10^{-8}$	Belle	[1054]
		$< 14.0 \cdot 10^{-8}$	BABAR	[1055]
$\Gamma_{160} = e^- K_S^0$		$< 2.6 \cdot 10^{-8}$	Belle	[1056]
		$< 3.3 \cdot 10^{-8}$	BABAR	[1057]
$\Gamma_{161} = \mu^- K_S^0$		$< 2.3 \cdot 10^{-8}$	Belle	[1056]
		$< 4.0 \cdot 10^{-8}$	BABAR	[1057]
$\Gamma_{174} = e^- f_0(980)$	lS^0	$< 3.2 \cdot 10^{-8}$	Belle	[1058]
$\Gamma_{175} = \mu^- f_0(980)$		$< 3.4 \cdot 10^{-8}$	Belle	[1058]
$\Gamma_{164} = e^- \rho^0$	lV^0	$< 1.8 \cdot 10^{-8}$	Belle	[1059]
		$< 4.6 \cdot 10^{-8}$	BABAR	[1060]
$\Gamma_{165} = \mu^- \rho^0$		$< 1.2 \cdot 10^{-8}$	Belle	[1059]
		$< 2.6 \cdot 10^{-8}$	BABAR	[1060]
$\Gamma_{168} = e^- K^*(892)^0$		$< 3.2 \cdot 10^{-8}$	Belle	[1059]
		$< 5.9 \cdot 10^{-8}$	BABAR	[1060]
$\Gamma_{169} = \mu^- K^*(892)^0$		$< 7.2 \cdot 10^{-8}$	Belle	[1059]
		$< 17.0 \cdot 10^{-8}$	BABAR	[1060]
$\Gamma_{170} = e^- \bar{K}^*(892)^0$		$< 3.4 \cdot 10^{-8}$	Belle	[1059]
		$< 4.6 \cdot 10^{-8}$	BABAR	[1060]
$\Gamma_{171} = \mu^- \bar{K}^*(892)^0$		$< 7.0 \cdot 10^{-8}$	Belle	[1059]
		$< 7.3 \cdot 10^{-8}$	BABAR	[1060]
$\Gamma_{176} = e^- \phi$		$< 3.1 \cdot 10^{-8}$	Belle	[1059]

Table 279 – continued from previous page

Decay mode	Category	90% CL Limit	Exp.	Ref.
$\Gamma_{177} = \mu^- \phi$		$< 3.1 \cdot 10^{-8}$	<i>BABAR</i>	[1060]
		$< 8.4 \cdot 10^{-8}$	Belle	[1059]
		$< 19.0 \cdot 10^{-8}$	<i>BABAR</i>	[1060]
$\Gamma_{166} = e^- \omega$		$< 4.8 \cdot 10^{-8}$	Belle	[1059]
		$< 11.0 \cdot 10^{-8}$	<i>BABAR</i>	[1061]
		$< 4.7 \cdot 10^{-8}$	Belle	[1059]
$\Gamma_{167} = \mu^- \omega$		$< 10.0 \cdot 10^{-8}$	<i>BABAR</i>	[1061]
$\Gamma_{178} = e^- e^+ e^-$	<i>lll</i>	$< 2.7 \cdot 10^{-8}$	Belle	[1062]
$< 2.9 \cdot 10^{-8}$		<i>BABAR</i>	[1063]	
$\Gamma_{181} = \mu^- e^+ e^-$		$< 1.8 \cdot 10^{-8}$	Belle	[1062]
$\Gamma_{179} = e^- \mu^+ \mu^-$		$< 2.2 \cdot 10^{-8}$	<i>BABAR</i>	[1063]
		$< 2.7 \cdot 10^{-8}$	Belle	[1062]
		$< 3.2 \cdot 10^{-8}$	<i>BABAR</i>	[1063]
$\Gamma_{183} = \mu^- \mu^+ \mu^-$		$< 2.1 \cdot 10^{-8}$	Belle	[1062]
		$< 3.3 \cdot 10^{-8}$	<i>BABAR</i>	[1063]
		$< 4.6 \cdot 10^{-8}$	LHCb	[1064]
$\Gamma_{182} = e^- \mu^+ e^-$		$< 1.5 \cdot 10^{-8}$	Belle	[1062]
		$< 1.8 \cdot 10^{-8}$	<i>BABAR</i>	[1063]
		$\Gamma_{180} = \mu^- e^+ \mu^-$	$< 1.7 \cdot 10^{-8}$	Belle
		$< 2.6 \cdot 10^{-8}$	<i>BABAR</i>	[1063]
$\Gamma_{184} = e^- \pi^+ \pi^-$	<i>lhh</i>	$< 2.3 \cdot 10^{-8}$	Belle	[1065]
		$< 12.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
		$\Gamma_{186} = \mu^- \pi^+ \pi^-$	$< 2.1 \cdot 10^{-8}$	Belle
$\Gamma_{188} = e^- \pi^+ K^-$		$< 29.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
		$< 3.7 \cdot 10^{-8}$	Belle	[1065]
		$< 32.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
$\Gamma_{194} = \mu^- \pi^+ K^-$		$< 8.6 \cdot 10^{-8}$	Belle	[1065]
		$< 26.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
		$\Gamma_{189} = e^- K^+ \pi^-$	$< 3.1 \cdot 10^{-8}$	Belle
$\Gamma_{195} = \mu^- K^+ \pi^-$		$< 17.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
		$< 4.5 \cdot 10^{-8}$	Belle	[1065]
		$< 32.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
$\Gamma_{192} = e^- K^+ K^-$		$< 3.4 \cdot 10^{-8}$	Belle	[1065]
		$< 14.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
		$\Gamma_{198} = \mu^- K^+ K^-$	$< 4.4 \cdot 10^{-8}$	Belle
		$< 25.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
$\Gamma_{191} = e^- K_S^0 K_S^0$		$< 7.1 \cdot 10^{-8}$	Belle	[1056]
$\Gamma_{197} = \mu^- K_S^0 K_S^0$		$< 8.0 \cdot 10^{-8}$	Belle	[1056]
$\Gamma_{185} = e^+ \pi^- \pi^-$		(L)	$< 2.0 \cdot 10^{-8}$	Belle
	(L)	$< 27.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
$\Gamma_{187} = \mu^+ \pi^- \pi^-$	(L)	$< 3.9 \cdot 10^{-8}$	Belle	[1065]

Table 279 – continued from previous page

Decay mode	Category	90% CL Limit	Exp.	Ref.
$\Gamma_{190} = e^+ \pi^- K^-$	(L)	$< 7.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
	(L)	$< 3.2 \cdot 10^{-8}$	Belle	[1065]
$\Gamma_{196} = \mu^+ \pi^- K^-$	(L)	$< 18.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
	(L)	$< 4.8 \cdot 10^{-8}$	Belle	[1065]
$\Gamma_{193} = e^+ K^- K^-$	(L)	$< 22.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
	(L)	$< 3.3 \cdot 10^{-8}$	Belle	[1065]
$\Gamma_{199} = \mu^+ K^- K^-$	(L)	$< 15.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
	(L)	$< 4.7 \cdot 10^{-8}$	Belle	[1065]
	(L)	$< 48.0 \cdot 10^{-8}$	<i>BABAR</i>	[1066]
$\Gamma_{211} = \pi^- \Lambda$	BNV	$< 3.0 \cdot 10^{-8}$	Belle	[1067]
		$< 5.8 \cdot 10^{-8}$	<i>BABAR</i>	[1068]
$\Gamma_{212} = \pi^- \bar{\Lambda}$		$< 2.8 \cdot 10^{-8}$	Belle	[1067]
		$< 5.9 \cdot 10^{-8}$	<i>BABAR</i>	[1068]
$\Gamma_{213} = K^- \Lambda$		$< 4.2 \cdot 10^{-8}$	Belle	[1067]
		$< 15. \cdot 10^{-8}$	<i>BABAR</i>	[1068]
$\Gamma_{214} = K^- \bar{\Lambda}$		$< 3.1 \cdot 10^{-8}$	Belle	[1067]
		$< 7.2 \cdot 10^{-8}$	<i>BABAR</i>	[1068]
$\Gamma_{215} = p \mu^- \mu^-$		$< 44.0 \cdot 10^{-8}$	LHCb	[1069]
$\Gamma_{216} = \bar{p} \mu^+ \mu^-$		$< 33.0 \cdot 10^{-8}$	LHCb	[1069]

9.7 Combination of upper limits on τ LFV branching fractions

Combining upper limits is a delicate issue, since there is no standard and generally agreed procedure. Furthermore, the τ LFV searches published limits are extracted from the data with a variety of methods, and cannot be directly combined with a uniform procedure. It is however possible to use a single and effective upper limits combination procedure for all modes by re-computing the published upper limits with just one extraction method, using the published information that documents the upper limit determination: number of observed candidates, expected background, signal efficiency and number of analyzed τ decays.

We chose to use the CL_s method [1070] to re-compute the τ LFV upper limits, since it is well known and widely used (see the Statistics review of PDG 2013 [18]), and since the limits computed with the CL_s method can be combined in a straightforward way (see below). The CL_s method is based on two hypotheses: signal plus background and background only. We calculate the observed confidence levels for the two hypotheses:

$$\text{CL}_{s+b} = P_{s+b}(Q \leq Q_{obs}) = \int_{-\infty}^{Q_{obs}} \frac{dP_{s+b}}{dQ} dQ, \quad (253)$$

$$\text{CL}_b = P_b(Q \leq Q_{obs}) = \int_{-\infty}^{Q_{obs}} \frac{dP_b}{dQ} dQ, \quad (254)$$

where CL_{s+b} is the confidence level observed for the signal plus background hypothesis, CL_b is the confidence level observed for the background only hypothesis, $\frac{dP_{s+b}}{dQ}$ and $\frac{dP_b}{dQ}$ are the probability distribution functions (PDFs) for the two corresponding hypothesis and Q is called the test statistics. The CL_s value is defined as the ratio between the confidence level for the signal plus background hypothesis to the confidence level for the background hypothesis:

$$\text{CL}_s = \frac{\text{CL}_{s+b}}{\text{CL}_b}. \quad (255)$$

When multiple results are combined, the PDFs in Equations 253 and 254 are the product of the individual PDFs,

$$\text{CL}_s = \frac{\prod_{i=1}^{N_{\text{chan}}} \sum_{n=0}^{n_i} \frac{e^{-(s_i+b_i)} (s_i + b_i)^n}{n!} \prod_{j=1}^n s_i S_i(x_{ij}) + b_i B_i(x_{ij})}{\prod_{i=1}^{N_{\text{chan}}} \sum_{n=0}^{n_i} \frac{e^{-b_i} b_i^n}{n!} \prod_{j=1}^{n_i} B_i(x_{ij})}, \quad (256)$$

where N_{chan} is the number of results (or channels), and, for each channel i , n_i is the number of observed candidates, x_{ij} are the values of the discriminating variables (with index j), s_i and b_i are the number of signal and background events and S_i , B_i are the probability distribution functions of the discriminating variables. The expected signal s_i is related to the τ lepton branching fraction $B(\tau \rightarrow f_i)$ into the searched final state f_i by $s_i = N_i \epsilon_i B(\tau \rightarrow f_i)$, where N_i is the number of produced τ leptons and ϵ_i is the detection efficiency for observing the decay $\tau \rightarrow f_i$. For e^+e^- experiments, $N_i = 2\mathcal{L}_i \sigma_{\tau\tau}$, where $\mathcal{L}_i$ is the integrated luminosity and $\sigma_{\tau\tau}$ is the τ pair production cross section $\sigma(e^+e^- \rightarrow \tau^+\tau^-)$ [1071]. In experiments where τ leptons are produced in more complex multiple reactions, the effective N_i is typically estimated with Monte Carlo simulations calibrated with related data yields.

The extraction of the upper limits is performed using the code provided by Tom Junk [1072]. The systematic uncertainties are modeled in the Monte Carlo toy experiments by convolving the S_i and B_i PDFs with Gaussian distributions corresponding to the nuisance parameters.

Table 280 reports the re-computed limits for the B factories results as well as the corresponding HFAG combination. Since there is negligible gain in combining limits of very different strength, the combinations do not include the CLEO searches and we do not combine results for modes where the best limit is more than an order of magnitude better than the other limits. Figure 184 reports the re-computed τ LFV searches upper limits and their combination.

Table 280: Combinations of upper limits on lepton flavor violating τ decay modes. The table includes, for each experimental result, the published information that has been used to re-compute the limit with the CL_s method, *i.e.* the integrated luminosity, the cross-section for τ lepton pairs production, the detection efficiency, the number of expected background events and the number of observed events. Since the LHCb collaboration published a limit determined with the CL_s method, in this case only the published limit is reported. The table finally reports the combined limit that is obtained by combining the re-computed limits. The modes are grouped according to the particle content of their final states. Modes with lepton number violation are labeled with “(L)”, modes with baryon number violation are labeled with “(BNV)”.

Decay mode	Cat.	90% CL Limit	Exp.	L (fb ⁻¹)	$\sigma_{\tau\tau}$ (nb)	efficiency (%)	N_{bkg}	N_{obs}
$\Gamma_{156} = e^- \gamma$	$l\gamma$	$< 5.4 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	535	0.919	3.00 ± 0.10	5.14 ± 3.30	5
$\Gamma_{157} = \mu^- \gamma$		$< 5.0 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	535 524	0.919 0.919	5.07 ± 0.20 6.10 ± 0.50	13.90 ± 5.00 3.60 ± 0.70	10 2
$\Gamma_{160} = e^- K_S^0$	lP^0	$< 1.4 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	671 469	0.919 0.919	10.20 ± 0.67 9.10 ± 1.73	0.18 ± 0.18 0.59 ± 0.25	0 1
$\Gamma_{161} = \mu^- K_S^0$		$< 1.5 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	671 469	0.919 0.919	10.70 ± 0.73 6.14 ± 0.20	0.35 ± 0.21 0.30 ± 0.18	0 1
$\Gamma_{164} = e^- \rho^0$	lV^0	$< 1.5 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	854 451	0.919 0.919	7.58 ± 0.41 7.31 ± 0.20	0.29 ± 0.15 1.32 ± 0.17	0 1
$\Gamma_{165} = \mu^- \rho^0$		$< 1.5 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	854 451	0.919 0.919	7.09 ± 0.37 4.52 ± 0.40	1.48 ± 0.35 2.04 ± 0.19	0 0
$\Gamma_{168} = e^- K^*(892)^0$		$< 2.3 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	854 451	0.919 0.919	4.37 ± 0.24 8.00 ± 0.20	0.29 ± 0.14 1.65 ± 0.23	0 2
$\Gamma_{169} = \mu^- K^*(892)^0$		$< 6.0 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	854 451	0.919 0.919	3.39 ± 0.19 4.60 ± 0.40	0.53 ± 0.20 1.79 ± 0.21	1 4
$\Gamma_{170} = e^- \bar{K}^*(892)^0$		$< 2.2 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	854 451	0.919 0.919	4.41 ± 0.25 7.80 ± 0.20	0.08 ± 0.08 2.76 ± 0.28	0 2
$\Gamma_{171} = \mu^- \bar{K}^*(892)^0$		$< 4.2 \cdot 10^{-8}$	HFAG Belle <i>BABAR</i>	854 451	0.919 0.919	3.60 ± 0.20 4.10 ± 0.30	0.45 ± 0.17 1.72 ± 0.17	1 1
$\Gamma_{176} = e^- \phi$		$< 2.0 \cdot 10^{-8}$	HFAG Belle	854	0.919	4.18 ± 0.25	0.47 ± 0.19	0

Table 280 – continued from previous page

Decay mode	Cat.	90% CL Limit	Exp.	L (fb ⁻¹)	$\sigma_{\tau\tau}$ (nb)	efficiency (%)	N_{bkg}	N_{obs}
$\Gamma_{177} = \mu^- \phi$		$< 6.8 \cdot 10^{-8}$	<i>BABAR</i>	451	0.919	6.40 ± 0.20	0.68 ± 0.12	0
			HFAG					
$\Gamma_{166} = e^- \omega$		$< 3.3 \cdot 10^{-8}$	Belle	854	0.919	3.21 ± 0.19	0.06 ± 0.06	1
			<i>BABAR</i>	451	0.919	5.20 ± 0.30	2.76 ± 0.16	6
$\Gamma_{167} = \mu^- \omega$		$< 4.0 \cdot 10^{-8}$	HFAG					
			Belle	854	0.919	2.92 ± 0.18	0.30 ± 0.14	0
			<i>BABAR</i>	451	0.919	2.96 ± 0.13	0.35 ± 0.06	0
			HFAG					
			Belle	854	0.919	2.38 ± 0.14	0.72 ± 0.18	0
			<i>BABAR</i>	451	0.919	2.56 ± 0.16	0.73 ± 0.03	0
$\Gamma_{178} = e^- e^+ e^-$	<i>lll</i>	$< 1.4 \cdot 10^{-8}$	HFAG					
$\Gamma_{181} = \mu^- e^+ e^-$		$< 1.1 \cdot 10^{-8}$	Belle	782	0.919	6.00 ± 0.59	0.21 ± 0.15	0
			<i>BABAR</i>	472	0.919	8.60 ± 0.20	0.12 ± 0.02	0
$\Gamma_{179} = e^- \mu^+ \mu^-$		$< 1.6 \cdot 10^{-8}$	HFAG					
			Belle	782	0.919	9.30 ± 0.73	0.04 ± 0.04	0
$\Gamma_{183} = \mu^- \mu^+ \mu^-$		$< 1.2 \cdot 10^{-8}$	<i>BABAR</i>	472	0.919	8.80 ± 0.50	0.64 ± 0.19	0
			HFAG					
$\Gamma_{182} = e^- \mu^+ e^-$		$< 4.6 \cdot 10^{-8}$ $< 8.4 \cdot 10^{-9}$	Belle	782	0.919	6.10 ± 0.58	0.10 ± 0.04	0
			<i>BABAR</i>	472	0.919	6.40 ± 0.40	0.54 ± 0.14	0
$\Gamma_{180} = \mu^- e^+ \mu^-$		$< 9.8 \cdot 10^{-9}$	HFAG					
			Belle	782	0.919	7.60 ± 0.56	0.13 ± 0.20	0
			<i>BABAR</i>	472	0.919	6.60 ± 0.60	0.44 ± 0.17	0
			HFAG					
			Belle	782	0.919	11.50 ± 0.89	0.01 ± 0.01	0
			<i>BABAR</i>	472	0.919	12.70 ± 0.70	0.34 ± 0.12	0
$\Gamma_{211} = \pi^- \Lambda$	BNV	$< 1.9 \cdot 10^{-8}$	HFAG					
			Belle	906	0.919	4.39 ± 0.36	0.31 ± 0.18	0
$\Gamma_{212} = \pi^- \bar{\Lambda}$		$< 1.8 \cdot 10^{-9}$	<i>BABAR</i>	237	0.919	12.20 ± 8.50	0.56 ± 0.56	0
			HFAG					
$\Gamma_{213} = K^- \Lambda$		$< 3.7 \cdot 10^{-9}$	Belle	906	0.919	4.80 ± 0.39	0.21 ± 0.15	0
			<i>BABAR</i>	237	0.919	12.28 ± 8.50	0.42 ± 0.42	0
$\Gamma_{214} = K^- \bar{\Lambda}$		$< 2.0 \cdot 10^{-9}$	HFAG					
			Belle	906	0.919	3.16 ± 0.27	0.42 ± 0.19	0
			<i>BABAR</i>	237	0.919	9.47 ± 0.66	0.12 ± 0.12	1
			HFAG					
			Belle	906	0.919	4.11 ± 0.35	0.31 ± 0.14	0
			<i>BABAR</i>	237	0.919	10.63 ± 0.74	0.26 ± 0.26	0

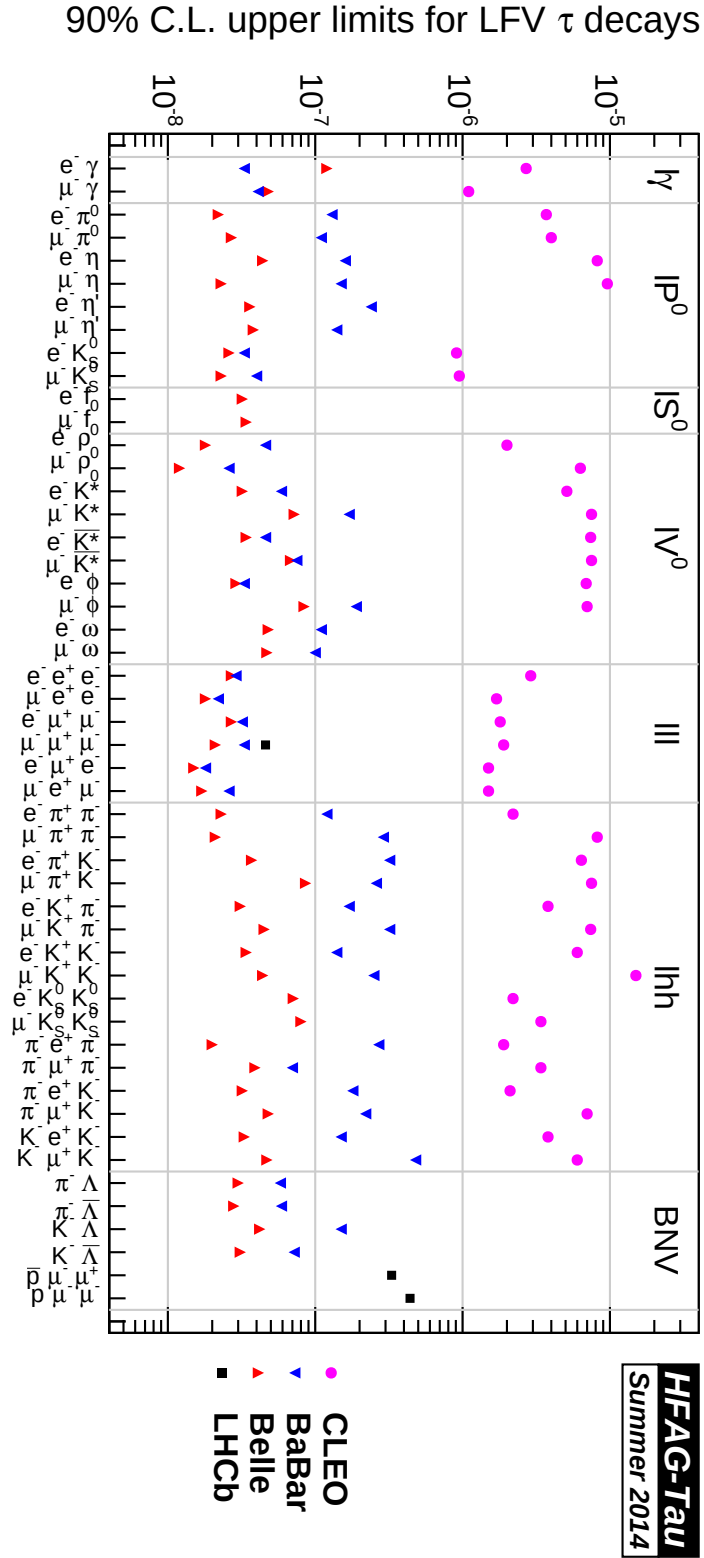


Figure 183: Tau lepton-flavor-violating branching fraction upper limits summary plot.

90% C.L. upper limits for LFV τ decays

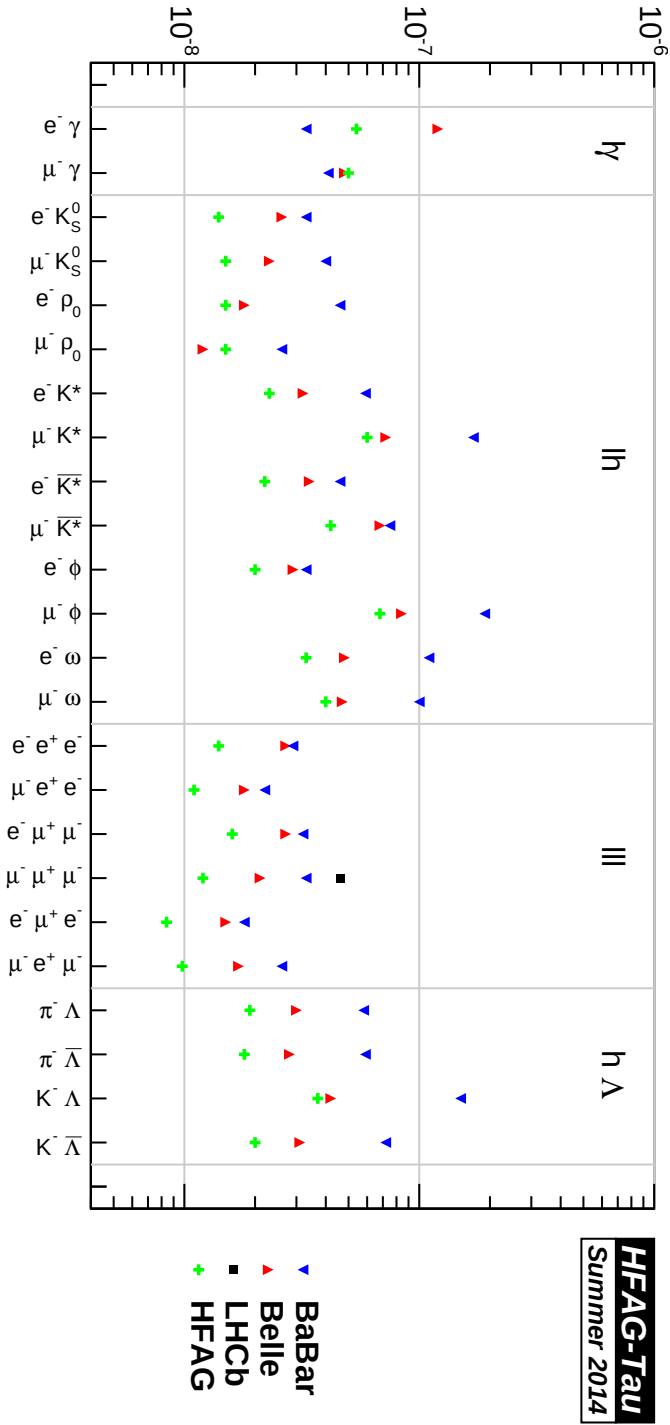


Figure 184: Tau lepton-flavour-violating branching fraction upper limits combinations summary plot. For each channel we report the HFAG combined limit, and the experimental published limits. In some cases, the combined limit is weaker than the limit published by a single experiment. This arises since the CL_s method used in the combination can be more conservative compared to other legitimate methods, especially when the number of observed events fluctuates below the expected background.

10 Summary

This article provides updated world averages for b -hadron properties using results available through summer 2014. A small selection of highlights of the results described in Sections 3-9 is given in Table 281.

Table 281: Selected world averages.

b-hadron fractions f^{+-}/f^{00} in $\Upsilon(4S)$ decays f_s in $\Upsilon(5S)$ decays f_s, f_{baryon} in Z decays f_s, f_{baryon} at Tevatron	1.059 ± 0.027 $0.200^{+0.030}_{-0.031}$ $0.100 \pm 0.008, 0.080 \pm 0.010$ $0.100 \pm 0.010, 0.199 \pm 0.044$
b-hadron lifetimes $\tau(B^0)$ $\tau(B^+)$ $\bar{\tau}(B_s^0) = 1/\Gamma_s$ $\tau(B_{sL})$ $\tau(B_{sH})$ $\tau(B_c^+)$ $\tau(\Lambda_b^0)$ $\tau(\Xi_b^-)$ $\tau(\Xi_b^0)$ $\tau(\Omega_b^-)$	1.520 ± 0.004 ps 1.638 ± 0.004 ps 1.509 ± 0.004 ps 1.422 ± 0.006 ps 1.607 ± 0.010 ps 0.507 ± 0.009 ps 1.467 ± 0.010 ps 1.559 ± 0.037 ps 1.465 ± 0.031 ps $1.57^{+0.23}_{-0.20}$ ps
B^0 and B_s^0 mixing / CP violation parameters Δm_d $\Delta\Gamma_d/\Gamma_d$ $ q/p _d$ Δm_s $\Delta\Gamma_s$ $ q/p _s$ $\phi_s^{c\bar{c}s}$	0.510 ± 0.003 ps $^{-1}$ 0.001 ± 0.010 1.0009 ± 0.0013 17.757 ± 0.021 ps $^{-1}$ $+0.081 \pm 0.006$ ps $^{-1}$ 1.0038 ± 0.0021 -0.015 ± 0.035
Parameters related to Unitarity Triangle angles $\sin 2\beta \equiv \sin 2\phi_1$ $\beta \equiv \phi_1$ $-\eta S_{\phi K_S^0}$ $-\eta S_{\eta' K^0}$ $-\eta S_{K_S^0 K_S^0 K_S^0}$ $-\eta S_{K^+ K^- K_S^0}$ $\phi_s(\phi\phi)$ $-\eta S_{J/\psi \pi^0}$ $S_{K^* \gamma}$ $S_{\pi^+ \pi^-}$ $C_{\pi^+ \pi^-}$ $S_{\rho^+ \rho^-}$ $a(D^{*\pm} \pi^\mp)$	0.682 ± 0.019 $(21.5^{+0.8}_{-0.7})^\circ$ $0.74^{+0.11}_{-0.13}$ 0.63 ± 0.06 0.72 ± 0.19 $0.68^{+0.09}_{-0.10}$ $-0.17 \pm 0.15 \pm 0.03$ rad 0.93 ± 0.15 -0.16 ± 0.22 -0.66 ± 0.06 -0.31 ± 0.05 -0.05 ± 0.17 -0.039 ± 0.010

Selected world averages – continued from previous page.

$A_{CP}(B \rightarrow D_{CP+} K)$	0.19 ± 0.03
$A_{\text{ADS}}(B \rightarrow D_{K\pi} K)$	-0.54 ± 0.12
$R_{\text{ADS}}(B \rightarrow D_{K\pi} K)$	0.0153 ± 0.0017
Semileptonic B decay parameters	
$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu})$	$(4.93 \pm 0.11)\%$
$\mathcal{B}(B^- \rightarrow D^{*0} \ell^- \bar{\nu})$	$(5.69 \pm 0.19)\%$
$\eta_{\text{EW}} \mathcal{F}(1) V_{cb} $	$(35.81 \pm 0.45) \times 10^{-3}$
$ V_{cb} $ from $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$	$(38.94 \pm 0.49_{\text{exp}} \pm 0.58_{\text{th}}) \times 10^{-3}$
$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu})$	$(2.19 \pm 0.12)\%$
$\mathcal{B}(B^- \rightarrow D^0 \ell^- \bar{\nu})$	$(2.27 \pm 0.11)\%$
$\eta_{\text{EW}} \mathcal{G}(1) V_{cb} $	$(42.65 \pm 1.53) \times 10^{-3}$
$ V_{cb} $ from $\bar{B} \rightarrow D \ell^- \bar{\nu}_\ell$	$(39.45 \pm 1.42_{\text{exp}} \pm 0.88_{\text{th}}) \times 10^{-3}$
$\mathcal{B}(\bar{B} \rightarrow X_c \ell^- \bar{\nu}_\ell)$	$(10.65 \pm 0.16)\%$
$\mathcal{B}(\bar{B} \rightarrow X \ell^- \bar{\nu}_\ell)$	$(10.86 \pm 0.16)\%$
$ V_{cb} $ from $\bar{B} \rightarrow X \ell^- \bar{\nu}_\ell$	$(42.46 \pm 0.88) \times 10^{-3}$
$\mathcal{B}(\bar{B} \rightarrow \pi \ell^- \bar{\nu})$	$(1.45 \pm 0.05) \times 10^{-4}$
$ V_{ub} $ from $\bar{B} \rightarrow \pi \ell^- \bar{\nu}$	$(3.28 \pm 0.29) \times 10^{-3}$
$ V_{ub} $ from $\bar{B} \rightarrow X_u \ell^- \bar{\nu}$	$(4.45 \pm 0.16_{\text{exp}} \pm 0.22_{\text{th}}) \times 10^{-3}$
Rare B decays	
$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$	$2.8^{+0.07}_{-0.06} \times 10^{-9}$
$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-)$	$0.39^{+0.16}_{-0.14} \times 10^{-9}$
Different observables in $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays in bins of $q^2 = m^2(\mu^+ \mu^-)$	See Sec. 7.5
Up-down asymmetry in $B^+ \rightarrow K^+ \pi^- \pi^- \gamma$ decays in bins of $m(K^+ \pi^- \pi^-)$	See Sec. 7.5
$\mathcal{B}(B \rightarrow X_s \gamma)$	$(3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$
$\mathcal{B}(B^+ \rightarrow \tau^+ \nu)$	$(1.14 \pm 0.22) \times 10^{-4}$
$A_{CP}(B^0 \rightarrow K^+ \pi^-)$	(-0.082 ± 0.006)
$A_{CP}(B^+ \rightarrow K^+ \pi^0)$	(0.040 ± 0.021)
$A_{CP}(B_s^0 \rightarrow K^- \pi^+)$	(0.26 ± 0.04)
D^0 mixing and CP violation parameters	
x	$(0.41^{+0.14}_{-0.15})\%$
y	$(0.63^{+0.07}_{-0.08})\%$
A_D	$(-0.71^{+0.92}_{-0.95})\%$
$ q/p $	$0.93^{+0.09}_{-0.08}$
ϕ	$(-8.7^{+8.7}_{-9.1})^\circ$
x_{12} (no direct CP violation)	$(0.43^{+0.14}_{-0.15})\%$
y_{12} (no direct CP violation)	$(0.60 \pm 0.07)\%$
ϕ_{12} (no direct CP violation)	$(0.9^{+1.9}_{-1.7})^\circ$
a_{CP}^{ind}	$(0.01 \pm 0.05)\%$
$\Delta a_{CP}^{\text{dir}}$	$(-0.25 \pm 0.10)\%$
Semileptonic/Leptonic D decays	
f_D	$(203.7 \pm 4.9) \text{ MeV}$
f_{D_s}	$(257.4 \pm 4.6) \text{ MeV}$

Selected world averages – continued from previous page.

$ V_{cd} $	0.219 ± 0.006
$ V_{cs} $	0.998 ± 0.020
τ parameters, lepton universality, and V_{us}	
g_μ/g_e	1.0018 ± 0.0014
g_τ/g_μ	1.0029 ± 0.0015
g_τ/g_e	1.0029 ± 0.0015
$\mathcal{B}_e^{\text{uni}}$	$17.814 \pm 0.023\%$
R_{had}	3.6315 ± 0.0081
$ V_{us} $ from inclusive sum of strange branching fractions	0.2176 ± 0.0021
$ V_{us} $ from $\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau)/\mathcal{B}(\tau^- \rightarrow \pi^- \nu_\tau)$	0.2232 ± 0.0019
$ V_{us} $ from $\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau)$	0.2212 ± 0.0020
$ V_{us} $ τ average	0.2204 ± 0.0014

The b -hadron lifetime and mixing averages have made substantial progress in precision in the past two years, since the previous version of this writeup [4]. In total 60 new results (of which almost two thirds are from LHCb, with the rest from CDF, D0, Belle, ATLAS, CMS and *BABAR*) have been incorporated in these averages. Our knowledge of the lifetime of each individual weakly decaying b -hadron species has improved by a factor 2–3, with the exception of the Ξ_b^- (factor 7 improvement), the Ξ_b^0 (first measurement), and the b baryons with more than one heavy quark (no measurement yet). Impressive precisions of 0.3% are achieved for the most common species, and the hierarchy $\tau(B^+) > \tau(B_{sH}) > \tau(B^0) \sim \tau(B_s^0) > \tau(\Lambda_b^0) > \tau(B_{sL}) > \tau(B_c^+)$ is established. Similarly the precision on the decay width differences in the B^0 and B_s^0 systems has improved by about a factor of two, confirming a sizable value of $\Delta\Gamma_s$ in agreement with the Standard Model (SM) prediction. The results are not yet precise enough to distinguish the small (as expected) value of $\Delta\Gamma_d$ from zero. As to the mass differences, only the uncertainty on Δm_s has substantially decreased (by a factor 2) reaching now the per mil level. Several new results on CP violation in mixing have been published, including the final like-sign dimuon and inclusive muon analyses from D0 (with a claimed 3.6σ overall deviation from the SM), but the averages of the individual B^0 and B_s^0 asymmetries remain well consistent both with zero and with their small SM predictions. Very impressive progress has been achieved in the measurement of CP violation induced by B_s^0 mixing in the $b \rightarrow c\bar{c}s$ transition, through the time-dependent angular analysis of tagged $B_s^0 \rightarrow J/\psi K^+ K^-$ and $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ decays, as well as the time-dependent analysis of tagged $B_s^0 \rightarrow D_s^+ D_s^-$ decays; despite the large improvement in sensitivity, the measured weak phase $\phi_s^{c\bar{c}s} = -0.015 \pm 0.035$ remains compatible with zero and with the SM prediction ($-0.0363^{+0.0012}_{-0.0014}$), but its experimental uncertainty (which has reduced by a factor 2.5) is now reaching the level of the SM central value. Many measurements are still dominated by statistical uncertainties and will improve once new data from the LHC becomes available.

The measurement of $\sin 2\beta \equiv \sin 2\phi_1$ from $b \rightarrow c\bar{c}s$ transitions such as $B^0 \rightarrow J/\psi K_s^0$ has reached $< 3\%$ precision: $\sin 2\beta \equiv \sin 2\phi_1 = 0.682 \pm 0.019$. Measurements of the same parameter using different quark-level processes provide a consistency test of the Standard Model and allow insight into possible new physics. All results among hadronic $b \rightarrow s$ penguin dominated decays of B^0 mesons are currently consistent with the Standard Model expectations. Recently, first measurements of CP violation parameters in $B_s^0 \rightarrow \phi\phi$ have become available, allowing a similar comparison to the value of $\phi_s^{c\bar{c}s}$; again, results are consistent with the SM expectation (which

in this case is very close to zero). Among measurements related to the Unitarity Triangle angle $\alpha \equiv \phi_2$, results from the $\rho\rho$ system allow constraints at the level of $\approx 6^\circ$. These remain the strongest constraints, although recent results from all of *BABAR*, Belle and LHCb have led to significantly improved precision of the CP violation parameters in $B^0 \rightarrow \pi^+\pi^-$ decays. Knowledge of the third angle $\gamma \equiv \phi_3$ also continues to improve. The world average values of the parameters in $B \rightarrow DK$ decays now show significant direct CP violation effects, and determinations of γ from the individual experiments now approach the level of 10° precision. The precision is expected to improve further as more data and more decays modes, for example time-dependent analysis of $B_s^0 \rightarrow D_s^\mp K^\pm$ decays and Dalitz plot analysis of $B^0 \rightarrow DK^+\pi^-$ decays, are added.

Regarding semileptonic B -meson decays, only a few new experimental results have appeared since the last update. The changes in the numerical values are mainly due to improvements in the theoretical calculations: the form factor normalization in $B \rightarrow D^*\ell\nu$ is now known with 1.5% precision, and calculations of inclusive decays $B \rightarrow X_c\ell\nu$ have reached NNLO precision. For both $|V_{cb}|$ and $|V_{ub}|$, the discrepancy between determinations from exclusive and inclusive decays is now at the level of about 3σ – a puzzle that must be addressed by the next generation of flavour experiments. Finally, the anomalous results obtained in $B \rightarrow D^{(*)}\tau\nu$ decays await experimental confirmation.

The most important new measurements of rare b hadron decays are coming from the LHC. Precision measurements of B_s^0 decays are particularly noteworthy. CMS and LHCb have both been constantly improving their restrictive limits for the decays $B_{(s)}^0 \rightarrow \mu^+\mu^-$. They recently published a combined analysis that allowed the first observation of the $B_s^0 \rightarrow \mu^+\mu^-$ decay to be obtained, and provided three standard deviations evidence of the $B^0 \rightarrow \mu^+\mu^-$ decay. These results are compatible with the SM predictions, and yield constraints on the parameter space of new physics models. Atlas, CMS and LHCb have also performed angular analyses of the $B^0 \rightarrow K^{*0}\mu^+\mu^-$ decay, complementing and extending earlier results from *BABAR* and Belle. One of the observables measured by LHCb, P'_5 , differs from the SM prediction by 3.7 standard deviations in one of the $m_{\mu^+\mu^-}^2$ intervals. Updated measurements from LHCb, and results from other experiments on this observable, are keenly anticipated. Among the CP violating observables in rare decays, the “ $K\pi$ puzzle” persists, and important new results have appeared in three-body decays. LHCb has produced many other results on a wide variety of decays, including b baryon decays. Belle and *BABAR* continue to produce new results though their output rates are dwindling. It will still be some years before we see new results from the upgraded SuperKEKB B factory and the Belle II experiment.

Many b to charm results from LHCb are included in our report for the first time this year, combining with results from *BABAR*, Belle and CDF to yield a total of 632 measurements reported in 216 papers. The huge combined sample of b hadrons allows measurements of decays to states with open or hidden charm content with unprecedented precision. Since our previous report, there is a dramatically increased number of results on decays of the B_c^+ meson and of b baryons.

In the charm sector, D^0 – $\bar{D}^0$ mixing is now well-established. Measurements of 45 separate observables from five experiments are input into a global fit for 10 underlying parameters, and the no-mixing hypothesis is excluded at a confidence level exceeding 11.5σ . The mixing parameters x and y differ from zero by 2.4σ and 9.4σ , respectively. The central values are consistent with mixing arising from long-distance processes, as predicted by theory; thus it will be difficult to identify new physics from mixing alone. The WA value for the observable y_{CP}

is positive, which indicates that the CP -even state is shorter-lived as in the $K^0-\bar{K}^0$ system. However, x also appears to be positive, which implies that the CP -even state is heavier, unlike in the $K^0-\bar{K}^0$ system. In the $D^0-\bar{D}^0$ system, there is no evidence for CP violation arising from mixing ($|q/p| \neq 1$) or from a phase difference between the mixing amplitude and a direct decay amplitude ($\phi \neq 0$). An initial evidence for direct CP violation in $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ decays is currently unconfirmed. Inputting the relevant measurements into a global fit gives $\Delta a_{CP}^{\text{dir}} \neq 0$ with a significance of 2.4σ .

The τ branching fraction fit has been updated using results from two *BABAR* papers and one from Belle, and updating the constraints on the experimental branching fraction measurements. The precise Belle measurement of the τ lifetime reduced the uncertainty on the PDG 2013 average by a factor two. We have updated the lepton universality tests, which give significantly improved precision thanks to the smaller uncertainty on the τ lifetime. We have also updated the three determinations of $|V_{us}|$ using the τ HFAG averages; there is no significant change with respect to the HFAG 2012 report. We updated the list of upper limits on τ Lepton-Flavour-Violating decays and, for the first time, we combine the available experimental information to compute world-average limits.

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