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CORRECTION

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Correction to: Lie–Poisson Methods for Isospectral Flows

Klas Modin[1] · Milo Viviani[2]

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Correction to: Foundations of Computational Mathematics (2020) 20:889–921 https://doi.org/10.1007/s10208-019-09428-w

Motivation The authors of [1] have found an error in the proof of Proposition 1. Specifically, the proof presented there implicitly assumed that the matrices $\nabla H(Q(t)^\dagger P(t))^\dagger$ commute for any $t \geq 0$. The statement of the Proposition 1 has now been reformulated in a suitable way to fit with the discussion below Proposition 1 in [1]. Here below, we present the corrected version and proof of Proposition 1.

Proposition 1 *Consider a solution* $(Q(t), P(t))$ *of Hamilton's equations* (12) *defined for* $0 \leq t \leq T$ *and a given initial point* (Q_0, P_0)*, such that* $Q_0^\dagger P_0 \in S$*, for* $S \subseteq \mathfrak{gl}(n, \mathbb{C})^*$ *a linear subspace as before, and* Q_0 *is invertible. Then, there exists* $G \in GL(n, \mathbb{C})$ *such that* $Q(t)^\dagger G \in N(S)$*, for any* $0 \leq t \leq T$*,*[1] *if and only if* $\nabla H(Q(t)^\dagger P(t))^\dagger \in \mathfrak{n}(S)$*, for any* $0 \leq t \leq T$*. Furthermore, if one of the two condition holds, then* $Q^\dagger(t)P(t) \in S$*, for any* $0 \leq t \leq T$*.*

Proof Let us first prove the equivalence of the two conditions. Let us assume that $\nabla H(Q(t)^\dagger P(t))^\dagger \in \mathfrak{n}(S)$, for any $0 \leq t \leq T$. We have

$$Q(t)^\dagger G = \exp\left(\int_0^t \Theta(s) ds\right) Q_0^\dagger G,$$

where Θ is a solution to

$$\frac{d\Theta(t)}{dt} = \mathrm{dexp}_{\Theta(t)}^{-1} \nabla H(Q(t)^\dagger P(t))^\dagger \quad \Theta(0) = 0,$$

[1] This could be replaced by assuming $Q(t)^\dagger \in N(S)$ for any $0 \leq t \leq T$, provided that $Q_0^\dagger$ is already in $N(S)$.

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✉ Klas Modin
klas.modin@chalmers.se

Milo Viviani
viviani@chalmers.se

1 Chalmers University of Technology and University of Gothenburg, Gothenburg, Sweden

2 Scuola Normale Superiore, Pisa, Italy

where dexp^{-1} is defined as in [2]. Hence, $\Theta(t) \in \mathfrak{n}(S)$, for any $0 \le t \le T$, being defined as the Magnus expansion of $\nabla H(Q(t)^\dagger P(t))^\dagger$ (see [2]). This proves the statement, since $N(S) \supseteq \exp(\mathfrak{n}(S))$.

Conversely, let us assume that there exists $G \in GL(n, \mathbb{C})$ such that $Q(t)^\dagger G \in N(S)$ for any $0 \le t \le T$. By the formula above, we have

$$Q(t)^\dagger Q_0^{-\dagger} = \exp\left(\int_0^t \Theta(s)ds\right).$$

Since the left-hand side is in $N(S)$ for any $0 \le t \le T$, we have $\Theta(t) \in \mathfrak{n}(S)$. This implies that also $\frac{d\Theta(t)}{dt} \in \mathfrak{n}(S)$. Therefore, since $\mathrm{dexp}_{\Theta(t)} \frac{d\Theta(t)}{dt} = \nabla H(Q^\dagger P)^\dagger$, we get the thesis.

Finally, we have

$$Q(t)^\dagger P(t) = \exp\left(\int_0^t \Theta(s)ds\right) Q_0^\dagger P_0 \exp\left(-\int_0^t \Theta(s)ds\right).$$

Hence, if one of the two condition holds, $Q^\dagger(t)P(t) \in S$, for any $0 \le t \le T$, by the definition of $N(S)$. □

References

1. Modin K. and Viviani M., *Lie–Poisson Methods for Isospectral flows*, *Found Comput Math* 20, 889–921 (2020).
2. Iserles A., Munthe-Kaas H.Z., Nørsett S. P. and Zanna A., *Lie-group Methods*, *Acta Numer* 9, 215–365 (2000).