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On boundaries, charges and Fermi fields

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ABSTRACT

We address some general issues related to torsion and Noether currents for Fermi fields in the presence of boundaries, with emphasis on the conditions that guarantee charge conservation. We also describe exact solutions of these boundary conditions and some implications for string vacua with broken supersymmetry.

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1. Introduction

String compactifications have been widely explored during the last decades, but almost exclusively with closed internal manifolds [1], so that the boundary conditions needed for Fermi fields when the manifold has a border have received little attention. Two notable exceptions are the Neveu–Schwarz–Ramond (NSR) open string [2] and the Horava–Witten link [3] between the $E_8 \times E_8$ heterotic string and the Cremmer–Julia–Scherk [4] eleven–dimensional form of Supergravity [5]. Boundaries, however, have played so far a prominent role in vacuum configurations for orientifolds [6] with "brane supersymmetry breaking" [7,8], whose prototype is the nine–dimensional Dudas–Mourad solution of [9]. This involves regions of strong coupling, but is classically stable [10] and the tension from branes and orientifolds, which signals the breaking of supersymmetry, renders the length of its internal interval finite. This compactification also concerns the $U(32)$ non–supersymmetric orientifold of [11], while a variant [9] applies to the non–supersymmetric heterotic model of [12]. These examples motivate, in our view, a closer look at their Fermi fields.

For definiteness, we choose a coordinate system such that the boundary $\partial\mathcal{M}$ of the D–dimensional manifold $\mathcal{M}$ lies at $r = 0$ and the metric takes nearby the form

$$ds^2 \equiv g_{MN}\, dx^M\, dx^N = g_{rr}\, dr^2 + ds^2_\perp \,. \tag{1.1}$$

The variation of the Dirac action for a spinor λ yields boundary terms, which can be removed provided[1]

$$\left(\bar\lambda \gamma^r \delta\lambda - \delta\bar\lambda \gamma^r \lambda\right)\big|_{\partial\mathcal{M}} = 0 \,. \tag{1.2}$$

Any boundary condition

$$(1 - \Lambda)\,\lambda\big|_{\partial\mathcal{M}} = 0 \,, \tag{1.3}$$

with a Hermitian matrix Λ such that

$$\Lambda^2 = 1 \,, \qquad \left\{\Lambda \,,\, \gamma^0\gamma^r\right\} = 0 \tag{1.4}$$

solves eq. (1.2). Different choices are possible, however, depending on the symmetries to be preserved: for example, $\Lambda = \gamma^r$ and $\Lambda = i\gamma^0$ are two solutions, and there are more options. One of our aims is to connect the allowed choices of Λ to the conservation of Noether Killing charges.

In Sections 2 and 3 we discuss the matter and gravity Bianchi identities related to diffeomorphisms and local Lorentz symmetries, taking into account that the back–reaction of Fermi fields includes in general the emergence of torsion. In Section 4 we connect diffeomorphisms and local Lorentz Bianchi identities to Noether Killing currents for global isometries, whose normal components should vanish on the boundary $\partial\mathcal{M}$ to grant charge conservation. This places further constraints on Λ, which we explore in Section 5 with an eye to string models with broken supersymmetry.

2. Bianchi identities and Bose fields

Let us begin by reviewing briefly the behavior of Bose fields with reference to the simplest case, a real scalar ϕ. If the metric takes the form (1.1) near the boundary $\partial\mathcal{M}$ of a D–dimensional manifold $\mathcal{M}$, the variation of the standard kinetic term yields the boundary condition

$$\delta\phi\, \partial_r\phi|_{\partial\mathcal{M}} = 0 \,, \tag{2.1}$$

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[1] We use a "mostly plus" signature, so that γ^0 is antihermitian while the other γ–matrices are hermitian.

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which is solved by the familiar Neumann ($\partial_r\phi = 0$) or Dirichlet ($\delta\phi = 0$) choices. Notice that the latter only implies that ϕ is a fixed function on $\partial\mathcal{M}$. Similar remarks apply to forms and to the metric tensor, up to Gibbons–Hawking terms [13].

Let us now explore whether eq. (2.1) suffices to guarantee the conservation of Noether Killing charges, which are built from *symmetric* energy–momentum tensors T^{MN} defined via the metric variations

$$\delta\mathcal{S}_m = \int_{\mathcal{M}} d^D x \sqrt{-g}\, \delta g_{MN}\, T^{MN} . \tag{2.2}$$

A consistent coupling to gravity demands that $\delta\mathcal{S}$ vanish for the metric variations

$$\delta\, g_{MN} = D_M \xi_N + D_N \xi_M , \tag{2.3}$$

which describe the effect of diffeomorphisms $\delta x^M = \xi^M$ when keeping fixed the coordinates in fields, and with ξ^M of local support a partial integration leads to the Bianchi identity

$$D_M\, T^{MN} = 0 . \tag{2.4}$$

Continuous symmetries of g_{MN} are generated by Killing vectors ζ^M, solutions of (2.3) with $\delta g_{MN} = 0$, and lead to the covariantly conserved Noether currents

$$\mathcal{J}^M = T^{MN}\, \zeta_N . \tag{2.5}$$

The combinations $\sqrt{-g}\,\mathcal{J}^M$ satisfy the ordinary conservations law $\partial_M\left(\sqrt{-g}\,\mathcal{J}^M\right) = 0$, and in the absence of a boundary the charges $Q(t)$, which we write for brevity in the form

$$Q(t) = \int_{\mathcal{M}} d^D x\, \delta(x^0 - t)\, \sqrt{-g}\, \mathcal{J}^0 , \tag{2.6}$$

are conserved. However, when $\mathcal{M}$ has a boundary $\partial\mathcal{M}$

$$\frac{d\,Q(t)}{dt} = \int_{\partial\mathcal{M}} d^{D-1} x\, \delta(x^0 - t)\, \sqrt{-g}\, \mathcal{J}^r , \tag{2.7}$$

and the condition

$$\mathcal{J}^r\big|_{\partial\mathcal{M}} \equiv T^{rN}\, \zeta_N\big|_{\partial\mathcal{M}} = 0 , \tag{2.8}$$

is needed to prevent charge flow across the boundary. It involves off–diagonal components of the energy–momentum tensor since ζ^r should vanish on $\partial\mathcal{M}$ in order not to affect it. For the bosonic actions of interest, the boundary conditions like (2.1) that emerge from the equations of motion must be supplemented in general by eq. (2.8). For instance, Killing translation symmetries on $\partial\mathcal{M}$ require for a Dirichlet scalar ϕ that

$$\zeta^M\, \partial_M \phi\big|_{\partial\mathcal{M}} = 0 , \tag{2.9}$$

whereas for a Neumann scalar eq. (2.8) is identically satisfied.

3. Bianchi identities and Fermi fields

When Fermi fields are present, local Lorentz transformations also acquire a key role, and there are consequently a few novelties. The metric tensor leaves way to the vielbein $e_M{}^A$ and the spin connection $\omega_M{}^{AB}$, while the variation of the matter action,

$$\delta\,\mathcal{S}_m = \int_{\mathcal{M}} d^D x\, e\, \left[\delta e_M{}^A\, \mathcal{T}^M{}_A + \delta\omega_M{}^{AB}\, \mathcal{Y}^M{}_{AB}\right] , \tag{3.1}$$

now defines generally a *non–symmetric* energy–momentum tensor $\mathcal{T}^M{}_A$ and a new tensor $\mathcal{Y}^M{}_{AB}$. In the following, early Latin labels describe flat indices, while late Latin labels describe curved ones. The vielbein is covariantly constant,

$$D_M\, e_N{}^A \equiv \partial_M\, e_N{}^A + \omega_M{}^{AB}\, e_{NB} - \Gamma^P{}_{MN}\, e_P{}^A = 0 , \tag{3.2}$$

and this condition defines the $\Gamma^P{}_{MN}$, whose antisymmetric part

$$S^P{}_{MN} = \Gamma^P{}_{MN} - \Gamma^P{}_{NM} \tag{3.3}$$

is the torsion tensor.

A local Lorentz transformation with parameters $\epsilon^{AB} = -\epsilon^{BA}$ acts as

$$\delta\, e_M{}^A = \epsilon^{AB}\, e_{MB} , \qquad \delta\,\omega_M{}^{AB} = -\,D_M\,\epsilon^{AB} . \tag{3.4}$$

Rephrasing the argument reviewed for Bose fields, eq. (3.1) yields

$$\delta\,\mathcal{S}_m = \int d^D x\, e\, \left[\epsilon^{AB}\, e_{MB}\, \mathcal{T}^M{}_A - D_M\,\epsilon^{AB}\, \mathcal{Y}^M{}_{AB}\right] , \tag{3.5}$$

and after a partial integration one obtains the Bianchi identity

$$D_M\, \mathcal{Y}^M{}_{AB} - S^P{}_{PM}\, \mathcal{Y}^M{}_{AB} = \frac{1}{2}\left(\mathcal{T}_{AB} - \mathcal{T}_{BA}\right) . \tag{3.6}$$

This step entails a small subtlety, since in the presence of torsion the covariant derivative of a vector V^M, equal to $\epsilon^{AB}\mathcal{Y}^M{}_{AB}$ in this case, does not lead to a total derivative, but

$$D_M\, V^M = S^M{}_{MN}\, V^N + \frac{1}{e}\,\partial_M\left(e\, V^M\right) . \tag{3.7}$$

Up to a local Lorentz rotation, diffeomorphisms act on $e_M{}^A$ and $\omega_M{}^{AB}$ as

$$\delta\, e_M{}^A = D_M\,\xi^A - S^A{}_{MN}\,\xi^N , \qquad \delta\,\omega_M{}^{AB} = -\,R_{MN}{}^{AB}\,\xi^N , \tag{3.8}$$

when keeping fixed the coordinates in fields, where we define the Riemann tensor, following the conventions in [17], as

$$\begin{aligned} R_{MN}{}^{AB} &= \partial_M\,\omega_N{}^{AB} - \partial_N\,\omega_M{}^{AB} + \omega_M{}^{AC}\,\omega_{NC}{}^B - \omega_N{}^{AC}\,\omega_{MC}{}^B \\ &= e^{PB}\, e^{QA}\left(\partial_N\,\Gamma^P{}_{MQ} - \partial_M\,\Gamma^P{}_{NQ}\right. \\ &\quad \left. + \Gamma^P{}_{NR}\,\Gamma^R{}_{MQ} - \Gamma^P{}_{MR}\,\Gamma^R{}_{NQ}\right) . \end{aligned} \tag{3.9}$$

Resorting again to (3.7), a partial integration now leads to a second Bianchi identity,

$$D_M\,\mathcal{T}^M{}_N + S^P{}_{MN}\,\mathcal{T}^M{}_P - S^P{}_{PM}\,\mathcal{T}^M{}_N = -\,R_{MN}{}^{AB}\,\mathcal{Y}^M{}_{AB} . \tag{3.10}$$

For a spin–$\frac{1}{2}$ Fermi field λ the Hermitian Dirac action

$$\mathcal{S}_m = -\,\frac{i}{2}\int d^D x\, e\, \left[\bar\lambda\,\gamma^M\, D_M\,\lambda - D_M\,\bar\lambda\,\gamma^M\,\lambda\right] \tag{3.11}$$

determines

$$\begin{aligned} \mathcal{T}^M{}_A &= \frac{i}{2}\left[\bar\lambda\,\gamma^M\, D_A\,\lambda - D_A\,\bar\lambda\,\gamma^M\,\lambda\right] , \\ \mathcal{Y}^M{}_{AB} &= -\,\frac{i}{4}\,\bar\lambda\,\gamma_{ABC}\,\lambda\, e^{MC} , \end{aligned} \tag{3.12}$$

where we have kept in $\mathcal{T}$ only terms that do not vanish on shell. The boundary condition is now eq. (1.2), and in this case $\mathcal{Y}$ is totally antisymmetric, so that the traces $S^M{}_{MA}$ are absent in eqs. (3.6), (3.7) and (3.10). However, they play a role for a spin–$\frac{3}{2}$ Fermi field ψ_M, since the Hermitian Rarita–Schwinger action

$$\mathcal{S}_m = -\frac{i}{2}\int d^D x\, e\left[\bar\psi_M\,\gamma^{MNP}\,D_N\,\psi_P - D_N\,\bar\psi_M\,\gamma^{MNP}\,\psi_P\right] \tag{3.13}$$

determines

$$\begin{aligned}\mathcal{T}^M{}_A &= \frac{i}{2}\left[D_A\,\bar\psi_N\,\gamma^{MNP}\,\psi_P - \bar\psi_N\,\gamma^{MNP}\,D_A\,\psi_P\right],\\ \mathcal{Y}^{MAB} &= \frac{i}{4}\,\bar\psi_N\,\gamma^{MNPAB}\,\psi_P - \frac{i}{4}\big[\bar\psi^A\,\gamma^M\,\psi^B + \bar\psi_N\,\gamma^N\,\psi^A\,e^{MB}\\ &\quad + \bar\psi^A\,\gamma^P\,\psi_P\,e^{MB} - (A\leftrightarrow B)\big],\end{aligned} \tag{3.14}$$

and consequently

$$\mathcal{Y}^M{}_M{}^B = i\,\frac{(D-2)}{4}\left(\bar\psi_M\,\gamma^M\,\psi^B - \bar\psi^B\,\gamma^M\,\psi_M\right). \tag{3.15}$$

In $\mathcal{T}$ we have kept again only terms that do not vanish on shell, and the counterpart of the boundary conditions (1.2) and (2.1) is now

$$\left(\bar\psi_M\,\gamma^{MrP}\,\delta\,\psi_P - \delta\,\bar\psi_M\,\gamma^{MrP}\,\psi_P\right)\Big|_{\partial\mathcal{M}} = 0\,. \tag{3.16}$$

In a similar fashion, varying the vielbein and the spin connection in the Einstein–Hilbert action

$$\mathcal{S}_{EH} = \frac{1}{2k^2}\int\limits_{\mathcal{M}} d^D x\, e\, e^M{}_A\, e^N{}_B\, R_{MN}{}^{AB} \tag{3.17}$$

yields

$$\delta\,\mathcal{S}_{EH} = -\frac{1}{k^2}\int\limits_{\mathcal{M}} d^D x\, e\left[\delta\,\omega_N{}^{AB}\,\Theta^N{}_{AB} + \delta\,e_M{}^A\,G^M{}_A\right], \tag{3.18}$$

where

$$\begin{aligned}G^M{}_A &= \left(e^M{}_C\,e^P{}_A - \frac{1}{2}\,e^M{}_A\,e^P{}_C\right)e^Q{}_D\,R^{CD}{}_{PQ}\\ &= R^M{}_A - \frac{1}{2}\,e^M{}_A\,R\end{aligned} \tag{3.19}$$

is generally a *non–symmetric* Einstein tensor, and

$$\Theta^N{}_{AB} = -\frac{1}{2}\left(S^P{}_{PA}\,e^N{}_B - S^P{}_{PB}\,e^N{}_A\right) - \frac{1}{2}\,S^N{}_{AB}\,. \tag{3.20}$$

Retracing the preceding arguments leads to the Bianchi identities

$$\begin{aligned}&D_M\,\Theta^M{}_{AB} - S^P{}_{PM}\,\Theta^M{}_{AB} = \frac{1}{2}\,(G_{AB} - G_{BA})\,,\\ &D_M\,G^M{}_N + S^P{}_{MN}\,G^M{}_P - S^P{}_{PM}\,G^M{}_N = -\,\Theta^M{}_{AB}\,R_{MN}{}^{AB}\,,\end{aligned} \tag{3.21}$$

that reflect the invariance of the Einstein–Hilbert Lagrangian under local Lorentz transformations and diffeomorphisms, while putting together matter and gravity sectors leads to the equations of motion

$$G^M{}_A = 2k^2\,\mathcal{T}^M{}_A\,, \qquad \Theta^M{}_{AB} = 2k^2\,\mathcal{Y}^M{}_{AB}\,, \tag{3.22}$$

which are manifestly compatible with the Bianchi identities of eqs. (3.6), (3.10) and (3.21). Notice, finally, that eqs. (3.21) would follow directly from the Bianchi identities for the Riemann tensor,

$$\begin{aligned}&R_{[MNP]}{}^A = D_{[M}\,S^A{}_{NP]} - S^R{}_{[MN}\,S^A{}_{P]R}\,,\\ &D_{[M}\,R_{NP]}{}^{AB} = S^R{}_{[MN}\,R_{P]R}{}^{AB}\,,\end{aligned} \tag{3.23}$$

here expressed in terms of covariant derivatives including the torsion contribution, under which the vielbein is covariantly constant.

4. Killing vectors and Fermi fields

In the presence of Fermi fields, continuous symmetries and Killing vectors are to be defined with reference to diffeomorphisms, with parameters ζ^M, and local Lorentz rotations, with parameters θ^{AB}, whose combined effects leave both e and ω invariant. These two conditions read

$$\begin{aligned}&\delta\,e_M{}^A \equiv D_M\,\zeta^A - S^A{}_{MN}\,\zeta^N + \theta^{AB}\,e_{NB} = 0\,,\\ &\delta\,\omega_M{}^{AB} \equiv -\,R_{MN}{}^{AB}\,\zeta^N - D_M\,\theta^{AB} = 0\,,\end{aligned} \tag{4.1}$$

and the first determines

$$\theta^{AB} = D^A\,\zeta^B - S^{BA}{}_C\,\zeta^C\,, \tag{4.2}$$

while the antisymmetry of θ^{AB} translates into the *modified Killing equation*

$$D_M\,\zeta_N + D_N\,\zeta_M = \left(S_{MN}{}^P + S_{NM}{}^P\right)\zeta_P\,. \tag{4.3}$$

Moreover, using eq. (4.2), the second of eqs. (4.1) can be cast in the form

$$\begin{aligned}D_M\,D_A\,\zeta_B &= \left(D_M\,S_{BA}{}^N\right)\zeta_N\\ &\quad + S_{BA}{}^N\,D_M\,\zeta_N - R_{MNAB}\,\zeta^N\,,\end{aligned} \tag{4.4}$$

which generalizes the usual result for the second derivatives of Killing vectors.

Noether currents should now satisfy the *modified conservation laws*

$$D_M\,\mathcal{J}^M - S^M{}_{MN}\,\mathcal{J}^N = 0\,, \tag{4.5}$$

a subtlety whose origin we already highlighted in eq. (3.7). Given a Killing vector ζ^A solving eq. (4.3), one can indeed verify that

$$\mathcal{J}^M = \mathcal{T}^M{}_N\,\zeta^N - \mathcal{Y}^M{}_{AB}\,\theta^{AB}\,, \tag{4.6}$$

with θ^{AB} given by eq. (4.2), satisfies the modified conservation law (4.5). To this end, notice that the Bianchi identities of eqs. (3.6) and (3.10) give

$$\begin{aligned}D_M\,\mathcal{J}^M - S^P{}_{PM}\,\mathcal{J}^M &= -\left(S^P{}_{MN}\,\mathcal{T}^M{}_P + R_{MN}{}^{AB}\,\mathcal{Y}^M{}_{AB}\right)\zeta^N\\ &\quad + \mathcal{T}^{AB}\,(D_A\,\zeta_B - \theta_{AB}) - \mathcal{Y}^M{}_{AB}\,D_M\,\theta^{AB}\,,\end{aligned} \tag{4.7}$$

while using the definition of θ^{AB} this expression reduces to

$$D_M\,\mathcal{J}^M - S^P{}_{PM}\,\mathcal{J}^M = -\,\mathcal{Y}^M{}_{AB}\left(R_{MN}{}^{AB}\,\zeta^N + D_M\,\theta^{AB}\right), \tag{4.8}$$

whose right–hand side vanishes on account of the second of eqs. (4.1). Repeating considerations made in Section 2 one can now conclude that, if the modified conservation laws (4.5) are supplemented by the boundary conditions

$$\mathcal{J}^r\big|_{\partial\mathcal{M}} = 0\,, \tag{4.9}$$

the corresponding charges are conserved even in the presence of a boundary $\partial\mathcal{M}$.

5. Lower–dimensional spinors from an interval

In [14] we shall explore families of D–dimensional warped metrics of the type

$$ds^2 = e^{2B(r)}dr^2 + e^{2A(r)}\, g_{\mu\nu}(x)\, dx^\mu\, dx^\nu + e^{2C(r)}\, g_{ij}(y)\, dy^i\, dy^j\,, \tag{5.1}$$

where $g_{\mu\nu}$ is typically a Minkowski metric $\eta_{\mu\nu}$ of dimension d and g_{ij}, the metric of an internal compact space of dimension N, is typically δ_{ij}. Examples of this type were also recently described in [15], and a wide portion of these solutions involve, just as the ones in [9], r-intervals of finite length. When $g_{\mu\nu}$ and g_{ij} are flat metrics, the relevant Killing symmetries are translations in space-time and along an internal torus, together with spacetime Lorentz rotations. The former correspond to constant ζ^μ or ζ^i, so that

$$\mathcal{J}^M = \mathcal{T}^M{}_\mu\, \zeta^\mu + \mathcal{T}^M{}_i\, \zeta^i\,, \tag{5.2}$$

while the latter correspond to $\zeta^\mu = \theta^{\mu\nu} x_\nu$, with constant anti-symmetric $\theta^{\mu\nu}$, so that

$$\mathcal{J}^M = \mathcal{T}^M{}_\nu\, \theta^{\nu\rho}\, x_\rho - \mathcal{Y}^M{}_{\mu\nu}\, \theta^{\mu\nu}\,. \tag{5.3}$$

For the currents in eqs. (5.2) and (5.3), the conditions in eq. (4.9) therefore demand that

$$\mathcal{T}^r{}_\mu\big|_{\partial\mathcal{M}} = 0\,, \qquad \mathcal{T}^r{}_i\big|_{\partial\mathcal{M}} = 0\,, \qquad \mathcal{Y}^r{}_{\mu\nu}\big|_{\partial\mathcal{M}} = 0\,. \tag{5.4}$$

For a spin–$\frac{1}{2}$ fermion, $\mathcal{T}$ and $\mathcal{Y}$ are given in eq. (3.12), and the first two sets of conditions are implied by eqs. (1.3) and (1.4). The last set puts on Λ the additional constraints

$$\bar\lambda\, \gamma^r\, \gamma_{\mu\nu}\, \lambda\big|_{\partial\mathcal{M}} = 0\,, \tag{5.5}$$

which are also solved by a matrix Λ in eqs. (1.3) and (1.4), provided

$$\big[\Lambda, \gamma_{\mu\nu}\big] = 0\,. \tag{5.6}$$

In settings of interest for Supergravity and String Theory, Λ is often subject to further restrictions. If the dimension D of $\mathcal{M}$ is even and λ is a Weyl spinor, one should demand that

$$\big[\Lambda\,,\, \gamma_\chi\big] = 0\,, \tag{5.7}$$

where γ_χ is the chirality matrix of $\mathcal{M}$, while if λ is a Majorana spinor one should demand that

$$C^{-1}\, \Lambda^T\, C = -\,\gamma^0\, \Lambda\, \gamma^0\,, \tag{5.8}$$

where C is the charge–conjugation matrix of $\mathcal{M}$. When D is *odd*, with no other internal manifold, the choice $\Lambda = \gamma^r$, which rests on the chirality matrix of $\partial\mathcal{M}$, satisfies eqs. (1.3), (1.4), (5.8) and commutes with all spacetime Lorentz generators of $\partial\mathcal{M}$. This case is central to the Horava–Witten construction [3]. When D is *even*, similar settings obtain with non–chiral spinors. For example, in type–IIA supergravity the choice $\Lambda = \gamma^r$, used in [16], respects all Lorentz symmetries in nine dimensions while connecting the two chiralities on $\partial\mathcal{M}$, and the Neveu–Schwarz–Ramond open string [2] was a first example of this type. The situation becomes less conventional when starting from chiral spinors, which is the case for the solutions in [9]. Now the choice $\Lambda = \gamma^r$ violates the Weyl constraint (5.7), so that no solutions exist that respect the whole nine–dimensional Lorentz symmetry. However, when a compact internal manifold is also present, the Weyl constraint can be solved combining γ^r with an *odd* number of internal γ's, and a first option also compatible with the Majorana constraint (5.8), as needed in [9], is $\Lambda = \gamma^6\gamma^7\gamma^8\gamma^r$. It respects the six–dimensional Lorentz group, which suffices when $\mathcal{I}$ combines with a three–torus.

In general, in D–dimensional spacetimes of "mostly plus" Minkowski signature,

$$(i)^{\frac{n(n-1)}{2}}\, \gamma^{A_1,\ldots A_n}\,, \qquad n = 0, \ldots, \tilde D\,, \tag{5.9}$$

with $\tilde D = D$ if D is even or $\tilde D = \frac{(D-1)}{2}$ if D is odd, are a basis for $2^{\left[\frac{D}{2}\right]} \times 2^{\left[\frac{D}{2}\right]}$ matrices. The matrices in eq (5.9) are self-adjoint and square to one when all $A_i \neq 0$, and otherwise they are self-adjoint and square to one when multiplied by i. One can distinguish the two sets

$$(i)^{\frac{n(n+1)}{2}}\, \gamma^{r i_1,\ldots i_n} \qquad \text{and} \qquad i(i)^{\frac{(m+d-1)(m+d)}{2}}\, \gamma^{01\ldots(d-1) i_1,\ldots i_m}\,, \tag{5.10}$$

with $n \leq \min(N, \tilde D - 1)$ and $m + d \leq \min(N + d, \tilde D)$, which we call *n–type* and *m–type* matrices, all of which satisfy the constraints (1.4). When D is even, one can also start from a Weyl fermion, but eq. (5.7) then demands that $n + 1$ and/or $m + d$ be even. Moreover, when $D = 2, 3, 4$ modulo 8, the Majorana constraint is possible, and eq. (5.8) then demands that $n = 0, 3, 4, 7$ modulo 8 or $m + d = 2, 3, 6, 7$ modulo 8. Alternatively, when $D = 2, 8, 9$ modulo 8 the pseudo–Majorana constraint is possible and allows the same options. Finally, when $D = 2$ modulo 8 the Weyl-Majorana constraint is possible [20], and eq. (5.8) then demands that $n = 3, 7$ modulo 8 or $m + d = 2, 6$ modulo 8. In particular, the example given above eq. (5.9) rests on an *n–type* Λ with $n = 3$. In conclusion, when starting in $D = 11$ with a Majorana spinor, there are *n–type* Λ's with $n = 0, 3, 4$, and *m–type* Λ's with $m + d = 2, 3$, because $\tilde D = 5$. Moreover, when starting in $D = 10$ with a Weyl spinor, there are *n–type* Λ's with n odd and *m–type* Λ's with $m + d$ even. Finally, when starting in $D = 10$ with a Majorana–Weyl spinor, there are *n–type* Λ's with $n = 3, 7$ and *m–type* Λ's with $m + d = 2, 6$. These solutions are compatible with the Lorentz symmetry in six or fewer dimensions.

A gravitino ψ_M contains lower–dimensional spin–$\frac{3}{2}$ modes ψ_μ in its space–time components, which are selected by the additional constraint

$$\gamma^\mu\, \psi_\mu = 0\,, \tag{5.11}$$

to which the preceding considerations apply almost verbatim. There are also internal spin–$\frac{1}{2}$ components that mix, in general, with other spinor modes. For example, the internal component of a Majorana–Weyl gravitino in nine dimensions yields a spinor of chirality opposite to the one present in the ten–dimensional $(1, 0)$ supergravity multiplet. The two build a Majorana spinor, so that at the ends of $\mathcal{I}$ one can relate them with $\Lambda = \gamma^r$, but the other Fermi modes of the Sugimoto model [7] do not satisfy the boundary conditions (1.3) compatibly with the full Lorentz symmetry of more than six non–compact dimensions. Notice, finally, that different choices of Λ at the two ends of $\mathcal{I}$ could be used, in general [18], to induce Scherk–Schwarz deformations [19].

These considerations have counterparts in AdS_{2n}, which have a boundary at infinity, so that, in view of the preceding discussion, chiral fermions are not compatible with their isometries. The chiral limit of a massive fermion propagator is indeed singular in AdS_4, while the order parameter $\langle\bar\psi\psi\rangle$ acquires a vacuum value inversely proportional to the AdS radius [21].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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