

Improved determination of $|V_{us}|$ with τ decays

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$|V_{us}|$ is computed using the exclusive τ lepton branching fractions $\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau)$ and $\mathcal{B}(\tau^- \rightarrow \pi^- \nu_\tau)$ and using the inclusive τ branching fraction to all “strange” final states, $\mathcal{B}(\tau^- \rightarrow X_s^- \nu_\tau)$, computed as the sum of all the relevant exclusive branching fractions. Assuming the Standard Model, the kaon branching fractions measurements $\mathcal{B}(K^+ \rightarrow \ell^+ \nu_\ell)$ and $\mathcal{B}(K^+ \rightarrow \ell^+ \pi^0 \nu_\ell)$ with $\ell = e, \mu$ are used to improve the experimental determination of $\mathcal{B}(\tau^- \rightarrow X_s^- \nu_\tau)$ and $|V_{us}|$.

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1. Introduction

$|V_{us}|$ determinations using τ measurements provide additional experimental information to the precise evaluations based on kaon measurements and lattice QCD estimates of form factors and decay constants. When the total branching fraction of the τ decaying to final states with strangeness [$\mathcal{B}(\tau \rightarrow X_s \nu)$ or \mathcal{B}_s] is used, $|V_{us}|$ can be computed without using lattice QCD results and therefore free of the related systematic uncertainties. \mathcal{B}_s is computed as the sum of all exclusive τ branching fractions into any strange final state. Its precision can be improved by using kaon measurements to predict some τ branching fraction [1], obtaining the most precise measurement of $|V_{us}|$ that is independent of lattice QCD techniques.

The τ branching fractions obtained in the HFLAV 2018 global fit of τ measurements [2] are used. That fit includes recent measurements made public by *BABAR* in 2018 [3, 4, 5].

2. $|V_{us}|$ determinations

We compute $|V_{us}|_{\tau s}$ using the total branching fraction of the τ to strange final states, \mathcal{B}_s , following Ref. [6]:

$$|V_{us}|_{\tau s} = \sqrt{R_s / \left[\frac{R_{\text{VA}}}{|V_{ud}|^2} - \delta R_{\text{theory}} \right]} = 0.2195 \pm 0.0019 ,$$

where $|V_{ud}| = 0.97420 \pm 0.00021$ [7], $R_s = \Gamma_s / \Gamma_e^{\text{uni}} = \mathcal{B}_s / \mathcal{B}_e^{\text{uni}}$, $R_{\text{VA}} = \Gamma_{\text{VA}} / \Gamma_e^{\text{uni}} = \mathcal{B}_{\text{VA}} / \mathcal{B}_e^{\text{uni}}$, \mathcal{B}_{VA} is the inclusive τ branching fraction to non-strange final states, $\mathcal{B}_e^{\text{uni}}$ is the universality-improved branching fraction $\mathcal{B}(\tau \rightarrow e \nu \bar{\nu}) = \mathcal{B}_e^{\text{uni}} = (17.814 \pm 0.022)\%$ [8, 9], and the SU(3)-breaking term $\delta R_{\text{theory}} = 0.242 \pm 0.033$ is computed using inputs from Ref. [6] and $m_s = (95.00 \pm 6.70)$ MeV [10] (the uncertainties on m_s have been symmetrized).

We compute $|V_{us}|$ using the ratio $\mathcal{B}(\tau \rightarrow K \nu) / \mathcal{B}(\tau \rightarrow \pi \nu)$ as

$$|V_{us}|_{\tau K/\pi} = |V_{ud}| \frac{f_{\pi^\pm} m_\tau^2 - m_\pi^2}{f_{K^\pm} m_\tau^2 - m_K^2} \sqrt{\frac{\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau) R_{\tau/\pi}}{\mathcal{B}(\tau^- \rightarrow \pi^- \nu_\tau) R_{\tau/K} R_{\tau K/\tau\pi}}} = 0.2236 \pm 0.0015 ,$$

where $f_{K^\pm} / f_{\pi^\pm} = 1.1932 \pm 0.0019$ from the FLAG 2019 lattice QCD averages with $N_f = 2 + 1 + 1$ [11, 12, 13, 14]. The radiative correction terms are $R_{\tau/K} = 1 + (0.90 \pm 0.22)\%$, $R_{\tau/\pi} = 1 + (0.16 \pm 0.14)\%$ [15, 16, 17, 18], $R_{\tau K/\tau\pi} = 1 + (-0.69 \pm 0.17)\%$ [19, 20, 21]. The other parameters are taken from the Review of Particle Physics (RPP) 2018 [10].

We compute $|V_{us}|$ using $\mathcal{B}(\tau \rightarrow K \nu)$ as

$$|V_{us}|_{\tau K} = \sqrt{\frac{\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau) 16\pi\hbar}{f_{K^\pm}^2 \tau m_\tau^3} \frac{m_\tau^2}{G_F^2} \frac{1}{m_\tau^2 - m_K^2} \frac{1}{R_{\tau/K} R_{K\mu 2}}} = 0.2234 \pm 0.0015 ,$$

where $f_{K^\pm} = 155.7 \pm 0.3$ MeV from the FLAG 2019 lattice QCD averages with $N_f = 2 + 1 + 1$ [11, 12, 22, 13], $R_{\tau/K} = 1 + (0.90 \pm 0.22)\%$ [15, 16, 17, 18] and $R_{K\mu 2} = 1 + (1.07 \pm 0.21)\%$ [20, 23, 24], which includes short and long-distance radiative corrections. The physical constants have been taken from RPP 2018 (which uses CODATA 2014 [25]).

The average of the three $|V_{us}|$ determinations is $|V_{us}|_\tau = 0.2221 \pm 0.0013$. All correlations documented in the HFLAV 2018 report have been included. The correlation between f_{K^\pm} and $f_{\pi^\pm} / f_{\pi^\pm}$ has been assumed to be zero.

Table 1: Deviations of $|V_{us}|$ computed with τ data with respect to $|V_{us}|$ obtained with CKM unitarity. The third row reports the $|V_{us}|_{\tau S}^K$ determination performed in this paper. The HFLAV Spring 2017 did not include the determination of $|V_{us}|_{\tau K}$ with $\mathcal{B}(\tau \rightarrow K\nu)$.

	$\Delta V_{us} _{\tau S}$ [σ]	$\Delta V_{us} _{\tau K/\pi}$ [σ]	$\Delta V_{us} _{\tau K}$ [σ]
HFLAV Spring 2017	-3.0	-1.0	
HFLAV 2018	-2.9	-1.2	-1.3
HFLAV 2018 + kaon predictions	-2.7		

3. τ branching fraction predictions from kaon measurements

Assuming the validity of the Standard Model (SM), three τ branching fractions have been computed using the precisely measured $K_{\ell 2}$ and $K_{\ell 3}$ branching fractions and the measured $\tau^- \rightarrow (K\pi)^- \nu_\tau$ spectra [1]:

$$\begin{aligned} \mathcal{B}(\tau^- \rightarrow K^- \nu_\tau) &= (0.713 \pm 0.003)\% , \\ \mathcal{B}(\tau^- \rightarrow K^- \pi^0 \nu_\tau) &= (0.471 \pm 0.018)\% , \\ \mathcal{B}(\tau^- \rightarrow K^0 \pi^- \nu_\tau) &= (0.857 \pm 0.030)\% . \end{aligned}$$

The uncertainties on the last two results are fully correlated. It has been observed [1, 19] that all the above indirect values are higher than the corresponding directly measured τ branching fractions. If the indirect values replace the direct ones, $|V_{us}|_{\tau S} = 0.2207 \pm 0.027$ [1].

We add the kaon-indirect determinations of the three above τ branching fractions to the dataset of the HFLAV 2018 global fit and we perform a new fit for the τ branching fractions. Using the results of this second fit, we obtain an improved calculation of $|V_{us}|_{\tau S}^K = 0.2202 \pm 0.0018$.

4. Consistency of $|V_{us}|$ with the CKM matrix unitarity

Assuming the CKM matrix unitarity,

$$|V_{us}|_{\text{uni}} = \sqrt{1 - |V_{ud}|^2 - |V_{ub}|^2} = 0.22565 \pm 0.00089 ,$$

using $|V_{ud}| = 0.97420 \pm 0.00021$ [7] and $|V_{ub}| = (0.3940 \pm 0.0360) \cdot 10^{-2}$ [10]. Table 1 summarizes the residuals, expressed as numbers of standard deviations, of the $|V_{us}|$ determination from τ data with respect to $|V_{us}|_{\text{uni}}$. The Table includes the $|V_{us}|$ values described in this work and the $|V_{us}|$ values listed in the HFLAV Spring 2017 report [8]. The discrepancies reported in the HFLAV Spring 2017 report persist with a moderate reduction in the slightly more precise values described here.

Figure 1 reports the $|V_{us}|_{\tau S}$ determinations described above together with the three results published in two recent papers [26, 27].

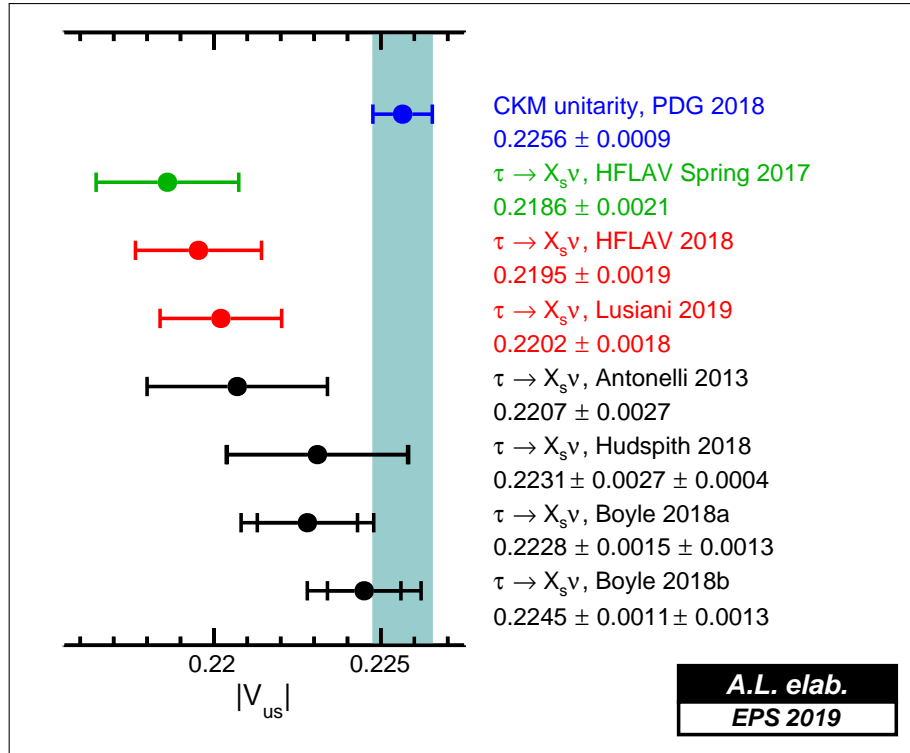


Figure 1: $|V_{us}|_{\tau_S}$ determinations. From the top: $|V_{us}|_{\text{uni}}$ as reported in RPP 2018 [10], $|V_{us}|_{\tau_S}$ with HFLAV Spring 2017 results, $|V_{us}|_{\tau_S}$ with HFLAV 2018 results, $|V_{us}|_{\tau_S}$ computed here with the results of a new fit on HFLAV 2018 data and the three τ branching fractions predicted from the kaon measurements, (Lusiani 2019), $|V_{us}|_{\tau_S}$ from Ref. [1] (Antonelli 2013), from Ref. [26] (Hudspith 2018) and from Ref. [27] (Boyle 2018a and Boyle 2018b). Please refer to the references for the details on the experimental measurement that have been used.

5. Conclusions

$|V_{us}|$ has been computed in several ways using τ lepton and kaon measurements. Recent new results made public by *BABAR* have improved the precision on the $|V_{us}|$ calculations based on τ measurements. When $|V_{us}|$ is obtained using the total inclusive τ width into “strange” final states, the value is significantly lower than the value of $|V_{us}|$ obtained from $|V_{ud}|$ and $|V_{ud}|$ assuming the CKM matrix unitarity. The significance of the anomaly moderately decreases when one includes the τ branching fractions measurements made public by *BABAR* in 2018 and when one includes three τ branching fractions predictions obtained using kaon measurements. Two recent studies point out that modifications of the $|V_{us}|$ τ -inclusive extraction procedure produce values that are consistent with the CKM matrix unitarity. More precise experimental data would be instrumental for a better understanding of the discrepancy.

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