

RECEIVED: March 18, 2025

REVISED: July 19, 2025

ACCEPTED: August 28, 2025

PUBLISHED: March 26, 2026

The Science of the Einstein Telescope

The Einstein Telescope collaboration

Full author list at the end of the paper

E-mail: marica.branchesi@gssi.it, archisman.ghosh@ugent.be,
michele.maggiore@unige.ch

ABSTRACT: Einstein Telescope (ET) is the European project for a gravitational-wave (GW) observatory of third-generation. In this paper we present a comprehensive discussion of its science objectives, providing state-of-the-art predictions for the capabilities of ET in both geometries currently under consideration, a single-site triangular configuration or two L-shaped detectors. We discuss the impact that ET will have on domains as broad and diverse as fundamental physics, cosmology, early Universe, astrophysics of compact objects, physics of matter in extreme conditions, and dynamics of stellar collapse. We discuss how the study of extreme astrophysical events will be enhanced by multi-messenger observations. We highlight the ET synergies with ground-based and space-borne GW observatories, including multi-band investigations of the same sources, improved parameter estimation, and complementary information on astrophysical or cosmological mechanisms obtained combining observations from different frequency bands. We present advancements in waveform modeling dedicated to third-generation observatories, along with open tools developed within the ET Collaboration for assessing the scientific potentials of different detector configurations. We finally discuss the data analysis challenges posed by third-generation observatories, which will enable access to large populations of sources and provide unprecedented precision.

KEYWORDS: gravitational waves / experiments, gravitational waves / sources, gravitational waves / theory, gravitational wave detectors

ARXIV EPRINT: [2503.12263](https://arxiv.org/abs/2503.12263)



Contents

1	General introduction	3
2	Fundamental Physics with ET	5
2.1	Introduction	5
2.2	Testing the fundamental principles of the gravitational interaction	6
2.3	Testing the nature of compact objects & horizon-scale physics	19
2.4	Searches for dark-matter candidates & new fields	30
2.5	Synergies with other aspects of the ET Science Case	38
2.6	Executive summary	39
3	Cosmology with ET	41
3.1	Stochastic gravitational-wave backgrounds	42
3.2	Probing the early Universe	60
3.3	Probing the late Universe with Einstein Telescope	93
3.4	Probing the large scale structure of the Universe	118
3.5	Executive summary	127
4	Population studies and astrophysical background	130
4.1	Introduction: formation channels of binary compact objects	130
4.2	Merger rate density of CBC across cosmic time	143
4.3	Mass function of BHs and its evolution with redshift	155
4.4	Constraining the mass function of neutron stars	164
4.5	Spins of stellar-origin BHs and NSs	168
4.6	Primordial versus stellar-origin BHs	174
4.7	Revealing Population III stars with the first BHs	182
4.8	Intermediate-mass BHs (IMBHs): formation channels and merger rate	191
4.9	The host galaxies of binary compact object mergers	199
4.10	Populations backgrounds	202
4.11	Executive summary	213
5	Multi-messenger observations in the ET era	216
5.1	State of the art	216
5.2	Modeling the EM counterparts of ET detected CB mergers	226
5.3	Observational facilities and strategies	244
5.4	Neutrinos from Compact Binary Coalescence	250
5.5	Multi-messenger infrastructure challenges	255
5.6	Executive Summary	256
6	Synergies of ET with other gravitational-wave observatories	259
6.1	Introduction	259
6.2	Compact object binaries	262

6.3	Nuclear physics	270
6.4	Fundamental physics	275
6.5	Hubble tension and cosmography	279
6.6	The origin of supermassive black holes	283
6.7	Early Universe cosmology	291
6.8	Executive summary	297
7	Subatomic Physics with ET	299
7.1	Introduction	299
7.2	Current status of microphysics properties	302
7.3	Prospects for constraints on microphysics with ET data	319
7.4	Uncertainties and degeneracies in our measurements	355
7.5	Executive summary	367
8	Stellar collapse and rotating neutron stars	370
8.1	Introduction	371
8.2	Stellar collapse	372
8.3	Neutron stars	404
8.4	ET observational prospects	413
8.5	Executive Summary	436
9	Waveforms	439
9.1	Introduction	439
9.2	Waveform systematics and accuracy requirements for 3G	440
9.3	Techniques for waveform modeling: Current state and advances	443
9.4	Waveform Models for Specific Sources	464
9.5	Waveform Acceleration Techniques	488
9.6	Executive summary	491
10	Tools for assessing the scientific potentials of detector configurations	493
10.1	Basic formalism	494
10.2	Software tools for CBC sources	500
10.3	Metrics for CBCs	514
10.4	Tools for the ringdown phase of binary mergers	531
10.5	Stochastic searches	534
10.6	Tools for the null stream	545
10.7	Conclusions	547
10.8	Executive summary	549
11	Data analysis	550
11.1	Introduction	550
11.2	Challenges	551
11.3	Innovative methods: machine learning applications	556
11.4	Signal detection method	558

11.5	Parameter estimation methods	565
11.6	Peculiarities of a triangular ET	574
11.7	Simulations and Mock Data Challenges	581
11.8	Challenges	583
11.9	Synergies in data analysis developments	583
11.10	Conclusion	585
11.11	Executive summary	586
12	Contributions	588
13	List of Acronyms	600
	The Einstein Telescope collaboration	885

1 General introduction

Thanks to the extraordinary discoveries of the LIGO, Virgo and KAGRA (LVK) Collaboration in the last few years (e.g. [1–9]), the field of gravitational-wave (GW) science is blossoming. However, the current second-generation detectors are nearing the limits of their capabilities, constrained by their infrastructures. As a result, the GW community is preparing the jump toward third-generation (3G) GW detectors, new observatories that are designed to detect GW sources along the cosmic history up to the early Universe. The 3G European observatory project is Einstein Telescope (ET) [10, 11], while the United States community effort is represented by the Cosmic Explorer (CE) project [12–14].

In recent years the ET project has undergone a significant acceleration, with the successful proposal in 2020 for including ET in the European Strategy Forum on Research Infrastructures (ESFRI) Roadmap (the roadmap which identify priority European research infrastructures), and the formal creation of the ET Collaboration at the XII Einstein Telescope Symposium in 2022.¹ The ET Collaboration (ETC) currently counts over 1800 members, representing 264 Institutions in 31 different countries. The ETC activities have been structured in four boards: the Instrument Science Board (ISB), for the design of the instrument; the Site Characterization Board (SCB) for the geological/geographic characterization of the site(s); the e-Infrastructure Board (eIB) for computing and services; and the Observational Science Board (OSB), whose goal is to develop the science case of ET, along with the associated data analysis that will enable the full exploitation of its observations. In addition, the OSB aims at developing the synergies with other GW observatories and with multi-messenger observations. In parallel, the ET Organization (ETO) has the mandate to coordinate the ET Project’s preparatory phase, which will conclude with the selection of the site, the decision on the configuration of the detector and the governance for the future organization, and to lay the foundations of the future legal entity that will be responsible for the operation of the ET on technical and administrative level.

The present paper summarizes work performed within the OSB (which currently counts more than 700 members) over the last few years, and provides an updated and comprehensive

¹See <https://indico.ego-gw.it/event/411/>.

study of the science that can be achieved with ET. In the context of the recent developments of ET, a first systematic discussion of the science case (building on many previous works) was presented in [15], which was part of the ESFRI submission. In a subsequent work [16] the ET science case was further developed, also in relation to the two possible geometries that are currently considered for ET, i.e. a single-site triangular geometry made of 3 nested detectors, and a network made of two identical L-shaped detectors in two distant sites (“2L” in the following). The present paper significantly extends these studies, thanks to the large community work performed within the OSB. While the objective of ref. [16] was to compare different ET configurations and designs by establishing a common set of modeling and astrophysical assumptions to build comparison metrics, the objective of the present work is to present a comprehensive overview of the current knowledge of GW sources detectable by ground-based observatories, identify the major open questions and the predicted impact of ET on the widest range of science cases.

The paper is organized as follows. Section 2 discusses how ET can be used to probe fundamental physics, such as testing fundamental aspects of gravitational interactions, compact objects and near-horizon physics, and dark matter candidates. Section 3 is devoted to cosmology; ET can probe early-Universe cosmology through various mechanisms that can produce stochastic GW backgrounds, and late-Universe cosmology through “standard sirens”. Section 4 discusses the impact that ET will have on our understanding of astrophysical populations; with its vast distance reach and large detection rates, $\mathcal{O}(10^5)$ binary black hole (BBHs) and binary neutron stars (BNSs) per year, ET will characterize these populations across the whole cosmic history, including the possibility of identifying black holes from the first stars (Population III stars) and primordial black holes, and the access to low frequencies will make it possible to observe population of intermediate-mass black holes up to $10^4 M_\odot$. Section 5 is dedicated to multi-messenger astronomy, where ET will force a change of paradigm on electromagnetic searches and characterization, making the rare multi-messenger observations of today a routine with possibly hundreds of detections per year. The synergies of ET with other GW detectors, such as Cosmic Explorer, LISA, deciHz detectors, or PTA are investigated in section 6. Section 7 is devoted to what can be learned on nuclear physics and matter under extreme conditions from ET observations of neutron star mergers, while section 8 discusses the perspective for detection for core-collapse supernovae, burst sources, continuous and long-transient sources. Sections 9, 10 and 11 address different aspects related to the development of forecasts and of data analysis for 3G detectors. In particular, section 9 discusses the requirements and advances in waveform modeling at the level of accuracy required by ET; section 10 presents public tools developed within the OSB to assess the scientific potential of ET in different configuration or networks, such as tools for performing relatively fast forecasts on large populations of compact binaries, ringdown studies or stochastic searches. Section 11 presents the challenges and current progresses toward setting up the data analysis for ET. Each section ends with an executive summary of its key science takeaways.

Forecasts will in general be presented using the same ET sensitivity curve already used in [16],² which are shown in figure 177. For parameter estimation forecasts that depend on astrophysical populations we use in general the same BBH and BNS catalogs as in [16].³ The comparison between the triangle and 2L geometry will be performed whenever significant.

2 Fundamental Physics with ET

In parallel with the anticipated diverse astrophysical applications of gravitational-wave astronomy in the forthcoming years, next-generation detectors hold the promise to explore foundational problems in gravity, high-energy, and particle physics. These explorations encompass scrutinizing the fundamental principles of the gravitational interaction, unraveling the nature of compact objects and horizon-scale physics, and identifying novel gravitational-wave signatures of dark matter surrounding or constituting compact sources. In this context, owing to its superior sensitivity and larger bandwidth, ET will be a unique discovery machine that will open the precision-gravity frontier, while intersecting various scientific domains and establishing profound synergies with waveform modeling, data analysis, population studies, nuclear and high-energy physics, as well as other branches of astrophysics and cosmology. A summary of the main — potentially game-changer — deliverables is discussed in this section.

2.1 Introduction

Gravity is the weakest and arguably the most enigmatic among the fundamental interactions. Its understanding is paramount to comprehend the evolution of the Universe and is intertwined with some of the deepest longstanding puzzles in physics, from the tiniest to the largest scales. Thanks to the advent of GW astronomy and observational BH physics, the last decade has witnessed the rapid increase in our understanding of the gravitational interactions of extreme sources. This momentum is far from being exhausted, with the forthcoming decades promising an even more dramatic improvement.

One of the fundamental science objectives of ET is to test General Relativity (GR), Einstein’s celebrated theory of gravity, the nature of GW sources, such as compact objects, and our understanding of matter and particle physics under the most extreme conditions. The output of ET observations will therefore have profound implications for diverse fields ranging from astrophysics, cosmology, nuclear and particle physics, to perhaps even the realms of quantum gravity. In parallel, the discovery potential in GW science has reinvigorated the ongoing efforts to construct high-accuracy waveform models, either through numerical simulations or using sophisticated analytic methods from particle physics that have been instrumental for “new physics” searches at particle colliders through various precision-data programs. The science of ET will then usher us into the precision-gravity frontier, leading the path forward to several tests which are inaccessible with present-day GW detectors, greatly impacting our understanding of the Universe. Devising these tests and describing the

²These curves have been computed in the context of the activities of ISB and are publicly available at <https://apps.et-gw.eu/tds/ql/?c=16492>, see in particular the Annex files; see also section 10 of this paper for more details. Some of the results presented have rather been obtained with the somewhat older ET-D sensitivity curve, which however will not affect significantly the results obtained.

³These catalogs are publicly available at <https://apps.et-gw.eu/tds/?content=3&r=18321>.

theoretical repercussions of GW observations is the main motivation of this section, developed in the context of the ET OSB “Fundamental Physics” Division.

This section will be dealing with several topics carrying a unique discovery potential, thus complementing the numerous deliverables for the science of ET discussed throughout other sections. It is divided into three broad areas: *Testing the fundamental principles of the gravitational interactions* (section 2.2); *Testing the nature of compact objects and horizon-scale physics* (section 2.3); and *Searches of dark-matter candidates and new fields* (section 2.4). For each of them we will discuss ET’s ability to detect (or constrain) fundamental physics beyond our present knowledge of the laws of nature, in the advent of a new era of explorations of the cosmos. Synergies with other OSB Divisions are briefly covered in section 2.5.

2.2 Testing the fundamental principles of the gravitational interaction

GR is an extremely successful theory describing the gravitational interaction in geometric terms. It is rooted in Einstein’s strong equivalence principle and based on a certain number of assumptions. Lovelock’s theorem states that GR is the only theory in four spacetime dimensions with second-order field equations that are diffeomorphism-invariant and built solely from the metric tensor [17]. Relaxing any of these hypotheses (e.g., considering extra dimensions, higher-order field equations, a massive graviton, or including new gravitational degrees of freedom) gives rise to multiple pathways toward alternative theories of gravity [18, 19]. The minimal and most explored deviation from GR is the inclusion of extra fields which, depending on the specific theory, can violate one or more of the pillars upon which GR is built. Therefore, searching for the effects of extra fields in gravitational systems provide a way to test GR in various regimes.

The emission of GWs from binary systems offers a unique possibility to test fundamental aspects of the gravitational interactions which are also responsible for the binding of compact objects, such as BHs and NSs, in Nature. This can be achieved from an accurate reconstruction of the early stages of an inspiral phase, or from the strongly-coupled regime of the merger, or by an accurate characterization of the coalescence’s remnant. In all these phases, GR deviations may manifest themselves in additional propagating degrees of freedom and dissipative channels, as well as other subtle nonlinear effects. The quest towards high-precision waveform models also unlocks new challenges in our understanding of the two-body dynamics that deserve further study. This is a fundamental problem on its own, independently of the phenomenological implications associated with GW science. For instance, both precision tests of the inspiral regime as well as numerical simulations for beyond-GR theories are crucial if one wishes to be agnostic about the origin of the GW signals. These topics are naturally framed within the context of fundamental aspects of gravitational interactions and covered in this section.

2.2.1 Tests of the inspiral dynamics

A model-independent approach to quantifying GR violations in the GW phase evolution is to allow for modifications in a limited number of post-Newtonian (PN) coefficients in the perturbative expansion [20–27]. Deviations in the amplitude can also be searched for, which is especially interesting in the presence of detectable higher harmonics of the basic

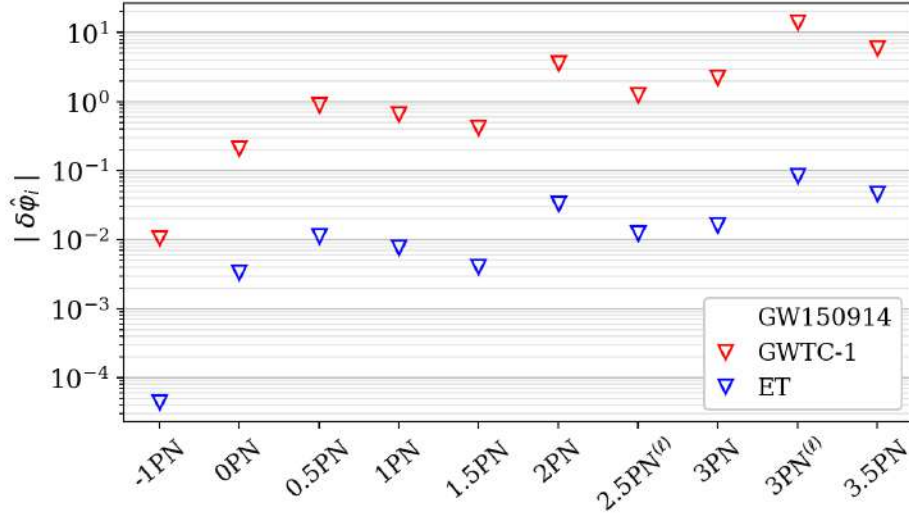


Figure 1. 90% upper bounds on the beyond-GR parameters $\delta\hat{\varphi}_n$ and $\delta\hat{\varphi}_n^{(l)}$ in the inspiral phase of GW150914, from -1PN to 3.5PN order. Red triangles represent bounds from Advanced LIGO observations [34], while blue triangles show bounds from simulated GW150914-like signals injected into synthetic noise for a single triangular ET.

signal [28–31]. More concretely, one considers the frequency-domain phase in the stationary phase approximation:

$$\Psi(f) = 2\pi f t_c - \varphi_c - \frac{\pi}{4} + \frac{3}{128\eta} \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^7 \left[\varphi_n + \varphi_n^{(l)} \log\left(\frac{v}{c}\right) \right] \left(\frac{v}{c}\right)^n, \quad (2.1)$$

where $v/c = (\pi M f)^{1/3}$, with M the total mass of the binary in the detector frame, t_c and φ_c are respectively the time and phase at coalescence, and $\eta = m_1 m_2 / M^2$, with m_1, m_2 the component masses. Within GR, the coefficients φ_n and $\varphi_n^{(l)}$ depend on the masses and spins of the component objects [32], and a series of tests is done on them [8]. For example, each of the φ_n are replaced by $(1 + \delta\hat{\varphi}_n) \varphi_n$, and the fractional deviation $\delta\hat{\varphi}_n$ is measured as a variable together with all other parameters entering the waveform; similarly for the $\varphi_n^{(l)}$. Very broad classes of modified theories of gravity can be mapped, to leading PN order, to specific PN deviations [23, 33]. For example, theories causing dipole radiation manifest themselves to leading order at $n = -2$ in the notation of eq. (2.1) (i.e., a -1PN correction). Since this term is absent in GR, one does a test with a coefficient $\delta\hat{\varphi}_{-2}$ multiplying the corresponding power of v/c ; in this case the testing coefficient represents an absolute rather than a relative deviation. The same holds at 0.5PN , since $\varphi_1 = 0$ in GR.

To get an idea of what to expect with ET compared to existing detectors for signals from binary BH coalescences, it is convenient to use phenomenological inspiral-merger-ringdown waveforms such as IMRPhenomPv2 [35–37], which includes the phase of eq. (2.1) as part of its description of the inspiral. Figure 1 shows 90% confidence bounds that were obtained on the $|\delta\hat{\varphi}_n|$ and $|\delta\hat{\varphi}_n^{(l)}|$ from Bayesian analyses of the first detected GW signal, GW150914, which indeed happened to be from a binary BH. These analyses also yielded maximum-likelihood values for the testing parameters along with the other parameters (masses, spins, sky position,

distance, ...) that characterize the signal [34]. Subsequently, simulated signals for those same parameters were “injected” into stationary, Gaussian noise following the noise power spectral density of ET, and the analyses were repeated to predict 90% confidence bounds one would obtain with the latter when observing a signal such as GW150914. As one can see, for the given signal, ET improves over Advanced LIGO in its first observing run by about two orders of magnitude for all the testing coefficients. However, it should be noted that it is possible to combine results from *all* the binary BH detections made (as is already being done with current detectors [8, 34, 38]). ET may observe $\mathcal{O}(10^5)$ such signals and, assuming the GR deviations are the same for all sources, measurement accuracies on the $|\delta\hat{\varphi}_n|$ will scale roughly as $1/\sqrt{N_{\text{det}}}$, where N_{det} is the number of detections. Therefore, ET will enable upper bounds that are well below 0.01 — or reveal a GR violation of such size, if present.

2.2.2 Extra polarizations

Metric theories of gravity allow for up to six different polarization modes, which can be categorized into tensor modes (the two polarizations present in standard GR), two vector modes, and two scalar modes [39, 40], which below will be denoted by T, V, and S, respectively.

Extra polarizations occur in a wide variety of alternative theories of gravity [41–53], and are thus an important science target for fundamental physics with ET. In the notation of Newman and Penrose [54], the scalar, vector and tensor polarizations can be written in terms of the Riemann tensor as

$$\begin{aligned}
 \Phi_{22} &= -R_{0x0x} - R_{0y0y} & (s=0), \\
 \Psi_2 &= -\frac{1}{6}R_{0z0z} & (s=0), \\
 \Psi_3 &= -\frac{1}{2}R_{0x0z} + \frac{i}{2}R_{0y0z} & (s=\pm 1), \\
 \Psi_4 &= -R_{0x0x} + R_{0y0y} + 2iR_{0x0y} & (s=\pm 2).
 \end{aligned} \tag{2.2}$$

However, they are not necessarily equivalent to dynamical degrees of freedom, and as pointed out in [39], the latter can be fewer or more than the number of polarization modes. For example, in various scalar-tensor theories of gravity, the number of dynamical degrees of freedom (three) is smaller than the number of polarization modes (four) [44]. On the contrary, in other models, where different fields produce the same polarization mode, the number of dynamical degrees of freedom is larger than the number of polarizations [55]. In general, the metric perturbation $h_{\mu\nu}$ can be decomposed into irreducible pieces according to their properties under spatial rotations, which can then be recombined into six quantities invariant under infinitesimal coordinate transformations (i.e., gauge-invariant at linear order) and which, under spatial rotations, transform as two scalars, two components of a transverse vector and two components of a transverse traceless tensor [56–60]. Examples of the polarization contents of some modified gravity theories considered in the literature are the following:

- (i) Scalar-tensor theories (in the metric formalism) add an extra mode associated to the scalar (propagating) field [41–43], an example of this being $f(R)$ gravity [44, 45].
- (ii) Einstein-aether theory predicts two tensor modes, two vector modes and one scalar mode [46, 47].

- (iii) The ghost-free massive gravity proposed by de Rham-Gabadadze-Tolley (dRGT) [61], in which gravity is mediated by a massive field m_s of spin $s = 2$, propagates $2s + 1 = 5$ polarizations [48].
- (iv) In Bumblebee gravity the polarization content is anisotropic, and depends on the angle between the background vector field and the propagation direction of GWs [49].
- (v) Randall-Sundrum braneworld models yield five bulk graviton polarizations [50].
- (vi) In the version of non-local massive gravity free of ghosts on Minkowski spacetime [62], one has five perturbative degrees of freedom. However, this model still has a cosmologically unstable mode [63].
- (vii) In the non-local infrared modification of gravity proposed in [64] (see [65, 66] for reviews) the non-local term can be localized using auxiliary fields that do not propagate, so only the two tensor modes propagate, while the non-propagating conformal mode becomes massive.
- (viii) A class of UV-complete (unitary and renormalizable) non-local theories of quantum gravity features eight non-perturbative degrees of freedom [67], just like non-unitary local Stelle gravity [68]. Out of these degrees of freedom, only two modes corresponding to the massless graviton propagate in Minkowski spacetime but the others can become non-trivial on curved backgrounds.

Given the fact that the existence of extra polarizations means extra energy emission channels, this can accelerate the orbital decay of binary systems. There are many studies on constraining modified gravity with observations of such decays, e.g. [69–71], as well as the phase evolution in the tensor modes, e.g. [19, 72, 73]. In what follows we discuss some general features of using measurements of the GW polarization to test deviations from GR.

2.2.2.1 Detector response to extra polarizations. Polarization tests typically rely on the fact that interferometers have different antenna responses to the different polarizations. When multiple, differently oriented detectors are available then the presence of non-standard polarizations can in principle be established. Currently, there are three major types of beam detectors: ground-based interferometric detectors, space-based interferometric detectors, and pulsar timing arrays. They have different response to different polarizations, and we refer to [74–80] for details.

For the case of stochastic GW backgrounds, cross correlation of the outputs of multiple detectors is needed [81, 82]. The sensitivity to the stochastic background of the detector pair depends on the antenna response of each detector, and it gets reduced due to the separation and non-optimal orientations of the detector pair. The reduction of the sensitivity is characterized by the overlap reduction function [80, 83] (see section 10.5.1). For different polarizations, the overlap reduction functions of various of ground-based detector network configuration are discussed in refs. [74, 75]. Different polarizations are projected on the strain via the antenna pattern function. Thus, for a network of multiple non-coaligned detectors, we get different linear combination of polarizations. Theoretically, for a transient signal,

the number of non-coaligned detectors should be at least the same as the number of the polarization modes in order to distinguish each of them. It should be observed, however, that the detector pattern functions of the two scalar modes are degenerate in the limit $f \ll f_*$, where $f_* = c/(2\pi L)$ (with L the interferometer arm-length) [74]; still, this degeneracy is lifted and the signals from the two scalar modes can in principle be disentangled at $f > f_*$ (see figure 24 in [75]). For $L = 10$ km (as in the 10 km triangle), $f_* \simeq 4.7$ kHz; for $L = 15$ km, as in the 2L configuration of ET, $f_* \simeq 3.2$ kHz; for a 40 km CE, $f_* \simeq 1.2$ kHz. Thus, for a stochastic background, only the part of the spectrum above such frequencies can be used to disentangle the two scalar modes.

The capability of the second generation detector network to separate the polarization modes for the inspiraling compact binary was studied systematically in ref. [84]. ET has great sensitivity at lower frequency, and thus it can observe the early inspiral stage of the binary neutron star for a long time. In this sense, a single ET detector can be regarded as a virtual detector network due to the Earth's rotation [85], which helps distinguishing the scalar, vector and tensor polarizations among them. Note, however, that for CBCs disentangling further the two scalar modes among them is more challenging since, particularly for a 10 km arm-length, basically all of the observable signal is at $f \ll f_*$ (and the Earth's rotation in this case does not help, since the degeneracy between the pattern functions of the two scalar modes at $f \ll f_*$ is not affected by the Earth's rotation).

2.2.2.2 Current and future constraint. Preliminary tests were carried out with GW events seen in both LIGO detectors and Virgo, e.g. GW170814 [86] and GW170817 [2]. For the first detected GW event, GW150914 [1], there was no conclusive constraint on the polarization because there were only two detectors and they have similar orientation [87]. In contrast, for the first two GW events detected by three detectors, GW170814 [86] and GW170817 [2], it is found that the purely tensor polarization is strongly favored over purely scalar or vector polarizations [86, 88]. The same test was also performed to GW170818 in GWTC-1 [34]. The test was based on the method proposed in [89], which applied coherent Bayesian analysis using GR waveform template but with different antenna response. For future studies, the precise waveform template in modified gravity is needed; for more details we refer to section 2.2.1 for inspiral, section 2.2.5.1 for merger and section 2.3.2 for ringdown. The Bayesian model selection method is also applicable for persistent sources, such as continuous waves from isolated neutron star [90] and stochastic gravitational wave background [91].

If the sky location of the GW source is known through an electromagnetic counterpart, one can linearly combine the multiple outputs from the detector network to construct a so-called *null stream*, in which the two tensor modes are eliminated completely [72, 92].⁴ The analysis of polarizations for signals like GW170817 with null stream was studied in refs. [93, 94], and then developed in refs. [95, 96]. The null stream method does not require precise waveform templates for the extra polarizations; one only needs to check whether or not the null stream is statistically consistent with the presence of noise only. Such analyses have been applied to the GW events in the GWTC-2 catalog of LIGO and Virgo [38], and in

⁴For a given signal with known localization, the null stream can be constructed also in the 2L geometry of ET. In the triangular geometry, one can construct a null stream simultaneously for all (tensor) signals, independently of the knowledge of the sky position; see sections 10.6 and 11.6.1 for extended discussion.

Tensor+Vector+Scalar	ET Δ + CE	ET 2L $\alpha = 0^\circ$ + CE	ET 2L $\alpha = 45^\circ$ + CE
$h_0^2 \Omega_{\text{GW}}^T$	2.80×10^{-10}	2.63×10^{-11}	1.24×10^{-10}
$h_0^2 \Omega_{\text{GW}}^V$	1.82×10^{-9}	3.24×10^{-11}	1.18×10^{-10}
$h_0^2 \Omega_{\text{GW}}^S$	1.13×10^{-9}	2.37×10^{-11}	8.17×10^{-11}

Table 1. Detectable energy density (SNR= 5 and $T = 5$ yr, integrated in the sense of the PLS), for a SGWB made of combination of tensor, scalar and vector modes with ET and CE in different configurations. In particular, for ET we consider the triangle configuration, 2L parallel, and 2L at 45° ; for CE we consider a single 40-km detector located at the Hanford site. The parameter ξ_s , which is used to characterize the contribution from breathing mode and longitudinal mode, is fixed to $\xi_s = 1/3$.

the GWTC-3 catalog [8], though the resulting constraints are not yet very informative. For ET, combined with other next-generation detectors, it has been demonstrated that one can probe the relative ratio of the amplitude of the scalar mode to the tensor mode at the level of $A_T^S \simeq 0.045$, and $A_T^V \simeq 0.014$ for the vector modes, from a single BNS merger located at 100 Mpc with optimal sky location [97]. Considering multiple events uniformly distributed in volume within a sphere with a large radius corresponding to a luminosity distance of ~ 1 Gpc, the detection limit can reach $A_T^S \simeq 0.05$ and $A_T^V \simeq 0.02$ on average; see figure 4 in [97].

Though stochastic GW backgrounds have not yet been found in the LIGO-Virgo-KAGRA band [98–100], they too offer opportunities for studying polarization content. The latest upper limits on the energy density spectrum (normalized by the critical density) are $\Omega_{\text{GW}}^S(25 \text{ Hz}) \leq 2.1 \times 10^{-8}$ for scalar backgrounds, $\Omega_{\text{GW}}^V(25 \text{ Hz}) \leq 7.9 \times 10^{-9}$ for vector backgrounds, and $\Omega_{\text{GW}}^T(25 \text{ Hz}) \leq 6.4 \times 10^{-9}$ for tensorial ones [100]. For ET together with CE as a network of next generation detectors, the optimal geometric configuration (location and orientation) to GW backgrounds of different polarizations is discussed in detail in [75]. In general, the sensitivity can reach $\Omega_{\text{GW}}^A \simeq 10^{-12} - 10^{-11}$ ($A = T, V, S$), for five years of observations [75], constituting two to three orders of magnitude improvement; see table 1 for details.

2.2.2.3 Polarization studies with lensed GW events. Returning to transient sources of GWs, even if an EM counterpart is available to localize a source on the sky, the current network of ground-based detectors is limited in its ability to disentangle the six polarizations, since this requires the number of detectors in the network to be at least as large as the number of polarizations to probe (and, even in this case, we have seen above that the two scalar modes can only be disentangled at $f > f_*$). While in the ET/CE era we will not have that many detectors, Nature offers us a way to artificially extend our detector network through *strong lensing* [101]. GW lensing is similar to that of light, where a massive object on the GW’s travel path affects its propagation. A lensed GW signal is observed as multiple ‘images’ that arrive on Earth at different times and have different (de)magnifications, but the same frequency evolution [101–104]. In between the arrival of two images, the Earth rotates and the detector network gets re-oriented, effectively becoming a different network. Thus, N detectors observing M images is equivalent to a network of $N \times M$ detectors observing a single signal. Note that strongly lensed images are separated on the arcsecond scale, while GW detectors have a precision of several degrees. So, the separation between the images is

completely swamped in the relatively low sky location accuracy from the interferometers, and the M images are effectively associated to the same direction in space.

If a BBH signal is strongly lensed, leading to e.g. 4 images, even a single triangular ET observatory becomes equivalent to a 12-detector network observing a single signal, with potentially excellent sky localizability. Since in the ET era one expects to observe tens to hundreds of strongly lensed events per year [105–107], strong lensing indeed offers the possibility of probing the polarization contents of GWs without the need for an EM counterpart.

To demonstrate the capacity of lensing to identify extra polarizations with future detectors, we perform Bayesian parameter estimation runs in which we inject signals with all six polarizations present. In figure 2, we show example posterior probability distributions for the relative amplitudes of the non-GR polarizations with a triangular ET, for the lensed and unlensed cases. For this event, the detector frame component masses are $73.6 M_\odot$ and $71.7 M_\odot$, and the signal-to-noise ratio is 100 (150) for the unlensed (lensed) case, the difference being indicative of what one can expect from the multi-image nature of lensed events. The relative amplitudes of the polarizations were picked randomly between 0 and 1. It is evident that in the lensed case, these relative amplitudes are well measurable, and zero is excluded for the given parameter choices, while in the unlensed case, the posterior distributions are essentially uninformative.

2.2.3 Anomalous gravitational-wave propagation, tests of Lorentz violation and minimal length

A feature of several extensions of the Standard Model (SM) of particle physics is that the dispersion relations of massive and massless particles gets modified. Furthermore, several arguments suggest that quantum gravity features a minimal length [108–113] and dispersion relations can be modified also by models of quantum gravity that incorporate a minimal length [114–116].

A possibility to test physics beyond the SM in the gravitational sector utilizes the GW signal from the coalescence of compact binary systems [117–124]. Another option is the induced anomalous GW propagation and its imprint on the waveform [125]. Violations of fundamental symmetries, such as Lorentz invariance, is also predicted in beyond-GR theories of gravity, which may also induce a modified dispersion relations [126–128]. In figure 3, we show the results of a simulation where BBH signals are injected into stationary, Gaussian noise, and analyzed in a Bayesian way. We show upper bounds on $A_0 = m_g^2$, with m_g the graviton’s mass, for (a) a network consisting of LIGO Livingston, LIGO Hanford, and Virgo (LVK), and (b) a triangular ET. The ET bounds improve on LVK by nearly two orders of magnitude, with only weak dependence on distance to the source. At relatively low luminosity distance this results from the higher SNRs of ET signals, whereas at larger distances we see the advantage of the longer path traveled by GWs. Other possibilities to perform this test are offered by the signal from deformed rotating NSs, and the search for continuous GW sources is ongoing [129, 130]. Overall, it is expected that the next-generation of GW detectors, in particular ET, will allow us to set stringent bounds on parameters characterizing violations of fundamental symmetries with an unprecedented level of accuracy.

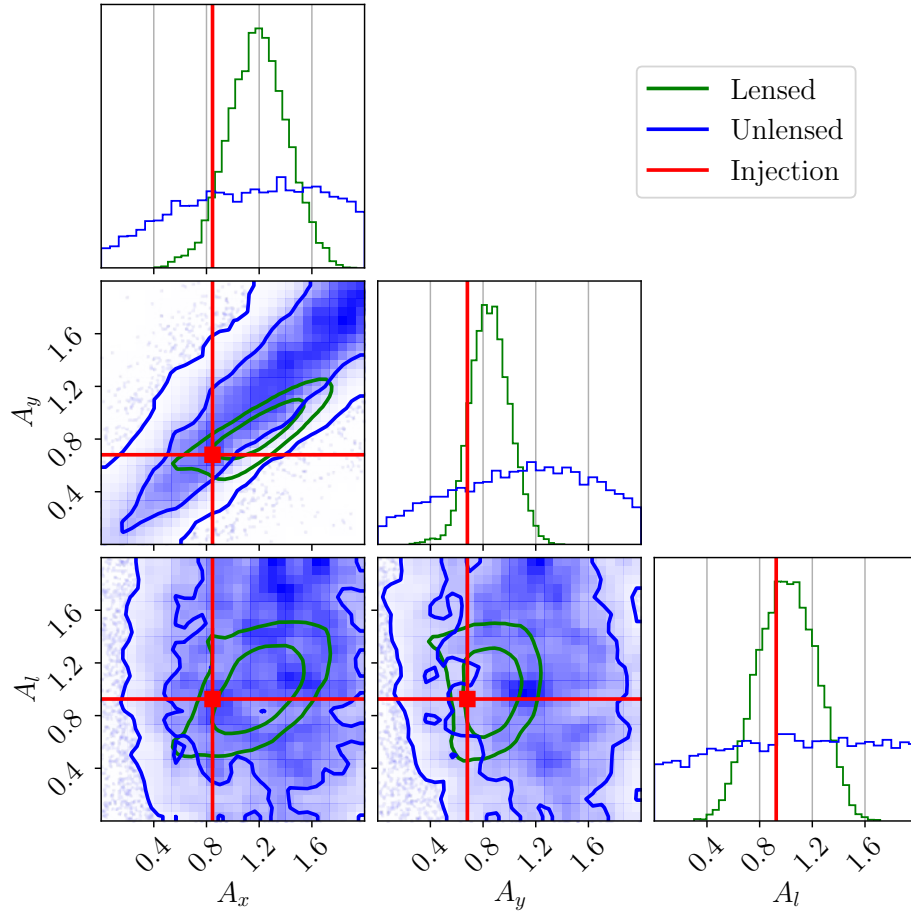


Figure 2. An illustration of strongly lensed GWs used to disentangle polarization states with a single triangular ET. Apart from the tensor polarizations we consider the two vector polarizations, x and y , and the longitudinal scalar polarization l . Posteriors are shown for the amplitudes of the vector polarizations relative to the $+$ and $\times$ amplitudes, which are denoted A_x and A_y , and similarly for the scalar polarization, where the relative amplitude is denoted A_l . Green indicates lensing, blue unlensed; solid lines show true values. Without lensing, posteriors are uninformative, while lensing enables individual polarization studies.

2.2.4 Gravitational-wave memory

A key aspect of GR is the expectation that GWs contribute also through non-oscillatory terms, known as the memory effect [133–137], often described as a displacement that manifests as a permanent offset between test particles after the passage of GWs. The gravitational memory represents a pivotal aspect of the so-called “infrared triangle”, connecting it with Weinberg’s soft graviton theorem [138] and the Bondi-Metzner-Sachs (BMS) symmetry group of asymptotically-flat spacetimes [139–141]. Later studies of BMS symmetries proposed more relaxed fall-off conditions for the spacetime metric near null infinity and have been shown to lead to new symmetries and new GW memory terms. In particular, one such null memory term is known as the spin memory [142]. These studies reveal the profound interrelations among disparate aspects of long-distance physics, offering new insights into gravity’s universal

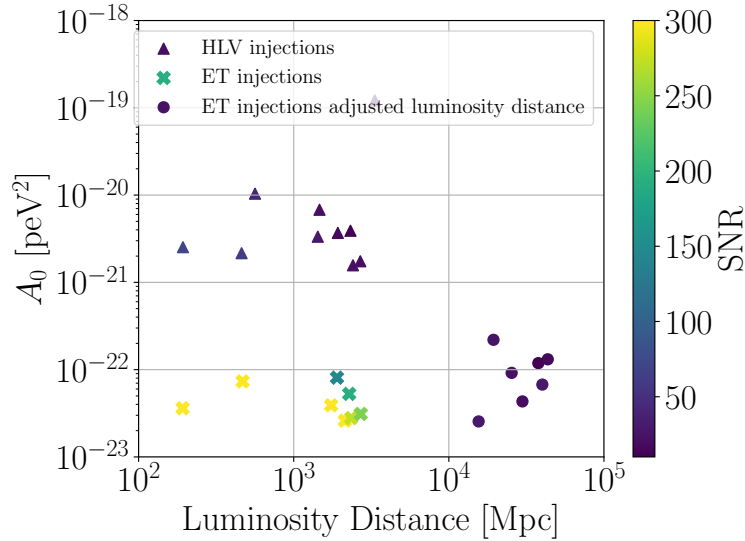


Figure 3. 90% confidence interval for the graviton mass, $A_0 = m_g^2$, as a function of luminosity distance for three GW injection sets. Triangles represent analyses with HLV, crosses and circles with a triangular ET; for the circles, luminosity distance was increased to match SNRs of LVK. The results demonstrate that ET improves bounds on A_0 both due to increased SNR for close-by events, and its ability to access events at larger distances despite their lower SNR.

characteristics [143]. The GW memory may be then used to experimentally unveil Nature’s true asymptotic symmetries [132].

The detection of the displacement memory may be within reach of currently operating ground-based detectors [132, 137, 144, 145], and efforts on this direction are underway [146, 147]. The displacement memory may further assist in breaking degeneracies in parameter estimation [132, 148], identifying low-mass BBHs [149], and help distinguish between BHs and NSs [150–152]. Next-generation ground-based detectors, such as ET, may also be able to detect spin memory effects [132, 137]. In figure 4, we show how evidence for the displacement and spin memory accumulates as a function of the observation time for various ground-based detectors and ET [132]. In short, operating at design-sensitivity, ET may be able to detect displacement memory with only a handful of BBHs, and it may be able to detect spin memory in less than a year of uninterrupted observations.

2.2.5 Fundamental aspects of the two-body problem

2.2.5.1 Numerical simulations beyond General Relativity. Numerical simulations are needed for the purpose of performing model selection and comparing the beyond-GR with the standard GR hypothesis in the nonlinear regimes probed by merging binaries or by the gravitational collapse. This topic is discussed in detail in section 9. However, even before attempting to model the signal to an accuracy level matching the ET sensitivity, just performing simulations beyond GR requires significant theoretical advances, notably in terms of viable theories and of a more rigorous understanding of the numerical evolution beyond GR. We discuss some paradigmatic examples below.

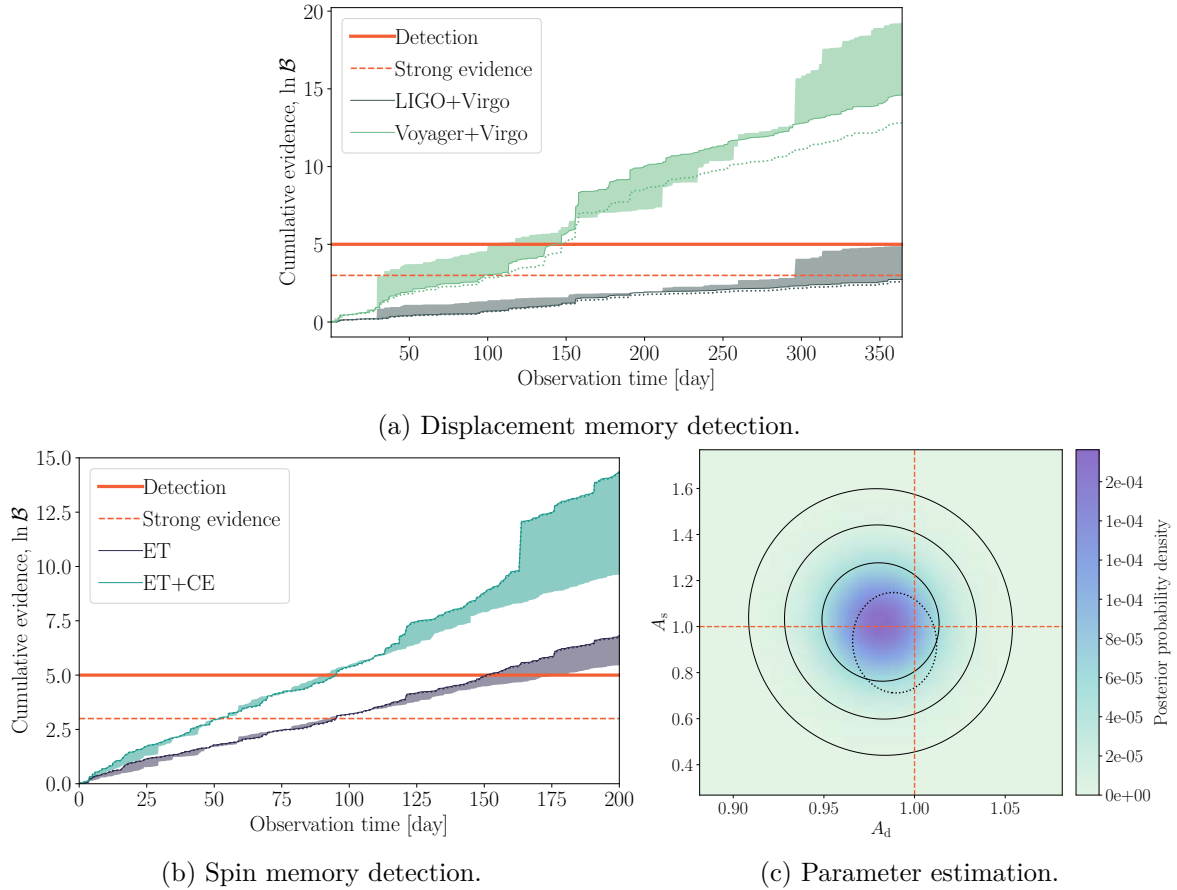


Figure 4. Panel (a): evidence (solid lines) or absence (dotted lines) for displacement memory based on multiple BH GW observations with the LIGO-Virgo network, as well as in a LIGO-Voyager network (where Voyager is a proposed upgraded of LIGO making use of the same LIGO facility [131]). Panel (b): spin memory evidence using ET. Filled regions in both panels show differences between expected and pessimistic BH merger rates. Evidence is plotted against observation time at design sensitivity. Panel (c): simulated measurements of displacement and spin memory strains A_d and A_s , from 1000 optimally-chosen BH mergers observed by ET. Solid curve denote the 1, 2, 3- σ credible levels, whereas the dotted curve denotes the 1- σ credible level up to redshift $z \approx 1$. We assume a 10 km triangular configuration for ET and a 40 km configuration for CE. See [132] for more details. Reprinted figure with permission from [132], Copyright (2024) by the American Physical Society.

BBH mergers. Tracking the merger of two BHs requires fully nonlinear numerical simulations. While these are viable in GR (see e.g. [153] and references therein), performing such simulations beyond GR can be challenging, not only because of the technical difficulties but also because of the need of a well-posed formulation. In fact, for many conjectured extensions, even the existence of such formulation remains an open issue. Nonetheless, in some cases, for instance Bergmann-Wagoner scalar-tensor theories [154], a well-posed formulation has been found [155] and BBH simulations have been carried out, e.g., in [156]. In this class of theories, however, stationary BHs are still described by the Kerr metric [157], yielding deviations from GR which are difficult to probe with BBH mergers. This limitation may be overcome if the scalar field is supported by time-dependent boundary conditions or spatial gradients [158].

Other type of scalar-tensor theories, like Einstein-Maxwell-dilaton gravity [159] and cubic Horndesky gravity [160], have also been proven to allow for a well-posed formulation, although hyperbolic evolution is guaranteed only in the so-called weak-coupling regime for the latter. Numerical simulations have been performed [161–163], and in some cases, if the scalar field is massive, a long-lived cloud may survive long enough to significantly affect the waveforms [159, 164]. Furthermore, numerical simulations of coalescing BBHs have also been performed for theories for which stationary BH solutions are different from those of GR [163, 165–167]. However, such simulations have shown the formation of elliptic regions, signaling the lack of predictability, when curvature invariants become large [168, 169].

An alternative approach consists of expanding the field equations in a small-coupling constant parameterizing deviations from GR. In principle, this can potentially circumvent the well-posedness issues, provided other constraints (such as, e.g., causality [170]) are enforced. Along these lines, an initial value formulation has been studied in various theories [171, 172], and simulations have also been performed through an expansion of the coupling constant [173–178]. However, these simulations show a secular growth in the inspiral phase, which signals the breakdown of the perturbative expansion at late times. These developments clearly illustrate the need to further strengthen the theoretical backbone of modifications of GR, and in particular a consistent embedding into a complete framework.

Another interesting aspect of scalar-tensor theories is the presence of so-called *spontaneous scalarization* [179–182], where BHs with a non-trivial scalar field configuration are also solutions of the theory. For certain values of the mass and/or spin, the Kerr solution becomes unstable, and spontaneously grows a scalar field. The dynamics of this mechanism was studied in the context of BBH merger, both in the limit of small coupling constant [183–185], and for finite couplings, within the modified harmonic formulation [163, 166, 167, 186].

In conclusion, despite recent progress, theoretical control over modifications of GR, and the accuracy of corresponding waveforms for the merger phase of BBHs, is not yet at the required level, and further development is required, to be ready when ET will become operational.

BNS mergers. Numerical simulations beyond GR have also been explored for BNS mergers, within the context of parameterized tests of GR (e.g. [187]) or theory-specific effective models for waveform building (e.g. [188]). It is worth mentioning that probes of modifications of GR with NSs almost inevitably suffer from uncertainties in the nuclear EoS, that are often degenerate with modified-gravity effects, see section 2.3.3 and section 7. While BBHs are cleaner from the point of view of matter “contamination”, BNS may display very different phenomenology [19]. An example of a strong-field effect that has been unveiled by numerical simulations is dynamical scalarization [189–192], originally discovered by Damour and Esposito-Farèse [193], who demonstrated that NSs may develop a nontrivial scalar charge. Observations of radio pulsars strongly constrain the scalar charge, and the parameter space for spontaneous scalarization [194–196]. Yet, BNS simulations have shown that large scalar charges are possible during the late inspiral and plunge phases, which can significantly impact the waveforms [189–191]. This is evident, for example, in figure 5 where the scalarized NS merge faster compared to GR. Theory-agnostic models [197] suggest that this effect is expected whenever the theory predicts scalarization of isolated compact objects, which has been confirmed, for instance, by numerical simulations of NS mergers in a class of theories

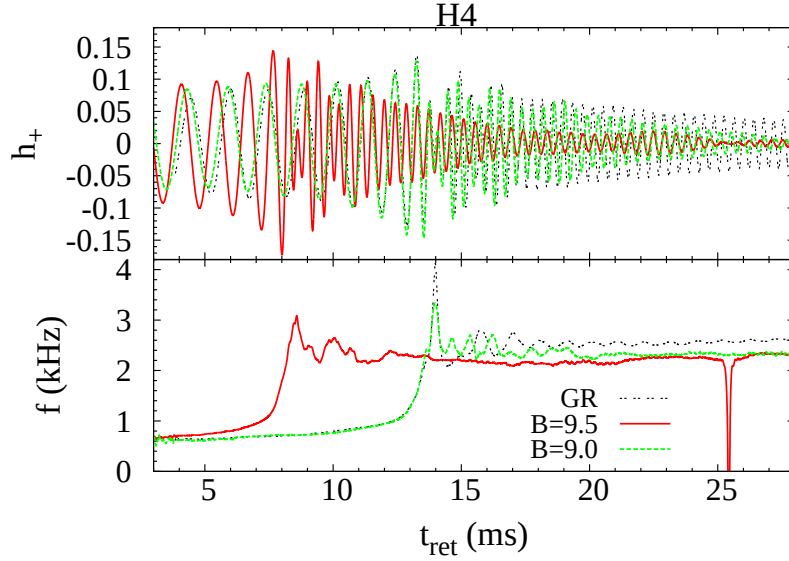


Figure 5. GWs from the coalescence of equal-mass NSs in a scalar-tensor theory compared to GR. *Upper panel:* plus polarization along the axis perpendicular to the orbital plane. *Lower panel:* GW frequency vs. retarded time. Simulations assume a binary total mass $m = 2.7M_{\odot}$ and the AH4 EoS, with B denoting the scalar-Ricci coupling strength. Reprinted figure with permission from [191], Copyright (2014) by the American Physical Society.

that admit scalarization [198]. The high sensitivity of ET in the kHz region makes it ideal for capturing (or constraining) such effects.

Numerical simulations in these theories have also revealed that the post-merger BNS signal may be appreciably different from GR both in scalar-tensor theory [191] and in theories of gravity involving higher curvature corrections [199]. For instance, the post-merger dynamics affects the lifetime of the remnant hypermassive NS, as well as the amount of ejected material which can source electromagnetic counterparts. The oscillation spectrum of the newly formed hypermassive remnant can be significantly altered by the presence of a scalar field and new types of scalar-field-induced modes can be excited. Since the post-merger signal is known to depend significantly on the nuclear EoS, many of the effects observed in these simulations may be degenerate with the unknown EOS in the NS core. Systematic investigations are therefore needed in order to identify distinct signatures of modifications to GR, such that the observation of the post-merger spectrum by ET will impact not only the understanding of nuclear matter but also putative new degrees of freedom.

Stellar core collapse. Stellar core collapse beyond GR was first studied in the Damour-Esposito-Farèse scalar-tensor model [200, 201]. The final outcome of the collapse strongly depends on the theory parameters, and it has been shown that three possibilities for the remnant are possible [201]: a GR (i.e non-scalarized) NS, a scalarized NS, or a short-lived proto-NS followed by the formation of a nonscalarized BH. An important advance was the extension to the case of massive scalar-tensor theories. The presence of a scalar mass reconciles the theory with binary pulsar observations for a larger range of parameters [202, 203], while providing interesting consequences for the GW signal [204–209]. The most prominent feature is the

possible existence of an inverse-chirp signal that can last for years with a near monochromatic signature on time scales of the order of one month. The results indicate that for a source located at $D = 10$ kpc, the resulting GW signal, observed for 2 months, can reach an SNR of over 1000 for ET [207]. Stellar core collapse has also been considered in other modified theories, e.g. [210], with rich phenomenology. For instance, the proto-NS oscillation modes, potentially detectable with ET, can be affected significantly by the presence of a scalar field [211, 212].

2.2.5.2 Analytical approaches. Time and again, the rich history of the two-body problem in gravity has shown that with any major advance, beyond just accessing the next level in the perturbative scheme of choice, post-Newtonian (PN) [32, 213, 214], post-Minkowskian (PM) [215], or self-force (SF) [216] approximations, the resolution of new fundamental aspects of the problem was needed. For instance, the complete understanding of the 4PN dynamics in GR was only possible after the interplay between near- and far-zone fields was clarified [217–220], allowing for a systematic resolution of the IR/UV divergences which (separately) plagued the potential and radiation regions [221]. Furthermore, recent developments in the study of scattering processes using techniques inspired by collider physics, e.g. [222–236], have also opened new venues towards simultaneously tackling near- and far-zone dynamics, while at the same time incorporating an infinite tower of velocity corrections at each order in Newton’s constant. To quote an example, the study of hereditary and radiation-reaction terms [227–230, 237–239] were instrumental in the determination of the imprint of nonlinear gravitational effects in the dynamics at 5PN order [240], as well as in the resolution of apparent discrepancies with the previous literature.

One of the key aspects in the computations of gravitational observables is the role of time nonlocality, already present at 4PN, due to tail effects [217–220]. These are nonlinear contributions associated with the emitted GWs interacting with the background geometry, yielding orbit-dependent results. At the same time, nonlocality may also be present for memory terms at 5PN order, due to the interaction with the earlier radiation instead, as well as in corrections stemming from second-order effects in the leading radiation-reaction force [240]. All of these subtleties illustrate the intricate connection between several physical effects, as we move forward in the perturbative expansion, in a nonlinear theory such as GR. Fortunately, many of these issues are naturally handled by importing ideas from particle physics — such as the effective field theory approach [213, 214, 241–244] — which provide a systematic framework to tackle the two-body problem in gravity [220, 240, 245–248].

Moving onto the case of scattering events, the state-of-the-art results in the PM expansion [215, 230, 233], endowed with a resummation scheme [249, 250], have proven to be very successful in matching numerical simulations with exquisite level of precision [251–253], see figure 6. For the case of bound states, on the other hand, fundamental challenges arise connecting between observables for unbound/bound motion. The relevance of scattering dynamics for the understanding of the bound problem was originally triggered by the observation that data from asymptotic states can also provide information about an effective potential [254, 255], as well as by the “boundary-to-bound” dictionary relating observables for hyperbolic (deflection angle) and elliptic (periastron advance) orbits through analytic continuation of the radial action [249, 250, 256]. The challenge, however, is to properly separate the universal part of the dynamics (local-in-time) from the orbit-dependent part (nonlocal-in-time) [218, 257, 258].

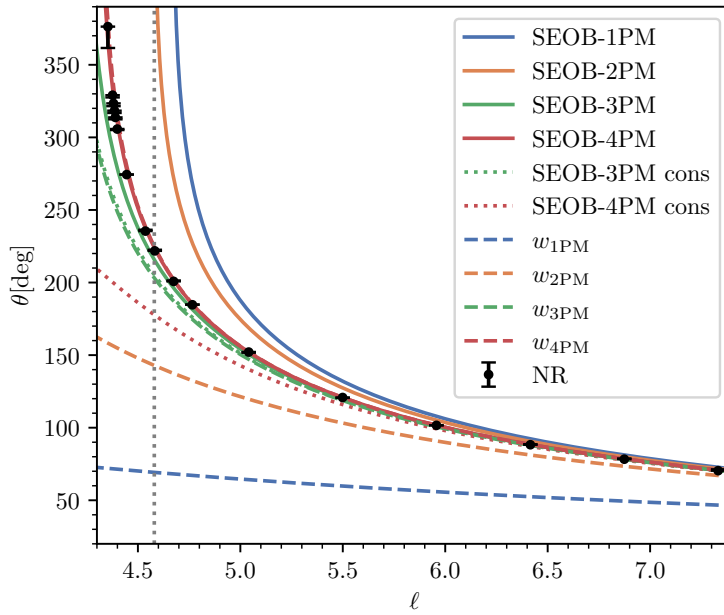


Figure 6. Comparison of the scattering angle in various PM approximations vs. numerical simulations. See [252] for details. Reprinted figure with permission from [252], Copyright (2024) by the American Physical Society.

While the latter is amenable to the boundary-to-bound connection [256], the former depends on the type of trajectory, and therefore cannot be directly extrapolated from hyperbolic to elliptic motion. This observation also directly impacts the expected connection between the mass-dependence of the scattering result and bound observables [249, 250, 259], which becomes even more subtle at higher PN/PM orders [235, 240].

Progress in the direction of capturing the universal part of scattering data has already appeared in [235], where the local-in-time contribution to the scattering angle was obtained at 4PM order, and a total bound Hamiltonian derived using a hybrid PN/PM scheme for the remaining nonlocal-in-time effects. However, due notably to the challenging “integration problem” [215, 234], a scalable approach, likely merging several perturbative (PN/PM/SF) schemes and novel ideas, will be sorely needed to achieve the level of precision required by ET. As expected, the hunt for accuracy will inevitably lead to a deeper understanding of the gravitational interactions, and perhaps to novel (non-perturbative) descriptions of Einstein’s theory, which will be put to the test in the new area of GW science. See section 9 for more details.

2.3 Testing the nature of compact objects & horizon-scale physics

Current electromagnetic and gravitational observations are compatible with BHs as predicted by GR [8, 34, 38, 87, 260–262]. An important open issue is to which level these observations confirm that dark compact objects have an event horizon concealing a curvature singularity, where it is also known that Einstein’s classical field equations must break down. From the theoretical standpoint, the possibility that quantum effects might not only resolve the

singularity but also modify the physics at horizon-size scales is attracting more interest. Several recent investigations also suggest that quantum gravity effects may show up at distances larger than the Planck scale [263–269], and make predictions that may be tested using GW astronomy [270–279] (see [280] for an overview). Another possibility is the presence of exotic compact objects (ECOs), which are regular (horizon-less) objects predicted in various gravitational models (e.g., fuzzballs [281], gravastars [282]), as well as produced by dark matter and/or exotic fields (e.g., boson stars [283], wormholes [284]). In the following, we will discuss the phenomenology of horizon-scale physics and ECOs, emphasizing their detectability with ET in a model-agnostic way, but also occasionally referring to specific models.

2.3.1 Inspiral tests

2.3.1.1 Model-agnostic tests of the multipolar structure. The multipolar structure of a Kerr BH is uniquely determined by its mass and spin [285], $M_\ell^{\text{BH}} + iS_\ell^{\text{BH}} = M^{\ell+1} (i\chi)^\ell$, where M_ℓ and S_ℓ are the mass and current multipole moments, respectively, $\chi := J/M^2$ is the dimensionless spin and M the mass. One can then parametrize deviations from the Kerr multipole moments for exotic compact objects in the following way:

$$M_\ell^{\text{ECO}} = M_\ell^{\text{BH}} + \delta M_\ell, \quad S_\ell^{\text{ECO}} = S_\ell^{\text{BH}} + \delta S_\ell, \quad (2.3)$$

with δM_ℓ and δS_ℓ model-dependent corrections. This parametrization can also be extended beyond axisymmetry [286–290]. Generically, the dominant multipolar effect comes from the quadrupole moment, M_2 . Figure 7 shows the projected constraints on the dimensionless parameter $\delta\kappa_s$ (defined as $\delta\kappa_s = \kappa_s - \kappa_s^{\text{Kerr}}$, where $\kappa_s = (\kappa_1 + \kappa_2)/2$ and $\kappa_i := -M_2/(M^3\chi^2)$ for each i -th binary component) from a typical ET observation and compared with LVK bounds. The constraints were obtained assuming aligned binaries, using an augmented TaylorF2 waveform, and both using the Fisher-matrix code GWFISH [291] and through a Bayesian inference with BILBY [292]. Interestingly, the Fisher-matrix and Bayesian results are in fairly good agreement between each other, especially for ET, as expected given the higher SNR. Overall, ET bounds can improve by more than an order of magnitude compared to future LVK observations, and the constraints with the 15 – km 2L configuration are better than those with a 10 – km triangular configuration by a factor of a few, in agreement with the analysis in ref. [16].

2.3.1.2 Model-agnostic tests of tidal effects. Since the BH’s of GR have been shown to have tidal deformability that vanishes in the static limit [293–296], the detection of a nonzero value would constitute a smoking-gun signature of new physics [297], either in the form of deviations from GR [298, 299], ECOs [298], or condensates of ultralight dark matter particles, e.g. [300]. Alternatively, it might signal dense environments around merging BHs [301, 302]. The largest effect is described by the quadrupolar, electric-type, Love number that parameterizes the proportionality between the gravitational field and the objects’ deformation. The Love numbers of various ECOs, including boson stars, were computed in [298, 303–309]. General prescriptions for computing quantum mechanical contribution to the Love numbers of astrophysical BHs were suggested in [273], and the classical limit was discussed in [275]. For instance, the area quantization [310] may induce nonzero Love

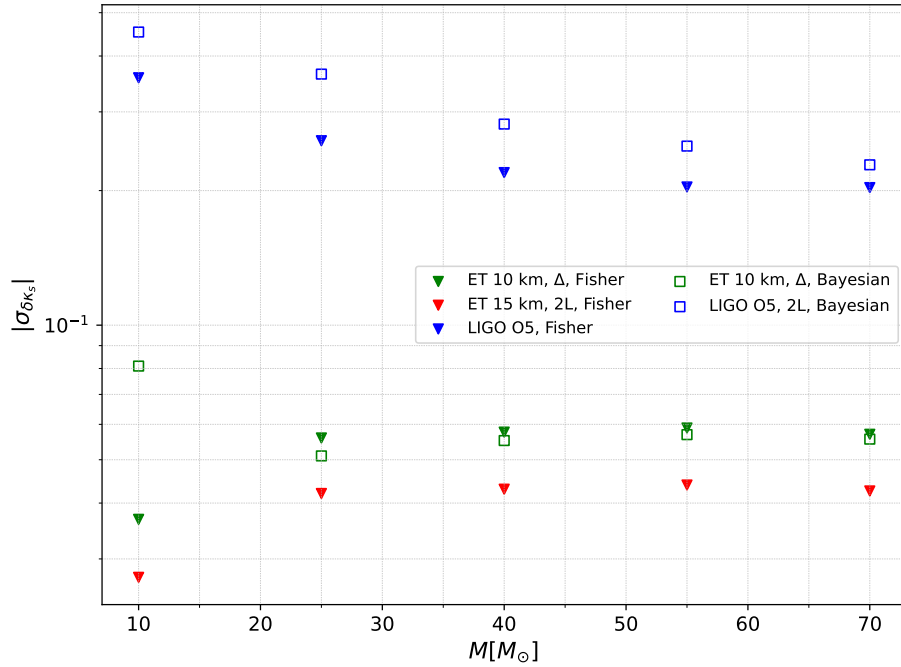


Figure 7. 68% constraints on δk_s , quantifying deviations from the Kerr quadrupole moment, as a function of binary total mass ($d_L = 400$ Mpc). We assume mass ratio $q = 3$, aligned spins $\chi_1 = 0.9$ and $\chi_2 = 0.8$, and inclination $\iota = \pi/3$. Blue/green triangles represent LIGO O5 and ET constraints (Fisher analysis); empty squares show Bayesian inference results. We show results for both a 10 – km triangular configuration and a 15 – km 2L configuration of the ET observatory.

numbers [278], and a model for dynamical Love numbers of ECOs was recently developed in [279]. Despite the differences among models, an overall common feature of ECOs, as well as regular (devoid of singularities) [311] or quantum [312] BHs is the presence of internal structures. As a result, their tidal response is expected to scale qualitatively similarly to those of NSs. It is thus straightforward to develop model-agnostic tests of the tidal deformability in compact binaries, by including tidal corrections to the waveform (starting at 5PN order) [298, 307]. Figure 8 shows the prospects for measurability of the Love numbers with ET and compares it with future LVK bounds (see [313] for a recent analysis with current LVK data). Also in this case we adopted an augmented TaylorF2 waveform, assuming aligned spins, and performed the analysis both with a Fisher matrix and with a Bayesian inference, finding good agreement and better constraints with the 15 – km 2L configuration [16]. As we see in the plot, the bounds with ET improve by roughly an order of magnitude relatively to LVK O5. For the case of stellar-mass objects (which can be confused with ordinary NSs) approximately universal relations between different tidal Love numbers [314] can be exploited to break the degeneracy among different models [309]. This requires measuring both the leading quadrupolar electric Love number, and a subdominant one (either the magnetic quadrupolar or the electric octupolar, entering at 6.5PN and 7PN, respectively), a challenging task that is possibly achievable only with next-generation GW detectors [315].

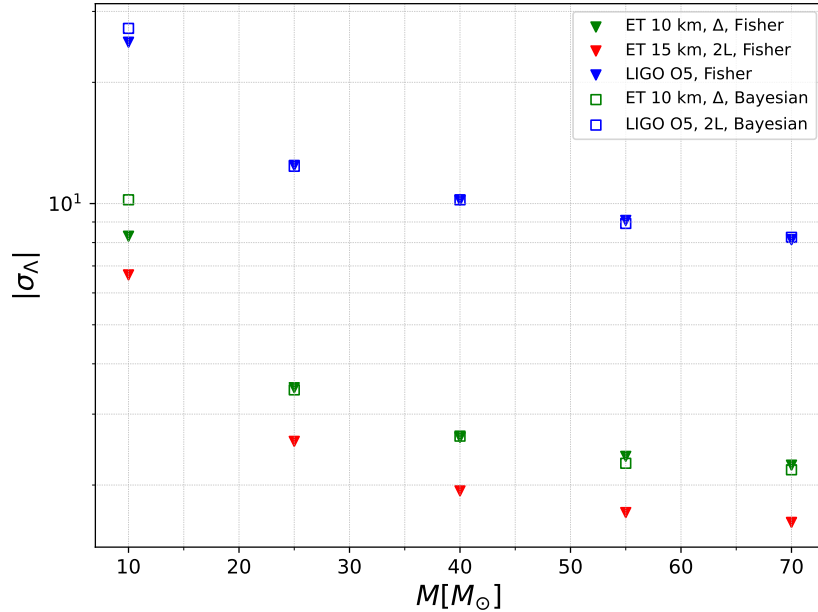


Figure 8. Same as in figure 7 but for the tidal deformability parameter Λ as a function of the total mass of the binary ($d_L = 400$ Mpc). In this case we assume mass ratio $q = 3$, zero spins, and inclination $\iota = \pi/3$. We show results for both a 10-km triangular configuration and a 15-km 2L configuration of the ET observatory.

Another peculiar property of classical BHs is the so-called tidal heating [243, 244, 316, 317], which may be also modified for quantum BHs, or suppressed for horizon-less compact objects. This effect is particularly significant for highly spinning BHs, and in the latest stages of the inspiral, since it enters the GW phase at 2.5PN (4PN) order for spinning (nonspinning) binaries. At the same time, it is also mildly correlated with the time and phase of coalescence in the waveform, and therefore hard to measure. Constraints of the amount of dissipation introduced by tidal heating are stronger for eccentric [318] or highly spinning binaries with large mass ratios [319, 320] and are particularly stringent for EMRIs/IMRIs [317, 320–327]. For comparable mass binaries, generic PN deviations are searched for with test-of-GR pipelines [8], whereas a specific analysis with LIGO data was performed in [328, 329]. EMRI/IMRIs are among the main sources of space missions such as LISA [330], but they may also be detectable by ET if subsolar-mass compact objects exist [331].

2.3.1.3 Constraining couplings to scalar fields. In addition to model-agnostic inspiral tests discussed above, for concrete models one can attempt to construct inspiral waveforms from first principles. In this case the deviations from the standard BH waveforms are not independent, but eventually dictated by the fundamental coupling constants of the model. For instance, in the case of a boson star, the tidal deformability of a spherically symmetric configuration can be shown to depend directly on the couplings of the theory [307], which in turn can be used to discern between fundamental interactions from measurements of finite-size effects in the waveforms [332].

Figure 9 presents a subset of posterior distributions illustrating the analysis of a boson-star binary signal injected in both ET and O4 noises. Since the fundamental coupling constant

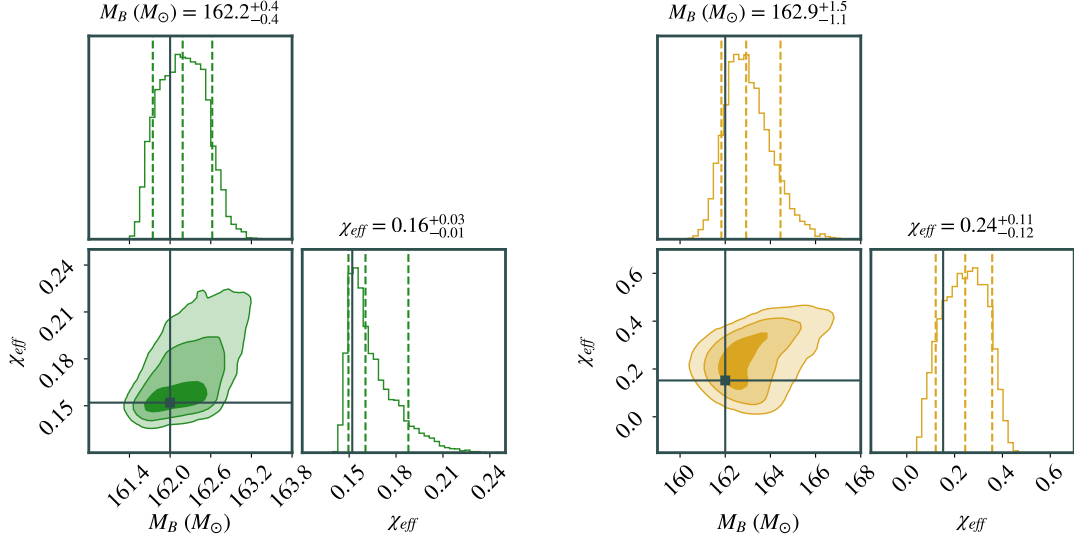


Figure 9. Posterior distributions for the effective spin parameter $\chi_{\text{eff}} = (m_1\chi_1 + m_2\chi_2)/(m_1 + m_2)$ and $M_B \equiv \sqrt{\lambda}/\mu^2$, with (μ, λ) the boson’s mass and self-interaction coupling, from a mock boson-star binary signal with ET (left) and LVK O4 (right) at $d_L = 400$ Mpc. The binary has chirp mass $\mathcal{M} = 8M_\odot$, mass ratio $q = 0.6$, and component (aligned) spins $(\chi_1, \chi_2) = (0.2, 0.1)$. The inspiral cutoff frequency is fixed at the Roche frequency $f_{\text{RO}} = 120$ Hz, corresponding to the binary tidal disruption.

M_B is related to the tidal deformability only logarithmically [332], the constraints improve only by a factor three. Nevertheless, ET capability of constraining the fundamental couplings of a scalar field forming a boson star at the subpercent level will help discriminate between various classes of compact objects and ultimately facilitate model selection.

2.3.2 Ringdown and post-merger tests

The ringdown waveform, corresponding to the phase when the binary coalescence settles down and approaches stationarity, can be modeled as a superposition of quasinormal modes (QNM), writing $h(t) = h_+(t) + ih_\times(t)$, with

$$h_{+,\times}(t) = \sum_{lmn} \mathcal{A}_{lmn} \cos\left(2\pi f_{lmn}t + \phi_{lmn}^{+,\times}\right) e^{-t/\tau_{lmn}} \mathcal{Y}_{+,\times}^{lm}(\iota), \quad (2.4)$$

where $\phi_{lmn}^\times = \phi_{lmn}^+ - \pi/2$, ι is the spin inclination angle of the remnant with respect to the line of sight, and $\{f_{lmn}, \tau_{lmn}, \mathcal{A}_{lmn}, \phi_{lmn}^+ \equiv \phi_{lmn}^+\}$ are the frequency, damping time, amplitude, and phase of the (lmn) quasinormal mode, respectively, and the $\{+, \times\}$ polarizations should be defined using a spin-weighted spheroidal harmonics basis. The integers (lmn) refer to the multipolar, azimuthal, and overtone index, respectively, where $n = 0$ corresponds to the fundamental tone and $(lmn) = (220)$ is the dominant mode in a non-precessing quasicircular coalescence. The amplitude scales as $\mathcal{A}_{lmn} \sim M_f/d_L$, where M_f is the mass of the remnant BH and d_L is the luminosity distance from the source [333].

The (complex) QNM frequencies, $f_{lmn} - i/\tau_{lmn}$, depend only on the remnant’s properties. If the latter is a Kerr BH, the entire spectrum depends only on the mass and spin. Hence, observing multiple ringdown modes from a single source provides a null-hypothesis test of

the nature of compact objects. Furthermore, most modified gravity theories and models of ECOs predict extra modes not present in GR [334] and different amplitudes and phases compared to the standard BH binary case in GR [335], so both effects can be studied in a model agnostic way.

The prospects for accurately observing ringdown signals with ET are extremely promising [336, 337]. ET will allow us to transition from observing such events occasionally, and with moderate SNR, to becoming almost daily occurrences extracted with great accuracy. For this reason, testing GR and the nature of the remnant with ringdown signals is a key science objective of ET, as we detail below.

2.3.2.1 Black-hole spectroscopy. Using stellar mass BBH population estimates calibrated to LVK catalogues up to O3 [338], ET is expected to observe $\sim 10^4$ events with a ringdown-only SNR (ρ_{RD}) bigger than 10 of which $\sim 5/\text{yr}$ will have $\rho_{\text{RD}} > 100$ [16, 337]. Furthermore, with ET we expect a ringdown-horizon up to $z \sim 10$ [339]. Numerical simulations show that, in non-precessing quasi-circular BBH ringdowns, the dominant mode is the $(2, 2, 0)$ mode, while the most excited subdominant modes are the $(3, 3, 0)$, $(2, 1, 0)$ and $(4, 4, 0)$ modes [333, 335, 340, 341]. The frequency and damping time of the dominant mode can be inverted to infer an independent measurement of the remnant’s mass and spin, while any subdominant mode can be used as a null-hypothesis test of GR, since they are fully determined once the fundamental mode is fixed. Ref. [337] quantifies ET’s measurability of subdominant modes and prospects of BH spectroscopy. ET is expected to measure $\mathcal{O}(10^3)$ ringdowns/yr with $\mathcal{O}(10)\%$ precision for deviations of the subdominant frequencies from the GR predictions, whereas $\mathcal{O}(10)$ ringdowns/yr are expected to have $\mathcal{O}(1)\%$ precision, see left panel of figure 10. Constraints on deviations of the damping times are less precise and therefore will produce weaker tests of the BH spectrum. Apart from stellar mass BH binaries, ET will also be able to perform a reasonably good BH spectroscopy with a measurability $\sim 10\%$ for $\sim 5 - 20$ events/yr from light seed supermassive BH population [342], thus providing complementary information to LISA [330]. The possibility of using pre-merger information for ringdown based tests has also been explored in [343] which shows promising prospects for the SNRs attainable by ET. Furthermore, ET will be able to put strong constraints using consistency tests based on ringdown mode amplitudes and phases [335], see right panel of figure 10.

In order to demonstrate the impact of ET in performing BH spectroscopy and the large improvement with respect to present GW detectors, figure 11 (left) shows that ET will allow measuring multiple ringdown frequencies with $\mathcal{O}(1\%)$ precision from individual events, and (right) that $\mathcal{O}(10\%)$ constraints will be placed on the magnitudes of the ringdown amplitudes and phases, thus allowing for novel tests of the BH nature [335, 344]. These results are in agreement with a Bayesian analysis on the detectability of a secondary (330) mode (see figure 12), which is very challenging with current detectors, mostly due to the limited SNR [30] (see also figure 11).

Exceptionally loud events with $\rho_{\text{RD}} > 100$ are expected in ET on a yearly basis and they will deliver the tightest constraints as individual events. Regarding the ability of distinguishing a ringdown waveform in GR from that in an alternative theories of gravity, a study in ref. [345] shows that percent-level deviations from GR can be identified with SNR $\rho_{\text{RD}} \sim \mathcal{O}(150 - 500)$, which is well within the ET capabilities. These results are also

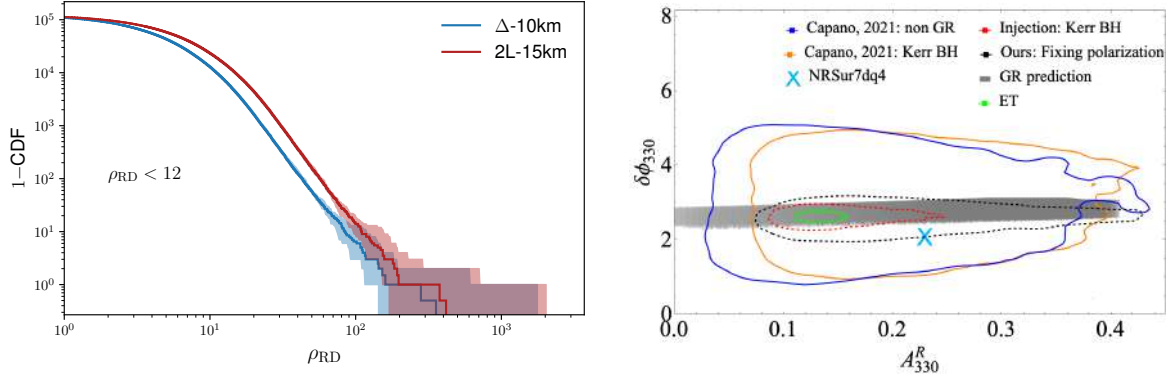


Figure 10. *Left:* inverse cumulative distribution of ringdown SNRs ρ_{RD} for different detector configurations. The shaded band corresponds to signals with $\rho_{\text{RD}} < 12$. Reprinted figure with permission from [337], Copyright (2023) by the American Physical Society. *Right:* forecasts for measuring the amplitude and phase of a secondary ($lmn = 330$) ringdown mode in a GW190521-like event detected by ET, compared with current uncertainties and with the GR prediction. Extended from [335]. Reprinted figure with permission from [335], Copyright (2023) by the American Physical Society.

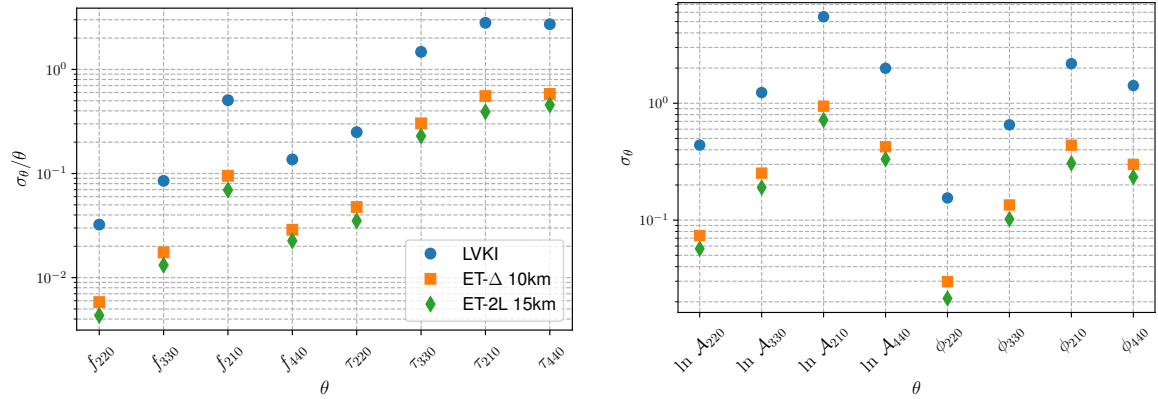


Figure 11. BH spectroscopy with a GW150915-like event using LVKI or ET. *Left:* bounds on the frequencies f_{lmn} and damping times τ_{lmn} . *Right:* bounds on the excitation amplitudes $\mathcal{A}_{lmn}$ [333, 340] and phases ϕ_{lmn} .

corroborated by specific case studies in modified theories of gravity [346–350]. They rely on the QNM spectra in modified theory which are currently available for Kerr-Newman BHs in Einstein-Maxwell theories [351], slowly-spinning BHs in theories with quadratic curvature corrections [348, 349, 352–354], Horndeski gravity [355], and higher-order effective-field-theory gravity [299] to different orders in spin and coupling constants.

Finally, at large SNR the ringdown analysis might be affected by next-to-leading order effects in perturbation theory [356–360]. A recent analysis showed that the dominant nonlinear mode may be observable by next generation detectors for few tens of events per year, although such results depend on the event rates [361]. The loudest signals can provide constraints on the quadratic mode frequency and damping times of the order of few and ten percent, respectively.

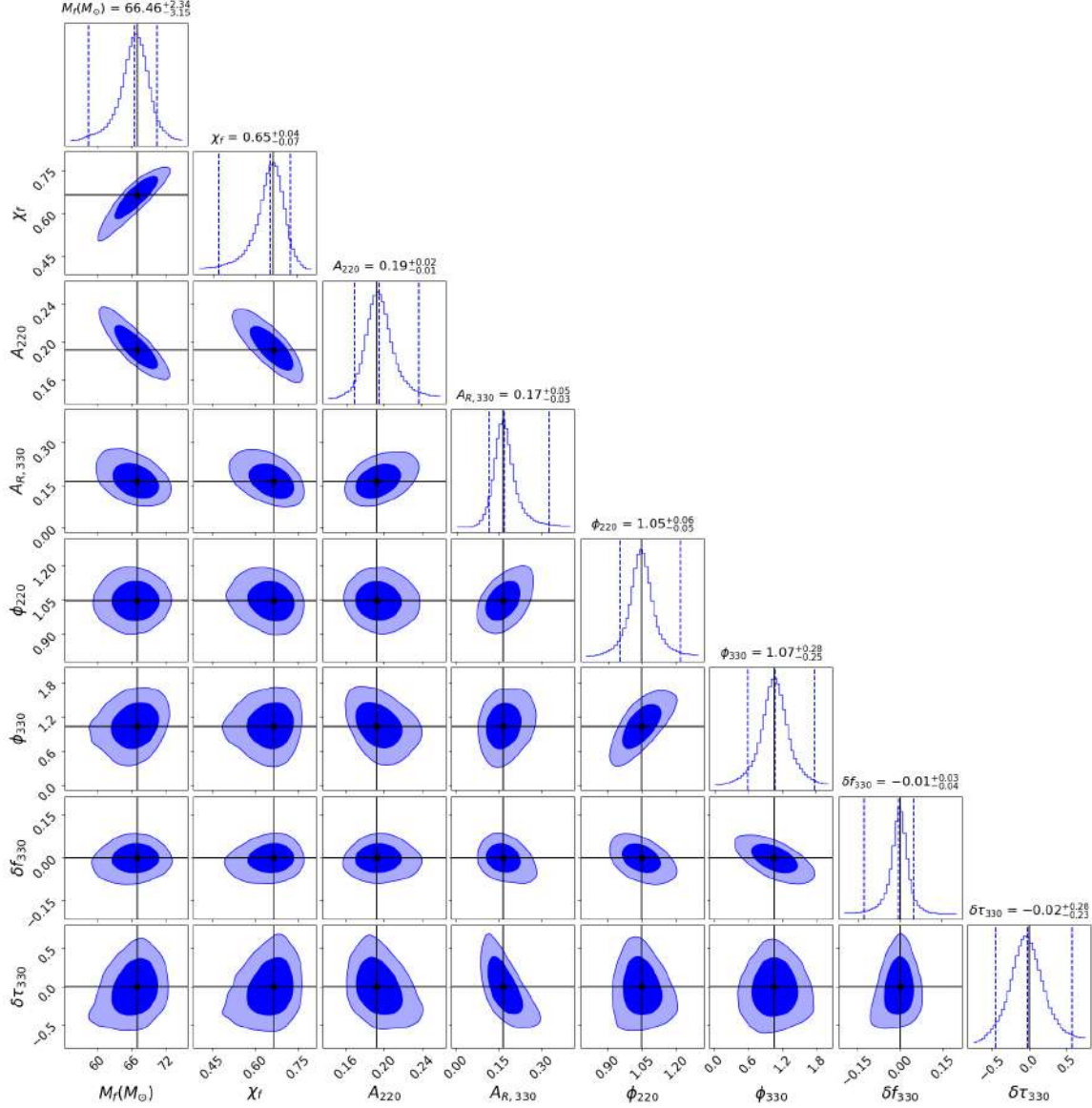


Figure 12. Posteriors of the final mass, spin, and amplitudes/phases of the 220 and 330 modes, and of the 330 GR deviations (δf_{330} and $\delta \tau_{330}$) for a GW150914-like remnant, detected with ET in the standard 10-km triangular configuration. Here, A_R is the relative mode amplitude. The 2D distributions show 68% and 90% credible levels, while the dashed lines in the 1D distributions mark the 90% credible intervals. Black lines indicate the injected values.

2.3.2.2 Tests of exotic compact objects with quasinormal modes and echoes. If the remnant is an exotic compact object (ECO), the amplitude of the QNM will be different, and new modes associated with matter fields may be excited. For a nonspinning object the perturbations of the remnant can be classified as axial or polar, depending on their parity. For BHs, axial and polar modes are isospectral, whereas for a horizon-less compact object, axial and polar QNMs form a doublet [362]. In the limit in which the ECO is almost as compact as a BH, the deviations from the BH QNM spectrum are increasingly large and

the QNMs are low-frequencies and long-lived [363, 364]. These modes are trapped in the cavity of the effective potential formed between the light ring and the compact object interior. Furthermore, the internal structures of regular [311] or (integrable) quantum [312] BHs can induce modifications in the outer geometry predicted by GR [365]. These modifications will affect the geodesic motion and ringdown phase after the merger. For example, by modeling the external geometry with a quantum coherent state, one expects deviations as large as 5% from the QNMs of the pure Schwarzschild geometry [266]. Measuring QNMs may also enable tests of black hole area quantization [272, 366]. This is being explored through analytical methods incorporating a minimum-length metric to study the impact of area quantization on the ringdown [367–369].

In the time domain, the prompt ringdown signal of ultracompact objects would be nearly indistinguishable from that of a BH since it is due to the excitation of the light ring [363]. If the object does not absorb GW radiation, a train of modulated pulses – known as GW echoes – would be emitted in the late post-merger stage in addition to the ringdown expected from BHs [363, 364, 370, 371]. The amplitude of GW echoes depends on the boundary conditions describing the compact object and the initial conditions of the perturbation [372–377]. In particular, the echo amplitude depends on the reflectivity of the compact object to ingoing perturbations, whereas a BH is described by null reflectivity at the horizon. Several searches for GW echoes have been performed based on the superposition of generalized wavelets adapted from burst searches [378, 379], Fourier windows using the fact that GW echoes should pile up at specific frequencies [380, 381], and the coherent WaveBurst pipeline [382]. Following some debated claim [379–381, 383–388], the LVK Collaboration found no evidence of echoes in GW data [8, 38], both using a template-based search [383, 387] in the second GW transient catalog and a morphology-independent search [378, 379] in the third catalog. The ET observatory has the potential to observe (or rule out) GW echoes emitted by horizon-less compact objects in a much larger region of the parameter space relative to the one currently available [376, 389–391]. It is estimated that models with perfect reflectivities can already be detected or ruled out by Advanced LIGO and Virgo; whereas excluding or detecting echoes for models with generic reflectivities as small as a few percent requires $\text{SNR} \gtrsim 100$ in the ringdown, which will be achieved by ET [389, 390]. Ref. [16] assessed the measurability of a GW150914-like event with different ET configurations, particularly the triangular-10 km and 2L-15 km designs. The parameters of the system are final mass $M = 70M_\odot$ and final spin $\chi = 0.68$ placed at a fiducial distance $d_L = 1$ Gpc. If measured by ET, the first GW event would have had a ringdown SNR between 141 and 196 for the triangular-10 km and 2L-15 km configurations, respectively. Figure 13 shows the fractional percentage errors on the reflectivity of compact objects expected for a GW150914-like event with ET compared to the Advanced LIGO and Virgo noise curve used for O4 simulations [392]. The accuracy on the reflectivity is affected by an order of magnitude between the O4 LIGO-Virgo and ET configurations (see table 2).

2.3.2.3 Nonlinear dynamics of exotic compact objects. Previous tests focused on specific waveform parts (inspiral or ringdown) as these regimes are conveniently modeled perturbatively, forming a useful basis for model-independent parametrizations. More information can be extracted from the full coalescence signal (inspiral, merger, ringdown) by

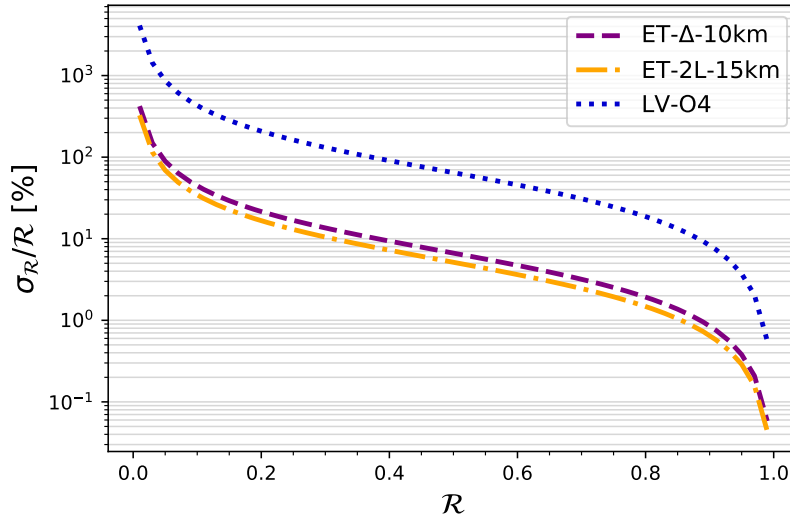


Figure 13. Fractional percentage errors on the reflectivity of compact objects for a GW150914-like event [16]. Reproduced from [16]. The Author(s). CC BY 4.0.

Detector configuration	$\sigma_{\mathcal{R}}/\mathcal{R}[\%]$		
	$\mathcal{R} = 0.01$	$\mathcal{R} = 0.5$	$\mathcal{R} = 0.99$
LIGO-Virgo O4	4104	67	0.56
ET Δ -10 km	422	6.9	0.058
ET 2L-15 km	327	5.3	0.044

Table 2. Fractional percentage errors on the measurability of the reflectivity of compact objects for small ($\mathcal{R} = 0.01$), medium ($\mathcal{R} = 0.5$) and large ($\mathcal{R} = 0.99$) reflectivity of a GW150914-like event [16]. Reproduced from [16]. The Author(s). CC BY 4.0.

specifying an ECO model. This requires NR simulations which are primarily available for consistent models like boson stars [283]. For instance, for the case of boson stars, simulations of binary mergers have shown that collisions may result in a more massive star or a BH [393]. The stars' intrinsic properties affect the emitted GW energy and waveform [393, 394]. For example, phase opposition between stars could lead to a repulsive force forming a dipole configuration [395]. Orbital mergers of boson stars have confirmed that more compact stars may collapse into a BH, emitting a similar signal to a BH binary merger [396, 397]. GW templates from Proca star collisions have been compared to events like GW190521, suggesting that ECO hypotheses can sometimes fit observations slightly better than traditional binary BH models, though more studies are needed to extend these models [398, 399]. While overall it is challenging to distinguish ECOs from BBH mergers with current observations, the ET improved sensitivity, especially at low frequencies, could ultimately reveal smoking guns of ECOs during the coalescence. Furthermore, if ECOs are present in Nature, they could form hybrid binaries with BHs and NSs, possibly producing an electromagnetic counterpart [400]. Highly compact boson stars are also expected to eventually decay into a spherically symmetric state, with gravitational emission serving as a distinguishing signal [401].

2.3.3 Neutron stars and fundamental physics

Modified gravity versus EoS. NSs provide the densest accessible medium to carry out tests of the coupling of gravitational degrees of freedom to matter fields. Testing gravity with NSs, however, faces the problem of the degeneracies between the EOS of matter in the NS core (still unknown, although increasingly constrained, see section 7), and modified-gravity theories [402–404]. In particular, measurements of the tidal deformability is not sufficient to constrain/discard a modified theory of gravity, since varying the EOS can mimic beyond-GR effects, as will be discussed in more detail in section 7.4.3. In principle, the large number of BNSs detectable with ET, and the accuracy in the measurement of tidal parameters [405], can help reduce the degeneracy. In addition, reconstructing the dependence of the tidal deformation with the mass from future ET data one can also constrain some of the parameter space of modified-gravity theories [406, 407]. Another promising (model-agnostic) approach to test gravity with BNSs is by exploiting quasi-universal relations, for example between the tidal deformability and the spin-induced quadrupole moment [408] or between different tidal Love numbers [314] (see [409] for a review). ET will be crucial in this context, because measuring the quadrupole moment or subleading tidal terms requires a level of precision which is not achievable with current detectors. The same approach can be used to distinguish NSs from ECOs [309].

Finally, it is relatively common in the numerical simulation community to employ simple polytropic EOS because of their smooth behaviour that avoids propagating errors in an evolution code. This common practice however disregards progress in nuclear and particle physics [410], and leaves out the possibility of first-order phase transitions (non-analyticities) in NS matter. Although this further complicates tests of gravity with NSs, it would be important to systematically include such effects in future investigations (see section 7). New physics parameters whose effect is a strengthening of gravity of order 10% are allowed and degenerate with variations of the EoS. ET data should supply ample opportunity to test best practices in the EOS inference, and modified gravity could also be used to benchmark the uncertainties in nuclear matter.

Dark Matter versus EoS. The possibility of DM detection in NSs has been already emphasized in the literature, e.g. [411]. Due to the large uncertainties, the impact of DM on the NS’s properties are model dependent. Figure 14 shows the mass-radius and the tidal-deformation/mass diagrams for three EOS (black lines) and for EOS with added DM (see also figure 138 for a comparison with the effect of some specific modified gravity theories). Since ET will substantially increase the population of observable NS-participated binaries, potentially detecting events near galactic centers where the DM density may be larger, one possible strategy is thus to employ BNSs to constrain the parameter space of DM models.

Another test of DM-admixed NSs is directly from the merger phase. These can be simulated using standard codes, for instance by adding a new fluid component [413, 414]. The presence of DM can impact the properties of the remnant. For example, for a sufficiently large boson-to-fermion ratio ($\sim 10\%$), the quadrupole symmetry of the massive remnant may be broken by the bosonic cores. This would lead to a suppression of the $l = m = 2$ emission, and give rise to a new peak, at $f \sim 1$ kHz, related to the $m = 1$ mode. Compact dark cores

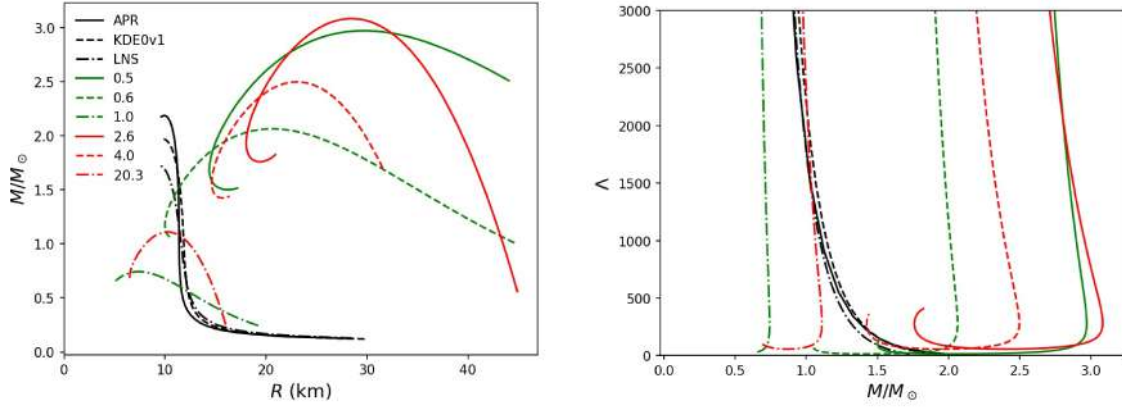


Figure 14. Mass-radius diagram (left) and dimensionless tidal deformability (Λ) and mass diagram (right) of a NS with a DM core. Stars containing fermionic DM (green lines) are labeled by the particle mass μ (in GeV). The ones with bosonic DM (red lines) are labeled by $\rho_0 \hbar^3$ (in 10^{-4} GeV^4). Reprinted figure with permission from [412], Copyright (2022) by the American Physical Society.

in NSs have also been modeled as point particles in [415]. In this case, the compact dark objects undergo orbital motion within the merger remnant, generating a GW signal with a frequency of a few kHz. The signal is expected to be weak, but long lasting, and may be detectable if integrated during the remnant lifetime (assumed to be sufficiently long) [415].

2.4 Searches for dark-matter candidates & new fields

There is overwhelming evidence for the existence of DM, coming from various observations, such as measurements of galaxy rotation curves, the CMB, gravitational lensing, and galaxy cluster dynamics (see e.g. [416–418] for reviews). However, although the existence of DM is by now a nearly irrefutable fact, its nature remains one of the biggest unresolved puzzles in fundamental physics. At the same time, GW observations provide a unique and novel way to search for DM signatures, potentially unveiling its ultimate character [419]. In this section we summarize some of the most promising prospects to search and constrain DM candidates and new fundamental fields with the ET observatory. Notably, the possibility to detect new *ultralight* bosons (much below the eV scale), either directly interacting with the GW detectors or surrounding BHs in binaries, as well as other type of “environmental” effects due to DM such as dynamical friction. As we shall see, these are complementary to searches at particle colliders [419, 420], or with lower-frequency GW observatories such as LISA [391], or pulsar timing arrays [421].

2.4.1 Direct detection of ultralight dark matter with interferometers

Searches for the imprint of ultralight DM fields in interferometric GW detectors have mainly focused, so far, on vector particles in the form of dark photons. The latter would cause time-dependent oscillations in the mirrors, leading to a differential strain on the detector [422, 423] (a similar behavior has been predicted for tensor particles [424].) Dark photons cause a small differential change in the interferometer’s arm length in two ways. First, the mirrors are subject to slightly different phases of the dark photon field [422]. Second, an equivalent

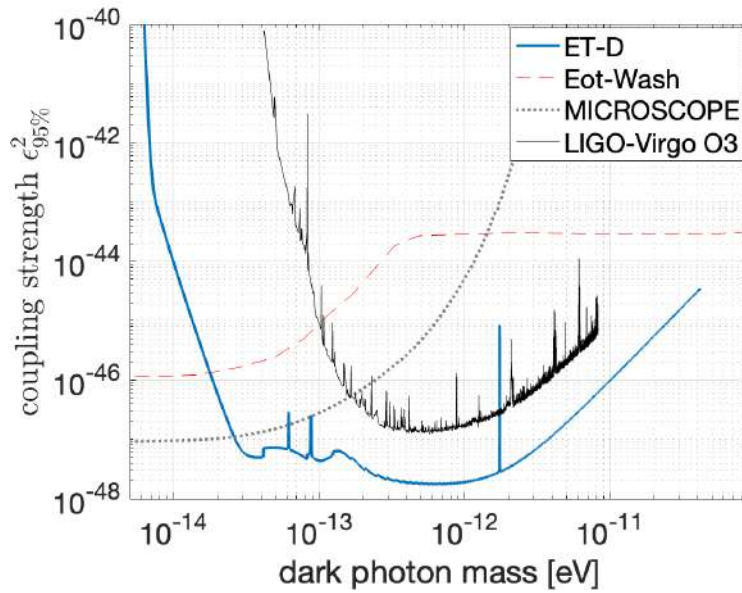


Figure 15. Predicted constraint on dark photon/baryon $U(1)_B$ coupling from ET-D (blue continuous), compared to LIGO O3 upper limit (black continuous) and current MICROSCOPE (dotted) [428] and Eöt-Wash torsion balance experiment [429] (dashed) limits. Adapted from [427]. Reprinted figure with permission from [427], Copyright (2022) by the American Physical Society.

differential strain is produced by the finite travel time of the laser light in the arms [425]. The overall signal at the detector’s output is stochastic and narrow-band, with an amplitude proportional to the strength of the dark photon/baryon (or baryon-lepton) coupling, and a frequency spread of the order of $10^{-7} f_0$, with $f_0 = m_b c^2/h$, and m_b the dark photon’s mass which, in range 10^{-14} – 10^{-11} eV, corresponds to frequencies in the ET band.

The dark photon signal requires the same type of methodology typically used for stochastic (or continuous) GW searches [422, 425, 426]. In particular, cross-correlation and excess power methods have been used to establish competitive upper limits in the frequency range 25–2000 Hz [427]. Figure 15 is adapted from [427] and shows upper limits on the dark photon to baryon coupling $U(1)_B$ obtained analyzing O3 LIGO data, compared to MICROSCOPE constraints on scalar dilaton [428] and Eöt-Wash torsion balance experiment on scalar particles [429], and to the predicted constraint from ET-D. As shown in [430–432] differences in mirror materials (as it happens in KAGRA detector) can significantly enhance the sensitivity to $U(1)_{B-L}$ gauge symmetry. As a consequence of the variation of fermion masses and electromagnetic fine-structure constant induced by the presence of new interactions, extra scalar fields can also change the size and refractive index of objects. Constraints on scalar couplings, based on the variation of the refraction index and thickness of the beam-splitter of GEO600 detector, have been described in [433, 434].

2.4.2 Environmental effects

2.4.2.1 Boson clouds from superradiance. Among the most suitable DM candidates for which GW detectors are particularly relevant are ultralight bosons. The existence of such

particles has been proposed in a multitude of Beyond Standard Model theories, e.g. [435–443], with the earliest candidate perhaps associated with the Peccei and Quinn’s solution to the strong CP problem [435, 436], implying the existence of a new light (pseudo)-scalar particle: the “axion” [437, 438]. Bosonic fields with masses in the range $m_b \simeq 10^{-21}$ – 10^{-11} eV have Compton wavelengths that are of the order of the size of known astrophysical BHs, with masses ranging from stellar-mass, $M_{\text{BH}} \simeq 10M_\odot$, to supermassive BHs, $M_{\text{BH}} \simeq 10^{10}M_\odot$, respectively. These ultralight fields can then render spinning BHs superradiantly unstable — a process in which the BH’s rotational energy is transferred to the boson field, which in turn condensates in a macroscopic “boson cloud” around the BH (see ref. [444] for a review).

The possible formation of such clouds has been shown to produce a number of GW signatures. For example, given that rotational energy is extracted from the BHs, a robust observable for this effect is the lack of BHs spinning above a certain threshold, which depends on M_{BH} and m_b [445–447]. Therefore BH spin measurements, either from GW observations or other probes, can be used to constrain the mass of ultralight bosons [446, 448, 449]. In this respect 3G detectors will be particularly useful, since they will both provide accurate measurements of the binary BH spins and will detect a large number of events, allowing for an accurate reconstruction of the “gaps” in the BH mass-spin diagram [441, 445] and, in turn, for a measurement of the putative field mass [447, 450–452]. Besides these indirect signatures, the formation of boson clouds also leads to the emission of GWs from the cloud itself [446, 450, 453, 454]. Moreover, various modifications with respect to the evolution in vacuum occur when the BH surrounded by a boson cloud is a member of a binary system [300, 455–457]. We discuss these possibilities in more detail below.

Continuous waves. Once formed, boson clouds will in general dissipate via annihilation into gravitons, emitting a quasi-monochromatic signal (over a time scale much longer than the scale of superradiant instability) with frequency [458]

$$f_{\text{gw}} \simeq 483 \text{ Hz} \left(\frac{m_b}{10^{-12} \text{ eV}} \right) \left[1 - 7 \times 10^{-4} \left(\frac{M_{\text{bh}}}{10M_\odot} \frac{m_b}{10^{-12} \text{ eV}} \right)^2 \right], \quad (2.5)$$

where m_b is the boson mass and M_{bh} is the black hole mass. The second term in parenthesis is a small correction to the leading term. In general, scalar boson clouds [446, 450, 453] emit longer but weaker signals, with respect to vector bosons [449, 454] or light tensor bosons [459]. Moreover, the signal is characterized by a spin-up, due to the cloud contraction over time [449, 460] and, possibly, to the boson self-interactions [461].

Searches for continuous gravitational waves from both scalar and vector boson clouds have been proposed [462–464] and carried on LIGO-Virgo data, and they allowed setting constraints on the existence and properties of ultralight bosons with mass in the range 10^{-13} – 10^{-11} eV. Specifically, scalar clouds have been subject to all-sky searches (namely, without making any assumption about source position and signal parameters). This allows for the exclusion of a portion of the $m_b - M_{\text{BH}}$ plane, given assumptions on the distance, spin, and age of the BHs [458, 465]. A directed search toward a specific potential source, i.e. Cygnus X-1, has also been performed [466].

The ET sensitivity will allow us to either make a direct detection, or set much more stringent constraints on the boson cloud mechanism. As an example, figure 16 shows the

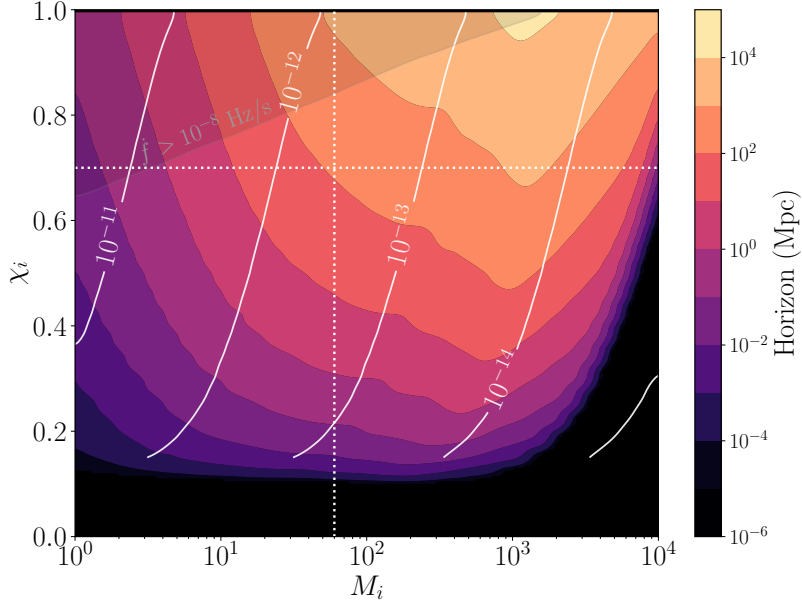


Figure 16. Maximum detectable luminosity distances for optimal scalar clouds around BHs with initial mass M_i and dimensionless spin parameter χ_i for ET, assuming one year of observation and ET in the triangle configuration (results for the 2L configuration are almost indistinguishable). White contour lines indicate the values of the corresponding boson rest-energy in units of eV. The dotted white lines highlight the mass and spin of a GW150914-like merger remnant. Adapted from ref. [463] and obtained using the python package gwaxion [467]. We are indebted to Max Isi for providing the code. Reprinted figure with permission from [463], Copyright (2019) by the American Physical Society.

maximum detectable luminosity distance for direct searches over one year of observation, assuming the triangle configuration for ET. The distance reach is well beyond the Galaxy for a portion of the parameter space, and this makes remnants of BH-BH mergers, for which the final BH mass and spins can be measured to good accuracy, potentially interesting sources. All-sky searches will be able to set constraints for galactic scalar boson clouds, as shown in figure 17, where the ET exclusion regions in the $m_b - M_{\text{BH}}$ plane are plotted assuming a BH distance of 15 kpc, an initial spin $\chi_i = 0.6$ and three different BH ages of 10^4 , 10^7 and 10^9 years.

It is worth stressing that strong bosonic self-interactions, couplings of the ultralight boson with Standard Model particles, or other environmental effects such as nearby objects, could also affect the GW signal, since such effects could slow down the superradiance extraction process or even destroy the cloud [300, 461, 468–472]. One therefore expects that any constraints on ultralight bosons based on continuous GW searches to hold provided that ultralight bosons have weak self-interactions and/or interact weakly with other particles.

Stochastic background. Besides the continuous GW searches mentioned above, one also expects that the incoherent superposition of all non-resolved GW signals emitted by boson clouds produces a stochastic GW background. This background has been computed in refs. [450, 473–475] both for scalar and vector clouds and considering both galactic and extra-

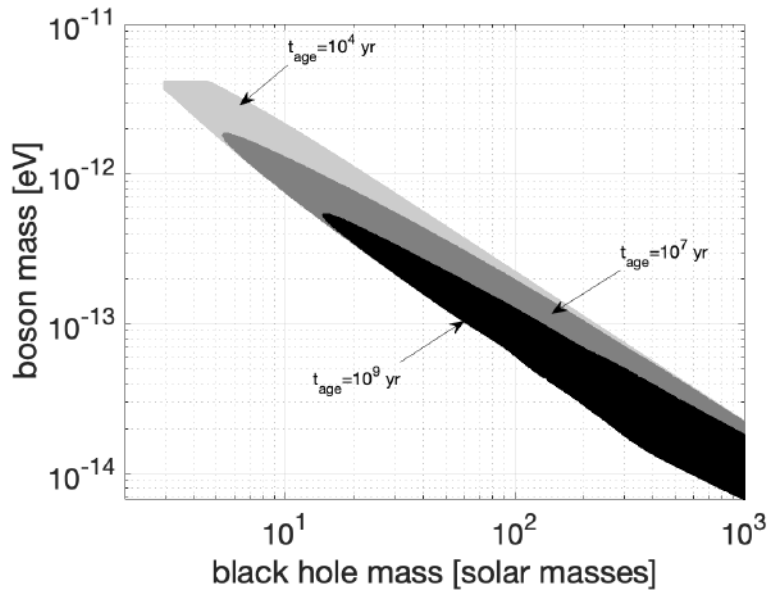


Figure 17. ET exclusion regions for scalar boson clouds assuming a BH distance of 15 kpc, an initial spin $\chi_i = 0.6$ and three different BH ages of 10^4 , 10^7 and 10^9 years.

galactic sources. Constraints based on the lack of observation of a stochastic background in LIGO and Virgo have been set [474, 476, 477] showing that, within reasonable astrophysical assumptions, current observations rule out boson clouds formed by ultralight bosons with masses in a range around $m_b \sim 3 \times 10^{-13}$ eV. Such constraints, or potential detections, will be significantly improved with ET [475]. In particular, the improved sensitivity at low-frequencies with respect to current detectors will allow us to probe stochastic backgrounds in a wider boson mass range compared to LIGO-Virgo, potentially probing the entire range $m_b \sim 9 \times [10^{-13}, 10^{-12}]$ eV [475, 477].

Binary systems. Boson clouds can reach a mass density much higher than other astrophysical environments. As a result, they are able to dramatically affect the gravitational dynamics during the inspiral regime of a binary system containing a BH surrounded by it. There are two prominent types of cloud-binary interactions, orbital resonances and dynamical friction, alongside a number of associated effects. Orbital resonances [300, 455, 478–483] are excited whenever the binary’s orbital frequency is equal to (or is an integer multiple of) the energy difference between the bound state initially occupied by the cloud and other eigenstates in the spectrum, often described (due to similarities with the hydrogen atom) in terms of the (n, ℓ, m) *quantum* numbers [300]. Resonances may be classified as *hyperfine* (only $\Delta m \neq 0$), *fine* ($\Delta \ell \neq 0, \Delta n = 0$), and *Bohr* ($\Delta n \neq 0$), respectively [300, 455]. Overtones of a resonance can also be excited for eccentric orbits [456, 457, 484]. For equatorial orbits, the main resonant Bohr frequency for GW emission, is given by

$$f_{\text{GW}}^{\text{res}} \simeq \frac{26 \text{ Hz}}{|m_2 - m_1|} \left(\frac{10 M_\odot}{M_{\text{BH}}} \right) \left(\frac{\alpha}{0.2} \right)^3 \left| \frac{1}{n_1^2} - \frac{1}{n_2^2} \right|, \quad (2.6)$$

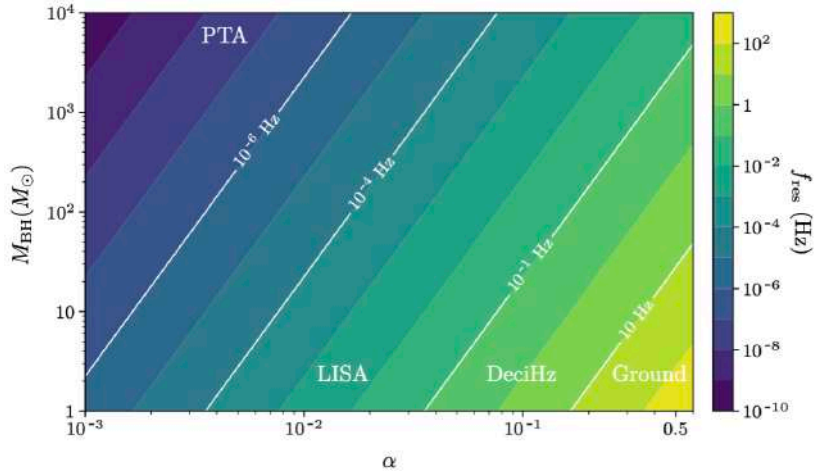


Figure 18. Resonance frequency of the (main) Bohr transition [see eq. (2.6)], as a function of the BH mass, M_{BH} , and α . See ref. [455]. Reprinted figure with permission from [455], Copyright (2020) by the American Physical Society.

where $\alpha = M_{\text{BH}} m_b$ (in $G = \hbar = c = 1$ units) is the gravitational coupling, $n_{1,2}$ and $m_{1,2}$ are the principal and magnetic quantum numbers of the two resonating states of the cloud, see figure 18.

During the orbital evolution, the cloud undergoes a *Landau-Zener* type of transition between resonant states [455], resembling similar phenomena in atomic physics. The energy required for the process comes at the expenses of the binary systems, which in turn experiences *sinking* or *floating* orbits, depending on the type of transition [455–457]. During floating, the orbital frequency and associated GWs emitted remain at an approximately constant value (with a pre-factor which may vary for different resonances). The floating time scales as

$$\Delta t_{\text{float}} \sim 0.2 \text{ s} \left(\frac{M_{\text{BH}}}{10 M_{\odot}} \right) \frac{(1+q)^{2/3}}{q^2} \left(\frac{\alpha}{0.2} \right)^{-3}, \quad (2.7)$$

where q is the binary mass ratio, which significantly alters the evolution of the GW phase [455]. On the other hand, sinking orbits introduce a temporary speed-up of the frequency chirp.

The low-frequency sensitivity of ET will be crucial to directly observe resonant transitions through the changes in the orbital evolution. Detecting *multiple* resonances would provide strong evidence for the existence of the cloud, as the ratio of successive resonant frequencies is predicted and constrained by the cloud’s energy spectrum. Another prominent effect, relevant for Bohr resonances, is the dynamical friction exerted by the cloud on the secondary object moving through it [469, 485, 486]. This is the backreaction of the excitation of the cloud to gravitationally unbound states, which carry mass, energy and angular momentum away from the system; by analogy with atomic physics, the process is also known as “ionization”. Energy can be dissipated through ionization much more efficiently than through GW radiation. As a result, when the separation of the binary is of order $\mathcal{O}(M_{\text{BH}} \alpha^{-2})$, the dynamics is driven by ionization rather than GW emission. During this phase, the frequency chirps according to a

universal shape [487], which includes, remarkably, quasi-discontinuous features at frequencies

$$f_{\text{GW}}^{(g)} \simeq \frac{6.45 \text{ Hz}}{g} \left(\frac{10 M_{\odot}}{M_{\text{BH}}} \right) \left(\frac{\alpha}{0.2} \right)^3 \left(\frac{2}{n} \right)^2, \quad g = 1, 2, \dots \quad (2.8)$$

The distinctive frequency evolution due to ionization makes the boson cloud distinguishable from other environments [488]. For high enough mass ratios q , however, ionization becomes efficient enough to disrupt the cloud significantly. Numerical investigations suggest that the total mass lost by the cloud throughout the inspiral is roughly $\sim q M_{\text{BH}}$, independent of α . Only clouds more massive than the smaller compact object will then survive and still undergo ionization in the late stages of the inspiral.

Due to its relatively large extent (of the order $1/\alpha^2$ of the BH's size), the presence of a boson cloud can also be inferred through the measurement of non-vanishing finite size effects, such as larger-than-expected spin-induced quadrupole moments and nonzero tidal Love numbers [300, 455, 489, 490]. This possibility is further enhanced by the intriguing result that, within GR in four spacetime dimension, the Love numbers of vacuum, isolated, BHs vanish [293–295, 297, 491–494]. The existence of a boson cloud then provides an effective dressing of astrophysical BHs, yielding large tidal effects. That is because Love numbers of BHs surrounded by a bosonic scalar-field condensate scale as $\frac{1}{\alpha^n} \left(\frac{M_{\text{cloud}}}{M_{\text{BH}}} \right)$, where M_{cloud} is the mass of the cloud (typically $M_{\text{cloud}} \simeq \alpha M_{\text{BH}}$), whereas $n = 8$ for scalar, vector and spin-2 odd tidal perturbations [300, 489, 490], and $n = 10$ for the dominant spin-two even perturbations [495].

Tidal effects may ultimately be unraveled by the gravitational interactions with a companion in binary systems towards the late stages of the inspiral phase. As the companion enters the cloud, finite size effects are suppressed (due to Gauss' law). This process can then be modeled by introducing a time-dependent tidal Love number, that smoothly approaches zero at higher frequency, leading to tidal effects that fade away at late times towards the merger phase [489, 490]. The time dependence of Love numbers is also manifest during resonance transitions, in which the cloud transitions into a *decaying* mode and is ultimately reabsorbed into the BH [300, 455]. In refs. [489, 490], the detectability of Love numbers through GW observations was estimated (see also [313]). The main results are plotted in figure 19, which shows that ET could measure scalar field masses in the range $m_b \in [10^{-14}, 10^{-11}]$ eV with an accuracy of a few percent, for stellar mass binaries at distances of a few Gpc. Interestingly, these constraints extend within the 10^{-14} eV regime, which is harder to probe with other superradiance-driven observations. Finally, let us stress that an accurate measure of time-dependent Love numbers allows for the possibility to disentangle dressed BHs from neutron stars in the (sub)solar mass range and from exotic compact objects (see section 2.3.1.2), since the latter are characterized by tidal effects which persist until the last stages of the merger [496, 497].

Besides the effects discussed above, the cloud-binary system exhibits a number of other potentially observable phenomena, notably through the backreaction on the orbital parameters during a resonant transition. For instance, the presence of a cloud in the earlier cosmological history of the binary's evolution, even much prior to entering the detector's band, can induce larger-than-expected values for the orbital eccentricity that are later observed via GW emission [456, 457]. This can have a big impact in the interpretation of future GW

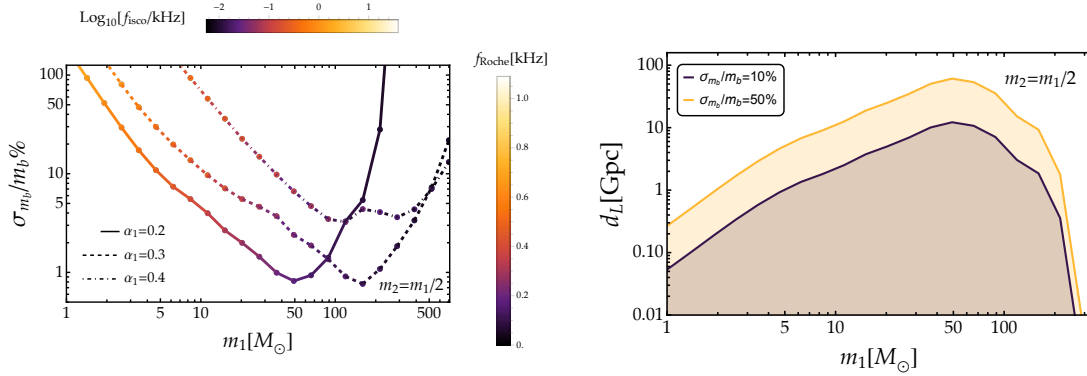


Figure 19. *Left panel:* relative percentage error on the scalar field mass m_b for a BH binary system with mass ratio $m_2/m_1 = 1/2$, detected by ET at $d_L = 1$ Gpc. Bars show the values of the innermost stable compact orbit (ISCO) and Roche frequencies. We fix the scalar cloud mass to the upper bound $M_{\text{cloud}} = 0.1m_i$ for both objects ($i = 1, 2$). *Right panel:* maximum luminosity distance for which the scalar field mass can be constrained by ET with a relative percentage accuracy of 10% (magenta) and 50% (yellow), assuming $\alpha_1 = 0.2$. See ref. [490]. Reprinted figure with permission from [490], Copyright (2023) by the American Physical Society.

observations, in particular inferring the origin (e.g. dynamical or in isolation) of binary compact objects based on the eccentricity and mass distributions derived from pure vacuum evolution. Other potentially observable effects include accretion on the companion [469], backreaction of the cloud on the geometry [498, 499], binary outspiral due to the cloud decay in GWs [500], and the termination of superradiance due to level mixing [471, 501].

2.4.2.2 Dynamical friction, accretion, and overdensities. Among the type of environmental effects that can play a significant role is dynamical friction, which emerges as a fundamental mechanism in dense environments [502]. The gravitational interactions between the binary and surrounding matter can lead to a gradual loss of orbital energy and angular momentum. This process, governed by the density and velocity distribution of the medium, plays a key role in altering the orbital parameters and phase evolution of compact binaries [488, 503–509]. Accretion stands as another notable environmental effect, particularly prevalent in scenarios involving gas-rich environments or surrounding accretion disks [488, 503, 510]. As the binary interacts with these regions, matter accretes onto one or both binary components, inducing changes in their masses and influencing the orbital dynamics and phase evolution [488, 503, 510]. The intricacies of accretion processes, ranging from Bondi-Hoyle-Lyttleton accretion to collisionless accretion, dictate the degree of impact on the binary’s evolution, further complicating the interplay between environmental factors and binary dynamics [488, 503, 505, 510, 511].

Most studies of environmental effects on GW signals have typically focused on sources relevant for LISA [391, 512], mainly due to the fact that environmental effects tend to be more important in the early inspiral and more easily detectable in intermediate or extreme mass-ratio systems [488, 504–510, 513–515]. In ref. [516], it was estimated that ET will be sensitive to the effect of dynamical friction in a GW170817-like event evolving in mediums

with densities $\gtrsim 10^{-3} \text{ g/cm}^3$. Such densities would be possible for compact binaries merging within dense thin accretion disks [503], but it is unlikely that DM environments can reach densities $\gtrsim 10^{-3} \text{ g/cm}^3$ around such sources. A more promising possibility, however, is that of primordial BHs [517] (see section 4). If they do not make up all of the DM, which is likely on the mass range that ET is able to probe, it has been shown both analytically and numerically that DM particles should form dense spikes around them [518–521]. It is expected that DM spikes around equal-mass systems will be disrupted or destroyed before the objects are close enough for the dynamical friction effect to be observable in the GW signal [522]. Yet, in the regime where the mass ratio of the binaries is $q \lesssim 10^{-2.5}$, the smaller BH will move through the DM, causing a dephasing with respect to an equivalent system inspiralling through vacuum [506, 513]. This effect can then be observed by ET with week-long duration signals, and failing to include dynamical friction may lead to significant signal-to-noise ratio losses [523]. Furthermore, for BHs with sub-solar masses, ET will provide exquisite measurements of the binary parameters [331], which are also sensitive to dynamical-friction feedback [506, 507, 510], and secondary accretion [524]. Finally, let us emphasize that the detection of primordial BHs, through their imprint in GW observables, would in turn also put among the strongest constraints one could set on dark matter candidates and extensions of the standard model [525–527], see section 4.

2.5 Synergies with other aspects of the ET Science Case

Here we discuss a list of outstanding ET science cases related to fundamental physics which require an interdisciplinary approach with topics discussed in other chapters. Such an approach is ubiquitous in most aspects of ET science, but topics related to fundamental physics are, by their own nature, particularly cross-cutting. We envisage strong synergies in the following aspects:

- **Improve (beyond) GR waveforms to match ET requirements.** The unprecedentedly loud signals to be detected by 3G detectors will require an accuracy improvement of the waveforms to be used to extract signals from data. A necessary condition for two waveforms to be indistinguishable is that their mismatch must be smaller than $1/(2\text{SNR}^2)$, so that louder events exacerbate any possible modeling systematics (see [528] for a recent overview). When available, coalescence waveforms in theories beyond GR or for ECO models are far less accurate than the prototypical binary BH waveforms in GR, both in absolute terms and in terms of coverage of the multidimensional parameter space. An outstanding task is to select the best motivated beyond-GR models and develop them at the level of accuracy required in the ET era. This will be achieved by extending various perturbative and numerical techniques discussed in section 9 also for alternative theories and models, most likely on a case-by-case basis.
- **Improve data analysis techniques beyond GR.** As discussed in section 11, ET poses new urgent challenges for data analysis, mostly due to the long duration of certain signals, the possible overlap of several sources, and the huge number of expected detections. While overlapping signals should not be a significant concern both in terms of detection [529, 530] and parameter estimation [531–534], it would be advisable to

optimize current data-analysis pipelines for this scenario. Any fundamental physics effect that one wishes to search for in the datastream should be properly included in the pipeline. In the near future, synergy among data analysis (section 11), waveform modeling (section 9), and the fundamental physics topics discussed in this section will be needed, for example, to introduce beyond-GR effects in the ET mock data challenge (discussed in section 11).

- **Fundamental physics implications of primordial relics, cosmic strings, and primordial BHs.** Early Universe cosmology is strongly tied with fundamental physics, GR extensions, and Beyond Standard Model physics. In particular, a putative GW detection of primordial BHs, cosmic strings, or any primordial relics (see section 3), or any direct signature of inflation, would have a huge impact on foundational physics. For example, if microscopic horizon-less relics form in the early Universe either directly through gravitational collapse or as stable remnants of the Hawking evaporation of primordial BHs, they could evade cosmological constraints arising from evaporation [535] and could explain the entirety of the DM. The stochastic GW background associated with the formation of microscopic DM relics in various scenarios is at most marginally detectable with current interferometers but could be measured by ET [536]. This strengthens the connection between fundamental compact-object physics and cosmological GW sources, and calls for a synergy between the topics developed in this section and those in section 3.
- **Identifying the origin of merging binaries across cosmic history.** Likewise, detecting a new population of compact objects other than BHs or NSs, or BH binaries of primordial origin, would have a strong impact on fundamental physics. Indeed, it would imply the existence of Beyond Standard Model and/or beyond-GR effects, which should then be further developed and could be possibly explored by other complementary means. Except for the smoking gun signatures of deviations from standard theory discussed in this section, evidence for new families of compact object could come from population studies (see section 4), underscoring a strong connection between the latter and fundamental physics.
- **Fundamental physics implications of NS mergers.** As the densest known objects in the Universe, NSs are a unique laboratory for all fundamental interactions. As previously explained, NS-based tests of gravity are tantalizing but severely limited by the degeneracy with the unknown EOS in the NS core (see section 7). Finding ways to break this degeneracy calls for a synergy among nuclear matter and fundamental physics studies. As discussed, a promising possibility is to make use of approximately universal relations among the NS parameters that are almost EOS insensitive. ET will be instrumental for these tests, since they require very high SNR to measure subleading parameters such as quadrupole moments or non-standard Love numbers.

2.6 Executive summary

We conclude this section by summarizing the advancements that ET will bring in Fundamental Physics, in comparison to the current state of the art, specifically emphasizing the distinct

features that ET will allow us to explore. The following are the major areas where ET's capabilities surpass the current generation of GWs detectors.

Fundamental principles of the gravitational interaction

ET will probe the spacetime dynamics, the generation, and the propagation of GWs in Einstein's theory of gravity like never before, including:

- *Tests in the inspiral phase*, with up to two orders of magnitude improvements compared to current detectors;
- *Searches for extra GW polarizations*, which are inaccessible with current detectors;
- *Tests of the mass of the graviton and dispersion relations*, with up to two orders of magnitude improvements compared to current detectors;
- *Tests of GW (displacement) memory effects*, for the first time;
- *High-precisions tests of the fully nonlinear (perturbative/non-perturbative) structure of GR*.

Consequently, ET will be able to tell whether the gravitational interactions driving the coalescence of compact objects are described by GR, or if the data will point instead to alternative theories which manifest themselves in regions of parameter space that are not reachable by any other experiment.

The nature of compact objects and horizon scale physics

ET will provide high accurate measurements of the nature of GW sources, including:

- *Ringdown tests and BH spectroscopy* at unprecedented level, with $\mathcal{O}(10)$ event/yr expected to have $\mathcal{O}(1)\%$ precision in the measurement of multiple ringdown modes;
- *Tests of intrinsic multipole moments and no-hair theorems*, with almost an order of magnitude improvement compared to LVK at design sensitivity;
- *Tests of tidal deformability effects and absorption*, with almost an order of magnitude improvement compared to LVK at design sensitivity;
- *Tests of the post-merger phase and of horizon-scale physics*, with more than an order of magnitude improvement compared to current detectors.

Consequently, ET will be able to tell whether the dark massive compact objects in the sky producing ripples of spacetime are indeed the BHs of GR, or whether ET will allow us to detect novel types of compact objects for the first time.

Dark-matter particles and new fields

ET has a unique potential to discover the existence of new fields/particles in nature, including:

- *Searches of ultralight particles* using BH mergers and the stochastic GW background at extra galactic distance, exploring the region between 10^{-14} eV and 10^{-11} eV of putative new bosonic matter;
- *Tests of dark-matter environmental effects, boson clouds, and dynamical friction*, in regions currently inaccessible by GW detectors.

Consequently, even if Einstein's theory withstands all the aforementioned scrutiny, ET will then turn into a precision machine for searches of new particles and fields through GW precision data, opening entirely new windows of exploration. These searches will be synergetic with complementary experimental efforts, including space-based detectors such as LISA.

3 Cosmology with ET

With its unprecedented sensitivity, ET will detect GWs from events such as merging black holes, neutron star collisions, and potential signals from the early Universe. By probing these cosmic events, ET will provide new insights into the fundamental properties of gravity, the expansion of the Universe, and the nature of dark matter and dark energy, serving as a powerful tool to explore previously uncharted aspects of cosmology.

One of the main targets for ET is the stochastic gravitational wave background (GWB), which is a diffuse signal composed of many unresolved gravitational wave sources, encompassing both astrophysical (AGWB) and cosmological (CGWB) components. Section 3.1 discusses the general formalism for GWBs, including anisotropies, parity violation, the separability of different contributions, and the impact of correlated noise on detectability. As we will discuss in section 3.2, stochastic background of cosmological origin are probes of the early Universe, and potential sources in the ET band include inflationary models beyond single-field slow-roll scenarios, first-order phase transitions occurring at energy scales of up to 10^8 GeV, corresponding to timescales of 10^{-12} to 10^{-6} seconds after the Big Bang; topological defects, such as cosmic strings and domain walls, as well as individual GW bursts from cosmic string cusps and kinks; or signals related to the formation mechanism of primordial BHs.

ET will also offer unique opportunities for probing the late Universe and contributing to cosmography, by detecting BBH mergers at high redshifts, up to $z \sim 20$ and higher. This will allow ET to observe black hole formation in the early Universe, tracing back to the population of primordial black holes. A key feature of GWs from compact binary coalescences is that they are 'standard sirens', i.e. they serve as self-calibrating distance indicators, enabling reconstruction of the distance-redshift relation and providing insights into the expansion history of the Universe. However, GWs alone generally cannot determine redshift of the binary. For bright standard sirens, which have an electromagnetic counterpart, the redshift can be directly measured from the electromagnetic observations, enabling constraints on

cosmological parameters. Most GW events, however, are ‘dark sirens’, i.e. do not have an observed electromagnetic counterpart, and necessitate the use of correlation with galaxy catalogs or population inference techniques to perform statistical inference on the relation between luminosity distance and redshift. The potential of bright and of dark sirens for cosmology and for probing the late Universe with ET will be discussed in section 3.3.

Finally, ET observations, particularly when combined with future galaxy surveys, are expected to help address various cosmic tensions, such as discrepancies in the Hubble constant H_0 and potential differences in cosmic dipole measurements from electromagnetic and large-scale structure (LSS) observations. The potential of the ET observations for probing LSSs of the Universe is discussed in section 3.4.

3.1 Stochastic gravitational-wave backgrounds

Gravitational-wave backgrounds are of great interest in cosmology. The detection of a CGWB would be a smoking-gun probe of the physics of the early Universe, as it would allow for unique constraints on cosmic inflation, phase transitions, or cosmic strings. The anisotropies of the CGWB are a probe of the cosmological perturbations in the early Universe. Moreover, the AGWB, resulting from the superposition of numerous unresolved CBC signals, can also be used as a cosmological probe, as its frequency spectrum is sensitive to cosmological parameters, and its anisotropies are tracers of the large scale structures (LSS) of the Universe. Finally, the detection of a polarized CGWB could serve as key evidence for parity-violating mechanisms in the early Universe (even though the AGWB is also expected to be polarized and constitutes an unavoidable foreground for a polarized CGWB). Indeed, the frequency spectrum, anisotropies, and polarization of the GWB are crucial observables for distinguishing between the CGWB and AGWB, as various cosmological and astrophysical sources are expected to produce GWBs with distinct and unique signatures.

3.1.1 Definition and characterisation

A GWB is a diffuse signal formed by the incoherent superposition of numerous unresolved GW sources. Some GWB components exhibit stochastic properties due to their generation process, while others result from the limited sensitivity of the specific detectors used for their observation. A variety of sources are expected to produce a gravitational-wave background, with different properties and frequency profiles (see [60, 537–540] for reviews). Traditionally, we distinguish between an astrophysical GW background (AGWB) and a cosmological GW background (CGWB). In a first approximation, the GWB is assumed to be homogeneous and isotropic across the sky. This contribution (i.e., the monopole) is the dominant contribution and it is characterized through its frequency dependence. However, the GWB exhibits also some level of anisotropy due to inhomogeneous distribution of sources, line-of-sight effects accumulated during the propagation across the large-scale structure (LSS) [541–555], and kinematic anisotropies arising from the observer’s velocity relative to the rest frame of the GWB [556–558].

Let us recall here the basic definitions for the characterization of stochastic backgrounds (we follow the conventions in [559]). A continuous superposition of GWs coming from all

directions, with propagation directions labeled by a unit vector $\hat{\mathbf{n}}$, can be written as

$$h_{ij}(t, \mathbf{x}) = \int_{-\infty}^{\infty} df \int_{S^2} d^2 \hat{\mathbf{n}} \sum_{A=+, \times} \tilde{h}_A(f, \hat{\mathbf{n}}) e_{ij}^A(\hat{\mathbf{n}}) e^{-2\pi i f(t - \hat{\mathbf{n}} \cdot \mathbf{x}/c)}, \quad (3.1)$$

where the spatial indices i, j take values 1, 2, 3 and e_{ij}^A are the polarization tensors, normalized as (the sum over the spatial indices i, j is understood)

$$e_{ij}^A(\hat{\mathbf{n}}) e_{ij}^{A'}(\hat{\mathbf{n}}) = 2\delta^{AA'}. \quad (3.2)$$

In a given detector, located at the position $\mathbf{x}_a$, the observed signal $h_a(t)$ is obtained convolving each polarization with the pattern functions $F_a^A(\hat{\mathbf{n}})$ that encode the detector response,⁵

$$h_a(t) = \int_{-\infty}^{\infty} df \int_{S^2} d^2 \hat{\mathbf{n}} \sum_{A=+, \times} \tilde{h}_A(f, \hat{\mathbf{n}}) F_a^A(\hat{\mathbf{n}}) e^{-2\pi i f(t - \hat{\mathbf{n}} \cdot \mathbf{x}_a/c)}, \quad (3.3)$$

and therefore the Fourier transform of the signal is given by

$$\tilde{h}_a(f) = \int_{S^2} d^2 \hat{\mathbf{n}} \sum_{A=+, \times} \tilde{h}_A(f, \hat{\mathbf{n}}) F_a^A(\hat{\mathbf{n}}) e^{2\pi i f \hat{\mathbf{n}} \cdot \mathbf{x}_a/c}. \quad (3.4)$$

If the signal is given by a stochastic background of GWs that is stationary, isotropic and unpolarized, its two-point correlator is given by

$$\langle \tilde{h}_A^*(f, \hat{\mathbf{n}}) \tilde{h}_{A'}(f', \hat{\mathbf{n}}') \rangle = \delta(f - f') \frac{\delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}')}{4\pi} \delta_{AA'} \frac{1}{2} S_h(f), \quad (3.5)$$

where the brackets $\langle \cdot \rangle$ denote the ensemble average and $\delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}')$ is a Dirac delta over the two-sphere, $\delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}') = \delta(\phi - \phi') \delta(\cos \theta - \cos \theta')$, where (θ, ϕ) are the polar angles that define $\hat{\mathbf{n}}$. Equation (3.5) defines the (single-sided) spectral density of the signal, $S_h(f)$. The factor $1/(4\pi)$ in eq. (3.5) is a standard choice of normalization, chosen so that

$$\int_{S^2} d^2 \hat{\mathbf{n}} d^2 \hat{\mathbf{n}}' \langle \tilde{h}_A^*(f, \hat{\mathbf{n}}) \tilde{h}_{A'}(f', \hat{\mathbf{n}}') \rangle = \delta(f - f') \delta_{AA'} \frac{1}{2} S_h(f), \quad (3.6)$$

so that, for each polarization, the right-hand side has the normalization used also for the (single-sided) noise spectra density, see eq. (10.2) below.⁶

However, more generally, a stochastic GW background can be polarized and anisotropic. Assuming again stationarity (which, in Fourier space, implies that the correlator is proportional to $\delta(f - f')$), the most general form of the two-point correlator is then [80, 540, 560, 561]

$$\langle \tilde{h}_A^*(f, \hat{\mathbf{n}}) \tilde{h}_{A'}(f', \hat{\mathbf{n}}') \rangle = \delta(f - f') \frac{\delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}')}{4\pi} \frac{1}{2} H_{AA'}(f, \hat{\mathbf{n}}). \quad (3.7)$$

Note that we still assume that signals coming from different directions are uncorrelated, so that the correlator is still proportional to $\delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}')$; however, anisotropy is encoded in the fact

⁵We assume here that the size of the detector is much smaller than the GW wavelength, so we can neglect the spatial variation of the GW over the extension of the detector. This condition is very well satisfied by ground-based interferometers: as the GW frequency f ranges between 10 Hz and 1 kHz, the wavelength c/f ranges between 3×10^4 km and 300 km, much larger than the 10 km or 15 km length of the arms of ET.

⁶Here we follow the conventions used in refs. [537, 559], and in most of the literature. Observe, however, that these differ by a factor of 2 from those used in [80], see their eq. (2.14).

that the function $H_{AA'}(f, \hat{\mathbf{n}})$ could depend on $\hat{\mathbf{n}}$, and the possibility of a polarization in the fact that $H_{AA'}(f, \hat{\mathbf{n}})$ is a 2×2 matrix in the polarization indices, not necessarily proportional to $\delta_{AA'}$. The reality of $h_a(t)$ implies that $\tilde{h}_A^*(f, \hat{\mathbf{n}}) = \tilde{h}_A(-f, \hat{\mathbf{n}})$ so, taking the complex conjugate of eq. (3.7), one sees that $H_{AA'}(f, \hat{\mathbf{n}})$ is a Hermitian matrix in the polarization indices. As any 2×2 Hermitian matrix, it can then be decomposed (with real coefficients) into the basis of the identity matrix and the three Pauli matrices,

$$H_{AA'}(f, \hat{\mathbf{n}}) = I(f, \hat{\mathbf{n}}) \delta_{AA'} + U(f, \hat{\mathbf{n}}) \sigma_{AA'}^1 + V(f, \hat{\mathbf{n}}) \sigma_{AA'}^2 + Q(f, \hat{\mathbf{n}}) \sigma_{AA'}^3, \quad (3.8)$$

where the Pauli matrices are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

In matrix form

$$H(f, \hat{\mathbf{n}}) = \begin{pmatrix} I(f, \hat{\mathbf{n}}) + Q(f, \hat{\mathbf{n}}) & U(f, \hat{\mathbf{n}}) - iV(f, \hat{\mathbf{n}}) \\ U(f, \hat{\mathbf{n}}) + iV(f, \hat{\mathbf{n}}) & I(f, \hat{\mathbf{n}}) - Q(f, \hat{\mathbf{n}}) \end{pmatrix}. \quad (3.9)$$

The coefficients of this decompositions define the Stokes parameters of the GW stochastic background, and are real functions of f and $\hat{\mathbf{n}}$ describing intensity (I), linear polarization (U and Q) and circular polarization (V). Therefore,⁷

$$\begin{aligned} & \left(\begin{aligned} & \langle \tilde{h}_+^*(f, \hat{\mathbf{n}}) \tilde{h}_+(f', \hat{\mathbf{n}}') \rangle \langle \tilde{h}_+^*(f, \hat{\mathbf{n}}) \tilde{h}_\times(f', \hat{\mathbf{n}}') \rangle \\ & \langle \tilde{h}_\times^*(f, \hat{\mathbf{n}}) \tilde{h}_+(f', \hat{\mathbf{n}}') \rangle \langle \tilde{h}_\times^*(f, \hat{\mathbf{n}}) \tilde{h}_\times(f', \hat{\mathbf{n}}') \rangle \end{aligned} \right) = \\ & = \frac{1}{8\pi} \delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}') \delta(f - f') \begin{pmatrix} I(f, \hat{\mathbf{n}}) + Q(f, \hat{\mathbf{n}}) & U(f, \hat{\mathbf{n}}) - iV(f, \hat{\mathbf{n}}) \\ U(f, \hat{\mathbf{n}}) + iV(f, \hat{\mathbf{n}}) & I(f, \hat{\mathbf{n}}) - Q(f, \hat{\mathbf{n}}) \end{pmatrix}. \end{aligned} \quad (3.10)$$

If the GWB is unpolarized, $U = V = Q = 0$ and eq. (3.10) reduces to

$$\langle \tilde{h}_A^*(f, \hat{\mathbf{n}}) \tilde{h}_{A'}(f', \hat{\mathbf{n}}') \rangle = \delta(f - f') \frac{\delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}')}{4\pi} \delta_{AA'} \frac{1}{2} S_h(f, \hat{\mathbf{n}}), \quad (3.11)$$

with $S_h(f, \hat{\mathbf{n}}) = I(f, \hat{\mathbf{n}})$. If, furthermore, the background is isotropic, we get back eq. (3.5) with a spectral density $S_h(f) = I(f)$ independent of $\hat{\mathbf{n}}$. If we assume that the GWB is Gaussian, then the 2-point correlation function, eq. (3.10) in the most general case or eq. (3.11) in the unpolarized case, is sufficient to fully describe its statistical properties.

An important property of a GWB is its energy density per unit logarithmic frequency interval, which is conventionally normalized to the critical energy density for closing the Universe ρ_c ,

$$\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f}, \quad (3.12)$$

⁷Here, again, one must be aware of the existence of different conventions in the literature, in which a factor 4π or 8π in eq. (3.7) is reabsorbed into the definition of the Stokes parameters. Our conventions are the same as in [562] (see, e.g., also [561, 563]).

where $\rho_c = 3c^2 H_0^2 / (8\pi G)$ and H_0 is the present-day value of the Hubble parameter. Note that H_0 , in the denominator of eq. (3.13), entered through the normalization factor ρ_c , so the observational uncertainty in this factor H_0 is not related to an intrinsic uncertainty of the energy density of the stochastic background considered, but rather of the quantity that we have chosen for normalizing it. In order to get rid of this uncertainty, it is often convenient to work in terms of the quantity $h^2 \Omega_{\text{GW}}(f)$, where the reduced Hubble constant h is defined from $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$.

For a stationary, isotropic and unpolarized background, described by eq. (3.6), the energy density can be written in terms of the spectral density as (see eq. (7.202) of [559])

$$\Omega_{\text{GW}}(f) = \frac{4\pi^2}{3H_0^2} f^3 S_h(f). \quad (3.13)$$

The same expression holds for a polarized background (as long as it is stationary and anisotropic), since the Stokes parameters related to linear and circular polarization do not contribute to the energy density, and only the intensity $I(f) \equiv S_h(f)$ contributes [561]. For a generic anisotropic and polarized GW background (but still assuming stationarity), one can show that the energy density is given by [561]

$$\rho_{\text{gw}} = \frac{\pi c^2}{2G} \int_0^\infty d \log f f^3 \int_{S^2} \frac{d^2 \hat{\mathbf{n}}}{4\pi} I(f, \hat{\mathbf{n}}), \quad (3.14)$$

so that

$$\frac{d\rho_{\text{gw}}}{d \log f d^2 \hat{\mathbf{n}}} = \frac{c^2}{8G} f^3 I(f, \hat{\mathbf{n}}). \quad (3.15)$$

Note that this expression is obtained using the full correlator given in eqs. (3.7) and (3.8). However, in the computation the traceless terms associated to polarizations give zero [561], and only the intensity Stokes parameter contributes to the energy density. Then, we have

$$\begin{aligned} \Omega_{\text{GW}}(f, \hat{\mathbf{n}}) &\equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(f, \hat{\mathbf{n}})}{d \log f d^2 \hat{\mathbf{n}}} \\ &= \frac{\pi}{3H_0^2} f^3 I(f, \hat{\mathbf{n}}). \end{aligned} \quad (3.16)$$

In the typical situation in which the anisotropic part is a small perturbation of the isotropic part, we can write

$$\Omega_{\text{GW}}(f, \hat{\mathbf{n}}) = \bar{\Omega}_{\text{GW}}(f) + \delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}}), \quad (3.17)$$

i.e., the energy density is split into a homogeneous component $\bar{\Omega}_{\text{GW}}(f)$, and an anisotropic perturbation, $\delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}})$, which is generally small compared to the monopole. Under the assumption of statistical isotropy, the sky distribution of the GWB energy density can be characterised in terms of a frequency-dependent angular power spectrum, $C_\ell(f)$, defined through

$$\langle \delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}}) \delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}}') \rangle = \sum_{\ell=1}^{\infty} C_\ell(f) P_\ell(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}'), \quad (3.18)$$

where $P_\ell(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}')$ are the Legendre polynomials and the multipoles ℓ are inversely proportional to the angular scale of the perturbation.

A useful tool in the study of the sensitivity of a network of detectors to an unpolarised and isotropic GWB is the power-law sensitivity (PLS) curve [564]. We refer the reader to section 10.5.2 for details on its construction, its physical meaning and some properties, and here we limit ourselves to a brief reminder of the main ingredients needed. When constructing the PLS curve, one considers the family of power-law GW backgrounds

$$\Omega_{\text{GW}}(f; \beta) = \Omega_\beta \left(\frac{f}{f_{\text{ref}}} \right)^\beta, \quad (3.19)$$

where the exponent β is the family parameter, which can take any real value, and f_{ref} is an arbitrary fixed reference frequency (whose change simply amounts to a redefinition of Ω_β). The dependence of the amplitude coefficient Ω_β on β is determined by requiring that, in a given coincident observation time T , the integrated signal-to-noise ratio (SNR) in the frequency range $[f_{\text{min}}, f_{\text{max}}]$ probed by a given detector network takes an assigned value ρ . This leads to

$$\Omega_\beta = \rho \left[2T \int_{f_{\text{min}}}^{f_{\text{max}}} df \Omega_{\text{eff}}^{-2}(f) \left(\frac{f}{f_{\text{ref}}} \right)^{2\beta} \right]^{-1/2}, \quad (3.20)$$

where the function $\Omega_{\text{eff}}(f)$ depends on the noise properties of the detectors (their PSD or noise correlations, if present) and by the geometry of the detector configuration via the overlap reduction functions for each pair of detectors, see eq. (10.64) for the explicit expression.

The PLS curve is then defined as the envelope of the family of curves (3.19), with Ω_β as in eq. (3.20), and it is given by

$$\Omega_{\text{PLS}}(f) = \max_{\beta} \Omega_{\text{GW}}(f; \beta). \quad (3.21)$$

By construction, in a log-log plot any line tangent to the PLS curve represents a power-law GW background with integrated SNR equal to the value of ρ used in the PLS definition.

3.1.2 Anisotropies of the GWB

3.1.2.1 Anisotropies of the Cosmological Background. The anisotropies of the CGWB can be described by introducing a distribution function of the primordial gravitons (in the high frequency approximation, where anisotropies are typically studied) and solving the Boltzmann equation in cosmological perturbation theory [541, 553, 554, 565]. The solution of the Boltzmann equation shows that the anisotropies are generated by any inhomogeneity and anisotropy in the production mechanism that sourced the background (i.e., initial conditions) and by the metric perturbations around a Friedmann-Lemaître-Robertson-Walker background crossed by gravitons along their geodesics. In addition, kinematic anisotropies, in particular a dipole, are induced by the peculiar motion of the observer with respect to the rest frame of the CGWB [544, 556].

The anisotropies due to the propagation of gravitons and kinematic effects are proportional to $4 - n_{\text{gwb}}$, where n_{gwb} is the spectral tilt of the CGWB monopole. So, the spectral tilt may have an impact on the amplitude of the anisotropies. Such a possibility has been

discussed in [566, 567] for the case of sharply peaked CGWB spectra generated at second order by curvature perturbations, making these anisotropies a potential probe of primordial black-holes physics.

The contribution due to the initial conditions largely depends on the model considered and could dominate the spectra in a number of different scenarios. For instance, if the CGWB is produced by the decay of a field which gives the dominant contribution to the curvature perturbation of the Universe, the initial conditions are expected to be adiabatic, in analogy with the CMB. For the CGWB generated by quantum fluctuations of the metric during inflation a large non-adiabatic contribution in the initial conditions is expected, which can increase the anisotropies by an order of magnitude [568]. In the case of scalar-induced gravitational waves (SIGWs), the initial conditions are sensitive to any amount of local primordial non-Gaussianity, parametrized by f_{NL} [569], while for GWs from phase transitions large isocurvature perturbations due to a decoupled sector at the production time could enhance the spectrum [570]. The angular power spectrum of the CGWB for all these mechanisms and the corresponding predictions for ET, can be computed with the code `GW_CLASS`⁸ [571].

Another important source of anisotropies in the form of a contribution to the initial condition term arises in inflation models predicting a sizable tensor or mixed (scalar-tensor) bispectrum in the squeezed configuration. A non-trivial correlation between two short-wavelength modes and one long-wavelength mode of the primordial fluctuations would indeed result in a modulation of the gravitational wave two-point function, giving rise to anisotropies in the energy density of the GW background [572]. If observed, these signatures would be a smoking gun for inflation models beyond the basic single-field slow-roll paradigm [573]. Another interesting aspect of this class of anisotropies is that they may be tested through cross-correlations with the cosmic microwave background: the same large-scale fluctuations modulating the GW energy density at interferometer frequencies, would contribute to large-scale CMB anisotropies, making GW-CMB cross-correlations a new probe of squeezed primordial non-Gaussianity, complementary to CMB bispectrum measurements [574–576].

Since gravitational interactions decouple at the Planck scales, the CGWB could be used to test the geometry and the particle content of the Universe at early times. In [555] it was shown that the presence of exotic (beyond Standard Model) relativistic and decoupled particles could modify the amplitude of the anisotropies on large scales. In [577] the dependence of the anisotropies on the equation of state parameter of the Universe was analysed, finding a universal behavior for adiabatic primordial fluctuations. It was shown that deviations from this behavior (which could be in the form of an amplification of the anisotropies) would point to the presence of isocurvature fields during inflation. In [571] a joint Bayesian analysis of the CMB and CGWB anisotropies for ET+CE has been performed, exploiting the large correlation expected between the CMB and the CGWB [578]. It has been shown how the CGWB could be used to estimate cosmological parameters, breaking some degeneracies which are present in CMB measurements, such as the one between the amplitude of the primordial

⁸The code is publicly accessible as a new branch (`GW_CLASS`) on the `CLASS` repository www.github.com/lesgourg/class_public.

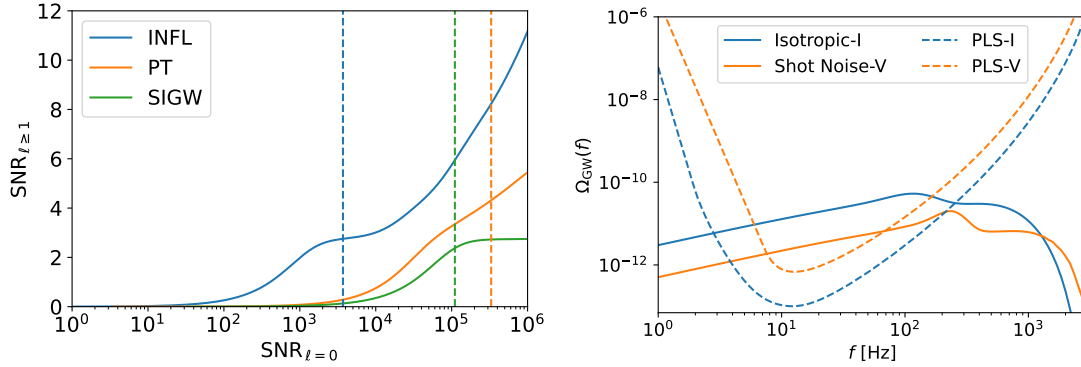


Figure 20. *Left:* plot of the cumulative SNR of the anisotropies as a function of the SNR of the monopole for ET (triangular) + CE (40 km) for five years of observation. The dashed lines represent the LVK upper bounds on the amplitude of the monopole for the three mechanisms considered (inflation, PT and SIGW). *Right:* plot of the monopole of the intensity (solid blue), circular polarization (solid orange) of the AGWB, and of the PLS of the intensity and circular polarization (dashed blue and orange respectively) for ET+CE in one year of observation.

scalar power spectrum and the reionization epoch of the CMB photons. The parameter in this case is the angular power spectrum of the CGWB. A Fisher matrix analysis of the detectability of the angular power spectrum from the case of phase transitions has been done in [579].

In the left panel of figure 20 we plot the cumulative SNR of the angular power spectrum of the auto-correlation of the CGWB and cross-correlation between the CGWB and CMB as a function of the SNR of the monopole. We consider as detector network ET in the triangular configuration in combination with one CE of 40 km, for 5 years of observation. Because of the limited angular resolution of the network, the cumulative SNR of the anisotropies is sensitive up to $\ell_{\text{max}} \approx 10$ (for the largest values of the monopole in the figure). We show the result for the CGWB from inflation with $n_T = 0.35$, phase transitions with $f_* = 1$ Hz and a Gaussian SIGW spectrum peaked at $f_* = 100$ Hz. The dotted lines identify the maximum values of the SNR of the monopole consistent with the upper bounds of LVK [100], which are distinct for the three signals, because of the different dependence on the frequency of the monopoles. For low values of the SNR of the monopole, the dominant contribution to the SNR of the anisotropies is given by the kinetic dipole, which is orders of magnitude larger than the cosmological anisotropies, while for larger monopoles the cosmological anisotropies become comparable with the instrumental noise, giving rise to detectable signals.

3.1.2.2 Anisotropies of the Astrophysical Background. The distribution of the number of sources, and thus the AGWB, is expected to exhibit a uniform and dominant contribution across all directions in the sky (i.e., a monopole component). Superimposed on this dominant term, small anisotropies of the order of $10^{-2} - 10^{-5}$ are anticipated. These deviations from a homogeneous and isotropic spectrum arise due to cosmological perturbations (i.e., intrinsic anisotropies), which also account for the overdensity and peculiar velocity of galaxies, as well as the anisotropies in the CMB. Additionally, Poisson fluctuations result from the finite number of sources contributing to the background within a limited timeframe and volume (i.e.,

shot noise), and the observer’s peculiar motion relative to the sources’ rest frame introduces kinetic effects, such as Doppler boosting and aberration in the AGWB.

Following [580, 581], the monopole energy density of the AGWB generated by the superposition of many, unresolved, astrophysical sources can be computed by integrating the product of the energy spectrum and the source rate over the entire parameter space describing the population of GW events (see also section 4.10),

$$\bar{\Omega}_{\text{GW}}(f) = \frac{f}{\rho_c c^2} \int \frac{dz}{(1+z)H(z)} R(z) \int d\vec{\theta} p(\vec{\theta}) \frac{dE}{df d\Omega}(f, z, \vec{\theta}). \quad (3.22)$$

Here, z is the redshift of the source, $\vec{\theta}$ represents the intrinsic parameters of the binary, e.g., in the case of BBH, the masses, spins and inclination angle of the orbit (i.e. the angle between the angular momentum of the binary and the direction of observation); $dE/df d\Omega$ denotes the energy spectrum per solid angle and frequency emitted by individual binaries [580, 581]. The merger rate per comoving volume $R(z)$ has been assumed here independent of the distribution of the intrinsic parameters of the binaries $p(\vec{\theta})$; see section 4.3 for a more detailed discussion of the evolution of the mass distribution with the redshift.

The other Stokes parameters of the AGWB, introduced in eq. (3.10), can be computed by replacing the energy density of the individual binaries in eq. (3.22) with an equivalent “polarized energy spectrum”, which has the same amplitude as the intensity, but varies differently with respect to the inclination angle; see, e.g., [582].⁹ The circular polarization of the AGWB has been discussed in the previous subsection, while cosmological models which produces polarized signals will be discussed in section 3.1.3.

The intrinsic anisotropies have been computed in [542, 543, 546–548, 583, 584] and can be classified into density anisotropies, which are related to the distribution of GW sources in the sky; Kaiser and Doppler anisotropies, which are related to the peculiar velocities of the hosts; and anisotropies imprinted by gravitational potentials along the graviton’s path. A fully relativistic, gauge-invariant computation using the *cosmic rulers* formalism [585] has been performed in [547]. The monopole and anisotropies of the AGWB generated by BBH, BNS and BHNS have been fully characterized in [551, 552]. The public available code `CLASS_GWB` [552] allows computing the angular power spectrum of the intrinsic and shot noise anisotropies for different populations of CBC. The relative perturbation in the energy density of the AGWB at the observer can be written as

$$\frac{\delta\Omega_{\text{GW}}^{\text{intr}}}{\bar{\Omega}_{\text{GW}}}(f) = \int \frac{d^3k}{(2\pi)^3} \zeta(\vec{k}) \int_0^{\eta_0} d\eta \left[\Delta^{\text{den}}(\eta, k, f) + \Delta^{\text{rsd}}(\eta, k, f) + \Delta^{\text{gr}}(\eta, k, f) \right], \quad (3.23)$$

where ζ is the primordial curvature perturbation and η_0 is the conformal time today. The expression for the source functions Δ^i can be found e.g. in [547, 552]. The cross-correlation of the intrinsic anisotropies with other LSS tracers will be discussed in section 3.4.1.2, focusing in particular on the dependence of the intrinsic anisotropies on the bias of the GWs. The intrinsic anisotropies are typically considered as Gaussian random variables of zero mean and covariance proportional to the power spectrum of the primordial curvature perturbation,

⁹Note, however, that only the Stokes parameter associated to the intensity, I , corresponds to an energy density, and no actual energy density is associated to the Stokes parameters describing linear and circular polarizations [561].

although the current Planck bounds admit the possibility of the presence of a non-negligible amount of primordial non-Gaussianity in the cosmological perturbations [586]. In section 3.4.2 we will discuss the effect of primordial non-Gaussianity on the anisotropies of the AGWB in cross-correlation with the CMB.

The number of events that contribute to the AGWB fluctuate according to a Poisson distribution, inducing a shot noise term in the AGWB [561, 587–589]. In the formalism adopted here, consistently with [552, 557, 582], the shot noise of the energy density is a Gaussian random variable $\delta\Omega_{\text{GW}}^{\text{SN}}(\hat{n}, f)$, with zero mean and covariance given by

$$C^{\text{SN}}(\hat{n}, f; \hat{n}', f') = \int dz d\vec{\theta} \frac{R(z)p(\vec{\theta})\delta(\hat{n} - \hat{n}')}{(1+z)^2 H^2(z) \frac{dV}{dzd\Omega}(z) T_{\text{obs}}} \prod_{\tilde{f}=f, f'} \frac{\tilde{f}}{\rho_c c^2} \frac{dE}{df d\Omega}(\tilde{f}, z, \vec{\theta}), \quad (3.24)$$

where we have neglected Poisson fluctuations proportional to the square of the number of GW events, suppressed by the low number of GWs per host galaxy. The term $\delta(\hat{n} - \hat{n}')$ indicates that Poisson fluctuations are uncorrelated for objects in different regions in the sky, thus the angular power spectrum is constant in the multipole space.

Kinematic anisotropies. The motion of the observer relative to the rest frame of the sources induces kinematic anisotropies in the AGWB on the order of $10^{-3}c$, consistent with measurements of Doppler boosting in the CMB [590, 591]. The velocity of the Local Group (LG) relative to the large-scale structure (LSS) generates a dipole in the anisotropies [557], while the motion of the Earth relative to the LG induces a Doppler boosting of the signal, resulting in a dipole of order one in the velocity and in a mode coupling between different multipoles, linear in both the dipole and the intrinsic anisotropies [556, 592]. In [557], a comprehensive discussion of all contributions to the dipole of the AGWB is provided, demonstrating how the Einstein Telescope (ET) in combination with the Cosmic Explorer (CE) could be used to accurately measure the velocity of the Local Group (LG), yielding results competitive with large-scale structure (LSS) observations. Ref. [558] showed that prior knowledge of the direction of the LG velocity (e.g., from CMB observations) can enhance the sensitivity to the AGWB kinematic dipole. Additionally, it showed that the Earth’s peculiar motion must be accounted for to avoid biases of up to $\sim 10\%$ in the inferred dipole amplitude.

Component separation of the anisotropies. The anisotropies of the AGWB are expected to be dominated by the shot noise fluctuations, which are of the order of $10^{-1} - 10^{-3}$, because of the low number of CBC mergers in the Universe per year. Therefore, the detection of the kinetic and intrinsic anisotropies could be very challenging. Several attempts have been made in order to reduce the impact of the shot noise, for instance by cross-correlating the AGWB with galaxies [589] or lensing [551], but, because of the low angular resolution of ET to the anisotropies of the GWB, it is challenging to get a SNR larger than one for the intrinsic and kinetic components using standard techniques. In [557], it has been realized that the three contributions to the angular power spectrum exhibit different frequency dependencies for $f \gtrsim 100$ Hz. Therefore, the distinct frequency scaling of these anisotropies could potentially be used for component separation. In the left panel of figure 21 we plot the three dipoles at different frequencies, while in the right one we show the SNR of the anisotropies with ET+CE in 5 years of observations. In the right plot, the blue line is the SNR of the kinetic

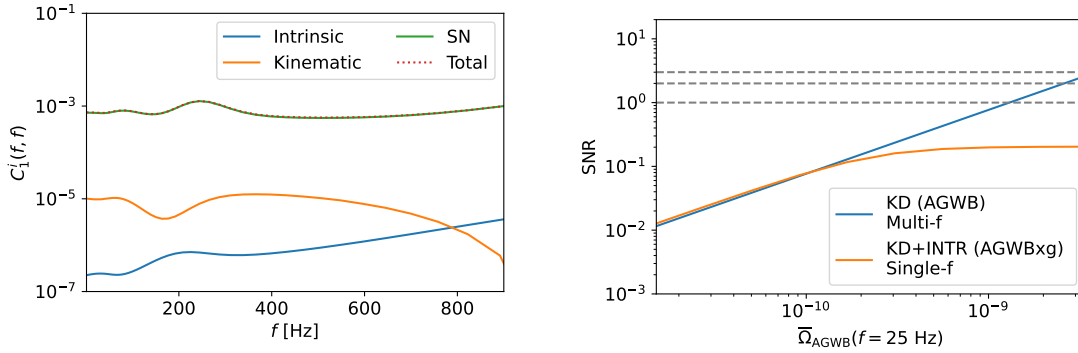


Figure 21. *Left:* plot of the intrinsic, shot noise and kinetic contributions to the dipole of the AGWB as a function of the frequency. *Right:* plot of the SNR of the kinetic dipole of the AGWB, obtained with the multi-frequency analysis of the anisotropies, and of the intrinsic and kinetic anisotropies of the AGWB, in cross-correlation with the galaxy survey SKAO2, as a function of the amplitude of the monopole of the AGWB. Here a network with the triangular ET configuration plus CE with the two interferometers (of 20km and 40km) placed in Hanford and Livingston for 5 yrs has been considered.

dipole of the AGWB with the multi-frequency analysis. When the instrumental noise is taken into account, it becomes evident that a kinetic dipole in an AGWB with an amplitude consistent with the current LVK bound could be detected with an $\text{SNR} \simeq 3$. In order to show the power of the multi-frequency analysis, we also plot the SNR of the intrinsic and kinetic anisotropies of the auto- and cross-correlation of the AGWB with the SKAO2 galaxy catalogue [593], up to $\ell_{\text{max}} = 100$ (orange line), without considering the frequency dependence of the anisotropies. When instrumental noise does not dominate the AGWB signal, it become evident that multi-frequency analysis seems to be the most efficient technique for addressing the shot noise issue, improving by orders of magnitude the SNR.

3.1.3 Polarization of the GWB and parity violation

3.1.3.1 Polarization of the AGWB. In GR, the AGWB is expected to be unpolarized because the average over polarization angles set to zero linear polarizations and the average over inclination angles set to zero the circular polarization (see [561] for the explicit computation). However, because of the effect of sampling a realization, shot noise can generate a net amount of both linear and circular polarization (even for isotropic source distributions [561]). In particular, circular polarization could be observed by ET+CE with an SNR greater than 2 in one year of observations [582]. A computation analogous to eq. (3.24) for the V Stokes parameter indicates that the amplitude of the circular polarization is indeed comparable to the shot noise in the intensity and exhibits an angular power spectrum that is constant in multipole space. In the right panel of figure 20 we plot the monopole (intensity) and amplitude of the shot noise as a function of frequency, along with the PLS of the intensity (monopole) and circular polarization (monopole plus anisotropies). The circular polarization of the AGWB represents a foreground for the detection of polarized cosmological signals, and component separation techniques based on the frequency and angular dependence of the signal should be adopted [582].

Various mechanisms in the early Universe can generate parity violation, leading to an uneven production of left- and right-handed circularly-polarized GWs. Since parity-violating GWs are expected to be cosmologically sourced, polarization could be used together with the intensity to identify the primordial origin of the signal, although the circular polarization of the AGWB discussed is an unavoidable foreground for polarized cosmological backgrounds. Searches for polarized GWBs can allow one to place constraints on parity-violating theories, as we discuss next.

3.1.3.2 Chiral GW Sources. A number of mechanisms can result in parity violation in the early Universe [594]. Two compelling, well-studied, classes of parity-violating inflationary models that could lead to a circularly polarized GW background include Chern-Simons gravity [595–598], and axion-gauge field inflation [599–623]. More details on axion inflation mechanism can be found in section 3.2.1.2. Other sources such as GWB from turbulence in the primordial plasma induced either from cosmological first-order (electroweak or QCD phase transition beyond the Standard Model) phase transitions [624–626] and from the primordial magnetic fields that are coupled to the cosmological plasma [627–631] can be chiral. Although an AGWB is typically expected to be unpolarized, polarization can occur when considering shot noise fluctuations, as we discussed in section 3.1.2.2.

3.1.3.3 Detection Formalism for Chiral Gravitational Waves. We now discuss the formalism used in detecting parity-violating backgrounds, following [560, 632]. This formalism was used in practice in the LVK data in [633, 634], and in simulated third-generation networks in [633, 635]. We use the circularly polarized bases $e^R = (e^+ + ie^\times)/\sqrt{2}$ and $e^L = (e^+ - ie^\times)/\sqrt{2}$ (with $+$ and $\times$ the plus and cross polarizations, respectively) to obtain the right- and left-handed modes $h_R = (h_+ - ih_\times)/\sqrt{2}$ and $h_L = (h_+ + ih_\times)/\sqrt{2}$, respectively. Using eq. (3.10), the right-right and left-left correlators can then be written as

$$\begin{pmatrix} \langle h_R^*(f, \hat{\mathbf{n}}) h_R(f', \hat{\mathbf{n}}') \rangle \\ \langle h_L^*(f, \hat{\mathbf{n}}) h_L(f', \hat{\mathbf{n}}') \rangle \end{pmatrix} = \frac{1}{8\pi} \delta(f - f') \delta(\hat{\mathbf{n}}, \hat{\mathbf{n}}') \begin{pmatrix} I(f, \hat{\mathbf{n}}) - V(f, \hat{\mathbf{n}}) \\ I(f, \hat{\mathbf{n}}) + V(f, \hat{\mathbf{n}}) \end{pmatrix}, \quad (3.25)$$

We use the standard cross-correlation estimator [80, 636–638]

$$\begin{aligned} \langle \hat{C}_{d_1 d_2} \rangle &= \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \delta_T(f - f') \langle s_{d_1}^*(f) s_{d_2}(f') \rangle \tilde{Q}(f') \\ &= \frac{3H_0^2 T}{10\pi^2} \int_0^{\infty} df \frac{\Omega'_{\text{GW}}(f) \gamma_I^{d_1 d_2}(f) \tilde{Q}(f)}{f^3}, \end{aligned} \quad (3.26)$$

where

$$\begin{aligned} \Omega'_{\text{GW}} &= \Omega_{\text{GW}} \left[1 + \Pi(f) \frac{\gamma_V^{d_1 d_2}(f)}{\gamma_I^{d_1 d_2}(f)} \right], \\ \gamma_I^{d_1 d_2}(f) &= \frac{5}{8\pi} \int d\hat{\Omega} (F_{d_1}^+ F_{d_2}^{+*} + F_{d_1}^\times F_{d_2}^{\times*}) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x}}, \\ \gamma_V^{d_1 d_2}(f) &= -\frac{5}{8\pi} \int d\hat{\Omega} (F_{d_1}^+ F_{d_2}^{\times*} - F_{d_1}^\times F_{d_2}^{+*}) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x}}, \end{aligned} \quad (3.27)$$

with H_0 the Hubble parameter, T the observing time, $\delta_T(f) = \sin(\pi f T)/(\pi f)$, $\tilde{s}_{d_1}(f)$ and $\tilde{s}_{d_2}(f)$ the Fourier transforms of the strain time series of two GW detectors (denoted by

d_1, d_2). We apply the usual $\tilde{Q}(f)$ as the optimal filter taking into account detectors' strain power spectral densities, and $F_n^A = e_{ab}^A d_n^{ab}$ stands for the contraction of the tensor modes of polarization A to the n th detector's geometry. We denote by $\gamma_I^{d_1 d_2}$ the standard overlap reduction function of two detectors d_1, d_2 , and by $\gamma_V^{d_1 d_2}$ the overlap function associated with the parity violation term. The polarization degree, $\Pi(f) = V(f)/I(f)$, takes on values between -1 (fully left polarization) and 1 (fully right polarization), with $\Pi = 0$ being an unpolarized isotropic GWB. We build the Gaussian log-likelihood for a network of N detectors

$$\log p(\hat{C}(f)|\boldsymbol{\theta}) \propto \sum_{i,j>i}^N \sum_f \frac{[\hat{C}_{d_i d_j}(f) - \Omega'_{\text{GW}}(f, \boldsymbol{\theta})]^2}{\sigma_{d_i d_j}^2(f)}, \quad (3.28)$$

where $\hat{C}_{d_i d_j}(f)$ is the frequency-dependent cross-correlation estimator of the GWB calculated using data from detectors d_i, d_j , and $\sigma_{d_i d_j}^2(f)$ is its variance, and $\boldsymbol{\theta}$ are the associated parameters to the tested GWB model.

3.1.3.4 Detection Prospects for Chiral Gravitational Waves. In the context of ET, this formalism has a number of ramifications on parity violation detection. We plot the standard and parity-violating associated overlap reduction functions in figure 22. One can see that the parity violating associated overlap reduction function between two detectors of an assumed triangular design¹⁰ would be approximately 0 due to their co-planarity to one another. From eq. (3.27) this would imply that $\Omega_{\text{GW}} \approx \Omega'_{\text{GW}}$ for even entirely chiral GWs. Thus, parity- violation inferences on GW data with this formalism is difficult with a triangular ET configuration alone. However, ET in combination with other detectors (such as CE) should have $\gamma_V^{d_1 d_2}(f) \neq 0$ due to their non-coplanarity — thus allowing for parity violating inferences in extended networks whilst taking advantage of ET's sensitivity. A 2L design for ET, and any additional detectors in the network, would not face such problems for similar reasons. Reconsidering the previously described formalism while taking into account the full response functions and noise curves across the entire frequency band (for planar detectors), it is found that $\mathcal{O}(1)$ net polarization can be detected in the GWB, with a magnitude of $h^2 \Omega_{\text{GW}} \simeq 10^{-11}$ and an SNR of order one [639]. Fully analytical and covariant details of these parity-sensitive response functions, which are critical for parity violation detection, can be found in [639]. Extensions to search for GWBs with anisotropies and circular polarization have been developed and studied in the context of ground-based detectors [640]. Many results are in agreement with [634] in an LVK context; however, an ET+CE network can improve this by about three orders of magnitude. As discussed in section 3.1.2.2, the presence of a circularly polarized AGWB may provide an additional challenge for CGWB parity violation detection [582]. A simplified example showed that, by taking advantage of different background features, such as their frequency dependence and features in the sky maps, it

¹⁰We follow the terminology of the 2020 ET Design Report Update, <https://apps.et-gw.eu/tds/?r=18715>: the high-frequency (HF) and low-frequency (LF) interferometers that make the so-called “xylophone” configuration are indeed referred to as “interferometers”. The combination of a HF interferometer and a LF interferometer (whether in a L-shaped geometry, or with arms at 60° as in the triangle configuration) is called a “detector”. The whole ensemble of detectors is called an “observatory”. So, ET in the triangle configuration is made of three detectors for a total of six interferometers, while in the 2L configuration it is made of two detectors, for a total of four interferometers.

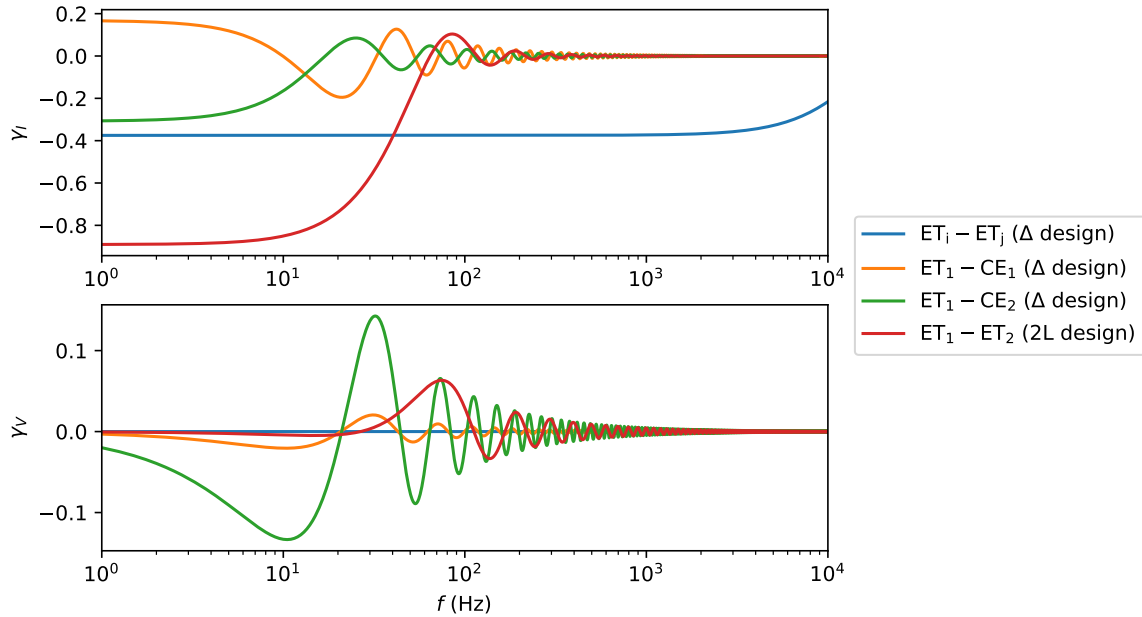


Figure 22. Standard and parity-violating overlap reduction functions plotted over ground-based detector frequency range. When assuming a triangular configuration for ET, we plot the result for a pair of ET detectors (out of the three composing the triangle), and for one ET detector (out of the three composing the triangle) and a Cosmic Explorer (CE), with CE taken either in the current Hanford site or in the current Livingston sites. When assuming the 2L configuration for ET, we plot the result for this pair of L-shaped detectors. The triangular ET design is assumed in Virgo location. We denote the triangular pairing with i, j as all pairings produce the same overlap reduction functions.

is possible to reconstruct the primordial background with good precision using ET+CE, primarily limited by instrumental noise, as in the case of the intensity. That said, separation corrections of the order of 20% to the total SNR must be carefully taken into account.

3.1.4 Source separability

Once a GWB is detected, the next important question will be to relate the signal to the sources that contribute to it. In particular, one will be interested in separating compact binary coalescences (namely a pure astrophysical contribution) from possible sources of a cosmological origin (for instance inflation, cosmic strings or first-order phase transitions). A Bayesian analysis has shown that current terrestrial interferometers (i.e., Advanced LIGO and Advanced Virgo network), even once operating at design sensitivity will not allow for separation of the sources [641]. Assuming one is able to do an individual source subtraction [642], the residual CBC contribution to the GWB will be dominated by unresolved binary neutron star mergers at the level of $\sim 10^{-11}$ at 10 Hz.¹¹ Third generation (3G) detectors, like Einstein Telescope, and in particular a network of 3G detectors will allow simultaneous separation of astrophysical and cosmological GWBs, provided reasonable levels of individual source subtraction can be achieved [641].

¹¹A more detailed analysis shows that the result actually depends on the shape of the cosmological spectrum that is being searched, see [643] and section 10.5.3.

The separability of sources in the ET frequency band has several challenges given that multiple components contribute to the overall signal. This makes the parameter space large and complex. This complexity, along with degenerate characteristics, complicates such a task making it also computationally challenging. A computationally less demanding approach could be performed in a two-step process: first, reconstruct the GWB spectrum agnostically (as in LISA data analysis techniques [644–647]); second, convert the spectrum constraints into population and cosmological parameter constraints (see e.g., [648] for a recent application). This method allows to balance accuracy and feasibility and can help identify relevant parameter spaces for full Bayesian techniques.

Another limitation in the detection of the GWB signal is the impact of significant residuals, primarily from uncertainties in coalescence phase and luminosity distance, when resolved sources are subtracted [642, 643]. Enhanced signal subtraction methods are necessary to improve the observation of other astrophysical and cosmological GWBs.

3.1.5 Impact of correlated noise on GWB

To be able to resolve the weak signal from the GWB, one needs to rely on cross-correlation methods to suppress independent noise sources at the different detectors. In the absence of correlated noise, this implies that any remaining correlated signal is from GWs. However, past research has shown that such a correlation analysis is prone to noise contamination from correlated noise sources, both for co-located as well as widely separated detectors. As a first order approach one should consider to reduce the noise couplings, so to lower or entirely remove such correlated noise disturbances. If this is deemed impossible one could consider performing a joint parameter estimation taking both the correlated noise and the GWB signal into account (see section 3.1.6). Finally, if the correlated noise has a much larger amplitude and/or is not known with high accuracy, one has to consider whether it is more efficient to remove all the contaminated frequency bins/time segments and only use the part of the data with the highest data quality.

In the next sections we will discuss two noise sources, which have been considered to potentially contaminate the search for an isotropic GWB with the ET. Here we only focus on fundamental, environmental noise sources and their impact. Earlier work has indicated that co-located detectors could have additional correlated noise from detector infrastructure, which could potentially have significantly higher effects [649]. However, more work is needed to understand the levels of correlation of infrastructural noise in a realistic ET environment.

3.1.5.1 Noise correlations from electromagnetic phenomena. On the Earth, every second about hundred lightning strikes occur. This vast amount of electromagnetic discharges creates standing waves in the cavity formed by the Earth’s surface and the ionosphere, known as the Schumann resonances [650, 651]. With the fundamental mode and subsequent harmonics occurring at ~ 7.8 Hz, ~ 14 Hz, ~ 21 Hz, etc., this broadband noise source falls within ET’s low frequency sensitivity range. Due to the global nature of this noise source, it has the potential to couple coherently to GW interferometric detectors, no matter their separation. In past work the Schumann resonances were coherently observed in the magnetometers at both the LIGO sites as well as at Virgo [652–656].

Earlier work has highlighted how correlations due to electromagnetic fluctuations can impact the search for an isotropic SGWB up to ~ 30 Hz if one assumes similar levels of magnetic coupling at the ET compared to 2G detectors such as LIGO and Virgo [649]. The coupling should be reduced by three to four orders of magnitude per detector around 5 Hz to remove any noise contamination [649]. Furthermore, above 30 Hz there is potential of additional contamination in narrow frequency bands around resonant features in the magnetic coupling. More work is needed to understand whether the magnetic coupling to ET can be reduced compared to 2G detectors to (partially) deal with this noise contamination. Also the effectiveness of noise subtraction, as described in [653–655], or joint estimation, as discussed in [657], in case of such a loud noise sources should be further investigated. However, it is to be noted the correlated signal from magnetic origin has a steeply declining behavior as a function of frequency, whereas many of the expected GW backgrounds have a flat or slowly increasing Ω_{GW} signature. This characteristic behavior originates from the steeply declining magnetic coupling function observed at 2G detectors.

3.1.5.2 Noise correlations from seismic phenomena. More recent work has also investigated the effect of coherent seismic noise and the subsequent Newtonian Noise (NN) [658, 659].¹² This noise source will only appear correlated between different ET detectors if they are located close to each other. In this context, the detectors are considered to be located close to each other if their input and/or end towers are separated by a distance of several hundreds of meters up to several kilometers. This will be the case for ET in the triangular configuration. The exact distance at which the coherence of seismic noise decreases enough in the relevant frequency band for the ET is likely to be somewhere between 2 km and 10 km; however, additional research is needed to confirm this.

Depending on the specific seismic environment of ET’s site, the coherent NN noise originating from seismic body-waves will contaminate the sensitivity to an isotropic SGWB up to 20 Hz–40 Hz [658, 659], as shown in figure 23. Similar to the magnetic noise scenario, the lowest frequencies are affected more seriously and the NN is predicted to have a declining behavior as a function of frequency (when expressed in terms of Ω_{GW}). This might facilitate spectral separation if one is looking for a GWB which has a flat or increasing Ω_{GW} spectrum as function of frequency. In section 3.1.6 we will discuss the recovery of a cosmological signal in the presence of such correlated noise.

3.1.6 Reconstruction of GWBs in presence of correlated noise

As discussed in section 3.1.5, a critical challenge in detecting the GWB is the precise characterization of correlated detector noise, which is essential for distinguishing the signal from spurious instrumental and environmental fluctuations. We have seen that the triangular geometry suffers from correlated noise due to the relative proximity of interferometer components, leading to potential contamination in GWB searches.

The effect of correlated noise on the detection and characterization of the GWB with ET has been recently studied in [660], where a Bayesian framework has been introduced

¹²NN is a force directly exerted on GW test-masses caused by density fluctuations in the surrounding medium.

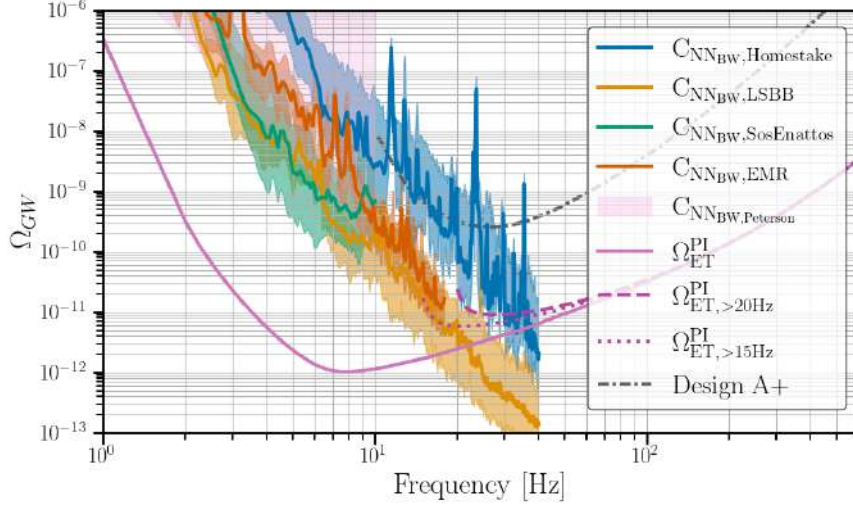


Figure 23. The projected impact from correlated NN from body-waves on the search for an isotropic GWB. We present budget based on observed seismic correlations from four different seismic sites. Note that many parameters, such as depth, horizontal separation, etc., vary for the different sites; see [659] for an in-depth discussion. Reprinted figure with permission from [659], Copyright (2024) by the American Physical Society.

to simultaneously reconstruct the parameters of the GWB and the correlated noise, both modeled as power laws in the frequency domain. For detectors I and J , the covariance of the noise is defined as the two-point correlation function, $N_{IJ}(f) \equiv \langle \tilde{n}_I(f) \tilde{n}_J^*(f') \rangle$. For the triangle configuration, in ref. [660] is made the assumption that all diagonal terms in the PSD are given by the same function $N_d(f)$, and that all off-diagonal terms are also all given by the same function $N_o(f)$, so that

$$N_{IJ}(f) = \begin{pmatrix} N_d(f) & N_o(f) & N_o(f) \\ N_o(f) & N_d(f) & N_o(f) \\ N_o(f) & N_o(f) & N_d(f) \end{pmatrix}, \quad (3.29)$$

and it is assumed that the off-diagonal terms can be parametrized by a simple power-law

$$N_o(f) = N_o(2.75 \text{ Hz}) \left(\frac{f}{2.75 \text{ Hz}} \right)^{n_{\text{noise}}}. \quad (3.30)$$

Defining $r = N_o(2.75 \text{ Hz})/N_d(2.75 \text{ Hz})$, so that r is the correlation coefficient at the reference frequency 2.75 Hz, we then write

$$N_o(f) = N_d(2.75 \text{ Hz}) r \left(\frac{f}{2.75 \text{ Hz}} \right)^{n_{\text{noise}}}, \quad (3.31)$$

The value of the tilt expected from Newtonian noise is $n_{\text{noise}} = -8$ [658, 659, 661], while different values of the correlation coefficients, in the physical range $r \in [-0.5, 1]$, have been tested.

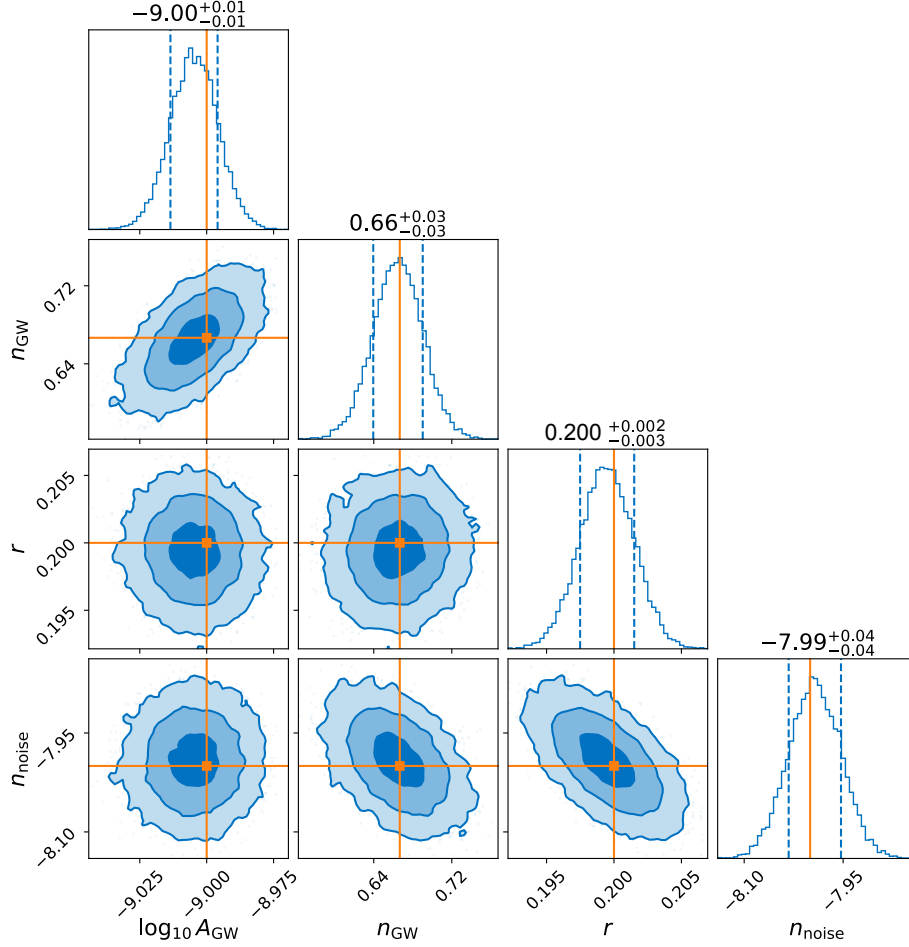


Figure 24. Corner plot of the posterior distributions for the log-amplitude and tilt of the GWB, $\{\log_{10} A_{\text{GW}}, n_{\text{GW}}\}$ for the correlation coefficient and tilt of the noise, $\{r, n_{\text{noise}}\}$, when the injected signal has $\log_{10} A_{\text{GW}} = -9$ and $n_{\text{GW}} = 2/3$, while the injected noise has $r^{\text{inj}} = 0.2$, $n_{\text{noise}} = -8$. The shaded areas represent the 1-, 2-, and 3- σ credible regions. The orange lines indicate the injected values for the four parameters. Reproduced with permission from [660].

The stochastic GW signal is also modeled as a simple power-law,¹³

$$\Omega_{\text{GW}}(f) = A_{\text{GW}} \left(\frac{f}{25 \text{ Hz}} \right)^{n_{\text{GW}}} \quad (3.32)$$

Under these assumptions, ref. [660] derives a general likelihood for the GWB that accounts for correlated noise and performs parameter estimation for the whole set of parameters $\{A_{\text{GW}}, n_{\text{GW}}, r, n_{\text{noise}}\}$ that characterize the signal and the correlated noise.

The quality of the reconstruction depends, of course, on the amplitude of signal with respect to the noise. Figure 24 shows the result of the parameter reconstruction when the

¹³For the GW signal, it is customary to take a reference frequency closer to the minimum of the PLS, such as 25 Hz here, while for the correlated noise in eq. (3.31) is used a reference frequency 2.75 Hz closer to the region where it is more relevant. Of course, different choices of the reference frequency can be reabsorbed into different definitions of the corresponding amplitudes.

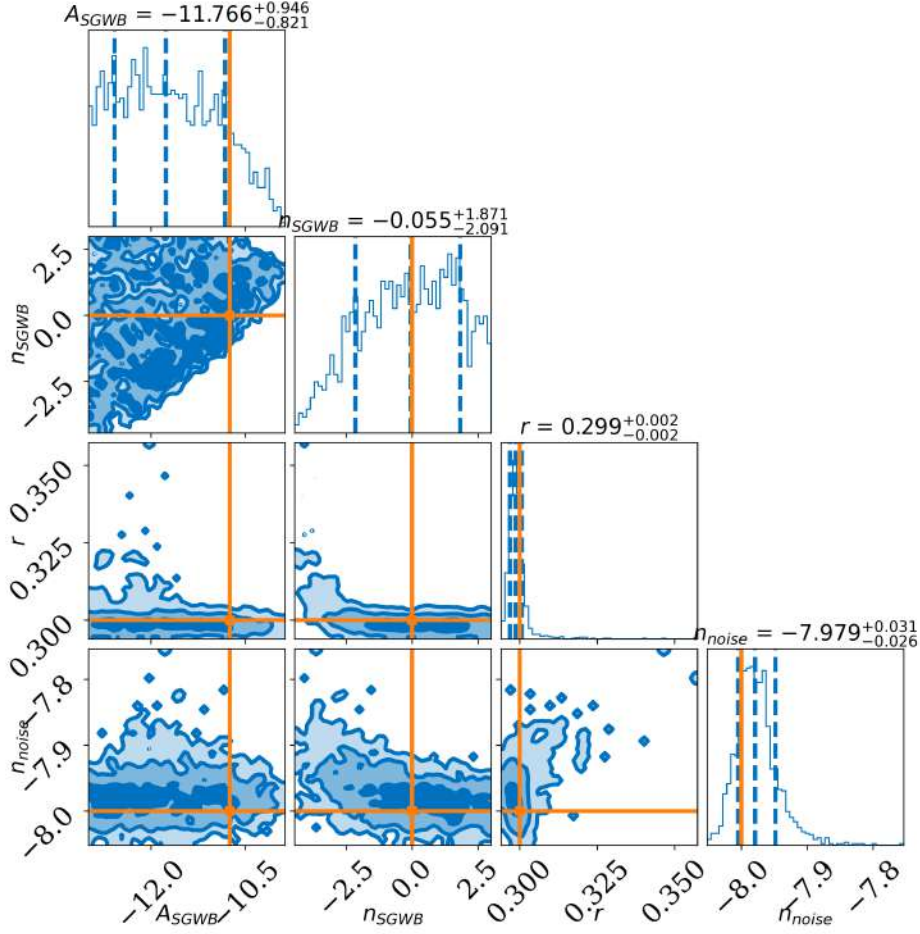


Figure 25. As in figure 24 when the injected signal has $\log_{10} A_{\text{GW}} = -11$ and $n_{\text{GW}} = 0$, while the injected noise has $r^{\text{inj}} = 0.2$, $n_{\text{noise}} = -8$.

injected correlated noise has the correlation coefficient $r^{\text{inj}} = 0.2$ and a tilt $n_{\text{noise}}^{\text{inj}} = -8$, while the injected GW signal is taken to have $A_{\text{GW}}^{\text{inj}} = 10^{-9}$, $n_{\text{GW}}^{\text{inj}} = 2/3$, which are the amplitude and tilt expected for the stochastic background generated by the superposition of coalescing binaries in the inspiral phase (see figure 205). This injected stochastic GW background accumulates an SNR of about 67 in just 1 day of observation (restricting the analysis to frequencies below 200 Hz, where the CBC background can be modeled as a power law with $n_{\text{GW}}^{\text{inj}} = 2/3$), while the correlated noise, if interpreted as a signal, has $\text{SNR} \sim 135$ (again for 1 day of observation).¹⁴ Since the SNR for a two-detector (or multiple-detector) correlation grows with the integration time as $T^{1/2}$, see eq. (10.59), these spectra correspond to an SNR of about 1280 and 2580, respectively, for 1 yr of integration.

In this case, where the GW signal has a very large SNR and a mild blue slope, quite different from the steep red slope of the noise, we see from the figure that, already with

¹⁴These are the signal-to-noise ratio computed using only the diagonal term N_d for the noise, i.e. we are considering the off-diagonal term as a competing correlated signal.

one day of observation, both the GWB and noise parameters could be reconstructed with percent-level accuracy.

We have repeated the analysis for a GW background with $A_{\text{GW}}^{\text{inj}} = 10^{-11}$, $n_{\text{GW}}^{\text{inj}} = 0$. Such a background would produce a signal-to-noise ratio $\text{SNR} = 3$ in 1 day of observation (i.e., $\text{SNR} \simeq 57$ in 1 yr). For the noise we still take $r^{\text{inj}} = 0.2$ and a tilt $n_{\text{noise}}^{\text{inj}} = -8$. The results of the parameter reconstruction with 1 day of data are shown in figure 25. We see that in this case the quality of reconstruction of the signal is less good (and in particular the tilt cannot be reconstructed), as expected from the fact that it has a smaller SNR, while the parameters of the noise, which is the same as before and therefore has a very large SNR (again, about 135 in just 1 day), can still be reconstructed well. Note that this reconstruction is obtained from just 1 day of data, but this is due to the fact that also the injected GW signal reaches an SNR equal to 3 in just 1 day. The effect of the same correlated noise will be significantly more important in the reconstruction of GW backgrounds with even lower amplitude, such as a background that would accumulate, say, $\text{SNR} = 3$ in 1 yr of data, i.e. near the nominal sensitivity limit determined by the diagonal terms in eq. (3.29).

This shows that the presence of correlated noise in the triangle configuration is a relevant limitation, although it could still be possible to reconstruct the GWB parameters, provided that the SNR of the signal is sufficiently large and that a reliable noise model is available. This conclusion, however, holds when the injected data are generated using the same power-law noise model that is then employed in the recovery of the signal. When dealing with real data, more sophisticated noise models, possibly informed by seismographic data taken on-site, will be necessary to achieve the required level of precision in noise reconstruction. The 2L configuration, where correlated noise is (to a first approximation) negligible, is immune to this problem.

3.2 Probing the early Universe

3.2.1 GWs from inflation

3.2.1.1 The single-field slow-roll paradigm: vacuum fluctuations. In its simplest realization, inflation is driven by a single scalar field slowly rolling down its potential [662–665]. There exist several such “minimal” inflationary models that fare well compared to observations, such as the Starobinsky model [663] or Higgs inflation [666]. The general prediction associated with single-field slow-roll models is that of a slightly red-tilted GW spectrum sourced by vacuum fluctuations [667, 668]. Typically, CMB probes are then expected to provide the most stringent constraints on this class of inflationary models.

Let us recall that, in a generic inflationary model, the primordial spectrum of tensor perturbations $\mathcal{P}_{T,\text{in}}(k)$ is defined by¹⁵

$$\langle h_{ij}^{\text{TT}}(\eta_{\text{in}}, \mathbf{x}) h_{ij}^{\text{TT}}(\eta_{\text{in}}, \mathbf{x}) \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_{T,\text{in}}(k), \quad (3.33)$$

where η_{in} is chosen deep into radiation dominance, and such that all modes of interest are super-horizon, and is taken as the initial value of conformal time for the subsequent

¹⁵We follow the definitions and notation of chapter 19 of [60].

evolution until the present epoch. A useful parametrization for the primordial GW spectrum in single-field slow-roll (SFSR) models is a power law

$$\mathcal{P}_{T,\text{in}}(k) = A_T(k_*) \left(\frac{k}{k_*} \right)^{n_T}, \quad (3.34)$$

where k_* is a suitably chosen pivot scale (for CMB studies typical values are $k_* = 0.05 \text{ Mpc}^{-1}$, or $k_* = 0.002 \text{ Mpc}^{-1}$), while n_T is the tensor spectral index, which in general is a function of k . In similar way, can be parameterized the primordial scalar power spectrum, with an amplitude $A_{\mathcal{R}}(k)$, where $\mathcal{R}$ is the primordial curvature perturbation. Using the equivalent notation $A_S(k) \equiv A_{\mathcal{R}}(k)$ for the amplitude of scalar perturbations, the ratio of the amplitude of the tensor and scalar power spectra allows to define the tensor-to-scalar ratio $r(k) = A_T(k)/A_S(k)$.

In the SFSR paradigm, for $f \gtrsim 10^{-4} \text{ Hz}$ (and therefore at the frequencies relevant for ground-based detectors, as well as for LISA and even PTA) the GWB produced by the amplification of quantum vacuum fluctuations can then be written as

$$h_0^2 \Omega_{\text{gw}}(f) \simeq 1.44 \times 10^{-16} \left(\frac{r(k_*)}{0.1} \right) \left(\frac{A_S(k_*)}{2.14 \times 10^{-9}} \right) \left(\frac{f}{f_*} \right)^{n_T} \left(\frac{106.75}{g_*(T_k)} \right)^{1/3}, \quad (3.35)$$

(see, e.g., eq. (21.357) of [60]).¹⁶ CMB observations constrain the amplitude of scalar perturbations to $A_S(k_*) \simeq 2.14 \times 10^{-9}$ at a pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$, and put an upper bound on $r(k_*)$ of about 0.03, when the consistency relation $r = -8n_t$ is assumed [669] (see also [670] for a constrain at $k_* = 0.01$). If the consistency relation is not assumed the bounds are less stringent with $r_{0.01} < 0.066$ and $-0.76 < n_T < 0.52$ at 95%CL [671]. The frequency f_* corresponding to the pivot scale k_* is extremely low, compared to the frequencies explored by ground-based detectors. For instance, a pivot scale $k_* = 0.002 \text{ Mpc}^{-1}$ corresponds to $f_* \simeq 3 \times 10^{-18} \text{ Hz}$. Therefore, in eq. (3.35), at the frequencies of ground-based detectors (or of LISA), in the term $(f/f_*)^{n_T}$, the factor f/f_* is very large, and the prediction for $h_0^2 \Omega_{\text{gw}}(f)$ at frequencies relevant for ET depends crucially on the sign of n_T . For single-field slow-roll models of inflation, $n_T = -r(k_*)/8$, so the tilt is negative (and small in absolute value, given the observational bound $r(k_*) < 0.1$). Then, at the frequencies relevant for ET, $h_0^2 \Omega_{\text{gw}}(f)$ is below 10^{-16} , and is not detectable by 3G detectors, see figure 26.

3.2.1.2 Multi-field scenarios. Remarkably, there exist a non-trivial number of multi-field inflationary scenarios supporting a detectable signal at interferometer scales. Amongst these, a promising class of models that has gained considerable attention is axion inflation (see [672] for a review). In these models the axion potential has an (approximate) shift symmetry protecting its light mass from large quantum fluctuations. When the axion-like particle plays the role of the inflaton the symmetries of the theory addresses directly the η -problem. Well studied single-field realisation of axion inflation (such as natural inflation [673]) have been recently ruled out [674]. Remarkably, multi-field realizations exist [599–623, 675, 676] with intriguing gravitational wave phenomenology and the added feature of a sub-Planckian axion decay constant, a very desirable feature from the top-down perspective [677]. In

¹⁶The expression of $h_0^2 \Omega_{\text{gw}}(f)$ for $f_{\text{eq}} \lesssim f \lesssim 10^{-4} \text{ Hz}$, where $f_{\text{eq}} \simeq 3 \times 10^{-17} \text{ Hz}$, is also quite similar, see eq. (19.291) of [60].

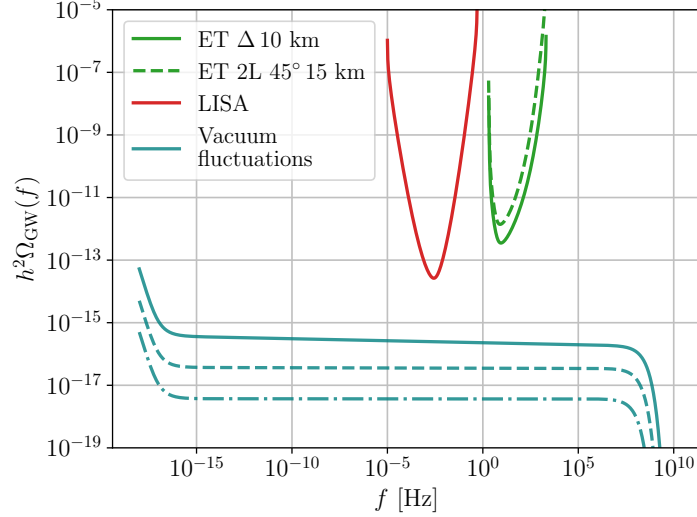


Figure 26. Spectrum of the single-field slow-roll inflation as a function of frequency, together with the PLS of LISA and ET in two different configurations. The solid, dashed, dot-dashed standard inflation curves are computed by adopting $r(k_*) = 0.1$, $r(k_*) = 0.01$ and $r(k_*) = 0.001$, respectively.

multi-field constructions the axion is typically coupled to a (hidden) gauge sector via a Chern-Simons term. Note that models with multiple axion-like particles (ALPs) have also been explored [678, 679], with ALPs populating also spectator sectors of the inflationary Lagrangian. The GW signatures of theories rich in spectator fields are very similar to those of their more minimal counterparts.

Let us consider the example of a pseudo-scalar inflaton field ϕ and $\mathcal{N}$ gauge fields A_μ^I (see e.g. [599–601, 603, 604, 675, 680, 681]):

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}^I F_I^{\mu\nu} - V(\phi) - \frac{\lambda^I}{4f}\phi F_{\mu\nu}^I \tilde{F}_I^{\mu\nu}, \quad (3.36)$$

with scalar potential V typically of the natural inflation type $V(\phi) = \Lambda^4[1 + \cos(\phi/f)]$, and where $F_{\mu\nu}^{Ia} = \partial_\mu A_\nu^{Ia} - \partial_\nu A_\mu^{Ia} - g\epsilon^{abc}A_\mu^{Ib}A_\nu^{Ic}$ is the field strength tensor. Notice that the gauge group index “a” is suppressed in eq. (3.36). The field strength reduces to the Abelian case for vanishing g . The dual field strength tensor is $\tilde{F}_I^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}^I/(2\sqrt{-g})$ and f is the mass scale suppressing higher dimensional operators, whilst λ parameterizes the strength of the Chern-Simons coupling. This setup leads to exponential gauge boson production, with potentially detectable GWs and possibly amplified scalar perturbation sourced by the gauge field.

Common to Abelian and non-Abelian scenarios is the following dynamics: the axion-inflaton rolls down its potential and dissipates some of its kinetic energy into the gauge sector. The parity-violating nature of the Chern-Simons term results in only one polarisation of the gauge fields being amplified when sourcing GWs (and scalar fluctuations). Such a sourcing mechanism may give rise to a blue or bump-like features in the GW and scalar spectra depending also on the inflaton/spectator role played by the axion(s).

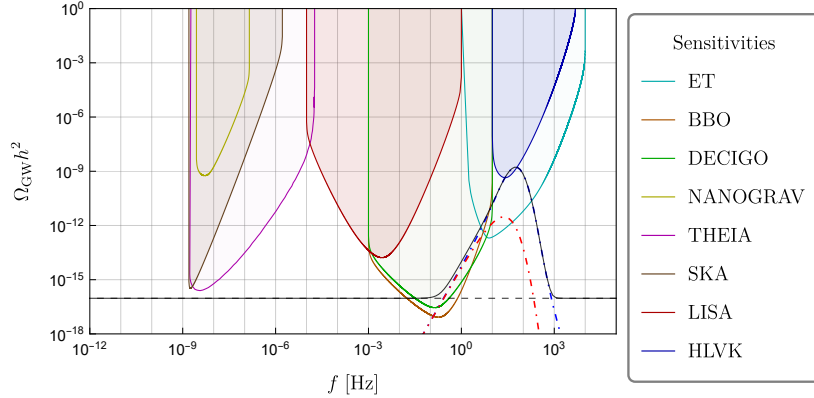


Figure 27. The GW density Ω_{GW} obtained in the case of a spectator axion superimposed with the sensitivity curves of GW detectors, including ET. The blue (red) dashed line corresponds to the strongly (weakly) amplified polarization. The continuous black line gives the total signal.

The corresponding GWB can be characterised as [601, 614, 682]

$$\Omega_{\text{GW}} \simeq \begin{cases} \frac{1}{12} \Omega_{R,0} \left(\frac{H^2}{\pi^2 M_{\text{Pl}}^2} \right) \left(1 + 4.3 \times 10^{-7} \frac{H^2}{M_{\text{Pl}}^2 \xi^6} e^{4\pi\xi} \right), & \text{for } U(1) \\ \frac{1}{12} \Omega_{R,0} \left(\frac{H^2}{\pi^2 M_{\text{Pl}}^2} \right)_{\xi=\xi_{\text{cr}}} \left[1 + \frac{1}{2} \xi_{\text{cr}}^6 \left(\frac{2^{7/4} H}{g \sqrt{\xi}} e^{(2-\sqrt{2})\pi\xi} \right)^2 \right]_{\xi=\xi_{\text{ref}}}, & \text{for } SU(2) \end{cases} \quad (3.37)$$

where M_{Pl} is the reduced Planck mass, the radiation energy density today $\Omega_{R,0} = 8.4 \times 10^{-5}$ and the parameter ξ is proportional to the inflaton velocity

$$\xi \equiv \frac{\lambda \dot{\phi}}{2fH}. \quad (3.38)$$

where $\xi_{\text{cr}} = \xi(x=1)$ and $\xi_{\text{ref}} = \xi(x=(2+\sqrt{2})\xi_{\text{cr}})$, with $x = -k\tau$ for conformal time η . The result for the $SU(2)$ case is valid when $0.008e^{2.8\xi_{\text{cr}}} \gtrsim 1/g$.

The exponential sensitivity of the gravitational wave spectrum is a key feature that allows for promising detection prospects. The GWB in eq. (3.37) is expected to be also non-Gaussian and fully chiral for large ξ [603, 614, 683], two features that can assist in the separation of sources in the GW data.

The ability of gauge fields to significantly amplify GW notwithstanding, the GW observational prospects can be limited, for Abelian models, by the risk of overproducing scalar modes [682, 684]. The non-observation of primordial black holes (PBH) places upper bounds on the scalar power spectrum. PBHs are produced when large curvature fluctuations re-enter the horizon during the radiation epoch. These PBHs must satisfy constraints on their abundance, derived from cosmological and astrophysical data [535, 685]. The statistics of the comoving curvature perturbation plays a key role in this upper bound [682, 684]: a Gaussian assumption places an approximate upper bound of $\mathcal{P}_{\mathcal{R}} \leq 10^{-2}$, whereas an assumed χ^2 -statistics places a tighter upper bound of $\mathcal{P}_{\mathcal{R}} \leq 10^{-4}$.

Enforcing PBH constraints may result in a GWB sourced by axion inflation that is undetectable by ET. There are however several models that can generate a detectable GW

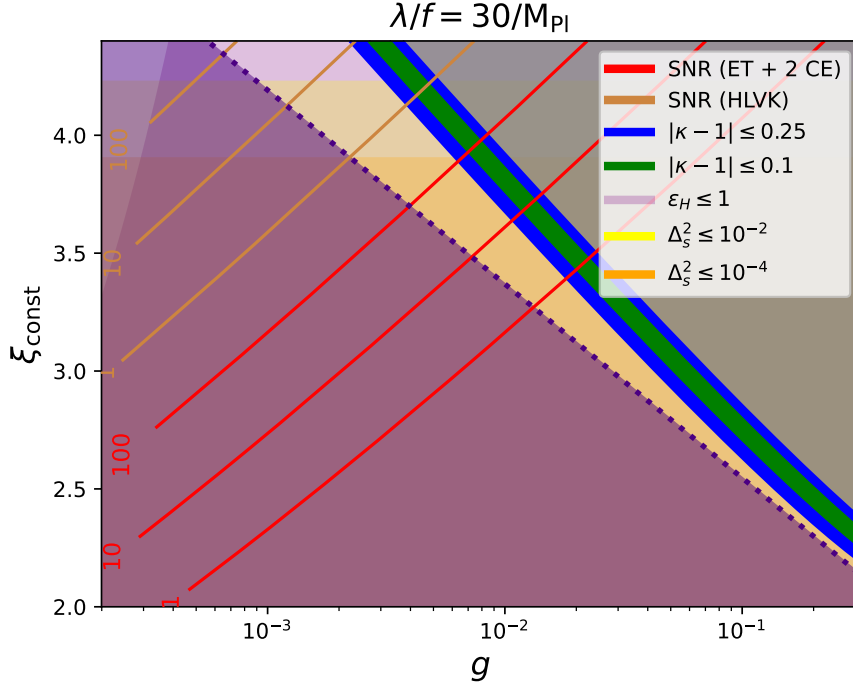


Figure 28. Backreaction and SNR prospects as a function of the gauge coupling constant g and the velocity parameter $\xi(f) = \xi_{\text{const}}$ for two values of the coupling constant, $\lambda/f = 30/M_{\text{Pl}}$. The Hubble rate on the CMB scale is assumed to be $H_{\text{CMB}} = 2.1 \times 10^{-5} M_{\text{Pl}}$. The brown and red lines are the SNR contours calculated assuming 1 year of observation with the design LVK and ET + 2 CE network sensitivities, respectively. The blue and green bands represent $|\kappa - 1| \leq 0.25$ and 0.1 , where we expect that the PBH overproduction bound could be relaxed in the strong backreaction regime. The light purple region represents the slow-roll parameter satisfying $\epsilon_H \leq 1$, where inflation is still ongoing. Yellow and orange regions correspond to the parameter space where the primordial curvature spectrum stays below the PBH upper limit assuming Gaussian ($\mathcal{P}_{\mathcal{R}} \leq 10^{-2}$) and χ^2 ($\mathcal{P}_{\mathcal{R}} \leq 10^{-4}$) statistics. Additionally, we shade the Abelian regime in magenta (below the black dotted line) and the strong backreaction regime in gray (upper region of the $|\kappa - 1|$ band), in both of which the estimation of the SNR is not reliable. Reproduced from [689]. CC BY 4.0.

signal all the while satisfying PBH bounds. For example, $\mathcal{N} > 1$ coupled $U(1)$ gauge fields can suppress the scalar power spectrum by $1/\mathcal{N}$ in the $\xi \gg 1$ regime. Such an extension was shown to create a working linear potential for $\mathcal{N} = 6$ couplings and a working quadratic model for $\mathcal{N} = 10$ couplings [635, 684]. Recent lattice simulations of axion $U(1)$ inflation assuming a quadratic potential showed that strong backreaction close to the end of inflation can suppress non-Gaussian behavior in comoving curvature perturbation, softening the PBH bound in ground detector frequency regime [686] (see also [687] for lattice studies on Abelian models). This relaxed bound would allow this model to be detectable by GW detectors, including ET, LISA and even advanced LIGO. Extending axion inflation models with an additional pseudoscalar field different from the inflaton can provide a localised bump in the tensor and scalar spectrum. It can also create detectable GW signal that conform to PBH and CMB related bounds [684, 688], see figure 27.

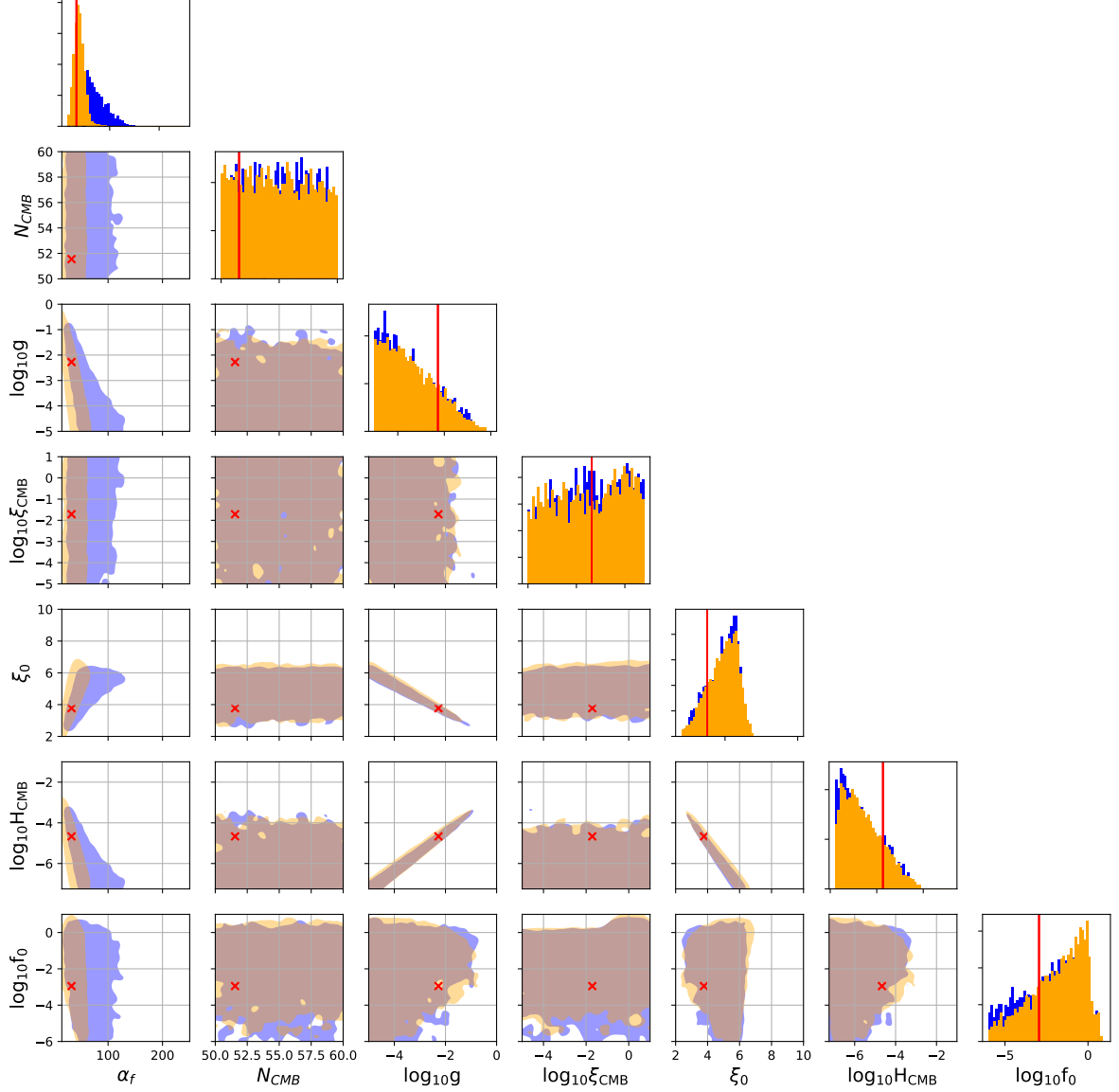


Figure 29. Posterior distributions from Bayesian inference assuming a piecewise, toy model potential, with inflaton velocity ξ_{CMB} before pivot frequency f_0 , and velocity ξ_0 after. This search is done assuming χ^2 statistics. The shaded regions correspond to the 95% CL. The different colors used are: blue for ET and orange for a network consisting of ET+2CE. The red cross and line correspond to the injected values. Reproduced from [689]. CC BY 4.0.

The GW sourcing mechanism for non-Abelian models is rather different: GWs are sourced already at the linear level by tensor degrees of freedom in the gauge sector [605]. The sourcing mechanism for scalars being less tied to the fate of tensors amplification, PBH bounds are typically less stringent in these scenarios.

A PBH constraint in non-Abelian field contexts can limit, but does not exclude GWB detection prospects [623, 689]. A general parameter space study performed in [689] found that assuming, conservatively, a constant inflaton velocity,¹⁷ a GWB from non-Abelian fields

¹⁷Inflaton velocity ξ generally increases exponentially during inflation.

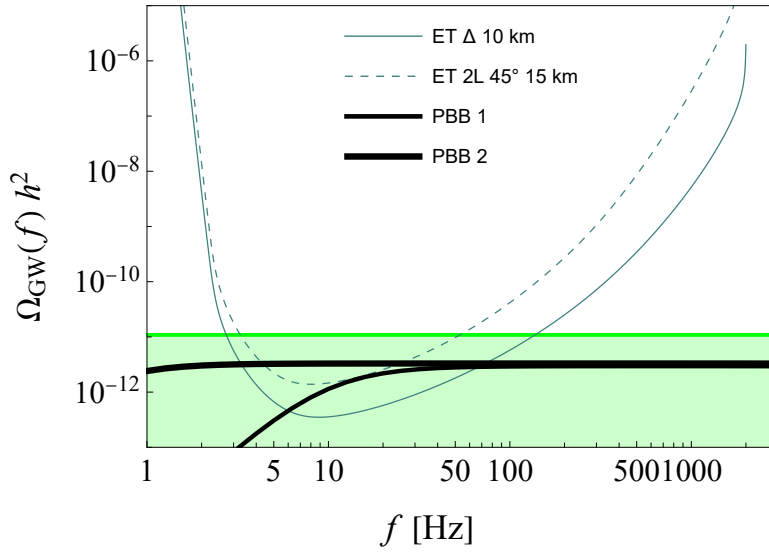


Figure 30. GWB predicted by pre-big-bang cosmology, compared with the PLS of the considered ET configurations. Here we assume an observation time of one year and $\text{SNR} = 1$. The shaded green area is the range of the GWB allowed by a set of consistency conditions on the parameter space of the model [690]. The GWB spectral shape is computed assuming $z_s = 5 \times 10^6$, $z_d = 10^3$, $z_\sigma = 2.2$ (“PBB 1” curve, broken power law within the ET sensitivity window) and $z_s = 10^8$, $z_d = 10^3$, $z_\sigma = 2.2$ (“PBB 2” curve, almost flat plateau within the ET sensitivity window). Here $z_s = \eta_s/\eta_1$, $z_d = \eta_d/\eta_1$ and $z_\sigma = \eta_\sigma/\eta_1$ are ratios of conformal-time scales marking four different regimes: de Sitter evolution for $-\eta_s < \eta < -\eta_1$, radiation domination for $-\eta_1 < \eta < \eta_\sigma$, matter domination for $\eta_\sigma < \eta < \eta_d$ and another radiation-domination phase for $\eta > \eta_d$.

can be 3G detectable — even with more constraining power in the χ^2 -statistical assumptions, as reported in figure 28. An injection of a GWB sourced by a piece-wise scalar potential analysed using a triangular ET detector and a ET+2CE network yielded a logBayes factor for an GWB versus noise greater than 79, a confident detection. The posterior distribution of the non-Abelian and of the cosmological parameters is shown in figure 29 for different detector configurations.

3.2.1.3 Pre-big-bang cosmology. As we have seen in the previous subsection, the inflationary prediction from single-field slow-roll inflation is in general too small to be detectable by third-generation detectors. From eq. (3.35) we see that the key to having a detectable spectrum is to have $n_T > 0$ (i.e., a “blue” spectrum), at least up to the ET frequencies. Eventually, the slope will have to flatten until it reaches a cutoff, in order not to violate upper bounds on $h_0^2 \Omega_{\text{gw}}(f)$, such as the Big-Bang Nucleosynthesis (BBN) bound, that requires that the integral of $h_0^2 \Omega_{\text{gw}}(f)$ over $d \log f$ will be smaller than $\mathcal{O}(10^{-6})$ [691].

A model that displays such a behavior is the so-called “pre-big-bang” cosmology [692, 693], which emerges quite naturally from the low-energy effective action of string theory, and predicts a blue tensor spectrum [694, 695]. The GWB of this model roughly has a bell-shaped form characterized by a maximal flat plateau whose length in frequency can vary from zero (maximum peak) to a wide interval, depending on the value of the theoretical parameters [690].

The portion of this shape falling into the ET frequency range is either a flat signal (the plateau at maximal amplitude) or a broken power law; see figure 30. In the latter case, the GWB raises as f^3 at low frequencies and then, at some frequency f_s , it changes slope to f^0 ; f_s must be taken as a free parameter, since it depends on the details of the cosmological evolution in the high-curvature phase of the model [696, 697]. The spectrum then drops down at higher frequencies not shown in the plot.

3.2.2 GWs from phase transitions

One of the possibly strongest sources of GWs from the early Universe is a cosmological first-order phase transition [624] that could lead to large observable signals at next-generation GW observatories. The peak frequency of the signal is fixed by the temperature of the Universe at the time of the phase transition. Particularly interesting is the case of the electroweak phase transition that would give rise, if it is first-order, to a GW signal in the milliHertz range observable at LISA [537, 698–702]. ET would be a window on phase transitions happening much earlier, and would therefore probe particle physics that is well beyond the reach of particle colliders.

While the Standard Model of high energy physics does not exhibit any first order phase transitions, numerous Beyond Standard Model (BSM) scenarios do. Quantum field theory and string theory typically feature a complicated vacuum manifold with several coexisting minima. The unknown evolution of the early Universe might have populated, possibly via the mechanism of symmetry restoration, false vacua and lead to a series of phase transitions (PTs) between the minima of the manifold. The larger the latent heat released during the transition, the stronger is the resulting GW signal. In this context, first-order PTs (FOPTs) proceed via the stochastic nucleation of spherical bubbles of the new phase in the old phase, with the probability of nucleation per unit of time and volume controlled by stochastic processes

$$\mathcal{P}_{\text{nucleation}} \propto A(t)e^{-\mathcal{S}_c(t)}, \quad (3.39)$$

where $\mathcal{S}_c(t)$ is the euclidean action of the critical bubble. The bubbles subsequently grow and fill the space until the total conversion of the plasma to the true minimum. In this process they produce GWs that can be observed at ET. There are two crucial steps on mapping the BSM physics parameters (couplings, mass scales) to the induced GW signal. The first step consists in mapping the BSM parameters to the thermal description in terms of energy budget parameters. The energy released during the PT can be quantified by the ratio between the energy stored in the vacuum and in the plasma at the moment of the transition [700, 701, 703]

$$\alpha(T) \equiv \frac{\Delta V(T) - T \frac{\partial \Delta V}{\partial T}}{\rho_{\text{rad}}} \quad (\text{Strength parameter}), \quad (3.40)$$

which has to be evaluated at the percolation temperature. A second important parameter is the duration of the phase transition. The duration from the nucleation to the percolation can be approximated by expanding the rate of nucleation at first order and leads to the definition

$$\tilde{\beta} \equiv \frac{\beta}{H} = -\frac{d(\mathcal{S}_c/T)}{H dt} \quad (\text{Duration of the transition}). \quad (3.41)$$

The normalised speed of nucleation $\tilde{\beta} \propto t_{\text{expansion}}/t_{\text{transition}}$, with $t_{\text{transition}}$ the typical time the transition takes to complete and $t_{\text{expansion}}$ the Hubble time, measures how fast a bubble nucleates with respect to the expansion of the Universe, giving an estimate of the speed of completion of the transition. This value can be related to the size of the bubbles at collision via [701, 704, 705]

$$R_{\text{coll}} = \frac{(8\pi)^{1/3}}{H_{\text{reh}}\beta}. \quad (3.42)$$

Note that in the case in which the phase transition is seeded by topological defects [625, 706, 707], the duration of the transition and the bubble size are instead related to the number of defects per Hubble volume, affecting also the GW signal [708–712].

Assuming that the transition occurs in less than an Hubble time, we can apply conservation of energy arguments to obtain the *reheating temperature* T_{reh} :

$$(1 - \Omega_{\text{GW}})(\Delta V + \rho_{\text{rad}}|_{T=T_{\text{per}}}) = \rho_{\text{rad}}|_{T=T_{\text{reh}}} \Rightarrow T_{\text{reh}} \approx (1 + \alpha)^{1/4} T_{\text{nuc}}, \quad (3.43)$$

where we neglected the energy lost in the GW signal.

The terminal velocity of the bubble wall v_w of the bubble has been the subject of intense investigation, as it requires to solve a very non-trivial set of coupled equations. Estimates of the terminal wall velocity are usually based on kinetic theory [713–716], improvement of it [717–719], or holographic methods for strongly coupled theories [720–724] (see ref. [725] for a quasiparticle (quasiglons) method). Typically, calculating the terminal wall velocity is a very challenging task, and has only been performed for a limited number of models [714–716, 719, 726, 727]. In such computations, one needs to solve the scalar field equation of motion (EoM) coupled to the Boltzmann equations describing the particles in the plasma. Simplifications can be obtained by assuming local thermal equilibrium (LTE) [728–735] or in the ultrarelativistic regime [736–742]. Schematically, this mapping can be written

$$\boxed{\text{BSM parameters}(\text{couplings, masses} \dots) \Rightarrow (\alpha, \beta, T_{\text{reh}}, v_w)}. \quad (3.44)$$

The second matching consists of the prediction of the features of the GW signal, namely, the shape of GW spectrum, the peak amplitude $\Omega_{\text{GW,peak}}$, and the peak frequency f_{peak} , from a set of $(\alpha, \beta, T_{\text{reh}}, v_w)$. Schematically

$$\boxed{(\alpha, \beta, T_{\text{reh}}, v_w) \Rightarrow (f_{\text{peak}}, \Omega_{\text{peak}}, \text{GW spectrum})}. \quad (3.45)$$

The spectrum of the GW signal depends strongly on the source and requires to be simulated. The process of expansion and collision of those bubbles of true phase will generate GWs via three distinct mechanisms: i) from the *bubble collision*, where the energy collected in the scalar shell on the bubble wall is released at collision [743–748], ii) from the *sound waves* where the energy injected in the plasma propagates under the form of sound waves [705, 749–751, 753–755], iii) from the *turbulence* occurring when the sound waves decay [756] (see also [752] for another source of GWs). It is typically found to be well fitted by a *broken power-law* of the type

$$\Omega_{\text{GW}}(f) \propto \frac{(a+b)^c f_{\text{peak}}^b f^a}{(b f_{\text{peak}}^{\frac{a+b}{c}} + a f^{\frac{a+b}{c}})^c}, \quad (3.46)$$

Reference and group	IR tail	UV tail
Envelope simulation [743] Ω_ϕ	3	1
Envelope simulation [744] Ω_ϕ	$2.9^r - 2.95$	$0.9 - 1$
Envelope analytic [745, 746] Ω_ϕ	3	1
Bulk flow [744] Ω_ϕ	$0.95 - 0.9^r$	$2.9 - 2.1^r$
Lattice Simulation [747] Ω_ϕ	3	~ 1.5
Lattice Simulation [748] Ω_ϕ	$0.68 - 1.56$	$1.35 - 2.25$
Sound Shell model [749, 750] Ω_{sw}	9	4
Analytic [746] Ω_{sw}	3	1
Hybrid simulation [751] Ω_{sw}	3	3
Lattice Simulation [705] Ω_{sw}	3	4
Analytic and simulations [752] Ω_{FS}	1	2

Table 3. Shape of the GW signals from some of the different groups working on GW from FOPT in the UV and the IR. Results are shown under the form of $(f/f_{p,0})^n$ for the IR and $(f/f_{p,0})^{-n}$ for the UV tail. The suffix r means that the result is specific to relativistic walls. The mismatch between numerical simulations and the current models available (Sound shell and Bulk flow) motivates us to investigate further the analytic understanding of GW from bubble collision.

with parameters a, b, c given by simulations. In table 3 we report the slope in the UV and in the IR for different simulations for the Ω_ϕ (bubble collision source), the Ω_{sw} sound wave sources and from the free-streaming source Ω_{FS} . Even if it seems that the spectra from the sound waves and the bubble collisions are very different for small α , it was recently argued that the spectra from the sound-waves in the regime of large α resembles the spectrum of the bubble collision [757].

The typical frequency peak f_{peak} is controlled by the temperature T_{reh} after the completion of the PT and the size of bubbles at collision R_{coll} :

$$f_{\text{peak}} \approx \mathcal{O}(1) \times 10^{-5} \frac{1}{H_{\text{reh}} R_{\text{coll}}} \frac{T_{\text{reh}}}{100 \text{ GeV}} \text{ Hz}, \quad (3.47)$$

where H_{reh} is the Hubble constant after the completion of the PT and a $\mathcal{O}(1)$ factor depending on the details of the source. Figure 31 shows the expected spectra from sound waves, while figure 32 shows the parameter spaces of FOPT that can be probed by ET.

Testing particles physics model would consist in inverting the steps that we have made so far. Assuming an observed stochastic signal, we have to perform the inverse problem

$$\boxed{(\text{GW spectrum}, f_{\text{peak}}, \Omega_{\text{GW,peak}}) \Rightarrow (\alpha, \beta, T_{\text{reh}}, v_w) \Rightarrow \text{BSM parameters}.} \quad (3.48)$$

Supercooled phase transitions in particular lead to very strong signals (large α , small β/H). This arises in particular naturally in models featuring nearly-conformal dynamics [760–762]. Such supercooled PTs and their associated signals were studied extensively in the context of new TeV physics [763–765] but the same physics can be readily applied to strong dynamics occurring at higher energies that could be connected to dark matter and that would lead to signals in the ET window [766].

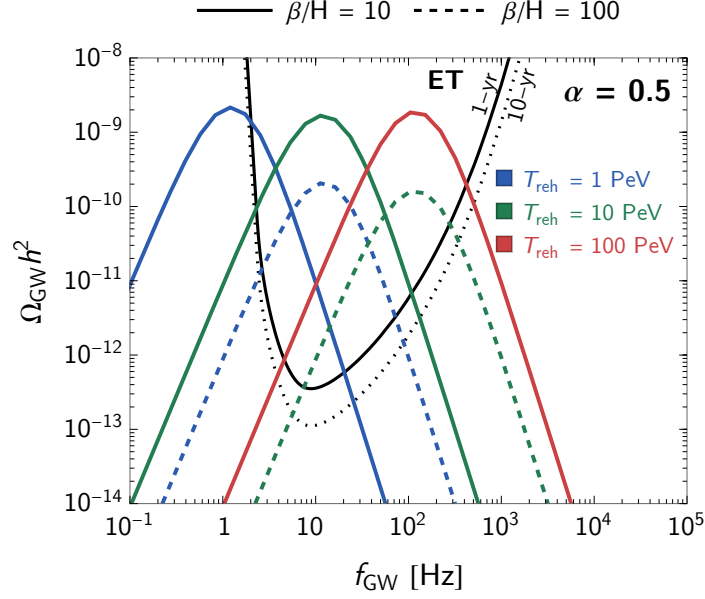


Figure 31. GW spectra from sound waves produced by first-order FOPTs at temperature T_{reh} [701], with the wall velocity that gives the maximal energy transfer to sound waves [758, 759]. The black curves are the power-law integrated sensitivities of ET assuming a triangular xylophone configuration with 10km arms and the 1 and 10 years observation time with SNR = 1.

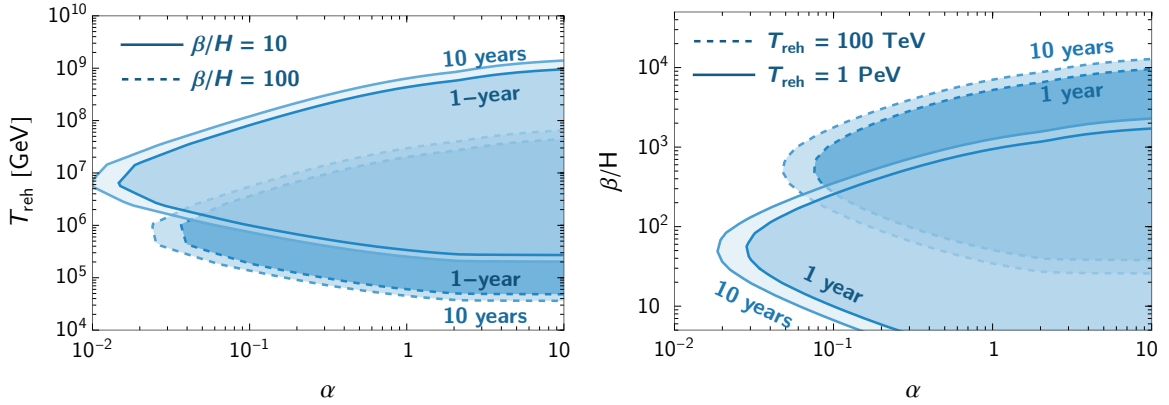


Figure 32. Regions of FOPT parameter spaces that can be probed by ET assuming triangular xylophone configuration with 10km arms for two values of β/H (left) and T_{reh} (right), assuming GW from sound waves [701] shown in figure 31. The darker regions can be probed with ET at SNR ≥ 1 and 1 year of observation time, while the lighter regions require 10 years.

Going back to eq. (3.47), as can be seen from figure 31, this GW signal would roughly lie in the detection range of the ET detector, f between a few Hz and a few 10^3 Hz, if the ratio $(H_{\text{reh}}R_{\text{coll}})^{-1}(T_{\text{reh}}/100\text{GeV})$ is between a few times 10^5 and a few times 10^8 , i.e. for PTs largely above the EW scale. In this detection range, the LVK collaboration could also constrain large regions of parameter space, specially in the case of models featuring large supercooling, such as the conformal models [767–769], Pati-Salam models [770], high energy $B - L$ breaking PT [771], SUSY breaking [772, 773], Peccei-Quinn symmetry breaking [774, 775], flavour models [776, 777], models with extended gauge groups [778–781], and the remnant of the confinement of a hypothetical composite Higgs [765, 782–784] (see also [785–787] for more general studies). Other physical phenomena related to signals in ET have also been envisioned, such as bubble wall enhanced phenomenon like baryogenesis [788–790], leptogenesis [791, 792], Dark Matter production [766, 779, 793–795], and could be in the reach of detection of ET. This exciting possibility motivates us to strengthen the relation between model building parameters and observables in ET telescope.

3.2.3 GWs from cosmic strings

3.2.3.1 Local and global strings. Topological defects of various kinds — including cosmic strings and domain walls (see section 3.2.4) — may form in symmetry-breaking phase transitions in the early Universe [796]. In particular, if the vacuum manifold contains non-contractible loops then a network of strings can be produced, and (if they are dynamically or energetically stable) they will still be present in the Universe until today. Cosmic strings are referred to as *local/global* depending on the nature of the underlying symmetry groups (see [797–800] for reviews on cosmic strings). Local cosmic strings are ubiquitous in supersymmetric grand unified theories: indeed all models which solve the monopole problem, lead to baryogenesis after inflation and are consistent with proton lifetime measurements form local strings [801]. Global strings on the other hand are characteristic of axion models, and are formed for instance when the Peccei-Quinn symmetry is broken after inflation [802–804]. The global strings in these models can become unstable at a later stage when a subsequent symmetry breaking phase transition leads to the formation of domain walls [802–807] (see also section 3.2.4).

Both local and global string networks (and indeed domain-wall networks) can source gravitational waves (GWs), and this production occurs throughout the lifetime of the network, from the initial phase transition to the present day. As a result, strings produce a GWB spectrum which generally covers a very broad range of frequencies (see e.g. [808]), and hence can be probed by experiments in different frequency bands — from the nHz and pulsar timing array (PTA) experiments [421, 809, 810], to the mHz and LISA [808], up to the $1\text{--}10^4$ Hz band of ET and other ground-based interferometers. Furthermore, if a string is sufficiently close to us, the characteristic bursts of GWs it emits [811, 812] can be searched for individually [129]. Both type of observations (bursts or GWB) can put constraints on the energy scale of the phase transition in which strings are formed, η , and can thus reveal the properties of the phase transition and particle physics at that energy scale.

Local strings have a characteristic width $w \sim 1/\eta$ and finite energy per unit length which (in units $\hbar = c = 1$), is given by

$$G\mu \sim 10^{-6} \left(\frac{\eta}{10^{16} \text{ GeV}} \right)^2. \quad (3.49)$$

Local strings predominantly lose energy into GWs. In contrast, global strings dominantly radiate Nambu-Goldstone bosons, and the existence of this Nambu-Goldstone radiation leads to a time-dependent, logarithmically-divergent energy per unit length

$$G\mu(t) \sim 10^{-6} \left(\frac{\eta}{10^{16} \text{ GeV}} \right)^2 \log \left(\frac{H^{-1}(t)}{w} \right), \quad (3.50)$$

where $H(t)$ is the Hubble parameter.

In the remainder of this section, focusing on the ET frequency band, we summarise the burst and GWB signatures of both local and global strings, taking into account existing observational constraints. Beyond the minimal cosmic-string models — depending only on $G\mu$ (or equivalently η) — discussed here, there are several other well-motivated cosmic-string models that depend on additional parameters, modifying the string-network dynamics and leading to interesting features on GWB. These include the superstrings [813–818], the metastable cosmic strings [797, 819–829], Hubble-induced strings [830] and superconducting strings [831–834]. Although their GWB and features are relevant for ET, these models are subjected to some uncertainties and will need more numerical studies in order to draw robust predictions.

3.2.3.2 Gravitational waves from local strings. Due to the vast separation of scales between the length and the width of a cosmic string, it is often convenient to describe their dynamics in the zero-width limit. For local strings, this results in a Nambu-Goto action characterised purely in terms of $G\mu$. Cosmic string networks can also be studied using lattice field theory simulations (see, e.g., [835–837]). The results from these simulations have sparked a long-standing debate on how to reconcile these two types of simulations. Recent studies however suggest that indeed field theory string loops coming from cosmic string network simulations evolve as Nambu-Goto dictates [838]. On the other hand, it is clear that more work in this direction is needed to conclusively show the agreement between these two approaches. Here we will concentrate of the predictions from Nambu-Goto strings.

This Nambu-Goto approximation has been used extensively in the literature to investigate the dynamics of local string networks (see, e.g., refs. [797, 798, 808, 839] for reviews). The picture that emerges is that each Hubble patch contains $\mathcal{O}(1)$ ‘long’ superhorizon strings, which continually produce closed string loops by self-intersecting. These loops evolve periodically over time, expanding and contracting at relativistic velocities due to their tension. This relativistic dynamics results in strong GW production from loops, particularly via pronounced features such as cusps (string segments which move instantaneously at the speed of light), kinks (discontinuities in the tangent vector along the loop), and kink-kink collisions [811, 840, 841]. As a result, loops decay in size and energy via GWs long kinky strings [842] at a rate $dE/dt \approx -\Gamma_{\text{GW}} G\mu^2$ (with $\Gamma_{\text{GW}} \approx 50$ a dimensionless coefficient [841, 843, 844]), radiating at increasingly high frequencies until they vanish from the network. This decay of individual

loops, combined with the continuous production of new loops from the self-intersection of ‘long’ strings, leads to an attractor solution in which the distribution of loop sizes and the total energy contained in the network scale with the expansion of the Universe [845]. This is the so-called *scaling solution*. Evidence of this attractor solution has been found in many of the numerical simulations of string networks (see, e.g., [839, 846–848]).

Early attempts to detect local strings focused mostly on their imprints on the cosmic microwave background (CMB) due to the deflection of CMB photons, resulting in constraints of $G\mu \lesssim 10^{-7}$ [849–852]. However, the advent of GW astronomy has enabled much more sensitive searches, and therefore much more stringent constraints on the string tension, as we discuss below. The main theoretical uncertainty in these searches is the loop size distribution reached by the network in the scaling regime. The two most well-established models for this distribution — which we refer to here as model A [853] and model B [854] (based on [855]) — predict different abundances of small loops, and therefore different GW signatures, particularly at the higher frequencies relevant to ground-based interferometers such as ET and LIGO/Virgo/KAGRA. For a discussion of the origin of this discrepancy on the loop number density of non-self-intersecting loops computed from different simulations see [856, 857].

Another important theoretical uncertainty on the computation of the GW signatures of cosmic strings comes from the role played by gravitational back-reaction on the loop’s evolution. The emission of gravitational radiation from loops modifies their shape altering their power spectrum. This effect has been recently studied analytically as well as numerically from a small number of loops [858–862]. However, its impact on the final gravitational wave signals coming from strings has only been discussed on simple toy model description in [844].

3.2.3.3 GWs from global strings. The dynamics of global strings can be described effectively as Nambu-Goto strings coupled to the Goldstone-boson field using the Kalb-Ramond action [863–865]. This additional coupling affects GW production in two ways. First, the evolution of the global-string network is modified by the back-reaction from Goldstone emission [866–868] in competition with the loop-chopping; this affects the loop number density. Second, a global-string loop only lives for a short time after its production because it decays quickly into Goldstone bosons at the rate $dE/dt \approx -\Gamma_{\text{gold}}\mu/\log(H^{-1}/w)$ with $\Gamma_{\text{gold}} \approx 65$ [864], which is $\mathcal{O}(1/(G\mu))$ more efficient than GW emission. Similarly to local strings, long global strings emit Goldstone bosons [869]. As a result, the GW production from loops is roughly the same as the time of loop production, and is therefore diluted more by Hubble expansion than in the local-string case. The resulting GWs are highly suppressed compared to those from local networks formed at the same symmetry-breaking scale [870–873]. In the case of axion strings, the spectrum is cut off at frequencies below a threshold set by the axion mass due to the strings decaying. However, for axion mass below 10^{-22} eV and axion decay constant $\sim 10^{15}$ GeV that do not cause dark matter overproduction [805–807, 873–878], this cutoff is well below the ET frequency band, so can be neglected here. (This cutoff is relevant however for low-frequency GW observations, e.g. with PTAs [873, 879].)

GW predictions for global strings must be treated with care, as there are several unresolved theoretical uncertainties. Many lattice simulations have attempted to verify whether the global string network actually reaches the scaling regime or has its string correlation length

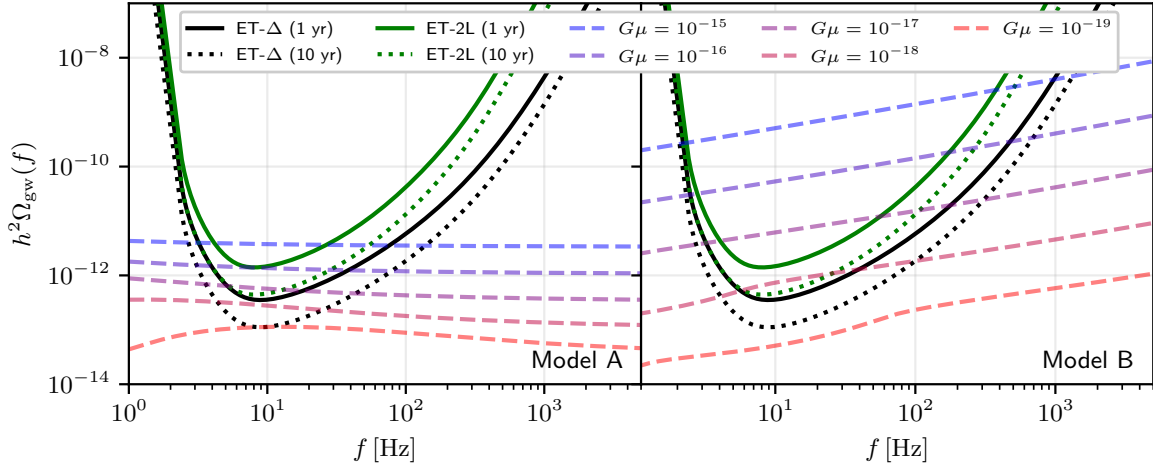


Figure 33. Forecast power-law-integrated GWB sensitivity of ET, compared with signals from local Nambu-Goto strings of various tensions $G\mu$. We consider both the triangular configuration for ET with 10 km arms, and the 2L configuration with 15 km arms, misaligned as in [16]; the solid black curves corresponds to one year of observations, while the dotted black curve corresponds to 10 years. The left and right panels show the predictions for models A [853] and B [854] of the loop network, respectively. Both models predict that, in the triangle configuration, ET will be sensitive to $G\mu \gtrsim 10^{-18}$ after one year of observations with $\text{SNR} \geq 1$.

corrected with $\log(H^{-1}/w)$ [866, 868, 873, 875–878, 880–884]. Moreover, the recent debate on the powers of emissions into GW, Goldstone bosons, and massive radiation in different numerical simulations is open [873, 885–888], e.g., the amplitude of the GW signal can be smaller by $\log^2(H^{-1}/w)$ than the previous expectation [888].

In addition to GWs, the massless Goldstone production from global strings is subject to strong constraints on the number of relativistic degrees of freedom present at BBN and CMB scales, N_{eff} . Accounting for uncertainties in the global-string network evolution, this constraint bounds the energy scale of the string network formation to be $\eta \lesssim (1 - 3) \times 10^{15}$ GeV [872, 873]. The bound from deflection of CMB photons ($G\mu(t_{\text{CMB}}) \lesssim 10^{-7}$) also gives similar constraints [870–873].

3.2.3.4 GW background from strings. The incoherent superposition of GWs from many string loops gives rise to a GWB signal, whose amplitude and spectral shape are determined by the symmetry-breaking scale η (which in the case of local strings corresponds one-to-one with the string tension $G\mu$), as well as by the loop network modelling as discussed above. As mentioned above, the signal spans many decades in frequency, due to the very broad range of loop sizes and emission epochs involved in the network. As a result, it is possible to set complementary constraints on string scenarios with multi-band GW observations. For local strings, results from the LVK O3 stochastic search rule out $G\mu \gtrsim 10^{-8}$ for model A and $G\mu \gtrsim 4 \times 10^{-15}$ for model B [129], while the most recent results from NANOGrav and EPTA each rule out $G\mu \gtrsim 10^{-10}$ for both models A and B [421, 809, 810]. (This assumes that the GWB signal detected by these PTAs is due to inspiralling supermassive

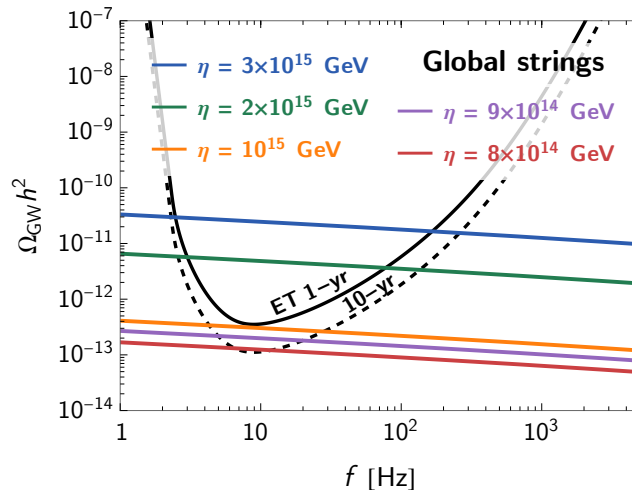


Figure 34. Spectra of GWB from global strings formed at energy scale η are shown in solid colored lines. The black lines are the same ET-triangle sensitivity curves as shown in figure 33. We used the semi-analytic calculation in [871]; see also [870, 872].

binary black holes [889]; several more exotic explanations for this signal have been proposed, including cosmic strings.)

These constraints will improve drastically with next-generation GW observations in the 2030s. The space-based interferometer mission LISA, which has recently undergone ESA adoption [330], is expected to be sensitive to local string networks with $G\mu \gtrsim 10^{-17}$ (for both models A and B) [808]. As shown in figure 33, we can expect even stronger constraints from ET. In the triangular 10km configuration we can probe string tensions $G\mu \gtrsim 10^{-18}$ with just one year of data both for models A and B, while with the 2L 15km aligned configuration we can probe $G\mu \gtrsim 10^{-19}$, for model A, and slightly smaller for model B [16].

For global strings, as shown figure 34, we can expect ET to probe $\eta \gtrsim 8 \times 10^{14}$ GeV. This sensitivity is comparable to that of LISA [871–873], while for the most recent PTA data from e.g., NANOGrav, one obtains $\eta \gtrsim 3 \times 10^{15}$ GeV [879], which is comparable to the current constraints from Nambu-Goldstone radiation. ET will therefore improve upon these current constraints; however, these other constraints are also likely to improve by the time ET is operational, e.g., due to stage-4 CMB surveys [890].

3.2.3.5 GW bursts from strings. Strong bursts from nearby cosmic string loops may be resolved in GW detectors and searches are currently undertaken with the hope to detect individual cusps or kinks in LVK datasets [129, 891] as well as PTA data [892]. Similar searches will also be carried out in the future for LISA [330, 893, 894]. Here we forecast the capabilities of ET to detect individual cusps assuming a single detector with a triangular xylophone configuration with 10km arms (ET LF+HF) [16].

As a first approximation, we compute the combined SNR of individual bursts using the response function for a triangular detector [529] and claim a detection if the SNR is above 20. Accurate data analysis would rely on match-filtering methods or new methods such as machine learning [895]. Similarly to the procedure already described for LISA [894],

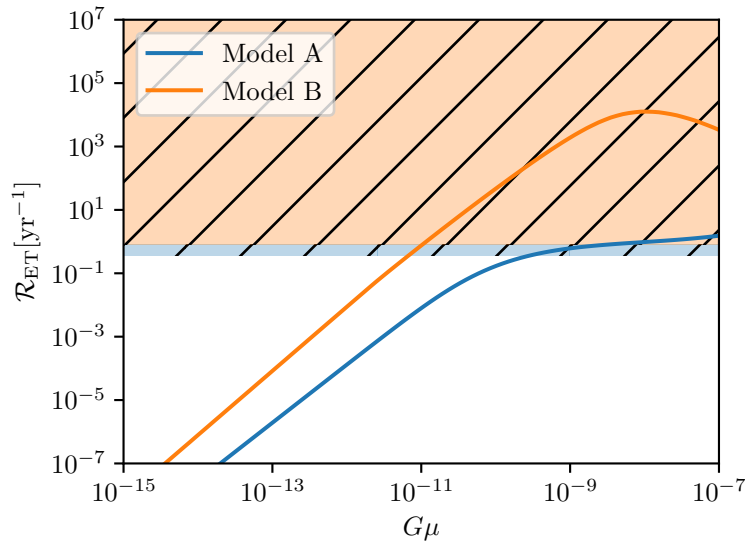


Figure 35. Expected rate of detected bursts in Einstein Telescope as a function of the string tension for models A and B. In case ET does not detect bursts from cosmic string cusps, the orange hatched region is excluded after 4 years of observations and the blue hatched region is excluded after 8 years.

we simulate a set of sources with random polarization and sky-localization, and obtain the average response function of ET to cosmic string bursts as a function of their amplitude. We then combine the response function of ET with the rate of bursts predicted for different loop population models to obtain the expected rate of bursts detected in ET.

The expected burst rates for both models A and B are presented in figure 35. For $G\mu \approx 10^{-11}$ compatible with the PTA signal [421, 809, 810], one expects to detect about one burst per year for B and 10^{-2} for A. If ET does not detect bursts from cosmic string cusps, it would place constraints on the abundance of loops in our cosmic neighbourhood, and constrain the string tension to $G\mu \lesssim 10^{-11}$ for B and $G\mu \lesssim 10^{-10}$ for A.

Note however that the computation of these rates have not taken into account the effect of gravitational back-reaction. This could not only affect the typical number of cusps per loop but also their persistence in time.

3.2.4 GWs from domain walls

Domain walls (DWs) are two-dimensional defects (surfaces) that separate regions of space belonging to physically different vacua of the theory. These vacua are typically related by a global discrete symmetry, so that they are exactly degenerate in energy and the DWs are topologically stable. As the DW configuration interpolates between discrete minima, the corresponding field is forced to leave the vacuum manifold at the center of the wall, so that its energy density can in fact be very high. Domain walls can possibly lead to a cosmological problem if they are absolutely stable and eventually dominate the energy density of the Universe. Solutions to this problem come with interesting phenomenological consequences for the domain wall network, as we will discuss.

Domain walls arise in many beyond Standard Model theories where the symmetry of the SM is extended and a discrete group (or subgroup) of this symmetry is spontaneously broken during the cooling of the Universe. The list of motivated models is large and includes: discrete flavor symmetries [896, 897], two-Higgs [898, 899], twin-Higgs [900, 901], composite Higgs [902, 903] models, non-Abelian flavor symmetries [896, 897], axion monodromy [904, 905], supersymmetry [906–910], grand unification [911–913], Hubble-induced phase transitions [914, 915] and discrete spacetime symmetries [916–921].

QCD axion models [435–438] are of particular interest. While solving the strong CP problem of the Standard Model, they lead naturally to DWs [802, 803, 922]. The underlying Peccei-Quinn (PQ) mechanism introduces a new global $U(1)_{\text{PQ}}$ symmetry that is spontaneously broken at a high scale F_a and it is anomalous under QCD [435, 436]. At low energies, the model consists of the (pseudo-)Goldstone boson, the axion, coupled to topological defects: global strings, and DWs. See also [923–927] and [439, 441, 928–939] for recent reviews on several aspects related to axion models and phenomenology.

In post-inflationary scenarios, the cosmological PQ phase transition leads to the formation of a network of $U(1)_{\text{PQ}}$ global strings. At much lower temperature, $T \sim \Lambda_{\text{QCD}}$, QCD further breaks $U(1)_{\text{PQ}}$ to a discrete subgroup $Z_{N_{\text{DW}}}$, and DWs appear. The integer N_{DW} is the number of DWs attached to each string and is a model dependent parameter (it relates to the PQ charges of SM and/or heavy quarks). If $N_{\text{DW}} = 1$, the string network is quickly annihilated at $T \sim \Lambda_{\text{QCD}}$ and negligible GWs are produced. Instead, if $N_{\text{DW}} > 1$ a string-wall network is formed and some additional $U(1)_{\text{PQ}}$ breaking sector enforces DW annihilation.

As a result, QCD axion models with $N_{\text{DW}} > 1$ naturally lead to a DW epoch and so to potentially a sizeable GW signal. Numerical simulations in this scenario were performed in [940]. The signal is strongest in heavy QCD axion, which are themselves of particular interest because they address the axion ‘quality’ problem [941–960], see also [961–966]. Importantly, in these models the amplitude of GW observables is correlated with the proton and neutron electric dipole moments [961, 962], which is a welcome feature towards possible degeneracies in interpretations.

The relevance of DWs in cosmology has been pointed out a long time ago [796, 967]. The typical setup involves the spontaneous breakdown of a discrete symmetry during the thermal history of the Universe. Regions of space separated by distances that are larger than the correlation length of the field around the temperature of the phase transition will randomly select one among the possible vacua of the theory. DWs will then form at the boundaries between these causally-disconnected regions where the field remains trapped at the maximum of the potential.

The resulting DW network is characterized by the tension σ , which measures the DW energy (or mass) per unit surface, and the correlation length, R , indicating the typical distance between the walls as well as their average curvature radius. The evolution of the network in a cosmological setting is determined by the tendency of the walls to reduce their energy by reducing their surface, as well as by the Hubble expansion and the effect of friction with the surrounding plasma. When particle friction can be neglected, the network is known to approach an attractor solution independently of the initial conditions in which the correlation length is set by the Hubble horizon, $R = \xi H^{-1}$, with constant $\xi = \mathcal{O}(1)$

indicating the average number of DWs per Hubble patch. In this so-called scaling regime, the energy density of the network is

$$\rho_{\text{DW}} = 2\mathcal{A}\sigma H, \quad (3.51)$$

with $\mathcal{A} = \mathcal{O}(1)$. It appears from the scaling in (3.51) that the DW energy density will eventually take over the energy in matter and radiation leading to an inconsistent cosmology [803, 967, 968]. This conclusion can be avoided if the DWs still remain a subdominant component of the energy budget today. This can occur if the DW tension is small enough, or in case a period of inflation has provided sufficient dilution of the network. These two options inevitably suppress most of the signals coming from the DWs.¹⁸ We will instead consider a third possibility in which the DW network is actually metastable and it undergoes annihilation at a temperature T_{ann} when its energy density is still far from domination.

As mentioned before, DWs related to exact discrete symmetries are topologically stable and cannot annihilate. To allow for annihilation one then needs to introduce an explicit breaking of the underlying discrete symmetry so that the degeneracy of the vacua is lifted by a small bias ΔV . This introduces an additional force acting on the network, namely a vacuum pressure towards the true vacuum of the theory. At the early stage of the evolution, the bias has no effect and the network can still reach the scaling regime. However, at the temperature T_{ann} when the vacuum pressure balances the tension force σ/R , the friction with the plasma as well as the Hubble expansion, the network begins to collapse. One estimates this temperature to be related to ΔV as [940, 972–974]

$$T_{\text{ann}} \simeq 270 \text{ MeV} \left(\frac{\sigma^{1/3}}{100 \text{ TeV}} \right)^{-3/2} \left(\frac{\Delta V^{1/4}}{100 \text{ MeV}} \right)^2 \left(\frac{g_*}{10} \right)^{-1/4}. \quad (3.52)$$

Annihilation at T_{ann} results in the copious production of gravitational waves, particle excitations and (as discussed in section 3.2.5.3) possibly additional relics such as primordial black holes. The energy budget available from the DW annihilation can be parameterized by α_{ann} given by

$$\alpha_{\text{ann}} \equiv \frac{\rho_{\text{DW}}}{\rho_{\text{tot}}} \Big|_{\text{ann}} \simeq 0.02 \left(\frac{\sigma^{1/3}}{100 \text{ TeV}} \right)^3 \left(\frac{T_{\text{ann}}}{100 \text{ MeV}} \right)^{-2} \left(\frac{g_*}{10} \right)^{-1/2}, \quad (3.53)$$

with $\rho_{\text{tot}} = 3H^2 M_{\text{Pl}}^2$ the total energy density, and $\alpha_{\text{ann}} < 1$ to avoid DW domination. The corresponding spectrum of GWs is characterized by a peak amplitude

$$\Omega_{\text{peak}} \simeq 1.64 \times 10^{-6} \left(\frac{\tilde{\epsilon}_{\text{gw}}}{0.7} \right) \left(\frac{\mathcal{A}}{0.8} \right)^2 \left(\frac{g_*(T)}{10} \right) \left(\frac{g_{*s}(T)}{10} \right)^{-4/3} \alpha_{\text{ann}}^2, \quad (3.54)$$

with $\tilde{\epsilon}_{\text{gw}}$ an $\mathcal{O}(1)$ efficiency factor, and the peak frequency is

$$f_{\text{peak}} \simeq 1.15 \times 10^{-9} \text{ Hz} \left(\frac{g_*(T)}{10} \right)^{1/2} \left(\frac{g_{*s}(T)}{10} \right)^{-1/3} \left(\frac{T_{\text{ann}}}{10 \text{ MeV}} \right). \quad (3.55)$$

¹⁸Another possibility is that the domain wall tension decreases with time, eventually leading to the melting of the DW network [969–971].

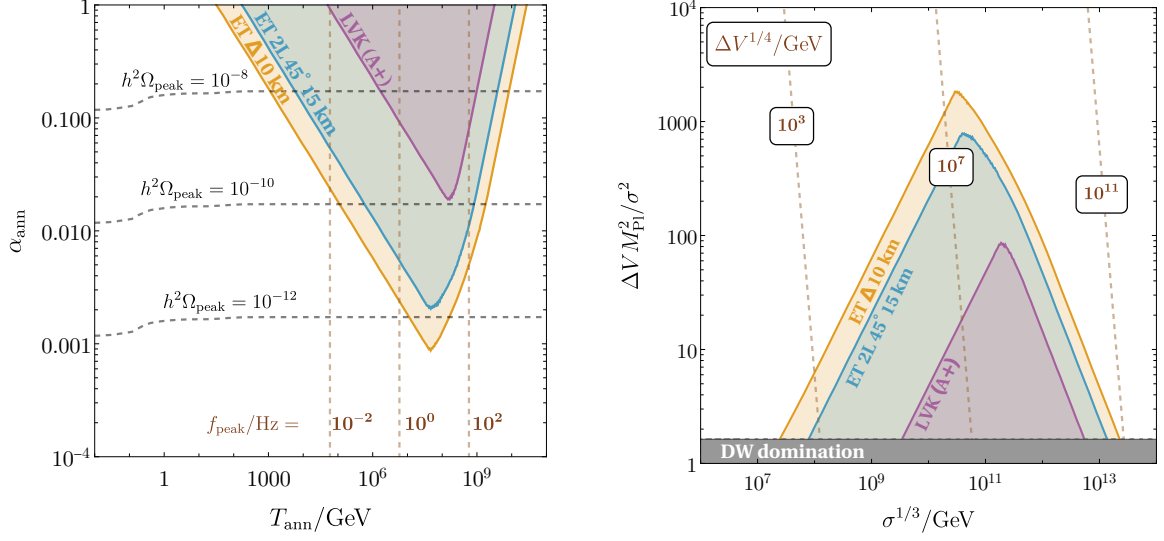


Figure 36. *Left:* $(\alpha_{\text{ann}}, T_{\text{ann}})$ parameter space which can be probed by ET, assuming a triangular xylophone configuration with 10km arms (orange), a 2L misaligned configuration with 15km arms (blue), both for an observation time of $T = 1$ year and $\text{SNR} = 1$. For comparison, we show the sensitivity region of LVK A+ (purple) with $T = 1$ year and $\text{SNR} = 1$. Contours of $h^2\Omega_{\text{peak}}$ and f_{peak} are shown by black and brown dashed lines respectively. *Right:* bias vs. tension parameter space with similar sensitivity regions as on the left plot. Brown dashed lines indicate the size of the bias. The gray region corresponds to a forbidden DW dominated Universe, for which $\alpha_{\text{ann}} \geq 1$.

As expected, the GW amplitude scales like α_{ann}^2 while the peak frequency is set by the correlation length of the network just before annihilation begins, which for the scaling solution corresponds to the Hubble horizon at T_{ann} .

Figure 36 shows the reach of ET to these signals, in terms of the model independent parameters, $\alpha_{\text{ann}}, T_{\text{ann}}$, and of the parameters in the minimal Z_2 model. We choose to show the energy scales associated to the DW tension and the bias, $\sigma^{1/3}$ and $(\Delta V)^{1/4}$, which in the simplest model mark the scales of spontaneous and explicit breaking of the Z_2 symmetry. The breadth of the ranges of scales is remarkable.

It is worth mentioning that GWs are actually continuously radiated by the DW network from the moment of its formation. However, since the DW energy density grows with time with respect to the critical density, the contribution to the GW spectrum is maximal at $T = T_{\text{ann}}$ and it outshadows the previous emission. Let us also mention that our expressions for the amplitude and peak frequency correspond to the spectrum coming from the last moment of scaling, and does not include the contribution from the actual collapse of the network. This has been recently investigated in refs. [975, 976], see also [977–979]. Additional effects may arise if the friction from the thermal plasma on the DW network is not negligible around the time of annihilation, [964, 965].

Finally, note that collapsing DW networks also can lead to substantial PBH formation, see section 3.2.5.3. This adds signatures to the DW models that are correlated with GW observables, which should be useful to discriminate among different candidate sources.

3.2.5 Primordial black holes

Primordial Black Holes (PBHs) can form in the very early Universe through a variety of mechanisms and have been studied as a possible candidate to explain the dark matter (see e.g., [980] for a recent review). Nevertheless, even if they accounted for a subdominant contribution to the current energy density in the Universe, they could have played a crucial role in the cosmological history, leading to various GW signatures. As the mass of these objects is linked by model-dependent $\mathcal{O}(1)$ factors to the mass of the Hubble scale at formation, they could have formed within a very large range of masses, potentially even lighter than $m_{\text{PBH}} \lesssim 10^{-18} M_{\odot}$ corresponding to unstable objects through Hawking evaporation within a timescale comparable to the age of the Universe [981]. The Einstein Telescope has the opportunity to constrain this putative population of BHs by searching for the associated GW signatures.

A population of PBHs is expected to form binaries which could merge in the relatively late-time Universe and lead to GWs signatures.¹⁹ These signatures of PBH binaries falling within the ET frequency band would come from PBH of masses around the stellar mass and thus need to be distinguished from the expected signatures of astrophysical origins, i.e. coming from stars and astrophysical black holes. This is discussed in detail in chapter 4, dedicated to GW population studies.

In this section, instead, we will focus on the truly cosmological signatures of PBH formation scenarios, i.e., the cosmological GWB associated with the physical mechanisms that lead to the collapse of large overdensities in the early Universe. As the formation of PBHs requires extreme overdensities, it is generically associated with the emission of sizeable GWB. To gain an intuition on the typical mass range probed by ET, we can consider the relation between the present-day GW frequency and the mass contained in the Hubble sphere at Hubble crossing time (taken to be a proxy for the PBH mass)

$$f_{\text{GW}} = 4.1 \text{ Hz} \left(\frac{\kappa}{2.51} \right) \left(\frac{g_*}{106.75} \right)^{-1/12} \left(\frac{M_H}{10^{-17} M_{\odot}} \right)^{-1/2}, \quad (3.56)$$

where $\kappa = kr_m$ relates the perturbation size r_m (see, e.g., [983]) to the wavenumber k and g_* is the number of effective degrees of freedom in the energy density (see, e.g., [60, 537]), that in eq. (3.56) has been normalized to the value that it has in the Standard Model at high energy. From this relation, we immediately see that ET will probe the formation of ultra-light PBHs, potentially narrowing the open window where no bounds prevent PBHs from being the dark matter (see, e.g., [535] for a recent review).

3.2.5.1 PBHs from primordial perturbations. The simplest formation scenario assumes the collapse of extreme inhomogeneities during the radiation-dominated era [984–987]. The PBH mass resulting from the collapse is related to M_H by an order-unity factor controlled by the critical collapse parameters [988] (see [989] for a recent review). Adopting threshold

¹⁹Note, however, that for PBHs formed at the early Universe, the expansion of the Universe cannot be neglected, questioning the choice of metric that most adequately describes PBHs. The choice of the appropriate metric will have implications for the conditions for two PBHs to form a decoupled binary, and therefore for calculating the merger rate [982].

statistics, we compute the mass fraction β assuming Gaussian primordial curvature perturbations (see e.g., [990–995] for extensions alleviating this assumptions) and accounting for the nonlinear relationship between curvature and density perturbations [996–998]. One obtains

$$\beta = \mathcal{K} \int_{\delta_l^{\min}} d\delta_l \left(\delta_l - \frac{3}{8} \delta_l^2 - \delta_c \right)^\gamma P_G(\delta_l), \quad (3.57)$$

where P_G is the Gaussian distribution for the linear component of the density contrast δ_l , related to the primordial curvature perturbations by $\delta_l = -4r\partial_r\zeta(r)/3$. We indicate with $\sigma(r_m)$ the variance of the linear density field computed at horizon crossing time and smoothed on a scale r_m (see e.g., [999] for more details), while δ_c is the threshold for collapse derived by dedicated numerical relativity simulations [1000–1003]. We also introduced the parameters $\mathcal{K}$ and γ to include the effect of critical collapse [988, 1004–1007].

The same large adiabatic perturbations required in this scenario of PBH formation are also inevitably responsible for the GW emission at second order in perturbation theory [1008–1015]. The current energy density of these scalar-induced GWs (SIGW, whose effect on anisotropies of the CGWB we already discussed in section 3.1.2.1) is given, in terms of the curvature power spectrum $\mathcal{P}_\zeta$, by (see e.g., [1016] for a recent review)

$$\Omega_{\text{GW},0} = c_g \Omega_{R,0} \int_0^\infty dv \int_{|1-v|}^{1+v} du \mathcal{T}(u,v) \mathcal{P}_\zeta(ku) \mathcal{P}_\zeta(kv). \quad (3.58)$$

We introduced the parameter c_g , which accounts for the suppression of SIGW amplitude induced by the evolution of the degrees of freedom in thermal equilibrium, with $c_g \simeq 0.4$ assuming that the Standard Model remains valid up to the relevant energy, while $\mathcal{T}(u,v)$ is the so-called transfer function [1017, 1018] which accounts for the time evolution of the source. For a sufficiently narrow spectrum of curvature perturbations, the SIGW spectrum features a low-frequency tail that scales as $\Omega_{\text{GW}} \sim f^3$, due to the causality limited efficiency of the super-Hubble emission [1019–1023], while the high-frequency tail that depends on the drop-off of the curvature spectrum. The peak amplitude is expected to be $\Omega_{\text{GW}} \simeq 10^{-5} A^2$, where A parametrizes the enhanced amplitude of $\mathcal{P}_\zeta$ at small scales.

In figure 37, we show the region of parameter space that ET will be able to constrain by searching for SIGW signals (orange dashed line). This plot assumes the curvature power spectrum to feature a lognormal enhancement at around k_* with a width $\Delta = 1$ (see [769] for more details). The green curve indicates the amplitude required to generate PBH abundance that saturates the current constraints in the mass range shown in the upper edge of the plot.

The parameter space on the left of the dashed diagonal line in figure 37 (see also the range between the dashed lines in figures 38 and 39) indicates the asteroid mass window where the PBHs can constitute all of dark matter. For lower masses, i.e., to the right side of the dashed line, Hawking evaporation leads to the emission of particles that could be detected with different probes (mainly BBN, CMB anisotropies and distortions and cosmic rays, among others [535]). However, these bounds may be avoided in scenarios where black hole evaporation deviates from the semi-classical result [1024–1027]. Furthermore, microscopic horizonless relics could form in the early Universe either directly through gravitational collapse or as stable remnants of the Hawking evaporation of primordial black holes. In both cases they completely or partially evade cosmological constraints arising from Hawking evaporation

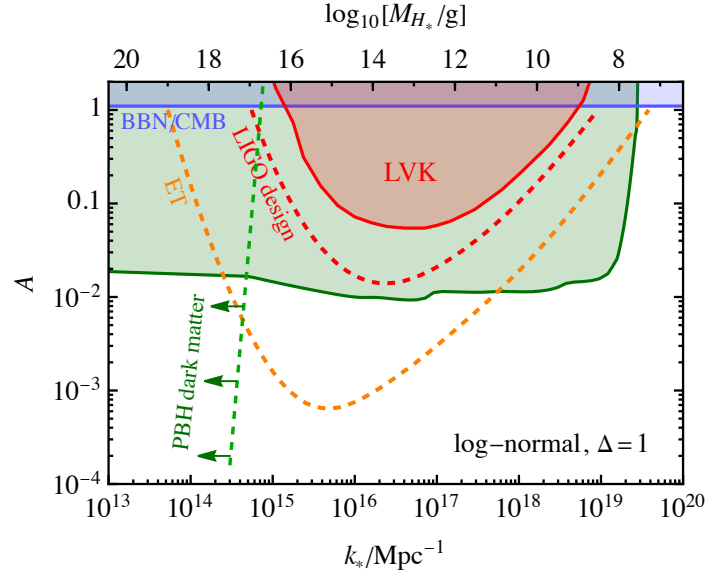


Figure 37. Upper bound on the amplitude of the curvature power spectrum at small scales A as a function of the wavenumber k_* in case of null detection of a SIGW. The green and blue regions at the top show, respectively, the exclusions from PBH constraints and from BBN/CMB constraint on the number of relativistic degrees of freedom. In red the bound from LVK is shown for comparison. Adapted from [769]. Reprinted figure with permission from [769], Copyright (2022) by the American Physical Society.

and they could still explain the entirety of the dark matter in certain mass ranges, a scenario that will be tested by ET [536, 1028, 1029].

3.2.5.2 PBHs from phase transitions. Cosmological phase transitions are typically associated with the breaking of global or local symmetries in a process where the vacuum expectation value of a scalar field changes. They can be either continuous transitions where the scalar field rolls down to the new minimum or first-order transitions that proceed by nucleation and expansion of bubbles inside which the field is in the true vacuum, see the discussion in section 3.2.2. Either way, the transition can lead to the formation of large inhomogeneities, GWs and even PBHs. For the latter, the transition typically needs to be so strongly supercooled that the vacuum energy of the scalar field in the false vacuum causes a period of thermal inflation. The formation of PBHs can occur during the transition in collisions of several bubbles [1030–1032] or after it when the scales that exited the cosmological horizon during the thermal inflation reenter the horizon [1033–1037]. In terms of the phase transition parameters defined in section 3.2.2, the first-order phase transitions where PBH formation is efficient correspond to $\alpha \gg 1$ and $\beta/H_* \lesssim 8$.

In case of a first-order phase transition, the distribution of the density contrast δ generated for the modes that exited horizon during the thermal inflation has a negative non-Gaussianity [1037]. For comparison, models of primordial inflation including an ultra-slow-roll period during which the fluctuations can become very large predict a positive non-Gaussianity [1038]. This implies that the SIGW signature (3.58) of the PBH formation from fluctuations generated during a first-order phase transition is stronger because the

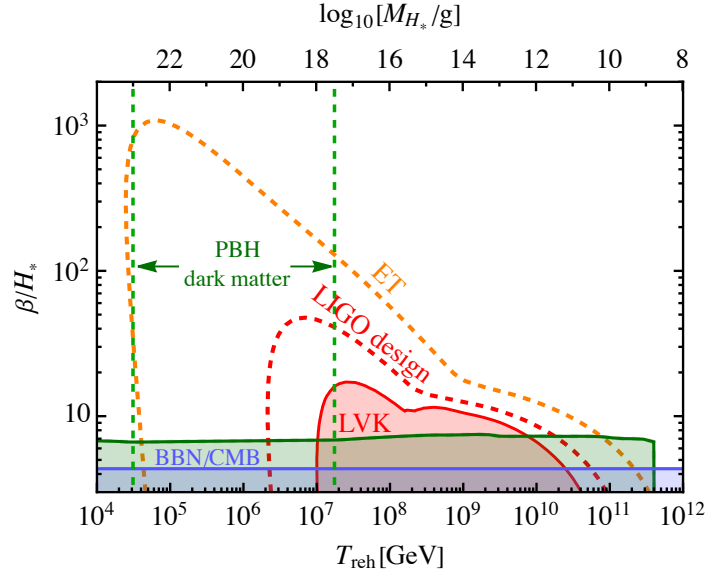


Figure 38. Lower bound on the inverse time duration of the phase transition β/H_* as a function of the temperature T_{reh} right after the phase transition in case of null detection of a SIGW. The green and blue regions at the bottom show the exclusions from PBH constraints and from BBN/CMB constraint on the number of relativistic degrees of freedom.

PBH abundance is sensitive to the large δ tail of the distribution that is suppressed by the negative non-Gaussianity, whereas the SIGW abundance is sensitive to the typical values of δ , i.e. the width of the distribution. Moreover, as discussed in section 3.2.2, also the bubble collisions source GWs. Consequently, the GW spectrum related to PBH formation in a strongly supercooled first-order phase transition has two peaks [1037]: the SIGW peak at the scale corresponding to the horizon size at the end of the thermal inflation, and the bubble collisions GW peak at the scale corresponding to the typical bubble size.

In figure 38 we show the parameter space ET will be able to probe by searching the GWB from a strongly supercooled ($\alpha \gg 1$) first-order phase transition. For $\beta/H_* \lesssim 20$ the double-peak structure of the spectrum can be detectable, whereas for $\beta/H_* \gtrsim 20$ the component sourced by bubble collisions fully dominates the spectrum. The range of ET extends up to PBH masses $\simeq 10^{-10} M_\odot$, covering the asteroid mass window where PBHs can constitute all DM. At these masses ET will probe the high-frequency tail of the GW spectrum sourced by the bubble collisions.

3.2.5.3 PBHs from collapsing domain wall networks. We have summarized the motivation and observational consequences of Domain Walls in section 3.2.4. In short, DW networks have a strong impact on cosmology and so they are an excellent means to probe models with spontaneously broken discrete symmetries. The strong impact translates into abundant production of cosmological relics, like a GW background.

The ability of DW networks to form PBHs has been re-examined recently [976, 1039–1046]. The PBH formation process is tied to the annihilation regime, the end of the DW epoch. As in section 3.2.4, we specialize the discussion to the simplest annihilation mechanism, a small explicit breaking of the discrete symmetry. This uplifts the degeneracy in the vacuum

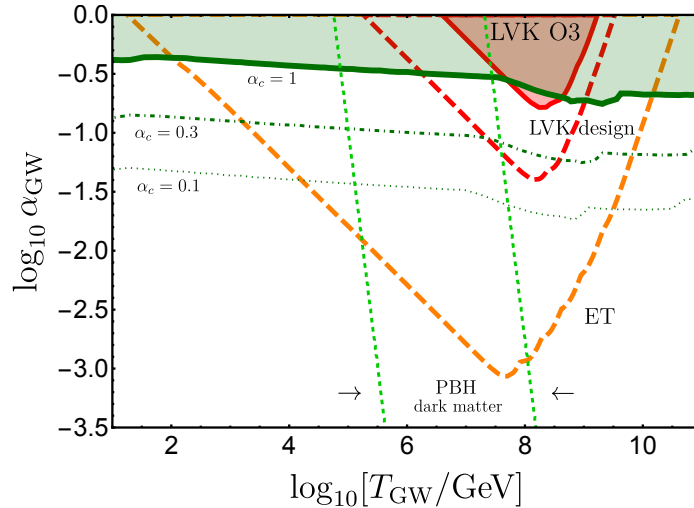


Figure 39. Constraints from PBH overproduction as a function of temperature T_{GW} and DW abundance at the peak of GW emission (adapted from [976]). Superimposed, the detectable regions with ET and LKV design as well as the exclusion from LVK O3 data. The shaded region above the solid line is excluded from the PBHs formed at t_{PBH} or later. The dash-dotted and dotted lines refer to sub-Hubble PBHs, for representative values of α_c , giving a reasonable sense of current systematic uncertainties. The vertical dashed lines indicate the asteroid mass range (assuming $\alpha_c = 1$). Reproduced from [976]. The Author(s). CC BY 4.0.

energies of the vacua, ΔV , which acts as a pressure that pushes the walls to increase the true vacuum volume. During annihilation most DWs collapse due to tension σ_{DW} and vacuum difference ΔV . The annihilation time t_{ann} is estimated from $\sigma_{\text{DW}} H_{\text{ann}} \simeq \Delta V$. At that time the fraction in DWs is given by α_{ann} , see (3.53), which must satisfy $\alpha_{\text{ann}} < 1$ in order not to run into a DW problem.

The PBH abundance is mainly governed by two factors [1042]: the likelihood that Hubble-sized false vacuum pockets shrink within their own Schwarzschild volume R_S^3 ; and the number of Hubble-sized pockets at each time. The first factor is small at t_{ann} (for $\alpha_{\text{ann}} \ll 1$), but it increases in time [1042]. This leads to a rather sharp notion of ‘PBH formation time’, t_{PBH} , defined as when the Schwarzschild radius of a Hubble-sized pocket coincides with the Hubble length. Surviving Hubble-sized pockets at $t \gtrsim t_{\text{PBH}}$ convert into BHs efficiently, independently of shape and angular momentum (and have a baby-Universe wormhole topology, see [1039, 1040]). Notice that there is a parametric delay between the time of GW emission (set by t_{ann} up to a numerical factor [976]) and PBH formation: $t_{\text{PBH}} \sim t_{\text{GW}}/\alpha_{\text{ann}}^{1/2}$ [976, 1042].

The second factor (the abundance of Hubble sized pockets) requires a dedicated computation of how fast the network decays in time, which is challenging from the point of view of numerical simulations. Recent work combining numerical simulations and semi-analytical methods [976] concluded that this abundance decays like $\exp[-(t/\tau)^{3/2}]$, see also [1047]. For Hubble-sized pockets the decay time τ is shorter than t_{ann} by a factor 2 – 3 [976]. This leads to a very strong dependence of the abundance, of the form $\exp[-c\alpha_{\text{ann}}^{-3/4}]$ with c a constant.

False vacuum pockets entering before t_{PBH} can also form BHs, though they require some amount of sub-Hubble contraction. Their collapse probability depends significantly

on asphericities, angular momentum, etc, but on the other hand these pockets are more abundant. A proper computation of this population is challenging and, at present, lacking. A pragmatic way to account for these sub-Hubble BHs is to introduce a lower collapse criterion, α_c (the minimum value of $R_S H$ when the pocket enters the Horizon).

The resulting constraints are shown in figure 39 as a function of temperature T_{GW} and DW abundance at the peak of GW emission, together with the detectable regions in ground based observatories. Remarkably, ET will be able to test whether DM is composed of asteroid mass PBHs produced during the collapse of a DW network.

3.2.6 GWs as probes of the early Universe expansion history

When studying modifications to the standard cosmological model based on GR and Λ CDM, it is convenient to distinguish between the early Universe evolution, defined as the evolution from the earliest instants of the Universe until, say, Big Bang Nucleosynthesis (BBN), and the late Universe evolution, from BBN until the present epoch. The late Universe evolution is strongly constrained by observations, and the role of GWs in probing it will be discussed in details in section 3.3. Early Universe cosmology is much less constrained. Deviations from the predictions of GR and Λ CDM could appear as a consequence of modifications of gravity relevant at high energies, which indeed emerge in different contexts [1048], such as scalar-tensor theories [1049–1051], braneworld theories [1052, 1053], compactified extra dimensions and Kaluza-Klein models [1054, 1055], $f(R)$ theories and theories with higher-order terms in the curvature invariants [1056, 1057], string loop effects [1058] or string cosmology [695, 1059]. The evolution predicted by the standard Λ CDM cosmological model can also be affected, in the early Universe, by physics beyond the Standard Model, phase transitions, etc. In the previous subsections we have seen for instance how such physics beyond the Standard Model could lead to GW signals produced by cosmic strings, domain walls, or related to primordial BH formation.

Another way in which GWs can be used to probe the early Universe, and in particular modifications of its expansion history, is through the imprint that the expansion history leaves on primordial GWs (PGWs) produced in the early Universe, as they propagate until the present epoch. This information is contained in the transfer function, that connects the primordial power spectrum to the power spectrum observed today. To fix the notation, let us recall that the primordial power spectrum $\mathcal{P}_{T,\text{in}}(k)$ of the tensor perturbations was defined in eq. (3.33). The simplest form of the spectrum, at least in a range of comoving wavenumbers k close to a “pivot scale” k_* , is just a power-law, see eq. (3.34), characterized by an amplitude A_T (which depends on the value chosen for k_*) and a tilt n_T (which to a first approximation can be taken as constant, or else can also be made to “run” with k). A typical choice for k_* is 0.05 Mpc^{-1} , corresponding to the wavenumbers probed by the CMB. The tensor amplitude $A_T(k_*)$ can be traded for the tensor-to-scalar ratio $r(k_*) = A_T(k_*)/A_S(k_*)$, where $A_S(k_*)$ is the analogous amplitude for scalar perturbations, whose value is fixed by CMB observations: e.g., $A_S(k_* = 0.05 \text{ Mpc}^{-1}) \simeq 2.14 \times 10^{-9}$.

Given a cosmological model, such as Λ CDM, the evolution in time of the Fourier modes of the GW perturbations over a FRW background can be computed, and the result is encoded

in the transfer function $T_{\text{GW}}(k)$,

$$\tilde{h}_A(\eta_0, k) = T_{\text{GW}}(k) \tilde{h}_A(\eta_{\text{in}}, k), \quad (3.59)$$

(where $A = +, \times$ labels the two polarizations and, at least in GR as well as in any modifications of it that preserves parity, the transfer function is the same for both polarizations). Then, the power spectrum observed today, $\mathcal{P}_{T,0}(f)$, is related to the primordial tensor power spectrum by

$$\mathcal{P}_{T,0}(k) = |T_{\text{GW}}(k)|^2 \mathcal{P}_{T,\text{in}}(k). \quad (3.60)$$

Instead of the comoving wavenumber k , we can equivalently express all results in terms of the GW frequency observed today, f , which (in units $c = 1$) is related to k by $f = k/(2\pi)$. The relation between $\Omega_{\text{GW}}(f)$ and $\mathcal{P}_{T,0}(k)$ is given by [see eq. (19.288) of ref. [60]]

$$\Omega_{\text{GW}}(f) = \frac{\pi^2}{3H_0^2} f^2 \mathcal{P}_{T,0}(f), \quad (3.61)$$

and therefore

$$\Omega_{\text{GW}}(f) = \frac{\pi^2}{3H_0^2} f^2 |T_{\text{GW}}(f)|^2 \mathcal{P}_{T,\text{in}}(f), \quad (3.62)$$

where we have used $f = k/(2\pi)$ as the argument of the functions. In particular, for modes that re-entered the horizon during radiation dominance, i.e. modes with $f \gg f_{\text{eq}} \sim 1.7 \times 10^{-17}$ Hz (and therefore for modes relevant for pulsar timing arrays, space-borne and ground based GW interferometers), we have

$$h_0^2 \Omega_{\text{GW}}(f) \simeq \frac{1}{24} h_0^2 \Omega_{\text{R}} \left(\frac{g_*(T_k)}{3.363} \right) \left(\frac{3.909}{g_*^S(T_k)} \right)^{4/3} \mathcal{P}_{T,\text{in}}(f), \quad (3.63)$$

where Ω_{R} is the energy fraction of radiation (numerically $h_0^2 \Omega_{\text{R}} \simeq 4.184 \times 10^{-5}$); T_k is the temperature at which a mode with comoving wavenumber k re-enters the horizon; and $g_*(T)$ and $g_*^S(T)$ are the effective number of species in the energy density and in the entropy, respectively, at temperature T . For the frequencies relevant for space-borne and ground-based interferometers we have $T_k \gg 100$ GeV and, in the Standard Model, $g_*(T_k) = g_*^S(T_k) \simeq 106.75$, and therefore

$$h_0^2 \Omega_{\text{GW}}(f) \simeq 6.73 \times 10^{-7} \mathcal{P}_{T,\text{in}}(f), \quad (3.64)$$

where we have used the numerical value $h_0^2 \Omega_{\text{R}} \simeq 4.184 \times 10^{-5}$. Equivalently, using eq. (3.34) for the primordial power spectrum, and writing $A_T(k_*) = r(k_*) A_S(k_*)$, $h_0^2 \Omega_{\text{GW}}(f)$ can be written as in eq. (3.35).

If the cosmological evolution is modified, with respect to that obtained assuming the validity of GR and Λ CDM, the transfer function in eq. (3.59) will be different and, given a mechanism that produces a known primordial spectrum $\mathcal{P}_{T,\text{in}}(f)$, the stochastic GW background observed today, given by eq. (3.62), will be different from that expected in GR and Λ CDM. Stochastic GW backgrounds can therefore be a probe of the expansion history of the Universe, and in particular of the early Universe, for which it is in general more difficult to obtain observational constraints.

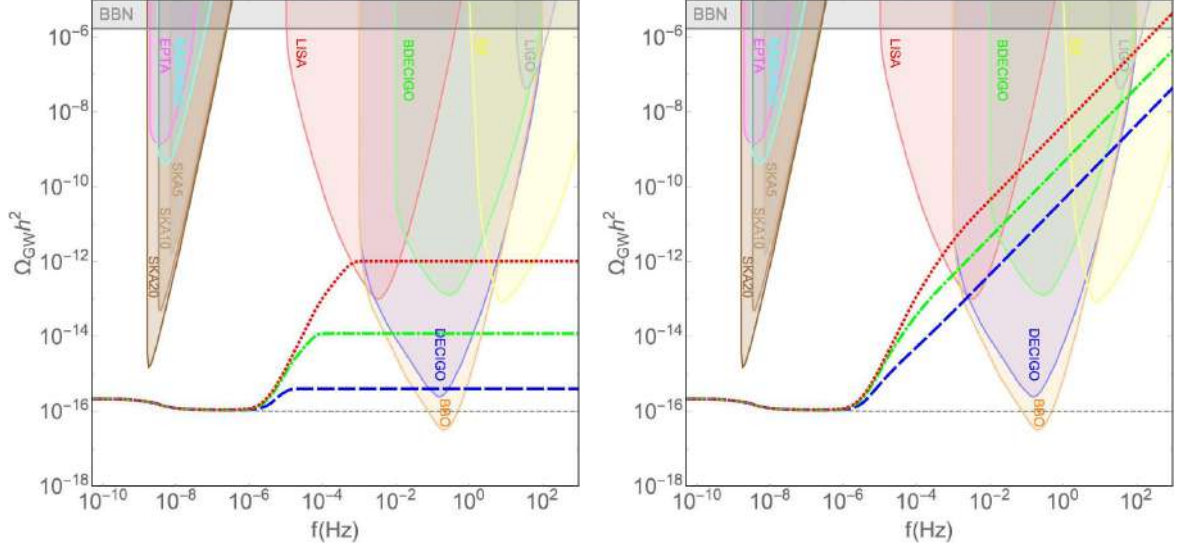


Figure 40. $h^2\Omega_{\text{GW}}(f)$ for the amplification of vacuum fluctuations in standard single-field slow-roll inflation. The gray dashed lines corresponds to the result in GR+ Λ CDM, assuming a tensor-to-scalar ratio $r = 0.07$ and a scale-invariant primordial tensor spectrum, $n_T = 0$. The colored lines show the result obtained with the modified cosmological discussed in the text, for fixed $f_* \simeq 2.5 \times 10^{-6}$ Hz, and $A_* = 1$ (blue dashed lines), $A_* = 10$ (green dot-dashed lines), $A_* = 100$ (red dotted lines), and $\nu = 0$ (left panels), $\nu = 1$ (right panels). The colored regions refer to the BBN constraint and the projected sensitivities of various GW observatories. Reproduced from [1066], © 2020 IOP Publishing Ltd and Sissa Medialab. All rights reserved.

Power-like phenomenological modifications of the expansion rate. At the level of background evolution, modification of the expansion rate can be described writing $H(t)$ as $H_{\text{MC}}(T) = A(T)H_{\text{GR}}(T)$ [1060–1062], where $H_{\text{MC}}(T)$ refers to a given modified cosmology while $H_{\text{GR}}(T)$ refers to the standard cosmological model (GR+ Λ CDM). The factor $A(T)$, which depends on the temperature and accounts for the underlying modified cosmological model, can be parameterized phenomenologically in different ways [1060, 1061]. In all cases, to avoid conflict with the successful predictions of BBN [1063–1065], the parametrization must satisfy the conditions $A(T) \rightarrow 1$ at the onset of the BBN period. At early times, however, one can have $A(T)$ significantly different from one. A number of models, in this regime, can be described by a power-law function $A(T) \simeq A_*(T/T_*)^\nu$ [1060, 1061], with $\{A_*, \nu\}$ dimensionless constants and T_* a reference temperature. In particular, $\nu = 2$ corresponds to a Randall-Sundrum type II brane cosmology; $\nu = 1$ to kination models in GR; $\nu = 0$ gives a model with a constant rescaling of the Hubble expansion rate; $\nu < 0$ in scalar-tensor cosmology; and $\nu = 2/n - 2$ in $f(R)$ cosmology, with $f(R) = R + \alpha R^n$. This power-law behavior is then smoothly interpolated toward $A(T) = 1$ at some temperature T_* [1060, 1061]. For the analysis of the PGWs spectrum, it is convenient to refer to the frequency f_* , instead of the corresponding temperature T_* . For $f \ll f_*$ and $f \gg f_*$, the following general cases can be analyzed [1066, 1067]:

- $f \ll f_*$. In this range of frequencies, the cosmology converges to GR, $H(a) = H_{\text{GR}}(a)$, so the transfer function is the same and, for a given primordial spectrum $\mathcal{P}_{T,\text{in}}(f)$, the

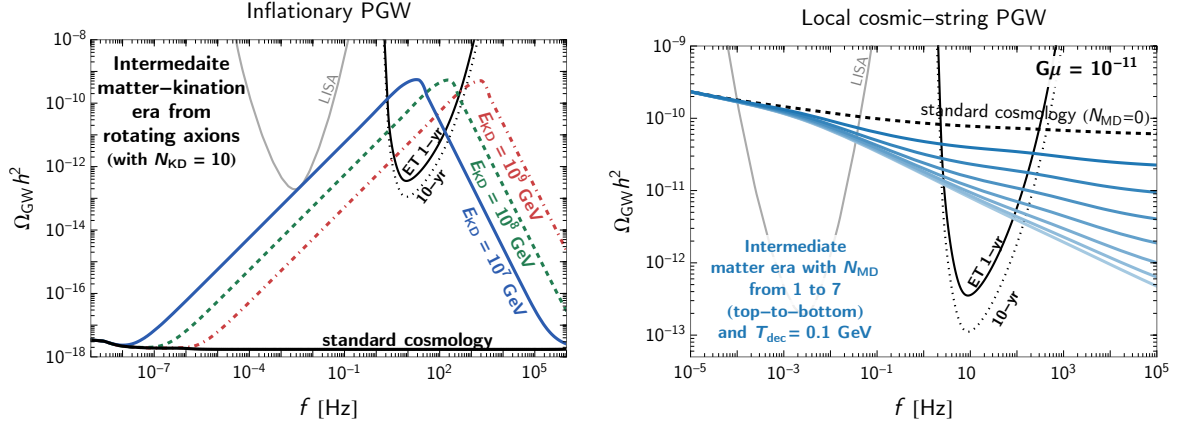


Figure 41. (left) Inflationary spectra induced by the intermediate matter-kination era are shown as colored lines, while the prediction from standard Λ CDM cosmology is the black line. Following the period of matter domination, the intermediate kination era starts at energy scale E_{KD} and lasts for N_{KD} e-folds of the scale-factor expansion. We assume the scale invariant tensor perturbation with inflationary energy scale $E_{\text{inf}} \simeq 1.6 \times 10^{16}$ GeV. The sensitivity curves of ET assume respectively 1 and 10 years of observation with $\text{SNR} = 1$, and the LISA sensitivity comes from [647]. (right) Effect of an intermediate matter era — lasting for N_{MD} e-folds and ending at temperature T_{dec} — on the prediction for $h^2\Omega_{\text{GW}}(f)$ from local cosmic strings, shown by the blue lines [871, 1067–1069]. Reproduced from [1067], © 2020 IOP Publishing Ltd and Sissa Medialab. All rights reserved.

spectrum $\mathcal{P}_{T,0}(f)$ observed today is the same. Therefore $h_0^2\Omega_{\text{GW}}(f) \propto \mathcal{P}_{T,\text{in}}(f)$, see eqs. (3.63) and (3.64).

- $f \gg f_*$. In this range of frequency, the factor $A(f)$ plays a major role. The regimes where $\nu > 0$, $\nu = 0$, and $\nu < 0$ are separately discussed:
 - $\nu > 0$: if ν takes positive values, the PGW relic density reads $\Omega_{\text{GW}}(f) \propto \mathcal{P}_{T,\text{in}}(f) f^{\frac{2\nu}{1+\nu}}$, in which appears an extra factor $f^{\frac{2\nu}{1+\nu}}$, that generates a blue-tilted modification to the original tensor power spectrum. This enhancement follows from the reduction of the friction term entering the equation of GWs.
 - $\nu = 0$: In this case, the Hubble rate is enhanced by a constant factor $A = 1 + A_*$, and the PGW spectrum is therefore not distorted and is modified by an overall shift $\Omega_{\text{GW}}(f) \propto (1 + A_*)^2 \mathcal{P}_{T,\text{in}}(f)$.
 - $\nu < 0$: for negative values of ν , the factor $A \rightarrow 1$ both for low ($f \ll f_*$) and high frequencies ($f \gg f_*$), with a maximum at $f = \bar{f} \gtrsim f_*$ given by $A(\bar{f}) = A_*$. In these limits the PGW spectrum presents the same tilt as the original tensor power spectrum, with a characteristic bump at $f = \bar{f}$.

Figure 40 shows $h^2\Omega_{\text{GW}}(f)$ for the amplification of vacuum fluctuations in standard single-field slow-roll inflation. The result in GR+ Λ CDM is given by the gray dashed line, while the colored lines shows the results for $\nu = 0$ (left panel) and $\nu = 1$ (right panel) for different values of A_* . The colored regions refer to projected sensitivities for different GW experiments.

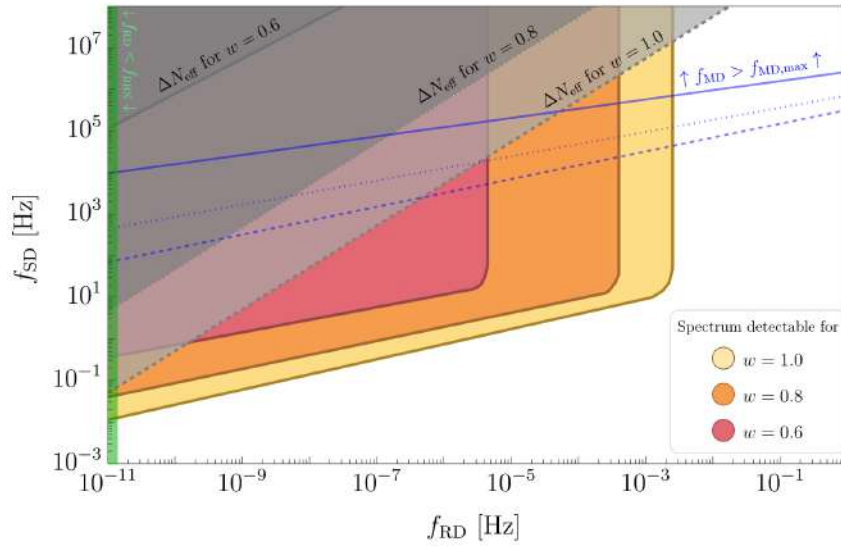


Figure 42. The $(f_{\text{RD}}, f_{\text{SD}})$ parameter regions accessible by ET are shown. The colored regions represent the detectable parameter spaces for three different values of w : red for $w = 0.6$, orange for $w = 0.8$, and yellow for $w = 1.0$ (corresponding to kination). The gray regions indicate the parameter space excluded by indirect limits from BBN and CMB, with the darkest gray for $w = 0.6$ and the lightest gray for $w = 1$. We again assume the scale invariant tensor perturbation with inflationary energy scale $E_{\text{inf}} \simeq 1.6 \times 10^{16}$ GeV. The parameter f_{MD} is chosen as a function of f_{RD} , f_{SD} and w , as described in the text, and we denote the parameter region where $f_{\text{MD}} > f_{\text{MD,max}}$ with blue lines (dashed for $w = 1$, dotted for $w = 0.8$ and thick for $w = 0.6$). The sensitivity curve of ET assumes 1 year of observation with $\text{SNR} = 1$. The green area on the left is excluded, since there $f_{\text{RD}} < f_{\text{BBN}}$, which is not allowed. Reprinted figure with permission from [1074], Copyright (2024) by the American Physical Society.

Modified equation of state in the early Universe. As another example of modified expansion history, if there were some pre-BBN epochs in the cosmological history with a modified equation of state associated for instance with either a matter domination or a stiff era, this would lead to a tilt in $\Omega_{\text{GW}}(f)$ with spectral index $\beta = -2(1 - 3w)/(1 + 3w)$, with w the equation of state of the Universe, at the frequencies corresponding to the modes which re-entered the horizon in this non-standard era [1062, 1070, 1071]. For example, the kination era right after inflation can result in the enhanced GW spectrum shown in the right panel of figure 40, although any observable signal at ET would be strongly constrained by ΔN_{eff} bound, as shown in [1070, 1071]. The most striking case comes from an intermediate matter-kination era, well-motivated by rotating axions [1071–1073], that largely amplifies the inflationary signal and induces the “triangular signature” [with $\beta = 1$ during kination era ($w = 1$) and $\beta = -2$ slope during matter era ($w = 0$)] shown in the left panel of figure 41. This amplification can make the signal from slow-roll inflation observable at ET, for kination energy scales of order $(10^6 - 10^9)$ GeV, that connect to axion mass m_a and axion decay constants f_a in the range $m_a \lesssim 6.5 \mu\text{eV}(10^9 \text{ GeV}/f_a)(\Omega_a/\Omega_{\text{DM}})$, where the last bracket is the energy fraction of axions as dark matter [1071].

Furthermore, one can study scenarios with a stiff epoch weaker than kination ($1/3 < w < 1$) [1070, 1074]. As a case study, we focus on a cosmological history similar to the one studied above, where after inflation there is an intermediate radiation-matter-stiff era (denoted with RD1, MD and SD respectively), followed by the conventional radiation dominated Universe. We assume instantaneous transitions between the different epochs. This scenario also leads to a peak-like spectrum that can be testable at Advanced LIGO A+ sensitivity and by third-generation detectors [1074]. The frequency of the stiff-to-radiation transition is denoted by f_{RD} , and the one of matter-to-stiff transition by f_{SD} . The parameter space of the model that is testable at ET is illustrated in figure 42, for three different values of the equation of state w during the stiff period. Assuming no entropy variation, the frequency at the RD1-to-MD transition can be expressed as $f_{\text{MD}} = f_{\text{RD}}^{\frac{1-3w}{1+3w}} f_{\text{SD}}^{\frac{6w}{1+3w}}$. This transition cannot occur earlier than the end of inflation, implying a maximum value of $f_{\text{MD,max}} = 1.8 \times 10^8 \text{ Hz}$. This limit is indicated by the blue lines in figure 42.

The non-standard era beyond the radiation era at high temperatures not only impact the inflationary PGW, but it can also lead to significant spectral distortions in GW from other cosmological sources, in particular, from cosmic strings [870–872, 1068, 1069, 1071, 1073, 1075–1077], and causality tails [828, 1021, 1071, 1078, 1079]. The frequencies at which the spectral slope gets modified correspond to the scales at which the GW is being produced; the slope depends on the equation of state of the Universe and can differ from the case of inflationary PGW; see e.g., [1062, 1080] for reviews. This is illustrated for the case of local cosmic strings in the right panel of figure 41.

Oscillation features in the GWB. Interferometers such as ET have the capability not only to observe the general coarse-grained trend of a primordial GWB, but also its fine-grained features. In particular, periodic oscillatory patterns in the GWB have been identified as distinctive signatures of primordial features [1081–1083]. These imprints reflect early-universe dynamics that deviate significantly from scale invariance and arise across a broad range of theoretical frameworks, including both inflationary models and their alternatives (see [1084–1086] for reviews on the subject). The GWB provides a powerful observational probe of primordial features at scales well beyond the reach of the CMB and galaxy surveys, where most searches have traditionally been concentrated. Detecting such signatures would unveil new physics beyond the standard cosmological model and opening an unprecedented window into the dynamics of the primordial universe.

Primordial features often manifest as oscillations in the scalar power spectrum. These have been found to be of two types. *Sharp features* arise from brief deviations in background parameters from their attractor behavior, producing transient, linear oscillations in the scalar power spectrum. This linear oscillatory shape typically appears in inflationary models with a step-like feature in the potential [1087–1096], turns in the multi-field trajectory [1091, 1097–1100] or sudden transition in the speed of sound [1101–1104]. *Resonant features* result from periodic background oscillations that resonate with quantum modes of the primordial curvature fluctuations, leading to sustained oscillations at constant frequency across a wide log-scale range. These log-oscillations can be produced in inflationary scenarios where the inflaton potential has oscillatory bumps [1105], multi-field dynamics [1091, 1106–1111],

other settings such as axion monodromy inflation [1112–1115] and axion Chern-Simons gravity [1116]. A hybrid of both types appears in models known as classical primordial standard clocks [1106, 1107, 1110, 1111, 1117–1127].

These features in the scalar sector are imprinted in the GWB through nonlinear interactions [1081–1083, 1100, 1128, 1129], where scalar perturbations source tensor modes, if their amplitude is sufficiently large. A broad class of oscillatory patterns in the GWB can then be captured by the following generic templates, either of linear oscillations,

$$\Omega_{\text{GW}}(f) = \bar{\Omega}_{\text{GW}}(f) \left\{ 1 + \sum_{l=1}^{l_{\text{max}}} \left[A_l^c \cos \left(l\omega \frac{f}{f_*} \right) + A_l^s \sin \left(l\omega \frac{f}{f_*} \right) \right] \right\}, \quad (3.65)$$

or of log-periodic ones,

$$\Omega_{\text{GW}}(f) = \bar{\Omega}_{\text{GW}}(f) \left\{ 1 + \sum_{l=1}^{l_{\text{max}}} \left[A_l^c \cos \left(l\omega \ln \frac{f}{f_*} \right) + A_l^s \sin \left(l\omega \ln \frac{f}{f_*} \right) \right] \right\}, \quad (3.66)$$

where $\bar{\Omega}_{\text{GW}}(f)$ is the envelope or main spectral shape, f is the observed frequency of the GW signal, f_* is the pivot frequency scale of the observatory, l_{max} denotes the maximum harmonic determined by the sourcing mechanism. The coefficients $A_l^{c,s}$ represent (not necessarily constant) amplitudes, and ω is the angular frequency of the oscillations — not to be confused with the frequency f .

Sharp features during inflation lead to the template in eq. (3.65) in its simplest form, i.e. $l_{\text{max}} = 1$ [1081, 1082], for GWs produced when scalar fluctuations — affected by the feature — re-enter the horizon during the post-inflationary era.²⁰ In this context, the characteristic frequency is $\omega \sim 2/f_{\text{f}}$, where f_{f} corresponds to the scale that exited the horizon at the time of the feature during inflation. Typically, the resulting GW spectrum exhibits $O(10\% - 20\%)$ modulations, arising from the resonant amplification of $O(1)$ -level oscillations in the sourcing scalar power spectrum. These oscillations arise from moderate to copious particle production induced by the sharp feature [1081]. This also leads to a characteristic relation between the frequency at which Ω_{GW} peaks, denoted f_* , and the feature scale f_f , typically given by $f_* \sim O(10-100) f_f$. Consequently, for a signal within the ET band, linear oscillations can be probed in the range $0.1/\text{Hz} < \omega < 10/\text{Hz}$. Additionally, both the amplitude of the oscillations and the shape of the envelope are influenced by the background equation of state at the time of GW production [1128].

Resonant features, on the other hand, lead to a template as given in eq. (3.66), with $l_{\text{max}} = 2$. In this case, the GW spectrum exhibits a superposition of two oscillatory components, with frequencies ω_{log} and $2\omega_{\text{log}}$ [1083]. Here, ω_{log} corresponds to the logarithmic oscillations in the primordial power spectrum, often linked to a mass parameter in units of the Hubble scale. The relative amplitudes of these two harmonics depend on ω_{log} , the shape of the peak in the scalar power spectrum, and the background equation of state at the time of GW production. The templates that have been derived in [1081–1083, 1128] for the post-inflationary contribution to the GWB encapsulate these dependencies.

²⁰For the template of primordial GWs sourced directly at the time of the feature during inflation, see ref. [1130].

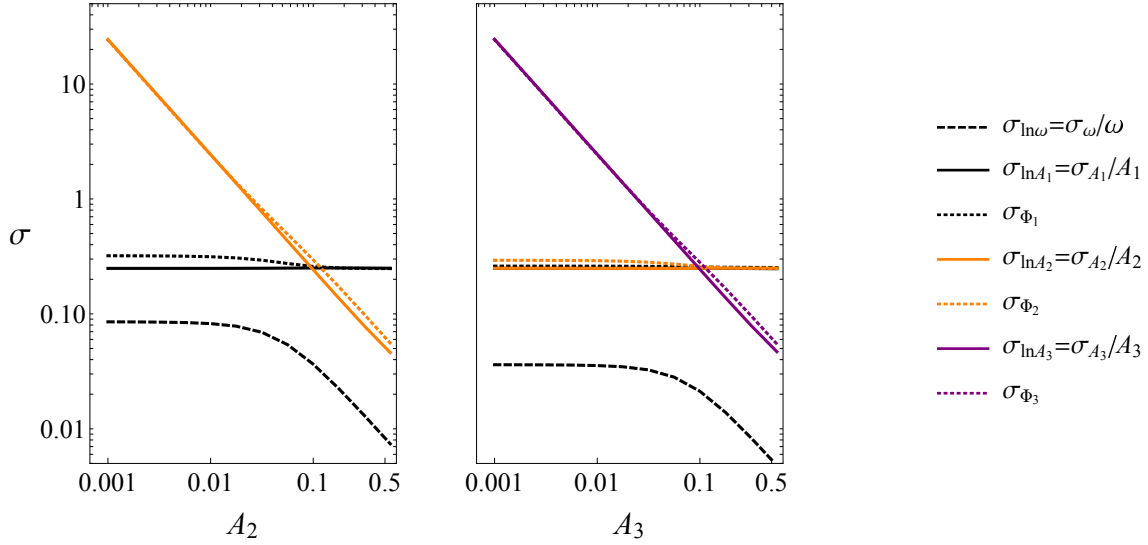


Figure 43. Dependence of the expected relative errors σ for $l_{\max} = 2$ (left) and $l_{\max} = 3$ (right) harmonics. The oscillation amplitude A_2 and A_3 vary, while the other parameters are fixed as follows. For $l_{\max} = 2$: $\Omega_0 = 10^{-11}$, $n_t = 0$, $\omega = 10$, $A_1 = 0.1$, $\Phi_1 = 0$. For $l_{\max} = 3$: $A_2 = 0.1$, $\Phi_2 = 0$ and $\Phi_3 = 0$. The black, orange and purple curves correspond to the oscillation parameters (oscillation amplitude A_l and phase Φ_l) for $l = 1$, $l = 2$, and $l = 3$, respectively. Reproduced from [1131], © The Author(s). Published by IOP Publishing Ltd. CC BY 4.0.

The log-oscillatory profile in eq. (3.66) can also be produced directly in the tensor sector as a manifestation of a broken discrete scaling symmetry in scenarios of quantum gravity [1131, 1132]. In this case, the energy spectrum in (3.66) is given by eq. (3.63) with

$$\mathcal{P}_{T,\text{in}}(k) = \bar{\mathcal{P}}_{T,\text{in}}(k) \left[1 + A_1^c \cos \left(\omega \ln \frac{k}{k_*} \right) + A_1^s \sin \left(\omega \ln \frac{k}{k_*} \right) \right], \quad (3.67)$$

where the prefactor $\bar{\mathcal{P}}_{T,\text{in}}(k)$ depends on the model. A function F is invariant under a discrete scale symmetry if $F(\lambda_\omega^m f) = F(f)$, where $\lambda_\omega := e^{\frac{2\pi}{\omega}}$ and m is an integer. The oscillatory modulation of (3.66) enjoys this symmetry but the envelope $\bar{\Omega}_{\text{GW}}(f)$ in general does not. A discrete scale invariance in frequency as the one above corresponds to a symmetry in momentum space, mirrored by an invariance under discrete dilations $x^\mu \rightarrow \lambda_\omega x^\mu$ in position space. Such a micro-structure of spacetime geometry would correspond to the one of a deterministic (multi-)fractal [1133–1137] replicated at all scales. This everpresence of the same structure transcending a UV/IR divide is responsible for the appearance of log oscillations at all frequencies in (3.66).

Expressions (3.65) and (3.66) can serve as templates to fit any signal displaying linear or logarithmic oscillations, regardless of its physical origin [1131]. To check the detectability of the log oscillations in ET, in [1131] the Fisher matrix for the cross-correlation analysis was calculated to estimate the expected error on the parameters in (3.66) for the ET-D sensitivity curve assuming a three-year observation run. It has been shown that, for oscillation amplitudes $A_l \sim 0.1$ and a frequency $\omega \sim 10$, we need a detection with signal-to-noise ratio $\gtrsim 100$ to determine all the oscillation parameters with an $O(10\%)$ precision. If the GWB is

primordial and the envelope is parameterized by $\bar{\Omega}_{\text{GW}} = \Omega_0(f/f_*)^{n_t}$, the minimum GWB amplitude Ω_0 and the minimum tensor tilt n_t needed to achieve such SNR at the ET frequencies satisfying the current upper bound $r < 0.036$ of the tensor-to-scalar ratio at the CMB scale $k_0 = 0.05 \text{ Mpc}^{-1}$ [674] are roughly

$$\Omega_0 \gtrsim 10^{-11}, \quad n_t \gtrsim 0.28. \quad (3.68)$$

Relative errors on oscillation frequency, oscillation amplitude, and the oscillation phase scale as $\sigma_{\ln \omega} \propto 1/(A_l \omega)$, $\sigma_{\ln A_l} \propto 1/A_l$ and $\sigma_{\Phi_l} \propto 1/A_l$, respectively, and they all decrease as $\propto 1/\text{SNR}$. Notably, the error is insensitive to the inclusion of higher-order harmonics, for the reason that oscillation frequencies are ordered hierarchically as integer multiples of ω and each term in the harmonic expansion can be constrained independently. The $l_{\text{max}} = 2$ and $l_{\text{max}} = 3$ cases are shown in figure 43. The range of ω accessible by ET is roughly

$$10 < \omega < 10^3, \quad (3.69)$$

where the lower bound is fixed by requiring to have enough oscillation cycles inside the sensitivity band of ET and the upper bound is determined by $\omega < f/\Delta f$, where Δf is the frequency resolution of the instrument. Our estimation is obtained by $f \sim 10 \text{ Hz}$ and $\Delta f \sim 0.01 \text{ Hz}$, which is a similar value used in the LVK stochastic analysis as a result of coarse graining.

3.3 Probing the late Universe with Einstein Telescope

In the previous section we have discussed how ET can probe the early Universe. The relevant observable, in that case, is the stochastic GW background of cosmological origin that, as we have seen, can be generated by many different early Universe mechanisms. For probing the late Universe with GWs, in contrast, the most important signals are those due to the coalescence of compact binaries. A crucial feature of GWs for late time Universe studies is the fact that CBC sources are self-calibrating distance indicators. When the redshift of the source can also be somehow inferred, one can reconstruct the distance-redshift relation, tracking the expansion history of the Universe and constraining cosmological parameters such as the Hubble constant [1138, 1139], similarly to the use of standard candles. In this context, CBC are then called “standard sirens”. This reconstruction is, however, non-trivial, since from a GW alone one can not in general infer the redshift, which is degenerate with the chirp mass. While breaking this degeneracy is possible in special cases, like for *Love sirens* [1140], or for matter structure/cosmological effects considered in [1141], in general direct redshift measurements are not possible with GW detectors. On the other hand, for *bright standard sirens*, which are those events followed by electromagnetic counterparts, direct redshift measurements are enabled by host galaxy identification. However, most GW events seen so far by LVK have not been followed by electromagnetic counterparts, hence the name of *dark (standard) sirens*. With dark sirens, the most common techniques for cosmological parameter estimation rely on combinations with galaxy catalogs from which redshift information can be extracted [1138, 1142], allowing to track the cosmic expansion history [1143–1147]. Additionally to these methods, one can make use of known merger redshift distribution [1148–1150] or features in the mass distribution of BHs and NS to obtain redshift

information [1151–1157], a technique called *spectral sirens*. We highlight, however, that cosmological and astrophysical population inference are deeply intertwined [1145, 1147, 1158–1161]. In this section we discuss various types of methods for doing cosmography with ET and their prospects.

3.3.1 Cosmography with coalescing binaries

3.3.1.1 Bright sirens. Bright sirens are expected to have two main types of prompt electromagnetic counterparts: kilonova (KN) explosions, and short GRBs, as we know from the celebrated example of GW170817 [3, 4]; see section 5 for a detailed discussion of the perspective for multi-messenger observations with ET.

Estimates of the accuracy on cosmological parameters that can be obtained from bright sirens depend on the BNS and BHNS rates, which are currently less constrained than the BBH rate, and also on uncertainties on the network of electromagnetic observatories that will operate at the time of ET, as well as on the amount of telescope time that they will devote to the EM follow-up of GW events. Here we assume the two different scenarios for the joint GW+EM detection, already considered in [16]. In the first one, the merger of a BNS is followed by a Kilonova (KN) emission that is potentially observed electromagnetically by the Vera Rubin Observatory (VRO) [1162]. In the second scenario, under proper circumstances, a short Gamma Ray Burst (GRB) detection by THESEUS [1163, 1164] is associated to the GW event from a BNS. For the ET+VRO study we consider the multimessenger sources detected in one year of observations, while for ET+THESEUS, in order to have a statistically similar sample, we consider a 5 years catalog; the resulting dataset of bright sirens is the same as in tables 6 and 10 of [16]. We also use the same BNS population as in [16].²¹

As shown in [16] (see tables 28 and 29), the resulting marginalized error on H_0 in the GW+GRB case (and 5 yr of observations) is 5.7% for the 10-km triangle and 4% for 2L-15km-45° configuration, while for GW+KN (and 1 yr of observations) it is 0.9% for the 10-km triangle and 0.6% for 2L-15km-45°.

We have repeated the analysis showing also the impact of using the Planck 2018 [591] posterior distributions as a prior. The corresponding forecasts for the accuracy on H_0 and Ω_M are shown in figure 44 for GW+KN, and in 45 for GW+GRB. In particular, from these results we find that, when adding the Planck prior, the relative error on H_0 in the GW+GRB case (and 5 yr of observations) becomes 1.1% for both the 10-km triangle and for 2L-15km misaligned (so in this case the result is clearly dominated by the Planck prior), while for GW+KN (and 1 yr of observations) it becomes 0.7% for the 10-km triangle and 0.4% for 2L-15km misaligned.

We stress, however, that on top of the uncertainties related to the BNS rate and to the facilities and telescope time that will be available for the electromagnetic follow-up of GW events, the analysis performed in the GW+GRB case has been conducted under the assumption that the relative error on the luminosity distance can be approximated as $2/\text{SNR}$, as assumed also in previous studies [16, 1165]. While the SNR^{-1} scaling theoretically represents a good

²¹This catalog is obtained from state-of-the art population models tuned to the O3 LVK data. The absence, to date, of BNS candidates in the currently ongoing O4 run implies that the rate used for BNS is now toward the upper range, rather than the mean value.

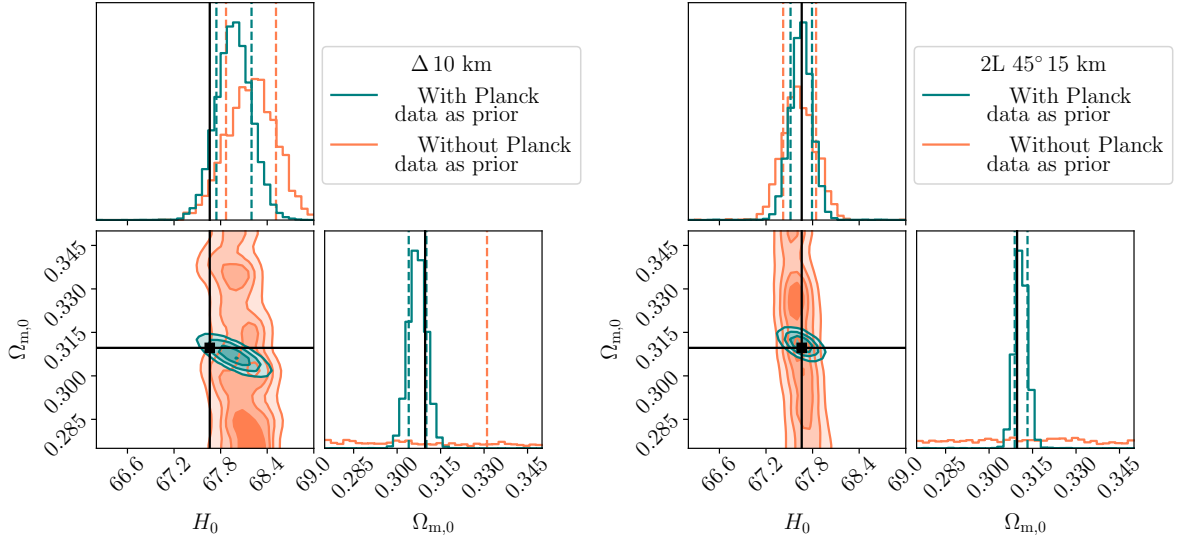


Figure 44. Results of the joint inference on the cosmological parameters H_0 , $\Omega_{m,0}$, employing GW+KN events detected in one year of observations by the 10 km triangular (left panel) or the 2L-15km-45° (right panel) ET configurations, with the EM counterpart detected by the Vera Rubin Observatory. Vertical dashed lines represent the 68% CI of each distribution, while the black solid lines label the fiducial values.

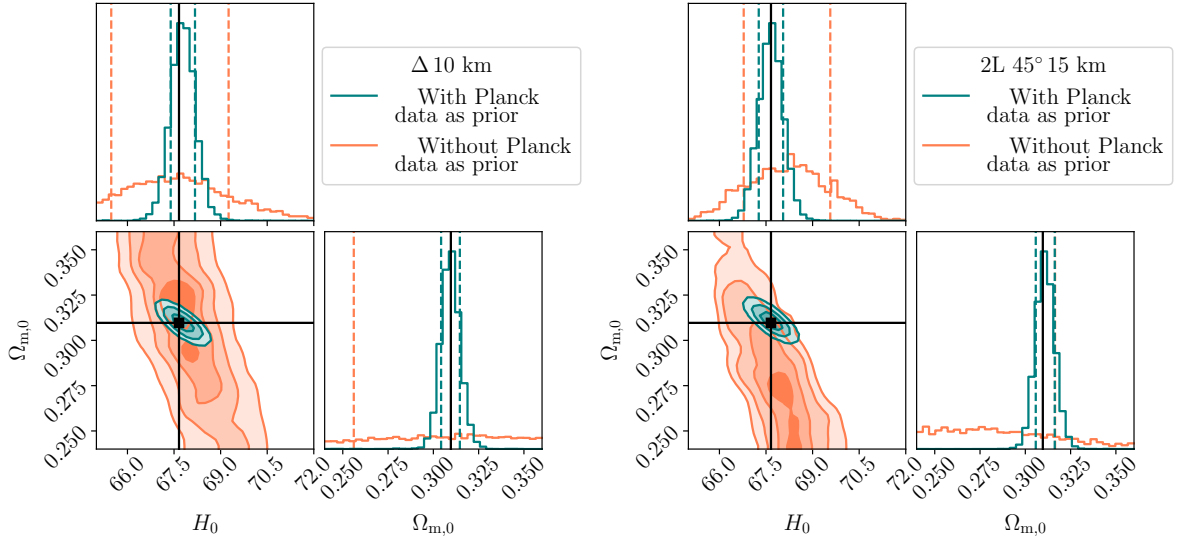


Figure 45. Results of the joint inference on the cosmological parameters H_0 , $\Omega_{m,0}$ employing GW+GRB events detected in 5 years of observations by the 10 km triangular (left panel) or the 2L-15km-45° (right panel) ET configurations, with the EM counterpart detected by THESEUS. Vertical dashed lines represent the 68% CI of each distribution, while the black solid lines label the fiducial values.

proxy for most of the parameter space, it reaches important limitations in some specific corners [1166]. In particular, the GRB counterpart associated to a GW event can be detected only for small inclination angles, since the emission mechanism typically ejects the material in the same direction of the angular momentum of the binary. This circumstance introduces two non-trivial challenges in the determination of the luminosity distance uncertainty: in the limit of face-on/face-off binaries, the Fisher Matrix becomes ill-conditioned, leading to an unfaithful reconstruction of the infamous d_L - ι degeneracy [1167, 1168] and to a regime where the $2/\text{SNR}$ approximation is not appropriate anymore; additionally, this intrinsic correlation gets stronger as the inclination angle approaches smaller values. The two above-mentioned effects, which inevitably appear in the framework of GW+GRB observations, represent non-negligible sources of bias that can jeopardize the whole inference if not taken into account for. A comprehensive description of how to properly deal with such issues has been recently proposed in [1169], where the Fisher Matrix limitations are overcome and the d_L - ι correlation is mitigated by the underlying fact that the EM detection implicitly selects a specific corner of the parameter space, see also the discussion in section 10.2.3.

Besides model-based cosmological parameter estimation with bright sirens, one can also employ model-independent methods to reconstruct the distance-redshift relation and test for deviations of the standard cosmological model or some of its basic assumptions. That can be done for example with the use of Gaussian Process as in [1170–1172], or by combining GW distances with other distance indicators, such as standard candles and angular diameter distances from the large-scale structure [1172, 1173]. In ref. [1172], it was shown that fewer than 40 GW-GRB detections, in combination with forthcoming cosmological surveys, will enable unprecedented precision on H_0 and Ω_m , and accurately reconstruct the DE density evolution. Taking the ratio of different distances is an interesting approach as it avoids the intrinsic degeneracies present in the distance-redshift relation between standard cosmological parameters, notably Ω_m , and effects that can modify the GW luminosity distance or other cosmic distances, such as violations of the transparency of the Universe (cosmic opacity) or modified gravity (see section 3.3.2). In [1173], it was shown that, in combination with other distance measurements expected for the near future with surveys like Euclid, LSST, DESI and Roman, bright sirens from ET would allow tests of cosmic opacity together with deviations in the GW propagation due to modified gravity models that can be achieved with percent level precision, in various redshift bins up to $z = 0.9$, with no assumption on the redshift evolution of the mechanisms causing deviations from standard physics.

As shown in [1174], bright sirens will be useful tracers of both the density and velocity fields in cosmology. In particular, a combination of ET sirens with EM counterparts with LSST allows for a 30% improvement on the measurements of H_0 with ET alone, but also percent-level measurements of Ω_{k0} (an improvement of over one order of magnitude). We can also get good constraints on the growth of structure in particular on σ_8 and the growth-rate index γ . In [1174] it was also proposed the use of preliminary GW distances, obtained for the alerts for follow-ups, to inform the optimal telescope exposure time. It was shown that this technique reduces the necessary telescope time to obtain the same kilonova completeness by around 25%. This could be useful since in the ET era the bottleneck for bright sirens catalogs will possibly be due to available telescope time.

3.3.1.2 Dark sirens. While bright sirens provide the most direct method of using resolved sources for late-time cosmology, the other side of the coin is the difficulty in the observation of the EM counterpart. Events with direct EM counterparts will constitute only a tiny fraction of the total detected events, with a strong selection from the EM side limiting these sources to redshifts generally of order $z \lesssim \mathcal{O}(1)$ [1165, 1175]. On the other hand, an unprecedented opportunity offered by ET will be the detection of compact binary mergers across all cosmic history, in particular BBH mergers up to redshift $\mathcal{O}(20)$ [16, 1167]. At redshifts below the star formation peak, ET can in particular detect almost 100% of the population [16, 1167]. The vast majority of these events will not have EM counterparts but contain precious information about the evolution of the Universe, especially at redshifts that are not accessible to the current generation of GW detectors, not even at their most advanced sensitivity. This will open a window on the evolution of the Universe well before the Dark Energy dominated era and beyond the star formation peak, possibly even in region where compact objects formed by standard stellar-evolution scenarios are expected to be absent [1176, 1177]. Even if their actual high-redshift nature will have to be carefully assessed on an event-by-event basis [1178–1181], inferring their statistical distribution might lead to clearer signatures [1182]. Therefore, methods that target the use of the full sample of detected events will be crucial for cosmography with ET. Several of those are actively being developed by the community, and are generally referred to as “dark siren” methods. The general principle behind dark sirens is the use of prior information on the properties of the underlying distribution of sources, of which the observed catalog is a biased sample, to break the degeneracy between source-frame masses and redshift in the GW waveform at the statistical level [1159].

The line-of-sight redshift prior. The GW observations provide a direct measurement of the luminosity distance. For a dark siren measurement of H_0 or the other cosmological parameters, the GW distance information needs to be convolved with some “prior” information on the redshift. This prior comes from the underlying cosmology and binary merger rate evolution (uniform part), as well as potential host galaxies (non-uniform part). Cosmology inference algorithms allow us to construct this prior along each line-of-sight. This line-of-sight redshift prior essentially consists of an in-catalogue part and an out-of-catalogue part. The in-catalogue part is constructed using redshifts and luminosities of the galaxies in the region. The luminosity (in a certain observation band) is used to effectively weight the galaxy in proportion to the probability of sourcing a merger. Astrophysical assumptions go into this construction, given that the luminosity in a certain band may trace a certain aspect of the astrophysics. The out-of-catalogue is used to model the galaxies which are beyond the catalogue, due to their distance or due to low intrinsic brightness. This part usually depends on galaxy luminosity parameters (or Schechter function parameters) in addition to the assumed cosmological model. A detailed prescription of the construction of the line-of-sight redshift prior can be found in [1161]. In the following paragraphs we describe the galaxy catalogs and surveys which can be used to construct this prior in the 3G era.

Galaxy catalogs. The lack of an EM counterpart poses a tough challenge in GW cosmology. However, as first proposed by Schutz in 1986 [1138], statistical approaches that rely on available galaxy catalogs can counterbalance the missing source’s redshift information. Specifically,

the potential host galaxies that fall in the localization error volume of a detected GW event display an inhomogeneous distribution that can be employed as redshift prior in the Bayesian framework. Contrarily to bright sirens, a single or few of such events are likely unable to deliver tight constraints on the cosmological parameters. As a statistical tool, the efficiency of this approach strongly relies on the size of the sample of dark sirens used, and we expect the precision to shrink with the square root of the number of events. In the past decade, this methodology has successfully been applied both to simulated [1142, 1145, 1147, 1160, 1183–1185] and real data with different galaxy catalogs, including GLADE/GLADE+ [9, 1143, 1161, 1186–1190], the Dark Energy Survey [1191, 1192], the DESI Legacy Survey [1193], the DECam Local Volume Exploration Survey [1194], DESI [1195], and to the PSZ2 and eRASS galaxy cluster catalogs [1196].

Spectral Sirens. A complementary possibility (also called the “spectral sirens” method) is the use of features in the source-frame mass distribution [1151–1156]. The population properties, in particular, the mass and redshift distributions, are given in source-frame; for example, one can predict the probability that a black hole has a certain mass, or forms at a certain redshift, from population synthesis codes [1197–1199]. On the other hand, GW detections measure detector-frame quantities (redshifted masses and luminosity distance). The population properties reconstructed in source-frame from detector-frame observations will thus be reshaped by a change of the cosmology. In case source-frame distributions are informative, the degeneracy between source-frame masses and redshift in the GW waveform can be broken by combining an ensemble of sources. This can drive constraints on cosmology, as the presence of preferential scales breaks the mass-redshift degeneracy at the statistical level.

Potential systematic effects. Recent advances have led to the development of pipelines incorporating information from both the mass distribution and galaxy catalogs [1147, 1160, 1161]. Thus one no longer needs to assume a fixed mass distribution for an inference using galaxy catalogs, which may otherwise have biased the measurement. Taking care of this leading order systematics exposes us to smaller sources of potential systematic uncertainties. These can come from the GW data, the EM data, or the underlying assumptions. It is crucial to understand and address even the smallest of the potential systematic effects as the statistical power of the data increases, in order to converge towards an unbiased measurement. On the GW side, mis-modelling the source population or its redshift evolution could pose a problem [1200]. On the EM side, it is important to understand redshift measurement errors [1201]. Uncertainties in the measurement of galaxy luminosities and the measurement of the Schechter luminosity function parameters (and their evolution) come next [1202]. It has recently been shown that the results could be biased due to incorrect assumptions on the luminosity weighting of galaxies [1203, 1204]. A large body of work is underway largely in the context of upcoming LVK observations [1205], to understand and address small systematic effects and ensure an accurate measurement as we move towards precision. Targeted studies to determine the potential of these techniques for ET will be necessary in the future.

The 3G era is expected to bring $\mathcal{O}(10^{4-5})$ BBH detections per year. Albeit computationally challenging, this abundance naturally meets the conditions that make the galaxy catalog approach a powerful statistical tool. Indeed, current forecasts predict that a sub-percent

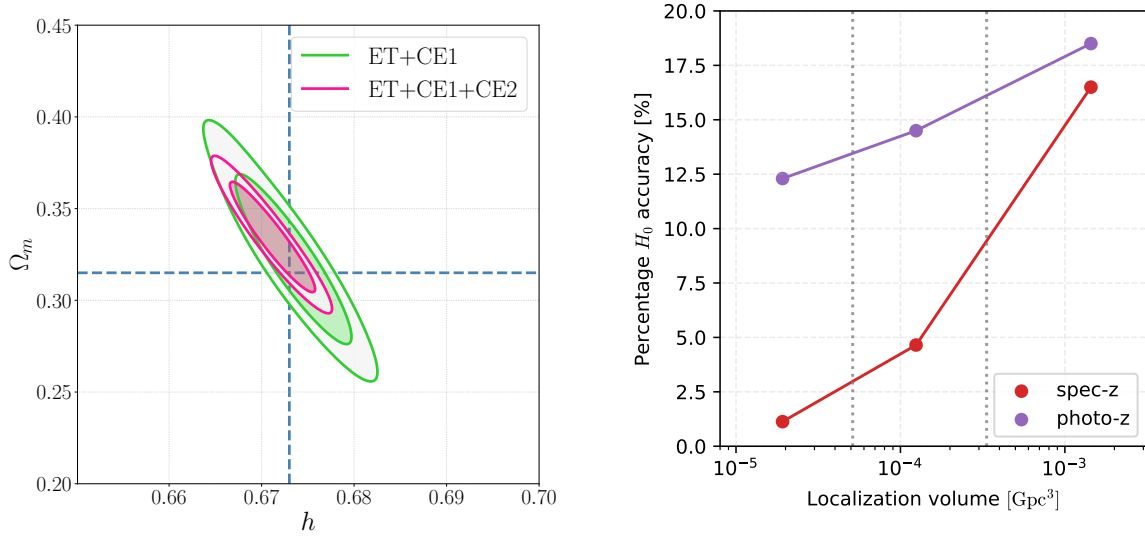


Figure 46. *Left panel:* posterior joint distribution on H_0 and $\Omega_{m,0}$ obtained by employing BBHs as dark sirens and cross correlating the loudest GW event localization volumes with a galaxy catalog. The contours, which refer the 68% and 90% confidence level, are shown for two networks of 3G detectors assuming one year of continuous observations. Here ET is in a 10 km-triangular configuration and CE1 and CE2 correspond to 40 km and 20 km Cosmic Explorer, respectively. Reprinted figure with permission from [1185], copyright (2023) by the American Physical Society. *Right panel:* dependence of the accuracy on H_0 on the GW event localization volume with a photometric (photo-z) or spectroscopic (spec-z) galaxy catalog.

measurement of H_0 and a $\sim 10\%$ measurement of $\Omega_{m,0}$ can confidently be attained by 3G detectors by employing only the loudest dark sirens observed in one year [1185, 1206]. Yet, as shown in figure 46, reaching this precision requires a mandatory joint effort between more than a single 3G detector, as the localization power plays a crucial role in the statistical machinery.

We stress, however, that these predictions are subject to variability, which depends on the properties of the galaxy catalog adopted in these studies. A promising candidate will be the database created from the observations of the Vera Rubin Observatory. It is predicted to contain 10^{10} galaxies up to a redshift $z \leq 6$, covering 20,000 square degrees. For about 4×10^9 galaxies, six-band photometric measurements and shape measurements will be available, making the estimation of photometric redshifts much more precise [1207]. Early estimations show that a faint sample of all types of galaxies having photometric information in 6 bands would have a statistical photo- z error of $\Delta z \sim 0.05(1+z)$ per galaxy [1208]. Even assuming improved photometric redshift estimating techniques, the errors from these measurements would be an important source of uncertainty for the dark siren results. As demonstrated in [1147], the accuracy in redshift determination plays a crucial role in the estimate of cosmological and astrophysical parameters, since already at O5 sensitivities the accuracy on the determination of H_0 can improve down to the percent level when a spectroscopic catalog is considered (see figure 46, right panel). Hence, a spectroscopic survey would be crucial to maximising the potential of this method.

The galaxy database from the Vera Rubin Observatory, therefore, should be complemented with other galaxy catalogues to provide spectroscopic full-sky coverage as deeply as possible. From this point of view, several ongoing and planned galaxy surveys can be exploited. Currently, the largest spectroscopic galaxy surveys that are taking data are the Dark Energy Spectroscopic Instrument survey (DESI, from the ground, [1209]) and the ESA mission Euclid (from space, [1210]). DESI is planned to observe order of 10^7 galaxies at $0 < z < 3.5$ over ~ 14000 square degrees, at a depth of 24, 23.4, and 23 for AB magnitudes g , r , and z , respectively; on the other hand, covering ~ 14000 square degrees, Euclid will provide 1.5 billions photometric redshifts in the range $0 < z < 2$ and order of 30 millions spectroscopic redshifts at $0.84 < z < 1.88$, down to a Euclid H_E magnitude of 24. In the meanwhile, future spectroscopic surveys are already being planned and, pushing significantly deeper in redshift and completeness, they could constitute a fundamental resource to exploit the dark sirens provided by ET; in this respect, the most promising ones are the Wide-Field Spectroscopic Telescope (WST [1211]) and Spec-S5²² facilities.

Besides utilising existing data, another possibility is performing dedicated galaxy surveys for the best localised GW events without a counterpart to maximise their impact on the dark siren method. In parallel to that, dedicated spectroscopic follow-up surveys could be conducted for the best-localized events.

Formally, dark sirens analyses are conducted with a hierarchical Bayesian approach [1212–1214], which allows the propagation of uncertainties on the individual events to the population properties. Crucially, when using the population properties to infer cosmology, one must take into account the fact that those are actually unknown, but determine the selection bias of the experiment, which needs to be carefully deconvolved to obtain unbiased estimates of the cosmological parameters. This is usually done with large injections campaigns to determine the sensitivity of the instrument to the targeted population [1215–1217]. The scaling of this technique to ET needs to be assessed, both in terms of accuracy and computational cost. Furthermore, the likelihood evaluation, involving the computation of multi-dimensional Monte Carlo integrals at each iteration to marginalize over the individual source properties and/or the construction of redshift priors from the interpolation of large galaxy catalogs, is computationally demanding. Alternative approaches such as simulation-based [1218] and/or machine-learning leveraged pipelines [1219] and/or GPU-accelerated codes [1220], might be beneficial for tackling these challenges.

We can identify several milestones that ET can bring in this context, regardless of the technical details. The construction of a redshift prior from galaxy catalogs benefits in particular from the accurate sky localization.²³ Figure 47 shows the localization capabilities of ET in the triangular and 2L configurations.²⁴ In particular, we show the 90% sky localization area as a function of redshift, while the color refers to the number of galaxies contained in the localization volume, assuming a constant galaxy density of 10^{-2} galaxies/Mpc³. In particular, ET has the potential of localizing $\mathcal{O}(10^3)$ sources per year to better than 10 deg^2 [16]. Remarkably, figure 47 also shows that ET alone could allow pinpointing the host galaxy of

²²See <https://www.spec-s5.org/pubs/>.

²³Similar arguments hold for the cross-correlation of resolved sources with other tracers of the LSS, see section 3.4.1, as well as for the identification of a direct counterpart, see section 3.3.1.1.

²⁴We refer to ref. [16] for the details on the catalogs, software, and detector shape and orientations used.

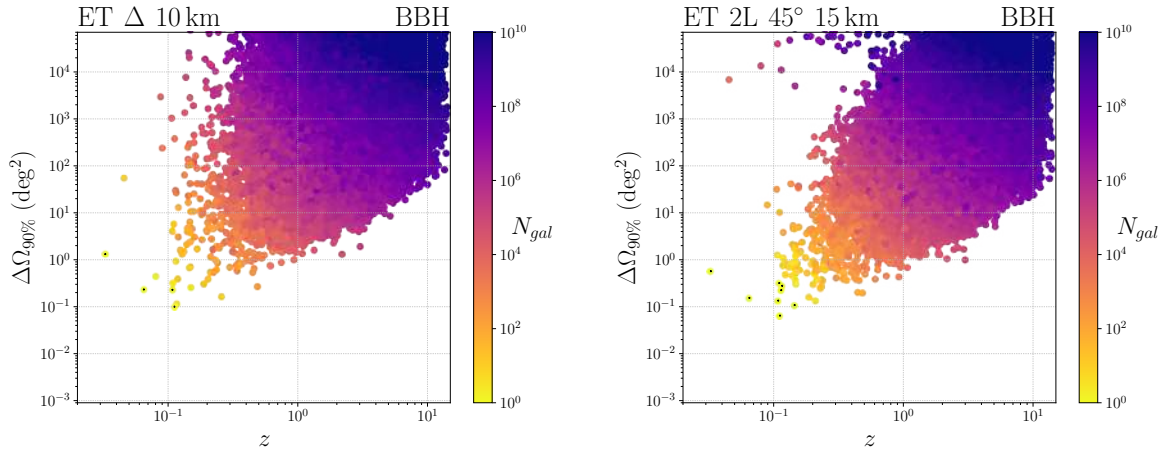


Figure 47. Localization capabilities for BBHs of ET in its triangular (left panel) and 2L-15km-45° (right panel) configurations. The color scale denotes the number of galaxies expected in the 90% localization volume. Events marked with black dots are localized to one galaxy only.

the merger at a rate of a few per year without the need of a direct EM counterpart. Events with a single host galaxy are further marked with a black dot in figure 47. This scenario alone would lead to a determination of the Hubble constant, and is a unique possibility of ET. Figure 48 shows the number of galaxies enclosed in the GW localization volume of ET golden BBH events (i.e., with $\Delta\Omega < 2 \text{ deg}^2$) in the triangular and 2L configurations using the catalog of massive MICEcat galaxies, as in [1147]. In particular, ET has the potential of localizing $\mathcal{O}(10^2)$ sources per year to better than 2 deg^2 [16]. We note that, besides cosmology, this could bring invaluable information about the relation between galaxy properties and GW emission [1203]. Indeed, assigning a weight to galaxies according to their properties such as mass or star formation rate can help increasing the accuracy on H_0 when using also GW sources with a large number of galaxies in their localization volume [9, 1143]. However, the dependence on the galaxy properties will have to be correctly modeled and marginalized over, in order to avoid systematics [1203].

Besides subsamples of “golden” events, as already remarked, it is important to stress that ET will detect almost the entire population of BBHs below the star formation peak, and more than 90% of BBHs of stellar origin across all redshifts [16, 1167]. In this large-statistics regime, information from sources with poorer sky localization will still contribute to reconstruct precisely the source-frame mass distribution, hence to determine cosmology [1157]. In this context, the possible systematics related to the correct modeling of source-frame features [1200], and their correlation to redshift evolution [1221], will need to be addressed. However, we stress that the presence of features in the mass spectrum can in principle be inferred directly from data, without the need of a specific modeling, using non-parametric approaches [1221–1223]. This will allow a robust measurement of the cosmic expansion history.

Love Sirens. Neutron stars taking part in compact binary coalescences, whether as BNS or NSBH systems, undergo tidal deformation during the later stages of the inspiral process. The tidal deformability parameter (Λ) is related to the compactness ($\mathcal{C}$) of the neutron star

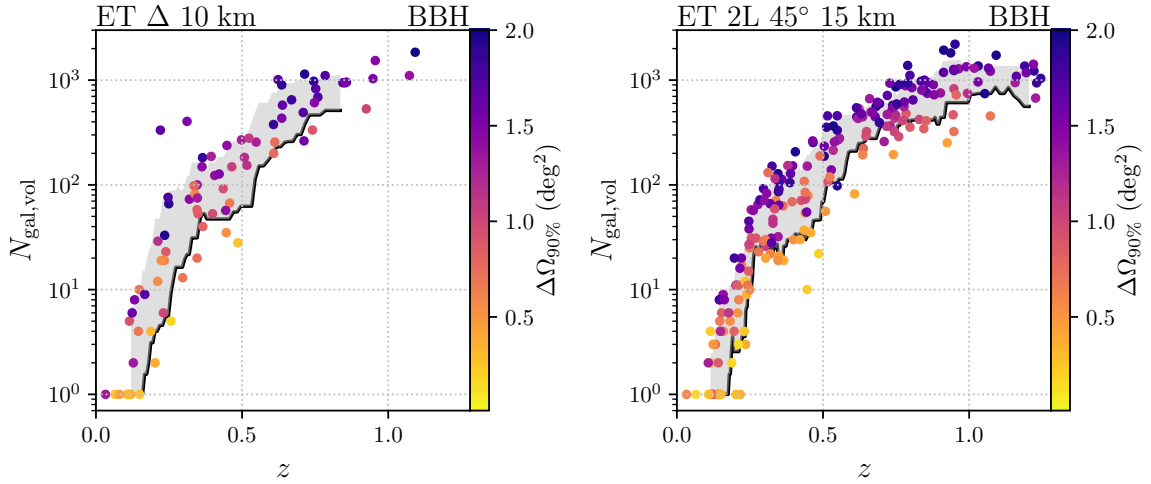


Figure 48. Number of galaxies inside the localization volume of ET golden BBH events ($\Delta\Omega < 2 \text{ deg}^2$) for the triangular (left) and 2L (right) configuration. The color bar represents the 90% sky localization area. The black line and shaded region indicate the 16th and 84th percentiles, respectively.

via the second tidal Love number (k_2) by,

$$\Lambda = \frac{2}{3} k_2 \mathcal{C}^{-5}, \quad (3.70)$$

where $\mathcal{C} = m/R$, where R is the radius of the NS and m its source-frame mass [1224]. The parameters k_2 and $\mathcal{C}$ depends on the equation of state governing the NS (see section 7 for details). These deformations leave an imprint on the GW amplitude and phase; in particular, the dominant effect of tidal deformation appears in the phase at the fifth post-Newtonian order, characterized by a combination of the individual tidal deformability parameters (note that $\Lambda = 0$ for BHs) and the mass ratio of the binary [1225]. This combination, called the reduced tidal deformability $\tilde{\Lambda}$ (see eq. (7.5) or eq. (10.18) for the explicit expression), is the best-measured tidal parameter using GW. The next-to-leading order contribution to the GW phase is characterized by another linear combination of individual Λ s and the mass ratio, denoted by $\delta\tilde{\Lambda}$, see eq. (10.18). The contribution of $\delta\tilde{\Lambda}$ to the phase is subdominant compared to that of $\tilde{\Lambda}$, even at the sixth post-Newtonian order, which is why it is not expected to be constrained very well with GW observations. Furthermore, if the neutron stars are spinning, the individual Λ s also contribute to the phase from spin-induced quadrupolar deformations which appear at the 2.5 post-Newtonian order and beyond [1226–1229]. Given these effects, one can constrain the individual tidal deformability of the two compact objects using GW detections, as we detail in section 7.

With an established NS equation of state, inferred Λ can be converted to constraints on source-frame mass using eq. (3.70). As GW observations also provide precise constraints on detector-frame mass, the measured detector-frame mass can be compared to the inferred source-frame mass, giving constraints on the redshift associated with the source [1140]. Together with the measurement of the luminosity distance d_L obtained from the GW signal, one can then obtain measurements of the cosmological parameters. Furthermore, exploiting

the post-merger signals of BNS mergers can enable the resolution of redshift-mass degeneracy by exploiting spectral features dependent on the source-frame masses of the binary [1230]. This method, which leverages the constraints on tidal deformability parameters, knowledge of the neutron star equation of state, and, possibly, the spectral features from neutron star post-merger, to break the mass-redshift degeneracy and obtain a redshift estimate, is referred to as the Love siren approach.

If using only the inspiral part of the signal, the redshift constraints obtained using the Love siren method depend on the precision with which $\tilde{\Lambda}$ and the detector-frame masses are inferred. The sensitivity improvement in ET at both low and high frequencies greatly improves these two measurements. Specifically, for $\mathcal{O}(100)$ BNS events detected with the triangular ET configuration every year, the fractional error in $\tilde{\Lambda}$ is expected to be less than 10% and that in chirp mass is expected to be less than $\mathcal{O}(10^{-5})$ [1167]. With the 2L-45° geometry, this number can increase to $\mathcal{O}(10^3)$ every year [1231]. With the triangular ET geometry, the redshift can be measured with a precision of $\mathcal{O}(10\%)$ for systems up to $z \sim 1$ [1140, 1232, 1233]. With the post-merger spectra, this can be achieved for systems up to $z \sim 0.04$ [1230]. In fact, the post-merger signal for a BNS system can be detected with an SNR of 10 for systems up to ~ 90 Mpc with ET alone, resulting in $\mathcal{O}(1\%)$ measurement of the post-merger peak frequency f_2 [1234]. Together with CE, the redshift can be resolved to $\sim 1\%$ for $\mathcal{O}(10^4)$ yearly detections, which translates to 0.1% error in H_0 and 0.61% error in Ω_m in a year of observation [1235] (see also [1236, 1237]).

The Love siren method represents an approach to cosmological parameter estimation that only relies on the GW signal, requiring a robust understanding of the neutron stars equation of state, expected to be significantly enhanced by forthcoming observatories [107, 1231, 1238, 1239]. An equation of state agnostic approach can also be used, that jointly estimates the equation of state and the cosmological parameters [1240]. However, caution is warranted in post-merger spectrum-based forecasts, which require improved modeling of gravitational waves from hypermassive neutron star remnants.

3.3.1.3 Cosmic dipole tension. The Earth’s motion relative to the Hubble flow generates a dipole anisotropy in both the temperature of the CMB [590, 591, 1241] and in the angular distribution of electromagnetic sources observed at cosmological distances [1242–1244]. This type of angular anisotropy is typically called the “cosmic dipole”. There is currently a $\sim 5\sigma$ tension between the estimation of the cosmic dipole from the early Universe (CMB) and the one inferred from the angular distribution of active galactic nuclei (AGN) [591, 1242]. It has been argued that this tension might be alleviated by modeling the redshift evolution of radio sources [1245], however, nowadays this tension is still open for debate (see also the discussion in [1246] and the references therein).

GW sources observed with 3G detectors offer a new avenue to solve this cosmic dipole tension. The cosmic dipole could be estimated from the stochastic GW backgrounds generated by cosmological and astrophysical sources (see section 3.1.1 for more details), but can also be estimated from the angular distribution of GW sources [1247, 1248]. ET embedded in a detector network with CE will provide $\sim 3 \cdot 10^4$ BBH sources per year with a 90% localization area lower than 10 deg^2 . The angular distribution of these well-localized sources can be used to detect and estimate the cosmic dipole.

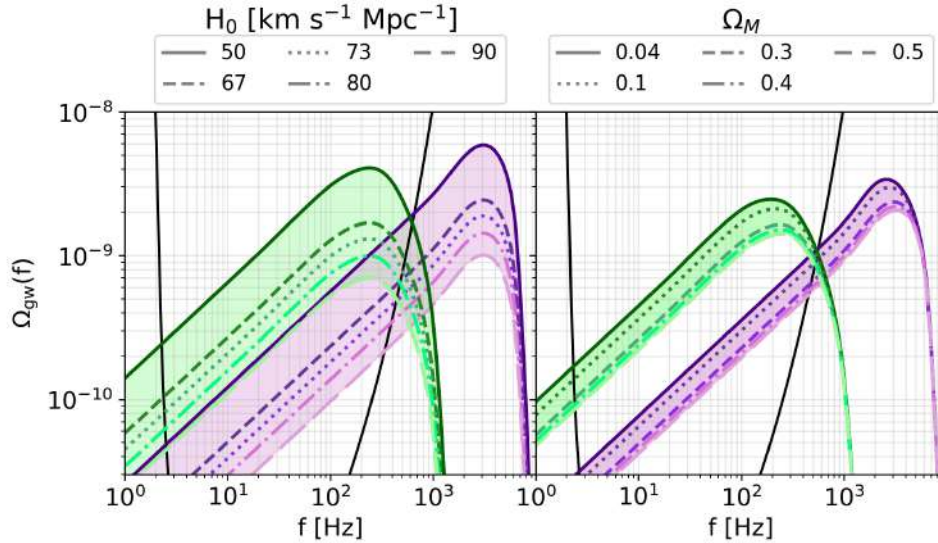


Figure 49. Effect of varying the value of H_0 and Ω_M on the energy density of the stochastic background from BBHs (green) and BNSs (purple). The astrophysics is fixed to the models described in [551, 1249]. The black curve is the PLS of ET for the triangular 10 km configuration [16].

By considering the angular number density of GW sources, with just 2–3 years of observations, 3G detectors have a 75% probability of detecting the cosmic dipole if its value is consistent with the one inferred from AGN [1247]. The probability of detecting the dipole increases to 100% if we consider 10 years of joint observations. Therefore, with just few years of observations, there is the concrete possibility of ruling out the AGN value of the cosmic dipole and solving this tension. The possibility of detecting and measuring the cosmic dipole are enhanced if also binary neutron star (BNSs) sources are considered. For this class of sources, it is also possible to inspect the presence of a cosmic dipole by studying the angular dependence of the detector-frame mass spectrum of BNSs, which is also expected to be modified by the cosmic dipole. Including BNSs and the angular distribution of the detector-frame mass spectrum, it will be possible to enhance the probability of detecting the cosmic dipole by 30–50% [1248] for a given observing time, thus making the detection of an AGN cosmic dipole in just 2–3 years of observation almost certain.

Furthermore, as discussed in detail in section 3.1.2.2, in [555] it has been shown that a detection of the dipole of the AGWB at different frequencies with ET and CE would allow to measure the dipole of the AGWB emitted by BBH and to distinguish between the kinematic, intrinsic and shot noise contributions, see figure 21 for more details. The component separation of the dipole of the AGWB which could be done by ET and CE is crucial to understand if the cosmic dipole tension is due to a relative velocity between the CMB and LSS frame (by comparing the CMB and AGWB kinetic dipoles) or to an anomalous intrinsic dipole of the CMB or LSS.

3.3.1.4 The astrophysical background as a cosmological probe. Another observable that can be used to constrain cosmological parameters with ET is the astrophysical GW background (AGWB) produced by coalescing binaries. The frequency spectrum of the

AGWB is determined by the complex interplay of various astrophysical factors, as discussed in section 4, but also by the underlying cosmology. The region of the spectrum which contains most of the information is the high-frequency peak, whose shape and position are very sensitive to all the various astrophysical and cosmological factors at play. For instance, in figure 49 we show the effect of varying the value of the Hubble parameter and the matter density parameter on the peak of the AGWB produced by BBHs and BNSs. With the sensitivity of ET in the 1–100 Hz range, the frequency spectrum of the astrophysical GWB will be measured precisely. This, in turn, will open the possibility for a Bayesian reconstruction of both astrophysical and cosmological parameters, in line with the methodology outlined in [1250] for the specific case of the GWB from BNSs. This approach complements existing methods such as GW sirens, discussed in the previous sections, which constitute the main strategy to constrain cosmological parameters with GWs. By employing these techniques concurrently, it becomes feasible to mitigate degeneracies and potentially refine parameter constraints further, particularly if posterior contours are orthogonal.

3.3.2 Modified GW propagation

3.3.2.1 Observables for the dark-energy sector. In Λ CDM, on sufficiently large scales where cosmological perturbation theory applies, one performs a separation between a homogeneous Friedmann-Lemaître-Robertson-Walker (FLRW) background, and scalar, vector and tensor perturbations over it. Whenever gravity is modified on cosmological scales, both the background evolution and scalar and tensor perturbations are modified (while, in Λ CDM and in most modified gravity models, vector perturbations usually only have decaying modes and are irrelevant). At the level of the background evolution, the effect of modified gravity is encoded in a dark energy (DE) density $\rho_{\text{DE}}(z)$ and a DE pressure $p_{\text{DE}}(z)$. One can then introduce the DE equation of state $w_{\text{DE}}(z)$, defined by $p_{\text{DE}}(z) = w_{\text{DE}}(z)\rho_{\text{DE}}(z)$; Λ CDM is recovered for $w_{\text{DE}}(z) = -1$. Using the conservation of the DE energy-momentum tensor, one finds that the DE density is given as a function of redshift by

$$\rho_{\text{DE}}(z) = \rho_0 \Omega_{\text{DE}} \exp \left\{ 3 \int_0^z \frac{d\tilde{z}}{1+\tilde{z}} [1 + w_{\text{DE}}(\tilde{z})] \right\}, \quad (3.71)$$

where $\Omega_{\text{DE}} = \rho_{\text{DE}}(0)/\rho_0$ is the DE density fraction and $\rho_0 = 3H_0^2/(8\pi G)$ is the critical density today. At the background level, the properties of DE are therefore encoded just in one function $w_{\text{DE}}(z)$. The corresponding expression for the luminosity distance is

$$d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{d\tilde{z}}{\sqrt{\Omega_{\text{M}}(1+\tilde{z})^3 + \Omega_{\text{R}}(1+\tilde{z})^4 + \rho_{\text{DE}}(\tilde{z})/\rho_0}}, \quad (3.72)$$

where Ω_{M} is the present matter fraction and Ω_{R} that of radiation (which we have written for completeness, but is completely negligible at the redshifts where astrophysical compact binaries merge; we also write for simplicity our equations for a flat cosmology, so $\Omega_{\text{M}} + \Omega_{\text{R}} + \Omega_{\text{DE}} = 1$). When $w_{\text{DE}} = -1$, ρ_{DE} in eq. (3.71) becomes a constant, that we denote ρ_{Λ} , and eq. (3.72) reduces to the Λ CDM expression

$$d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{d\tilde{z}}{\sqrt{\Omega_{\text{M}}(1+\tilde{z})^3 + \Omega_{\text{R}}(1+\tilde{z})^4 + \rho_{\Lambda}}}, \quad (3.73)$$

where $\Omega_\Lambda = \rho_\Lambda/\rho_0$. At low redshift, both eq. (3.72) and eq. (3.73) reduce to Hubble’s law $d_L(z) \simeq (c/H_0)z$, so standard sirens at low redshift allows us to measure H_0 . At the redshifts accessible to ET, however, we can also test the DE equation of state, and any deviation from the Λ CDM value $w_{\text{DE}} = -1$ would provide evidence for a dynamical dark energy. In general, however, it is difficult to extract from the data a full function of redshift, such as $w_{\text{DE}}(z)$, and a parametrization in terms of a small number of parameters is useful. For the DE equation of state a standard choice is the (w_0, w_a) parametrization [1251, 1252],

$$w_{\text{DE}}(z) = w_0 + \frac{z}{1+z} w_a. \quad (3.74)$$

The evolution of the cosmological background can be tested both with standard electromagnetic cosmological probes, such as CMB, Supernovae, Baryon Acoustic Oscillations (BAO) or structure formation, and by GW observations, combining the GW measurement of the luminosity distance of coalescing binaries with information on the redshift, obtained either with an electromagnetic counterpart, if it is observed (“standard sirens”) or with statistical methods (“dark sirens”), as discussed in section 3.3.1. A first observable for GW observations, in the context of dark energy and modified gravity, is therefore the DE equation of state $w_{\text{DE}}(z)$ or, more simply, the two parameters (w_0, w_a) . Modifications of the cosmological model in the sector of scalar perturbations are less directly relevant for GW experiments, and rather manifest themselves in cosmological structure formation; these modifications are among the targets of current galaxy surveys. However, for GW observation, there are further quantities that are accessible, to test modified gravity: GW observation can in fact access modifications in the tensor perturbation sector (i.e., GWs propagating over a FLRW background). In GR the equation that governs the propagation of tensor perturbations over FLRW is given by

$$\tilde{h}_A'' + 2\mathcal{H}\tilde{h}_A' + c^2 k^2 \tilde{h}_A = 0, \quad (3.75)$$

where $\tilde{h}_A(\eta, \mathbf{k})$ is the Fourier-transformed GW amplitude, $A = +, \times$ labels the two GW polarizations, the prime denotes the derivative with respect to cosmic time η [defined by $d\eta = dt/a(t)$], $a(\eta)$ is the scale factor, and $\mathcal{H} = a'/a$. In modified gravity theories, this propagation equation is modified, as has been found on many explicit modified gravity models [66, 1253–1262]. A modification of the coefficient of the $k^2 \tilde{h}_A$ term induces a speed of GWs, c_{gw} , different from that of light. After the observation of GW170817, this is now excluded at a level $|c_{\text{gw}} - c|/c < \mathcal{O}(10^{-15})$ [3] (at least for scale-independent modifications, see [1263]). However, the modified gravity models that pass this constraint still, in general, induce a change in the above equation, that becomes

$$\tilde{h}_A'' + 2\mathcal{H}[1 - \delta(\eta)]\tilde{h}_A' + c^2 k^2 \tilde{h}_A = 0, \quad (3.76)$$

for some function $\delta(\eta)$ that encodes the modifications from GR. As a result, the amplitude of a GW propagating across cosmological distances, from the source to the observer, is modified, because of this different friction term; one can then show that the amplitude of the GW from a coalescing binaries no longer allows us to obtain the standard luminosity distance $d_L(z)$ of the source [that, in this context, we will denote by $d_L^{\text{em}}(z)$, since this is the quantity that would be measured, for instance, using the electromagnetic signal from

a counterpart]. Rather, the quantity extracted from GW observation is a “GW luminosity distance” $d_L^{\text{gw}}(z)$ [1257], related to $d_L^{\text{em}}(z)$ by [1257, 1259]

$$d_L^{\text{gw}}(z) = d_L^{\text{em}}(z) \exp \left\{ - \int_0^z \frac{dz'}{1+z'} \delta(z') \right\}, \quad (3.77)$$

where the function δ that appears in eq. (3.76) has now been written as a function of redshift.

Of course, to perform a full inference on the function $\delta(z)$ from future GW data is difficult (although it can be performed, to some extent, with the technique of Gaussian process reconstruction [1170]) and, similarly to what is done for the DE equation of state, it is convenient to introduce a parametrization. A very convenient choice, proposed in [1259], takes the form

$$\frac{d_L^{\text{gw}}(z)}{d_L^{\text{em}}(z)} = \Xi_0 + \frac{1 - \Xi_0}{(1+z)^n}, \quad (3.78)$$

in terms of two parameters (Ξ_0, n) . This parametrization interpolates, with a power-law in the scale factor $a = 1/(1+z)$, between $d_L^{\text{gw}}/d_L^{\text{em}} = 1$ at $z = 0$ (i.e. when there is no propagation), and a constant value large redshifts; the latter limit is motivated by the fact that, in typical modified gravity models, the deviations from GR only appear in the recent cosmological epoch, so $\delta(z)$ goes to zero at large redshift, and the integral in eq. (3.77) saturates to a constant; GR corresponds to $\Xi_0 = 1$ (for any n). This simple parametrization has been shown to work remarkably well for practically all best-studied modified gravity models [1262] (except bigravity, where $d_L^{\text{gw}}/d_L^{\text{em}}$ displays some oscillations).

Another parametrization of modified GW propagation which has been used in the literature is given by

$$\alpha_M(z) \equiv -2\delta(z) = c_M \frac{\Omega_{\text{DE}}(z)}{\Omega_{\text{DE}}}, \quad (3.79)$$

where c_M is a constant and $\Omega_{\text{DE}}(z) = \rho_{\text{DE}}(z)/\rho_0$. This parametrization was introduced in [1264] to study the scalar sector of Horndeski theories and, in these theories, it turns out that the same function also describes the modification of the tensor sector. A drawback of the parametrization (3.79) is that it requires to postulate an expression for $\Omega_{\text{DE}}(z)$, i.e. for $\rho_{\text{DE}}(z)$, so it is less suitable for model-independent studies. In practice, when applied to modified GW propagation, this parametrization has been used assuming that the dark energy density is constant, $\rho_{\text{DE}}(z) = \rho_{\text{DE}}(0)$ [1265, 1266] (which, however, is not expected to be the case, since if the tensor sector of the theory is modified with respect to Λ CDM, also the background evolution will be different). In any case, as long as $\Omega_{\text{DE}}(z)$ is such that the integral in eq. (3.77) saturates to a finite value (which is also the case of a constant Ω_{DE}), this parametrization is still qualitatively of the same form as eq. (3.78), since the ratio $d_L^{\text{gw}}(z)/d_L^{\text{em}}(z)$ goes from unity at $z = 0$ to a constant Ξ_0 at large z . However, this parametrization is less general than the (Ξ_0, n) parametrization, since it only has one parameter, c_M , so it implicit assumes a relation between Ξ_0 and n (see appendix F of [1154] for detailed comparison). Other proposed parametrizations include

$$\Xi(z) = \sqrt{\frac{1 + \Omega_0}{1 + \Omega_0 a^\beta}}, \quad (3.80)$$

from [1267], which has the same behavior as that of eq. (3.78) when the deviations from GR are small ($|\Omega_0| \ll 1$), with the correspondence $\Xi_0 = 1 + \Omega_0/2$ and $n = \beta$; also

$$\alpha_M(z) = \alpha_{M0} a^\beta \quad (\beta > 0), \quad (3.81)$$

used in the context of the EFT of dark energy [591, 1268]; and the one of [1269].

It is interesting to note that, with electromagnetic observations, the background evolution and the scalar perturbations have been tested to a few percent level. For instance, combining many different cosmological datasets (and setting $w_a = 0$), one finds $w_0 = -1.031^{+0.030}_{-0.027}$ [1270], consistent with Λ CDM at the 3% level (although the recent DESI results [1271], even if consistent with a flat Λ CDM model, when combined with other datasets prefer an evolving dark energy equation of state with phantom crossing below $w = -1$ at redshift $z \simeq 0.4$), while for the parameters Σ_0 and μ_0 that parametrize deviations in the scalar perturbation sector (with $\mu_0 = \Sigma_0 = 0$ in GR) one finds $\mu_0 = -0.04 \pm 0.25$ and $\Sigma_0 = -0.024 \pm 0.054$ [1272], so the latter corresponds to a test of GR and Λ CDM at the 5% level. Naively one would then imagine that, if a modified gravity model complies with existing bounds in the background evolution and in the scalar perturbation sector, even in the tensor perturbations its deviation from GR will not exceed a few percent. Indeed, this is true for some important classes of models, like the $f(R)$ or several scalar-tensor theories, where the GW friction is the time variation of the effective Planck mass,

$$\alpha_M(z) = \frac{d \log M_*^2}{d \log a}. \quad (3.82)$$

There, α_M is also sourcing gravitational slip $\eta \equiv \Phi/\Psi = 2\Sigma/\mu - 1$ [1253, 1264, 1267], therefore linking tests of gravity based on standard sirens with those from large-scale structure. However, this link is not generally true; as an explicit example, the model originally proposed in [64] complies with all observational limits in the background evolution and in the scalar sector but, in the tensor sector, predicts a value of Ξ_0 that (depending on a free parameter related to initial conditions) can be as large as 1.8 [66], corresponding to a 80% deviation from GR. Figure 50, from ref. [66], shows the prediction for $\delta(z)$ (left panel) and $d_L^{\text{gw}}/d_L^{\text{em}}$ in this non-local model, for different values of a free parameter of the theory (see [66] for details).

This shows that the window of cosmological tensor perturbations, that has been opened for the first time by the LVC and LVK observations, and that will be deeply explored by ET, is extremely interesting and could reserve us remarkable surprises.

3.3.2.2 Current limits on Ξ_0 . Using the various methods for bright and for dark sirens discussed previously in this chapter, several limits on modified GW propagation have already been obtained from the current dataset of GW detections.

A first limit was obtained in [1259] using GW170817 as a standard siren with counterpart (see also [1265]). In this case the electromagnetic luminosity distance of the host galaxy can be obtained from surface brightness fluctuations. Because of the very small redshift of GW170817, what is obtained is really a limit on $\delta(z=0)$, independently of the parametrization used, and the result from [1259] is $\delta(0) = -7.8^{+9.7}_{-18.4}$. In terms of Ξ_0 , this corresponds to a rather broad limit $\Xi_0 \lesssim \mathcal{O}(10)$. This limit is not stringent because GW170817 has a very small redshift, $z \simeq 0.01$, and the effect of modified GW propagation disappears as $z \rightarrow 0$.

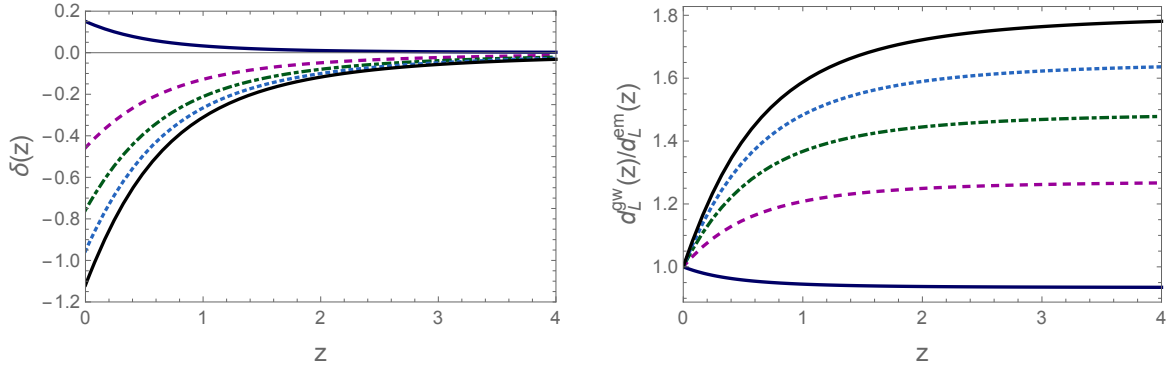


Figure 50. The functions $\delta(z)$ (left panel) and $d_L^{\text{gw}}(z)/d_L^{\text{em}}(z)$ (right panel), for the non-local gravity model proposed in [64], for different values of a free parameter of the theory. Reproduced from [66], © 2020 IOP Publishing Ltd and Sissa Medialab. All rights reserved.

A stronger bound was obtained in [1143] using BBH dark sirens from the O1, O2 and O3a LIGO/Virgo run, and performing a correlation with the GLADE [1186] galaxy catalog. This gives (at 68% c.l.)

$$\Xi_0 = 2.1_{-1.2}^{+3.2}. \quad (3.83)$$

A more stringent bound can be obtained if one accepts the tentative identification of the flare ZTF19abnhr as the electromagnetic counterpart of the BBH coalescence GW190521, as proposed in [1273].²⁵ Then, the analysis in [1143] (keeping fixed H_0 to the Planck value) gives

$$\Xi_0 = 1.8_{-0.6}^{+0.9}, \quad (3.84)$$

(68% C.L.). A similar result is obtained in [1275], again assuming the identification of ZTF19abnhr with GW190521, and combining this with GW170817. Using again a narrow prior on H_0 the most stringent limit obtained in [1275] (for different choices of the waveform) is $\Xi_0 < 2.9$.

Another approach is to constrain modified GW propagation using the BBH mass distribution, following the strategy proposed in [1153]. Using the BBH mass function and the GWTC-3 catalog, ref. [1154] finds

$$\Xi_0 = 1.2_{-0.7}^{+0.7}, \quad (3.85)$$

with a flat prior on Ξ_0 , and $\Xi_0 = 1.0_{-0.8}^{+0.4}$ with a prior uniform in $\log \Xi_0$, while using the GWTC-2 catalog of GW events and the parametrization (3.79) with a constant ρ_{DE} , ref. [1266], finds $c_M = -3.2_{-2.0}^{+3.4}$ (see appendix G of [1154] for a discussion of the technical differences between the analyses of refs. [1266] and [1154]).

²⁵In [1274] it was observed that the volume localization of GW190521 is not statistically sufficient for a confident identification of ZTF19abnhr as the electromagnetic counterpart. However, this assumes no modified GW propagation. Introducing the possibility of modified GW propagation has precisely the effect of restoring the consistency between the volume localization of GW190521 and the flare ZTF19abnhr.

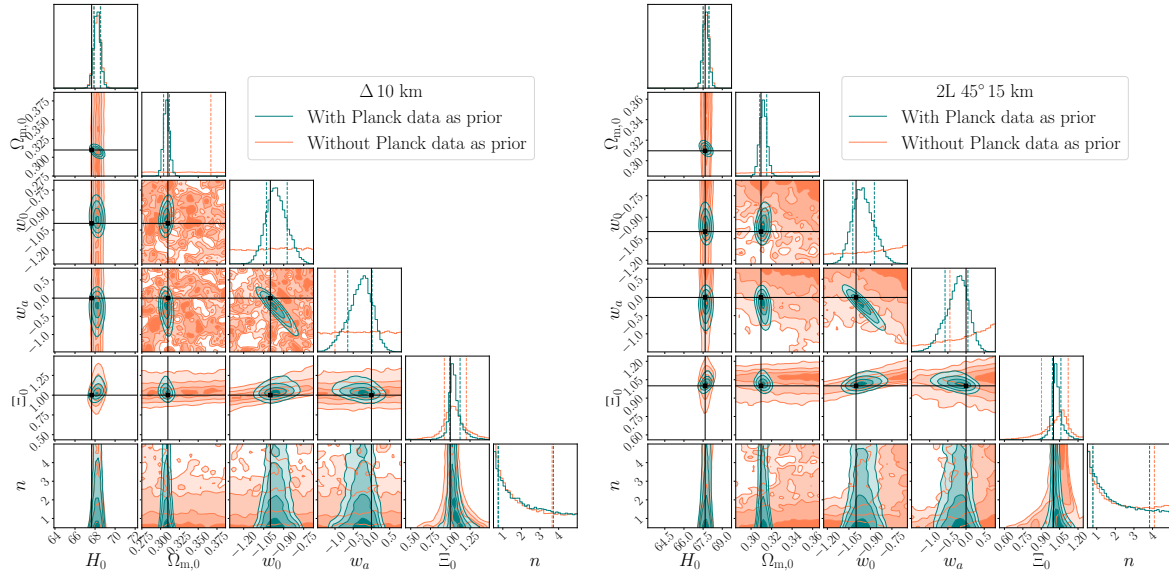


Figure 51. Results of the joint inference on the cosmological parameters H_0 , $\Omega_{m,0}$, w_0 , w_a , Ξ_0 and n employing GW+KN events detected in one year of observations by the 10 km triangular (left panel) and the 2L-15km-45° (right panel) ET configurations, together with the Vera Rubin Observatory. Vertical dashed lines represent the 68% CI of each distribution, while the black solid lines label the fiducial values.

3.3.2.3 Forecasts for ET. Constraints on deviations from General Relativity and Λ CDM, as those parametrized by the dark energy equation of state and by modified GW propagation, are among the primary targets of the ET science case. Information on these parameters can come from both bright sirens and dark sirens, as we discuss next.

Constraints from bright sirens. Forecasts on the accuracy of the cosmological parameter H_0, Ω_M that can be obtained from bright sirens in the framework of Λ CDM have been presented in section 3.3.1.1. Here, using the same methodology, we enlarge the analysis to modified gravity, extending the analysis performed in [16] — where the inference was carried out on two cosmological parameters at a time — to a more comprehensive investigation. Specifically, this study is conducted on the same dataset of bright sirens (see tables 6 and 10 of [16]), but aims to extract a measurement for all the 6 considered Λ CDM+DE+modified GW propagation parameters H_0 , $\Omega_{m,0}$, w_0 , w_a , Ξ_0 and n simultaneously. While this choice inevitably leads to broader constraints, we stress that this wider approach is necessary if no other complementary survey’s results are readily available to fix one or more parameters in the inference. Furthermore, it represents a standalone benchmark for the additional science that GW astronomy can deliver regardless of the available previous measurements.

We report these results in figures 51, 52 and in table 4, where we also show the impact of using the Planck 2018 [591] posterior distributions as a prior. As expected, extending the analysis to all 6 cosmological parameters worsen the precision on Ξ_0 by a factor of order ~ 5 with respect to the results reported in [16], while n remains undetermined in both cases due to the strong degeneracy with Ξ_0 . On the other hand, however, using the early Universe from Planck as priors strongly increase the precision on Ξ_0 , which ranges from $\sim 12\%$ in

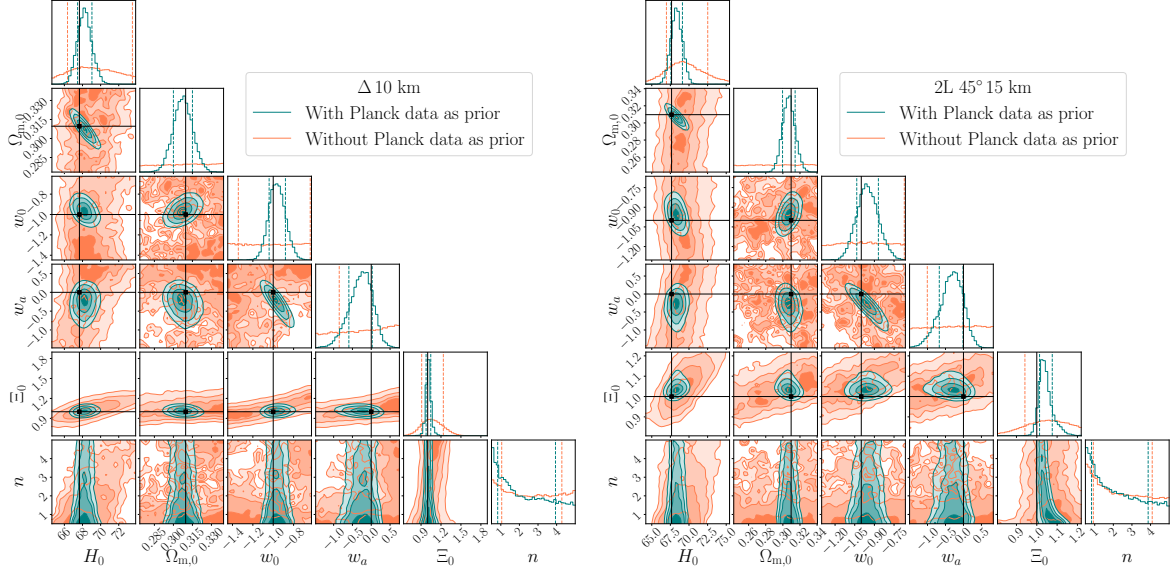


Figure 52. Results of the joint inference on the cosmological parameters H_0 , $\Omega_{m,0}$, w_0 , w_a , Ξ_0 and n employing GW+GRB events detected in 5 years of observations by the 10 km triangular (left panel) and the 2L-15km-45° (right panel) ET configurations, together with THESEUS. Vertical dashed lines represent the 68% CI of each distribution, while the black solid lines label the fiducial values.

the worst scenario, to $\sim 6\%$ in the best case. The error on H_0 , on the other hand, increases only marginally compared to the case when we restrict to Λ CDM. For instance, as we found in section 3.3.1.1, for GW+KN (and 1 yr of observations) in Λ CDM, without using the Planck prior, the relative error on H_0 was 0.9% for the 10-km triangle and 0.6% for 2L-15km misaligned, and these numbers become, respectively, 1.0% and 0.7% when enlarging the parameter space to $\{H_0, \Omega_{m,0}, w_0, w_a, \Xi_0, n\}$. It is also interesting to observe that, as we saw in section 3.3.1.1, in Λ CDM the constraint on H_0 from GW+GRB are much less stringent than from GW+KN. However, we see that this is no longer the case for the Ξ_0 parameter. This is due to the fact that KN observation are restricted to smaller redshifts compared to GRB, while modified GW propagation is a cumulative effect that increases with the distance to the source, and therefore is better probed by high-redshift sources.²⁶

Further improvement might come when combining these observations with other distance measurements expected from future standard candles and large-scale structure surveys, in which case the precision on Ξ_0 could reach the $\sim 1\%$ level, while keeping model-independence [1173]. In the context of EFT of dark energy, when identifying Ξ as the effective Planck mass ratio of eq. (3.82), also LSS and CMB data will help constraining Ξ_0 when combined with bright sirens, since in this case the modification in the tensor perturbation sector are related to those in the scalar perturbation sector, and can therefore be constrained also by structure formation data. The price one has to pay is of course to be more model-dependent in order to describe the modifications of gravity in the scalar sector. In the analysis of [1267], it

²⁶We also stress again that, as in the discussion in section 3.3.1.1, in this study the relative uncertainty on the luminosity distance has been approximated as $2/\text{SNR}$, since the Fisher matrix approach breaks down for face on/face off sources. This approximation could be improved in future studies along the lines recently proposed in [1169] and discussed in section 10.2.3.

EM Source	ET configuration	Without Planck data priors		With Planck data priors	
		$\Delta\Xi_0/\Xi_0$	Δn	$\Delta\Xi_0/\Xi_0$	Δn
KN	Δ 10 km	0.27	> 1	0.12	> 1
	2L 45° 15 km	0.21	> 1	0.07	> 1
GRB	Δ 10 km	0.31	> 1	0.07	> 1
	2L 45° 15 km	0.27	> 1	0.06	> 1

Table 4. 68% CI relative uncertainties on the modified GW propagation parameters Ξ_0 and n obtained from multimessenger observations and adopting the two shown ET configurations.

was shown that the addition of 500 bright sirens with the triangular 10 km configuration of ET would improve current Planck constraints on Ξ_0 by $\sim 40\%$, for typical late time modifications of gravity where the same Ξ is generating gravitational slip. The further combination with galaxy clustering and weak lensing from an Euclid-like survey would allow reaching an error of 0.3% (more precisely, $\sigma(\Omega_0) = 0.6\%$ with the equivalent parametrization of eq. (3.80)).

Constraints from dark sirens. As we have seen in section 3.3.2.2, the dark siren method based on features on the mass function can provide stringent limits on modified GW propagation. Currently, the best limits are obtained from the BBH mass function, given that the current BNS sample is very small and limited to small redshifts (where the effect of modified GW propagation is suppressed). However, ET will detect a large number of BNS, of order $10^4 - 10^5$ per year, and up to large redshifts of order 2–3. This will allow to use this statistical method also on BNSs, which has the advantage that the BNS mass function is quite narrow, compared to the BBH mass function. The logic behind the possibility of bounding (or observing) modified GW propagation from the BNS mass function can be described as follows [1155]. From a GW detection of a BNS we obtain its GW luminosity distance, d_L^{gw} . If Nature is described by a modified gravity theory with $\Xi_0 \neq 1$, d_L^{gw} is different from the electromagnetic luminosity distance d_L^{em} ; therefore, the source redshift z_{GR} that would be inferred from the measured d_L^{gw} assuming (incorrectly) the validity of GR and ΛCDM , would differ from the true value z_{true} . The effect is shown in the left panel of figure 53, for some values of Ξ_0 consistent with the limits given in eqs. (3.83)–(3.85) at about the $(1-2)\sigma$ level.²⁷ We see that the effect can become very significant at large redshifts: for instance, if Nature were not described by GR but rather by a modified gravity theory with $\Xi_0 = 1.8$, a source whose true redshift is $z_{\text{true}} = 1$, would be assigned a wrong redshift $z_{\text{GR}} \simeq 1.45$, if the redshift is inferred assuming $\Xi_0 = 1$.

In turn, this would produce a systematic (and redshift-dependent) bias in the inference of the actual (“source-frame”) masses m_i ($i = 1, 2$) of the component neutron stars, since these are obtained from the measured “detector-frame” masses $m_{(\text{det}),i}$ through the relation $m_i = m_{(\text{det}),i}/(1+z)$. Getting z wrong would then result in a wrong estimate of m_i . More

²⁷In this plot, we use for illustration the same values of H_0 and Ω_M in the modified gravity theory and in GR, setting them to $H_0 = 67.9 \text{ km}/(\text{s Mpc})$ and $\Omega_M = 0.30$. In a full analysis, all cosmological parameters will have to be determined self-consistently in GR and in modified gravity, by fitting the prediction of each theory to the cosmological observations.

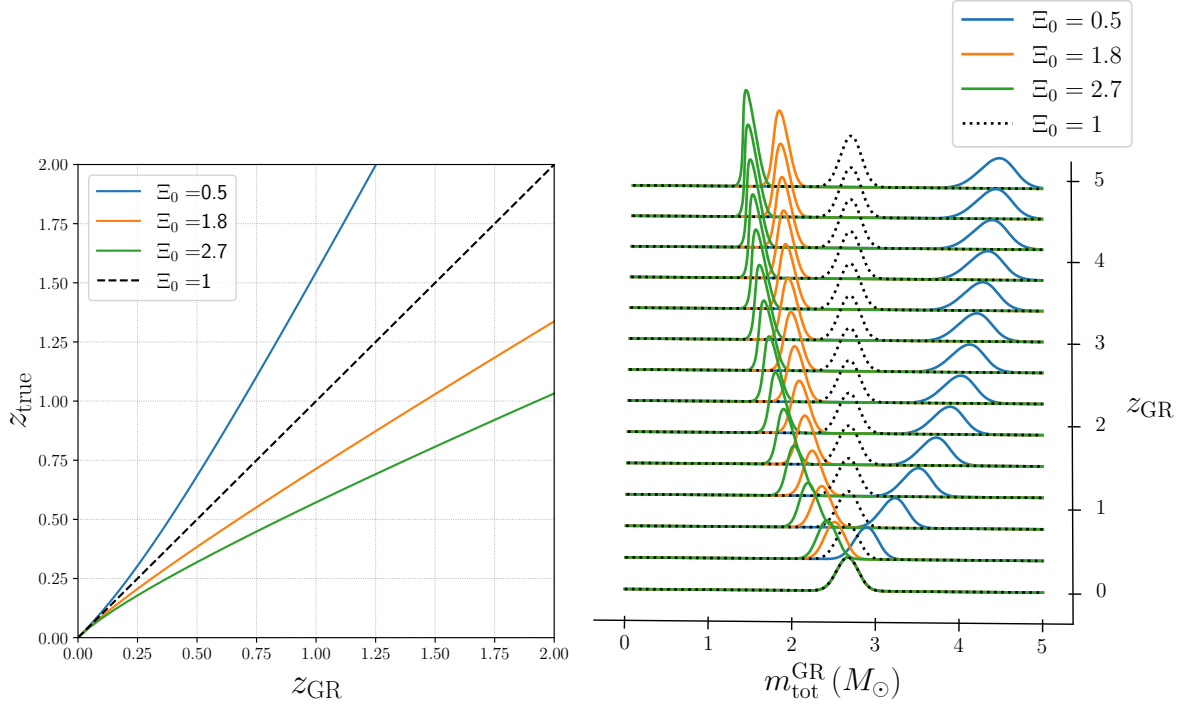


Figure 53. *Left:* the redshift z_{true} of a source, as a function of the value z_{GR} that would be incorrectly inferred using GR if Nature is described by a modified gravity theory with $\Xi_0 \neq 1$, for different values of Ξ_0 . *Right:* the effect on the distribution of the total mass of the binary from a ‘wrong’ reconstruction using GR, $m_{\text{tot}}^{\text{GR}}$, assuming that the true distribution of the source-frame total mass of the binary is a (redshift-independent) Gaussian, with mean $2.66M_{\odot}$ and standard deviation $0.13M_{\odot}$. Adapted from [1155]. Reproduced from [1155]. CC BY 4.0.

precisely, if Nature is described by a modified gravity theory with $\Xi_0 \neq 1$, the true values of the source-frame masses, $m_{\text{true},i}$ are obtained from the observed detector-frame masses by $m_{\text{true},i} = m_{(\text{det}),i}/(1 + z_{\text{true}})$, while the values incorrectly inferred assuming GR would be $m_{\text{GR},i} = m_{(\text{det}),i}/(1 + z_{\text{GR}})$, and therefore

$$m_{\text{true},i} = \left(\frac{1 + z_{\text{GR}}}{1 + z_{\text{true}}} \right) m_{\text{GR},i}. \quad (3.86)$$

For instance (assuming again, as an example, $\Xi_0 = 1.8$), for a NS with $m_{\text{true}} = 1.35M_{\odot}$ at $z_{\text{true}} = 1$, the mass incorrectly inferred from GR would be $m_{\text{GR}} \simeq 1.10M_{\odot}$, while for $z_{\text{true}} = 2$, in GR one would infer $m_{\text{GR}} \simeq 0.99M_{\odot}$. The effect is quite striking because it would bias, in the same way, all NS masses at a given z (so, in particular, both masses of a BNS, contrary to what could be expected from evolutionary paths leading to accretion of matter on one of the two neutron stars), shifting the whole distribution of NS masses in a characteristic redshift-dependent manner. This is shown (for the distribution of the total mass of the binary) in the right panel of figure 53.

Given the large number of BNS that ET will detect, this method can in principle reach a very good accuracy on Ξ_0 . The effect can be tested directly on the chirp mass of the system, which is very accurately measured, so in the end, with this technique, the main error on $d_L^{\text{gw}}/d_L^{\text{em}}$ from a single BNS is expected to be of order of the relative width of the BNS mass

function. With N detections at sufficiently large redshift, where the effect of modified GW propagation can be sizable, the error scale approximately as $1/\sqrt{N}$, so sub-percent errors on Ξ_0 could be obtained, as long as other competing sources of biases, such as lensing or a possible redshift evolution of the BNS mass function [1276], can be adequately accounted for.

3.3.3 GW lensing

According to GR, if a GW encounters a massive object on its travel path from source to observer, it can undergo gravitational lensing. Depending on the lens mass and the lens-source alignment, one can get different effects; for the lowest masses and more misaligned systems, one has microlensing, where the GW undergoes frequency-dependent modulations [101, 1277]. On the other hand, for more massive lenses and better-aligned systems, one is in the geometric optics regime, and the wave is split into multiple copies of the original system, with an overall magnification, and overall phase shift and a time delay [101, 1278]. If the system is less aligned and/or the lens lighter, the various images have a small time delay and overlap, leading to a single non-trivial signal in the detector, called millilensing [1279]. For larger masses, the images are individually resolvable, generally referred to as strong lensing [1278]. While rates for micro and millilensing are uncertain — due to their strong dependency on the nature of dark matter — strong lensing is expected to be already observable for BBH signals in current detectors [106, 1280]. Hundreds of such lensed signals [107] and tens of lensed BNSs should be observed in next-generation detectors every year [1281]. A fraction of the latter could also have an observable lensed EM counterpart, making their correlation easier to do [1282].

3.3.3.1 GW lensing as probe of cosmological parameters. In the geometric optics, according to GR, GW and light are bend the same way, which can be leveraged to study our Universe, with unique possibilities at high redshift. Indeed, if a GW coming from a BBH is lensed, then the EM emission from its host galaxy is also lensed. One can then match both to use their complementary information to probe various physical phenomena [1283, 1284]. Matching a lensed BBH and its host galaxy is also easier than for an unlensed signal because lensing offers a much more precise sky localization [103]. In addition, GW measurements lead to millisecond precision measurements of the arrival times, and one can use time delay cosmography to constrain cosmological parameters [1285], provided the EM observations complete the observations by giving access to the lens and source redshifts. Moreover, one can also have extra constraints via the reconstructed magnifications of the images. The total measure obtained for the Hubble constant is relatively accurate, even for a single observation [1283]. Additionally, in next-generation detectors, several observations can be stacked to improve the final constraints, similarly to what is already done nowadays for dark sirens [9]. When the BBH host galaxy cannot be identified, one can still develop a dark sirens-like approach for lensing, which accounts for a list of possible counterparts and the probability to miss the genuine system in the EM. While less precise than with a direct access to the counterpart in the EM, the list of candidates will be more restrained than for the usual dark siren method thanks to the selection done using the observed lens characteristics [1283]. Moreover, these events would also allow for measurements at higher redshifts than unlensed systems, giving extra insights on the early Universe.

Furthermore, lensed events without EM counterpart can be used to constraint cosmology based on their statistical properties, using the fact that the expected number of lensed events and the distribution of the time delay between lensed images depend on the cosmology. In particular, in ref. [1286] is shown how the expected distributions of lensed GW events can be used to constrain the cosmological parameters. Denoting by N the number of lensed BBHs events observed over an extended period of time T_{obs} (each one assumed to produce two copies of the observed GW signal), and by $\{\Delta t_i\}, i = 1, \dots, N$ their corresponding time delays, one can construct a probability distribution for the cosmological parameters $\vec{\Omega}$ as

$$p(\vec{\Omega}|N, \{\Delta t_i\}, T_{\text{obs}}) = \frac{p(\vec{\Omega})p(N|\vec{\Omega}, T_{\text{obs}})p(\{\Delta t_i\}|\vec{\Omega}, T_{\text{obs}})}{Z}, \quad (3.87)$$

where one uses the conditional independency between the time delays and the number of lensed BBH detections given the parameters and the observation time T_{obs} ; $p(\vec{\Omega})$ is the prior and Z is the normalization constant. By performing various simulations on a coarse grid of values for the cosmological parameters, it is then possible to construct the two probability functions. Then, one can evaluate those with a given observation time and find a probability distribution for the cosmological parameters. With this approach, using 10 years of observation and a BBH merger rate of $5 \times 10^5 \text{ yr}^{-1}$ with 3G detectors (made of CE and ET), one obtains $H_0 = 67.8 \pm 1.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_m = 0.3142 \pm 0.0056$ at $1\text{-}\sigma$ C.I. for the standard Λ CDM cosmology. Figure 54 shows the posterior distribution for the cosmological parameters found with this methods. The bounds obtained depend on the detection rate, with broader constraints for lower rates. Besides, the approach also works for other cosmological models, such as w CDM, where the same observational conditions lead to competitive bounds compared to other traditional approaches (see right plot in figure 54) [1286]. An interesting feature of these observations is that (a fraction of) such BBHs can come from high redshift, and, therefore, such studies could also probe cosmological parameters in a regime that is not explored by other methods.

As discussed in section 3.3.2, modified GW propagation can be a smoking gun for modifications of gravity at cosmological scales, and lensing can also be used to study it [1287, 1288]. Basically, if one observes a strongly lensed GW and its EM counterpart, and performs the lens reconstruction, one can access two different measures of the luminosity distance: one via GWs — by correcting the measured luminosity distance by the magnification, and one by the EM redshift measurements. If GR is the correct theory of gravity, the two values should match. However, in the presence of modified GW propagation, the two quantities will be different, and the difference can be used to constrain the parameter(s) linking the two quantities. This can be done for the Ξ_0 parametrization mentioned above [1287] but also for other theories such as large extra spatial dimensions or a time-varying Planck mass [1288]. This method can be used for lensed BNS, NSBH, and BBH events. The former have the advantage of offering a better sky location thanks to the use of Earth rotation in addition to the multiple images [1289], while the latter would lead to better constraints thanks to the fact that, for a given SNR, they can be detected to larger distance.

Another way to look into possible modifications in long-distance propagation is to test the speed of gravity by comparing the arrival time of photons and GWs [3]. For unlensed events, the intrinsic difference in emission time between photons and GWs is one of the main

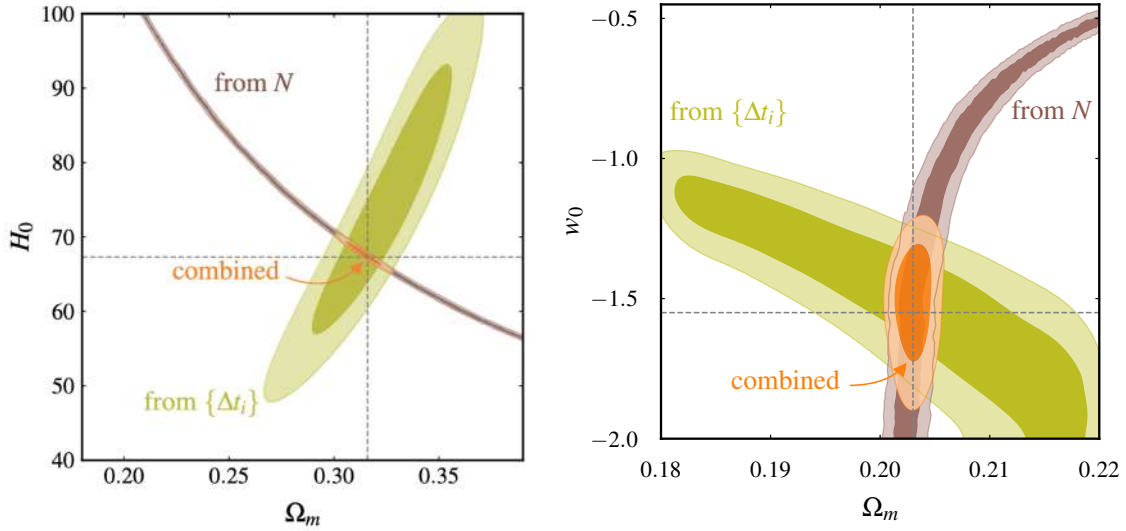


Figure 54. Constraints obtained from the statistical method relying on lensed BBH events to constrain the cosmological parameters presented in [1286]. *Left:* H_0 vs. Ω_m . *Right:* w_0 vs. Ω_m . These results are obtained for a ten year observation period, assuming that a generic network of 3G detectors will have a nominal BBH detection rate of $5 \times 10^5 \text{ yr}^{-1}$ upto a typical redshift of $z_{\text{max}} \sim 15$ (exact value dependent on the cosmology and population models), and that their time of arrivals will be measured precisely. The bounds obtained are competitive with other studies, and the BBH signals originate from further away, potentially carrying information from earlier periods in the Universe. Figures adapted from [1286]. Reprinted figure with permission from [1286], Copyright (2023) by the American Physical Society.

sources of uncertainty, and varies from one model to the other [3]. As mentioned above, if a GW is lensed, the light emitted by its source is also lensed. Therefore, we would now have multiple measures of the difference in arrival time between GW and EM. Moreover, by comparing the differences between EM and GW measured for the various images, one can cancel out several nuisance terms in the measure, and in particular the intrinsic difference in emission time [1290, 1291]. Therefore, one may expect a more precise measurement of the speed of gravity, depending on the source of EM emission. Also, this method suffers from different sources of uncertainties (related to the lens itself and not the time of emission), so it can be used to cross-check the measurements made with other observations.

3.3.3.2 GW lensing and dark matter. When a GW is lensed, the lensing effect encodes the mass and density profile of the lens. Therefore, it can be used to probe the lens itself. In particular, millilensing and microlensing originate from smaller-scale objects, which can potentially match some dark matter candidates such as dark matter halos or primordial black holes. A particularly interesting case is when one searches for micro- or millilensing effects on top of strong lensing; if a GW is deflected by a galaxy or a galaxy cluster, one image or more can interact with smaller objects in the macrolens. These can be stars or dark matter [1292–1294]. In particular, relying on different mass models, and looking how the mass is distributed in the lens, it is possible to probe various dark matter models [1295]. The same is true for millilensing, where objects are heavier than for microlensing, corresponding

to the expected mass regime of dark matter halos. However, the number of images, their magnifications and time delays inform us about the objects within the galaxy that lead to the extra effect [1279]. Knowing more about the scale helps constraining dark matter models since, from one model to the other, different scales are favored or prohibited [1296, 1297]. Alternatively, isolated objects can also lead to such lensing effects, and they can be used to study the abundance of intermediate-mass and primordial black holes [1298, 1299]. For these studies, micro- and milli-lensing effects from baryonic substructures need to be carefully accounted for. Effects similar to microlensing can stem from other sources, such as orbital precession, missing waveform features or noise artefacts. Recent and ongoing work attempt to break the degeneracies in order to distinguish true microlensing from such false positives [1300–1302]. For micro- and millilensing searches, ET offers the interesting advantage of enabling the detection of more subtle features thanks to its enhanced sensitivity.

The above assumes the detection of micro or millilensing. However, non-detection can also be particularly useful to constrain the fraction of dark matter made of compact objects such as MACHO’s [1303]. Indeed, depending on the model for dark matter, the fraction of it made of compact objects, and the BBH distribution, one can expect a different number of detected microlensed events. So, in the absence of detection, one can constrain the dark matter fraction made of compact objects. This is done by simulating many realisations with different fractions and checking which ones are compatible with the non-observation, leading to an upper bound on this fraction [1303, 1304]. If a microlensed GW is detected, it is also possible to use this to put a lower bound on the dark matter fraction. For this, ET offers extra opportunities thanks to its additional sensitivity within the volume already probed, but also thanks to its larger horizon distance, increasing the probability for lensing.

3.3.3.3 Impact of lensing on standard sirens measurements. In the previous paragraphs we have discussed the prospects of lensing as an extra source of information. On the other hand, it is important to highlight that lensing can also act as an additional source of nuisance or systematic uncertainty in standard siren measurements. Besides unrecognized strongly lensed events, the phenomenon of weak lensing due to the gravitational potential of large-scale structures in the Universe is ubiquitous, as it accumulates along the propagating paths of all GWs [1139, 1305–1307]. Within the geometric optics approximation, lensing induces frequency-independent (de)magnification of the GW amplitude, which directly translates to the inference of a biased luminosity distance of the GW source:

$$d_L^{\text{obs}}(z, \mu) = \frac{d_L^{\text{true}}(z)}{\sqrt{\mu}}, \quad (3.88)$$

where d_L^{true} is the true luminosity distance of the GW source and μ is the lensing magnification. The probability distribution function of the lensing magnification depends on the redshift of the source; it is sharply peaked at $\mu \approx 1$ [1308], such that the majority of events will suffer very small modification, but it has a tail towards high magnifications that becomes more relevant for increasing redshifts. Moreover, lensing selection effects, i.e. the fact that magnified events are more likely to be detected than demagnified ones, tend to worsen its impact [1309–1311]. This effect is usually accounted as an additional source of noise in the luminosity distance uncertainty [1312], worsening the estimation of the cosmological parameters, particularly

those relying on distant events. Focusing on the case of bright standard sirens, lensing can also act as a systematic uncertainty in the estimation of such cosmological parameters. Using idealized mock catalogs of bright standard sirens events, relevant for ET, [1308] shows that in some scenarios the lensing bias on the cosmological parameters can be comparable or greater than the expected statistical uncertainty. In this regard, [1313] highlights the importance of properly modeling the lensing magnification probability distribution function when evaluating the expected lensing uncertainty. Furthermore, in the absence of an EM counterpart, and when the mass-redshift degeneracy cannot be broken by any other external information, lensing can also impact the inference of the source-frame masses of the GW sources [1282, 1314–1316]. In particular, in the case of BNS mergers, the masses can be magnified above the intrinsic maximum mass of the distribution, thus falling into the lower mass gap. In the era of ET, it would be important to account for this effect, to properly infer the maximum mass of NSs and to avoid misinterpretation of NS events as BH ones, spoiling for example the multi-messenger searches. In the coming years, the development of mitigation strategies in the data analysis will be necessary to properly account for lensing in ET events.

3.4 Probing the large scale structure of the Universe

3.4.1 Cross-correlation GWxLSS

As a cosmic tracer of the underlying matter distribution, resolved GWs and the GWB offer the unique possibility to map the large scale structure (LSS) of the Universe through gravitational radiation [547, 1317, 1318]. In the ET era, the anisotropic distribution of GWs will provide an independent cosmological probe and effectively become a useful LSS tracers in a complementary way to other LSS surveys (that trace cosmic structures through redshift measurements), as well as to emerging techniques such as Line Intensity Mapping or HI intensity mapping. Since all those map the same underlying density field via complementary observables, their cross-correlation offers a new opportunity for Cosmology and probing modified gravity (see also [1317, 1319–1327]).

3.4.1.1 Cross-correlation of GW resolved events with LSS. Besides information that can be extracted from individual detections (such as in the case of bright sirens) or from ensemble of GW detections (such as from dark sirens), large GW catalogs of resolved events open more general possibilities. They naturally trace structure in a luminosity distance space [1318], whereas galaxy surveys probe the redshift space [1328]. The expected number count exhibits different effects in both spaces, and the mapping of one into the other depend on both cosmology and gravity theory. The expression for the relativistic number count in luminosity distance space was initially considered in [1329] (see [1174, 1318] for subsequent studies), and then calculated in full generality in [1307, 1330]. Importantly, these works showed that the result is different with respect to its analogue in redshift space, with contributions such as lensing including additional terms. Those differences will become relevant for ET, which could allow using tomographic redshift bins up to $z \sim 2$ [1307]. Ref. [1331] further models the evolution and magnification biases for 3G detectors which affect those contributions.

Interestingly, similarly to supernovae, bright sirens are useful tracers not only of the density but also of the velocity field [1332, 1333], because the latter introduces correlations in

the distances which are well modelled by linear theory [1334]. Combining both measurements, one can improve precision substantially by breaking the degeneracy between fundamental parameters such as σ_8 and the tracer bias. Moreover, combining with a galaxy catalog covering the same area of the sky, one gains access to a total of one velocity and two density tracers. One can thus probe six power spectra with the different combinations, and perform a 6×2 pt analysis [1174, 1335]. In particular, it was shown in [1174] that, using GW signals from BNS with ET with kilonova follow-ups from the Vera Rubin Observatory,²⁸ together with DESI galaxies²⁹ in the range $0 < z < 0.5$, such an analysis could, when compared to a standard BNS Hubble diagram one, improve H_0 precision by 30%, improve precision in Ω_{k0} by a factor of more than 10 (reaching $\sigma(\Omega_{k0}) < 0.02$ in 5 years), and constrain well both σ_8 and the growth-rate index simultaneously. Therefore, a joint analysis of GWs and LSS will help address both H_0 and σ_8 tensions. Figure 55 shows forecasts obtained in [1174] for BNS with KN obtained with a Vera Rubin Observatory follow-up, together with DESI BGS galaxies in an area of 14000 deg^2 , assuming a maximum wavenumber $k_{\text{max}} = 0.1 \text{ h/Mpc}$ and $z_{\text{max}} = 0.5$. The forecast is for $\{\sigma_8, \gamma, \Omega_{m0}, \Omega_{k0}, H_0\}$, where γ is the growth-rate index, which is assumed constant as a simple parametrization of beyond-GR gravity, marginalized over two free bias parameters in each of 5 redshift bins and three global non-linear redshift-space distortion parameters.

Basic formalism. The anisotropic distribution of resolved GW events is described by the relative GW number count fluctuation in a direction $\mathbf{n}$ and at redshift z defined as

$$\Delta(\mathbf{n}, z) = \frac{N - \langle N \rangle}{\langle N \rangle}. \quad (3.89)$$

Given that GW interferometers are effectively all-sky experiments, it is convenient to study the n -point functions of this stochastic field in a base that reflects the intrinsic symmetry of the sky. Therefore, following the treatment presented in [1320], we decompose the number count fluctuation in a spherical harmonics base and we study the angular power spectrum given by

$$C_\ell^{XY}(z_i, z_j) = \frac{2}{\pi} \int \frac{dk}{k} \mathcal{P}(k) \Delta^{X, z_i}(k) \Delta^{Y, z_j}(k), \quad (3.90)$$

where we allow for the possibility of including different tracers X, Y (GWs and/or galaxies) in different redshift bins, and $\mathcal{P}(k)$ is the (almost-scale invariant) primordial scalar power spectrum. The number count fluctuation $\Delta^{X, z_i}(k)$ reads as

$$\Delta^{X, z_i}(k) = \int_0^\infty dz \frac{dN_X}{dz} w(z, z_i) \Delta^X(k, z), \quad (3.91)$$

where dN_X/dz is the *observed* source number density per redshift interval, $w(z, z_i)$ a normalized window function centered at redshift z_i , and $\Delta^X(k, z)$ encodes contributions from density, velocity, lensing and gravity effects [552, 1328].

²⁸<https://www.lsst.org/>.

²⁹<https://www.desi.lbl.gov/>.

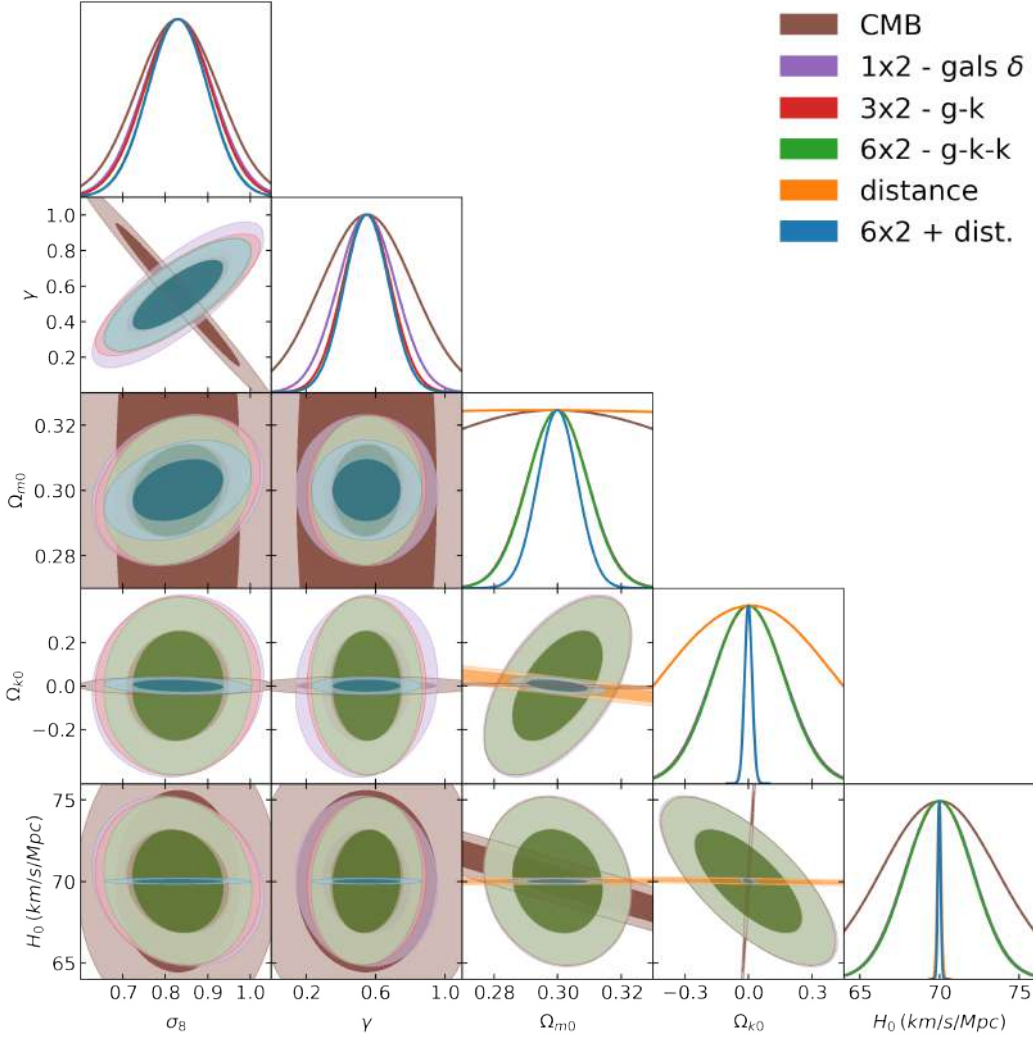


Figure 55. Marginalised constraints (1σ and 2σ confidence regions) combining ET KN and galaxies for $z \leq 0.5$, separated according to the method used. Purple: only DESI BGS galaxies; Red: DESI+KN velocities (3×2 pt); Green: DESI+KN in the full 6×2 pt; Orange: KN distances only; Blue: 6×2 pt + distances; Brown: CMB. Reproduced with permission from [1174].

Noise, dependence on the population model and detector sensitivity. Since GWs are a discrete tracer of the LSS, their angular power spectrum measurement has a shot noise contribution analogous, for instance, to that of galaxies. Assuming the noise of different experiments and in different redshift bins to be uncorrelated, the shot noise reads as $\mathcal{N}_\ell^{XY} = \delta_{XY} \delta_{ij} (dN_X/d\Omega)^{-1}$, where $dN_X(z_i)/d\Omega$ is the *total* number of *observed* sources per steradian in the i -th redshift bin. This quantity depends on the population properties, in particular on the total merger rate and on detector sensitivity. Conversely, we note that the signal in eq. (3.91) depends on a normalized window function, hence the dependence on the population model is through its shape rather than through its overall amplitude [1320].

For GW sources, an important difference with LSS studies is the limited angular resolution of GW detectors. The best angular resolution determines a limiting scale in the angular

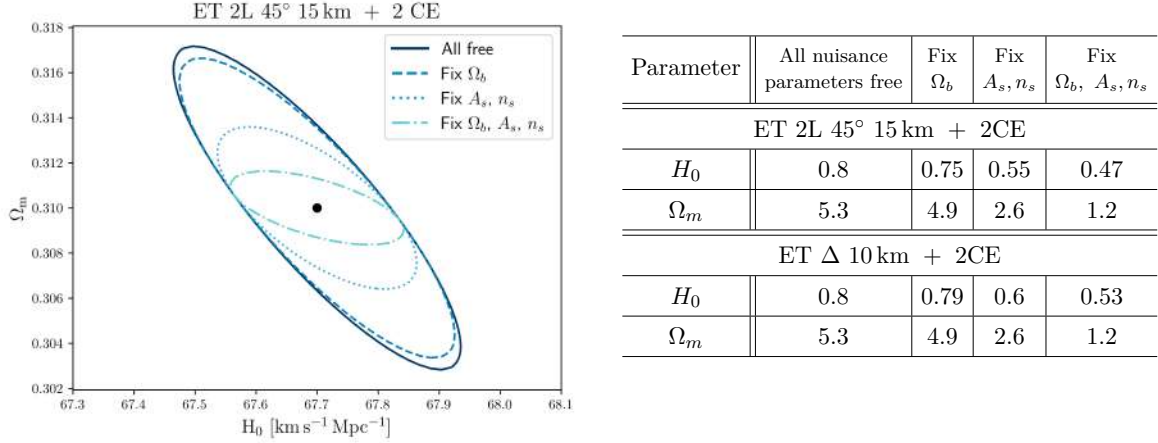


Figure 56. *Left:* 1σ constraints in the $H_0 - \Omega_m$ plane from the tomographic angular auto- and cross-correlation of resolved BBHs with a Euclid-like photometric galaxy catalog, for ET in the 2L configuration in combination with two CE detectors. Different lines correspond to fixing other cosmological parameters (baryon density Ω_b , amplitude A_s and spectral index n_s of the primordial power spectrum), representing the case where prior knowledge is assumed on those, while still marginalising over the tracers’ bias. The case of a triangular ET is not displayed since almost indistinguishable. *Right:* relative errors (in %) on H_0 , Ω_m for the 2L and triangular configurations of ET. Results from [1338]. Reproduced from [1338]. CC BY 4.0.

power spectrum [1318, 1319, 1323, 1324], $\ell_{\max} \sim 180^\circ / \Delta\Omega_{\max}^{1/2}$ with $\Delta\Omega_{\max}$ being the best 1σ sky localization of the GW catalog. From figure 47, we see that the best angular resolution of ET alone corresponds to roughly $\Delta\Omega_{\max} \sim 0.1 \text{ deg}^2$, hence a maximum multipole of at most ~ 300 in a local redshift shell (up to $z \sim 0.2$), while for higher redshifts the range will be further limited. The effect of limited spatial resolution can be effectively modeled as a smoothing of the underlying overdensity field by an observational “beam”, similarly to CMB experiments. A common choice in the literature is to parametrize this effect with a Gaussian beam, inducing an exponential damping in the GW-GW correlation signal [1318, 1323, 1324, 1336]. Moreover, error in distance/redshift localization can be accounted as showed in refs. [1318, 1319, 1324, 1336], which suggested to implement a probabilistic treatment similar to those employed in weak lensing analysis, where photometric redshifts carry a non-negligible uncertainty.

Cosmography. In presence of tracers of the same density fields in distance and redshift spaces, the cross-correlation signal depends on the parameters of the distance-redshift relation. Here we consider in particular GW sources correlated with a galaxy catalog, as well as the respective auto-correlations. The window function in eq. (3.91) takes the schematic form $w_i(z|\text{bin in } z)$ for the galaxy survey and $w_i(d_L(z, H_0, \dots)|\text{bin in } d_L)$ for the GW catalog. That is, the binning is performed in the spaces where sources are observed — redshift for galaxies and luminosity distance for GWs, but GW sources distributed according to the source-frame distribution $dN_{\text{GW}}(z_i)/dz$ are observed in different distance bins depending on the value of the parameters determining the distance-redshift relation, with the likelihood of the observed signal being maximised for the correct cosmology [1319, 1337, 1338].

Figure 56 shows the forecasts in the $H_0 - \Omega_m$ plane obtained in [1338] for the auto- and cross-angular power spectrum in tomographic redshift/distance bins [see eq. (3.90)] of GW sources with a galaxy catalog with the specifics of the Euclid photometric survey [1210, 1339]. While the application of this technique to ET alone is possible, the angular resolution highly benefits from the presence of a network of detectors. So, we focus on 10 years of observations of ET in combination with two CE detectors. We refer to [16] for a description of the BBH population used. We employ a Fisher matrix analysis with 20 equally populated redshift bins for the galaxy catalog, converted into luminosity distance bins for GW events assuming a fiducial cosmology. We account for the dominant contributions to the number count fluctuation, namely density and lensing [1307]. We also include the effect of the limited angular resolution with an exponential damping in the signal, with a redshift dependence computed in each redshift bin from the median localization in the simulations shown in figure 47. Furthermore, we restrict to a maximum multipole $\ell_{\max} = 300$ in any case, and for GW sources we impose a stricter hard cut on the maximum multipole in each bin (denoted with the index i) at $\ell_{\max,i} = \pi/\sqrt{\Delta\Omega_{\min,i}}$ being $\Delta\Omega_{\min,i}$ the best sky localization in the bin. The result is marginalised over the amplitude and spectral index of the primordial power spectrum, the baryon density, and the amplitude of the GW and galaxy bias in each bin, for a total of 43 nuisance parameters. Therefore, importantly in light of the Hubble tension, this constraints are independent on any other external probe or prior (e.g. BBN, CMB, SNe). We also show the result obtained fixing the baryon density Ω_b and primordial power spectrum amplitude A_s and spectral index n_s to their fiducial values, corresponding to the case where external priors are added. The relative error is reported in the table in the right panel of figure 56. In the best case, we forecast a 0.8% measurement of H_0 , which can be reduced to $\sim 0.5\%$ when assuming prior knowledge of Ω_b , A_s , n_s , in which case a $\sim 1\%$ measurement of Ω_m is also obtained. In conclusion, the cross-correlation technique for ET in combination with CE can lead to a promising sub-percent determination of H_0 , with percent-level precision already reached after one year of operation.

Reconstructing the Hubble diagram from cross-correlations of GW sources and tracers in redshift space can also be done directly through full-sky cosmological galaxy mock simulations. In [1340], the authors employ thousands of relativistic galaxy simulations together with binary black-hole sky maps populating the same large-scale structures of the galaxy maps, taking into account various systematic sources of anisotropies: the direction dependent sensitivities of both 2G and 3G GW networks, galaxy mask, the lensing magnification of luminosity distances, and modeling BBHs as extended sources due to the much larger errors in the determination of their 3D positions (RA, Dec, D_L) as compared to the ones for galaxies. Ref. [1340] shows that the method is robust to all these effects and to the choice of fiducial cosmology behind the simulations. This means that the position of the crest in the (z, D_L) plane illustrated in figure 57 is a cosmic standard ruler, not depending strongly on the details of the individual large-scale structures in the two spaces (distance or redshift) and rather on their matching. With such technique, one can get sub-percent measurements of the Hubble constant with 5 years of ET+2CE, while measuring also other cosmological parameters, which is one order of magnitude more precise than what is expected from the next observing run of LVK (O5). Moreover, if LVK is still active at the time of ET, LVK would help ET on

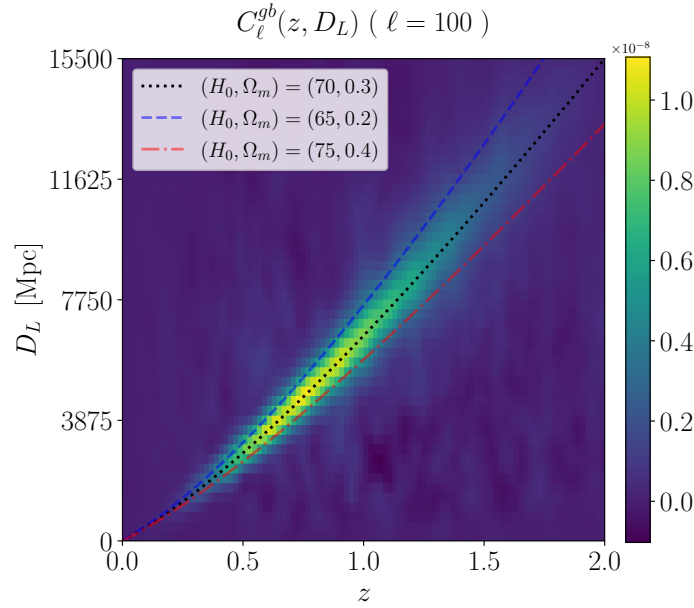


Figure 57. The cross-correlation between the harmonic modes of galaxies (g) at z and BBHs (b) at D_L leads to a cross-angular power spectrum $C_\ell^{gb}(z, D_L)$. We show the average cross-spectrum for 1000 light-cone simulations of galaxies and BBHs, for $\ell = 100$ ($\sim 2^\circ$). When a shell in z coincides with a shell in D_L , the correlation between the galaxy and BBH maps is maximal. The red, blue and black lines correspond to the Hubble diagrams $D_L(z)$ of three different cosmologies — in the fiducial model $H_0 = 70.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_m = 0.3$. Reproduced from [1340], © 2025 IOP Publishing Ltd and Sissa Medialab. All rights reserved.

localizing binary sources, even when those are below the detection thresholds for the 2G network, also potentially leading to sub-percent level measurements of H_0 .

Modified gravity. One of the goals of all next generation cosmological experiments is to test General Relativity in the late Universe. In particular, General Relativity has not been fully tested on scales comparable to today’s cosmological horizon. At those scales projection effects have the largest impact on the clustering of LSS tracers, while, at the same time, being very sensitive to deviations from General Relativity. Therefore GW clustering offer the unique possibility to fulfill this task in a complementary fashion to galaxy surveys or line intensity mapping experiments.

Two possible extensions that can be tested are: an equation of state for dark energy $w_{\text{DE}} \neq -1$, and a modified growth of structure, often parametrised via the (possibly) time- and scale-dependent μ, η functions defined as [1341]

$$k^2 \Psi(k, z) = -4\pi G a^2 \mu(k, z) \bar{\rho} D, \quad \Phi(k, z) = \eta(k, z) \Psi(k, z), \quad (3.92)$$

where Ψ, Φ are the Bardeen potentials, a is the scale factor, $\bar{\rho}$ is the background energy density, and D is the gauge-invariant energy density fluctuation. This parametrization allows to test a broad class of models, from scalar-tensor theories [1342–1345] to models that invoke extra dimensions [1346], by choosing an appropriate time- and scale-dependence of the μ, η variables.

Many different authors employed different parametrisations to forecast sensitivity of next generation GW observatory, such as ET, on deviations from General Relativity, both as unique probe or by cross-correlating this datasets with galaxy clustering, weak lensing and HI line intensity mapping [1323, 1324, 1347]. Constraints on these parameters are heavily dependent on the chosen parametrization, however the general consensus is that in all the cases ET alone, or in synergy with Cosmic Explorer, will provide unparalleled constraints on extensions of General Relativity.

Primordial black holes. The clustering properties of PBH binaries can be substantially different from those of traditional stellar-origin BH binaries [1325, 1348, 1349]. While astrophysical BH binaries are the result of intense stellar formation history in galaxies, and thus they trace the galaxy (anisotropic) spatial distribution, PBH binaries trace a different kind of environment, depending on their formation channel (see section 4 for detailed discussion). In particular, there are two well-understood formation channels that are worth mentioning: early-time [1350] and late-time binaries [1348, 1351]. In the former channel, binaries start forming in the early Universe, during radiation domination, and therefore they are thought to evenly trace all sort of dark matter structure that form in the late Universe. In the latter formation channel, binaries are expected to form in the late Universe in low-mass dark matter halos ($M_{\text{halo}} \simeq 10^2 - 10^3 M_{\odot}$) via a direct capture process because of the low relative velocity of halo particles.

Clustering properties of GWs depend mainly on the bias parameter b_{GW} . Since astrophysical binaries trace luminous galaxy distributions, we expect them to have a bias close to that of galaxies, i.e., $b_{\text{GW}} \sim b_{\text{gal}} \sim 1.2 - 3.0$. On the other hand, early-time binaries, since they effectively trace the entire dark matter distribution, are expected to have $b_{\text{GW}} \sim 1$. Finally, late-time binaries, because of the peculiarity of their formation mechanism, trace only low mass halos, mostly in filaments, thus they have $b_{\text{GW}} \sim 0.5$. Therefore, it is possible to disentangle PBHs from astrophysical binaries by measuring the GW bias with sufficient precision. Even in this case, the exact numbers depend on the modeling of PBHs; however the consensus reached by different studies is that by measuring the bias with a precision of order few percent it is possible to constraint PBH abundance at the ten percent level [1320, 1324, 1325, 1347, 1349]. Interestingly, reaching that level of precision will be well within the reach of ET in combination with many current and future LSS surveys, making it potentially capable of resolving the long-standing issue of what the nature of dark matter is [1325].

3.4.1.2 Cross-correlation of AGWB with LSS. In section 3.1.2.2 it has been discussed that the inhomogeneous distribution of sources and the line-of-sight effects accumulated during propagation contribute to the anisotropies of the AGWB. These intrinsic anisotropies are generated by the primordial perturbations in the Universe on large angular scales, and therefore constitute a tracer of the LSS. This implies that the cross-correlation of the anisotropies of the AGWB with other tracers of the LSS, such as galaxy number counts [546, 549, 557, 589, 1352, 1353], weak gravitational lensing [546, 549], CMB [578], and CMB lensing [1354], could be very large in light of the common seeds of the perturbations which generate the anisotropies of these signals.

It is possible indeed to characterize the anisotropies of the AGWB as a tracer of the LSS by computing the redshift distribution, the bias, and the magnification bias of the AGWB energy density, as shown in [547, 551, 552]. Among these three functions, the bias is of particular interest, since it contains information about GW clustering, quantifying how effectively the AGWB traces the overdensity of CDM. The analysis of the bias of the AGWB could be complementary to the one of the anisotropies of the resolved GW sources discussed in section 3.4.1.1. The bias of the AGWB energy density can be theoretically predicted starting from the bias of the host galaxies as

$$b_{\text{GW}}(z) = \frac{\int d\theta_{\text{gal}} \frac{d\bar{\Omega}_{\text{GW}}}{dzd\theta_{\text{gal}}} b_{\text{gal}}(z, \theta_{\text{gal}})}{\int d\theta_{\text{gal}} \frac{d\bar{\Omega}_{\text{GW}}}{dzd\theta_{\text{gal}}}}, \quad (3.93)$$

where θ_{gal} is the key parameter describing the properties of the host galaxies. It could be the star-formation rate ψ as in [1354], or the total stellar mass $M_{\star}$ as in [552]. In [552] an analog expression for the bias has been given in terms of the mass function and the bias of the host halos. Since the dominant contribution to AGWB anisotropies is given by the clustering term [547, 549], even a measurement of the first few multipoles of the angular power spectrum can yield informative constraints on b_{GW} . The astrophysical implications of the cross-correlation of the AGWB with other probes are discussed in section 4.10.6.

In addition, the cross-correlation of the AGWB with other LSS probes could play a crucial role in the detection of the anisotropies. As discussed in section 3.1.2.2, the shot noise and instrumental noise make the direct measurement of the anisotropies very challenging with current standard techniques. However, cross-correlating the GWB with other LSS tracers [546, 549, 557, 578, 589, 1352–1354], could help increasing the SNR by a significative factor. Thus, the cross-correlation with electromagnetic probes represents one of the best tools to detect GWB anisotropies with ET, if used together with the multi-frequency analysis of the anisotropies discussed in section 3.1.2.2 or other data analysis techniques. Indeed, even detecting the first few multipoles of the cross-correlation angular power spectrum would provide insights into the aforementioned underlying astrophysical and cosmological processes.

3.4.2 Cross-correlation of AGWB with CMB

Cross-correlations of GW and CMB anisotropies are an important probe for characterising the GW background and possibly disentangling its various components (e.g., the cosmological versus astrophysical contributions). The two backgrounds are indeed affected by the same large-scale (scalar and tensor) perturbations, both at production and during propagation through the Universe towards the Earth [541, 553–555, 566, 568–575, 577–579]. An overview of GW background anisotropies and their properties in the context of different early Universe scenarios can be found in section 3.1.2.1.

As discussed before, a possible source of anisotropies for the GW signal comes from the distribution of GW sources in the sky. Astrophysical GWs events, in fact, are expected to be hosted in galaxies, whose distribution follows the underlying dark matter one. Hence such sources reside in the large-scale gravitational potentials. Among the different anisotropies imprinted on the CMB signal, instead, the relevant one for the cross-correlation with GW sources is the late-times integrated Sachs-Wolfe effect [1355]. This effect arises from the

evolution of gravitational potential in times: the energy of a photon going through the potential is changed leaving an imprint on the CMB signal. The cumulative change of energy during the photon path in the dark-energy era gives rise to the late-Integrated Sachs-Wolfe (ISW) effect. This is not only a late-time effect but happens to be also relevant on the largest scales. The physical explanation for the cross-correlation, hence, becomes clear: the large-scale gravitational potential giving rise to the late-ISW are the same potentials hosting GW sources, hence tracing their distribution in the sky. The dominant contribution to the GWxCMB cross-correlation is thus expected from the density anisotropies of the former with the ISW of the latter.

The cross-correlation between GW and the CMB could also provide a complementary way to constrain primordial non-Gaussianity. As first shown by [1356–1358], the presence of local non-Gaussianities induces an additional scale-dependent term in the bias of the tracer considered. This correction affects the density term in (3.23), being the only one proportional to the GW bias [547, 552]. The scale dependence scales as $1/k^2$, hence it dominates on the largest scales. As explained above, on those scales the cross-correlation of GWs and the CMB is expected to be important and hence to be affected by the presence of non-Gaussianities, suffering of an enhancement or a suppression which can be probed by 3G detectors [576].

3.4.3 Probing LSS with GWs alone

The study of the LSS of the Universe is performed by constructing a two-point correlation function of galaxies, which describes their clustering properties at large scales ($\gtrsim 10$ Mpc). These studies are usually performed using galaxy catalogues constructed from large EM surveys. One result of these studies is the identification of the Baryon Acoustic Oscillation (BAO) peak at $\approx 100 h^{-1}\text{Mpc}$, which puts constraints on the density parameter Ω_{m0} of the cosmological model and on the equation of state of dark energy models. We expect that the 3D localization of coalescing compact binaries could serve as another probe providing complementary information, as these compact objects trace the underlying DM distribution in a possibly different way than the luminous matter of galaxies does, due to the astrophysical processes leading up to the formation of CBCs. This discrepancy is quantified by the clustering bias. Compact binary source localizations thus probe the *GW bias* b_{GW} , which is inherently different from the galaxy bias b_{gal} probed by luminous matter distribution. One should note that b_{GW} is potentially dependent on redshift as well as the astrophysical environment for compact binary formation and merger. It goes without saying that GW observations are needed to probe the GW bias. Earlier in this section, we have discussed the prospects of measuring b_{GW} using a combination of GW and EM tracers. It is worth exploring the possibility of measurement of b_{GW} using GW probes alone.

The proposed paradigm comes with certain challenges. Firstly, instead of point-like objects, the new method has to deal with extended GW detection volumes, which are currently of the same size or larger than the clustering scale itself. Secondly, the current GW detection rates are too low to perform this kind of statistical analysis. Hence, this endeavor requires a new detector network with enhanced sensitivity. Einstein Telescope and the 3G detector network in general open up the unique prospect of probing the LSS of the universe using GW alone.

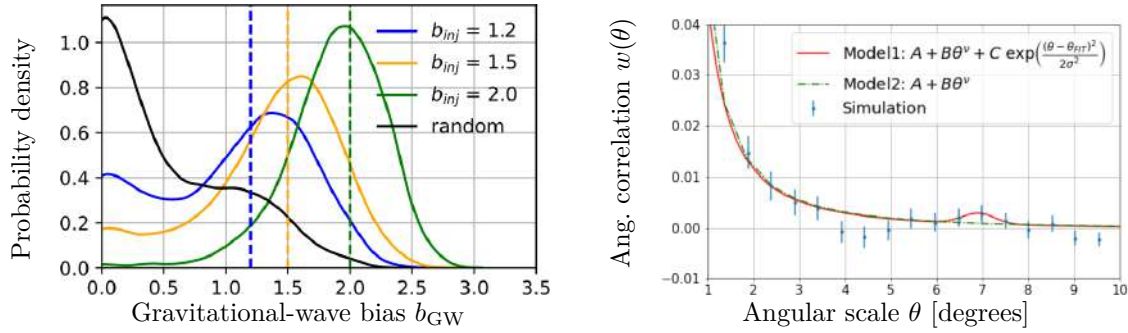


Figure 58. *Left:* recovery distributions of the GW bias b_{GW} correspond to the injected values [1359]. These results are for simulated BBH observations at a redshift $z = 0.5$ (shell thickness $350 h^{-1}$ Mpc) and with 10 years of observation by a worldwide ET+2CE network (ET 10 km Δ + CE 40 km US + CE 40 km Australia). Reproduced from [1359]. CC BY 4.0. *Right:* recovery of the angular two-point correlation function $w(\theta)$ reveals the BAO peak at the injected angular scale, $\theta = 6.9^\circ$ [1360]. These results are for simulated BNS observations at redshift $z = 0.3$ (shell thickness $150 h^{-1}$ Mpc) observed by the same detector network and over similar observation times. Reproduced from [1360]. CC BY 4.0.

Using third-generation ground based detectors consisting of ET and CE, we expect to detect thousands of mergers per year with sky localization areas within a square degree. It has been shown that with 5 to 10 years of observation time with such a network, we can extract the clustering bias of the BBH population [1359] (figure 58, left panel). In that same observation time, it is possible to detect the BAO peak solely from GW observations [1360] (figure 58, right panel). This reconstruction constrains the value of H_0 . Therefore, the separate galaxy and GW measurements of BAO peak are complementary to each other, and combining their data will enable us to constrain cosmological parameters using a standard ruler, i.e. the BAO scale [1361]. The results of these preliminary studies are the first indications that there are indeed prospects in probing the LSS with GW alone and more-realistic follow-up studies are in order.

Another method for probing LSS with third-generation GW observatories is by leveraging its impact on the event rate of BBHs and other CBCs [1362]. In particular, the rate of high-redshift BBHs is a sensitive tracer of the abundance of low-mass dark matter halos that form at early times. Measuring this rate therefore allows us to test for suppression in the matter power spectrum on small scales compared to Λ CDM, as predicted by various alternatives, such as warm dark matter, fuzzy dark matter, and interacting dark matter [1363]. Ref. [1362] found that a 3G network composed by ET+2CE could improve current constraints on these dark matter alternatives by as much as two orders of magnitude.

3.5 Executive summary

Here we summarize the advancements that ET will bring in Cosmology, in comparison to the current state of the art, specifically emphasizing the distinct features that the ET will allow us to explore. The following are the major areas where ET's capabilities surpass the current generation of GWs detectors.

Early Universe Cosmology with Einstein Telescope

- **Gravitational Wave Background (GWB)**
 - **Sensitivity:** Einstein Telescope will provide a strain sensitivity improvement of up to a factor of 10 compared to Advanced LIGO-Virgo, achieving a targeted sensitivity of $S_h^{1/2}(f) \simeq 10^{-24} \text{ Hz}^{-1/2}$ at 10 Hz and a broader frequency range extending down to 3 – 5 Hz, compared to the $\sim 10 - 20 \text{ Hz}$ lower limit of current detectors (see figure 177 for a plot of the strain sensitivity in different configurations). This expanded range will allow ET to capture more sources and trace features of the GWB that are inaccessible today.
 - **Detailed Study of GWB Components:** With its enhanced sensitivity and wider frequency range, ET will be able to:
 - * **Cosmological GWB:** Detect the CGWB with a sensitivity reaching energy density values of $\Omega_{\text{GW}} \sim 10^{-12}$, allowing us to probe early Universe events like inflation, phase transitions, and cosmic strings.
 - * **Astrophysical GWB:** Extract astrophysical population properties more precisely, tracing large-scale structures with better sensitivity.
 - **Anisotropies and Parity violation:** with its improved sensitivity and new layout ET will have better resolution to probe anisotropies of the GWB, and will be also sensitive to chiral GWBs, which are indicators of new early Universe physics.
- **Inflation, Phase Transitions and Cosmic Strings**
 - **Inflation:** ET is expected to be sensitive to GWs originating from inflationary models beyond standard single-field slow roll and from multifield models, providing an opportunity to test fundamental physics and the presence of extra field in the early Universe.
 - **First-Order Phase Transitions (PTs):** ET is expected to be sensitive to gravitational waves originating from PTs, with an observable energy density Ω_{GW} potentially as low as 10^{-11} for strong PTs at temperatures of $\sim 10^3 \text{ GeV}$, corresponding to earlier epochs of the primordial Universe with respect to those probed by LISA.
 - **Cosmic Strings:** ET will probe local string networks with tensions as low as $G\mu \sim 10^{-18}$, which is an improvement of several orders of magnitude compared to existing bounds ($G\mu \gtrsim 10^{-10}$ from current PTA experiments).

Late Universe Cosmology with Einstein Telescope

- **Cosmography and Cosmological Parameters**
 - **Bright Sirens:** ET will detect hundreds of bright standard siren events (neutron star mergers with kilonovae/short GRB counterparts) per year, allowing direct measurements of the Hubble constant H_0 with sub-percent precision.
 - **Dark Sirens:** For dark sirens (BBHs without EM counterparts), ET will utilize galaxy catalog information, or correlation with the mass function, to statistically infer redshifts, enabling a $\sim 10\%$ determination of H_0 and other cosmological parameters.
 - **Distance-Redshift Relation:** The precision in measuring the distance-redshift relation will enable model-independent tests of the expansion history of the Universe, addressing current tensions in H_0 measurements.
- **Modified Gravity and New Physics**
 - **Gravitational Wave Propagation:** The ability to compare GW and EM luminosity distances will allow ET to test deviations in GW propagation that might arise from modifications to General Relativity, potentially detecting differences of order 10^{-2} .
 - **Cosmic Dipole and H_0 Tensions:** ET will also contribute to resolving cosmic dipole discrepancies by comparing dipoles in the GW and EM sectors with a precision that is unachievable with current second-generation detectors.
- **Detection of Individual Cosmological Events**
 - **Gravitational Wave Bursts from Cosmic Strings:** ET will be capable of detecting GW bursts from cosmic string cusps if $G\mu \sim 10^{-11}$, while non-detection could constrain $G\mu$ below this level for different string loop models.
 - **Primordial Black Holes:** With its improved sensitivity, ET could detect signatures from PBH mergers, originating not only from inflationary mechanisms but also from domain walls or phase transition. In this way, it will allow to shed light on Dark Matter.
 - **Ultralight dark matter:** ET will probe the existence of scalar or vector ultralight dark matter, that can form clouds around spinning black holes.

Einstein Telescope will provide a significant step forward in our ability to study our early and late Universe. It will offer deeper insight into processes occurring at high redshifts and across a broader range of frequencies than ever before, making it an essential tool in the pursuit of our understanding of cosmology.

4 Population studies and astrophysical background

The first detection of gravitational waves (GWs) in 2015 is often compared to Galileo’s first astronomical observation with an optical telescope. Moving from LIGO-Virgo-KAGRA to the Einstein Telescope (ET) and Cosmic Explorer marks an equally transformative leap, comparable to advancing from Galileo’s crude instrument to the James Webb Space Telescope, which now reveals the cosmos in unparalleled detail. ET will observe approximately 10^5 binary black hole (BBH) and $10^4 - 10^5$ binary neutron star (BNS) mergers every year, up to a redshift of 100, and for a few thousand BBH events per year and several tens to $\mathcal{O}(10^2)$ BNS per year, the signal-to-noise ratio will exceed 100 [16, 1167]. With such capabilities, ET will unveil the mass function and spin distribution of BHs and neutron stars (NSs) in exquisite detail, allowing us to probe their evolution with redshift. While LIGO and Virgo have already paved the way for studying intermediate-mass BHs ($10^2 - 10^5 M_\odot$) with GWs, ET will observe systems with masses of $\approx 10^3 M_\odot$ up to a redshift of ≈ 8 , possibly via joint multi-band observations with LISA, the first space-borne GW detector. This will be pivotal for understanding the formation of supermassive black holes in the Universe.

At high redshift, Population III stars, the very first stars formed in the Universe, continue to elude our observations, even with the James Webb Space Telescope. The Einstein Telescope will routinely observe mergers of black holes born from Population III stars, with the main challenge being distinguishing such high-redshift messengers from other events. The individual observation of a binary BH merger at redshift $\gtrsim 30$, or of a sub-solar mass non-tidally deformable object would be a smoking gun for the existence of primordial black holes, with profound implications for our understanding of dark matter. This will ultimately pave the way for interpreting the formation channels of binary compact object mergers.

4.1 Introduction: formation channels of binary compact objects

After three observation runs, the LVK detected ~ 100 compact binary coalescences (CBCs, [6]), a number that may double or triple by the end of the ongoing fourth observation run. The number of detected CBCs enabled the scientific community to start placing constraints on the properties of the underlying population of compact objects. Nonetheless, the origin of CBCs is still unknown and widely debated. Broadly speaking, we can distinguish two main formation channels: the isolated channel, according to which compact binaries are the byproduct of binary stellar evolution; and the dynamical channel, in which compact binaries — or their progenitors — pair up through gravitational interactions in dense stellar environments. ET will contribute key improvements to reveal the formation channels and distinguish between them, for example through golden (high SNR) events that will pin down properties currently difficult to measure like spin, as well as through multidimensional constraints, such as the redshift dependence of masses and spins. Here below, we discuss this in detail.

4.1.1 Isolated channel: compact binary mergers from pairs and multiples in galactic fields

4.1.1.1 Binary evolution with mass transfer. For most close binaries, it can become unavoidable for the two stars to interact at some point during their life to eventually form a GW source. A circular binary with two compact objects can only experience a merger due to

GW emission within a Hubble time if the orbital separation is less than a few solar radii (for NS+NS) to less than several tens of solar radii (for BH+BH) [1364]. However, the radius of their progenitor stars is already larger than a few solar radii at birth, and easily reaches tens to hundreds of solar radii during the giant phases [1365]. For that reason, the story of the formation of GW sources in stellar binaries is one of stellar interactions and mass transfer.

A mass transfer phase strongly affects the stellar components (i.e., their masses and mass ratio, their spin properties), as well as the orbital separation, and consequently the delay time (i.e. the time elapsed from binary formation down to the merger). Progenitors of CBCs typically undergo two mass transfer interactions: a first one in which the initially most massive (primary) star is stripped of its hydrogen envelope, and the second one where the companion star is stripped. If the process occurs in a self-regulating manner, it is referred to as “stable mass transfer”, while “unstable mass transfer” is often assumed to lead to a common-envelope (CE) phase [1366–1369]. Interestingly, the process of stable and unstable mass transfer affects the system in different ways, leaving different imprints on the system. These imprints provide us with a wonderful opportunity to distinguish between (or potentially determine the) GW progenitors from the source properties. Here we mention the two main channels in binary evolution with mass transfer and discuss the corresponding source properties. Other channels likely make up only a few per cent of the total yield (e.g., [1370, 1371]).

Common envelope channel. A well-known evolutionary pathway is the common-envelope channel (hereafter CE channel: an example is given in figure 59, left). This channel leads to plenty of BBH mergers [1372, 1373], BHNS mergers [1370, 1374, 1375], as well as BNS mergers [1374, 1376–1378]. Initially on the zero-age main-sequence (ZAMS), the stars are not interacting (stage 1). After expanding, the primary star fills its Roche lobe and the first phase of mass transfer commences (stage 2). It proceeds in a stable manner, typically leading to widening of the orbit. The donor star is slowly stripped of its envelope, and at the end of its nuclear evolution, it will collapse to a compact object (i.e. BH or NS) at stage 3 (see also section 4.3.3.1). The next stage in the channel is driven by the evolution of the secondary star, leading to a second phase of mass transfer at stage 4 (see also section 4.4.1). For this channel, the second phase of mass transfer occurs in an unstable manner due to the extreme mass ratio and the response of the donor to mass loss, and a common-envelope phase is expected to develop. The name of the channel is derived from this second phase of mass transfer. During the common-envelope phase, the companion plunges into the envelope of the secondary star, and continues spiraling in due to friction. A merger is avoided if and when the envelope becomes unbound and is ejected from the system. The remaining binary has a much reduced orbital separation. In case of BHNS and NSNS formation, the secondary star may fill its Roche lobe a second time as a hydrogen-poor helium-star (i.e. case BB Roche Lobe Overflow), before collapsing to a NS. At stage 5, the system consists of two compact objects in a close orbit. Due to GW emission, the orbit shrinks further, eventually leading to a double compact object merger (stage 6).

Stable mass transfer channel. The stable mass transfer channel differs from the CE channel in only one key aspect [1375, 1379–1381]; the second phase of mass transfer (stage 4) does

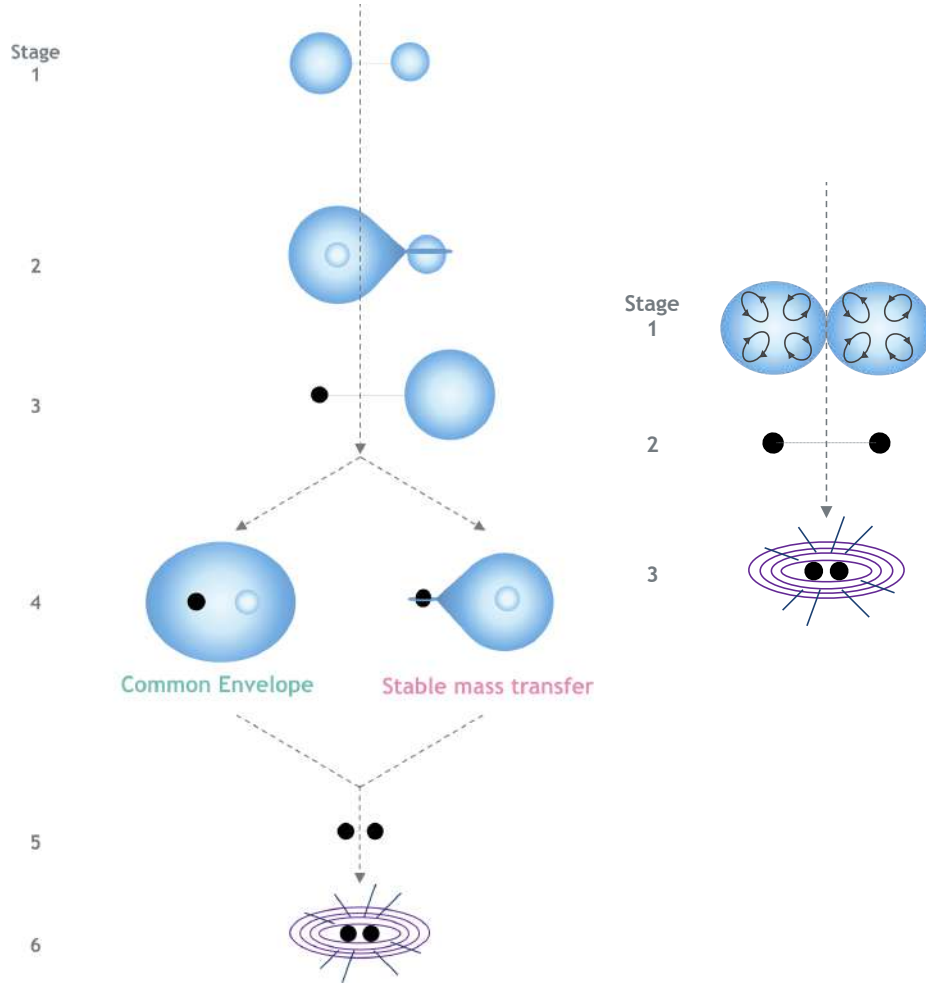


Figure 59. Evolutionary pathways to form a GW source for isolated binary evolution. *Left:* the common-envelope channel and the stable mass transfer channel. Stage 1) Zero-Age-Main-Sequence (ZAMS) 2) First phase of mass transfer 3) Formation of BH or NS 4) Second phase of mass transfer 5) Formation of double compact object 6) GW merger. *Right:* chemically homogeneous evolution. Stage 1) ZAMS 2) Formation of binary BH 3) GW merger.

not lead to a CE-phase, but in fact the mass transfer is self-regulating and therefore proceeds in a stable manner (stage 4 in figure 59).

It is important to realise that in order to form a GW merger, this second phase of mass transfer should lead to a shrinkage of the orbit. This occurs for those systems that have relatively high mass ratios (i.e. the secondary star is more massive than the compact object) and that experience relatively large angular momentum loss compared to their mass loss, as expected for a BH accretor. As the compact object binaries tend to be wider at formation (stage 4 in figure 59) than for the classical CE-channel, the stable channel typically leads to GW events with longer time delays [1382, 1383].

GW signatures. As the GW sources from isolated binary evolution can be formed through multiple sub-channels (e.g., CE channel and stable mass transfer channel), one can ask: what is the relative contribution of each channel? Historically, binary evolution calculations

have found that the CE channel dominates the total merger rate [1371, 1384]. However, recently there are indications that its contribution to the BBH merger rate may have been overestimated [1385–1389]. These works suggest that the common envelope may not be successfully ejected (stage 4 in figure 59) in many binaries, but instead leads to a merger or to stable mass transfer. The level of mixture between the channels therefore depends on the assumptions made when modeling binary evolution [1371, 1390, 1391].

ET can play an important role in distinguishing formation channels, since the intrinsic level of mixture is very likely redshift-dependent. As the stable channel typically leads to binary compact objects with relatively wide orbits (see above), its GW sources have longer delay times ($t_{\text{delay}} \gtrsim 1\text{Gyr}$) merging predominantly in the local Universe. Correspondingly, binary compact objects from the CE channel are born with more compact orbits, and merge with shorter delay times ($t_{\text{delay}} \lesssim 1\text{Gyr}$) and higher redshifts on average. It is found that the BBH merger rate from isolated binary evolution is dominated by sources from the CE channel at large redshifts, while in the local Universe the contributions from the CE and stable channel are similar [1383, 1392].

In addition ET can put constraints on the physics of the CE phase that is poorly understood [1368] by inferring from the merger rate the size of the post-CE orbits that depends on the efficiency of the deposition of energy into the envelope, in particular for BNS³⁰ mergers [1393]. This is not possible with current instruments, given the limited redshift coverage, even with the help of precise redshift determinations from electromagnetic counterparts.

There are also correlations between the formation channel and the compact object masses that ET can uncover. For example, the typical mass ratio of BBHs formed through the stable mass transfer channel range between 0.6–0.8, whereas the CE channel provides sources with a relative flat mass ratio distribution between³¹ 0.2–1.0 [1374, 1375, 1382, 1383, 1394]. Similarly, the mass ratio of BHNS ($M_{\text{NS}}/M_{\text{BH}}$) peaks at ~ 0.3 for the stable channel, and spans from 0.1 to 0.5 for the CE channel [1370, 1374]. In addition, there is a link between the BH mass, the delay time and the formation channel of BBH mergers [1383, 1390, 1395, 1396]. The CE channel preferentially produces BHs with masses below about $30M_{\odot}$, while the stable channel primarily forms systems with BH masses above $30M_{\odot}$. As BBH mergers from the stable mass transfer channel typically merge with long delay times, mergers of massive BHs are expected to predominantly occur at low redshifts and be relatively less common at higher redshifts. This counter-intuitive result holds despite the fact that massive BHs are easier to form at low metallicities which are more common at higher redshifts [1383, 1395, 1397].

4.1.1.2 Chemically homogeneous evolution (CHE). An example of non-mass-transfer binary evolution is chemically homogeneous evolution [1398–1400]. In the context of GW progenitors, the CHE scenario considers compact binaries that are near contact or even in contact at the start of the hydrogen burning phase (see stage 1 in figure 59, right panel [1401–1405]). In such binaries, tidal interaction will enforce rapid stellar rotation, which in turn induces meridional currents in the radiative layers of the rotating star [1406–1408]. The

³⁰It is advantageous to use BNS over BBH or NSBH, as the BNS merger yield is virtually independent of metallicity [1374, 1375, 1377].

³¹The shape of the mass distribution depends on the CE efficiency [1374, 1382].

mixing currents prevent the build-up of a chemical gradient inside the star and keep the star chemically homogeneous during most of the core hydrogen burning phase. Contrary to classically evolving (i.e. slowly rotating) stars, chemically homogeneously evolving stars do not experience significant expansion during the post-main-sequence phase. After the core hydrogen burning phase ends, the stars will become helium-burning stars, before collapsing to a black hole (stage 2). The binary black hole then merges due to GW emission (stage 3).

GW signatures. For the CHE scenario to produce mergers in a Hubble time, stellar winds need to be weak, as is expected at low metallicity [1409, 1410]. The reason for this is threefold: 1) stellar winds cause a spin-down of the star which halts rotational mixing, 2) stellar winds cause a widening of the orbit in which a tidally locked star has to spin down, 3) the widened orbit may be too wide to merge within a Hubble time due to GW emission alone. Therefore we expect the CHE channel to take place only at low metallicities. In addition, as the wide orbits give rise to long delay times, mergers at high redshifts are less likely to be observed for this channel in comparison with the classical channels (e.g. [1392]). Lastly, naively one may expect large effective spins from this channel due to the high rotational velocities required for the stars. However, Marchant et al. [1403] argue for moderate spins (~ 0.4) due to stellar winds during the Wolf-Rayet phase.

Other signatures that may be used to distinguish events from the CHE channel from other channels are related to the expected masses. In the CHE channel, there is a strong preference for high mass black holes ($\gtrsim 25M_\odot$) as CHE is not effective for lower mass progenitors [1411]. Therefore, it only produces binary black hole mergers, and not binary neutron star or BH-NS mergers. Secondly, there is a preference for detecting events with nearly equal-mass components [1402, 1403]. ET will thus be able to determine if a population of BBH with these properties exists or not.

The CHE channel is competitive with the classical isolated binary evolutionary channel with respect to rates [1402–1405] (see also section 4.2 for a discussion on rates and their large uncertainties). In particular, Riley et al. [1405] showed that the CHE channel may contribute 50–75% of all aLIGO BBH detections from isolated binary evolution. For the highest chirp masses ($M_{\text{chirp}} > 30M_\odot$), this percentage may be as high as 80%.

ET can also measure the effective inspiral spin parameter, $\chi_{\text{eff}} = (m_1 \vec{s}_1 + m_2 \vec{s}_2) / (m_1 + m_2) \cdot \hat{\mathbf{L}}$, where $\vec{s}_1$ and $\vec{s}_2$ are the dimensionless spins of the two components, while $\hat{\mathbf{L}}$ is the orbital angular momentum versor. For isolated binary evolution, at first sight it does not strongly constrain the formation channels in isolated binary evolution. This is because the effective spin is predominantly determined by the natal BH spin and the orientation of the BH spins [1412, 1413]. If angular momentum is transported efficiently through the progenitor star, the resulting BH natal spin is low, as preferred by current-day GW observations (e.g., [1414, 1415]). Low effective spin is also possible if the BH spins are misaligned. It is possible to form GW sources with high spins in isolated binaries if the stellar angular momentum transport is inefficient, or by increasing BH spins through accretion [1382, 1416–1418] or through tidal interactions [1392, 1419–1421]. Interestingly, the latter implies a relation between the effective spin and the redshift of the merger that should become visible with third-generation instruments [1392]. The fraction of highly spinning BBHs from isolated binary evolution is expected to increase with redshift, as low metallicity stars experience less

wind mass loss and less radial expansion, enabling the formation of more compact binaries with stronger tidal interaction and prompt GW mergers. At low redshift ($z \sim 0.5$ i.e., in the currently observable population) highly spinning BHs are expected to be dominated by BHs formed through chemically homogeneous evolution. In addition, for low CE efficiencies that translate to tighter post-CE orbits and more strongly spun up BHs, the highly spinning BHs in the CE channel merge at high redshifts ($z \sim 2$) close to the peak of the cosmic star formation rate, but outside current detector horizons [1382]. The advent of third-generation instruments would finally enable us to detect the redshift dependence of the intrinsically highly spinning BBH population from isolated binaries. This would enable us to clearly identify the population of BBHs that are of isolated origin.

4.1.1.3 Evolution in higher order systems. Observations of massive stars in the Milky Way and the Magellanic Clouds suggest that their companion fraction is high; typically massive stars not only have a stellar companion, but two or three companions [1422–1427]. It is therefore interesting to ask if and how this may change the GW signatures compared to isolated binary evolution. In the following section, we consider the evolution of small and isolated stellar systems.

Non-interacting triples. If both the inner and outer orbit are sufficiently wide that mass transfer will not take place, the stars evolve as if they were single stars. After a triple compact object has been formed, the system experiences strong three-body dynamical effects such as von Zeipel-Lidov-Kozai cycles [1428–1430]. During these cycles, the eccentricity of the inner binary can reach high values, which enhances the GW emission and can lead to a merger [1431–1439]. The predicted BBH merger rate for this channel events is estimated at $\sim 0.5 - 5 \text{ yr}^{-1} \text{ Gpc}^{-3}$ for solar metallicity and $25/\text{yr}/\text{Gpc}^3$ at low metallicity. Due to the strong impact of supernova kicks, the rate for NSBH mergers is more uncertain, $10^{-4} - 25 \text{ yr}^{-1} \text{ Gpc}^{-3}$ [1438, 1439].

Interacting triples. When the orbits are closer, triples do experience mass transfer at some point in their evolution (right column in figure 60, see [1440–1443]). The existence of a tertiary star can lead to an earlier interaction in the inner binary, a merger of the inner binary turning the triple into a binary system, or even multiple mergers in case of higher-order systems. In a third of the triple systems, the tertiary plays a fundamental role in the evolutionary pathway, leading to configurations that are not predicted for isolated binaries. The predicted merger rate in this channel can reach $25 \text{ yr}^{-1} \text{ Gpc}^{-3}$ (e.g. [1444]).

GW signatures. The GW signatures of these mergers are high eccentricity [1433, 1445] and a mass ratio q -distribution that is significantly flatter than for isolated binary evolution or for dense stellar clusters [1436, 1444, 1446, 1447]. Multiple mergers can produce masses in the upper and lower mass-gap [1448–1452], see also section 4.3. Moreover, in triples with chemically homogeneously evolving inner binary stars, commonly the third star will transfer mass to the inner binary black hole [1447], possibly leading to “wet” mergers in which the orbital decay is driven by gas drag instead of GW emission [1453]. Of particular interest is the effect of triple stars on the effective spin of BBH mergers. Although complicated by GR effects, many studies have found that, independently of their magnitude, the coupling of

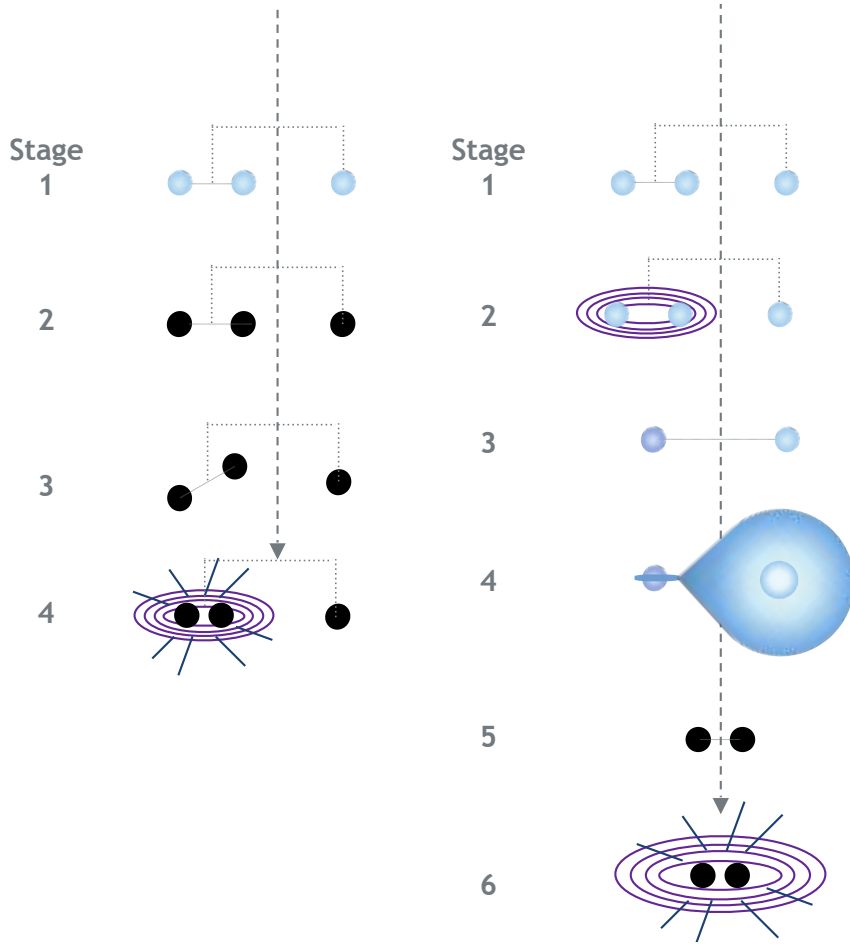


Figure 60. Evolutionary pathway to form GW sources a) according to the non-interacting triple channel, b) in an interacting triple with a stellar merger in the inner binary. On the left: Stage 1) ZAMS 2) Formation of triple BH 3) ZLK cycles and orbital dissipation 4) GW merger. On the right: Stage 1) ZAMS 2) Stellar merger in the inner binary 3) Formation of binary with a rejuvenated star 4) Various phases of mass transfer 5) Formation of binary BH 6) GW merger.

triple dynamics with GW energy loss leads to misaligned spins and $\chi_{\text{eff}} \sim 0$ [1435, 1437, 1439, 1445, 1454–1457]. ET will be able to uncover a significant population of mergers originating in triple systems (e.g., [1458]).

4.1.2 Dynamical channel: stellar systems as factories of merging compact objects

With densities much larger than galactic fields, star clusters represent ideal factories for the production of dynamical CBCs. Theoretical models and observations generally gather clusters into three main categories: open and young massive clusters, with typical masses in the range $10^2 - 10^5 M_\odot$ [1459], globular clusters ($\sim 10^5 - 10^7 M_\odot$) [1460], and nuclear clusters ($10^6 - 10^8 M_\odot$) [1461]. A primary difference among different cluster types is the typical density, which varies from $< 10^4 M_\odot \text{ pc}^{-3}$ in open clusters [1459, 1462] and can reach extreme densities of $10^8 M_\odot \text{ pc}^{-3}$ in the densest nuclear clusters [1463]. Additional differences pertain

to the possible presence of gas in young clusters, the development of multiple populations in globular clusters, and the peculiar star formation history or the possible presence of a central supermassive black hole in nuclear clusters.

Within the dynamical scenario, there is a large variety of processes that can lead to the formation of CBCs, as we briefly describe in the following. The branching ratio of such processes intrinsically depends on the binary properties and the cluster type and structure.

Dynamical formation in a star cluster is expected to be more effective for BBHs than for both BNSs and NSBHs, because BHs dominate the dynamics in the cluster central regions, preventing the segregation of NSs and their progenitors and suppressing their interaction rate.

As the star cluster evolution proceeds, though, BHs are slowly evacuated from the cluster in what is called the BH burning process [1464]. As the number of BHs diminishes, the probability for NSs to interact with other BHs and among each other increases. Therefore, theoretical models predict that the number of NSBH and BNS mergers with a dynamical origin is much smaller than that of BBHs, and their occurrence requires that the host cluster lost a significant fraction of its original BH population [1465–1469].

Despite their rarity, a subpopulation of dynamical NSBH mergers may exhibit characteristic features that differentiate them from the isolated ones. These include BHs heavier than $10M_{\odot}$ [1467, 1470], a chirp mass $> 4M_{\odot}$ [1468], the absence of an electromagnetic counterpart [1468, 1471, 1472], and longer delay times, with most of them likely forming over a cluster relaxation time [1466, 1470].

BNS mergers are even rarer than NSBH and their occurrence in star clusters is expected to be several orders of magnitudes smaller than in the field [1466, 1473]. Given their rarity, it is also unclear whether dynamical BNS mergers can exhibit distinctive features compared to those forming in galactic fields [1467, 1474]. Most likely, the only way to infer a dynamical origin is via the association of the resulting EM counterpart with a dense cluster.

Moreover, the low detection rate of both BNS and NSBH by current ground-based detectors, alongside with the limited number of BNS detected through other means of observation, make it hard to assess the existence of a “lower mass-gap” separating NSs and BHs, in the mass range $2 - 5M_{\odot}$. In this sense, the recent discovery of a CBC by the LVK Collaboration involving a NS and an object with a mass in the range $2.5 - 4.5M_{\odot}$ [1475] could make the case for a dynamical origin [1467, 1474, 1476] or push new studies to explore more some poorly understood mechanism of stellar evolution, like the common envelope physics [1477].

4.1.2.1 How can CBCs form in star clusters and merge within a Hubble time.

Primordial binaries. The simplest way to form CBCs in a star cluster is from the evolution of binary stars already paired at birth, the so-called “primordial binaries”. Observations of stellar nurseries and young star clusters suggest that up to 50 – 70% (and even 100% in some extreme cases) of stars are born in binary systems and higher-order multiples [1423, 1424]. The binary fraction seems to increase with the primary mass [1423], thus suggesting that many, if not all, compact object progenitors may have been part of a “primordial” binary.

At the simplest level, the binary star can evolve into a CBC via stellar evolution making it indistinguishable from the same binary evolving in isolation. If, however, the timescale

for binary–single stellar interactions is shorter than the binary delay time, the perturbations induced by stellar flybys and close encounters on the orbit can significantly alter the binary evolution, either shortening its merging time or delaying it, leading to a swap between an interloper and one of the binary components, or causing its ionization.

Even in the extreme case that no stars are born in a binary system, there are several ways to form CBCs via dynamics.

Gravitational wave captures. If the two CBC components, initially unbound, undergo a close hyperbolic encounter, GWs emitted during the passage at pericentre (GW bremsstrahlung) can carry away enough energy to bind the two objects. This process, called gravitational wave capture, requires an environment with densities typical of galactic nuclei [1478, 1479] and dense globular clusters [1480, 1481]. In this scenario, the pericentre distance between the two objects must be so small that CBCs forming through this channel merge minutes to hours after the interaction [1478]. Given their short formation time, their merger rate will directly follow the binary formation rate, which is expected to be in the range $0.2 - 150 \text{ Gpc}^{-3} \text{ yr}^{-1}$ [1482]. Up to 90% of mergers formed this way can retain an eccentricity $e > 0.9$ at 10 Hz [1478, 1483–1485].

Three-body binary formation. Another way to form tight binaries is via three-body binary formation. In this case, three initially unbound objects find each other and closely interact up to the point that two objects pair up and the third carries away the extra energy. The timescale of this process is extremely sensitive to the cluster velocity dispersion, being $\propto \sigma^9$ [1486], and generally exceeds a Hubble time for the densest globular clusters and nuclear clusters. Nonetheless, three-body binary formation represents a primary channel to build-up hard binaries, i.e. binaries with a binding energy larger than the average kinetic energy of the environment [1487], containing at least one compact remnant [1488, 1489], but see also [1490]. Such binaries can either merge outside the cluster or harden via binary–single and binary–binary interactions.

Binary–single interactions. If the cluster harbours a population of binaries, either primordial or formed via three-body interactions (see paragraph above), CBCs can form through interactions between the binary and a third compact object (binary–single interactions) depending on the binary dynamical status (i.e. if it is hard or soft) and the star cluster structure and evolution [1491, 1492]. This type of interaction can account for up to 20 – 70% of all mergers occurring in Galactic nuclei [1493], and is a primary mechanism for forming CBCs with large eccentricity (e.g., [1492, 1494–1496]) or with at least one component in the lower or upper mass gap by triggering the merger of the inner binary system [1497–1499].

When the distance between a binary and an incoming third body falls below a few times the binary semimajor axis, the three bodies can undergo a so-called *resonant* interaction, a chaotic phase during which the three bodies pair up and exchange continuously (e.g., [1500–1502]). This process proceeds up to the point in which one of the objects, generally the least massive, is expelled and the other two form a hard binary [1465, 1468, 1490, 1496, 1502].

Gas-driven dynamics. Star clusters rich in gas, such as embedded open clusters or young massive clusters, can nurture the development of CBCs as the gaseous medium can alter

the orbit of a binary, for example by inducing a dynamical friction term on the components that shrinks the binary and can enhance the probability for the binary to merge within a Hubble time [1503].

Gas-driven CBC formation can also occur in the dense disks that surround AGNs [1504–1506]. In such extreme environments, the disk can dissipate the orbital energy of compact objects passing through its medium and capture them, thus their orbits settle in the plane of the disk and undergo inward migration that steepens the radial density profile around the SMBH [1507]. The aforementioned process acts faster than relaxation in AGN disks, and can nurture the formation of CBCs in the plane of the disk aided by a combination of gaseous accretion, gas dynamical friction, and stellar scattering [1482, 1505].

In the case of BBHs, it has been suggested that mergers occurring in an AGN can power an EM counterpart [1508]. GW190521, the first GW source to produce a final mass in the intermediate-mass black hole (IMBH) range ($\sim 150M_\odot$) [1509], has been associated with a high-energy transient detected by the Zwicky Transient Facility [1273], although it is rather hard to assess whether this association can be robustly confirmed [1274].

Secular mechanisms. Secular effects generally develop when the binary is perturbed by a third object or a series of close passages by different objects. Depending on the perturber orbital properties, the binary can undergo Kozai-Lidov oscillations [1429, 1430], a mechanism that could boost the merger probability of CBCs in dense clusters [1510–1512] and galactic nuclei [1513–1518]. Weak perturbations acting on soft binaries in loose star clusters, for example, can induce a steady increase of the binary eccentricity on a timescale shorter than the binary evaporation time, up to the point that the binary coalescence occurs within a Hubble time [1519, 1520].

4.1.2.2 Properties of dynamical CBC. Dynamical CBCs exhibit peculiar properties, some of which may be unique marks of this formation channel, as we briefly discuss in the following.

Eccentricity of dynamical CBCs. Several works have pointed out that orbital eccentricity may represent a key parameter to identify dynamical CBCs, as according to most stellar evolution theories, isolated CBCs are expected to circularise before reaching the Hz frequency band [1521]. Moreover, eccentricity could be used to untangle the population of mergers occurring after their ejection or inside a cluster, either triggered by repeated interactions or owing to hierarchical secular dynamics [1495, 1511, 1522–1524]. In general, we expect that in-cluster mergers are more eccentric than mergers occurring after the ejection of the CBC from the parent cluster [1522, 1523].

The different eccentricity distribution for in-cluster and ejected mergers may be particularly evident in the 1–10 Hz frequency band. In-cluster mergers forming from three-body and binary–single interactions have typical eccentricity in the $10^{-4} - 10^{-2}$, whilst those forming from GW captures have very high eccentricities, > 0.9 , and merge on timescales of days or years. In the case of ejected binaries, instead, their eccentricity may be too small $< 10^{-5}$ to be accurately measured in the 1 – 10 Hz band, and a detector sensitive at lower frequencies could be more suited to detect them. In these regards, the possibility to access the 0.001 – 0.1 Hz fre-

quency band, for example with LISA, DECIGO, or the LGWA, could enable us to observe the eccentricity evolution of the binary weeks to months prior to the merger, helping us to place constrain on the origin of stellar mass CBCs. However, particular attention is needed when modeling the signal, owing to the spin precession-eccentricity degeneracy [1525, and reference therein], which can significantly hinder eccentricity measurements (e.g., [1525–1527]).

Stellar interactions, hierarchical mergers, and spins. A binary traveling in a stellar environment is subjected to binary–single interactions that can impart on its motion a Newtonian recoil and remove part of the binary orbital energy. As a consequence, the binary progressively shrinks (*hardening* process), making the new interactions rarer but more violent [1487]. At some point, GW emission takes over and drives the binary to coalescence inside the cluster, provided that the binary shrinking rate due to GWs is larger than the binary–single interaction rate. Otherwise, a new interaction can, on the one hand, increase the binary eccentricity and possibly trigger a rapid GW coalescence, and, on the other hand, transfer enough kinetic energy to expel the binary, which then will merge outside the cluster.

Using the population synthesis code B-POP [1528], we show in figure 61 how the fraction of in-cluster mergers varies with the cluster escape velocity, calculated at the time of the binary merger for different assumptions about the initial binary semimajor axis distribution. Specifically, we either assume a Gaussian distribution peaked around the hard-binary separation [1502] with dispersion $\sigma_a = 0.1, 0.3$, or a flat distribution between 0.1–0.2 AU [1480]. B-POP models are compared against direct N -body [1524, 1529] and Monte Carlo simulations [1495]. Note that for all the simulation database considered, the fraction of in-situ merger is calculated taking into account only dynamically assembled binaries, and the range of simulated velocity dispersion is shown, generally limited to within a factor of a few from the average value. Despite B-POP relies on an approximated method to simulate BBH mergers, their results agree surprisingly well with both few-body and self-consistent simulation of star clusters. In general, it is evident that the fraction of in-cluster mergers sensibly increases with the cluster escape velocity, especially in the case of tight BBHs [1481, 1530–1533]. This aspect can have crucial implications for the development of multiple mergers and the formation of higher-generation BHs, as we discuss in the next section.

Asymmetries in the emitted GW signal impart a relativistic kick on the merger remnant [1534–1536]. If the amplitude of such recoil is smaller than the cluster escape velocity, the remnant can remain in the cluster and merge again, leaving behind a second-generation remnant. The development of repeated mergers is usually referred to as the hierarchical merger mechanism [1491]. Relativistic kicks can be as large as 10^4 km s^{-1} , therefore hierarchical mergers are expected to develop only in the most massive and dense star clusters [1481, 1491, 1528, 1537–1546], especially because the escape velocity of star clusters decreases over time owing to relaxation processes, which cause mass-loss and expansion [1528, 1542, 1547].

Higher-generation BHs are expected to be characterised by clearly different spins compared to first-generation mergers. For example, second-generation mergers are expected to have dimensionless spin distribution tightly peaked around $\chi \sim 0.7$ [1548–1550]. The ability to identify a second-generation merger thus depends on the natal spin distribution of their progenitors [1528, 1551, 1552]. For higher generation remnants, instead, the older the

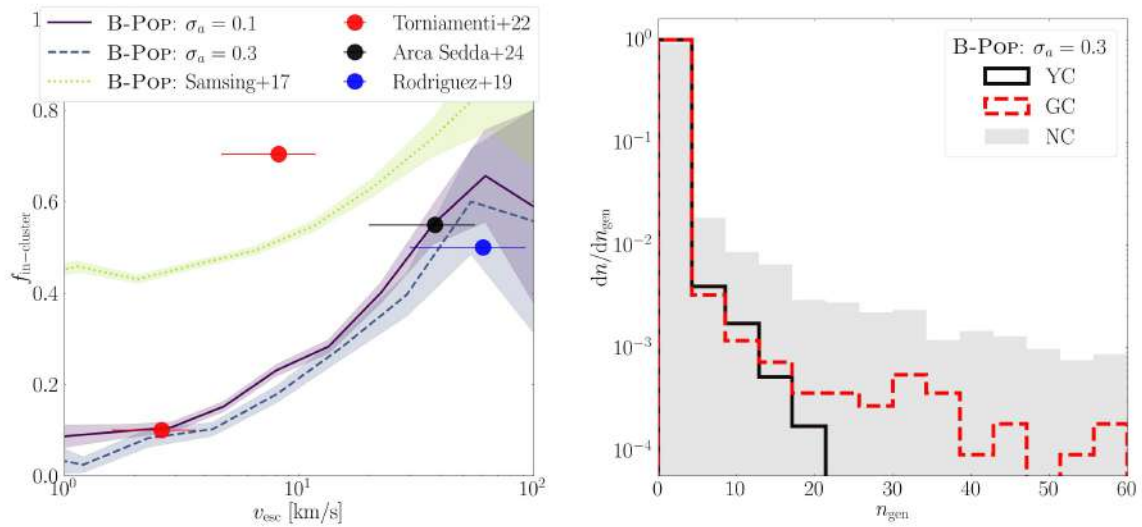


Figure 61. *Left panel:* fraction of BBHs that merge inside the cluster for different assumptions about the binary semimajor axis sampling: assuming a Gaussian distribution peaked around the hard binary separation with $\sigma_a = 0.1$ (purple straight line), or with $\sigma_a = 0.3$ (blue dashed line), or following the assumption of [1480], i.e. that a is distributed between 0.1 – 0.2 AU according to a flat distribution. Shaded areas encompass the Poissonian error associated with the samples. The points represent data from self-consistent N -body and Monte Carlo simulations of star clusters. *Right panel:* distribution of merger generation for different cluster types. All models are performed with the B-POP population synthesis tool [1528].

generation the smaller the spin [1538, 1553]. Higher generation merger remnants also feature typical precessing spin values $\chi_p = 0.5 - 0.8$, which are hardly achievable with isolated and 1st generation mergers [338, 1542, 1554, 1555].

The mass, spin, and generation of a hierarchical merger product encode information about the host cluster and the properties of the previous chain of mergers. For example, the GW recoil is generally smaller if the merging components have small spins, therefore the development of a long merger chain requires a natal spin distribution that favors low spins [1481, 1532]. The amplitude of the kick requires a host environment with a large escape velocity, generally $> 100 \text{ km s}^{-1}$, thus favoring dense and massive globular and nuclear clusters as preferred environments for higher generation merger products [1528, 1530, 1543, 1553, 1556].

The hierarchical merger mechanism represents a viable way to form IMBHs (see section 4.8 and references therein) and BHs in the upper mass gap [1488, 1491, 1499, 1524, 1539, 1540, 1544, 1557–1560]. Upper mass-gap BHs can also form via stellar evolution under particular conditions [1561–1564] or via stellar collisions [1395, 1499, 1524, 1565–1567]. Mergers involving upper mass-gap BHs can thus carry information about their formation origin. Precise measurements of masses and spins could help untangle different formation channels for mergers involving one or two BHs with masses in the gap. In this sense, at masses below $O(500M_\odot)$ ET will enable to place stringent constraints on the primary spin, chirp mass, location and distance [16, 107, 1167, 1181, 1568, 1569], offering a unique opportunity to infer the origin of mergers with one or both components in the upper mass gap.

4.1.3 Fingerprints of different formation channels

As we have seen in the previous sections, CBCs forming from the isolated and dynamical channels exhibit some peculiar features that could be used to infer the contribution of different channels to the cosmic population of CBCs [338, 1528, 1570]. With the prospective detection of 10^5 BBH and BNS events per year [16, 1167], it will be possible to characterise in detail the impact of the isolated and dynamical channels on the formation of CBCs. This can be achieved by constraining crucial quantities like merger rates, redshift dependencies, masses and spins of merging CBCs.

Merger rates are highly uncertain, as they depend on the cosmic star formation history, the metallicity evolution, and the star cluster formation process. Isolated binary models predict a BBH merger-rate density $\mathcal{R} \sim (0.5 - 5,000) \text{ yr}^{-1} \text{ Gpc}^{-3}$ at redshift $z \lesssim 1$, while dynamical models suggest $\mathcal{R} \sim (10^{-3} - 10^2) \text{ yr}^{-1} \text{ Gpc}^{-3}$ (for a comprehensive discussion about rates from different channels, [1482, 1571]). A similar difference in the expected merger rates is also estimated for BHNS mergers, while for BNS most models suggest that isolated mergers outnumber dynamical mergers by one order of magnitude, despite the large uncertainties [1571]. Note that the upper limits to theoretical predictions are already in tension with the local merger-rate densities inferred from LVK data [7]. See also section 4.2 for further details about CBC merger rates. With ET, we will be able to verify the possible evolution with redshift of the merger rate [7] and other quantities, like the median mass or the primary mass of merging BBHs [1528, 1542, 1572], and their mass ratio [7].

The binary mass, primary mass, and mass ratio distributions can be diagnostic quantities to unveil features from different channels. For example, mergers with large primary mass and low-mass ratios are most likely byproducts of dynamical processes [1466–1468, 1481, 1491, 1528, 1537–1546], while isolated mergers predominantly favor the formation of lower mass mergers [1370, 1395, 1415, 1477, 1573–1577], although uncertainties in binary stellar evolution and pair-instability physics leave room for massive BH formation also through this channel [1394, 1578].

Spin magnitude and alignment are other quantities that ET can accurately measure, at least for the closest BBH mergers, to provide crucial insights about the unknown distribution of BH natal spins and the origin of the merging binary, despite the large uncertainties in stellar dynamics and evolution, and in the physics of stellar explosions [1421, 1579–1581]. Isolated BBHs are predicted to have mildly aligned spins, depending on the impact of SN explosions on the binary orbit [1579, 1582, 1583], and the establishment of precessional instabilities that can completely change the initial spin configuration (e.g., [1413, 1580]), whilst dynamical BBHs should have randomly oriented spins, unless some effect efficiently aligns them, e.g. like disk torques in AGN disks [1584].

Eccentricity probably is the most representative parameter of the dynamical channel. The isolated channel largely produces nearly circular mergers at $f > 1$ Hz. The eccentricity of dynamically assembled CBCs, gravitational-wave captures, and hierarchical triple systems, instead, can attain high values ($e > 0.9$) depending on the processes that affected the binary evolution. ET can measure eccentricities above $e > 0.01$ with accuracy at 1% level at redshift $z \lesssim 0.1$ [16, 1585], making it possible to identify mergers coming from high-energy dynamical

interactions in dense star clusters, e.g. GW captures or collisions from resonant interactions in dense star clusters, or from dynamics in isolated triples.

4.2 Merger rate density of CBC across cosmic time

A common metric of the capabilities of GW detectors for population studies, which demonstrates the promise of next-generation detectors, is the fraction of the population that can be detected above a certain signal-to-noise (SNR) threshold as a function of redshift. The upgraded network of current detectors (Advanced LIGO, Advanced Virgo, and KAGRA with sensitivity curves as shown in [1586]) will be able to detect complete population of stellar BBH mergers with $\text{SNR} \geq 10$ up to $z \sim 0.1$ and only up to $z \sim 0.01$ for the lower mass BNS mergers [107, 1586]. Therefore, even the upgraded network of current GW detectors will provide limited insight into the evolution of CBC properties through cosmic history. As we discuss in this section, this dependence encodes valuable information about the formation and evolution of stars and stellar multiples in environments very different from our cosmic neighborhood. It also probes the cosmic history of star clusters and galaxies. In addition, constraints on the evolution of the merger rate at redshifts $z > 2$ (above the peak of the cosmic star formation history [1587]) may prove key to overcoming current degeneracies in the astrophysical interpretation of the population properties of BBH mergers.

Although efforts are made to constrain the evolution of the CBC merger rate with current telescopes through sources such as gamma-ray bursts, many challenges (such as understanding selection effects, source classes, and environments) limit our ability to constrain the CBC merger rate even at small redshifts, let alone at $z \gtrsim 2$ [1571]. Such constraints can be obtained with next generation GW detectors. Iacovelli et al. [1167] estimate that ET alone can detect 100% of the BBH (BNS) mergers with $\text{SNR} \geq 12$ up to $z = 1$ ($z = 0.2$). For the network of at least two next generation GW detectors, the complete BBH (BNS) population can be detected up to $z \approx 2$ ($z \approx 0.3$) [107, 1167]. Moreover, more than 90% of the BBH population can still be detected with $\text{SNR} \geq 12$ out to $z = 20$. This redshift is more than a factor of two higher than that of the most distant known electromagnetic signal associated with the formation of a stellar compact object [1588–1590] and further back in the history of our Universe than any galaxy has ever been observed [1591]. ET will allow us to map the BBH merger rate beyond the current limits on the onset of the cosmic star formation and to access the unexplored epoch of the formation of the first stars and the first galaxies with BBH mergers.

4.2.1 The key ingredients of compact binary coalescence rates

With the exception of primordial BH mergers (see section 4.6), CBCs detectable by ground-based GW detectors originate from compact objects left over from the evolution of massive stars ($\gtrsim 8M_\odot$). Given the short evolutionary timescales of such stars (a few $10^6 - 10^7$ yr), the formation rate of stellar BHs and NSs follows the overall cosmic star formation history. However, only a small fraction of those compact objects evolve into observable CBC. Furthermore, depending on the efficiency of the processes that bring the two compact objects to merger (see section 4.1), these events can occur with a wide range of time delays, t_{delay} , with respect to the formation of their progenitor stars. These delays can range from a few Myrs to the age of the Universe $T_{\text{Hubble}} \sim 14$ Gyr. Consequently, the observable CBC population

contains a mixture of systems formed throughout the Universe: at different times, with different chemical compositions, and in very different environments. Both the birth chemical composition and the environment in which stars evolve strongly influence the formation of CBCs and their population properties.

Birth chemical composition leaves imprint on stellar afterlives (CBC properties). Metallicity (i.e. the abundance of elements heavier than helium, Z) is a necessary initial condition for modeling the evolution of massive stars [1592] and stellar binaries/multiples, and influences the properties of stellar-origin compact objects such as masses and spins. Especially important is the abundance of iron, which determines the mass loss in stellar winds [1409, 1410, 1593–1595]. Iron-poor stars can end their lives as heavier black holes [1596], whose mergers lead to stronger GW signals. Weaker wind mass loss reduces the amount of angular momentum removed from the stars, possibly allowing the formation of compact objects with higher spin magnitudes. Stellar winds also remove mass and angular momentum from the system’s orbit, typically leading to wider binaries at higher metallicities. This can change the type of interaction the binaries undergo later in their evolution, which has a non-trivial effect on the formation of double compact objects and CBC rates. Metallicity appears to be particularly important for the formation of mergers involving stellar BHs, whose formation efficiency in isolated channels may be greatly enhanced at low metallicity [1199, 1371, 1374, 1575–1577, 1597]. In addition, certain CBC formation scenarios (e.g., chemically homogeneous evolution; see section 4.1.1.2) may operate exclusively for metal-poor progenitors. Modeling the CBC population requires specifying the birth metallicity distribution of the progenitor stars forming at different times.

Environment plays a crucial role in the formation of CBCs. Other differences in the environments in which stars evolve allow for unique merger formation channels [1598]) (see section 4.1). In sparse stellar environments (galactic field), CBC formation involves exchanges of mass and angular momentum between stellar companions in isolated binary/multiple systems. In regions with a sufficiently high density of stars and compact objects, dynamical effects can further influence the formation of CBCs. In the densest environments (e.g., globular/nuclear clusters), compact object binaries can efficiently assemble dynamically and possibly undergo multiple mergers. Further differences in the CBC formation are expected in the presence of gas (e.g., in AGN disks), or in the case of extremely metal-poor/metal-free progenitors (see section 4.7). Therefore, modeling the CBC population requires specifying the cosmic formation history of the progenitors in the considered type(s) of environment.

CBC population — a mixed bag of everything. The overall observable CBC population is likely a superposition of subpopulations formed in multiple channels, whose relative contributions are expected to vary over cosmic time. The resulting cosmic merger rate density for a certain type of double compact objects (either BBH, BHNS or BNS binaries) is the sum over the contributions from different environments:

$$\begin{aligned}
 R_{\text{merger, DCO}}(t_{\text{merger}}) &= \frac{d^2 N_{\text{merger, DCO}}}{dt_s dV_c}(t_{\text{merger}}) = \\
 &= \sum_{\text{env}} \int dZ \int_0^{t_{\text{merger}}} dt_{\text{delay}} \left(\frac{dM_{\text{SFR}}}{dt_{\text{form}} dV_c dZ}(t_{\text{form}}, Z) \right)_{\text{env}}^{(i)} \times \left(\frac{d^2 N_{\text{form, DCO}}}{dM_{\text{SFR}} dt_{\text{delay}}}(Z) \right)_{\text{env}}^{(ii)} \quad (4.1)
 \end{aligned}$$

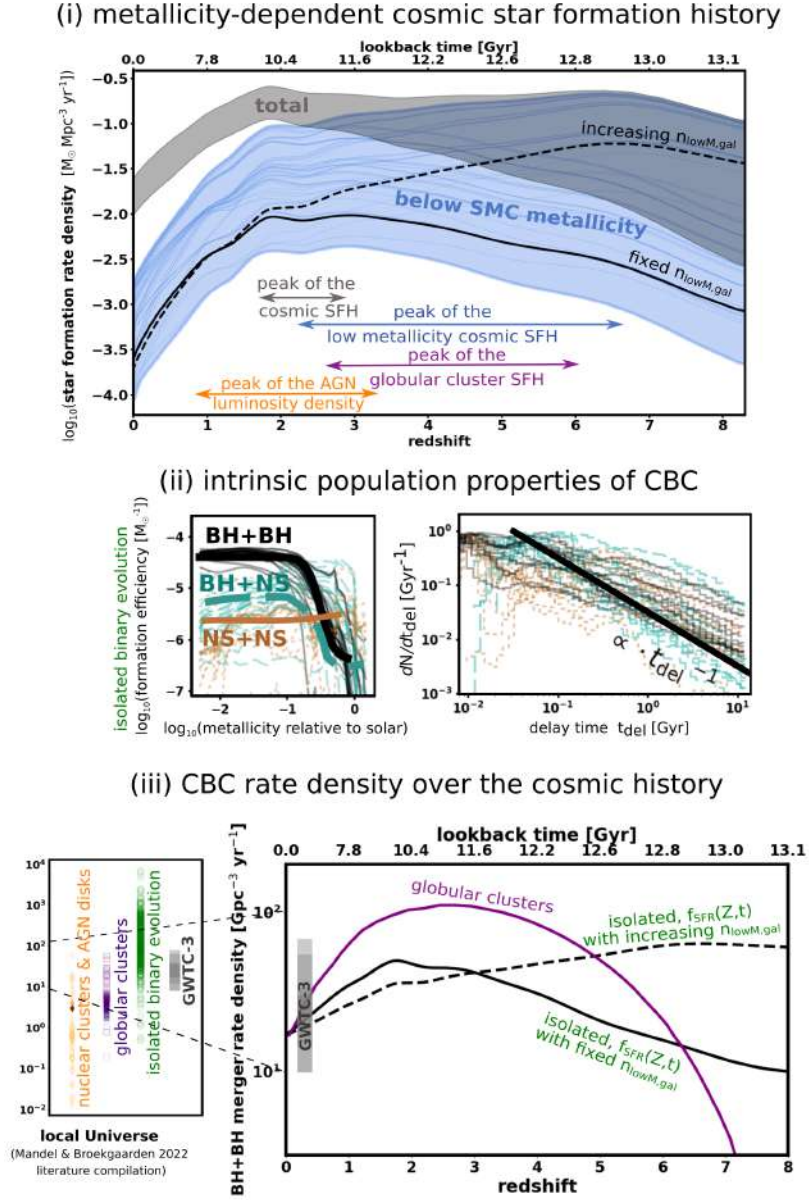


Figure 62. The key ingredients of compact binary coalescence rates. *Top:* star formation rate density (gray — total, blue — at metallicity lower than in the Small Magellanic Cloud) as a function of redshift/lookback time, spanned by observation-based $f_{\text{SFR}}(Z, t)$ models from [1599, 1600]. Solid and dashed black lines show variation in the evolution of the low-metallicity Star Formation History (SFH) due to the uncertain high redshift evolution in the number density of low-mass galaxies ($n_{\text{lowM,gal}}$). The arrowheads indicate the predicted range of redshifts for the peak of the star formation history in various types of environments and peak of the AGN luminosity density. *Middle:* η_{form} and delay-time distribution (DTD) illustrative of merging BBH (black), BHNS (turquoise) and BNS (brown) formed in isolated binary evolution channel. Thick lines: simplified main trends found in the literature. Thin lines: η_{form} and DTD for example binary population synthesis model variations from [1371] (left) and [1601] (right, normalised to the same value at peak) showing a diversity of shapes. *Bottom left:* literature compilation of the local BBH R_{merger} from [1571]. *Bottom right:* BBH R_{merger} redshift evolution examples normalised to the same local rate. Black dashed/solid lines — calculated using $f_{\text{SFR}}(Z, t)$ variations corresponding to those shown as black dashed/solid lines in the top panel and the η_{form} and DTD shown as black lines in the middle panel. Purple line — example BBH R_{merger} for globular cluster channel from [1602].

where t_s is the time in the source frame of the merger, V_c is the comoving volume, $t_{\text{merger}} = t_{\text{form}} + t_{\text{delay}}$ and Z stands for metallicity. The term (ii) in the second parenthesis describes the *intrinsic* properties of the CBC populations. Specifically, $d^2 N_{\text{form, DCO}}/dM_{\text{SFR}}dt_{\text{delay}}$ is the number of binary compact objects that form per unit stellar mass and merge per unit delay time with $t_{\text{delay}} < T_{\text{Hubble}}$. In general, term (ii) depends on the formation channel/environment (e.g. field, globular cluster with certain properties). We can call merger efficiency the term $\eta_{\text{form, DCO}} = dN_{\text{form, DCO}}/dM_{\text{SFR}}$ and delay time distribution (DTD) the term $\text{DTD} = dN_{\text{form, DCO}}/dt_{\text{delay}}$.

For a population of field stars formed with the same chemical composition, η_{form} and DTD depend on factors such as the fraction and initial parameters of stellar binaries/multiples, their evolution and interactions (mass transfer, tides), core collapse physics [1371, 1383, 1597, 1603, 1604]. For the CBC population formed in dense environments, the initial cluster properties (density profile, mass, binary fraction) can override the effects of stellar evolution on η_{form} and DTD [1547]. The term (i) in the first parenthesis of eq. (4.1) describes the cosmic volume-averaged star formation rate density at a given time and metallicity that ends in the considered type of environment (section 4.2.2). Its role is to weight the intrinsic population properties according to the chemical and star formation history representative of that environment. Term (i) is sensitive to factors such as the chemical evolution of galaxies, the physics of star formation and cluster formation history [1604].

The interpretation challenge. Many of the factors that go into the $R_{\text{merger, DCO}}$ calculation are subject to large uncertainties. Taken together, this leads to a wide range of model CBC rate predictions in the literature (see [1571] and references therein). Strikingly, these predictions span a range that is orders of magnitude larger than the current GW-based constraints on the local R_{merger} . This is the case for all CBC types, but is particularly striking for the BBH merger rate $R_{\text{merger, BBH}}$, which is the best measured. The lower left panel of figure 62 shows that the GW constraints already rule out most current models, which tend to overestimate $R_{\text{merger, BBH}}$ (especially when considering the possible contribution from multiple formation channels). However, the astrophysical interpretation of this fact, and hence the lesson to be learned from it, is unclear. The interpretation of these constraints is challenging because, even when considering a single CBC formation channel, $f_{\text{SFR}}(Z, t)$ (term i), η_{form} and DTD (term ii) degenerate into R_{merger} . In particular, different combinations of modeling assumptions about terms (i) and (ii) and the relative contributions of merger formation channels operating in different environments can lead to similar R_{merger} in the local Universe. The accuracy of local R_{merger} measurements for each type of CBC will continue to improve with ongoing and future GW observations, narrowing the overall parameter space of viable models. However, information on the redshift dimension is likely to be required to resolve the degeneracies in the interpretation. The role of ET constraints on the CBC rate in addressing this challenge is twofold:

- First, ET will provide joint constraints on the merger rate of all BBH, NSBH and BNS out to at least $z \sim 1$, mapping the rate evolution for all CBCs over the bulk of the cosmic history. This joint measurement is key: although BBH, BNS and BHNS are expected to be sensitive to different modeling uncertainties (e.g. due to the mass

dependence of stellar evolution or the efficiency of dynamical merger formation), one must be able to reproduce $R_{\text{merger}}(z)$ of all three from the same metallicity dependent cosmic star formation history.

- Second, ET will constrain the BBH merger rate well beyond the peak of the cosmic star formation history, where the uncertain factors associated with the environment (term (i)) are likely to dominate and lead to divergent $R_{\text{merger, BBH}}(z)$ predictions, and they may affect the mass-specific rate even more strongly, see section 4.3 on mass distribution. These factors, discussed further in the following subsection 4.2.2, could potentially be constrained with the Einstein Telescope.

4.2.2 Part (i): cosmic star formation history as a function of metallicity and environment

Each subsequent generation of stars forms with broadly different metallicities. Their exact chemical composition depends on the complex interplay between the timing and metal yields of enrichment events, the mixing of metals in the interstellar medium, metal-poor inflows and metal-rich outflows. These processes shape the chemical evolution of individual galaxies. Their combined evolution determines the metallicity dependent cosmic star formation history $f_{\text{SFR}}(Z, t)$, which characterises term (i) in eq. (4.1) for stars in the field and young/open clusters. The fraction of $f_{\text{SFR}}(Z, t)$ forming in massive bound clusters (the progenitors of today’s globular clusters), and the properties of these clusters, are expected to vary with the physical conditions in the local star-forming environment. Such factors further affect term (i) in eq. (4.1) characterising CBC progenitor formation in dense environments. The possibility of enhanced dynamical BBH merger formation in galactic nuclei (nuclear star clusters, gaseous AGN disks) introduces yet another class of environments with uncertain cosmic formation history. In principle, all environments characterised by different cosmic star formation histories can leave a unique imprint on the redshift evolution of the BBH rate, which can potentially be unravelled with future GW data.

4.2.2.1 Metallicity-dependent cosmic star formation history. Metallicity-dependent cosmic star formation history can be derived by combining statistical galaxy properties constrained by galaxy surveys [1249, 1599, 1600, 1605, 1606] or extracted from cosmological simulations of galaxy evolution [1607–1611]. Mixed approaches and simplified analytical prescriptions for the birth metallicity distribution of stars normalised to the chosen cosmic SFH are often used in the context of GW source population modeling (see [1604] for a recent discussion and comparison of literature assumptions).

The limitations of current methods result in $f_{\text{SFR}}(Z, t)$ being particularly poorly constrained at low metallicity and high redshift (figure 62, top panel). Due to limited resolution, low-mass and starburst galaxies, which are crucial for low-metallicity star formation, are currently not accurately described in cosmological simulations. Many processes that are key to chemical evolution (e.g., enrichment by different types of supernovae, feedback processes) rely on prescriptions with poorly constrained parameters. Studies of the effects of such parameter choices in the full cosmological volume are limited by the high computational cost. Empirical bounds on $f_{\text{SFR}}(Z, t)$ can be derived by combining observational distributions

of galaxy properties (star formation rate, gas phase metallicity, stellar mass) and number statistics [1249, 1599, 1600, 1605, 1606]. These are increasingly incomplete at high redshift and low mass, i.e. for typically metal-poor galaxies. Constraints on the gas-phase metallicity are typically inferred from the oxygen abundances, which have large systematic uncertainties [1612, 1613] that translate directly into the uncertainty in $f_{\text{SFR}}(Z, t)$ (see blue range in figure 62, which is based on oxygen abundance measurements). These systematics are one of the dominant factors driving the $f_{\text{SFR}}(Z, t)$ uncertainty at low redshifts. Furthermore, the metallicity dependence of stellar evolution (and CBC formation) is thought to be primarily driven by iron. While constraints on gas-phase oxygen abundances are rapidly becoming available for much larger samples and redshifts, constraints on iron abundances in the star-forming material are scarce and likely to remain a challenge. Consequently, oxygen is often used as a proxy for iron abundance. However, the two elements are produced by sources operating on different timescales and the link between them is not straightforward — failure to account for that can lead to significant errors [1614].

The case for next generation GW detectors. Combined, the issues discussed above lead to a wide range of observationally allowed $f_{\text{SFR}}(Z, t)$. This uncertainty cannot be ignored when interpreting the GW-based CBC population constraints, as the assumed $f_{\text{SFR}}(Z, t)$ can strongly affect the model CBC properties, especially R_{merger} [1249, 1371, 1375, 1390, 1597, 1603, 1604, 1615, 1616]. This is particularly important for BBH mergers, where $f_{\text{SFR}}(Z, t)$ alone can affect the local merger rate by more than an order of magnitude. If the strong low-metallicity preference of the BBH η_{form} seen in isolated channels is correct (black line, middle left panel in figure 62), the low-metallicity cosmic star formation history becomes the determining factor for the high redshift $R_{\text{merger, BBH}}$ of such formed BBH mergers. This part of the $f_{\text{SFR}}(Z, t)$ is shaped by the uncertain properties of low-mass and faint galaxies [1600], which map to a distinct $R_{\text{merger, BBH}}$ evolution at redshifts >2 (compare the upper and lower panels of the figure 62). Constraints in this regime will provide a potential new way, complementary to electromagnetic observations, to study cosmic star formation, the early (iron) enrichment history and the properties of low-mass galaxies in the reionisation epoch [1604, 1617–1619].

Different types of mergers are expected to have a different η_{form} dependence on metallicity (figure 62, middle left panel), and hence their $R_{\text{merger}}(z)$ may probe different parts of $f_{\text{SFR}}(Z, t)$. Given the exciting prospect that the properties of BBH mergers will be mapped to $z > 10$ by the next generation GW detectors, these systems received the most attention in the literature as probes of cosmic history. However, the interpretation of the redshift evolution of BBH mergers is complicated by the potential competing contributions of different formation channels. This is less likely to be the case for mergers involving NS (see section 4.1), where isolated binary evolution is thought to dominate [1571, 1598]. While the GW constraints will not be available for the lower-mass BNS and BHNS at such high redshifts as for BBH mergers, their R_{merger} will be measured for most of cosmic history. Due to the expected weak metallicity dependence of BNS formation, their $R_{\text{merger}}(z)$, modulo the effect of the DTD, is likely to track the total cosmic SFH (i.e. $f_{\text{SFR}}(Z, t)$ integrated over all metallicities). BHNS formation is found to be enhanced at low metallicity, although less so than BBH. This makes the relative rate of BNS and BHNS mergers another promising probe of $f_{\text{SFR}}(Z, t)$ [1371, 1604].

4.2.2.2 Cluster formation history and properties. Both observational [1620–1622] and theoretical studies [1623–1625] indicate that the efficiency of cluster formation and their properties depend on the star forming conditions. The emerging consensus is that denser and more actively star-forming regions (such as those found in local starbursts [1626] and in the early Universe [1627, 1628]) favor the formation of massive and dense star clusters that can remain bound and evolve into today’s globular clusters. Conversely, unclustered star formation leads to stellar systems that dissolve on short timescales and/or evolve into less dense open clusters. However, the full picture, from star formation through young clusters (where isolated CBC formation dominates) to the potential globular cluster stage (where dynamical interactions drive the CBC), remains fragmentary. This evolution is challenging to model because of the variety of spatial and temporal scales that need to be considered as clusters co-evolve with their host galaxies (but see [1629, 1630], for the recent efforts in the GW context). Related, the observational link between globular clusters and their progenitors is unclear because of their long history during which clusters can lose mass or dissolve completely.

Nevertheless, it is clear that only a fraction of $f_{\text{SFR}}(Z, t)$ will eventually end up in environments where dynamical CBC formation becomes efficient. Given the properties of local globular clusters [1631, 1632] and the conditions that favor the formation of their progenitors, this fraction is expected to be higher at higher redshifts and lower metallicities. Furthermore, the efficiency with which CBCs can be dynamically produced depends on the uncertain distributions of initial cluster properties [1547, 1633–1636]. In other words, term (ii) in eq. (4.1) is sensitive to cluster characteristics, such as their masses and densities/half-mass radii. Calculating the total R_{merger} from cluster environments requires summing over the contributions from clusters with different characteristics.

Finally, we note that while dynamical formation in globular clusters can make a substantial contribution to the overall BBH merger rate, this is less likely to be the case for BNS and BHNS mergers. Their lower masses make them less likely to reside in cluster central regions where dynamical interactions are frequent [1459] and natal kicks associated with NS formation cause many of them to escape the cluster [1637] (see section 4.1.2). The potential imprint of the dynamical formation on other population properties of BBH mergers (mass, effective spin and eccentricity distributions) further motivates the focus on this class of CBC in the literature in the context of this channel (see section 4.1.3).

The case for next generation GW detectors. Both the possibility that the majority of today’s globular cluster progenitors formed within the first Gyr of the Universe, and the possibility that the majority formed near the peak of cosmic star formation are considered in the literature. Such scenarios could potentially be discriminated with the $R_{\text{merger, BBH}}$ constraints provided by the next generation GW detectors [1638, 1639].

Several authors discuss the promise of future GW data to jointly constrain the formation history of globular clusters and some of their initial properties [1547, 1639]. It is important to note that GW data alone cannot constrain all these uncertain properties, since many of them are degenerate in BBH R_{merger} (e.g. the distribution of virial radii and the cosmic formation rate of globular clusters). However, it can provide complementary information to improve constraints from other studies, at least as long as the subpopulation of cluster-formed

mergers among the detected CBCs can be identified (e.g. by using GW-based information on masses, eccentricities and spins, see section 4.1.3).

4.2.2.3 Nuclear star clusters. A somewhat separate class of cluster environments are nuclear star clusters. These are the densest stellar systems in the Universe, and as such were proposed as promising sites for efficient dynamical CBC formation [1482]. They are found in the centres of different types of galaxies, are more common in massive star-forming objects, and can make up $\sim (0.1 - 1)\%$ of the stellar mass of the galaxy [1461]. They are thought to form from a combination of in-situ star formation [1461, 1640] and migration of globular clusters that spiral to the galactic center due to dynamical friction. These two scenarios imply different cosmic star formation histories for nuclear clusters, which could range from following the total $f_{\text{SFR}}(Z, t)$ in the second case, to resembling the globular cluster formation history in the first case. They would lead to different properties of the stellar and compact object populations in the cluster [1482]. Recent observational studies support an intermediate scenario, in which globular cluster accretion dominates the mass build-up of the nuclear cluster in dwarf galaxies, while in situ star formation dominates the formation of the most massive nuclear clusters in massive galaxies [1640, 1641]. It is currently unclear whether the history of nuclear cluster formation can lead to a distinguishable imprint on CBC R_{merger} evolution [338].

4.2.2.4 AGN disks. The absolute contribution to R_{merger} from the AGN channel is particularly uncertain and depends strongly on factors such as the number density of galaxies hosting an AGN, the lifetime and the characteristics of the AGN disk [1482, 1505, 1541, 1642]. Current population predictions for the AGN channel focus mostly on the local Universe, and the discussion of trends in the high redshift regime that will be probed by ET is lacking. Nonetheless, because of the short BBH merger delay times expected from this channel, the resulting $R_{\text{merger}}(z)$ is expected to be proportional to the number density of AGNs with the appropriate properties. The dominant AGN property influencing the BH merger rate appears to be the mass accretion rate onto the central supermassive BH [1506], also powering the AGN luminosity. However, the exact dependence is different for several processes that can drive BBH mergers in AGN disks [1505]. If the dominant processes are strongly enhanced at high accretion rates, then the BBH merger rate will scale with the abundance of the most luminous AGNs. Conversely, if such a dependence is weak, then low luminosity AGNs will contribute a substantial fraction of the total BBH merger rate, and the rate will scale better with the total AGN number density. Since the number density of faint AGNs appears to peak at lower redshifts than that of bright ones [1643], in the latter case the AGN-driven BBH merger rate would peak at lower redshifts. This has been found by Yang et al. [1506], who use the AGN luminosity function [1643] to derive AGN-driven BBH merger rate at $z < 2$. Their $R_{\text{merger}}(z)$ shows a milder evolution with a peak at lower redshifts ($z < 1$) than expected from other BBH merger formation channels and when compared to the peak of the cosmic SFH and of the cosmic luminosity density of AGNs [1644]. However, there are considerable uncertainties in the number density and luminosity distribution of (particularly faint) AGNs as a function of redshift [1644] which need to be explored further in future studies. Such a distinct $R_{\text{merger}}(z)$ evolution would help to identify the CBC subpopulation formed in

AGN disks. Next-generation detectors will allow to select such subpopulations, even for subdominant channels, by dramatically increasing the number of detections and the accuracy of their estimated astrophysical parameters. Information from the spatial correlation between observed GW events and AGN already constrains this formation channel at low redshifts and high AGN luminosities [1645–1647]. Future GW data could help to constrain it in the faint and high redshift regimes, where the approach based on the AGN luminosity function described above is challenged by the incompleteness of galaxy catalogs.

4.2.3 Part (ii): intrinsic properties of the CBC populations set by binary compact object formation efficiencies and delay times

The binary compact object formation efficiency and delay time distributions play a key role in setting the properties and rates of CBC populations across cosmic time. Binary compact objects are a rare outcome of massive star evolution — to form a binary compact object that can merge in a Hubble time the progenitor system typically has to survive many phases of massive star evolution and/or dynamical interactions. The physics underlying these processes impact the fraction of progenitor stars that can form binary compact objects: the so-called formation efficiency. This can be written down as a ‘Drake equation’: a multiplication of factors that a population of stars needs to satisfy to form a CBC. For the formation of binary compact objects through the isolated massive binary star channel, the key factors can be written as something like:

$$\eta_{\text{form}} = f_{\text{init}} \times f_{\text{survive SNe}} \times f_{\text{mass transfers}} \times f_{\text{merge}}, \quad (4.2)$$

where each factor f is a factor selecting only a subset of a given initial population of stars that has the properties to become a CBC source, as we will describe in more detail for the channels below.

The delay time distribution (DTD) spans a broad range of values (from many Myr to Gyr) and as a result, for binary compact objects formed at a given redshift in our Universe, can significantly impact at which redshift these sources can contribute to the merger population and thus what sources GW detections probe as they reveal the CBC populations across cosmic time [1604]. This is because the orbital angular momentum loss by GW radiation is typically very small. For a binary consisting of two compact objects with masses M_1 , M_2 , eccentricity e , and semi-major axis a the change in separation due to GW radiation is given by [1364]

$$\frac{da}{dt} = -\frac{64G^3}{5c^5} \frac{M_1 M_2 (M_1 + M_2)}{a^3} \frac{1}{(1 - e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 \right), \quad (4.3)$$

which, for a circular orbit and a system with masses $M = M_1 = M_2$, simplifies to $t_{\text{insp}} \approx 0.1 \text{ Gyr} (a/R_\odot)^4 (M/M_\odot)^{-3}$. This means that the delay times depend strongly on the separation, as $t \propto a^4$. If the distribution of orbital separations at the moment of binary compact object formation is given by some power law distribution $dn/da \propto a^{-\gamma}$, we can use this to derive that the DTD is approximately described as:

$$\frac{dn}{dt} \propto \frac{dn}{da} \frac{da}{dt} \propto a^{-\gamma} a^{-3} \propto t^{-(\gamma+3)/4} \quad (4.4)$$

If we take that, based on observational evidence, binaries are born with a separation distribution described by $\gamma \approx 1$ [1424, 1648–1650], the equation reduces to $dn/dt \propto t^{-1}$, which is a commonly used distribution and also found by observations (e.g., [1651–1654]).

The formation efficiency and DTD are still uncertain and can vary for different formation channels based on different underlying physical processes leading to binary compact object formation. The details of these ingredients play a major role in shaping the CBC properties as a function of redshift. Observations with Einstein Telescope of the CBC merger rate as a function of redshift will probe the formation efficiency and DTD in enough detail to enable inferring the underlying physics [10, 12, 15, 107, 1655, 1656]. We discuss this in more detail below.

4.2.3.1 Isolated massive stars (the field). In eq. (4.2), the key factors for determining the formation efficiency of CBCs in the field and their DTDs are the initial conditions, and those relating to mass transfer episodes, stellar winds, and supernovae — fundamental phases in the lives of massive stars that are poorly understood and can be uniquely probed with the Einstein Telescope. The key factors entering the CBC efficiency depend on the specific formation channel within the field (only stable mass transfer, classic common-envelope, CHE, triples, see the overview in section 4.1). Here below, we list some of the most important among such factors.

First, the initial conditions of the stars need to be sufficient for binary compact object formation, (f_{init}). The system needs to have stars that are massive enough to form NSs or BHs, the system typically needs to be born with a small enough separation to initiate a mass transfer phase during its lifetime (as mass transfer interactions are expected to be required to bring the binary compact object to a tight enough orbit to merge within a Hubble time) but not too small for it to undergo a stellar merger at ZAMS, and the system needs to be born as a binary or a triple star system (typically described by the “multiplicity factor”). The initial conditions and binary fractions of massive stars are uncertain and can be informed by observations [1424, 1657–1664]. Second, the binary (or hierarchical) system typically disrupts during the supernova by the significant natal kicks of hundreds of km/s that the system can receive when the NS (or BH) forms (e.g., [1665, 1666]). This likely disrupts the majority of the systems during the supernovae (especially the first supernova when the system typically has a wide orbit), though uncertainties remain [1378, 1667, 1668]. This probability is expressed by the factor $f_{\text{survive SNe}}$. Third, the system is typically thought to undergo several mass transfer phases, either stable or an unstable common-envelope phase. The stability of mass transfer and the mass and angular momentum loss during these phases might well be one of the most uncertain aspects of massive star evolution. What stars and what fraction of stars can successfully eject the envelope and form a tight system is uncertain and adds an important factor in the formation efficiency equation $f_{\text{mass transfer}}$ [1371, 1374, 1375, 1378, 1391, 1577, 1603, 1669]. Last, even once the binary compact object has formed it needs to have a tight enough orbit to merge within the age of the Universe so that it can merge and be observable today, likely removing another half of the systems described by the factor f_{merge} [1364]. Other field-based channels, including CHE, triples and hierarchical systems have similar ingredients impacting the DTD and formation efficiency, but often include extra Drake factors, corresponding to questions such as: What

is the metallicity at which CHE formation channel operates [1401, 1402, 1404]? How do interactions with a triple or hierarchical companion contribute to binary compact object formation efficiency and DTD [1440–1443, 1670]? How do Population III stars form, evolve, and produce CBCs [1034, 1415, 1671–1675]? These factors combined with the DTD decide the merger rate of CBCs. Observing the CBC merger rate across redshift with the Einstein Telescope will be crucial to infer the formation efficiency and DTD and study the underlying physical processes in massive star evolution [1617, 1619, 1676].

4.2.3.2 Cluster formation (dynamical). The formation efficiency and DTD distributions for binary compact objects formed in dense star clusters is still under debate and depends on the dynamical formation channel at play and the expected binary compact object properties, as described in detail in section 4.1. Important open questions that ET will probe by inferring the CBC merger rate include the following. First, what are the CBC host cluster properties? The formation rate and history of globular cluster populations across cosmic time is still under debate as well as the (initial) mass distribution of globular clusters, which is important as the formation efficiency and DTD are found to be significantly impacted by the cluster mass [1528, 1539, 1540, 1546, 1547, 1624, 1633–1636, 1677, 1678]. What fraction of binary compact objects can be retained in the cluster and undergo multiple interactions thought to be required to harden the binary enough to form a CBC [1679, 1680]? What fraction of merger products receives low enough recoil kicks such that it can be retained in the cluster center and form hierarchical mergers [1437, 1551, 1681, 1682]? How does the presence of a supermassive black hole impact the formation and merger rate of CBCs [1540, 1683, 1684]? What is the role of accretion disks around supermassive black holes and potential migration traps in forming CBC sources across cosmic time [1504, 1531, 1642, 1685–1688]? Einstein Telescope is poised to detect the CBC merger rate with sub-percent precision; this, together with measuring the properties of CBC across cosmic time, is expected to unveil the formation channels and uniquely inform the key ingredients contributing to the CBC rate: the star formation histories, formation efficiencies, and DTDs [10, 12, 15, 107, 1551, 1639, 1655, 1656].

4.2.4 Future outlook

- *CBC merger rate is expected to evolve as a function of redshift.* Its evolution is not expected to follow directly the cosmic SFH. The simple link to cosmic SFH is modified by the delay time distribution (relevant for all types of mergers), further affected by the metallicity dependence of the CBC formation efficiency (BH-NS and BBH mergers), and possibly by contributions from multiple formation channels (expected primarily for BBH mergers) whose relative importance may vary with redshift. $R_{\text{merger}}(z)$ encodes a wealth of astrophysical information, especially at high redshifts that will be accessible to ET, where much of the uncertainty in current models lies.
- *There is a large parameter space to explore with ET, and a challenge for the astrophysical interpretation.* There are substantial uncertainties due to factors related to both the formation environment (metallicity dependent cosmic star formation history, cluster formation history and initial properties) and intrinsic properties of the CBC merger population (delay time distribution and formation efficiency for different formation

channels). All of these factors affect the predicted evolution of the CBC merger rate with redshift. However, many of them are degenerate in R_{merger} . Future constraints on the joint redshift dependence of all CBC and on the high redshift evolution of BBH merger rate will be key to break such degeneracies in the interpretation. These constraints will be most informative if subpopulations of mergers formed in specific channels/environments can be identified, and if improving electromagnetic constraints are used to place bounds on the population models. From this perspective, the greatest promise of ET lies in the use of future GW constraints to complement the electromagnetic studies.

- *It is unclear whether the redshift-dependent merger rate of the entire BBH population will show multiple features (peaks) or not.* The former case can be expected if different formation channels dominate or give similar contributions to the merger rate at different redshifts. The latter case could occur if one of the formation channels dominates the BBH formation, or if multiple channels lead to similar merger rate evolution over cosmic history. Both scenarios seem possible within the uncertainties of current models and ET will provide constraints to distinguish between them. More detailed astrophysical inference requires: (a) on the theoretical side, reliable models for $R_{\text{merger, BBH}}(z)$ from various formation channels/environments, or the ability to confidently identify the corresponding subpopulations in the data based on other properties, and (b) on the empirical side, accurate $R_{\text{merger, BBH}}(z)$ measurements up to high redshifts and the ability to accurately infer the redshift of sources and associated features in population properties [1180, 1602, 1676]. For the latter, a network of next-generation GW detectors is advantageous. In general, a network of GW detectors leads to better redshift measurements for individual sources and thus to tighter constraints on the evolution of CBC population properties compared to a single detector (which requires longer observing times to reduce statistical uncertainties). The fraction of GW detections at $z > 1$ with accurate luminosity distance measurements increases by almost an order of magnitude for a network of 3 next-generation detectors compared to a single ET [107, 1167].
- *Full population inference for CBC catalogs observable by a detector with ET capabilities is needed.* As discussed in this section, some of the poorly-constrained astrophysical assumptions lead to divergent predictions of the evolution of the BBH merger rate only at high redshift $z > 3$. These are of particular interest in the context of next-generation detectors, which are going to provide unique constraints on the rate in this regime. To better quantify which astrophysical assumptions we can expect to constrain with ET, it is important to quantify (i) what is the minimum factor to which we can constrain the CBC merger rate, especially at high redshifts and (ii) what is the minimum amplitude/width of a feature in the BBH merger rate that can be confidently recovered from the population analysis of ET-like data (as a function of redshift).

The scalability of currently employed population inference schemes to the numbers of detections and accuracy on the reconstruction of the single events' parameters achievable at ET is limited by the high computational cost. Work in this direction is ongoing (see

also the discussion in section 10.2.4), extending methodologies to more flexible models, improving their efficiency, as well as exploring new tools, such as machine learning techniques.

- *ET is essential to make significant progress in observing the CBC merger rate.* In the coming years updating current ground-based detectors (O4 and O5) will likely bring the number of BBH mergers to around ~ 500 with detections out to $z \sim 1$ [392], but this is not enough to reveal important parts of the CBC merger rate such as the peak of the merger rate (expected around $z \sim 2$), multiple features or peaks in the merger rate especially expected at $z \gtrsim 2$ as different formation channels might be dominating the rate at different redshifts. ET will detect stellar-mass black hole mergers out to (and beyond) redshifts $z \gtrsim 10$, and BNS and BHNS beyond $z \gtrsim 2$, which will drastically change the field by increasing our understanding of the redshift behavior of CBCs [10, 15, 107, 1655, 1656].

4.3 Mass function of BHs and its evolution with redshift

4.3.1 The scientific potential of the mass function

GW astronomy is well-suited to measure the masses of binary components. From the physics encoded in the observed gravitational waveform, we can infer the source properties [1689, 1690]. The inspiral portion of a binary coalescence is determined, to leading order in the post-Newtonian expansion, by the chirp mass of the binary, with the mass ratio contributing at the next order [1691–1694]. The transition to merger and the subsequent ringdown are principally determined by the binary’s total mass [1695–1697]. The sensitivity of next-generation detectors means that for the closest (and hence highest SNR) sources, these mass measurements will have unprecedented precision [16, 1167, 1586, 1698, 1699].

The masses of BHs are a key tracer of their evolutionary history. Considering a star in isolation, its remnant mass is primarily set by its initial mass [1592]. Given the initial mass function of stars, we therefore expect more low-mass BHs [1664, 1700]. However, the mapping between initial and final mass is not monotonic: different evolutionary histories can lead to different amounts of mass loss, e.g., through stellar winds and supernova explosions. The amount of mass loss also varies with metallicity [1701–1703]. Stellar winds are stronger for higher metallicity stars, meaning that lower mass remnants are typically formed.

With GWs, we observe BHs in merging binaries. Therefore, we must consider the processes that form binaries, and drive them to be close enough to merge. As discussed in section 4.1.1.1, periods of mass transfer between binary stars will alter the orbital configuration and remnant masses. Dynamical exchanges may preferentially select the most massive objects in a cluster to form a merging binary [1481, 1502, 1704]. Accretion for binaries embedded within active galactic nucleus disks will grow BHs [1505, 1705]. This wealth of diverse astrophysical processes leaves imprints on the masses of observed BHs —both on the individual component masses and their mass ratio— and through reconstructing the mass distribution we may uncover the underlying physics that shaped these binaries.

The component masses are one piece of information about the origins of BHs. Further information is encoded in other measurements. The overall merger rate (section 4.2), the spins

(section 4.5), the distribution in redshift, and the orbital eccentricity all contain information about the origins of binaries. Crucially, looking at the correlations between parameters (such as whether spin alignment is correlated to mass ratio) can reveal information that is overlooked when considering each parameter in isolation [1706–1708]. Only by combining all this information can we untangle the complicated physics of how BHs form (section 4.1), distinguishing primordial from stellar BHs (section 4.6), and BHs that formed from Population III or from later stars (section 4.7).

Understanding the distribution of masses also has implications beyond understanding the sources themselves. As we discussed in section 3.3, GW observations can be used to measure the expansion of the Universe [9, 1157, 1709]. While the GW measurement provides the distance [1138, 1142], the redshift has to be obtained through other means. If no electromagnetic counterpart is detected (which is expected to be the case for the majority of GW sources) we can link the GW source to its host galaxy (and thus merger redshift) through a statistical association with galaxy catalogs, as has been done in recent years for both simulated data [1142, 1145, 1147, 1160, 1183–1185] and actual data [9, 1143, 1186–1189, 1191–1195]; see section 3.3.1.2 for extended discussion. Alternatively, features in the mass distribution, such as mass gaps or peaks, can be used to determine the redshift [1151–1156], see again section 3.3.1.2; however, this requires that we understand how these features of the mass distribution evolve with redshift [1200, 1710]. In either case, calculation of the distance is influenced by mass measurement: a signal can potentially be from a louder source (higher mass) at greater distance or a quieter source (lower mass) at small distance. Therefore, when performing an inference to measure cosmological parameters, we must simultaneously infer the underlying astrophysical source population [9, 1158, 1161].

Next-generation ground-based GW observatories such as ET are well-designed to uncover the details of the mass distribution of BHs. Their sensitivity means that we can detect a large number of BH binaries, and make detections out to large distances [16, 1167, 1181, 1699]. This sensitivity compared to current-generation GW detectors is illustrated in figure 63, which shows the time necessary to obtain a significant number of detections to constrain the merger rate as a function of redshift and primary mass. This is computed based on the POWER LAW+PEAK parametric model determined from GWTC-3 [7], although the upper limit on BH masses of the model has been increased to $m_{\text{max}} = 250M_{\odot}$ to highlight the expected sensitivity of ET at high masses.³² The total merger rate has been taken to follow the cosmic star formation history as given by [1587], with the rate at $z = 0.2$ fixed to the value inferred from GWTC-3 [7]. The precision of population measurements typically scales with the square root of the number of detections [1711, 1712], so the increased sample size will enable a revolutionary advancement compared to existing GW observatories. The long-distance reach enables reconstruction of the mass distribution over cosmic time — although figure 63 was constructed assuming it is redshift independent, we expect this distribution to change with time due to the evolution of the source population, e.g., through changes in the metallicity distribution [1198, 1375, 1416], in the balance of star formation in

³²The median value derived for m_{max} from GWTC-3 is poorly constrained, with large support up to $m_{\text{max}} = 100M_{\odot}$ which is the edge of the uniform prior used [7].

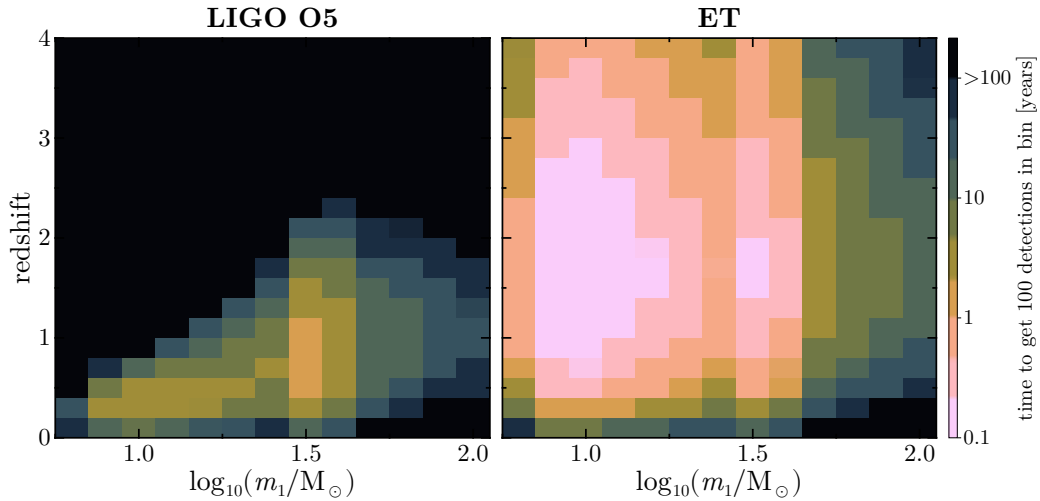


Figure 63. Time to detect 100 merging binary BHs in bins of size $\Delta z = 0.2$ in redshift and $\Delta \log m_1 = 0.1$ in primary mass, indicative of the number of measurements required to provide a constraint on the rate with an error of 10% in each bin. The distribution of masses is taken to be the POWER LAW+PEAK parametric model derived from GWTC-3 [7] using the median values for all parameters (which include a localized Gaussian peak at $34M_\odot$), and increasing the upper cut on BH mass to $m_{\text{max}} = 250M_\odot$. This figure assumes the sensitivity curves adopted in [16] for a 2L configuration with 15 km arm-length.

the field compared to clusters [1570, 1656], and due to the prevalence of primordial compared to stellar-origin BHs [1182, 1713].

The combination of the large number of detections with the increased detection horizon means that with these observations we can precisely reconstruct not only the overall mass distribution, but the distribution as a function of redshift, and explore correlations at different points in the Universe’s history [1714]. As summarised in table 5, this is precisely the information needed to unravel the intricate mysteries of how (the progenitors of) BHs form and evolve. Throughout the rest of this section, we link the scientific outcomes listed in table 5 to the science enabled by ET.

4.3.2 Reconstructing the mass distribution

To uncover the astrophysics of the population of GW sources, we must perform a sequence of analyses. First, signals must be identified in the data stream, and their source properties inferred [6, 1715]. The sensitivity of next-generation detectors like ET means that we can detect sources at large distances, and that the closest sources (as currently detected by the LVK [6]) will be measured with high SNR, meaning that the source properties will be precisely measured [16, 1167, 1586, 1691, 1698, 1699]. To map from the observed sample to the underlying astrophysical population, we must correct for selection effects [1213]. Some signals are easier to detect than others, hence the observed distribution is biased compared to the true source distribution. This is the case for current GW observations [6]. However, the sensitivity of next-generation detectors means that there will be a range of masses and redshifts where the probability of detection is effectively equal to 1; hence all these sources

Scientific outcome	High SNR	High redshift	Large sample	Complete low-mass census	Complete local census	High-mass binaries
Measurement of different formation channels	✓	✓	✓			
Reconstructing correlations between binary parameters	✓	✓	✓			
Identification of rare binaries	✓		✓		✓	✓
Evolution of mass distribution across cosmic time		✓	✓	✓		
Tracing stellar evolution with metallicity		✓	✓		✓	✓
Inferring the remnant masses of stars between $10\text{--}100 M_{\odot}$		✓	✓	✓		
Inferring the remnant masses of stars with $>100 M_{\odot}$		✓	✓			✓
Measurement of mass gaps	✓		✓	✓		✓

Table 5. The interdependence of observational capabilities of next-generation GW detectors and how they may uniquely answer key scientific questions about the masses of BHs. Shades of green indicate how relevant an observational capability is for the envisioned science goal, while check marks designate essential capabilities.

will be detected, and the observed population is a complete, unbiased census of all the merging compact-object binaries. To consider the full ranges of masses and redshifts, selection effects will still need to be accounted for. Finally, we can combine the inferred properties of the observed sources with the calculated selection effects to constrain the astrophysical population: this requires an assumption about the underlying distribution.

There are multiple approaches to modeling the underlying distribution when performing a population inference. Broadly, these may be categorised into three approaches:

- *Using predictions driven by population models.* We can use predictions from theoretical models of different formation channels to match observations [1716–1718]. The observed population is a mix of different channels, and so it is necessary to consider a combination of channels in this inference (omitting channels leads to biased results [1570]). The observations can be used to infer the branching ratio between different channels, and any uncertain model inputs (section 4.1). Hence, this approach provides the most direct probe of the underlying physics. However, this approach also requires accurate model predictions [1719], and the calculation of (potentially computationally expensive) models across a range of input parameters.
- *Using parametric models.* We can alternatively use a flexible parametric model, such as a power-law, to fit the population [1720–1722]. This does not rely upon theoretical predictions for the inference, making it easier to identify unexpected features in the data. However, it is still necessary to define a model, and if the model is not a good fit

to the underlying data, the inferred fit will not be a good representation of the truth. The parameters of the model need not have direct physical meanings, making their interpretation more difficult than if using theoretical predictions. However, the model features, such as the position of a peak in the mass distribution, may drive development of theoretical models to explain them.

- *Using non-parametric models.* All models require parameters to define them, but some approaches allow the number of parameters to be adapted during the fitting (potentially growing to an infinite number). These are referred to as non-parametric models. Examples include using a Dirichlet process Gaussian mixture model [1221, 1222] or an autoregressive process [1723]. The flexibility means that they can recover unexpected features in the observations. The result of the inference is a reconstruction of the underlying population, but this does not directly encode information about the source physics.

In each case, the expected measurement precision on the population properties scales with the square root of the number of detections. The high detection rate of next-generation detectors will mark a significant advancement in our understanding of these populations.

In practice, we expect a combination of different approaches to be used to understand results. This may be illustrated in considering the evolution of LVK results. The first inference of the astrophysical population used a simple power-law for the masses — given the small number of detections at the time, that was the most complicated model that was considered justified [1690]. As the population has grown, more complicated parametric models have been introduced, adding peaks and dips to the model [1724–1727]. Flexible features have been added to the parametric models (such as using a variable number of Gaussian kernels to reconstruct the mass distribution [1728], using a binned distribution [1729–1731], or adding a spline to model additional features in the mass distribution [1732]), starting to bridge the gap to non-parametric models [7]. These allow for checks of whether the parametric models are capturing all of the features in the astrophysical population. With the current population of order ~ 100 observations, it is still difficult to understand if a lack of detections is the sign of a physical dip in the population, or the result of a Poisson fluctuation [1733]. Despite the relatively small sample size, non-parametric approaches have also been applied, showing that some features found by the parametric models are robust (such as a peak at low masses) [1221, 1723, 1734–1736]. Direct comparison with population-synthesis predictions have also been made [1704, 1737–1739], demonstrating that there is probably a mix of different formation channels [1570, 1740, 1741]. As the number of detections has grown, we have gone from identifying general trends, to starting to identify features and pointing out limitations in our current theoretical models.

Over the coming years, we expect the current generation of GW detectors will yield hundreds of new detections [392]. Our understanding of the astrophysics of these sources will therefore advance greatly. This knowledge, however, will be eclipsed by that resulting from next-generation detectors, as demonstrated in figure 63. Detectors like ET will detect large numbers of sources ($\sim 10^5 \text{ yr}^{-1}$), allowing us to perform the same types of analyses as we

do currently for individual redshift bins — we will not only be able to map out the mass spectrum in high detail, but also see how it varies across cosmic time [1714].

4.3.3 BH mass spectrum and outstanding questions

The BH mass spectrum holds crucial information about the evolution and final fate of stars. Here, we address BHs of stellar origin only (primordial BHs are discussed in section 4.6) and the richness of the physics that could be inferred from the mass distribution. The masses of such BHs depend on the final moments of the life of their stellar progenitors and whether they explode in a supernova, collapse directly into a BH, or experience partial explosion and fallback of mass onto the forming compact remnant, possibly giving rise to a BH. The question of the explodability of stars relates to the supernova mechanism (e.g., electron-capture supernovae, core-collapse supernovae and pulsational pair-instability supernovae) and the final structure of stars, which is set by their evolutionary history. This evolution depends strongly on the metallicity of the stars and the formation channel of the binaries (i.e., their interactions with their environment or stellar companion). For example, for single stars, there is a critical metallicity beyond which stars lose so much mass by stellar winds that virtually no star can form a BH anymore; see [1742] for a recent review of many of such aspects related to the formation of BHs in stars.

As shown in figure 63, ET will reconstruct the BH mass spectrum at various redshifts, and hence at different average metallicities. Therefore, it will be possible to directly probe theoretical models and a wide range of basic physics, as highlighted in table 5. In the following, we describe possible features in the BH mass spectrum that will be observable with ET, and explain the essential physics that governs them. In essence, ET can help solve the complex question “Which stars form BHs?”. This question encompasses aspects ranging from stellar explodability to the physics of pair-instability pulsations, but also concerns the formation channels (i.e., stellar collisions and hierarchical mergers). We start the discussion with the explodability of stars (section 4.3.3.1) and then dive into various features of the BH mass spectrum. We consider the transition from the neutron star to the BH mass spectrum and the possibility of the neutron star-BH mass gap (section 4.3.3.2); we explain theoretical ideas and observations of global peaks in the BH mass spectrum (section 4.3.3.3), and we finally move to the upper mass region of the BH mass spectrum, where is expected an upper mass gap with a pile-up due to pulsational pair-instability supernovae (section 4.3.3.4).

4.3.3.1 Supernova or direct collapse. Stars with low iron-core masses and those undergoing electron-capture supernovae are thought to leave behind neutron star remnants, discussed in section 4.4.1. We now focus on iron-core collapse situations within the framework of neutrino-driven supernovae (pulsational pair-instability supernovae are discussed in section 4.3.3.4). The pre-supernova stellar structure is often summarized by the so-called compactness parameter $\xi_M = (M/M_\odot)/(R(M)/1000 \text{ km})$, which is the dimensionless mass-to-radius ratio at a given mass coordinate M , usually taken to be $2.5 M_\odot$ to measure the compactness just outside the iron core [1743]. High values of compactness ($\xi_{2.5} \gtrsim 0.45$) signal BH formation, while neutron star formation may occur for lower values. Over the last decade, various groups reported a qualitatively similar landscape of compactness showing clear peaks and troughs as a function of the core mass of stars. The peaks could indicate islands of BH

formation, but given the uncertainties in the supernova explosion mechanism and core-collapse dynamics, it remains an open question which pre-supernova stellar structures lead to BH formation. If there was a direct link between high compactness and BH formation, an almost universal picture would emerge with probable BH formation at carbon-oxygen core masses of $7\text{--}8 M_{\odot}$ and $>13 M_{\odot}$: a qualitatively similar landscape is found for different stellar-evolution codes [1592, 1744–1748], largely independent of evolutionary histories of stars (i.e., single stars, binary-stripped stars, accretors of binary mass transfer and even stellar mergers) [1747, 1749], metallicities [1750–1752], and uncertain physics such as convective boundary mixing [1753]. Since pre-supernova red supergiants follow a tight carbon-oxygen core mass-to-luminosity relation, the high compactness at carbon-oxygen core masses of $7\text{--}8 M_{\odot}$ predicts a luminosity range of red supergiants that shall collapse into BHs and not explode in a supernova. The best candidate red supergiant to have collapsed into a BH, N6946-BH [1754, 1755], had an inferred luminosity in general agreement with this prediction [1749, 1753].

On top of the general compactness landscape, there is considerable variability, and the connection of the pre-supernova stellar structure to BH formation is still uncertain. While three-dimensional simulations of iron core collapse supernovae seem to converge on the explosion mechanism [1756, 1757], the exact outcome remains uncertain because it is computationally challenging to reach a conclusive statement regarding compact-object formation and possible fallback of envelope material onto the proto-neutron star. Starting from one realisation of one-dimensional simulations of core collapse, a complex picture emerges that reproduces the general picture described above, but additionally shows sub-structure in the form of smaller islands of BH formation [1758, 1759]. Another set of such computations gives opposite results in terms of explodability and hence compact remnant masses [1760]. With ET, there will finally be a direct observation of the BH mass spectrum that will allow us to put constraints on the question of BH formation in stars and the explosion mechanism of core-collapse supernovae (section 4.3.3.2, section 4.3.3.3 and section 4.3.3.4).

4.3.3.2 The neutron star-BH mass gap. More than two decades ago, studies on the mass distribution of BHs based on X-ray binary observations suggested an absence of BHs in the $(3\text{--}5) M_{\odot}$ range [1761–1764]. This idea of a gap between the most massive neutron stars and the least massive BHs, also known as the neutron star-BH mass gap (or sometimes, the “lower-mass gap”), hinges on the mass measurement in one of the X-ray binaries (GRO J0422+32), and has been a topic of active debate ever since it was first suggested, due to expected observational and evolutionary selection biases [1765–1768]. Nonetheless, multiple theoretical models have been proposed to explain this gap as the result of a discontinuous remnant-mass distribution, driven by either the supernova engine [1769, 1770] or the progenitors’ density profile, which determines if a supernova fails [1771, 1772].

The first catalogs of GW observations showed tentative support for a relative dearth of merging BBHs with component masses between $2.5\text{--}5 M_{\odot}$ [1727, 1773, 1774]. Studies have linked this dearth of low-mass BHs observed with GWs to the underlying supernova engine (e.g., [1477, 1775]). However, the population of GW sources is a heavily biased population in terms of detected events and should receive equal, if not more scrutiny over its evolutionary selection effects as X-ray bright sources. Binary evolution alone can also produce a dearth of low-mass BHs without invoking a discontinuous remnant-mass distribution [1380].

Moreover, at the time of writing, there has been at least one confident detection of a GW source with a primary component falling into this neutron star-BH mass gap (GW230529_181500) [1475], while there have been two detections with tentative secondary component masses detected in this mass range (GW190814, GW200210_092254) [7].

How sparsely populated is this mass range? Is this the result of the remnant mass function or of evolutionary selection effects? Is this dearth related to the metallicity and/or redshift of star formation? ET will be perfectly positioned to answer these questions, since the improvement of ET over current LVK detections will be most significant in the relevant mass regime of this purported mass gap (figure 63).

4.3.3.3 Global peaks of the BH mass spectrum. There are several lines of evidence that point towards an abundance of $\sim 9 M_\odot$ BHs. First, there seems to be a peak at about this mass in BH mass distribution of X-ray binaries [1765]; second, a similar peak is found in BBHs observed with GWs [7], and third, BHs found in field binaries often have a similar mass (e.g., Gaia BH1 and BH2 [1776–1778]). The current GW data further reveals a significant peak in the primary BH mass distribution at $\sim 35 M_\odot$ and a tentative peak at $\sim 16 M_\odot$ [7, 1779]. While the significance of these features still needs to be assessed [1724, 1733, 1736], we discuss the possible origins of these and other features, and what information they may hold for our understanding of BH formation in light of the capabilities of ET.

The isolated binary evolution channel for BBH mergers makes the prediction of peaks in the BH mass spectrum at $\sim 9 M_\odot$ and $\sim 16 M_\odot$ [1380, 1752], and the $\sim 9 M_\odot$ peak is even predicted to exist because of two different mechanisms. Both mechanisms probe different physics, and revealing such features in observed populations of BHs would allow us to directly constrain this physics.

First, the stable mass transfer channel has been proposed to naturally lead to a dearth of low-mass compact objects [1380]. The stability of mass transfer depends on the mass ratio of the systems involved in the mass transfer. This places strong constraints on the progenitor population masses that can lead to merging BBH systems. This should lead to a minimum primary compact-object mass that can follow from the stable mass transfer channel. If this is the case, then the global peak at $9 M_\odot$, and a decrease towards lower masses could be the transition from the stable mass transfer channel as the dominant formation channel, to the common-envelope channel (cf. [1385, 1387–1389, 1780]). This is in line with recent work that suggest that the dominant formation channel for binary neutron star mergers is the common-envelope channel, while BBH formation has been suggested to be dominated by the stable mass transfer channel [1574, 1781]

Second, both BHs in BBH mergers from isolated binary evolution are from progenitor stars that have been stripped of their hydrogen-rich envelopes. Envelope removal is known to affect the core structures at the pre-supernova stage: almost independently of metallicity, binary-stripped stars with carbon-oxygen core masses of $\sim 7\text{--}8 M_\odot$ achieve stellar structures at core collapse that are difficult to explode within the framework of neutrino-driven supernovae, and may thus give rise to direct collapse BHs. Since binary-stripped stars at a large range of metallicities follow a carbon-oxygen core mass-final stellar mass relation, they are predicted to form BHs of similar mass around $\sim 9 M_\odot$ and largely independent of metallicity [1752]. A similar reasoning holds for higher carbon-oxygen core masses of $\gtrsim 13 M_\odot$ and binary-stripped

stars with such cores are found to exhibit core structures that appear likely to directly collapse into BHs [1750–1752]. Indeed, one-dimensional, core-collapse simulations show that these stars likely produce BHs of $\sim 9 M_\odot$ and $\gtrsim 12 M_\odot$ [1782]. The models clearly predict the existence of such BH masses largely independent of metallicity and this can be directly probed by ET. The physics governing these BH masses is a combination of stellar, nuclear and SN physics [1752]. On the stellar physics side, one can, e.g., test binary evolutionary channels leading to the formation of BBH mergers and convective boundary mixing during core helium burning. In terms of nuclear physics, the characteristic carbon-oxygen core masses are set by the complex interplay between nuclear burning processes during the late stages of massive star evolution and thermal neutrino losses, and particularly the generally important $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction [1783]. Regarding supernovae, a broad range of physics is relevant to establish the connection of pre-supernovae core structures that appear difficult to explode and the actual explodability of stars. Another important question is how much of the final stellar mass may form the resulting BH and how much is still ejected in the collapse of the star.

From data of the third GW Transient Catalog [7], a clear peak at $\approx 35 M_\odot$ has been inferred. Notably, also the third BH discovered by Gaia, Gaia BH3, with a mass of about $33 M_\odot$ falls into this peak [1784]. As of today, there is no consensus on the origin of this feature. Among other scenarios, mergers in globular clusters [1785] and repeated mergers of initially lower-mass BHs from other peaks in the BH mass spectrum have been suggested [1779], as well as a pile-up of BHs from pulsational pair-instability supernovae [1724, 1786, 1787] (see also section 4.3.3.4 for more details).

Peaks in the BH mass spectrum should persist across a wide range of metallicities and redshifts. These features are expected to evolve with redshift due to a combination of cosmological redshift, and the redshift-dependence of their formation channels. As long as these two effects can be disentangled, the redshift evolution of features in the mass distribution can serve as “spectral sirens”, see the discussion in section 3.3.1.2. This offers an independent measurement of the Hubble constant, and will help resolve the Hubble constant tension [591, 1788], as well as the possibility of constraining or revealing deviations from the standard cosmological model, see section 3.3.2.

4.3.3.4 Upper mass gap and (pulsational) pair-instability supernovae. Stellar theory predicts a gap in the BH mass function between approximately $45\text{--}80 M_\odot$ and $120\text{--}160 M_\odot$, known as the pair-instability supernova mass gap [1592, 1750, 1789, 1790]. The progenitor stars of these BHs reach temperatures and densities in their centers that allow for electron-pair production, triggering a premature collapse of the carbon-oxygen core that results in an explosion that completely disintegrates the star [1791, 1792]. For helium cores of $M_{\text{He}} \gtrsim 130 M_\odot$, photo-disintegration prevents the disruption of the star and we expect BHs to form again, see e.g. [1789]. Pair-instability theory thus predicts a mass gap in the distribution of BH masses. The expected location of this gap is robust against most uncertainties in stellar evolution, but is only sensitive to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate [1793–1795]. This has renewed interest in constraining this rate [1796–1798], and it has conversely been suggested that an observed mass gap in GW sources can be used to assess its value [1783, 1799].

ET is particularly promising in this regard because its lower frequency range will increase our sensitivity from $\sim 100 M_\odot$ to up to $\sim 500 M_\odot$, allowing us to observe both the lower, *and*

the upper edge of the PISN mass gap. Initial GW detections supported a dearth of BHs with masses above $45M_{\odot}$ [1721, 1725], but more recent observations have shown that this proposed gap is not empty [7]. Since pair-instability theory predicts that isolated binaries cannot form BHs in this mass range, other formation channels have been suggested to explain the low but non-zero rate of mergers in the gap, including hierarchical mergers of BHs. Confirming the location of the PISN mass gap is thus linked to fundamental questions, such as “What is the most massive BH we can form from stars?” (also closely related to IMBH formation, see section 4.8), “Which merging BBHs are formed from isolated binaries vs. dynamical environments?”.

Have we detected the lower edge of the PISN gap? Just below the lower edge of the PISN mass gap, stars exhibit pulsations that are not strong enough to completely disrupt the star, but instead map a range of core masses to similar BH masses. We thus expect a pile-up of BH masses just below the lower bound of the PISN mass gap [1800]. Over the past six years, the feature observed in the primary BH-mass distribution at $35M_{\odot}$ has consistently been attributed to this pile-up resulting from pulsational-PISN [1415, 1724, 1786, 1787, 1801]. However, recent studies seem to pile on evidence that the location of this feature is in tension with the expected location of PISN [1797, 1802, 1803], and whether the lower edge of the PISN mass gap has been observed remains an open question [1804, 1805].

Addressing this requires detecting *both* edges of the PISN mass gap. The lower frequency range of ET provides a unique chance to measure the *full* range of the PISN gap, thereby enabling us to answer this question and distinguish between different BH formation channels.

4.4 Constraining the mass function of neutron stars

4.4.1 Observations of radio pulsars and accreting NSs

Neutron star (NS) properties [1806] are determined by a complex interplay between stellar evolution, supernova physics, binary interactions (mass transfer), plasma physics, and ultra-dense matter behavior. Therefore, powerful insights into these processes can be obtained from macroscopic observables such as NS masses, radii, moments of inertia, spins, and magnetic fields. NS masses in particular have been proven to be an exceptionally informative probe of NS physics.

Because BNS mergers are intrinsically much fainter than BBHs, second-generation detectors do not provide a detailed view of the NS mass spectrum. Instead, currently the only detailed insights are provided by binary radio pulsars within our Galaxy, which provide the most precise mass measurements [1807]. Most of these are derived from long-term monitoring of pulse arrival times, which are modulated by binary motion, thereby giving access to the orbital dynamics. In tight binaries and/or favorably oriented systems, constraints on leading-order post-Keplerian (pK) parameters such as the Shapiro delay, orbital precession, and orbital decay often become possible [1808]. Component masses can be measured in systems for which at least two pK parameters can be determined [1809]. For systems in which the pulsar companion is optically bright, spectroscopy and photometry can also be used to constrain the mass ratio and the companion mass [1810–1812]. Despite significant progress in recent years, acquiring precise pulsar mass measurements still presents a formidable challenge.

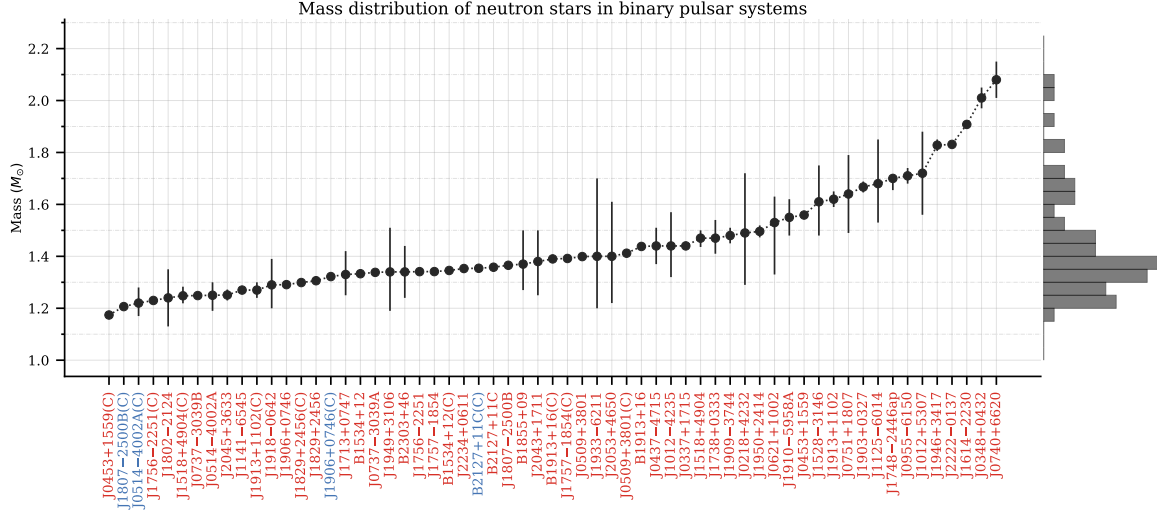


Figure 64. NS mass measurements in binary pulsar systems, sorted by increasing mass. Names followed by (C) indicate companions to radio pulsars that are consistent with being NSs. The names in blue show systems in which the companion could also be a massive WD. The error bars indicate the 68% confidence intervals. The histogram on the right is based on the median of the mass PDFs. Figure reproduced with permission from V.V. Krishnan and P. Freire, from https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html (up-to-date versions).

Although more than 600 binary radio pulsars are now known, only 56 masses in 46 binaries have been constrained with a precision better than 15% (see figure 64). Most of these objects are millisecond pulsars (MSPs) with low-mass companions, with only 22 being double NSs (DNSs). It is therefore unclear whether the general conclusions drawn from these systems are directly applicable to merging DNS binaries.

Determination of NS masses in X-ray binaries is also possible due to the eclipsing and accreting, or even pulsating, nature of such binaries where a NS orbits a visible companion star. In such double-lined spectroscopic binaries, for which the orbital inclination angle can be determined, one can determine the mass of the NS [1813]. Unlike the situation for binary radio pulsars, the empirically determined masses of NSs in X-ray systems are typically subject to uncertainties of order $\sim 10\%$, or more [1814, 1815]. For this reason, individual NSs in X-ray binaries can rarely be used to constrain NS structure and formation theories. The mass distribution of NSs in X-ray binaries appears to roughly follow the spectrum of NS masses obtained from radio pulsar binaries.

4.4.2 Overall shape of empirical NS mass distribution

As a first guess, the masses of NSs are expected to spread between the Chandrasekhar limit and the Tolman-Oppenheimer-Volkoff limit. In principle, from an EOS point of view, the NS mass distribution could be wider: for instance, if part of the iron core is somehow ejected during the supernova (SN), or if very fast differential rotation prevents a massive (e.g. post-merger) NS from collapsing [1816].

The empirical NS mass distribution for pulsars in our Galaxy is overall consistent with this picture. The lightest known in a DNS system, J0453+1559, has a mass of $m = 1.174(4) M_{\odot}$,

while the most massive known NS is PSR J0740+6620 with a mass of $m = 2.08(7) M_{\odot}$ [1817, 1818], although there are candidates such as PSR J0952–0607, that may have even higher NS masses [1811, 1819]. More recently, a binary NS was discovered [1820] with a total mass of $3.887(4) M_{\odot}$ in NGC 1851. The compact companion has a mass between 2.09 and $2.71 M_{\odot}$ (95% confidence), but it is not yet clear whether it is a massive NS or a low mass BH.

The observed masses are not uniformly distributed between the two extremes, but rather seem to cluster around a small number of characteristic values. Approximately one third of objects appear to have masses greater than $1.6 M_{\odot}$ (figure 64). Several studies suggest that the mass spectrum is multimodal, with at least two groups clustered around $\sim 1.4 M_{\odot}$ and $\sim 1.8 M_{\odot}$ [1815, 1821, 1822]. There is also marginal evidence for a high-mass cutoff around $2.1 M_{\odot}$, that could signify the dividing line between NSs and BHs [1821–1823]. However, the presence of such a cutoff appears to be sensitive to modeling choices and requires more measurements to be constrained with confidence.

Differences between the mass spectra of different NS populations also appear to be present. For example, DNS systems seem to cluster with symmetric NS masses around $(1.35 - 1.40) M_{\odot}$, thus generally having smaller masses than MSPs in binaries that host the most massive NSs. However, recent discoveries of asymmetric DNS systems challenge this picture. Progress on this front requires robust quantitative and qualitative comparisons with the DNS merger population, which will become possible in ET era.

4.4.3 Theoretical expectations

Several theoretical studies suggest that the complexity of the NS mass spectrum can be largely explained by the nonlinear nature of late stellar evolution and an element of stochasticity in SN dynamics [1824]. For instance, subtle differences in the way late shell burning phases proceed may give rise to large explodability fluctuations between stars that had very similar properties at the beginning of their evolution [1751, 1782, 1825, 1826]. In turn, these fluctuations seem to significantly influence not only the nature of the final remnant (NS or BH), but also the amount of matter ejected and falling back during the SN.

Additional complexity may be introduced by more elusive core-collapse events such as electron-capture supernovae and accretion-induced collapse. For example, for stars which evolve to develop low-mass metal cores, the exact boundary in outcome between an iron core-collapse SN and an electron-capture SN depends on the location of ignition of the off-center neon and oxygen burning and the propagation of the neon-oxygen flame [1827]. The location and narrow range ($\sim 0.2 - 0.6 M_{\odot}$) of initial ZAMS masses that produce electron capture SNe, and thus the total number of low-mass NSs produced, are a function of metallicity [1828, 1829]. However, it should be noted that the lowest mass NSs ($< 1.25 M_{\odot}$) are not expected from electron-capture SNe but from low-mass iron core-collapse SNe [1830]. The reasons for this is that the cores of stars with ZAMS masses $\geq 12 M_{\odot}$ during and beyond carbon burning are not degenerate, and therefore their collapse is not directly related to the Chandrasekhar mass, as is the case for electron-capture collapse. As a consequence of this, NSs with the very lowest masses are expected to be below even the Chandrasekhar limit. In electron-capture SNe, the core will collapse rapidly and thus produce a NS accompanied by a dim SN explosion. The related NS formation channel of an accretion-induced collapse (AIC) of a massive WD may

also leave behind a NS. Because of the promptness of these core collapses, caused by the steep density gradient at the edge of the core in the progenitor stars, there is therefore little time for asymmetries to develop and it is therefore believed that both electron-capture SNe and AIC events produce small NS kicks and NS masses very close to the Chandrasekhar limit.

Mass transfer from the binary companion after the NS formation may also affect the mass spectrum towards higher masses. Interestingly enough, the mass difference between non-recycled and recycled NSs need not be dominated by accretion of matter after the formation of the NS. As argued by [1376], the first-born NSs in NS+NS systems probably only accrete $\lesssim 0.02 M_\odot$, based on an analysis of the various accretion phases and observational data of both mass and spin periods. While there is no doubt that NSs in low-mass X-ray binary (LMXB) systems (producing MSPs with He WD companions or H-rich “spiders”) are able to accrete significantly more material than the first-born NSs in DNS systems, because the timescale of evolution in the former systems is much longer, there are still many examples of fully recycled MSPs which have small masses of $(1.2 - 1.4) M_\odot$. Since the original birth masses of these NSs could not be much smaller than $1.2 M_\odot$, these systems show clear evidence of very inefficient accretion. (For a reference, it only takes $\sim 0.1 M_\odot$ of accreted material to spin up a NS to a rotation period of 2 ms [1831].) Therefore, the general differences in masses between non-recycled and recycled NSs is likely caused by a difference in the birth masses of NSs born first or second in a binary system. Thus, these mass differences are likely to be the result of the progenitor star evolution. At the upper end of the mass scale, it is possible that heavy accreting NSs may reach a critical mass and collapse to produce low-mass BHs [1832], even with masses below this critical threshold due to loss of gravitational binding energy when the BH is formed.

4.4.4 Anticipated impact from ET data

A high-resolution NS mass spectrum (and ultimately its dependence on redshift) is a key ET objective. ET BNS detection rate is expected to be up to two orders of magnitude higher compared to second-generation detectors [16]. For a BNS population following the Galactic DNS mass distribution, this could translate to several thousands of events per year, detected up to redshifts $z \sim 5$. A considerable fraction of these events (few hundred per year) will be recorded with sufficient precision during both the inspiral and post-merger phases to accurately separate the component masses. Hence, these data will provide a completely new probe of the NS mass distribution and its evolution with redshift. In turn, this will allow for the first time a direct link to physical processes before and after the NS formation. For example, the different nature of core collapse SNe (iron core collapse vs. electron capture) is guaranteed to produce a distinct signature in a high-resolution NS mass spectrum obtained at different metallicities [1833].

Synergies with existing (e.g. the Five-hundred-metre Aperture Spherical radio; FAST) or upcoming (e.g. the Square Kilometre Array; SKA) facilities that will provide an order of magnitude more mass measurements for galactic binary pulsar systems [1834], will also help resolve degeneracies between the effects of the EoS, core-collapse physics, and binary evolution [1821]. Likewise, nearby golden events that will be observable for several days to weeks will also allow for joint EM monitoring and follow-up. Such events will further help to

identify non-equilibrium effects (see section 7 for details) during the inspiral and use them to obtain more accurate estimates of NS macroscopic properties. It is important to note that the LF instrument will be critical for achieving the sensitivity required to meet these objectives.

4.5 Spins of stellar-origin BHs and NSs

4.5.1 BH spins — theoretical expectations

The origin of spin in astrophysical BHs remains an active research topic. However, from first principles, BH spin is related to four factors [1835]: (i) the angular momentum content of the BH’s immediate progenitor star, (ii) the angular momentum of material accreted onto the BH after its formation, (iii) an instability during the core collapse that transfers significant amount of angular momentum to the core of the collapsing star, while a small amount of material carrying the opposite angular momentum is ejected from the system, and (iv) the conversion of orbital angular momentum into spin angular momentum in a binary BH (BBH) system. In the next paragraphs we examine each of these four aspects in the process of forming coalescing BBHs and NSBHs.

In the last several years, it has been argued that a BH formed from a single star or the first-born³³ BH in an isolated BH binary systems is likely to have very slow spin as long as there is efficient angular momentum transport within the star [1414, 1836]. As the BH progenitor star is expanding in its post-main-sequence evolution, angular momentum is transported to its outer layers, depleting the core from angular momentum. The outer layers of the BH progenitor star are subsequently removed via stellar winds and binary interactions, typically leaving behind a slowly spinning stripped star which eventually forms a slow-spinning BH. For single stars, the exact BH spin depends on the initial rotation of the star and the details of the assumed angular momentum transport mechanism, but in any case it is expected to be $\lesssim 0.1$ [1414, 1415, 1837]. However, when member of a binary system, the progenitor star of the first-born BH may maintain a slightly higher angular momentum content, due to tidal interactions during its evolution, resulting in BH spins up to 0.2 [1838]. One caveat to the argument above is the possibility for chemically homogeneous evolution (CHE) [1365]. The rapid rotation of a massive star, which can be either its initial rotation or induced by a binary interaction, can induce strong internal mixing, maintaining a uniform chemical composition throughout the star. CHE stars do not expand to become supergiants, and thus can avoid angular momentum losses, especially at very low metallicity where stellar winds are significantly reduced, and can form BHs with higher spins. It is believed that CHE occurs for low-metallicity, fast rotating, massive stars, but the exact conditions that lead to CHE remain uncertain (e.g., [1404, 1839–1841]).

In all formation channels emerging from the evolution of isolated binaries, the progenitor of the second-born BH is a stripped He star. The spin of the second-born BH is determined by the net effect of the stellar wind and the tidal interaction of the BH–He-star binary system. Because of the efficiency of the angular momentum transport, as explained above, the He-star

³³One should note that the terms “first-born” and “second-born” here refer to the order in evolution of the BBH progenitor system that the two BHs were formed, and should not be used interchangeably with the terms “primary” and “secondary”, often used in observational GW astronomy, referring to the more and the less massive BH, respectively, at the time of merger.

emerges from the second mass-transfer event with a negligible spin. If the orbital separation is small enough, and stellar winds do not widen the system significantly, the He-star, however, can be spun up by tides. These conditions are typically met at low metallicities for BBHs formed through the common-envelope formation channel. In contrast, in the case of stable mass-transfer, the orbits shrink less efficiently leading to less tidally spun-up second-born BHs (e.g. [1414, 1415, 1419–1421, 1837, 1842]).

Accretion of material onto a BH can alter its spin. While it is relatively easy to spin up a non-degenerate star (only $\sim 1\%$ of the star’s initial mass needs to be accreted in order to significantly alter its spin [1843]), a BH mass must be roughly doubled due to accretion to produce a rapidly spinning object. A more accurate calculation (e.g., [1836, 1844–1846]) shows that 20% of the BH’s mass must be added to bring the spin of a non-spinning BH to 0.5, and the initial BH mass must be more than doubled to bring the spin to 0.99. The mass doubling timescale for a BH accreting at the Eddington limit is of order 100 million years — far longer than the $\lesssim 10^5$ year lifetime of high-mass X-ray binaries composed of a BH and its mass-shedding companion.

Stable Roche-lobe overflow accretion onto a BH can result in mass-transfer rates that can exceed the Eddington rate by many orders of magnitude. In the context of binary evolution and binary population synthesis modeling, a common assumption is to limit the accretion onto the BH to the Eddington, or a few times the Eddington, rate. However, hypercritical accretion has been claimed as a way to allow for a significant amount of matter to be accreted at super-Eddington rates [1847], and radiation GR-MHD simulations of super-critical accretion disk (e.g., [1848]) partially support this argument. Binary population synthesis models that consider super-Eddington accretion show that BHs can indeed increase significantly their masses during a stable mass-transfer episode (e.g., [1390]).

The question of angular momentum gained during such a super-Eddington mass-transfer episode is a more delicate one. The common assumption that the accreted material carries the angular momentum of the ISCO may not be valid at highly super-Eddington rates. Radiation GR-MHD simulation of super-Eddington accretion disk around BHs show that, if significant structured magnetic fields are present in those disks, then the disks may enter in a magnetically arrested disk state, where the specific angular momentum of the material accreted by the BH is much less than that of the ISCO [1849]. Furthermore, magnetically arrested disks can launch relativistic jets, which extract rotational energy from the BH [1850]. In that case accretion can lead to spin down of the BH instead of spin up. In the absence of significant structured magnetic fields, the assumption that the accreted material carries the specific angular momentum of the ISCO is closer to what numerical simulations predict [1851].

It is plausible that core collapse itself is a process that spins up a BH. If during a supernova explosion the ejecta somehow were to carry away a significant amount of angular momentum, while the collapsing part of the star acquired the conjugate angular momentum, then a spinning BH could be formed. However, black holes more massive than $\gtrsim 15M_\odot$ are likely to form through complete fallback [1769]. In this context, it has been suggested that black holes could spin up if part of the ejecta were torqued by the binary companion before falling back, effectively converting part of the orbital angular momentum to BH spin. For this process to be efficient, however, a fine-tuning of the ejecta velocity distribution is

required, making it unlikely to lead to significant spin-up in most cases [1852, 1853]. Finally, the standing accretion shock instability during core collapse and gravity waves during the late evolutionary stages of massive stars have been suggested as potential mechanisms that can stochastically transfer angular momentum to the stellar core right before or during its collapse, potentially explaining the spin distribution of young NSs. However, it is not expected that either of these two mechanisms can significantly affect the spin of a BH [1854, 1855].

While all the above processes were discussed in the context of the evolution of isolated binaries, they are also relevant for most other formation channels, e.g. dynamical formation in triple and higher multiplicity systems (e.g., [1432, 1433]), globular clusters (e.g., [1856, 1857]), nuclear clusters (e.g., [1540]), young star cluster (e.g., [1559]) or AGN disks (e.g., [1505]). In such environments, dynamics and stellar/binary evolution act together to form coalescing double compact objects. Dynamical encounters are expected to lead to a perfectly isotropic orientation of BH spins in the densest environments, where each binary system experiences multiple close gravitational interactions and exchanges [1858]. Only in open clusters and low-mass young clusters, spin orientation can retain some memory of the initial alignment with the orbital angular momentum of the binary system [1859].

In the densest stellar systems, globular and nuclear clusters, the merger product of a coalescing BBH can be retained. This second generation BH retains part of the orbital angular momentum of the progenitor BBH, such that even if the two progenitor BHs were non-rotating, the resulting BH would have a spin of ~ 0.7 [1860]. Although a kick is imparted onto the BH merger product, second generation BHs can sometimes be retained in the clusters and be involved in the formation of a new BBH. Merging BBH that contain at least one second generation BH, although account only for a small fraction of the total BBH population, can leave a distinct imprint in the BH spin distribution inferred from BBH mergers (e.g., [1481, 1543]).

4.5.2 BH spins — empirical evidence from X-ray binaries and GWs

The first efforts to constrain the spins of stellar-mass BHs were done using electromagnetic observations of BH X-ray binaries, and specifically by studying either the spectrum of the emitted X-ray radiation or the X-ray timing variability of the systems. There are three widely used methods for estimating the spins of stellar-mass BHs observed in X-ray binaries [1861]: (i) fitting the thermal continuum spectrum of the accretion disk, (ii) modeling the reflection spectrum of the disk and specifically the distortion of the Fe K line due to GR effects, and (iii) modeling high-frequency ($\sim 100 - 450$ Hz) quasi-periodic oscillations (HFQPOs). While for the former two methods there are well-understood physical models underpinning them, there is no established model for the origin of HFQPOs. Over the last decade, the results from disk continuum fitting and the reflection line fitting methods have broadly converged. The picture that emerges is that BHs in low-mass X-ray binaries have spins that cover the entire range, from 0 to 1, while BHs in high-mass X-ray binaries tend to be fast, almost maximally, spinning (figure 65).

Gravitational-wave observations have provided insights into the spins of black holes. Spins leave an imprint of the gravitational waveform, enabling measurement of spin magnitudes (between 0 and 1) and orientations (typically defined relative to the orbital angular momen-

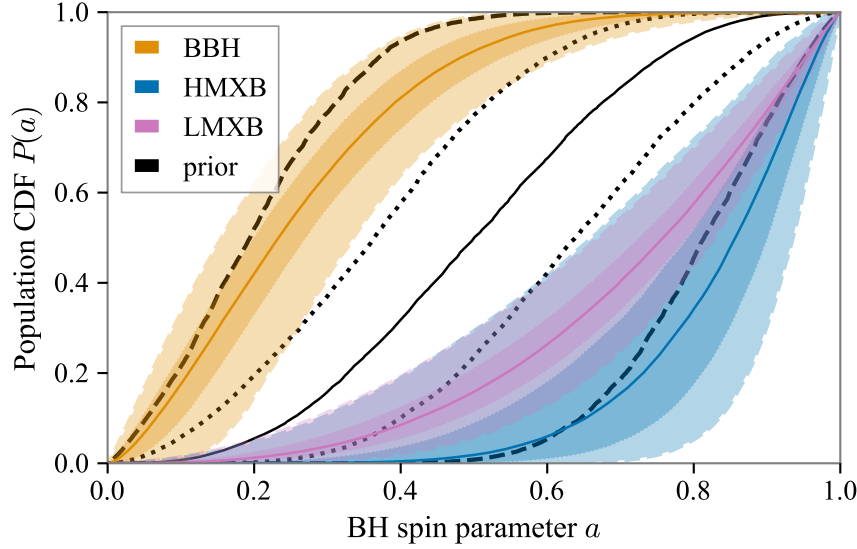


Figure 65. Cumulative distributions of BH measured from BBHs (yellow), low-mass X-ray binaries (blue) and high-mass X-ray binaries (magenta). The distribution of prior CDFs is shown by the unfilled black bands [1866]. The choice of priors is discussed in [1726]. Reproduced from [1866]. The Author(s). CC BY 4.0.

tum) [1689, 1690]. These imprints are more subtle than the effects of the masses, making them more difficult to precisely measure [1693, 1862–1864]. The low-frequency sensitivity of ET may be especially important for measuring spin precession, that takes place when the spins are misaligned with the orbital angular momentum [1865].

In contrast to X-ray observations, the majority of the detected gravitational-wave sources are consistent with having zero spin [6]. The more massive component of the source GW190814 has the most precise spin-magnitude measurement, $\chi \leq 0.08$ [1867, 1868]. Several sources, such as for GW151216 [1868, 1869], have been observed to have non-zero spin [6]. Considering the observed population, half of spin magnitudes are below $\chi = 0.25$ [7]. The statistical analysis by ref. [1866] shows that the spins of BBHs from gravitational waves and those of BHs in X-ray binaries cannot be drawn from the same underlying distribution (figure 65).

Spins are often discussed using effective spin parameters: the effective spin parameter $\chi_{\text{eff}} = (m_1\chi_1 + m_2\chi_2)/(m_1 + m_2)$ is a mass-weighted combination of the spins aligned with the orbital angular momentum [1870, 1871], and the effective precession spin χ_p characterises a mass-weighted in-plane that contributes to spin precession [1872]. To date, most detected sources are consistent with $\chi_{\text{eff}} = 0$ [6], which could indicate either spin magnitudes are close to zero, spins are anti-aligned with each other, or spins are primarily in the orbital plane. The systems with non-zero χ_{eff} currently all have $\chi_{\text{eff}} > 0$ indicating a preference for spins aligned, rather than antialigned, with the orbital angular momentum [6, 7]. GW190814’s source has the best measured value of χ_p , with $\chi_p \leq 0.07$ consistent with having a low-spin primary component. Identifying an individual source showing spin precession has been more difficult: GW200129_065458 is potentially the best candidate [6, 1873, 1874], but this result is sensitive to the noise subtraction procedure used [1875, 1876] and it has been suggested

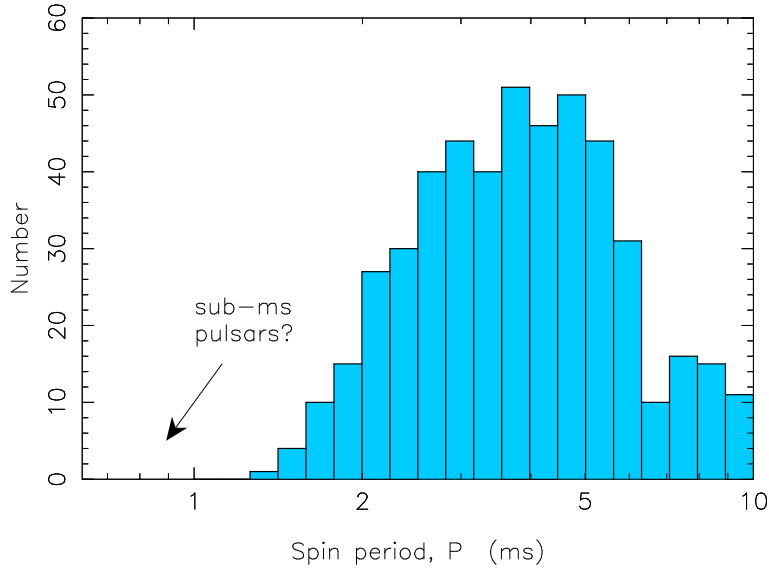


Figure 66. Distribution of spin periods of 485 radio MSPs with $P < 10$ ms. Despite intense searches, no sub-ms MSP has been discovered thus far. ET may reveal such a sub-ms MSP in a GW merger — see text. Reproduced with permission from [1813].

that the signs of precession could be a misdiagnosis of the influence of orbital eccentricity on the signal [1877]. Considering the entire population, however, there is evidence that some sources have non-zero χ_p [7, 1726]. Therefore, black hole spins can be misaligned with their orbital angular momentum.

4.5.3 NS spin periods — theoretical expectations and limitations

NS spin periods measured so far span over 7 orders of magnitude [1813]: from ~ 1.4 ms for the radio millisecond pulsar J1748–2446ad [1878] to more than 5 hr for the symbiotic X-ray binary 4U 1954+31 [1879]. The possible existence of sub-ms pulsars ($P < 1$ ms) is a long standing and intriguing question. The distribution of MSP spins is shown in figure 66. The discovery of a sub-ms MSP is very important for constraining the equation-of-state of NSs [1880]. Given the physical mass shedding frequency limit of about 1500 Hz ($P \simeq 0.67$ ms), it is perhaps puzzling why no sub-ms pulsars have been found so far in light of the intense efforts to detect these objects in both radio and X-rays. Nowadays, there seems to be no instrumental or computational selection effects that would prohibit such a discovery. From a theoretical point of view, it is an open question whether the lack of detected sub-ms pulsars might be due to magnetospheric conditions [1813] or GW radiation during the accretion phase that halts the spin period above a certain level [1881–1884]. A GW discovery of an accreting sub-ms pulsar would thus be a major discovery. The emitted GW signal would have a frequency above 2000 Hz, and although this is not optimal for ET where the sensitivity peaks at 200–300 Hz, any GW strain above $10^{-24} \text{ Hz}^{-1/2}$ @ 2000 Hz is potentially detectable.

4.5.4 Spins of double NS systems: evidence from Galactic sources and expectations for mergers

Radio pulsar data on recycled and young pulsars in Galactic field NS+NS systems reveal that the spin values of the second-born (i.e. non-recycled) pulsars are equivalent to those of single young pulsars ($\sim 0.1 - 3$ s) and that the recycling process of the first-born NS is rather inefficient: the most rapidly spinning NS measured in a Galactic NS+NS system has a spin period of $P \simeq 21$ ms. This is expected [1813] because NS+NS systems descend from high-mass X-ray binaries (HMXBs), which means that the relatively short lifetime of the massive (ultra-stripped) progenitor of the second-born NS prevented sufficient supply of material to spin up the first-born accreting NS [1885]. However, the situation is different for NS+NS produced in dense stellar environments, like globular clusters [1886], because binary exchange encounters may pair up an MSP (formed in a low-mass X-ray binary systems) with another NS — a process which is evident from exactly such MSP+NS pairs detected in globular clusters [1887]. Thus we do expect ET to eventually detect mergers of NS+NS systems in which one NS component (or even both) may be an MSP, although their rate will likely only be a small fraction of the total detected NS+NS merger rate [1466, 1888].

A natural question is what the spin rate will then be at the moment of the merger since NS+NS system may have long delay times of several Gyr, during which magnetodipole radiation and magnetospheric plasma currents give rise to loss of rotational energy. It turns out that many MSPs will lose only a small amount of rotational energy [1831] because their surface B-fields are very low ($10^7 - 10^8$ G) as evident from their $\dot{P}$ values down to $\sim 10^{-21}$. Thus we do expect ET to detect some NS+NS mergers with NS component spin periods of order ms, although the rate is likely low and the resulting post-merger GW signal from such NS+NS mergers with one high-spin component is slightly weaker than for slower spinning NSs [1888].

4.5.5 BH/NS spin-axis direction and core collapse

Whereas the spin magnitudes of BHs and NSs are expected to be determined by progenitor star evolution and binary interactions (mass accretion and tides) after their formation, the directions of BH and NS spins are of uttermost importance for probing core collapse and understanding SN explosion physics. Measurements of individual spin components will still remain challenging with ET. However, the distribution of measured effective spins (χ_{eff} , i.e. spins projected along the orbital angular momentum vector during in-spiral) can teach us not only about the formation channel of GW mergers [1579, 1858, 1889, 1890] but also about SN explosion physics. Evidence has recently been suggested for spin-axis tossing (flipping the rotation axis in a random direction) at birth based on empirical data from LVK observing runs O1–O3 [1891] and theoretical investigations of this phenomenon (at least for NSs and low-mass BHs) has been put forward [1892]. A large increase in GW merger data from ET will significantly boost the empirical statistics to firmly infer both progenitor formation channels and the question of tossing the spin axis at birth.

4.5.6 BH/NS spins in light of ET

ET offers a significant increase in detector sensitivity compared to LVK. This improvement enhances the possibility of detecting continuous GW signals at high frequencies from Galactic spinning NS with sufficiently large ellipticities [1893–1896], see section 8.3.2 for extended discussion. Detection of continuous GWs from a spinning NS would be a major breakthrough and open up a new path to study the structure and evolution of NSs. If GWs from a spinning NS are detected in a tight binary, from which LISA will be able to detect the orbital motion in low-frequency GWs, i.e. a dual-line GW source [1897, 1898], the prospects for investigating the interior of these NSs are even brighter.

4.6 Primordial versus stellar-origin BHs

As we already discussed in section 3.2.5, primordial black holes (PBHs) are a putative population of BHs that could have formed in the early Universe, much before the formation of structures and stars. The simplest formation scenario assumes the collapse of extreme inhomogeneities during the radiation-dominated era [984–987] (although various alternative mechanisms were suggested, e.g. based on early matter dominated eras [1899, 1900], cosmological phase transitions [1032, 1034, 1036, 1901, 1902], the collapse of domain walls [976, 1046, 1903] and others [1904–1908]). While remaining one of the most studied candidates to explain the dark matter, even a subdominant population of PBHs could serve as the seed of supermassive black holes at high redshift [1909–1912], explain small scale structure features [517, 1913, 1914], and lead to boosted signatures of other dark matter candidates (such as WIMPs) [525–527, 1915].

The characteristic natal mass of these objects is linked by model-dependent factors to the mass within the Hubble sphere m_H at formation. For example, if the formation takes place in a radiation dominated Universe, it was shown to follow a critical scaling as a function of the collapsing overdensity above the threshold. For typical perturbations, m_{PBH}/m_H takes values of order unity, although different hierarchies can be attained. Overall, PBHs can generically be formed in a range of masses which spans several orders of magnitude [1916–1919], from the ultra-light masses $m_{\text{PBH}} \lesssim 10^{-18} M_\odot$ that are unstable through Hawking evaporation, up to supermassive scales. There are many constraints on the PBH abundance $f_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}}$ (see [535] for a comprehensive review). In this landscape, ground-based detectors will be able to provide strong bounds in the mass range that goes from planet size masses, $m_{\text{PBH}} \sim \mathcal{O}(10^{-6}) M_\odot$, up to intermediate masses $m_{\text{PBH}} \sim \mathcal{O}(10^4) M_\odot$, by searching for the GW signatures of PBH mergers, exceeding in most of this mass range the existing bounds derived from microlensing surveys and various phenomena related to particle emission from accreting PBHs. The complementary constraints that will be set by searching for GWs associated with the formation mechanisms are discussed in section 3.2.5.

While current LVK observations are still compatible with a fraction of the events to have primordial origin [1738] (see also refs. [999, 1920] for the most recent analyses, and [980, 1921] for reviews on PBH mergers as GW sources), it is exciting that next-generation GW detectors, such as ET, would greatly enhance our capabilities to search for GW signatures of PBHs [1922–1924], leading to unprecedented tight bounds on their abundance, as low as $f_{\text{PBH}} \lesssim 10^{-10}$ in some extreme scenarios [1925].

PBHs form in the early Universe. Their distribution is random unless specific correlations are induced in the primordial perturbations [1926–1932], and they can pair to form binaries through different processes [1933]:

- i)* via gravitational decoupling from the Hubble expansion, which typically happens before the matter-radiation equality [1934, 1935]. Such binaries are initially highly eccentric, with a semi-major axis distribution allowing for a large fraction to merge in the late-time Universe;
- ii)* through gravitational capture within present-day small scale structures and galaxies [1348, 1351];
- iii)* through three-body interactions in Poisson-induced PBH small scale structures [1678, 1936]. It is also possible that GW bursts are produced via PBH hyperbolic encounters [1937], although with an expected subdominant rate within most of the ground-based observable band [1938].

Out of those channels, it was shown that the dominant contribution comes from binaries formed in the early Universe [1936, 1939–1941].³⁴

It is important to stress that understanding clustering properties, and in particular clustering evolution at low redshift, is crucial in this context, as non-trivial PBH correlation beyond what is induced by Poisson initial conditions can enhance binary formation in both the early- and late-time Universe, while at the same time inducing stronger suppression factors due to disruption of binaries through more frequent binary-PBH interactions. More broadly, a precise description of the clustering evolution of PBHs after their formation is challenging, especially at low redshift. Numerical simulations of PBH cosmologies were able to describe only high redshift ($z \gtrsim 100$) [1942] or individual clusters of PBHs [1943–1945], while the impact of clustering on the merger rate was investigated with advanced analytical studies [1936, 1941]. The detection of PBH mergers would provide valuable insights and allow to narrow down theoretical uncertainties. Furthermore, due to the range of scales involved with the formation of PBHs of masses that can be probed by ET, synergies with other observables are possible. On the one hand, scenarios induced by large stellar-mass PBH clustering require non-trivial correlations at the scales probed by CMB distortions [1946] with future experiments (e.g. PIXIE and Voyage 50 [1947]), while the enhanced density perturbations required to form PBHs would fall in the range probed by PTA experiments.

Searching for evidence on the existence of PBH binaries is very challenging due to large uncertainties on their properties, such as their mass distribution, which is inherently model-dependent and linked to the early Universe initial conditions. An additional challenge is posed by the expected astrophysical BBHs that act as foreground, and trying to distinguish the two contributions is mostly limited by existing theoretical uncertainties and degeneracies between the astrophysical and primordial channels. However, it is important to stress here that some distinctive features exist, and in the following, we will present what are considered the smoking-gun signatures of PBH binaries.

³⁴This conclusion was also tested with dedicated N-body simulations for relatively narrow mass functions [1940] (up to 2 orders of magnitude in width). Whether disruption of binaries is more or less efficient for broader mass distributions is still subject to debate (see also [517]).

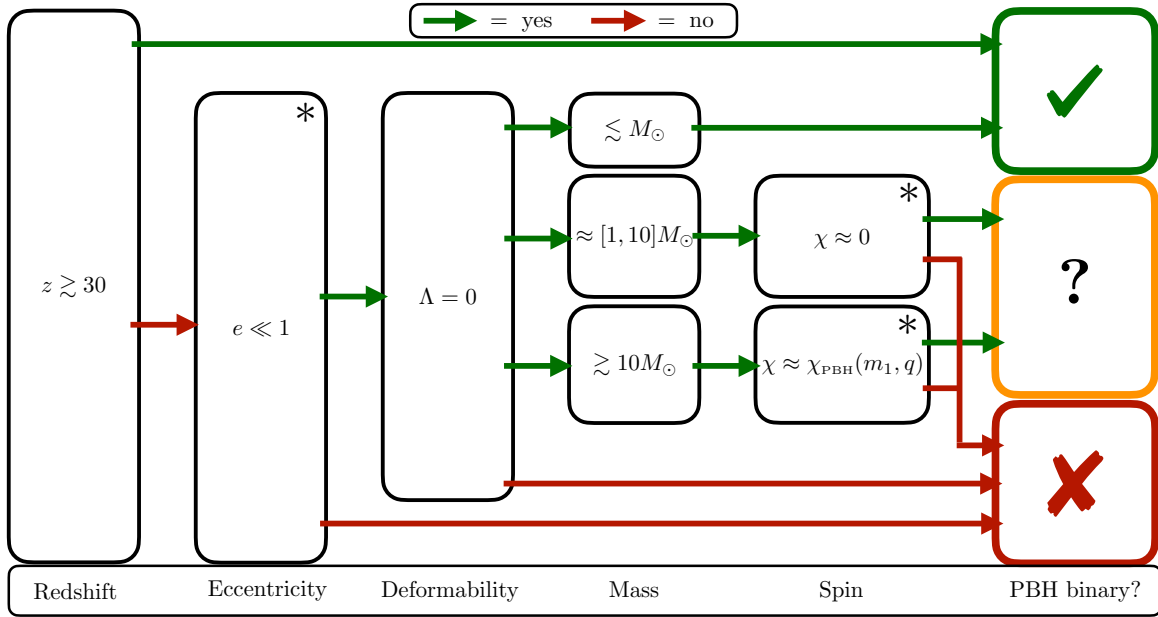


Figure 67. A flowchart to test the primordial nature of a binary. Green and red refer to the condition in the box being met or violated, respectively. Adapted from ref. [1948]. The asterisks indicate that the condition is evaluated in the scenario where PBHs are formed in a radiation-dominated Universe, and a model-dependent alternative is possible. Reprinted figure with permission from [1948], Copyright (2022) by the American Physical Society.

N^{tot}	N^{SS}	$N^{z>10}$	$N^{z>30}$	N^{LMG}	N^{UMG}
1920000	708487	1400384	795904	300220	7774
13347	1650	336	17	2638	235

Table 6. The total number of PBH (N^{tot}), subsolar events (N^{SS}), high redshift ($z > 10$ and $z > 30$) events, and “mass gap” events (LMG and UMG for the lower and upper mass gap, respectively), for the intrinsic numbers of PBH mergers per year predicted by maximizing current LVK bounds [1713] (second row) and for the events per year detected with ET at $\text{SNR} \geq 12$ (third row).

In figure 67 we summarize the main properties of PBH binaries formed in the radiation-dominated early Universe. As we will see, detecting high redshift mergers and/or subsolar binaries would allow for a confident discovery of a PBH population. In most scenarios, one also expects specific mass-spin correlations that could be searched for in the GW data. In this endeavor, ET will be a game-changer compared to current GW detectors [1948]. These smoking guns could be exploited on an event-by-event basis or through population studies. We will discuss both approaches below. In section 4.10.3 we will discuss the potential contribution from PBH mergers to the stochastic GW background.

Current LVK observations set the most stringent bound on the merger rate of primordial binaries. However, next-generation detectors would boost our capabilities to constrain this scenario. In table 6 we report the intrinsic and detectable number of mergers in the interesting mass and redshift range (assuming the PBH abundance saturates current PBH upper bounds, see ref. [1948] for more details).

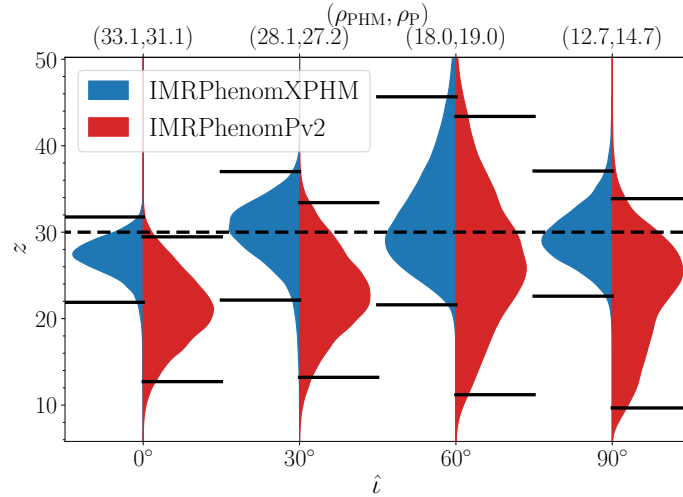


Figure 68. Redshift posteriors for sources with $(M_{\text{total}}, z, q) = (40M_{\odot}, 30, 1)$ at $\hat{i} = 0^\circ, 30^\circ, 60^\circ$ and 90° , obtained with a waveform with (blue) and without (red) higher-order modes and for a detector network composed of two CE and one ET (see ref. [1179] for more details on the assumed detector network). The solid horizontal lines show the 95% credible intervals, whereas the dashed lines mark the injected value $z = 30$. The top axis shows the optimal SNR of the two waveforms. Reprinted figure with permission from [1179], Copyright (2023) by the American Physical Society.

4.6.1 Constraints on high-redshift merger rate evolution

The unique feature of the PBH merger rate density is that it is monotonically increasing with redshift as $\mathcal{R}_{\text{PBH}}(z) \propto t^{-34/37}(z)$ [1939, 1943, 1949], extending up to early epochs at $z \gtrsim \mathcal{O}(10^3)$. Interestingly, this property is dictated by how pairs of PBHs decouple from the Hubble flow before the matter-radiation equality. Therefore, it is a robust feature of the PBH channel.³⁵

As high-redshift mergers are difficult to produce by astrophysical channels, one can set a conservative threshold above which no astrophysical mergers are expected to exist within standard cosmologies [1951]. One can adopt the conservative redshift threshold $z \gtrsim 30$ to be a smoking gun for primordial binaries [1178, 1179, 1182, 1923, 1951–1953].

The superior sensitivity and wider bandwidth of ET at low frequencies are crucial to detect putative mergers at $z \gtrsim 30$, and up to hundreds of them per year are expected when the PBH abundance saturates current bounds [16, 1923]. However, an important aspect is the measurement uncertainty on the source redshift z associated with a putative detection. In simple terms, this is because an individual detection that is compatible with $z \approx 10$ within uncertainties could be confused with black holes originating from Pop III stars, see section 4.7.1 [1178–1181, 1948]. Figure 68 shows the posterior of the redshift in various cases [1178, 1179] for an example network composed of ET+2CE. Note that analogous conclusions are reached with similar detector configurations. For a correct recovery of the true redshift, it is important to include higher-order modes in the waveform [1178, 1179], which alleviate the distance-inclination degeneracy and allow for a sufficiently accurate measurement

³⁵In the case of initially strongly clustered PBHs, the merger rate evolution with cosmic time is only slightly modified to $\mathcal{R}_{\text{PBH}} \sim t^{-1}(z)$ [1925, 1940, 1950].

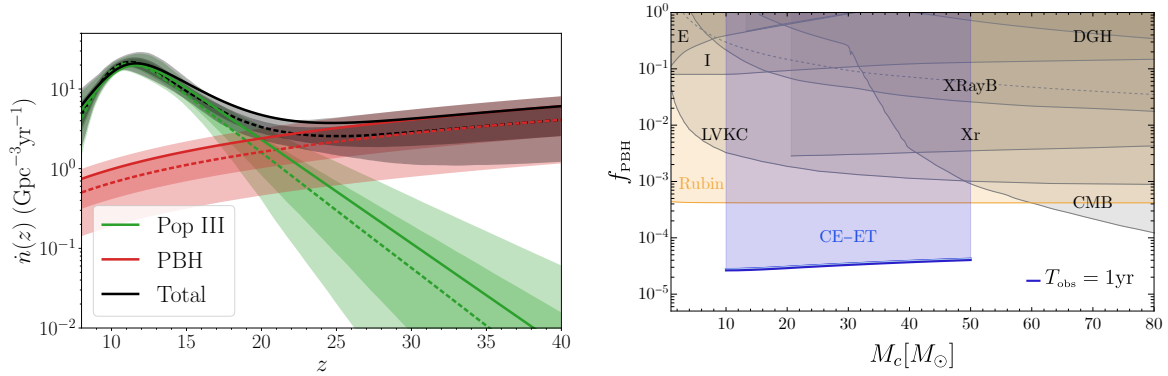


Figure 69. *Left panel:* reconstructed merger rate at $z > 10$ assuming ET+CE and two populations of events: astrophysical (Pop III) and primordial. We consider the most conservative scenario in which both populations have the same mass function, we neglect PBH accretion, and we adopt an optimistic Pop III merger rate [1954]. *Right panel:* a null observation for a primordial population with ET+CE can be translated into an upper bound on the PBH abundance. Reproduced from [1182]. The Author(s). CC BY 4.0.

of z . Still, uncertainties could remain as large as $\Delta z/z \sim 30\%$ for a source at $z = 30$, depending on the source properties, and would quickly degrade at larger distances.

Further information can be gathered by reconstructing the merger rate as a function of redshift for a population of events. Figure 69 (left panel) shows the posterior predictive distributions at $z > 10$ reconstructed from a hierarchical Bayesian inference of 4 months of synthetic data, assuming two populations of mergers: a representative astrophysical merger channel from Pop III stars and a primordial one with abundance saturating current upper bounds and a mass function centered around $M_c \sim 30M_\odot$. In the right panel, we report the corresponding upper bound that will be set on f_{PBH} from a non-detection of PBHs in 1 year of data taking.

4.6.2 Subsolar PBH binary searches

The detection of a subsolar object in a compact binary merger is regarded as one of the smoking gun signatures of a population of PBHs.³⁶ Current bounds from LVK observations are already forcing the PBH abundance to be smaller than unity in the subsolar mass range [1966–1968]. Also, microlensing constraints are setting comparable bounds in the nearly-stellar mass range [1969–1973]. Even complying with current bounds, however, the number of potential subsolar mergers in 1 yr of observation of ET could be large, as indicated in table 6, given the much higher sensitivity of the future detector. In case one candidate detection is found, it would be crucial to:

³⁶Alternative new physics scenarios could also lead to the formation of subsolar BHs. For example, dissipative dark matter models allow for the generation of dark black holes, with similar properties compared to the formation from Population-III stars with, however, potentially different masses [1955, 1956]. Additionally, nearly solar-mass BHs can form out of NS transmutation in certain scenarios [1957–1964]. If one observed both the NS and the transmuted BH populations, one could infer implosion timescale, and eventually the properties of the dark matter particle that induced the transmutation, such as mass and interaction cross section [1965].

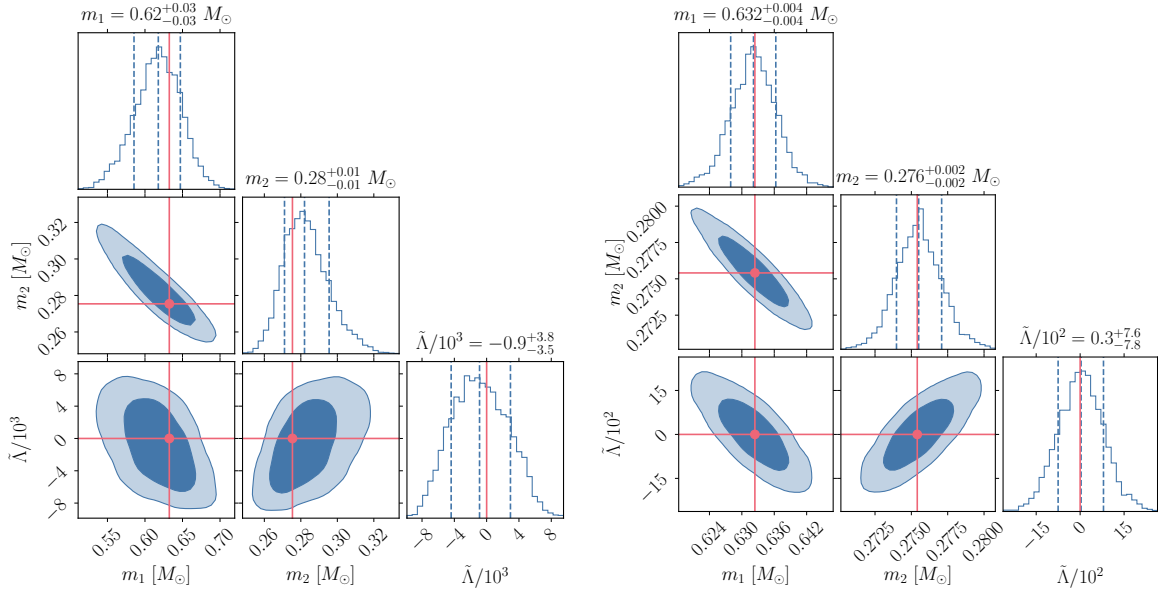


Figure 70. Posterior distribution of the masses (m_1, m_2) and the tidal deformability $\tilde{\Lambda}$ parameter for LVK O5 (left) and ET+2CE (right) (see ref. [496] for more details on the assumed detector network). The red lines represent the injected values of a typical subsolar event. In ET+2CE case, the upper bounds on $\tilde{\Lambda}$ would exclude neutron stars or more exotic alternatives [280]. Reproduced from [496]. CC BY 4.0.

- i) check that the supersolar regime is excluded within uncertainties;
- ii) exclude possible confusion with other subsolar objects, such as light NSs or more exotic compact objects.

PBH binaries could be distinguished from stellar binaries based on the measurability of tidal effects (i.e. tidal Love number as well as tidal disruption of the compact objects) [496, 1974]. At variance with PBHs, the GW signal from stellar binaries is affected by tidal effects (see also sections 2 and 7), which dramatically grow for moderately compact stars, as those expected in the subsolar range. The prospects of measuring tidal effects in subsolar binaries are good already with the LVK network, but only the next generation of GW detectors such as ET will allow to claim a putative subsolar detection at ($> 5\sigma$) level [496]. This is shown in figure 70, summarizing the results of a model-agnostic Bayesian analysis [496] of a simulated GW event from a PBH binary with masses $m_1 = 0.62 M_\odot$ and $m_2 = 0.27 M_\odot$ at $z = 0.02$, and in table 7. This example is motivated by the low significance trigger found by LVK [1967], see also [1975].

In particular, table 7 shows two types of analyses, assuming the event was either a BNS (including tidal deformability in the waveform and tidal disruption frequency $f_{\max}$ beyond which the signal shuts off, parametrized by the effective tidal deformability $\tilde{\Lambda}$ and $\tilde{\lambda}_f \equiv f_{\max}/f_{\text{ISCO}}$, respectively, where f_{ISCO} is the frequency corresponding to the innermost-stable circular orbit) or a BBH (assuming an injection with zero tidal effects but including $\tilde{\Lambda}$ in the recovery). While the errors on the masses are sufficiently small to be confidently ascribed to the subsolar range already for LVK O4, in the BNS case the errors on the tidal

Network	LVK O3	LVK O4	LVK O5	ET+2CE
BNS ($\tilde{\Lambda} = 1.5 \cdot 10^5, \delta\tilde{\Lambda} = 4.9 \cdot 10^4, \tilde{\lambda}_f = 0.075$)				
SNR	7.90	12.8	22.4	398
$\Delta m_1/m_1$	0.47	0.22	0.082	0.0017
$\Delta m_2/m_2$	0.39	0.19	0.070	0.0015
$\Delta\tilde{\Lambda}/\tilde{\Lambda}$	0.86	0.66	0.55	0.047
$\Delta\tilde{\lambda}_f/\tilde{\lambda}_f$	0.38	0.24	0.13	0.015
BPBH ($\tilde{\Lambda} = \delta\tilde{\Lambda} = 0, \tilde{\lambda}_f = 1$)				
SNR	8.38	13.4	23.9	403
$\Delta m_1/m_1$	0.20	0.13	0.044	$6.6 \cdot 10^{-3}$
$\Delta m_2/m_2$	0.17	0.11	0.037	$5.6 \cdot 10^{-3}$
$\Delta\tilde{\Lambda}$	$9.1 \cdot 10^3$	$5.8 \cdot 10^3$	$3.0 \cdot 10^3$	$7.5 \cdot 10^2$

Table 7. Fisher parameter estimation uncertainties with current and future GW experiments. We inject a system with $m_1 = 0.62M_\odot$ and $m_2 = 0.27M_\odot$, assuming the binary was either a BNS (top rows) or a BPBH (bottom rows). Adapted from ref. [496]. Reproduced from [496]. CC BY 4.0.

deformability are relatively large, so that $\tilde{\Lambda} = 0$ cannot be excluded with a high confidence level. Only next-generation detectors such as ET will allow to exclude $\tilde{\Lambda} = 0$ at least at 3σ level (at $> 5\sigma$ level, in fact). Nonetheless, the cut-off frequency for tidal disruption (parametrized by $\tilde{\lambda}_f$) would be constrained away from unity at 4σ level already by O4, which would still support the interpretation of the observed signal as coming from a BNS. Likewise, if the subsolar event is a binary PBH, model-agnostic measurements of $\tilde{\Lambda}$ with ET+2CE (or similarly with ET alone) will exclude the NS hypothesis and also certain classes of exotic compact objects, such as compact boson stars [496].

Even lighter binaries could be searched for using ad-hoc techniques [1976]. As light PBH binaries would only appear in ET frequency band during the inspiral phase, the signal evolution is very slow, and appears as a nearly monochromatic signal. While the current LVK sensitivity is still not sufficient to derive constraints which are competitive with microlensing upper bounds on f_{PBH} [1973] for $m_{\text{PBH}} \lesssim 10^{-2}M_\odot$ [1976], ET will drastically improve the sensitivity leading to much tighter bounds [1977, 1978], tightening the ones currently derived from microlensing surveys.

4.6.3 Spins of primordial black holes

A broad class of models predicts the Kerr dimensionless spin parameter at PBH formation to be very small, of the order $a \lesssim 10^{-2}$ [1979]. This includes the most studied formation mechanism where the collapse of extreme inhomogeneities takes place during the radiation-dominated Universe. However, alternative formation scenarios [1908], such as collapse during an early matter-dominated phase, or through the subhorizon assembly of massive objects, can lead to larger natal spins [1980]. This model dependence is inevitably linked to the primordial scenario due to the currently poor knowledge of the initial phases of cosmological evolution.

After formation, PBHs may spin up due to the accretion of baryonic matter throughout the cosmological evolution, in a process that is expected to quench at the onset of the reionization era, due to both enhanced PBH peculiar velocities within structures as well as gas heating up [1981, 1982]. The dominant contribution to the PBH merger rate comes from binaries formed in the early Universe, through decoupling from the Hubble expansion, and thus no spin alignment is expected at binary formation. Spin induced by accretion, which is only expected to be efficient for masses larger than $\mathcal{O}(10)M_\odot$, leads to a correlation between large total binary masses and large spins which can be used to distinguish this scenario from others.

4.6.4 Tests based on mass and spin distributions

PBHs can be formed in a wide range of masses and, at odds with the astrophysical scenarios, there are no forbidden mass gaps where they cannot form. In particular, as the formation mechanisms rely on the collapse of large density perturbations in the early Universe, they are not affected, for example, by the physics dictating pair-instability supernova [1791, 1792, 1800]. While large uncertainties on the exact location of the astrophysical mass gaps remain (see section 4.3.3.2 and 4.3.3.4), the presence of events in such regions would potentially require different astrophysical mechanisms (e.g. astrophysical hierarchical mergers) or point towards the primordial nature of the binary.

Let us focus first on the so-called lower mass gap, the putative gap in the distribution of masses separating astrophysical NS and BH binaries in the range $\approx [3, 5]M_\odot$. If PBH binaries were contaminating the NS range, the mass distribution would also contaminate the lower mass gap, due to the difficulty of generating a sharply peaked PBH mass distribution in the early Universe. In figure 71 (left panel) we show an example of this, following ref. [999]. The green band indicates the mass distribution assuming the primordial scenario to be responsible for the light events in the GWTC-3 catalog [6], to be compared with the NS predictions in purple. It shows the larger width necessarily induced by the QCD effects and critical collapse [1920, 1983–1985], which would then correspondingly predict events both in the adjacent subsolar and lower mass gap range. Similar considerations apply to the upper mass gap, in the range approximately between $50 - 130 M_\odot$ where, as discussed in section 4.3.3.4, a dearth of BHs is theoretically expected due to the pair-instability supernova mechanism [1791, 1792, 1800]. As contamination of the gap by astrophysical binaries is possible (see section 4.1 for more details), it is important to devise robust tests for the primordial nature of the binary [1984, 1987].

One possibility to distinguish between these two populations is based on searching for features in the mass distribution, or searching for correlations between masses and spins. In a general sense, remnants of second-generation BHs, which are the product of a prior merger, are identified by a final spin of around $\chi_f \simeq 0.68$ [1988], assuming that the spin of first-generation BHs is negligible or isotropic distribution of BH spin directions [1550]. Additionally, it is expected that the mass ratio will be less than unity. Conversely, the relationship between masses and spins in the PBH scenario exhibits a different pattern. By concentrating on the formation in a Universe dominated by radiation (which is generally applicable even in extended scenarios [1989]), the main predictions of the primordial scenario for the distribution of the effective spin parameter χ_{eff} are as follows [1948] (see also figure 71):

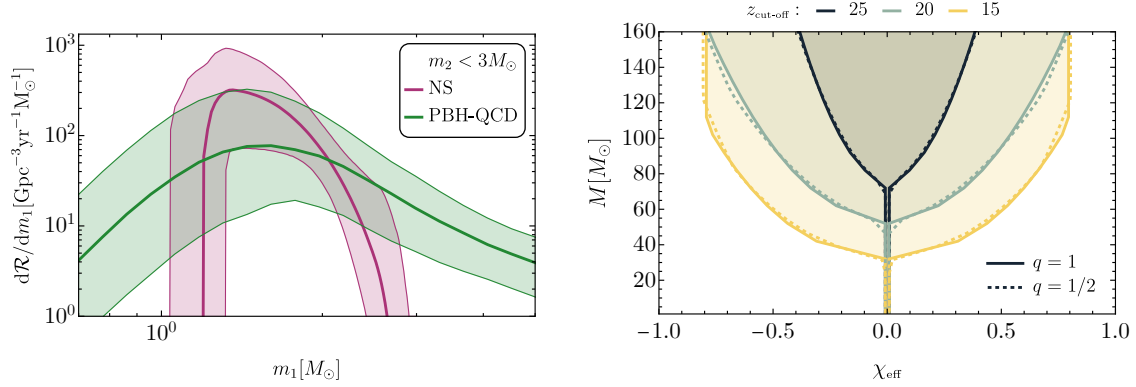


Figure 71. *Left panel:* posterior predictive distribution for events within the NS range based on the LVK GWTC-3 catalog [999]. *Right panel:* distribution of χ_{eff} expected for PBH binaries as a function of the total mass M depending on the accretion efficiency (see [1981, 1986] for more details). We stress that this prediction applies to PBHs born in a radiation-dominated Universe or to alternative formation scenarios for which the natal spin is negligible, and therefore represents a weaker test of the primordial nature of the binary. Reprinted figure with permission from [999], Copyright (2022) by the American Physical Society.

- i) Binaries possess negligible χ_{eff} at low masses within the observable range of ground-based detectors;
- ii) At larger masses, one expects a correlation between large binary total masses and wide distributions of χ_{eff} induced by accretion effects;
- iii) The distribution of χ_{eff} is only dependent on the total mass of the binary, with negligible dependence on the mass ratio (see also figure 71). Therefore, for fixed primary mass m_1 , the χ_{eff} distribution widens for larger q . In other words, there is a positive correlation between larger spin magnitude and mass ratios close to unity. This trend with q is *opposite* to the one expected for dynamically formed astrophysical BBH, where more massive, higher-generation mergers tend to have smaller mass ratios, i.e. negative correlation between large spin magnitude and q .

While current GWTC-3 observations allow for a positive correlation between binary mass and width of the χ_{eff} distribution [1990], poorly measured spins and the low statistics do not allow to confirm this trend and to distinguish between the PBH and dynamical astrophysical channels. ET is expected to gather a much larger number of detections with more precise spin measurements, potentially reaching sub-percent precision on individual spin magnitudes for a large fraction of detections (see e.g. [16, 1713]), and allowing for tests of the presence of mass-spin correlation consistent with the PBH scenario.

4.7 Revealing Population III stars with the first BHs

4.7.1 The nature of Pop III stars

The formation of the first (Pop III) stars marks a primary transition phase of cosmic evolution. Their light ends the so-called ‘dark ages’, and they play a key role in cosmic metal enrichment

and reionization, thereby shaping the Universe at large and the present-day galaxy population. The field of Pop III star formation and how Pop III stars impact cosmic evolution has been rapidly growing in the past 25 years, and it is therefore a relatively young field of research. As a consequence, there is still considerable debate about the physical processes that govern stellar birth at high redshift, and the overall properties of the first stars. Here we provide a very brief summary of the latest findings and highlight if and how ET observations of the GW emission from their BH remnants may help us improve our understanding of the first stars. For the interested readers, we suggest some of the latest excellent reviews which have appeared on this subject [1176, 1177, 1991–1993].

4.7.1.1 Cosmological context of Pop III star formation. Initially, primordial star formation was proposed to be very simple, governed by well defined initial conditions provided by Gaussian fluctuations of the cosmic density field. In this context, the formation of the first stars occurs in regions where the cosmic fluid fulfils two conditions. First, it needs to decouple from the global expansion and begin to contract due to the self-gravity of dark matter, in a so-called dark matter halo. Second, the gas within the dark matter halo needs to be able to cool and go into runaway collapse to eventually reach stellar densities. For a gas of primordial composition, the only available coolant are atomic and molecular hydrogen. The combination of Λ CDM and primordial gas cooling leads to an initial collapse phase driven by H_2 cooling in dark matter (DM) mini-halos with masses $M_h \sim 10^5 - 10^6 M_\odot$ which form at the intersect of cosmic web filaments, at $20 \lesssim z \lesssim 30$ [1994–1996].

At these high redshifts, electromagnetic observations of individual Pop III forming regions are very challenging even for the James Webb Space Telescope (JWST) [1997–2000]. However, Pop III stars forming in pockets of metal-free gas at $z \lesssim 10$ have been predicted by simulations [2001, 2002] and may have already been tentatively detected by JWST [2003, 2004]. In addition, their cumulative emission leaves an important signature on the 21cm signal from cosmic dawn and the epoch of reionization [2005], which will provide important constraints on the onset and efficiency of Pop III star formation [2006–2010].

We shall also note that other important physical processes affect the onset of Pop III star formation and the minimum dark matter halo mass where this can occur. One example is the existence of streaming velocities, a relative motion between dark matter and baryons which stems from second-order cosmic perturbation theory and decreases linearly as the Universe expands [2011]. Simulations that include streaming velocities suggest that their presence delays the onset of cooling, and leads to a larger critical mass for collapse to set in [1177]. Another important aspect that may affect the minimum halo mass to host star formation is the nature of dark matter (DM). Warm DM or fuzzy DM models suppress structure formation on small scales that depends sensitively on the mass of DM particles, which therefore affects the predicted minimum scale for structure formation and the onset of Pop III star formation [2012–2016]. This may result in a delay of Pop III star formation into more massive (atomic-cooling) halos ($M_h \gtrsim 10^7 M_\odot$), where the primordial gas can cool by the collisionally excited Lyman- α line.

Finally, once the first Pop III stars form, there are other additional effects which control Pop III star formation, such as the presence of radiation fields in the LW UV band, which photo-dissociate H_2 , and hence suppress cooling, or in the X-ray band, which instead promote

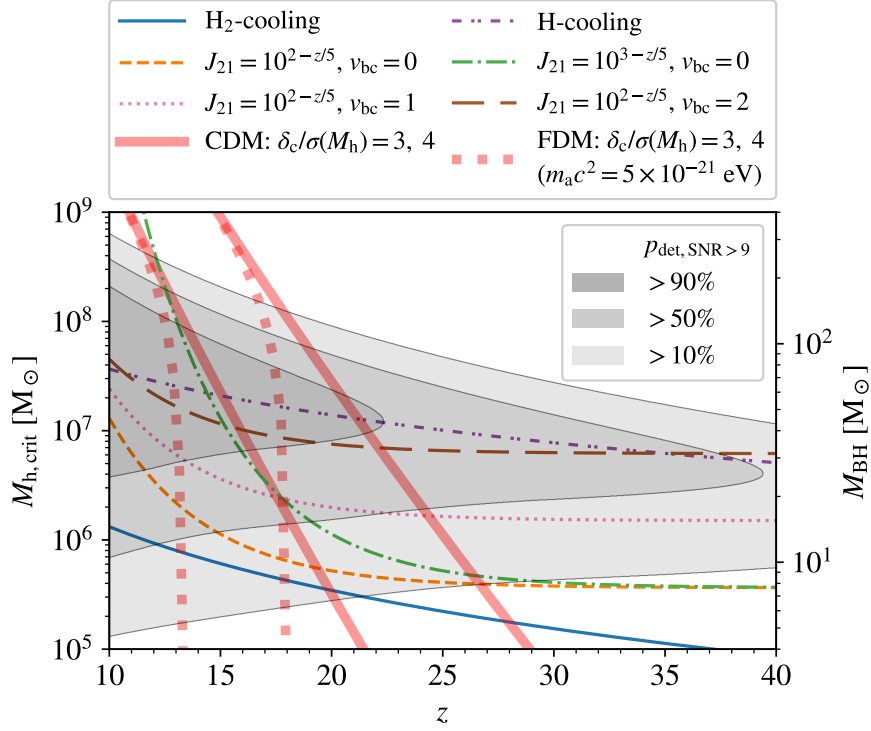


Figure 72. Redshift evolution of the critical halo mass threshold above which Pop III star formation can happen (left axis) and the BH mass range in BBH mergers detectable by ET (right axis). We show $M_{h,crit}$ for efficient H₂- and H-cooling from [1995] with the thin solid and dash-dot-dotted curves. We plot $M_{h,crit}$ regulated by Lyman-Werner (LW) radiation and baryon-DM streaming motion in four cases based on [2017] (see also [1996]), adopting different models for the LW background intensity J_{21} (in the unit of $10^{-21} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ sr}^{-1}$) and streaming velocities v_{bc} (in the unit of the cosmic root-mean-squared velocity). The thin dashed, dotted, and long-dashed curves show the results for $v_{bc} = 0, 1$, and 2 , respectively, under the model of J_{21} from [2018, 2019], while J_{21} is boosted by a factor 10 under $v_{bc} = 0$ for the dash-dot-dotted curve. We also show the halo masses corresponding to 3- and 4- σ density peaks for the standard Λ CDM model and a fuzzy DM (FDM) model given a boson particle mass of $m_a c^2 = 5 \times 10^{-21} \text{ eV}$ with the thick solid and dotted curves. For the right axis, the inner, intermediate, and outer shaded regions show the BH mass ranges in which more than 90%, 50%, and 10% of *equal-mass* BBH mergers can be detected at $\text{SNR} > 9$ by ET.

H₂ formation by increasing the free electron fraction and hence favor H₂ cooling. These additional physical processes however, depend on the properties of the radiation sources, hence on their mass spectrum and multiplicity, which we will discuss below.

In figure 72 we illustrate how some of the above effects may be constrained by ET by detecting the high- z BBH mergers formed as remnants of Pop III stars. The left axis shows the redshift evolution of the halo mass threshold, $M_{h,crit}$ above which Pop III star formation can happen. In particular, we report $M_{h,crit}$ for efficient H₂- and H-cooling from [1995], and $M_{h,crit}$ regulated by LW radiation and baryon-DM streaming motion in four cases based on [2017]. We also show the halo masses corresponding to 3- and 4- σ density peaks for the standard Λ CDM model and a fuzzy DM model with a boson particle mass of $m_a c^2 = 5 \times 10^{-21} \text{ eV}$. The onset of Pop III star formation can be identified by the intersects of these halo masses

with $M_{\text{h,crit}}$, which span a broad range of redshifts ($z \sim 12 - 30$). To appreciate these results in the context of GW observations, we show the BH mass ranges in which more than 90%, 50%, and 10% of *equal-mass* BBH mergers can be detected at $\text{SNR} > 9$ by ET with the right axis of figure 72. It is clear that ET has the potential to probe the formation history of Pop III stars up to $z \sim 30$ via mergers of Pop III BH remnants (at least in the mass range of $\sim 20 - 40 M_{\odot}$) (see e.g. [1674] for detailed discussions on the relation between the detection rate of Pop III BBH mergers by ET and the Pop III star formation history), which will provide interesting constraints on the factors that regulate the onset of Pop III star formation, such as the nature of DM, baryon-DM streaming motion, and UV radiation fields.

4.7.1.2 Initial mass function of Pop III stars. Starting from a pristine dark-matter mini-halo, the sequence of events which characterize the initial collapse phase is dictated by the physics of H_2 . The formation of the dense cores, which are hotter and more massive (typically a few $1000 M_{\odot}$) than those in present-day conditions, corresponds to the point where H_2 cooling becomes inefficient.³⁷ From this point on, the cloud continues to collapse without further fragmentation until it becomes optically thick and forms the central protostar. Early simulations were stopped at this point, when the object was surrounded by only a small disk-like structure which was more strongly supported by pressure than by rotation [2020–2029]. The authors of these studies argued that all the inflowing mass would end up in one single high-mass star.

In addition, due to the larger gas temperatures in the star-forming cores, the accretion rates onto Pop III protostars is expected to be several orders of magnitude higher than for present-day star formation and at these high rates the evolution was predicted to continue up to the arrival of the growing protostar onto the Zero-Age-Main-Sequence, where its mass was expected to be already of $M_{\text{ZAMS}} \sim 100 M_{\odot}$ [2030].

Because the luminosity increases rapidly with protostellar mass, radiative feedback is expected to regulate the mass accretion and ultimately shut off the accretion flow, setting the final mass to a few tens of $M_{\odot}$ [2030, 2031], although this value is likely to depend on environmental conditions, with values ranging from just under ten to more than one thousand solar masses [2032, 2033]. It is important to stress that these results were based on 2D radiation-hydrodynamic simulations that only follow the evolution of the central collapsing core. Hence, by construction, they describe a mass spectrum of solitary Pop III stars, one for each halo.

In the last two decades, the simple scenario of solitary formation of one massive Pop III star per minihalo has significantly evolved into a more complex picture that primordial star-forming gas is highly susceptible to fragmentation and Pop III stars typically form with broad mass distributions in (small) clusters where binary and multiple systems are common. In figure 73 we show some illustrative examples of the Pop III stellar initial mass function (IMF) predicted by numerical studies with and without the inclusion of radiative feedback [1177]. Despite the large variation in range (due to numerical limitations in resolution and time coverage), the predicted distributions are rather flat (in log space) compared to the highly

³⁷This occurs when the gas temperature is $\simeq 200\text{K}$ and the gas density is $\simeq 10^4\text{cm}^{-3}$, and the H_2 levels are no longer populated according to non-local thermodynamic equilibrium (NLTE). As a result, the cooling becomes less efficient [1992].

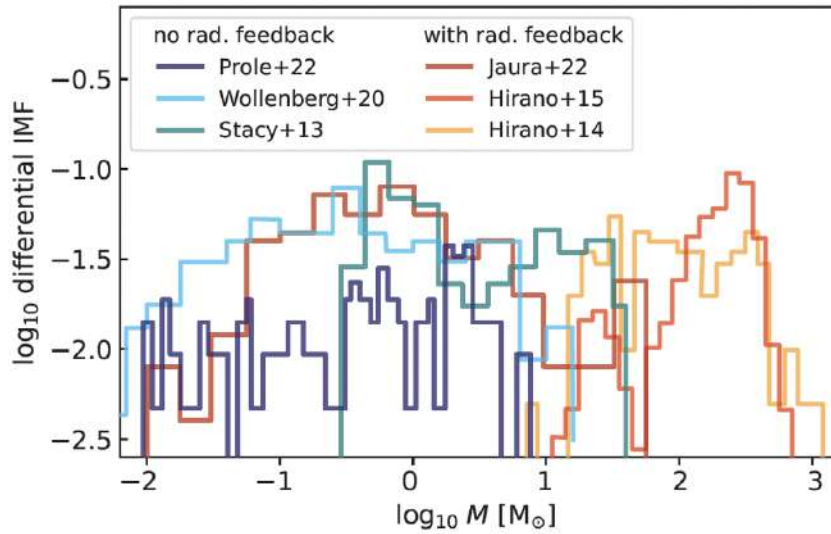


Figure 73. Illustrative examples of the Pop III stellar IMF predicted by three high-resolution models without stellar feedback [2034–2036] and three simulations with radiative feedback included [2032, 2033, 2037]. This figure is adapted from figure 6 in [1177]. Reproduced from [1177]. CC BY 4.0.

bottom-heavy present-day IMF, and can extend up to $\sim 1000 M_{\odot}$. Pop III stars may therefore leave behind black holes with a variety of masses, ranging from remnants of core-collapse SN explosions, to black holes formed by failed explosions, up to black holes with masses above the PISN mass gap, reaching the mass range expected for intermediate-mass black holes (IMBHs, see section 4.3.1 and 4.8). In this new picture, fragmentation has significant implications not only for the resulting Pop III stellar mass spectrum but also for the formation of binary/multiple systems of stars and BHs, as we discuss below.

4.7.1.3 Pop III star clusters and binaries. In the standard pathway of Pop III star formation, a star-forming disk forms at the center following the monolithic collapse of pristine gas by runaway molecular cooling in a minihalo ($M_h \sim 10^5 - 10^6 M_{\odot}$). The disk fragments to produce multiple protostars, which eventually become a small cluster of a few to few tens stars and a total mass of $\sim 10 - 1000 M_{\odot}$ when the disk is evaporated by stellar feedback. The initial configurations of Pop III star clusters and statistics of binary/multiple systems are shaped by poorly understood processes of protostar formation and evolution [1954, 2038]. Suffering from uncertainties due to limited resolution and imperfect modeling of stellar feedback, some simulations predict that Pop III binary stars typically have large separations $a \sim (2 \times 10^2 - 4 \times 10^4)$ AU due to outward migrations of nearly equal-mass protostars by rapid accretion of gas with high angular momentum [2039–2042], while others favor the formation of closer binaries ($a \lesssim 10^3$ AU down to the resolution limit of a few AU) with inward migrations by, e.g., dynamical friction in dense gas structures [2043, 2044].

The situation becomes even more complex in non-standard pathways where Pop III star formation is delayed to occur in more massive (atomic-cooling) halos ($M_h \gtrsim 10^7 M_{\odot}$) by, e.g., baryon-DM streaming motion and LW radiation (see figure 72 and section 3.3 in [1177]).

The rapid gas inflows ($\dot{M} \sim 0.1 - 10^3 M_\odot \text{ yr}^{-1}$) in such massive halos are expected to produce massive ($\gtrsim 10^4 M_\odot$), dense star clusters, where competitive accretion and runaway stellar collisions lead to the formation of a small number of very/super massive stars with $M_{\text{ZAMS}} \gtrsim 300 M_\odot$ up to a few $10^5 M_\odot$ [2045–2053], which are promising progenitors of IMBHs (see section 4.8). However, the abundance and properties of such massive Pop III clusters (e.g., mass, size, virial factor, internal structure, and IMF of stars) are also uncertain in current theoretical models.

In summary, our knowledge of Pop III star clusters and binaries is still incomplete due to computational limitations of simulations (e.g., resolution, time coverage, volume, treatment of feedback) and the lack of direct observations (which will remain challenging in the near future). Nevertheless, as discussed below, the properties of Pop III star clusters and binaries, as well as Pop III stellar evolution, are encoded in the properties of compact object mergers from Pop III stars. Therefore, GW observations by ET will bring us new insights into the physics of Pop III star formation/evolution.

4.7.2 The evolution of Pop III stars and binary black holes

Compared with metal-enriched Population I/II (Pop I/II) stars, Pop III stars are more compact and experience negligible mass loss from radiation-driven winds because radiation escapes from them more easily in the lack of metal-line opacity (see figure 13 of [1177]). The special metal-free stellar evolution, massive nature (section 4.7.1.2), and common multiplicity (section 4.7.1.3) of Pop III stars indicate that they are efficient progenitors of BBHs. Similar to BBH mergers from Pop I/II stars, the evolution of Pop III BBHs to mergers can also be classified into the isolated (section 4.7.2.1) and dynamical (section 4.7.2.2) channels. As summarized in figure 74 and discussed in detail below, these two channels are closely connected to the special cosmological context and pathways of Pop III star formation, and they can potentially produce BBH mergers³⁸ with unique signatures identifiable in GW observations by ET, which makes ET an indispensable probe of Pop III stars (section 4.7.3).

4.7.2.1 Pop III binary black hole mergers from isolated binary stellar evolution.

BBH mergers from Pop III stars have been studied most intensively for the isolated binary stellar evolution channel (see section 4.1.1) [1671–1675, 1954, 2055, 2057–2064]. The isolated binary stellar evolution channel is expected to be suitable for the standard pathway of Pop III star formation that produces small Pop III clusters with short dynamical and relaxation timescales, where most dynamical interactions happen in the early stage (within $\lesssim 1 \text{ Myr}$), and Pop III binaries³⁹ evolve effectively in isolation thereafter. Although well-motivated, the merger efficiency and properties of Pop III BBH mergers from isolated binary stellar evolution are highly sensitive to the obscure initial statistics of Pop III binary stars, binary interaction processes, and Pop III stellar evolution.

³⁸Pop III stars also produce BH-NS and NS-NS mergers but at much lower efficiencies compared with BBH mergers (see, e.g., [1954, 2054–2056]). Such less massive mergers are also more difficult to detect at high z . Therefore, we focus on Pop III BBH mergers.

³⁹Triples and quadruples also form in small Pop III star clusters [2038, 2065, 2066]. The BBH mergers produced by such higher-order systems can be different from those from standard isolated binary stellar evolution (see section 1.1.3). However, this aspect has not been explored for Pop III stars.

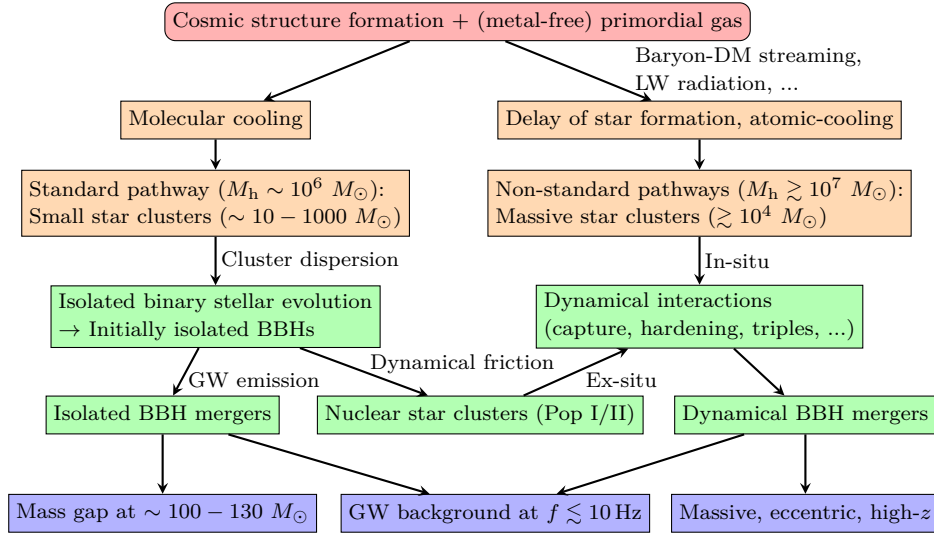


Figure 74. Connections between the cosmological context (red) and pathways (orange) of Pop III star formation (section 4.7.1), evolution of Pop III BBHs (green, see section 4.7.2), and identifiable signatures of Pop III BBH mergers in ET observations (blue, see section 4.7.2 and 4.7.3).

For instance, if the initial binary statistics of Pop III stars is dominated by wide binaries with $a \sim 10^2 - 10^4$ AU as found in the simulations from [2036], rather than close binaries with $a \sim 0.1 - 10$ AU like in observations of present-day stars [1424], the merger efficiency of Pop III BBHs will be reduced by up to a factor of ~ 20 , and the dominant evolutionary pathway changes from stable mass transfer to common envelope evolution [1672, 1674, 1675]. The evolution of individual stars has strong effects on the BH mass distribution in Pop III BBH mergers, in particular for massive BHs inside or above the PISN mass gap⁴⁰ $\sim (50 - 130) M_\odot$. In general, models with stronger stellar expansion disfavor the formation of massive Pop III BBH mergers via isolated binary stellar evolution, since larger stars are more likely to suffer strong mass loss and undergo stellar mergers during binary interactions. If massive Pop III stars with $M_{\text{ZASM}} \gtrsim 100 M_\odot$ become red super giants with radii $R \gtrsim 10^3 R_\odot$ when they approach the end of main sequence (MS) or during post-MS evolution, Pop III BH mergers mostly contain BHs below $50 M_\odot$ [1674, 1675]. In contrast, if stellar expansion is suppressed, for example, by inefficient convective overshooting, close ($a \lesssim 1$ AU) binaries of Pop III stars with $M_{\text{ZAMS}} \sim (50 - 100) M_\odot$ can produce BBH mergers involving BHs of similar masses inside the PISN mass gap via stable mass transfer with little mass loss, and binaries of more massive ($M_{\text{ZAMS}} \gtrsim 240 M_\odot$) Pop III stars can produce BBH mergers with BHs above the PISN mass gap ($\gtrsim 130 M_\odot$) avoiding stellar mergers [1673, 2061–2064].

⁴⁰Pop III single stars with initial masses $\sim (50 - 100) M_\odot$ can avoid pair instability and retain most of their hydrogen envelopes before collapse to produce BHs of similar masses in the PISN mass gap [2067, 2068]. If their spins remain small ($\lesssim 0.2$) after birth [1414, 1573, 2069], Pop III BHs in the mass gap can be distinguished from the merger products of smaller BHs [2070], although it is challenging to tell them from PBHs with similar masses and spins (see section 4.6). How such Pop III BHs show up in BBH mergers is still in debate [2061].

Moreover, these results point out the intriguing possibility that the BH mass distribution in Pop III BBH mergers from isolated binary stellar evolution shows a unique⁴¹ narrow mass gap around $\sim (100-130) M_\odot$, under the peculiar condition that (1) close ($a \lesssim 10$ AU) binaries of massive Pop III stars are common, and (2) massive Pop III stars do not expand significantly before collapse. The identification or non-detection of this feature in GW observations of BBH mergers (up to $z \sim 20$ by ET, see figure 72) will place novel constraints on the binary statistics of Pop III stars and Pop III (binary) stellar evolution.

4.7.2.2 Dynamical Pop III binary black hole mergers in dense star clusters. Other than isolated binary stellar evolution, dynamical Pop III BBH mergers can also occur in dense star clusters in two scenarios. (1) In-situ: Pop III BBHs form in massive ($\gtrsim 10^4 M_\odot$), dense clusters of Pop III stars/remnants themselves [2049, 2050, 2071] hosted by atomic-cooling halos ($M_h \gtrsim 10^7 M_\odot$). (2) Ex-situ: Pop III BBHs (initially born in isolation) fall into nuclear star clusters made of Pop I/II stars by dynamical friction when their natal minihalos grow/merge into larger halos [2056, 2072, 2073]. The main difference between the two scenarios is that Pop III mergers in nuclear star clusters mostly happen at relatively low redshifts $z \lesssim 6$ because only massive ($M_h \gtrsim 10^9 M_\odot$) halos can possibly host nuclear star clusters, while a large fraction ($\sim 0.4-0.8$) of in-situ dynamical Pop III mergers can happen at $z \sim 6-20$ providing that massive Pop III clusters are common. Both scenarios produce mergers of more massive BHs with lower mass ratios compared with the case of isolated evolution, especially for BHs above $\sim 50 M_\odot$ including IMBHs that can be extremely rare in isolated mergers [1674, 1675, 2071, 2073]. Besides, a significant fraction (up to $\sim 30\%$) of dynamical Pop III BBH mergers will have large enough eccentricities $e \gtrsim 10^{-3}$ detectable by ET at the observer-frame frequency $f = 10$ Hz [1585].

These results are consistent with the common properties of dynamical BBH mergers (see sections 4.1.2 and 4.1.2.2), although the predicted merger rates are subject to large uncertainties in the abundance, initial conditions, and long-term environments of massive Pop III clusters, as well as the assumptions on high- z nuclear star clusters and dynamics of Pop III BBHs in high- z galaxies. It will be challenging to identify Pop III BBH mergers in nuclear star clusters where they are likely overwhelmed by dynamical mergers of Pop I/II BBHs with similar masses [1543]. However, observations of massive, eccentric mergers (with low mass ratios) at very high redshifts ($z \gtrsim 10$) by ET will strongly constrain the in-situ Pop III scenario, since BBH mergers from other origins either occur at lower redshifts (Pop I/II BBHs) or on quasi-circular ($e \lesssim 10^{-5}$) orbits (isolated Pop III BBHs and PBHs).

4.7.3 Gravitational waves from Pop III remnants

Due to large uncertainties in the cosmic star formation history of Pop III stars (see figure 2 of [1177]), properties of Pop III clusters and binaries, as well as Pop III (binary) stellar evolution, theoretical predictions Pop III BBH mergers are highly uncertain. Their merger rate (density) is expected to peak around $z \sim 6-15$ with $\mathcal{R}_{\text{PopIII}}^{\text{max}} \sim (0.1-200) \text{ yr}^{-1} \text{ Gpc}^{-3}$, and the local rate is $\mathcal{R}_{\text{PopIII}}(z=0) \sim (10^{-3}-100) \text{ yr}^{-1} \text{ Gpc}^{-3}$ [1671-1674, 1954, 2049, 2050,

⁴¹BBH mergers from extremely metal-poor ($Z \lesssim 10^{-4}$) stars have similar properties as Pop III mergers if given the same conditions [1675, 2063]. However, it is unknown whether such stars can be as abundant and massive as Pop III stars to have noticeable signatures in GW observations.

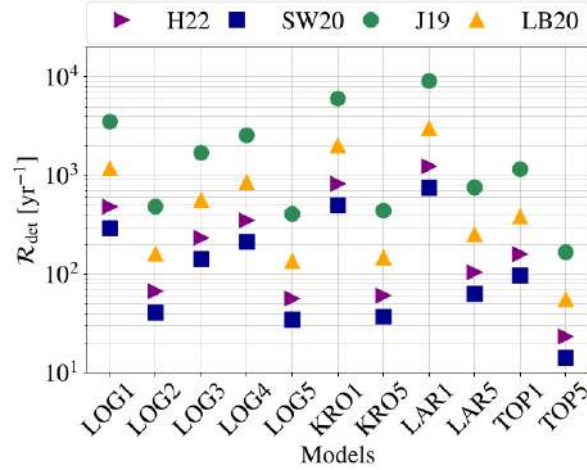


Figure 75. Detection rates of Pop III BBH mergers from isolated binary stellar evolution by ET with $\text{SNR} > 9$ for different models of the initial statistics of Pop III stellar binaries (x axis) and cosmic star formation histories of Pop III stars (denoted by different markers) adapted from figure 4 in [1674]. The rates predicted by other studies for isolated mergers and dynamical mergers in dense star clusters are within the range shown here. Reproduced with permission from [1674].

[2055, 2056, 2058, 2060, 2062–2064, 2071, 2073, 2074]. In general, Pop III BBH mergers will dominate the merger rate at least for $z \gtrsim 20$ (unless PBH mergers are abundant), and they typically contain BHs above $20 M_{\odot}$, more massive than the low-mass ($\sim 10 M_{\odot}$) BHs that dominate the observed local merger rate. Depending on the Pop III IMF and (binary) stellar evolution parameters, Pop III BBH mergers in isolation may contain massive ($\gtrsim 50 M_{\odot}$) BHs inside and above the PISN mass gap, while such BHs will be common in dynamical Pop III BBH mergers in dense star clusters. The high sensitivity of ET, especially at low-frequencies, makes it an ideal instrument for observing Pop III BBH mergers at very high redshifts ($z \sim 30$), reaching the smallest structures hosting the first generation of stars in the Universe (figure 72). A promising detection rate of $\sim 10 - 10^4$ events per year is estimated from current theoretical models, as illustrated in figure 75.

More importantly, even if Pop III BBH mergers remain a sub-dominant component in the detectable events, they can be identified with their potentially unique features, such as a narrow mass gap $\sim 100 - 130 M_{\odot}$ in the BH mass distribution and high eccentricities ($e \gtrsim 10^{-3}$ at $f = 10 \text{ Hz}$) at $z \gtrsim 10$, and with the aid of machine learning [1585, 1739, 1799, 2075]. Beyond resolved merger events, under certain conditions, Pop III BBH mergers can possibly make significant contributions to the GW background, producing a spectral feature at $f \lesssim 10 \text{ Hz}$ accessible to ET observations, in particular for the residual background where individual detected sources are properly subtracted, see also section 4.10.2 [2073, 2076–2078]. Since it is incredibly difficult to observe individual Pop III stars directly via electromagnetic waves [1998, 1999], GW observations by 3rd-generation detectors like ET, combined with other indirect probes of Pop III stars [2079–2081], will be extremely valuable for revealing the elusive nature of Pop III stars and early structure formation.

4.8 Intermediate-mass BHs (IMBHs): formation channels and merger rate

Intermediate-mass black holes (IMBHs) are an elusive class of objects that may bridge stellar-mass black holes and supermassive black holes (SMBHs). IMBHs are expected to densely populate the $10^2 - 10^5 M_\odot$ mass range, although there is little knowledge about their properties due to scarce observational evidence.

During the first three observing runs, the LVK collaboration identified 8 IMBH candidates (3 of them at 90% credibility level) formed from stellar BH mergers [1509, 2082, 2083]. Several IMBHs, generally heavier than $50,000 M_\odot$, have been observed in the nucleus of dwarf galaxies [2084–2091], and in the center of massive systems in the outskirts of massive galaxies, suggesting they are the nucleus of a dwarf galaxy that was stripped during a minor merger (e.g., [2092–2096]).

IMBHs are one possible engine powering the so-called ultra-luminous X-ray sources (ULXs) (e.g., [2097, 2098]), off-centered sources with an X-ray luminosity above the Eddington limit for a stellar BH, $L_X \sim 10^{39} \text{ erg s}^{-1}$ (see the review [2099]). Most of the known population of ULXs are associated with emitting NSs, but the most energetic ones — also known as hyper-luminous X-ray sources (HLXs) — with $L_X > 10^{41} \text{ erg s}^{-1}$, can be powered only by an object with mass $> 10^3 M_\odot$ [2100, 2101].

In the $(10^3 - 10^4) M_\odot$ mass range, putative IMBHs have been also observed in a few galactic and extragalactic star clusters (e.g., [2102–2106]). However, most observations of these mid-range IMBHs are controversial (e.g., [2107–2110]). The absence of compelling evidence for IMBHs with masses $(10^3 - 10^4) M_\odot$ leads to a natural, fundamental question about IMBHs: do IMBHs constitute a unique class of astrophysical BHs? Or, rather, do they populate the high end of the stellar BH mass distribution and the low end of the SMBH mass distribution?

Several scenarios have been proposed to explain IMBH formation. We can broadly divide them into three main ensembles: direct collapse, Pop III stellar evolution, and dynamical formation.

4.8.1 Formation Scenarios

4.8.1.1 Direct collapse. Direct collapse BHs can have masses of $\sim (10^4 - 10^5) M_\odot$ and result from the collapse of a supermassive star of $\sim 10^5 M_\odot$ [2111–2117]. Such supermassive stars form via a rapid inflow of dense gas in metal-free, or very metal-poor, protogalaxies, in which an intense Lyman-Werner radiation field prevents gas fragmentation and star formation. This Lyman-Werner radiation can be provided by close star-forming galaxies [2118–2120]. The high supersonic turbulence in the primordial gas reservoirs could however also yield the formation of direct collapse BHs without the need for a strong Lyman-Werner field [2121]. Because of all these constraints, direct collapse BHs are thought to be less abundant than IMBHs formed via other mechanisms (e.g. Pop III stars) and their mass range is out of ET operating band. They could however be detected at near-infrared wavelengths at $z \gtrsim 20$ [2122, 2123] and in the radio regime at $z \sim 8 - 10$ [2124, 2125].

4.8.1.2 Pop III stars. One possible pathway to form IMBH seeds with masses in the range $(10^2 - 10^3) M_\odot$ is via single and binary stellar evolution of Pop III stars. This channel

has been discussed in detail in section 4.7.1.3. In this scenario, IMBHs form in the early Universe, at a redshift $z \sim 8 - 20$ (for a discussion about the various uncertainties in Pop III star formation history and their impact on BH formation, see [1674]) in the center of protogalaxies. Given their mass, IMBH from Pop III stars are usually referred to as “light seeds”, in contrast to the “heavy seeds” formed from direct collapse discussed above. In fact, light seeds can, in principle, further grow via gaseous accretion and reach final masses that cross the IMBH-SMBH boundary. However, theoretical and numerical models suggest that the IMBH mass growth is not sufficiently rapid to explain the observation of SMBHs at redshift $z > 10$, as recently discovered by JWST [2126, 2127], favoring the direct collapse scenario.

A viable way to increase the IMBH mass is via repeated mergers with smaller BHs and accretion of stellar material. This “dynamical” process is poorly explored for Pop III stars, owing to our poor knowledge of the physics of clustering in the first stages of life of the Universe, but it is quite robustly explored in the case of Pop I and Pop II star clusters, as we discuss in the next section.

4.8.1.3 Dynamical formation of IMBHs. Several galactic and extragalactic GCs exhibit peculiar features that may be ascribed to a central IMBH with masses from a few $10^2 M_\odot$ to $\sim 10^4 M_\odot$ (e.g., [2106]). Three main mechanisms can support the seeding and growth of an IMBH in a star cluster: stellar collisions, stellar feeding, and hierarchical BH mergers (for a recent review, see [2128]).

Stellar collisions. The combination of mass segregation, by which the most massive objects in a stellar ensemble lose orbital energy to the lighter ones, and relaxation, which is driven by close dynamical interactions, causes over time a continuous increase of the central density of star clusters up to a critical point, referred to as “core-collapse”, at which the density reaches a peak and interactions become so violent to favor the formation of tight binaries. The binaries act as a heating source, scattering off stars and causing the expansion and cooling of the cluster core. During this complex phase, binaries can get so tight that their components merge, and densities become high enough to favor close hyperbolic interactions.

If the timescale of core-collapse is shorter than the typical stellar evolution time for massive stars, this process can favor stellar mergers and collisions. This generally happens in clusters with a mass $\lesssim 10^5 M_\odot$ and half-mass radii in the range $\sim (0.2 - 1)$ pc [2129, 2130]. In such low-mass and dense clusters stellar collisions can proceed in a runaway fashion, rapidly building up a very massive star (VMS) with a mass $\sim (10^2 - 10^3) M_\odot$ [1560, 2129, 2131–2134]. The VMS ultimately collapse to an IMBH, which possibly grows later, at a slower pace, by capturing passing by stars or merging with smaller BHs, see figure 76.

There may be a threshold value of the cluster core density above which the stellar collision process is favored over others to build-up an IMBH, likely in the range $\rho_{\text{thre}} \sim (3 \times 10^5 - 10^7) M_\odot \text{ pc}^{-3}$ [1560, 2129, 2131]. The time over which this runaway process onset depends critically on several quantities that characterise the cluster structure, like possible primordial mass segregation, primordial binaries, or the stellar mass function.

The possibility of forming an IMBH seed through this mechanism depends on the mass of the VMS, which must overcome the mass range affected by PISN and pulsational pair-instability supernova (PPISN). Typically, the onset of PISN, which completely disrupts the

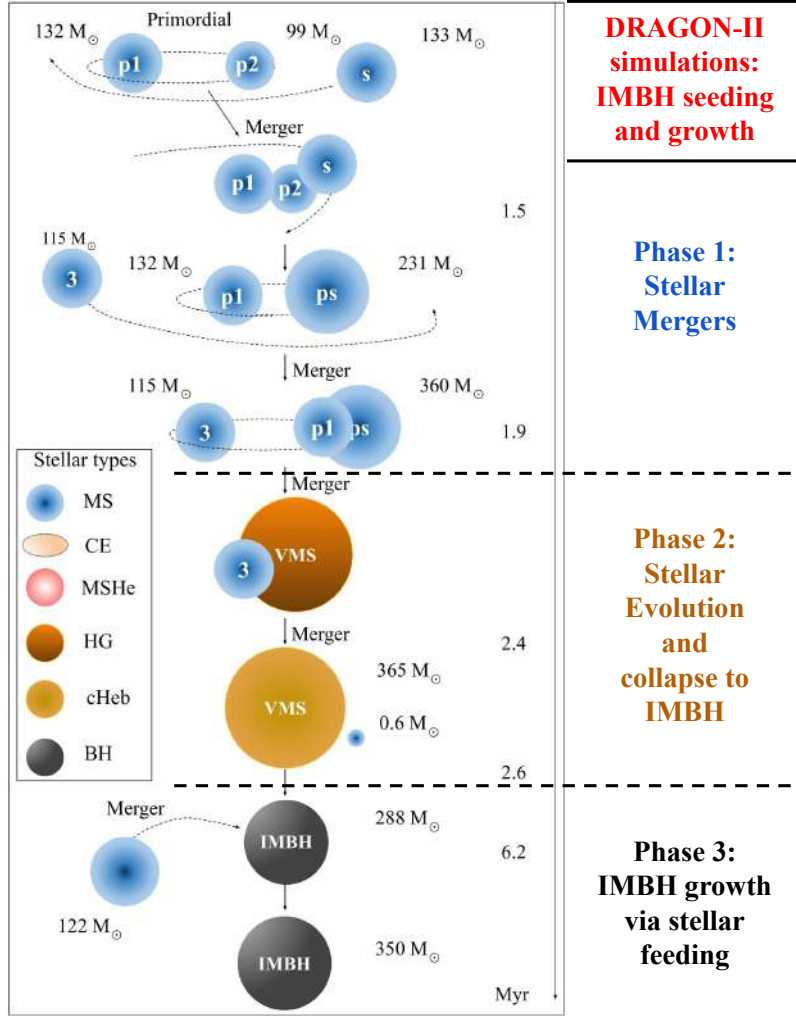


Figure 76. Stellar mergers leading to the formation of an IMBH in one of the DRAGON-II numerical simulations. Reproduced with permission from [1560].

star, becomes inefficient at $m_{\text{VMS}} > 240 M_{\odot}$, although the minimum star mass above which PISN significantly increases at increasing the metallicity (e.g., [2135]). Unfortunately at metallicities $Z \geq 0.008$, the VMS collapse would result in a BH of $\sim (20 - 30) M_{\odot}$, regardless its mass [2135], thus suggesting that metal-rich clusters are unlikely site of formation of IMBHs via stellar collisions, unless the internal structure of the VMS significantly differs from that of a single massive star owing to the material reshuffling promoted by the stellar collision process. Accretion of gaseous material during the phase in which clusters are still embedded in their progenitor clouds can significantly contribute to the VMS growth, possibly favoring the formation of supermassive stars as massive as $(10^3 - 10^4) M_{\odot}$ (e.g., [2052, 2136]), that would likely undergo direct-collapse.

Stellar feeding. In dense stellar environments, close encounters between stars and BHs can trigger star-BH collisions, with consequent fall of stellar material onto the BH. This can

sustain the IMBH growth, especially for massive ($> 50 M_{\odot}$) stars. Whether, and in what amount, such material can be fed to the BH is unclear.

Little accretion is expected in the case of a normal star colliding with a stellar BH, as in such a case most of the energy is expected to be radiated away via jets [2137–2140]. Nonetheless, some models suggest that during the common envelope phase, a BH can accrete the star core and expel the envelope [2141], a process that can significantly spin up the BH. If the accreting star is a byproduct of previous stellar collisions, the merger product should be characterised by a tenuous envelope and dense core [2142]. In such a case, the BH can accrete most of the star mass.

IMBHs forming through this mechanism may be preceded by SN-like [2141] and other high-energy transients [2140]. These IMBHs may be characterised by peculiar spins, depending on their formation history. If most of its mass comes from the accretion of one massive star, the accretion process can significantly spin-up the IMBH up to extremal values. If, instead, the IMBH mass growth proceeds through episodic accretion from random directions, the cumulative effect should lead the IMBH to spin-down.

Hierarchical BH mergers. IMBHs can also form via repeated mergers among BHs, or *hierarchical* mergers [1491, 1538, 1539, 1556, 1560]. The growth of an IMBH through repeated mergers with smaller BHs can be hindered by both Newtonian and Relativistic recoil, which can expel the IMBH from the host cluster (see also section 4.1). Newtonian kicks on IMBHs with $m < 10^3 M_{\odot}$ can be as large as 10^2 km s^{-1} [1560, 2143]. Relativistic kicks typically exceed 10^2 km s^{-1} for binary mass ratio > 0.1 ⁴² [1552]. Owing to the typical mass and density of young massive and globular clusters, IMBHs in these clusters are expected to reach maximum masses in the range $(100 - 1000) M_{\odot}$ (e.g., [1528, 1538, 1542, 2144]). Heavier IMBHs could form in nuclear clusters, where escape velocities can reach values $(10^2 - 10^3) \text{ km s}^{-1}$ [1482, 1540, 1543, 1556].

Long merger chains can leave observable imprints on the properties of the IMBH population. The increase in mass through successive mergers and the reduction in spin driven by the accretion of smaller BHs from isotropic directions can lead to a peculiar mass-spin-redshift relation (e.g., [1538]), although such complex relation is severely affected by the uncertainties in theoretical models. Accurate measurements of spin and masses can thus possibly help to discriminate between IMBHs that grew prevalently via BH mergers and those that accreted a significant amount of stellar matter.

Dynamical formation in AGN disk. An alternative formation scenario for IMBHs is via repeated mergers and gaseous accretion in the disk of AGN [1705, 2145]. In this framework, orbits of stars and BHs crossing the disk are damped by dissipation forces and settle in the disk plane. Additionally, massive stars can form directly in-situ from fragmentation in marginally stable discs [2146, 2147], although the slope and extent of the mass function may depend on the SMBH mass and environment [2148].

The reduced relative velocities in the disk plane, the impact of gaseous dynamical friction, and the possible emergence of regions of the disk where inward and outward forces cancel

⁴²Although depending on the spin alignment the kick can remain in the range $10 - 100 \text{ km s}^{-1}$ for a binary mass ratio value > 0.7 .

out (so-called migration traps) favor scatterings and collisions that can rapidly build up an IMBH [1498]. This channel can leave peculiar imprints in the distribution of merger masses and spins, especially if the IMBH accretes gas from the disk after the formation (see the recent review by [1482]).

This type of process could, in principle, favor the formation of an IMBH binary, although it is hard to assess the likelihood for this process to occur.

Multiple scenarios. The aforementioned scenarios do not work necessarily exclusively. For example, stellar mergers can lead to the formation of an IMBH seed that subsequently grows via repeated BH mergers. If the formation of an IMBH depends on the cluster structure, the cosmic distribution of IMBH mass should encode information on the structure and cosmic formation history of star clusters. The possibility to observe $\sim (10 - 10^3)$ mergers/yr involving an IMBH with ET [1538] will give us insights on the global IMBH mass distribution, at least at masses $< 10^3 M_\odot$ [2149], and their nursing environments [1639].

4.8.2 IMBH binaries

4.8.2.1 Massive star clusters. In dense star clusters, forming an IMBH-IMBH binary requires that multiple merger chains separately seed the two BHs (e.g., [2150, 2151]). This process is unlikely in young massive and globular clusters, where typically one massive star or BH grows and soon dominates the dynamics cannibalizing the population of BHs or their progenitors and preventing the onset of a new merger chain (e.g., [2131, 2133, 2152]). Two chains may develop if the cluster is sufficiently massive, dense, and rich in binaries [2150].

The merger of massive, Pop III, binaries can represent a viable way to form IMBHs with mass $< 10^3 M_\odot$ in high-redshift star clusters (see also section 4.7 for further details on this scenario).

Alternatively, star cluster collisions can aid IMBH binary formation (e.g., [2153–2155]). In a similar scenario, the collision of molecular clouds and protoclusters in starburst environments can lead to the rapid build-up and pairing of IMBHs with nearly equal masses [2152, 2156].

The detection of a merging IMBH binary with mass $< 10^3 M_\odot$ by ET can help place constraints on the properties of the star clusters that can host such systems.

4.8.2.2 Dwarf galaxies. According to the Λ CDM scenario for structure formation, dwarf galaxy mergers are expected to be relatively frequent, as also supported by cosmological simulations [2157, 2158]. This process can aid IMBH binary formation (e.g., [2159–2161]), although it is unclear whether dynamical friction is efficient enough as to prevent the IMBHs from staling and not reaching coalescence [2160].

Observing IMBH mergers in the mass range $\gtrsim 10^3 M_\odot$ with ET could thus provide us with crucial insights about dwarf galaxy formation processes.

4.8.3 Can ET decipher the origin of IMBHs?

An instrument like ET has the potential to place stringent constraints on the mass distribution of double IMBH binaries with total masses $m_{\text{IMBH}} \lesssim 3 \times 10^3 M_\odot$ at redshift $z > 1$ [16, 1181]. Compared to the equal-mass binary case, the horizon redshift slightly changes if an IMBH paired with a smaller compact object is considered.

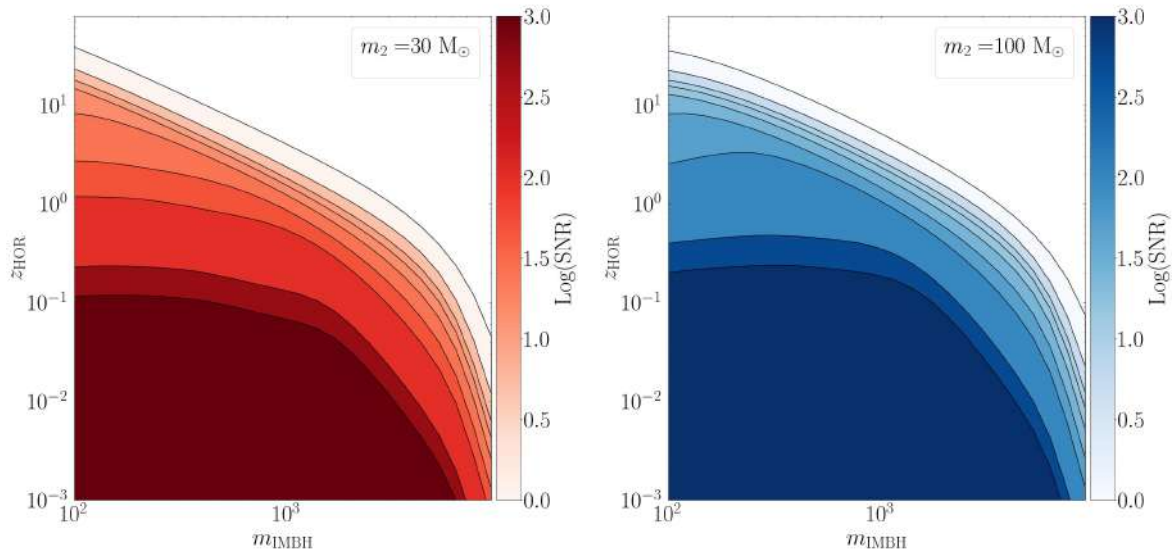


Figure 77. Horizon redshift for IMBH-BH mergers assuming a companion mass of $30M_{\odot}$ (left panel) and $100M_{\odot}$ (right panel). From brighter to darker colours, contours encompass regions with an $\text{SNR} > 1, 5, 10, 15, 25, 50, 100, 500, 1,000$, respectively. Calculations are performed through the GWFISH package [291], assuming a PhenomD waveform approximant.

Figure 77 shows ET horizon redshift for different values of the SNR and assuming a companion BH with a mass m_2 equal to $30M_{\odot}$ or to $100M_{\odot}$. ET can potentially detect merging IMBHs with masses in the mass range $(10^2 - 10^3)M_{\odot}$ with a stellar (upper-mass gap) BH out to a redshift $\lesssim 3(6)$ with an $\text{SNR} \gtrsim 15$. Golden events at $z < 1$ (with an $\text{SNR} > 100$) could help place stringent constraints on the mass of both the spin and mass of the merging components. ET can thus significantly boost our understanding of IMBHs and their cosmic evolution. We also compare a 2L ET configuration with 15 km arm length and 45 deg inclination and a triangle configuration with 10 km arm length and found minimal differences, see figure 78.

Essentially, ET will be able to detect IMBH binaries and intermediate-mass ratio inspirals (IMRIs). However, while the theory about SMBH-BH mergers, i.e. extreme mass-ratio inspirals (EMRIs), is robustly defined, despite the main unsolved uncertainties, the same theoretical framework is hard to apply to IMRIs. SMBHs remain anchored at the host galaxy center, whilst IMBHs wander in the cluster core. Therefore, the EMRI formation is driven by relaxation of stellar motion in the cusp, and most EMRIs are expected to appear highly eccentric in LISA band. For IMBHs, the wandering motion can have a non-trivial effect on the global distribution of the eccentricity at the moment of IMRI’s formation [2162]. Depending on the measurement accuracy of IMRI eccentricities, ET could potentially help to place constraints on the IMRIs eccentricity distribution. For example, an excess of high-eccentricity IMRIs in ET band would support the idea that IMRIs form in massive and dense star clusters via close single-single and single-binary encounters (e.g., [2163]).

Whether IMBHs are spinning depends on many uncertain quantities, like the natal spin distribution of stellar BHs, or the impact of mass transfer on the spin evolution.

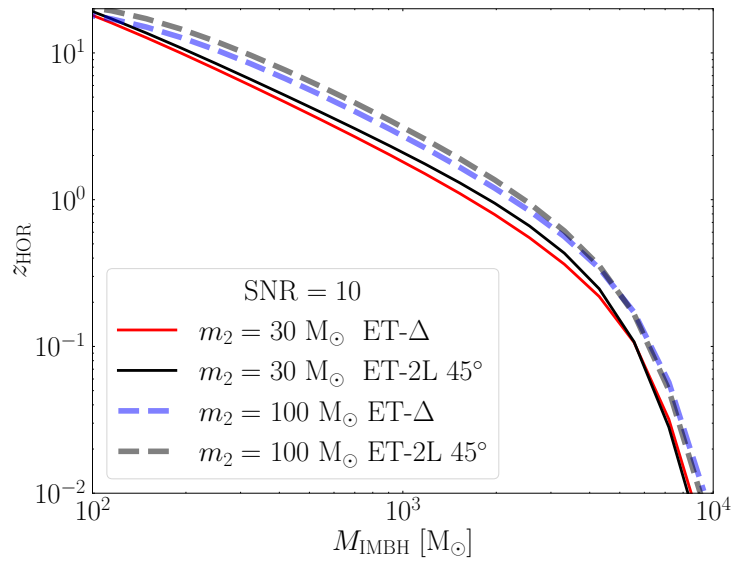


Figure 78. Horizon redshift for IMBH-BH mergers assuming a companion mass of $30M_{\odot}$ (solid line) and $100M_{\odot}$ (dashed line) for a fixed SNR= 15 and comparing a 15km 2L ET configuration with 45 deg inclination (black) and a 10km triangle ET configuration (red and blue for the 30 and 100 $M_{\odot}$ companion, respectively). The sensitivity curves come from [16]. We used the GWFISH package [291], assuming a PhenomD waveform approximant, i.e. the same as figure 77.

Recent numerical simulations based on both stellar dynamics and evolution suggest that young massive clusters can nurture the formation of IMBHs with masses in the range $(100 - 500) M_{\odot}$ [1560, 2133, 2134, 2164] and expected spins $\chi > 0.6$ [1560]. More massive IMBHs, $m_{\text{IMBH}} > 10^3 M_{\odot}$, forming through long chains of repeated BH mergers, instead, are expected to develop a final spin relatively small < 0.1 , leading to an anti-correlation between the IMBH spin and its mass [1538, 1553]. Stellar feeding can also have a significant effect: coherent accretion, for example, represents a viable way to spin up the IMBH while feeding on stellar material, similar to what is expected for SMBHs (for a recent review on the subject, see [2165]). This would be the case in which a single, very massive star, collides with a stellar-mass BH, leading the resulting IMBH to gain most of its mass in a single event. Differently, when the mass growth occurs through slow accretion of small stars (e.g., [2131, 2166]), the IMBH should spin-down. This may lead to a variety of IMBH sub-populations characterised by peculiar spin distributions at different cosmic times — as the timescale regulating the IMBH accretion can vary by several orders of magnitudes — depending on the process. Repeated mergers with BHs can damp the IMBH spin once the IMBH mass reaches a given threshold, i.e. the binary mass ratio falls below a certain value [1538]. If the IMBH forms in a gaseous medium, like an AGN disk, the accretion of gaseous material can also spin up the IMBH [1584].

Depending on the assumptions, models predict that ET could observe $10 - 10^3$ IMBH-BH mergers per year involving IMBHs with masses up to $\lesssim 10^3 M_{\odot}$ mass range [1538]. This is somehow supported by the population of mergers observed during the O1-O3 runs by

z	m_1	m_2	$\chi_{1,2}$	SNR	σ_{m_1}/m_1	σ_{χ_1}/χ_1
IMRI						
1	100	30	0.5	117	0.006	0.005
1	500	30	0.5	209	0.0006	0.002
1	1000	30	0.5	67	0.006	0.01
1	3000	30	0.5	6	0.04	0.07
4	100	30	0.5	39	0.02	0.04
4	500	30	0.5	33	0.01	0.02
4	1000	30	0.5	4	0.1	0.3
4	3000	30	0.5	0.1	3.2	10.1
IMBH binaries						
1	100	100	0.5	90	0.04087	0.08624
1	500	500	0.5	564	0.03564	$7.186e-07$
1	1000	1000	0.5	242	0.01833	$2.578e-07$
1	3000	3000	0.5	16	0.07471	$3.146e-06$
4	100	100	0.5	46	0.1018	0.001492
4	500	500	0.5	44	0.03316	$7.034e-07$
4	1000	1000	0.5	4	0.1561	$3.331e-06$

Table 8. Col. 1: redshift. Col. 2-3: primary and companion mass. Col. 4: component spins. Col. 5: SNR. Col. 6-7: relative error on the IMBH mass and spin. Simulations are carried out with GWFISH [291], assuming a PhenomD waveform approximant.

LVK, which lead to a predicted rate for equal-mass binaries with mass $200 M_\odot$ of $0.056 \text{ Gpc}^{-3} \text{ yr}^{-1}$ [2083]. Assuming that ET can detect all IMBH mergers and a constant merger rate up to redshift 8 leads to ~ 160 detections per year. Since ET can measure the IMBH spin down to $\sigma_\chi \simeq 10^{-3}$ [1568], the large number of possibly detectable IMBHs can provide crucial clues on the spin distribution of merging IMBHs.

For the sake of comparison, we show in table 8 the SNR and the relative error in measuring the IMBH mass and spin for different assumptions of the IMBH-BH binary properties. From our analysis, performed with the GWFISH package, we find that ET could observe IMBH-BH binaries with mass $< 10^3 M_\odot$ out to redshift $z < 4$ with an $\text{SNR} \gg 1$.

Therefore, ET has the unique possibility to deeply probe the IMBH mass range within $m_{\text{IMBH}} < 10^3 M_\odot$, a region of the phase space that cannot be accessed with detectors operating at lower frequencies, like LISA. In some cases, the emitted signal could also be observed in different bands, depending on the binary location, distance, and orbital parameters. An example of multiband sources is shown in figure 79, where the signal coming from different IMBH-IMBH binaries is compared against the SNR of several detectors. A network of terrestrial detectors could also represent a viable way to place strong constraints on IMBH properties, allowing to precisely measure masses, spins, and redshift within $\mathcal{O}(0.1 - 10)\%$ error at redshift $z < 10$ (see for example figures 6 and 7 in [1569]). Therefore, exploiting the

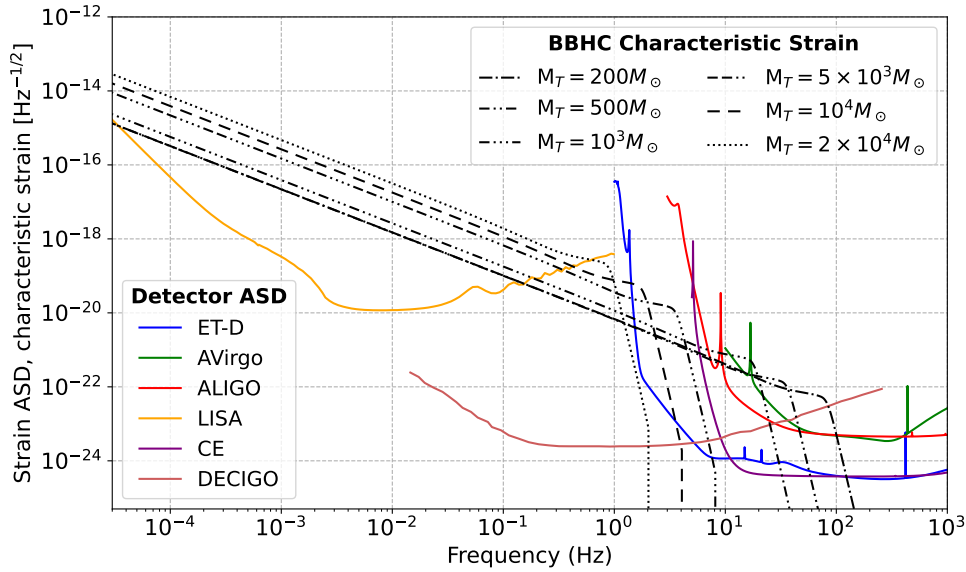


Figure 79. Characteristic strain amplitude and frequency for IMBH-IMBH binaries with different masses, assuming a luminosity distance $d_L = 1$ Gpc. The signal is overlaid to the sensitivity curves of various detectors at low-frequency (LISA), mid-range (DECIGO), and high-frequency (ET, CE, AVirgo, and ALIGO).

synergies among ET and lower-frequency detectors can enable us to obtain a full picture of the IMBH mass range (see section 6 for a more detailed discussion on these topics).

4.9 The host galaxies of binary compact object mergers

The properties of host galaxies where binary compact objects form and merge provide complementary information to understand how these binary systems evolved. Identifying the host galaxies can be challenging and, in principle, requires an electromagnetic counterpart emission after the GW event. While some of the GW events from the coalescence of binary neutron stars (BNS) and black hole-neutron star binaries (BHNS) would be accompanied by an electromagnetic counterpart detection, this is possibly not the case for binary black hole (BBH) mergers, which are likely to be EM-dark (although BBH mergers in dusty regions such as an AGN disk is a possible scenario for an EM counterpart event, see [1273]).

To date, the sole unequivocally identified host is NGC 4993, an early-type galaxy with a stellar mass of $(0.3 - 1.2) \times 10^{11} M_\odot$, a mean stellar age of 3 Gyr and a metallicity of $\sim (0.2 - 1.0) Z_\odot$ [2167–2170]. The merging binary neutron star, observed as the GW event GW170817, occurred in this galaxy [3, 128, 2171–2180]. This event coupled with its electromagnetic counterpart shed light on a wide range of physical phenomena. It confirmed that GWs travel at the speed of light and validated binary neutron star mergers as both progenitors of short gamma-ray bursts [2] and prolific sites for the formation of heavy elements [2181].

Studying the host galaxies of binary compact object mergers is crucial for several reasons. First, the identification of the most likely host galaxy can help to improve the electromagnetic counterpart detection of GW events (section 4.9.1) which, as discussed in

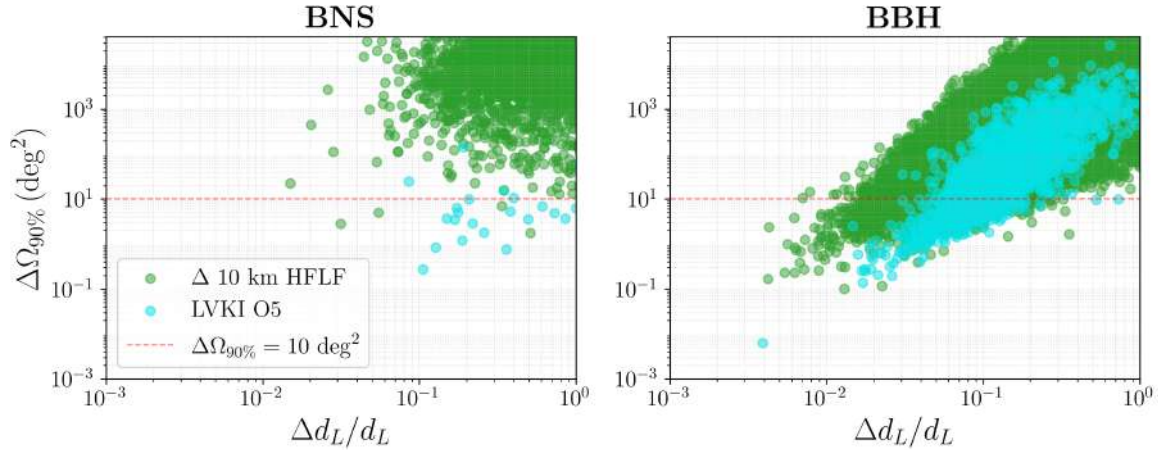


Figure 80. The joint accuracy on luminosity distance ($\Delta d_L/d_L$) and angular resolution ($\Delta\Omega_{90\%}$) for BNS (left) and BBH (right). Green shows results from the triangular ET detector (10-km arms) with both high- and low-frequency instruments (HFLF). Light blue indicates forecasts for LVKI O5, the most advanced 2G detector network (LIGO Hanford, LIGO Livingston, Virgo, KAGRA, and LIGO India) [392]. The dashed red line represents a 10 deg^2 sky localization error. Reproduced from [16]. The Author(s). CC BY 4.0.

detail in section 3.3.1, provides an alternative measurement of current methods to constrain the cosmological parameters such as the Hubble constant. Second, galaxy formation and evolution throughout the history of the Universe affects stellar populations, thereby shaping the likely pathways for compact object mergers (section 4.9.2). Here, we explore these aspects in anticipation of future detections with ET.

4.9.1 Multi-messenger detections and sky localization of the host galaxies

The large sky localization inferred with current GW detectors, spanning hundreds of square degrees [392], represents a challenge for host galaxy identification. The impact of this uncertainty on the efficiency of host galaxy identification is substantial, as it determines the number of pointings required by electromagnetic instruments to cover the GW signal location and the corresponding observational time to be used. This challenge, however, holds promising prospects with the deployment of ET. Its low-frequency sensitivity promises to observe binary compact objects for a longer period of time before the merger occurs [16]. For instance, a BNS signal analogous to GW170817 can be visible by ET > 20 hours before merger [1167] (see also figure 198 in section 11). This aspect will significantly increase the number of BHNS and BNS mergers with a sky localization of less than 10 square degrees [16, 1167, 2182]. Figure 80 shows that the angular resolution expected for the most advanced 2G network at full O5 sensitivity (LVKI O5: LIGO Hanford, LIGO Livingston, Virgo, KAGRA, and LIGO India) is similar to that of a single 10-km ET triangular observatory. However, ET detects over 10 times more events, including those with high SNR, resulting in a nearly tenfold reduction in the error on luminosity distance [16].

Despite the improved sky localization achieved with ET, the electromagnetic follow-up process still typically entails scanning through multiple galaxies (although in some case

the angular resolution could be sufficiently good that there will be only one galaxy in the localization volume, see figure 47 in section 3). Therefore, optimized follow-up strategies can be accomplished through simulation-informed ranking of galaxies within a given sky localization. We can derive the probability that a galaxy hosts a compact object coalescence based on various properties, such as its stellar mass and star formation rate (see e.g. [2183–2186]). The refinement of simulation-informed ranking, where multiple formation scenarios of compact object mergers are taken into account, is particularly crucial for identifying the host galaxies of compact objects where electromagnetic emission is impossible or improbable, such as BBH mergers and BHNS mergers [2187].

As we saw in section 3.3.1, host galaxy identification is also crucial in the use of compact binary mergers as “bright” sirens, in order to infer cosmological parameters. As discussed in particular in section 3.3.1.2, even with the lack of an EM counterpart detection, galaxies with known redshift within the sky localization can be ranked and used to infer the cosmological parameters with statistical methods, particularly if the localization volume contains a limited number of possible host galaxies. The thousands of GW events that ET will detect with its improved sky localization will provide a significant step forward in determining cosmological parameters.

4.9.2 Formation channels and their link with galaxy properties

As we have discussed, the improvements expected for third-generation detectors will provide a number of simultaneous identification of GW events and their host galaxies at different cosmic times [16], either through the detection of an electromagnetic counterpart (at least for BNSs) or, for BBHs, through the statistical association with a limited number, or possibly just a single host galaxy in the localization volume. These concurrent detections (or statistical associations) offer a unique opportunity to gain additional insights into the underlying physics governing the formation channels and evolution of compact objects (see e.g. [2188–2195]).

As mentioned previously (see section 4.1 and reference therein), the two main proposed pathways are the isolated formation channel, which includes processes like common envelope evolution, chemically homogeneous evolution, and stable mass transfer; and the dynamical formation channel taking place in dense stellar clusters (e.g. galactic center, open star clusters, globular clusters and young star clusters). Current detections suggest that multiple formation channels are needed to explain the origin of BBHs (see e.g. [1570]). Furthermore, most binary compact object systems are formed in galaxies. Therefore, the host galaxy properties are an excellent complement to understand the role of the different formation channels of gravitational-wave events across cosmic time.

Galaxy formation and evolution is regulated by a large diversity of baryonic processes shaping their star formation history, chemical enrichment, stellar mass, gas abundance, among others [2196]. These factors impact the stellar populations’ properties and, in turn, the most favorable formation channels of binary compact object mergers.

For instance, galaxies in high-dense regions often undergo major mergers increasing the probability of hosting a large number of stellar clusters [2197, 2198]. These galaxies, in turn, would produce more BBHs through dynamical interactions [1630]. Notably, observations from the local Universe indicate a strong dependence of the number and mass of globular

clusters on the total stellar mass of the galaxies, and a tendency to be more numerous in early-type galaxies [2199–2201]. Furthermore, stellar clusters and active galactic nuclei are considered potential factories of BBH mergers in the pair-instability mass gap and above (see e.g. [1498, 1541, 1630]).

The delay time, defined as the time for compact objects to merge, is important for connecting galaxy properties with binary compact object formation channels. It depends on the outcome of specific physical processes experienced by stars in binary systems such as the common envelope phase in the isolated scenario (see section 4.1). In particular, the number of mergers that occur in massive and passive galaxies tends to increase when the delay time is longer [2184, 2202]. The wide redshift range expected for ET detections [10, 12, 15], together with their electromagnetic counterparts will play a fundamental role in deciphering the contribution of the different formation channels across cosmic time.

The chemical abundance of the stellar populations within galaxies is another key piece of information. Theoretical inferences from binary population synthesis simulations suggest the efficient formation of BBHs at sub-solar metallicities [1198, 1371, 1597, 1603, 2203]. Consequently, low-mass galaxies ($< 10^8 M_\odot$) emerge as promising sites for detecting these mergers [1249, 1606]. However, their faintness and high-redshift nature ($z > 4$) make them challenging to constrain through EM observations. As a result, ET emerges as a promising tool for delving into the chemical evolution of galaxies [1617, 2204]. As host galaxies leave distinctive imprints on compact object mergers throughout cosmic time, the identification capability of ET becomes crucial. If these galaxies have features like low mass and low metallicity, it implies a relatively short delay time for BBHs on average. This breakthrough will effectively resolve the degeneracy between the impacts of formation metallicity and delay time distribution [1604], addressing a significant challenge in gravitational-wave astrophysics. This is also shown in figure 81 schematically.

4.10 Populations backgrounds

The incoherent superposition of the GW signal from all astrophysical sources in our Universe creates a stochastic background whose amplitude and spectral shape reflect the properties of each source population contributing to it. The stochastic GW background (SGWB) is thus a powerful astrophysical probe, complementary to the detection of individual astrophysical sources.

As discussed in section 3.1.1 a SGWB, either of astrophysical or cosmological origin, is characterised in terms of the dimensionless energy density parameter, defined in eq. (3.12), that we recall here,

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(f)}{d \ln f}, \quad (4.5)$$

where ρ_c is the critical energy of the Universe. The astrophysical component can be schematically parametrised as:

$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c c^2} \sum_i \int_0^{z_{\text{max}}} \int_\lambda \frac{R_i(z, \lambda)}{(1+z) H(z)} \frac{P_i(\lambda)}{d f_s} \frac{dE_{\text{GW},i}(f_s, \lambda)}{d f_s} d\lambda dz, \quad (4.6)$$

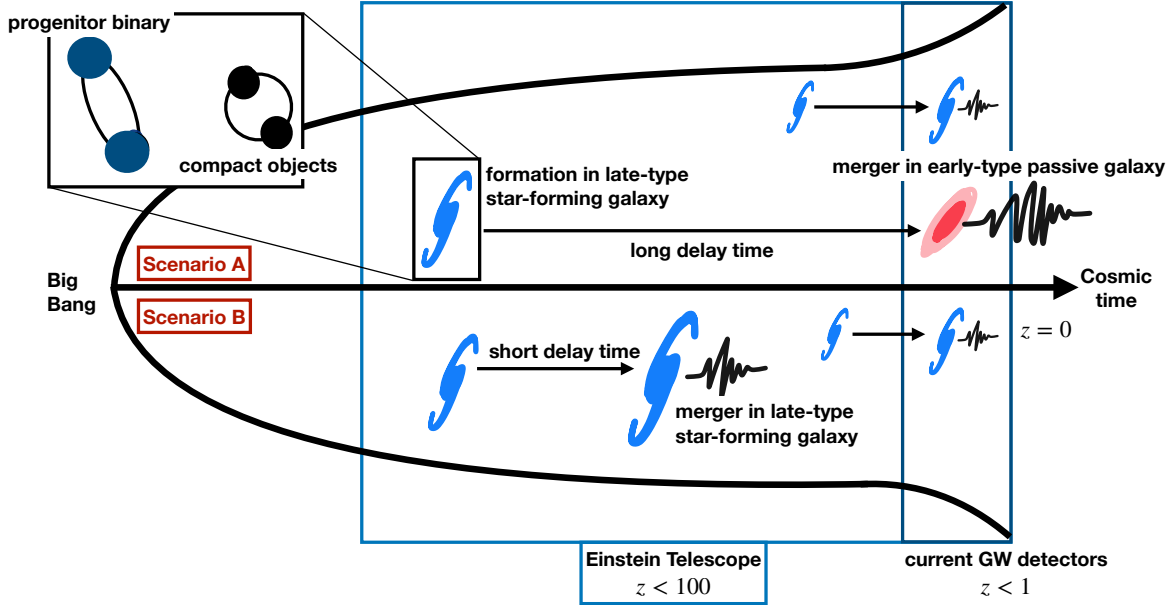


Figure 81. ET will explore the interplay between the physics of compact objects, originating from massive stars, and the galaxies hosting their mergers. Presently, with current GW detectors, exploration is confined to the local Universe ($z < 1$), where mergers can occur in early-type passive galaxies, like NGC 4993 which hosted GW170817. In contrast, ET will be able to observe BBH mergers with sky localization error of less than 10 deg^2 up to $z \sim 6$ (see figure 10 in [16]), opening to completely new perspectives in host galaxy identification.

where $H(z)$ is the Hubble parameter, the index i runs over different source classes (such as BBH, BNS, supernovae, neutron stars) and the parameter λ encodes all the source properties of each class (such as masses and spins for CBC, ellipticity for rotating neutron stars etc.). Finally, $R_i(z, \lambda)$ is the rate of each source and $dE_{\text{GW},i}(f_s, \lambda)/df_s$ is the energy spectrum it emits.

We will focus our discussion here on CBCs and the unique information that can be obtained with ET. The main challenges are studying BBH formation channels, searching for Population III stars and detecting the residual BNS background. We will also mention other sources and their contribution to the SGWB and discuss synergies with electromagnetic observations.

We note that the background defined in eq. (4.6) is the total background. In the case of CBCs, it can be relevant to study the residual background obtained after subtracting individually detected sources. On the one hand this background is complementary from individual detection and can provide statistical information about the faintest sources at the largest distances, and on the other hand subtracting sources can help observing backgrounds that are masked by the total background. For LIGO-Virgo-Kagra, the number of sources detectable individually is sufficiently small so that the distinction between total background and residual one (obtained after resolvable sources are filtered out), is very mild. However, with its highly improved sensitivity, ET will detect a very large number of individual sources: their accurate subtraction is crucial and will pose itself an interesting challenge. Indeed, as

discussed in section 10.5.3, suboptimal individual sources subtraction will manifest itself as an additional stochastic component. In the rest of this section we will assume that source subtraction is completely noiseless and discuss the scientific questions that SGWB observations can address. We will discuss both the total background (without subtraction) and the residual background (obtained after noiseless subtraction).

We note that, even if most of the BBHs are expected to be detectable individually by ET, this will not be the case for a population of BNSs: since they are fainter than BBH and with a longer time in band, they are expected to produce superposed signals, and the bulk of the population is expected to be subthreshold (see figure 14 of [1167] or figure 107 in section 10). A background approach will provide us with information on this population that cannot be achieved with a catalog (individual detections) approach alone. However, even for BBHs, using a background approach to reconstruct parameters such as merger rate and mass distribution is useful to perform a cross-check with the same quantities extracted from a catalog. A discrepancy might be the indication of the presence of a faint subpopulation, that is not captured when looking at individual events. A population of PBHs extends to redshift higher than the instrument horizon, and the properties of a high- z population can be extracted only with a background approach. Finally, since astrophysical backgrounds represent a foreground for sources of primordial origin, understanding their properties is crucial for accurately subtracting their contribution from an observed background map and looking at cosmological components.

4.10.1 Study of BBH formation channels

Detection and characterization of the SGWB will provide additional information on source population properties. One of the main contributions to the signal in ET is expected to be from BBH mergers occurring at relatively low redshifts ($z < 5$), with masses in the range $\sim (10 - 10^3) M_\odot$ [16]. The formation channels of these BBH, as discussed in section 4.1, can be broadly divided into the isolated channel (including formation from isolated field pairs and multiples), and the dynamical channel. The latter including formation in globular (GC), nuclear (NSC) or young star clusters (YSC), and gas-driven formation in AGN disks. These formation channels leave an imprint on the masses, spins and eccentricities of the resulting BBH, as well as the merger rate, the distribution of time delays and the evolution of the mass spectrum with redshift. These differences will manifest themselves in the SGWB detectable with ET.

As an example, in figure 82 we show the contributions to the SGWB from BBH formed via the isolated channel (Field) and three variants of the dynamical channel: YSC, NSC and GC. All the computations were performed with the public code `PRINCESS`⁴³ using population models from [338]. We show both the total and residual backgrounds, where the latter are computed by removing all sources having an individual SNR $\rho_{\text{thrs.}} > 20$ (neglecting the error in the subtraction; see section 10.5.3 for the treatment of the subtraction error) and assuming a random inclination and position in the sky. The shaded areas shown for the Field, YSC and GC channels reflect the uncertainty in the merger rates for each population model, and are derived from table 2 in [338]. The model uncertainty of the NSC channel is relatively small

⁴³See <https://gitlab.com/Cperigois/Princess>.

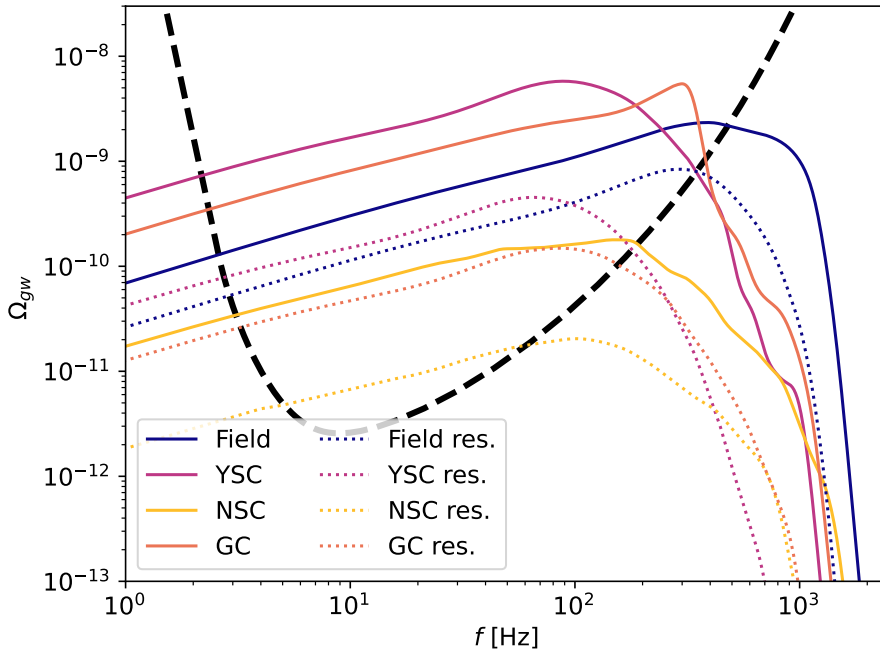


Figure 82. Background energy density spectrum for BBHs assuming four evolution channels, isolated (Field), Young Star Cluster (YSC), Globular Cluster (GC) and Nuclear Star Cluster (NSC). Uncertainties, depicted by the shaded areas, are derived from the merger rates given in table 2 of [338]. The black dashed curve shows the sensitivity of ET for the triangular configuration.

(up to 30%). We also show the power-law integrated sensitivity curve of ET to stochastic backgrounds, computed for the triangular configuration and assuming SNR of 2 and 1 year of observing time with duty cycle of 50%. As discussed in [16], this sensitivity depends on the configuration: choosing two L-shaped detectors decreases the sensitivity above ~ 100 Hz because the overlap reduction function drops significantly. However, L-shaped configurations where the detectors are co-aligned perform better at low frequencies than the triangular ones.

As can be seen in figure 82, the spectral shape at low frequencies is dominated by the inspiral phase of each BBH and therefore has a constant slope close to $f^{2/3}$. At higher frequencies, however, we start observing features specific to each formation channel that reflect the distribution of masses and redshifts of each population [2076, 2205]. In particular, the frequency at which the spectrum deviates from a power-law corresponds to the minimal mass in the BBH population (the peak is relatively wide due to redshifting effects). In figure 82, this deviation starts at a few hundred Hz for the Field formation channel, and at ~ 100 Hz for the YSC and GC channels. Thus, observing the shape of the SGWB can provide important clues as to the formation scenario of the sources.

4.10.2 Study of Population III stars

As discussed in section 4.7.1, Pop III stars are the first stars that formed in the Universe. Similarly to Pop I/II stars, they form BBHs via the isolated or dynamical channels. However, Pop III remnants are expected to have unique properties that may help to distinguish them

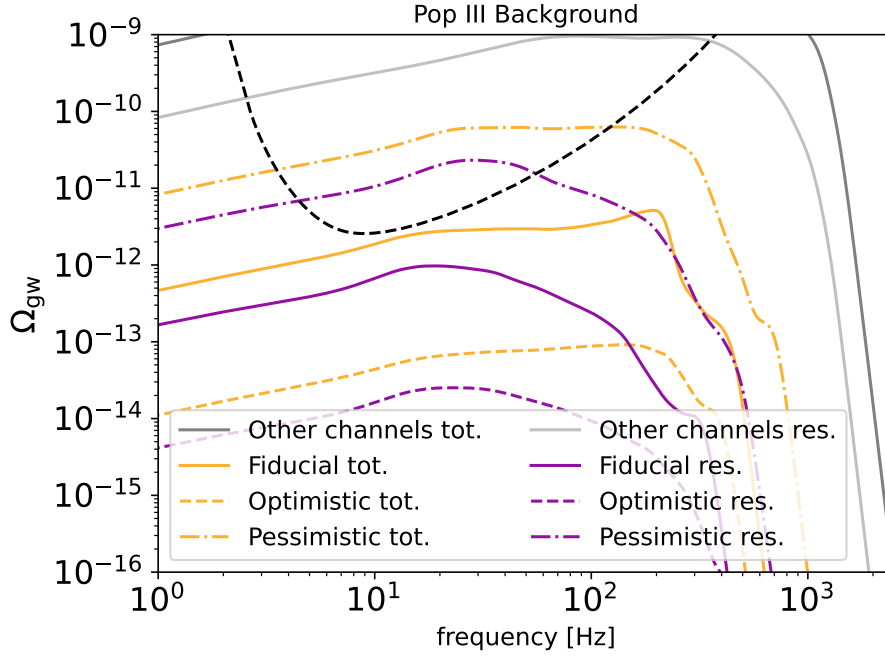


Figure 83. Background energy density spectra for BBHs born from Pop. III stars. Fiducial, Optimistic and Pessimistic models are the LOG_H22, LAR_H22 and LAR_J19 respectively populations models from [1674]. The residuals correspond to $\rho_{\text{thrs.}} > 20$, and the grey lines correspond to the background from Pop. I/II channels (see section 4.10.1).

from Pop I/II remnants, in particular merger rate evolution with redshift and mass spectrum. These properties are also imprinted on the SGWB they produce [2077, 2078, 2080, 2206].

As an example, in figure 83 we show the total and residual SGWB, where the source populations are taken from [1674]. The residual signal was calculated assuming all sources with an individual signal to noise ratio $\rho_{\text{thrs.}} > 20$ are subtracted noiselessly. Note that the SGWB from Pop III remnants starts deviating from a power-law at lower frequencies compared to Pop I/II remnants, as a consequence of their higher masses and higher merger redshifts.

We note that the residual signal from Pop I/II remnants in the model shown in figure 83 is more than two orders of magnitude above that of Pop III remnants due to their low merger rate. Therefore a better understanding of Pop III remnants is needed in order to efficiently extract information from the SGWB. Nevertheless, even if the residual Pop III signal is indeed much below the one from Pop I/II, SGWB observations will contribute to putting upper limits of high- z merger rates. In addition, the predictions for Pop III remnant merger rates and masses are much more uncertain than those for Pop I/II. Finally, we note that the ability to accurately subtract the signal from individually detected Pop III remnants highly depends on ET performance at low frequencies.

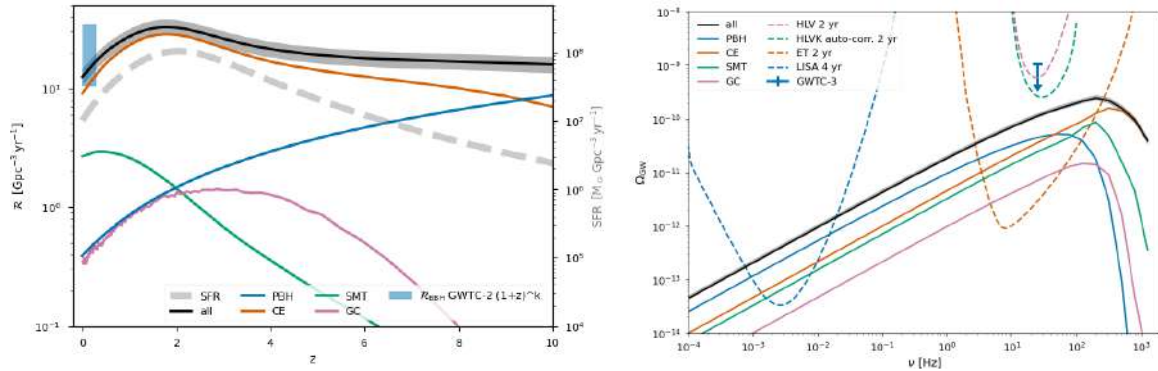


Figure 84. *Left panel:* merger rate evolution for representative astrophysical models and the PBH channel. As for the astrophysical models, CE, SMT, and GC refer to common envelope mass transfer, stable mass transfer, and globular clusters, respectively. The normalization comes from the Bayesian inference on the GWTC-2 catalog in ref. [1382]. The blue band reports the local merger rate bound from LVK while the dashed gray band the star formation rate (SFR). *Right panel:* total SGWB (black line) coming from the various channels shown in the left panel, while we report with colored lines their contributions. We do not show the residual SGWB. Figures adapted from ref. [2205]. Reproduced with permission from [2205]. S.S. Bavera et al., A&A, 660, 2022, reproduced with permission, © ESO.

4.10.3 Primordial black hole contribution

Primordial black hole mergers may lead to a sizeable SGWB [1922, 2205, 2207–2210].⁴⁴ As indicated in eq. (4.5), the SGWB results from the incoherent superposition of stellar and BH mergers, and it gets contributions from the entire merger history. In other words, it is sensitive to the rate evolution with redshift of the channels responsible for GW emission. Crucially, the PBH scenario is characterized by a merger rate that grows monotonically with redshift [1939, 1943, 1949] $\mathcal{R}_{\text{PBH}}(z) \propto t^{-34/37}(z)$, extending up to early epochs at $z \gtrsim \mathcal{O}(10^3)$, see also discussion in section 4.6.1.⁴⁵

In figure 84 (left panel), we show the merger rate distribution of PBHs as a function of redshift (blue line), compared to other representative astrophysical models both from isolated (common envelope, stable-mass transfer) and dynamical (globular cluster) formation scenarios from ref. [1570] (see [1382] for more details). The chosen normalization reflects the result found in ref. [1738] with a multi-channel Bayesian inference of the GWTC-2 catalog. While we do not discuss the mass distribution of the populations assumed in these plots, we highlight the large contribution from PBHs at low frequencies even if they only contribute to a subdominant fraction of the local merger rate density in this specific example. This property allows testing the presence of PBH binaries, adding an independent constraint to the model selection, as a cross-correlation between local rate and SGWB amplitude would provide valuable information on the merger rate distribution of the various channels at a relatively high redshift. The degeneracy between the PBH models and other astrophysical scenarios would then be broken by the fact that the former predicts a monochromatic growth

⁴⁴The SGWB associated with the PBH formation mechanisms is discussed in section 3.2.5.1.

⁴⁵We do not consider the contribution to the SGWB from PBH hyperbolic encounters, since it is typically subdominant [1938].

of the merger rate density with redshift, thus the integrated SGWB would be larger for the same local (i.e. low redshift) observed rate.

4.10.4 Astrophysical Uncertainties in background description

While the recent detections of CBC by ground-based interferometers have provided a wealth of information on the BBH, BNS and NSBH populations, many uncertainties remain. These uncertainties can have an important impact on our predictions of the SGWB and can be divided into two categories: CBC source properties, which can be modelled with stellar models and constrained with current observations; and galaxy evolution, including SFH, Initial Mass Function (IMF) and metallicity evolution, which are taken from observations. On the other hand, observations with ET offer an opportunity to lift these uncertainties and thus obtain indirect constraints on these physical processes.

As was shown in [2211], uncertainties in the mass distribution of stellar-mass BBH affect not only the overall amplitude of the SGWB (see also [2212]), but also the spectral shape around the peak and the location of the peak. Moreover, current instruments are sensitive only to relatively low redshifts ($z \sim 0$ for BNS and $z \lesssim 1$ for BBH). Therefore in order to compute the SGWB we need to extrapolate our models up to $z \sim 10$. However, not only the merger rate may evolve with redshift, but also source parameters, such as mass distribution, which would strongly influence the SGWB, as shown in [2211].

Another major source of uncertainty is the SFH, which serves as the basis for SGWB modeling since it determines the total number of stars formed at each epoch [2202]. The SFH can be measured from the UV and IR luminosity functions of galaxies; however, in addition to various observational biases, such as dust attenuation, surveys often suffer from incompleteness at higher redshifts, beyond $z \sim 3$ (see e.g. [1249]). A closely related parameter is the IMF, which describes the number of *massive* stars formed in a given population. The IMF is typically assumed to be universal across environments and redshifts. However, some recent studies show that this may not be the case, which could strongly influence the number of compact objects formed and therefore the SGWB. Finally, the metallicity evolution is highly important for modeling the SGWB, since, as discussed in section 4.2, the efficiency of forming CBCs strongly depends on metallicity (see e.g. [1375, 1597, 2202, 2213]). The reason for this dependence is two-fold: first of all, stellar winds become stronger at high metallicity, and can remove a large fraction of the initial stellar mass before the star reaches core-collapse. This process leads to reduced masses of the resulting BHs. Second, metallicity can impact the efficiency of mass transfer phases during binary star evolution (like the common envelope phase), which can result in less efficient mergers and a lower probability of forming BBHs. In addition, metallicity influences the properties of supernovae that produce BHs through fallback. Measuring the metallicity evolution remains challenging, especially for $z \gtrsim 3$ (e.g., [1249]) and these observational uncertainties lead to large variations in the predicted SGWB [2076, 2211].

We expect several of these uncertainties to be lifted in the next few years: the number of detected CBCs is expected to increase, leading to a better determination of the mass spectrum; the reach of electromagnetic observations, in particular with JWST, is also extending, and a better measurement of the SFH at high redshift is foreseeable. Nevertheless, some of

the open questions will remain until the deployment of ET, such as the mass distribution and merger rate of CBCs at high redshifts. The metallicity evolution and IMF universality also remain challenging targets to study. Thus, future observations of the SGWB with ET will help constraining not only the properties of CBCs but also several crucial aspects of galactic evolution. In this regard, studying the SGWB is complementary to the direct catalog approach: combining both resolved and unresolved populations will give the most complete picture of the compact binary evolution across cosmic times.

4.10.5 Sources other than CBCs

While CBC are the most studied class of GW sources, and the only one detected so far, other sources are expected to be detectable with ET, including stellar collapses and isolated neutron stars that are described in detail in section 8. Recent works have studied the SGWB produced by core-collapse SNe [2214, 2215] and rotating neutron stars [581, 2216]. In the case of core-collapse supernovae (CCSN), individual detections with ET would be possible only for the local Universe, whereas the SGWB includes also the contribution from more distant sources. Ref. [2215] studied a large range of CCSN explosion models and calculated the stochastic GW signal. They showed that for most models the expected SGWB is below the sensitivity of ET, with $\Omega_{\text{GW}} \simeq 10^{-16}$ at $f \sim 10$ Hz and $\Omega_{\text{GW}} \simeq 10^{-14}$ at $f \sim 100$ Hz. The only exception are extreme models that feature rapidly rotating progenitors with a low ratio of rotational to gravitational potential energy that produces a rotational instability (low- $T/|W|$ instability). This instability produces strong GW emission between 200 and 300 Hz. Therefore, if a large fraction of CCSN progenitors share this property, the resulting stochastic background would be much stronger, with $\Omega_{\text{GW}} \simeq 10^{-10}$ at $f \sim 200$ Hz and possibly detectable with ET. It should be noted that such extreme models are considered unrealistic, since only a small fraction of massive stars are rapidly rotating. On the other hand, there are currently large uncertainties in the predictions of the GW signal from CCSN, and the resulting GW background differs by several orders of magnitude between models.

Rotating non-axisymmetric neutron stars also emit GW due to time varying mass quadrupole. This signal is expected to be much weaker than that of merging binary compact objects, but to last for much longer times. The loudest sources will thus be possibly detectable individually, but the contribution from unresolved sources will constitute a confusion background. This signal depends on the highly uncertain equation of state of neutron stars, their deformability, the distribution of rotation periods and magnetic fields (that contribute to the deformation of the star) and possible accretion from a companion. In ref. [581], the SGWB from rotating neutron stars was estimated as $\Omega_{\text{GW}} \simeq 2 \cdot 10^{-8}$ at $f = 760$ Hz, which would be potentially detectable with ET.

In both cases, CCSN and isolated neutron stars, the modeling uncertainties are still very large and accurate predictions are challenging. More theoretical work is needed, in particular in the areas of hydrodynamical simulations of core-collapse SNe and the nuclear equations of state of neutron stars.

4.10.6 Anisotropies and cross-correlation with electromagnetic observables

The astrophysical SGWB is not perfectly homogeneous across the sky, but exhibits some level of anisotropy. As we already saw in eqs. (3.17) and (3.18), these can be addressed introducing a directional dependence in eq. (4.5), writing

$$\Omega_{\text{GW}}(f, \hat{\mathbf{n}}) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(f, \hat{\mathbf{n}})}{d \ln f d \hat{\mathbf{n}}} = \bar{\Omega}_{\text{GW}}(f) + \delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}}), \quad (4.7)$$

where $\hat{\mathbf{n}}$ is the direction of observation, $\bar{\Omega}_{\text{GW}}(f)$ is the homogeneous component of the energy density, and $\delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}})$ is the anisotropic perturbation. The statistical properties of the anisotropies can be characterised in terms of the frequency-dependent angular power spectrum:

$$\langle \delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}}) \delta\Omega_{\text{GW}}(f, \hat{\mathbf{n}}') \rangle = \sum_{\ell} C_{\ell}(f) P_{\ell}(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}'), \quad (4.8)$$

where $P_{\ell}(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}')$ are Legendre polynomials. The SGWB anisotropies are generated by the intrinsic clustering of sources and the line-of-sight effects accumulated during the propagation through the large-scale structure [541–545, 547, 548]. Moreover, kinematic anisotropies arise from the motion of the observer with respect to the rest frame of the SGWB [556–558]. The theoretical expression of the SGWB anisotropies and their detection prospects have been subject of intense study in the last years; see section 3.1.2 for a review of this topic, for both the cosmological and astrophysical background components.

The astrophysical SGWB is given by the superposition of a large yet finite number of unresolved signals. In the ET band, the GW events from BBHs are expected to be separated in time and with a limited time overlap (while some overlap is expected for BNS mergers). Hence, the sources of the SGWB are discrete in space and time, and their number fluctuates according to a Poisson distribution, resulting in a significant shot noise. The total SGWB angular power spectrum is therefore $C_{\ell}^{\text{tot}} = C_{\ell} + N^{\text{shot}}$, where the shot noise contribution is flat in the ℓ -space and dominates over the intrinsic anisotropies. Indeed, the intrinsic anisotropies are typically at the level of $\delta\Omega_{\text{GW}}/\Omega_{\text{GW}} \sim 10^{-3} - 10^{-2}$ [546, 549–552], while the shot noise is around one order of magnitude larger [587–589, 1354]. Given the sensitivity of ET to various multipoles (see e.g. the analysis of [2217]) and the presence of shot noise, directly detecting the anisotropies due to clustering and line-of-sight effects will be extremely challenging. However, shot noise is expected to be measured with good precision, at least within the range of amplitudes predicted by most astrophysical scenarios. Despite being insensitive to the sky distribution of the sources, the shot noise still carries astrophysical information as its amplitude depends on the overall number of sources contributing to the SGWB and thus reflects the underlying astrophysics.

An effective way to overcome the shot noise issue is to cross-correlate the SGWB with electromagnetic observables. Indeed, the SGWB anisotropies are correlated with other cosmic fields that trace the large-scale structure, such as galaxy number counts [546, 549, 589, 1352, 1353], weak gravitational lensing [546, 549], CMB [578], and CMB lensing [1354]. All of these studies have shown that the SNR of the cross-correlation outperforms that of the auto-correlation by several orders of magnitude. Measuring the angular power spectrum of the cross-correlation between the SGWB and an electromagnetic observable can provide

constraints on various astrophysical and cosmological quantities. Moreover, cross-correlating with a redshift-dependent observable (such as the number counts of galaxy, at different redshifts), allows one to have a tomographic reconstruction of redshift information, i.e. to extract the contribution to the total background from sources in a specific redshift band.

In [1353], the cross-correlation between LVK data and the Sloan Digital Sky Survey catalog was exploited to derive upper limits on a set of effective astrophysical parameters describing the local process of GW emission at galactic scales. As the sensitivity of the GW detector network improves, the approach presented in [1353] will become more effective, enabling more stringent constraints on the astrophysical kernel and its effective parameters. ET will improve the sensitivity to the angular power spectrum of the cross-correlation by more than a factor ~ 1000 : ET is expected to reach and explore the astrophysically interesting region of the parameter space. Yet another approach could be to use the hierarchical Bayesian approach to estimate the BBH SGWB anisotropy [2218], which could also lead to ~ 1000 times sensitivity improvements relative to the approach presented in [1353], hence resulting in a very significant improvement when applied to ET data. This is a promising approach which might help to set stringent constraints on the parameter space of the underlying astrophysical models. The cosmological significance of cross-correlating the astrophysical SGWB with electromagnetic observables, such as measuring the bias of the SGWB and constraining non-gaussianities in the large-scale structure, is discussed in section 3.4.1.2.

Given a generic anisotropic signal, an important challenge is mapping its angular distribution [567]. For an unpolarized stochastic signal, the stochastic search with ET aims to reconstruct the SGWB power spectrum $S_h(f, \hat{\mathbf{n}})$ defined in eq. (3.11), while in the more general case of a polarized signal we want to reconstruct the quantity $H_{AA'}(f, \hat{\mathbf{n}})$ defined in eq. (3.8), which also includes the Stokes parameters describing circular and linear polarization. For most of the astrophysical backgrounds, the frequency and the angular dependence of the power spectrum of the SGWB in eq. (3.11) are factorized.⁴⁶ Then, restricting to the unpolarized case

$$S_h(f, \hat{\mathbf{n}}) = S_h(f)E(\hat{\mathbf{n}}) . \quad (4.9)$$

It is then convenient to decompose the angular part of the power spectrum in spherical harmonics⁴⁷

$$E(\hat{\mathbf{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \delta_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}) , \quad (4.10)$$

and then reconstruct the value of the anisotropic (map) coefficients $\delta_{\ell m}$ from the cross-correlation of the instruments' data streams that are available [640, 2222]. The reconstruction is performed in the framework of Bayesian analysis, by writing the likelihood function of

⁴⁶Although Doppler anisotropies induced by the motion of the detector with respect to the rest frame of the SGWB induce a mixing between frequency and angular dependence, see [556]. For a more general treatment we refer to the formalism developed in [2219].

⁴⁷The so-called pixel domain is an alternative basis that can be employed to decompose the angular dependence of the power spectrum [2220, 2221]. However, for the astrophysical and most of the cosmological models of a SGWB the spherical harmonics choice is the most natural one.

the cross-correlators, expressing it as a function of the coefficients $\delta_{\ell m}$ and performing a maximum likelihood estimation, which can be done by a Markov-chain Monte Carlo (MCMC) or by a Fisher analysis.

Several factors may affect the quality of the reconstruction of the anisotropic map. Firstly, the instrumental noise puts a threshold on the detectability of the anisotropy coefficients $\delta_{\ell m}$. In particular, the higher values of ℓ in the expansion of eq. (4.10) produce a signal in the high-frequency side of the power spectrum. This is due to the fact that the anisotropic overlap reduction functions of ground-based interferometers (which quantify the sensibility of a cross-correlator to a specific anisotropy) show a peak at a frequency $f \propto \ell b/c$, where b is the baseline between the pair of instruments taken into account.

Furthermore, up to today most of the map-making analyses have been performed for a Gaussian SGWB [100, 640], with stationary statistics in its reference frame. Under this assumption, the frequency domain is a natural choice to perform the data analysis.⁴⁸ The signal-to-noise ratio (SNR) grows with the square root of the time and it is dominated by the frequencies at which the overlap reduction functions, for each anisotropic coefficient of the map, display a peak. In fact, the astrophysical GW background is expected to be nonstationary due to the finite number of sources generating it [551, 587–589]. More sophisticated analyses that aim to work beyond the frequency domain must be employed if we want to tackle the challenge of extracting a weak anisotropic signal, as expected by both the astrophysical and cosmological models. A joint analysis of the SGWB along with the information given by the angular distribution of the resolved GW sources and the electromagnetic surveys will certainly help in that direction.

Another aspect that must be considered when studying the SGWB with ET (and in general in next generation detectors) is the nuisance effect due to the intrinsic variance of the signal [2223]. This effect impacts the measurement of the anisotropy coefficients in the limit where the instrumental noise is lower in absolute value with respect to the total effect of the SGWB on the interferometers. In fact, the current analyses work in the opposite regime, i.e. the so-called noise-dominated assumption. It has been shown [640] that current detectors live in the noise-dominated regime, while for ET and the new generation interferometers, the intrinsic variance of the signal will no longer be a negligible effect when compared to the low instrumental noise power spectrum.

As a last important remark, the mapping of the high ℓ values for the SGWB angular power spectrum and all the odd ℓ values is suppressed when the baseline between the instruments is small. This results in enhanced sensitivity for the proposed configuration [16] of 2 L-shaped Einstein Telescopes in two different sites, improving the detectability for those anisotropies of several orders of magnitude with respect to the original triangular-shaped design. In a similar way, the analysis obtained by cross-correlating ET with other new-generation instruments (such as Cosmic Explorer) will result in a significantly greater sensitivity to the SGWB anisotropies.

⁴⁸The rotation of the Earth results in an observed GW background which is cyclo-stationary in the reference frame of the detectors. In [640, 2222] a folding method is employed in order to take into account this effect in the data analysis process.

4.10.7 Spectral shape reconstruction

As already mentioned in section 4.10.2, the SGWB spectral shape contains plenty of information on the underlying astrophysical populations from which the sources responsible for the SGWB generation are drawn. Techniques for SGWB spectral shape reconstruction are thus crucial for extracting this information from the data. While it is known that the astrophysical background is not perfectly isotropic, stationary, or unpolarized (see, e.g., [546, 551, 582–584, 587, 589]), in the remainder of this section, we neglect these features and focus on the spectral shape only. In this limit, all the information on the SGWB is contained in the function $S_h(f)$ in eq. (4.9), which fully characterizes the spectrum. This quantity is uniquely determined, through eqs. (3.13) and (4.5), by the population properties, and inherits the dependence on the population (hyper)parameters, say Λ_i . These (hyper)parameters characterize, among others, the mass function and the merger rates for all the subpopulations contributing to the SGWB, and impact the overall amplitude and the position and sharpness of the high-frequency cutoff in the frequency spectrum [2205, 2224].

In a consistent Bayesian approach, all the Λ_i would have to be included in the analysis, and their parameter space would have to be sampled simultaneously.

There are several challenges to be faced while following the above procedure in the context of ET. For example, since several components contribute simultaneously to the overall signal, the dimensionality of the parameter space may be significant (if not prohibitive), and all signal components will effectively act as an additional nuisance to each other. Sampling such a parameter space, which might be highly degenerate and non-Gaussian, can be quite a non-trivial problem from a numerical point of view. For this reason, an alternative (and computationally less demanding) avenue could be to separate the data analysis problem into two steps. In an initial step, one would attempt an agnostic reconstruction of the SGWB spectrum (for some techniques developed in the context of LISA data analysis, see, e.g., [644–647, 2225, 2226]) to capture the overall frequency shape. In a second step, the constraints on the spectrum are converted into constraints on the population (hyper)parameters. Such an approach has the merit of compromising between accuracy and feasibility, and it could be used, e.g., to identify the relevant part of the parameter space to be sampled with the full Bayesian techniques described in the previous paragraph.

4.11 Executive summary

Here we summarize the advancements that ET will bring in population studies and astrophysical backgrounds, in comparison to the current state of the art, specifically emphasizing the distinct features that the ET will allow us to explore. The following are the major areas where ET’s capabilities surpass the current generation of GWs detectors.

Astrophysical CBCs across cosmic time

- **Formation channels.** Current models describe a remarkable zoo of CBC formation channels, including (stable and unstable) mass transfer in massive binaries, chemically homogeneous evolution, dynamics in star clusters and gas rich AGNs. With the prospective detection of 10^5 BBH and BNS events per year, it

will be possible to characterise in detail the impact of the isolated and dynamical channels on the formation of CBCs. This can be achieved by constraining crucial quantities like merger rates, redshift dependencies, masses and spins of merging CBCs.

- **Merger rate density evolution.** Current detectors enable us to infer the merger rate density of BNSs and BHNSs at redshift zero, and to reconstruct the BBH merger rate density up to a redshift of $z \approx 1.5$, under several assumptions. ET will be a paradigm shift for measuring the merger rate evolution of astrophysical CBCs: we will estimate the intrinsic BBH merger rate up to a redshift of $z \approx 100$ and the BNS merger rate at least up to cosmic noon. This will yield unique opportunities to constrain the formation of CBCs, their possible dependence on stellar metallicity, their delay time, the evolution of their stellar progenitors, and the link with their host galaxies.
- **CBC masses.** From current detections, several peaks have already emerged in the mass function of primary black holes. ET will not only constrain these features to an exquisite level of accuracy, but will also probe their evolution with cosmic time and the existence of the claimed pair-instability mass gap.
- **CBC spins.** Current gravitational-wave data suggest a tension between the spin of black holes in gravitational-wave detections and X-ray binaries. Our ability to interpret this claimed tension in terms of formation history of X-ray and gravitational-wave black holes is hampered by large observational uncertainties. Moreover, hierarchical merger models predict a peak of the precessing spin at $\chi_p \sim 0.7$ for second-generation black holes. ET will yield an accurate reconstruction of the spin distribution of black holes, providing a conclusive answer about the claimed tension and the existence of moderate to high-spin peaks.
- **Population III stars.** ET will yearly observe between 10 and 10^4 binary black hole mergers from the first generation of stars that appeared in the Universe (Population III stars). The main challenge will be to disentangle such population from primordial black holes and black holes born from other stellar progenitors.
- **Intermediate-mass black holes.** Intermediate-mass black holes are crucial to explain the formation of supermassive black holes observed at the center of most galaxies, but have largely eluded our efforts to characterize them with electromagnetic surveys. GW190521 demonstrated the potential of gravitational-wave detectors to probe the intermediate-mass black hole regime. ET will enable us to study a rich population of intermediate-mass black holes, up to a redshift $z \approx 10$. This will offer unique opportunities for multi-band gravitational-wave observations with space-based detectors.

Primordial Black Holes

PBHs are BHs which could have formed in the early universe and might compose a non negligible fraction of the dark matter. ET will allow to pin down the presence of PBHs to unprecedented precision through its capability to search their GW signatures. On one side, thanks to its superior sensitivity and wider bandwidth at low frequencies, ET will test the unique feature of the PBH merger rate density, its monotonically increase with redshift, up to redshifts larger than 30 where astrophysical sources are not present. Secondly, ET will drastically improve the sensitivity leading to a possible detection of subsolar PBHs, a smoking gun for their existence given the fact that subsolar astrophysical BHs may not exist.

Population backgrounds

- **Sources.** Most BBHs can be detected individually by ET. However, using a background approach for parameter reconstruction is valuable for cross-checking results against catalog data, as any discrepancies may indicate faint subpopulations that individual analyses might miss. For BNSs, the bulk of the population will be sub-threshold, and a background study provides insights that catalog approaches cannot offer. ET is also expected to detect sources other than CBCs, such as stellar collapses and isolated neutron stars (see section 8). Finally, a population of PBHs extends at redshifts beyond ET's detection horizon, making a background approach crucial for studying them.
- **Astrophysical information.** Different formation channels for BBHs will influence their masses, spins, and eccentricities, as well as the merger rate, distribution of time delays, and the evolution of the mass spectrum with redshift. These variations are expected to be reflected in the stochastic gravitational wave background (SGWB) observable by ET. Pop III remnants are expected to possess unique characteristics that could be detected in the SGWB they generate.
- **Detection prospects.** The angular power spectrum of the cross-correlation between the SGWB and electromagnetic observables is a promising observable for a first detection of anisotropies. ET is expected to enhance sensitivity to this angular power spectrum by over a factor of approximately 1000, allowing access to astrophysically significant regions of parameter space. Similar improvements are expected from the hierarchical Bayesian approach.
- **Mapping.** The configuration of two L-shaped detectors at different sites greatly enhances the detectability of anisotropies compared to the triangular design. Moreover, cross-correlating ET with new-generation instruments like Cosmic Explorer will further improve angular sensitivity to the SGWB.
- **Spectral shape.** An agnostic reconstruction of the frequency shape allows converting constraints on the spectrum into limits on population (hyper)parameters.

5 Multi-messenger observations in the ET era

The cosmological reach of the Einstein Telescope is such that it will detect extremely large numbers of compact object mergers which should also create luminous electromagnetic counterparts. If successfully paired, this can lead to substantial samples of multi-messenger sources that can be used to understand heavy element nucleosynthesis, probe physics at high density and extreme velocity, measure cosmological parameters, and constrain alternative theories of gravity. These objects will typically lie in the mid-to high-redshift Universe, and so while their detection in gravitational waves may be straightforward, we must enhance and optimize our electromagnetic capabilities if we are to exploit the treasure trove of scientific results that can flow from multi-messenger observations. Here we review the likely signatures occurring in coincidence with ET-detected mergers. We construct both population models and up-to-date implementations of the emission physics for electromagnetic counterparts to determine their recovery with both electromagnetic and neutrino facilities that may be available in the 2030s. We highlight the requirements of localisation capability, and that EM capability should be co-developed along with ET to ensure that the critical science questions enabled by multi-messenger observations can be answered.

5.1 State of the art

5.1.1 Multi-messenger Astronomy

Traditionally, we have studied the Universe with light, from millennia of observations with the naked eye to the insights enabled by the discovery of the telescope and its continued refinement. In the 20th century, astronomers made spectacular progress with the development of multiwavelength astronomy. Multiwavelength astronomy led to the realisation that physical conditions in the Universe enabled some astrophysical sources to emit light across the electromagnetic spectrum. A myriad of astrophysical insights stem from multiwavelength observations, from the cosmic microwave background [2227] and its many applications [591] to new exotic source classes, including the radio-emitting pulsars [2228] and the high-energy dominated gamma-ray bursts [2229].

Throughout the twentieth century, there was also a realisation that distant astrophysical sources may also create signals that are not electromagnetic in origin. The journey to understanding these alternative carriers of information (messengers) began with the discovery of cosmic rays in 1912 and flows through solar neutrinos in the 1960s [2230] to the increasingly complex and sophisticated detectors for gravitational waves, which discovered the first sources in 2015 [1].

Multi-messenger astronomy is the combination of two or more of these astrophysical messengers, although one of these is typically electromagnetic light. The value of such detections is immense; the non-photonic messengers often carry information from regions that are essentially invisible to light. Gravitational waves cannot be shielded and so provide information about the behaviour of mass in kilometre-sized regions where compact objects such as neutron stars and black holes merge. Neutrinos can escape from the cores of collapsing stars, hidden from electromagnetic view by the star itself. Therefore, multi-messenger astronomy provides a direct probe of matter in conditions far exceeding those available in any Earth-bound laboratory. Strikingly, many objects that emit panchromatic electromagnetic light

are also likely to be multi-messenger emitters, and this combination of messengers is critical both in studying the full details of the events in question and in using them as probes of extreme astrophysics and cosmology (see figure 85).

The first multi-messenger detection was of thermal neutrinos, coincident with SN 1987A in the Large Magellanic Cloud [2231]. However, supernovae neutrinos have not been seen in the 35 years since, despite the substantial improvements in detectors. The gravitational wave multi-messenger era began in 2017 with the discovery of gravitational waves from the binary neutron star merger GW170817 [2] and its subsequent identification from the gamma-ray to radio regime [4]. This detection enabled a raft of new scientific insights, which we describe in detail in section 5.1.2. To date, there has yet to be a further gravitational wave-electromagnetic source, despite intensive efforts. These endeavours will likely unveil further electromagnetic counterparts as the current generation of interferometers reaches their design sensitivities in the next few years. However, the rates of such detections mean that building samples beyond a handful of events will be challenging, and these events will all be at low redshift, ideal for detailed study of extreme physics in compact objects but of less utility in understanding source evolution or in deriving cosmological parameters.

Large samples of electromagnetic bright sources will require next-generation gravitational wave detectors such as the Einstein Telescope (ET), whose astrophysical reach is sufficient to uncover populations of thousands of likely electromagnetically bright events. This dramatic rate increase is possible because of the increased horizon of ET. It comes at the cost that the events will be more distant and will require novel approaches for their identification and characterisation, combined with the next generation of electromagnetic observatories that will be active in the 2030s. The realisation of the multi-messenger promise of ET will rely on developing electromagnetic capability in tandem with the gravitational wave detectors.

This section outlines the science enabled by multi-messenger observations. It reviews the progress made to date via studies of both GW170817 and similar systems, perhaps seen so far only in electromagnetic light. We subsequently construct detailed source population models to make predictions for both the rates of events and, critically, the expected electromagnetic signals. We then use these, along with a census of likely telescopes available to the community in the mid-2030, to make robust, if broad, predictions for the recovery of electromagnetic counterparts.

5.1.2 GW170817

To date, there is only a single unambiguous event in which gravitational waves and electromagnetic light have been observed in tandem, GW170817 [2, 4]. This binary neutron star merger provided an exceptionally rich set of data that continues to provide new scientific insight. Observations of GW170817 hence offer a template for what future observations may enable and also an opportunity to learn lessons and improve follow-up.

Both the LIGO detectors detected GW 170817, while an upper limit from VIRGO was decisive in breaking location degeneracies and providing an error box of only ~ 30 sq. degrees [4]. Only 1.7 seconds after the merger of the two neutron stars, a short-duration gamma-ray burst, GRB 170817A, was also detected by both the Fermi [2171] and INTEGRAL [2172]

satellites. The rate of short GRB detection by these satellites is approximately one per week, so the time coincidence alone made identifying the GRB a highly significant event.

The source location was non-optimal for electromagnetic observations, lying only ~ 50 degrees from the Sun, with very limited possibilities for follow-up. This location was also not promptly visible to any observatories at the point at which it was disseminated. Targeted electromagnetic observations did not begin immediately, but several hours later, when observatories in South America could observe the sky localisation, and the *Swift* satellite started its search [2232]. These searches were rapidly successful, with [2174] reporting a new transient, subsequently named AT2017gfo, in the galaxy NGC 4993 in observations taken 11 hours after the merger. Indeed, before this first announcement, five more observatories had also observed the source [2175, 2233–2236]. Initially, it was unclear if this source was related to GW170817 or was an unrelated supernova explosion. However, imaging and spectroscopy over the next 48 hours revealed a spectral evolution unlike any previously observed transient and cemented the association [2176, 2179, 2237–2239]. This spectral evolution was extremely rapid, with the source spectrum moving from a spectral peak at $\sim 5000\text{\AA}$ to $\sim 15000\text{\AA}$ on a timescale of only a few days. Strikingly, this signature was qualitatively extremely similar to what had been predicted for kilonova emission [2240, 2241], in which the high opacity of heavy elements synthesised in the explosion effectively blocks the optical light. IR and optical observations of the source continued for as long as allowed by the encroaching Sun, and on a timescale of 10–20 days, the counterpart was also detected in both X-ray [2173, 2242] and radio [2243] observations. Indeed, while a faint source was detectable by the Hubble Space Telescope 100 days after the merger [2244], the X-ray and radio afterglows continued to brighten, reaching their peaks on timescales of hundreds of days before decaying [2245–2248].

The comprehensive observational campaign for GW170817 led to a myriad of scientific advances, which we highlight below.

5.1.2.1 The kilonova and the origin of the heavy elements. The optical and IR light created in GW170817 was caused by the decay of newly synthesized elements in the merger. During the merger, neutrons are ejected both in tidal tails from the merger and a range of additional processes, such as the wind from a newly formed accretion disc. These neutrons then undergo neutron capture reactions via the so-called r -process, which builds extremely neutron-rich nuclei far from the valley of stability [2249]. The energy released as these unstable isotopes decay then powers the faint and fast transient that we know as a kilonova [2241] while also populating the high-mass regions of the periodic table. Indeed, it is thought that the r -process is responsible for the creation of around half the elements heavier than iron and is the dominant source of elements with significant social, geological, and even biological importance, such as gold, thorium, and iodine.

Before the discovery of GW170817 and its associated kilonova (AT2017gfo) several candidate kilonovae had been uncovered in the afterglows of (short) GRBs [2250, 2251]. However, AT2017gfo is by far the best-studied event, with a comprehensive set of both imaging and spectroscopic observations that are outlined in figure 87. These observations provide strong evidence for the production of r -process elements.

The abundance patterns of heavy elements show three apparent “peaks”, often referred to as the first, second, and third peak elements. These are broad regions of higher elemental

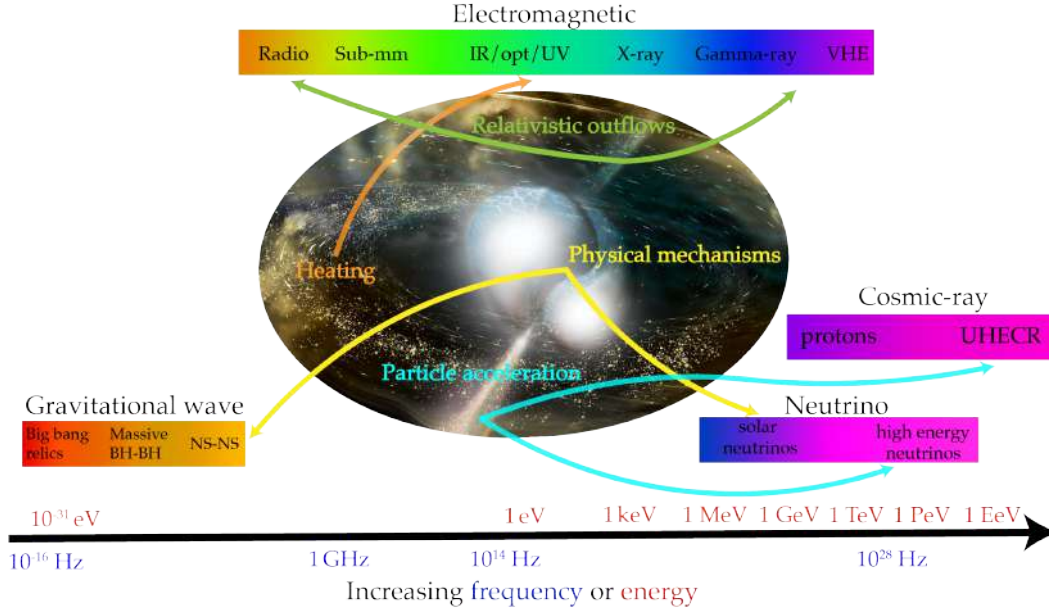


Figure 85. The promise of multi-messenger astrophysics. While much astronomy to date has been undertaken in the electromagnetic bands, in particular in optical light, many extreme systems in the Universe emit light across the electromagnetic spectrum and also produce additional messengers in the form of cosmic rays, neutrinos, or gravitational waves. Understanding the details of these systems and the insight they offer into central questions in astronomy, cosmology, and fundamental physics is something that can be done via multi-messenger observations.

abundance centered at atomic numbers around krypton (atomic number 36, first peak), xenon (atomic number 54, second peak), and platinum (atomic number 78, third). The production of elements around each peak depends on the nuclear physics ongoing within the material forming the heavy elements (in the case of a compact object merger, this is the ejecta). In particular, different nucleosynthetic pathways are available depending on how neutron-rich the material is. Perhaps counterintuitively, the neutron-richness of the ejecta is defined by its so-called electron fraction (Y_e), which is equivalent to the proton-fraction for material that is neutral on a bulk scale. In broad terms, access to the more massive elements becomes increasingly straightforward for decreasing Y_e . In general, the production of material in the second and third r -process peaks is thought to require $Y_e < 0.15$ [2241].

Within the merger of a binary system of neutron stars, or a neutron star with a black hole, material can be ejected via various mechanisms. Firstly, the initially less massive object is larger and will be tidally stretched and disrupted as the gravitational wave-driven inspiral reaches its final phases. Material from these (or plausibly both) stars is ejected in the plane of the orbit in so-called tidal dynamical ejecta. Having been stripped directly from a neutron star, this material is extremely neutron-rich and should be able to undergo r -process reactions to the most massive elements. As the merger proceeds, the vast majority of the total mass remains with the stars and, ultimately, the central merged object. However, some also form a disc around this object. This disc may be a source of r -process synthesis itself [2253], and also drives a wind which creates further ejecta in the polar directions. Because of strong radiation

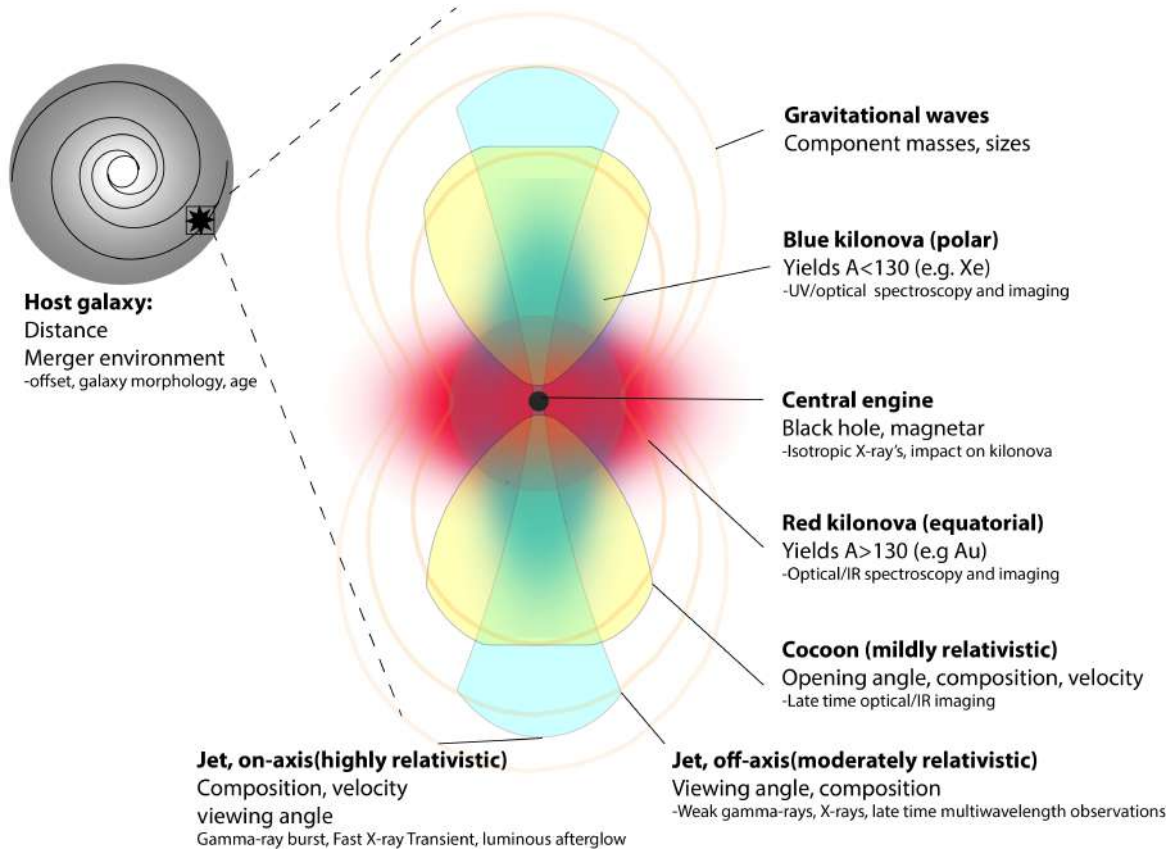


Figure 86. The physical process ongoing in the merger of a binary system of neutron stars (or a neutron star and a black hole), their multi-messenger observational signals and the scientific insight enabled from them (updated from an original concept by [2241]). Gravitational wave observations provide robust measurement of the component masses, and constraints on their sizes and spins. Many possible electromagnetic signals are also possible, including the detection of associated GRBs, cocoon emission, and kilonova signatures. Reproduced from [2241]. CC BY 4.0.

from this disc (in particular in the form of neutrinos which trigger inverse beta-decays), the material in this polar ejecta is less neutron-rich than in the tidal ejecta and likely yields intermediate mass elements around the first r -process peak.

While the elemental abundances within the ejecta are determined by nuclear physics, the observed light in the kilonova is determined by the atomic physics of the newly synthesized material. While this atomic physics, and in particular the energy levels of a given isotope, are well known for low-mass elements, there are substantial gaps and uncertainties for the most massive nuclei due to the immensely complex electron structures. Of particular importance for the appearance of kilonovae are the so-called lanthanides and actinides that have open electron f -shells, and extremely high numbers of electron transitions throughout the UV and optical region. The effect of this is to provide extremely high opacity to outgoing light rendering kilonovae which produce these elements extremely red [2240].

This work, much of which was undertaken prior to the discovery of AT2107gfo (but has been refined since), provided a broad expectation for what a kilonova should look like. It will

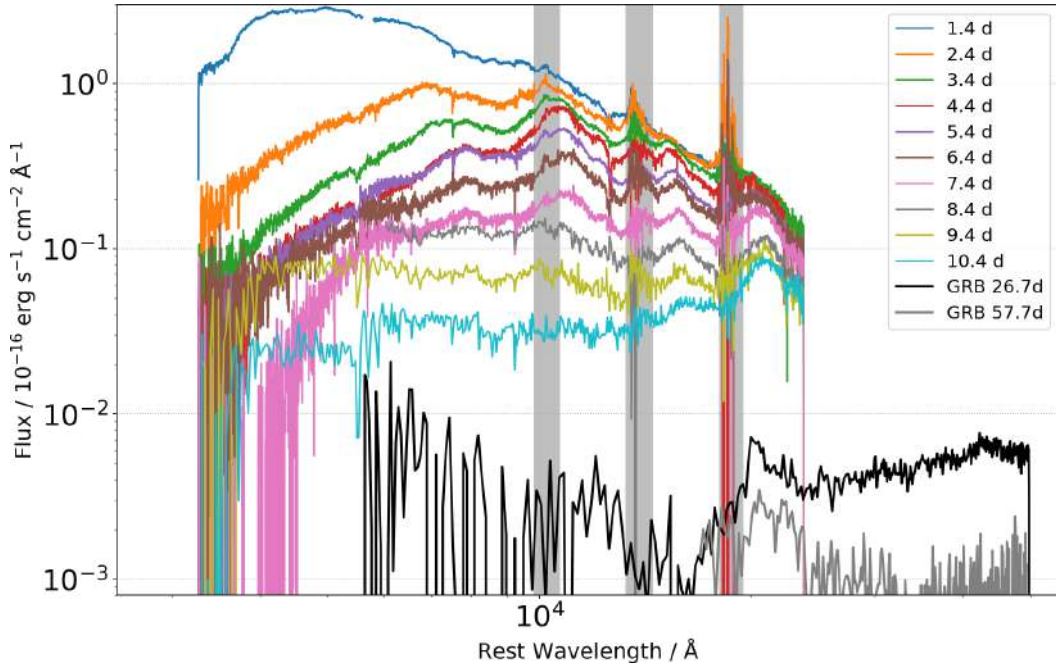


Figure 87. The spectroscopic evolution of kilonovae. The upper (colored) lines show the X-shooter spectral sequence for AT2017gfo [2179, 2237], demonstrating the early blue emission with the subsequent transition to a much redder spectrum. The lower (grey/black) lines show much later time spectra of the kilonova in GRB 230307A (AT 2023vfi) obtained with *JWST* [2252]. The spectra contain numerous spectra features both early in absorption and later in emission, and these have been linked to several different *r*-process elements.

have a low luminosity at peak (compared to most transients), evolve quickly because of the low ejecta mass, and either be very red or transition from blue to red on a timescale of a few days.

In bulk terms, this is exactly what was observed in AT2017gfo, and it was the similarity of the observed properties to the pre-existing theoretical predictions that gave strong evidence that AT2017gfo was indeed the counterpart of GW170817. However, it is also true that when investigated in detail there were many aspects of the observed event that did not match the models, and demonstrated the requirement of more sophisticated approaches to fully exploit the potential of kilonovae in probing heavy element enrichment. In part, this explains why continuing analysis of the AT2017gfo observations reveals new insight 7 years after its discovery. Indeed, the kilonova, AT2017gfo has been the most widely studied component of GW170817, with several hundred articles dedicated to its study. While many of these studies disagree on the details of the source, often because of the use of disparate sets of models, the broad picture appears to be generally agreed upon. The source emitted in the range 0.02–0.1 $M_{\odot}$ of *r*-process material [2176, 2179, 2181, 2233, 2235, 2237, 2254]. The composition of this material is a mixture of lighter *r*-process elements which are responsible for the early bluer emission, and heavier elements, responsible for the redder, later emission [2254, 2255].

Most of these conclusions are drawn from the study of the multi-wavelength lightcurves of the kilonova. However, very substantial effort has also gone into the evolving spectral series, which shows significant variations, even on the sub-hour timescale at early times [2256].

These observations reveal the presence of strontium at early times [2257], an identification now generally agreed upon in the community [2258] and supported by theoretical expectations [2259]. These early observations also potentially contain several further spectral features, such as yttrium [2260], although the challenge in such identifications is substantial given the uncertainties in the atomic physics of heavy elements.

There has also been substantial interest in the later phases of the spectrum, which because of the low ejecta mass and high velocities rapidly become optically thin. At this point, it is plausible that emission lines may be observed. In the later IR spectra of AT2017gfo a prominent line at 2.2 microns has been interpreted as tellurium [2261, 2262], a line also seen in the JWST spectrum of a further KN in a GRB [2252, 2262].

The rich dataset for AT2017gfo continues to be exploited as models improve. The complex spectral and temporal evolution of the kilonova provides the opportunity to dissect the details of its composition and ejecta mass, although are currently limited substantially by model uncertainties. As these improve it should be possible to robustly infer the contribution of kilonova to the heavy element budget of the Universe. However, doing so concretely is likely to require samples of tens of kilonovae to understand source diversity and pinpoint rates. The current paucity of detections means that it is unclear if such observational samples can be built in a reasonable time with the current generation of gravitational wave detectors.

5.1.2.2 The gamma-ray burst and its afterglow. In addition to diagnostics from the core of the merging binary, electromagnetic observations also probe the relativistic outflow that created the GRB. In particular, GW observations (in particular when combined with the EM distance to the galaxy) provide a route to measuring both the energetics of the outflow, the inclination angle, and the structure of the relativistic jet. These provide substantial insight into the physics of GRB jets, as well as potentially into larger physics questions relating to the acceleration of material to ultra-relativistic velocity.

GRB 170817A was observationally a rather typical, if faint, short duration GRB, with a duration of $T_{90} = 2.0 \pm 0.5$ [2171] and a spectral shape consistent with that seen for other GRBs. In the absence of a distance measurement, it would not have stood out as exceptional. However, at a distance of only 40 Mpc the inferred energetics of the GRB are at the extremes of the distribution. In particular, the so-called isotropic equivalent energy release (the energy from the burst on the (false) assumption that it emits equally in all directions) is $E_{iso} = 10^{46}$ erg [2171, 2172]. This is four orders of magnitude lower than typical for short-GRBs [1652, 1653], which are typically found at much larger distances (these closest short-GRB after GRB 170817A is GRB 080905A at $z = 0.125$, or ~ 600 Mpc [2263], a factor of > 15 more distant than GRB 170817A, although there are some merger-related GRBs likely at lower redshift).

This likely explains why GRB 170817A was so underluminous compared to the bulk of the short GRB population — it was viewed off-axis. In turn, studies of the GRB afterglow provide a route to studying the jet structure and imply that the jet in GRB 170817A was highly structured, with a lower Lorentz factor in the wings, but a luminous, highly collimated event to an on-axis observer [2244, 2246, 2247, 2264–2267]. Indeed, in the years following the merger, deep observations utilizing radio interferometry [2268, 2269] and the *Hubble Space Telescope* [2270] could directly resolve the motion of the relativistic GRB jet. The implication

is that GRB 170817A was viewed ~ 25 degrees from its jet-axis. It was an extremely faint afterglow, only just detectable to the premier facilities in the X-ray (*Chandra/XMM-Newton*), optical (*HST*) and radio (JVLA). Indeed, it may be that multi-messenger observations are one of the most effective routes of identifying GRB jets off-axis, and in doing so to study the structure of the GRB jets, itself of great importance in understanding the details of ultra-relativistic motion and particle acceleration.

5.1.2.3 The host galaxy and environment. In addition to the light from the transient itself, we can also glean much information about a given event from the environment in which it is born. Indeed, early in the history of studies of supernovae, it was apparent that type II events occurred exclusively in young, star-forming galaxies, while SN Ia also occurred in older populations, implying different progenitor systems. There is now a long history of using the environments of supernovae, gamma-ray bursts [1653, 2271–2277] and other transients [2278–2280] as a route to understanding the stars that form them.

The host galaxy of GW170817/AT2017gfo is NGC 4993, a massive lenticular galaxy. This has also been studied in great detail on both the scale of the galaxy [2167, 2168, 2170, 2281], and the local environment [2167, 2282]. These studies generally concur that the host galaxy is dominated by an old stellar population (> 1 Gyr, [2167, 2281, 2283]), with evidence for minor merger tens of millions of years ago [2170, 2284]. The vast majority of the stellar mass resides in the older populations, suggesting it is most likely the binary was formed in this population [2285]. There is no evidence for any underlying globular cluster to limits that probe the majority of the globular cluster luminosity function [2170, 2244]. In principle, observations of these locations are highly diagnostic of progenitor production, natal kicks, and delay times, since the host of GW 170817 is much closer than those of short GRBs for which such studies are typically undertaken [1653, 2286, 2287]

5.1.2.4 Tests of cosmology and fundamental physics. Observations of GW170817, GRB170817A, and the kilonova AT2017gfo enabled a unique route of probing the extreme physics at play in compact object mergers. For example, gravitational wave observations measure (or place constraints) the so-called tidal deformability of the neutron stars during the merger. The degree of deformability places restrictions on plausible equations of state (or mass-radius relations) for the matter at nuclear densities within neutron stars [2288]. Consistency checks on the results from these analyses can be run in tandem with electromagnetic observations; for example, are the allowed equations of state consistent with the mass ejected in the kilonova, or even how the equations of state rule in (or out) via tidal deformability studies compare with those inferred for different neutron stars via high-resolution X-ray spectroscopy of millisecond pulsars [2288, 2289].

A critical use of combined EM and GW observations is a novel one-stop route (distance ladder independent) to measure cosmological parameters. In particular, the waveforms of merging compact objects encode their masses and are so-called standard sirens [1138], where the measurement of the gravitational wave amplitude directly provides the distance to the source. The Hubble constant can be determined if paired with a measure of the velocity, for example, in the form of a redshift of either the electromagnetic counterpart or its host galaxy. For GW 170817, initial estimates were of $H_0 = 70^{+12}_{-8}$ km s $^{-1}$ Mpc $^{-1}$ [1709], but improved

to $H_0 = 71.9 \pm 7.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [2290] using improved estimates of the distance to NGC 4993 or to $H_0 = 70.3_{-5.0}^{+5.3}$ via the determination of the inclination angle due to superluminal motion measurements of the GRB afterglow [2291].

Finally, the joint observations can constrain core questions in fundamental physics. In particular, the time delay between the gravitational waves and the first electromagnetic emission provides robust tests on any deviation between the speed of gravitational waves and the speed of light, which are several orders of magnitude better than previously available, suggesting any variations are smaller than one part in $\sim 10^{15}$ [3]. In turn, this rules out numerous alternatives to General Relativity which predict a difference between the two speeds. While more complex scenarios remain plausible (e.g., frequency-dependent speed of GWs), the observations are a strong demonstration of the ability to multi-messenger observations to pose new tests for the validity of General Relativity.

5.1.2.5 Limits on other messengers associated with GW170817. GW170817 was the first source to be identified in both gravitational waves and light, but there is also significant interest in the prospect that it also created further messengers, such as high-energy neutrinos accelerated in the relativistic jet. Several such searches were undertaken using time windows around GW170817 including both close to the merger time [2292] and over a much longer ~ 2 week time frame [2293]. The detection of such neutrinos would be a major step forward in our understanding of the details of particle acceleration in compact object mergers, although despite the relatively close distance to GW170817 the limits were generally significantly shallower than expected for the majority of models [2293, 2294].

5.1.3 GRBs and KN as counterparts of compact binary mergers

While we have, to date, only uncovered one electromagnetic counterpart of a binary neutron star merger detected in gravitational waves, we likely studied hundreds of such events in electromagnetic light only. In particular, the detection of a GRB in GW170817 strongly links at least some short-duration gamma-ray bursts to the mergers of compact objects. Indeed, this joint detection cemented a growing body of evidence that already linked short-GRBs to such mergers. For example, the host galaxies of the bursts appear to span a broad range of ages, from young, star-forming dwarfs to giant ellipticals [2295]. Such a broad range of ages is inconsistent with any origin in young, massive stars. Furthermore, the locations of the short-GRBs in and around their host galaxies were very different from that seen in long-duration gamma-ray bursts [2272] or in supernovae [2275]. In particular, a substantial fraction of short-GRBs arise at large offsets from their host galaxies [2296–2298], making it difficult to explain their locations without invoking progenitors that have received some kind of “kick” (though see [2299, 2300]). Such kicks are naturally expected to occur in compact object systems in which the combination of mass loss at the time of supernova (the so-called Blaauw-kick) and natal kicks to neutron stars on formation [1665, 1666] can propel the binary from its birth site at tens to hundreds of km s^{-1} . Finally, the first suggestions of kilonova emission were in the short-GRB 130603B [2250, 2251] and several possible examples had been uncovered before 2017 [2301, 2302]. Combined, this evidence suggests that many, if not all, short-duration GRBs arise from compact object mergers and provides us with a much larger sample of events from which we can hone our expectations for future gravitational wave sources.

A remarkable realisation in the past two years has been the identification of kilonova signatures in long-duration gamma-ray bursts. In particular, two events, GRBs 211211A [2303–2306], and GRB 230307A [2252, 2262, 2307], both have durations which place them in the bulk of the long duration GRB population, but also appear to contain kilonovae remarkably similar to AT2017gfo. The implication is that mergers powering long-GRBs may be comparably common in the local Universe to mergers powering short-GRBs, and motivates using both categories for GRB-GW coincidence searches.

A complexity in using GRBs to predict the properties of gravitational wave counterparts is that they are (mostly) selected as events in which we view down the relativistic jet. The GRB and its afterglow are bright in this scenario, and we cannot expect off-axis events to be such luminous high-energy emitters. Indeed, even the kilonovae themselves are expected to have a varying appearance depending on the viewing angle and if we observe primarily the polar (light r -process) or equatorial (heavy r -process) material.

In principle, the kilonovae associated with mergers can be identified without any pre-selection from either gravitational wave or GRB identification. A suite of small telescopes now routinely scan essentially all the visible sky every few nights (these telescopes include the Zwicky Transient Factory (ZTF, [2308]), the ATLAS telescopes [2309], the MASTER telescopes [2310] and the Gravitational Wave Optical Transient Explorer (GOTO) [2311]), while other, slightly narrower but deeper telescopes also conduct surveys of much of the sky (e.g. DECCAM at CTIO, PanSTARRS, BlackGEM). These predominantly optical surveys can uncover kilonovae and potentially have unrecognised events within their large catalogues of transients. However, there are substantial challenges in moving from detection to discovery for these events. In particular, the rate of very nearby events is likely low, with even the current complete range of uncertainties from the GWTC-3 predicting between 0.05 and 7 events per year at 100 Mpc [6]. Given some loss of events is to be expected (direction too close to the Sun or behind the Galactic plane, poor weather, technical problems, some events too far from host galaxies to be associated at < 100 Mpc), it is perhaps unsurprising that these searches have, to date, not yielded a robust kilonova detection. Indeed, this problem is compounded because the number of sources that may mimic the peak luminosities and timescales of kilonovae likely substantially outnumber the kilonovae. Indeed, intensive searches for faint fast transients have been run for the past several years and have identified many such sources. While none show the combined hallmarks of rapid evolution and fast transition from blue to red, substantial follow-up resources have been necessary to demonstrate this. Indeed, most of these faint, fast sources appear to be some combination of outbursts from Luminous Blue Variables, unusual white dwarf explosions (e.g., SN Iax), or ultra-stripped supernovae [2312]. These are all of substantial interest in their own right but complicate the identification of kilonovae. Indeed, with the imminent arrival of the Vera Rubin Observatory (VRO) and its Legacy Survey of Space and Time (LSST), the ability to isolate candidate kilonovae, either found blindly or in the error box of gravitational wave searches, becomes an urgent challenge to overcome.

5.1.4 Alternative signatures of mergers

The vast majority of effort in the detection of multi-messenger signals has been directed at the mergers of CBs that contain a neutron star, or more specifically that contain a neutron star

and a mass ratio such that material is expected to remain outside the innermost stable orbit of the merged compact object [1471, 2313]. This is natural because it is in these scenarios where baryonic matter is left outside the merger remnant producing luminous electromagnetic emission. The merger of a BBH binary in a vacuum should yield no detectable EM emission. Despite this, it is striking to note that the first BBH detection, GW150914 [1], was associated with a weak detection of a short GRB [2314] that remains controversial [2315].

Partly motivated by this coincidence, several works explored the possibility of deriving light from BBH mergers. These critically consider that BBH mergers do not happen in a true vacuum, but contain material in the form of the surrounding medium which may take a variety of forms. For example, it may be an ionized plasma [2316], a remnant disc around one of the black holes [2317, 2318] or a BBH embedded in the dense disc around a supermassive black hole [1642].

It is the latter of these suggestions that has received the most interest to date, with the suggestion from [1273] of an AGN outburst consistent in time and space with the extremely high mass BBH mergers GW190521 [1509]. This was suggested to be an unusual AGN outburst triggered by the merger of the two components within the disc of the AGN. Subsequent analysis of a large sample of BBH mergers that received follow-up from the Zwicky Transient Factory revealed evidence of a larger number of unusual AGN outbursts in the error boxes than expected randomly, perhaps implying a substantial fraction of BBH mergers that could be formed in the discs of AGN [2319]. Although the rates of such mergers are expected to be much lower than for those formed in the field, the higher masses result in a larger horizon. Through studies of GRBs, there is now some limited evidence for neutron star mergers in AGN discs [2320, 2321], and so BBH mergers may naturally be expected as well.

However, this result has also proved to be controversial. In particular, the challenge of associating a given AGN outburst with a BBH merger in a large error box is non-trivial [1646, 2322] and the details of the wide range of AGN variability remain poorly understood. Nonetheless, these claims suggest that we should remain alert to the possibility of multi-messenger signals from BBH mergers, and the science they may enable.

5.2 Modeling the EM counterparts of ET detected CB mergers

This section presents predictions of the observational properties of the electromagnetic counterparts of compact binary mergers, namely BNS and BHNS binaries, detectable by the Einstein Telescope.

We consider the kilonova and the jet-related emission, the latter including both the prompt emission (appearing as a short GRB when the jet is observed close to on-axis) and the broadband afterglow. Four state-of-the-art population synthesis models are considered, with predicted gravitational and electromagnetic signals for each binary. Despite the considerable knowledge advancement made possible by GW170817, these model predictions are still based on a variety of astrophysical assumptions which need to be tested with new detections in the years to come until ET operations. Therefore, any description of the expected EM properties of the GW sources detected by ET is subject to a considerable amount of uncertainty. To partly account for this modeling uncertainty, we adopted four models published in the literature, each of which provides an end-to-end simulation of the GW and EM signals. We

highlight that the kilonova properties could be impacted if some of the compact objects are quark stars instead of neutron stars [2323–2331]. An overview of the main assumptions of our models is provided in sections 5.2.2 and 5.2.2.1. Specifically, the parameter estimation analysis of the GW signal is presented in section 5.2.2.2, while the models adopted for the prediction of the electromagnetic signals are described in section 5.2.2.3.

For each binary class and for each emission component, we consider the subsample of events detectable by ET. For these we provide in section 5.2.3 two kind of information:

- the properties (e.g., light curves, energetics, characteristic timescales) of the KN and jet-related GRB emission components;
- the annual joint detection rates as a function of the individual EM component flux at specific frequencies or within selected energy bands.

While there are a number of present or planned major facilities for the search and follow-up of GW-detected sources that are expected to operate in synergy with ET, the specific evolution of the observational landscape is hard to predict. This section, therefore, aims to provide a few general tools that can be used to estimate the expected joint detection rates by scaling for basic facility/instrument configurations.

Figure 88 shows a possible 1 year of multi-messenger detections by ET and compares BNS (left) with BHNS (right). The jet-related emission (cyan double cones) and the kilonova (red butterflies) show the EM horizons reachable by a typical GRB satellite (with a sensitivity comparable to that of Fermi/GBM) and an optical search facility (reaching a 26 magnitude limit), respectively. Jets of BNS are brighter than BHNS systems and can be detected out to and above redshift $z = 1$. For BNS systems an EM horizon at $z \sim 0.5$ is identified. Within this horizon most of the multi-messenger events will be KN-GW systems and a fraction will have GRB-KN-GW signals. At $z > 0.5$ most of the BNS joint detections will be GRB-GW signals. This selection effect is mainly due to the orientation of the jet, with off-axis jets being detectable only within relatively smaller distances. Compared to the BNS case, a much smaller number of joint detections is expected for the BHNS channel and the relative EM horizons are much closer to the Earth.

5.2.1 Dynamics and emission components of compact binary mergers

In figure 89 we present a schematic representation of the dynamics of BNS and BHNS mergers and their respective multi-messenger emissions. For BNS systems, during the final phase of the inspiral, the development of tidal forces leads to the partial disruption of the stars, producing an emission of neutron-rich material. This material can be divided into two main components: a gravitationally unbound component, called dynamical ejecta, which leaves the merger region, and a bound component, which forms an accretion disk around the remnant. Compression and shock heating lead to additional emissions when the two stars' surfaces collide. The emitted neutron-rich material is an ideal site for r-process nucleosynthesis, which produces the heaviest elements in the universe. The synthesized nuclei are unstable, and their radioactive decay can power a KN emission. On longer timescales, other emissions originate from the disk: some ejecta are produced by magnetic pressure and neutrino-matter

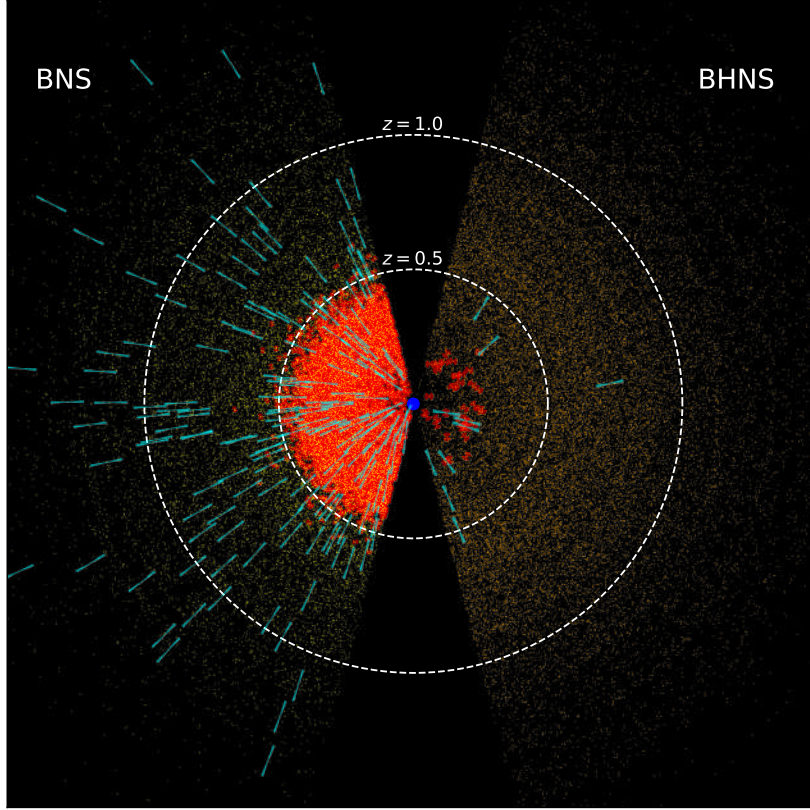


Figure 88. Representation of two of our reference multi-messenger synthetic populations in a geocentric Universe. Compact binaries in the synthetic populations of C22 and C23 that are detected by ET- Δ are represented by yellow (BNS — left semicircle) and orange (BHNS — right semicircle) dots. The distance of each dot from the Earth (blue circle) is proportional to the redshift of the corresponding compact binary. If the simulated merger produces a relativistic jet whose prompt ($> 4 \text{ ph cm}^{-2} \text{ s}^{-1}$) or afterglow emission ($> 0.01 \text{ mJy}$, radio band) is detectable, a cyan jet is plotted centered on the dot, with its axis inclined by the actual viewing angle with respect to the line of sight to the Earth. If a kilonova is also produced, and if it is detectable ($< 26 \text{ mag}$, g band), then a red butterfly shape is also plotted. The total number of binaries is representative of one year of ET operation.

interactions, called wind ejecta, and some ejecta are produced by viscous processes inside the disk, called secular ejecta.

When the NS is disrupted and some of the released material forms a disk, its accretion onto the remnant can induce the launch of a relativistic jet. Internal shocks inside the jet and/or magnetic reconnection phenomena lead to the dissipation of a fraction of the jet’s kinetic energy, which is radiated, giving rise to the prompt emission of the GRB. The jet then continues its expansion into the interstellar medium, where it begins to decelerate. This causes the formation of a shock near which electrons are accelerated, producing synchrotron emission: the afterglow emission of the GRB.

BHNS mergers can also be accompanied by an EM counterpart, as in the BNS case. The condition for ejecta production is that the tidal disruption distance d_{tidal} , at which tidal disruption occurs, must be larger than the BH’s innermost stable circular orbit R_{ISCO} .

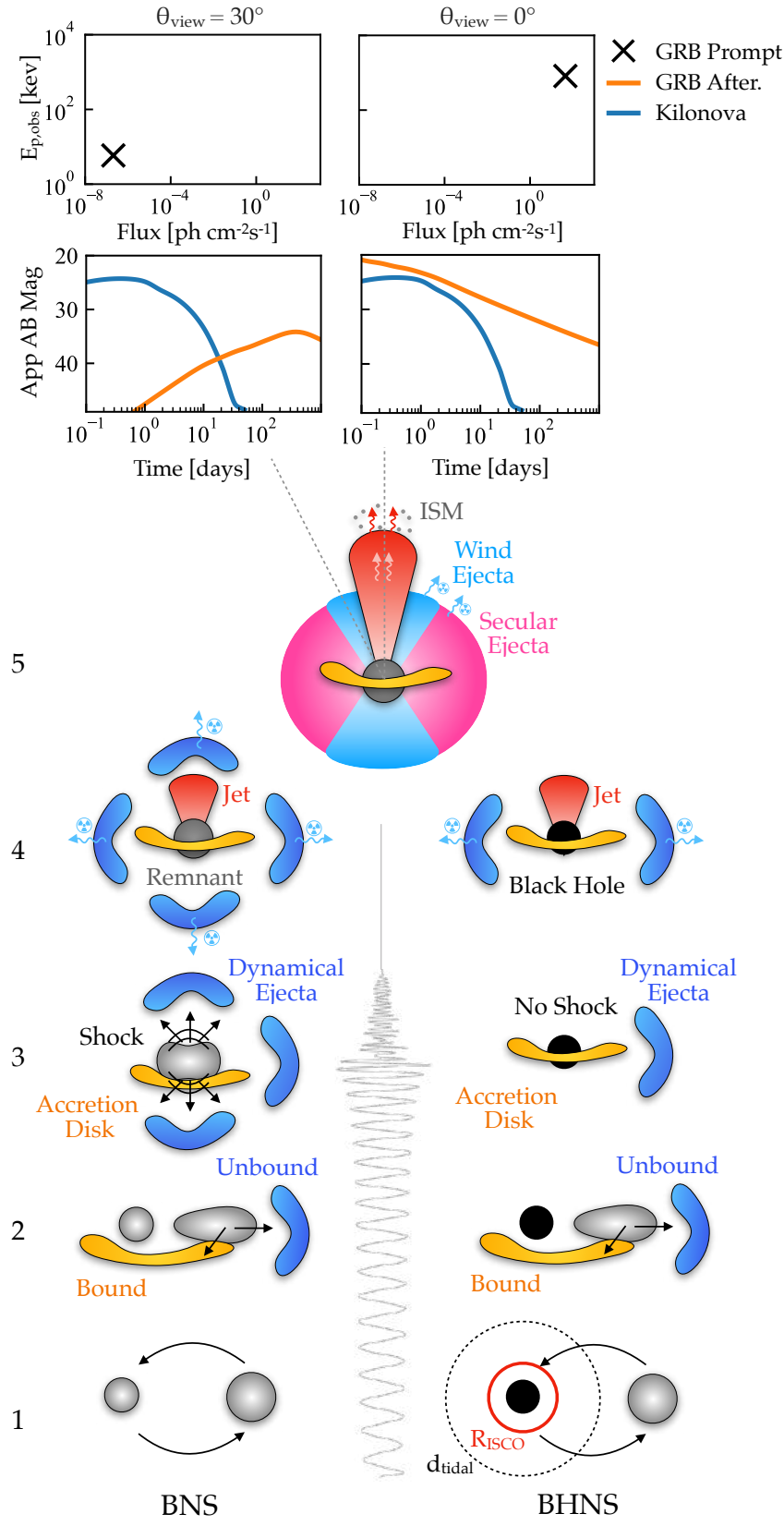


Figure 89. Schematic representation of BNS and BHNS mergers and their multi-messenger emissions.

This condition is favored if the BH is of low mass and/or rapidly spinning. Otherwise, the NS directly plunges into the BH, and no ejecta are produced (and, consequently, the event has no EM counterpart). Unlike BNS mergers, where the impact of the two surfaces provokes additional emissions making the distribution of the dynamical ejecta much more isotropic, BHs do not have a crust, so no shock occurs. Therefore, these systems produce dynamical ejecta close to the equatorial plane and mainly in about half of the azimuthal angle. Moreover, the absence of a hypermassive or supramassive NS phase can cause differences in the properties of the wind and secular ejecta.

In figure 89 we present (in the middle of the stages labeled 1–4) a representation of the GW waveform of a BNS merger.⁴⁹ At the top, we show a representation of the expected EM emission. Specifically, in the first-row panels we show the observed peak energy and photon flux for the GRB prompt emission, assuming a viewing angle of 0 and 30 degrees. In the second-row panels we present the optical light curve of the KN (blue) and the GRB afterglow (orange), again assuming viewing angles of 0 and 30 degrees. The EM emission was calculated using the C22 model for a BNS of 1.4 and 1.4 $M_{\odot}$ at the median redshift of the population, $z \sim 0.22$. It can be noted that the KN emission is much more isotropic, not showing substantial differences in the light curve when moving to larger viewing angles. In contrast, the GRB emission, being collimated, is much more sensitive to variations in this parameter. When shifting from 0 to 30 degrees, the GRB prompt flux decreases by several orders of magnitude, and the afterglow light curve peaks several months after the merger at a much lower apparent magnitude compared to the on-axis case.

5.2.2 Multi-messenger BNS & BHNS population models

We considered three population models for BNS: Colombo et al. 2022 (C22), Ronchini et al. 2022 (R22) [1175] and Loffredo et al. 2025 (L25) [2334]. For the BHNS population we consider the model of Colombo et al. 2023 (C23) [2187]. These models are based on different assumptions of the hyperparameters describing the population of binaries, their evolution and their EM emission components. Table 9 lists the main assumptions of the models providing the key references. In this section, we provide a brief description of these models and compare their different assumptions. The reader is referred to the relevant publication for a more in-depth description of each model. For C22 and C23 updated populations are given in [2335].

5.2.2.1 Progenitor compact binary population properties. The population models considered here predict different local merger rates, $\mathcal{R}_0$, compatible with current constraints provided by the Advanced LIGO and Virgo detectors in GWTC3 [6] ($\mathcal{R}_0 \in [10, 1700]$ Gpc⁻³ yr⁻¹ for BNS and $\mathcal{R}_0 \in [7.8, 140]$ Gpc⁻³ yr⁻¹ for BHNS). Given the lack of BNS merger detections in the ongoing run (O4) of LVK, we tune our populations on a fiducial local merger rate closer to the lower bound of the GWTC3 constraint. In particular, for BNS the

⁴⁹The time domain GW signal corresponds to the BNS simulation BAM:0095 [2332] in the CORE database [2333].

⁵⁰The value for the local merger rate corresponds to the fiducial population in L25, while the value for the pessimistic population is provided in parenthesis. These populations are consistent with the BNS mergers observed by LVK, as detailed in [2334].

	C22 [2336]	R22 [1175]	L25 [2334]	C23 [2187]
CBC System	BNS	BNS	BNS	BH-NS
CBC Pop.	R0+delayed SFR	Santoliquido+21	Iorio+23	Broekgaarden+21
$\mathcal{R}_0$ [$\text{Gpc}^{-3} \text{ yr}^{-1}$]	347^{+536}_{-256}	365	$107(23)^{50}$	149^{+153}_{-87}
Mass distr.	Analytic	Uniform	Gaussian	Broekgaarden+21
EOS	SFHo	—	BLh	SFHo
Fisher analysis	GWFast	GWFish	GWFish	GWFast
SNR cut	12	8	8	12
EM Transients	KN, GRB	GRB	KN	KN, GRB
KN ejecta model	[2337–2339]	—	[2334, 2337, 2339]	[2339, 2340]
KN emission model	[2341, 2342]	—	[2343]	[2338, 2341]
Jet launch & breakout	Yes	No	—	Yes
Cocoon Emission	Yes	No	—	No

Table 9. List of the main characteristics of the four different models used to compute EM emission from BNS and BH-NS mergers detectable by ET.

local rates considered correspond to $\mathcal{R}_0 = 347, 365, 107 \text{ Gpc}^{-3} \text{ yr}^{-1}$ for C22, R22, and L25, respectively. We adopt $\mathcal{R}_0 = 149 \text{ Gpc}^{-3} \text{ yr}^{-1}$ for the BHNS population of C23.

The mass distribution of NSs (isolated and in binaries) is still uncertain, therefore we explore three possibilities for BNS systems. C22 models the mass distribution of BNSs via an analytic fit to both Galactic BNS and the two BNS mergers detected through GWs (GW170817 and GW190425). R22 adopts a uniform NS mass distribution between $1.0 M_\odot$ and $2.5 M_\odot$, which is consistent with the GW detections, while L25 considers a Gaussian distribution centred at $1.33 M_\odot$ with a standard deviation of $0.09 M_\odot$, which fits the mass of Galactic NSs in binaries. The NS and BH mass distributions in C23 are derived from the population synthesis model of [1370] ensuring that the mass distribution of the population of BHNS is compatible with the constraints imposed by GW observations [1774]. NSs in binaries are predicted to have a negligible spin, therefore we assume spinless NSs in all our populations. BHs might have a non-negligible spin before the merger. However, since larger BH spins in BHNS binaries favour a stronger EM emission, we choose a conservative approach imposing a BH spin parameter $\chi_{\text{BH}} = 0$. R22 and L25 compute the redshift distribution of BNS mergers using the semi-analytic code **COSMORATE** [2344], which models the star formation rate density and the average metallicity evolution of the Universe as in [2345]. Conversely, C22 models the BNS redshift distribution analytically, using a delay time distribution compatible with the results of population synthesis models [1576]. For BHNS C23 assumes the fiducial metallicity specific star formation rate density from [1370], based on the phenomenological model described in ref. [1375].

The EOS of NSs determines the tidal deformability of NSs in binaries and affects the properties of the EM emission. Given the present uncertainties on the NS EoS, we assume two different nuclear EOSs, namely SFHo [2346] (for C22 and C23) and BLh [2347, 2348] (for L25). These EOSs are built within very different theoretical frameworks, in particular, SFHo is based on relativistic mean field theory while BLh is a microscopic EOS based on

the Brueckner-Bethe-Goldstone many-body theory. However, these EOSs predict similar values for the NS maximum mass and corresponding radius, specifically $M_{\text{max}} = 2.06M_{\odot}$ and $R_{\text{max}} = 10.3\text{ km}$ for SFHo, while $M_{\text{max}} = 2.10M_{\odot}$ and $R_{\text{max}} = 10.4\text{ km}$ for BLh. Both the EOSs are compatible with current experimental constraints on nuclear matter properties and astrophysical constraints on NS maximum mass, radius, and tidal deformability. They both produce quite compact NSs, with SFHo predicting slightly more compact NSs compared to BLh. By selecting the SFHo and BLh EOSs we made a conservative choice for the modeling of the KN emission since more compact NSs in binaries are associated with fainter KNae. A different approach is followed in R22, which does not make any assumption on the NS EoS. Accordingly, C22, L25, and C23 compute the NS dimensionless tidal deformability, Λ , from the mass and the EOS of each NS, while R22 samples Λ uniformly in the interval $[0, 2000]$ and independently for each NS. In all the considered models, the BNS and NS-BH mergers are distributed isotropically in the sky with a random inclination of the orbital plane to the line of sight.

5.2.2.2 Gravitational wave detection and parameter estimation simulation. The parameter estimation analysis of the BNS population for L25 and for R22 was carried out using the Fisher matrix code **GWFish** [291]. For the BNS and BHNS populations of C22 and C23, respectively, **GWFast** [1167, 2349] is adopted. These codes have been cross-checked within the activities of the *Common Tools* OSB division and yield comparable results. For a comprehensive description of the codes, we refer to section 10.

The settings of the gravitational analysis are common across the three BNS population models and the two different codes. We used the **IMRPhenomD_NRTidalv2** waveform approximant [36, 1229], which allows us to include tidal effects. We used the full cryogenic (HFLF-cryo) sensitivity curve for ET, including both low and high-frequency,⁵¹ with the starting frequency corresponding to 2 Hz.

We analyzed the two following detector configurations [16]:

1. the reference triangular ET design of 10 km located in Sardinia (Sos Enattos) with the HFLF-cryo sensitivity curves;
2. two L-shaped interferometers of 15 km arms misaligned at 45 deg located, respectively, in Sardinia and in the Meuse-Rhine region.

For BHNS systems of C23, the GW analysis, we consider the same configurations, sensitivities, and starting frequency of 2 Hz as for the BNS case, but we adopt the waveform approximant **IMRPhenomNSBH**, which is more suitable for this kind of systems [1229, 2350]. In particular, it is tuned up to mass ratios of $q \sim 100$, it includes tidal effects and the signatures on the signal due to different possible outcomes of the merger (plunge or disruption).

⁵¹All the available curves, including HFLF-cryo, can be found at <https://apps.et-gw.eu/tds/?content=3&r=18213>.

5.2.2.3 Electromagnetic emission models.

Kilonova. C22 computes the BNS KN light curves from 0.1–50 days, using the anisotropic three-component model of [2341, 2342]. The model accounts for the emission from three different outflow components:

- dynamical ejecta, produced by tidal interactions on a timescale of few milliseconds;
- winds ejecta, produced by the accretion disk, due to neutrino matter-interactions and magnetic pressure on a time scale of tens of milliseconds;
- secular ejecta, also produced by the accretion disk, due to viscous processes of magnetic origin throughout the duration of the accretion.

The ejecta are assumed to be in homologous expansion and thermal emission is computed following [2351, 2352]. The expected ejecta mass, ejecta average velocity and accretion disk mass are computed using numerical relativity-informed fitting formulae [2337, 2339, 2353], assuming the SFHo EOS [2346] and the masses of the wind and secular ejecta are computed as fractions of the disk mass: $\xi_w = 0.05$ and $\xi_s = 0.2$, respectively. The KN model parameters, such as the ejecta geometry, opacity, and the velocity of the wind and secular ejecta, are assumed as the best-fit parameters from [2338].

For the BHNS population in C23, the KN light curves are computed in the same way as for the BNS in C22, accounting for the expected differences in the KN model parameters [2338]. In this case, they compute the expected mass m_{dyn} and average velocity v_{dyn} for the dynamical ejecta and the mass m_{out} remaining outside the remnant BH, using the numerical relativity-informed fitting formulae from [2313, 2339, 2340]. For the binaries where $m_{\text{dyn}} > 0.5m_{\text{out}}$, they assume $m_{\text{dyn}} = m_{\text{dyn}}^{\text{max}}(m_{\text{out}}) = 0.5m_{\text{out}}$ [2354]. Finally, the mass of the accretion disk is computed as $M_{\text{disk}} = m_{\text{out}} - m_{\text{dyn}}$.

L25 models the BNS KN emission from 1 hour to 10 days assuming a two-component anisotropic ejecta. The first ejecta component (C1) includes matter ejected on a dynamical timescale (dynamical ejecta) and matter unbound from the disk through spiral-wave winds. The second component (C2) contains matter unbound from the disk through viscous processes on a secular timescale (secular ejecta). The mass and velocity of the dynamical ejecta and the amount of mass in the disk are computed employing numerical relativity-informed fitting formulas reported in [2334], assuming the BLh EOS [2347, 2348]. These formulas model the possibility of the prompt collapse of the remnant to a BH, which is favoured for more massive and asymmetric binaries. A prompt collapse to BH affects the ejecta properties, reducing the amount of dynamically ejected matter and preventing the formation of spiral wave winds. Therefore, if the remnant undergoes prompt collapse to BH, the amount of matter unbound through spiral-wave winds is set to zero, while it corresponds to a fraction $\xi_{sw} = 0.15$ of the disk mass, otherwise. The total amount of mass in the first component, $m_{C1} = m_{\text{dyn}} + m_{sw}$, is distributed as $\sin \theta$, where θ is the polar angle. The corresponding velocity, v_{C1} , is obtained as a weighted average of the velocity of the dynamical ejecta, v_{dyn} , and the velocity of the spiral-wave wind $v_{sw} = 0.17c$ [2355]. The mass in the second component, m_{C2} , is modelled as a fraction $\xi_s = 0.3$ of the disk mass and the velocity is set to $v_{C2} = 0.06c$. The ejecta

properties such as opacity, entropy, expansion timescale, and floor temperatures are chosen as in [2334], ensuring that the KN light curves of the BNS in the population with chirp mass compatible with GW170817 reproduce AT2017gfo data. The KN light curves are computed using the semi-analytic model `xkn-diff` [2343], based on a semi-analytic solution of the radiative transfer equation for homologously expanding matter.

Jet-related emission. C22 and C23 adopt, for both the BNS and the BHNS population, a jet angular structure inspired by GRB170817A [2268] with an half-opening angle of 3.4 deg. They compute the GRB afterglow light curves from 0.1 to 1000 days in the radio (1.4 GHz), optical (g band) and X-rays (1 keV) bands, fixing the median interstellar medium density $n = 5 \times 10^{-3} \text{ cm}^{-3}$ [1652] and assuming the microphysical parameters characteristic of GW170817, $\epsilon_e = 0.1$, $\epsilon_B = 10^{-3.9}$ and $p = 2.15$.

In C22 and C23, the methodology for GRB prompt emissions is based on models similar to those in [2338] and [2356]. They assume that a constant fraction $\eta_\gamma = 0.15$ of the jet energy density, confined to regions with a bulk Lorentz factor $\Gamma \geq 10$, is emitted as photons. The resulting observed spectrum is the integration of this radiation over the jet's solid angle, incorporating relativistic beaming effects. Finally, photon fluxes in specific energy bands are calculated, assuming a standard burst duration of 2 seconds.

In C22 they assume the launch of a relativistic jet just for BNS mergers where the remnant mass $M_{\text{rem}} \geq 1.2M_{\text{TOV}}$, which suggests either an immediate or delayed collapse into a black hole. For BHNS mergers in C23, they consider the formation of a relativistic jet with energy E_c if the disk mass $M_{\text{disk}} > 0$. In both works, if the jet energy exceeds a specific threshold (as defined by [2357]), it successfully breaks through the ejecta, resulting in GRB prompt and afterglow emissions.

For BNS systems observed within a viewing angle $\theta_v \leq 60^\circ$, C22 also incorporates a cocoon shock breakout component, modeled in line with the properties of GRB170817A, namely a luminosity $L_{\text{SB}} = 10^{47} \text{ erg s}^{-1}$, a cutoff power-law spectrum with νF_ν peak photon energy $E_{\text{p,SB}} = 185 \text{ keV}$ and low-energy photon index $\alpha = -0.62$. In BHNS systems, C23 does not include this additional emission due to the absence of observational constraints for this type of source.

For the prompt emission, R22 considers a universal jet structure, corresponding to the profile derived for GW170817. The jet is assumed to have a core with an opening angle of 3.4 deg. The profile outside the jet core follows a power-law with index $s = 4$. The values of jet energy, on-axis rest-frame peak energies and pulse duration are extracted from eqs. (6), (7) and (8) of R22, respectively. The parameters of the distributions, including also the probability that a BNS produces a successful jet, are calibrated performing a comparison with existing samples of short GRBs observed by Fermi.

For the computation of the afterglow emission, the `afterglowpy` model is adopted [2358]. The isotropic-equivalent kinetic energy of the jet is derived starting from the posterior distribution of the isotropic-equivalent prompt energy taken from R22. The micro-physical parameters are $\epsilon_e = 0.1$, $p = 2.2$, $\log_{10} \epsilon_B$ uniformly distributed in the range $[-4, -2]$ and $n \in [0.25, 15] \times 10^{-3} \text{ cm}^{-3}$.

5.2.3 EM properties of the GW-detectable sub-population

In this section we describe the observed properties of the EM emission of sources detected in GW by ET. We also provide the rates of joint detectable sources as a function of a variable detection limiting flux in different energy bands for each emission component. It is important to distinguish between detectable and detected joint EM-GW events. The former are those sources, detected as GW signals by ET, which are above a given sensitivity limit, i.e., they are in principle bright enough to be detectable by any facility reaching that sensitivity. We will consider this condition in defining detectable joint events. However, the rate of joint detected sources may be smaller than the rate of detectable ones. Indeed, the latter should be scaled by any EM facility property other than its sensitivity, such as the field of view, the observability constraints, the duty cycle etc. In this section, we will only report the joint detectable event rates. We will provide more realistic detected rates in section 5.3 by considering specific mission/facilities.⁵²

5.2.3.1 Kilonova. Figure 90 shows the distributions of the KN lightcurves in the g band of the subsample of sources (BNS and BHNS) detected as GW events and localized with a sky accuracy of up to 100 deg^2 by ET in two different configurations (10 km triangle and 15 km 2L — left and right panels, respectively). There are no appreciable differences in the light curves detected by the two configurations for either the BNS or BHNS populations. BNS kilonovae typically exhibit AB magnitudes in the range of 20–27 (99% confidence region) at approximately 2.4 hours post-merger, and their brightness remains almost constant (or slightly increases) until about 1 day, and then decreases rapidly over the 10-day timescale. Despite still being perfectly consistent within their dispersion, the BNS-KNae lightcurves of L25 show a shallower decay with respect to those of C22 owing to the different KN model parameters, as well as the different ejecta distribution in the two populations.

BHNS systems (bottom panels of figure 90) produce KN light curves, which are typically fainter (AB magnitudes in the range 22–29 at 2.4 hours post-merger) than BNS and evolve towards a faster decline within 1 day from the merger. This may be explained, respectively, by the larger GW horizon of BHNS systems with respect to BNS and the different ejecta properties resulting from the assumed mass and spin distributions and EoS.

Figure 91 depicts the joint detection rate as a function of different AB magnitude detection limits for the BNS populations in L25 (red) and C22 (blue), and for the BHNS population in C23 (green). The panels on the right assume the ET triangle configuration, while those on the left assume ET-2L. Solid lines indicate all detectable KN+GW systems, while dashed and dotted lines represent only events with a sky localization $\Delta\Omega_{90\%} < 100 \text{ deg}^2$ and $\Delta\Omega_{90\%} < 40 \text{ deg}^2$, respectively.

The colored band represents the uncertainty due to the local merger rate, which propagates as a constant relative fraction to all other rates. Despite different assumptions for the initial BNS population and the KN model, L25 and C22 predict rates that differ by only a factor of about 2–3, and when considering the uncertainty due to the local merger rate, they are entirely compatible.

⁵²We adopt the convention to distinguish between “missions” and “facilities” referring to space-based detectors and ground based telescopes, respectively.

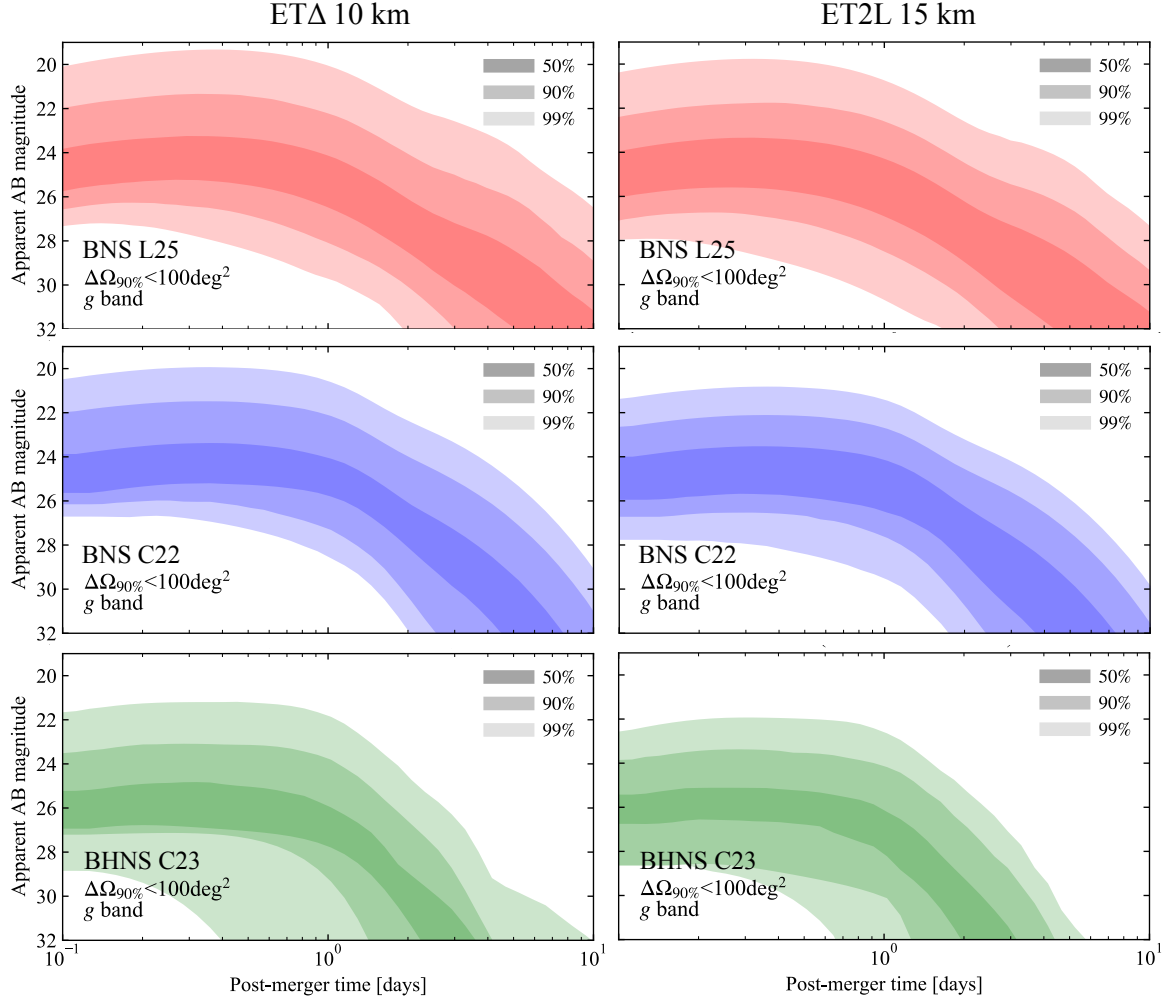


Figure 90. Distribution of ET optical KNe as a function of time assuming the BNS population from L25 (red, first row) and C22 (blue, central row), and the BHNS population from C23 (green, last row). The left column assumes the ET- Δ configuration, the right column is the 2L configuration. The plots represent the apparent AB magnitude versus post-merger time (days) for our simulated KNe light curves in the g (484 nm) band, restricting to ET GW-detectable sources with $\Delta\Omega_{90\%} < 100 \text{ deg}^2$. The shaded regions contain 50%, 90%, and 99% of the KN light curves.

Using this plot, given an instrument with specific sensitivity, one can estimate the rate of GW events with a KNe brighter than this limit, thus estimating the rate of potentially detectable KNe. For instance, the Vera Rubin Observatory can reach an AB magnitude of 26 in the g band with an exposure time of 180s. The majority of KNe lightcurves are brighter than this limiting magnitude up to 1–2 days post-merger, see figure 90). Considering this representative limit and events with $\Delta\Omega_{90\%} < 100 \text{ deg}^2$, L25 yields a rate of detectable KNe $100(21) \text{ yr}^{-1}$,⁵³ while C22 yields a rate of $191^{+295}_{-141} \text{ yr}^{-1}$.

For BHNS events, the rate is $5.1^{+5.2}_{-3.0} \text{ yr}^{-1}$. In general, the BHNS rates are systematically lower than those of BNS events. The reasons, as explained in C23, are due to the fact

⁵³The rate of detectable into parenthesis refers to the pessimistic population of L25 [2334].

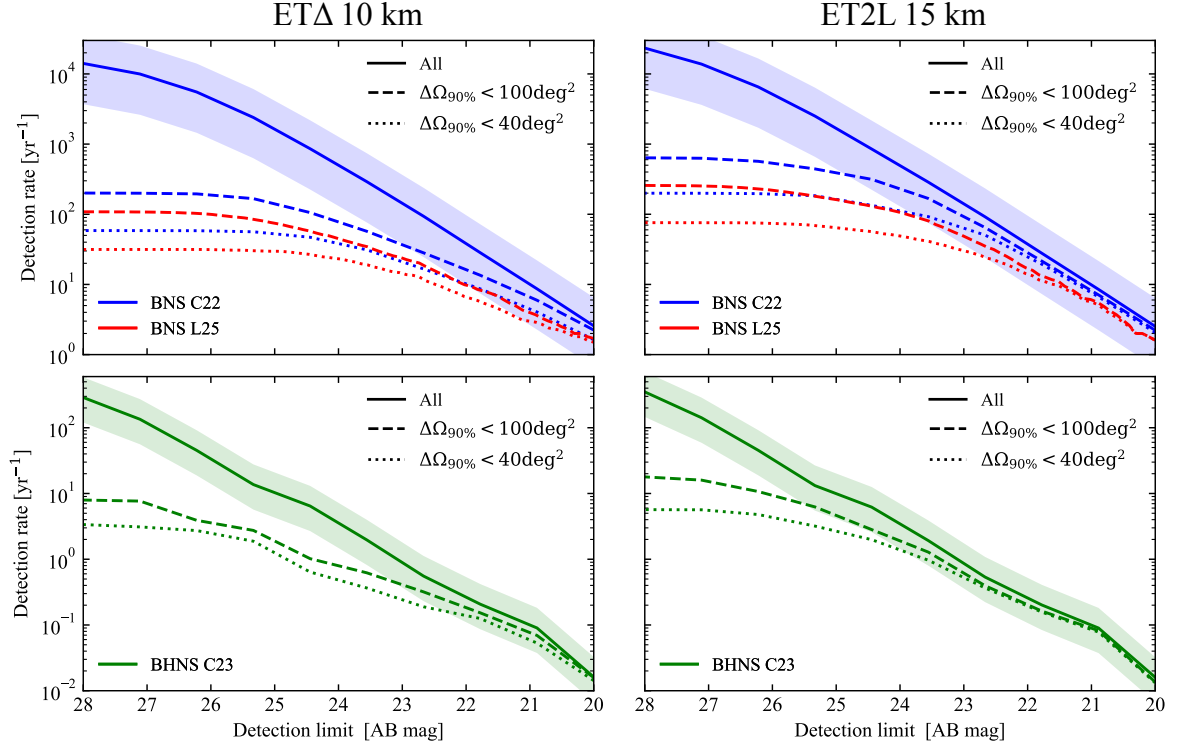


Figure 91. Detection rate as a function of detection limit threshold for the KN BNS population presented in L25 and C22 (*upper panels*, red and blue, respectively) and for the BHNS population from C23 (*lower panels*, green). The left column assumes the ET- Δ configuration, and the right column is the 2L configuration. The lines indicate the KN+GW (g band) detectable binaries, assuming all the BNSs (solid line from C22 *upper panels* and C23 *lower panels*), the ones with $\Delta\Omega_{90\%} < 100\text{deg}^2$ (dashed lines) and the ones with $\Delta\Omega_{90\%} < 40\text{deg}^2$ (dotted lines). The shaded region indicates the uncertainty due to the local merger rate.

that only a small fraction (about 2%) of events can emit ejecta, and the KN population is generally less luminous.

5.2.3.2 Gamma-Ray Burst prompt emission. Figure 92 presents the observed peak energy $E_{p,\text{obs}}$ as a function of the photon peak flux for the BNS populations of R22 (*upper panels*) and C22 (*mid panels*), and for the BHNS population of C23 (*lower panels*). The colored regions include events that are both GW and GRB prompt-detectable. For prompt detection, Fermi/GBM was used as a reference instrument, with threshold values chosen to replicate the short GRB observations of Fermi/GBM: $0.5 \text{ ph cm}^{-2} \text{ s}^{-1}$ for R22 and $4 \text{ ph cm}^{-2} \text{ s}^{-1}$ for C22. Note that R22 considers the 50–300 keV band, while C22 assumes the 10–1000 keV band.

Generally, GW+GRB prompt events extend from an $E_{p,\text{obs}} \sim 30 \text{ keV}$ to $E_{p,\text{obs}} 10 \text{ MeV}$, concentrating around $E_{p,\text{obs}} \sim 300 \text{ keV}$. For C22, the events do not extend beyond 1 MeV due to different assumptions regarding the jet peak energy. Concerning the photon flux, all models reach up to $100 \text{ ph cm}^{-2} \text{ s}^{-1}$, while the 50% of events concentrates between $0.5\text{--}10 \text{ ph cm}^{-2} \text{ s}^{-1}$, depending on the flux cut assumption.

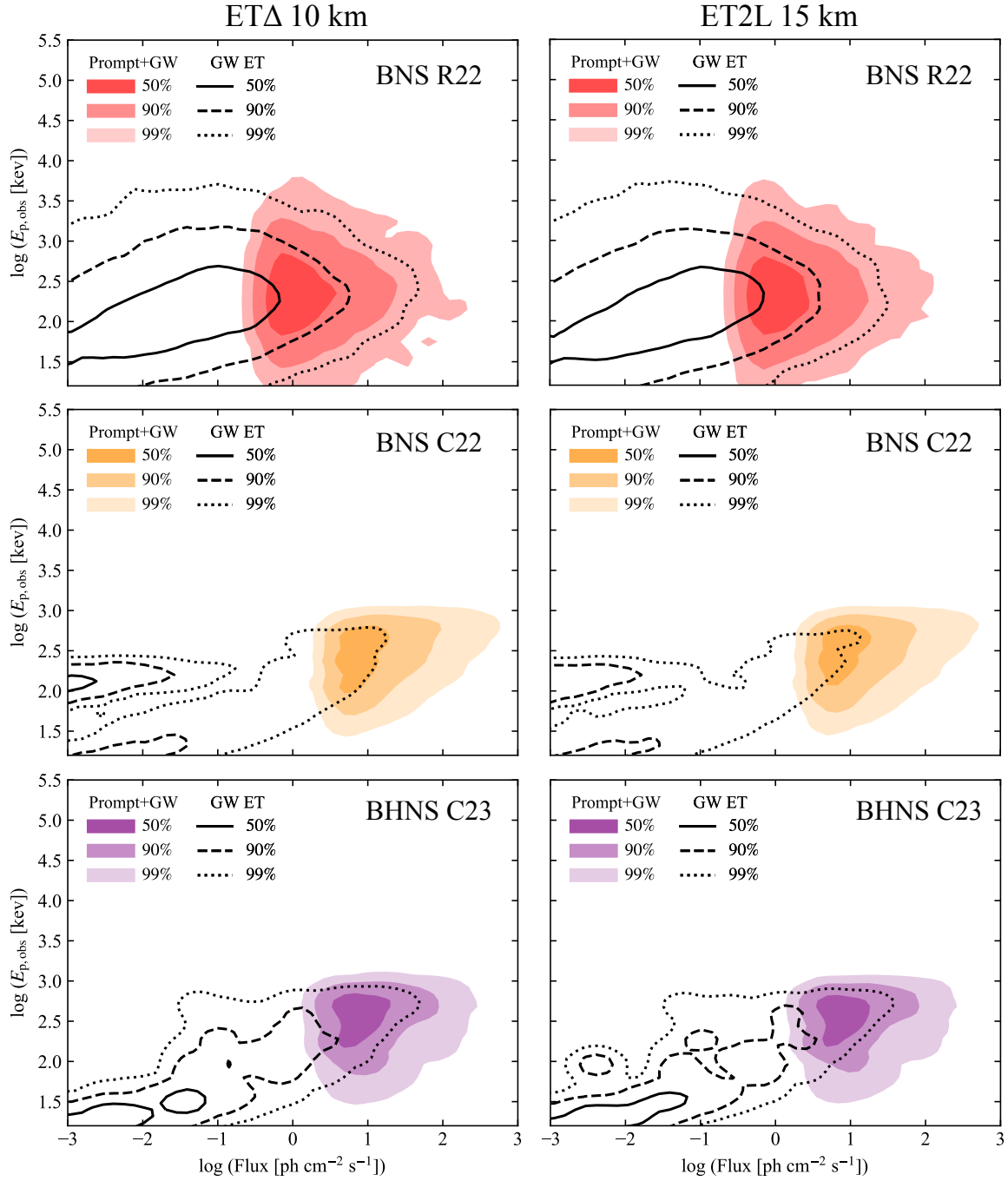


Figure 92. Observed spectral energy distribution peak photon energy $E_{p,obs}$ vs. photon flux for the GRB prompt BNS population presented in R22 (first row) and C22 (central row), and for the BHNS population from C23 (last row). The colored-filled regions contain 50%, 90%, and 99% of the binaries, both GRB Prompt and ET detectable. The black lines contain 50%, 90%, and 99% (solid, dashed, and dotted, respectively) of the ET-detectable binaries.

The black lines enclose 50% (solid), 90% (dashed), and 99% (dotted) of events detectable by ET, without the requirement of prompt detection. In this case, the population extends to much lower fluxes and peak energies and becomes dominated by off-axis events. While these events are out of the reach of Fermi/GBM mainly due to its sensitivity and low energy threshold, next-generation GRB detectors may extend their energy band below 10 keV with an improved sensitivity, thus capturing the bulk of the prompt emission.

The BNS C22 population shows two clusters when considering only the events detected by ET. This effect is due to the assumption that for events with a viewing angle less than 60 degrees, there is also a shock breakout component whose properties mimic those of GRB170817A. Events dominated by the cocoon shock breakout component form the upper cluster and become insignificant when also considering the detection by Fermi/GBM, which is dominated by events with a higher photon flux.

Figure 93 shows the detection rate as a function of different photon flux detection limits for the BNS populations of R22 (red), C22 (orange), and the BHNS population of C23 (purple). The solid curve refers to all BNS events, while the dashed and dotted lines consider events with sky localization smaller than 100 deg^2 and 10 deg^2 , respectively. The colored bands represent the uncertainty in the local merger rate for C22 and C23, and the Poisson uncertainty due to variations in hyperparameters for R22. Considering the flux cuts assumed by the respective studies using Fermi/GBM, the rates obtained are $57_{-35}^{+56} \text{ yr}^{-1}$ and $59_{-44}^{+91} \text{ yr}^{-1}$. Additionally, these rates can be scaled by a factor accounting for the duty cycle and field of view of the instrument. We consider the latter two parameters yielding, for Fermi/GBM, a scaling factor 0.6 so that ET(triangle)-Fermi/GBM joint detection rate for the prompt emission would be $34_{-21}^{+34} \text{ yr}^{-1}$ and $35_{-26}^{+55} \text{ yr}^{-1}$. This means that the majority of short GRBs currently detected by Fermi/GBM would have a GW counterpart detected by ET.

5.2.3.3 Gamma-Ray Burst afterglow emission. Figure 94 shows the average properties of the afterglow lightcurves of all the BNS and NSBH mergers detected by ET, considering the delta (left panel) and 2L (right panel) configurations. The figure reports the 50% (solid line) and 90% (dashed line) contours containing the peak fluxes of afterglow light curves in the radio (red), optical (green), and X-ray (blue) bands for the BNS populations of R22 (upper panels) and C22 (mid panels), and for the BHNS population of C23 (lower panels). The gray curves in the background are a sample of optical light curves for the respective populations.

Generally, all populations have peaks concentrated after 10^2 days, with a tail extending to shorter timescales. Specifically, the R22 population is concentrated at around $10^{2.5}$ days, while the C22 and C23 populations show a slower growth, with peaks concentrated around 10^3 days.

Most populations exhibit a subset of bright optical and X-ray events with peaks at very short timescales (hours). These are the on-axis events of the population, as can be seen from the monotonous gray light curves in the background, representing light curves observed at viewing angles comparable to the jet opening angle. This subset is always present, even when the population is dominated by off-axis events, which do not show such curves. Overall, there are no noticeable differences between the delta and 2L configurations, in terms of peak brightness of the afterglow and distribution of the peak times.

Figure 95 represents the detection rates as a function of different flux limits in mJy for the BNS populations of R22 (upper panels) and C22 (mid panels), and for the BHNS

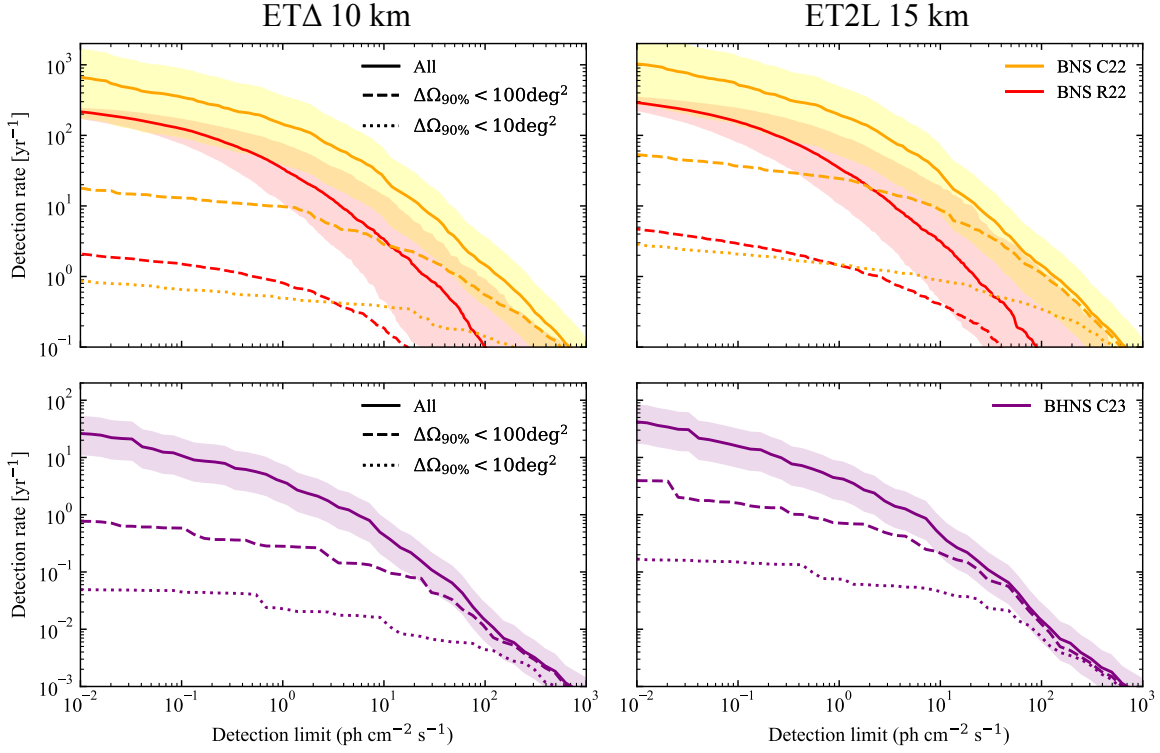


Figure 93. Detection rate as a function of detection limit threshold for the GRB prompt BNS population presented in R22 and C22 (*upper panels*, red and orange, respectively) and for the BHNS population from C23 (*lower panel*). The left column assumes the ET delta configuration, and the right column assumes the 2L configuration. The lines indicate the GRB prompt+GW detectable binaries, assuming all the BNSs (solid lines), the ones with $\Delta\Omega_{90\%} < 100\text{deg}^2$ (dashed lines) and the ones with $\Delta\Omega_{90\%} < 10\text{deg}^2$ (dotted lines). The shaded region indicates the uncertainty at 90% credible level. For C22 and C23, it indicates the uncertainty due to the local merger rate.

population of C23 (lower panels). The lines represent the detectable GW+GRB afterglow events, assuming all binaries (solid line), binaries with sky localization within 100deg^2 (dashed line), and within 10deg^2 (dotted line). In the X-rays, for a representative limiting flux of $10^{-14}\text{ erg cm}^2\text{ s}^{-1}$, R22 predicts around 10^3 detectable BNS per year, while C22 predicts around 200 detectable BNS per year. For both R22 and C22, the detectable sources reduce to ~ 10 and less than 1, considering the ones localized better than 100deg^2 and 10deg^2 . The difference can be attributed to the different GW SNR detection threshold and modelling between R22 and C22. Overall, the 2L configuration gives approximately a factor two more BNS and NSBH mergers with detectable afterglow.

5.2.4 GW sky localization of multi-messenger events

Figure 96 shows the $\Delta\Omega_{90\%}$ sky localization distribution as a function of redshift for the BNS population from C22 (upper panel, blue) and the BHNS population from C23 (lower panel, green), considering only events capable of emitting an EM counterpart, thus having non-zero ejecta and/or accretion disk. The colored bands represent the 50%, 90%, and 99% credibility

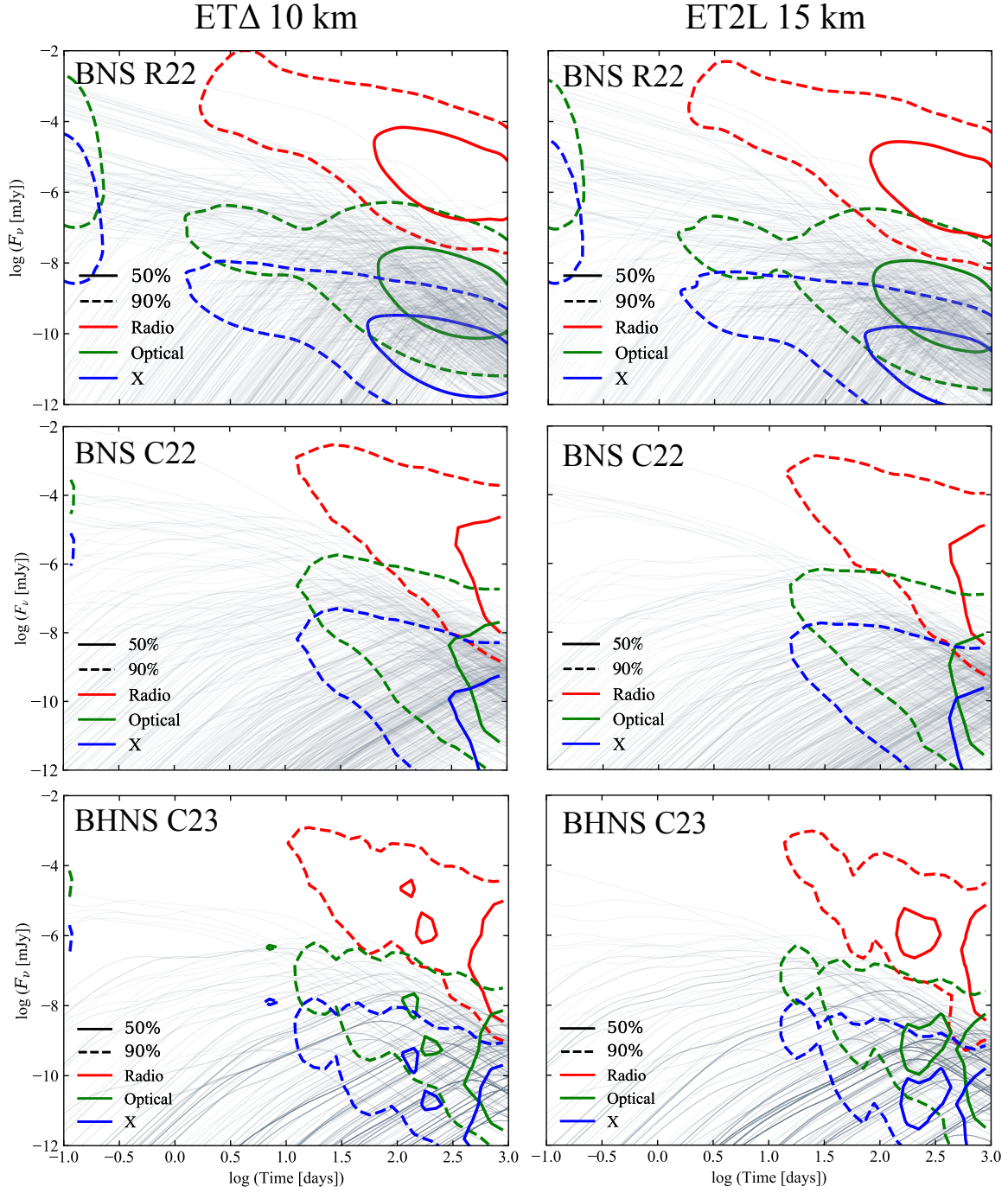


Figure 94. Afterglow lightcurves in Radio (red), Optical (green) and X-ray (blue), for the delta (left) and 2L (right) configurations, assuming the BNS population from R22 (first row) and from C22 (central row), and the BHNS population from C23 (last row). The dashed and solid lines are the 50% and 90% containment regions of the afterglow peaks, in the respective bands. The grey lines in the background are randomly sampled optical light curves.

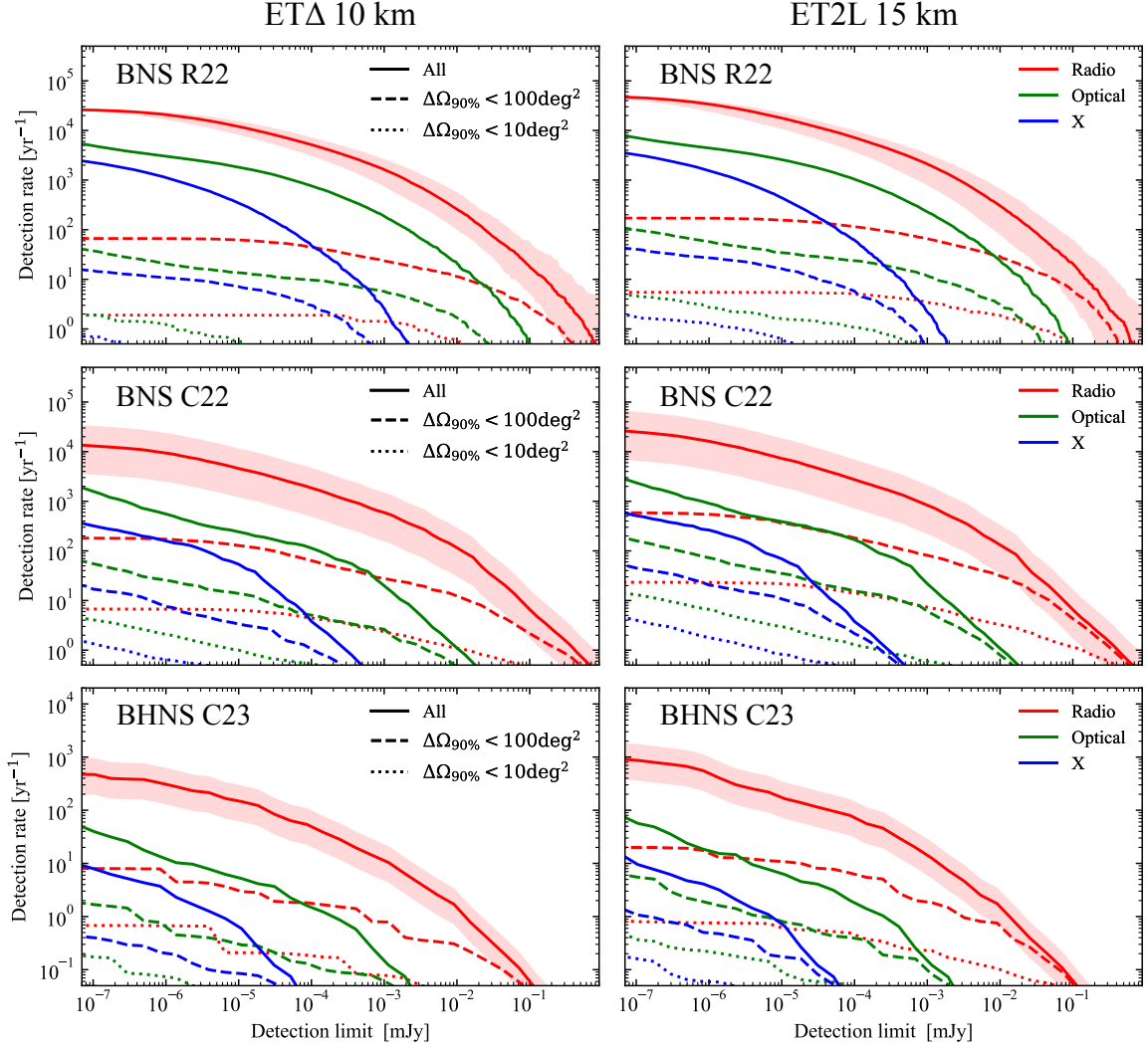


Figure 95. Detection rate as a function of detection limit threshold for the GRB afterglow BNS population presented in R22 (first row) and C22 (central row), and for the BHNS population from C23 (last row). The left column assumes the ET delta configuration, the right column the 2L configuration. The lines indicate the GRB afterglow+GW detectable binaries in the radio (red), optical (green) and X (blue) bands, assuming all the BNSs (solid line), the ones with $\Delta\Omega_{90\%} < 100\text{deg}^2$ (dashed lines) and the ones with $\Delta\Omega_{90\%} < 10\text{deg}^2$ (dotted lines). The shaded region indicates the uncertainty at 90% credible level. For C22 and C23, it indicates the uncertainty due to the local merger rate.

regions, while the solid line indicates the median value. The left column assumes the ET configuration with a triangle, while the right column assumes the 2L configuration.

The figure illustrates the gradual broadening of sky localization with increasing redshift. For the triangle configuration, within a redshift of approximately $z \lesssim 0.05$, 50% of the events have a sky localization between 10 and 100 deg^2 . Beyond a redshift of $z \gtrsim 0.2$, 50% have a sky localization exceeding 1000 deg^2 . In contrast, for the 2L configuration, there is a general decrease in the median value by about an order of magnitude. Specifically, within a redshift of $z \lesssim 0.05$, most events have a sky localization between 1 and 10 deg^2 , while 50% have

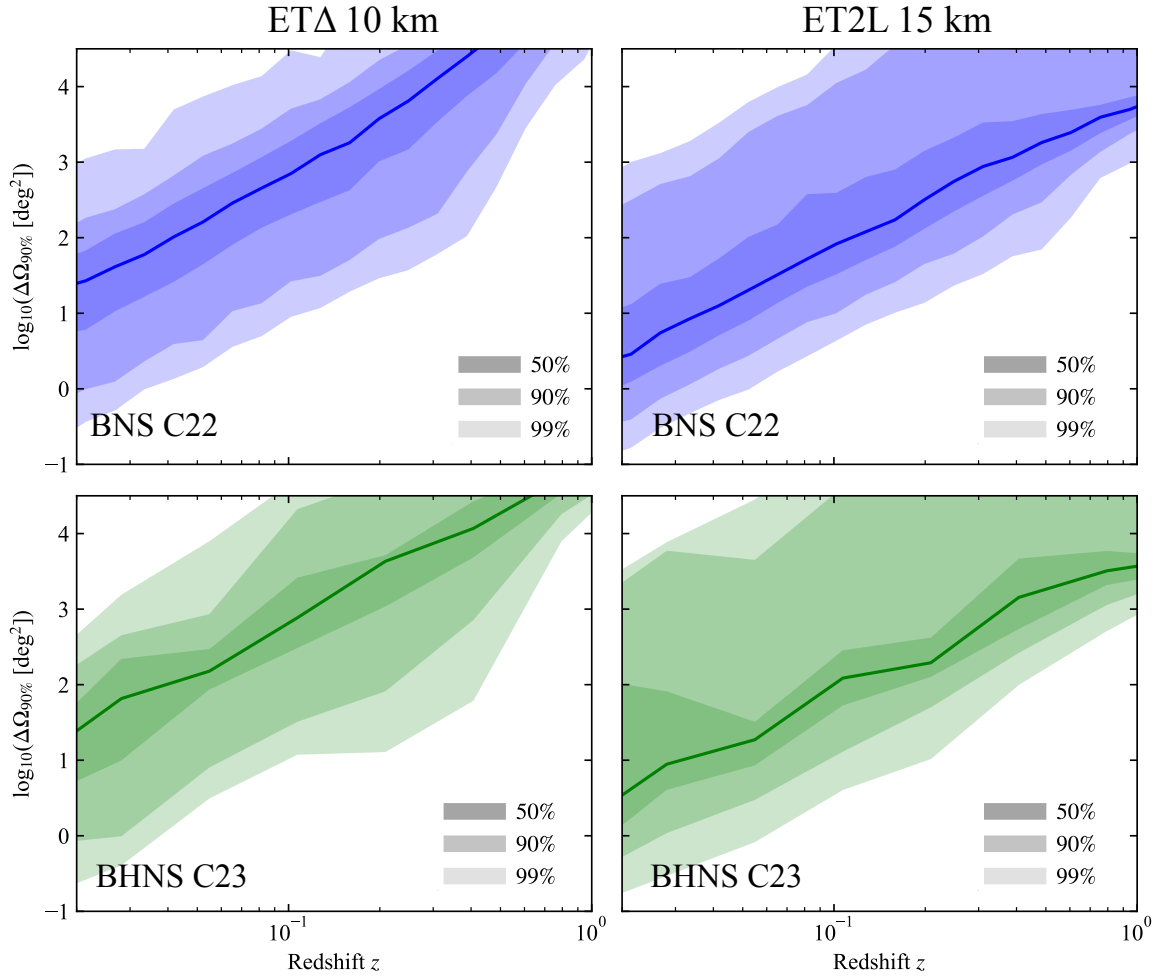


Figure 96. Distribution in redshift of the sky localizations of BNS events (*upper panel*) and BHNS events (*lower panel*) that can power and EM counterpart, for the delta (left) and 2L (right) configurations. The line indicates the median value, the colored bands are the 50%, 90% and 99% credible regions.

a sky localization exceeding 1000 square degrees for redshifts greater than $z \gtrsim 0.5$. The greater broadening of the distribution for the 2L configuration as compared to the triangle can be traced mainly to events for which only one of the detectors is operational due to our choice of the duty cycle. For a discussion regarding the comparison of the sky localization capabilities of the triangular and 2L ET configurations within the context of Fisher forecasts we refer the reader to section 3 of [16].

5.2.5 ET in a network with Cosmic Explorer

ET may operate together with another 3G detectors such as Cosmic Explorer. Predicted performances of an ET-CE network have been explored in a number of works (e.g. [16, 1167, 1175, 2334, 2359]), and the synergies between ET and CE will be further discussed in section 6.

To compare ET as a standalone observatory with ET as part of a network of other 3G detectors, figure 97 shows the KN detection rates as a function of detection limits for the

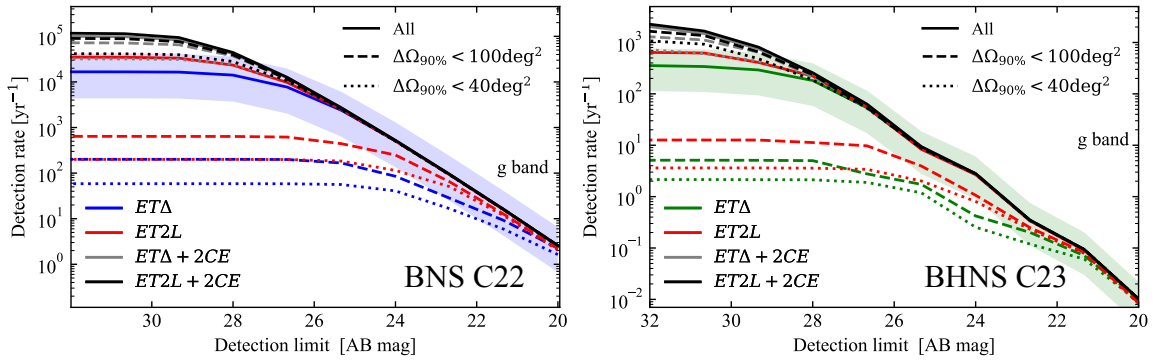


Figure 97. Detection rates as a function of the detection limit threshold for the KN BNS population from C22 (left panel) and the BHNS population from C23 (right panel). The different colors represent various ET configurations and detector networks: blue (or green) for $ET\Delta$, red for $ET2L$, gray for $ET\Delta + 2CE$, and black for $ET2L + 2CE$. The lines correspond to KN+GW (g -band) detectable binaries: solid lines indicate all BNSs (or BHNSs), dashed lines represent events with $\Delta\Omega_{90\%} < 100\text{deg}^2$, and dotted lines represent events with $\Delta\Omega_{90\%} < 40\text{deg}^2$. The shaded region illustrates the uncertainty arising from the local merger rate.

BNS population from C22 (left plot) and the BHNS population from C23 for different GW detector networks. The different line colors represent various ET configurations and network setups. Specifically, the blue (or green) lines indicate $ET\Delta$, while the red lines correspond to $ET2L$. The gray and black lines represent the networks $ET\Delta + 2CE$ and $ET2L + 2CE$, respectively. In this context, we assume the 2 CE network as the most optimistic scenario, consisting of two L-shaped detectors with arm lengths of 40 km and 20 km located in the USA. Solid, dashed, and dotted lines represent, respectively, all BNS (or BHNS) events, events with $\Delta\Omega_{90\%} < 100\text{deg}^2$, and events with $\Delta\Omega_{90\%} < 40\text{deg}^2$, respectively.

The comparison between the two ET configurations has been previously discussed; however, it is noteworthy that in a network including 2 CE detectors, the number of joint detections significantly increases due to the enhanced precision in sky localization. Specifically, considering all events with $\Delta\Omega_{90\%} < 100\text{deg}^2$, the number of detectable events (assuming a detection limit of AB magnitude 26) increases by more than an order of magnitude. In a complete network, the differences between the two ET configurations become negligible up to a magnitude of 26. For deeper limits, the increased EM horizon allows for the observation of more distant events, which results in a small but noticeable difference in detection rates between the two configurations.

5.3 Observational facilities and strategies

5.3.1 The ET change of paradigm

ET will revolutionize the field of multi-messenger astrophysics, particularly the study of BNS and BHNS mergers. It will transform the field from being observationally constrained to the local universe and to a poor number of events into a domain exploring more than 10 billion years of cosmic history, encompassing hundreds of thousands of objects.

This paradigm shift necessitates the development of new observational strategies to detect, identify, and characterize the EM counterparts of GW events detected by ET. Different strategies should be developed depending on the scientific goals, and taking into account four intrinsic factors well illustrated in the previous sections: (i) the large localization uncertainty for the majority of the GW events; (ii) the large number of ET detections; (iii) the faintness of the KN and GRB off-axis afterglow counterparts; (iv) the brightness of on-axis GRBs and afterglows at any distances explored by ET. Indeed, while studies involving cosmic evolution should take advantage of the entire volume covered by the ET sensitivity, the study of KNe or off-axis afterglows may rely on the numerous nearby events detected by ET with a relatively good localization.

The precision of GW localization, although introducing biases favoring nearby events, will be a key selection criterion for the EM observations, because of the extremely large observing time required to scan large sky region at the necessary depths, and of the extremely high number of ET events. About hundred ET-detected BNS events per year at $z < 0.5$ (a few tens at $z < 0.2$) will have a GW localization to within a few tens of square degrees (see figure 185 in section 10.3.3 and ref. [2334]). However, if a scenario with ET + CE observations is considered, more than ten thousand events will have such a sky-localization, several thousands of which will have localization uncertainty smaller than 10 deg^2 , decreasing to less than 1 deg^2 for a few hundreds of them (see figure 107 in section 6.2.1). In such a scenario, additional selection criteria will therefore be crucial to optimize the use of the limited observational time. Prioritization of the alerts and observational strategies will have to be adapted for each specific scientific case.

5.3.2 Kilonovae

The best spectral range to look for KNe is the UV-optical-(near)infrared one. Through photometry it is possible to obtain images of the sky where to look for unidentified transients, and to follow their lightcurves in search of the time and color evolution signatures expected for KNe. However, for a certain KN identification, spectroscopic observations provide more precise (r-process element) features and are often required. Considering the intrinsic faintness of such EM counterparts (see section 5.2.3.1) instruments with a high sensitivity are required. Unfortunately, to date, the best instruments to obtain deep images have a small Field of View (FoV), i.e. they are able to observe only a very limited area of the sky, much smaller than the typical GW sky-localization regions. Spectroscopic observations are even more limited because the fibers or slit cover only few arcsec, unless Integral Field Spectroscopy (IFS) is used. This technique enables the possibility to obtain a spectrum for each pixel of the image of a field, but nowadays sensitive IFS are limited to FoV of the order of the arcmin². The ideal facility for the most efficient multi-messenger observations of KN would be a highly sensitive integral field spectrograph with a very large FoV (several square degrees) and (sub-)arcsecond spatial resolution.

The simulations presented in section 5.2 on the predicted brightness of the KN (and GW error regions) imply that, with the most advanced facilities expected to be available by the late 2030s (see the table in figure 100), KN detections will remain limited to redshifts $z < 0.5$. Among the planned instruments, photometric and spectroscopic facilities with unprecedented

sensitivity and coverage, such as the Vera Rubin Observatory and the Wide Survey Telescope (WST), will be particularly well-suited for detecting KNe up to $z < 0.4 - 0.5$ from the events localized to within a few tens of square degrees [1211, 2334]. Target of Opportunity observations by the Vera Rubin Observatory will provide the photometric depth and spectral constraints needed to detect candidate KN counterparts. Realistic simulations including ToO multi-filter and multi-epoch observations (more effective to identify the counterparts among large number of contaminants), mosaic of the sky-localization region, Rubin sky coverage, and Galactic absorption predict several tens of KN per year associated with BNS detected by ET [2334]. This numbers may improve by a factor of ~ 10 , providing to dispose of enough observing time, if ET will operate in the network with current detectors LIGO and Virgo at the design sensitivity and including a detector in India. A further increase of detections by Rubin is obtainable when ET operates with CE but it is limited to a factor of a few because of the Rubin detection efficiency drop already at z larger than 0.3 [2334].

Facilities such as WST could work in synergy with Rubin or independently and detect about a hundred KN counterparts per year at a signal-to-noise ratio (S/N) > 10 (thousands at lower S/N), if that observations are collected within $\sim 12 - 24$ hours after the merger (see figure 98). It is evident that, in both cases, the observing time available will be the main limitations to the number of events for which KN can be detected. Therefore, in addition to the distance and position indications provided by ET, other parameters (such as the inclination angle) will have to be considered to select the events triggering the search for a KN counterparts. For more details on the synergies between ET and WST, see [2360].

Current and forthcoming sky surveys providing galaxy redshift catalogs could enable photometric and/or spectroscopic targeted observations, leveraging distance information from ET detections. However, this would not be an efficient strategy to find KNe beyond $z \sim 0.1$. In fact, in this case, the number of galaxies requiring observation quickly becomes vast — thousands, within the expected sky localizations of ten — few tens square degrees. Targeted spectroscopic searches at higher redshifts could be useful in a scenario where ET operates with two next-generation observatories distributed around the globe, for example a CE(40 km) in USA and one in Australia (or also with the addition of an interferometer in China, e.g. [2361]), as, for $\sim$ a thousand events per year, the sky localizations would have error regions less than 1 deg^2 also for events at $z \sim 0.5$, and the errors on the distances will be reduced by a factor of ten, allowing targeted searches (with IFS / multi-object spectrographs) up to $z \sim 0.3$. We stress, however, that the Rubin Observatory can easily cover such sky regions in just one exposure. In this case, redshift catalogues will also be useful to discard contaminant transients from KN candidates.

Detecting EM candidate counterparts is only the first step in multi-messenger studies. Proper characterization of KN counterparts will require the largest ground-based observatories and space telescopes, such as the *James Webb Space Telescope* (JWST), which offers unprecedented sensitivity and spatial resolution. From ground, 30-meter-class telescopes as the *Extremely Large Telescope* (ELT) will be fundamental to track spectroscopically the evolution of KNe and study their role in the chemical enrichment of the universe, because in most cases 10-meter-class telescopes will lack the sensitivity for spectral studies. Given the high-demand of such kind of telescopes, it is likely that such deep studies will be performed

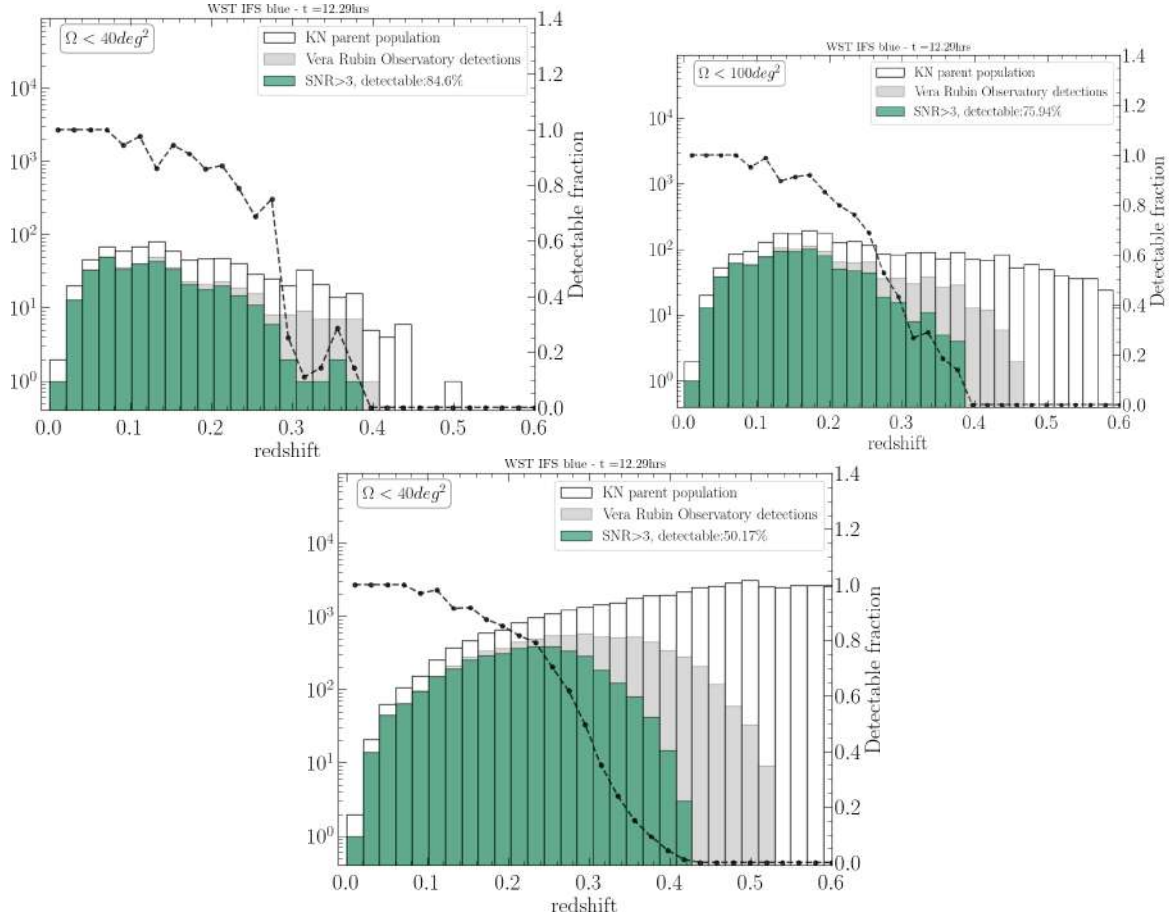


Figure 98. Example of redshift distribution of simulated KNe detected by Vera Rubin and WST, associated with BNS detected by ET in the 2L (top panels) and 2L + CE(40 km) (bottom panel) configurations, and having error regions of $< 40 \text{ deg}^2$ (top left and bottom panels), and $< 100 \text{ deg}^2$ (top right panel). The background distribution in white corresponds to the parent BNS+KN population. The green colored distribution corresponds to the KNe that are detectable with WST IFS ($\text{SNR} > 3$). KNe detectable with Vera Rubin are shown in grey. Black points refer to the y-axis scale on the right-hand side and show the fraction of detectable KN with WST with respect to those that are detectable with Vera Rubin. From Bisero et al., in preparation.

only for a limited number of detected KN. In any case, it is of the highest importance for science that multi-messenger studies are well integrated in the highest priority observations of such facilities.

5.3.3 Off-axis afterglows

Another EM counterpart of BNS and BHNS mergers whose detection will also be limited to the low-redshift regime (for viewing angles larger than $\sim 15^\circ$) is the off-axis afterglow of GRBs associated with GW events. Observing strategies for these counterparts partially align with those for KNe but, in this case, the search at lower (radio) and higher (X-rays) frequencies will play a significant role. Radio frequencies result in being very appropriate for off-axis afterglow detections. SKAO-Mid, with its unprecedented sensitivity and survey speed

(see the table in figure 100), will be the ideal facility to detect (and follow, in synergy with the ngVLA) off-axis afterglows (see section 5.2.3.3). It is of great importance that specific SKAO GW follow-up programs are established. X-rays can also be an efficient way to look for off-axis afterglows, provided that suitable facilities joining sensitivity and FoV will be operational at the ET epoch. Missions projects such as THESEUS and *NewAthena* at ESA can be key to the identification and characterization of these EM counterparts, detecting $\sim$ dozens of them per year (e.g. [1175, 2362, 2363]).

5.3.4 On-axis GRBs

As stated, while KN and off-axis afterglows will be likely too faint to be detected and studied at $z > 0.5$, ET will be able to detect BNS and BHNS mergers up to redshifts beyond the cosmic star formation peak. To exploit the entire volume probed by ET for multi-messenger studies, the detection of on-axis (or slightly off-axis, i.e., viewing angles $< 10^\circ$) GRBs and afterglows will be indispensable. Realizing this opportunity will depend on the availability of facilities capable of detecting and localizing GRBs. Missions like the current *Neil Gehrels Swift Observatory* or the SVOM satellite have the required capabilities, but ensuring that similar missions are operational beyond 2035 is critical. As of now, no approved missions are slated to fly in the late 2030s or beyond. The proposed THESEUS satellite, currently competing for selection as ESA's M7 mission, could fulfill these requirements and offers the additional advantage of onboard redshift determination for a significant fraction of GRBs.

5.3.5 ET pre-merger detection and ET and early warnings alerts

The Einstein Telescope's access to low frequencies makes it possible to observe signals, such as the inspirals of BNS systems, for a long time (hours), and thus to use the Earth rotation imprint in the GW signal itself to detect and localise BNS systems before the merger [16]. This allows early warnings to be sent to the astronomical community hours to minutes before mergers, with an increasingly precise sky-localization as the event approaches merger. Several studies have been conducted to assess the pre-merger capabilities of ET. ET is capable of localizing several tens to hundreds of BNS mergers per year with a sky localization of less than $\sim 100 \text{ deg}^2$ minutes before the merger [2364–2366]. This number increases to a hundred of BNS mergers per month for the network of ET operating with CE [2367]. Figure 99 shows the improvement in sky localization capabilities as a function of the redshift and how increasingly distant events can be localized as they get closer to the time of merger.

Detecting localized BNS pre-merger opens the possibility of following the BNS during the inspiral phase with the EM observatories and being on the source before and during the merger. Indeed, the rapid ($\sim$ minute scale) pointing capabilities of optical transient survey facilities will allow for the exploration of the very early emission associated with KNe, which is fundamental for probing the first ejecta and processes at play in KN emission. Furthermore, GW early warnings will allow the search for associated very high energy (VHE) emission by VHE observatories such as CTAO [2365] that will have suitable rapid pointing, FoV and sensitivity. As recently demonstrated by *Swift*, high-energy satellites can react to triggers by ground-based observatories on $\sim$ minute scale. Wide FoV X-ray instruments

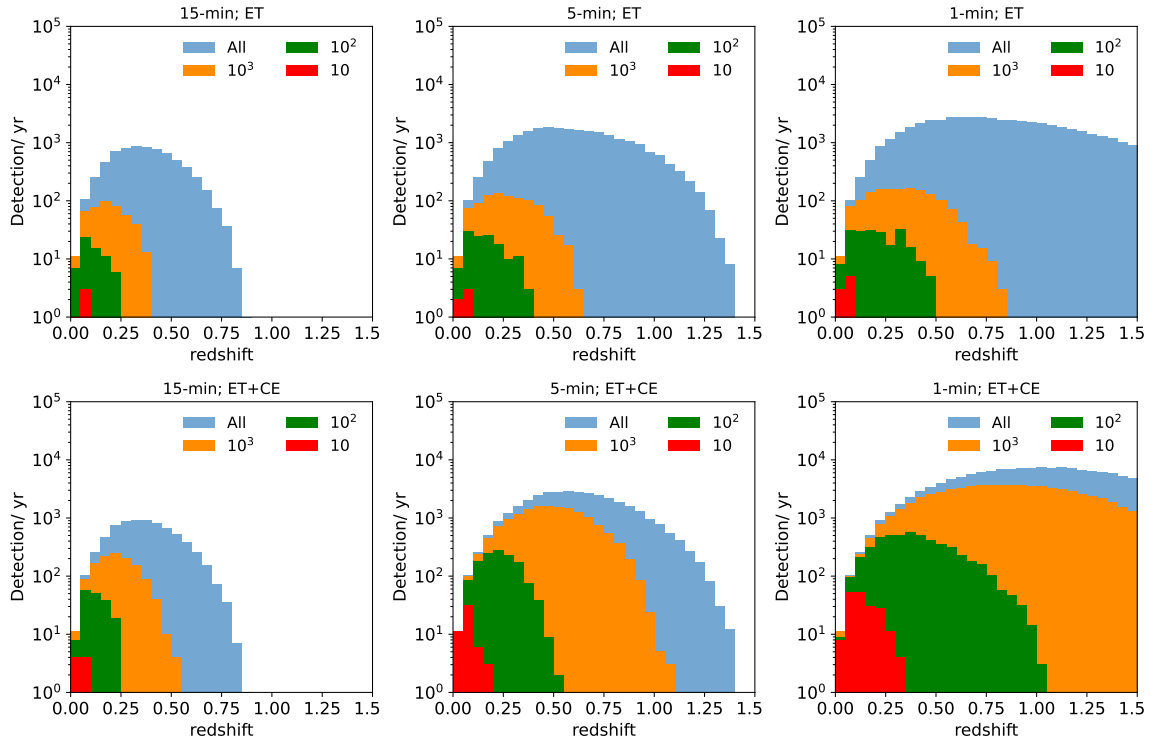


Figure 99. Redshift distribution of the sky-localization uncertainty (given as 90% credible region) for ET and ET+CE configurations. The panels show the BNS detections and the corresponding sky-localizations as a function of the redshift 15, 5, and 1 minute(s) before the merger. The blue histogram represents all detected sources, while the other colors indicate sources with sky localizations more precise than 10^3 deg^2 (orange), 10^2 deg^2 (green), and 10 deg^2 (red). Reproduced from [2365]. CC BY 4.0.

with such rapid pointing could detect the early X-ray emission of off-axis GRBs in absence of the prompt gamma-ray detection [1175].

5.3.6 ET as an alert receiver

Finally, we should not forget that ET will also operate as alert *receiver*. Due to the huge amounts of events, it will be computationally prohibitive to perform a complete parameter estimation for all the GW detections, especially in low-latency. However, EM facilities will regularly detect short GRBs (space-based missions), afterglows (space-based missions and ground-based facilities, scanning the sky at different wavelengths covering the entire EM spectrum, from VHE, as CTAO, to radio, as SKAO) or KN (especially optical and near-infrared sky surveys). ET will *listen* to these alerts from the EM community, and use them to perform an informed search and parameter estimation for the corresponding GW signal. It will be crucial to prioritize the GW analysis of these events and send in low-latency refined parameter estimations for them to inform the astronomical community and optimize the use of the observational resources.

5.3.7 Available and expected EM facilities characteristics

In figure 100 we report the results of a survey within the ET observational science board on the best EM current facilities and future projects for multi-messenger observations (column 1), as well as their corresponding characteristics, in terms of Field of View (FoV; column 2), sensitivity (column 3), spectral range (column 4), resolution (column 5), capability and time delay to perform Target of Opportunity (ToO) (column 6), survey strategy (column 7), data policy and data access (columns 8 and 9). Each facility may be of good use to look and/or to follow-up and characterize one or more type of EM counterparts (GRBs, off-axis afterglows, KNe). This is indicated from column 10 to 15. Projects that are still at the proposal stage and not yet adopted are indicated with an asterisk.

5.3.8 Requirements for ET BNS and BHNS multi-messenger observations based on simulations.

Based on the simulations presented in this work, in figures 101–103 we report the minimal requirements to detect and study the EM counterparts (on-axis GRB, off-axis afterglow, KN) of BNS and BHNS detected by ET, for EM facilities working in different ranges of the EM spectrum. The evaluation of the requirements takes into account a balanced amount of time to be required to the EM observatories. We consider the FoV, sensitivity, spectral range, resolution, and capability and time delay to perform ToOs. We separate the *Search* for the EM candidate counterparts (orange columns) from their in depth study for the *Characterization* of their properties (green columns). For on-axis GRBs, the search requirements are based on the assumption of a GRB prompt detection by a satellite with arcmin localization capability. For the VHE the requirements take into account the possibility to point pre-merger to detect the emission during the prompt/early phase. We stress that, except for the scenario of a well-localized GRB detection in temporal coincidence with a GW signal which can drive the multi-wavelength search and characterization, also in the ET era sensitive wide FoV instruments ($> 1 \text{ deg}^2$) will be necessary for the search. In addition, to increase the chance to detect off-axis afterglows also observations at late time (months to years) might be required. Except for on-axis GRBs, very sensitive instruments are required at any wavelength for the characterization.

5.4 Neutrinos from Compact Binary Coalescence

5.4.1 Common environments for neutrinos and GWs

The existence of a common astrophysical source of neutrinos and GWs is theoretically well-motivated. One such example is compact binary mergers. The most promising ones are binary systems with at least one neutron star component, such as binary neutron stars (BNS), and neutron star-black hole (NSBH) systems [2368–2370]. Otherwise, binary black holes (BBH) embedded in Active Galactic Nuclei (AGN) disks could also have a favourable environment for neutrino production [2371, 2372].

Based on theoretical predictions, following the merger, a central compact object is formed. Consequently, powerful relativistic jets efficiently eject the remaining accretion disc matter into the surrounding environment after the merging phase. There, non-thermal charged

	FOV (deg. diam)	Sensitivity	Spectral range	Resolution (spectral / spatial)	ToO? delays?	Survey strate	Data policy	Data access	search/characterization	On-axis GRB/afterglow	off-axis GRB/afterglow	KN
SKA-MID	1 degree ² (1.4 GHz/observing freq)	1.4 uJy in one hour	0.35-15.4 GHz	Channel width 13-80 kHz up to 200 Hz 'zoom' Angular: 0.4 arcsec ² (1.4 GHz/frequency)	Transient buffers 288 sec / ToO or DDT likely	not yet clear	not yet published, but imminent	X	X	X	X	
SKA-LOW	2 degrees ² (300 MHz/observing freq)	14-26 uJy in one hour	0.05-0.35 GHz	Channel width 5 kHz, up to 220 Hz 'zoom' Angular: 6-300 arcsec	Likely transient buffers / Likely ToO or DDT policy	not yet clear	not yet published, but imminent			X	X	
ngVLA	0.6 - 25 arcmin (from top to bottom of spectral range)	0.15-40 uJy/beam in one hour	1.2-50.5 GHz and 70-116 GHz	Narrowest channel < 1 kHz; Angular: 1 milliarcsec - 1 arcsec; LBA to 0.1 milliarcsec	Likely ToO and/or DDT policy	TBD	not yet published	NRAO archive	X	X	X	
	19° at 300 GHz for a 12-m antenna and a 337 for a 7-m antenna for single pointings	Continuum sensitivity in one hour: 2 uJy/beam and 0.3 mJy/beam at 40 and 550 GHz, respectively.	35-950 GHz	Spectral: 13.5 kHz resolution over full receiver bandwidth (velocity resolution of 0.12 (35 GHz)/km s ⁻¹ , where v is the observed frequency in GHz). Ultra-high resolution of 10 m/s with zoom mode at all frequencies. / Spatial:	Y, reaction time can be less than 24 hours but depends on several factors	Proprietary	Public archive	X	X	X	X	
ALMA 2030 (WSU)		Continuum (over delta/nu=18 GHz bandwidth) = 180 microJy/beam in 60seconds at 300 GHz observing frequency (assuming 25th percentile weather conditions and 60 deg elevation), beam is 4.3" at this frequency.	30 - 950 GHz (over approximately 8 broad windows where the Earth's atmosphere is transparent)	Spatial: under diffraction-limited assumption, FWHM arcsec = 1300/(obs_frequency(GHz) or FWHM arcsec = lambda/(microns*230). Spectral: depends on instrument, from resolutions of R 10 ³ (heterodyne) to R 300 for (sub-mm-IFUs, and to broadband (~40 GHz per band) continuum multi-choic bolometerKID arrays.	ToO yes	Proprietary for 1 year	Public archive	X	X	X	X	
ALIST	2 degrees (circular, 2 instrument slots with this FoV) or 1 degrees (4 instrument slots with this FoV). Note the higher frequencies (>400 GHz) will be closer to < 1 degree with the optical corrections.	q(white) = 23 is 300s (dark)	350-1000nm	arcsec								
BLACKGEM	8.1 deg ² (3 x 2.7 deg)	q(white) = 23 is 300s (dark)	350-1000nm	arcsec								
GOTO	2 x 100 deg ² (North/South)	q(white) = 23 is 300s (dark)	350-1000nm	arcsec								
Rubin Observatory	1.75 degrees (0.6 sq. deg. effective FoV)	~24.0 mag in 30s	upgrt filters (~3,500-10,000A)	arcsec	ToO yes	TBD	public alerts, images available after 80 hours to member countries	via Alert Brokers, Rubin Science Platform https://data.rubin-observatory.org/	X	X	X	X
WFST	~6 sq. deg. effective FoV	~22.84 mag in 30s	upgrt filters (~3,500-9,000A)	=	ToO yes	Deep High-contrast Survey and Wide-field Survey	Proprietary	private pipeline	X	X	X	X
ELT/HARMONI	Four different FoVs: 0.61"x0.82", 1.52"x0.04", 3.04"x4.08", 4.56"x6.12"	Shr on-source integration, SNR=5: R_AB=23 (4mas/pix scale), 25 (30mas/pix); H_AB=26.3 (4mas/pix, R=3100), 24.3 (4mas/pix, R=17000), 26.8 (30mas/pix, R=3100), 25.2 (30mas/pix, R=17000); K_AB=25.7 (4mas/pix, R=3100), 24.4 (4mas/pix, R=17000), 25 (30mas/pix, R=3100), 24.4 (4mas/pix, R=17000).	(0.47) 0.8 - 2.45 um		TBD	N/A	TBD	ESO archive	X	X	X	X
ELT/MICADO-Ima	LR: 50 arcsec (0.014 deg diameter), HR: 15 arcsec (0.0053 diameter)	5 hr, 5 sigma limiting point source AB mag: J=28.6, H=29.5, K=29.1	I,Y,J,H,K filters	lambda/(microns)*230.	N/A	TBD	ESO archive	X	X	X	X	
ELT/MICADO-Spec	Slit (width x length): 16masx3", 20masx15", 48masx3"	similar to imaging mode	1.4-2.46 um or 0.96-1.65 um	Spectral: ~10000 or ~20000	TBD	N/A	TBD	ESO archive	X	X	X	X
ELT/MOSAC	Patrol field of 40 arcmin ² at 3 observing modes: - MOS-VIS: 140 VIS apertures of 0.7" at LR, 85 apertures at HR - MOS-NIR: 200 NIR apertures of 0.6" at both LR and HR - NIR-nRIF: 8 hexagonal NIR IFUs of 2.2arcsec diameter. Possible parallel observations mixing VIS and NIR apertures/IFUs in the patrol field	Performance currently in revision/estimation following a recent evolution of the architecture. Examples of objectives: - point sources at LR with MOS modes: I(AB)=24.5 with S/N >20, V(AB)=22 with S/N>=50, H(AB)=25.4 with S/N>=20 - extended sources at LR with MOS modes: R(AB)=23.5 with S/N >20A - extended sources at LR with IFUs: J(AB)=26-27 with S/N>=3 per spatial in emission lines	VIS: 380-950 nm in three bands obtained in one shot NIR: 550-1600 nm in two bands obtained in one shot	VIS-LR at R=4,000 NIR-LR at R=4,000 4 VIS-HR windows at R=18,000: 390-450, 510-590, 610-710, 800-940 (all TBC) 1 NIR-HR window at R=18,000: 1520-1070 (TBC) For the NIR-IFUs: spaxel scale = 150mas	TBD	TBD	TBD	ESO archive	X	X	X	X
ELT/ANDES seeing-limited	1 square-arcsec	AB>20 in 1 hr SNR=10	400-1800 nm	R=100,000	yes	N/A	ibid	ESO archives	X	X		
ELT/ANDES IFU	0.5 square-arcsec	AO requires m(AB)=6-15mag	980-1800 nm	R=100,000	yes	N/A	ibid	ESO archives				
ELT/MUSE-GLAO or seeing limited	1 arcmin (0.017 deg diameter)	VIAB=22, 1hr, SNR=5 or (B550nm)	465,000 - 930,000 nm or 480,000 - 930,000 nm	Spatial: 250 mas per pixel, Spectral: 3000-3500	yes	N/A	0 to 12 months proprietary time	ESO archives	X	X	X	X
ELT/MUSE-LTAO	7.5 arcsec (0.0021deg diameter)	VIAB=21.5, 1hr, SNR=5 (B550nm)	480,000 - 930,000 nm	Spatial: 25 mas per pixel, Spectral: 3000-3500	yes	N/A	0 to 12 months proprietary time	ESO archives	X	X	X	X
VLTX-a shooter	Slit length=13", several slit widths (e.g. 0.5", 0.8", 1.0", 1.3", 1.6", 5.0") depending on the arm	m_AB=22.4 @500nm, m_AB=21 @800nm, m_AB=20.6 @1500nm, m_AB=19 @2000nm in 1hr and with SNR=10	300-2500 nm split in 3 arms	Spectral resolution depends upon the selected slit, and the considered arm wavelength. Generally, it ranges from min 4300 to max 19400	yes	N/A	0 to 12 months proprietary time	ESO archives	X	X	X	
WST*	3 deg ² 2 MOS (3x3 arcmin IFU)	m_AB=24.5 MOS, m_AB=25.2 MOS in 1hr and with SNR=10	350-970nm to be extended to 1600nm	LR mode R=40000, 20000 fibers; HR mode R=40000, 2000 fibers	yes, ~hours	TBD	ibid	ibid	X	X		X
MSE*	1.5 deg ² 2	m_AB=24, 1hr	360-950 nm, 360-1800nm	3000 (NIR extension), 6500, with 20000 fibers; HR mode R=20000	foreseen, ?	TBD	ibid	ibid				
HST	variable by instrument (up to ~10 sq. arcmin)	q(F606w) ~ 29(AB)	100-1600 nm imaging and spectroscopy	0.1" resolution in imaging (wavelength dependent), low resolution prism (R=100) in optical/IR. Higher resolution available via COS and STIS (up to R=1600 / << 1 arcsec	Non-disruptive ToO: >21 days, Disruptive ToO: 3-21 days, ultra-disruptive <3 days							
JWST/NIRCam	9.7 arcmin ²	m_AB=22 (grism), m_AB=29-31 depending on the configuration; S/N=10 in 1hr	2400-5000nm (grism), 600-5000nm covered by different filters	R=100 / << 1 arcsec	Non-disruptive ToO: > 14 days, Disruptive ToO: between 3 and 14; Ultra-disruptive ToO: < 3	N/A	0 to 12 months proprietary time	archive	X	X	X	X
JWST/NIRSpec	3"x3" in IFU mode	m_AB=23 (grism), m_AB=22-24 depending on the configuration; S/N=10 in 1hr	400-5000nm (grism), 700-5000 divided in 3 bands	R=100,1000, 2700 depending on the obs. mode	Non-disruptive ToO: > 14 days, Disruptive ToO: between 3 and 14; Ultra-disruptive ToO: < 3	N/A	0 to 12 months proprietary time	archive	X	X	X	X
JWST/MIRI	112.6"x73.5"	m_AB=23 (all @8000nm), m_AB=24 (imaging @13000nm); S/N=10 in 1hr	5000-12000nm (all); 5600-28000nm (imaging - covered by different filters)	R=100 / < 1 arcsec	Non-disruptive ToO: > 14 days, Disruptive ToO: between 3 and 14; Ultra-disruptive ToO: < 3	N/A	0 to 12 months proprietary time	archive	X	X	X	X
JWST/NIRISS	133" x 133"	m_AB=25, m_AB=28 (imaging covered by different filters)	600-2200nm, 800-5000nm (imaging covered by different filters)	R=150 / << 1 arcsec	Non-disruptive ToO: > 14 days, Disruptive ToO: between 3 and 14; Ultra-disruptive ToO: < 3	N/A	0 to 12 months proprietary time	archive	X	X	X	X
ROMAN	0.281 sq. arcmin	m_AB=28 depending on the configuration	500-~2000nm	R=461 R=80-180 depending on the filter	TDMM program	TBD	0 to 12 months	archive	X	X	X	X
SVOM/GRM	~5.6 sr combined, 3 modules of 2.6 sr	0.22 ph/cm2/s	15 keV-5 MeV	<16% @60 keV	Y, from <5 hours to 48 hours depending on urgency		Data are proprietary, scientific products public	X	X			
SVOMECLAIRS	2 sr	1.6 ph/cm2/s	4-120 keV	1.3 keV @60 keV, 12 arcmin (loc. accuracy)	Y, from <5 hours to 48 hours depending on urgency		Data are proprietary, scientific products public		X	X	X	
SVOMVT	26"x26"	22.5 in 300 seconds	450-1000nm	~ arcsec	Y, from <5 hours to 48 hours depending on urgency		Data are proprietary, scientific products public	X	X			
SVOMMKT	~1 deg ²	~10 ⁻¹² erg/cm2/sec	0.2-10 keV	75 eV ~ tens of arcsec	Y, from <5 hours to 48 hours depending on urgency		Data are proprietary, scientific products public	X	X	X	X	
SwiftBAT	1.4-1.6 steradians	0.3 ph/cm2/s **	15-150 keV	7 keV, 20 arcmin	yes, as soon as schedulable	Public	Public archive	X	X	X	X	
SwiftXRT	15'	8 x 10 ⁻¹⁴ erg cm-2s-1 in 104"	0.3-10 keV	190 eV (10keV), 18 arcsec	yes, as soon as schedulable	Public	Public archive	X	X	X	X	
Einstein Probe	WXT: ~1-1 steradians; FXT: 60 arcmin (diam)	WXT: 2.5x10 ⁻¹¹ erg/cm2/s in 10 ³ s FXT: ~10 ⁻¹³ erg/cm2/s in 1500 s	WXT: 0.5-4.0 keV FXT: 5-15 arcsec (spatial)	WXT: ~3 arcsec (spatial) FXT: 5-15 arcsec (spatial)	yes, as soon as schedulable	iling, scan region	Proprietary	X	X	X		
AXIS*	24 arcmin (diameter)	1.5x10 ⁻¹⁵ erg cm-2/s in 1hr	0.3-10 keV	< 2 arcsec	yes, ~2 hours	TBD	TBD	TBD	X	X	X	
NewATHENA*	40x40 arcmin ² (WFU); 4 arcmin (diameter, IFU)	1e-16 erg/cm2/s (WFU) in 10 ⁴ s	0.2-12 keV	4 eV @ 7 keV / 5'	iling, scan region	TBD	TBD	TBD	X	X	X	
THEUESUSXSP*	0.5 steradian	1e-10 erg/cm2/s (100 sec)	0.3-5 keV	<2 arcmin	Y, 0-15 min	TBD	Public archive	X	X	X	X	
THEUESUSXSPS	2 steradian	1e-8 erg/cm2/s	30keV-2MeV	15 arcmin	X	TBD	Public archive	X	X	X	X	
THEUESUSRPT*	15x15 arcmin	210) to 20.80(H) (150s, SNR=5)	0.7-1.8 mu	R=400	Y, 0-15 min	TBD	Public archive	X	X	X	X	X
FERMI/GBM	~8 steradian	2 ph/cm2/s	8keV-40MeV	1-100 keV (0.01-10MeV), 10 deg		Public	Public archive	X	X	X	X	
FERMILAT	2.5 sr	1e-6 erg/cm2/s	20 MeV - 300 GeV	3.5-0.15 deg (100 MeV-10GeV)		Public	Public archive	X	X	X	X	
HERMES Pathfinder	5.2 sr	5 ph/cm2/s	3keV-2MeV	No		TBD	Public archive	X	X	X	X	
HERMES Confs*	All sky	1 ph/cm2/s	3keV-2MeV	No		TBD	Public archive	X	X	X	X	
CTAO	3.5-11	1e-11 erg/cm2/s @100 GeV, 50hrs, 5e-11 erg/cm2/s, MAGIC.	20 GeV - 300 TeV	5-15%	20-40 s	iling, scan region	TBD	Public interface X (gammasymp)				
LST	3.5	1e-12 erg/cm2/s @100 GeV, 50hrs, 5e-11 erg/cm2/s, MAGIC.	20 - 300 GeV	15-10%	20s	iling, scan region	Proprietary	private pipeline	x			
ASTRI-MA	10	1e-10 erg/cm2/s @1TeV 100s	1-300 TeV	<10% >1 TeV, 0.05 deg	40s	iling,	Proprietary	private pipeline	x			

Figure 100. Characteristics of most advanced EM facilities expected to be available by the late 2030s. Also available as an Excel table at <https://apps.et-gw.eu/tds/ql/?c=17760>. Asterisk indicates facilities and missions that are still in the project phase and have not yet been approved or adopted.

ON-AXIS GRB and AFTERGLOW										
SEARCH						CHARACTERIZATION				
	FoV	Sensitivity	Spectral range (nm)	Spectral / Spatial Resolution	ToO delay	FoV	Sensitivity	Spectral range (nm)	Spectral / Spatial Resolution	ToO delay
Radio		0.1 mJy	> 1 GHz	-- / ~ 1"	hours	arcsec ²	0.01 mJy	> 1 GHz	--/~arcsec	monitoring Log scale
NIR	~ few arcmin ²	21 mag	1000 - 1800 nm	-- / ~ 1"	minutes	arcsec ²	24 mag (phot.) 22 mag (spectr.)	1000-1800 nm	low-mid / 1"	< 12 hr
Optical	~ few arcmin ²	22 mag	500 - 900 nm	-- / ~ 1"	minutes	arcsec ²	26 mag (phot.) 22 mag (spectr.)	300-900 nm		
X-ray	~ tens of arcmin ²	10 ⁻¹³ erg cm ⁻² s ⁻¹	0.2-10 keV	a few arcsec	< hours	~ tens of arcmin ²	10 ⁻¹⁶ erg cm ⁻² s ⁻¹	0.2-10 keV	a few arcsec	< hours
γ-ray	~10 sr	~1 ph cm ⁻² s ⁻¹	10 keV - 10 MeV	arcmin						
VHE	>1 deg ²	< 10 ⁻¹⁰ erg cm ⁻² s ⁻¹	10 GeV - 30 TeV	DE/E ~20% 0.1-1 deg	< hours	1 deg ²	< 10 ⁻¹¹ erg cm ⁻² s ⁻¹ @100 GeV	10 GeV - 30 TeV	DE/E ~20% 0.1 deg	< hours

Figure 101. Minimal requirements for detecting and characterizing EM counterparts of BNS and BHNS mergers detected by ET, considering on-axis GRB and afterglow emission and EM facilities operating across different wavelength ranges. The gamma range in the table extends to the X-ray band considering the characteristics of the gamma-rays instruments for GRB detection.

OFF-AXIS AFTERGLOW										
SEARCH						CHARACTERIZATION				
	FoV	Sensitivity	Spectral range (nm)	Spectral / Spatial Resolution	ToO delay	FoV	Sensitivity	Spectral range (nm)	Spectral / Spatial Resolution	ToO delay
Radio	≈ 1 deg ²	< 10 ⁻² mJy	> 1 GHz	--/~arcsec	Few days to years!	Some arcsec ²	< 10 ⁻³ mJy	> 1 GHz	--/~arcsec	logarithmic monitoring for days, months, years
NIR	≈ 1 deg ²	> 24 mag (AB) < 10 ⁻³ mJy	> 1000 nm	--/~arcsec	Few days to years!	~ arcsec ²	> 24 mag (AB) < 3x10 ⁻⁴ mJy	1000-1800 nm	low-mid / ~1"	ToO at first then logarithmic monitoring
Optical	≈ 1 deg ²	> 26 mag (AB) < 10 ⁻⁴ mJy	500 - 900 nm	--/~arcsec	Few days to years!	~ arcsec ²	> 27 mag (AB) < 5x10 ⁻⁵ mJy	500 - 900 nm	low-mid / ~1"	ToO at first then logarithmic monitoring
X-ray	≈ 1 deg ²	< 10 ⁻¹³ erg cm ⁻² s ⁻¹ < 10 ⁻⁷ mJy	0.2-10 keV	--/~few arcsec	Few days to years!	~ arcsec ²	< 10 ⁻¹⁷ erg cm ⁻² s ⁻¹ < 10 ⁻⁸ mJy	0.2-10 keV	--/~few arcsec	logarithmic monitoring for days, months, years
γ-ray	~10 sr	0.01-0.1 ph cm ⁻² s ⁻¹	3 keV - 10 MeV	--/arcmin						

Figure 102. Minimal requirements for detecting and characterizing EM counterparts of BNS and BHNS mergers detected by ET, considering off-axis afterglow emission and EM facilities operating across different wavelength ranges. In this case, we consider a field of view equal to 1, deg² (for radio, NIR, optical, and X-ray), in contrast to figure 103, where a larger field of view is required. This choice accounts for the possibility of detecting gamma-ray or X-ray emission from slightly off-axis systems, which could enable source localization. The gamma range in the table extends to the X-ray band considering the characteristics of the gamma-rays instruments for GRB detection.

particles are accelerated in the presence of strong magnetic field. The accelerated protons in and around the jet actively participate in hadronic interactions. They interact with the strong ambient photon fields, leading to photo-pion interactions. As one of the secondary particles, high-energy neutrinos (HENs) are produced via charged pion decay. It could happen during the prompt phase [2373], and also later, coincident with the longer afterglow radiation [2374]. It is worth mentioning that HENs can also be produced via hadronuclear (pp) interactions. Such processes also result in the production of charged pions and kaons, which subsequently decay into charged leptons and neutrinos. Furthermore, neutral pions,

KILONOVA											
SEARCH						CHARACTERIZATION					
	FoV	Sensitivity (AB)	Spectral range (nm)	Spectral / Spatial Resolution	ToO delay	FoV	Sensitivity (AB)	Spectral range (nm)	Spectral Res.	ToO delay	
NIR	>1 deg ²	>22 mag	1000-1800	~1000 / ~ 1"	1 day	~arcsec ²	>24 mag	1000-1800	>1000	1 day	Ideally nightly monitoring of the evolution for ~1week. However at such late time most counterparts will reach extremely faint magnitudes (m~30 AB)
Optical	>1 deg ²	>22 mag	500-900	~1000 / ~ 1"	12 hr	~arcsec ²	>24 mag	500-900	>1000	12 hr	Ideally nightly monitoring of the evolution for ~1week. However at such late time most counterparts will reach extremely faint magnitudes (m~30 AB)
UV	>1 deg ²	>22 mag	300-500	~1000 / ~ 1"	< 1 hr	~arcsec ²	>24 mag	300-500	>1000	< 1 hr	Ideally nightly monitoring of the evolution for ~2-3 days.

Figure 103. Minimal requirements for detecting and characterizing EM counterparts of BNS and BHNS mergers detected by ET, considering KN emission and EM facilities operating across different wavelength ranges.

produced from the same primary hadrons, decay into high-energy photons. These photons, if within the observable range from Earth, strengthen the connection between high-energy neutrinos and gravitational waves, reinforcing the multi-messenger approach.

5.4.2 Current and future neutrino telescopes

In view of promising theoretical predictions, since the last decade, multiple neutrino detectors have been looking for the correlation between the detection of gravitational waves and any possible detection of astrophysical neutrinos. At the time of writing, three major installations, IceCube, KM3NeT, and GVD-Baikal, are active and taking data although at different stages of realisation. All of them come with a target instrumented volume of the size of 1 km³.

The IceCube Neutrino Observatory [2375, 2376] completed its installation in 2013. It is a cubic kilometer ice-Cherenkov detector located under the Antarctic ice in the geographic South Pole. Motivated by its rich science output, further enhancements offering better sensitivity across a vast energy range are envisioned. The sensitivity of IceCube in the sub-TeV range will improve with the installation of IceCube Upgrade [2377, 2378], i.e. seven new densely packed strings, planned for 2025–26. Another extension of the existing IceCube infrastructure in planning is IceCube-Gen2 [2379]. It will be an 8 km³ and multi-component detector with better sensitivity to HENs with energies above 1 PeV.

Although already operational, KM3NeT and GVD-Baikal are still in construction stages in the abyssal site of the Mediterranean Sea and at the bottom of Baikal Lake in Siberia, respectively. In the Northern Hemisphere, the KM3NeT neutrino telescope [2380] is composed of two undersea detectors, namely ORCA and ARCA. KM3NeT/ARCA, with a target instrumented volume of 1 km³, is optimised for the detection of astrophysical neutrinos from 10 TeV up to hundreds of PeV. KM3NeT/ORCA is designed to study the physical properties of neutrinos, such as the mass hierarchy and the oscillation parameters, by detecting the atmospheric neutrinos in the sub-GeV up to the TeV energy range. Early in 2025, the KM3NeT Collaboration published the discovery of a record-breaking neutrino detected by

KM3NeT/ARCA with an estimated energy of about 220 PeV [2381], demonstrating the excellent capabilities of underwater neutrino telescopes.

Baikal-GVD [2382] is located at a depth of ~ 1400 m in the Baikal Lake. Promising results of the measurement of the astrophysical neutrino diffuse flux above 100 TeV were recently published [2383].

A group of next-generation TeV-PeV neutrino telescopes are under design. In some cases, the prototypes of the new technologies are already being tested. They are concentrated in the Northern Hemisphere in different sites around the World. P-ONE [2384], currently at a prototypal stage, aims at the realisation of a 1 km^3 array of multi-PMT optical modules off Canada in the Pacific Ocean. The Chinese project TRIDENT [2385] is studying the feasibility of a neutrino telescope in the South China Sea, aiming at an instrumented volume of 8 km^3 . In China, other ambitious projects are under discussion (e.g. NEON [2386] or HUNT [2387]) to realise very large telescopes of 10 km^3 or 30 km^3 .

Binary neutron star mergers are potential sources of ultrahigh-energy neutrinos (above 1 PeV), and next-generation radio detectors such as GRAND [2388], IceCube-Gen2 Radio array, and RNO-G [2389] are designed to detect these elusive particles.

5.4.3 Multi-messenger neutrino and GW frameworks

Despite all the decade-long observational efforts, coincident detection of HEN and GWs is still missing [2293, 2390–2393]. To date only upper limits on the HEN content of GRBs [2394] and observed GW sources have been obtained, owing, most likely to the fact that the current neutrino experiments are not sensitive enough to the neutrino flux of individual sources.

At present, both IceCube and KM3NeT are following up on the existing individual GW events to identify their neutrino counterparts. The current GW follow-up strategy consists of both real-time and archival searches for individual GW events, using different event topologies, ranging from MeV to PeV energies [2395–2399]. Sub-threshold GW events from the Ligo-Virgo-KAGRA third observation run (O3) are also separately followed up by IceCube in an archival search to identify potentially interesting multi-messenger sources [2400]. Similar studies would play a crucial role in the era of next-generation GW detectors. For GeV-TeV neutrinos, the typical observation time window is selected to be ± 500 s concerning the GW event detection time. We expect the neutrino signal from the source to dominate over the background within this time window [2401, 2402]. However, in IceCube, for BNS mergers, an extended time window of two weeks is also considered, to account for certain model predictions [2403]. Additionally, real-time analysis to identify MeV neutrino bursts with IceCube within four different time windows: 0.5 s, 1.5 s, 4 s, and 10 s associated with individual GW events is under development [2395]. On the other hand, in KM3NeT, an additional $[-500 \text{ s}, +6 \text{ h}]$ time window is considered for real-time searches of GeV-TeV neutrinos, while a 2 s window is used for MeV neutrinos. So far, no significant correlation has been observed. An alternative archival search strategy could also be explored in the future, aiming to identify neutrino counterparts from a population of GW candidates via stacking analysis. This approach enhances the detection potential by accumulating signal neutrinos within an optimal time window, allowing them to stand out against the background neutrino distribution.

Such observational prospects will surely increase with next-generation GW like Einstein Telescope (ET) and HEN detectors, specifically with KM3NeT, and Baikal-GVD. Once they reach their nominal size and with the active participation of P-ONE and IceCube-Gen2, the combined performance of all these advanced neutrino detectors can offer maximum sensitivity to detect neutrino sources all over the sky, which would benefit GW follow-up efforts [2404]. According to the current plan, the Einstein Telescope [15, 16] and the US-lead Cosmic Explorer (CE) [13, 14] will be able to detect BNS mergers out to $O(10)$ Gpc. During their active operation time, IceCube and KM3NeT will continue actively participating in the GW follow-up efforts. As neutrinos do not get absorbed or deflected while propagating toward us from the respective astrophysical sources, the prospect of coincident detections from the furthest of sources in the future remains a key objective. With the expected boosts in the sensitivities of future neutrino detectors, extending the current searches for coincident detection of HEN and GWs for next-generation detectors is crucial for the future of multi-messenger astronomy. Ref. [2405] investigates the prospects of performing triggered-stacking searches for coincident high-energy neutrinos from BNS mergers, using next-generation GW detectors in synergy with IceCube-Gen2. The study quantifies optimal distance thresholds and time windows to minimize background, and highlights that precise sky localization, achieved by the combined network of ET and CE, is essential for enabling either neutrino detections or placing meaningful constraints on emission models within approximately 10 years. A fraction of BNS mergers is expected to produce long-lived magnetars as remnants, which, according to some models, can lead to enhanced electromagnetic and neutrino emission. Ref. [2406] shows that GW-triggered stacking searches, using the ET and CE network together with IceCube-Gen2, offer a realistic chance of detecting neutrinos from these events within an operational timescale of about 20 years. Considering ultrahigh-energy neutrinos, instruments such as GRAND and IceCube Gen2 Radio, using triggers by the ET+CE network, have a significant probability of detecting at least one coincident GW/neutrino within 15 years [2407].

While such a joint detection can be very informative about the physical processes happening in extreme conditions, it will also improve the sky localisation of the event by orders of magnitude compared to the localisation with GWs only. Considering the huge data we will receive daily with ET, identifying neutrino counterparts for GW events before triggering EM follow-up searches will allow us to improve GW event localisation by identifying joint best-fit GW + ν source location. This will enable us to schedule EM follow-up search initiatives more efficiently, using telescopes with narrow fields of view. All these prospects motivate us to question further the probability of finding a common source of GW and HEN in the era of ET.

5.5 Multi-messenger infrastructure challenges

The large number of alerts generated by ET during both the inspiral phase and the merger of compact objects in binary systems poses significant computational, organizational and logistical challenges. There is a need for dynamic databases capable of efficiently managing a high number of candidate events and the flow of updated information for each of them to send to the astronomical community. These systems must also be able to automatically prioritize events to be characterized with precise parameter estimation. Additionally, ET will

need to operate as an EM trigger listener, requiring infrastructure also capable of receiving and processing a large number of external alerts detected by the EM observatories, in order to search for potential temporal and spatial coincidences with GW signals.

The search for gravitational wave signals will not rely solely on the match filtering technique but will also incorporate innovative methodologies aimed also at identifying new classes of signals (see section 8). In this context, advanced multi-messenger analyses that simultaneously integrate spatial and temporal information will be extremely valuable to optimize the identification, characterization, and interpretation of gravitational wave signals.

Multi-messenger astronomy presents critical challenges in terms of interoperability and the application of FAIR (Findable, Accessible, Interoperable, and Reusable) principles. The rapid exchange of information is enabled by the development and adoption of shared standards, facilitating seamless integration and data sharing across heterogeneous research infrastructures. The standardization of processes will be fundamental to allowing scientists to efficiently work with more and more complex datasets from diverse observational sources, including gravitational waves, neutrinos, and traditional electromagnetic signals.

Founded in 2002, the International Virtual Observatory Alliance (IVOA) is a freely accessible consortium that brings together scientific communities, research institutions, and software developers. It functions as an active community and a collaborative forum where members define and discuss protocols for the management, exchange, and analysis of astronomical data. The IVOA protocols currently used by the scientific community for transient events provide a valuable basis for advancing the handling of data and computational demands on a much larger scale. LSST, with its live stream of millions of transients detected every night, will define the protocols and standards for decades to come. A key element of the ET integration in mainstream of multi-messenger astronomy will be the implementation and adoption of open and shared standards, essential for ensuring interoperability across different observational platforms and facilitating data integration and analysis. The use of common protocols will help overcome technological barriers, optimizing the exchange of information among the various components of the multi-messenger network and enabling an effective response to fundamental scientific questions that only a truly integrated approach can address. The ET GW community is collaborating with the EM communities to include the peculiarities of gravitational transients in the upcoming standards and infrastructures.

This section of the paper shows how the realization of ET's full multi-messenger potential will depend on the coordinated development of electromagnetic observational capabilities. Here, we highlight that to maximize scientific return of next-generations multi-messenger observatories, it will also be critical to develop infrastructures, protocol and standards in close coordination.

5.6 Executive Summary

Einstein Telescope will, each year, identify thousands of gravitational wave signals from sources that also emit electromagnetic light. This rich stream of data will provide the opportunity to build the first substantial samples of multi-messenger events. In turn, these objects will enable transformative science across many of the core areas of ET science.

Discovery of sources with multiple messengers

- ET-alone is capable of localizing a few hundreds of BNS and NSBH mergers with a localization of tens to hundred of square degrees in size. Although these regions may contain thousands of galaxies, they are sufficiently informative for the electromagnetic observatories to discover an electromagnetic counterpart able to pinpoint the GW source on the sky, identify host galaxies and provide measurements of the distance (redshift) to the source.
- There are many plausible electromagnetic signals that a merging compact object binary may create during or after the merger, including high-energy transients such as **gamma-ray bursts (GRBs)**, **multi-wavelength afterglows** seen from the TeV range through X-ray/Optical/NIR to the radio band, and radioactively powered **kilonovae** emitting dominantly in the optical to mid-infrared. In addition to these EM signals which are dealt with in this chapter, ET may also open new science cases with the joint association of GW to high energy neutrino.
- The identification of these sources in the large gravitational wave sky-localizations is not straightforward, and in this chapter we utilize two state of the art approaches to provide self-consistent estimates of event rates and detectability to the current and next generation of electromagnetic observatories for two proposed configurations of the Einstein Telescope (triangle and 2L) and for ET in a network of 3G detectors. These simulations show that, while ET may identify $> 10,000$ compact object mergers that create electromagnetic light, the majority will remain undetectable to near-term electromagnetic facilities. However, tens to hundreds of events per year will be accessible, providing the first opportunity to build substantial **samples of object detected both in gravitational waves and electromagnetic light**.

Enabling new science

Knowledge of the precise source locations offers a route to addressing a wealth of astrophysical questions, many of which are described in other chapters of this book. In particular, these observations will enable ET data, in concert with electromagnetic observations to:

- Determine the contribution of compact object mergers to **heavy element enrichment** via studies of their kilonovae.
- Study the launching, collimation and emission processes of **relativistic jets** from the post-merger remnant.
- Use the properties of the associated electromagnetic signals to probe the **equation of state of matter at nuclear densities** (see section 7).

- Combine electromagnetically determined redshifts with gravitational wave measured distances to measure key **cosmological parameters** (see section 3).
- Use source locations and the demographics of the galaxies which host them to inform the **evolution of the stellar populations** that create compact object binaries (see section 4).

A new paradigm for multi-messenger astronomy

Focus in the current era has been on identifying multi-messenger signals in the relatively nearby Universe. In contrast, the cosmological reach of ET will identify them across cosmic time. This dramatically changes the most likely electromagnetic counterparts and the associated search techniques.

- Our simulations suggest that the best route to growing the multi-messenger samples in the ET era will arise via the detection of coincident gamma-ray burst (or associated high-energy transient) signals. **Realising the scientific return of ET will critically rely on the availability of high-energy instrumentation with good localisation capabilities.**
- **Future observational facilities** will play a crucial role in identifying and characterizing EM counterparts. Photometric (e.g. Vera Rubin) and spectroscopic (e.g. WST) instruments will be pivotal for detecting KNe up to $z \sim 0.5$ while large ground and space based telescopes (e.g. ELT, JWST) will be essential for spectroscopic characterization.
- **Radio and X-ray** follow-up will be critical for detecting off-axis afterglows with facilities such as SKA, ngVLA, NewAthena and THESEUS. GW with on-axis GRB counterparts, detected up $z > 2$, will provide valuable insights into the cosmic star formation history of double compact systems.
- The capability of ET to provide several minutes of advanced warning prior to a given merger raises the prospect of sensitive (but still wide field) electromagnetic observations observing the merger in real-time. This opens new EM observational windows for studying **precursor emission, kilonova shock physics and VHE counterparts**. The latter could be efficiently detected by CTAO and other next-generation gamma-ray observatories.
- The identification of EM-signals (e.g. gamma-ray bursts) may be crucial as a route of identifying the GW signals of events of particular interest within the ET data flow.
- **High-energy neutrino observatories** such as IceCube-Gen2, KM3NeT, and Baikal-GVD will play a key role in the multi-messenger network, enhancing the probability of detecting coincident GW–neutrino–EM events.

6 Synergies of ET with other gravitational-wave observatories

This section aims to explore the scientific synergies that arise from combining gravitational waves detected by ET with those from other GW observatories. Synergies with other ground based detectors operating between a few Hz and a few kHz, such as the Cosmic Explorer, are crucial for consolidating the science case of ET [15, 16, 1699, 1714]. Synergies with GW observatories at frequencies that are not accessible from Earth-based detectors ($\lesssim$ Hz) broaden instead the scientific reach of ET, allowing for connections to be drawn with different GW sources. This chapter investigates key themes that highlight the scientific advances that are most impactful for the science of ET. The consequences of these synergetic observations span astrophysics, fundamental physics, nuclear physics, late-Universe cosmography, and early-Universe cosmology, offering immense potential for breakthroughs across these fields.

6.1 Introduction

The synergies of ET with other GW observatories enable a *deeper, sharper, and wider* exploration of our Universe, enhancing our understanding of the physical laws that govern it. In certain areas, ET can benefit from complementary observations by other GW observatories, while in others it can uniquely contribute to them, thanks to its high sensitivity and extended observational horizon, especially in the lower range of the high-frequency spectrum around a few Hz.

With ET operating as part of a network of other third-generation (3G) ground-based observatories, such as the Cosmic Explorer (CE) [14], the accuracy of measurements for physical parameters, luminosity distance, and sky localization of GW sources will improve significantly [16, 107, 1167]. The scientific advantages include enhanced cumulative sensitivity, longer observation periods and duty cycles, an extended reach to distant sources, and larger sample sizes of detected events. The baseline configuration considered here includes ET (in either a triangular or 2L configuration) alongside two CE detectors, covering a frequency range from approximately 3 Hz to the kHz region. We will also consider a 3G network made by ET (either in the triangular or in the 2L configuration) together with a single 40 km CE detectors in the US and a LIGO-India detector. In the “high-frequency” range, we also highlight synergies with the proposed Neutron Star Extreme Matter Observatory (NEMO) [2408], a GW interferometer optimized for studying nuclear physics at frequencies above a kHz.

At lower frequencies, we consider different GW observatories in space: the Laser Interferometer Space Antenna (LISA) [330], recently adopted as a flagship mission by ESA in collaboration with NASA, and the concept designs Tian Qin [2409] and Taiji [2410], all of which are sensitive to GWs from below 10^{-4} Hz to about 0.1 Hz. In the decihertz band, we consider the Lunar Gravitational Wave Antenna (LGWA) [2411, 2412] as an example of Moon-based projects (see also LILA; [2413]) and the proposed space mission DECIGO [1952, 2414–2416], with sensitivities extending from about 0.01 Hz up to Hz frequencies. Finally, at even lower frequencies, around the nHz band, we will discuss possible synergies with Pulsar Timing Array (PTA) experiments, using expected radio observations from the Square Kilometre Array (SKA) [2417] as a primary example of a radio telescope infrastructure of interest for our considerations.

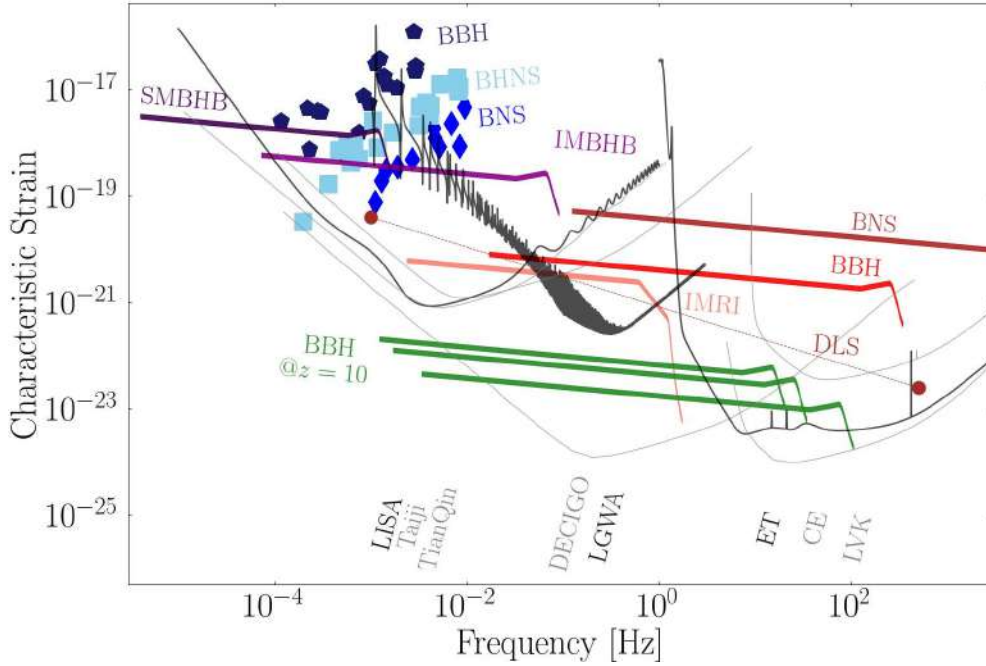


Figure 104. The GW landscape — a schematic representation of synergetic sources in the GW frequency-amplitude plane. The figure shows the sensitivities for ET and CE (including the current LVK detectors) at higher frequencies, and for LISA, Taiji, Tian Qin, DECIGO and LGWA at lower frequencies (sensitivities for the latter are from [330, 2412, 2414, 2416, 2432, 2433]). Tracks (left to right) represent: an equal mass black hole binary merger (labelled SMBHB) of $10^6 M_\odot$ at $z \sim 6$; an equal-mass intermediate mass black hole merger (IMBHB) of $10^5 M_\odot$ at $z = 1.5$; a multi-band source hosting an inspiralling equal-mass stellar black hole binary (BBH) of $60 M_\odot$ at $z \sim 0.1$ merging in the ET bandwidth; an intermediate mass ratio inspiral (IMRI) hosting a stellar black of $30 M_\odot$ orbiting around a black hole of $5 \times 10^3 M_\odot$ at $z = 1$. The “BBH@ $z=10$ ” track denote mergers at $z = 10$ of equal-mass black hole binaries forming in metal poor early galaxies, with a total mass, from bottom to top, of 20, 60, $100 M_\odot$. Squares, pentagons and diamonds refer to Galactic compact binaries detected by LISA, millions to billions of years away from merging. DLS indicates a dual line system. Each section of this chapter presents each of these sources and their scientific opportunities in detail.

Although we may mention other GW observatories that have been proposed in the literature (e.g. [131, 2418–2431]), at the present stage of developments the ones mentioned above represent the current standard for expected detection capabilities across different frequency ranges during the observational epoch of ET. We will therefore consider these as primary examples providing synergetic observations for ET. In figure 104, we show the sensitivities of some of the GW observatories mentioned above, along with the frequency tracks of few example GW sources.

In section 6.2 we quantify the scientific gains from combining observations of merging binary neutron star (BNS) systems and stellar-mass binary black holes (BBHs) from a network of 3G Earth-based observatories, including ET. These joint observations will deepen our understanding of the properties and evolution of BNSs and stellar-mass BBHs across cosmic epochs and will provide insights into the equation of state of nuclear matter, the dynamics of the Universe on large scales, and the possible existence of exotic compact objects. While

these scientific topics are covered in later sections, section 6.2 will focus on the improvements in detection and parameter estimation for these sources. The synergy of combining GW observations of compact binary mergers, such as BNSs and BBHs, can extend to observatories operating at frequencies below a few Hz, which are inaccessible to ground-based observatories. Section 6.2 also discusses nearby extragalactic stellar-mass BBHs, which serve as intrinsic *multi-band GW sources*. When far from coalescence, they emit GWs at frequencies twice their Keplerian orbital frequency, appearing as slowly chirping signals. These low-frequency signals contain unique information about their formation pathways, which is otherwise inaccessible. Detectors such as LISA, Tian Qin, and Taiji can capture these signals at frequencies as low as mHz, long before they enter the Deci-Hz band and eventually merge within ET's frequency range (above a few Hz). Deci-Hz detectors like LGWA and DECIGO will not only track the inspiral of stellar BBHs closely but also observe BNS inspirals, providing early estimates of their masses, distances, and sky locations. These early estimates will allow electromagnetic (EM) telescopes to target these sources long before merger, greatly improving the potential for EM follow-up observations and hence, significantly advancing multi-messenger science.

In section 6.3 we review the expected insights into nuclear physics. The combined measurements of a 3G network of Earth-based observatories will allow for unprecedented constraints on the tidal deformability parameters of NSs and in turn on the state and properties of nuclear matter at extreme densities. These scientific breakthroughs will complement those from nuclear physics experiments at lower energy as well as those from particle physics experiments at higher energy, yielding a complete panorama of the behavior of nuclear matter at all energy scales.

In section 6.4 we present the implications for fundamental physics obtained by the combination of ET with other GW observations. BHs are scale-invariant, vacuum solutions of Einstein's field equations. Probing the spacetime geometry of BHs, i.e. whether they are described by the Kerr metric or not, from the stellar-mass scale to the supermassive scale, is of paramount importance. The benefit of combining ET with CE or with LISA has far reaching consequences. Tests of the nature of black hole horizons, of the orbital evolution during the inspiral phase and of the propagation properties of GWs can be carried out at better precision, given the wider dynamic range that becomes accessible.

The synergy among different GW observatories has the potential to revolutionize late-time cosmology, as discussed in section 6.5. With ET operating as part of a 3G network, the larger dataset, and significantly improved sky localization, will enable better identification of the host galaxies of sources through the detection of EM counterparts of BNS and NSBH systems. These events will serve as unique cosmic tracers, allowing precise measurements of the Universe's expansion, delivering stringent constraints on the Hubble constant, and enabling new tests of the nature of dark energy. Even in the absence of an EM counterpart, the ET+CE network will provide accurate measurements of source masses, sky locations, and distances to enable powerful cosmological analyses. These include measuring the Hubble constant with dark sirens, cross-correlating localization data with galaxy catalogues and exploiting features in the mass distribution of black holes (BHs) and neutron stars (NSs). The large number and extended distance reach of detected GW sources will facilitate precise measurements of the Universe on large scales, offering new insights into current cosmological enigmas and serving as valuable probes at high redshifts.

Section 6.6 discusses an additional, albeit indirect, intersection of ET with LISA. This connection addresses an unresolved problem in astrophysics: the origin of supermassive black holes (SMBHs), which range from millions to billions of solar masses and reside at the centres of bright galaxies in the present-day Universe. The discovery of quasars near and beyond the epoch of cosmic reionization raises significant questions about the rapid growth of these massive, accreting black holes. One hypothesis is that these SMBHs form from *seed black holes* of about $10^2 M_\odot$ to $10^5 M_\odot$, and grow by accretion and mergers. Among these, light seeds — those lighter than about $10^3 M_\odot$ — are expected to be the relics of the first forming, metal-free massive stars present in the earliest dark matter halos assembling around $z \sim 20$, when the Universe was only a few hundreds of millions of years old. Observing the merger signals of black holes from this first pristine population of stars with ET would establish a connection with the population of merging massive black holes that LISA will detect at comparable and lower redshifts in the mass range around $(10^4 - 10^7) M_\odot$. Matching these two independent observations will provide invaluable clues into the physical origin of the seeds, their masses and the processes through which they grew to become supermassive during galaxy evolution. This connection is unique and is made possible since both ET and LISA have comparable cosmic horizon-reaches.

Section 6.7 explores potential synergies between ET, LISA and PTA in the detection of a stochastic GW background (SGWB) from the early Universe (see in particular section 3.2 for discussion of cosmological backgrounds and section 4.10 for discussion of astrophysical backgrounds). By combining these independent observations we can cover a much broader range of GW frequencies and achieve a precise characterization of the expected cosmological background(s). The detailed morphology of the SGWB encapsulates information about the mechanisms that originated the emission of GWs in the early Universe. Complementary observations at different frequencies will thus greatly enhance our ability to precisely characterise these early-Universe phenomena that have produced the detected SGWB. Additionally, the synergy with other Earth-based detectors is crucial for cross-correlating the data of ET and differentiating between noise and stochastic signal. A single interferometer alone will struggle to extract more than an upper bound on SGWBs from the observed data.

In each of the following sections, readers will find two boxes highlighting important text. Cyan boxes outline the main scientific questions associated to the section, while teal boxes summarize key take-home messages of the section.

6.2 Compact object binaries

THE STELLAR-MASS COMPACT OBJECT BINARIES IN THE UNIVERSE

- WHAT SCIENTIFIC QUESTIONS ON THE ASTROPHYSICS OF NEUTRON STARS AND BLACK HOLE BINARY SYSTEMS CAN ONLY BE ANSWERED WITH ET IN SYNERGY WITH A NETWORK OF NEXT GENERATION GROUND-BASED OBSERVATORIES?
- WHAT IS THE PROMISE OF MULTI-BAND GW OBSERVATIONS FOR UNDERSTANDING COMPACT OBJECT BINARIES?
- ARE THERE DUAL-FREQUENCY GW SOURCES AND HOW WILL THEY INFORM US ABOUT THE PHYSICS OF COMPACT OBJECTS?

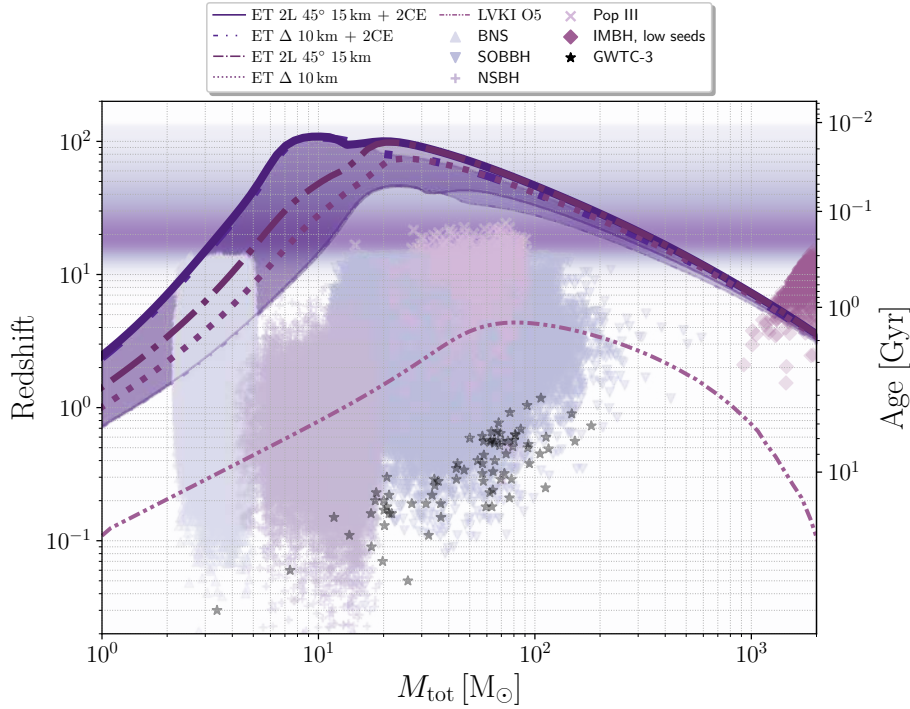


Figure 105. Detector horizons for the two baseline configurations of ET, alone and in conjunction with a network of two CE as described in the legend. The shaded bands, reported only for the combinations with CE, denote the range within which 50% of the sources are detected, when averaging over sky location. Known classes of target sources are reported for illustration: stellar-mass black holes (SOBBH) and double neutron star systems (catalogs from [16]), neutron star-black hole binaries (catalogs from [1167] based on [1370, 1374, 2434]), Population III stars [1674, 1675], and intermediate mass black holes (IMBH) [330]. The orange band is representative of a population of high-redshift primordial black holes (PBHs) [1182, 1738]. Triangles show the events detected by the LVK collaboration [6].

The majority of ET sources will consist of two stellar-mass compact objects with masses in the range $\sim 1\text{--}1000 M_{\odot}$. ET is expected to detect hundreds of thousands of these sources throughout the Universe each year [107, 1167, 1586]. The properties and origin of the population of compact object binaries have only started to be explored, with many formation pathways suggested (see section 4). There is no doubt that ET on its own will enable very significant advances in our understanding of the population and origin of compact object binaries. However, in synergy with other GW detectors, several additional science results are expected, which are highlighted here. We discuss both synergy with high-frequency detectors and with mid-frequency detectors.

6.2.1 Synergy with ground-based detectors

There are several other detectors in the frequency range of ET, from a few Hz to a few kHz, that could provide synergy with ET. The most important one is Cosmic Explorer (CE) [14], a US project for a third generation detector. Einstein Telescope and Cosmic Explorer strengthen and complement each other in important ways. An important difference is that

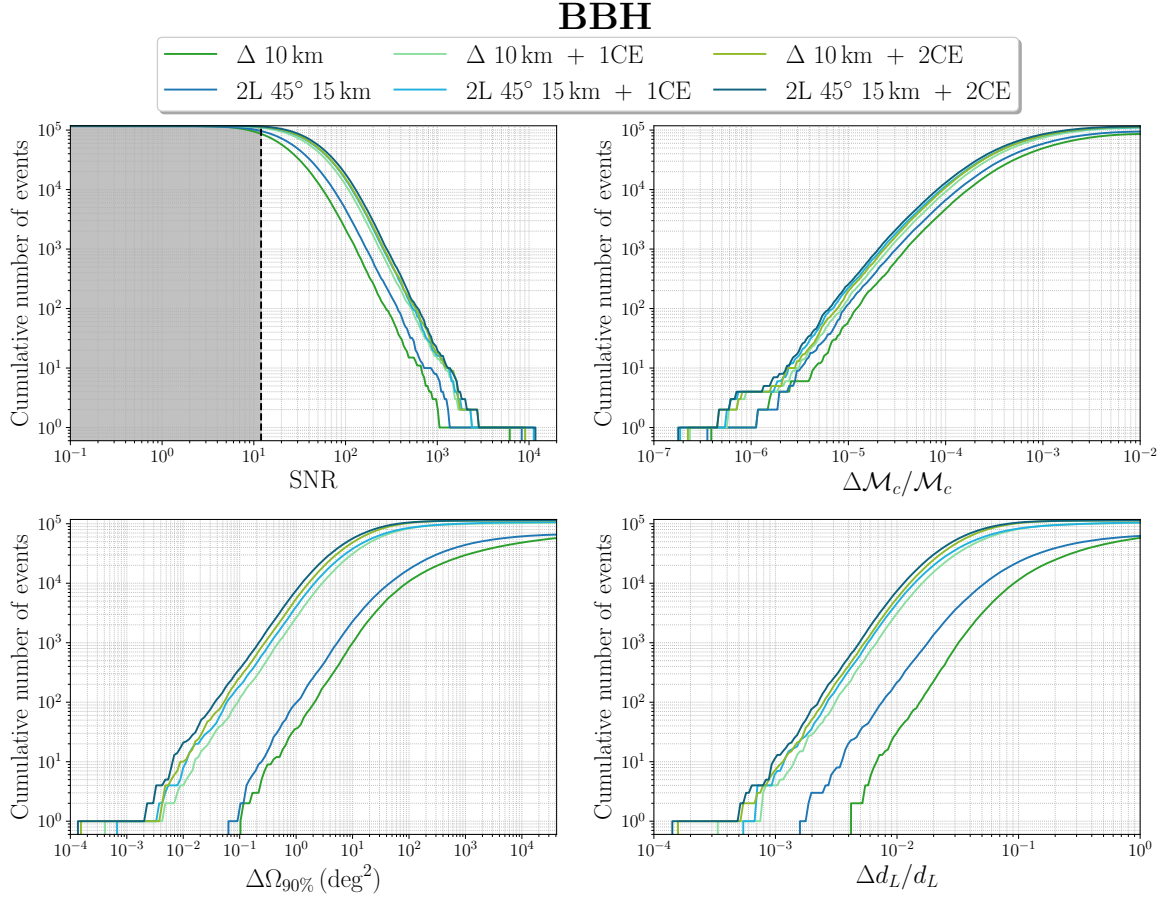


Figure 106. Comparison of SNR and parameter estimation error for the chirp mass, angular localization and luminosity distance for BBHs. We show the results for ET alone, for ET +1CE and for ET+2CE and, in all cases, for ET we consider both the triangle 10 km and the 2L-45° configurations. The results are obtained with the **GWfast** code [2349]; technical details as in [16]. Reproduced from [16]. The Author(s). CC BY 4.0.

CE is expected to be slightly more sensitive in the 10–100 Hz range which will complement the ET sensitivity curve [14], whereas ET is expected to have better sensitivity compared to CE at lower frequencies as a result from ET’s underground location that suppresses seismic disturbances, reducing the Newtonian noise that limits CE at low frequencies. To highlight this synergy, figure 105 shows the detection horizons for equal-mass, non-spinning binaries, with a network $\text{SNR} \geq 8$, for ET in its triangular and 2L configurations, both alone and in a network of two CE detectors.

From just a point of view of detection, ET alone is expected to detect more than 90% of BBHs of stellar origin (SOBBHs) and about 26%(15%) of the binary neutron stars in the 2L (triangular) configuration [16]. This corresponds to $\mathcal{O}(10^5)$ BBH and BNSs detected per year. ET and CE operating in network will allow us to significantly improve range and number of BNS detections. In the ET2L+2CE configuration, the number of BNS sources detectable per year will be $> 4 \times 10^5$ (in the population model considered). Moreover, a network of next generation GW observatories with both ET and CE is crucial to provide orders-of-magnitude

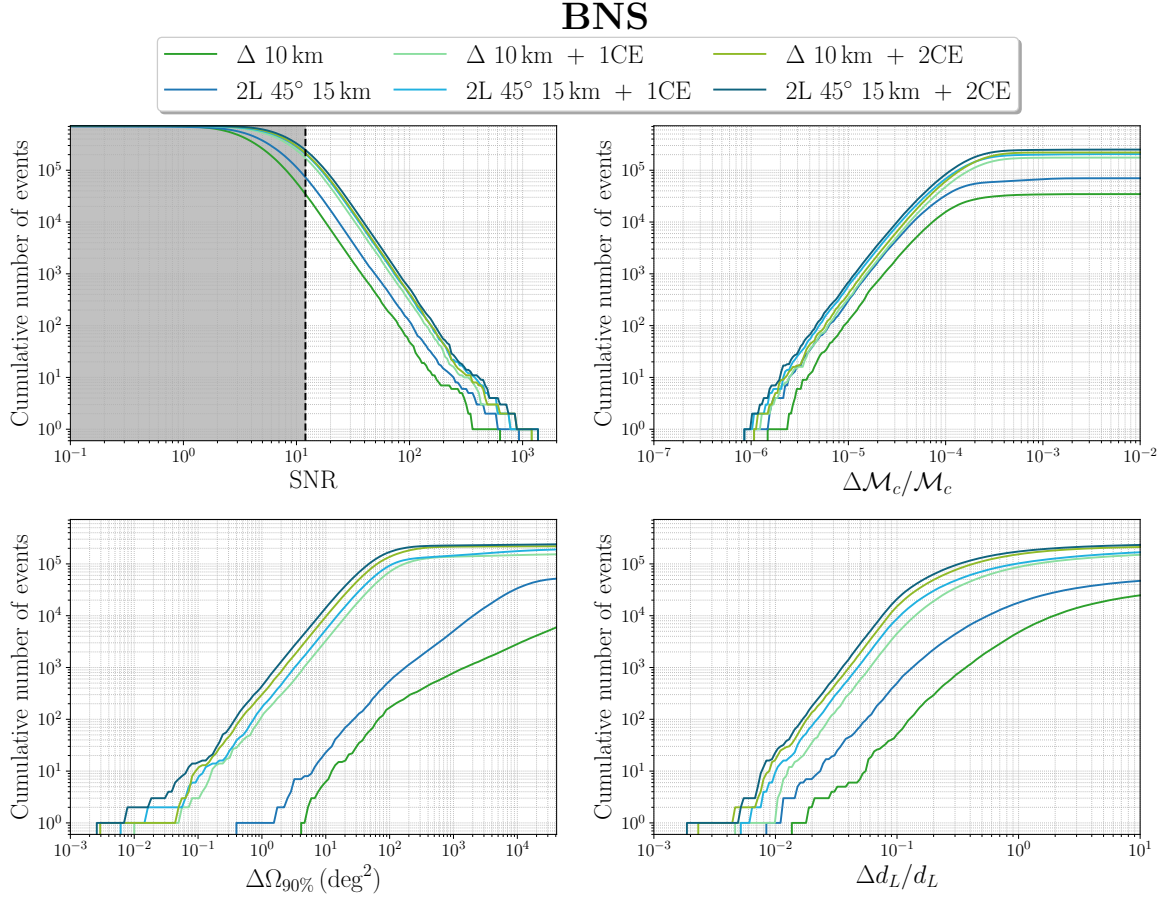


Figure 107. As in figure 106, for BNSs. Reproduced from [16]. The Author(s). CC BY 4.0.

improvement in parameter estimation for coalescing binaries, with significant impact on most of the science goals outlined in the ET and CE studies. As an example, figure 106 shows the SNR distribution as well as the parameter estimation error for the chirp mass, the angular localization and the luminosity distance for the BBHs mergers corresponding to one year of data (using the same population as in [16]), while figure 106 shows the same results for BNS. In both plots we show the results for ET alone, for ET together with a 40 km CE (labeled ET+1CE) and for ET together with a 20 km and a 40 km CE (ET+2CE). In all cases, for ET we consider both the 10 km triangle and the configuration with 2L of 15km at a relative angle of 45° . We see that the improvement is particularly remarkable for angular localization and for luminosity distance (while the improvement in the accuracy of the mass measurement is less dramatic).

Examples of scientific results that can only be found by the combination of ET and CE are: (i) detections of BBHs (BNSs) with 90%-credible sky area less than 0.01 (0.1) deg^2 , important for multi-messenger observations [16, 107, 1167]; (ii) regular detection of BNS (BBH) systems at $z > 1$ ($z > 10$) with mass uncertainty $\delta m_1 / m_1 < 0.3$ (0.2) [107], important for understanding BH and NS properties across cosmic time, probing fundamental physics and performing precision cosmology; (iii) detection (or strong upper limits on the rate of)

primordial black hole mergers at $z \gtrsim 25$ with $\delta z/z < 0.2$ and population III binary black holes at $z \gtrsim 10$ with $\delta z/z < 0.1$, which will be crucial to inform BH formation channels [107, 1178]. Importantly, by being located in the USA, CE will provide a longer baseline that will improve sky localizations and parameter estimations [107, 1167, 1586], as we see from figures 106 and 107. Having both CE and ET will also be beneficial for increasing the completeness of the observed population: for example, a network with both ET and CE will achieve 50% completeness of BNS sources up to $z \sim 2$, as can be seen in figure 105, whereas an (A+)-like network will only achieve that for $z \sim 0.2$ [1586]. Furthermore, having both ET and CE will increase the time that at least one detector is always online to assure measurements are made during exceptional events such as a galactic supernova [107, 1167, 1586].

Overall, the presence of a next-generation detector network, including ET and CE, will enhance the science obtainable from ET observing alone. Fields such as those related to the measurement of cosmological parameters, unraveling the nuclear equation of state, and making new discoveries such as the presence of dark matter around NSs or BHs, multi-messenger merger observations can largely benefit from the improved capabilities of a network of observatories [14, 107, 1167, 1586]. Similar synergies can be outlined between ET and the Neutron Star Extreme Matter Observatory (NEMO) [2408], a proposed GW interferometer optimized to study nuclear physics with merging neutron stars which, together with ET, could contribute to inform the physics of BHs and NSs and their progenitors.

Another network configuration which could be possible in the next-generation era is made of ET (either in its triangular or in its 2L configuration) together with a single 40 km CE detector in the US and LIGO-India. This configuration has been recently studied in [107, 2435] (restricting however to ET in its triangular configuration). LIGO-India, currently under construction in Aundha, India, will feature 4 km arms as its US counterparts and is expected to be operational in the 2030s at A+ sensitivity [392], with upgrades at A[#] sensitivity levels [2436]. As shown in [107, 2435], the results obtained with this network are quite interesting. Indeed, with respect to a network made by ET together with a 40 km CE and a 20 km CE one finds that, for angular localization and accuracy on luminosity distance, replacing the 20 km CE detector with LIGO-India operating at A[#] sensitivity gives nearly identical performances, since the factor of five in arm length is offset by the increase in baseline relative to a second CE (although other aspects of the science case are limited by the shorter arm length and narrower sensitivity band).

Here we perform again the analysis for a network made by ET together with a single 40 km CE detector and LIGO-India, extending the analysis of [2435] by considering both the triangular and the 2L configuration of ET, and assuming A[#] sensitivity for LIGO-India. For uniformity with the results shown in figures 106 and 107, as well as with similar analysis presented elsewhere in this paper (see in particular section 10) we use the same population model, sensitivity curve and technical details as in [16].⁵⁴ The results are shown in figures 108

⁵⁴To compare with the results presented for BNS in [2435] one must take into account some technical differences. Our population model, restricting (as in [2435]) to $z < 0.5$, contains 1.1×10^4 BNS (and corresponds to a local merger rate $R_0 = 250 \text{ Gpc}^{-3} \text{ yr}^{-1}$), to be compared with 1.6×10^4 BNS (and $R_0 = 320 \text{ Gpc}^{-3} \text{ yr}^{-1}$) in [2435]. Furthermore, we assume an uncorrelated 85% duty cycle in each L-shaped detector, and in each of the three instruments composing the triangle, while [2435] uses 100% for the duty cycle; this affects in particular the number of rare events with the very best accuracy of reconstruction. We have checked that,

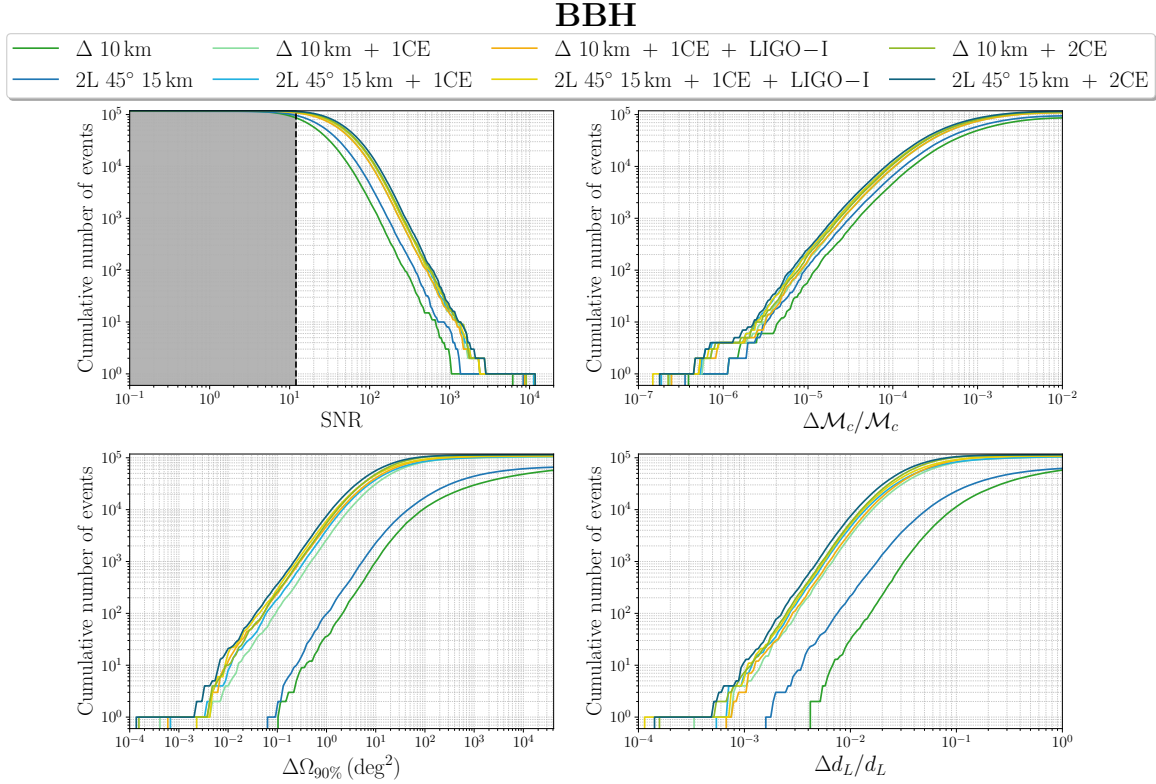


Figure 108. Comparison of SNR and parameter estimation error for the chirp mass, angular localization and luminosity distance for BBHs for different networks (as in the legend), including networks involving LIGO-India. The results are obtained with the `GWfast` code [2349]; technical details as in [16]. Reproduced from [16]. The Author(s). CC BY 4.0.

for BBHs and in figure 109 for BNSs, and show that a network made by ET together with a single 40km CE detector and LIGO-India can be quite competitive.

6.2.2 Synergies with space-borne detectors

The synergies with space-borne detectors, in particular LISA, the space mission recently adopted by ESA in collaboration with NASA [330], and operating in the “mid-frequency” band $\sim 10^{-4} - 10^{-1}$ Hz, come in three distinct ways: i) observations of *different instances* of a class of compact object binaries at lower frequency, that provide complementary insight in their population properties and their origin; ii) observations of *multi-band* sources, that are first observed at mid-frequency and then later in the ET band; and iii) *dual-line sources* that emit simultaneously in both mid-frequency band and the ET band.

Observations of the same source class. Different origins of compact object binaries (from isolated binaries, from dynamical processes, or from primordial black holes) give rise to different population properties that can be used to disentangle their origin, e.g. [2437]. Orbital eccentricity is thought to be a significant indicator of formation via dynamical

if we use 100% duty cycle and take into account the different BNS rate, the results are consistent, within a factor of 2, with those in [2435].

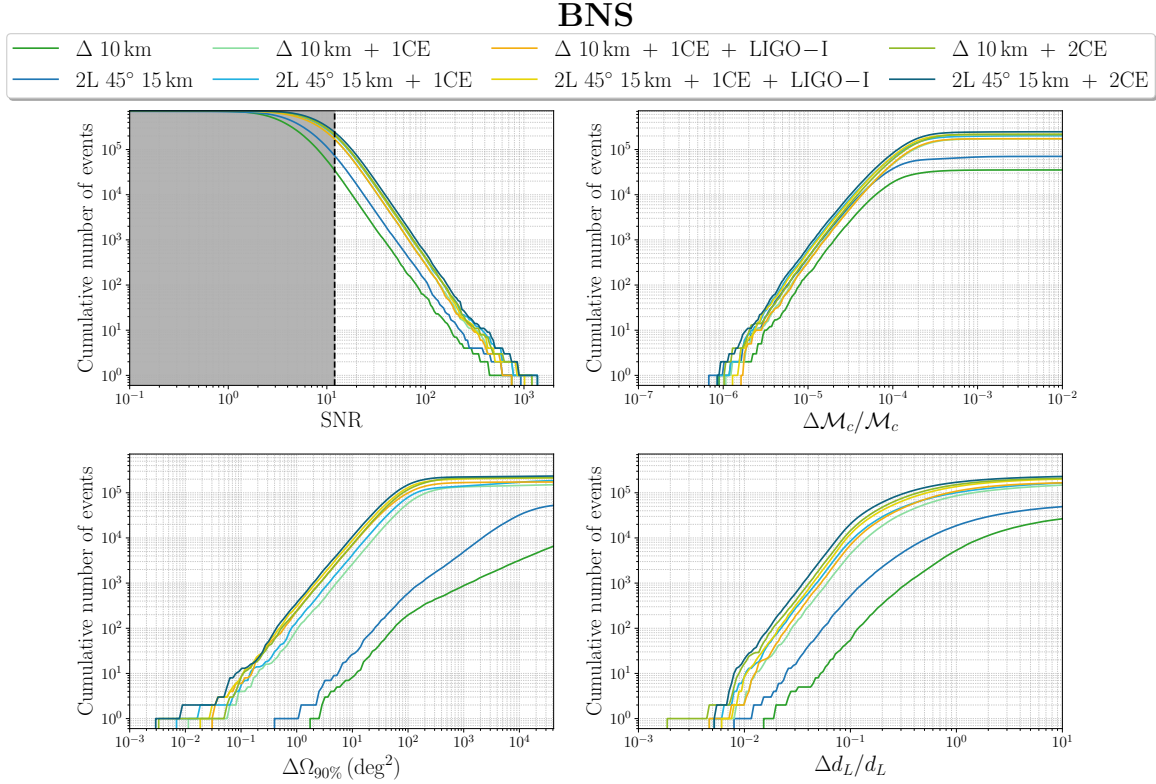


Figure 109. As in figure 108, for BNSs. Reproduced from [16]. The Author(s). CC BY 4.0.

processes and can be detected by LISA, but cannot easily be measured by high-frequency detectors, since the eccentricity is radiated away by GW and it very small by the time the signals enter the detectable band, even for large initial eccentricity [1521, 2437]. Observing the same type of sources at much lower frequency is an excellent way to constrain the importance of dynamical formation, albeit only in the nearby Universe, since LISA and similar detectors typically observe stellar-mass binary black holes in the Milky Way and nearby galaxies [2438], unless they are at very high frequency when they can be observed to several tens or at most hundreds of Mpc, see [512]. Deci-Hz observatories such as DECIGO would in fact be even better at detecting eccentricity given the large number of sources they can observe up to cosmological distances [1458].

Multi-band sources. These are binary black holes that are observed in the LISA band and then later, on human time scales (years), merge in the high-frequency band [2439]. In order for this to happen the binaries must have heavy black holes of around $30 M_{\odot}$ or higher. Such heavy sources have been observed already with the current detectors. However, the expected number of multi-band systems that will be observed with LISA is likely small [2440, 2441]. The scientific value of these measurements would be high: the LISA measurements can make a very accurate prediction of the time and sky localisation of the merger in the ET band and the properties of these systems would be determined to very high accuracy, see e.g. [2442]. Another proposed special class of multi-band sources is a low-frequency binary black hole orbiting around a super-massive black hole, where high induced eccentricity could move the

system temporarily into the ET band [2443]. Multi-band sources would as well be ideal targets for Deci-Hz detectors, as these observatories will be able to closely follow their inspiral almost until the merger, providing a strong complementarity with ET measurements and excellent pre-localisation accuracy for multi-messenger studies [2412, 2444–2447].

Dual-line sources. These are proposed sources emitting simultaneously in two frequency bands. An example are ultra-compact X-ray binaries consisting of a rapidly rotating NS (with spin frequencies in the hundreds of Hz) orbiting a companion star [1897]. GW from the orbital motion can be detected by LISA while continuous GWs can be detected in the ET band due to the presence of accretion-built mountains, toroidal magnetic fields, and/or r-mode oscillations [1898]. A dual-line detection could provide percent-level constraints on the mass, radius, and internal magnetic field strength of the neutron star. With ET, at least four of the known ultra-compact binaries become potentially dual-line detectable [1898].

6.2.3 Stochastic background from compact binaries

Finally, there is added value in correlating GW observations between different instruments for the detection of broadband stochastic signals that make up the astrophysical GW background, e.g. [100, 2448] to both characterize the population of astrophysical GW sources and distinguish these from cosmological backgrounds, as discussed in section 10.5.3. In the ET frequency band, the astrophysical GW background is the collective signal produced by unresolved black hole and neutron star binaries, and can be detected by ET with significant confidence (see section 4.10). However, its broadband features imply it may be difficult to distinguish from instrumental noise, particularly when the latter is not very well known, see e.g. [540]. Cross-correlating measurements of different, independent instruments, such as ET and CE will thus increase detection confidence.

In addition, there is synergy with detection efforts in other frequency bands: the same binary background extends to lower frequencies and can in principle be detected by detectors such as LISA and Tian Qin. A multi-band detection and characterization effort would provide additional constraints on compact binary population features, including information on compact binary progenitors, see e.g. [2205, 2211, 2212, 2449], complementary to those coming from resolved source analyses. Furthermore, low frequencies such as those monitored by LISA will be dominated by a broadband stochastic foreground due to unresolved white dwarf binaries (that, due to the relatively large size of the white dwarfs, does not contribute to the ground-based detector GW band). Recent studies suggest that the white dwarf background may well dominate LISA data, hindering detection prospects for the stellar-mass compact object background [2450, 2451]. In this scenario, detection of this background signal with ET will be all the more valuable, and will be used to inform analyses at lower frequencies.

A notable data analysis challenge in this context will be component separation. Differentiating between resolved events and background and between background contributions of various nature requires novel analysis techniques, and in particular the residual subtraction error could dominate stochastic measurements [2452, 2453]. However, recent work that performed explicitly the correlation between data streams of different detectors, after reconstruction and subtraction of CBCs from the individual detector data streams, show that the enhanced resolving power of a network made of ET+CE can reduce significantly the errors

in the reconstruction of resolved sources, to the extent that the sensitivity to cosmological stochastic background will only be marginally affected, see [643] and section 10.5.3.

TAKE HOME MESSAGE

- ET IN A 3G NETWORK ALONGSIDE OTHER GROUND-BASED DETECTORS, SUCH AS COSMIC EXPLORER, WILL EXTEND THE COSMIC HORIZON REACH, ENABLE DEEPER AND MORE ACCURATE SURVEYS, AND YIELD SIGNIFICANTLY MORE PRECISE PHYSICAL MEASUREMENTS OF STELLAR-MASS COMPACT OBJECT BINARIES.
- JOINT OBSERVATIONS WITH A GROUND-BASED GW NETWORK (ET WITH CE OR NEMO) WILL ENABLE US TO MAP NEUTRON STARS ACROSS COSMIC TIME.
- SYNERGIES BETWEEN ET AND LOWER-FREQUENCY SPACE-BASED DETECTORS, SUCH AS LISA, WILL ENABLE MULTI-BAND OBSERVATIONS OF THE SAME SOURCE CLASSES, OFFERING UNIQUE INSIGHTS INTO THEIR FORMATION PATHWAYS.
- MULTI-BAND OBSERVATIONS OF THE SAME SOURCE ARE ESSENTIAL TO DISCOVER DUAL-LINE.
- ASTROPHYSICAL, STOCHASTIC GW SIGNALS FROM COMPACT BINARIES OF DIVERSE TYPE AND IN DIFFERENT EVOLUTIONARY STATES ARE EXPECTED TO BE DETECTED BY ET AND LISA. THESE OBSERVATIONS WILL PROVIDE UNIQUE STATISTICAL INFORMATION INTO THE UNDERLYING SOURCE POPULATIONS.

6.3 Nuclear physics

NUCLEAR PHYSICS IN THE REALM OF GW SOURCES

- HOW MANY MORE BNS AND NSBH MERGERS WE CAN OBSERVE WITH ET+CE? HOW DOES IT TRANSLATE TO CONSTRAINTS ON EOS?
- HOW MUCH BETTER CAN WE OBSERVE THE POST-MERGER PHASE OF A BNS MERGER? HOW DOES IT TRANSLATE TO QCD PHASE TRANSITION CONSTRAINT?

A principal goal of heavy-ion collision experiments such as Relativistic Heavy Ion Collisions (RHIC) [2454] and Facility for Rare Isotope Beams (FRIB) [2455] is to understand the nature of hot and dense hadronic matter. These experiments have improved our understanding of the quark-gluon plasma, a state of matter thought to have existed in the early moments after the Big Bang. However, much about the properties of this matter remains unknown. At the other end of the spectrum, neutron star cores are believed to contain matter at even higher densities, though at much colder temperatures than those explored in heavy-ion collision experiments.

During the inspiral phase of a neutron star merger, the properties of the cold supranuclear matter are encoded in the tidal dephasing of the signal. The supranuclear matter can reach a temperature of 50 MeV [2456–2458] during the post-merger phase. Therefore, the observation of a binary neutron star merger provides a unique opportunity to explore the QCD phase

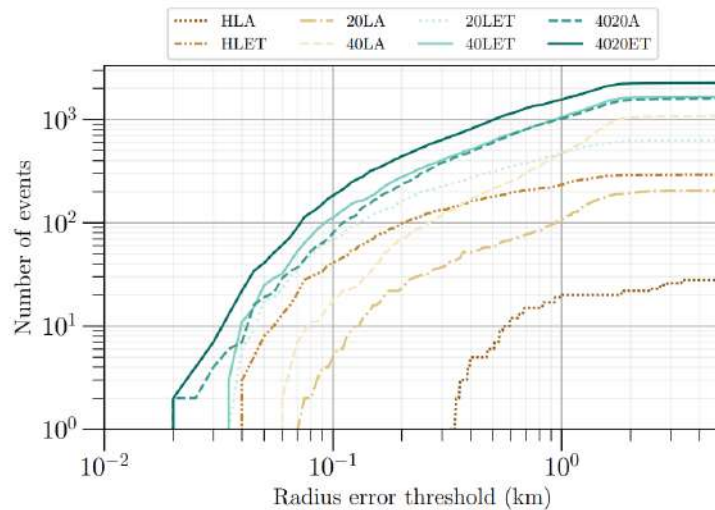


Figure 110. The cumulative number of GW detections (vertical axis) for which it will be possible to constrain NS radii below a specified threshold (horizontal axis) is shown. The lines represent different GW detector configurations: HLET, 20LA, and 40LA indicate networks with one next-generation detector, while 4020A, 20LET, and 40LET represent networks with two next-generation observatories. Figure reproduced with permission from [107].

diagram comprehensively, spanning both cold and hot regimes of dense matter, as we discuss in detail in section 7.

In this section, we showcase how the synergy between ET and CE can: (1) constrain the cold supranuclear matter with the thousands of neutron star merger detections; (2) detect the currently undetectable post-merger signal; (3) gain insights about the supranuclear matter with the post-merger signal.

6.3.1 Population approaches for nuclear physics

Next-generation GW observatories will detect thousands of binary coalescences involving a neutron star (NS) [16, 107]. These mergers are rich laboratories to study the behavior of matter under extreme conditions [2459], as also demonstrated by the binary neutron star GW170817 [4, 2288]. When the NS approaches its companion, in particular a BH, it may undergo tidal deformation and be disrupted. This process imprints a distinctive signature on the GW waveform that is parametrized with a tidal deformation parameter Λ for the NS (see eqs. (7.4) and (7.5) below). From the knowledge of Λ and the NS mass, it becomes possible to estimate the NS radius. Finally, a joint estimation of the NS mass and radius is crucial for determining the Equation of State (EoS) that governs the star mass-radius relation [2460]. However, the EOS for neutron stars remains unknown and measuring NS radii is extremely challenging as the Λ parameters are not often well constrained. The BNS merger GW170817 currently provides the best constraint on the NS radii, with a precision of a few kilometres [2460]. This precision was only sufficient to exclude the stiffest EoSs.

Next-generation observatories will detect thousands of GW events like GW170817, including hundreds of sources with higher signal-to-noise ratio [16, 107, 1167]. Figure 110

shows the forecasted constraints on the NS radii achievable in the next-generation GW era. For a GW detector network that includes at least one next-generation GW detector, it is anticipated that NS radii can be measured with a precision of 1 km (approximately 10% of the nominal NS radius) for about 200 detections per year [107]. In a network with at least two next-generation GW observatories, this number increases to around 1,000 detections annually. These observations would provide the robust data necessary to precisely test and refine proposed EoSs for NSs [1231, 1239]. The ability of 3G observatories to constrain the NS EOS from populations of compact binary coalescences is not significantly affected by the specific geometries of the detectors but rather depends on the number of 3G observatories available to capture these signals [1231].

6.3.2 Measurability of the post-merger phase signal

The post-merger phase explores a different regime in the QCD phase diagram, characterized by significantly higher densities and temperatures compared to the inspiral phase. During the inspiral, the neutron stars remain largely intact, and the densities reached are limited to their central densities, approximately 3–4 times the nuclear saturation density. On the other hand, the post-merger phase probes densities exceeding five times nuclear saturation density [2461]. At temperatures of approximately 50 MeV, the effects of different transport coefficients become prominent, leaving noticeable imprints on the data [2456–2458]. However, the amplitude of the post-merger phase signal is weaker than that of the inspiral phase due to the reduced quadrupole component [2462–2466]. This challenge is compounded by the reduced sensitivity of current detectors at the higher frequencies where the post-merger signal lies. However, a next-generation GW detector significantly improves access to the post-merger phase. As shown in table 10, for three sources taken at the same distance of 68 Mpc,⁵⁵ but different values of the masses and of the tidal deformability, the post-merger phase SNR is five times higher in the next-generation GW detector network as compared to current generation networks.

To translate the SNR into physically meaningful parameters, parameter inference must be conducted within a next-generation GW detector network. Figure 111 illustrates that including the post-merger phase and linking its parameters to those of the inspiral phase using quasi-universal relations (QU-PM) results in significantly tighter posteriors for tidal deformability compared to analyses that exclude the post-merger phase (NO-PM). This demonstrates the potential of the post-merger phase to enhance studies of cold supranuclear matter. Additionally, as shown in figure 112, the parameters of the post-merger phase are only measurable with a next-generation GW detector network. This highlights the critical role of the post-merger phase in studying hot supranuclear matter, contingent on the availability of a next-generation GW detector network.

A network of next generation GW observatories is essential for breaking degeneracies among waveform parameters. Adding interferometers such as NEMO [2408], which are

⁵⁵This specific value of the distance was chosen in [2467] because, as we see from the row labeled LHV in table 10, in a network made by Advanced LIGO+ and Advanced Virgo+, it corresponds to a signal-to-noise ratio of 100 for Source 1 (which is defined by the masses and tidal deformability given in the table caption).

	Source1		Source2		Source3	
	Total	PM	Total	PM	Total	PM
LHV	100	2.0	94	2.5	100	2.7
ET-CE	1267	10.2	1190	12.3	1268	13.3

Table 10. The SNRs for three source located at a distance of 68 Mpc and with chirp mass, mass ratio and tidal deformability as follows. Source 1: $\{\mathcal{M}_c = 1.17524 M_\odot, q = 0.8, \tilde{\Lambda} = 604\}$; Source 2: $\{\mathcal{M}_c = 1.08819 M_\odot, q = 1.0, \tilde{\Lambda} = 966\}$; Source 3: $\{\mathcal{M}_c = 1.17524 M_\odot, q = 1.0, \tilde{\Lambda} = 607\}$. The table includes the SNR for the entire waveform (labeled as “Total”) and for the post-merger phase specifically (labeled as “PM”). See ref. [2467] for details. Reprinted figure with permission from [2467], Copyright (2023) by the American Physical Society.

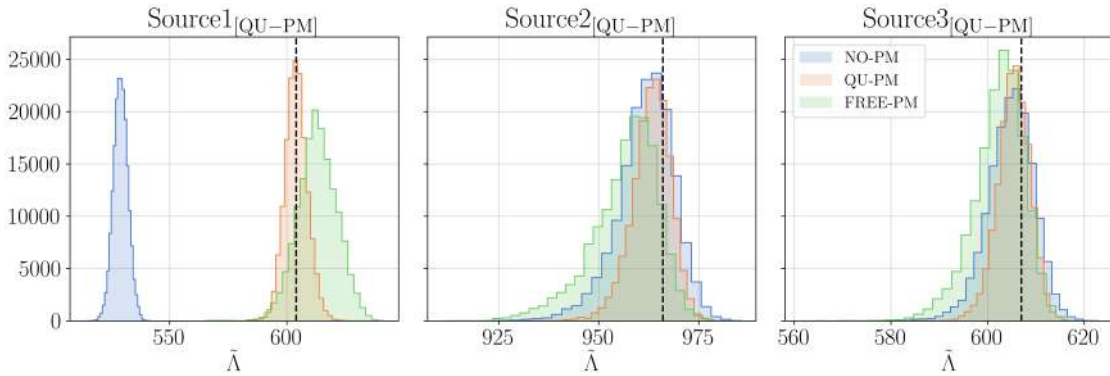


Figure 111. Posterior probability density for $\tilde{\Lambda}$ (see eq. (7.5) for definition) for sources observed by the ET+CE network, recovered with three different models NO-PM (without post-merger phase), QU-PM (with post-merger phase and employing quasi-universal relation), and Free-PM (with post-merger phase and its parameters freed from that of the inspiral phase), in blue, orange, and green, respectively. The black dashed lines correspond to the injected values. The parameters (distance, masses, tidal deformability $\tilde{\Lambda}$) of the three sources are the same as in table 10. Reprinted figure with permission from [2467], Copyright (2023) by the American Physical Society.

optimised for the higher-end of the high-frequency GW spectrum, would provide even stronger constraints on the EoS, thanks to a more precise characterisation of the post-merger phase.

6.3.3 Inferring exotic nuclear phenomena with binary neutron star mergers

The merger of BNS systems can result in a dense NS remnant that can resist gravitational collapse for timescales ranging from several milliseconds to a few minutes after merger [2468, 2469]. The post-merger evolution of these remnants results in GW emission [2470], making them promising candidates for understanding the behavior of matter at extreme densities. These remnants have been shown to potentially exhibit several exotic phenomena, such as phase transitions to deconfined quarks [2471, 2472], deconfined quarks *via* quark-hadron crossover [2473, 2474], formation of absolutely stable strange quark matter [2327, 2475], the appearance of hyperons [2476, 2477], and thermal effects [2478, 2479]; see section 7.3.2.1 for more detailed discussion.

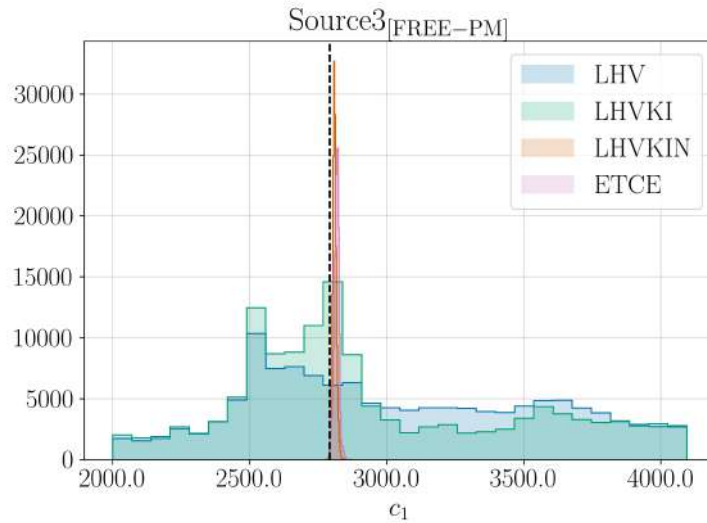


Figure 112. Posterior probability density for c_1 , one of the post-merger phase Lorentzian parameter for different detector networks. The dashed vertical line indicates the injected value. Reprinted figure with permission from [2467], Copyright (2023) by the American Physical Society.

Among other outcomes, these effects can alter the compactness of the remnant (compared to the cold hadronic equation of state), thereby affecting the value of the peak post-merger frequency, f_2 . Hence, precise measurement of f_2 is important for inferring the nuclear processes occurring within the dense remnant. Ref. [1234] finds that deviations in f_2 caused by phase transitions can be detected with 90% confidence for a post-merger SNR of 10. This corresponds to an inspiral SNR of approximately 600, achievable by ET alone at distances ~ 90 Mpc. The 20 km CE detector, when optimized towards post-merger frequencies, can accumulate the same post-merger SNR at ~ 120 Mpc. For the crossover scenario, ref. [2480] employs a Bayesian model selection to claim that a crossover to the quark phase will be detectable with ET and CE observatories for a few BNS mergers every year. Further, the deviation in f_2 due to thermal effects can be detected with 90% credibility for a post-merger SNR of 15, achievable for by ET for events at 50 Mpc [2479]. Together with CE, such events can be detected with the same SNR up to higher distances, which will increase the rate of detection of these events.

While phase transitions to quark matter are expected to affect post-merger signals due to their relevance at high densities, several studies suggest that signatures of strong phase transitions may also appear in the inspiral phase. Single or sequential phase transitions can lead to significant deviations in the mass-radius ($M - R$) relationship compared to their hadronic counterparts [2481, 2482]. EoSs with first-order phase transitions may exhibit small deviations in the tidal deformability parameter Λ while causing variations in the radius on the order $\mathcal{O}(100)$ m [2483]. Detecting such changes due to phase transitions using GWs alone requires precise measurements of both radius and Λ , which can only be achieved with next-generation GW observatories. Using a non-parametric approach, ref. [2484] finds that a catalog of several hundred BNS mergers would be necessary to confidently detect phase

transitions, a capability that would only be feasible in the next-generation GW detector era. However, ref. [2485] demonstrates that if the phase transition occurs at twice the nuclear saturation density, it could be confidently inferred from a single BNS event at a distance of $\mathcal{O}(100)$ Mpc with next-generation GW observatories.

Hence, precise measurement of NS parameters in both pre- and post-merger GW signals from BNS mergers will allow the inference of exotic nuclear processes. Such detailed and numerous estimates will only be achievable with a network that includes both ET and CE observatories.

TAKE HOME MESSAGE

THE SYNERGIES BETWEEN ET AND OTHER EARTH-BASED GW OBSERVATORIES ARE ESSENTIAL FOR ACHIEVING A COMPREHENSIVE AND ROBUST UNDERSTANDING MATTER AT SUPRANUCLEAR DENSITIES.

6.4 Fundamental physics

Stellar- and intermediate-mass BBH systems serve as exceptional laboratories for testing General Relativity (GR). Stellar-mass BBHs evolve over long timescales within the LISA frequency band, where their dynamics can be accurately described using the post-Newtonian (PN) approximation [32]. As they gradually inspiral out of LISA's range and transition into the frequency band of ET, their dynamics enter a highly non-linear regime that eventually necessitates numerical relativity (NR) simulations. Multi-band observations of these systems thus provide a unique avenue to probe both lower-order PN predictions and the strong-field effects captured by NR simulations, offering a comprehensive test of GR across a wide range of physical regimes.

In contrast, intermediate-mass BBH systems have shorter lifespans in both the LISA and ET bands, but their significantly larger signal strains and SNRs enable unique tests of GR. In the LISA band, these systems allow exploration of mildly non-linear dynamics during the inspiral phase, while in the ET band, their post-merger signals probe the strongly non-linear regime. Additionally, concurrent observations by ET and CE will provide an unprecedented opportunity to test the polarization properties of the waves, further advancing our understanding of gravitational phenomena in the strong-field regime. The synergetic fundamental physics exploration with ET are enumerated in the box below.

SYNERGETIC FUNDAMENTAL PHYSICS

- MULTI-BAND OBSERVATIONS OF THE INSPIRAL, MERGER AND RINGDOWN PHASES OF COMPACT BINARY COALESCENCES OFFER A UNIQUE OPPORTUNITY FOR PRECISION TESTS OF GR.
- JOINT OBSERVATIONS BY ET AND CE CAN HELP BREAK DEGENERACIES.

As we have discussed in detail in section 2, one of the fundamental science objectives of ET is to test General Relativity (GR). ET alone will be able to test GR to incredible precision with the detection of black hole binaries like GW150914 with an SNR of more than

1000. With multi-band observations with space interferometers the precision to which the theory can be tested increases by a order of magnitude. Additionally, combining ET and CE data will help test the theory in new regimes such as polarization states and lensing of the observed gravitational waves as they would require accurate sky pointing and distance measurements. In the reminder of this subsection we discuss how ET can achieve greater sensitivity to test GR with data from other terrestrial and space-based GW observatories.

6.4.1 Tests of the inspiral phase

In GR, the phase evolution of a binary system is driven by a balance equation between the change in orbital energy and the energy flux emitted in GWs. During the inspiral, this evolution is well represented by the post-Newtonian approach [32], that permits to derive a perturbative series for the phase evolution written in terms of the small PN parameter $v = (GM\omega/c^3)^{1/3}$ for a total mass M and orbital frequency ω , with v^2 denoting the PN order parameter. In GR, the dominant term in the phase evolution is driven by the quadrupole radiation and is $\propto v^{-5}$; it is often called Newtonian, or 0PN, term. High and low (or negative) PN orders affect most the late or early inspiral, respectively.

In alternative theories of gravity, this structure can be modified due to either the existence of extra fields or a modification of the gravitational dynamics. One well-known example is dipolar radiation, present generically in theories featuring scalar or vector fields [2486], which appears at leading order at the -1 PN order with respect to the quadrupole radiation. Because dipole radiation appears at a relative lower order of -1 PN with respect to the quadrupole radiation, it affects most prominently the early inspiral of a binary and represents a valuable window to look for clues of new physics in the gravity sector [2487]

Among proposed modifications from GR (see e.g. table VII in [2488]), time-dependent variations of the gravitational constant itself create a negative -4 PN correction; modifications of black hole evaporation in large extra dimensions also yield a -4 PN term; scalar-tensor theories such as Jordan-Fierz-Brans-Dicke and Einstein-Dilaton-Gauss-Bonnet theories, as well as Einstein-Aether theories will all feature dipolar radiation with a -1 PN leading term; introducing a graviton mass yields a 1 PN term; parity-violating Dynamical Chern-Simmons theory and some models of non-commutative gravity will introduce a modification at the 2 PN order.

In this landscape, negative PN orders will affect most prominently the early inspiral, while positive PN orders will affect the late inspiral the most. This creates a strong trend across instruments: observations sensitive to the late inspiral and merger of a binary system, such as massive BH binaries (MBHBs) in LISA or stellar-mass BH binaries (SBHBs) in ET, are typically best to constrain high PN orders, while observations sensitive to the early inspiral, such as SBHBs for LISA or binary pulsar timing, are best to constrain negative PN orders. Notably, dipolar radiation might even dominate over the quadrupolar radiation at low enough frequencies. However, if multi-band GW observations are made, the high-frequency band data can also help a lot in inferring parameters that are otherwise degenerate if only low-frequency data are available.

This synergy has been explored in many recent works. Ref. [73] proposed a theory-agnostic test on black-hole dipolar radiation, combining the LISA band and the band of

ground-based instruments. Ref. [2489] explored this multi-band synergy for parity-violating gravity, ref. [2490] for parity-violating gravity as well as scalar-tensor theories, ref. [2491] in the form of a theory-agnostic test at every PN order. These studies relied on the Fisher matrix approximation for parameter recovery. Ref. [2492] used Bayesian inference for the LISA data with priors from ground-based instruments to simulate an analysis of dipolar radiation and graviton mass. The emerging view from these studies is clear: multi-band observations would significantly improve over the constraints that could be achieved with LISA or ET alone.

Beyond multi-band observations with LISA, a similarly synergy would exist with instruments in the Deci-Hz band. Ref. [2493] demonstrated that by combining data from ground-based observatories such as ET and CE with data from milli-Hertz observatory (e.g., LISA) or Deci-Hertz observatory (e.g. DECIGO) the parameter precision on the dipolar-radiation parameter indeed improves by several orders of magnitude. Similar improvements were also found [2494] by performing joint parameter estimation using ET and atom-interferometer gravitational-wave observatories that are also investigated for operating in the Deci-Hertz band [2431, 2495].

In cases where dipolar GW radiation happens, it is common to simultaneously have a varying gravitational constant, e.g. in the scalar-tensor theory [2486, 2496]. Ref. [2497] showed that a synergy between ET and other deci-hertz gravitational-wave observatories will be able to simultaneously test the effects from dipolar radiation and a linear-in-time evolving gravitational constant, thus constraining the underlying gravity theory from multiple angles.

A somewhat different scenario has been proposed where multi-band observations would be valuable: in presence of ultralight boson clouds around black holes, a GW detection in the LISA band could direct a search in ET for a continuous wave generated by clouds orbiting the remnant of the merger [2498].

6.4.2 Tests of the merger and ringdown

Multi-band observations with LISA and ET would provide a different view for model-agnostic tests checking the consistency of the signal in its inspiral and merger-ringdown phases, by providing independent information on the inspiral and improving the overall parameter estimation [2499]. This would be particularly important in providing a view of the inspiral of massive systems only detectable at merger in the ET band [2489, 2500].

Multi-band observations also provide an important operational input for ringdown analyses: SBHBs observed by LISA before their merger will be among the loudest targets of ground-based instruments, and an advance warning would allow us to make sure that the ground network will be fully operational and at optimal sensitivity by the time the merger occurs. It is also possible, in principle at least, to tune ground-based interferometers to achieve an increased sensitivity in a narrow frequency band that can be chosen to maximize the precision of ringdown tests for a particular binary, achieving an improvement by a factor of 2 [2501].

6.4.3 Tests of frequency-dependent speed of GWs

A particularly important consequence of the observation of the bright siren event GW170817 has been the determination of the propagation speed of GWs to be the speed of light within

10^{-15} [2, 4]. This bound applies to GWs in the frequency range probed by ground-based interferometers. However, in an effective field theory approach to dark energy, the GW frequencies probed by ground-based interferometers lie around the typical strong coupling scale of the EFT, leaving the possibility that an appropriate ultraviolet completion of dark energy scenarios could avoid present constraints on the GW speed, while still showing deviations at lower frequencies [1263, 2502]. Refs. [2502] and [2503] showed however that a single multi-band observation of a GW150914-like event by LISA and a ground-based interferometer would allow to constrain the speed of GWs to match the speed of light within 10^{-15} across a much wider frequency range.

6.4.4 Tests of the GW polarization

In modified theories of gravity, GW signals can typically feature additional modes of polarization beyond the usual transverse-traceless tensor modes $h_+, h_\times$ [2486]. In the most general case, the GW can comprise up to 6 modes, two transverse tensorial modes, two vectorial modes, a breathing scalar mode and a longitudinal scalar mode [72, 2486]. Although the dominant effect of modifications to GR tends to be the contribution to the orbital phasing, and thereby to the phase of tensorial modes, the GW itself will also be modified by the addition of these extra modes of polarization, for instance due to the additional energy loss in dipolar radiation. Since this is in essence an amplitude correction rather than a phase correction, constraints on the parameters of specific modified gravity theories are typically weak. However, checking the tensorial nature of GWs can also be seen as a theory-independent verification of the consistency of GR with the data, and different polarization hypotheses can be compared using null-stream methods [8, 96].

Because each detector (or TDI channel in the LISA case) is only sensitive at any given time to a projection of the gravitational wave, constraining additional modes of polarization requires at least as many detectors as targeted modes [84], although, for long-lived signals, the detector motion can also help in breaking degeneracies by changing the detector's pattern function over time [85]. Theory-independent null-stream methods benefit from combining multiple detectors as well [95, 97]. Overall, this test presents a strong synergy between detectors, where combining detectors in a network, and in particular combining ET with one or two CE observatories, would directly improve our ability to disentangle additional polarizations [85].

TAKE HOME MESSAGE

- SYNERGETIC OBSERVATION OF STELLAR AND INTERMEDIATE BBH SYSTEMS BY ET AND LISA CAN BOUND GR BY AN ORDER OF MAGNITUDE BETTER THAN EITHER OBSERVATORIES.
- JOINT OBSERVATION BY ET AND CE IS A UNIQUE OPPORTUNITY TO MEASURE POLARIZATION MODES OF THE WAVES AND CONSTRAIN ALTERNATIVE GRAVITY THEORIES.
- DIPOLAR-RADIATION TEST IS BOOSTED BY COMBINING ET WITH MILLI-HERTZ AND DECI-HERTZ GW OBSERVATORIES.

6.5 Hubble tension and cosmography

COSMOGRAPHY AND COSMOLOGY: SYNERGIES WITH CE AND LISA

- WHAT ARE THE KEY IMPROVEMENTS THAT A 3G NETWORK OF EARTH-BASED GW OBSERVATORIES CAN BRING TO COSMOLOGY?
- CAN THE SYNERGY AMONG GW EXPERIMENTS BE ALSO CRUCIAL FOR OTHER OBSERVATORIES, PARTICULARLY IN THE CONTEXT OF MULTI-MESSENGER ASTRONOMY?

The determination of the expansion history of the Universe largely benefits from the combination of different datasets, as in the current “concordance” scenario. The use of GW sirens provides a new player in this landscape, which will be further bolstered by the synergy among future GW observatories.

The discrepancy between local [1788, 2504–2507] and CMB-based [591] measurements of the Hubble constant H_0 either points to unresolved systematics or new physics. As such, the field of cosmology greatly benefits from new classes of cosmological sources for which it is possible to measure the Universe’s expansion. The presence of a network of next-generation GW observatories will give us access to a plethora of sources at cosmological scales, thus allowing us to corroborate the origin of the H_0 tension. In the case of new physics, the possibility of determining H_0 to sub-percent level with tracers complementary to EM and CMB observations would be of paramount importance.

Furthermore, the increased detection horizon of 3G observatories, as shown in figure 105, will give us access to thousands of GW events from the dark matter-dominated era (beyond redshift 0.3). Contrary to current generation GW observatories at their designed sensitivity, 3G generation observatories will allow us to constrain cosmological parameters beyond H_0 such as the dark matter fraction Ω_m . GWs can further allow us to probe the presence of dark energy, through the imprint of its EOS on rate of expansion of the Universe, and more importantly the phenomenon of “modified GW propagation” taking place whenever GR is modified at cosmological scales, as discussed in section 3.3.2. The coordination of ET with a network of next-generation GW observatories will significantly boost current techniques for GW cosmology and also allow for studies of GW sources as large-scale structure tracers that are currently impossible. In general, we can identify several common benefits for GW cosmology: (i) the improved distance reach allowing to target high-redshift sources; (ii) the improved localization capabilities of a network of GW observatories; (iii) the overall boost in the number of detections per observation time and (iv) the possibility of multi-band observations and constraints. We discuss how these common benefits will impact future prospects for GW cosmology.

6.5.1 Bright sirens cosmology

Localization is of crucial importance for GW cosmology. A precise localization of GW events can allow us to promptly identify a possible EM counterpart but also statistically identify the source galaxy from galaxy surveys.

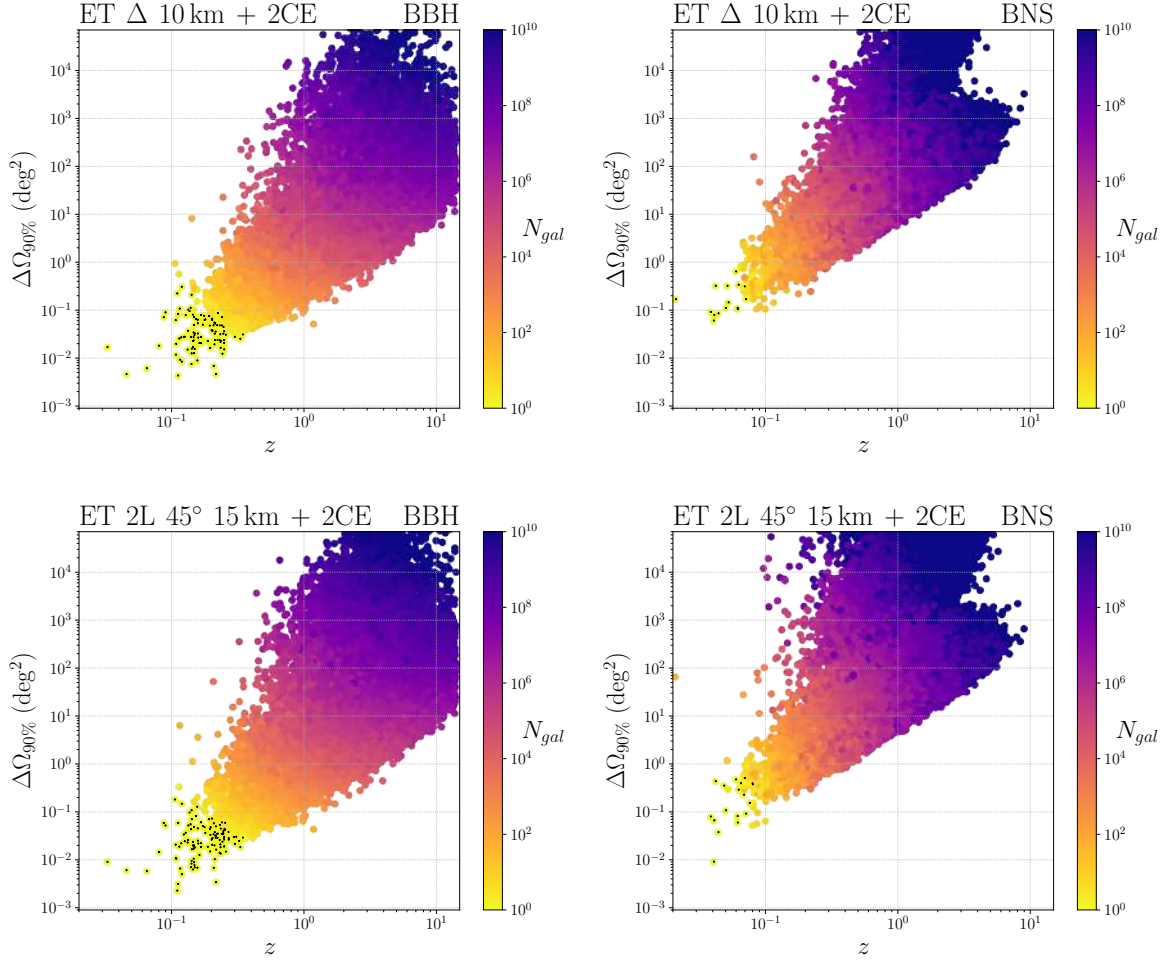


Figure 113. Localization capabilities for BBH (left panels) and BNS (right panels) of ET in its triangular and 2L configurations (first and second rows respectively), in a network with two CE. The color scale denotes the number of galaxies expected in the 90% localization volume. Events marked with black dots are localized to one galaxy only.

Figure 113 shows the localization capabilities of ET in synergy with two CE observatories, for BBH and BNS respectively.⁵⁶ In particular, we show the 90% sky localization area as a function of redshift, while the color refers to the number of galaxies contained in the localization volume, assuming a constant galaxy density of 10^{-2} galaxies/Mpc³, where the galaxy density is derived integrating the Schechter function of ref. [2508] from 10^9 to 10^{12} solar masses. Events with a single host galaxy in their localization volume are further marked with a black dot. Specifically, ET triangle (2L) operating with 2CE is able to localize within a volume containing a single galaxy (for galaxy in the above mass range), 47 (55) BBH mergers per year at redshifts below 0.4 and 17 (16) BNS mergers per year at redshifts below 0.1.⁵⁷

⁵⁶We refer to ref. [16] for the details on the catalogs and software used.

⁵⁷We underline again that these numbers are subject to uncertainties dominated by the limited knowledge of the local merger rate of compact objects. For BNS mergers, taking into account the absence, to date, of detection in the current O4 run, we can consider these numbers as an upper limit, see also footnote 21 on page 94.

In particular, BNS systems are targets for follow-up multimessenger campaigns, where sky localization plays a major role. The synergy with CE brings a dramatic improvement in localization: ref. [16] finds that the number of sources localized to better than 10 deg^2 increases from 8 to ~ 6500 for ET in the triangular configuration, and from 25 to ~ 9800 for ET in the 2L configuration; for localizations better than 100 deg^2 the number of sources increases from 184 to $\sim 10^5$ for ET in the triangular configuration, and from 559 to $\sim 1.5 \times 10^5$ for ET in the 2L configuration. As shown in figure 113 there is subset of $\mathcal{O}(10)$ well-localized GW events for which it will be possible to identify the host galaxy solely from the GW localization volume, without the need of any transient EM counterpart. This is not currently possible with current-generation GW detectors.

For the impact on the measurement of H_0 , it would already be possible to obtain sub-percent accuracy with one year of observations of ET alone (see section 6.4 of [16]), if a precise redshift measurement following the localization of the source through the kilonova emission produced by BNS mergers in the optical band can be obtained. In particular, this can be possible in synergy with the Vera Rubin Observatory [16] for sources up to redshift $z \sim 0.3 - 0.4$. The addition of two CE detectors would boost this accuracy to an unprecedented level, as well as decreasing the time required to reach such a goal (even to just a few days of observations). Even more interestingly, we note that besides H_0 , other parameters affecting the distance-redshift relation might not be constrained to better than a few percent with ET alone. In particular, ref. [16] finds that the matter density Ω_M would be constrained at $20 - 30\%$, the dark energy EOS parameter w_0 at $\geq 10\%$ and the parameter Ξ_0 describing modified GW propagation [1257, 1259] at $\sim 3\%$. Crucially, the addition of CE would allow to reach percent or sub-percent level accuracy for all these parameters, allowing to reconstruct the local expansion history in full generality (as well as to provide a stringent test of Λ CDM) rather than just the local value of the Hubble constant [1172].

6.5.2 Dark sirens cosmology

3G interferometers are expected to detect most of the stellar BBHs merging up to $z \sim 100$ [16, 1167]. Albeit no associated EM counterpart is emitted, the abundance of such dark sirens allows for statistical methods, discussed in section 3.3.1.2, to become efficient. Within this framework, the localisation power plays a crucial role in the precision that this approach can reach, as galaxy catalogues are employed to counterbalance the lack of the source's redshift [1143–1145, 1147, 1160, 1161, 1185, 1206, 2509–2511]. While ET alone might not be able to pin down the sky position of a large number GW source accurately enough, a network of ET and CE can restrict the potential hosts to a few candidates for a large number of events. A sub-percent H_0 measurement can confidently be attained by employing only the loudest dark sirens observed in 1 yr of observations [1185]. Furthermore, we expect to localise a few tens of GW events per year accurately enough that only one host galaxy falls inside the credible region (left panels of figure 113). These unique events, which contain most of the information, would constrain H_0 down to $\mathcal{O}(1\%)$ precision alone [1185, 2511] if provided with complete galaxy catalogues.

Dark sirens detected by 3G detectors will play a crucial role in cosmology even in the absence of any galaxy survey information. As discussed in section 3.3.1.2, by exploiting the

relation $m_{\text{det}} = (1 + z)m$ between detector-frame mass m_{det} and the actual (“source-frame”) mass m , it is possible to obtain a statistical redshift estimation of the source [1151, 1152] and jointly fit for the source mass spectrum and cosmological expansion parameters [1147, 1153, 1156, 1158, 1160, 1161]. Dark sirens that are employed for this type of analysis have been recently named “spectral sirens” [1156]. To reach a percent-level estimate of H_0 , we would require at least thousands of spectral sirens [1147, 1153, 1158] that could be provided at the end of the planned observing runs in 2027 for current GW detectors. While current GW detectors could already provide in the next years a percent-level measure of H_0 with spectral sirens, they will not provide a high enough number of sources to consistently check for systematics in the H_0 measure that could arise from poor calibration of mass distribution models [1200]. In contrast, 3G observatories will make accurate and precise cosmology with spectral sirens possible; 3G observatories will provide $\mathcal{O}(10^5)$ GW sources at all the cosmic times in just one year of observation, as we see also from figure 105. The large number of GW detections will allow us to perform spectral siren analyses with independent subsets of thousands of GW sources. The use of an independent subset of GW observations, divided accordingly to their mass and distance from us, will provide an in-depth understanding on the most critical assumptions that could possibly introduce systematics in the estimate of H_0 .

Another possibility is the use of GWs as tracers of the Large Scale Structures, and the extraction of cosmological information from their two-point cross-correlation with galaxy surveys [1318, 1319, 1323, 1324, 1337, 1338, 1361, 2512–2515], see the discussion in section 3.4.1. The main limitation of this method comes from the poor angular resolution of GW sources as compared to galaxies. In particular, the minimum resolution of the GW survey determines a limiting scale in the angular power spectrum [1318, 1319, 1323, 1324], $\ell_{\text{max}} \sim 180^\circ / \Delta\Omega_{\text{max}}^{1/2}$ with $\Delta\Omega_{\text{max}}$ being the maximum resolution of the instrument. While the angular resolution of ET alone corresponds to roughly $\Delta\Omega_{\text{max}} \sim 0.1 \text{ deg}^2$, hence a maximum multipole of at most ~ 300 only in a local redshift shell (up to $z \sim 0.2$), from figure 113 we see that the addition of CE could allow to reach $\Delta\Omega_{\text{max}} \lesssim 0.01 \text{ deg}^2$, corresponding to a maximum multipole of ≥ 1800 at low redshift, and a maximum multipole of ≥ 300 up to redshift $z \sim 1$. In practice, going to high multipoles would require to model non-linear scales; however, refs. [1319, 1338] find that sub-percent accuracy on H_0 could already be achieved with a cut $\ell_{\text{max}} = 300$ which is reachable with ET in combination with CE.

Finally another dark sirens method which can be applied to 3G observatories, consists in the simultaneous estimation of cosmological parameters and the EOS of NSs, as discussed in section 3.3.1.2. A network of 3G observatories will have the potential to determine the EOS of NSs with great accuracy, in turns providing an associated redshift measurements for all compact binary mergers including a NS. Such a method does not require an EM counterpart and could deliver constraints on H_0 comparable to other siren methodologies [1140, 1230, 1233, 2516].

6.5.3 Multi-band sources

The presence of multiple GW observatories covering complementary frequency ranges will open further unique possibilities. ET detection horizon is complementary, in terms of source masses, to LISA, as well as to the Deci-Hz detectors discussed in section 6.1. This

complementarity is crucial for GW cosmology for two reasons. These low-frequency detectors could provide early-warning alerts to prepare ET for the detection of the merger of BNSs and SOBBHs [2510]. They will scan the Universe for a population of BBHs with masses $\gtrsim 10^3 M_\odot$, on which we currently lack knowledge. If this population of GW sources exists, we will be able to perform GW cosmology with an independent set of data that can be used to corroborate possible systematics present in ET [2517].

In addition, there are classes of compact objects for which detections of the same source in different experiments will be possible. In particular, BBHs originating from massive stars, which exceed the pair-instability mass gap and with masses in the range $120 M_\odot - 300 M_\odot$, are considered prime dark siren candidates for GW observations across multiple frequency bands [2510, 2518, 2519]. The prolonged inspiral signals emitted by these systems can be observed by LISA, or even better by Deci-Hz detectors, while the later stages of inspiral and merger will be detected by ET. The combination of the observations from LISA and ET will improve the sky localization of these sources by a factor $\mathcal{O}(10 - 100)$ compared to those obtained with the same sources using only ET. Ref. [2510] finds that with 4 years of LISA observation this multiband approach may allow for a $\mathcal{O}(2\%)$ estimate of the Hubble constant H_0 with $\mathcal{O}(10)$ detected sources. An even better precision is expected from Deci-Hz detectors, which can follow the inspiral up to minutes before merger [2520, 2521].

TAKE HOME MESSAGE

THE SYNERGY AMONG DIFFERENT EARTH-BASED GW OBSERVATORIES CAN BE A GAME-CHANGER FOR LATE-TIME COSMOLOGY WITH RESOLVED SOURCES, DUE TO THE IMPROVED LOCALIZATION CAPABILITIES WHICH ARE CRUCIAL FOR BOTH BRIGHT AND DARK SIREN COSMOLOGY. THE INTERPLAY WITH GW OBSERVATORIES IN OTHER FREQUENCY BANDS WILL FURTHER ALLOW TO ENLARGE THE SPECTRUM OF GW SOURCES USED FOR COSMOLOGY AND TO TARGET WELL-LOCALIZED MULTI-BAND SOURCES.

6.6 The origin of supermassive black holes

THE LINK BETWEEN STELLAR AND SUPERMASSIVE BLACK HOLES

- WHAT CAN JOINT OBSERVATIONS BY ET AND LISA REVEAL ABOUT THE NATURE OF THE SEED BLACK HOLES THAT EVENTUALLY GROW SUPERMASSIVE? ARE STELLAR-MASS BLACK HOLES, FORMED AT COSMIC DAWN, THE SOLE BUILDING BLOCKS OF THE SUPERMASSIVE BLACK HOLES POWERING QUASARS OBSERVED AS EARLY AS REDSHIFT $z \sim 11$, AND NOW RESIDING ALMOST UBIQUITOUSLY IN THE CENTERS OF QUIESCENT GALAXIES?
- WHAT CAN ET AND LISA TELL US ABOUT INTERMEDIATE-MASS BLACK HOLES IN THE RANGE $10^2 - 10^5 M_\odot$, WHICH BRIDGES THE GAP BETWEEN THE STELLAR AND THE SUPERMASSIVE BLACK HOLES?

The discovery of rare quasars at redshifts as high as $z \sim 11$ [2126, 2522–2529], of the large population of active galactic nuclei (AGN) emerging at $z \lesssim 6$ and extending to

$z \sim 1$ [2530], and of dark massive objects in the nuclei of today’s quiescent galaxies [2531] provides evidence of the ubiquity of supermassive black holes (SMBHs) in the near and far Universe. Understanding the origin of SMBHs is important and goes hand in hand with understanding the formation and evolution of stars and galaxies, which is a process rooted in our current model of cosmic structure formation [2532].

The most massive BH binary currently detected by GW observations, GW231123, resulted in a final BH with a mass of about $225 M_{\odot}$ [2533], providing evidence of hierarchical assembly of heavy BHs from stellar-mass parents. ET on Earth and LISA from space will probe an immense volume of the Universe as both observatories have sensitivities to detect GW signals from merging binaries out to $z \sim 15 - 20$. This capability enables the measurement of black hole masses across the largely unexplored segment of the spectrum, ranging from about $10 M_{\odot}$ up to $10^7 M_{\odot}$, and over cosmic time, offering crucial insights into when, where and how SMBHs form from their seeds (see [2532, 2534, 2535], for detailed reviews). ET will have the capability of discovering the earliest black holes, relics of the first population III stars. Inside gas-rich regions of the host halos, some of these black holes can rapidly grow via accretion and mergers becoming viable seeds for SMBHs [2536–2539]. This establishes a close connection between stellar-mass and supermassive black holes. While ET can reveal the existence of the first seeds, LISA will trace their subsequent growth, uncovering the evolving population of accreting light seeds over cosmic time [2540].

Independent, synergetic observations of the two different populations, the stellar and massive ones, will help breaking degeneracies among different evolution pathways. This is achieved by matching, in a multi-dimensional space and within a defined theoretical framework, the information resulting from the measurements of the black hole masses, spins (when possible) and redshift of individual sources, and by confronting the relative rates of coalescence events detected by ET and LISA, respectively.

6.6.1 Seed black holes

In section 4.7 we presented a detailed overview of the latest advances in understanding the properties of the first stars, known as population III stars, forming in the earliest collapsing dark matter halos of $10^{5-6} M_{\odot}$ at $z \lesssim 30$. Their initial mass spectrum is unknown and theoretical investigations indicate that it is in the range from $1 M_{\odot}$ to several $10^2 M_{\odot}$. This population is expected to dominate at $z > 13$ and to persist down to $z \sim 6$, albeit in subdominant fractions compared to the growing population II stars. Despite the increasing number of hints, recently provided by the JWST telescope, on possible indications of population III star’s signatures in the spectra of high redshift galaxies or in lensed sources [2003, 2004, 2541, 2542], direct EM observations of single stars are beyond foreseen capabilities.

The properties of population III stars are encoded in those of their relics, i.e. the black holes that ET can uncover, see section 4.7. The role of ET is therefore central in inferring the properties of these first stars, but not only. These early black holes are expected to represent the channel of formation of the light seeds of up to a few to several $10^2 M_{\odot}$ [2532]. These may not be the only seeds to grow SMBHs. Heavy seeds of $10^{3-5} M_{\odot}$ have been also hypothesized, resulting either from direct collapse of massive gas clouds, via runaway stellar mergers or hierarchical black hole mergers [1551, 2537]. LISA can observe these heavy

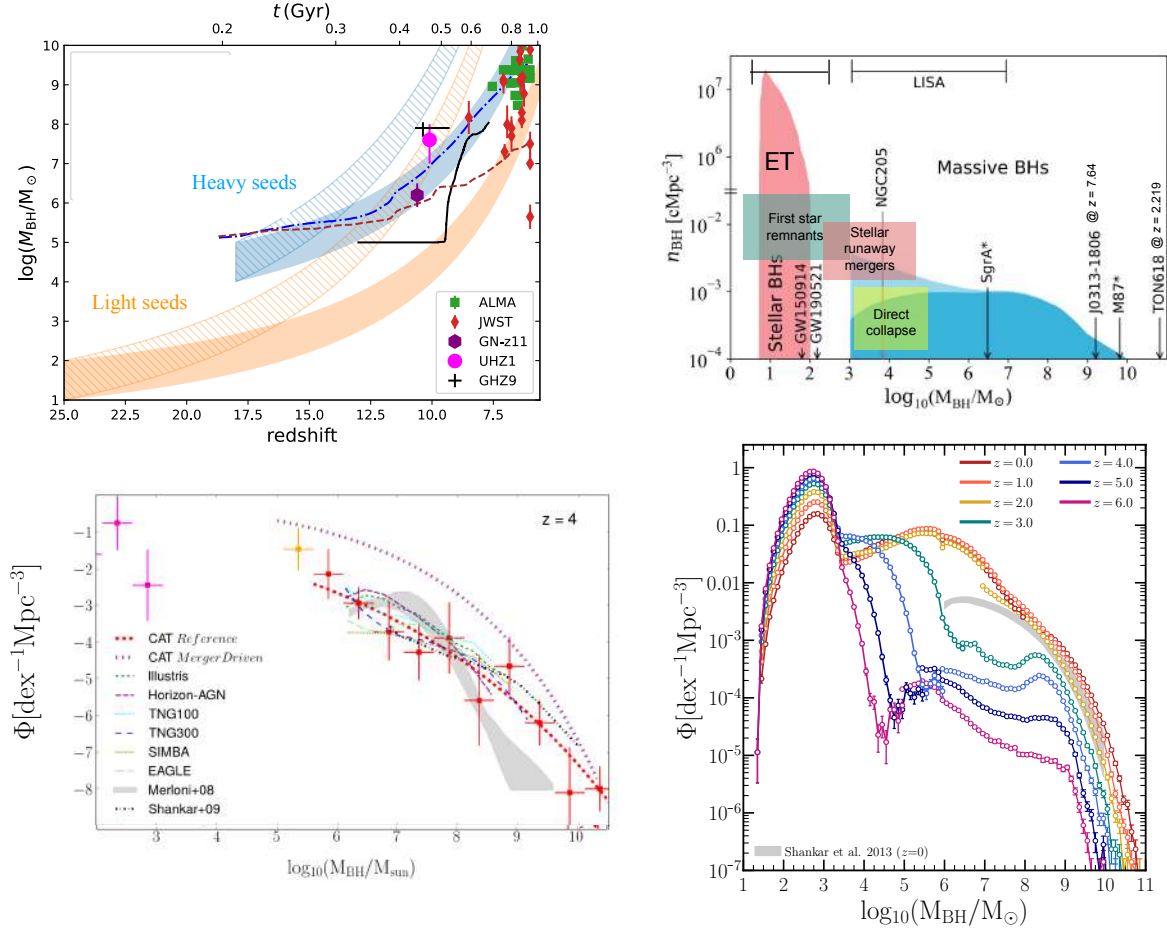


Figure 114. *Upper left panel:* tracks of BH growth, for different initial *seed* masses assuming uninterrupted accretion at the Eddington rate and at rates twice above this limit. The different colored symbols labeled in the figure refer to $z > 6$ observations of quasars by ALMA (green squares [2543]) and JWST (red diamonds [2524, 2528, 2544–2546]), including the $z \sim 10$ massive black holes in UHZ1 (magenta circle [2126]), GHZ9 (black cross [2527]) and the candidate super-Eddington accreting black hole in GN–z11 (violet hexagon [2127]). Light seeds can explain these quasars if they accrete at a super-Eddington rate. Heavy seeds, forming at lower redshifts, do not necessarily need sustained super-Eddington accretion. There is a debate whether sustained accretion above the Eddington limit is likely in high-redshift galaxies [2529, 2539, 2547–2550]. The black, blue and red tracks refer to state-of-the-art high-resolution cosmological zoom-in simulations [2550, 2551] where a $10^5 M_{\odot}$ black hole was implanted at $z = 10$ and let evolve including accretion and feedback. *Upper right panel:* cartoon sketching the mass spectrum of black holes and the different formation paths. Arrows and labels indicate selected quasars, the Milky Way black hole, Sgr A*, [2552, 2553] M87 [2554], and the post-merger mass of GW150914 and GW190521, the first and the most massive coalescence events detected to date by LIGO, respectively [1509, 2555]. Shown are the windows of exploration of ET and LISA (adapted from [2532]. Reproduced from [2532], with permission from Springer Nature.). *Lower left panel:* BH mass spectrum at $z = 4$ from [2539] as inferred from a suite of semi-analytical models and numerical simulations to show the still large theoretical uncertainties in predicting BH evolution. Data points show the distribution of light (magenta) and heavy (red, orange) BH seeds, with the best fit for the distribution of heavy seeds in red and violet dotted lines, and the results from large-scale cosmological simulations: Illustris (dark green, cyan, and blue [2556, 2557]), SIMBA (olive [2558]), and EAGLE (light green [2559]). The observational constraints by [2560] and [2561] are shown in the grey shaded area and black dashed-dotted lines. Reproduced with permission from [2539]. *Lower right panel:* evolution of the BH mass function from the semi-analytical model L-Galaxies [2562]. The figure shows how the intermediate-mass range is being progressively filled due to the growth of light seeds from population III relics (Courtesy of Izquierdo Villalba).

seeds, whose relative contribution in explaining the formation of high- z quasars ($z > 4$) is under debate [2534, 2535, 2563].

The upper left panel of figure 114 illustrates the concept of black hole growth from light seeds and heavy seeds in selected mass ranges. In the same figure (upper right panel) we show the extension of the BH mass spectrum, known as of today, indicating the main formation pathways. Note the presence of a desert around $10^{2-3} M_{\odot}$, and the uncertainty in the rise or decline of mass distribution around $10^{3-5} M_{\odot}$, where intermediate mass black holes (IMBHs) sit [2090, 2091]. IMBHs are in the midst of the high-end tail of the stellar BH mass distribution and the low-end tail of the SMBH mass distribution. At present, whether a discontinuity exists in the families of stellar and supermassive BHs, related to the existence of uncorrelated physical processes, is not known. In this regard, the lower right panel of figure 114 shows the evolution of the BH mass distribution as a function of redshift from the semi-analytical cosmological model L-Galaxies [2562] that describes the formation and evolution of SMBHs from light and heavy seeds. This model indicates that the IMBH segment is being progressively filled by growing light seeds which evolve to account for the vast population of AGN, with heavy seeds partially explaining the rare high- z quasars.

There are large theoretical uncertainties related to the way the seeding process and seed growth is treated in inferring the BH mass spectrum as a function of redshift. This is illustrated in the left lower panel of figure 114, taken from [2539]. Different models give different predictions on the shape and normalization of the mass function [2539, 2564–2568]. Therefore, future GW observations will be key to help reducing these uncertainties, jointly with forthcoming deeper studies and observations of quasars and AGN in the EM sector.

6.6.2 Synergies between ET and LISA

Participating in the formation, growth, and assembly of cosmic structures, massive black hole in merging galaxies form binaries that coalesce, becoming sources of low-frequency GWs [330, 512]. LISA, operating from below 0.1 mHz to about 1 Hz, aims at detecting the signals from these sources in the segment of the mass spectrum between about $10^4 M_{\odot}$ and $10^7 M_{\odot}$, out to redshift $z \sim 10 - 15$. The LISA’s cosmic horizon together with ET’s horizon are shown in figure 115.

Signals. The GW signals detectable by LISA encode key information on the black hole source-frame masses, luminosity distance, and when sufficiently loud the spins [330]. GW signals from binaries of $\sim 10^4 - 10^5 M_{\odot}$ feature an inspiral phase lasting several months before merger, with median signal-to-noise ratios (SNRs) typically reaching up to $\lesssim 100$. The source-frame masses in this case can be determined with uncertainties from 10% to 100%. ET will measure with comparable accuracies the physical parameters of potential light seeds, whose signal display only few inspiral-cycles before final plunge [1181]. Below $z \sim 8$, the inspiral, merger and ringdown signal of LISA’s binaries contains all information to enable accurate measurements of the source-frame masses at the level of 10% and effective spin with accuracy of 1%. At $z \sim 2$, SNRs are as large as a few thousands, with black hole masses and luminosity distances measured with accuracy at the level of 0.1%, and the spin

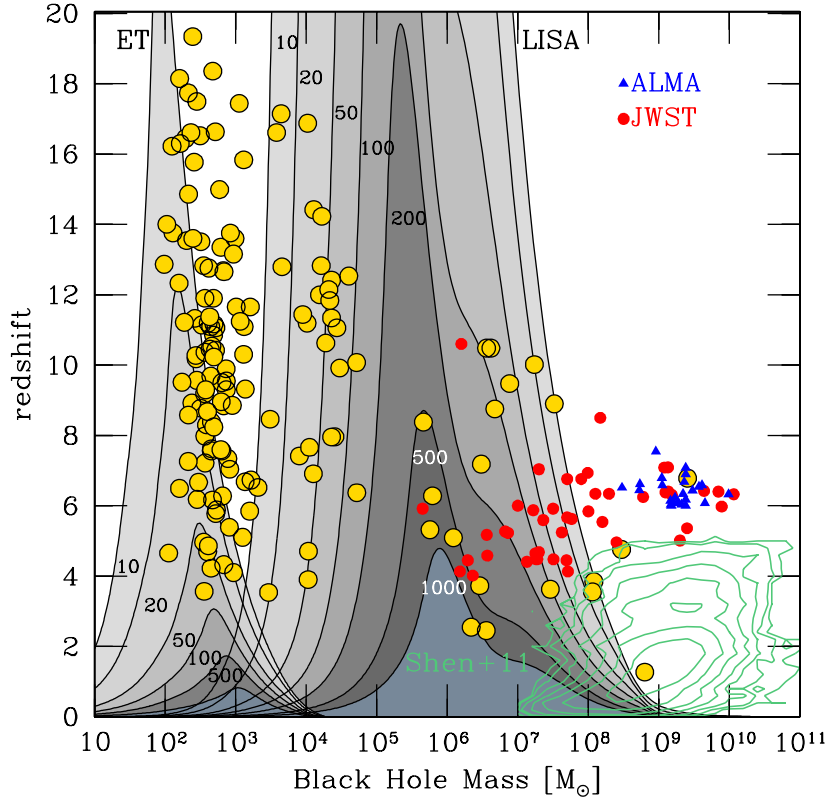


Figure 115. The ET and LISA cosmic horizons. The figure shows contour lines of constant signal-to-noise ratio (SNR) in the plane redshift versus total mass of the black hole binaries, as measured in the source frame. Binaries have all mass ratio 0.5. Yellow dots denote loci of BH mergers extracted from the semi-analytical model of [2540], which tracks the evolution of light and heavy seeds forming in merging halos. Light seeds form at $z \sim 20 - 30$ as relics of population III stars in molecular cooling dark matter halos of $10^6 M_\odot$. Heavy seeds form later ($z \sim 10 - 20$) in atomic cooling halos of $10^8 M_\odot$ under contrived conditions (see the review by [2535] and references therein). The bulk of the merging black holes in LISA come from the growing light seed population that ET can detect when seeds form in binaries. Red dots and blue triangles denote the quasars observed at $4 < z < 11$, from JWST [2524, 2528, 2544–2546] and ALMA [2543], respectively. Green contour lines depicting the vast population of AGN are from [2530].

of the primary black hole, which provides clues on the way accretion proceeded [512, 1550], with an absolute error of 1 part in 10^3 [330].⁵⁸

Rates. The ET merger rate density associated to population III stars (and in turn to potential seeds) has been estimated to range from 0.1 to about $200 \text{ yr}^{-1} \text{ Gpc}^{-3}$, extending from $z \sim 15$ to 6, corresponding to a detection rate of about ten to hundreds of events per year in ET

⁵⁸It should be observed that many LISA massive black hole mergers are expected to occur in gas-rich (“dirty”) environments. Gas-induced effects, such as those from accretion disks, generally introduce waveform dephasing measurable during the end of the inspiral, the merger, and post-merger phases. This is known to be critical for extreme mass ratio inspirals (EMRIs), which however are sources unique to LISA. For the high-redshift massive black holes observed by LISA during their long inspiral, and which could then later merge in the ET band, these environmental effects are smaller than the statistical uncertainties. For detailed discussions on environmental effects in GW astrophysics, see [2569].

(e.g. [1673, 1674, 2049, 2056, 2063, 2071, 2073, 2570]). The LISA merger rates range between a few to several tens per year, with the uncertainties in these values rooted in the largely unknown process of pairing of massive black holes down to the milli-parsec scale below which GWs drive the late inspiral and final plunge [2540, 2562, 2565, 2571–2575]. The physics of binary formation in LISA is completely different from that of ET stellar binaries, as it occurs via galaxy mergers: orbit’s sinking in a galactic collision is controlled by the complex interaction black holes have with their surrounding dark matter, stars and gas, during a major or/and minor galaxy merger [512]. LISA rates at $z \gtrsim 8$ depends largely on how short is the sinking time compared to the “then” Hubble time, and depending on the model rates range again from a few to several per year [2562, 2574, 2575].

ET and LISA together. In figure 115 we show the synergy between ET and LISA. In [2540], a semi-analytical cosmological model was developed, aimed at reproducing typical quasars observed at redshift 8 and 2. The seeding mechanisms include both light and heavy seeds and merging binaries (yellow dots in the figure) form during halo-halo mergers, under the hypothesis that dynamical interactions lead to their formation and pairing. The model shows that most of the LISA events are rooted in the process of formation of stellar black holes from population III stars, with only a few resulting from heavy seeds.

Comparing GW merger events of $\sim 10^2 M_\odot$ binaries at $z \sim 10 - 14$ detected by ET, with events of $\sim 10^4 - 10^5 M_\odot$ at comparable or lower redshifts with LISA (just on the edge of the left side of the LISA waterfall plot shown in figure 115) paves the way for identifying light seeds as progenitors of SMBHs. Lack of ET high- z events around a few $10^2 M_\odot$ could indicate that either they rarely form, or that they rarely pair in binaries which merge shortly after. From the LISA perspective this might indicate that heavy seeds only are the progenitor of the SMBHs or that light seeds grow at a very fast (super-Eddington) rate, following their formation without experiencing mergers. All of this needs to be quantified through careful modeling of black hole cosmic formation and evolution.

6.6.3 IMBH populations across frequency bandwidths and detectors

Intermediate mass black holes (IMBHs) in the mass range between a few hundreds $M_\odot$ and $\mathcal{O}(10^5) M_\odot$, bridging the known stellar and supermassive BHs, are ideal sources to exploit the advantages of having more than one detector in different frequency bands. As described in section 4.8, IMBHs are expected to form as outcome of close and repeated dynamical encounters among stars or stellar black holes inside dense and massive clusters [1491, 2129, 2131]. IMBHs can also form in the collapse of gaseous clouds in protogalaxies [2576], or from gaseous accretion onto massive stellar remnants originating from the first stars [2577]. Despite the plethora of formation scenarios and the growing evidence of IMBH candidates in dwarf galaxies [2090, 2091], the existence of IMBHs and their role in galaxies, whether ubiquitous or rare, is still unclear and actively debated. In these regards, the detection of GWs from merging IMBHs is key to understanding where, when, and how IMBHs form along cosmic history.

ET can detect equal-mass IMBH binaries with source-frame masses of $\lesssim 10^3 M_\odot$ at redshift $z \lesssim 7$ [16], and IMBH-BH systems having a stellar BH and a primary of $\sim 10^3 M_\odot$, at redshift $z \gtrsim 1$ [1538]. Detecting IMBH and IMBH-BH binaries (the last referred also as

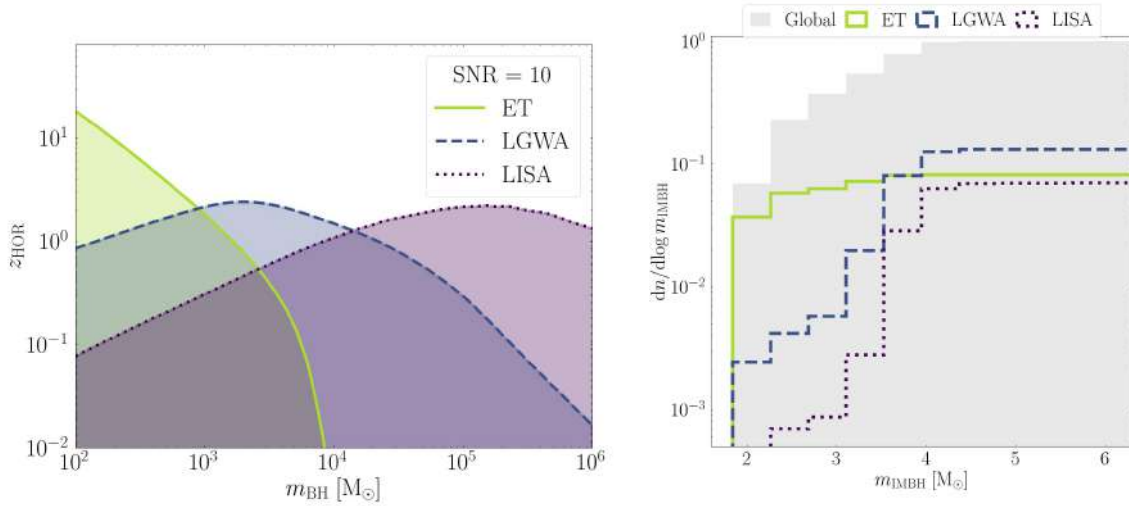


Figure 116. *Left panel:* horizon redshift for IMRIs involving a stellar BH of $30M_{\odot}$ as a function of the mass of the more massive primary m_{IMBH} . Detection is set at $\text{SNR}=15$ using the GWFISH tool [291] for ET (green straight curve), LGWA (blue dashed curve), and LISA (purple dotted curve). *Right panel:* cumulative mass distribution of IMRI detection versus the IMBH mass. These systems are dynamically formed in young, globular, and nuclear clusters and their overall population is shown by the filled grey histogram. The detectable sub-population of IMRIs are shown for ET, LGWA and LISA (as in the legend). Models are obtained from combined population synthesis simulations performed with the BPOP code [1528] and parameter estimation performed with GWFISH. Adapted from Arca Sedda (in prep).

intermediate-mass ratio inspirals or IMRIs) will provide us information about the efficiency of dynamical processes in population I, II, and III star clusters in producing IMBHs.

As discussed above (see also figure 115), the horizon reach of ET for IMBHs of mass close to $(1 - 5) \times 10^2 M_{\odot}$ extends up to $z \sim 10$ [16, 1181, 1569]. However, binary IMBHs can also be present at lower redshifts. With ET operating with two CEs in network, we can measure the mass of an equal-mass IMBH binary of $\sim 10^3 M_{\odot}$ with accuracy of $\lesssim 0.1\%$ (1%) at redshift $z = 0.5$ (2) [1569]. Such a network can localise binaries at $z = 0.1 - 0.5$ with mass ratio $q \sim 0.1$ and primary mass of $\sim (5 \times 10^2 - 10^3) M_{\odot}$ within an angular localization $\Delta\Omega \sim (0.01 - 1) \text{ deg}^2$ [1569]. Good localisation and accurate measurements of the properties of merging components can help distinguishing both the environment nurturing the IMBH growth and the IMBH formation channel.

At $z \lesssim 1$, ET can detect signals from binaries made of an IMBH and a stellar BH, which could in principle also be detected by low-frequency observatories such as LISA or Deci-Hz detectors. This is illustrated in figure 104. IMRI binaries are exquisite probes of the physics of formation of IMBH in dense environments as these systems form exclusively from the dynamical channel. Combining different detectors operating independently at different frequencies makes it possible to cover the IMBH mass range fully and to place stringent constraints on the IMBH mass distribution, shedding light on the origin of IMBHs. As an example, the right panel of figure 116 shows the mass distribution of a population of IMRIs resulting from dynamical interaction in dense young, globular, and nuclear clusters modeled

with the B-POP population synthesis model [1528] and the corresponding distribution of detectable mergers in ET and other detectors, while the left panel of the figure shows the horizon redshift for ET, LISA, and LGWA in the case of an IMRI hosting a stellar BH of $30 M_{\odot}$ inspiralling/merging with a primary in the mass-range $(10^2 - 10^6) M_{\odot}$. As shown in figure 116, an IMRI can be detected during the inspral phase in LISA and at merger in LGWA. The cumulative mass distribution for this selected IMRI (right panel of figure 116) shows the saturation mass in the three detectors. Finally, it is interesting to remark that with LGWA and Deci-Hz detectors we have the possibility to probe the region of the IMBH mass spectrum that is poorly covered by ET or LISA, i.e. roughly $(5 \times 10^3 - 10^4) M_{\odot}$ [1470, 2412, 2578].

Joint multi-band observations of the same event will be possible for $(10^2 - 10^4) M_{\odot}$ binaries out to redshift $z \sim 2 - 4$, enhancing the ability to carry on precise measurements of the source parameters at such high redshifts [2579]. Multi-band detection of distant lower (higher) mass binaries would be instead limited by the sensitivity for LISA at frequencies around and above 0.1 Hz (for ET/CE at frequencies around and below 3 Hz).

TAKE HOME MESSAGE

- ET WILL BE UNIQUELY CAPABLE OF DETECTING BLACK HOLE RELICS FROM THE FIRST GENERATION OF MASSIVE STARS. ITS SENSITIVITY WILL ALLOW GOOD MEASUREMENTS OF THEIR MASSES AND REDSHIFTS, OFFERING VALUABLE INSIGHTS INTO THE FORMATION OF PRISTINE STARS IN BINARY SYSTEMS. SOME OF THESE EARLY BLACK HOLES MAY SERVE AS “LIGHT” SEEDS IN THE HIERARCHICAL GROWTH OF MASSIVE AND SUPERMASSIVE BLACK HOLES.
- BY COMPARING MERGER EVENTS OBSERVED BY ET AND LISA, EACH SENSITIVE TO DIFFERENT MASS INTERVALS — AROUND $100 M_{\odot}$ FOR ET AND $10^5 M_{\odot}$ FOR LISA —IT WILL BE POSSIBLE TO STATISTICALLY PROBE THE MECHANISMS OF BLACK HOLE SEEDING AND GROWTH.
- INTERMEDIATE-MASS BLACK HOLES OF $10^3 - 10^4 M_{\odot}$ PAIRED WITH STELLAR BLACK HOLE COMPANIONS, ARE UNIQUE PROBES OF THE DYNAMICAL FORMATION CHANNEL. ET, IN SYNERGY WITH LOWER-FREQUENCY DETECTORS, OFFERS A DISTINCTIVE OPPORTUNITY TO DETECT THESE STILL ELUSIVE BLACK HOLES.

6.7 Early Universe cosmology

COSMOLOGICAL BACKGROUNDS

- WHAT PARTICLE PHYSICS MODELS PREDICT EARLY UNIVERSE PHASE TRANSITIONS OR THE FORMATION OF COSMIC STRINGS? CAN WE PRECISELY PIN DOWN THE PROPERTIES OF SUCH MODELS BY MEASURING THE GWs THEY PRODUCE, USING ET IN SYNERGY WITH OTHER GW OBSERVATORIES?
- WHAT IS THE SOURCE OF COSMIC INFLATION? CAN IT GENERATE A BROAD SPECTRUM OF SGWB WITH A SUFFICIENTLY LARGE AMPLITUDE TO BE DETECTED BY ET, AND BY OTHER GW EXPERIMENTS OPERATING AT LOWER FREQUENCIES?
- CAN WE EXTRACT INFORMATION ON THE EARLY UNIVERSE COSMIC HISTORY (BEFORE BIG BANG NUCLEOSYNTHESIS) BY PRECISELY MEASURING THE FREQUENCY DEPENDENCE OF THE STOCHASTIC GW BACKGROUND PRODUCED BY PRIMORDIAL SOURCES? DOES A SYNERGY BETWEEN ET AND OTHER INSTRUMENTS OFFER ANY ADVANTAGE IN ANSWERING THIS QUESTION?
- WHAT IS THE SOURCE OF THE GW BACKGROUND FOR WHICH EVIDENCE WAS RECENTLY FOUND BY SEVERAL PTA COLLABORATIONS? IF IT HAS A PRIMORDIAL ORIGIN IN TERMS OF COSMIC PHASE TRANSITIONS, INFLATION, OR COSMIC STRINGS, CAN A FUTURE DETECTION BY ET HELP IN CHARACTERIZING ITS ORIGIN?

Stochastic GW backgrounds can be detected across various frequency bands and have an astrophysical or cosmological origin [60, 537–540]. In particular, as discussed in detail in section 3.2, the early Universe immediately after the Big Bang could create a stochastic GW background (SGWB), through mechanisms such as the amplification of vacuum fluctuations in models beyond the single-field slow-roll paradigm, first-order phase transitions, cosmic strings, or domain walls. From the observed spectrum of the backgrounds, ET could learn about the physical processes that generated them and hence obtain a picture of the Universe at the earliest moments of its creation. The sensitivity to stochastic backgrounds of ET as a single observatory, both in the triangle and in the 2L configurations, are discussed in detail in section 10.5, see in particular figure 195, and in section 11.6 where, for the triangle configuration, is also discussed the use of the null stream, see section 11.6.1, as well as the potential caveat of the masking effect of correlated noise, see section 11.6.2. The correlation of ET with other ground-based observatories would help rule out whether the observed backgrounds originate from instrumental or environmental disturbances. For example, ET in conjunction with Cosmic Explorer will be able to identify and characterize most binary coalescence events, subtract them from the data, and potentially measure the underlying primordial background (although the accumulation of the errors in the subtraction of resolved sources will also produce an important foreground [642, 643, 2453], see also section 10.5.3).

Several mechanisms that generate stochastic GW background produce a spectrum that extends over many decades in frequency. The use of observatories operating in very different frequency bands is then crucial for characterizing them (as well as for increasing the confidence in a detection). Working with PTAs, the 10-year long GAIA database, LISA, as well as with Deci-Hz detectors, ET could help understand stochastic backgrounds over six orders of magnitude in frequency range from nanohertz to kilohertz. In this section, we explore the synergies between ET and LISA (section 6.7.1), and ET and PTA (section 6.7.2) towards the aim of a joint detection of the cosmological SGWB.

6.7.1 Synergies between ET and LISA

Although ET and LISA operate across different frequency ranges, both are expected to have comparable integrated sensitivities to the amplitude of a SGWB. In this subsection we explore the potential synergies between these two observatories for analyzing GW sources from the early Universe that generate a SGWB with a high amplitude and a broad frequency spectrum. In combining the results of LISA and ET, we are able to cover a much larger range of frequencies, of order

$$\mathcal{O}(10^{-5}) \text{ Hz} \leq f \leq \mathcal{O}(10^3) \text{ Hz}. \quad (6.1)$$

As discussed in section 3.2, SGWB sources that could produce an observable stochastic GW signal in this range include first order phase transitions, cosmic strings, and inflation (beyond single-field slow roll). For such broad signals, combining data from both LISA and ET offers a clearer signal characterisation and more precise extraction of physical properties than using either instrument alone, since we can study the signal across the full frequency band in eq. (6.1).

We analyse this possibility using Fisher forecasts to assess the detectability of the SGWB shape in synergy among the two instruments. We consider a Gaussian likelihood on the GW energy density Ω_{GW} with a covariance matrix given by a combination of the nominal sensitivity curves for LISA [647] and ET in its triangle configuration. The Fisher plots are produced using the `GetDist` package [2580]. For the theoretical SGWB frequency profile, a useful SGWB template is the broken power law (BPL) function. This model provides a good first approximation for GW spectra generated by various early Universe phenomena [2581]. We follow ref. [702] for the form of the BPL function

$$\Omega_{\text{GW}}^{\text{th}}(f) = \Omega_{\star} \left(\frac{f}{f_{\star}} \right)^{n_1} \left[\frac{1}{2} + \frac{1}{2} \left(\frac{f}{f_{\star}} \right)^{\sigma} \right]^{\frac{n_2 - n_1}{\sigma}}, \quad (6.2)$$

where $f_{\star}$ determines the break position, and $\Omega_{\star}$ is the amplitude of the spectrum at the break. The parameters n_1 and n_2 correspond to the spectral indices before and after the break, while σ controls the break's smoothness. We use the BBN bound $\Omega_{\text{GW}} \leq 1.7 \times 10^{-6}$ [691] as the reference for the SGWB's maximal amplitude, even when considering post-BBN sources. The previous profile will be applied to investigate GW from phase transitions and cosmic strings; a different, log-normal Ansatz will be considered for the case of inflation. We show in figure 117 the benchmark scenarios for the examples we are going to discuss next, together with the nominal and power law sensitivity curves for LISA, and for ET in the triangle

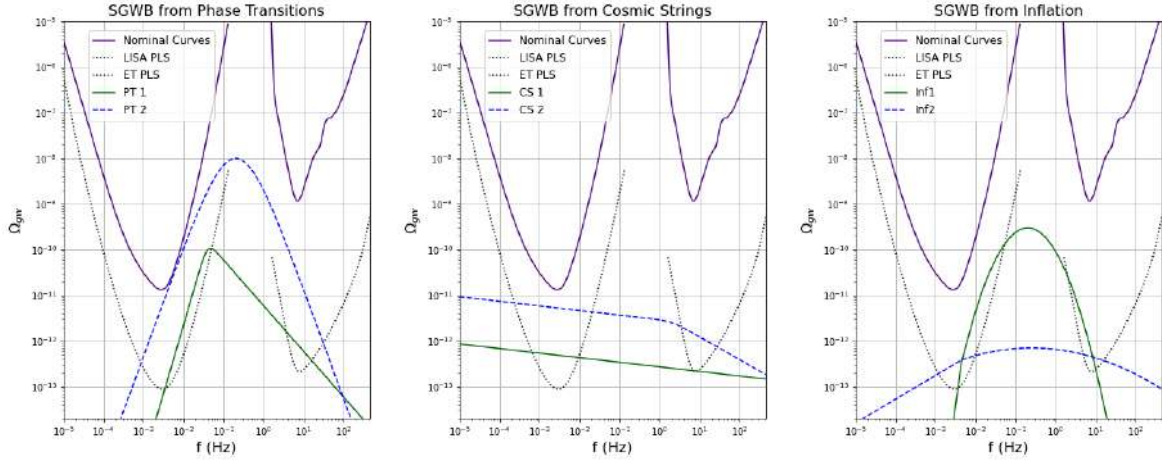


Figure 117. *Left panel:* examples of SGWB from phase transitions (PT). *Middle panel:* examples of SGWB from cosmic strings (CS). *Right panel:* examples of SGWB from cosmic inflation. Besides LISA and ET sensitivity curves (PSD) and the corresponding PLS (see eq. (3.21) and section 10.5.2 for definitions), in blue and green lines are represented the benchmark scenarios discussed in each of the aforementioned sections. See ref. [2605]. Reprinted figure with permission from [2605], Copyright (2025) by the American Physical Society.

	$\Omega_\star$	n_1	n_2	σ	$f_\star$
PT1	1×10^{-10}	3	-1	7.2	0.04
PT2	1×10^{-8}	2.4	-2.4	1.2	0.2
CS1	4×10^{-13}	-0.1	-0.1	—	0.02
CS2	2.5×10^{-12}	-0.1	$-\frac{1}{2}$	3	2

	$\Omega_\star$	$f_\star$	ρ
Inf1	3×10^{-10}	0.2	0.45
Inf2	7×10^{-13}	0.25	1.6

Table 11. *Left:* benchmark values associated with Ansatz (6.2) for the scenarios corresponding to cosmological phase transitions and cosmic string models. *Right:* benchmark values for cosmological inflation scenarios described by Ansatz (6.3). See ref. [2605]. Reprinted figure with permission from [2605], Copyright (2025) by the American Physical Society.

configuration (using the ET-D sensitivity curve).⁵⁹ For more details about our scenarios and our methods of analysis, see ref. [2582].

Phase transitions. The production of stochastic GW background from phase transitions has been discussed in section 3.2.2. In particular, in the GWs produced by bubble collisions, the SGWB follows the BPL profile [702], as in eq. (6.2) or similar parametrizations such as eq. (3.46). Other mechanisms that produce GWs from PTs, such as sound waves in primordial plasma [2583, 2584] and turbulent motion [2585–2587], rather follow a double broken power-law profile, see [702]. However, the focus here is on the overall structure across the broad frequency band in eq. (6.1), and so we use the BPL template eq. (6.2) as a reasonable approximation.

⁵⁹See figure 195 for the ET PLS with the updated sensitivity curve used in [16] as well as for the PLS of the 2L configurations, with different relative angles and arm lengths.

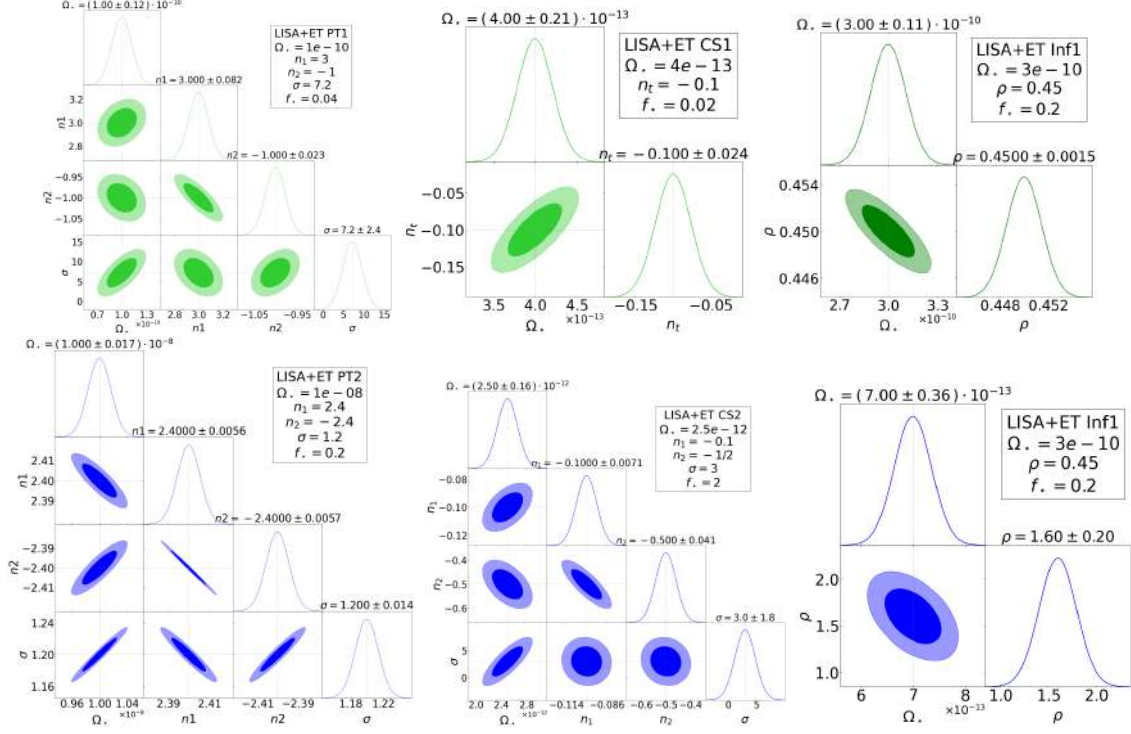


Figure 118. First and second column: Fisher forecasts for the phase transition and cosmic string scenarios. Third column: Fisher forecasts for the inflation benchmark scenarios. See table 11. See ref. [2605]. Reprinted figure with permission from [2605], Copyright (2025) by the American Physical Society.

We consider the scenarios in table 11: scenario PT1 has spectral index $n_1 = 3$ in the infrared [1019] and $n_2 = -1$ in the UV. This scenario matches cases where GWs are produced by sound waves of the bubble surrounding plasma, or by effects of turbulent behavior in the fluid. For the second benchmark scenario, PT2, we consider the model of ref. [757] in the context of highly relativistic fluid shell dynamics, which finds $n_1 = -n_2 \simeq 2.4$ and $\sigma \simeq 1.2$. These scenarios are shown in the left panel of figure 117. Notice that the SGWB amplitude is well below the nominal sensitivity curves (i.e. the Power Spectral Density, PSD): nevertheless, the SGWB can be detected with a sufficient signal-to-noise ratio by integrating over frequencies. In fact, in all panels of figure 117, besides the PSD, we also include the power-law integrated sensitivity curves (PLS) introduced in ref. [564] (see also section 10.5.2), which exploit the aforementioned integration over frequencies. In plotting the curves, we assume three years of observation, and we take a threshold of detection corresponding to $\text{SNR} = 5$.

For both scenarios PT1 and PT2 in figure 118, LISA and ET individually can accurately measure only one of the spectral indices n_1 or n_2 (for more details and additional Fisher plots comparing different cases, see ref. [2582]). However, the two experiments together can measure both quantities with at least 10% accuracy. The parameter σ , which controls the smoothness of the transition, can also be measured more precisely with combined data. This highlights that using LISA and ET together provides richer insights into high-temperature PT physics, also reducing parameter degeneracies.

Cosmic strings. As discussed in section 3.2.3, cosmic strings form networks where interactions create loops of varying sizes, which decay and generate a SGWB spanning a broad frequency range. Determining the spectral tilts n_1 , n_2 and the break position will provide insights into both the cosmic string properties and the Universe’s evolution before BBN. In fact, by analysing the spectrum’s frequency profile, we can also be able to constrain non-standard pre-BBN cosmic epochs. For normal cosmological expansions, we expect the SGWB from cosmic strings to approximately follow a power law profile in the band of eq. (6.1): this corresponds to our benchmark scenario CS1 of table 11 (see e.g. ref. [2581]). We obtain the values for the benchmark scenario CS2 from [2588], corresponding to a spectral tilt being dependent on the equation of state w in a non-standard cosmological expansion, where $n_2 = -2(3w - 1)/(3w + 1)$ for values where $w \geq 1/4$. The middle panel of figure 117 illustrates the benchmark values used in our CS cases. Only with both instruments together can the full set of parameters be measured within 10% accuracy. See figure 118, middle column, and ref. [2582] for more details.

Cosmic inflation. As discussed in section 3.2.1, see in particular figure 26 while the most minimal inflation models based on single-field slow-roll predict a SGWB amplitude too small for both ET and LISA to detect, several well-motivated scenarios can boost the amplitude to observable levels within the frequency band given by eq. (6.1). These scenarios include dynamics of vector and axion fields [601, 603]; spontaneous space-time symmetry breaking [2589–2591]; and effects associated with the production of primordial black holes [1013, 1014]. Generally, a SGWB from inflation exhibits a more complex frequency profile than those from other sources, potentially featuring log-normal shapes, multiple peaks, or oscillatory patterns (see ref. [648] for a discussion of various inflation templates.). We consider the so-called log-normal peak Ansatz:

$$\Omega_{\text{GW}} = \Omega_{\star} \exp \left\{ \left[-\frac{\ln^2(f/f_{\star})}{2\rho^2} \right] \right\}, \quad (6.3)$$

where $\Omega_{\star}$, $f_{\star}$, and ρ control the amplitude, position, and smoothness of the peak. This scenario can be the result of axion or axion spectator inflation models [608, 611, 688]. Table 11 gives the values used in our benchmark scenarios, which are also shown in the right panel of figure 117. The corresponding Fisher forecasts are presented in figure 118, third column. Particularly for scenario Inf2 we learn there are significant benefits from the combining the results of the two instruments for reducing the errors in the measurements (again see ref. [2582] for a more detailed analysis).

6.7.2 Synergies between ET and PTAs

A primordial SGWB can span a large range of frequencies, from Cosmic Microwave Background frequencies of 10^{-17} Hz, Pulsar Timing Array (PTA) frequencies of around 10^{-9} Hz, LISA frequencies in the milli-Hertz range as well as ground-based observatories above a few Hz and up to a few kHz. ET observations can thus be studied in synergy with CMB, PTA and LISA observations. Specific scenarios are however different for each type of cosmological background, e.g. inflation, first-order phase transitions, cosmic strings, domain walls or second-order scalar perturbations associated with primordial black holes: see for example figure 17 of [2592].

Single field slow-roll inflation is detectable by CMB instruments such as Planck at the lowest frequencies, but its red tilt implies that it cannot be detected by either PTAs, LISA or ET, see figure 26. However, as we discussed in section 3.2.1 other, more complex inflationary scenarios can produce a GW spectrum with a positive spectral index n_t (blue tilt), making them potentially detectable by PTAs, LISA and ET. Ref. [2593] used actual CMB, PTA and LIGO measurements to place constraints on a SGWB of inflationary origin, in particular on n_t and the tensor-to-scalar ratio r (characterizing the amplitude of a primordial GW signal as compared to the amplitude of scalar perturbations as detected in the CMB). This is an example of how different instruments across the GW spectrum can be used to place constraints on a SGWB.

More recently, in 2023, PTA results across the globe have shown evidence for a SGWB [2594–2598]. Work is in progress to confirm these results with a 5σ significance. If confirmed as a true SGWB, the most likely source is a background of supermassive black holes. In this case, because of the completely different masses, there would be little synergy between PTAs and ET (but there would be synergy between PTAs and LISA in the sense that PTA sources can inform expected LISA findings on massive black hole binaries). However, another possible source is a cosmological background. In refs. [421, 809], the observed signal was interpreted as a cosmological background and the spectral index n_t and tensor-to-scalar ratio r were estimated. While, in principle, these estimates could be used to search for this same background in the ET data, the observed spectral index in PTA data is quite high and not compatible with the high frequencies of ET if the spectral index is assumed to be fixed; however, in the case of a running index across GW frequencies, a PTA background could be compatible with a background in ET. Another possibility is a non-conventional cosmological history with a period characterised by a stiff equation of state [1074]. Additionally, a CMB detection of a GW background would be groundbreaking; a combined detection of a primordial SGWB in both the CMB and PTA data could make it easier to find the same signal in ET. However, it is much more likely that in PTA data, the cosmological background is present as a secondary background with a foreground of supermassive black hole binaries. Work is ongoing to model a search for both backgrounds. Cosmic strings, domain walls, first-order phase transitions and second-order perturbations or Primordial black holes (PBH) could also be detected by PTAs, LISA and ET. The recent PTA results [421, 809] are not compatible with a cosmic string signal, but are potentially compatible with a first-order phase transition or PHBs. Ref. [2599] looked at the combined NANOGrav and EPTA results and their possible interpretation as a cosmological background. Many papers on the interpretation of PTA results as a cosmological background have in fact been published after the June 2023 release of results by the main PTA collaborations, and trying to summarize them all is a research avenue in and of itself.

In conclusion, PTA data can help inform an ET search for a cosmological background. On the one hand, PTA data can help constrain a SGWB in the nanohertz GW frequency range, as the sensitivity of PTAs improves, in particular in light of the upcoming Square Kilometre Array. Recent results (from 2023) provide evidence for a correlated common red noise that could be a SGWB that is compatible with a background generated in exotic inflationary scenarios, first-order phase transitions or second-order scalar perturbations, but

requires in each case a surprisingly large effect. More probably, the cosmological background is only secondary to a foreground of supermassive black holes, in which the spectral index of the cosmological background would be lower and thus more compatible with a similar index at high ET frequencies. Studying scenarios of running spectral indices across the GW frequency range as well as differing equations of state are good avenues for linking the PTA frequencies with the ET frequencies and thus finding synergies.

TAKE HOME MESSAGE

- POSSIBLE EARLY UNIVERSE GW SOURCES CAN PROVIDE A SGWB PROFILE SPANNING MANY DECADES IN FREQUENCY: COSMIC PHASE TRANSITIONS, COSMIC STRINGS, AND COSMIC INFLATION.
- SGWB FROM PRIMORDIAL SOURCES HAVE A RICH FREQUENCY DEPENDENCE, WHOSE CHARACTERIZATION IMPROVES UP TO A 10% ACCURACY BY MEANS OF A MULTIBAND DETECTION BY ET AND BY OTHER GW DETECTORS.
- SYNERGIES BETWEEN ET AND OTHER GW INSTRUMENTS THUS PROVIDE INVALUABLE INFORMATION ON PRIMORDIAL SOURCES OF THE SGWB, AND ON THE EARLY HISTORY OF OUR UNIVERSE EXPANSION.

6.8 Executive summary

In this section we have explored how the science case of ET is enhanced by exploiting synergies with other GW observatories, operating either in a similar frequency band, as Cosmic Explorer, or in completely different bands, as LISA, deciHz detectors such as LGWA, or PTA.

Joint observations of ET with Cosmic Explorer will consolidate the science of this emerging and transformative field. The synergy with lower-frequency GW observatories, such as PTA at nHz, LISA in the mHz bandwidth, and design concepts like LGWA in the deciHz interval, will broaden the scientific case for ET. These synergies allows for connections to be made with other GW sources also in different astrophysical environments, enabling the investigation of some of the most compelling questions in astrophysics, physics and cosmology. In particular:

Synergies of ET with other GW observatories:

- Synergies between CE and ET enable a more comprehensive exploration of the Universe by extending further the cosmic horizon (see figure 105), enlarging the search for intermediate-mass black holes — an entirely uncharted territory — and enhancing the accuracy of measurements of the source’s physical parameters, resolving degeneracies and obtaining a comprehensive and robust understanding of the nature of neutron stars and black holes.
- Joint observations of ET with CE offer a unique opportunity to measure the GW polarization modes and constrain alternative gravity theories. The dipolar radiation test is significantly enhanced by combining ET with milli-Hertz and Deci-hertz gravitational wave detectors.

- 3rd-generation observatories like ET and CE will be the only observatories capable of discovering the relics of the first massive stars expected to form during cosmic dawn. These observatories will provide better measurements of the masses and redshifts of these black holes, shedding light on the formation of pristine stars in binaries. Some of these black holes could serve as “light” seeds for the growth of massive black holes.
- The collaboration between ET and CE represents a transformative advancement for late-time cosmology with resolved sources, due to the improved localization capabilities that are critical for both bright and dark siren cosmology. Furthermore, the synergy with other gravitational wave observatories, such as LISA, will expand the range of GW sources available.
- Multi-wavelength observations of merging black holes, captured by LISA in their early inspiral phase and by ET at the point of merger, are crucial for understanding the formation pathways of stellar-mass and intermediate-mass binary black holes. The combined observations of these systems by ET and LISA can further constrain General Relativity (GR) with an order of magnitude better precision than either observatory could achieve alone.
- BBHs originating from massive stars with masses in the range $120M_{\odot} - 300M_{\odot}$, are prime candidates for GW observations across multiple frequency bands. The prolonged inspiral signals emitted by these systems can be observed by LISA and by Deci-Hz detectors, while the later stages of inspiral and merger will be detected by ET. The combination of the observations from LISA or from Deci-Hz detectors such as LGWA, with the ET observations will dramatically improve the sky localization of these sources, making them effective dark sirens for the determination of cosmological parameters.
- By comparing merger events detected by ET and LISA, which are sensitive to different mass ranges (around $100 M_{\odot}$ for ET and $10^5 M_{\odot}$ for LISA), we can gain statistical insight into the mechanisms behind black hole seeding and growth.
- Intermediate mass ratio inspiral, where a stellar mass BHs merges with an intermediate mass black hole (IMBH), are exquisite probes of the physics of formation of IMBH in dense environments. Combining observations from ET, LGWA and LISA makes it possible to cover the IMBH mass range fully and to place stringent constraints on the IMBH mass distribution, shedding light on their origin, see figure 116.
- GW sources from the early Universe, such as cosmic phase transitions, cosmic strings, and cosmic inflation, may provide a stochastic gravitational wave background (SGWB) profile across multiple frequency bands. The SGWB from primordial sources has a rich frequency dependence, which can be characterized with up to 10% accuracy through multi-band detection by ET and other GW

detectors such as LISA, detectors operating in the deciHz such as LGWA, and PTA. Consequently, synergies between ET and other instruments offer invaluable information about primordial sources of the SGWB and the early history of the Universe’s expansion.

- A confident detection of a stochastic GW background requires control of correlated noise among the detectors involved in the correlation (the two distant detectors in the ET-2L configuration, the three colocated detectors in the ET-triangle configuration, and the three independent channels of LISA). Bringing in fully independent observatories, such as CE on the ground, or TianQin and Taiji in space, provides more robust detection prospects. Cross-correlating data from distant sites eliminates (or, on ground, further reduces) the possibility of correlated environmental or instrumental disturbances (see sections 3.1.5, 3.1.6 and 11.6.2 for discussions), yielding a far more robust claim of correlated signal, rather than just an excess of correlated broadband noise. At the same time, each new baseline adds signal-to-noise, improving overall sensitivity.

7 Subatomic Physics with ET

The first detection of GWs from a merging neutron star binary system and the accompanying observations of electromagnetic counterparts in 2017 demonstrated the enormous potential of multi-messenger astronomy for nuclear physics. Neutron stars comprise the highest densities of matter that can stably exist in the Universe. The gravitational-wave signals emitted from events involving neutron stars encode detailed and clean information on the dense matter in their interiors and, for a subset of binary neutron star mergers, on the subatomic microphysics in regimes of yet higher density and temperature. Such invaluable insights into states of subatomic matter under extreme conditions are inaccessible to any other probes and, in conjunction with electromagnetic counterparts, enable a multitude of additional opportunities, such as understanding the nucleosynthesis of very heavy elements. This section emphasizes the advances in nuclear and neutron star physics poised to open up with future gravitational-wave detections by ET, which are synergistic yet go beyond what can be accomplished with other astrophysical and terrestrial experiments. We also advocate progress in microscopic nuclear and hadron theory to fully exploit ET and test gravity in the regime of largest achievable densities and pressures, that of neutron stars.

7.1 Introduction

Astrophysical environments involving neutron stars (NSs) — objects where gravity compresses matter to several times the normal nuclear density — are unique testbeds for a rich variety of subatomic physics. A NS encompasses different phases of matter over a large range in density: from neutron-rich nuclei and free electrons in the outer parts to a uniform liquid of primarily neutrons, until the densities become so high in the very cores that other particles may be populated such as hyperons or mesons, which could also form condensates, and/or a transition to degenerate quark matter. This is illustrated in figure 119 and makes NSs

exquisite probes for unexplored regimes of strongly interacting matter, as well as for nuclear many-body phenomena and their variations with density and isospin asymmetry.

Among the messengers probing NSs and related phenomena, gravitational waves (GWs) are unique because they are purely based on gravitational interactions, and thus travel nearly uncontaminated from the densest regions of their sources to the detectors. The potential for nuclear physics with GWs has already become evident with the spectacular first detection of a binary NS (BNS) merger, GW170817 [2]. For this event, only the inspiral epoch, when the NS and its companion are still orbiting and matter is near its ground state, was discernible in the GW data. This enabled inferring constraints on the cold equation of state (EOS) above nuclear density from measurements of the NS's tidal deformability, a characteristic parameter that depends on the properties of matter and — together with the electromagnetic counterpart — provided the first definite identification of the central engine of a kilonova and the association of a BNS merger with a GRB. Since then, at least four more events likely involving one or two NSs in a binary were discovered, but apart from the masses of the coalescing objects, no new information relevant for nuclear physics could be extracted [1475, 2600, 2601].

Current detectors are mainly sensitive to the inspiral of NS binary coalescences. The outcomes beyond that vary depending on the type of binary system and its parameters. For mergers of NSs with a black hole (BH), the NS either becomes tidally disrupted, which leads to a strong suppression of the GW signal, or plunges into the BH, or is partially disrupted. In all such scenarios, the final remnant is a BH, and cases with tidal disruption are accompanied by electromagnetic counterparts. The mergers of two NSs have different outcomes depending on the initial systems: prompt collapse to a BH or the formation of a rotating massive NS heated by shocks during the merger process, which either collapses to a BH after shedding enough angular momentum or forms a stable NS remnant. In the latter two cases, the post-merger behavior is highly dynamical, involving differential rotation, oscillations, and dissipation, along with various microphysical processes, small-scale magnetohydrodynamic instabilities, and higher temperatures. This aftermath of a BNS merger provides a window onto a completely unexplored swath of the parameter space of subatomic physics that represents a complex nonperturbative regime in-between our current understanding of matter at low baryon densities from nuclear physics and heavy-ion collisions, and the regime of unreachable high densities where first principle calculations within quantum chromodynamics (QCD), the theory of the strong interaction, are valid.

The coalescences of NS binaries are also among the key production sites of elements heavier than iron in the cosmos. Heavy elements can be synthesized from the neutron-rich material expelled during BNS mergers or the tidal disruption of NSs by BHs and winds from the remnant accretion disks. The subsequent radioactive decay of these elements powers a kilonova electromagnetic transient.

In addition to binary mergers, future multi-messenger observations include the possible discoveries of signals from (i) a nearby (galactic) core-collapse supernova (CCSN), where the GWs and neutrinos contain imprints of the yet poorly understood explosion mechanism, dense matter properties in regimes complementary to heavy-ion collisions and BNS mergers, and the possible hot proto-NS, and (ii) rotating isolated NSs with a deformation (e.g. elastically- or magnetically-induced mountains on their surface or unstable oscillation modes) encoding