

Generating quantum discord between two distant Bose-Einstein condensates with Bell-like detection

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We propose a technique that enables the creation of quantum discord between two distant nodes, each containing a cavity consist of the Bose-Einstein condensate, by applying a non-ideal Bell-like detection on the output modes of optical cavities. We find the covariance matrix of the system after the non-ideal Bell-like detection, showing explicitly that one enables manipulation of the quantum correlations, and particularly quantum discord, between remote Bose-Einstein condensates. We also find that the non-ideal Bell-like detection can create entanglement between distant Bose-Einstein condensates at the two remote sites.

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I. INTRODUCTION

Nonlocal entanglement is one of the most fundamental features of quantum mechanics and it has variety of applications in quantum information processing [1–4], such as secret key distribution [5], quantum teleportation [6–8] and superdense coding [9–11]. One possible way to establish nonlocal entanglement is to perform a projection measurement on the state of two particles onto an entangled state. This measurement does not necessarily require a direct interaction between the two particles. If each of the particles is entangled with one other partner particle, an appropriate measurement (such a Bell-state measurement) on the partner particle will collapse the state of the remaining two particles onto an entangled state. This Bell projection, which establishes nonlocal correlations between distant systems that have never interacted is a key ingredient in quantum computation and communication protocols such as teleportation [6] and entanglement swapping [3, 12–14].

Entanglement swapping is the central principle used in quantum repeaters [15], whose main aim is to overcome photon loss in long-range quantum communication. Entanglement swapping can be used as a realistic protocol to generate entanglement between quantum memories at distant nodes in hybrid quantum repeaters; it has been implemented in several discrete [16] and continuous variable (CV) [17–24] systems.

Entanglement swapping, however, is able to generate the other types of quantum correlations such as quantum discord between distant sites. Quantum discord as a measure of the quantumness of correlations is important because entanglement does not describe all the non-classical properties of quantum correlations. Quantum discord has been proposed as a measure of all quantum correlations including, but not restricted to, entanglement [25, 26]. It is known that the quantum discord can offer a new model of quantum information processing which requires very little or no entanglement [27]. Therefore, the quantum discord proposes that the separable

or weakly entangled states could be used as a powerful tools for quantum information processing (such as quantum remote state preparation) as they are much easier to prepare and control even in dissipative environment.

Motivated by the above mentioned studies, in this paper we propose a scheme for generating quantum correlation between two distant nodes by applying a non-ideal Bell-like detection. In such scheme each node contains a Bose-Einstein condensate (BEC) [28, 29] trapped inside an optical cavity [see Fig. 1.(a)]. The output of the optical cavity of each node (which shows robustness against thermal noise) is sent to an intermediate site where a Bell-like detection is performed on these optical pairs [see Fig. 1.(b)]. We show that the immediate consequence of this Bell-like detection is the creation of quantum discord between two distant BECs. Moreover, our system can be realized with state-of-the-art technology, and is suited to such potential applications as quantum remote state preparation and quantum state teleportation between distant BECs.

This paper is organized as follows. In Sec. II we provide the theoretical description of the cavity containing a BEC. In Sec. III we introduce our proposal for producing quantum discord between two distant nodes by using a non-ideal Bell-like detection. We find the covariance matrix after Bell measurement and then we explicitly show that the Bell-like detection can produce quantum discord between remote BECs. In Sec. IV we discuss the generation of bipartite entanglement between two distant BECs. Finally, our conclusions are given in Sec. V.

II. FORMULATION AND THEORETICAL DESCRIPTION OF THE SYSTEM AT EACH NODE

Our main goal is to establish the quantum discord between two distant nodes that never interacted (see Fig. 1b). In order to construct such quantum link first we need to study the dynamics of the system at each node.

The schematic of the system at each node is depicted

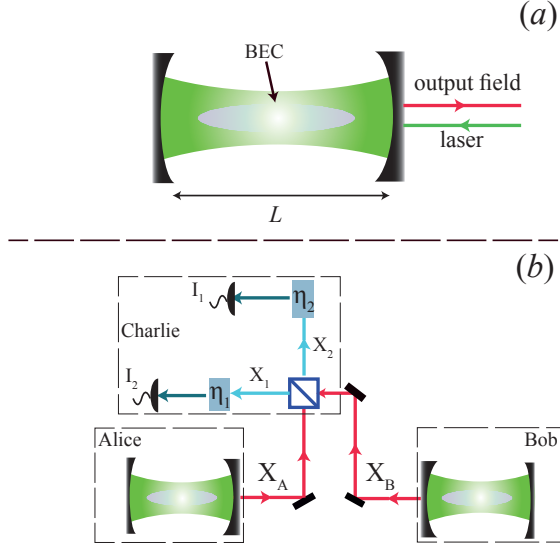


FIG. 1. (Color online) (a) Scheme of a trapped ultracold atoms inside the cavity where the BEC atoms are coupled with a single mode of the optical cavity. We also assume the optical cavity is driven by an intense laser. (b) A possible protocol for generating the quantum correlations between two BECs by applying a non-ideal Bell-like detection.

in Fig. 1(a). We consider a cigar-shaped gas of N ultracold bosonic two-level atoms with transition frequency ω_a and mass m_a inside an optical cavity with length L . The optical cavity is driven by an intense laser at frequency ω_d with amplitude $E_d = \sqrt{2\mathcal{P}\kappa/\hbar\omega_c}$, where $\mathcal{P}$ is the power of the input laser and κ shows the damping rate of the optical cavity with the bare frequency ω_c . In the dispersive regime where the laser pump is far detuned from the atomic resonance ($\Delta_a = \omega_d - \omega_a$), the excited electronic state of the atoms can be adiabatically eliminated and spontaneous emission can be neglected [30]. Therefore, the many-body Hamiltonian in the rotating frame with respect to the laser frequency ω_d takes the following form [28]

$$\hat{H} = \hbar\Delta_c\hat{a}^\dagger\hat{a} + i\hbar E_d(\hat{a}^\dagger - \hat{a}) + \int_{-L/2}^{L/2} dx \hat{\psi}^\dagger(x) \left[\frac{-\hbar^2}{2m_a} \frac{d^2}{dx^2} + \hbar U_0 \cos^2(kx) \hat{a}^\dagger\hat{a} + \frac{1}{2} U_s \hat{\psi}^\dagger(x) \hat{\psi}(x) \right] \hat{\psi}(x), \quad (1)$$

where $\hat{a}$ ($[\hat{a}, \hat{a}^\dagger] = 1$) is the annihilation operator for the cavity mode, and $\Delta_c = \omega_c - \omega_d$ is the detuning between the laser and a cavity mode. The parameter $U_0 = g_0^2/\Delta_a$ is the height of potential barrier of the optical lattice per photon and represent the backaction on the field, g_0 is the vacuum Rabi frequency, $U_s = \frac{4\pi\hbar^2 a_s}{m_a}$, and a_s is the two-body s-wave scattering length [31, 32].

In equation (1) we assume only the first two symmetric momentum side modes with momenta $\pm 2k$ are excited by the atom-light interaction [29]. This approx-

imation is valid in the weakly interacting regime i.e., $U_0\langle a^\dagger a \rangle \leq 10\omega_R$ ($\omega_R = \hbar k^2/2m_a$ is the recoil frequency of the condensate atoms and $k = \omega_d/c$ is the wavenumber). However, because of the parity conservation and considering the Bogoliubov approximation, the atomic field operator can be expanded as

$$\hat{\psi}(x) = \sqrt{\frac{N}{L}} + \sqrt{\frac{2}{L}} \cos(2kx) \hat{c}, \quad (2)$$

where $\hat{c}$ ($[\hat{c}, \hat{c}^\dagger] = 1$).

By substituting the atomic field operator Eq. (2), into the Hamiltonian (1) one obtains [28]

$$\begin{aligned} \hat{H} = & \hbar\delta_c\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\Omega_c(\hat{P}^2 + \hat{Q}^2) \\ & + \hbar G\hat{a}^\dagger\hat{a}\hat{Q} + \frac{1}{2}\hbar\omega_{sw}\hat{Q}^2 + i\hbar E_d(\hat{a}^\dagger - \hat{a}), \end{aligned} \quad (3)$$

where we define the Bogoliubov position $\hat{Q} = (\hat{c} + \hat{c}^\dagger)/\sqrt{2}$ and momentum $\hat{P} = (\hat{c} - \hat{c}^\dagger)/i\sqrt{2}$ quadratures. However, $\delta_c = \Delta_c + \frac{NU_0}{2}$ is the effective Stark shift detuning. $\omega_{sw} = \frac{8\pi\hbar N}{ML\nu^2}$ is the s-wave scattering frequency and ν is the waist of the optical potential. $\Omega_c = 4\omega_R + 1/2\omega_{sw}$ is the effective detuning of BEC and $G = \frac{\omega_c}{L} \sqrt{\frac{\hbar}{4\omega_R m_s}}$ is the coupling strength of the optical cavity mode with Bogoliubov mode, where $m_s = \frac{\hbar\omega_c^2}{L^2 N U_0^2 \omega_R}$ is effective mass of side mode.

The quantum Langevin equations (QLEs) for BEC and the cavity variables are obtained by adopting the dissipation-fluctuation theory [33]

$$\begin{aligned} \dot{\hat{P}} = & -(\Omega_c + \omega_{sw})\hat{Q} - \gamma_c\hat{P} - G\hat{a}^\dagger\hat{a} + \sqrt{2\gamma_c}\hat{Q}_{in}, \\ \dot{\hat{Q}} = & \Omega_c\hat{P} - \gamma_c\hat{Q} + \sqrt{2\gamma_c}\hat{P}_{in}, \\ \dot{\hat{a}} = & -(i\delta_c + \kappa)\hat{a} + iG\hat{Q}\hat{a} + E_d + \sqrt{2\kappa}\hat{a}_{in}, \end{aligned} \quad (4)$$

where γ_c is the dissipation of the collective density excitations of the BEC, and $\hat{a}_{in}(t)$ is the cavity input noise which obeys the white-noise correlation functions [33]

$$\begin{aligned} \langle \hat{a}_{in}(t) \hat{a}_{in}^\dagger(t') \rangle &= \delta(t - t'), \\ \langle \hat{a}_{in}^\dagger(t) \hat{a}_{in}(t') \rangle &= 0, \end{aligned} \quad (5)$$

where we have set $N = [\exp(\hbar\omega_c/k_B T) - 1]^{-1} \approx 0$, since $\hbar\omega_c/k_B T \gg 1$ at optical frequency, where k_B being the Boltzmann constant, and T is the temperature of the reservoir. On the other hand, $\hat{Q}_{in}$ and $\hat{P}_{in}$ are the thermal noise inputs for the side mode of BEC which satisfy the Markovian correlation functions $\langle \hat{P}_{in}(t) \hat{P}_{in}(t') \rangle = \langle \hat{Q}_{in}(t) \hat{Q}_{in}(t') \rangle = 2\gamma_c(n_c + 1/2)\delta(t - t')$ [34, 35], where we have assumed that the BEC has been trapped and isolated from its environment so that its effective temperature $T_c \simeq 0.1\mu\text{K}$ and $n_c = [\exp(\hbar\omega_B/k_B T_c) - 1]^{-1}$ is the number of thermal excitations for the Bogoliubov mode which oscillates with frequency $\omega_B = \sqrt{\Omega_c(\Omega_c + \omega_{sw})}$.

One simple way to represent the dynamics of the system is to use the vector $\mathbf{u} = [\hat{Q}, \hat{P}, \hat{X}, \hat{Y}]^T$, where the quadratures of the optical field cavity are defined as

$\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ and $\hat{Y} = i(\hat{a}^\dagger - \hat{a})/\sqrt{2}$. Since the cavity is driven by an intense laser then one can linearize the QLEs given in Eq. (4) around the semiclassical mean values, i.e., $u_j = u_{j,s} + \delta u_j(t)$, where $u_{j,s}$ are the classical mean values and $\delta u_j(t)$ are zero-mean fluctuation operators. The mean values are obtained by setting the time derivatives to zero, resulting in

$$\begin{aligned} Q_s &= -\frac{G\alpha_s^2}{\Omega_c + \omega_{sw} + \frac{\gamma_c^2}{\Omega_c}}, \\ P_s &= \frac{\gamma_c}{\Omega_c} Q_s, \\ \alpha_s &= \frac{E_d}{\sqrt{\Delta^2 + \kappa^2}}, \end{aligned} \quad (6)$$

where $\Delta = \delta_c + GQ_s$ is the effective detuning and $\alpha_s = \alpha_s^*$ is the mean field value of the optical cavity mode.

The dynamics of the quantum fluctuations, $\delta \mathbf{u}(t) = [\delta \hat{Q}, \delta \hat{P}, \delta \hat{X}, \delta \hat{Y}]^T$, are given by the linearized QLEs which one can write them as

$$\delta \dot{\mathbf{u}}(t) = \mathbf{A} \delta \mathbf{u}(t) + \mathbf{n}(t) \quad (7)$$

where

$$\mathbf{A} = \begin{pmatrix} -\gamma_c & \Omega_c & 0 & 0 \\ -(\Omega_c + \omega_{sw}) & -\gamma_c & -\sqrt{2}G\alpha_s & 0 \\ 0 & 0 & -\kappa & \Delta \\ -\sqrt{2}G\alpha_s & 0 & -\Delta & -\kappa \end{pmatrix}, \quad (8)$$

is the drift matrix and $\mathbf{n}(t) = [\gamma_c(2n_c + 1), \gamma_c(2n_c + 1), \kappa, \kappa]^T$ describes the vector of the noises. We note that the current system is stable and reaches a steady state after a transient time if all the eigenvalues of the drift matrix $\mathbf{A}$ have negative real part. These stability conditions can be obtained by using the *Routh-Hurwitz* criterion [36].

The output mode of the optical cavity is given by the standard input-output relation $\hat{a}_{\text{out}} = \sqrt{2\kappa}\hat{a} - \hat{a}_{\text{in}}$. One can also define the selected output mode by means of the causal filter function $\hat{a}^{\text{filt}} = \int_{t_0}^t F(t-s)\hat{a}_{\text{out}}(s)ds$, where the causal filter function $F(t) = \sqrt{2/\tau}\exp[-(1/\tau + i\Omega)t]\Theta(t)$ is characterized by central frequency Ω , bandwidth $1/\tau$, and the Heaviside step function $\Theta(t)$ [37–39]. In the frequency domain, the stationary covariance matrix for the quantum fluctuations of the BEC, and the output mode of the optical cavity variables, $\mathbf{u}^{\text{filt}}(t) = [\hat{Q}, \hat{P}, \hat{X}^{\text{filt}}, \hat{Y}^{\text{filt}}]^T$, takes the form

$$\begin{aligned} \mathbf{V} &= \lim_{t \rightarrow \infty} \frac{1}{2} \langle u_i^{\text{filt}}(t) u_j^{\text{filt}}(t) + u_j^{\text{filt}}(t) u_i^{\text{filt}}(t) \rangle \\ &= \int d\omega \mathbf{\Upsilon}(\omega) (\tilde{\mathbf{M}}(\omega) + \mathbf{P}) \\ &\quad \times \mathbf{D} (\tilde{\mathbf{M}}(\omega)^\dagger + \mathbf{P}) \mathbf{\Upsilon}^\dagger(\omega), \end{aligned} \quad (9)$$

where $\tilde{\mathbf{M}}(\omega) = (i\omega\mathbf{I} + \mathbf{A})^{-1}$, $\mathbf{P} = \text{Diag}[0, 0, \frac{1}{2\kappa}, \frac{1}{2\kappa}]$, $\mathbf{D} = \text{Diag}[\gamma_c(2n_c + 1), \gamma_c(2n_c + 1), 2\kappa, 2\kappa]$,

and $\mathbf{\Upsilon}(\omega)$ is the Fourier transform of

$$\mathbf{\Upsilon}(t) = \begin{pmatrix} \delta(t) & 0 & 0 & 0 \\ 0 & \delta(t) & 0 & 0 \\ 0 & 0 & \mathcal{R} & -\mathcal{I} \\ 0 & 0 & -\mathcal{I} & \mathcal{R} \end{pmatrix}, \quad (10)$$

where $\mathcal{R} = \sqrt{2\kappa}\text{Re}[h(t)]$ and $\mathcal{I} = \sqrt{2\kappa}\text{Im}[h(t)]$ are determined by the causal filter functions.

III. GENERATING QUANTUM DISCORD BETWEEN TWO DISTANT BECS WITH BELL-LIKE DETECTION

In this section, we use the results obtained in previous section in order to create a quantum link between two remote BECs at different places by performing Bell-like detection. First we find covariance matrix of the system after Bell-like detection and then we quantify the generated bipartite quantum discord.

A. Bell-like detections with arbitrary quantum efficiencies

The overarching goal of this subsection is devoted to finding the covariance matrix of the system after Bell-like detection. More information about general aspect of this approach can be found in Ref. [40].

The whole system, as sketched in Fig. 1(b), is composed of two independent bipartite bosonic modes (include BEC, and optical field modes), where each of them possessed by Alice and Bob at different stations. Note that the bipartite bosonic modes at each site is fully characterized by covariance matrix (9). Alice and Bob prepare a bipartite state and each shares one travelling optical mode (the outputs of optical cavities) with Charlie, who is located for simplicity halfway between them. The covariance matrix of the whole system, composed of two independent bipartite bosonic modes, can be written in the blockform

$$\mathbf{V}_{ACB} = \begin{pmatrix} \mathbf{A}' & \mathbf{C} \\ \mathbf{C}^T & \mathbf{B}' \end{pmatrix}, \quad (11)$$

where the 4×4 matrix $\mathbf{A}'$ shows the reduced covariance matrix of the BECs modes,

$$\mathbf{C} = (\mathbf{C}_1 \quad \mathbf{C}_2),$$

is a real matrix, describing the correlations between BECs and the output of optical cavities, and

$$\mathbf{B}' = \begin{pmatrix} \mathbf{B}_1 & \mathbf{W} \\ \mathbf{W}^T & \mathbf{B}_2 \end{pmatrix},$$

is the reduced covariance matrix of the two optical modes received by Charlie. For simplicity let us write down the

reduced covariance matrix $\mathbf{B}'$ of the optical modes by setting

$$\mathbf{B}_1 := \begin{pmatrix} \alpha_1 & \alpha_3 \\ \alpha_3 & \alpha_2 \end{pmatrix}, \quad \mathbf{B}_2 := \begin{pmatrix} \alpha'_1 & \alpha'_3 \\ \alpha'_3 & \alpha'_2 \end{pmatrix},$$

$$\mathbf{W} := \begin{pmatrix} \beta_1 & \beta_3 \\ \beta_3 & \beta_2 \end{pmatrix}.$$

Note that matrices $\mathbf{A}'$, $\mathbf{B}'$, and $\mathbf{C}$ are determined by combination of two covariance matrices $\mathbf{V}$ (see Eq. (9)) of the each remote side.

Now, Charlie applies a Bell-like detection on the received optical modes. As depicted in Fig. 1(b), this detection consists in applying a beam splitter of transmissivity T , which transforms the optical modes X_A and X_B into the output modes X_1 and X_2 , followed by two conjugate (I_1 and I_2) homodyne detections. Here, we assume the realistic detectors with quantum efficiencies $0 < \eta_1 \leq 1$ and $0 < \eta_2 \leq 1$. Means that two homodyne detectors modelled by inserting two beam-splitters with transmissivities η_1 and η_2 , which mix the signal modes with vacuum. As a result, after Bell-like detection the final covariance matrix of the remaining modes, i.e., two BECs is given by [40]

$$\mathbf{V}_{AB} = \mathbf{A}' - \frac{1}{\det \mathbf{\Gamma}} \sum_{i,j=1}^2 \mathbf{C}_i \mathbf{K}_{ij} \mathbf{C}_j^T, \quad (12)$$

where

$$\mathbf{K}_{11} := \begin{pmatrix} (1-T)\gamma_2 & \sqrt{T(1-T)}\gamma_3 \\ \sqrt{T(1-T)}\gamma_3 & T\gamma_1 \end{pmatrix},$$

$$\mathbf{K}_{22} := \begin{pmatrix} T\gamma_2 & -\sqrt{T(1-T)}\gamma_3 \\ -\sqrt{T(1-T)}\gamma_3 & (1-T)\gamma_1 \end{pmatrix},$$

$$\mathbf{K}_{12} = \mathbf{K}_{21}^T := \begin{pmatrix} -\sqrt{T(1-T)}\gamma_2 & (1-T)\gamma_3 \\ -T\gamma_3 & -\sqrt{T(1-T)}\gamma_1 \end{pmatrix},$$

with

$$\mathbf{\Gamma} := \begin{pmatrix} \gamma_1 & \gamma_3 \\ \gamma_3 & \gamma_2 \end{pmatrix},$$

and

$$\gamma_1 := (1-T)\alpha_1 + T\alpha'_1 - 2\sqrt{T(1-T)}\beta_1 + \frac{1-\eta_1}{\eta_1},$$

$$\gamma_2 := T\alpha_2 + (1-T)\alpha'_2 + 2\sqrt{T(1-T)}\beta_2 + \frac{1-\eta_2}{\eta_2},$$

$$\gamma_3 := \sqrt{T(1-T)}(\alpha'_3 - \alpha_3) - (1-T)\beta_3 + T\beta_4.$$

Note that the covariance matrix (12) includes all information about the quantum correlations between two BECs after Bell-like detection. The rest of the paper devotes mainly to study these quantum correlations and we explicitly show that the Bell-like detection ables to generate both quantum discord and quantum entanglement between distant BECs.

B. quantum discord between two BECs after Bell-like detection

In this subsection, we focus on quantum discord and we show that how Bell-like detection is able to create the quantum discord between two remote BECs.

The quantum discord between BEC A and BEC B , is defined as [25, 26]

$$\mathcal{D} = \mathcal{I}(AB) - \mathcal{J}(B|A). \quad (13)$$

where $\mathcal{I}(AB)$ represents the quantum correlation between BEC A and BEC B and $\mathcal{J}(A|B)$ is interpreted as the information gain about BEC B by measuring the BEC A . According to [41] on the optimality of Gaussian discord, the quantum discord Eq. (13) can be computed in the term of elements of covariance matrix (12), reads [42–44]

$$\mathcal{D} = f(\sqrt{s_1}) - f(\lambda_-) - f(\lambda_+) + f(\sqrt{\epsilon}), \quad (14)$$

where

$$f(x) = \left(\frac{x+1}{2}\right) \log_2 \left(\frac{x+1}{2}\right) - \left(\frac{x-1}{2}\right) \log_2 \left(\frac{x-1}{2}\right), \quad (15)$$

and if $(s_4 - s_2 s_1)^2 \leq (1 + s_1) s_3^2 (s_2 + s_4)$ then

$$\epsilon = \frac{2s_3^2 + (s_1 - 1)(s_4 - s_2) + 2|s_3| \sqrt{s_3^2 + (s_1 - 1)(s_4 - s_2)}}{(s_1 - 1)^2},$$

otherwise

$$\epsilon = \frac{s_2 s_1 - s_3^2 + s_4 - \sqrt{s_3^4 + (s_4 - s_2 s_1)^2 - 2s_3^2(s_4 + s_2 s_1)}}{2s_1},$$

where $s_1 = 4 \det \mathbf{V}_1$, $s_2 = 4 \det \mathbf{V}_2$, $s_3 = 4 \det \mathbf{V}_3$, $s_4 = 16 \det[\mathbf{V}_{AB}]$. Here we have rewritten the two-modes covariance matrix (12) as

$$\mathbf{V}_{AB} = \begin{pmatrix} \mathbf{V}_1 & \mathbf{V}_3 \\ \mathbf{V}_3^T & \mathbf{V}_2 \end{pmatrix}, \quad (16)$$

where $\lambda_{\pm} = 2^{-1/2} [s_{\Delta} \pm \sqrt{s_{\Delta}^2 - 4s_4}]^{1/2}$ and $s_{\Delta} = s_1 + s_2 + 2s_3$.

C. Results

We now utilize the results obtained in the previous subsection to show that how Bell-like detection can generate quantum discord between two distant sides. The bipartite quantum discord between subparts of the remote systems, i.e., BEC-BEC is determined by Eq. (14). The quantum discord $\mathcal{D}$ after Bell-like detection as a function of the filtering bandwidths $\varepsilon_1 = \Omega_1 \tau_1$ and $\varepsilon_2 = \Omega_2 \tau_2$ has been plotted in Fig. 2, where we have taken the experimentally feasible parameters [45, 46] i.e., an optical cavity of length $L = 1$ mm with wavelength of

$\lambda = 1046$ nm, finesse $\mathcal{F} = 1.15 \times 10^5$, and damping rate $\kappa = \pi c/L\mathcal{F}$. The optical cavity is driven with pump amplitude $E_d = 3\kappa$ and its detuning is $\Delta = \Omega_c$. The recoil frequency of BEC is $\omega_R/2\pi = 3.57$ kHz with dissipation of collective density excitations $\gamma = 0.001\kappa$, atom-cavity coupling constant $G = 0.5\omega_B$, and temperature $T_c = 0.1\mu\text{K}$. As seen, the Bell-like detection successfully generates the quantum discord among the elements of two distant BECs. Fig. 2, however, shows that the quantum discord of BEC-BEC is considerable around $\varepsilon_1 = \varepsilon_2 \simeq 8$, which means that, in practice, by appropriately filtering the output fields one can generate stronger quantum discord with Bell-like detection.

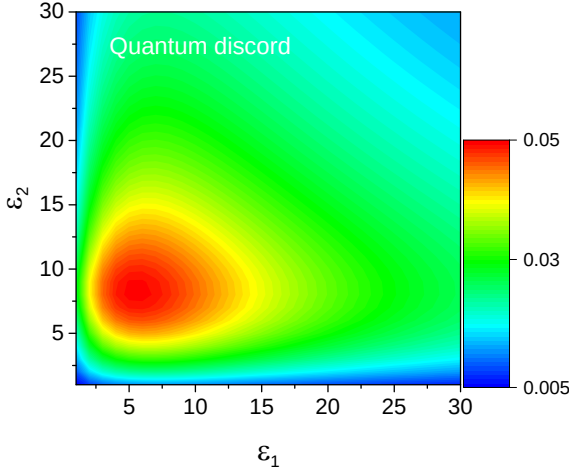


FIG. 2. (Color online) Quantum discord between remote BECs after Bell-like detection with respect to the filtering bandwidths $\varepsilon_1 = \Omega_1\tau_1$ and $\varepsilon_2 = \Omega_2\tau_2$, where $\eta_1 = \eta_2 \simeq 1$. The central frequencies of detected output fields are $\Omega_1 = \Omega_2 = -\omega_B$. Here we assume an optical cavity of length $L = 1$ mm with wavelength of $\lambda = 1046$ nm, finesse $\mathcal{F} = 1.15 \times 10^5$, and damping rate $\kappa = \pi c/L\mathcal{F}$. The optical cavity is driven with pump amplitude $E_d = 3\kappa$ and its detuning is $\Delta = \Omega_c$. The recoil frequency of BEC is $\omega_R/2\pi = 3.57$ kHz with dissipation of collective density excitations $\gamma_c = 0.001\kappa$, atom-cavity coupling constant $G = 0.5\omega_B$ and temperature $T_c = 0.1\mu\text{K}$.

As is noted, we consider non-ideal Bell-like detection in which we assume the realistic homodyne detectors with quantum efficiencies $\eta_1 < 1$ and $\eta_2 < 1$. The effect of the quantum efficiencies of the detectors on the quantum discord between distant nodes is plotted in Fig. 3 where we assume $\eta := \eta_1 = \eta_2$. This figure confirms that in order to establish strong quantum discord between two remote BECs one should use close to perfect detectors with quantum efficiencies $\eta_1 \simeq 1$ and $\eta_2 \simeq 1$.

Moreover, Fig. 4 shows the quantum discord between two BECs with respect to Ω_1/ω_B and Ω_2/ω_B for $\omega_{sw} = 0$, where the discord is maximum around the particular resonance frequency of BECs mode $\Omega_i = -\omega_B$ (with $i = 1, 2$). Finally, the effect of atomic collisions has been

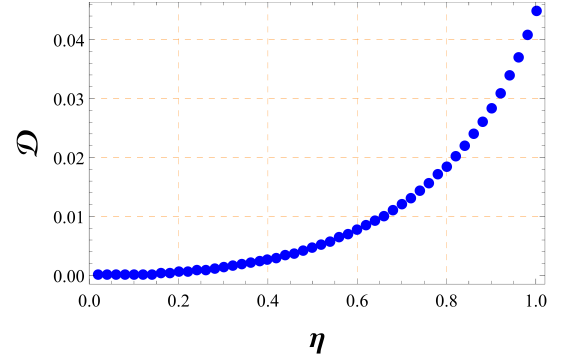


FIG. 3. (Color online) Effect of the detectors quantum efficiencies $\eta := \eta_1 = \eta_2$ on the quantum discord between distant BECs where we assume $\varepsilon_1 = \varepsilon_2 = 8$. The other parameters are the same as Fig. 2.

plotted in Fig. 5. It is clear that the effect of the atomic collisions on the quantum discord is inevitable. The atom-atom interactions in the BEC strongly degrades the bipartite quantum discord between remote nodes.

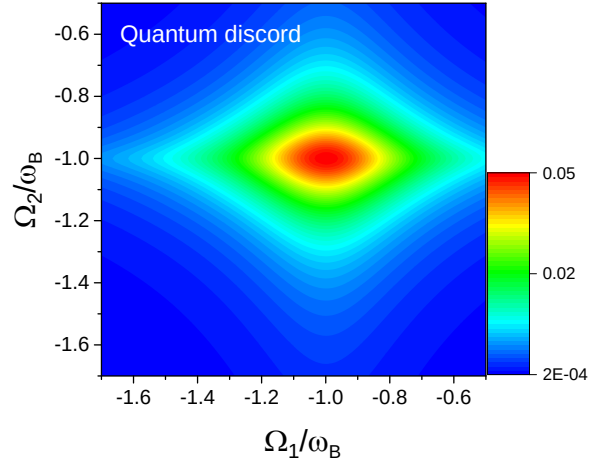


FIG. 4. (Color online) Quantum discord between two remote BECs with respect to the normalized frequencies Ω_1/ω_B and Ω_2/ω_B , where we assume $\eta_1 = \eta_2 \simeq 1$ and $\varepsilon_1 = \varepsilon_2 = 8$. The other parameters are the same as Fig. 2.

IV. ENTANGLEMENT BETWEEN TWO DISTANT BECS

Finally, we focus on the the entanglement after Bell-like detection. The bipartite entanglement between two remote BECs after Bell-like detection can be characterized by the logarithmic negativity [47]

$$E_{\mathcal{N}} = \max[0, -\ln(2\eta_-)], \quad (17)$$

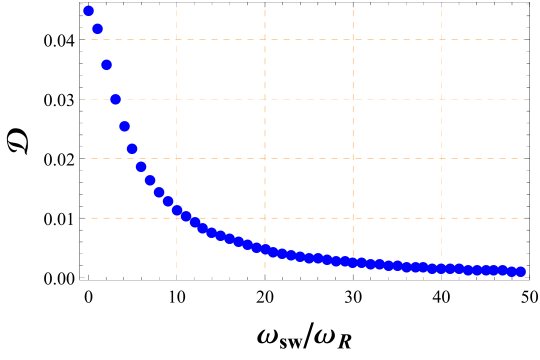


FIG. 5. (Color online) Quantum discord between remote BECs with respect to the collision parameter ω_{sw}/ω_R for $\varepsilon_1 = \varepsilon_2 = 8$. The other parameters are the same as Fig. 2.

where $\eta_- = 2^{-1/2} [\sigma - \sqrt{\sigma^2 - 4\det\mathbf{V}_{AB}}]^{1/2}$ is the least symplectic eigenvalue of the covariance matrix (16) and $\sigma = \det\mathbf{V}_1 + \det\mathbf{V}_2 - 2\det\mathbf{V}_3$.

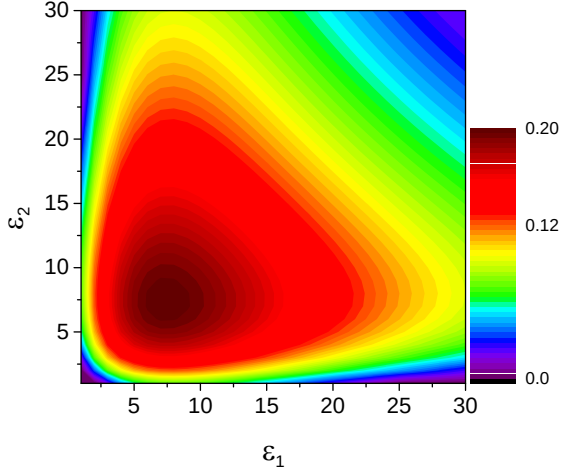


FIG. 6. (Color online) Bipartite entanglement after Bell-like detection between remote BECs with respect to the filtering bandwidths $\varepsilon_1 = \Omega_1\tau_1$ and $\varepsilon_2 = \Omega_2\tau_2$, where $\eta_1 = \eta_2 \simeq 1$. The central frequencies of detected output fields are $\Omega_1 = \Omega_2 = -\omega_B$. The other parameters are the same as Fig. 2.

The logarithmic negativity E_N after Bell-like detection as a function of the filtering bandwidths $\varepsilon_1 = \Omega_1\tau_1$ and $\varepsilon_2 = \Omega_2\tau_2$ has been plotted in Fig. 6. As seen, the Bell-like detection successfully generates entanglement between distant BECs. However, as we expected this figure shows that the entanglement between two remote BECs is considerable around $\varepsilon_1 = \varepsilon_2 \simeq 8$. Furthermore, the effect of the quantum efficiencies of the detectors on the entanglement between distant nodes is plotted in Fig. 7.

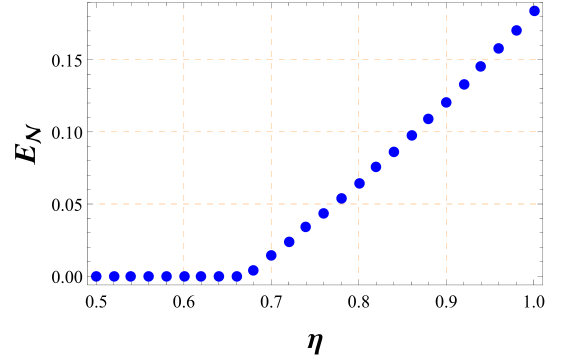


FIG. 7. (Color online) Effect of the detectors quantum efficiencies $\eta := \eta_1 = \eta_2$ on the generated entanglement between distant two BECs where we assume $\varepsilon_1 = \varepsilon_2 = 8$. The other parameters are the same as Fig. 2.

V. CONCLUSIONS

In this paper, we have studied the possibility of producing the quantum correlations between two distant Bose-Einstein condensates simply by performing a non-ideal Bell-like detection. We have shown that by applying a Bell-like detection on the travelling optical fields of two optical cavities each containing a Bose-Einstein condensate, one can generate quantum discord between distant Bose-Einstein condensates. The covariance matrix of the Bose-Einstein condensate modes after Bell-like detection has been found. We then have evaluated the quantum discord between two distant Bose-Einstein condensates at different nodes. The effect of the filtering bandwidths and quantum efficiencies of the detectors have been discussed. Moreover, the effect of the atom-atom interaction in the Bose-Einstein condensates on the quantum discord has been investigated. We have also studied the entanglement in the system that is created by the Bell-like detection. The effect of the detection bandwidths and efficiencies of detectors on the entanglement have been studied. The promising results of the paper show that the Bell-like detection is able to build up the quantum correlations (discord and entanglement) between distant BECs that never interacted. However, we should note that the degree of these correlations that can be achieved with this technique still is not very large.

In view of significant recent progress in quantum state engineering, the Bell-like detection protocol discussed here may offer interesting perspectives for future applications in optical implementations of hybrid quantum repeaters and quantum communication networks. Finally, we note that the Bell-like detection scheme discussed here can be implemented experimentally by employing the standard protocols of entanglement swapping [12, 14, 48].

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