

Let  $\mathcal{L}_1, \dots, \mathcal{L}_k$  be left-invariant differential operators on a Lie group  $G$  which induce essentially self-adjoint operators on  $L^2(G)$  (with initial domain  $C_c^\infty(G)$ ) with commuting self-adjoint extensions. Then, by means of the spectral theorem it is possible to define  $m(\mathcal{L}_1, \dots, \mathcal{L}_k)$  for every Borel measurable function  $m: \mathbb{R}^k \rightarrow \mathbb{C}$ . When  $m$  is bounded,  $m(\mathcal{L}_1, \dots, \mathcal{L}_k)$  is a bounded operator of  $L^2(G)$  which commutes with left translations, so that it has a (right) convolution kernel  $\mathcal{K}(m) \in \mathcal{D}'(G)$ .

When  $G = \mathbb{R}^n$ , we have  $\mathcal{L}_j = P_j(-i\partial)$  for some polynomial  $P_j$ ,  $j = 1, \dots, k$ , so that  $\mathcal{K}(m) = \mathcal{F}^{-1}(m \circ P)$ , where  $\mathcal{F}$  is the Euclidean Fourier transform. Given the analogy between  $\mathcal{K}$  and  $\mathcal{F}^{-1}$ , one may wonder whether some of the basic properties of  $\mathcal{F}$  hold, suitably interpreted, for  $\mathcal{K}$ . On the one hand, A. Martini proved in 2011 that, if  $\mathcal{L}_1, \dots, \mathcal{L}_k$  form a weighted subcoercive system of operators, then  $\mathcal{K}$  induces an isometry from  $L^2(\beta)$  into  $L^2(G)$  for some ('Plancherel') measure  $\beta$  on  $\mathbb{R}^k$ ; furthermore, when  $G$  is a group of polynomial growth, then  $\mathcal{K}$  maps  $\mathcal{S}(\mathbb{R}^k)$  into  $\mathcal{S}(G)$ . On the other hand, it is not hard to run into reasonably well-behaved examples where  $\mathcal{K}(m) \in L^1(G)$  but  $m$  is not continuous, or where  $\mathcal{K}(m) \in \mathcal{S}(G)$  and  $m$  is not a Schwartz function (or even continuous).

I will present some results concerning  $\mathcal{K}$  in various contexts, focusing on homogeneous groups.