The Incomplete Revolutions of String Theory

Dedicated to the memory

of my dear friend and colleague

Yassen S. Stanev, 1962-2017

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If one tried to get a palatable picture of Electromagnetism, it would be natural to hear from an expert about charges, flux lines, potentials and waves. These are subtle concepts, and yet they can convey some valuable intuition on these phenomena, despite the intricacies of the underlying Mathematics. Similar questions about General Relativity would probably bring up falling bodies, the Equivalence Principle and deformations of the fabric of spacetime where planets slide along their orbits. String Theory, however, is still a very different matter, and experts have a hard time defining it. One could say that strings replace particles, as we shall try to explain, but in our current view particles emerge from fields, whose geometry underlies their interactions. As we shall see, appealing to such geometrical principles within the low-energy Supergravity has provided unprecedented glimpses of a unified view of Nature that transcends not only strings but our very picture of spacetime. Yet, we lack somehow satisfactory answers to some basic questions, which ought to have preceded all this. What replaces the principles of General Relativity in String Theory? What do strings tell us about spacetime at short distances? Why is our Universe the way it seems? String Theory is today an awesome unfinished monument, whose roots remain elusive despite decades of intense effort. While this brings about some distress, it also makes the subject mysterious, challenging and highly fascinating. In the following, I shall describe the origin of this unusual situation, while also trying to address some future prospects.

1. Quantum Fields and Particles

Our current understanding of the Fundamental Interactions rests on Quantum Field Theory. This setting affords a novel incarnation of the point particle idea, which has nurtured Physics since the time of Galileo Galilei and the subsequent birth of Newtonian Mechanics. Rigid bodies, strings, membranes and fluids are but variations on this theme, and point particles have also provided important clues on light and waves. When Quantum Mechanics found its place in a relativistic picture, in the 1930's, particle-wave duality found a concrete realization in Quantum Field Theory, where masses and momenta qualify field quanta together with a novel attribute, spin. Atomic Physics had indeed provided, since the early 1920's, compelling reasons for electron spin, but this intrinsic angular momentum soon became a label for "polarizations" of general particle-waves. Its emergence brought along the link between spin and statistics, with a sharp distinction between "bosons'', integer-spin particles like photons, and "fermions", half-odd-integer-spin ones like electrons or neutrinos that are subject to the Pauli exclusion principle. One can indeed observe electromagnetic fields without perceiving that they are streams of photons, while fermions have strikingly different manifestations. A finer taxonomy distinguishes spin-0 particles like the Higgs boson from spin-1 particles like the photon, the gluons of the Strong Interactions or the vector bosons of the Weak Interactions, or from the graviton, a spin-2 quantum that ought to lie behind the recently discovered gravitational waves. In addition, one might conceive a role for elementary Bose and Fermi particles of spin larger than 2 or 1/2, and massive composites of this type recur among the short-lived hadronic resonances copiously produced in particle accelerators.

A particle-wave can also possess one or more types of charges, and for example positrons and electrons, with charges ±e, interact with photons in Quantum Electrodynamics via the minimal substitution

$$-\partial \rightarrow -\partial \pm \frac{ie}{\hbar c} A \,, \qquad (1)$$

where A is the quantum vector potential. Striking accounts of subtle phenomena in Atomic Physics vindicated, in the late 1940's, the novel renormalization techniques capable of taming the infinities that had vexed this theory since its inception. Quantum fluctuations then revealed surprising effects that have been tested to unprecedented levels of precision, thanks to the small value of the "fine structure constant''

$$\alpha_e = \frac{e^2}{\hbar c} \cong \frac{1}{137} \,, \qquad (2)$$

and thus to the mild interaction strength among electrons, positrons and photons. These developments also brought to due prominence the local $U(1)_{em}$ symmetry that underlies eq. (1), paving the way to the Yang-Mills extension that became eventually a backbone of the Standard

Model. However, the same methods proved initially inadequate for the Strong Interactions of nuclear matter, and much effort was thus devoted, in the 1960's, to characterizing "a priori" S-matrix amplitudes that could capture at least some aspects of hadronic diffusion processes.

2. The Birth of String Theory

String Theory is somehow an unexpected offspring of these attempts. It emerged in 1968, when Gabriele Veneziano proposed his renowned amplitude for the elastic diffusion of a pair of scalar particles,

$$A = \frac{\Gamma(-1-\alpha' s)\Gamma(-1-\alpha' t)}{\Gamma(-2-\alpha'(s+t))} \ . \qquad (3)$$

In this expression originally meant for mesons, which we now recognize as quark-antiquark composites, the poles introduced by the Euler Γ functions readily unveil, to an expert eye, the presence of a zoo of other particles in the theory. Their squared masses and spins lie on linear "Regge trajectories" determined by the "Regge slope" α', and adjusting it led to patterns in rough agreement with the known distribution of hadronic resonances. The Veneziano formula in eq. (3) displays a striking symmetry under the interchange of s and t, two "Mandelstam variables" that characterize the total energy and the diffusion angle in the center-of-mass frame. This property is usually called "planar duality". There are actually three Mandelstam variables, s, t and u, which satisfy the linear constraint

$$s + t + u = \sum {m_i}^2 \qquad (4)$$

involving the masses m_i of the particles that enter 2→2 diffusion processes, and Joel Shapiro and Miguel-Angel Virasoro soon discovered a similar, albeit more complicated, amplitude also based on Euler's Γ function. This second amplitude has again infinitely many poles, with squared masses and spins that lie on different linear "Regge trajectories" also determined by the "Regge slope" α', but is symmetric under all interchanges of s, t and u. This property is usually called "non-planar duality".

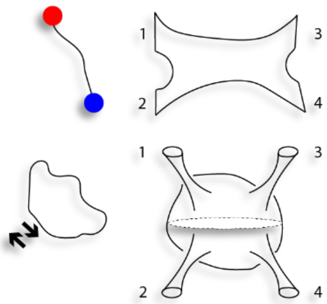

Fig. 1. The Feynman diagrams of open and closed string can provide a rationale for (non)planar dualities.

Within about two years, strings took center stage: open strings for the Veneziano amplitude and closed strings for its Shapiro-Virasoro counterpart, thanks to Yoichiro Nambu, Holger Nielsen and Leonard Susskind (fig. 1). Strings could clearly explain the origin of (non)planar dualities: their Feynman diagrams involve rubber-like bands or tubes, whose deformations can subsume contributions that in ordinary models of Quantum Field Theory would arise from genuinely different terms. However, consistency requirements readily introduced some surprising restrictions. Massless spin-1 modes and a single massless spin-2 mode thus accompanied inevitably, in open and closed strings, their infinitely many massive excitations, and the resulting long-range forces were clearly foreign to hadron physics. Moreover, the dynamics was apparently consistent only in 26 dimensions, where the open (and closed) would-be mesons were actually tachyons, particles of imaginary mass that signal vacuum instabilities. Experts might object that there would be tachyons also in the Standard Model, around the unstable vacuum with unbroken gauge symmetry, but shifting the Higgs doublet field suffices to eliminate them. We are still unable to perform this type of operation for complete string spectra, and indeed the tachyon problem for these "bosonic strings" is still unsolved, despite some important progress spurred by Ashoke Sen in the late 1990's. Finally, strings interacted at high energies less efficiently than particles, which clashed with the deep inelastic scattering experiments that in the late 1960's revealed point-like constituents inside the proton.

These apparent failures, however, soon proved a blessing in disguise. An enticing and far more ambitious picture potentially encompassing all Fundamental Interactions emerged from their ashes, where the massless spin-1 modes of open strings could be photons and other Yang-Mills vector bosons, while the single massless spin-2 mode of closed strings could be the graviton, the purported quantum responsible for the gravitational force. Moreover, gravity emerged inevitably, because closed strings soon proved ubiquitous ingredients in String Theory, and allowed the extra dimensions to curl up according to an old proposal of Theodor Kaluza and Oskar Klein, yielding a palatable four-dimensional interpretation. When Joel Scherk, John Schwarz and Tamiaki Yoneya came this far in the mid 1970's, a few diehard enthusiasts started to struggle with the tachyons, trying to remove them consistently from string spectra. These efforts led to what remains, in my opinion, the most important contribution to String Theory, the GSO projection of Ferdinando Gliozzi, Scherk and David Olive. Some deep novelties of the 1970's, however, were instrumental for this step, and therefore it is time to return to Quantum Field Theory and Particle Physics in this survey.

3. Supersymmetry, Supergravity and String Theory

In the 1970's, several decades of research converged into the "Standard Model". This Quantum Field Theory accounts for the Strong Interactions of quarks, the Weak Interactions of quarks and leptons and the Electromagnetic Interactions of quarks and charged leptons, combining gauge symmetries with vacua that respect only part of them. The underlying phenomenon, "spontaneous symmetry breaking", had surfaced originally in Ferromagnetism and made the headlines, at the beginning of this decade, with the discovery of the Higgs boson. The Standard Model involves three gauge groups, SU(3) for the Strong Interactions and SU(2)xU(1), spontaneously broken to the $U(1)_{em}$ of Electromagnetism, for the Electroweak Interactions. Spontaneously broken gauge symmetries readily granted a setup compatible with the renormalization procedures that had proved successful for Quantum Electrodynamics. Moreover, even if the short-range Weak Interactions are chiral, and thus violate parity (and also time reversal), the known patterns of elementary particles conspire remarkably to eliminate all anomalies, dangerous violations of classical symmetries that quantum effects could have induced in the Standard Model.

These momentous developments left aside gravity, which also rests on a gauge principle, related in this case to diffeomorphisms or local Lorentz rotations in spacetime. Gravity need not be present in Quantum Field Theory and appears largely irrelevant in the microscopic domain, where other interactions are far more intense. However, it dominates large-scale phenomena in the Universe via cumulative effects, and its quantum behavior presents an ultraviolet problem that makes it puzzlingly incompatible with the renormalization techniques at work in the Standard Model. One can get a feeling of this from the similarities between Coulomb's law of Static Electricity and Newton's law of Gravity, which suggest the replacements

$$\alpha_e = \frac{e^2}{\hbar c} \rightarrow \frac{G\, m^2}{\hbar c} \rightarrow \alpha_G = \frac{G\, E^2}{\hbar c^5} \, , \qquad (5)$$

in view of the mass-energy correspondence. These simple steps identify a gravitational ``fine-structure function'', which grows however quadratically with energy and becomes of order one at

$$E_{Planck} = \sqrt{\frac{\hbar c^5}{G}} \sim 10^{19}\, GeV \, . \qquad (6)$$

Hence, according to General Relativity gravity should become strong at this Planck energy scale. These considerations convey a feeling of why quantum effects of "virtual" particles of arbitrarily high energies ought to be very singular in General Relativity, and an explicit analysis confirms all this, which however is not all. The Compton wavelength for a particle of mass m_{Planck},

$$\lambda_C = \frac{\hbar}{m_{Planck}\, c} \, , \qquad (7)$$

which characterizes typical quantum uncertainties, is comparable to the Schwarzschild radius of a corresponding black hole, and the very nature of spacetime is thus called into question at the Planck scale.

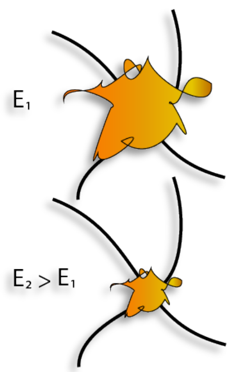

Fig. 2. A cartoon linking the gravitational interactions of strings at high energies to their partial overlaps.

One can simply argue that strings grant milder high-energy interactions. Referring to fig. 2, in this limit their effective pairwise coupling ought indeed to result from the energy carried by the portions of their length l_s that are bound to overlap, so that

$$\alpha_G = \left(\frac{E}{E_{Planck}}\right)^2 \Rightarrow \left(\frac{E}{E_{Planck}}\right)^2 \left(\frac{\Delta x}{l_s}\right)^2 \sim \left(\frac{\hbar c}{l_s E_{Planck}}\right)^2 , \quad (8)$$

since in view of the uncertainty principle

$$\left(\frac{\Delta x}{l_s}\right)^2 \sim \left(\frac{\hbar c}{E\, l_s}\right)^2 . \quad (9)$$

These simple considerations point in the right direction, but arguments that are more refined show that high-energy string interactions are actually far milder. One can thus foresee that String Theory can tame the ultraviolet problem of gravity, albeit at the non-negligible price of bringing along extra dimensions that are foreign to our senses. Moreover, gravity makes their features apparently unpredictable from first principles. I shall return to this important point in the following.

Let me also mention that generalizations of electric-magnetic duality, a familiar property of Maxwell's equations, can provide important clues on the behavior of theories where the counterpart of α_e is large. When combined with Quantum Mechanics, electric-magnetic duality leads in fact to Dirac's relation

$$\alpha_e\, \alpha_m = \frac{1}{4} \quad (10)$$

between minimal electric and magnetic "fine-structure" constants. This key result indicates that "dual" formulations, where "magnetic" degrees of freedom different from the naïve ones dominate the dynamics, can prove valuable to investigate strong-coupling regimes. As we shall see shortly, considerations of this type, together with other key simplifications, provide glimpses of an enticing picture for String Theory in the presence of space-time supersymmetry, a Bose-Fermi correspondence that we now turn to describe.

It all started in 1970, when Pierre Ramond (R) found an analogue of the Dirac equation describing a tower of fermionic string excitations, and André Neveu and John Schwarz (NS) found a similar setting that, however, gave rise to a tower of bosonic modes. Putting the two "sectors" together allowed, for the first time, to combine Bose and Fermi excitations in the NSR string. This required 10 dimensions, rather than 26 as the bosonic Veneziano and Shapiro-Virasoro constructions, but did not seem to improve matters, since tachyon instabilities were still present. At any rate, Quantum Chromodynamics soon put an end to the attempts of framing the Strong Interaction in this language.

The original NSR model had nonetheless a profound legacy for Quantum Field Theory, which began to surface in Western Europe when Jean-Loup Gervais and Bunji Sakita exhibited a striking Bose-Fermi symmetry, or supersymmetry, on the world-surfaces swept by NSR strings. In the Soviet Union, Yuri Golfand and Evgeny Likhtman, and Dmitry Volkov and Vladimir Akulov, had anticipated some of its features, but Julius Wess and Bruno Zumino opened a hectic and exciting period in 1974 presenting a four-dimensional supersymmetric scalar-spinor theory. Sergio Ferrara and Zumino and, independently, Abdus Salam and John Strathdee, obtained its Yang-Mills counterpart, and with these ingredients Pierre Fayet began to explore supersymmetric extensions of the Standard Model, coining also the familiar terms "photino" and "gaugino". In 1976 Daniel Freedman, Peter van Nieuwenhuizen and Sergio Ferrara made a major step forward discovering Supergravity, a deep extension of General Relativity based on Supersymmetry, where the metric tensor couples to spin-3/2 fields called "gravitini". Stanley Deser and Zumino complemented their work, and drawing some inspiration from Supergravity they proposed convenient action principles for strings, a key result that Lars Brink, Paolo Di Vecchia and Paul Howe also obtained independently. The following two years saw an outburst of activity in Supergravity, which culminated in two remarkable discoveries. These were the construction of the unique Supergravity in eleven dimensions by Eugene Cremmer, Bernard Julia and Joel Scherk, and the subsequent recovery from it, by the first two authors, of the maximal N=8 supergravity in four dimensions. This model of unprecedented complexity, with profound symmetries whose full import is still under scrutiny, describes the interactions of gravity with 8 gravitini, 28 vectors, 56 spinors and 70 scalar fields. Eleven dimensions is an upper bound for Supergravity, while a handful of options, now called type-IIA, type-IIB and type-I models, soon emerged in ten dimensions.

4. Ten-Dimensional Superstrings

In 1976 Gliozzi, Scherk and Olive realized that the NSR construction violated somehow the link between spin and statistics for part of the string excitations. Once they devised a projection that could remedy all this, the result was not only the removal of tachyons but also an enticing link between String Theory and ten-dimensional Supergravity, which encompassed its low-energy features! The new perspective, however, did not establish a complete correspondence between String Theory and Supergravity. It left aside the eleven-dimensional model and the variants of Supergravity with Yang-Mills interactions, whose vacua are typically anti de Sitter (AdS) spaces. Bernard de Wit and Hermann Nicolai built the deformed N=8 model in four dimensions in 1982, but the actual significance of this type of constructions remained unclear for a while.

In the early 1980's a wide scrutiny of higher-dimensional theories impinged on the generic tendency of the Kaluza-Klein mechanism, in its original form, to wash out chirality, even if initially present in higher dimensions. Moreover, anomalies can place stronger constraints on chiral theories in higher dimensions, and yet Luis Alvarez-Gaumé and Edward Witten showed that they were magically absent in type-IIB Supergravity. While chiral, that theory is however very remote from the Standard Model, since for one matter it does not include any Yang-Mills fields. A wide community was thus stunned when Michael Green and Schwarz showed that anomalies also cancel in the type-I string, whose massless open-string modes include Yang-Mills gauge fields and other matter along the lines of the Standard Model, for the special gauge group SO(32). They also identified, in the low-energy supergravity, a new cancellation mechanism that would allow two gauge groups, SO(32) and E_8xE_8, but only the first choice was available for type-I strings, as had been shown previously by Neil Marcus and myself. David Gross, Jeff Harvey, Emil Martinec and Ryan Rohm thus took a different route that led them, within a couple of months, to closed-string hybrids of the NSR and bosonic string models. They built two versions of these "heterotic" strings, with gauge groups E_8xE_8 and SO(32): the same type-I supergravity described, at low-energies, the latter and the type-I model.

5. Calabi-Yau Manifolds and Orbifolds

The next task was establishing a connection with our four-dimensional world relying on some variant of the Kaluza-Klein mechanism. Some authors had already explored in the 1970's products of circles, which are usually called tori. However, this simple option destroys chirality, even if originally present in ten dimensions. Still, when Kumar Narain took a closer look at the problem in 1986, he was confronted with much more freedom than originally expected: consistent string spectra existed for generalizations of products of circles depending on large continuous families of parameters. These parameters, which include shape and volume of the internal manifold, bear the somewhat abused name of "moduli": they affect the low-energy couplings that emerge in four dimensions precisely as dictated by N=4 supergravity, and this freedom leads to a complete loss of predictivity! One often hears that this fate is a problem of String Theory, but it is deeply rooted in Einstein gravity, which lacks a global minimum energy principle. The problem is rather, in my view, that insofar as we understand

String Theory, it does not seem to introduce significant restrictions. I shall return to this important point at the end, since it is now the object of renewed interest from a different perspective.

At any rate, Philip Candelas, Gary Horowitz, Andrew Strominger and Witten obtained classes of chiral spectra in four dimensions from the E_8xE_8 heterotic string, shortly after its discovery, relying on the low-energy Supergravity. This work brought to prominence Calabi-Yau manifolds, profound generalizations of products of circles where internal fluxes break the large gauge symmetry present in ten dimensions. They also reduce, at the same time, the amount of supersymmetry to the minimal N=1, which is compatible with the chiral nature of the Weak Interactions and yet grants vacuum stability. One should notice the change of perspective: in the original Kaluza-Klein mechanism one *symmetric* extra dimension gave rise to Electromagnetism, while in Calabi-Yau manifolds *asymmetric* extra dimensions are used to break the ten-dimensional groups, E_8xE_8 and SO(32), to smaller ones. These constructions connected for the first time ten-dimensional strings to supersymmetric GUT's, elegant extensions of the Standard Model that predict proton decay.

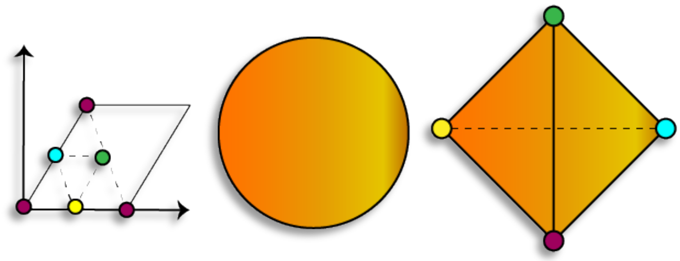

Fig. 3. The tetrahedron is an orbifold of a two-torus, here represented as a parallelogram with opposite sides identified. It can be obtained identifying points of the torus that differ by an inversion and captures the deformation of a sphere that confines its curvature to the four vertices. Closed strings behave simply on tori, and one can extend the description to orbifolds in two steps. The first is a projection of the spectrum to states symmetric under the inversion, while the second is the addition of "twisted" sectors that reside at "fixed points", the points that the inversion maps to themselves. In this example they are the four vertices of the tetrahedron.

Lance Dixon, Harvey, Cumrun Vafa and Witten then showed that, in special limiting instances where the internal curvature concentrates on lower-dimensional subspaces, Calabi-Yau compactifications afford exact realizations in String Theory via orbifolds, which also revealed important details of its inner workings. The tetrahedron in fig. 3 is a simple example of two-dimensional orbifold. In theories with closed string only, orbifold constructions begin with projections of simple toroidal spectra and, in general, additional (twisted) sectors that live precisely where the curvature concentrates must be present for consistency. "Modular invariance", a property of the bosonic string that the GSO projection had extended to the NSR framework, is the deep logic behind the emergence of twisted sectors. In short, there are infinitely many ways of selecting a time direction on the torus, the simplest vacuum amplitude for closed strings, and thus of identifying the spectrum flowing on it, which should be all equivalent (fig. 4). They correspond to the infinitely many (pairs of) elements of the SL(2,Z) group of 2x2 matrices with integer entries and unit determinant, which is usually referred to as the "modular group" of the torus amplitude. This strong consistency condition is, essentially, what determines consistent closed-string spectra. It also played a key role in fruitful cross-fertilizations with two-dimensional Statistical Physics in the 1980's.

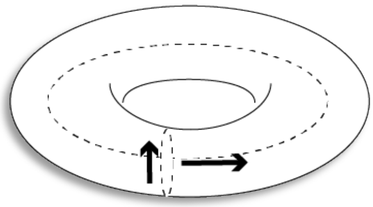

Fig. 4. The torus amplitude captures the one-loop vacuum energy for closed strings, after an integration over inequivalent shapes. There is no preferred direction of time on the torus (the figure displays two of them), and modular invariance demands that the same spectrum emerge from all corresponding Hamiltonians. The resulting integrands encode the detailed features of closed-string spectra.

Summarizing, five consistent ten-dimensional supersymmetric strings, or superstrings for short, which could provide ultraviolet completions of Einstein gravity, had emerged by the mid 1980's. The first two are the non-chiral type-IIA model, which reduces at low energies to type-IIA supergravity, and the chiral type-IIB model, which reduces at low energies to type-IIB supergravity. They only involve closed strings and no Yang-Mills fields are present in their massless spectra. As we have stressed, however, these fields play a prominent role in the Standard Model, and they are present among the massless modes of the SO(32) type-I model of open and closed strings and in the two heterotic models of closed strings. Systematic constructions leading to chiral models with N=1 supersymmetry in four dimensions soon became possible with closed strings only, and the algebraic approach pioneered by Ignatios Antoniadis, Costas Bachas and Costas Kounnas was most stimulating for my own research.

Surely, if one demands a close correspondence with the Standard Model, matters become more subtle, but there is something to meditate upon here. Why, pushing Theoretical Physics to its (current) limits, one finds a handful of options, and yet the resulting four-dimensional scenarios are manifold? The structure of our Universe might be an accident, as anthropic considerations have embarrassingly suggested for some time.

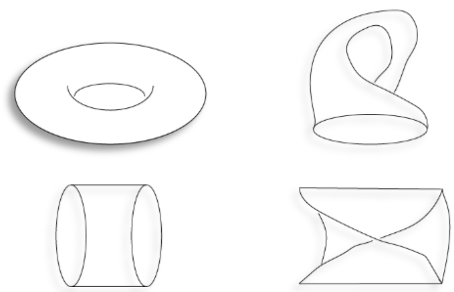

Fig. 5. The torus amplitude is accompanied, in the orientifold construction, by the Klein bottle, the annulus and the Möbius strip. In each of the additional surfaces there is one choice of time direction that manifests the closed-string spectrum.

6. Orientifolds and D-Branes

Open strings entered the preceding picture in 1987. String Theory had revealed, by then, that it reacts to projections of its spectra by the emergence of new sectors, and I realized that a type of operation now called Ω projection is what links the type-IIB and type-I strings in ten dimensions. The closed spectrum of the latter results indeed from a projection obtained mixing the left-moving and right-moving string modes of the former. These two families of waves propagate along elementary closed vibrating strings, and remain independent even in intricate string diagrams that describe quantum corrections. However, they combine at the ends of open strings, which play the role of a twisted sector. Tadpole conditions, Diophantine equations linking to one another the last three surfaces in fig. 5, all of which involve closed strings in special channels, determine the detailed nature of the resulting open spectra, and in particular the SO(32) gauge group of the type-I string in ten dimensions. This original intuition grew into a systematic construction of new classes of vacua in lower dimensions, now called generically "orientifolds", which revealed a number of unexpected features related to anomalies, gauge groups and scenarios for supersymmetry breaking. Long and extensive efforts of Massimo Bianchi, Gianfranco Pradisi and Yassen Stanev, who is highly missed by me and many other colleagues and, for briefer interludes, contributions of Carlo Angelantonj, Davide Fioravanti and Fabio Riccioni, were also instrumental to this effect. The late Joe Polchinski added a pervasive space-time picture, associating open sectors to extended solitons (D-branes) and closed-string projections to end-of-the-world mirrors (O-planes). This deconstruction gave D-branes a life of their own as extended solitons, beyond the perturbative limit where they appear infinitely rigid, and the whole setting has stimulated a large number of other developments.

7. String Dualities in Ten Dimensions

The late 1980's also witnessed a deeper appreciation of T-duality, a key link between large and small radii in String Theory that had played a prominent, if indirect, role in the formulation of the heterotic string. In Quantum Mechanics, a particle propagating on a circle of radius R must have momenta $p_m = \frac{m}{R}$, with m integer, for its wavefunction to be periodic, but a closed string can also wind around the circle an arbitrary number of times. As a result, the masses of states in the closed bosonic string involve the pair *(m,n)* of quantum numbers, and

$$M^2 \sim \left(\frac{m}{R}\right)^2 + \left(\frac{n\,R}{\alpha'}\right)^2 + O(\alpha') \,. \qquad (11)$$

The first two terms reflect the internal momentum and the overall winding of strings around the circle, whose contribution grows with R, and we have left implicit the contributions of string modes. This simple expression suffices to indicate that one can turn into one another closed-string spectra at large and small radii combining the interchange of momenta and windings with the inversion of the radius:

$$R \to \frac{\alpha'}{R} \quad . \qquad (12)$$

The familiar behavior of the Ising model under a similar discrete symmetry would suggest that something special ought to occur at the self-dual radius

$$R = \sqrt{\alpha'} \quad , \qquad (13)$$

and this is indeed the case, independently, for the left- and right-moving modes of closed bosonic strings. In Field Theory, as we have anticipated, the Kaluza-Klein mechanism on a circle grants the emergence of a vector, consistently with the fact that, say, a five-dimensional matrix g_{MN} contains the lower-dimensional metric $g_{\mu\nu}$, a vector $g_{\mu,4}$ and a scalar $g_{4,4}$. However, one can show that two sets of *three* vectors emerge from the left-moving and the right-moving modes of the bosonic string, if one tunes the radius to the self-dual value in eq. (13), where the contributions in eq. (11) become commensurable to the other string excitations. The corresponding gauge group extends from U(1) to SU(2), and for the bosonic string two SU(2) groups, each of rank one, arise in this fashion from left-moving and right-moving modes. Each special circle can thus enlarge by one unit the rank of the gauge group arising, say, from the left-moving modes, so that special combinations of 16 circles can yield at best gauge groups of rank 16. The reader may have noticed that 16, the difference between the critical dimensions of the bosonic and NSR string models, is also the rank of the two ten-dimensional gauge groups SO(32) and $E_8 x E_8$. This is what we alluded to before: the two heterotic strings rest indeed, in technically subtle ways, on tori with self-dual radii for their bosonic-string modes.

A closer look would actually reveal that T-duality involves a parity inversion acting on right-moving modes. This explains why there is no self-dual point for type-II strings while T-duality turns type-IIA and type-IIB strings into one another: a right-moving parity interchanges their chiral R sectors. A more sophisticated reasoning would show that T-duality interchanges the SO(32) and $E_8 x E_8$ heterotic models and, as we have seen, the orientifold construction links type-IIB and type-I strings, so that in the early 1990's one was left already with only two groups of ten-dimensional superstrings. Edward Witten completed the picture in 1995, supplementing these results with brilliant and well-motivated conjectures on non-perturbative links sufficient to join all five ten-dimensional superstrings.

The lessons that led further were deeply rooted in the low-energy Supergravity. We have already mentioned that a handful of choices exist in ten dimensions, while most notably a unique form of supergravity exists in eleven dimension. This theory involves only the metric g_{MN}, a gravitino ψ_M and a 3-form potential A_{MNP}. This type of generalizations of the vector potential had led Michael Duff, Paul Townsend and others to stress, over the years, their direct link to extended solitons or p-branes. Just as charged particles source the vector potential A_M of Electrodynamics, whose label M reflects the tangent vectors to their world lines, strings source potentials A_{MN}, whose labels reflect the tangent planes to their world surfaces, and indeed all low-energy manifestations of String Theory contain at least one potential of this type. However, membranes, rather than strings, source the potential A_{MNP} of eleven-dimensional supergravity, so that this theory appeared foreign to the picture that we have illustrated so far. However, the Kaluza-Klein mechanism complicates matters: in ten dimensions g_{MN} gives rise to a metric $g_{\mu\nu}$, a vector field $g_{\mu,10}$ and a scalar $g_{10,10}$, which is the dilaton ϕ of type-IIA String Theory, while A_{MNP} gives rise to $A_{\mu\nu\rho}$ and to $A_{\mu\nu,10}$, which does source strings. One can say, therefore, that strings emerge from membranes wrapped along the eleventh dimension!

The key to appreciate further developments is that our current understanding of String Theory is largely confined to power series in the string coupling

$$g_s = e^{\phi} \quad , \qquad (14)$$

where ϕ, the dilaton, is a universal low-lying excitation that accompanies the graviton. Supergravity does not suffer from this limitation, although it only captures the corresponding low-energy dynamics. Our tools and the resulting intuition on String Theory are thus effective only when the string coupling is small, and the lowest-order contributions link strings with General Relativity and Yang-Mills theory. Witten stressed, however, that the Kaluza-Klein radius of the circle connecting the eleven-dimensional and type-IIA supergravities grows as a power of g_s, and this has a striking consequence. The eleven-dimensional theory of Cremmer-Julia-Scherk does enter the big picture, albeit in the limit of infinite string coupling, where strings are no more central to it! As its ten-dimensional cousins, it ought to be the low-energy limit of something, but we have no idea of what lies behind or beyond it. The naïve guess would be membranes, but our control of their dynamics is too limited to come to a definite conclusion. However, one can also link the eleven-dimensional theory to the $E_8 x E_8$ heterotic model via a spacetime counterpart of the orientifold construction, which builds a compactification on an interval, and finally the SO(32) heterotic and type-I strings via an electric-magnetic duality, with couplings related according to eq. (10). All ten-dimensional string theories are thus equivalent, in such a way that the excitations of one map into branes, the solitons of another, but equivalent to what? What is the principle underlying this monumental setup? It is hard to resist the feeling that, after one Century, Physics has impinged again on something like the old Bohr-Sommerfeld rules. If this is indeed the case, the near future may have deep surprises in store (see fig. 6).

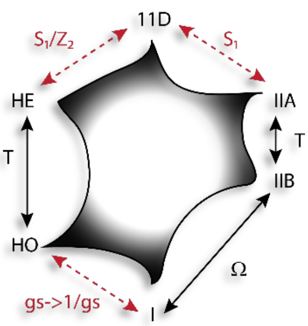

Fig. 6. Dualities among supersymmetric strings in ten dimensions also link them to 11D supergravity. The overall picture, usually called M-theory, transcends String Theory and our very notion of spacetime.

8. The AdS/CFT Correspondence

The AdS/CFT correspondence is one more facet of this enticing, mysterious and incomplete picture. I have already mentioned that Supergravity admits generically continuous deformations leading to non-abelian Yang-Mills fields and supersymmetric AdS vacua. What is their meaning? A key input into the story was the recognition that AdS spaces have a spatial boundary at infinity, which is somehow akin to an infinite stretching of Minkowski space. This fact, together with some stringent symmetry arguments, led Juan Maldacena to propose a non-perturbative correspondence between the bulk of five-dimensional AdS space and its four-dimensional boundary. More precisely, the AdS/CFT correspondence links, in its simplest setting, the type-IIB String Theory and its low-energy Supergravity, in the bulk, to the maximally supersymmetric Yang-Mills theory, which is conformally invariant and thus insensitive to stretching, in the boundary. This observation gave rise to an unprecedented wave of excitement, since it translated for the first time non-perturbative investigations of a class of gauge theories into accessible analyses in Supergravity. In addition, it materialized old holographic conjectures about gravity, stimulated originally by the Bekenstein-Hawking relation between black-hole entropy and horizon area. Still, the close link to special AdS spacetimes, at least asymptotically, limits in my opinion the actual insights that AdS/CFT can provide into the nature of Quantum Gravity.

9. Where Do We Stand?

Let me now come, in conclusion, to what I consider the real conceptual frontier for the whole program. It has to do with the deep and vexing issue of supersymmetry breaking. The key results that I have mentioned rest indeed heavily on supersymmetry, since for one matter the five theories in ten dimensions and the eleven-dimensional supergravity embody this principle, together with their simpler vacua, which are also stable thanks to supersymmetry. Supersymmetry introduces enormous simplifications that find a counterpart, if you will, in what Complex Analysis does in Mathematics. However, it must be broken somehow, since the spectrum of the Standard Model does not display any Bose-Fermi correspondence. Intriguingly, when supersymmetry is broken in the presence of gravity quantum effects tend generically to destabilize vacua. Even if the process were to terminate nearby, as I have stressed, we would be unable, in general, to follow it in String Theory with our current tools. We can do it to some extent in gravity, but how does one know, then, that the result still concerns String Theory? These questions have led to weak-gravity and other conjectures that are currently under scrutiny. Some mounting evidence indicates that gravity can constrain to some extent Physics at LHC energy scales, although its microscopic effects become large, if they do, many orders of magnitude beyond them. There is also some evidence that breaking Supersymmetry in String Theory is less harmful in cosmological scenarios, where it appears to provide even some clues on the onset of the inflationary phase. There are some analogies with Newtonian gravity, which grants stable dynamical patterns of planets, stars, galaxies and clusters in the Universe and yet does not allow any stable static configurations. This line of research builds on an important contribution of Emilian Dudas and Jihad Mourad that was stimulated, in its turn, by novel scenarios that the orientifold construction allows for supersymmetry breaking. Progress along these lines is clearly essential to connect more convincingly String Theory to Particle Physics. Perhaps even more importantly, however, it can potentially shed light on some long-standing mysteries that I have tried to address here.

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