SCUOLA NORMALE SUPERIORE
Dipartimento di Scienze Politico-Sociali
Corso di Perfezionamento in Civiltà del Rinascimento

ATOMISM AND MATHEMATICS IN THE THOUGHT OF GIORDANO BRUNO

Relatore: Prof. Michele CILIBERTO
Correlatore: Dr. Aurélien ROBERT
Supervisore: Prof.ssa Nicola PANICHI

Tesi di Perfezionamento di:
Paolo ROSSINI
Matr. n. 19689

Anno Accademico 2018 – 2019

SCUOLA NORMALE SUPERIORE

Dipartimento di Scienze Politico-Sociali

Corso di Perfezionamento in Civiltà del Rinascimento

SCUOLA
NORMALE
SUPERIORE

ATOMISM AND MATHEMATICS IN THE THOUGHT OF GIORDANO BRUNO

Relatore: Prof. Michele CILIBERTO

Correlatore: Dr. Aurélien ROBERT

Supervisore: Prof.ssa Nicola PANICHI

Tesi di Perfezionamento di:

Paolo ROSSINI

Matr. n. 19689

Anno Accademico 2018 – 2019

Table of Contents

Acknowledgements

Working on this thesis has been an incredible journey, both literally and metaphorically. It has taken three years or so and has given me the possibility to visit new places and meet new people. This alone has made my PhD experience worthwhile. It all started in November 2015 when, thanks to prof. Michele Ciliberto, I was given the opportunity to pursue a PhD at the Scuola Normale Superiore of Pisa. As my PhD supervisor, prof. Ciliberto was always very supportive, allowing me to follow my research interests wherever they brought me. I hope I put the freedom he granted me to good use.

Along the way, I joined forces with Aurélien Robert who, as my PhD co-supervisor, helped me work my way through the *mare magnum* of Bruno's sources. I greatly benefited from his extensive knowledge of medieval philosophy and atomism, and I would like to thank him for patiently answering my questions during our lunch breaks at the Bibliotèque nationale de France in Paris.

A grant from the Ermenegildo Zegna Foundation allowed me to spend two months at the University of Southern California in Los Angeles. Thanks to the generosity of David Albertson, I learned more about Nicholas of Cusa in those two months at USC than I did in my undergraduate studies. In addition to providing guidance on all sorts of Cusanus-related issues, David helped me strengthen the argument of the dissertation, starting with the introduction. I would also like to thank the staff of USC's School of Religion, above all Linda Wootton and Johnna Tyrell, for making me feel welcome and for their assistance in obtaining a visa.

A scholarship funded by the Erasmus Program gave me financial support to conduct research for two months at SPHERE, a research unit of the French National Center for Scientific Research (CNRS) located in Paris. There I had the chance to work with Vincenzo De Risi and David Rabouin, whose insightful comments led me to take a closer look at Bruno's dialogues on Mordente's compass. Speaking of Mordente, special thanks go also to the Director of the Museo Galileo in Florence, prof. Filippo Camerota, who kindly explained to me the functioning of Mordente's compass and how Bruno's view of this instrument diverged from that of its inventor.

During the three years of my PhD, I had the possibility to present my research at several conferences and meetings. In addition to providing a suitable forum to discuss my ideas on Bruno's atomism and mathematics, these meetings brought me in contact with scholars who, at various stages of my research, inspired and encouraged me to write this dissertation: Amir Alexander, Ovanes Akopyan, Angela Axworthy, Delphine Bellis, Susanna Berger, Paul Richard Blum, Guy Claessens, Stephen Clucas, Brian Copenhaver, Mordechai Feingold, Daniel Garber, Laura Georgescu, Guido Giglioni, Miguel A. Granada, Helen Hattab, Stephen Johnston, Dilwyn Knox, Laura Kotevska, Thomas Leinkauf, Lodi Nauta, Marco Panza, Doina-Cristina Rusu, Arielle Saiber, Andrea Sangiacomo, JB Shank, Maarten Van Dyck, Maude Vanhaelen.

Last but not least, I would like to thank both my Italian family—my parents Bianca and Luigi, my brother Luca and my grandmother Maria—and my Dutch family—Nel, Rob, Jos Ingrid and Lise—who have supported me in every way. Besides helping me with all sorts of tasks (including moving in and out of Pisa), their constant presence throughout the years has given me the stability I needed to focus on my studies. *Dulcis in fundo*, Ingeborg van Vugt deserves a special mention. As a PhD student herself, she knows the ups and downs of going through this experience. Her constant support and encouragement raised the level of my work, and made me a better researcher. For starters, she is the reason why this dissertation is written in English, although she speaks a perfect Italian.

Sommario

L'obiettivo di questa tesi è fornire un'analisi del pensiero matematico e della teoria atomistica di Giordano Bruno (1548-1600). Questi due aspetti della pensiero di Bruno sono strettamente intrecciati tra di loro nella misura in cui la matematica di Bruno è fondata sull'assunto atomistico che gli oggetti matematici (così come gli oggetti fisici) sono formati da parti indivisibili che Bruno chiama "minimi." La teoria bruniana dei minimi è stata già oggetto di analisi fin dalla fine del '800, quando Kurd Lasswitz vide in questa teoria il segno della rinascita dell'atomismo nell'età moderna. Tuttavia, nonostante questo riconoscimento dell'importanza di Bruno nella storia dell'atomismo e della scienza in generale, il giudizio di Lasswitz sulla teoria bruniana dei minimi evidenziava alcune criticità relative proprio alle sue applicazioni fisico-matematiche. A risentire di questo giudizio critico fu soprattutto la matematica di Bruno che continuò ad esser guardata come una teoria "obsoleta" anche nel corso del '900. Più di recente, gli studi di Aquilecchia, Bönker-Vallon e De Bernart, tra gli altri, hanno contribuito a ricalibrare questa immagine negativa della matematica bruniana. Tuttavia, molte rimangono le questioni aperte a proposito di questa teoria. Questa dissertazione rappresenta un tentativo di dare risposta a tre questioni e, nel far questo, intende fornire un'analisi della matematica di Bruno che tenga conto dei suoi punti di forza oltre che delle sue criticità. Le tre questioni affrontate in questo lavoro sono:

(1) Quali sono le fonti della matematica bruniana, ed in particolare delle idee di minimo e punto-atomo?
(2) In quale misura l'idea bruniana di minimo può essere considerata un antesignano del concetto moderno di grandezza infinitesimale?
(3) Bruno fu davvero un sostenitore del realismo matematico, come sostenuto da Hélène Védrine, la quale riteneva che questo fu il principale "ostacolo" che impedì a Bruno di sviluppare una teoria matematicamente corretta?

La prima parte della tesi affronta la prima domanda. Prima di individuare gli specifici testi ed autori che potrebbero aver inspirato Bruno a sviluppare la sua "geometria atomistica," la questione della fonti bruniane è affrontata in termini di "tradizione." La tesi è che la geometria atomistica di Bruno si inserisca nel solco tracciato da alcuni atomisti medievali, i quali, prendendo spunto da testi

Neopitagorici come *l'Institutio arithmetica* di Boezio, definirono il punto come atomo o "unità avente posizione." Come dimostrato recentemente da Aurélien Robert, tale definizione atomistica del punto divenne il punto di partenza per esplorare concezioni del continuo alternative a quella proposta da Aristotele. Com'è ben noto, Aristotele fu un convinto oppositore dell'atomismo, essendo dell'opinione che la divisione del continuo potesse procedere potenzialmente all'infinito. Tra gli autori medievali che, sfidando apertamente Aristotele, affermarono che il continuo fosse composto da punti indivisibili, vi era una nota fonte bruniana: Raimondo Lullo. Così come Bruno, Lullo era in linea con la tradizione dell'atomismo pitagorico. Il primo capitolo ricostruisce la storia di questa tradizione da un punto di vista storiografico, fornisce un'analisi del testo lulliano in cui le somiglianze tra le teorie matematiche di Bruno e Lullo sono più evidenti (il *Liber de geometria nova*), e si conclude con una sezione tesa ad accertare se Bruno possa effettivamente aver letto questo testo. A mia conoscenza, la *Geometria nova* di Lullo non è mai stata messa in relazione con la matematica di Bruno.

Il secondo capitolo prende in esame Niccolò Cusano, l'autore che Bruno stesso riconosce come fonte privilegiata della sue idee matematiche. Lo scopo di questo capitolo è duplice. Innanzitutto, esso intende chiarire se e in quale misura ci sia una connessione tra l'atomismo cusaniano e l'atomismo pitagorico. La tesi è che tale connessione sia fornita dai concetti di *explicatio* e *complicatio*, concetti che Cusano eredita dalla tradizione pitagorica (e in particolare dal commento di Thierry de Chartres sull'*Institutio arithmetica* di Boezio) e che rappresentano due elementi chiave della sua teoria atomistica. Per sostanziare questa tesi, il capitolo fornisce un'analisi di come il significato attribuito ai concetti di *explicatio* e *complicatio* evolva nel passaggio da Boezio a Thierry de Chartres e Cusano. Inoltre, il capitolo traccia lo sviluppo concettuale di *explicatio* e *complicatio* all'interno dell'opera di Cusano, prendendo come punti di riferimento il *De docta ignorantia* (1440) e il *De mente* (1450). Il capitolo si conclude con una sezione volta a mostrare le similitudini e, soprattutto, le differenze tra la concezione cusaniana di minimo (contrassegnata da un'inestinguibile ineffabilità) e la concezione bruniana di minimo (la quale, pur conservando un certo margine di indeterminatezza, sembra essere più alla portata dell'intelletto umano).

La seconda parte della dissertazione si concentra sul contenuto della matematica bruniana e tenta di dare una risposta alle ultime due domande. In particolare, il terzo capitolo affronta la questione se l'idea bruniana di minimo possa ritenersi un precursore del concetto di infinitesimale. Lo fa attraverso un'analisi dei primi scritti matematici di Bruno, i dialoghi sul compasso di Fabrizio Mordente, due dei quali furono scoperti solo nel 1957 da Giovanni Aquilecchia. Sia i dialoghi sul compasso di Mordente che la discussione che essi suscitarono tra lo stesso Mordente e Bruno non hanno ricevuto grande attenzione. Per questo, dopo aver dato alcuni ragguagli storici a proposito del compasso e della controversia tra Bruno e Mordente, il capitolo si sofferma soprattutto sul primo e terzo dialogo. L'analisi di questi dialoghi mostra innanzitutto come Bruno abbia cercato di imporre la propria interpretazione del funzionamento del compasso, compasso che, nella sua opinione ma non in quella di Mordente, confermava l'esistenza di grandezze minime delle quali si componevano gli oggetti matematici. Inoltre, una lettura dei dialoghi sul compasso di Mordente rivela come la teoria matematica originalmente sviluppata da Bruno in questi dialoghi fosse più coerente della teoria che egli presenta nei suoi scritti successivi. Nel criticare la matematica di Bruno, interpreti come Leonardo Olschki si sono concentrati quasi esclusivamente sugli scritti più tardi. Tuttavia, se avessero considerato i dialoghi sul compasso di Mordente, tali interpreti avrebbero intravisto una somiglianza tra l'idea bruniana di minimo e il concetto di grandezza infinitesimale. In questi dialoghi, Bruno concepisce il minimo come un ente esteso ma non avente una forma specifica. Questa concezione del minimo era compatibile con i principi della geometria euclidea, a differenza della concezione proposta nel *De minimo,* dove Bruno affermava che il minimo fosse un punto esteso di forma circolare.

Il quarto capitolo illumina due aspetti della concezione bruniana di matematica che sembrano essere stati dimenticati, se non addirittura fraintesi, dagli studi precedenti. Il primo aspetto è che la concezione bruniana di matematica evolve nel corso del tempo ed in parallelo con un concetto centrale nell'economia del pensiero di Bruno: l'infinito. Infatti, Bruno intraprende una riforma della matematica per fare spazio al concetto di infinitamente piccolo o minimo, concetto sul quale Bruno inizia a interrogarsi solo in una fase avanzata della propria riflessione. Il secondo aspetto riguarda la possibilità che Bruno abbia difeso una forma di realismo matematico, vale a dire la credenza che gli oggetti matematici esistano in natura indipendentemente dalla nostra mente. Vero è che nei dialoghi italiani, ed in particolare ne *La cena de le ceneri*, Bruno può essere considerato un realista in virtù della sua adesione al Copernicanismo. Tuttavia, già in queste opere, e ancor più evidentemente in quelle successive, Bruno si oppone all'idea che gli oggetti matematici esistano in natura ed esprime il proprio scetticismo nei confronti di quell'approccio che gli studiosi della Rivoluzione Scientifica hanno chiamato "matematizzazione della natura." Alla luce di questo, non sembra possibile parlare di Bruno come di un realista e, ancor di più, affermare che fu il realismo ad ostacolare il suo progetto matematico (come sosteneva Védrine). Al contrario, il quarto capitolo mostra come i problemi che affliggono la teoria bruniana dei minimi siano il risultato del tentativo di integrare in una sola teoria fisica, metafisica e matematica. Bruno infatti ritiene che tutti e tre questi aspetti della realtà siano analizzabili nei termini della sua teoria dei minimi. Il capitolo si conclude con una sezione sulla monadologia di Bruno. Lo scopo è mostrare che, contrariamente a quanto affermato da alcuni interpreti, la monadologia di Bruno non fu il modello della monadologia di Leibniz, in quanto l'unico elemento che sembra accomunare queste due teorie è il fatto che entrambe si basano su una concezione pitagorica della monade.

Summary

In the latter part of his career, Giordano Bruno (1548 – 1600) developed an innovative atomistic theory of mathematical objects. According to this theory, mathematical objects were composed of indivisible parts called "minima." In traditional accounts of early modern mathematics, indivisibles entered mathematics only in the seventeenth century with Bonaventura Cavalieri. Nowadays, Cavalieri's indivisibles are considered a forerunner of the infinitesimals and are associated to the invention of the calculus. On the contrary, Bruno has been regarded as a poor mathematician and his atomistic geometry has been neglected. It is the objective of this work to change the conventional image of Bruno's mathematics.

Part One deals with the source of Bruno's atomistic geometry. I claim that Bruno belonged to the tradition of Pythagorean atomism, that is the view that mathematical objects were composed of points. The Middle Ages witnessed a revival of Pythagorean atomism as several authors proposed alternative conceptions of the continuum based on the Pythagorean definition of the point as atom or "unit having position." Two Brunian sources were among these authors: Ramon Llull and Nicholas of Cusa. Bruno borrowed aspects of his atomistic geometry from both Llull and Cusanus. In particular, I claim that Bruno was indebted to Llull for his atomistic view of mathematical objects and to Cusanus for his idea of the minimum.

Part Two offers an account of Bruno's atomistic geometry. First, it provides an analysis of Bruno's first mathematical writings, four dialogues on the compass invented by Fabrizio Mordente. The analysis of these dialogues shows that the original version of Bruno's atomistic geometry was more coherent than the version of it presented in later works. In fact, one may claim that in the dialogues on Mordente's compass Bruno conceived the minimum in a way that seemed to anticipate the concept of infinitesimal magnitude. In addition, Part Two traces the development of Bruno's conception of mathematics from his vernacular works to his Latin poems. In doing so, it shows that Bruno was not a mathematical realist (as Védrine had him) and that Bruno's atomistic geometry was unsuccessful because it was part of a more ambitious plan: the integration of physics, metaphysics and mathematics into a single theory.

Introduction

Lost in the labyrinth: Bruno and the continuum problem

> There are two famous labyrinths where our reason very often goes astray: one concerns the great question of the Free and the Necessary, above all in the production and the origin of Evil; the other consists in the discussion of continuity and of the indivisibles which appear to be the elements thereof, and where the consideration of the infinite must enter in.[1]

With their elegant simplicity and evocative power, these words, taken from Leibniz' *Theodicy* (1710), fully capture the feeling of displacement with which scholars have faced the continuum problem ever since Antiquity. As a matter of fact, Leibniz borrowed the metaphor of the "labyrinth of the continuum" from the Louvain theologian and scientist Libert Froidmont, who in 1631 had published a book entitled *Labyrinthus sive de compositione continui* (*Labyrinth or on the Composition of the Continuum*).[2] In it, Froidmont raised the issue of the composition of the continuum from an Aristotelian perspective, criticizing the atomistic view that the continuum was composed of indivisible parts and those who had defended it throughout the centuries (from Democritus to John Wycliff).[3] In Leibniz' day, the discussion on the continuum problem had taken a mathematical turn due to the introduction of a new mathematical theory central

---

[1] Gottfried Wilhelm Leibniz, *Theodicy: Essays on the Goodness of God, the Freedom of Man, and the Origin of Evil*, ed. Austin Farrer, trans. E.M. Huggard (Eugene, OR: Wipf and Stock, 2007), 55.

[2] Libert Froidmont, *Labyrinthus sive de compositione continui liber unus* (Antwerp: Balthasar Moretus, 1631). On Lebiniz's debt to Froidmont, see Carla Rita Palmerino, "Geschichte Des Kontinuumproblems or Notes on Fromondus's Labyrinthus?: On the True Nature of LH XXXVII, IV, 57 R°-58v°," *Leibniz Society Review* 26 (2016): 63–98.

[3] See Carla Rita Palmerino, "Libertus Fromondus' Escape from the Labyrinth of the Continuum (1631)," *Lias: Journal of Early Modern Intellectual Culture and Its Sources* 42, no. 1 (2015): 3–36.

to which was the concept of indivisibles. To be fair, mathematical arguments had already been used in medieval controversies against atomism, and the idea that both the geometric and the physical continuum was composed of indivisible parts had its supporters as early as the ninth century in the Islamic world and the twelfth century in the Latin West.[4] However, leaving aside Archimedes (who made use of indivisibles in a treatise entitled *The Method* only rediscovered in 1906), for the first time in the seventeenth century the attempt was made to translate the concept of indivisibles into mathematical terms.

A theory of indivisibles was first proposed by the Italian mathematician and Galileo's pupil Bonaventura Cavalieri (1598-1647). Nowadays, Cavalieri's indivisibles are regarded as a forerunner of the modern infinitesimals, and thus as a pivotal moment in the history of the calculus. However, when Cavalieri's *Geometria indivisibilibus continuorum nova quadam ratione promota* (*Geometry, developed by a new method through the indivisibles of the continua*) was published in 1635, his contemporaries saw it as a threat to the Aristotelian orthodoxy.[5] Indeed, in book VI of the *Physics*, Aristotle claimed that the continuum could by no means be composed of indivisibles.[6] Paul Guldin and André Taquet, especially, argued against Cavalieri's theory on the grounds that it

[4] See Christoph Lüthy, John E. Murdoch, and William R. Newman, "Introduction: Corpuscles, Atoms, Particles, and Minima," in *Late Medieval and Early Modern Corpuscular Matter Theories*, ed. Christoph Lüthy, John E. Murdoch, and William R. Newman (Leiden: Brill, 2001), 1–38. On the history of atomism, see also Kurd Lasswitz, *Geschichte der Atomistik vom Mittelalter bis Newton* (Hamburg: Leopold Voss, 1890); Richard Sorabji, *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages* (London: Duckworth, 1983); Bernhard Pabst, *Atomtheorien Des Lateinischen Mittelalters* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1994); Andrew Pyle, *Atomism and Its Critics: From Democritus to Newton* (Bristol: Thoemmes Press, 1997); Christophe Grellard and Aurélien Robert, eds., *Atomism in Late Medieval Philosophy and Theology* (Leiden: Brill, 2009).

[5] Bonaventura Cavalieri, *Geometria indivisibilibus continuorum nova quadam rations promota* (Bologna: Monti, 1635).

[6] See Aristotle, "Physics," in *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, ed. J. Barnes (Princeton, NJ: Princeton University Press, 1995), bk. 6.

opened the doors of geometry to atomic entities (i.e. the indivisibles).[7] Cavalieri tried in vain to convince his critics that he made no assumption about the composition of the continuum.[8] In fact, he went as far as to claim that it was the philosopher's task (and not the mathematician's) to solve the continuum problem.[9] With the exceptions of Newton, Gregorie and Huygens who tried to provide the method of indivisibles with a rigorous foundation, seventeenth-century mathematicians followed Cavalieri's example and avoided entering the labyrinth of the continuum. That is to say, they adopted the indivisibles without questioning whether their use entailed an atomistic view of the continuum. The risk of getting lost in the twists and turns of that question was far too great.

Fortunately, in the early modern era there was no shortage of brave, even reckless innovators (in Latin *novatores*[10]), who were willing to spend decades in prison and sacrifice their lives to have their voices heard. Those were the years of the Inquisition, still today a recognized symbol of religious obscurantism because

---

[7] For Guldin's criticisms see Enrico Giusti, *Bonaventura Cavalieri and the Theory of Indivisibles* (Cremona: Edizioni Cremonese, 1980), 55–65, 73–76; Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (New York: Oxford University Press, 1996), 50–55; Egidio Festa, "Aspects de la controverse sur les invisibles," in *Geometria e atomismo nella scuola galileiana*, ed. Massimo Bucciantini and Maurizio Torrini (Florence: Leo S. Olschki, 1992). For Taquet's criticisms, see Dominique Descotes, "Two Jesuits Against the Indivisibles," in *Seventeenth-Century Indivisibles Revisited*, ed. V. Jullien (Basel: Birkhäuser, 2015), 249–74.

[8] Bonaventura Cavalieri, *Exercitationes geometricae sex* (Bologna: Monti, 1647), 199: "Apud eos enim, qui sustinent continuum ex indivisibilibus componi, descriptio dictorum indivisibilium erit descriptio superficiei. Apud eos vero, qui ulta haec indivisibilia ponunt aliquid aliud in ipso continuo, illud dicendum erit, describi in ipso motu."

[9] Antoni Malet, *From Indivisibles to Infinitesimals. Studies on Seventeenth-Century Mathematizations of Infinitely Small Quantities* (Bellaterra: Universitat Autònoma de Barcelona. Servei de Publicacions, 1996), 17.

[10] Seventeenth-century scholars used the term "*novatores*" to indicated those who challenged the Aristotelian and Scholastic philosophy. Most of the times, this term had a negative connotation. See Daniel Garber, "Descartes among the Novatores," *Res Philosophica* 92, no. 1 (2015): 1–19.

of its opposition to the development and dissemination of new knowledge. It was by the Inquisition, for instance, that Tommaso Campanella (1568-1639) was imprisoned for almost twenty-seven years, during which he wrote his most important work *La città del sole* (*The city of sun*[11]) and took pains to defend Galileo from the charges of heresy.[12] Others died at the hand of the Inquisition, as in the case of Giordano Bruno (1548-1600), who escaped from Italy because of his controversial views on religion, only to be tried and sentenced to death on his return fourteen or so years later. To commemorate his tragic fate, in 1889, a statue of him was erected in the place where he was burned at the stake on February 17, 1600: Campo de' Fiori in Rome.[13] By then, he had joined the ranks of early modern intellectuals such as Spinoza, having grown to become a champion of free thought.

Nowadays, Bruno is best known for his commitment to an infinitist view of the universe as well as for his inquisitorial trial. However, as flattering as this portrait of Bruno may be, it has tended to overshadow his other contributions, starting with his mathematical work. Few know that Bruno envisioned not only the infinitely large (the boundless universe), but also the infinitely small—what he called the "minimum;" and that, not unlike Cavalieri, he spent his last years developing a new mathematical theory in order to submit the concept of the minimum to mathematical analysis. Differently from Cavalieri, however, Bruno openly challenged Aristotle's authority by making it clear that the minima were the building blocks of the mathematical continuum.[14] To substantiate his claim,

[11] Tommaso Campanella, *La città del sole: e altri scritti*, ed. Germana Ernst and Luigi Firpo (Bari: Laterza, 1997).

[12] Tommaso Campanella, *A Defense of Galileo, the Mathematician from Florence: Which Is an Inquiry as to Whether the Philosophical View Advocated by Galileo Is in Agreement with, or Is Opposed to, the Sacred Scriptures*, ed. Richard J. Blackwell (Notre Dame: University of Notre Dame Press, 1994).

[13] For the history of the statue of Bruno, see Massimo Bucciantini, *Campo dei Fiori: storia di un monumento maledetto* (Torino: Einaudi, 2015).

[14] On Bruno and Cavalieri, see Angelika Bönker-Vallon, *Metaphysik und Mathematik bei Giordano Bruno* (Berlin: Akademie Verlag, 1995); Paolo Rossini, "Giordano Bruno and Bonaventura Cavalieri's Theories of Indivisibles: A Case of Shared Knowledge," *Intellectual History Review* 28, no. 04 (2018): 461–76.

Bruno had to come to terms with Aristotle and his epigones, who for centuries had raised objections against atomism. In other words, he had to enter the labyrinth of the continuum. Bruno showed courage in defending his heretic ideas before the tribunal of the Inquisition, and in envisioning a boundless universe at a time when the general consensus was that the universe was finite. In its way, the decision to address the continuum problem was also an act of bravery on Bruno's part, considering that as late as the 1630s mathematicians still avoided confronting that problem.

And yet, as we shall see in the next section, Bruno is considered an anti-mathematician or a poor mathematician even by Bruno scholars, and *a fortiori* is neglected in the traditional accounts of modern mathematics. Why is that so? As a matter of fact, the results achieved by Bruno were, in purely mathematical terms, inconsequential. In fact, his theory was flawed in several ways, as it made it impossible, for instance, to account for incommensurable magnitudes (such as the side and diagonal of a square). Moreover, Bruno's theory of minima had hardly any successful application, mainly because geometric indivisibles were postulated rather than used by Bruno. Cavalieri, on the other hand, deserved the credit for being the first to employ indivisibles in early modern geometry, developing a method capable of solving problems of measurement more quickly and directly than the methods handed down from the past (e.g. the method of exhaustion). Hence, from a mathematical perspective, Bruno's bad reputation as a mathematician seems to be justified, although it is the purpose of this thesis to show that this received image does not do full justice to Bruno's mathematical abilities.

However, if it is true that Bruno cannot be compared to Cavalieri and the other seventeenth-century indivisibilists on the basis of their mathematical success, there are other elements that can allow for a meaningful comparison. For instance, as I have tried to show elsewhere, a comparison between Bruno and Cavalieri can illuminate their common sources or better their 'shared knowledge,' conceived as the cultural milieu in which they *independently* came to conceptualize the idea of geometric indivisibles.[15] (I italicize the term "independently" because I do not mean to claim that Bruno played a role in

[15] See previous note.

shaping Cavalieri's theory of indivisibles). Here, the focus is on the continuum problem and how Bruno dealt with it. The words of Leibniz in the *Theodicy* give us an idea of what a tremendous challenge the continuum problem posed to early modern scholars. Seen in this light, it should be no surprise that Bruno encountered great difficulties in putting his theory of minima on a solid footing. One may well argue that his attempts to solve the continuum problem were doomed to fail, given the hardness of the problem itself. This begs the question: Why should we pay attention to Bruno's mathematical activities if we already know their negative outcome? Above all, because Bruno dared to venture where other, more skilled mathematicians did not even want to set foot on: the labyrinth of the continuum. It is true that he lost himself trying to find his way out of it, but this does not make his enterprise less interesting for us, who, like the spectator in the Lucretian poem, can see Bruno's shipwreck from the shore.[16]

The question of Bruno's modernity

Bruno owes his reputation to his infinitist cosmology. Indeed, he is known as the "philosopher of the infinite" and, additionally, as an unrepentant heretic because of his death at the hands of the Roman Inquisition. If this is true, then Bruno will be a perfect candidate for the role of modern philosopher, especially considering the emphasis placed on the concept of infinity in contemporary accounts of the so-called "passage to modernity."[17] Arthur Lovejoy was one of the first to express this idea in his classical book *The Great Chain of Being:*

> Through the elements of the new cosmography had, then, found earlier expression in several quarters, it is Giordano Bruno who must be regarded as the principal representative of the decentralized, infinite,

[16] Titus Lucretius Carus, *On the Nature of Things*, trans. Martin Ferguson Smith (Indianapolis: Hackett Pub, 2001), 35 (II, 1-4): "It is comforting, 1 when winds are whipping up the waters of the vast sea, to watch from land the severe trials of another person: not that anyone's distress is a cause of agreeable pleasure; but it is comforting to see from what troubles you yourself are exempt."

[17] Arguably, the most famous study on the "passage to modernity" is Louis Dupré, *Passage to Modernity: An Essay in the Hermeneutics of Nature and Culture* (New Haven: Yale University Press, 1993).

> and infinitely populous universe. for he not only preached it throughout Western Europe with the fervor of an evangelist, but also first gave a thorough statement of the grounds on which it was to gain acceptance from the general public.[18]

Nevertheless, with the passing of time, a more negative view of Bruno gradually emerged. There was no denying Bruno's role in changing the world-picture, but the way in which Bruno had brought about that change began to be questioned. Lovejoy already noticed that Bruno

> was not led to his characteristic convictions by reflection upon the implications of the Copernican theory or by any astronomical observations. Those convictions were for him primarily, and almost wholly, a deduction from the principle of plenitude, or from the assumption on which the latter itself rested, the principle of sufficient reason. [...] Bruno is, in short, precisely in those features of his teaching in which he seems most the herald and champion of a modern conception of the universe, most completely the continuer of a certain strain in Platonistic metaphysics and in medieval theology.[19]

It should be noted, however, that Lovejoy's purpose in writing *The Great Chain of Being* was to trace the history of the principle of plenitude, which might have led him to overestimate Bruno's dependence on that principle. This does not alter the fact that Bruno did not perform astronomical observations and, as we shall see in chapter 4, he was more of an instrumentalist than a realist in his approach to mathematics. A few years after Lovejoy, Alexandre Koyré took issue with Bruno's lack of a scientific method, while continuing to stress the importance of the new cosmological paradigm developed by the Italian philosopher:

> Giordano Bruno, I regret to say, is not a very good philosopher. The blending together of Lucretius and Nicholas of Cusa does not produce

[18] Arthur O. Lovejoy, *The Great Chain of Being: A Study of the History of an Idea ; The William James Lectures Delivered at Harvard University, 1933* (Cambridge, MA: Harvard University Press, 1982), 116.

[19] Lovejoy, 116–17.

> a very consistent mixture. [...] he is a very poor scientist, he does not understand mathematics, and his conception of the celestial motions is rather strange. [...] As a matter of fact, Bruno's world-view is vitalistic, magical; his planets are animated beings that mover freely through space of their own accord like those of Plato and Pattrizi. *Bruno's is not a modern mind by any means*. Yet his conception is so powerful and so prophetic, so reasonable and so poetic that we cannot but admire it and him. And it has—at least in its formal features—so deeply influenced modern science and modern philosophy, that we cannot but assign to Bruno a very important place in the history of the human mind.[20]

It is worth remembering that, when Koyré wrote these words, a new idea was in the making: the Scientific Revolution. At the turn of the twentieth century, Ernst Mach and Ernst Cassirer were among the first to support the view that, towards the end of the Renaissance, the world witnessed a radical change in the way scientific knowledge was created and developed.[21] In the 1930s, Koyré and other historians of his generation contributed to corroborate this view by highlighting how the rise of modern science coincided with the advent of a new scientific method. Among other things, this new method was characterized by a mathematical, experimentalist and mechanistic approach to the natural world. It was inevitable that Bruno, with his idea of a universe populated by animated planets and living suns, was regarded as a pre-modern thinker by those who, like Koyré, thought that the Scientific Revolution marked the beginning of the modern era. No matter how innovative Bruno's conception of an infinite universe

---

[20] Alexandre Koyré, *From the Closed World to the Infinite Universe* (New York: Harper Torchbooks, 1958), 54. Emphasis added.

[21] Ernst Mach, *Die Mechanik in ihrer Entwicklung: Historisch-Kritisch Dargestellt* (Leipzig: F.A. Brockhaus, 1883); Ernst Cassirer, *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* (Berlin: B. Cassirer, 1906). For an historical overview of the emergence of the idea of Scientific Revolution, see J.B. Shank, "Special Issue: After the Scientific Revolution: Thinking Globally about the Histories of the Modern Sciences," *Journal of Early Modern History* 21, no. 5 (October 27, 2017): 377–93.

was, it could not be considered the product of a modern mind because it did not result from the application of the scientific method.

Koyré's interpretation did not go unchallenged. In his *Die Legitimität der Neuzeit* (*The Legitimacy of the Modern Age*, 1966), Hans Blumenberg clearly affirmed that Bruno stood after what he called the "threshold of modernity:"

> There are no witnesses to changes of epoch. The epochal turning is an imperceptible frontier, bound to no crucial date or event. But viewed differentially, a threshold marks itself off, which can be ascertained as something either not yet arrived at or already crossed. Hence it is necessary; as will be done here for the epochal threshold leading to the modetn age, to examine at least two witnesses: the Cusan [Nicholas of Cusa], who still stands before this threshold, and the Nolan [Giordano Bruno of Nola], who has already left it behind.[22]

The reason why, for Blumenberg, Bruno was a modern thinker was that, unlike Cusanus, he rejected the medieval distinction between God's infinite power (*potentia Dei absoluta*) and its finite manifestation in the created world (*potentia Dei ordinata*).[23] Bruno overcome this distinction by assuming the existence of an infinite universe in which all possible beings could exist, and thus where God could give free course to his infinite power.[24] Indeed, Bruno thought that to say that the universe was finite was to set a limit to the divine omnipotence. It should be evident that Koyré and Blumenberg addressed the question of modernity from two different perspectives. While Koyré was "the dean of historians of the scientific revolution"—to borrow the words of Richard Westfall[25]—Blumenberg was in the tradition of Marx, Nietzsche, Husserl and Löwith. Löwith, especially,

---

[22] Hans Blumenberg, *The Legitimacy of the Modern Age*, trans. Robert M. Wallace (Cambridge, Massachusetts, and London, England: The MIT Press, 1999), 469.

[23] Blumenberg, pt. 4, chap. 3. See also Miguel A. Granada, "Il rifiuto della distinzione fra *potentia absoluta* e *potentia ordinata* di Dio e l'affermazione dell'universo infinito in Giordano Bruno," *Rivista di storia della filosofia* 49, no. 3 (1994): 495–532.

[24] See also Lovejoy, *The Great Chain of Being*, chap. 4.

[25] R. S. Westfall, "Newton and the Fudge Factor," *Science* 179, no. 4075 (February 23, 1973): 751–58.

had argued against the modernity of concepts such that of progress on the grounds that they were secularized version of original medieval ideas.[26] In the *The Legitimacy of the Modern Age,* Blumenberg took issues with Löwith by showing that not all medieval ideas found their way to modernity. This was well exemplified by Bruno, who saw himself as the herald of a new philosophy as opposed to that of, in particular, the Scholastics.

In the same years as Blumenberg defended Bruno's modernity in Germany, Frances Yates took Bruno's side in England. In her famous book *Giordano Bruno and the Hermetic Tradition* (1964), Yates made an effort to show "how shifting and uncertain were the borders between genuine science and Hermetism in the Renaissance"[27] in an attempt to challenge the narrative of the Scientific Revolution. In the 1950s, C. P. Snow's influential lectures on *The Two Cultures and the Scientific Revolution* had contributed to reinforce that narrative by presenting science and the humanities as two separate domains in need of connection.[28] By ruling out Bruno from the canon of modernity on account of his magical activities, Koyré failed to acknowledge that there was no clear demarcation between science and magic in the early modern period. It was not until recently that, following in the footsteps of Yates, historians of sciences have gradually recognized this lack of disciplinary boundaries. This, on the one hand, has led to a critical reappraisal of those bodies of knowledge which as late as the 1980s were still considered "pseudo-sciences," such as alchemy, astrology and magic.[29] On the other hand, it has become more and more evident that even the so-called "heroes" of the Scientific Revolution (e.g. Boyle and Newton) had an interest in fields that were far removed from what we now take to be science.[30]

---

[26] Karl Löwith, *Meaning in History* (Chicago: University of Chicago Press, 2006).

[27] Frances A. Yates, *Giordano Bruno and the Hermetic Tradition* (London: Routledge and Kegan Paul, 1964), 155.

[28] C. P Snow, *The Two Cultures and the Scientific Revolution* (Oxford: Oxford University Press, 1959).

[29] Shank, "After the Scientific Revolution," 389.

[30] Lawrence Principe, *The Aspiring Adept: Robert Boyle and His Alchemical Quest : Including Boyle's "Lost" Dialogue on the Transmutation of Metals* (Princeton, N.J.: Princeton University Press, 1998); William R. Newman and Lawrence Principe, *Alchemy Tried in the Fire: Starkey, Boyle, and the Fate of Helmontian Chemistry*

Yates' studies surely contributed to the popularity of Bruno in the English-speaking world. Nevertheless, her interpretation, with its emphasis on the Hermetic influence on Bruno's thought, had its critics. In a paper read at the Clark Library in 1974 and published in 1977, Robert Westman called for a reconsideration of what he termed the "Yates thesis."[31] As a matter of fact, Yates' book on Bruno of 1964 was the culmination of a process that had started in the 1930s with Paul Oskar Kristeller's studies on Marsilio Ficino[32]—the author of the first Latin translation of the *Corpus Hermeticum*. The joint efforts of André J. Festugiére and Arthur Darby Nock (who prepared a new edition of the *Corpus Hermeticum* that appeared between 1945 and 1954[33]) and the publication of Eugenio Garin's collection of hermetic texts in 1955[34] laid the groundwork for Yates' book. The 1970s witnessed a change in the attitude towards Renaissance Hermeticism, as the importance of this tradition began to be questioned. Westaman, for instance, disagreed with Yates on the role played by Hermeticism in Bruno's acceptance of Copernicanism. For Yates, Bruno's had a symbolic approach to Copernicus' *De revolutionibus* in that he read the Copernican text through the lens of Hermeticism.[35] On the contrary Westman, not unlike Lovejoy and Blumenberg, thought that "the key to Bruno's cosmology (if there is *a* key) has less to do with specific Hermetic ideals than with efforts to visualize the

---

(Chicago: University of Chicago Press, 2002); William R. Newman, *Newton the Alchemist: Science, Enigma, and the Quest for Nature's "Secret Fire"* (Princeton, N.J.: Princeton University Press, 2019).

[31] Robert S. Westman, "Magical Reform and Astronomical Reform: The Yates Thesis Reconsidered," in *Hermeticism and the Scientific Revolution*, ed. Robert S. Westman and J. E. McGuire (Los Angeles: University of California Press, 1977), 3–91.

[32] Paul Oskar Kristeller, "Marsilio Ficino e Lodovico Lazzarelli. Contributo alla diffusione delle idee ermetiche nel rinascimento," *Annali della R. Scuola Normale Superiore di Pisa. Lettere, Storia e Filosofia*, 2, 7, no. 2/3 (1938): 237–62; Paul Oskar Kristeller, "Ancora per Giovanni Mercurio da Correggio," *La Bibliofilía* 43, no. 1/2 (1941): 23–28.

[33] A new edition of the *Corpus Hermeticum* is now available: André-Jean Festugière, *La révélation d'Hermès Trismégiste* (Paris: Belles lettres, 2014).

[34] Eugenio Garin, ed., *Testi umanistici su l'ermetismo* (Roma: Bocca, 1955).

[35] Yates, *Bruno and the Hermetic Tradition*, 241.

creation of a rationally ordered universe by an all-powerful, all-knowing Creator."[36]

In recent years, the publication by a team of scholars led by Michele Ciliberto of a new edition of Bruno's Latin works has reopened the question of Bruno's modernity. In his introduction to Bruno's magical works, Ciliberto reconstructs the history of these works and their reception by modern interpreters. It was not until 1891 that Bruno's magical works were first published by Felice Tocco and Girolamo Vitelli in the third volume of the *Opera latine conscripta*—the national edition of Bruno's Latin works.[37] Bruno's unwillingness to publish his magical texts may explain why they have come down to us in an incomplete form. Ciliberto suggests that, instead of being meant for publication, these texts provided the teaching material for Bruno's lectures on magic.[38] These until then unknown texts confirmed Bruno's commitment to the view that nature was governed by magical forces. Despite (or perhaps because of) this fact, Tocco tried to reduce the importance of his discovery in his presentation of Bruno's unpublished works[39]. Likewise, Giovanni Gentile never mentioned the magical works in his influential book on *Giordano Bruno e il pensiero del Rinascimento* (*Giordano Bruno and the Renaissance Thought*, 1920).[40] The reason why, for Ciliberto, both Tocco and Gentile neglected Bruno's magic was because it did not sit well with their view that modernity started in the Renaissance with thinkers like Bruno:

[36] Westman, "Magical Reform and Astronomical Reform," 19. On Bruno and Copernicus, see also Ernan McMullin, "Bruno and Copernicus," *Isis* 78, no. 1 (1987): 55–74.

[37] Giordano Bruno, *Opera latine conscripta*, ed. F. Tocco and H. Vitelli, vol. 3 (Florence: Le Monnier, 1891).

[38] Michele Ciliberto, "Introduction," in *Opere magiche*, by Giordano Bruno, ed. Simonetta Bassi, Elisabetta Scapparone, and Nicoletta Tirinnanzi (Milan: Adelphi, 2000), xii.

[39] Felice Tocco, *Le opere inedite di Giordano Bruno: memoria letta all'Accademia di Scienze Morali e Politiche della Società Reale di Napoli* (Neaples: Tipografia della Regia Università, 1891).

[40] Giovanni Gentile, *Giordano Bruno e il pensiero del Rinascimento* (Florence: Vallecchi, 1920).

> All of this, in turn, implies the "crisis" of modernity as it is traditionally understood, and the consequent adoption of a new view of the modern centuries in which due consideration is given to those problems which sixteenth- and seventeenth-century philosophy has tried to obliterate. The new attention to magic, astrology, Hermeticism—which includes Bruno's magical works—is an integral part of the new, overall conception of modernity which needs to be considered to understand the "discovery," in this century, of Bruno's "unpublished works."[41]

Ciliberto's interpretation of Bruno's magical works was very much in line with that of Yates, as he himself acknowledges. However, he notices that, despite the emphasis placed on magic in Yates' account, she did not take into account Bruno's magical works, limiting herself to the analysis of the Italian dialogues in which magic had a secondary role. Moreover, like Westman before him, Ciliberto objects that Yates overestimated the role of Hermeticism in the formation of Bruno's thought, with the result that, in the wake of her studies, the prevailing view was that Bruno was only a hermetic magus. This, in turn, had led to neglect important aspects of Bruno's life and works, and especially his activities as a natural philosopher and mathematician. Indeed, although Yates underscored the lack of boundaries between magic and science in the early modern period, she also claimed that "the procedures with which the Magus attempted to operate have nothing to do with genuine science."[42] In the end, both Yates and Koyré seemed to agree that Bruno was not what we might call "a man of science." The fact that in her *Giordano Bruno and Renaissance Science* of 1999 Hillary Gatti felt compelled to attempt "an integration of the Hermetic Bruno into the scientific Bruno inherited from the nineteenth century"[43] tells us that at the turn of the twenty-first century Bruno was, and perhaps still is, considered more of a magus than a scientist.

Where does this leave us? The reader may have noticed that discussions about Bruno's modernity have tended to focus mainly on his cosmology.

[41] Ciliberto, "Introduction," xix. All translations are my own unless otherwise indicated.

[42] Yates, *Bruno and the Hermetic Tradition*, 449.

[43] Hilary Gatti, *Giordano Bruno and Renaissance Science* (Ithaca and London: Cornell University Press, 1999), 1.

Attention has been paid to Bruno's mathematics only to the extent that its flaws confirmed Bruno's inability to understand the technical aspects of the Copernican theory. Although the idea of infinitely small quantities would play a crucial role in the development of seventeenth-century mathematics (leading to the invention of the infinitesimal calculus), Bruno seemed unable to convert this idea into an acceptable mathematical doctrine. Even Bruno scholars have struggled to make sense out of his mathematics, which may explain why this aspect of Bruno's thought has gone largely unnoticed. Likewise, concerns have been voiced about Bruno's mathematics when considered in relation to his atomistic theory. The case has been made that the overall value of Bruno's atomistic theory was undermined by its mathematical applications. This was the opinion of Kurd Lasswitz, the first to systematically study Bruno's atomism in his classical book *Geshichte der Atomistik vom Mittelalter bis Newton* (1890).[44] Lasswitz credited Bruno with the revival of atomism in the early modern period. On the other hand, he could not help noticing the problems arising from the application of Bruno's atomistic theory to mathematics:

> The abstraction from the physical reality is impossible for Bruno because of the generality of the concept of monad. When one imagines a figure in the empty space, the figure must be imagined as composed of minima; the figure is rendered a physical entity by the very act of imagining it. […] There are no mathematical figures, but only physical ones.[45]

To understand Lasswitz's criticisms, we need to consider the concept of the minimum on which Bruno based his atomistic theory. In Lasswitz's view, Bruno took the mathematical minimum to be a substance, that is, an entity in its own right. This was a problem because, like Aristotle, Lasswitz held that mathematical objects were abstractions derived from the physical reality. Lasswitz's studies put

---

[44] Lasswitz, *Geschichte der Atomistik*. On Lasswitz's account of Bruno's atomism, see Francesca Puccini, "La 'Geschichte der Atomistik' di Kurd Lasswitz e la ricezione del materialismo di Bruno nella scienza tedesca del XIX secolo," *Bruniana & Campanelliana* 8, no. 2 (2002): 399–430.

[45] Lasswitz, *Geschichte der Atomistik*, II:383.

Bruno's atomism on the map and gave great impetus to the study of the history of atomism in general.[46] Yet his criticisms had a negative impact on the reception of Bruno's mathematics. For instance, Ernst Cassirer noticed that conceiving the mathematical minimum as a real object (and not as an abstraction) prevented Bruno from seeing those "laws and ideal relations whose value is independent from the nature of the existing things and of matter."[47] Likewise, Hélène Védrine, borrowing Bachelard's terminology, spoke of a "realistic obstacle" hindering Bruno's mathematics. By this term Védrine referred to the Platonic realist ontology of mathematical objects which Bruno would have been forced to adopt by his *a priori* rejection of Aristotle. However, Védrine argued, "Platonism does not contribute to the progress of mathematics."[48]

Recent attempts to gain a better understanding of Bruno's mathematical works have led a to a reappraisal of his mathematical thought. Giovanni Aquilecchia was the first to call for a new approach to Bruno's mathematics based on a careful analysis of his sources, his philosophical agenda and the historical circumstances under which he came to elaborate his atomistic geometry.[49] Furthermore, Aquilecchia must be credited with the discovery of two of Bruno's first mathematical writings (the dialogues on Fabrizio Mordente's compass),

---

[46] For Lasswitz's contribution to the historiography of atomism, see Lüthy, Murdoch, and Newman, "Introduction."

[47] Ernest Cassirer, "Il problema della conoscenza nella filosofia e nella scienza dall'Umanesimo alla scuola cartesiana," in *Storia della filosofia moderna*, trans. Pasquinelli, A., vol. I (Turin: Einaudi, 1961), 345.

[48] Hélène Védrine, "L'obstacle réaliste en mathématique chez deux philosophes du XVI siècle: Bruno et Patrizzi," in *Platon et Aristote à la Renaissance (XVI Colloque International de Tours)*, ed. J.-C. Margolin and M. Gandillac (Paris: J. Vrin, 1976), 247. My translation.

[49] Giovanni Aquilecchia, "Bruno e la matematica a lui contemporanea," in *Schede bruniane: 1950-1992* (Milan: Vecchiarelli, 1993), 311–17. For an overview of Bruno's "atomistic geometry" or "geometric atomism," see Marco Matteoli, "Materia, minimo e misura: la genesi dell'atomismo 'geometrico' in Giordano Bruno," *Rinascimento* 50 (2010): 425–49.

which remained unknown until the 1950s.[50] In the 2000s, the writings discovered by Aquilecchia provided the basis for Luciana De Bernart's monograph on Bruno's mathematics.[51] For De Bernart, Bruno's critics made the mistake of reading his mathematical writings through the lens of their own conception of mathematics, while they were best understood in the context of Renaissance mathematical practice.[52] This was confirmed by the dialogues on Mordente's compass in which Bruno tried to provide a theoretical explanation for the use of the instrument invented by his fellow countryman. A few years before De Bernart, Angelika Bönker-Vallon had attempted to develop new perspectives on Bruno's mathematics by analyzing it from the viewpoint of his metaphysics.[53]

It is worth mentioning that, as early as the 1920s, Ksenija Atanasijević offered a detailed account of Bruno's most important mathematical work in her *La doctrine métaphysique de Bruno exposée dans son ouvrage "De triplici minimo"* (*The metaphysical and geometrical doctrine of Bruno, as given in his work "De triplici minimo"* 1922; English translation 1972).[54] Although Atanasijević's interpretation was undermined by her belief that Bruno was precursor of the Serbian philosopher Branislav Petronijević (1875-1954), her book deserves a special mention because it was written at a time when scholars had no interest whatsoever in Bruno's mathematics. Also worth noting is that a new

---

[50] Giordano Bruno, *Due dialoghi sconosciuti e due dialoghi noti:* Idiota triumphans, De somnii interpretatione Mordentius, De Mordentii circino, ed. Giovanni Aquilecchia (Rome: Edizioni di storia e letteratura, 1957).

[51] Luciana De Bernart, Numerus quodammodo infinitus*: per un approccio storico-teorico al dilemma matematico nella filosofia di Giordano Bruno* (Rome: Edizioni di storia e letteratura, 2002).

[52] See the introduction to De Bernart.

[53] Bönker-Vallon, *Metaphysik und Mathematik bei Giordano Bruno*. See also Angelika Bönker-Vallon, "Giordano Bruno e la matematica," *Rinascimento* 39 (1999): 67–93; Angelika Bönker-Vallon, "Bruno e Proclo: connessioni e differenze tra la matematica neoplatonica e quella bruniana," in *La filosofia di Giordano Bruno. Problemi ermeneutici e storiografici*, ed. E. Canone (Florence: Leo S. Olschki, 2003).

[54] Ksenija Atanasijević, *The Metaphysical and Geometrical Doctrine of Bruno, as given in His Work De Triplici Minimo*, trans. G. V. Tomashevich (St. Louis, Missouri: W. H. Green, 1972).

edition and translation of Bruno's mathematical works is currently being prepared by Ciliberto's team, which will which make these works more accessible to the reader.

In spite of these developments, Bruno continues to be regarded as a poor mathematician.[55] In the belief that this received view does not do justice to Bruno, in this thesis I make an attempt to change it by dealing with the exegetical and historical problems posed by Bruno's mathematical texts. In particular, three questions are addressed in the following pages: What was the source of Bruno's mathematical ideas? Are Bruno's critics justified in claiming that Bruno by no means anticipated the concept of infinitesimals because his understanding of infinitely small quantities was mathematically unacceptable? Are Bruno's critics justified in claiming that Bruno was a mathematical realist? I shall show that Bruno's critics have misjudged his mathematical abilities because (1) they have regarded his atomistic geometry as an isolated event in the history of atomistic theories, while an inquiry into the sources of Bruno's mathematics shows that he belonged to a tradition going all the way back to the Middle Ages;[56] (2) they have

---

55 For instance, see John Henry, "Void Space, Mathematical Realism, and Francesco Patrizi Da Cherso's Use of Atomistic Arguments," in *Late Medieval and Early Modern Matter Corpuscular Theories*, ed. C. H. Lüthy, J. E. Murdoch, and W. R. Newman (Leiden: Brill, 2001), 145.

56 It should be noted that geometric atomism had at least another supporter in the sixteenth century: Petrus Ramus. Indeed, Robert Goulding has recently shown that, like Bruno, Ramus also claimed that the point was the minimum of magnitude and that lines were composed of points. He did so in a little known edition of his *Geometria* published in 1567, of which only three copies are extant. See Robert Goulding, "Five Versions of Ramus's *Geometry*," in *Et Amicorum: Essays on Renaissance Humanism and Philosophy in Honour of Jill Kraye*, ed. Anthony Ossa-Richardson and Margaret Meserve (Leiden: Brill, 2018), 355–87. The limited circulation of this work, along with the fact that Ramus took pains to expunge these atomistic claims from later editions of the *Geometria*, makes it unlikely that Bruno could have known of Ramus' short-lived sympathy for geometric atomism. Neither did Bruno have a high opinion Ramus' work in general, as attested by the fact that he called him a "French archpedant." See Giordano Bruno, *Cause, Principle, and Unity*, ed. Robert de Lucca and Richard J. Blackwell (Cambridge: Cambridge University Press, 1998), 54.

focused almost exclusively on Bruno's *De minimo* (1591) and disregarded his first mathematical writings (1586), which has prevented them from seeing that, in purely mathematical terms, Bruno's original theory was more coherent than the version of it presented in his later works. I pay special attention to Bruno's sources and to his first mathematical writings. In doing so, I do not put Bruno on a par with Regiomontanus, Cardano, Tartaglia and the leading mathematicians of the Renaissance, nor with Cavalieri, Newton and Leibniz and the inventors of the infinitesimal calculus. Rather, my aim is to offer an account of Bruno's mathematics that discusses its strengths as well as its weaknesses.

Between Pythagoras and Lucretius: On the sources of Brunian atomism

It is a common view that the early modern revival of atomism was a consequence of Poggio Bracciolini's 1417 rediscovery of Lucretius' *De rerum natura* (*On the Nature of Things,* I century BC). In fact, in his award-winning book *The Swerve,* Stephen Greenblatt goes as far as to claim that this event was the sparkle that ignited the Renaissance and the modern age.[57] As a matter of fact, the Lucretian poem was little known during the Middle Ages, as medieval scholars had no direct access to it but were acquainted with very few passages through secondary sources.[58] Thus, it is true that forgotten ideas were brought to light by the rediscovery of the poem in the fifteenth century. However, recent studies on the

[57] Stephen Greenblatt, *The Swerve: How the World Became Modern* (New York: W.W. Norton, 2011). On the same topic, see also Alison Brown, *The Return of Lucretius to Renaissance Florence* (Cambridge, MA: Harvard University Press, 2010); Ada Palmer, *Reading Lucretius in the Renaissance* (Cambridge, MA: Harvard University Press, 2014).

[58] On the reception of Lucretius in the Middle Ages, see Jean Philippe, *Lucrèce dans la théologie chrétienne du iiie au xiiie siècle et spécialement dans les écoles Carolingiennes* (Paris: E. Leroux, 1896); Ettore Bignone, "Per la fortuna di Lucrezio e dell'epicureismo nel medio evo," *Rivista di filologia e d'istruzione classica* 41 (1913): 230–62; Michael Reeve, "Lucretius in the Middle Ages and Early Renaissance: Transmission and Scholarship," in *The Cambridge Companion to Lucretius*, ed. Stuart Gillespie and Philip R. Hardie (Cambridge: Cambridge University Press, 2007), 205–13.

Renaissance reception of *De rerum natura* tend to reduce the importance of Poggio's rediscovery with regard to the revival of atomism. As made clear by Elena Nicoli, "it is certainly incorrect to regard Lucretius' doctrine, and especially his atomism, as the sole philosophical model that contributed to overturning Aristotle's natural philosophy and paved the way for the development of modern scientific thought."[59]

Like early modern atomism in general, Bruno's atomism was first regarded as an elaboration of Lucretian themes. This was the way in which it was presented by Dorothea Waley Singer, who had no doubt that "it was from Lucretius and certain Renaissance Lucretians such as Fracastor—whose name is given to a character in one of his dialogues—that Bruno drew his conception of what he calls the minima from which all things are formed."[60] By mentioning the minima, Singer clearly referred to Bruno's Latin works (1591). However, it is likely that her interpretation of Brunian atomism was based mainly on the Italian dialogues, and in particular on *De l'infinito, universo e mondi* (*On the Infinite, Universe and Worlds*, 1584) of which she published an English translation.[61] It was indeed in this work that Bruno first proposed the idea of an infinite universe in which atoms flowed from one body to another, in a way reminiscent of Epicurus and Lucretius. Nevertheless, in *De l'infinito,* there was no information about the shape of the atoms, their weight (or lack thereof), how they were arranged to form a body, and so on. This was because, although the seeds of Bruno's atomistic theory were found in *De l'infinito*, in was not until *De minimo* that this theory was fully developed. As soon as one pays attention to *De minimo*, one realizes that there were more differences than similarities between Bruno's and Lucretius' atomism.

In 1980, Carlo Monti published an Italian translation of Bruno's "Frankfurt trilogy", so called because it included three Latin poems (*De minimo,*

---

[59] Elena Nicoli, "The Earliest Renaissance Commentaries on Lucretius and the Issue of Atomism" (Unpublished dissertation, Radboud University, 2017), 243.

[60] Dorothea Waley Singer, "The Cosmology of Giordano Bruno (1548-1600)," *Isis* 33, no. 2 (1941): 189.

[61] Giordano Bruno, "On the Infinite Universe and Worlds," in *Giordano Bruno. His Life and Thought, with an Annotated Translation of His Work On the Infinite Universe and Worlds*, by Dorothea Waley Singer (New York: H. Schuman, 1950).

De monade and *De immenso*) that were printed in Frankfurt in 1591.[62] This work allowed Monti to a conduct a thorough analysis of Brunian atomism and its sources, the result of which were published in 1994.[63] In his account, Monti acknowledged that Lucretius was a "privileged source" for Bruno, who was indebted to the Latin poet (as well as to Epicurus) for his idea of an infinite space in which an infinite number of worlds composed of atoms were located.[64] He listed two other conceptual elements that the philosophical systems of Bruno and Lucretius had in common: the idea of a natural law and the anti-providentialist view that nature was able to reproduce itself without God's intervention. On the other hand, Monti noted that, unlike Lucretius and the ancient atomists Democritus and Leucippus, Bruno did not believe in the existence of absolute void, arguing that the space between atoms was filled by ether. Moreover, while for Lucretius and Epicures atoms moved downwards because of their weight, for Bruno atoms could move in all directions as the cause of their motion was an intelligent principle named the "world soul." Therefore, despite the similarities between their world-views, the atomistic doctrine of Bruno was far removed from that of Lucretius. This led Monti to conclude that:

> If, in sum, the Lucretianism is almost always in the background [of Bruno's philosophy], it is never a literal interpretation of Lucretius, but an important cultural motivation that plays different roles in the complex plot of Bruno's thought.[65]

---

[62] Giordano Bruno, *Opere latine*, trans. Carlo Monti (Torino: UTET, 1980).

[63] Carlo Monti, "Incidenza e significato della tradizione materialistica antica dei poemi latini di Giordano Bruno: la mediazione di Lucrezio," *Nouvelles de la République des Lettres* 2 (1994): 75–87.

[64] Previous studies on the influence of Epicureanism on Bruno's thought include: Fulvio Papi, *Antropologia e civiltà nel pensiero di Giordano Bruno* (Florence: La Nuova Italia, 1968), 91–125; Miguel A. Granada, "Epicuro y Giordano Bruno: descubrimiento de la naturaleza y liberación moral (una confrontación a través de Lucrecio)," in *Historia, lenguaje, sociedad. Homenaje a Emilio Lledó*, ed. Manuel Cruz, Miguel A. Granada, and Anna Pappol (Barcelona: Editorial Crítica, 1989), 125–41.

[65] Monti, "Incidenza e significato," 85.

It is a fact that, on his return to Paris from London in 1585, Bruno acquired a copy of Obert van Giffen's 1566 edition of Lucretius' *De rerum natura*.[66] This confirms that *De rerum natura* was a Brunian source. However, as shown by Amalia Perfetti, Bruno borrowed from Lucretius metaphors such as that of nature as *mater rerum* (mother of things) and general concepts such as that of *semina* (seeds).[67] Although, as argued by Hiro Hirai, "the concept of seeds can be regarded as a missing link in the chain which bridged between the medieval scholastic doctrine of substantial forms and the mechanistic corpuscular theories of the late seventeenth and eighteenth centuries,"[68] the concept itself seems unable to explain why Bruno's atomism possessed certain characteristics or, more importantly, why he provided an atomistic theory of geometric objects.

Nowadays, scholars agree that Bruno's atomism had no connection to Lucretius' and Epicurus'. In their exploration of the importance of Lucretius in the history of science, Monte Johnson and Catherine Wilson are adamant that "Bruno was by no means an orthodox Epicurean."[69] Yasmin Haskell is of the same opinion, as she writes that "the relationship between the two writers [i.e. Bruno and Lucretius] was more personal, more ideological, than strictly philosophical."[70] Likewise, in her study on the Renaissance reception of Lucretius, Nicoli argues that "while embracing Lucretius' atoms, Bruno conceived of them as living, dynamic, and monadic entities which constituted, as a sort of divine nuclei, the seeds of an endless cosmos made up of infinite worlds. This

---

66 Eugenio Canone, *Giordano Bruno: gli anni napoletani e la* peregrinatio *europea: immagini, testi, documenti* (Cassino: Università degli studi, 1992), 95.

67 Amalia Perfetti, "Motivi lucreziani in Bruno: la Terra come 'madre delle cose' e la teoria dei 'semina,'" in *Letture bruniane 1-2* (Pisa: Istituti editoriali e poligrafici internazionali, 2002), 189–208.

68 Hiro Hirai, "Seed Concept," in *Encyclopedia of Renaissance Philosophy*, ed. Marco Sgarbi (Cham: Springer International Publishing, 2015), 1.

69 Monte Johnson and Catherine Wilson, "Lucretius and the History of Science," in *The Cambridge Companion to Lucretius*, ed. Stuart Gillespie and Philip R. Hardie (Cambridge: Cambridge University Press, 2007), 133.

70 Yasmin Haskell, "Religion and Enlightenment in the Neo-Latin Reception of Lucretius," in *The Cambridge Companion to Lucretius*, ed. Stuart Gillespie and Philip R. Hardie (Cambridge: Cambridge University Press, 2007), 195.

doctrine seems very far from that presented in *De rerum natura*,"[71] We have already seen two aspects in which the atomistic theory advocated by Bruno was different from that of Lucretius: the existence of void (affirmed by Lucretius and denied by Bruno) and the atomic motion (downwards for Lucretius and in all directions for Bruno). Another difference was that Lucretius thought that the atoms had a vast but finite variety of shapes, while Bruno held that the atoms had only one shape: the circle or the sphere, depending on whether they were conceived in a two- or three-dimensional space (more on this in chapter 4). Finally, Bruno himself informs us about his commitment to Epicureanism early in his life and his subsequent disaffection:

> Democritus and the Epicureans, who claim that what is not body is nothing, maintain as a consequence that matter alone is the substance of things, and that it is also the divine nature, as an Arab named Avicebron has said in a book entitled Fount of Life. They also hold, together with the Cyrenics, the Cynics and the Stoics, that forms are nothing but certain accidental dispositions of matter. *I, myself, was an enthusiastic partisan of this view for a long time*, solely because it corresponds to nature's workings more than Aristotle's. But after much thought, and after having considered more elements, we find that we must recognize two kinds of substance in nature: namely, form and matter.[72]

The fact that Bruno did not consider Epicureanism a suitable philosophical option may have prevented him from adopting an Epicurean version of atomism. The only part of Epicurean atomism that may have attracted Bruno's attention was its theory of minima. This theory can be summed up as follows.[73] Unlike

---

[71] Nicoli, "The Earliest Renaissance Commentaries on Lucretius and the Issue of Atomism," 216.

[72] Bruno, *Cause, Principle, and Unity*, 55. Emphasis added.

[73] Different interpretations of the Epicurean theory of minima have been proposed over the years. My understanding of this theory is based mainly on Francesco Verde, *Elachista: la dottrina dei minimi nell'epicureismo* (Leuven: Leuven University

Democritus, Epicurus thought that atoms had parts called minima (in Greek *elachista*), which could not exist independently of the atom to which they belong and could not be further divided. The minima determined the characteristics of the atom (such as its weight, size and shape), which in turn affected the way in which atoms came together to form compound bodies. However, in the *Physics,* Aristotle argued against the view that an object (*qua* continuous) could be composed of indivisible entities such as the Epicurean minima, for these entities had no parts with which they could touch one another. (For Aristotle, an object was continuous only if its parts were in contact with each other).[74] We do not know for sure whether Epicurus knew Aristotle's *Physics*. Be that as it may, his theory of minima seemed to contain a reply to the Aristotelian objection that the continuum could not be composed of indivisibles. Epicurus invited us to consider the sensible minimum, that is the smallest perceivable thing. Despite the fact that sensible minima were so small that no part of them could be seen (it was as if they had no parts), they were arranged to form the object that contained them. "How then do they combine? 'In their own special way' is the most Epicurus ventures on the matter."[75] By analogy, the same could be said of atomic minima.

Bruno knew the Epicurean theory of minima as expounded by Lucretius in *De rerum natura*.[76] He held that the atom had "extremities" (in Latin *termini*), which could not be separated from the atom itself and were indivisible just like the Epicurean minima. Furthermore, he claimed that it was with their extremities that the minima (which for Bruno were synonymous with the atoms) touched one other, thus forming compound objects (see Chapter 4 for more details). It seems safe to conclude that Bruno borrowed from Epicurus this aspect of his atomistic theory, or at least used similar arguments to respond to Aristotle's criticisms of atomism. However, the fact remains that for the most part Brunian and

---

Press, 2013). See also Verde's commentary on Epicurus, *Epistola a Erodoto*, ed. Francesco Verde (Rome: Carocci, 2010), 150–76.

74 Aristotle, "Physics," bk. 6.

75 David Sedley, "Epicurean Physics," in *The Cambridge History of Hellenistic Philosophy*, ed. Keimpe Algra et al. (Cambridge: Cambridge University Press, 1999), 375.

76 Giordano Bruno, "De triplici minimo et mensura," in *Opera latine conscripta*, ed. F. Tocco and H. Vitelli, vol. I, pt. 3 (Florence: Le Monnier, 1889), 169.

Epicurean atomism were different theories, especially if we consider that Bruno applied the atomic model to geometric and physical objects alike. Therefore, Epicureanism may not have been the only source of Brunian atomism. Rather, it is more likely that Bruno's understanding of atomism resulted from the conflation of different sources. This begs the question: which authors and texts may have inspired Bruno to develop his atomistic theory, besides Epicurus and Lucretius? Did geometric atomism have supporters in Antiquity or in the Middle Ages? Indeed, Bruno was not the first to postulate the existence of geometric indivisible entities, but different versions of geometric atomism have been ascribed to two of the most famous philosophical schools of the past: the Platonic and the Pythagorean. As a matter of fact, it may have been that the Epicureans also pursued the project of a new geometry based on their theory of minima.[77] However, this hypothesis has not yet been confirmed, which is why I have limited myself to Plato and the Pythagoreans.

In the case of Plato, the term "geometric atomism" has been used to refer to the theory, expounded by Plato in the *Timaeus*, whereby the four elements were composed of parts that had the shape of a regular solid.[78] Each element was associated to one of the so-called Platonic solids: fire (tetrahedron), air (octahedron), water (icosahedron), earth (cube). Each solid, in turn, was composed of triangles, which were deemed to be to be the most basic geometric figures. This Platonic version of geometric atomism was far removed from that of Bruno. Over time, Bruno changed his mind about the shape of geometric indivisibles, which went from being indefinitely shaped to having a circular shape. Nevertheless, unlike Plato, Bruno never attributed a triangular shape to his minima. It is true that Xenocrates (396/395–314/313 BC), who was Plato's pupil and scholarch of the Platonic Academy, developed an atomistic theory

---

[77] See Verde, *Elachista*, chap. 3.

[78] On Plato's geometric atomism, see Benno Artmann and Lothar Schäfer, "On Plato's 'Fairest Triangles' (Timaeus 54a)," *Historia Mathematica* 20, no. 3 (August 1993): 255–64; John Visintainer, "A Potential Infinity of Triangle Types On the Chemistry of Plato's Timaeus," *HYLE: International Journal for Philosophy of Chemistry* 4, no. 2 (1998): 117–28; Gregory Vlastos, "Plato's Supposed Theory of Irregular Atomic Figures," *Isis* 58, no. 2 (July 1967): 204–9.

based on the concept of indivisible lines.[79] This Xenocretean theory was more similar to Bruno's atomistic doctrine, especially the original version of it presented in the dialogues on Mordente's compass where Bruno spoke of the indivisible minima of the lines (see Chapter 3). Nevertheless, Bruno's later mathematical works witnessed a change in his conception of the geometric minima, as they came to be conceived as circular, extended points. Thus, if, as proposed by Luciano Albanese,[80] Xenocrates' indivisibles lines provided a model for Bruno's minima, this would help explain the early stage of development of Bruno's mathematical thought. Yet the influence of Xenocrates cannot be used to account for the ultimate version of Bruno's atomistic theory, which revolved around the idea that the geometric minima were points (and not indivisible lines).

This leaves us with the Pythagoreans. Indeed, scholars have credited the Pythagoreans with a form of "point-atomism."[81] It all started with Paul Tannery who claimed that the target of Zeno's paradoxes was the Pythagorean view that geometric objects were composed of points placed side by side.[82] Tannery's thesis had supporters until the 1950s, when it was dismissed on the grounds that there was no evidence that the Pythagoreans were the authors of point-atomism. (The

[79] On Xenocrates' theory of indivisible lines, see John M. Dillon, *The Heirs of Plato: A Study of the Old Academy, 347-274 B.C* (Oxford: Oxford University Press, 2003), chap. 3; David Konstan, "Chapter One: Points, Lines, and Infinity: Aristotle's Physics Zeta and Hellenistic Philosophy," *Proceedings of the Boston Area Colloquium of Ancient Philosophy* 3, no. 1 (January 1, 1987): 1–32; Sylvia Berryman, "Ancient Atomism," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, Winter 2016 (Metaphysics Research Lab, Stanford University, 2016), para. 3, https://plato.stanford.edu/archives/win2016/entries/atomism-ancient/.

[80] Luciano Albanese, "Bruno e le linee indivisibili," *Bruniana & Campanelliana* 7, no. 1 (2001): 201–7.

[81] Francis M. Cornford, "Mysticism and Science in the Pythagorean Tradition," *The Classical Quarterly* 16, no. 3–4 (1922): 137–150; John Earle Raven, *Pythagoreans and Eleatics. An Account of the Interaction between the Two Opposed Schools during the Fifth and Early Fourth Centuries B.C.* (Cambridge: Cambridge University Press, 1948).

[82] Paul Tannery, *Pour l'histoire de la science hellène* (Paris: Felix Alcan, 1887).

general consensus was that point-atomism was developed by a later generation of Neo-Pythagorean authors, and not by the early Pythagoreans, as Tannery had it).[83] At the same time, scholars were aware that geometric atomism revived in the Middle Ages, in which period atoms were viewed as points instead of corpuscles. For a long time, the tendency was to regard these medieval theories of point-atoms as only a response to Aristotle's criticisms of atomism.[84] It was not until recently that historians started to explore the possibility that medieval atomists drew inspiration from Neo-Pythagorean authors such as Boethius, who expressed the idea that the point was an atom or a "unit having position."[85] It may also have been that the Pythagorean idea of point-atom found its way through the early modernity to become the cornerstone of Bruno's atomistic theory. If so, what was the channel through which this idea reached Bruno? Was Bruno familiar with authors who endorsed Pythagorean atomism?

In his introduction to two of Bruno's later mathematical writings, Giovanni Aquilecchia proposed that:

> The geometry of [Bruno's] *De minimo* is to be compared to Nicholas of Cusa's still understudied mathematics and, through this latter, to Ramon Llull's geometry, the importance of which has been recently claimed not only with regard to the Lullian Art, but also for interpreting particular aspects of the Renaissance civilization.[86]

---

[83] Gregory Vlastos, "Zeno of Elea," in *The Encyclopedia of Philosophy*, ed. Paul Edwards (New York: The Macmillan Company & The Free Press, 1967).

[84] John E. Murdoch, "Infinity and Continuity," in *The Cambridge History of Later Medieval Philosophy*, ed. Norman Kretzmann, Jan Pinborg, and Anthony Kenny (Cambridge: Cambridge University Press, 1982), 564–91.

[85] Aurélien Robert, "Atomisme pythagoricien et espace géométrique au Moyen Âge," in *Lieu, espace, mouvement: physique, métaphysique et cosmologie (XIIe-XVIe siècles). Actes du colloque international Université de Fribourg (Suisse), 12-14 mars 2015*, ed. Tiziana Suarez-Nani, Olivier Ribordy, and Antonio Petagine (Turnhout: Brepols, 2017).

[86] Giovanni Aquilecchia, "Introduction," in *Praelectiones Geometricae e Ars Deformationum. Testi Inediti*, by Giordano Bruno, ed. Giovanni Aquilecchia (Roma: Edizioni di Storia e Letteratura, 1964), xxiv–xxv.

Aquilecchia had reason to believe that a study of Cusanus' and Llull's mathematical theories could yield insights into Bruno's mathematics, although he left it to others to pursue this line of research. Following Aquilecchia's lead, De Bernart insisted on the importance of Cusanus' mathematical writings, especially those on the quadrature of the circle, which provided a framework for understanding the geometric aspects of Bruno's theory of minima.[87] Likewise, Jean Seidengart claimed that "it will be unfair and incorrect to see Bruno as the inventor of the metaphysics of the indivisible *Minimum* [...]. Therefore, we need to go back to the privileged source of these considerations [...]: the work of Nicholas of Cusa."[88] In addition, both Seidengart and, more systematically, David Albertson argued that Cusanus' conception of mathematics was deeply Pythagorean in that it derived from a close reading of Boethius' *Institutio arithmetica* and its commentary by Thierry of Chartres.[89] While scholars have tried to estimate the extent of Bruno's debt to Cusanus, they have not considered that Llull's mathematical writings could also have played a role in shaping Bruno's understanding of geometric minima. This may be due to the fact that, for the most part, Lullian mathematics is uncharted territory. Accordingly, the fact that Llull, like Bruno, claimed that geometric objects were composed of points and thus defended a version of Pythagorean atomism has gone unnoticed.

This study is in line with Aquilecchia's suggestion, its purpose being to show that both Llull and Cusanus were Bruno's sources for his mathematics. Special emphasis is given to Llull as his importance for Bruno's mathematics has been largely underestimated. On the other hand, this study offers a new approach to the question of Bruno's sources insofar as Llull and Cusanus (as well as Bruno himself) are regarded as belonging to the tradition of Pythagorean atomism. A

---

87 De Bernart, Numerus quodammodo infinitus, chaps. 2–3.

88 Jean Seidengart, "La metaphysique du minimum indivisible et la réforme des mathématiques chez Giordano Bruno," in *Atomismo e continuo nel XVII secolo*, ed. E. Festa and R. Gatto (Neaples: Vivarium, 2000), 55.

89 David Albertson, *Mathematical Theologies: Nicholas of Cusa and the Legacy of Thierry of Chartres* (Oxford: Oxford University Press, 2014); David Albertson, "*Boethius Noster*. Thierry of Chartres's *Arithmetica* Commentary as a Missing Source of Nicholas of Cusa's *De Docta Ignorantia*," *Recherches de Théologie et Philosophie Médiévales* 83, no. 1 (2016): 143–99.

methodological remark is in order. The fact that Pythagoreanism was a common source of knowledge for these authors may explain the similarities between their mathematical theories. However, as we shall see, these theories were not identical. In fact, there were major differences between the mathematical theories of Llull and Bruno on the one hand, and that of Cusanus on the other hand. (Unlike Llull and Bruno, Cusanus was against the view that geometric objects were composed of points). Yet, as I will argue in Chapter 2, in his later works, Cusanus endorsed a version of atomism central to which were concepts—enfolding and unfolding (*complicatio* and *explicatio*)—inspired by Boethian considerations. This means that, while the work of an author could be directly influenced by that of his predecessor(s), their common Pythagorean sources provided conceptual elements that could be reframed in a variety of ways, depending on the philosophical background of the authors themselves. For this reason, I believe that analyzing the mathematical theories of Llull, Cusanus and Bruno from the perspective of Pythagoreanism allows us to account for their differences as well as their similarities.

Outline of chapters

This thesis can be viewed as composed of two parts, each consisting of two chapters. The first part is concerned with the sources of Bruno's atomistic geometry, namely Ramon Llull (Chapter 1) and Nicholas of Cusa (Chapter 2). The premise is that both these authors (as well as Bruno) belonged to the tradition of Pythagorean atomism, a mathematical version of atomism attributed to the Pythagoreans in the early twentieth century. It should be noted that ancient and medieval scholars did not regard themselves as "Pythagorean atomists" in the same way as the Aristotelians did. Rather, the term "Pythagorean atomism" is used here to indicate those atomistic theories which were based on the teachings of Neo-Pythagorean sources, especially Boethius. In other words, Pythagorean atomism is more of a historians' that an actors' category. For this reason, in Chapter 1, a survey of studies on Pythagorean atomism is provided in lieu of a historical reconstruction of this tradition (§ 1.1). Historians have long debated the authorship of Pythagorean atomism. For if it is true that ancient sources (above all Aristotle) spoke of an atomistic theory of geometric objects developed by the

Pythagoreans, these sources could have referred to a generation of Pythagoreans other than the immediate disciples of the philosopher of Samos. In fact, it has been demonstrated that the Antiquity witnessed as "expansion of tradition"[90] of Pythagoreanism in the sense that, especially in the Platonic Academy, there was the tendency to invoke the authority of Pythagoras as a guarantee of the soundness of one's own views and, for the same reason, to call oneself a Pythagorean. Thus, it might well have been that in Antiquity a theory was conceived as Pythagorean, even though its origin dated later than Pythagoras. This is how scholars have viewed Pythagorean atomism since the 1960s.[91]

Boethius (477 – 544 AD) is a central element in this narrative of Pythagorean atomism because it was him who paved the way for the return of this theory in the Latin Middle Ages (§ 1.2). It must be said that Boethius was not an atomist and, as a good Aristotelian, he fought the view that geometric objects were composed of points. However, in addition to being an Aristotelian, Boethius was the Latin translator of Nicomachus of Gerasa's (c. 60 – c. 120 AD) *Arithmetike eisagoge* (*Introduction to Arithmetic*). This work was a compendium of Pythagorean mathematics and its translation provided the basis for Boethius' *Institutio arithmetica*. As is well known, in the *Institutio* Boethius presented the quadrivium, the four mathematical liberal arts (arithmetic, geometric, astronomy and music) that medieval students were required to learn before going on to study philosophy. More relevant to this study, in the *Institutio* Boethius described the point as an atom (because of its smallness and lack of parts) and a "unit having position." The fame of Boethius' *Institutio* in the Middle Ages gave visibility to this atomistic definition of the point and led to the exploration of non-Aristotelian conceptions of the continuum. An author who adopted an atomistic view of the mathematical continuum happened to be a Brunian source: Ramon Llull.

Ramon Llull (1232 – 1316) owes his reputation to his Art, a method designed to convert Jews and Muslims to Christianism which grew to become an

---

[90] Walter Burkert, *Lore and Science in Ancient Pythagoreanism* (Cambridge, Massachusetts: Harvard University Press, 1972), 2.

[91] Vlastos, "Zeno of Elea."

alternative to the Aristotelian logic (§ 1.3).[92] Bruno was fascinated by the Lullian Art, especially by its visual aspects, as attested by the fact that he wrote works on it.[93] It is on these works that scholars have focused their attention when assessing the influence of Llull on Bruno. The possibility that this influence extended beyond the boundaries of Bruno's Lullian works has remained unexplored. Yet a reading of Llull's *Liber de geometria nova* (*Book on the New Geometry*, 1299) reveals that he agreed with Bruno that geometric objects were composed of points. Thus, like Bruno, Llull appeared to defend a version of Pythagorean atomism. In fact, the pages of the *Geometria nova* show that he also adopted a form of Platonic atomism, that is the view that the four elements were composed of indivisible geometric figures (triangles, circles and squares, in the case of Llull). Hence, it can be assumed that Llull was a Bruno's source for his mathematics. To validate this hypothesis, we need to ask ourselves, could Bruno have read Llull's *Geometria nova*? This question arises because, to my knowledge, the *Geometria nova* was the only work in which Llull exposed his atomistic view of geometric objects.

Llull wrote the *Geometria nova* in Paris in 1299 (§ 1.4). An analysis of the eight extant manuscripts containing a copy of this work shows that none of these manuscripts could have ended up in Bruno's hands, either because they were produced at a later date or because their origin and provenance is not compatible with Bruno's wanderings across Europe. However, it could have been that Bruno had access to a copy of the *Geometria nova* which has not come down to us. To determine if this was the case, I examined the catalogues and inventories of Lullian works which mentioned the *Geometria nova*. The online Llull Database was an invaluable aid in this phase of the research.[94] The *Geometria nova* was listed, among others, in the catalogue of a manuscript known as the *Electorium*.[95] In addition to the catalogue, this manuscript contained a collection of Lullian

---

[92] For an overview of the Lullian Art, see Anthony Bonner, *The Art and Logic of Ramon Llull: A User's Guide* (Leiden: Brill, 2007).

[93] Giordano Bruno, *Opere lulliane*, ed. Marco Matteoli, Rita Sturlese, and Nicoletta Tirinnanzi (Milan: Adelphi, 2012).

[94] Anthony Bonner (dir.), *Ramon Llull Database*, Centre de Documentació Ramon Llull (University of Barcelona), http://orbita.bib.ub.edu/llull/).

[95] Paris, Bibliothèque Nationale, ms. lat. 15450, 89v-90.

texts put together by one of Llull's first disciples: Thomas Le Myésier (d. 1336).[96] The catalogue of the *Electorium* informs us that Le Myésier certainly owned a copy of the *Geometria nova*. What is more, since Le Myésier based his catalogue on the collection of Lullian manuscripts of the Charterhouse of Vauvert in Paris, it tells us that the Charterhouse also had a copy of the *Geometria nova*. The manuscript collection of the Charterhause of Vauvert had been established by Llull himself as part of a strategy to promote his work.[97] The Charterhouse was still an important center for the diffusion of Lullism at the beginning of the sixteenth century, when the French humanist and Llull admirer Jacques Lèfevre d'Étaples (c. 1450 – 1536) borrowed from the Charterhouse the manuscripts necessary to prepare his edition of Llull's work.[98] The Charterhouse remained active until the French Revolution when, as a consequence of the "dechristianization" of France, religious orders were suppressed and their goods (including libraries) confiscated.[99] As far as I known, all the documents concerning the Charterhouse are now kept at the Archives Nationales in Paris.[100] Unfortunately, among these documents, I was not able to find a proof that a copy of the *Geometria nova* was found at the Charterhouse as late as the 1580s, during which period Bruno visited Paris twice. Nevertheless, this possibility cannot be excluded on the basis of the evidence at hand, which leaves the door open for a possible dependence of Bruno on Llull for the atomistic character of his geometry.

The fact that Llull may have inspired Bruno to develop his atomistic geometry does not diminish the importance of Nicholas of Cusa (1401 – 1464) as

---

[96] On the *Electorium*, see Ramón de Alós y de Dou, *Los catalogos lulianos. Contribución al estudio de la obra de Ramón Lull* (Barcelona: Francisco J. Altés y Alabart, 1918), 14–17; J. N. Hillgarth, *Ramon Lull and Lullism in Fourteenth-Century France* (Oxford: Clarendon Press, 1971), 335–47.

[97] Lola Badia, Joan Santanach, and Albert Soler, *Ramon Llull as a Vernacular Writer. Communicating a New Kind of Knowledge* (Woodbridge: Tamesis, 2016), chap. 3.

[98] Eugene F. Rice, "Jacques Lèfevre d'Étaples and the Medieval Christian Mystics," in *Florilegium Historiale: Essays Presented to Wallace K. Ferguson*, ed. J. G. Rowe and W. H. Stockdale (Toronto: University of Toronto Press, 1971), 90–124.

[99] Alfred Franklin, *Les anciennes bibliothèques de Paris: églises, monastères, colléges, etc.*, vol. 1 (Paris: Imprimerie impériale, 1867), 326–28.

[100] Archives Nationales de France, S//3948-4160/2 and L//937-940.

a Brunian source, which is the subject of Chapter 2. Indeed, an analysis of the philosophical underpinnings of Bruno's mathematics reveals that it was based on two assumptions, both taken from Cusanus: the idea that there was a minimum as well as a maximum, and the idea that that minimum and maximum coincided. On the other hand, one may argue that it was not until the latter part of his career that Cusanus came to accept atomism and, even then, he continued to oppose the idea the idea that geometric objects were composed of points, which was central to Lullian and Brunian atomism. How does this affect our narrative of Pythagorean atomism? Can Cusanus be regarded as belonging to this tradition? If this were the case, the bridge between Cusan and Pythagorean atomism should be provided by an idea other than that on which Llull and Bruno built their atomistic theories. In fact, I believe that there were two ideas (which were part of the same theory) that could fulfill this function: enfolding and unfolding (*complicatio* and *explicatio*).

In *De docta ignorantia* (1440), arguably Cusanus' most famous work, enfolding and unfolding provided an explanation of how the unity of God was compatible with His being the creator of the world and of the multiplicity of things found therein (§ 1.1). Cusanus' answer to this question was that the multiplicity of things was enfolded in God's mind and unfolded in the created world, and that these two modes of existence were compatible with each other. It should be noted Cusanus' source for his account of enfolding and unfolding was a mathematical text that has been recently rediscovered by Irene Caiazzo: Thierry of Chartres' commentary of Boethius' *Institutio Arithmetica*.[101] This discovery enriches our knowledge of the sources of Cusanus' philosophy, but also sheds light on the conceptual development of enfolding and unfolding in his thought.[102] Thierry used these notions to bridge the gap between arithmetic and geometry, numbers and magnitudes. In Cusanus's view, enfolding and unfolding had a function similar to that attributed to them by Thierry. In *De mente* (1450), the bridging function of enfolding and unfolding was translated into a creative act of the human mind, and the two concepts were embedded into the atomistic theory that Cusanus was starting to develop. Hence, Cusanus was inspired by Boethius

---

[101] Thierry of Chartres, *The Commentary on the* De Arithmetica *of Boethius*, ed. Irene Caiazzo (Toronto: Pontificial Institute of Medieval Studies, 2015).

[102] Albertson, "*Boethius Noster*."

(via Thierry) to develop his account of enfolding and unfolding, which went from being part of a theological explanation (in *De docta ignorantia*) to being part of an atomistic theory (in *De mente*).

Chapter 2 continues with Cusanus' account of minimum and maximum (§ 2.2). There is no doubt that this aspect of Cusanus' thought deeply influenced Bruno. In particular, Cusanus might have been Bruno's source for his idea of the minimum as an infinitesimal quantity. In *De docta ignorantia*, Cusanus demonstrated that in arithmetic minimum and maximum could not be expressed numerically, while in geometry they were best understood as "absolute quantities," that is as quantities that were independent of any finite determination (e.g. small, big, etc.). This was due to the fact that numbers and magnitudes expressed the finitude and "contractedness" of the world, while minimum and maximum had an absolute character. In fact, Cusanus claimed that both minimum and maximum were infinite entities. Speaking of the maximum, Cusanus refereed to the infinity of God and the universe. On the other hand, he was silent about the infinity of the minimum, leaving it to the reader to infer the characteristics of the 'infinitely small.' I suggest that (1) the "contracted minimum" (the opposite of the "contracted maximum," i.e. the universe) coincided with the point; 2) the "absolute minimum" was ineffable just like God, the "absolute maximum." In an effort to put the ineffability of the absolute minimum and maximum into words, Cusanus used the metaphor of the inscribed polygon which could never coincide with the circumscribed circle, regardless of the number of its sides. In his first mathematical writings, Bruno also argued that the geometric minimum was affected by a certain degree of indeterminateness insofar as had no precise shape. Nevertheless, in his opinion, instruments such as Mordente's compass allowed us to visualize the minimum by revealing the minimum parts of lines and figures.

While the first part of the dissertation is centered on the sources of Bruno's mathematical theory, the second part focuses on the theory itself. Its main objective is to respond to the criticisms of early interpreters of Bruno's mathematics, especially Leonardo Olschki and Hélène Védrine. Chapter 3 takes issue with Olschki who claimed that Bruno's mathematics was too "concrete" to have a mathematical significance.[103] In the belief that this received view did not do justice to Bruno's mathematics, I started dealing with the exegetical problems posed by Bruno's texts and searching for hitherto neglected sources. However, as my research progressed, I realized that limiting myself to the analysis of Bruno's texts and its sources was not enough to change the conventional image of Bruno's mathematical abilities. I needed to look at Bruno's mathematics from a different perspective, and understand the social, political and cultural conditions under which Bruno came to develop his mathematical theory. This led me to study an event that has been relatively neglected by Bruno scholars, that is the controversy between Bruno and the Italian geometer Fabrizio Mordente. In reconstructing this controversy, I became aware that the "concrete" character of Bruno's mathematics, as Olschki had it, was due to Bruno's interest in mathematical practices and instruments, such as the proportional compass invented by Mordente. Indeed, Bruno's first mathematical writings were an attempt to give a theoretical explanation for the use of Mordente's compass.

Fabrizio Mordente (1532 – c.1608) was the inventor of one of the first proportional compasses, an instrument designed to measure the sections of a line or the area of an irregular figure (§ 3.1). Bruno met Mordente in Paris in 1586 and was immediately attracted to the compass, so much so that he decided to write four dialogues on it. However, it was not long before the two Italians engaged in a fight over the significance and authorship of the compass (§ 3.2). This controversy was long forgotten for two reasons: first, because, as already mentioned, the dialogues in which Bruno attacked Mordente were rediscovered only in the 1950s; second, because the only account of the controversy, given by Jacopo Corbinelli in his correspondence with Gian Vincenzo Pinelli, was published by Frances Yates in 1951.[104] As Bruno explained in the first dialogue, Mordente's compass allowed to divide geometric objects down to their "minimum" parts (§ 3.3). In this respect, Bruno continued, the theory behind the use of the compass was not unlike the Aristotelian theory of the *minima naturalia*. Developed by the followers of Aristotle in the Middle Age, this theory stated that there was a lower limit to the form of natural beings, beyond which

[103] Leonardo Olschki, *Giordano Bruno* (Bari: Laterza, 1927), 81.

[104] Francis A. Yates, "Giordano Bruno: Some New Documents," *Revue Internationale de Philosophie*, 1951, 174–99.

they would have been too small to retain their essence. If on the one hand Aristotelians admitted the existence of a "formal" minimum, on the other hand they denied the existence of a "material" minimum (i.e. an atom). For his part, Bruno believed that Mordente's compass confirmed the existence of atoms as it demonstrated that, when dividing straight and curved lines, a point was reached where both kinds of lines appeared to be composed of indefinitely shaped parts (§ 3.4). Indeed, these parts, which Bruno called "minima," were big enough for us to see them, but were too small to determine whether they were straight or curved.

The purpose of Chapter 4 is to provide an analysis of Giordano Bruno's conception of mathematics. Specifically, it intends to highlight two aspects of this conception that have been neglected in previous studies. First, Bruno's conception of mathematics changed over time and in parallel with another concept that was central to his thought: the concept of infinity. Specifically, Bruno undertook a reform of mathematics in order to accommodate the concept of the minimum, which was introduced at a later stage. Second, contrary to what Héléne Védrine claimed, Bruno was not a mathematical realist, but he believed that mathematical objects were mind-dependent.[105] To chart the parallel development of the conceptions of mathematics and infinity, a seven-year time span is considered, in which period Bruno published three works that are relevant for this study: *La cena de le ceneri* (1584), *Acrotismus camoeracensis* (1588) and *De minimo* (1591). *La cena de le ceneri* is usually taken to be the manifesto of Bruno's realism both because in this work he accepted the Copernican hypothesis of the motion of the earth, and because he took issue with Osiander's 'instrumentalist' reading of Copernicus' *De revolutionibus* as expressed in the anonymous letter *Ad lectorem* appended to the text (§ 4.1). It is true that, unlike the majority of his contemporaries, Bruno accepted the cosmological underpinnings of the Copernican theory. Nevertheless, it must be remembered that endorsing realism also required one's faith in the explanatory power of mathematics when applied to the study of nature. On the contrary, Bruno did not believe that mathematical models could be used to understand physical phenomena. The reason for Bruno's mistrust in mathematical physics was that he

[105] Védrine, "L'obstacle réaliste."

denied that mathematical objects existed independently of our mind, *pace* Vedrine.

The analysis of *Acrotismus camoeracensis* corroborates the idea that Bruno was far from being a mathematical realist. (§ 4.2). In addition, *Acrotismus* shows that Bruno's rejection of mathematical realism was motivated by his attempt to mathematize the concept of the minimum or, which amounts to the same, to provide an atomistic theory of mathematical objects. Indeed, after having established that there was a limit to the divisibility of the physical continuum, Bruno sought to demonstrate that the same was true of the mathematical continuum. In Bruno's view, Mordente's compass gave evidence that both geometric and physical objects were composed of minimum parts. However, one could argue that, no matter how small they were, geometric minima were not impenetrable nor possessed other characteristics that prevented them from being further divided. In response to this objection, Bruno argued that, although it was true that in line of principle the division of the mathematical continuum could go on to infinity, there was no point in performing a mathematical operation which exceeded the limits of nature. In other words, Bruno claimed that mathematics had to conform to nature instead of explain it. A similar argument was made in *De minimo*, where however one may argue that Bruno's 'campaign' against mathematical realism came to a halt (§ 4.3). The problem was that, in this work, Bruno claimed that the minimum (both physical and mathematical) had a circular shape, although he was adamant that geometric figures were nowhere to be found in nature. In my view, this problem was due to the fact that, with his theory of minima, Bruno attempted to integrate physics, metaphysics and mathematics into a single theory. The circular shape of the minimum was a result of this integration insofar as it derived from Bruno's fascination with theological and metaphysical concepts, such as the metaphor of the infinite sphere.

The thesis ends where it began—with a comparison between Bruno and Leibniz and their respective monadologies (§ 4.4). Early interpreters went as far as to claim that Bruno's monadology (which was related to his theory of minima) provided the model for Leibniz's. On the contrary, I argue that there were more differences than similarities between these two doctrines. If anything, both Bruno's and Leibniz's monadology were informed by a Pythagorean understanding of the monad, which, in the case of Bruno, confirms the importance of Pythagoreanism as a source of his mathematical thought.

PART ONE. SOURCES

1. Boethius, Llull and the legacy of Pythagorean atomism

1.1 Pythagorean atomism as a historians' category

It was the French mathematician and historian of mathematics Paul Tannery (1843 – 1904) who first attributed an atomistic theory to the ancient Pythagoreans. Also known for his edition of Descartes' work and correspondence (on which he collaborated with Charles Adam), Tannery was an engineer with a keen interest in the history of mathematics.[106] This interest led him to study the works of the ancient mathematicians, especially Diophantus, and to write a book on the science of the ancient Greeks in which he raised the issue of Pythagorean atomism.[107] Interestingly enough, no chapter of Tannery's book was devoted to the Pythagoreans, but the discussion on Pythagorean atomism was found in the chapter on Zeno of Elea. At the time Tannery wrote his book, the general consensus was that Zeno's paradoxes were directed against a common-sense view of multiplicity. Instead, Tannery thought that Zeno's purpose was to defend the monism of his master Parmenides from the attacks of the pluralist Pythagoreans. To this end, Zeno adopted the strategy of showing the inconsistencies of the Pythagoreans' own doctrines. For Tannery, those inconsistencies were reducible to a core belief of the Pythagoreans, a belief that, as we shall see, remained popular throughout the Middle Ages and the Renaissance:

> For the Pythagoreans, the point is the unit having a position or the unit in space. From this it follows that geometric bodies are a sum of points in the same way as numbers are a sum of units.[108]

In Tannery's opinion, not only did the Pythagoreans believe that geometric objects were composed of points, but they conceived the existence of physical points which served as the building blocks of physical bodies. This view was in accordance with the Pythagorean principle that all things were number, and it

[106] On Tannery as a historian of mathematics, see François Pineau, "Historiographie de Paul Tannery et receptions de son œuvre: sur l'invention du metier d'historien des sciences." (Doctoral dissertation, Université de Nantes, 2010).

[107] Tannery, *Pour l'histoire de la science hellène*.

[108] Tannery, 250.

was the target of Zeno's paradoxes. Indeed, Tannery argued, "when understood in this sense, the arguments of Zeno appear clear, compelling, undeniable, even those which may seem mere paralogisms."[109] It has to be noted that Tannery never used the term "Pythagorean atomism," but he claimed that there was a connection between the Pythagorean view that bodies were composed of points and the atomistic theories of Leucippus and Democritus. As a consequence of this and of the influence that Tannery had on the historiography of philosophy, the Pythagoreans came to be associated with atomism. This happened especially in the first half of the twentieth century, when the existence of a Pythagorean atomism was advocated by Francis Cornford and John Earl Raven.

Cornford's account of Pythagoreanism, as expounded in his article on "Mysticism and Science in the Pythagorean Tradition" (1922-3), was more complicated than Tannery's. First of all, Cornford contested the idea that there was only one line of thought within the Pythagorean school. On the contrary, he claimed that there were two Pythagorean systems: the mystical and the scientific. Furthermore, he identified the scientific system with an atomistic theory that the. Pythagoreans developed in response to Parmenides' criticism of their mystical system

> We can, in a word, distinguish between (1) the original sixth-century system of Pythagoras, criticized by Parmenides—the mystical system—and (2) the fifth-century pluralism constructed to meet Parmenides's objections, and criticized in turn by Zeno—the scientific system—which may be called 'Number-atomism.'[110]

Cornford returned to the issue of "Number-atomism" a few years later in his book on *Plato and Parmenides* (1939). The book opened with a discussion of the Pythagorean cosmogony, which was described as a process of derivation of physical bodies from numbers through geometric objects. For Cornford, the Pythagoreans developed two models to describe the derivation of geometric objects. According to the first model, geometric objects were constructed by adding a point to another. For instance, a line was formed by a row of points

[109] Tannery, 251.

[110] Cornford, "Mysticism and Science in the Pythagorean Tradition," 138.

placed side by side. In the second model, geometric objects were generated by the "flowing" of a point. Cornford noticed that the two models corresponded to two different stages in the evolution of the Pythagorean scientific system. The first model could be identified with Number-atomism, which however was affected by mathematical problems that could be solved by means of the second model. Thus, the "fluxion" model was an improvement on Number-atomism:

> The flowing of a single point into a line secures the continuity and infinite divisibility of magnitudes, and provides also for irrational quantities represented by incommensurable lines. The discovery of the irrational √2 and of the incommensurability of the diagonal of the square must have been made at a very early stage in geometry. It would follow upon the discovery of the Pythagorean theorem which may be due to Pythagoras himself, though the evidence is not conclusive. There can be little doubt that the earliest Pythagoreans, before these difficulties arose, simply built all geometrical magnitudes by adding unit-points.[111]

A pupil of Cornford, John Earle Raven, dared to question the interpretation of his master. This latter deemed Aristotle's account of Pythagoreanism unreliable because of its failure to distinguish the mystical from the scientific system. On the contrary, Raven claimed that "any account of Pythagoreanism that ignores the testimony of Aristotle is a house built upon sand."[112] Hence, Raven tried to show that the two systems were compatible with each other, and thus they could have been developed by the same generation of pre-Parmenidean Pythagoreans. Nevertheless, Raven agreed with Cornford that the Pythagoreans were the authors of an atomistic theory:

> In any case—and this is the important point—whether or not the Pythagoreans had actually spoken of ἄτομα μεγέθη a belief in their

[111] Francis M. Cornford, *Plato and Parmenides. Parmenides' Way of Truth and Plato's Parmenides Translated with an Introduction and a Running Commentary* (London: Routledge, 1964), 12–13.

[112] Raven, *Pythagoreans and Eleatics. An Account of the Interaction between the Two Opposed Schools during the Fifth and Early Fourth Centuries B.C.*, 6.

> existence does follow, as a logically inevitable consequence, from other propositions which they had undoubtedly accepted. If (1) bodies are composed of units, (2) the unit is indivisible (an axiom common to all Greek mathematics), and (3) units have size, it is impossible to evade the two conclusions that Aristotle voices, that indivisible magnitudes exist and that units have weight. To this extent, irrespective of Zeno's arguments, the supporters of the Number-atomism interpretation are justified.[113]

Cornford and Raven laid the foundation for a new understanding of Pythagoreanism. However, starting with the 1960s, their interpretation was challenged by scholars such as William Guthrie. Like his predecessors, Guthrie thought that the Pythagoreans developed an atomistic theory which predated Eleatic philosophy.[114] On the other hand, Guthrie raised doubts about the Pythagorean authorship of the fluxion theory (i.e. the second Pythagorean model for the derivation of geometric objects, according to Cornford). For Cornford and Raven, the fluxion theory was a Pythagorean achievement, as claimed by later sources such as Sextus Empiricus.[115] For his part, Guthrie relied on the testimony of Aristotle, who, in *De anima*, mentioned those who took the line to be the "flowing of a point" while discussing the conception of the soul as a "self-moving number."[116] Although Aristotle did not mention him, we know that this

---

[113] Raven, 54.

[114] William K. C. Guthrie, *A History of Greek Philosophy. Vol. I: The Earlier Presocratics and the Pythagoreans* (Cambridge: Cambridge University Press, 1962), 261–62.

[115] Sextus Empiricus, *Against the Physicists. Against the Ethicists*, trans. Robert Gregg Bury (Cambridge, MA: Harvard University Press, 1958), 346: "But some [Pythagoreans] assert that the body is constructed from one point; for this point when it has flowed produces the line, and the line when it has flowed makes the plane, and this when it has moved towards depth generates the body, which has three dimensions."

[116] Aristotle, "On the Soul," in *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, ed. J. Barnes (Princeton, NJ: Princeton University Press, 1995), 1428 (409a4-5): "Further, since they say a moving line generates a surface and a moving point a line, the movements of the units must be

conception of the soul belonged to Xenocrates. This led Guthrie to conclude that Xenocrates was also the father of the fluxion theory. In doing so, Guthrie made an assumption that became a cornerstone of later interpretations of Pythagoreanism: "Any modification of Pythagorean doctrine made in the Academy would have been freely accepted as Pythagorean by most Neopythagorean or later writers."[117]

Admitting that Platonism and Pythagoreanism crossed paths did not only explain why the fluxion theory, despite its Platonic origin, was reported by Sextus as a Pythagorean doctrine. More importantly, it meant that Pythagoreanism could not be viewed as a continuous tradition stemming from a single source, that is, Pythagoras and his school. Rather, there were more Pythagorean traditions, most of which were named after Pythagoras although they bore no relation to him. In the 1850s, Eduard Zeller already noticed that in ancient Pythagoreanism there was an "expansion of tradition," which however, in his opinion, amounted to a collection of "dogmatic preconceptions, partisan interests, dubious legends, and spurious writings."[118] This skeptical attitude changed over the years, with nineteenth-century commentators willing to give more credit to later Pythagorean sources. The most prominent example of this new tendency was Walter Burkert's *Lore and Science in Ancient Pythagoreanism* (1962-72). For Burkert, the early Academy represented a watershed in the history of Pythagoreanism. Speaking of a recently rediscovered fragment of Speusippus, he stated that:

> This [fragment] makes certain, what a careful analysis of the sources would in any case make likely, that a Platonizing interpretation of Pythagoreanism, which had a decisive influence on the later tradition, goes back to Plato's immediate disciples and differs sharply from the

---

lines (for a point is a unit having position, and the number of the soul is, of course, somewhere and has position)."

[117] Guthrie, *A History of Greek Philosophy*, 262. If Guthrie attributed the fluxion theory to Xenocrates, J. A. Philip argued that it was elaborated after the Academy. See J. A. Philip, "The 'Pythagorean' Theory of the Derivation of Magnitudes," *Phoenix* 20, no. 1 (1966): 32–50.

[118] Cited in Burkert, *Lore and Science*, 2.

> reports of Aristotle. The latter's evidence thus becomes more important than ever; for he alone warns us to separate Pythagorean and pre-Platonic from Platonic material.[119]

Returning to Tannery, the problem with his interpretation of Pythagoreanism was that it was an *a posteriori* reconstruction based on passages from Aristotle's *Metaphysics*. However, as noted by Gregory Vlastos, Aristotle's testimony was unreliable because there was no indication that "Aristotle had in view doctrines professed by Pythagoreans more than a hundred year before his own time rather than contemporary ones."[120] It may well have been that the atomistic theory which Tannery, following Aristotle, attributed to the *early* Pythagoreans was in fact developed much later. More recently, Aurèlien Robert argued that "if the historical aspect of Tannery's interpretation has to be rejected, its philosophical content has not entirely lost its relevance."[121] In Tannery's account, the Pythagoreans accepted the idea that the point was an atom or a unit having position. Robert noted that, in the Middle Ages, this idea was expressed by Boethius and other Neopythagorean sources such as Macrobius and Martianus Capella. Although not an atomist, Boethius conceded that the point could be viewed as an atom. In this way, he led medieval scholars to adopt the view that the continuum was composed of points, as happened in the case of William of Champeaux and Peter Abelard. [122]

1.2 Boethius and the revival of Pythagorean atomism in the Middle Ages

Boethius (born: 475–7, died: 526?) is best known as the author of *De philosophiae consolatione* (*The Consolation of Philosophy, c.* 524). In addition, he was a translator and influential commentator of Aristotle, and wrote on a wide range of different subjects, from logic to theology, from music to mathematics. Boethius' contribution to mathematics has been relatively little studied,

---

[119] Burkert, 13.

[120] Vlastos, "Zeno of Elea," 376.

[121] Robert, "Atomisme pythagoricien," 184.

[122] See Robert, 186–97.

notwithstanding the fact that his *Institutio arithmetica* (*c*. 500) grew to become "the standard reference book for arithmetic in the West for a millennium."[123] Conceived as a translation of Nicomachus of Gerasa's *Introduction to Arithmetic*, the *Institutio* inspired the work of theologians (such as Thierry of Chartres, who wrote a commentary on it which will be analyzed in the next chapter), philosophers (such as Nicholas of Cusa who referred to its author as "our Boethius"[124]) and even architects and engineers, who went as far as to construct cathedrals and churches based on the numerical ratios defined by Boethius.[125] Furthermore, the *Institutio* had a profound influence on the Renaissance when, despite the growing importance of Euclid's *Elements*, editions of it continued to be published and commented on, especially by the French humanist Jacques Lefèvre d'Étaples (1455 – 1536) and his circle.

At the outset of the *Institutio,* Boethius presented the four mathematical sciences (arithmetic, music, geometry, and astronomy) and their respective subjects (numbers, ratios, stable magnitudes, moveable magnitudes).[126] Borrowing an idea from Nicomachus and Martianus Capella, he coined the term *quadrivium* (which in Latin means the meeting of four roads) to describe the unity of the mathematical sciences. Later on, medieval scholars adopted the word

---

123 Jean-Yves Guillaumin, "Boethius's *De Institutione Arithmetica* and Its Influence on Posterity," in *A Companion to Boethius in the Middle Ages*, ed. Noel Harold Kaylor and Philip Edward Phillips (Leiden: Brill, 2012), 161.

124 Nicholas of Cusa, *De docta ignorantia*, ed. Paul Wilpert and Hans Gerhard Senger (Hamburg: Felix Meiner, 2002), bk. I, chap. 11, p. 42: Quem Platonici et nostri etiam primi in tantum secuti sunt, ut Augustinus noster et post ipsum Boethius affirmarent indubie numerum creandarum rerum 'in animo conditoris principale exemplar' fuisse."

125 See Otto Georg von Simson, *The Gothic Cathedral: Origins of Gothic Architecture and the Medieval Concept of Order* (Princeton: Princeton University Press, 1988); Robert Odell Bork, *The Geometry of Creation: Architectural Drawing and the Dynamics of Gothic Design* (Farnham: Ashgate, 2011).

126 Boethius, *Institutio arithmetica*, I, 1, 4, ed. Jean-Yves Guillaumin (Paris: Les Belles Lettres, 2002), 7: "Horum ergo illam multitudinem quae per se est arithmetica speculatur integritas, ilaam uero quae ad aliquid musici modulaminis temperamenta pernoscunt, immobilis uero magnitudinis geometria noticiam pollicetur, mobilis uero scientiam astronomicae disciplinae peritia uinidicat."

trivium to indicate the three literary disciplines (grammar, dialectic and rhetoric) which, together with the *quadrivium,* comprised the seven liberal arts.[127] Although all of the mathematical sciences were necessary to achieve true knowledge, Boethius explained that arithmetic had priority over geometry, music and astronomy because of the fact that God created the world using numbers as prototypes.[128] Accordingly, the existence of all created beings, including that of magnitudes and ratios, depended on numbers. "If you take away numbers, Boethius asked, in what will consist the triangle, quadrangle, or whatever else is treated in geometry?"[129] Furthermore, numbers provided the model for the derivation of magnitudes from the point, since the point was the principle of magnitudes just as the unit was the principle of numbers:

> Unity has the potential of a point, the beginning of interval and longitude; it is not itself capable of interval or longitude, just as the point is the beginning of the line and the interval, although it is itself neither interval nor line. Nor does a point put upon a point bring about an interval, any more than if you joined nothing to nothing. It is nothing and nothing comes from nothing. [...] Likewise, unity multiplied by itself begets nothing.[130]

---

[127] See Paul Oskar Kristeller, "The Modern System of the Arts: A Study in the History of Aesthetics Part I," *Journal of the History of Ideas* 12, no. 4 (October 1951): 496–527.

[128] *Institutio arithmetica*, I, 1, 8, ed. J.-Y. Guillaumin, 8-9: "[Arithmetica] enim cunctis prior est, non modo quod hanc ille huius mundanae molis conditor deus primam suae habuit ratiocinationis exemplar et ad hanc cuncta constituit quaecumque fabricante ratione per numeros adsignati ordinis inuenere concordiam. "

[129] *Institutio arithmetica*, I, 1, 9, ed. J.-Y. Guillaumin, 9: "Si enim numeros tollas, unde triangulum uel quadratum uel quicquid in geometria uersatur, quae omnia numerorum denominatiua sunt?" Translation: Micheal Masi, *Boethian Number Theory. A Translation of the* De Istitutione Arithmetica (Amsterdam: Rodopi, 1983), 74.

[130] *Institutio arithmetica*, II, 4, 4, ed. J.-Y. Guillaumin, 89: "Est igitur unitas uicem obtinens puncti, interualli longitudinisque principium, ipsa uero nec interualli nec longitudinis capax quemadmodum punctum principium quidem linae est atque

Later in the text, Boethius explained that magnitudes were derived from the point through a process that he called "unfolding" (*explicatio*).[131] Since the *Institutio* provided little information on the dynamic of this process, medieval commentators elaborated complex explanations to describe how magnitudes unfolded from the point. This was the case of Thierry of Chartres, who was the author of a theory of unfolding which will be analyzed in greater detail in the next chapter. In another passage of the *Institutio*, Boethius defined the point as "the principle of all intervals and indivisible by nature, which the Greeks call *atom* because it is so diminished and very small that parts of it cannot be found."[132] However, this should not lead us to conclude that Boethius was an atomist, as both in the *Institutio* and in his commentary on Aristotle's *Categories* he made it clear that magnitudes were not composed of points.[133] Like Euclid, Boethius took the point to be an extremity and not a part of the line. In spite of this, Robert has demonstrated that Boethius unwittingly contributed to the revival of Pythagorean atomism.[134] Indeed, especially in the fourteenth century, "the atoms of currency were extensionless, mere mathematical points, mere instants, mere moments of motion."[135]

---

interualli, impsum uero nec interuallum nec linea. Neque enim punctum puncto superpositum ullum efficit interuallum, uelut si nihil nulli iungas. Nihil enim est quod ex nullorum procreatione nascatur." Translation: Masi, *Boethian Number Theory*, 129.

[131] *Institutio arithmetica*, II, 4, 6, ed. J.-Y. Guillaumin, 90: "Ex hoc igitur principio, id est ex unitate, prima omnium longitudo succrescit quae a binarii numeri principio in cunctos sese numeros explicat, quoniam primum interuallum linea est."

[132] *Institutio arithmetica*, II, 4, 9, ed. J.-Y. Guillaumin, 91: "Omnium interuallorum esse principium et natura insecabile, quod Graeci atomon uocant, id est ita diminitum atque paurissimum ut eius pars inueniri non possit." Translation: Masi, *Boethian Number Theory*, 130.

[133] Boethius, *In Categorias Aristotelis commentaria*, ed. J.-P. Migne, Patrologia Latina 64 (Paris, 1847), 205 A-B: "Non autem hoe dicitur, quod linea constet ex punciis aut superificies ex lineis, aut solidum corpus ex superficiebus, sed quod et lineae termini puncta sunt, et superficiei linae, et solidi corporis superficies, nullasque res suis terminis constat."

[134] Robert, "Atomisme pythagoricien," 192–206.

[135] Lüthy, Murdoch, and Newman, "Introduction," 8.

The mathematical character of medieval atomism distinguished it from ancient atomism insofar as in Antiquity atoms were viewed as material particles instead of mathematical points. The first medieval supporters of mathematical atomism were found in the Islamic world among ninth-century Muta'zilite theologians,[136] while it was not until the fourteenth century that mathematical atomism was accepted in the Latin West, although both William of Champeaux and Peter Abelard spoke of points-atoms as early as the twelfth century.[137] According to John Murdoch, medieval atomism (both Islamic and Christian) raised as a response to Aristotle's critique of atomism.[138] This explained why Islamic and Christian thinkers held similar views on the continuum, without having to assume that the former influenced the latter. Murdoch's hypothesis has generally been accepted by historians of atomism. Minor adjustments have been made to it by Murdoch himself in one of his most recent papers.[139] Nevertheless, the view that "all the fourteenth-century atomists can be seen, indeed must be seen, as reacting to and criticizing the sixth book of Aristotle's *Physics*"[140] has remained unchallenged.

Robert's interpretation of medieval atomism is complementary to Murdoch's. For Robert, there is no doubt that the recovery of Aristotle's *Physics* in the thirteenth century had an impact on medieval discussions about the continuum. However, there may be other factors to be considered. In particular, it should be noted that medieval atomism was built on the Pythagorean

[136] On these two authors, see Robert, "Atomisme pythagoricien."

[137] On Islamic mathematical atomism, see Alnoor Dhanani, *The Physical Theory of Kalām: Atoms, Space, and Void in Basrian Muʿtazilī Cosmology* (Leiden: Brill, 1994).

[138] See Murdoch, "Infinity and Continuity," 576. See also John E. Murdoch, "Naissance et développement de l'atomisme au las moyen âge latin," in *La science de la nature: théories et pratiques* (Montreal: Bellarmin, 1974), 11–32.

[139] See John E. Murdoch, "Beyond Aristotle: Indivisibles and Infinite Divisibility in the Later Middle Ages," in *Atomism in Late Medieval Philosophy and Theology*, ed. Christophe Grellard and Aurélien Robert (Leiden; Boston: Brill, 2009), 15–38.

[140] Lüthy, Murdoch, and Newman, "Introduction," 9.

assumption that magnitudes were derived from the point.[141] Pythagoreanism, therefore, also played a role in the development of medieval atomism. It is in this sense that Robert uses the term "Pythagorean atomism" with regard to the medieval period.

According to Robert, the influence of Pythagoreanism on medieval atomistic theories is "evident in the twelfth century, but several elements invite us to think that the same framework can be also applied to the thirteenth and fourteenth century."[142] William of Champeaux, Peter Abelard, Robert Grosseteste, Henry of Harclay, and John Wyclif were among those who accepted the Pythagorean conception of the point-atom in the period covered by Robert.[143] In what follows, I will present the work of another medieval author who claimed that geometric objects were composed of points: Ramon Llull. The case of Llull is interesting because he could have been a Brunian source. Indeed, it is well known that Bruno was an expert in the Lullian Art and wrote several "Lullian" works.[144] Furthermore, Llull was an example of how Platonic and Pythagorean insights could be integrated into a single atomistic theory.

1.3 The *Geometria nova* of Ramon Llull

Ramon Llull (1232 – 1316) was born in Majorca, the son of a middle-class family originally from Barcelona. After having spent his youth on the island where he received his education and started a family, his life took a new turn at about the

---

[141] On the Pythagorean origin of this assumption, see Leonid Zhmud, *Pythagoras and the Early Pythagoreans* (Oxford: Oxford University Press, 2012); Gabriele Cornelli, *In Search of Pythagoreanism: Pythagoreanism as an Historiographical Category*. (Berlin: De Gruyter, 2013); Philip Sidney Horky, *Plato and Pythagoreanism* (New York: Oxford University Press, 2013).

[142] Robert, "Atomisme pythagoricien," 187.

[143] Aurélien Robert, "Atomisme et géométrie à Oxford au XIVe siècle," in *Mathématiques et connaissances du réel avant Galilée*, ed. Sabine Rommevaux (Paris: Omniscience, 2010), 15–85; Aurélien Robert, "Space, Imagination, and Numbers in John Wyclif's Mathematical Theology," in *Space, Imagination and the Cosmos from Antiquity to the Early Modern Period*, ed. Frederik A. Bakker, Delphine Bellis, and Carla Rita Palmerino (Cham: Springer, 2018), 107–31.

[144] Bruno, *Opere lulliane*.

age of 30, when he saw an apparition of Christ on the Cross. This inspired him to convert to Catholicism and become a missionary, in which capacity he travelled to Montpellier, Paris, Rome and Africa. In the midst of his wanderings, he developed the Art, which he conceived as an instrument to convert the infidels but also as an alternative to the Aristotelian logic. We do not know how well the Art served Llull's missionary purposes—if, and to what extent, it helped him win the hearts and minds of Jews and Muslims. What is certain is that it soon attracted the attention of theologians, philosophers and learned men who were seeking an alternative to the prevailing Scholasticism, and who devoted themselves to the study and diffusion of the Lullian works. Nicholas of Cusa, Jacques Lèfevre d'Étaples, Charles de Bovelles and Bernard de Lavinheta are among the best-known Lullists.[145] Thanks to their efforts, the Art continued to gain followers over the centuries, offering a model for early modern philosophical projects such as the mnemonics of Bruno and the combinatorics of Leibniz.[146]

Llull wrote more than two hundred works in Latin, Arabic and Catalan, including a number of treatises which were not closely related to the Art. Speaking of these treatises, Antony Bonner writes that:

> We must keep in mind that when Llull wrote on philosophy, it was not as a philosopher, and when he wrote on science, it was not as a scientist. He was less interested in those subjects for themselves than as tools to further his main purpose, the conversion of the unbelievers by means of a method based on the general principles that govern the natural order of the universe.[147]

Written in June 1299 during Llull's second stay in Paris (August 1298 – July 1299), the *Liber de geometria nova et compendiosa (Book on the New and Concise Geometry)* is a glaring example of Llull's attitude towards science. Llull's

---

[145] For an overview of the Lullists, see Hillgarth, *Ramon Lull*, 270ff.

[146] On the importance of Llull as a philosophical model, see Paolo Rossi, *Logic and the Art of Memory: The Quest for a Universal Language*, trans. Stephen Clucas (London: Continuum, 2006).

[147] Anthony Bonner, *Doctor Illuminatus: A Ramón Llull Reader*, (Princeton, N.J: Princeton University Press, 1993), 47.

purpose in this work was to propose a new conception of geometry in accordance with the principles of his Art. In the first book, he dealt with the operations that can be performed on geometric figures such as square, circle and triangle. In this context, he addressed the problem of squaring the circle by developing a method that, more than a century later, drew the attention of Nicholas of Cusa[148] More importantly, in the second book of the *Geometria nova,* Llull claimed that geometric objects were composed of points, thus adopting an atomistic view similar to that attributed to the Pythagoreans by Tannery. In addition, both Charles Lohr and José Higuera Rubio agree that Llull accepted the theory that the four elements were composed of indivisible geometric parts, namely circles, triangles and squares.[149] Since this theory was first proposed by Plato, it can be referred to as Platonic atomism.[150]

1.3.1 Platonic atomism[151]

In the Aristotelian theory of the elements, each element had two qualities, but

---

[148] Elena Pistolesi, "Quadrar el cercle després de Ramon Llull: el cas de Nicolau de Cusa," in *2n Col·loqui Europeu d'Estudis Catalans. La recepció de la literatura catalana medieval a Europa*, ed. Alexander Fidora and Eliseu Trenc (Péronnas: Tour Gile, 2007), 17–32.

[149] See Charles Lohr, "Ramon Lull's Theory of the Continuous and Discrete," in *Late Medieval and Early Modern Corpuscular Matter Theories*, ed. C. H. Lüthy, J. E. Murdoch, and W. R. Newman (Leiden: Brill, 2001), 75–90; José Higuera Rubio, "El 'atomismo' luliano y el problema del continuo: Una explicación lógico-geométrica de la constitución elemental de las sustancias," *Scintilla* 10, no. 1 (2013): 19–36; See also José Higuera Rubio, *Fisica y teologia (atomismo y movimiento en el Arte luliano)* (Madrid: Circulo Rojo, 2014).

[150] Plato, *Timaeus and Critias*, trans. Robin Waterfield (Oxford: Oxford University Press, 2008), 46: "The starting-point is, of course, universally accepted : that fire, earth, water, and air are material bodies. Now, this means that, like all bodies, they have depth, and anything with depth is necessarily surrounded by surfaces, and any rectilinear surface consists of triangles."

[151] In this section, I will limit myself to a brief account of the Lullian atomistic theory of the elements. For a more extensive analysis of this theory, see the studies quoted in note 36.

only one was dominant. For instance, fire had both heat and dryness, but only heat was dominant in fire, while dryness was dominant in earth.[152] By contrast, the Lullian theory of the elements was based on the distinction between "proper" (*propriae*) and "appropriated" (*appropriatae*) qualities. For instance, Llull distinguished between the heat proper to fire and the dryness appropriated by fire from the earth that burned.[153] The fact that a quality was possessed by one element or another not only determined whether that quality was proper or appropriated, but it also affected its structure. For proper qualities were continuous, while appropriated qualities were discrete, i.e. they were associated with a quantity which was divisible into a defined number of parts.[154] As we shall see, this provided the basis for Lullian atomism insofar the parts of which appropriated qualities were composed had, in Llull's view, a geometric shape. It should be noted that Urso of Salerno (d. 1225) held that the mixture of elemental qualities occurred by means of minimal parts (*minimae partes*), a view similar to the notion of discrete appropriated qualities proposed by Llull.[155] This seems to

---

[152] Aristotle, "On Generation and Corruption," in *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, ed. J. Barnes (Princeton, NJ: Princeton University Press, 1995), 1184 (331a1-5): "Nevertheless, since they [i.e. the elements] are four, each of them is characterized simply by a single quality: Earth by dry rather than by cold, Water by cold rather than by moist, Air by moist rather than by hot, and Fire [5] by hot rather than by dry."

[153] Ramon Llull, *Die neue Logik. Logica nova*, ed. Charles H. Lohr and Vittorio Hösle (Hamburg: Felix Meiner, 1985), 10: "Accidens aliud proprium, aliud appropriatum. Proprium, sicut caliditas ignis. Appropriatum, sicut sua siccitas, quam terra sibi appropriat."

[154] *Die neue Logik,* 103*:* "Qualitas propria est continua, sicut caliditas ignis. Et talis, qualis instantanea est ratione continuae quantitatis. Sed qualitas appropriata diffusa est successive per discretas quantitates, sicut caliditas aeris, aquae et terrae, piperis et cinamoni, et ceterorum elementorum."

[155] See Danielle Jacquart, "Minima in Twelfth-Century Medical Texts from Salerno," in *Late Medieval and Early Modern Corpuscular Matter Theories,* ed. Christoph H. Lüthy, John E. Murdoch, and William R. Newman (Leiden: Brill, 2001), 75–90. See also Maaike van der Lugt, "Chronobiologie, combinatoire et conjonctions élémentaires dans le *De commixtionibus elementorum* d'Urso de Salerne (fin XIIe

suggest a connection between Urso and Llull, but it is beyond the purpose of this study to explore this connection.

Continuing with the description of the Lullian theory of the elements, a compound object could possess more than one appropriated quality, thus it could contain parts of different qualities. The sum of these parts was the degree in which each quality was present in the object. For example, in the *Geometria nova*, Llull stated that "in a peppercorn, fire is in the fourth degree of heat, earth is in the third degree of dryness, air in the second degree of moisture and water in the first degree of cold."[156] In addition, he drew a diagram to describe the configuration of the elemental qualities in all plants in the fourth degree of heat (Figure 1). In this diagram, a different letter was used to refer to each elemental quality (a: heat, b: moisture, c: dryness, d. cold):

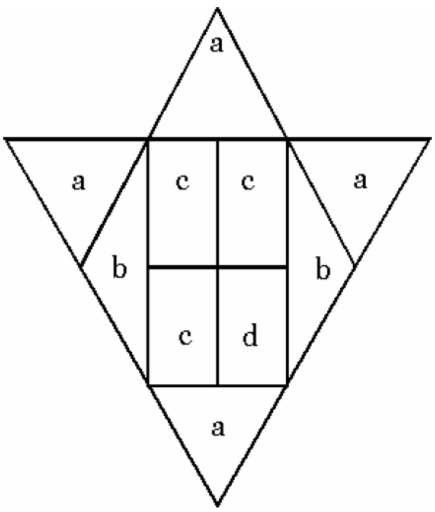

Fig. 1: The fourth degree of heat

siècle)," *La misura – Measuring, Micrologus. Natura, scienze e società medievali*, no. 19 (2011): 277–323.

[156] Ramon Llull, *El libro de la "Nova geometria,"* ed. José Maria Millás Vallicrosa (Barcelona: R. Torra, 1953), 69: "In grano piperis ignis est in quarto gradu caloris, terra in tercio gradu siccitatis, aer in secundo grado humiditatis, aqua in primo gradu frigiditatis." For the translation of Llull's *Geometria nova*, I rely on Yanis Dambergs' translation accessible online at http://lullianarts.narpan.net.

It was no coincidence that, in the above diagram, the single parts of the elemental qualities were represented by geometric figures. In the *Arbor scientiae* (*Tree of Science,* 1295-6), Llull stated that "because there are neither more nor less than four elements, they are aptly disposed to be configured in elemented (*elementatum*) things in square, circular and triangular figures, and these three figures must necessarily be situated in all elemented things.[157] The term *elementatum* was commonly used in twelfth-century discussion on the *Timaeus*.[158] For instance, it was used by William of Conches to distinguish between the element in its pure form (*elementum*) and the element as found in a mixture (*elementatum*).[159] This provides further evidence of the influence that Platonic commentators such as William of Conches had on the Lullian theory of the elements. In this context, the quadrature of the circle and the other operations presented in the first book of the *Geometria nova* took on a new significance. They proved that circle, triangle and square were equivalent figures that could be used interchangeably when analyzing compound objects. This, in turn, resulted in a considerable extension of the range of applications of geometry, as diagrams such as that representing the fourth degree of heat had multiple uses. Llull argued that by means of them

> geometers can know the composition of the angles, physiognomists can understand human forms, astronomers can know the figures of stars and the situation of the influences they transmit to things below,

---

[157] Ramon Llull, *Arbor scientiae. Volumen I. Libri I-VII*, ed. P. Villalba, vol. 24, Raimundi Lulli opera latina (Turnhout: Brepols, 2000), 34–35. "Sicut in pipere, in quo sunt omnes istae complexiones, et ei sufficit, quod sit subiectum differentiarum et concordantiarum et contrarietatum, quas praediximus, et adhuc, quia elementa sunt quattuor et non minus, neque plus, sunt disposita, quod sint figurata in elementatis in figura quadrangulari, circulari et triangulari. Translation by Yanis Dambergs accessible online at http://lullianarts.narpan.net.

[158] Theodore Silverstein, "*Elementatum*: Its Appearance Among the Twelfth-Century Cosmogonists," *Mediaeval Studies* 16 (January 1954): 156–62.

[159] Irene Caiazzo, "The Four Elements in the Work of William of Conches," in *Guillaume de Conches: Philosophie et science au XIIe siècle* (Florence: SISMEL Edizioni del Galluzzo, 2011), 3.

> and natural scientists can know the situations that the simple elements have in compounds.[160]

In addition to defending his atomistic theory of the elements, in the *Geometria nova* Llull proposed an atomistic conception of mathematical objects. As a matter of fact, this conception poses a number of interpretative problems as it is based on a notion—that of corporeal point—that Llull seems to describe rather than explain. At any rate, there is a close resemblance between this conception and that in which Bruno's atomistic geometry was grounded. Thus, in the next section, we take a closer look at Llull's atomistic conception of mathematical objects to determine if and to what extent this conception may have inspired Bruno to develop his atomistic geometry.

1.3.2 Pythagorean atomism

In the sixth book of the *Physics*, Aristotle ruled out that a line could be composed of points, because points could not be brought into contact with one another to form a continuum.[161] Likewise, Boethius made it clear that the points were the ends and not a part of the line (see § 1.2). By contrast, Llull claimed not only that "a point is an entity that is part of a line,"[162] but also that "the matter of the line is

---

[160] *Geometria nova,* 65: "Potuerunt geometri compositionem angulorum predictorum et fisiomantici figuras hominum cognoscere et astronomi figuras stellarum quas faciunt in celo et assituacionem influenciarum quas tramitunt ad hec inferiora, et naturales poterunt cognoscere assituaciones quas simplicial elementa habent in compositis."

[161] Aristotle, "Physics," in *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, ed. J. Barnes (Princeton, NJ: Princeton University Press, 1995), 861–62: "Nothing that is continuous can be composed of indivisibles: e.g. a line cannot be composed of points, the line being continuous and the point indivisible. For the extremities of two points can neither be one (since of an indivisible there can be no extremity as distinct from some other part) nor together (since that which has no parts can have no extremity, the extremity and the thing of which it is the extremity being distinct)."

[162] *Geometria nova,* 85: "Punctus est ens qui est pars linae"

made up of points."[163] This seems to suggest that Llull had a materialistic conception of the point, as confirmed by the fact that he postulated the existence of corporeal points.[164] Since corporeal points were extended, their existence enabled Llull to rebut the argument that surfaces could not be composed of points because surfaces were extended, while points were not.[165] This argument tells us that, in Llull's opinion, points were the building blocks of surfaces as well as lines. In fact, Llull seems to have thought that all mathematical objects were composed of points, as he claimed that surfaces consisted of wide points, lines of long points, circles of circular points, angles of obtuse and acute points, and so forth. Thus, for Llull, there were different species of points, as expressed in the following passage:

> The point is a genus that includes many species, like wide points and acute points, which are distinct due to distinct species of angles distinguished by the distinction between the triangle and the square.[166]

What did Llull mean by saying that points were corporeal and could have different shape? One possibility is that, in this context, Llull referred to sensible rather than intelligible mathematical objects, as he claimed that *sensible* lines were composed of points.[167] It should be noted that while an intelligible line (i.e. a breadthless length, as Euclid defined it in the *Elements*) is only extended in one dimension (length), a sensible line (e.g. a line drawn on a piece of paper) has also a certain breadth due to the instrument with which it was drawn. By the same token, when dividing a sensible line, the resulting parts will not be extensionless

---

[163] *Geometria nova*, 87: "Materia linee est de punctis"

[164] *Geometria nova*, 86: "Dictum est quod punctus qui est in centro est communis, unde sequitur quod omnis punctus communis sit corpus aut de natura corporis."

[165] Llull's counter-argument was found in the third part of the second book of the *Geometria nova* which Millás Vallicrosa omits in his edition without providing an explanation for his choice. However, this part of the text can be read in Damberg's English translation.

[166] *Geometria nova, 87:* "Punctus sit genus quails habet sub se species angulorum distinctorum per distinctionem quadranguli et trianguli, idcirco apparet quod species sunt essencia realia"

[167] *Geometria nova*, 86: "Omnis linea sensibilis est composita."

points, but they will have a certain extension and a certain shape. The shape of the points will depend on the object to which they belong: straight lines will be composed of 'straight' points, curved lines of 'curved' points, triangles of 'triangular' points, and so on. If Llull had this in mind when he spoke of corporeal points, the term "corporeal" should be taken as synonymous with "sensible." However, one may argue that, thus conceived, corporeal points were not points, but rather they were small portions of mathematical objects having different shapes. Moreover, they were not even corporeal as they were not three-dimensional, unless one considers the materiality of the support on which mathematical objects were made sensible (e.g. the piece of paper) as part of the objects themselves.

According to Boethius, the point was that which had no parts and for this reason he called it atom.[168] On the contrary, Llull believed that the points in common between lines, such as those in which two lines intersected to form an angle, were divisible and thus had parts.[169] Arguably, Llull meant that common points comprised part of the lines which passed through them. Moreover, since all the points of a line were potentially common points, they could all be divided into parts. Hence, we can conclude that those geometric entities which Llull called corporeal points were not points *stricto sensu*. Llull himself seemed to draw a distinction between corporeal and mathematical points, the latter being the only points, in his opinion, to be truly simple, i.e. without parts.[170] Furthermore, mathematical points were indivisible, incorporeal and unperceivable, that is to say, they were the exact opposite of corporeal points.[171]

What was the relationship between mathematical and corporeal points in Llull's thought? Speaking of mathematical points, Llull wrote that "the abstract

---

[168] *Institutio arithmetica*, II, 4, 6, ed. J.-Y. Guillaumin, 90.

[169] *Geometria nova*, 87: "Dictum est quod omnis punctus qui sit substanciam anguli communis est et divisibilis, unde concluditur quod alilcuis punctus sit corpus et divisibilis."

[170] *Geometria nova*, 86: "Solus punctus mathematicus est simplex."

[171] *Geometria nova*: 86-7: "Dictum est quod punctus qui non habet partem divisibilis esse non potest, unde sequitur quod talis punctus non potest esse corpus. [...] Dictum est quod nullus punctus indivisibilis sentitur nec habet partem, unde sequitur quod prima puncta indivisibilia, videri non possent, audiri nec tangi."

essences that are mathematical are simple, like one essence of fire, one essence of heat, or one potential form, and so with other things like these."[172] As is well known, Aristotle was the first to express the idea that mathematical entities were intelligible abstractions derived from sensible beings.[173] In doing so, Aristotle's purpose was to oppose the Platonic view that mathematical objects were intelligible entities in their own right, which stood in between sensible beings and pure ideas.[174] By claiming that mathematical points were "abstract" essences, Llull appeared to side with Aristotle and suggest that mathematical points were intelligible concepts derived from corporeal points. This is confirmed by the fact that Llull regarded the different species of corporeal points, and not their intelligible genus (i.e. the mathematical point), as real essences.[175] On the other hand, it may have been that in the *Geometria nova* corporeal points were named after their mathematical counterpart, which would explain why Llull used, albeit inappropriately, the term "points" to refer to the minimal parts of sensible mathematical objects.

More generally, the reason why the *Geometria nova* seems to frustrate attempts to understand the notion of corporeal points may be that it does not answer the question of the relationship between mathematical and physical objects. Llull's position on this question is not clear. Did he think that mathematics and physics were two separate realms? Or did he hold that

---

[172] *Geometria nova*, 87: "Essencia abstacta que sunt mathematica sunt simplicia sicut una essencia ignis, una essencia caloris aut una forma que est in potencia et sic de aliis rebus similibus istis."

[173] Henry Mendell, "Aristotle and Mathematics," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, Spring 2017 (Metaphysics Research Lab, Stanford University, 2017), para. 7.1, https://plato.stanford.edu/archives/spr2017/entries/aristotle-mathematics/.

[174] Plato, *The Republic*, ed. G. R. F. Ferrari, trans. Tom Griffith (Cambridge: Cambridge University Press, 2000), 217 (510d-e): "And you will also be aware that they [i.e. the mathematicians] summon up the assistance of visible forms, and refer their discussion to them, although they're not thinking about these, but about the things these are images of. So their reasoning has in view the square itself, and the diagonal itself, not the diagonal they have drawn."

[175] *Geometria nova*, 87: "Idcirco apparet quod species [puncti] sunt essencia realia."

mathematical objects were found in nature? Lacking this information, we can limit ourselves to conjectures based on a close reading of the *Geometria nova*. As far as I could see, the other main mathematical text written by Llull (the *Liber de quadratura*) does not help answer these questions, nor does the literature offer a full-blown discussion of Llull's mathematical thought.[176] It is also possible that Llull himself did not find it necessary to clarify the philosophical underpinnings of his mathematics, given that he regarded mathematics as a derivation from his Art.

To conclude, it cannot be said that Llull elaborated an atomistic theory of mathematical objects. Indeed, it should be noted that the *Geometria nova* did not provide a full account of how corporeal points were arranged to form mathematical objects, nor, to the best of my knowledge, did Llull address this issue in other works. As noted at the beginning of § 1.3, Llull had little interest in mathematics, which raises the question of why he wrote a work like the *Geometria nova*. Carla Compagno writes that, in the years prior to the *Geometria nova,* Llull viewed mathematics as only a science of sensible objects. However, this conception changed over the years, as Llull came to appreciate the soundness of mathematical demonstrations and the universality of mathematical language.[177] Unfortunately, the same cannot be said of the demonstrations and language of the *Geometria nova*, which has been criticized for its flawed arguments and its misuse of mathematical terminology.[178]

It cannot be denied, however, that in the *Geometria nova* Llull expressed the idea that mathematical objects were composed of points. Bruno could have borrowed this idea from Llull since he had an extensive knowledge of his works, although it must be said that the *Geometria nova* was never mentioned by Bruno. In addition, Bruno could have become familiar with Llull's mathematics through

---

[176] The only study on this topic is Charles Lohr, "Mathematics and the Divine: Ramon Lull," in *Mathematics and the Divine: A Historical Study*, ed. T. Koetzier and L. Bergmans (Amsterdam: Elsevier, 2005), 215–28. However, as the title suggests, this study focuses on the relationship between theology and mathematics.

[177] Carla Compagno, "Il *Liber de geometia noua et compendiosa* di Raimondo Lullo," *Ambitos* 31 (2014): 36.

[178] See José Maria Millás Vallicrosa, "Introduction," in *El libro de la "Nova geometria,"* by Ramon Llull, ed. José Maria Millás Vallicrosa (Barcelona: R. Torra, 1953), 13–52.

the work of a sixteenth century Lullist, Bernard de Lavinheta.[179] The work in question was Lavinheta's *Explanatio compendiosaque applicatio artis Raymundi Lulli* (*Concise Explanation and Application of the Art of Ramon Llull*, 1523).[180] Bruno must have known this work, as proved by the fact that he referred to it in his writings.[181] In the *Explanatio*, Lavinheta stated that "the point was the minimum part of the line, and it was indivisible like the atom. (…) The line is length composed of points *pro materia*, and of a certain flow *pro forma*"[182] Hence, like Bruno and his master Llull, Lavinheta thought that mathematical objects were composed of indivisible points. Furthermore, both Bruno and Lavinheta agreed that the line was generated by the flow of a point.[183] The mediation of Lavinheta makes it virtually certain that Bruno had at least a basic knowledge of Llull's mathematics. However, it was also possible that Bruno had a first-hand knowledge of Llull's mathematical works. For this reason, the rest of this chapter is devoted to examining whether there is evidence that Bruno could have read the *Geometria nova*.

---

[179] I would like to thank Marco Matteoli for this suggestion. For an overview of Lavinheta's life and work see Michela Pereira, "Bernardo Lavinheta e la diffusione del Lullismo a Parigi nei primi anni del '500," *Interpres. Rivista di studi quattrocentesci* 5 (1984): 242–65.

[180] Bernard de Lavinheta, *Explanatio compendiosaque applicatio artis Raymundi Lulli* (Lyon, 1523).

[181] A list of all the references to Lavinheta's *Explanatio* in Bruno's works can be retrieved using the search function of the online database http://bibliotecaideale.filosofia.sns.it/index.php.

[182] *Explanatio*, 112f.

[183] See Giordano Bruno, "De triplici minimo et mensura," in *Opera latine conscripta*, ed. F. Tocco and H. Vitelli, vol. I, pt. 3 (Florence: Le Monnier, 1889), 148: "Ergo linea nihil est nisi punctus motus, superficies nisi linea mota, corpus nisi superficies mota, et consequenter punctus mobilis est substantia omnium, et punctus manens est totum."

1.4 The *Geometria nova* as a Brunian source

1.4.1 The manuscript tradition[184]

There are eight extant manuscripts containing Llull's *Geometria nova*:

> ***Ma*[1]:** Palma de Majorca, Biblioteca Pública, ms. 1036, ff. 1r-56v.
> ***Mu*:** Munich, Bayerische Staatsbibliothek, Clm. 10544, ff. 214r-263v.
> ***V*:** Vatican City, Biblioteca Apostolica Vaticana, Ottob. Lat. 1278, ff. 109r-129r.
> ***S*:** Seville, Biblioteca Capitular y Colombina, 7-6-41, ff. 312ra-342va.
> ***Ma*[2]:** Palma de Majorca, Biblioteca Pública, ms. 1068, ff. 1r-51v.
> ***Mi*:** Milan, Biblioteca Ambrosiana, N 260 Sup., ff. 1r-54v.
> ***Ma*[3]:** Palma de Majorca, Societat Arqueològica Lul·liana, Aguiló 84, ff. 49r-112v.
> ***Mad*:** Madrid, Biblioteca Nacional de España, ms. 17714, ff. II, 1-58v.

Of these, Bruno could not have access to *Ma*[3] and *Mad* since these manuscripts were from the eighteenth century. *Ma*[1], *S* and *Ma*[2] must also be excluded because they appeared to have a Spanish origin and provenance, while Bruno never visited Spain. It remains to determine the origin of *Mu*, *V* and *Mi*, since their provenance is compatible with Bruno's wanderings over Europe.

In his description of *Mu*, Aloisius Madre states that the manuscript was composed in 1449-50 and that, before passing to the Bayerische Staatsbibliothek of Munich, it was possessed by the Palatine Library of Mannheim.[185] This latter information is relevant because Mannheim is close to Frankfurt, where Bruno published *De minimo* (1591) and spent his last year of freedom before returning to Italy. Thus, *Mu* would be a suitable candidate as a Brunian source, if it were not for the fact all the Lullian manuscripts of the library of Mannheim were

[184] The research presented in both this and the next section would not have been possible without the support of the Llull Database. See Anthony Bonner (dir.), *Ramon Llull Database*, Centre de Documentació Ramon Llull (University of Barcelona), http://orbita.bib.ub.edu/llull/).

[185] See Ramon Llull, *In monte pessulano anno mcccv composita*, ed. Aloisius Madre, vol. 9, Raimundi Lulli opera latina (Turhnolt: Brepols, 1981), xvii–xviii.

originally from Barcelona. The German scholar Ivo Salzinger (1669 – 1728) bought these manuscripts at the beginning of the eighteenth century with the purpose of using them to prepare his own edition of Llull's works. The manuscripts were first stored in Düsseldorf, then in Mannheim and finally in Munich.[186]

V also belonged to a Spaniard, Joan Marti Figuerola, as indicated on the title page of the manuscript.[187] All we know about Figuerola is that he was a priest in Valencia, had an interest in Lullism, and was still alive by 1530.[188] In the seventeenth century, *V* passed to the nobleman Gian Angelo d'Altaemps, and, along with the rest of d'Altaemps' personal library, entered the Vatican Library in 1748.[189] There is no reason to believe that Bruno could have read *V*.

Mi contains a copy of both the *Geometria nova* (ff. 1-54v) and the *Liber principiorum theologiae* (ff. 61-130). According to the annotations on ff. 55 and 131, both these works were copied by Joan Pla for Gaspar Sellés in 1566.[190] Joan Pla was also the copyist of another manuscript owned by Gaspar Selles which is now at the Biblioteca Ambrosiana of Milan (N 185 Sup). Both these manuscripts were listed in an inventory (Ambrosiana, P 217 Sup, ff. 26-26v.), which appeared to refer to a well-established Lullian collection since some of the manuscripts were indicated with a shelf-mark.[191] It is difficult to reconstruct the history of this Lullian collection, or to determine whether the whole of it (and not just the two above manuscripts) belonged to Selles. Surely, it was not until June 1603 that the first manuscripts were acquired by the Ambrosiana, while the library itself was inaugurated only in 1609. Bruno died on 17 February 1600. This, along with the Spanish origin of *Mi*, excludes that this manuscript could have been a Brunian source. As we have seen, this conclusion applies to all of the eight extent manuscripts containing the *Geometria nova* Let us now turn to examining the catalogues and inventories that listed Llull's work.

---

186 See Badia, Santanach, and Soler, *Llull Vernacular Writer*, 208.

187 See http://orbita.bib.ub.edu/ramon/ms.asp?262.

188 See http://orbita.bib.ub.edu/ramon/gent.asp?id=540.

189 See http://orbita.bib.ub.edu/ramon/gent.asp?id=1300.

190 See http://orbita.bib.ub.edu/ramon/ms.asp?440.

191 See http://orbita.bib.ub.edu/ramon/cat1.asp?AMB2.

1.4.2. The catologue of the **Electorium** *of Thomas Le Myésier*

The first catalogue that listed the *Geometria nova* was that included in Thomas Le Myésier's *Electorium* of 1325.[192] Both the catalogue and the *Electorium* are important to the history of Lullism. The catalogue is one of the oldest surviving catalogues of Lullian works, as it was drawn up while Llull was still alive (1311-4?). Furthermore, it provided the basis for later catalogues, such as those by Hauteville and Vernon to which we shall return later. The *Electorium* was a wide collection of Lullian texts compiled by Llull's disciple Thomas Le Myésier (d. 1336). As shown by J. N. Hillgarth in his classic book *Ramon Lull and Lullism in Fourtenth-Century France* (1971), Le Myésier had a key role in the diffusion of the Art, as his works influenced generations of Llull scholars across Europe.[193]

Besides providing a list of Lullian works, the catalogue of the *Electorium* served as the personal inventory of Le Myésier, who marked with a dot the works of the catalogue of which he owned a copy. The *Geometria nova* was among the works owned by Le Myésier. Unfortunately, Le Myésier's Lullian collection has been dispersed, save for six manuscripts. Of these, five manuscripts were acquired by Henry of Lewis (c. 1350) and, after his death, entered the library of the Sorbonne. They are now kept at the Bibliothèque nationale de France.[194] The sixth manuscript, instead, is kept at Arras.[195] None of these manuscripts seems to be helpful to reconstruct Le Myésier's lost manuscript of the *Geometria nova*. Nevertheless, other useful information can be gained from the catalogue of the *Electorium*.

Llull scholars agree that Le Myésier drew his catalogue from the Lullian collection of the Charterhouse of Vauvert in Paris. Therefore, we can assume that the *Geometria nova* was also among the Lullian manuscripts of the Charterhouse of Vauvert. The history of the Charterhouse—which occupied a portion of what is now called Jardin de Luxembourg—was deeply intertwined with that of Llull and

---

[192] Paris, Bibliothèque Nationale, ms. lat. 15450, 89v-90. The catalogue of the Electorium is accessible online at http://orbita.bib.ub.edu/ramon/cat1.asp?EL. See also de Alós y de Dou, *Los catalogos lulianos*, 14–17; Hillgarth, *Ramon Lull*, 335–47.

[193] Hillgarth, *Ramon Lull*. For Le Myèsier's legacy, see chap. 7.

[194] Paris, Bibliothèque Nationale, ms. lat. 16115, 16116, 16117, 16118, 16615.

[195] Arras, Bibliothèque Municipale, ms. 78 (Quicherat 100)

Lullism.[196] Since Llull did not belong to any institution (e.g. the church or the university), he had to develop a system of his own for the production and dissemination of his works.[197] As part of this system, the Charterhouse provided one of the three collections of Lullian works. The other two collections were located in Genoa and Palma de Majorca. This was clearly stated in the *Vita coetanea* (1311), a biography of Llull written by an anonymous monk of the Charterhouse:

> [Llull's] books are dispersed through the world, but he made three special collections; that is, in the monastery of Charthusians at Paris, and in the house of a certain noble of the city of Genoa [i.e. Perceval Spinola], and in the house of a certain noble of the city of Majorca [i.e. Pere de Sentmenat, Llull's son-in-law].[198]

The first contacts of Llull with the Charterhouse dated back to Llull's second stay in Paris (August 1298 – July 1299).[199] During the same visit, in June 1299, Llull also completed the *Liber de quadratura* and the *Geometria nova*. Besides being composed at the same time and place, these two mathematical writings had two elements in common. First, the *Liber de quadratura* was probably referred to in the *Geometria nova* (although with a different title).[200] Second, the *Liber de quadratura* and the *Geometria nova* followed each other in the catalogue of the *Electorium*.[201] This leads Hillgarth to conclude that the two writings could have been included in the same manuscript which was part of the Lullian collection of

---

[196] For the history of the Chartehouse and its library, see Franklin, *Les anciennes bibliothèques*, 1:323–28; Léopold Delisle, *Le cabinet des manuscripts de la Bibliothèque Nationale*, vol. 2 (Paris: Imprimerie impériale, 1874), 253; Paul Biver and Marie-Louise Biver, *Abbayes, Monastères et Couvents de Paris* (Paris: Editions d'histoire et d'art, 1970), 103–15.

[197] On this point, see Badia, Santanach, and Soler, *Llull Vernacular Writer*, chap. 3.

[198] B. de Gaiffer, ed., *Vita Beati Raymundi Lulli*, Analecta Bollandiana, xlviii, 1930, 175. Translation by Hillgarth (*Ramon Lull*, 142).

[199] Hillgarth, *Ramon Lull*, 142.

[200] See Compagno, "La *Geometria noua* di Lullo," 39.

[201] No. 140-141 of the online version of the catalogue.

Le Myésier.[202] In addition, since the catalogue of the *Electorium* was based on that of the Charterhouse, we can assume that both the *Geometria nova* and the *Liber de quadratura* were among the manuscripts possessed by the Charterhouse. This begs the question: was the manuscript of the *Geometria nova* still at the Charterhouse as late as the sixteenth century? If so, Bruno could have borrowed it during one his two stays in Paris (1581-3, 1585-6), which was also the time when he wrote his first mathematical writings on Fabrizio Mordente's compass (see Chapter 3).

1.4.3. The Lullian collection of the Charterhouse of Vauvert

There are two pieces of evidence that Llull's works were still found at the Charterhouse of Vauvert as late as 1516. First, we have the testimony of several professors at the Sorbonne who claimed that, in that year, "many books of Llull were to be found in the libraries of the Sorbonne and of the Paris Charterhouse."[203] This testimony was collected by Juan de Vera, a Spanish physician who was sent to Paris by Alfonso of Aragon, the archbishop of Saragossa, to inquire about the orthodoxy of the Lullian doctrine. At that time, this topic was hotly debated in Spain, while there was a revival of Lullism in France and especially in Paris.[204] The strongest advocate of Llull in Paris was Jacques Lèfevre d'Étaples, who made an effort to promote the teaching of the Lullian doctrine and published several of Llull's works.

Lèfevre d'Étaples was also asked by Juan de Vera to defend Llull's orthodoxy. As a response, Lèfevre dedicated his 1516 edition of Llull's *Liber proverbiorum* and *Arbor philosophiae amoris* to Alfonso of Aragon. Lèfevre's dedication provided further proof that that the Charterhouse still had a collection of Lullian manuscripts in 1516:

---

[202] Hillgarth, *Ramon Lull*, 347.

[203] For the source of this testimony, Jaume Custurer, *Disertaciones historicas del culto inmemorial del B. Raymundo Lulio* (Mallorca, 1700), 454. See also Rice, "Lèfevre and the Mystics," 93.

[204] See Joseph M. Victor, "The Revival of Lullism at Paris, 1499-1516," *Renaissance Quarterly* 28, no. 4 (December 1975): 504–34.

> Moreover, our libraries, especially those of the Sorbonne, that noblest home of famous theologians and public theological debate, and of the abbey of Saint-Victor have many of his works. The Carthusians, located just outside Paris, have an unusually fine collection, and these holy men read Llull's works constantly, gathering from them fruits of piety, lend them generously, and allow them to be printed.[205]

Thus, there can be no doubt that the Charterhouse was still active as a center for the diffusion of Lullism in the first decades of the sixteenth century. Furthermore, Lèfevre informs us that the Lullian collection of the Charthouse was readily accessible, as manuscripts could easily be borrowed and used for publication. This was a characteristic of the Charthouse, one that distinguished it from the Sorbonne and its strict lending policy. Indeed, to reduce the risk of thefts, books were chained in the Greater Library of the Sorbonne.[206] For that reason, it was likely that Lèfevre himself borrowed the manuscripts for his edition of Llull's works from the Charterhouse rather than from the Sorbonne.[207]

That being said, no catalogue of the library of the Charterhouse has come down to us from the sixteenth century. Hence, we know neither the amount nor the titles of the Lullian manuscripts which were kept at the Charterhouse at that time. Without this information, we cannot say whether the copy of the *Geometria nova* listed in the catalogue of the *Electorium* was still at the Charterhouse as late as the sixteenth century. By the same token, we cannot draw any conclusions about the possibility that Bruno could have read the *Geometria nova* while in Paris. Surely, the evidence does not exclude this possibility, for it is certain that the activities of the Charterhouse extended well into the beginning of the sixteenth century. As we shall see, the Lullian collection of the Charterhouse was completely dispersed by the end of the eighteenth century. However, it is difficult to believe that this occurred in just a few decades, and thus that the Lullian

---

205 Ramon Llull, *Proverbia Raemundi Philosophia amoris ejusdem doci Badii qui impressit tetrastichon...*, ed. Jacques Lefèvre d'Étaples (Paris: 1516), a ii. Translation by Rice (*Lèfevre and the Mystics*, 94)

206 See Hillgarth, *Ramon Lull*, 272.

207 See Dennis D. Martin, *Fifteenth Century Carthusian Reform: The World of Nicholas Kempf* (Leiden: Brill, 1992), 233.

collection of the Charterhouse had already been dispersed when Bruno reached Paris in the 1580s.

The outbreak of the French Revolution coincided with the end of the Charterhouse, which was closed in 1790 and demolished from 1796 to 1800. At the same time, the need to reallocate the resources of the Charterhouse, including its collection of books and manuscripts, produced a wealth of documents, some of which were directly concerned with the library. In *Les anciennes bibliothèques de Paris* (1867), Alfred Franklin gives a detailed account of the last days of the Charterhouse.[208] It is well-known that, as a result of the so-called "dechristianization" of France during the French revolution, religious orders were suppressed and their goods confiscated. This also affected the Charterhouse of Vauvert and its library. According to Franklin, the prior of the Charterhouse, Félix-Prosper de Nonant, tried to save the library from the confiscation by claiming that it consisted for the most part of books bought by himself. Thus, it was the prior and not the Charterhouse that had to be considered the owner of the library. Apparently, the officer in charge of the requisition took the prior at his word, and let the Carthusians keep their library. But this did not last long. Eventually, the library was seized and its books, like those of all the other religious libraries in Paris, ended up enriching the collections of what would soon be called Bibliothèque nationale.

A document dated September 30, 1791, states that the library of the Charterhouse owned 10,976 printed books, but no manuscript. [209] These figures are confirmed by another inventory that I have found among the documents belonged to the prior of the Charterhouse.[210] Hence, we can conclude that the manuscript collection of the Charterhouse was completely dispersed by the end of the eighteenth century. This is rather surprising and demands further investigation given the long-standing importance of the Charterhouse both as a religious institution and as a library. Hundreds of documents concerning the

---

[208] See Franklin, *Les anciennes bibliothèques*, 1:326–28.

[209] *Recensement déta document aillé par formats des livres des 88 Bibliothéques des Maisons d'hommes ecclesiastiques et relligieuses du département de Paris fait exactement jusqu'au 30 septembre 1791,* Archives Nationales de France, M//797

[210] *Etat des Bibliothèques particuliéres des Religieux*, Archives Nationales de France, T//583/1.

Charterhouse can be consulted at the Archives Nationales de France.[211] Unfortunately, among those documents, I could not find any catalogue or inventory of the library of the Charterhouse other than those mentioned. However, the hypothesis that the manuscript collection of the Charterhouse was gradually dispersed over the centuries is corroborated by two other elements.

First, of the only two extant manuscripts of the Lullian collection of the Chartehouse, one manuscript is kept at the Bibliothèque nationale de France (ms. lat. 3348 A). The manuscript contains a Latin translation of the *Llibre de contemplació en Déu* (1273-4?) which was donated by Llull himself to the Charterhouse in 1298. We can be sure that the manuscript was still at the Charterhouse in 1428, when Cusanus copied it.[212] However, it was already missing by the year 1505, when Lèfevre d'Étaples dedicated his edition of the *Liber contemplationis* to Gabriel, a Carthusian at Vauvert, to replace the original manuscript which had been lost.[213] The second extant manuscript, which is possessed by the Staatsbibliothek of Berlin (ms. Phill. 1911), was also donated by Llull to the Charterhouse. However, like the first manuscript, it left the Charterhouse in the sixteenth century when it was acquired by the French poet Philippe Desportes (1546 – 1606).

In addition, a French catalogue from the seventeenth century seems to suggest that the Lullian collection of the Charterhouse was dispersed at that time. The catalogue was compiled by Jean-Marie de Vernon and included in his *L'histoire véritable du bienheureux Raymond Lulle* (1668).[214] Vernon's catalogue was structured in two parts. The first part contained a list of Lullian works categorized according to their subject. The second part contained the catalogues

---

[211] See especially Archives Nationales de France, S//3948-4160/2 and L//937-940.

[212] See Gabriella Pomaro, "La tradizione latina del *Liber contemplationis*: il manoscritto Paris, Bibliothèque Nationale de France, lat. 3348A," in *Gottes Schau und Weltbetrachtung. Interpretationen zum »Liber contemplationis« des Raimundus Lullus*, ed. Fernando Domínguez Reboiras, Viola Tenge-Wolf, and Peter Walter (Turnhout: Brepols, 2011), 29.

[213] See Rice, "Lèfevre and the Mystics," 107.

[214] Jean-Marie de Vernon, *L'histoire véritable du bienheureux Raymond Lulle, martyr du tiers ordre S. François et la réparation de son honneur* (Paris: Renat Guignard, 1668). The catalogue is available online at http://www.ub.edu/llulldb/cat1.asp?VER

of the Lullian collections of the Sorbonne and the abbey of Saint-Victor. It is worth noting that neither of these two catalogues listed the *Geometria nova*, and that both the Sorbonne and Saint-Victor were reported by Léfevre d'Étaples as possessing two of the three Lullian collections in Paris. (The third collection was that of the Charterhouse). The fact that Vernon did not provide the Lullian catalogue of the Charterhouse can be taken as proof that this collection had been lost when Vernon compiled his catalogue.

On the other hand, it is true that Vernon's catalogue was based on another catalogue, that of Nicolas de Hauteville. Hauteville's catalogue was only two years older that Vernon's (1666).[215] More importantly, in the title of his catalogue, Hauteville claimed that he had drawn from several Lullian collections across Europe, including that of the Charterhouse. Unfortunately, Hauteville did not specify the provenance of any text in his catalogue, which includes the *Geometria nova*. Hence, Hauteville's catalogue does not enable us to determine whether the Charterhouse still possessed a Lullian collection as late as 1666. It is also possible that Hauteville's source of information about the Charterhouse was the catalogue of the *Electorium*. In that case, Hauteville could still mention the Lullian collection of the Charterhouse although this latter had already been lost.

To conclude, the manuscript of the *Geometria nova* has a long history. Unfortunately, the lack of documents makes it difficult to reconstruct the entire history of the manuscript. What is certain, however, is that a copy of the manuscript was at the Charterhouse of Vauvert at the beginning of the sixteenth century. This leaves the door open to the possibility that Bruno may have borrowed the copy of the Charterhouse during his two stays in Paris in the 1580s and, in doing so, he may have become acquainted with Llull's atomistic view of mathematical objects. This is important because Bruno's atomistic geometry has often been viewed as an original (albeit unfortunate) intuition. On the contrary, the discovery that Llull, whom we know to be a Bruno's source, also held an atomistic view of mathematical objects may lead to a different interpretation of Bruno's mathematics, one that does not regard it as a neglectable singularity. On the other hand, it should be noted that Bruno's atomistic view of mathematical objects differed in many respects from Llull's, starting with the fact that it was

[215] http://www.ub.edu/llulldb/cat1.asp?HAUT.

grounded in the concept of the minimum and not that of corporeal point. Indeed, Bruno borrowed from Nicholas of Cusa the concept of the minimum, among other things. For this reason, the next chapter turns to Cusanus and his importance for Bruno's mathematics.

2. Unfolding the point: The Pythagorean atomism of Nicholas of Cusa

Introduction

In his entry to the *Stanford Encyclopedia of Philosophy*, Clyde Lee Miller expresses the belief, to which I am confident that the majority of the historians of philosophy would subscribe, that Nicholas of Cusa (1401 – 1464) was "the most important German thinker of the fifteenth century."[216] The generic term "thinker" appropriately describes the intellectual activity of Cusanus, who at the height of his career was a cardinal, a renowned expert in canon law, and the author of a substantially original philosophy. Born in Kues (now Bernkastel-Kues), a little town on the Moselle river, Cusanus moved to the university of Padua at the age of twenty-two. There not only did he deepen his knowledge of canon law, but he met key figures for the future development of his thought, including the mathematician Paolo del Pozzo Toscanelli. Studying would remain a top priority of Cusanus throughout his entire life, even after he entered the church administration. In Cologne, where in the 1420s he was serving as secretary of the archbishop of Trier, Cusanus managed to find time to study the writings of Ramon Llull, to which he was introduced by Heimerich of Campo, another essential figure for the intellectual growth of Cusanus. And, by his own admission, it was during the return journey from Constantinople as a papal legate that Cusanus was inspired to write *De docta ignorantia* (1440).

In our days, Cusanus is widely acknowledged as a key figure in the history of thought. However, it was not until the beginning of the twentieth century that scholars began to investigate the life and works of Cusanus. For the most part, this renewed interest in Cusanus was the result of the efforts of the Neo-Kantian philosophers based in Marburg and Heidelberg.[217] This includes, *inter alia*,

[216] Clyde Lee Miller, "Cusanus, Nicolaus [Nicolas of Cusa]," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, Summer 2017, accessed April 17, 2018, https://plato.stanford.edu/archives/sum2017/entries/cusanus/.

[217] For an overview of twentieth-century German Cusanus scholarship, see Morimichi Watanabe, "The Origins of Modern Cusanus Research in Germany and the Foundation of the Heidelberg *Opera Omnia*," in *Nicholas of Cusa in Search of God*

Hermann Cohen, Heinrich Rickert, Ernst Hoffman, Raymond Klibansky and Ernst Cassirer. For these scholars, Cusanus had left a legacy of philosophical inquiries into theological, mathematical and political issues that was worth exploring. However, this was not how the immediate followers of Cusanus thought of him, with the only exception of Lefèvre d'Etaples and Bruno who did not conceal their admiration for the German cardinal.

Bruno was indebted to Cusanus for significant aspects of his mathematics, including his conception of the minimum. For this reason, Bruno scholars would argue, and rightly so, that an analysis of Bruno's mathematics requires an inquiry into Cusanus' "mathematical theology," to borrow the words of David Albertson.[218] On the other hand, Cusanus may pose a challenge to our narrative of Pythagorean atomism for not only was he not an atomist, but he considered the Pythagoreans to be opposed to atomism, as shown by the following passage of *De docta ignorantia* (1440):

> Was not the opinion of the Epicureans about atoms and the void-—an opinion which denies God and is at variance with all truth—destroyed by the Pythagoreans and the Peripatetics only through mathematical demonstration? I mean the demonstration that the existence of indivisible and simple atoms—something which Epicurus took as his starting point—is not possible.[219]

Later in his life, Cusanus seemed to change his mind about atomism. In *De mente* (1450), he stated that matter was not infinitely divisible but was composed of

and Wisdom. Essays in Honor of Morimichi Watanabe by the American Cusanus Society, ed. Gerald Christianson and Thomas M. Izbicki (Leiden: Brill, 1991), 17–44.

[218] Albertson, *Mathematical Theologies*.

[219] Nicholas of Cusa, *De docta ignorantia*, ed. Paul Wilpert and Hans Gerhard Senger (Hamburg: Felix Meiner, 2002), bk. I, chap. 11, para. 32: "Nonne Epicurorum de atomis et inani sententia, quae et Deum negat et cunctam veritatem collidit, solum a Pythagoricis et Peripateticis mathematica demonstratione periit? Non posse scilicet ad atomos indivisibiles et simplices deveniri, quod ut principium Epicurus supposuit." All translations are my own but I have consulted those of Jasper Hopkins accessible online at http://jasper-hopkins.info.

indivisible parts, i.e. atoms.[220] Both Ernest Cassirer and Hans Gerhard Senger agree that Cusan atomism was a *sui generis* theory. In his book on *Einstein's Theory of Relativity* (1921), Cassirer writes that in Cusanus' view the atom was "not an absolute minimum of being, but a relative minimum of measure."[221] In this way, Cassirer reveals the difference between Cusanus and ancient atomists such as Democritus and Epicurus. As conceived by Cusanus, the atom had a magnitude that varied in accordance with the object to be measured, it being "a *relative* minimum of measure." For Democritus and Epicurus, on the contrary, the atom was an indivisible unit that could not be smaller. Likewise, Senger claims that Cusanus did not accept the classical definition of atom, but he regarded it as the "contracted unity of being." For this reason, Senger argues that "Cusan atomism is in principle a mathematical theory, but in fact it is a metaphysical theory."[222] In addition Senger distinguishes the metaphysical atomism of Cusanus from the atomism of Bruno, which was at once a metaphysical, mathematical and physical theory.

It was possible that, over the years between *De docta ignorantia* and *De mente*, Cusanus came to accept atomism. But what was the nature of Cusan atomism? Certainly, it was different from the atomistic theory defended by Llull, for Cusanus never claimed that the line was composed of points, nor that the point was part of the line. Rather, in accordance with Euclid, Cusanus described the point as the end of the line, while he agreed with Boethius that "if you add a point to a point, you have nothing more than if you added nothing to nothing."[223]

---

[220] Nicholas of Cusa, *Idiota de mente*, ed. Renata Steiger and Ludwig Baur (Hamburg: Felix Meiner, 1983), bk. IX, para. 119: "Secundum mentis considerationem continuum dividitur in semper divisibile et multitudo crescit in infinitum, sed actu dividendo ad partem actu indivisibilem devenitur, quam atomum appello. Est enim atomus quantitas ob sui parvitatem actu indivisibilis."

[221] Ernst Cassirer, *Substance and Function, and Einstein's Theory of Relativity* (Chicago: Open Court, 1953), 360–61.

[222] Hans Gerhard Senger, "Metaphysischer Atomismus," in *Ludus Sapientiae: Studien Zum Werk Und Zur Wirkungsgeschichte Des Nikolaus von Kues* (Leiden: Brill, 2002), 140.

[223] Nicholas of Cusa, *De mente*, IX (118): "Concordas cum Boethio dicente: Si punctum puncto addas, nihil magis facis, quam si nihil nihilo iungas."

This chapter has two objectives. First, it deals with the issue of Cusan atomism and its relationship to the tradition of Pythagorean atomism. In my opinion, the notions of enfolding and unfolding (*explicatio* and *complicatio*) provided the bridge between Cusan and Pythagorean atomism. For this reason, the first part of the chapter is devoted to Cusanus' account of enfolding and unfolding in *De docta ignorantia* and *De mente*. The second part considers the importance of Cusanus as a Brunian source. It does so by analyzing Cusanus' account of minimum and maximum which informed Bruno's understanding of these two notions.

2.1 Cusanus' account of enfolding and unfolding

2.1.1 Variations on a Boethian theme: Enfolding and unfolding in De docta ignorantia

Arguably Cusanus' most famous book, *De docta ignorantia* (1440) has long been viewed as a monument to Dionysius the Areopagite and the Christian mystical tradition. Indeed, the theory that our knowledge of God could not be more than a learned ignorance was essentially a refurbishment of Dionysius' apophatic or negative theology. But there was more to it than that. To begin with, *De docta ignorantia* contained a thorough evaluation of the role of mathematics in theological thinking. Drawing inspiration from Llull and his theological reading of the problem of the quadrature of the circle, Cusanus started exploring the possibility of symbolically representing God and the coincidence of the opposites that characterized Him using geometric diagrams.[224] (The same diagrams were later reproduced in Bruno's works). The retrieval of Pythagorean doctrines as retained by authors such as Boethius and Thierry of Chartres added another layer of complexity to Cusanus' mathematical theology. Among those doctrines, one of the most important was that of enfolding and unfolding, which is the subject of this section.[225]

---

[224] Pistolesi, "Quadrar el cercle després de Ramon Llull: el cas de Nicolau de Cusa."

[225] On enfolding and unfolding, see T. P. McTighe, "The Meaning of the Couple '*Complicatio-Explicatio*' in the Philosophy of Nicholas of Cusa," *Proceedings of the American Catholic Philosophical Association* 32 (1958): 206–14; C. Ricatti, "Processio" *et* "explicatio"*: La doctrine de la création chez Jean Scot et Nicolas de*

It is generally acknowledged that Thierry of Chartres was the source of Cusanus' account of enfolding and unfolding. Until recently, however, it was thought that Cusanus drew inspiration from Thierry's *Lectiones* and *Glosa* on Boethius's *De trinitate*.[226] Indeed, these were the only known texts where Thierry spoke of enfolding or unfolding. Thanks to Irene Caiazzo's recent discovery of Thierry of Chartres' commentary on Boethius's *De arithmetica*, we now have a more complete picture of Thierry's works.[227] In his *Arithmetica* commentary, which presumably preceded both the *Lectiones* and *Glosa*, Thierry discussed at length the role of enfolding and unfolding in mathematics. Caiazzo has demonstrated that the *Arithmetica* commentary was the true origin of Thierry's theory of enfolding and unfolding.[228] Building on these findings, David Albertson has proposed that Thierry's *Arithmetica* commentary was also the source of Cusanus' account of enfolding and unfolding.[229] In what follows, I trace this account back to its origins, starting from Cusanus and going back to Thierry and Boethius. This is important to understanding the meaning attached by Cusanus to enfolding and unfolding and how this meaning changed in the passage from *De docta ignorantia* to *De mente*.

Cusanus

In *De docta ignorantia* II, 1, Cusanus presented enfolding and unfolding as primarily a model for understanding the relationship between the divine unity

Cues (Neaples: Bibliopolis, 1983); M. De Gandillac, "*Explicatio-Complicatio* chez Nicolas de Cues," in *Concordia discors. Studi su Niccolò Cusano e l'umanesimo europeo offerti a Giovanni Santinello* (Padua: Antenore, 1993), 77–106; Arne Moritz, *Explizite Komplikationen: der radikale Holismus des Nikolaus von Kues* (Münster: Aschendorff, 2006).

[226] Both these treatises are now published in N.M. Häring, ed., *Commentaries on Boethius by Thierry of Chartres and His School* (Toronto: Pontificial Institute of Medieval Studies, 1971).

[227] Thierry of Chartres, *Commentary on Boethius'* Arithmetica.

[228] Irene Caiazzo, "Introduction," in *The Commentary on the* De Arithmetica *of Boethius*, by Thierry of Chartres, ed. Irene Caiazzo (Toronto: Pontificial Institute of Medieval Studies, 2015), 64–67.

[229] Albertson, "*Boethius Noster*."

and the plurality of creation. The divine unity was the enfolding of all things, while all things are the unfolding of the divine unity. The reciprocity that characterized this relationship indicated that the existence of plurality did not affect the unity of God. In fact, Cusanus claimed that "plurality arises from the divine mind, in which there are many things without plurality since they are in enfolding unity."[230] Depending on whether plurality was considered in itself or as conceived in God's mind, plurality could exist in two ways, respectively, unfolded and enfolded. These two ways were fully compatible and, more importantly, they guaranteed that the existence of plurality did not deprive God of His unity.

In an effort to clarify the dynamics of enfolding and unfolding, in *De docta ignorantia* II, 3 Cusanus applied these two concepts to mathematics and physics. From reading this chapter, one has the impression that enfolding and unfolding were primarily theological concepts, and that their application to mathematics was for explanatory purposes only. Nevertheless, the analysis of Cusanus' sources shows just the opposite: Cusanus borrowed from a mathematical text, Thierry of Chartres's commentary on Boethius' *De arithmetica*, the concepts of enfolding and unfolding.[231] It is also true that Cusanus did not conceal the mathematical origin of enfolding and unfolding. In fact, as argued by Albertson, "[Cusanus] seems to assume that the case of numerical folding is not one among others, but the prime instance and indeed the very model of what enfolding and unfolding mean."[232] The passage of *De docta ignorantia* in which Cusanus defined enfolding and unfolding in mathematical terms is the following:

> In respect to quantity, which is the unfolding of unity, unity is called point, since nothing in quantity is found except the point. Just as a point is everywhere in a line, no matter where you divide it, so too is it in a surface and a body. Nor is there more than one point, which is nothing other than the infinite unity, because the infinite unity is the point, which is the limit, perfection and totality of the line and quantity

[230] *De docta ignorantia*, II, 3 (108)."Rerum pluralitas ex divina mente, in qua sunt plura sine pluralitate quia in unitate complicante."

[231] Albertson, "*Boethius noster*," 183.

[232] Albertson, 178.

> that it enfolds. The line is the first unfolding of the point, in which only the point is found.[233].

In analyzing this passage, one should bear in mind that enfolding and unfolding were Cusanus' response to the problem of unity and plurality, that is how the same thing could be both one and many. This problem, for Cusanus, affected theology and mathematics alike because the point was an instance of the infinite unity of God. God was found in all His creatures just as the point was found everywhere in lines, surfaces and bodies. All things were the unfolding of God just as the line was the unfolding of the point. Finally, there was only one God just as there was only one point, which was the limit, perfection and totality (*terminus, perfectio et totalitas*) of quantity. This latter claim seems more difficult to translate into mathematical terms. To understand it better, it may be useful to look at Cusanus' source, Thierry of Chartres.

Thierry of Chartres

There appears to be a connection between the above passage of *De docta ignorantia*, and the following passage taken from the second book of Thierry's *Arithmetica* commentary:

> But unity takes the place of the point, and rightly so. Because just as unity is the enfolding [*complicatio*] of number, and number is the unfolding [*explicatio*] of unity, so also the point is the enfolding of every magnitude, and magnitude is the unfolding of the point. [...]

---

233 *De docta ignorantia,* II, 3 (105): "Ipsa quidem unitas punctus dicitur in respectu quantitatis unitatem explicantis, quando nihil in quantitate reperitur nisi punctus; sicut undique in linea est punctus, ubicumque ipsam diviseris, ita in superficie et corpore. Nec est plus quam unus punctus, qui non aliud quam ipsa unitas infinita, quoniam ipsa est punctus, qui est terminus, perfectio et totalitas lineae et quantitatis, ipsam complicans; cuius prima explicatio linea est, in qua non reperitur nisi punctus."

> Therefore I say that the point is the limit and perfection of a given thing; it is the very totality of that thing. It delimits the entire thing wholly and perfectly, and there is properly speaking only [one] limit of one single thing.[234]

Like Cusanus, Thierry drew an analogy between point and unit, although Thierry referred to the arithmetic unit and not to the divine unity. Furthermore, not differently from Cusanus, Thierry used enfolding and unfolding to explain the relationship between unit and number on the one hand, and point and magnitude on the other hand. Finally, Thierry and Cusanus agreed that the point was the limit, perfection and totality of magnitude. In fact, Thierry made an even stronger claim, stating unequivocally that geometric objects had only one limit. In the *Elements*, Euclid made it clear that "the ends of a line are points" (Def. I, 3). Thierry could have read the *Elements* in the translation by Adelard of Bath, the first to translate the Euclidean text from the Arabic between 1126 and 1130. In Adelard's translation, we read that "the ends (*extremitates*) of a line are *two* points."[235] On the contrary, Thierry claimed that "when we say that a line has two limits (*terminos*), we misuse the term limit, since the limit is only one, the point is only one. [...] Nor is there another reason why we say that in a line there are limits, if not because the line is terminated here and there."[236] How should we interpret Thierry's words? One possibility is that, despite all evidence to the

---

[234] *Commentary on Boethius'* Arithmetica, 163–65 (230-32, 236-38): "Unitas vero locum obtinet puncti, et merito, quia sicut unitas est complicatio numeri, et numerus explicatio unitatis, ita quoque punctum est complicatio omnis magnitudinis, et magnitudo est explicatio puncti. [...] Dico igitur quod punctus est terminus rei ipsius et perfectio. Haec est ipsa rei totalitas, quae rem totam integre terminat et perfecte, nec est nisi unius rei proprie terminus". Translated by David Albertson.

[235] Hubert L. L. Busard, *The First Latin Translation of Euclid's Elements Commonly Ascribed to Adelard of Bath* (Toronto: Pontificial Institute of Medieval Studies, 1983), 31. "Extremitates [linae] quidem [sunt] duo puncta." Emphasis added.

[236] *Commentary on Boethius'* Arithmetica, 164–65 (262-3): "Cum dicimus duos terminus linae esse, abutimur dicendo terminus, cum unus si terminus, quia unum punctum solum est. [...] Nec ob aliud dicimus terminus in linea esse, nisi quia et hic terminatur linea et ibi."

contrary, Thierry meant that the line was terminated by only one point instead of two. Or it is possible that Thierry aimed to draw a distinction between the terms "limit" (*terminus*) and "end" (*extremitas*), meaning that the ends of a line had not to be called limits. If so, what meaning did Thierry attribute to the term "limit"?

From Thierry's perspective, the concepts of limit, perfection and totality could only be understood in relation to one another. In fact, Thierry seemed to use these three concepts as if they were synonymous. For instance, Thierry wrote that "the extension of a line unfolds its totality, which we call limit,"[237] or that "magnitude begins from totality, that is, from its own perfection."[238] At the same time, Thierry dealt with each concept individually. Speaking of perfection, Thierry argued that "perfection is prior to imperfection, since imperfection descends from perfection, and perfection is not caused by anything else. Thus, what begins from perfection must of necessity descend to imperfection. For nothing is beyond the perfection of a thing, but only below."[239] Hence Thierry viewed perfection as an insurmountable upper limit. In addition, perfection seemed to act as a starting point, since Thierry repeatedly mentioned that things began from their own perfection. Geometric objects could also be ordered according to their degree of perfection. Following this order, the point came first, the line second and the surface third. For "the point is the first perfection, while the line is imperfection: Not that the line is not the perfection of surface, but it is called imperfection in relation to the point."[240] The higher the number of dimensions, the lower the degree of perfection. This reflected the Pythagorean tendency, highlighted by

---

[237] *Commentary on Boethius'* Arithmetica, 164 (239-40): "Hanc ergo totalitatem, quam terminum nominamus, explicat ipsius linae extensio."

[238] *Commentary on Boethius'* Arithmetica, 164 (243-44): "Quoniam enim magnitudo a totalitate inchoat, id est ab ipsa rei perfection."

[239] *Commentary on Boethius'* Arithmetica, 164 (247-50):"Perfectio enim prior natura est imperfectione, quia imperfectio ab ipsa descendit, nec aliud causa perfectionis est. Qui ergo a perfectione inchoat, necesse esse in imperfectionem descendat. Ultra perfectionem namque rei nihil est, sed infra."

[240] *Commentary on Boethius'* Arithmetica, 165 (273-75): "Punctum est prima perfectio, linea vero est imperfectio, non quod non sit perfectio superficiei, sed imperfectio dicitur ad comparationem puncti."

Ruth Glasner, to define geometric objects moving from lower to higher dimensions.[241]

Why should the point be considered more perfect than the line and surface? Thierry's answer to this question is that "the point is the enfolding of every magnitude, and magnitude is the unfolding of the point."[242] It is worth noting that, at least on one occasion, Thierry replaced *explicatio* with *evolutio*, stating that "the line is the *evolutio* of the point."[243] In *De mente*, Cusanus also seemed to use *explicatio* and *evolutio* interchangeably. More precisely, Cusanus defined *evolutio* in terms of *explicatio*, which raises the question of how one concept was supposed to clarify the other. There was also the problem of translating *evolutio* in mathematical terms, since its proper meaning was the unrolling and reading of a book. We shall return to *evolutio* below. As for Thierry, we can assume that the point was the first perfection because all magnitudes were derived from the point by means of its unfolding. By the same token, the point set a limit to the perfection of magnitudes because nothing could be more perfect than that from which it was derived. Finally, the unfolding of one point was sufficient to generate all magnitudes. This would explain why Thierry claimed that there was only one point in all magnitudes, and the point was the "totality" of magnitude. It is true that Thierry did not explain in mathematical terms how the point "unfolded" into the line, surface, and so on. However, as we shall see in the next section, this problem also affected the passage of Boethius' *Institutio arithmetica* on which Thierry commented.

Boethius

As noticed by Caiazzo [244] Thierry developed his theory of enfolding and unfolding as an attempt to explain this passage from Boethius' *Institutio arithmetica:*

---

[241] Ruth Glasner, "Proclus' Commentary on Euclid's Definitions 1, 3 and 1, 6," *Hermes* 120, no. H. 3 (1992): 320–333.

[242] *Commentary on Boethius'* Arithmetica, 163 (231-32): "Punctum est complicatio omnis magnitudinis, et magnitudo est explicatio puncti."

[243] *Commentary on Boethius'* Arithmetica, 164 (240-41): "Linea namque est puncti evolutio."

[244] Irene Caiazzo, "Introduction," 64

> Therefore from this principle (i.e., from unity) arises the first length, which *unfolds* [*explicat*] from the principle of binary number into all numbers themselves, since line is the first extension.[245]

Rather than explaining the meaning of unfolding, Boethius used this concept to draw a parallel between number and magnitude. Building on this insight, Thierry went on to elaborate a model of unfolding that could account for both number and magnitude. More precisely, through unfolding Thierry explained the derivation of number and magnitude from their respective principles, the unit and the point. In light of this, Albertson argues that Thierry did not only give a more detailed explanation of what unfolding was, but he managed to "unify the bifurcated conceptual foundation of the ancient quadrivium."[246] Because of this bifurcation, arithmetic and music stood on one side of the quadrivium, while geometry and astronomy stood on the other side. Arithmetic and geometry were separate domains in Boethius, who in fact claimed that the former had priority over the latter because of the fact that number was the divine model for all created things, including magnitude (see § 1.2). Moreover, Boethius seemed to unable to explain the passage from unity to multiplicity, as in his account there was a gap between the arithmetic unit and the geometric point on the one hand, and number and magnitude on the other hand.

As argued by Albertson, unfolding offered a solution to the problem of bridging the gap between arithmetic and geometry. Another solution was to employ the fluxion theory, according to which magnitude was generated by the flux of a point. The Greek mathematician Hero of Alexandria (c. 10 AD – c. 70 AD) offered a glaring example of how the fluxion theory could be used in this context. In his *Definitiones*, Hero adopted the fluxion theory to prove that number and magnitude were derived from similar principles. For Hero, the point was the principle of magnitude just as the unit was the principle of number. Nevertheless, he argued that "the unit is part of number, whereas the point is not

[245] Boethius, *Institutio arithmetica*, II, 4 (6). "Ex hoc igitur principio, id est ex unitate, prima omnium longitudo succrescit quae a binarii numeri principio in cunctos sese numeros explicat, quoniam primum interuallum linea est."

[246] Albertson, *Mathematical Theologies*, 126–39; Albertson, "*Boethius Noster*," 182.

part of the line, but it is its mental presupposition."[247] How was the point the principle of the line if not by being one of its parts? Hero's answer to this question was that the line was generated by the "flow a point." This allowed him to say that the point was the principle of the line (and magnitude in general) without committing to the 'atomistic' view that the line was a row of points placed side by side.

Non unlike Hero, Thierry used the concept of unfolding to explain how the point was the beginning of the line, although the juxtaposition of two or more points did not produce a line. The only difference was that the fluxion theory implied a dynamic understanding of geometric objects, while unfolding was not explicitly associated with motion. On the other hand, one may argue that a dynamic element was implicit in the concept of *evolutio,* which both Thierry and Cusanus used in place of the concept of unfolding on a few occasions. This is worth noting especially in the case of Cusanus, who may not have relied on the fluxion theory to account for the derivation of magnitude from the point, and yet may have had a dynamic understanding of geometric objects, as suggested by his use of the concept of *evolutio*.

Thus we have seen that the concept of unfolding/*evolutio* bore a close resemblance to the fluxion theory. What was the relationship between the fluxion theory and the concept of unfolding/*evolutio* on the one hand, and geometric atomism on the other hand? Were they compatible? Or was the view of geometric objects underlying the fluxion theory and unfolding at odds with geometric atomism? To begin with, it should be noted that both the fluxion theory and unfolding offered an alternative to geometric atomism, as they could be employed against the view that geometric objects were composed of points. In fact, this was the way in which the concept of flux continued to be used in early modern times, as happened in the case of the controversy that surrounded Cavalieri's theory of

---

[247] Hero of Alexandria, *Opera quae supersunt omnia*, ed. J.L. Heiberg, vol. IV (Stuttgart: B.G. Teubner, 1903), 14 (13-26). Following Giardina's Italian translation, I translate the expression τὸ προεπινοούμενον τὲ αὐτῆς into "mental presupposition." For a more detailed analysis of the passage in question, see Giovanna R. Giardina, *Erone di Alessandria: le radici filosofico-matematiche della tecnologia applicata*. Definitiones. *Testo, traduzione e commento* (Catania: CUECM, 2003), 249–65.

indivisibles.[248] At the same time, it cannot be denied that atomistic theories of geometric objects (such as those proposed by Llull and Bruno) provided a framework for unifying arithmetic and geometry, for they were built on the assumption that both number and magnitude were composed of discrete parts. In other words, like the fluxion theory and the concept of unfolding, geometric atomism functioned as a bridge between arithmetic and geometry

Returning to Cusanus, the bridging function of unfolding was fundamental for the development of this concept in his later works. Indeed, both in *De docta ignorantia* and *De mente,* Cusanus described the derivation of magnitude from the point in terms of unfolding. What changed was the subject of unfolding, which in *De docta ignorantia* was conceived as an abstract mathematical operation, while in *De mente* was viewed as a creative act of the human mind. This shift reflected a more general change in the philosophy of the Cardinal, a change that, as we shall see, triggered a debate among Cusanus scholars about the modernity of his thought.

2.1.2 Embracing atomism: Unfolding in **De mente**

In 1450, Cusanus composed three dialogues centered on the figure of the *idiota: De mente, De sapientia* and *De staticis experimentis*. By choosing the *idiota* (illiterate in Latin) as the main character of these dialogues, Cusanus launched his attack on the medieval myth that the text was the authority, the only source of knowledge. On the contrary, in the three 1450 dialogues, knowledge came from a man, the *idiota*, who admittedly lacked formal education and represented the ideal of naïve knowledge.[249] It was the task of the Philosopher, the other protagonist of the dialogues along with the Orator, to seek a correspondence between the *idiota*'s beliefs and the texts of ancient and medieval authors. Following this pattern, the dialogues went on to address several questions and present a new formulation of Cusanus' philosophy. Among the issues that

[248] See Dominique Descotes, "Two Jesuits Against the Indivisibles," in *Seventeenth-Century Indivisi bles Revisited*, ed. V. Jullien (Basel: Birkhäuser, 2015), 249–74.

[249] On this point, see Richard J. Oosterhoff, "*Idiotae*, Mathematics, and Artisans: The Untutored Mind and the Discovery of Nature in the Fabrist Circle," *Intellectual History Review* 24, no. 3 (July 3, 2014): 301–19.

Cusanus touched on in *De mente,* there were also the concepts of enfolding and unfolding.

As suggested by the title, *De mente* was devoted to developing a theory of the human mind in accordance with the view that in Latin the term mind (*mens*) was etymologically derived from measurement (*mensura*).[250] The whole of *De mente* revolved around this idea of *mens/mensura*, including its conception of unfolding. For this reason, I shall first give a brief presentation of the idea of *mens/mensura* before moving on to the concept of unfolding and the issue of Cusan atomism.

Mens/mensura

Although at the beginning of *De mente* Cusanus equated knowing to measuring, in the first part of this work knowledge was described as a process of mental "assimilation" (*assimilatio*) to the object to be known. It was not until chapter IX that Cusanus answered the question of how the human mind "measured" things. At that point, he would dismiss the concept of assimilation, going on to describe knowledge only in terms of measurement. This conceptual shift did not occur arbitrarily. In chapter VI, Cusanus took pains to explain how number was derived from the human mind, which in turns allowed him to base knowledge on number and measurement. Knowledge by assimilation was therefore only a working definition, a first step in the development of the idea of *mens/mensura*. This was also confirmed by the fact that Cusanus had already dealt with the concept of mental assimilation in two previous works, *De filiatione dei* (1445) and *De genesi* (1449).[251] At the same time, the concept of assimilation was an integral part of the theory of mind presented in *De mente*. As such, it needs to be considered in reconstructing this theory.

---

[250] *De mente*, I (57): "Mentem esse, ex qua omnium rerum terminus et mensura. Mentem quidem a mensurando dici conicio."

[251] In *De genesi,* 4 (165:8-9) Cusanus interchanges *assimilatio* with *similitudo*. Cf. *De genesi,* 3 (164:12) with *De genesi,* 4 (165:4). See also *De filiatione dei*, 6 (87). Both these works are now published in Nicholas of Cusa, *Opuscula I: De deo abscondito, De quaerendo deum, De filiatione dei, De dato patris luminum, Coniectura de ultimis diebus, De genesi*, ed. Paul Wilpert (Hamburg: Felix Meiner, 1959).

In *De mente* IV, Cusanus recalled the definition of the human mind as an image of God (*imago Dei*), warning us not to confuse image with unfolding. His aim was to emphasize the superiority of the human mind over the rest of creation, for the human mind was an image of God, while "the creatures that lack mind are unfoldings rather than images of the divine simplicity."[252] By virtue of its being an image of God, the human mind had the power of assimilating itself to the things created by God. This assimilative power (*vis assimilativa*) was the source of our knowledge because it resulted in the creation of concepts. Our mind received information about the external world from the senses. Based on this information, it assimilated itself to the external objects by creating images of them, that is concepts. Perceptions were needed in order for our mind to exercise its assimilative power, which meant that our mind had to be embodied.[253] However, our mind was not a passive recipient of perceptions because these only provided the material out of which it created the concepts. As Cusanus made it clear in *De mente* VIII, concepts were not mere impressions of external objects on our mind.[254]

Cusanus returned to the issue of assimilation in *De mente* VII. The fact that our concepts were based on perceptions affected the quality of our knowledge. Indeed, no matter how hard the human mind strove to represent external objects as faithfully as possible, our mental representations were bound to be conjectural. The reason was that external objects were composed of matter and, as such, always changed. Therefore, as soon as an external object was mentally represented, the object in question was already changed and its mental representation was outdated. To have more stable representations, we needed to peer into God's mind, for, as Cusanus argued in *De mente* III, "if all things are present in the divine mind as in their precise and proper truth, all things are

[252] *De mente*, IV (76): "Creaturae mente carentes sunt potius divinae simplicitatis explicationes quam imagines"

[253] *De mente*, IV (77): "Vis mentis, quae est vis comprehensiva rerum et notionalis, non potest in suas operations, nisi excitetur a sensibilibus, et non potest excitari nisi mediantibus phantasmatibus sensibilibus. Opus ergo habet corpore organico, tali scilicet, sine quo excitation fieri non potest."

[254] *De mente*, VIII (110). "Igitur et conceptio passio? Non sequitur, ut per te vides."

present in our mind as in an image or a likeness of their proper truth."[255] But, as we have learned from *De docta ignorantia*, the infinity of God was beyond our grasp. Hence, as long as knowledge was acquired only by assimilation to external objects, it would remain conjectural.

In the following of *De mente* VII, Cusanus explored the possibility of acquiring knowledge independently of perceptions. He took the example of the circle which was defined as the set of all points equidistant from the center. Cusanus was aware that no material circle met this definition, because figures such as a perfect circle could not be constructed nor found in nature. Hence, he argued that the concept of perfect circle was not derived from perceptions but from the mind itself. In our mind, objects existed independently of matter and change. This meant that the concepts that our mind derived from itself were more certain than those created by assimilation to external objects. Cusanus called the concepts derived from the mind itself "abstract forms" (*formae abstractae*) and claimed that they were the subject of mathematical knowledge.[256] Therefore, mathematics provided a different kind of knowledge, one that was more certain than knowledge based on perceptions.

Cusanus discussed the epistemological status of mathematics in *De mente* VI. As already said, this chapter marked a turning point in the development of the idea of *mens/mensura*. Here, emphasis was placed on the fact that number was the mode of understanding (*modum intelligendi*) of the human mind, meaning that our knowledge was based on number. There has been much discussion about this claim. Scholars such as Ernst Cassirer and more recently Kurt Flasch have emphasized the modernity of the mathematical epistemology set forth in *De mente*.[257] The problem with this modernist reading of *De mente* is that it reduces the importance of previous works and especially of *De docta ignorantia*, suggesting that Cusanus was still under the influence of medieval authors when

---

[255] *De mente*, III (72). "Si omnia sunt in mente divina ut in sua precisa et propria veritate, omnia sunt in mente nostra ut in imagine seu similitudine propria veritatis."

[256] *De mente*, VII (104). "Et quia mens ut in se et a materia abstracta has facit assimilationes, tunc se assimilat formis abstractis."

[257] Ernst Cassirer, *Individuum und Kosmos in der Philosophie der Renaissance* (Leipzig: Teubner, 1927), 10–11; Kurt Flasch, *Nikolaus von Kues: Geschichte einer Entwicklung* (Frankfurt am Main: Vittorio Klostermann, 1998), 103–15.

writing these works.[258] Leaving this issue aside, I shall focus on how, in Cusanus' view, number was derived from the human mind, and how this affected his view of human knowledge.

Cusanus proceeded to assess the role of number in human knowledge in keeping with the idea that the human mind was an image of God. Like the plurality of things generated by God, the plurality of concepts derived from our mind presupposed the possibility of distinguishing one concept from the other. Thus conceived, concepts depended on number, because "without number things could not be understood to be different from one another and to be discrete."[259] It was in this sense that Cusanus took number to be the "exemplar" (*exemplar*) of our concepts. In fact, the whole of our mental activity relied on number, including our ability to assimilate, conceptualize, discriminate and measure. Number, in short, was the mode of understanding of the human mind because nothing could be understood independently of number.[260]

As is well known, Plato was one of the first to argue that mathematics was the foundation of human knowledge.[261] However, differently from Plato, Cusanus did not think that concepts were innate, but rather he thought that they were the result of a mental activity.[262] This also applied to mathematical concepts, with the only difference that these were not derived from the external world but from the mind itself. *De mente* VI contained an account of the derivation of number from the human mind. Assuming the Boethian definition of number as a collection of unities, Cusanus stated that number was derived from the human mind by multiplication of the arithmetic unit.[263] Likewise, in chapter IX, he argued that the line was generated by the unfolding or *evolutio* of a point. In mathematics, a

---

[258] For this criticism, see Albertson, *Mathematical Theologies*.

[259] *De mente*, VI (95): "Res enim non possunt aliae et aliae et discretae sine numero intelligi."

[260] *De mente*, VI (95): "Unde cum numerus sit modus intelligendi, nihil sine eo intelligi potest."

[261] See Plato, *The Republic*, bk. VII.

[262] *De mente*, IV (77): "Non est igitur credendum animae fuisse notiones concreatas."

[263] *De mente*, VI (94): "Non est aliud colligere quam unum et idem commune circa eadem multiplicare. Unde cum videas sine mentis multitudine binarium vel ternarium nihil esse, satis attendis numerum ex mente esse."

definition that describes an object in terms of its generation is called "genetic." The fact that Cusanus made use of genetic definitions attests that he viewed mathematical objects as constructs—mental constructs, to be more precise.

Unfolding as a creative act of the mind

As already mentioned, there is no agreement among Cusanus scholars on how to interpret *De mente*, whether as the place where Cusanus' thought took a modern turn or as a text in the spirit of medieval Scholasticism. Depending on the position taken by the interpreter on this issue, the same passage of *De mente* has been read in opposite ways. What none of the parties involved in this discussion has denied is that in *De mente* Cusanus described human knowledge as a creative process.[264] This stemmed from the analogy between human knowledge and divine creation made at the outset of *De mente*. However, there seems to be a contradiction between this creative view of knowledge and other accounts of knowledge that Cusanus gave in previous works and in *De mente* itself. Theo van Velthoven and Clyde Lee Miller propose two different solutions to solve this contradiction.

In his classical study on Cusanus' epistemology, Velthoven rephrases the problem of knowledge in terms of vision of God (*Gottesschau*) and human creativity (*menschliche Kreativität*).[265] By linking it to *De docta ignorantia,* he regards the vision of God as the passive act of contemplation whereby the human mind became aware of the impossibility of knowing God.[266] Thus conceived, the vision of God was incompatible with human creativity, which Velthoven associates to *De mente*. In this respect, Velthoven is one of those who views *De mente* as a watershed in the development of Cusanus' thought. On the other hand, Velthoven aims to establish a connection between vision of God and human

[264] This is the case of Jasper Hopkins, who points out to the medieval terminology used by Cusanus to describe the powers and activities of the human mind while emphasizing the active character of human knowledge. See Jasper Hopkins, *Nicholas of Cusa on Wisdom and Knowledge* (Minneapolis: Banning, 1996).

[265] Theo van Velthoven, *Gottesschau und menschliche Kreativität. Studien zur Erkenntnislehredes Nikolaus von Kues* (Leiden: Brill, 1977).

[266] Velthoven, 46–47.

creativity and, in doing so, he considers mathematics as a product of human creativity.[267] In Velthoven's opinion, the creation of mathematical objects also led to the vision of God, since this activity revealed our similarity to God.

Like Velthoven, Miller addresses the question of whether, in Cusanus' epistemology, the human mind had an active or a passive role in the acquisition of knowledge. Miller notes that *De mente* provided evidence in support of both answers insofar as it contained two accounts of knowledge: by assimilation and by measurement. When knowledge was acquired by assimilation, the human mind was the passive recipient of perceptions. On the contrary, when knowledge was acquired by measurement, the human mind was actively engaged in this task. The problem was how to reconcile these two apparently contradictory accounts of knowledge. Miller's answer is that "our minds are not limited to quantitative measures and thus can determine the conceptual measures or units that best fit or are adequate to the different sorts of things we want to know."[268] This was possible because the human mind had the power of assimilating itself to all external objects. Hence, there was no measurement without assimilation and, as claimed by Miller, "we cannot resolve the tension between the two but we must hold onto it."[269]

As insightful as Miller's contribution might be, it seems to me that it is not supported by the text. In *De mente,* Cusanus never spoke of the ability of the human mind to produce units of measurement that varied according to the object to be known. In fact, the idea of *mens/mensura* did not seem to require a real act of measurement. In *De mente* IX, when asked how the human mind measured things, the *idiota* answered that it did so by making (*facere*) the point and number; "hence multitude and magnitude are from the mind, and this measures all things."[270] This was in line with what Cusanus claimed in chapter VI about the number being the mode of understanding of human mind. Both chapters VI and

---

[267] Velthoven, 131–96.

[268] Miller, "Cusanus, Nicolaus [Nicolas of Cusa]," para. 2.2.3.

[269] Miller, para. 2.2.3.

[270] *De mente,* IX (116). "Mens facit punctum terminum esse lineae et lineam terminum superficiei et superficiem corporis, facit numerum, unde multitudo et magnitudo a mente sunt, et hinc omnia mensurat."

IX conveyed the idea that mathematics was the foundation of human knowledge. This was also, in my opinion, the meaning of the idea of *mens/mensura*.

As already mentioned, the fact that mathematical objects were the starting point of human knowledge did not mean that these objects were innate concepts. On the contrary, mathematical objects were mental constructs, as number was derived by multiplication of the arithmetic unit, and magnitude was the result of the unfolding or *evolutio* of the point. We have come across the concept of *evolutio* when discussing Thierry's *Arithmetica* commentary, in which *evolutio* and *explicatio* (unfolding) were used interchangeably to explain the derivation of quantity from the point. Likewise, in *De mente* IX, Cusanus first stated that the line was the *evolutio* of the point, and then he equated *evolutio* to *explicatio*.[271] In this way, both Thierry and Cusanus turned *evolutio*, which in Latin meant the unrolling and reading of a book, into a mathematical concept. The fact that the influence of Thierry's *Arithmetica* commentary is discernable in *De mente* confirms the importance of this text as a Cusan source. At the same time, it undermines the assumption, accepted especially by the advocates of Cusanus' modernity, that Cusanus' mathematical ideas were drawn from Proclus's Euclid commentary. Admittedly, the idea that mathematical objects were mental constructs was Proclean. Furthermore, thanks to Klibansky and other scholars, we know that Cusanus possessed some of the first Latin translations of Proclus' works.[272] However, Cusanus could not have accessed these texts before the late 1450s, which means that the influence of Proclus cannot be used to account for the mathematical ideas set forth in *De docta ignorantia* and in *De mente*.[273]

---

[271] *De mente*, IX (119). "Evolutionem id est explicationem."

[272] For Proclus as a source of Cusanus, see Raymond Klibansky, "Ein Proklos-Fund und seine Bedeutung," in *Sitzungsberichte der Heidelberger Akademie der Wissenschaften* (Heidelberg: Carl Winter Universitätsverlag, 1929); Werner Beierwaltes, "'Centrum tocius vite'. Zur Bedeutung von Proklos' 'Theologia Platonis' im Denken des Cusanus.," in *Proclus et la théologie platonicienne*, ed. A. Ph. Segonds and Carlos G. Steel (Leuven: Leuven University Press, 2000), 629–651; Jean-Marie Nicolle, "Introduction," in *Les écrits mathématiques*, by Nicholas of Cusa, ed. Jean-Marie Nicolle (Paris: Honoré Champion, 2007).

[273] For a more detailed discussion of this matter, see Albertson, *Mathematical Theologies*, chaps. 8–9.

In addition to being the place in which Cusanus rephrased unfolding in accordance with the idea of *mens/mensura*, *De mente* IX was also where he endorsed atomism. In fact, these two events did not occur independently of each other, as both the concepts of unfolding and atom were part of the answer to the same question:

> *Philosopher*: What do you mean [by saying that] a line is the "development" [*evolutio*] of a point?
> *Idiota*: [I mean that it is] the development, i.e., the unfolding, [of the point]—which [is to say] none other than the following: viz., that the point is present in the many atoms in such a way that it is present in each of them qua combined and connected. For there is one and the same point in all the atoms, just as there is one and the same whiteness in all things white.
> *Philosopher*: What do you mean by "atom"?
> *Idiota*: With respect to the mind's consideration a continuum is divided into what is further and further divisible, and the multitude increases ad infinitum. However, in actually dividing, we come to a part that is actually indivisible. This part I call an atom, for an atom is a quantity that, because of its smallness, is actually indivisible.[274]

In one stroke Cusanus connected the concepts of *evolutio*, *explicatio* and atom, while drawing a distinction between physical and mathematical indivisibles. According to this distinction, which was classical in ancient times, a physical indivisible (i.e. an atom) could contain several mathematical indivisibles (i.e. points).[275] Hence the division of the continuum could go on at the mathematical

---

[274] *De mente*, IX (119): Philosophus: Quomodo intelligis lineam puncti evolutionem?
Idiota: Evolutionem id est explicationem, quod non est aliud quam punctum in atomis pluribus ita quod in singulis coniunctis et continuatis esse. Est enim unus et idem punctus in omnibus atomis sicut una et eadem albedo in omnibus albis.
Philosophus: Quomodo intelligis atomum?
Idiota: Secundum mentis considerationem continuum dividitur in semper divisibile et multitudo crescit in infinitum, sed actu dividendo ad partem actu indivisibilem devenitur, quam atomum appello.

[275] Sorabji, *Time, Creation and the Continuum*, pt. 4.

level after having reached its end at the physical level. In fact, Cusanus thought that the division of the mathematical continuum could go on to infinity, which confirms that he was far removed from being a supporter of atomism in mathematics. Furthermore, the distinction between physical and mathematical indivisibles could help explain Cusanus' claim that there was one and the same point in all atoms. Arguably, Cusanus referred to the fact that atoms could vary in shape and size, while points were shapeless and thus there was only one species of point.

Once again, it is important to note that Cusanus did not consider the point to be an indivisible part of the line. Rather, the point had, for Cusanus, a double function. First, the point was the end of the line, a definition that was taken from Euclid's *Elements* (Def. I, 3). Second, the point was the generative principle of the line, this latter being the unfolding or *evolutio* of a point. This second definition was a borrowing from the Neopythagorean tradition and in particular from Thierry of Chartres. None of these definitions of the point implied an atomistic view of mathematical objects. In fact, we have seen how both those definitions were used to oppose the idea that mathematical objects were composed of indivisible parts. On the other hand, it should be noted that, in *De mente*, the infinite divisibility of the continuum was denied at the level of physical matter and three-dimensional bodies, but not at the level of mathematics. What is more, Cusanus saw a connection between the generative function of the point (i.e., its being the source of the unfolding of the line) and physical atomism. This connection, in my opinion, is the best possible way to describe Cusan atomism, a theory that defies definition and does not fall into a specific category.

2.2 Cusanus' account of minimum and maximum

Although Bruno was credited with the revival of atomism in early modern times, it should not be forgotten that his theory was centered on the concept of minimum (and not on that of atom). In fact, Bruno defined the atom as a species of the minimum, which in turn was the "substance of all things."[276] However,

[276] Giordano Bruno, "De triplici minimo et mensura," in *Opera latine conscripta*, ed. F. Tocco and H. Vitelli, vol. I, pt. 3 (Florence: Le Monnier, 1889), 138: "Minimum est sustantia rerum."

there has been a tendency in Bruno scholarship to reduce the minimum to the atom, which obscures Bruno's theory. Perhaps not coincidentally, most studies of 'Bruno's atomism' have focused on the inconsistencies and contradictions resulting from the application of this theory to the study of nature.[277] Disentangling the minimum from the atom may solve at least some of these difficulties. Moreover, reducing the minimum to the atom may have affected the search for Bruno's sources. Driven by the assumption that Bruno's theory was primarily a form of atomism, scholars have looked for Bruno's sources among ancient and medieval atomists, choosing Lucretius as the most likely candidate.[278] However, as I have tried to show in the Introduction, the difference between their atomistic theories prove that Lucretius was not Bruno's model in this respect. For this reason, I have suggested that Bruno stood in a different atomistic tradition: Pythagorean atomism. As soon as one starts placing emphasis on the minimum, the question arises of the source of this concept, and whether it was the same source as that of Bruno's concept of the atom.

As should be clear from the above, I believe that both Llull and Cusanus belonged to the tradition of Pythagorean atomism. Their views, however, diverged on the issue of the composition of the continuum. To my knowledge, Llull did not devote particular attention to the minimum. At the same time, he regarded the points as the atoms that composed geometric objects. Cusanus, on the other hand, rejected the idea that geometric objects were composed of point-atoms, while he gave a detailed account of minimum and maximum. It is important to underline this distinction between Llull and Cusanus because it highlights their different contribution to Bruno's mathematics. Bruno borrowed from Llull aspects of his atomistic geometry, while Cusanus was the source of Bruno's concept of the minimum.

The importance of Cusanus for Bruno's mathematics cannot be overestimated. Bruno himself acknowledged his debt to Cusanus, whom he hailed as the "inventor of geometry's most beautiful secrets."[279] For the most part, the relationship between Cusanus and Bruno has been studied with regard to their epistemology and cosmology, and to more specific issues such as their idea of

[277] For an overview of the literature on Bruno's atomism, see the Introduction.

[278] Singer, "The Cosmology of Giordano Bruno (1548-1600)."

[279] Bruno, *Cause, Principle, and Unity*, 97.

individual or their use of Platonism.[280] On the contrary, Cusanus' and Bruno's mathematics have rarely been compared. As a first step in this direction, Luciana de Bernart drew a parallel between Cusanus and Bruno based on the problem of the quadrature of the circle.[281] In addition, Jean Seidengart claimed that Cusanus was the source of Bruno's concept of the minimum.[282] However, Seidengart's insightful contribution overlooked the account of minimum and maximum that Cusanus gave in *De docta ignorantia*. In what follows, I try to fill this gap.

Gaining a better understanding of Cusanus' account of minimum and maximum is important to mark the distinction between the concepts of minimum and atom in Bruno's thought. For, as conceptualized by Cusanus, the minimum bore no relation to the atom. Instead, Cusanus dealt with the minimum always in relation to the maximum as the two coincided. Compared with other sources that Bruno had at his disposal, Cusanus offered without doubt the most comprehensive discussion of minimum and maximum. Furthermore, like Cusanus, Bruno argued for the coincidence of minimum and maximum, which provides further evidence of Bruno's dependence on Cusanus. If so, the idea of the minimum would have occurred to Bruno independently from the idea of atom, which indeed was inspired by Llull. Thus, it was not that the concepts of minimum and atom were originally connected, but it was Bruno who drew a connection between them, laying the foundations for his theory of minima.

Cusanus systematically addressed the issue of minimum and maximum in *De docta ignorantia*. For this reason, I have limited myself to the analysis of this work, although this was not the only place in which minimum and maximum were mentioned. In reconstructing Cusanus' argument, I have found useful to break it down into two parts. The first part is concerned with the problem of

---

[280] Paul Richard Blum, "'Saper trar il contrario dopo aver trovato il punto de l'unione': Bruno, Cusano e il platonismo," in *Letture bruniane I-II del lessico intellettuale europeo (1996-1997)* (Pisa: Istituti editoriali e poligrafici internazionali, 2002); Filippo Mignini, "La dottrina dell'individuo in Bruno e Cusano," *Bruniana & Campanelliana* 6, no. 2 (2000): 325–49; Pietro Secchi, *"Del mar più che del ciel amante." Bruno e Cusano* (Rome: Edizioni di storia e letteratura, 2006).

[281] Luciana De Bernart, *Cusano e i matematici* (Pisa: Scuola Normale Superiore, 1999). Reprinted in De Bernart, Numerus quodammodo infinitus.

[282] Seidengart, "La metaphysique du minimum."

expressing minimum and maximum in geometry and arithmetic. This paves the way for the discussion of the infinity of minimum and maximum, which is the subject of the second part.

2.2.1 Minimum and maximum in arithmetic and geometry

In Book I of *De docta ignorantia*, Cusanus spoke of minimum and maximum in mathematical terms to provide a framework for understanding these two fundamental concepts. As in the case of enfolding and unfolding (see § 4.1), it was no coincidence that Cusanus relied on mathematics to explain minimum and maximum. Like all the divine truths, minimum and maximum could only be understood through an intermediary, a symbol. Mathematical symbols were well suited for this task because they stood for objects that were more stable and certain than sensible objects.[283] This was why Cusanus firmly believed in the explanatory power of mathematics.

In *De docta ignorantia* I, 4, Cusanus initially presented minimum and maximum as two opposite concepts: the maximum was that which could not be greater, while the minimum was that which could not be smaller.[284] At the same time, minimum and maximum offered a clear example of what Cusanus famously called the "coincidence of the opposites" (*concidentia oppositorum*). The coincidence of minimum and maximum was explained by "contracting" them to quantity, the subject of geometry. "Contracted" was the term that Cusanus used to refer to the realm of finite and limited beings. It was the opposite of "absolute" which indicated the infinity and transcendence of God, but also of other entities. Speaking of minimum and maximum as contracted to quantity, Cusanus stated that:

---

[283] *De docta ignorantia*, I, 11 (31): "Abstractiora autem istis, ubi de rebus consideratio habetur, – non ut appendiciis materialibus, sine quibus imaginari nequeunt, penitus careant neque penitus possibilita|ti fluctuanti subsint – firmissima videmus atque nobis certissima, ut sunt ipsa mathematicalia"

[284] *De docta ignorantia*, I, 4 (11): "Et sicut [maximum] non potest esse maius, eadem ratione nec minus, cum sit omne id, quod esse potest. Minimum autem est, quo minus esse non potest."

> Maximum quantity is maximally large (*maxime magna*); and minimum quantity is maximally small (*maxime parva*). Therefore if you free (*absolve*) maximum and minimum from quantity—by mentally removing (*subtrahendo intellectualiter*) large and small—you will see clearly that maximum and minimum coincide. For maximum is a superlative (*superlativus*) just as minimum is a superlative.[285]

The argument presented in this passage requires further explanation. First, Cusanus invites us to think of minimum and maximum as contracted to quantity. The purpose of this was to show that, when contracted to quantity, minimum and maximum were two opposites entities, the smallest and greatest quantity. Then, Cusanus invites us to abstract minimum and maximum from quantity, which is the opposite of being contracted to quantity. In doing so, Cusanus aimed to show that, irrespective of quantity, the absolute minimum and maximum coincided. For, when considered separately from its quantitative determination (i.e. small), the minimum also appeared to be a kind of maximum. Therefore the contraction to quantity was only the first step in understanding minimum and maximum, which could only be understood when they were abstracted from quantity and revealed in their coincidence.[286] It should be noted that, when speaking of minimum and maximum as abstracted from quantity, Cusanus used the term "absolute quantity." As is well known, *absolutus* in Latin was the perfect

---

[285] *De docta ignorantia*, I, 4 (11): "Maxima enim quantitas est maxime magna; minima quantitas est maxime parva. Absolve igitur a quantitate maximum et minimum – subtrahendo intellectualiter magnum et parvum –, et clare conspicis maximum et minimum coincidere; ita enim maximum est superlativus sicut minimum superlativus."

[286] Gerda von Bredow has a different view of the coincidence of minimum and maximum. She argues that, in Cusanus' view, the minimum coincided with the maximum only when they were contracted to quantity. On the contrary, when minimum and maximum were abstracted from quantity, the maximum contained the minimum, but not the other way around. See Gerda von Bredow, "Die Bedeutung des Minimum in der *Coincidentia oppositorum*," in *Nicolò Cusano agli inizi del mondo moderno: atti del Congresso intemazionale in occasione de V centenario della morte di Nicolò Cusano, Bressanone, 6-10 Settembre, 1964* (Florence: G.C. Sansoni, 1970), 357–66.

participle of the verb *absolvo*, which meant to set something free. Hence it appeared that, like minimum and maximum, quantity could be abstracted from its determinations (e.g. great, small, etc.) and conceived as an absolute entity. The concept of absolute quantity was more suited to represent minimum and maximum in geometry. Indeed, Cusanus argued that "it is not that absolute quantity is more of a maximum that a minimum quantity, since in it minimum and maximum coincide."[287] We shall return to the idea of absolute quantity below.

After geometry, Cusanus moved on to arithmetic. As a preliminary remark, Cusanus stated that "if number is removed, the distinctness, order, proportion, and harmony of things cease, and so does the plurality of beings".[288] With these words, Cusanus seemed to accept the Platonic (but also Pythagorean) belief in a cosmic order based on number. It was in this light that Raymond Klibansky affirmed the existence of a Platonic tradition connecting the School of Chartres to Cusanus and the modern cosmology.[289] Following in the footsteps of Klibansky, Cusanus scholars have continued to use medieval Platonism as a framework for understanding Cusanus' thought.[290] On the other hand, as happened in *De docta ignorantia* II, 1, Cusanus also argued against the precision of human measurements.[291] For this reason, Sarah Powrie has recently proposed to consider another possible source of Cusanus' cosmology, namely, fourteenth-

---

[287] *De docta ignorantia*, I, 4 (11): "Absoluta quantitas non est magis maxima quam minima, quoniam in ipsa minimum est maximum coincidenter."

[288] *De docta ignorantia*, I, 5 (13): "Sublato enim numero cessant rerum discretio, ordo, proportio, harmonia atque ipsa entium pluralitas."

[289] Raymond Klibansky, *The Continuity of the Platonic Tradition during the Middle Ages* (London: The Warburg Institute, 1981), 28–29.

[290] For an overview of the literature on the School of Chartres and Cusanus, see Albertson, *Mathematical Theologies*, 12–17. Albertson's book itself is an attempt to rethink the relationship between Thierry of Chartres and Cusanus in the light of what Albertson calls "Christian Neopythagoreanism."

[291] See *De docta ignorantia*, II, 1 (61–63).

century natural philosophy.[292] The fact that representatives of this philosophical tradition (e.g. the so-called Oxford Calculators) placed special emphasis on issues such as the incommensurability of magnitudes may account for Cusanus' attention to mathematical inaccuracy and our inability to perform precise measurements.

In *De docta ignorantia* I, 5, Cusanus established that number must be finite to provide the basis for cosmic order. "For [if number were infinite] there would be no distinction between things; nor would order or plurality or greater and lesser be found in numbers; indeed, number itself would not exist."[293] One consequence of the finiteness of number was that minimum and maximum could not be conceived as finite numbers just as they could not be thought of as contracted quantities. To prove this point, Cusanus made the following argument. No number can be greater than the maximum number. Yet, no matter how great a number may be, it will always be possible to find a greater number by addition. Only the infinite cannot be greater than it is, but number must be finite. Hence, there cannot be a maximum number because it would be infinite. Likewise, if there were a minimum number, we should be able to subtract number by number until we reach a number that cannot be smaller. However, this would result in an infinite regress, because any number can be made smaller by subtraction. The concepts of minimum and maximum, therefore, contradicted the finiteness of number, and for this reason there could neither be a minimum nor a maximum number.

Cusanus identified minimum and maximum with the arithmetic unit instead of a finite number. The unit was not a number, but rather was "the beginning of all numbers *qua* minimum and the end of all numbers *qua* maximum."[294] Ancient and medieval philosophers of mathematics generally

---

[292] Sarah Powrie, "The Importance of Fourteenth-Century Natural Philosophy for Nicholas of Cusa's Infinite Universe," *American Catholic Philosophical Quarterly* 87, no. 1 (2013): 33–53.

[293] *De docta ignorantia*, I, 5 (12) "Quoniam nulla rerum discretio foret, neque ordo neque pluralitas neque excedens et excessum in numeris reperiretur, immo non esset numerus."

[294] *De docta ignorantia*, I, 5 (14): "[Unitas] est principium omnis numeri, quia minimum; est finis omnis numeri, quia maximum."

agreed that the unit was the beginning of number, without being itself a number. The underlying idea was that the unit was ontologically prior to number because it was its principle. For Cusanus, however, the unit was both the beginning and the end of number. More precisely, Cusanus first equated the unit to the minimum. Then, because of the coincidence of minimum and maximum, Cusanus claimed that the unit was also the maximum.[295] The result was that number was terminated at both ends by the unit, which raises the question of whether Cusanus had a 'circular' view of number. A similar view of number was held in Late Antiquity by Neopythagorean authors such as Moderatus of Gades (I century AD), who saw number as "a progression of multiplicity beginning from the monad, and a regression ending in the monad."[296] However, it was unlikely that Cusanus could have known the work of Moderatus, and thus that he could have been inspired by the Greek philosopher to adopt a circular view of number. Rather, it was more likely that Cusanus came to conceive the unit as both minimum and maximum as a result of the application of his own concept of *coincidentia oppositorum* to mathematics.

Cusanus viewed the arithmetic unit as "a minimum *simpliciter*, which coincides with the maximum."[297] Further on, Cusanus argued that "the unit cannot be a number, because number, which can be greater, cannot be either a minimum or maximum *simpliciter*."[298] The use of the Latin term *simpliciter*, which for Cusanus was synonymous with *absolute*, corroborates the idea that in arithmetic minimum and maximum did not belong to the domain of finite

---

[295] *De docta ignorantia*, I, 5 (13): "Quapropter necessarium est in numero ad minimum deveniri, quo minus esse nequit, uti est unitas. Et quoniam unitati minus esse nequit, erit unitas minimum simpliciter, quod cum maximo coincidit per statim ostensa."

[296] Quoted from John M. Dillon, *The Middle Platonists, 80 B.C. to A.D. 220* (Ithaca, N.Y: Cornell University Press, 1996), 350. On Moderatus, see Albertson, *Mathematical Theologies*, chap. 2.

[297] *De docta ignorantia*, I, 5 (13): "Et quoniam unitati minus esse nequit, erit unitas minimum simpliciter, quod cum maximo coincidit per statim ostensa."

[298] *De docta ignorantia*, I, 5 (14): "Non potest autem unitas numerus esse, quoniam numerus excedens admittens nequaquam simpliciter minimum nec maximum esse potest"

numbers. Likewise, in geometry the term "absolute quantity" was used to indicate that, to fully understand minimum and maximum, one had to go beyond the contraction to quantity. In both cases, Cusanus aimed to show that in arithmetic and geometry there was a concept of minimum and maximum, which however could not be expressed in terms of number and quantity. The reason was that number and quantity could only represent the finite and contracted, while minimum and maximum were instances of the absolute. It was on these grounds that, in Book II of *De docta ignorantia*, Cusanus conceptualized minimum and maximum as infinite entities.

2.2.2 Minimum and maximum as infinite entities

Cusanus' account of minimum and maximum continued in Book II of *De docta ignorantia*. Here, building on the mathematical explanation of minimum and maximum given in Book I, Cusanus shifted the focus from geometry and arithmetic to God and the universe. In chapters I, 4-5, the language used by Cusanus to describe minimum and maximum revealed the absolute and simple character of these two notions. In chapter II, 1, Cusanus went a step further and explicitly claimed that both minimum and maximum were infinite entities:

> Since neither an ascent to the maximum *simpliciter* nor a descent to the minimum *simpliciter* is possible, there could be no transition to infinity. As is evident in the case of number and the division of the continuum, given any finite entity, it is always possible to find a greater and a smaller. For maximum or minimum *simpliciter* are not given in (finite) things, nor could the progression go to infinity, as was just indicated.[299]

[299] *De docta ignorantia*, II, 1 (63-4): "Quoniam ascensus ad maximum et descensus ad minimum simpliciter non est possibilis, ne fiat transitus in infinitum, ut in numero et divisione continui constat, tunc patet, quod dato quocumque finito semper est maius et minus sive in quantitate aut virtute vel perfectione et ceteris necessario dabile – cum maximum aut minimum simpliciter dabile in rebus non sit –, nec processus fit in infinitum, ut statim ostensum est."

The impossibility of ascending to the maximum or descending to the minimum was a direct consequence of Cusanus' 'comparative' view of knowledge. This view of knowledge was a pillar of *De docta ignorantia*, and can be summed up in the statement that "every inquiry is comparative and uses the means of proportion."[300] By this Cusanus meant that the act of knowing consisted in comparing the unknown to the known. The smaller the gap between unknown and known, the easier the act of knowing. To clarify this point, Cusanus once again turned to mathematics, and in particular to mathematical proof. The complexity of a mathematical proof was proportional to the number of steps that need to be taken to trace the proof back to self-evident principles.[301] Likewise, an object was more or less difficult to know, depending on how far that object was from what was already known. This comparative view of knowledge imposed a constraint on the kind of objects that could be known. Indeed, only finite objects could be compared with one another, while infinite objects admitted no comparison. To explain the impossibility of comparing infinites, Cusanus took the example of two infinite lines composed of an infinite number of segments. Even if the segments composing one line were longer than those composing the other line, the two lines would still have the same infinite length.[302] For this reason, Cusanus stated that "the infinite as infinite is unknown because it escapes proportion."[303]

Returning to the topic of minimum and maximum, Cusanus first postulated the equivalence of the maximum and the infinite in *De docta ignorantia* I, 3.[304] In chapter II, 1, this equivalence was reframed to accommodate the difference between the two highest expressions of maximum:

---

[300] *De docta ignorantia*, I, 1 (5): "Comparativa igitur est omnis inquisitio, medio proportionis utens."

[301] *De docta ignorantia*, I, 1 (5): "Uti haec in mathematicis nota sunt, ubi ad prima notissima principia priores propositiones facilius reducuntur, et posteriores, quoniam non nisi per medium priorum, difficilius."

[302] *De docta ignorantia*, II, 1 (64): "Linea infinita ex infinitis bipedalibus esset minor linea infinita ex infinitis quadrupedalibus lineis"

[303] *De docta ignorantia*, I, 1 (6): "Infinitum ut infinitum, cum omnem proportionem aufugiat, ignotum est."

[304] *De docta ignorantia*, I, 3 (9): "Maximum vero tale necessario est infinitum."

God and the universe. Both God and the universe were infinite. Yet the universe was infinite privately (*privative*), while God was infinite negatively (*negative*).[305] Cusanus seemed to suggest that God was negatively infinite in the sense that He was the negation of the finiteness of creatures. On the other hand, Cusanus was more explicit about the privative infinity of the universe, which was associated with the limitlessness of the universe.[306] For Cusanus, the most fundamental difference between God and the universe was that God was an absolute maximum, while the universe was a contracted maximum. To use the words of Hans Blumenberg:

> [Contraction] is thus the general and thoroughgoing characteristic of the actual world and of what is actual within it. What is actual is this or that, which is to say that as this and not that, it is actual at the expense of possibilities no longer open. Nothing actual is what it can be. (…) The universe also, as a unique whole, the *universi prima generalis contractio*, which is followed by the further degrees of restriction into *genera*, *species* and *individua* does not exhaust the horizon of possibility, which is defined by God's omnipotence.[307]

For Blumenberg, Cusanus' distinction between absolute and contracted was a refurbishment of the medieval view whereby "the world was represented as [God's] absolute power's self-restriction to an arbitrary particle of what was possible for it."[308] For this reason, Blumenberg argues that Cusanus stood before what he called the "threshold of modernity."[309] More importantly here, the couple absolute-contracted may help clarify the difference between the infinity of God and the infinity of the universe. Cusanus argued that "with respect to God's

---

305 *De docta ignorantia*, II, 1 (64): "Solum igitur absolute maximum est negative infinitum; quare solum illud est id, quod esse potest omni potentia. Universum vero cum omnia complectatur, quae Deus non sunt, non potest esse negative infinitum, licet sit sine termino et ita privative infinitum."

306 Miller, "Cusanus, Nicolaus [Nicolas of Cusa]," para. 2.1.

307 Blumenberg, *The Legitimacy of the Modern Age*, 544.

308 Blumenberg, 563.

309 Blumenberg, 269.

infinite power, which is illimitable, the universe could have been greater."[310] This was because God's power included all the possibilities, while the universe included only those possibilities that were actualized. Nevertheless, there was no actualized possibility, that is no being, outside the universe. In this sense, the universe had reached its maximum extension and could be said to be infinite.

While Cusanus provided a wealth of detail on the infinity of the maximum, he did not specify how to conceive the infinity of the minimum. Yet Cusanus made it clear that if the ascent to the maximum went to infinity, so did the descent to the minimum. Despite this difficulty, an attempt can be made to reconstruct the infinity of the minimum based on the infinity of the maximum. Of the two models of infinity (privative versus negative) developed by Cusanus for the maximum, the privative model may also be applied to the minimum. Studying minimum and maximum in arithmetic has shown us that the minimum could not be expressed by a finite number. If numbers failed to represent the minimum, so did the other finite entities, for mathematical symbols were the most suited for visualizing concepts like the minimum (*De docta ignorantia*, I, 11). The minimum, therefore, could be said to be limitless or privatively infinite insofar as it could not be understood by means of finite beings such as numbers.

In *De docta ignorantia* II, 1, Cusanus described the privative infinity as an attribute of the contracted maximum, the universe. Thus, if the minimum could be described as privately infinite, it seems safe to assume that Cusanus acknowledged the existence of a contracted minimum. However, it must be said that there was no explicit mention of the term "contracted minimum" in Cusanus' works. Moreover, the case of geometry reminds us that the minimum was more of an absolute than a contracted quantity. This begs the question, was there a contracted entity resembling the minimum which could be used to visualize this otherwise ineffable concept? In *De thelogicis complementis* (1453), Cusanus argued that

> [God] made a point, which is almost nothing. For between a point and nothing there is no intermediary; for a point is to such an extent

[310] *De docta ignorantia*, II, 1 (65): "Licet in respectu infinitae Dei potentiae, quae est interminabilis, universum posset esse maius"

almost-nothing that if you added a point to a point, there would result no more than if you were to add nothing to nothing.[311]

By virtue of its being "almost nothing," the point was a suitable candidate for representing the contracted minimum. Indeed, the point was not a contracted quantity, since two or more contracted quantities could be added to form a greater quantity, while a point added to a point yielded nothing. (As already seen, this was also the reason why Cusanus did not subscribe to the atomistic belief that a line was a set of points placed side by side). On the other hand, the point existed and functioned as the end of the line. In sum, the point was on the threshold between nothingness and being. The contracted minimum had also a peculiar ontological status, since it was a contracted entity whose essence however exceeded that of finite beings. Given this similarity, the point could be used as a symbol of the contracted minimum.

Did Cusanus also conceive an absolute minimum? There was only one occurrence of the term "absolute minimum" in *De docta ignorantia* III, 1.[312] The passage in question can be regarded as a paraphrase of chapter II, 1, where Cusanus claimed the impossibility of ascending to the maximum or descending to the minimum. The only significant difference between the two passages was that Cusanus replaced *simpliciter* with *absolute*. As already mentioned, Cusanus used *simpliciter* as synonymous with *absolute*. Therefore, when inquiring about the concept of absolute minimum in Cusanus' thought, we should count the occurrences of the term "minimum *simpliciter*" as well as those of the term "minimum *absolute*." Since in *De docta ignorantia* alone this amounts to ten

[311] Nicholas of Cusa, *De theologicis complementis*, ed. Heide D. Riemann and Karl Bormann (Hamburg: Felix Meiner, 1994), bk. IX, para. 44: "Creator igitur duo fecisse videtur, scilicet prope nihil punctum – inter enim punctum et nihil non est medium; adeo enim prope nihil est punctus, quod, si puncto punctum addas, non plus resultat, quam si nihilo nihilum addideris."

[312] *De docta ignorantia*, III, 1 (119): "Non potest igitur ascensus vel descensus in contractis esse ad maximum vel minimum absolute."

occurrences (one of "minimum *absolute*," nine of "minimum *simpliciter*"),[313] we can conclude that Cusanus had a concept of absolute minimum.

To better understand Cusanus' concept of absolute minimum we need to return to *De docta ignorantia* II, 1. There Cusanus recalled the notion of equality (*equalitas*) with which he had already dealt in Chapter I, 3. The notion of equality offered a means to distinguish between God and the created world. Cusanus claimed that in the created world there were infinite degrees of equality. This meant that, as similar as two objects may appear, they would always remain separate objects and never coincide. By the same token, Cusanus argued that "the measure and the measured—however equal they are—will never be the same."[314] Cusanus conceived the act of measuring as a process whereby the measurement instrument came to coincide with the object to be measured. Nevertheless, he found it impossible to carry out this process in the created world because perfect equality did not belongs to created things but only to God. Accordingly, the measurement of created things would always imply a certain degree of approximation, as the gap between instrument and object could by no means be filled. As a metaphor of this approximation, Cusanus took the example of the polygon and the circle:

> The more angles the inscribed polygon has the more similar it is to the circle. However, even if the number of its angles is increased ad infinitum, the polygon never becomes equal [to the circle] unless it is resolved into an identity with the circle.[315]

---

313 This research can be easily conducted using the search function on the Cusanus-Portal accessible online at http://www.cusanus-portal.de.

314 *De docta ignorantia*, I, 3, (9): "Hinc mensura et mensuratum, quantumcumque aequalia, semper differentia remanebunt."

315 *De docta ignorantia*, I, 3 (9): "[Intellectus]habens se ad veritatem sicut polygonia ad circulum, quae quanto inscripta plurium angulorum fuerit, tanto similior circulo, numquam tamen efficitur aequalis, etiam si angulos in infinitum multiplicaverit, nisi in identitatem cum circulo se resolvat." The metaphor of the polygon and the circle reminds us of Cusanus' attempts to square the circle. On Cusanus and the quadrature of the circle, see Fritz Nagel, *Nicolaus Cusanus un die Entstehung der Exakten Wissenschaften* (Münster: Aschendorff, 1984).

In *De docta ignorantia* II, 1, Cusanus also acknowledged the impossibility of making precise measurements in the created world. In fact, in keeping with the idea that perfect equality only belonged to God, Cusanus launched an attack on what, in many other respects, provided the foundation for his mathematical theology, that is the Boethian quadrivium. Probably inspired by his efforts to reform the calendar, Cusanus demonstrated that the planetary motions could not be measured with precision.[316] Moreover, he claimed that it was impossible for two geometric figures or for two numbers to be perfectly equal, and that it was inappropriate to speak of musical harmony. By frustrating every human attempt at precise measurement, the four mathematical sciences of the *quadrivium* revealed that perfect equality could only be experienced negatively in the created world. It is worth noting that this departure from Pythagoreanism was aimed, in Cusanus' view, to reinforce the principle that perfect equality was exclusively a divine attribute.

Note that *De docta ignorantia* II, 1 was also the place where Cusanus denied the possibility of ascending to the maximum or descending to the minimum. In light of this, one may be tempted to draw a connection between the issue of the precision (or lack thereof) of measurement on the one hand, and the problem of how Cusanus conceptualized the absolute maximum and (more importantly for this study) the absolute minimum on the other hand. For Cusanus, one could not ascend to the maximum nor descend to the minimum, any more than one could make a polygon coincide with a circle by adding an endless number of sides. In fact, the circle could be used as a metaphor for the minimum and maximum, as both could be asymptotically approached but not reached. In turn, since minimum and maximum coincided in God, their ineffable nature substantiated the claim that our knowledge of God could only be a *docta ignorantia*, an ignorance aware of its necessity. This did not mean that our efforts to know God or to make precise measurements were in vain, as in making them we closed the gap with God and increased the degree of precision of our

[316] On Cusanus and the reform of the calendar, see Hans Gerhard Senger, *Die Philosophie Des Nikolaus von Kues Dem Jahre 1440. Untersuchungen Zur Entwicklung Einer Philosophie in Der Fruhzeit Des Nikolaus (1430-1440)* (Münster: Aschendorff, 1971).

measurements. And it was this quest for infinite preciseness that captured the essence of the absolute minimum and maximum.

As already mentioned in the Introduction, Bruno entered the labyrinth of the continuum and lost himself trying to find his way out of it. Unlike Bruno, Cusanus did not dare to set foot in the labyrinth of the continuum, but not because he thought that the continuum problem was more of a philosophical than a mathematical problem. (Seventeenth-century mathematicians used this argument to explain their decision not to deal with the continuum problem.) If anything, Cusanus viewed the continuum problem as more of a *theological* than a mathematical problem. Indeed, a reading of *De docta ignorantia* gives the impression that Cusanus regarded the labyrinth of the continuum as a temple, a sacred space. His account of minimum and maximum shows that he did not ignore the problems related to the conceptualization of the infinite, but he thought that the solution to these problems exceeded the limits of human understanding. Bruno would have thought of the minimum in the same terms, if it were not for Mordente's compass. It is true that the argument of Bruno's dialogues on Mordente's compass rested on the assumption that the minimum parts of mathematical objects had no defined shape (see next Chapter). Thus, like Cusanus', Bruno's definition of the minimum was characterized by a certain degree of indeterminacy—indeterminacy that would eventually disappear when in *De minimo* Bruno would claim that the minimum had a circular shape. Nevertheless, Mordente's compass allowed to divide and measure geometric quantities down to their minimum parts, sidestepping the problem of their indeterminacy. In this way, it not only provided an argument in favor of atomism, but it also turned the minimum from an ineffable reality to an understandable concept.

The aim of the first part of this thesis has been to provide an overview of the contribution of Llull and Cusanus to the development of Bruno's mathematics. In my opinion, the study of these two Bruno's sources is important because it yields insights into the origin of Bruno's mathematical ideas, the peculiarity of which has often led scholars to consider them individually and not in relation to a philosophical tradition. This is especially true of the atomistic view of mathematical objects that, as I have tried to show in Chapter 1, Bruno inherited from Llull and which inspired Bruno's project of a mathematical reform. As for

Cusanus, an analysis of his account of minimum and maximum reveals that, on building on this account, Bruno could not but be aware of the problematic status of these two concepts. On top of that, Bruno viewed the minimum as the indivisible part of which both mathematical and physical objects were composed. In doing so, he integrated the Cusan minimum into the atomistic framework derived from Llull, thus laying the foundation for his atomistic geometry.

PART TWO. BRUNO'S ATOMISTIC GEOMETRY

3. Bruno's first mathematical writings: The dialogues on Mordente's compass

Introduction

In October 1585, Giordano Bruno returned to Paris from London in the entourage of his patron, the French ambassador Michel de Castelnau. So ended the two and a half years that Bruno spent in England, a period in which he wrote eight works (including his six Italian dialogues), and had faced criticism and hostility from the English intellectual environment. Bruno's stay in England has been the subject of several studies, and yet we do not know with certainty the reasons that led to his return to France.[317] Saverio Ricci proposes that Bruno was not the only one who failed in England. The diplomatic mission of Castelnau also turned out to be unsuccessful.[318] With no one willing to support him in England, Bruno had no choice but to follow Castelanau when this latter decided to move back to France. What is certain is that Bruno did not feel welcome in Paris either, so much so that he left for Germany less than one year from his arrival.

Two events made it impossible for Bruno to stay in Paris longer: the controversy with the Italian mathematician Fabrizio Mordente (1532 – 1608), and the dispute against Aristotelian philosophy in which Bruno took part at the Collège de Cambrai (now part of the Collège de France) in May 1586. Bruno scholars have already provided a historical reconstruction of both these events.[319]

---

[317] On Bruno in England, see Hilary Gatti, *The Renaissance Drama of Knowledge. Giordano Bruno in England* (London; New York: Routledge, 1989); Giovanni Aquilecchia, "Giordano Bruno in Inghilterra (1583-1585). Documenti e testimonianze," *Bruniana & Campanelliana* 1, no. 1/2 (1995): 21–42; John Bossy, *Giordano Bruno and the Embassy Affair* (New Haven: Yale University Press, 2002); Mordechai Feingold, "Giordano Bruno in England, Revisited," *Huntington Library Quarterly* 67, no. 3 (September 2004): 329–46; Diego Pirillo, *Filosofia ed eresia nell'Inghilterra del tardo Cinquecento: Bruno, Sidney e i dissidenti religiosi italiani* (Rome: Edizioni di storia e letteratura, 2010).

[318] Saverio Ricci, *Giordano Bruno nell'Europa del Cinquecento* (Rome: Salerno editrice, 2000), 191–92.

[319] For a historical reconstruction of the Bruno-Mordente controversy, see Yates, "Bruno: New Documents." On the dispute at the Collège de Cambrai, see Amalia Perfetti,

However, less attention has been paid to the impact that the controversy with Mordente had on Bruno's mathematics. To my knowledge, Luciana De Bernart is the only scholar who has addressed this issue.[320] In writing this chapter, I have greatly benefited from her work.

There may be several reasons why the Bruno-Mordente controversy has been neglected so far. First of all, this may be the result of the low esteem in which scholars have held Bruno's mathematics in general.[321] For instance, Cassirer notices how the "concrete" character of Bruno's mathematics prevented him from seeing those "laws and ideal relations whose value is independent from the nature of the existing things and of matter."[322] Likewise, Védrine, borrowing Bachelard's terminology, speaks of a "realistic obstacle" hindering Bruno's mathematics (more on this in Chapter 4).[323] As a result of these criticisms, Bruno has been viewed as a poor mathematician. But when it comes to the Bruno-Mordente controversy, there are other factors to be considered, starting with the fact that this controversy remained unknown until the 1950s. Up to that time scholars were aware that Bruno and Mordente met in Paris in 1586, but they did not know about the controversy. It was thanks to the textual discoveries made by Frances Yates[324] and Giovanni Aquilecchia[325] (to which we shall return below) that the controversy became known, thus opening a new chapter in Bruno's already long history of conflicts and disagreements.

But what was the bone of contention between Bruno and Mordente? Mordente was the inventor of one the first proportional compasses (also known as sectors), an instrument constructed according to the principles of trigonometry

---

"Un nuovo documento sul secondo soggiorno parigino di Giordano Bruno (1585-1586)," in *Giordano Bruno: gli anni napoletani e la* peregrinatio *europea: immagini, testi, documenti*, ed. Eugenio Canone (Cassino: Università degli studi di Cassino, 1992).

320 De Bernart, Numerus quodammodo infinitus.

321 For more information on the reception of Bruno's mathematics, see the Introduction.

322 Cassirer, "Il problema della conoscenza nella filosofia e nella scienza dall'Umanesimo alla scuola cartesiana," 345.

323 Védrine, "L'obstacle réaliste," 247.

324 Yates, "Bruno: New Documents."

325 Aquilecchia's findings are published in Bruno, *Due dialoghi sconosciuti*.

to solve arithmetic and geometric problems (such as calculating the square root of a number or squaring a curved figure). Mordente's compass was almost unknown until the late 1800s, as its existence was overshadowed by that of another proportional compass, invented by a better-known Italian scientist: Galileo Galilei.[326] However, Mordente's compass did not go completely unnoticed by his contemporaries, catching the eye of technicians and mathematical practitioners, but also of speculative thinkers like Bruno. Puzzled by the novelty of Mordente's invention, Bruno offered to write a Latin exposition of the compass in the form of two dialogues (entitled *Mordentius* and *De Mordenti circino*). Mordente, however, must not have liked what Bruno had to say about his compass, as he tried to acquire and burn as many copies of Bruno's dialogues as possible. In response, Bruno wrote two more dialogues (entitled *De idiota triumphans* and *De somni interpreatione*), in which he accused Mordente of plagiarism and stupidity.[327]

The Bruno-Mordente controversy will be discussed at length below. Here, I would like to focus on what happened in the aftermath of that controversy. In the years from 1586 to 1591, Bruno would go on to develop the idea that geometric objects were composed of indivisible parts, turning it into a fully-fledged atomistic geometry. Ultimately, this project would result in the publication of *De minimo*, where Bruno theorized his atomistic geometry based on the concept of the minimum. But if *De minimo* marked the end of Bruno's mathematical odyssey, its starting point was found in the Italian dialogues that Bruno published in London in 1584, in particular in *De la causa, principio e uno* (*On Cause,*

---

[326] On Galileo's proportional compass, see Antonio Favaro, "Per la storia del compasso di proporzione," *Atti del Reale Instituto Veneto di Scienze, Lettere e Arti* 67 (1907): 723–39; Paul Lawrence Rose, "The Origins of the Proportional Compass from Mordente to Galileo," *Physis* 10 (1968): 53–69; Edward Rosen, "The Invention of the Reduction Compass," *Physis* 10 (1968): 306–8; Ivo Schneider, *Der Proportionalzirkel: ein universelles Analogrecheninstrument der Vergangenheit* (Munich: R. Oldenburg, 1970); Matteo Valleriani, *Galileo Engineer* (Dordrecht: Springer Netherlands, 2010), 27–40.

[327] All of the four dialogues on Mordente's compass are now published in Bruno, *Due dialoghi sconosciuti*.

Principle and Unity) and *De l'infinito, universo et mondi* (*On the Infinite Universe and Worlds*).

Indeed, as Angelika Bönker-Vallon[328] and Jean Seidengart[329] have demonstrated, it is in these dialogues that Bruno, building on the work of Nicholas of Cusa, laid the foundation for his atomistic geometry. Nevertheless, although Bruno first dealt with the issue of the composition of the continuum in *De la causa* and *De l'infinito,* Seidengart notices that in these works, Bruno's atomism was not yet fully developed.[330] The term "atoms" was mentioned several times especially in *De l'infinito*, and yet Bruno did not specify how these atoms came together to form an object, or if he subscribed to a specific kind of atomism (e.g. Democritean or Epicurean). Moreover, in the Italian dialogues, Bruno did not seem to conceive of the existence of a geometric minimum, as the atomic structure was only attributed to physical entities. It was not until the controversy with Mordente and the publication of the dialogues on his compass that Bruno claimed that geometric objects were composed of infinitely small indivisible parts. For this reason, an analysis of Bruno's dialogues on Mordente's compass may offer new insights into the development of Bruno's mathematical thinking. Also, it may show how Bruno's mathematics changed over time, highlighting the differences between the theory developed in the dialogues on Mordente's compass, and the theory presented in the later *De minimo*.

From an historical perspective, the Bruno-Mordente controversy is important for another reason. We have seen that early interpreters such as Cassirer and Védrine have argued against the modernity of Bruno's mathematics on account of its being more of a concrete than an abstract knowledge. On the one hand, the fact that Bruno's mathematics grew out of efforts to illustrate the use of Mordente's compass seems to corroborate this opinion, showing that indeed Bruno had an early interest in mathematical practices. On the other hand, if Bruno's mathematics is to be criticized for being outdated, we need to clarify what is meant by modern mathematics. Bruno's critics seem to assume that modern

---

[328] Bönker-Vallon, *Metaphysik und Mathematik bei Giordano Bruno*; Bönker-Vallon, "Giordano Bruno e la matematica."

[329] Seidengart, "La metaphysique du minimum."

[330] Seidengart, 63.

mathematics was characterized by its high level of abstractedness and theoretical speculation. In other words, their concept of modern mathematics seems to coincide with what has been called "pure knowledge" as opposed to "applied knowledge."[331]

But did Renaissance mathematics fall squarely within the domain of pure knowledge? Certainly, Renaissance mathematics was in the process of becoming pure, as one of the goals of those defending the certitude of mathematics at that time was to ensure the independence of mathematics from other forms of knowledge, especially physics.[332] However, one may argue that alongside 'theoretical' mathematicians (such as Cardano, Tartaglia, and Regiomontanus), there was a wide range of mathematical practitioners whose activities have gone almost unnoticed until recently. To be fair, exploring the world of mathematical practitioners is not an easy task, since their work was rarely converted into printed books or formalized in mathematical theories. Nevertheless, especially thanks to the pioneering studies of Eva G. R. Taylor,[333] scholars have gradually become aware of the importance of mathematical practitioners in establishing mathematics as a leading discipline during the Renaissance.[334] The Bruno-

331 For a discussion of the distinction between pure and applied knowledge in early modern mathematics, see Sophie Roux, "Forms of Mathematization (14th-17th Centuries)," *Early Science and Medicine* 15, no. 4–5 (2010): 319–37.

332 Here I am referring to the so-called *Quaestio de certitudine mathematicarum* which took place in the sixteenth century. Among those who made a case for the certitude of mathematics, there were Francesco Barozzi (1537-1604) and Christophorus Clavius (1538-1612). For more information on the *Quaestio,* see Anna De Pace, *Le matematiche e il mondo: ricerche su un dibattito in Italia nella seconda metà del Cinquecento* (Milan: Franco Angeli, 1993); Mancosu, *Philosophy of Mathematics*, 10–33; Emilio Sergio, *Verità matematiche e forme della natura da Galileo a Newton* (Rome: Aracne, 2006), 11–52.

333 Eva G. R. Taylor, *The Mathematical Practitioners of Tudor and Stuart England* (Cambridge: Cambridge University Press, 1954).

334 For a more recent analysis of early modern mathematical practitioners, see Lesley B. Cormack, Steven A. Walton, and John A. Schuster, eds., *Mathematical Practitioners and the Transformation of Natural Knowledge in Early Modern Europe* (Cham: Springer, 2017).

Mordente controversy provides yet another example of the interaction between theoretical and practical mathematics in this period.

Last but not least, the Bruno-Mordente controversy may shed new light on the question of to what extent Bruno's concept of the minimum may be considered a forerunner of the modern notion of infinitesimal. As is well known, the introduction of infinitely small quantities marked a turning point in early modern mathematics, leading to the development of the calculus.[335] Bruno's minimum was the smallest quantity which geometric objects were composed of. In spite of this, scholars have been reluctant to draw even the slightest connection between Bruno and the infinitesimals. The reasons for this behavior are well explained by Leonardo Olschki.[336] Arguably one of Bruno's harshest critics, Olschki claims that Bruno was prevented from seeing "the most basic version of the infinitesimal principle" by his denial of the coincidence of the minimum arc and the minimum chord.[337] In other words, the problem with Bruno's geometry, in Olschki's opinion, was that it envisaged two kinds of minima, one for the straight line and one for the curved line. On the contrary, in the theory of infinitesimals, every line—no matter whether straight or curved—was considered as composed of infinitely small, straight lines. Moreover, Olschki adds, "Bruno's concrete geometry would have taken on an evident significance, if it had been connected to a theory of motion."[338]

Writing in 1927, Olschki could not have read Bruno's last two dialogues on Mordente's compass, which were rediscovered only in 1957. If he could have done so, he would have probably realized that both his objections to Bruno's geometry were unwarranted. In fact, in *De idiota triumphans* (Bruno's third dialogue), he argued that both straight and curved lines were composed of the same minima (see § 3.4). Furthermore, in *De Mordentii circino*and more extensively in *De somni interpretatione* (respectively, Bruno's second and fourth dialogue) he envisioned the possibility of a law of motion that could account for both circular

[335] For the history of the calculus, see Carl B. Boyer, *The History of the Calculus and Its Conceptual Development* (New York: Dover, 1949); C. Henry Edwards, *The Historical Development of the Calculus* (New York: Springer, 1994).

[336] Olschki, *Giordano Bruno*.

[337] Olschki, 75.

[338] Olschki, 81.

and non-circular motions.[339] The main field of application of this law of motion was the study of planetary orbits. Thus, Bruno had an answer for both the objections raised by Olschki, although this latter could not have known it. This opens the possibility for a new assessment of Bruno's mathematics, which is what this chapter aims to carry out.

In the following, I will first give a brief description of Mordente's compass and of the controversy that originated from Bruno's decision to write about it. Then I will move on to analyze Bruno's first and third dialogues, where he explained how, working on Mordente's compass, he came to discover the geometric minimum. Since the analysis of the second and the fourth dialogue (those in which Bruno developed the idea of a law of planetary motion based on his theory of minima) would require a separate study, this analysis is left for future study (see note 23).

3.1 Mordente's proportional compass: Some historical remarks

To fully appreciate the value of Mordente's compass and its importance for the history of science, we need to understand the difference between the reduction and proportional compass in the first place. The reduction compass (Figure 1) was at least as old as the ancient Romans, one of its first examples having been discovered in the archeological site of Pompeii.[340] The main goal of a reduction

---

[339] *De Mordentii circino*, 58: "Is it not necessary that, in those things that are connected and related, the certain law of what moves away from and towards a center follows from the certain law of what rotates around a fixed center?" Considering that in the same text Bruno spoke of planetary motions, and that he mentioned the fact that "the stars happen to approach and move back from the sun and the earth" (*De Mordentii circino*, 57), one may argue that here Bruno seemed to postulate the existence of non-circular, planetary orbits. This claim would need to be substantiated by a thorough inquiry into Bruno's astronomy, a task that is beyond the purpose of this work. For this reason, as mentioned below, I will not address this issue here.

[340] The reduction compass found in Pompeii is now kept at the Museo Archeologico Nazionale of Naples (inv. 76684). For more information, see Filippo Camerota, *Il compasso di Fabrizio Mordente. Per la storia del compasso di proporzione* (Florence: Leo S. Olschki, 2000), 14.

compass was to reduce or enlarge a drawing. The proportional compass, on the other hand, allowed to perform several mathematical operations, such as dividing a segment or a circumference into equal parts, or squaring an irregular figure. It did so by exploiting the geometric property that similar triangles have proportional corresponding sides. As such, the proportional compass may be considered the first calculating instrument of the modern age. For a long time, Mordente's compass (Figure 2) had been seen as a reduction compass, a simplistic view that had not done justice to the Italian mathematician. Instead, as recently demonstrated by Camerota, Mordente's compass was a proportional compass in its own right.[341]

Fig. 1: An example of a reduction compass from the Medici Collections (Museo Galileo, Florence – Photography by Franca Principe)

341 Camerota, 5–7.

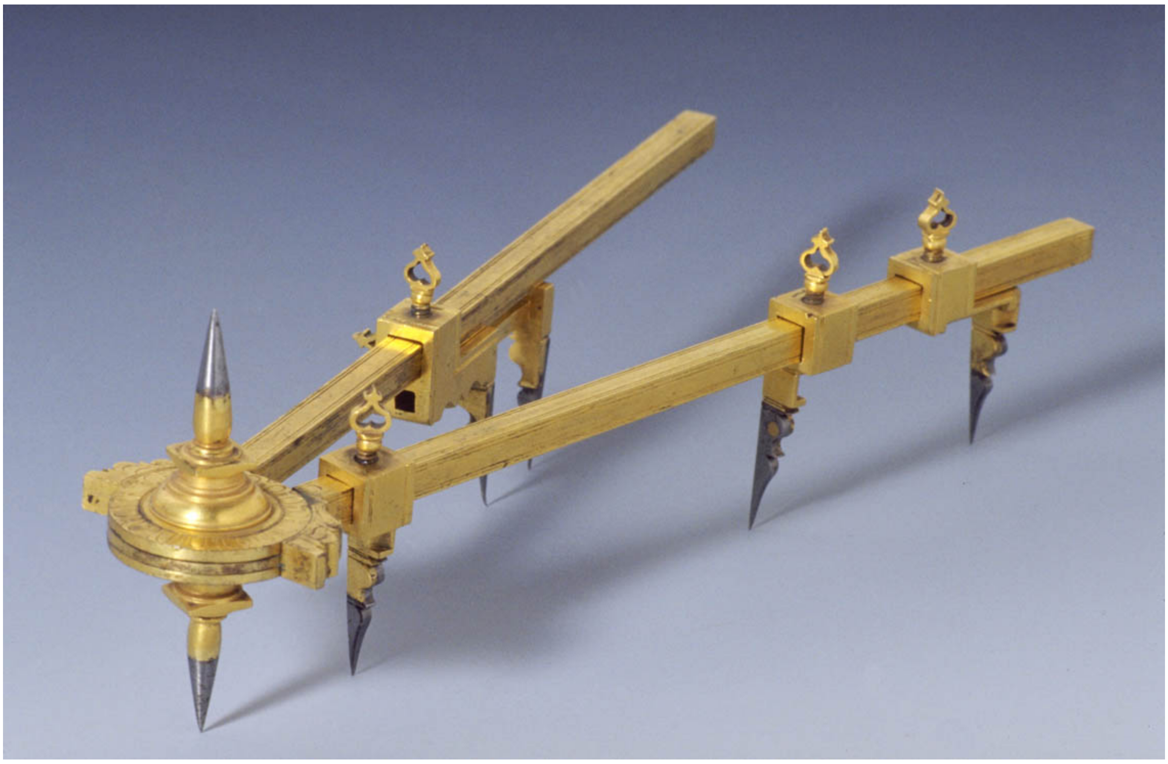

Figure 2: The compass of Mordente (Museo Galileo, Florence – Photography by Franca Principe)

The rediscovery of Mordente's compass in the late 1800s reopened the question of the authorship of the proportional compass. This question has been debated ever since Galileo published *Le operazioni del compasso geometrico e militare (The Operations of the Geometric and Military Compass,* 1606).[342] Galileo claimed to have constructed the first version of his compass (Figure 3) in 1597. However, there is evidence that other examples of proportional compass circulated in Europe even before 1597. A compass similar to that of Galileo had been constructed by the Flemish mathematician Michel Coignet as early as the 1580s.[343] Coignet in turn was familiar with Mordente's compass, which he had helped promote through the publication of several treatises.[344] In light of this

[342] Galileo Galilei, *Le operazioni del compasso geometrico e militare* … (Padoa: Pietro Marinelli, 1606).

[343] On Coignet's compass, see Ad Meskens, "Michel Coignet's Contribution to the Development of the Sector," *Annals of Science* 54 (1997): 143–60.

[344] Michel Coignet, *Della forma, et parti del compasso di Fabrizio Mordente Salernitano* … (Antwerp, 1608); Michel Coignet, *L'uso del compasso di Fabrizio Mordente*

intricate network of acquaintances and information exchanges, several reconstructions have been proposed to explain the genesis of the proportional compass. Despite all these efforts, however, it remains unclear whether Mordente may be considered the inventor of the proportional compass—as sustained by Boffito[345]—or whether Mordente's, Coignet's and Galileo's compass had different stories—as advocated, among others, by Favaro[346] and Rose.[347]

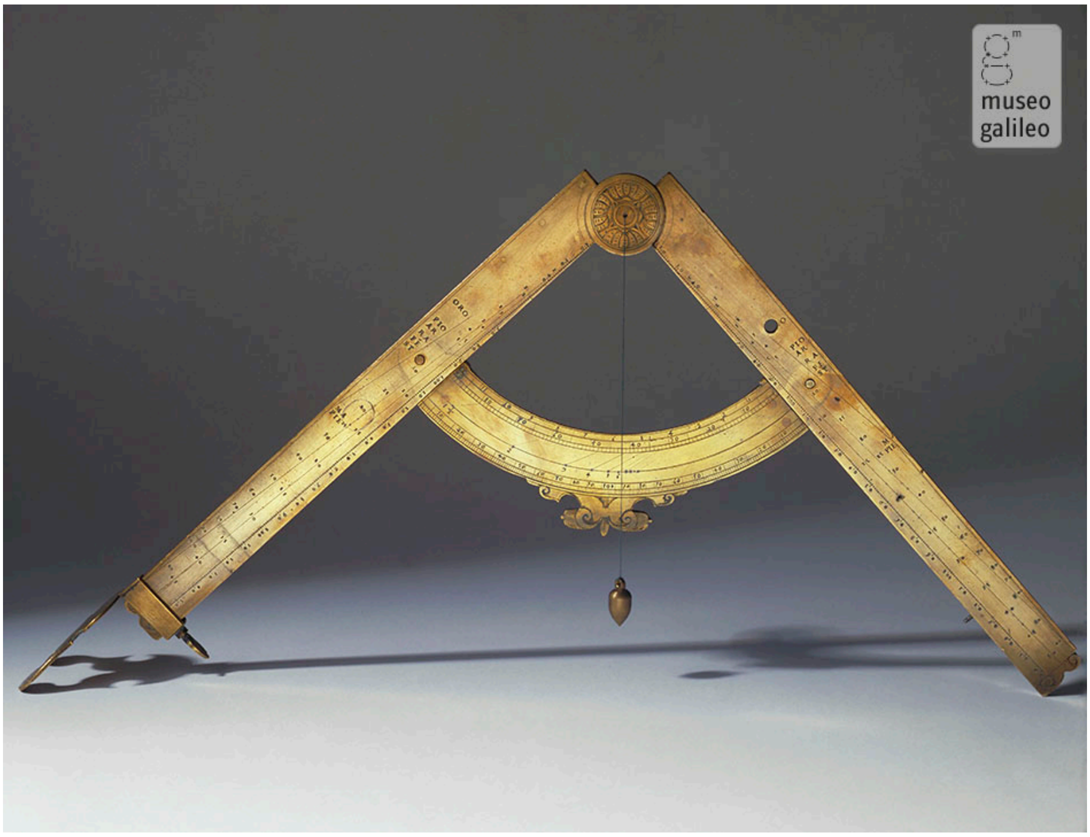

Fig. 3: The proportional compass of Galileo (Museo Galileo, Florence – Photography by Franca Principe)

Salernitano ... (Antwerp, 1608); Michel Coignet, *La geometrie reduite en un facile et briefve pratique* ... (Paris: Charles Hulpeau, 1626).

345 Giuseppe Boffito, *Paolo dell'Abbaco e Fabrizio Mordente. Il primo compasso proporzionale costruito da Fabrizio Mordente e la Operatio Cilindri di Paolo dell'Abbaco*, Il Facsmile 6 (Florence: Libreria internazionale, 1931).

346 Favaro, "Per la storia del compasso di proporzione."

347 Rose, "The Origins of the Proportional Compass from Mordente to Galileo."

But who was Fabrizio Mordente and under what circumstances did he invent his proportional compass? Born in Salerno in 1532, Mordente spent his youth travelling the world. During his explorations, he spent several months aboard Portuguese ships, an experience that would be crucial for the invention of the compass.[348] At the end of the sixteenth century, the Portuguese empire was still one of the largest colonial empires in the world, and its fleet was equipped with the most common astronomical instruments. In the age of world explorations, the issue of increasing the precision of astronomical instruments was of crucial importance not only for astronomy but also for navigation. The success of long-distance journeys across the oceans depended on instruments like the astrolabe, which were used by the sailors to determine their position in the open sea. However, the accuracy of these instruments was far from perfect. It is worth remembering that, in navigation, there is little room for error, as at sea level one minute of arc along the Earth's equator equals approximately one nautical mile (1.852 km or 1.151 mi). By closely studying the astronomical instruments, Mordente would have realized that their precision depended on the number of parts into which the degree of arc was divided. The more the parts of the degree, the more the precision of the instrument. Theoretically, Mordente's compass was capable of dividing the degree of arc into an infinite number of parts. For this reason, when Mordente published his first treatise, he presented the compass as a way to increase the precision of astronomical instruments.[349]

Mordente continued to advertise his compass by publishing three other treatises, the most important of which was *La Quadratura del cerchio, la Scienza de' residui, il Compasso et riga di Fabrizio, et di Gasparo Mordente fratelli salernitani* (*The Quadrature of the Circle, the Science of Remainders, The Compass and Ruler by Fabrizio and Gasparo Mordente, Salerno Brothers*).[350] Published in Antwerp in 1591, this treatise was the result of a joint effort by the two Mordente brothers, Fabrizio and Gasparo, and was composed of three parts.

---

[348] Camerota, *Il compasso di Mordente*, 25–26.

[349] Fabrizio Mordente, *Modo di trovare con l'Astrolabio, ò Quadrante, ò altro instrumento, oltre gradi, intieri, i minuti, et secondi, et ognaltra particella* (Venice: Paolo Forlani, 1567).

[350] Fabrizio Mordente and Gasparo Mordente, *La Quadratura del cerchio, la Scienza de' residui, il Compasso et riga* (Antwerp: Philippe Galles, 1591).

The first part presented an attempt to solve the problem of the quadrature of the circle. The second part was dedicated to defining what was called the "science of remainders" (*scienza de' residui*), whose objective was to measure even the smallest remainder of the division of a magnitude. Finally, the third part contained a description of the ultimate version of Mordente's compass. No adjustment would be made to the compass in the following years, probably because of the limited circulation of the instrument itself due to its high cost.[351]

As already mentioned, Mordente's compass would rapidly fall into oblivion, only to be rediscovered in the second part of the nineteenth century. Ironically, it was thanks to Bruno, a great admirer of Mordente at first but then one of his most severe critics, that modern-day scholars turned their attention to Mordente's compass. It all started with Berti, who drew attention to Bruno's first two dialogues on Mordente's compass, those in which Bruno praised Mordente for his invention.[352] Several decades later, Yates published the letters of Jacopo Corbinelli to Gian Vincenzo Pinelli, in which the Bruno-Mordente controversy was reported in detail.[353] This was six years before Aquilecchia published Bruno's last two dialogues, those written after Mordente tried to burn all the copies of the first two dialogues.[354] Hence, the encounter of Mordente and Bruno was important not only for the development of Bruno's atomistic geometry, but also for the history of the proportional compass.

3.2 The Bruno-Mordente controversy

In a letter to Gian Vincenzo Pinelli dated September 29, 1585, Jacopo Corbinelli reported that Fabrizio Mordente had arrived in Paris.[355] In his letter, Corbinelli mentioned two printings by Mordente: a single-sheet treatise showing an

---

351 Camerota, *Il compasso di Mordente*, 59.

352 Domenico Berti, *Vita di Giordano Bruno da Nola* (Turin: Paravia, 1868).

353 Yates, "Bruno: New Documents."

354 Bruno, *Due dialoghi sconosciuti*.

355 Rita Calderini De Marchi, *J. Corbinelli et les Erudits Francaise d'après la correspondance inédite Corbinelli-Pinelli (1566-1587)* (Milan: Hoepli, 1914), 240.

illustration of the compass,[356] and another work that has not yet been identified. Sheets like that mentioned by Corbinelli were distributed during the public demonstrations that Mordente organized to promote his compass. It is during one of these demonstrations that Bruno, who returned to Paris from London in October 1585, first became acquainted with Mordente's compass. From the beginning, he was very enthusiastic about the invention of his fellow countryman, as reported by the librarian of the abbey of Saint Victor, Guillaume Cotin, in his diary on February 2, 1586.[357] According to the librarian, Bruno hailed Mordente as the "god of geometers." Furthermore, since Mordente did not know Latin, Bruno offered to write a Latin exposition of his compass, as mentioned above.

The two dialogues entitled *Mordentius* (*Mordente*) and *De Mordentii circino*(*On Mordente's Compass*) were published shortly thereafter. For a long time, these two dialogues were the only known texts where Bruno spoke of Mordente's compass. This inevitably influenced early interpretations of the relationship between Bruno and Mordente, giving the impression that Bruno's opinion about Mordente was overall positive. For example, writing in 1927, Olschki argued that Bruno praised Mordente "more than any other thinker or mathematician, more than Paracelsus and Copernicus, Cusanus and Plato."[358] Olschki could not know that the two Italians engaged in a discussion as soon as Bruno started writing the first two dialogues, as recorded by Corbinelli in a letter to Pinelli dated February 16, 1586:

> I send you these two writings; our Fabritio is in a brutal rage against the Nolan and wishes to avenge himself in every way: but it does not seem to me that he has all the right on his side because, although the Nolan honors himself with the discourse of his, at the same time he also celebrates, and makes the author, him who is the author. The

[356] Fabrizio Mordente, *Il Compasso e Figura Di Fabrizio Mordente …* (Paris: Jean le Clerc, 1585).

[357] Vittorio Spampanato, *Documenti della vita di Giordano Bruno*, vol. 3 (Florence: Leo S. Olschki, 1933), 655.

[358] Olschki, *Giordano Bruno*, 76–77.

> other writing is considered mad by those who know and there are not many of them to be found. Of such, patience.[359]

The Pinelli-Corbinelli correspondence has been extensively studied, although a complete edition of it has not yet been published. Rather, scholars have tended to focus on micro-regions of this large correspondence, relying on it to shed light on specific historical events, such as the Massacre of St. Bartholomew[360] or the relationship between Corbinelli and the French intellectual environment.[361] Likewise, Yates limits herself to the analysis of the letters of Corbinelli to Pinelli containing references to Bruno with the aim of "anchor[ing] him [Bruno] in the contemporary scene."[362] In doing so, Yates manages to provide a chronology of the discussion between Bruno and Mordente.

The above letter shows that Bruno must have been on good terms with Pinelli, who defended him, contending that Mordente did not have "all the right on his side." The letter also offers insights into the reasons that led to the discussion between Bruno and Mordente. Pinelli had entrusted Corbinelli with the task of supplying books and manuscripts for the library he was establishing in Padua. In fulfilling this task, Corbinelli attached two writings to the above letter. Undoubtedly, the first writing had triggered the discussion between Bruno and Mordente. However, we cannot be sure that the writing in question was a printed copy of Bruno's first two dialogues, or only a draft of them, as assumed by Yates.[363] Be that as it may, Mordente possessed the same writing and certainly did not appreciate its content. The letter provides no information about the second writing that Corbinelli sent to Pinelli. Yates proposes that "Corbinelli is here being purposely vague and mystifying, as often in these letters when he is sending his employer something which he does not want to fall into inquisitorial

---

359 Yates, "Bruno: New Documents," 178.

360 Pio Raina, "Jacopo Corbinelli e la strage di S. Bartolomeo," *Archivio storico italiano,* no. 21 (1898), 54 ff.

361 Calderini De Marchi, *Corbinelli et les Erudits*.

362 Yates, "Bruno: New Documents," 176.

363 Yates, 178–79.

hands in Italy."[364] Hence, if Yates is correct, the second writing was unrelated to the discussion between Bruno and Mordente.

The first two dialogues that Bruno wrote on Mordente's compass must have been published prior to April 14, 1586. On that date, indeed, Corbinelli wrote to Pinelli:

> The Nolan has printed I know not what in which he extols to heaven Fabritio's compass, but as a philosopher it seems that he wants to regulate the judgement and the expression of the said Fabritio, as though to show him that he has need of someone who should expound his arguments better (that he can himself). Fabritio fulminated with rage and wanted to print, but he gets muddled both when he speaks and when he writes. And the Nolan, who knew this, was prepared to scold him well in the second dialogue. It seems to me that the affair is over, and that both of them are content to go no further. It has cost Fabritio several crowns to buy up the Nolan's dialogue and have it burned. If I can get hold of a copy I will send it to your excellence.[365]

When Yates first published this letter, she could not know that Bruno published two more dialogues on Mordente's compass in addition to the first two. For this reason, she assumed that the first two dialogues were published separately, and that Bruno was preparing the second dialogue by the time this letter was written. Yates' hypothesis was corrected by Aquilecchia once he rediscovered the other two dialogues.[366] As can be seen from the dates of Corbinelli's letters, the tension between Bruno and Mordente escalated very quickly. Within less than two months (from February 16 to April 14) Bruno published the first two dialogues on Mordente's compass. It was then that Mordente, annoyed by what Bruno had to say about his compass, sought to acquire and burn all the copies of Bruno's

---

[364] Yates, 179.

[365] Yates, 179–80.

[366] Giovanni Aquilecchia, "Introduction," in *Due dialoghi sconosciuti e due dialoghi noti:* Idiota triumphans, De somnii interpretatione Mordentius, De mordentii circino, by Giordano Bruno, ed. Giovanni Aquilecchia (Rome: Edizioni di storia e letteratura, 1957), xix.

dialogues. As a response, Bruno started working on two new dialogues to defend himself from Mordente's attacks.

The publication of Bruno's last two dialogues on Mordente's compass, titled *De idiota triumphans* (*The Triumphant Illiterate*) and *De somni interpretatione* (*The Interpretation of a Dream*), must have occurred before June 6, 1586, on which date Corbinelli wrote to Pinelli:

> The Nolan still against Mordente, and new dialogues. Now he is engaged in destroying the whole of the peripatetic philosophy, and, from what little I understand of it, it seems to me that he delivers his arguments very well. I think that he will be stoned by this University. But soon he is going to Germany. Enough that in England he has left very great schisms in those schools. He is a pleasant companion, an Epicurean in his way of life.[367]

Compared to the earlier letters, Corbinelli here says little about the Bruno-Mordente controversy, except that it is still going on. Rather, he draws attention to another event concerning Bruno, his public dispute against Aristotelian philosophy held at the Collège de Cambrai on May 28-29, 1586.[368] In contending that during the dispute Bruno had "deliver[ed] his arguments very well" and in calling him "a pleasant companion, an Epicurean in his way of life", Corbinelli once again expressed his sympathy for Bruno. He also spoke of Bruno's stay in England, which had caused "very great schisms in those schools." As noted by Yates,[369] Corbinelli was probably referring to the university of Oxford, where, in 1583, Bruno had taught for a few weeks before being charged of plagiarism. Corbinelli foresaw that Bruno would also be removed from the university of Paris because of the great clamor that had accompanied his anti-Aristotelian dispute. Bruno himself seemed to be aware of this, which explains why he was planning to go to Germany. However, the polemic with the Aristotelians may not be the only reason for Bruno's departure from Paris. Mordente may also have played a role,

---

[367] Yates, "Bruno: New Documents," 182.

[368] Cotin's diary informs us that the dispute took place on that date. See Spampanato, *Documenti*, 3:43–46..

[369] Yates, "Bruno: New Documents," 183.

having decided to abandon the circle of Henry of Navarre to support the Duke of Guise.[370] Bruno, on the other hand, had remained faithful to Henry of Navarre. Suddenly, the polemic between Bruno and Mordente had taken a political turn, and Bruno may have decided to retreat rather than engage in this sort of fight. He would be safe in Germany by the time the War of the Three Henrys broke out in 1587.

That of Corbinelli is the only extant account of the Bruno-Mordente controversy. Unfortunately, this account provides little information on how the controversy started, or why Mordente was outraged by Bruno. From the Pinelli-Corbinelli correspondence, one gains the impression that Mordente did not accept Bruno's interpretation of the compass. This is also confirmed by the 1591 treatise written by the two Mordente brothers wherein Bruno was defined as a "shadow of philosopher" because of his failure to understand the theory underlying the use of the compass.[371] In addition, Corbinelli informs us that the controversy started as soon as Bruno's first two dialogues began to circulate. Thus, Mordente's anger must have been provoked by something that Bruno had written in the first two dialogues. Given the lack of other documents, we can only turn to Bruno's dialogues to better understand the reasons for Mordente's anger, aware of the fact that the information gathered from Bruno's dialogues will be necessarily biased.

The first two dialogues that Bruno wrote on Mordente's compass were published together at the beginning of 1586 by Pierre Chevillot in Paris. The protagonists of the two dialogues were Giovanni Botero (the author of the *Reason of State*) and Mordente himself.[372] By the time Bruno wrote the two dialogues, Botero, Mordente and Bruno were all members of the circle surrounding Henry of Navarre (the future King of France). Only later, and probably as a result of the controversy with Bruno, would Mordente switched sides and went on to support

---

370 Yates, 186.

371 See Mordente and Mordente, *La Quadratura del cerchio*: "Ma se per sorte alcuna ombra di filosofo, per mostrare anch'ella di sapere…" Quoted from Camerota, *Il compasso di Mordente*, 54.

372 For the identification of Botero, the character of the first two Mordente dialogues, with the author of the *Reason of State*, see Aquilecchia, "Introduction," 1957, xiii–ix.

Henry of Navarre's rival, the Duke of Guise. In the preface to the dialogues, Bruno explained his decision to write about Mordente's compass by presenting its inventor as one of those "Mercuries" sent by the divine providence "to remedy the fatigue and indigence of mortals."[373] This was the best compliment that Mordente could receive from Bruno, who also saw himself as a Mercury, a divine messenger entrusted with the mission of revealing the truth.[374] The figure of Mercury, the Roman equivalent of the Greek god Hermes, was central to the Hermetic tradition that influenced many aspects of Bruno's thought—although the importance of this tradition as a Brunian source has been gradually reduced ever since Yates first drew attention to it.[375]

In addition, Bruno borrowed from the Hermetic tradition the idea that the history of human knowledge followed a circular path, which caused the truth to be regularly forgotten and rediscovered. For example, in the *Spaccio de la bestia trionfante* (*Expulsion of the Triumphant Beast*, 1584) Bruno argued against the Judeo-Christian tradition that had established its own religion at the expense of that of the old Egyptians. On the contrary, Bruno advocated a return to the religion of the old Egyptians who, worshipping animals and the living forces of nature, expressed the belief, to which Bruno subscribed, that "*natura est deus in rebus*."[376] Likewise, Bruno credited Mordente with the revival of a mathematical knowledge that was long forgotten.[377] However, he did not reveal the sources of Mordente's knowledge, arguably because he thought, as would be made clear in

---

373 *Mordentius*, 31: "Ut verum, ita et vulgatum satis est, Deum providentem certis quibusdam temporibus Mercurios, quibus mediantibus labori et inopiae mortalium succurrat, e caelo mittere."

374 For Bruno's self-identification with Mercury, see Michele Ciliberto, "Giordano Bruno, angelo della luce tra furore e disincanto," in *Dialoghi filosofici italiani*, by Giordano Bruno, ed. Michele Ciliberto (Milan: Arnoldo Mondadori, 2000).

375 Yates, *Bruno and the Hermetic Tradition*.

376 Giordano Bruno, *Dialoghi filosofici italiani*, ed. Michele Ciliberto (Milan: Arnoldo Mondadori, 2000), 631.

377 *Mordentius*, 31: "Fabricius Mordens Salernitanus inventionum mechanicarum parens non modo huiusce generis artes collapsas instaurat, emortuas revocat, mutilas perfecit: sed et quasdam pro impossibilitatis specie numquam intentatas exsuscitat."

De idiota thriumphans, that the idea of the compass had occurred to Mordente in an unconscious way. Lacking this information, it remains unclear on what basis Bruno claimed Mordente's mathematical knowledge to be old, raising the question whether he claimed so only in an attempt to give more credibility to Mordente's invention.

Having described the divine character of Mordente's invention and the forgotten knowledge that he had unearthed, Bruno went on to provide a portrait of the man behind the compass. As a matter of fact, this portrait was not entirely flattering. According to Bruno, Mordente was a quite person, who "speaks with facts, teaches by doing, and remaining silent goes further than anyone else could go by reasoning."[378] Bruno, however, was determined to break Mordente's silence, and translate into words what Mordente showed during his public demonstrations of the compass. The innovativeness of the compass, Bruno declared "with all due respect," was such that Mordente himself was not fully aware of it.[379] Probably, Bruno referred to the possibility of using Mordente's compass to demonstrate the existence of the geometric minimum. The fact that Mordente regarded his compass as 'only' a measurement instrument would have prevented him from seeing this possibility. But the truth was that Mordente was not interested in discovering the minimum. As mentioned earlier, his objective was to create an instrument that could measure the degree of arc down to its smallest fractions. In his works, there was no trace of the concept of the minimum nor of a theory of the use of the compass. Only in the wake of the controversy with Bruno, Mordente would make an effort to present his ideas in a more formal way, developing what he would call the "science of remainders."

It is sufficient to read these first lines to understand why Mordente tried to destroy all the copies of Bruno's first two dialogues. Despite the appreciation expressed for the work of the "divine" Mordente, Bruno showed little respect for his fellow countryman. Furthermore, Mordente was outraged by Bruno's attempt to impose his interpretation of the compass. But there was more to it. Not only

---

[378] *Mordentius*, 31: "[Mordens] actione loquitur, operatione docet: dum eo ipse silendo promovet, quo caeteri universi nunquam ratiocinando potuere."

[379] *Mordentius*, 32: "[In Mordentis geometriae partibus] adeo pregnans atque fecunda praxis continetur, ut illum mihi forte (quod citra iniuram dictum velim) plus quam putare et ipse possit invenisse."

did Bruno state that Mordente had not fully understood his own work. He also claimed that what the two Mordente brothers had written on the compass was "so inelegant, so rough, ordered in such a contorted way, and based on such an ignorant doctrine, that one can easily see how it is as if nothing has never been published."[380] By the time Bruno published his dialogues in 1586, three treatises on the compass were already circulating: the first by Fabrizio Mordente published in Venice in 1567, the second by Gasparo Mordente (Fabrizio's brother) published in Antwerp in 1584, the third by Fabrizio published in Paris in 1585. Camerota notes that Bruno's critique of Mordente's writings was all the more unfair, as he heavily relied on the 1584 treatise to describe the operations of the compass.[381]

3.3 Bruno's first dialogue: *Mordentius*

Bruno's first dialogue on Mordente's compass was entitled *Mordentius: sive de geometricis fractionibus ad exactam cosmimetriae praxim conducentibus*. (*Mordente: or on the Geometric Fractions Leading to the Exact Method for the Measurement of the Cosmos*). It was devoted to presenting the method developed by Mordente to measure the smallest fractions of geometric magnitudes. This method was based on two axioms. According to the first axiom, two magnitudes were in the same ratio as their corresponding parts. For example, if two segments were in a ratio of 1:3, this meant that the half of the shorter segment was three times shorter than the half of the longer segment. By the same token, if we knew that a circumference was divided in 16 equal parts, and we wanted to know the value of a fraction that was smaller than one-sixteenth of the circumference, we could take the length of that fraction and apply it 16 times to the circumference (The example is taken from chapter V of the *Mordentius*). Proceeding in this way, we would cover a portion of the circumference, equal to a certain number of entire parts. This number would be the value of the fraction. If there were a remainder, the same operation could be repeated indefinitely until no portion of

380 *Mordentius*, 32–33: "De circino autem aliquid editum extat, quod (per meam fidem) tam rude, tam crassum, tam contorto ordine, tam ignorante doctrina scriptum constat: ut ipsum certe tanquam non editum ideo quisque facilissime iudicare posset."

381 Camerota, *Il compasso di Mordente*, 90.

the line was left over.

If the first axiom of Mordente's method was mathematical in its character, the second axiom was more philosophical and could be traced back to medieval Scholasticism:

> The second is the common philosophical axiom that in natural and artificial objects a minimum and a maximum relative to their form are to be determined, which is why those who divide naturally as well as artificially do not happen to go to infinity.[382]

What Bruno presented as an axiom commonly accepted by philosophers was the cornerstone of the medieval theory of *minima naturalia*. Although different versions of this theory were developed especially in the thirteenth and fourteenth century, the idea of *minima naturalia* had only one source: Aristotle's *Physics*, Book I, Chapter IV (187b13 – 188a5). There, in arguing against Anaxagoras and his theory that everything was in everything, Aristotle claimed that the form of natural beings was confined with certain limits. The lower limit—the *minimum naturale*—indicated the smallest form that a natural being could assume without losing its essence. As John Philoponus put it in his commentary on Aristotle's *Physics*: "no man has the size of a fist or a finger or a grain, because if something is too small it cannot receive a form."[383] The corollary of this theory was that, at least as far as their form was concerned, natural beings could not be infinitely divided, otherwise there would be no limit to the smallness of their forms. This corollary was what gave Mordente (the fictional character created by Bruno, not to be confused with the instrument maker) confidence in the success of his method, assuring him that the division of a magnitude into its smallest fractions would come to an end. It is worth stressing that this was not the way Mordente

382 *Mordentius*, 38: "Secundum commune philosophorum axioma quod in subiectis phisicis et artificialibus determinatum est ad eorum formas maximum atque minimum: unde sicut vnon naturaliter ita nec artificose dividentibus accidit in infinitum facere progressum."

383 Quoted from Ruth Glasner, "Ibn Rushd's Theory of *Minima Naturalia*," *Arabic Sciences and Philosophy* 11, no. 1 (March 2001): 15.

conceived his compass, but it was Bruno who attributed this interpretation to him.

It is most likely that the axiom on *minima naturalia* was not included in the original method developed by Mordente, but it was added as a result of Bruno's intervention, for no reference was made to this axiom in Mordente's previous works. On the other hand, it should be noted that in *De idiota triumphans* Bruno would accuse Mordente of having misunderstood the theory of *minima naturalia,* showing how this theory ran counter to what Mordente aimed to demonstrate. For this reason, De Bernart argues that the axiom on *minima naturalia* was the work of Mordente, and that Bruno reported the axiom in the *Mordentius* only to criticize it in the *De idiota triumphans*.[384] In claiming so, De Bernart implicitly assumes that the project of writing *De idiota triumphans* dated back to the time when Bruno was composing the *Mordentius*. Yet this hypothesis is not supported by the Pinelli-Corbinelli correspondence, which instead informs us that Bruno decided to write the last two dialogues in response to Mordente's attacks on the first two. Rather, I believe that Bruno did not notice that Mordente's findings did not sit well with the theory of *minima naturalia* until a later stage, but then he laid the blame on Mordente instead of admitting that he had made a mistake. Thus, the axiom on *minima naturalia* should be regarded as a Bruno's addition to Mordente's method, despite the objections that Bruno himself would raise to this axiom in the later *De idiota triumphans*.

As one can see from the *dramatis personae* included in John Murdoch's paper on *The Medieval and Renaissance Tradition of Minima Naturalia,* several authors contributed to developing the theory of *minima naturalia* over the centuries.[385] Indeed, the theory of *minima naturalia* was discussed as late as the sixteenth century by the likes of Luis Coronel (d. 1531), Benedict Pereira (1536 – 1610) and Francisco de Toledo (1532 – 1596). As shown by Murdoch, different definitions of *minima naturalia* were given during the middle ages, each corresponding to a different group of authors. Among them, there were also those who associated the concept of *minima naturalia* with the issue of minimum

[384] De Bernart, Numerus quodammodo infinitus, 173–77.

[385] John E. Murdoch, "The Medieval and Renaissance Tradition of *Minima Naturalia*," in *Late Medieval and Early Modern Corpuscular Matter Theories*, ed. Christoph H. Lüthy, John E. Murdoch, and William R. Newman (Leiden: Brill, 2001), 99–101.

limits. Bruno himself established this connection in the *Mordentius*, claiming that the existence of a *minimum naturale* set a limit to the division of natural beings. However, it is hard to say whether Bruno was acquainted with what Murdoch calls the "limit decision literature."[386] Given Bruno's Dominican education, it is more likely that Thomas Aquinas and Averroes shaped his understanding of *minima naturalia*.

Aquinas discussed the theory of *minima naturalia* in his *Summa theologiae* rather than in his commentary on Aristotle's *Physics*. Murdoch notes that this choice reflects Aquinas's awareness of the relation between *minima naturalia* on one hand, and substantial forms on the other hand.[387] As for Averroes, he developed his theory of *minima naturalia* especially in the middle commentary on Aristotle's *Physics*.[388] A Latin translation of this text by Jacob Mantino (d. 1549) was included in the Junta edition of Aristotle's *Opera omnia*, a copy of which was possessed by the monastery of San Domenico Maggiore in Naples where Bruno received his education.[389] Averroes borrowed aspects of his theory of *minima naturalia* from the *mutakallimūn*, a group of ninth-century Islamic theologians that defended a form of geometric atomism similar to that of Bruno. In particular, Glasner has demonstrated that Averorres was indebted to the *mutakallimūn* for his idea of the minimum, which he adopted "taking it out of the atomistic context and adjusting it to the Aristotelian environment."[390]

Since the translation by Mantino only covered the first three books of Averroes's middle commentary on Aristotle's *Physics*, Bruno's knowledge of Averroes's theory of *minima naturalia* was bound to be limited. However, references to the theory were made in Book III of the middle commentary, where

---

386 Murdoch, 116–22.

387 Murdoch, 101.

388 For an overview of Averroes's physics, see Ruth Glasner, *Averroes' Physics* (Oxford: Oxford University Press, 2009).

389 For a reconstruction of the library of San Domenico Maggiore, see Eugenio Canone, "Contributo per una ricostruzione dell'antica 'libraria' di S. Domenico Maggiore," in *Giordano Bruno: gli anni napoletani e la* peregrinatio *europea: immagini, testi, documenti*, ed. Eugenio Canone (Cassino: Università degli studi di Cassino, 1992), 191–246.

390 Glasner, "Ibn Rushd's *Minima Naturalia*," 26.

we read that: "magnitude is infinitely divisible qua matter, not qua form; qua form its divisibility is limited."[391] Likewise, in the long commentary on the *Physics*, which was also included in the Junta edition, Averroes claimed that "a line as a line can be infinitely divided. But such a division is impossible if the line is taken as made of earth."[392] Reading these texts, Bruno would have thought that Averroes's theory was still Aristotelian in that it was grounded in the concept of 'formal' minimum. Seeking the minimum magnitude rather than the formal minimum, Bruno was more in line with the atomistic sources of Averroes, although it is unlikely that Bruno could have been familiar with the doctrines of the *mutakallimūn*.[393]

Again, it should not be forgotten that this discussion on *minima naturalia* had nothing to do with Mordente's compass. Indeed, as already mentioned, Mordente's objective was to create an instrument capable of dividing the degree of arc into a potentially infinite number of parts. As such, Mordente's compass did not challenge the Aristotelian view of the continuum, which in fact provided a theoretical justification for the use of the compass. Nor was Mordente committed to the theory of *minima naturalia*, as we have seen that there was no trace of this theory in Mordente's writings prior to his encounter with Bruno. Rather, it was Bruno who tried to use Mordente's compass against Aristotle, turning it on its head and taking it as an argument in favor of his atomistic view of the continuum.

3.4 Bruno's third dialogue: *De idiota triumphans*

De idiota triumphans was one of the last two dialogues that Bruno wrote in response to Mordente's attacks on the first two. Meanwhile, the tension between Bruno and Mordente had rapidly escalated, and Bruno's purpose in writing *De idiota triumphans* was to criticize Mordente's method. If, in the first part of *De idiota triumphans,* Bruno's criticism focused on more superficial aspects of Mordente's method (such as the actual number of operations that could be carried out with the compass), in the last part the focus shifted to its foundation.

---

391 Quoted from Glasner, 18.

392 Quoted from Glasner, 18.

393 For an overview of the doctines of the *mutakallimūn*, see Dhanani, *The Physical Theory of Kalām*.

In particular, Bruno took issue with what, in the *Mordentius*, he had presented as the second axiom of Mordente's method according to which natural and artificial beings had a minimum form and thus could not be infinitely divided in relation to their forms. Bruno started by noticing that this argument only applied to natural beings, and it could be extended to artificial beings insofar as these were considered as formal and not as artificial entities. Therefore, Bruno concluded, it was wrong to speak of artificial beings as distinguished from natural beings, because as formal entities they behaved in the same way. This remark tells us that Bruno was familiar with the Aristotelian theory of *minima naturalia*, for in what was considered the source of all the arguments on *minima naturalia* (Physics IV.1), Aristotle referred to natural and not to artificial beings.

For Bruno, the major flaw in the understanding of the theory of *minima naturalia* that he had attributed to Mordente was that it ignored the distinction between formal minimum and minimum magnitude. Bruno noted, and rightly so, that the supporters of *minima naturalia* did not consider "the minimum magnitude or the minimum continuous quantity, which for them cannot be found, but the minimum substance in which the form of each species can be retained."[394] The reason why especially medieval scholars emphasized the formal character of *minima naturalia* was to distinguish between two kinds of divisibility of natural beings, depending on whether they were viewed as continuous or as formal entities. The former case was associated with infinite divisibility, while the latter with finite divisibility. The source of this distinction was Aristotle, who in *Physics* IV (187b13 – 188a5) claimed that natural beings could not be infinitely divided without losing their form, while in *Physics* VI (231b14 – 15) he argued for the infinite divisibility of the continuum. As documented by Anneliese Maier[395] and John Murdoch,[396] medieval scholars

---

394 *De idiota triumphans,* 14: "Non intelligens quod dicit ratione respectuue formarum, declarare sensum illorum philosophorum non respicere minimum magnitudinis seu quantitatis continuae, quod numquam credunt incurri posse: sed minimum subietum in quo possit saluari forma cuiusque speciei."

395 Anneliese Maier, *Die Vorläufer Galileis im 14. Jahrhundert: Studien zur Naturphilosophie der Spätscholastik*. (Rome: Edizioni di Storia e Letteratura, 1966).

396 Murdoch, "Tradition of *Minima Naturalia*."

rephrased this Aristotelian distinction in different ways, speaking for example of natural beings as divisible in potency or in act.

In the first dialogue on the compass, Bruno had Mordente claim that the theory of *minima naturalia* provided the foundation for his method because this latter led to identify the formal minimum, or more specifically, the minimum fraction of a curved or a straight line. On the contrary, in *De idiota triumphans*, Bruno argued that Mordente's method showed the minimum magnitude, the existence of which was denied by the supporters of *minima naturalia*. Bruno's argument ran as follows:

> If one refers to the line or the surface to be divided, the assumption [of Mordente], which some philosophers accept as a principle, means that those who divide mechanically happen to lose first the perception of quality and then that of quantity or extension. For this reason, there is no difference in taking the minima or the almost minima of a curved or a straight line, of a regular and irregular figure. Hence, what is determined in its form is not limited in its matter. This is why Mordente should be considered a god.[397]

What Bruno meant was that when dividing a line down to its smallest fractions a point was reached where it was no longer possible to determine the shape of the fractions. As the size of the fractions decreased, we lost the ability to distinguish between curved and straight, and all the fractions ended up having the same indefinite shape to our eyes. This could pose a challenge to Mordente's method, insofar as if a fraction was too small it could not be measured with the compass. As reported by Bruno, Mordente solved this problem by simply measuring the remaining fraction, and then subtracting this value to the whole length of the

---

[397] *De idiota triumphans,* 14-5: "Quinimmo, si ad superficiem vel lineam dividendam respicere velit, illud acceptum pro principio a quibusdam philosophis: significat in proposito, quod mechanice dividentibus prius contingat perdere sensum qualitatis quam molis seu quantitatis, quia tandem non differt accipere minima seu prope minima lineae curvae atque rectae, regularis atque irregularis: et ideo determinatum secundum formam, nondum est terminatum secundum materiam: Unde Mordentius deificetur."

line.[398] Here, the fact that the smallest fraction turned out to have no defined shape was taken as proof that beyond the perceivable forms of curved and straight there was a common, shapeless minimum magnitude. This shapeless minimum was regarded as the matter of the line, which, in the above quotation, was defined as "determined in its form" (i.e. curved or straight) but "not limited in its matter."[399] The merit of Mordente's method was that it revealed this minimum, shapeless magnitude (or "ultimate fraction," as Bruno called it) standing on the threshold of perception, as it "teaches us to divide down to the ultimate sensible element and, with such ease as I have demonstrated in the specific dialogue, leads us to the ultimate fraction."[400]

A remark is in order. It is true that Bruno's argument worked insofar as curved and straight were considered as perceivable forms and not as abstract geometric determinations. For, in classical Euclidean geometry, curved and straight were not reducible to each other. However, it should be noted that one of the ancestors of the modern calculus, the method of exhaustion, was also based on the approximation of curved and straight. Traditionally ascribed to the ancient Greek mathematician Eudoxus of Cnidus, who in turn seemed to have borrowed the idea from Antiphon the Sophist, the method of exhaustion consisted in measuring the area of a circle by inscribing within it a regular polygon, the number of whose sides was progressively increased until the area of the inscribed polygon 'exhausted' that of the circumscribed circle. Yet, even when properly carried out, this procedure did not make the polygon coincide with the circle, but at best it reduced the difference between the two areas so that it could be neglected. To this extent, the method of exhaustion implied a certain degree of approximation and, as noted by Boyer, "the gap between the curvilinear and the rectilinear still remain[ed] unspanned."[401] The same can be said of the argument used by Bruno in *De idiota triumphans*.

---

[398] *De idiota triumphans,* 42.

[399] *De idiota triumphans*, 15.

[400] *De idiota triumphans,* 15: "[Mordenti] ad ultimum usque sensibile dividere doces, et tanta facilitate, quantam in dialogo proprio explicaui, ita ultimum fractionum insinuas."

[401] Boyer, *History of the Calculus*, 35.

What did Bruno's argument mean in geometric terms? Generally speaking, Bruno posited the existence of geometric minima, i.e., infinitely small quantities, which were extended but indefinitely shaped. Such minima were the building blocks of *all* geometric objects, regardless of whether they were regular or irregular polygons, curved or straight lines. In the years following the controversy with Mordente, Bruno would go on to develop this intuition into an atomistic geometry. However, differently from what he would do in *De minimo*, in the dialogues on Mordente's compass Bruno did not equate geometric minima to extended circular points.[402] This difference was of crucial importance, because claiming that geometric objects were composed of extended points caused several problems in geometry, such as the impossibility of accounting for incommensurable magnitudes. Therefore, in the dialogues on Mordente's compass Bruno developed a theory which, when compared to that set forth in *De minimo*, was more coherent from a geometric viewpoint. This raises the question of why Bruno changed his mind with regard to the status of geometric minima. Since this question will be answered in the next chapter, I will limit myself to the observation that the theory developed in *De minimo* was at the same time a geometric, metaphysical and physical theory. Thus, we can assume that Bruno was probably led astray from the geometric path taken in the dialogues on Mordente's compass by the necessity of combining different kinds of theoretical elements.

[402] As a matter of fact, the idea of a "minimum circle" could already be found in the fourth dialogue on Mordente's compass. See for instance, *De somni interpretatione,* 21: "An non individuum, (*quod minimus circulus est*) per sex puncta in plano tangibile erit, ni mathematica ratione infinitum progressum libeat adoriri?" (emphasis added). It may be no coincidence that, in this context, Bruno spoke of a "minimum circle" and not of a "circular minimum." Indeed, it should be noted that Bruno's objective in writing *De somni interpretatione* was to propose his solution to the problem of the quadrature of the circle. Furthermore, in the dialogues on Mordente's compass, Bruno did not explicitly claim that the minimum had a circular shape—as he would do in *De minimo*—while he was adamant that the minimum fractions of straight and curved lines were shapeless.

3.5 Bruno and the infinitesimals: A reappraisal

Most of the objections against Bruno's mathematics were raised in response to the version of it expounded in Bruno's *De minimo*. However, critics of Bruno's mathematics would probably have had a different picture of this theory, if they had also considered the dialogues on Mordente's compass. This is well exemplified by the case of Olschki. In the introduction, we saw that Olschki raised two objections to Bruno's mathematics. The first objection was that Bruno's mathematics envisaged different kinds of minima, while there should be only one sort of infinitesimal quantity. The second objection is that Bruno's mathematics was not linked to a theory of motion. For these reasons, Olschki concludes, the Brunian concept of the minimum cannot be considered a forerunner of the infinitesimals. If one reads *De minimo*, one cannot help but agree with Olschki. Nevertheless, as soon as the dialogues on Mordente's compass are brought in, one is forced to admit that Olschki's criticisms are unfair. In those dialogues, not only did Bruno claim that there was only one kind of minimum magnitude, but the reason why he claimed so was because he aimed to lay the foundations for the law of planetary motion which he had "dreamt of."[403]

Both the idea of infinitely small quantities and the attempt to account for natural phenomena such as motion belong to the historical development of the calculus. This, of course, does not mean that Bruno should be regarded as on a par with Leibniz, Newton, Cavalieri and all the other seventeenth-century mathematicians who contributed to the development of the calculus. For Bruno's theory of minima was not substantiated by any mathematical application, and it lacked a rigorous mathematical foundation. I am aware that the status of infinitely small quantities was debated in the mathematical community ever since

[403] As mentioned in the introduction to this chapter, Bruno first presented the idea of a law of planetary motion in the last part of the second dialogue on Mordente's compass, which is entitled *Insomnium* (*The Dream*). Then he elaborated on this idea, attempting to provide a mathematical foundation for it based on his geometry of minima, in the fourth dialogue, which was entitled D*e somni interpretatione* (*The Interpretation of the Dream*).

Bonaventura Cavalieri published his treatise on the indivisibles in 1635.[404] I am also aware that different kinds of infinitely small quantities were proposed throughout this period, ranging from the "heterogenous" invisibles of Cavalieri to the "homogenous" indivisibles of Pascal and Barrow.[405] In fact, if we want to draw a parallel between Bruno and seventeenth-century indivisibilists, we could say that he falls within the latter category, his indivisibles being homogeneous with the object they belong to. (Bruno thought that a line, no matter whether straight or curved, was composed of shapeless, two-dimensional extended parts. Cavalieri, on the contrary, thought that the indivisibles of a solid were planes, and the indivisibles of a plane were lines. Thus, Cavalieri's indivisibles had one dimension less than the object they belonged to, and in this sense they were said to be heterogenous).

That being said, I believe that a reading of the dialogues on Mordente's compass shows that, at least at the beginning of his mathematical career, Bruno had a mathematically correct understanding of infinitely small quantities. By this I means an understanding that was not ad odds with Euclidean geometry. This was not the case of the conception of invisibles that Bruno proposed in his *De minimo*, where, as already shown, he claimed that geometric objects were composed of extended points. On the other hand, I do not want to claim that Bruno developed a mathematically correct *theory* of infinitely small quantities, since the foundation of Bruno's theory was more philosophical (see his use of *minima naturalia*) than mathematical. As Rowland puts it: "he was moving toward the calculus himself, and could already outline what would become some of its fundamental ideas in theory, if he could not yet express them in usable

---

[404] Cavalieri, *Geometria indivisibilibus*; For an overview of seventeenth-century debates on indivisibles, see Mancosu, *Philosophy of Mathematics*, 34–64; Malet, *From Indivisibles to Infinitesimals*, 11–22; Amir Alexander, *Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World* (New York: Scientific American/Farrar, Straus and Giroux, 2014).

[405] V. Jullien, "Explaining the Sudden Rise of Methods of Indivisibles," in *Seventeenth-Century Indivisibles Revisited*, ed. V. Jullien (Basel: Birkhäuser, 2015).

equations."[406] To this extent, claiming that Bruno envisioned the infinitesimals would not be out of place.

---

[406] Ingrid D. Rowland, *Giordano Bruno: Philosopher/Heretic* (Chicago: University of Chicago Press, 2009), 194; See also Ingrid D. Rowland, "Giordano Bruno e la geometria dell'infinitamente piccolo," in *Aspetti della geometria nell'opera di Giordano Bruno*, ed. Ornella Pompeo Faracovi, (Lugano: Agorà, 2012), 53–70.

4. Changing conceptions of mathematics and infinity

Introduction: The “realist obstacle” to Bruno’s mathematics

This chapter takes issue with Hélène Vedrine’s view that Bruno’s mathematical failure was due to his belief in the actual existence of mathematical objects in nature, a belief that constitutes the core of what has been called “mathematical” or “Platonic realism.”[407] For this reason, Védrine speaks of a “realist obstacle” hindering Bruno’s mathematics. I challenge this received view of Bruno’s mathematics by charting the evolution of his conception of mathematics. Indeed, I claim that Bruno went from being a moderate realist to being an outright anti-realist over the seven years between the publication of his Italian dialogues (1584) and the publication of his Latin poems (1591). Contrary to what early interpreters have claimed, Bruno’s Latin poems were not a mere repetition of what he had already said in his Italian dialogues.[408] Rather, the years from 1584 to 1591 witnessed major changes in Bruno’s philosophy, changes that also involved his conception of mathematics. Furthermore, I believe that the evolution of Bruno’s conception of mathematics was related to the additions that he made to his theory of the infinite. In my opinion, it was no coincidence that while Bruno lost his (already little) faith in realism, his theory of the infinite gained a new element, namely, the concept of the minimum. It was to turn the concept of the minimum into a mathematical object that Bruno dismissed realism and started a reform of mathematics.

To be clear, this chapter discusses two versions of mathematical realism:

(1) The view that mathematical objects have a separate existence from our mind.

---

[407] Védrine, “L’obstacle réaliste.” For a definition of mathematical or Platonic realism, see Alexander Miller, “Realism,” in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, Winter 2016 (Metaphysics Research Lab, Stanford University, 2016), para. 2, https://plato.stanford.edu/archives/win2016/entries/realism/.

[408] Carlo Monti, “Introduction,” in *Opere latine*, by Giordano Bruno, trans. Carlo Monti (Torino: UTET, 1980), 9.

(2) The view that mathematical models can be used to explain physical phenomena.[409]

I shall argue that Bruno rejected both these versions of realism; more specifically, he denied (2) on the basis of (1). In other words, in Bruno's opinion, mathematical physics (or mathematical astronomy) had no explanatory power because mathematical objects were not found in the natural world. In order to show how this rejection of realism was connected to the introduction of the minimum in Bruno's mathematics, I shall examine three works: *La cena de le ceneri* (1584), *Acrotismus camoerancesis* (1588) and *De minimo* (1591). In addition, the last section of this chapter gives a brief account of Bruno's theory of monads and its relationship to Leibniz's monadology. Leibniz scholars are often struck by the similarities between these two theories, which has made especially early interpreters wonder whether Bruno's theory provided the model for Leibniz's. I believe that this was not the case and I try to show that the similarities between Bruno's and Leibinz's mondalogical doctrines were due to the fact that both these theories were informed by a Pythagorean understanding of the monad.

4.1 Realism vs instrumentalism in *La cena de le ceneri*

Ernan McMullin claims that "to call Bruno a Copernican requires one to empty the label of all content save the assertion that the earth and planets move around

---

[409] In a recent article on early modern astronomy, Çimen distinguishes mathematical realism from what he calls "physical realism," that is, "the belief that a true geometric description (or model) can be made out of a true physical theory, that is, of inquiries into physical reasons." Ünsal Çimen, "On Saving the Astronomical Phenomena: Physical Realism in Struggle with Mathematical Realism in Francis Bacon, Al-Bitruji, and Averroes," *HOPOS: The Journal of the International Society for the History of Philosophy of Science*, October 5, 2018, 3. As we shall see (especially in § 4.3), Bruno would have subscribed to this latter version of realism, insofar as he thought that mathematics should be modelled after physics. Mathematical realism has also associated to the Scientific Revolution and the "mathematization of nature." See John Henry, *The Scientific Revolution and the Origins of Modern Science* (Houndmills, Basingstoke, Hampshire: Palgrave, 2001), 15.

the sun."[410] In particular, McMullin notices how Bruno seemed unable to understand important aspects of the Copernican theory, especially when it came to technical issues. An example is provided by the explanation Bruno gave in *La cena de le ceneri* (*The Ash Wednesday Supper*, 1584) of the Copernican diagram showing the position of the planets in the solar system (Figure 1). The issue at stake was the position of the earth relative to the moon. According to Torquato, one of the two Oxford dons with whom Bruno (or better his fictional character) engaged in a conversation on the Copernican theory, Copernicus placed the earth on the third sphere, with the moon carried around it on an epicycle. On the contrary, Bruno thought that the earth and the moon were located on the same epicycle.[411]

---

[410] McMullin, "Bruno and Copernicus," 64.

[411] Giordano Bruno, *The Ash Wednesday Supper*, trans. Hilary Gatti (Toronto: University of Toronto Press, 2018), 161–65.

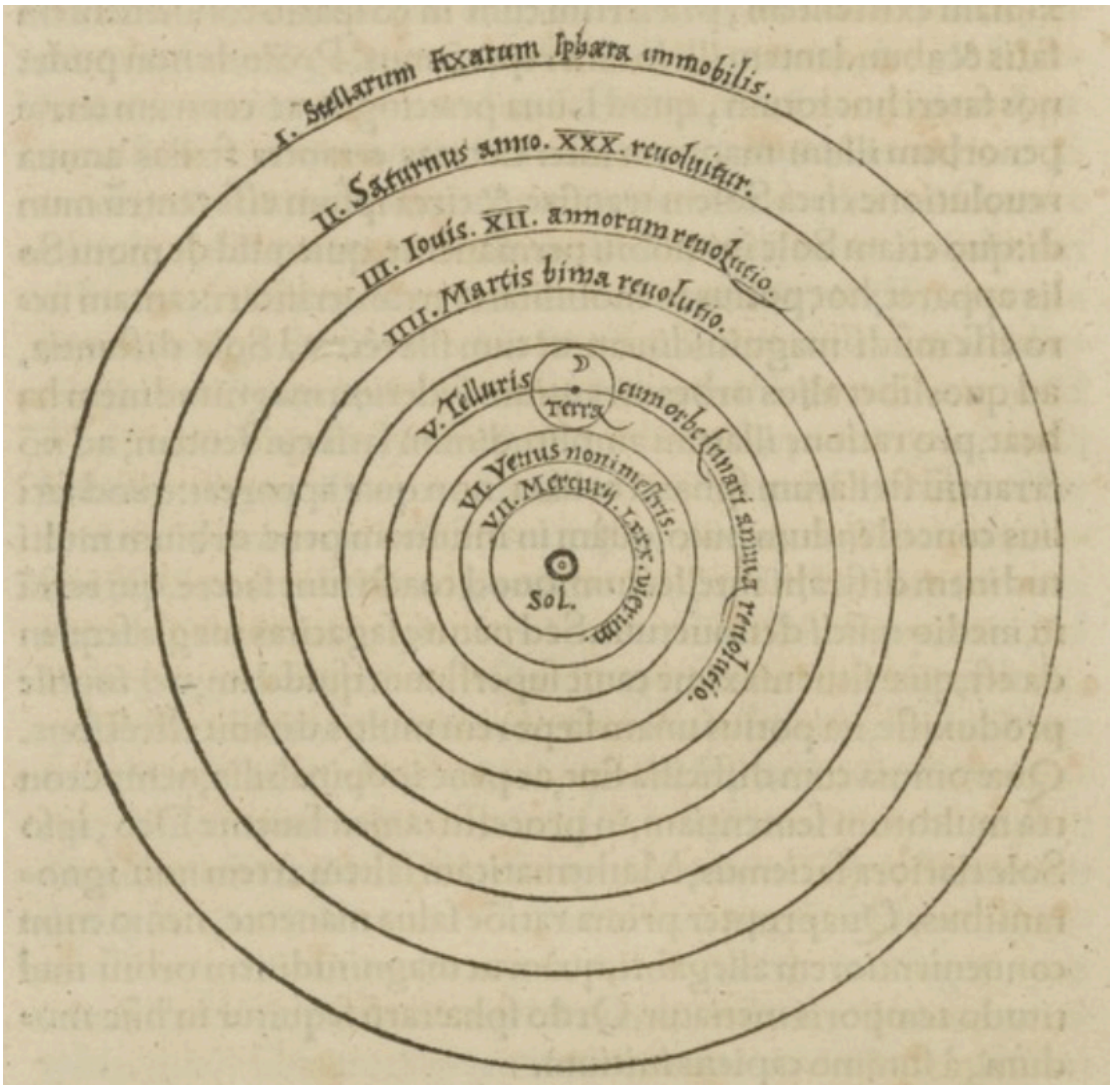

Figure 1: Diagram showing the position of the planets in the solar system. Copernicus, *De revolutionibus*, bk. I.

Although, in *La cena,* Torquato was forced to admit that he was wrong, it was Bruno who made a mistake. In response, scholars have tried to justify him by suggesting "external" causes for his misunderstanding of the Copernican theory. Frances Yates proposed that Bruno was reading Copernicus in a Hermetic way, which led him to see the Copernican diagram as more of a "hieroglyph" than an actual representation of the solar system.[412] More recently, Dario Tessicini has demonstrated that Bruno's view that the earth and the moon were on the same

[412] Yates, *Bruno and the Hermetic Tradition*, 241. For criticisms of the "Yates thesis," see Westman, "Magical Reform and Astronomical Reform"; McMullin, "Bruno and Copernicus."

epicycle could be traced back to the Pythagorean belief in the existence of a counter-earth.[413]

Be that as it may, Bruno's failure to understand technical aspects of the Copernican theory does not alter the fact that he was one of the first to explicitly endorse the idea that the earth moved, at a time when the vast majority of the astronomers and natural philosophers tended to accept only Copernicus' mathematical models, but not the underlying cosmological theory. What is more, as noted by McMullin, Bruno may have been the first to notice in print that the author of the anonymous letter entitled "Ad lectorem de hypothesibus huius operis" and appended to Copernicus' *De revolutionibus* (1543) was not Copernicus himself.[414] We now know that the author was Andreas Osiander (1498 – 1552), who took over the publication of Copernicus' work when Rheticus was forced to leave Nuremberg for Leipzig, where he had been appointed to the chair of mathematics. However, Bruno could not have been aware of this, as it was not until 1609 that Osiander was first identified as the author of "Ad lectorem" by Kepler.[415]

In "Ad lectorem," Osiander famously claimed that the ideas presented in *De revolutionibus* had to be regarded as mere hypotheses aimed at explaining astronomical phenomena in mathematical terms, and not as physical statements concerning the actual structure of the universe. In the belief that Copernicus was

---

413 Dario Tessicini, *I dintorni dell'infinito: Giordano Bruno e l'astronomia del Cinquecento* (Pisa: Fabrizio Serra, 2007), 9–58. See also Miguel A. Granada, "L'héliocentrisme de Giordano Bruno entre 1584 et 1591: la disposition des planètes inférieures et les mouvements de la Terre," *Bruniana & Campanelliana* 16, no. 1 (2010): 31–50; Miguel Ángel Granada, "Introduction," in *La cena de las cenizas*, by Giordano Bruno, ed. Miguel A. Granada (Madrid: Tecnos, 2015).

414 McMullin, "Bruno and Copernicus," 59. For an overview of the authors who first noted that 'Ad lectorem' was not the work of Copernicus, see Michel-Pierre Lerner and Alain-Philippe Segonds, "Sur un '"advertisement"' célèbre: L'*Ad lectorem* du *De revolutionibus* de Nicolas Copernic," *Galilaeana* 5 (2008): 118–20.

415 Lerner and Segonds, "L'*Ad lectorem* du *De revolutionibus*," 120–24. On Osiander, see also Bruce Wrightsman, "Andreas Osiander's Contribution to the Copernican Achievement," in *The Copernican Achievement*, ed. Robert S. Westman (Berkeley: University of California Press, 1975), 213–43.

not only trying his hand at new mathematical models but also proposing an alternative worldview, Bruno criticized the author of “Ad lectorem” for having betrayed Copernicus’ intentions. Early interpreters, especially Duhem, have labelled Osiander’s reading of *De revolutionibus* as instrumentalist, while Bruno and the other scholars who accepted the cosmological aspect of the Copernican theory have been defined as realist. Since the divide between instrumentalism and realism in early modern astronomy has attracted a great deal of scholarly attention, I shall give a brief account of the historiographical debate on these two epistemological stances before going on to address the question of Bruno’s realism.

In modern scholarship, Pierre Duhem was the first to acknowledge and oppose what, in his understanding, was Copernicus’ realism. In *To Save the Phenomena* (1908), Duhem claimed that Copernicus was a realist insofar as he believed that his astronomical hypotheses were both true and demonstrable. However, Duhem’s claim did not sit well with the findings of Noel Swerdlow and Otto Neugebauer,[416] which showed that Copernicus did not consider his mathematical proofs to be certain—a consideration that is confirmed by the fact that he was hesitant about publishing his work. In light of this, scholars have concluded that, rather than being based on textual evidence, Duhem’s hostility towards Copernicus was rooted in his own view of science, one that fiercely rejected “the philosophical-theological imperialism of the prevailing realism of the second half of the sixteenth century.”[417] This would also explain why Duhem was sympathetic with Osiander, who, in his opinion, had shown that “the hypotheses of physics are mere mathematical contrivances devised for the purpose of saving the phenomena.”[418]

A few decades after Duhem, Robert Westman demonstrated that Osiander’s reading of *De revolutionibus* was a variation on an established

---

[416] N. Swerdlow and O. Neugebauer, *Mathematical Astronomy in Copernicus’s* De Revolutionibus (New York: Springer, 1984), 19–21.

[417] André Goddu, “The Realism That Duhem Rejected in Copernicus,” *Synthese* 83, no. 2 (May 1990): 307.

[418] Pierre Maurice Marie Duhem, *To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo*, trans. Edmund Dolan and Chaninah Maschler (Chicago: University of Chicago Press, 1985), 117.

interpretation of the Copernican theory, which Westman called the "Wittenberg interpretation" and whose advocates were the followers of Philip Melanchthon (1497 – 1560).[419] This approach was characterized by an ambivalent attitude towards Copernicus, as members of the Melanchthon circle accepted the equantless models while rejecting the three types of terrestrial motion. For this reason, they tried to turn the Copernican models into computational devices that could fit into a geostatic view of the universe. These efforts must have been successful, because "the realist and cosmological claims of Copernicus's great discovery failed to be given full consideration."[420] On the other hand, in opposition to Duhem, Westman underlined that the Wittenberg interpretation was not instrumentalist in character, and that "it represented more than a position of epistemic resignation with regard to what one could know about actual celestial motions, while stopping short of a strong realist interpretation."[421]

In a later article, Westman took issue with what Duhem considered to be the origin of the realism-instrumentalism divide in early modern astronomy, namely, the presence of two competing disciplines in the realm of astronomical studies: natural philosophy and mathematics.[422] Roughly speaking, Duhem thought that natural philosophers were realist while mathematicians were instrumentalist. On the contrary, Westman claimed that "both [mathematical] astronomers and philosophers held realist ideals"[423] and that this disciplinary boundary within astronomical studies gradually faded away in the course of the sixteenth century, also because of the rise of the figure of the court astronomer.

More recently, Peter Barker and Bernard Goldstein warned against the use of the terms realism and instrumentalism in descriptions of sixteenth-century

---

[419] Robert S. Westman, "The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory," *Isis* 66, no. 2 (June 1975): 165–93.

[420] Westman, 168.

[421] Westman, 167.

[422] Robert S. Westman, "The Astronomer's Role in the Sixteenth Century: A Preliminary Study," *History of Science* 18, no. 2 (1980): 105–147.

[423] Westman, "The Astronomer's Role," 2.

astronomy.[424] Both these terms, the authors argued, were coined in the twentieth century in the context of a specific philosophical debate, hence "neither realism nor instrumentalism quite captures the predicament of the sixteenth-century astronomer."[425] Moreover, Baker and Goldstein noticed that in the sixteenth century, mathematical astronomy and natural philosophy were distinguished on the basis of their demonstrations. Following the Aristotelian theory, *propter quid* demonstrations (from causes to effects) were attributed to natural philosophy, while mathematical astronomy was limited to *quia* demonstrations (from effects to causes). The fact that astronomical knowledge could be probable at best was seen as a consequence of the impossibility of converting *quia* demonstrations into *proper quid*, for certain knowledge was achievable only through the latter. It is worth noting that the Aristotelian theory of demonstration was also invoked in another sixteenth-century epistemological debate: the *Quaestio de certitudine mathematicarum*.[426] This suggests a comparison between the *Quaestio* and contemporary discussions on the status of astronomical demonstrations such as those reported by Barker and Goldstein, which would require a separate study.

Seen against this background, the question of whether Bruno adopted a realist approach to the Copernican system may be phrased in the following terms: Did Bruno believe that the new mathematical models developed by Copernicus allow to gain a better understanding of the physical causes of astronomical phenomena? Did Bruno think that the physical universe had a mathematical structure? Scholars have tended to regard Bruno as a realist on the basis of two main elements, namely, his acceptance of the motion of the earth and his critique of Osiander. However, it should not be overlooked that adopting a realist attitude towards the Copernican system also required one's faith in the explanatory power

---

[424] Peter Barker and Bernard R Goldstein, "Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal," *Perspectives on Science* 6, no. 3 (1998): 232–258.

[425] Barker and Goldstein, 253.

[426] On the *Quaestio*, see Nicholas Jardine, "Epistemology of the Sciences," in *The Cambridge History of Renaissance Philosophy*, ed. Charles B. Schmitt, Quentin Skinner, and Eckhard Kessler (Cambridge: Cambridge University Press, 1988), 685–711; De Pace, *Le matematiche e il mondo*; Mancosu, *Philosophy of Mathematics*; Sergio, *Verità matematiche*.

of mathematics when applied to the study of the physical world. Quite interestingly, Bruno has been described as both a straightforward realist (in his commitment to the cosmological significance of Copernicus' innovations) and as a fierce opponent of the mathematics of his time. In fact, it was Bruno to convey this image of himself by endorsing Copernicus on the one hand, and by writing *One Articuli centum et sexaginta adversus mathematicos* (*Hundred and Sixty Articles against the Mathematicians,* 1588) on the other hand.

How did Bruno's realism fit with his mistrust in mathematics? Hélène Védrine claims that it was because of his realism that Bruno was skeptical about his contemporary mathematical research. For Védrine, Bruno was a realist in a Platonic sense insofar as he thought that "mathematical beings were in act in the sensible world."[427] On the contrary, sixteenth-century mathematic was, for the most part, based on the Aristotelian view that mathematical objects were intelligible concepts abstracted from sensible beings. Rejecting this Aristotelian ontology and the mathematics that was built on it, Bruno went on to propose his own "Platonic" version of mathematics, central to which was the concept of "real minima." However, Védrine notes, it was not Bruno but "the Paduan Aristotelians, Cardano, Tartaglia, Scipione del Ferro who contributed to the advancement of sixteenth-century mathematics."[428]

As mentioned above, I challenge Vedrine's view of Bruno's mathematics. I shall show that if it is true that traces of realism can be found in Bruno's vernacular works (and in particular in *La cena*), these works also contain the seeds of a different epistemology which will be fully developed in the Latin works. Let us start by reviewing the evidence in favor of Bruno's realism. Scholars, including Védrine, have regarded Bruno's critique of Osiander and the consequent defense of Copernicus' 'true' intentions as an endorsement of realism. In effect, this appears to be the most plausible explanation for the passages in question, which can be summarized as follows.

Not only was Bruno one of the first to note that the author of 'Ad lectorem' could not have been Copernicus, but he was probably the author of the first vernacular translation of the letter. A comparison with the original text shows

[427] Védrine, "L'obstacle réaliste," 240.

[428] Védrine, 241.

that Bruno faithfully translated its content, omitting only a section in which Osiander reiterated what he had already said about the use of hypotheses in astronomy—a use that was made necessary by the fact that astronomical reasoning could by no means yield knowledge of the physical causes of celestial phenomena. This was indeed the central point of "Ad lectorem" which ended with following words (taken from Bruno's translation):

> Let us then take advantage of the treasure of these suppositions only in so far as they render the art of calculation marvelously easy. For if anyone takes such fictions for real, he will leave this discipline more ignorant than when he entered it.[429]

As anticipated, Bruno utterly rejected this reading of *De revolutionibus* which he regarded as the work of a "ignorant and presumptuous ass."[430] In particular, what Bruno did not accept of "Ad lectorem" was its attempt to excuse Copernicus, as if the Polish astronomer wanted to defend himself from the charges of heterodoxy and heresy that theologians and Aristotelians could have pressed against his book. As a matter of fact, modern Copernicus scholarship has shown that this was precisely the case. Copernicus was afraid of how *De revolutionibus* could be received by the learned audience, and took steps to remove all the sensitive elements that could attract the attention of the ecclesiastical authorities. For this reason, Bruce Wrightsman writes that, far from endangering Copernicus' reputation (as Bruno and other more recent interpreters of *De revolutionibus* have argued)[431] "it is much more probable to claim that, for over a century, "Ad lectorem" *protected* the work […] during an extremely tense period of ideological and political conflict and thus, actually *permitted* the work to be used and

---

[429] *The Ash Wednesday Supper*, 93.

[430] *The Ash Wednesday Supper*, 91.

[431] For contemporary criticisms of Osiander, see Ernst Zinner, *Die Geschichte der Sternkunde: Von den Ersten Anfängen bis zur Gegenwart* (Berlin: Springer, 1931), 256–57; Edward Rosen, "The Ramus-Rheticus Correspondence," *Journal of the History of Ideas* 1, no. 3 (June 1940): 287–92; A. Rupert Hall, *The Revolution in Science, 1500-1750* (London: Longman, 1983), 55.

pondered during that period by those with such scruples."[432]

To dismiss Osiander's arguments and prove that it was Copernicus' purpose to claim that the earth moved, Bruno made the argument that *De revolutionibus* was a philosophical rather than a mathematical treatise. As we have seen, the distinction between natural philosophers and mathematicians was at the center of sixteenth-century astronomical debates. According to Bruno, Copernicus viewed himself as philosopher when, writing to Pope Paul III in the Preface to De *revolutionibus*, he underscored the importance of paying attention to the philosophers, and not to the "vulgar herd."[433] On the other hand, Bruno could not deny that Copernicus' work contained mathematical demonstrations, which however he regarded as a ploy to gain the support of the mathematical community, and not as an integral part of the Copernican theory. For this reason, Bruno concluded, Copernicus "not only acts as the mathematician who makes suppositions, but also at the physicist who demonstrates the movements of the earth."[434]

If *La cena* contained only this judgement of Copernicus and his theory, we should conclude that Bruno was a realist insofar as he not only considered the terrestrial motions to be more than a mathematical hypothesis, but also attributed this opinion to Copernicus himself. However, when earlier in the text Smithus asked Theophilus (Bruno's spokesman) what was his opinion about Copernicus, Theophilus answered that:

> His judgement in matters of natural philosophy was far superior to that of Ptolemy, Hipparchus, Eudoxus, and all the others who followed

[432] Wrightsman, "Andreas Osiander's Contribution," 240. Defenses of Osiander can also be found in Angus Armitage, *Copernicus, the Founder of Modern Astronomy*. (London: Allen and Unwin, 1938), 94; Lerner and Segonds, "L'*Ad lectorem* du *De revolutionibus*."

[433] *The Ash Wednesday Supper*, 93. On Copernicus' preface, see Robert S. Westman, "Proof, Poetics, and Patronage: Copernicus Preface to *De Revolutionibus*," in *Reappraisals of the Scientific Revolution*, ed. David C. Lindberg and Robert S. Westman (Cambridge: Cambridge University Press, 1990), 167–205; Geoffrey Blumenthal, "Diplomacy, Patronage, and the Preface to *De Revolutionibus*," *Journal for the History of Astronomy* 44, no. 1 (February 2013): 75–92.

[434] *The Ash Wednesday Supper*, 93.

> in their footsteps. [...] Yet he did not leave this philosophy far enough behind him; for, in so far as he was a student of mathematics rather than of nature, he was unable to penetrate those depths which would have allowed him to eradicate the useless and inappropriate principles from which it stems.[435]

Hence, Bruno had an ambivalent attitude towards Copernicus. At the outset of *La cena,* Copernicus was criticized for being more of a mathematician than a natural philosopher, whereas, in his defense of the reality of the Copernican model, Bruno emphasized the physical value of the ideas presented in *De revolutionibus*. How can this ambivalence be explained? McMullin writes that "Copernicus appears as a philosopher in search of the truth by contrast with other astronomers but as a "mathematician" by contrast with Bruno himself."[436] This may be true, but it is McMullin himself to notice that there were more differences than similarities between Bruno's and Copernicus' cosmological theories. An analysis of these differences will lead to a revaluation of Bruno's realism by shedding new light on the question of whether he considered mathematical objects to be in act in the sensible world (as Védrine has it) or not.

To begin with, there were Bruno's considerations about astronomical calculations. These calculations were the result of hundreds of years of astronomical observations. Being aware of this, Bruno gave credit to the generations of mathematicians whose work had laid the foundation for the cosmological theories of Ptolemy and Copernicus. Yet there was another side to Bruno's assessment of mathematical astronomy, as illustrated in the following passage:

> Such men [i.e. the mathematicians] are like interpreters who translate words from one language into another; yet it is not they but others who finally reach the heart of the matter. Again, they are like rustics who report the progress and fortunes of a battle to an absent captain;

[435] *The Ash Wednesday Supper*, 29.

[436] McMullin, "Bruno and Copernicus," 63.

> although they themselves are unable to understand the strategies, the causes and the design which have led to the victory.[437]

Two remarks are in order. First, in this quotation, Bruno once again emphasized that the "rustic" mathematicians did not know the causes of astronomical phenomena, whose knowledge instead belonged to the so-called "captain," whom we can assume to be the military counterpart of the natural philosopher. Hence, this quotation corroborates the idea that Bruno accepted the distinction, standard in his day, between mathematical astronomy and natural philosophy.

The second remark is that, for Bruno, mathematicians acted like interpreters, their task being to translate the language of nature into that of mathematics. Consider for a moment the Galilean metaphor of the book of nature written in the language of mathematics. Although there is no consensus on the meaning attached by Galileo to it,[438] it should be evident that such a metaphor stood at the opposite pole from the Brunian view that, in astronomical studies, mathematicians played the role of interpreters. The idea underlying this view was indeed that the language of mathematics was different from that of nature, otherwise there would be no need for an interpreter. At the same time, it should be noted that comparing mathematicians and interpreters did not amount to saying that it took a mathematician to *understand* nature, since Bruno made it clear that mathematicians lacked this ability which was only possessed by natural philosophers.

Hence, we can conclude that Bruno did not think that the universe had a mathematical structure. Rather, the universe could be described in mathematical terms, but this required a translation which, as faithful as it might be, could not entirely capture the essence of the physical reality. Thus, Bruno can hardly be said to be a realist in this respect. This is also confirmed by his rejection of circular devices, especially orbs, as a way of explaining planetary motions, which

[437] *The Ash Wednesday Supper*, 27.

[438] See Carla Rita Palmerino, "Reading the Book of Nature: The Ontological and Epistemological Underpinnings of Galileo's Mathematical Realism," in *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century*, ed. Geoffrey Gorham et al. (Minneapolis: University of Minnesota Press, 2016), 29–50.

was arguably the most significant difference between Bruno's and Copernicus' cosmological theories. In *La cena*, Bruno took issues with "those who want to imagine fillings and wadding of irregular orbs, [...] inventing plasters and other prescriptions in order to heal nature so that it can serve their master, Aristotle or someone else."[439] As is evident from the reference to Aristotle, Bruno here was attacking the advocates of the Aristotelian-Ptolemaic system. However, the same objection could be raised against Copernicus, who also conceived the existence of solid orbs in which heavenly bodies were embedded.

In Bruno's cosmological theory, orbs were dismissed in favor of "a single airy, ethereal, spiritual, and liquid body, a capacious place of motion and quiet, which reaches out into the immensity of infinity."[440] Having abandoned the solid orbs which carried around the heavenly bodies in a uniform circular motion, Bruno was forced to find an alternative physical explanation for planetary motions. To this end, he resorted to the original Platonic idea, whose major promoter in the Renaissance was Marsilio Ficino, that heavenly bodies were "animals," meaning that they were inhabited by a spirit called the "world-soul" (*anima mundi*).[441] It was this world-soul that was responsible for the motion of heavenly bodies, which wondered across the ethereal space and around their respective suns—Bruno envisioned infinite solar systems—to absorb the heat and light necessary to life.[442]

Let us now ask: on what grounds did Bruno reject the existence of solid orbs? Was this rejection caused by something more than his outspoken criticism of Aristotle and his epigones? In nature, Bruno argued in *La cena*, no body had a perfectly round shape, nor was there a body which moved along a perfectly circular trajectory. Hence, if this was the case on earth, why did we have to assume that perfect circularity was present in the heavens, and thus postulate the

---

439 *The Ash Wednesday Supper*, 115.

440 *The Ash Wednesday Supper*, 117.

441 See Ornella Pompeo Faracovi, "Tra Ficino e Bruno: Gli animali celesti e l'astrologia nel Rinascimento," *Bruniana & Campanelliana* 8, no. 1 (2002): 197–232.

442 For an overview of Bruno's 'biological' explanation of planetary motions, see Alfonso Ingegno, *Cosmologia e filosofia nel pensiero di Giordano Bruno* (Florence: La nuova Italia, 1978), 63–70.

existence of solid orbs?[443] This argument implied that, unlike what most of medieval cosmologists thought, there was no difference between the bodies which inhabited the super- and sublunary regions. Indeed, as is well known, such a distinction was absent from Bruno's homogenous universe in which all bodies were composed of the same four elements. Furthermore, this argument tells us that, contrary to what Védrine claims, Bruno did not believe in the actual existence of mathematical objects (such as perfect circles or circular motions) in the physical world. Based on the same argument, in the later Latin works, Bruno would go on to dismiss all circular astronomical devices, including eccentrics and epicycles (more on this in the next section). At that point, instead of criticizing the astronomers who "healed" nature so as to fit their mathematical models,[444] he would argue that it was mathematics that had to change in order to represent nature, thus advocating a reform of geometry. This reform was to make room for a new concept that meanwhile had entered Bruno's mathematics: the concept of the minimum.

4.2 *Acrotismus camoerancesis* and the making of Bruno's theory of minima

In essence, Bruno's theory of minima was an atomist doctrine, insofar as it rested on the assumption that all objects were composed of indivisible parts (i.e. minima).[445] Although it was not until the publication of *De minimo* (1591) that Bruno's theory of minima was fully developed, both the concepts of minimum and atom made their first appearance in the Italian dialogues. There, Bruno dealt with these concepts separately, speaking of minimum and maximum especially in the Fifth Dialogue of *De la causa, principio e uno* (*On Cause, Principle and Unity*, 1584), and making references to the atoms that "flowed" from a body to another in *De l'infinito universe et mondi* (*On the Infinite Universe and Worlds*,

[443] See *The Ash Wednesday Supper*, 115.

[444] *The Ash Wednesday Supper*,, 115.

[445] However, regarding Bruno's theory of minima *only as* an atomist doctrine may be simplistic, as shown in §§ 2.2 and 4.3.

1584).[446] It was no coincidence that the concepts of minimum and atom were treated separately, as, at least in the early stages of their development, Bruno viewed them as independent of each other. That is to say, differently from what would happen in *De minimo*, the atom was not regarded as a species of the minimum. Rather, by virtue of the coincidence of the opposites—a concept borrowed from Nicholas of Cusa (see Chapter 2)—the minimum was always treated in relation to the maximum, which was the main topic of discussion. After all, Bruno's purpose in writing the first three Italian dialogues was to promote his conception of an infinite universe – the highest expression of maximum after God.

As I have tried to show in Chapter 3, it was in the dialogues of 1586 devoted to the proportional compass of Fabrizio Mordente that Bruno first used the concept of the minimum in conjunction with the concept of atom. Building on what in *De l'infinito* he had said about atoms being exchanged among bodies, he went on to show how, using Mordente's compass, it was possible to demonstrate that both geometric and physical objects had an atomic structure. More precisely, in Bruno's understanding—but not in Mordente's, who vehemently protested against this interpretation of his instrument—the compass allowed to divide curved and straight lines down to their "minimum," indivisible fractions. This discovery must have inspired Bruno to further develop his atomistic theory, which in *De l'infinito* only consisted in a few scattered references to the topic. However, in order to do so, he needed to overcome what represented the major obstacle to the development of a full-fledged atomistic theory: the Aristotelian critique of atomism. He attended to this task in another work which was composed at about the same time as the dialogues on Mordente's compass, but was published two years later: the *Acrotismus camoerancesis* (1588).[447] Perhaps

---

446 In *De l'infinito*, there were twelve occurrences of the term "atoms", most of which were linked to the view that, in Bruno's infinite universe, all bodies underwent a process of growth and decay that could be explained in terms of gain and loss of atoms. A list of all the occurrences can be retrieved using the search function of the online database http://bibliotecaideale.filosofia.sns.it/index.php.

447 Giordano Bruno, "Acrotismus camoeracensis," in *Opera latine conscripta*, ed. F. Fiorentino, vol. I, pt. 1 (Neaples: Morano, 1879), 53–190.

not coincidentally, the *Acrotismus* also yielded insights into the issue of Bruno's realism.

The *Acrotismus* contained the Aristotelian theses, and Bruno's arguments against them, which had been the subject of a public dispute held at the Collège de Cambrai (now part of the Collège de France) in Paris in 1586. The title of the work seemed to hint at that dispute, as the term *Acrotismus* was derived from the Greek *akrorais*, which meant "to listen" and was included in the title of Aristotle's *Physics* (*Physike akroasis*), while the term *camoerancesis* was a neologism coined by Bruno to refer to the Collège de Cambrai.[448] On occasion of the dispute, Bruno had the theses printed under the title *Centum et viginti articuli adversus Peripateticos* (*One hundread and twenty articles against the Peripatetics*, 1586). The *Acrotismus* bore a close relationship to the *Articuli*, as the former may be viewed a revised and extended version of the latter.

Already in *La cena,* Bruno maintained that "in physics, division of a finite body cannot progress to infinity expect for those who are mad, whether you think of it in act or in potencial."[449] However, neither in *La cena* nor in the other Italian dialogues, Bruno explained why it was so "mad" to believe in the infinite divisibility of the physical continuum, thus postulating rather than rationally justifying the existence of the atoms. To a certain extent, the *Acrotismus* filled this gap. The structure of the book, which followed the model of sixteenth-century academic discussions, was designed to address, one by one, the central issues raised in Aristotle's *Physics*. To the problem of whether or not the continuum was infinitely divisible (which was mainly discussed in Book VI of Aristotle's *Physics*) Bruno devoted the 42nd article of the *Acrotismus*, which reads as follows:

> Before assuming the continuum to be infinitely divisible, Aristotle should have specified how the whole universe was divisible in the same way as this earth, and the whole of this globe in the same way as this apple; how these things, which *qua* finite beings are of different size,

---

[448] For more information on the title, see Barbara Amato, "Introduction," in *Acrotismo cameracense: le spiegazioni degli articoli di fisica contro i peripatetici*, by Giordano Bruno, ed. Barbara Amato (Pisa: Fabrizio Serra, 2009), 13–15.

[449] *The Ash Wednesday Supper*, 107.

> through division become equal. (...) How are they equal in potency, but unequal in act?[450]

For Bruno, to say that that two things were infinitely divisible was to say that they were composed of an infinite number of parts. Infinity, however, admitted no difference, as it was impossible for "an infinite to be bigger than another infinite neither in potency nor in act."[451] By the same token, it was impossible to say which of two infinitely divided things was bigger than the other, since after the division both would turn out to be composed of the *same* infinite number of parts. Nor did it matter that the parts of one thing were bigger in size than the parts of the other, "for the bigger parts taken only once from the bigger [thing] would be necessarily equal to the smaller parts taken multiple times from the smaller [thing]."[452] The Aristotelians would have replied to these objections by pointing out—as done in the pseudo-Aristotelian treatise *De insecabilibus lineis* (*On indivisible lines*)—that the fact that a bigger object could contain an infinite number of smaller objects told us nothing about the containing object itself, for one object could be contained in another without being part of it.[453]

---

450 *Acrotismus camoeracensis,* 151-52: "Priusquam Aristoteles supponeret, continuum in infinitum esse divisibile, dividique in semper divisibilia, indicare debuisset, quomodo totum universum aequaliter sit divisibile cum terra ista, et totus iste globus cum hoc pomo, quomodo haec, licet finita sint inaequalia, per divisionem in infinitum sunt aequalia. [...] quomodo aequalia sunt in potentia, actu vero inaequalia?' 'Acrotismus camoeracensis."

451 *Acrotismus camoeracensis*, 153: "Quomodo unum infinitum est maius alio in potentia vel in actu?"

452 *Acrotismus camoeracensis*, 152: "Quia partes quas semel accepisti a maiori maiores, acceptis iterum atque iterum a minori mole minoribus, adaequabuntur necessario."

453 Aristotle, "On Indivisible Lines," in *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, ed. J. Barnes (Princeton, NJ: Princeton University Press, 1995), 3305 (972b16-229): "Further, since the smallest of the things contained in a house is so called, without in the least comparing the house with it, and so in all other cases:—neither will the smallest of the constituents in the line be determined by comparison with the line." My understanding of this Aristotelian argument relies mainly on Henry, "Void Space, Mathematical Realism," 148–49.

This counter-argument did not seem to bother Bruno, who instead focused on another Aristotelian distinction—that between potential and actual infinity. In the *Physics*, Aristotle viewed the infinite divisibility of the continuum as a potential infinity, meaning that there was no limit to the number of parts into which the continuum could be divided; no matter how small a part was, it could always be further divided.[454] In addition, Aristotle made it clear that the divisibility of the continuum was never infinite in an actual sense, since, in his opinion, actual infinity was impossible.[455] Bruno challenged this claim, as he did not see how it was possible for a potency to exist without a corresponding act. Bruno assumed that an Aristotelian would reply that such potencies were found in the realm of mathematics, where it was allowed to perform operations which were impossible to perform in the real world. In response, Bruno noted that even the mathematicians did not take the line to be "absolutely infinite, as it would be useless to do so, but they consider it to be infinite in a certain respect, as, for them, infinite means 'as big as you want'"[456]. On this point, Bruno was in complete agreement with Aristotle, who in the *Physics* wrote that mathematicians made use of arbitrary large and not infinite magnitudes.[457]

It is beyond the purpose of this chapter to discuss the effectiveness of Bruno's arguments against the Aristotelian physics. Rather, I am interested in what these arguments can tell us about Bruno's conception of mathematics and its relationship to the idea of infinity. We have seen that Bruno (as well as

---

454 Aristotle, "Physics," in *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, ed. J. Barnes (Princeton, NJ: Princeton University Press, 1995), 780 (207b10-15): "Hence this infinite is potential, never actual: the number of parts that can be taken always surpasses any definite amount. But this number is not separable, and its infinity does not persist but consists in a process of coming to be, like time and the number of time."

455 See previous note.

456 *Acrotismus camoeracensis,* 152–53: "Mitto quod neque mathematici accipiunt infinitam sempliciter lineam, cuius nullus usus esse potest, sed secundum quid infinitam accipiunt, quia illis infinitum est quantumcunque."

457 *Physics*, 781 (207b30): "In point of fact they [i.e. the mathematicians] do not need the infinite and do not use it. They postulate only that a finite straight line may be produced as far as they wish."

Aristotle) thought that, strictly speaking, it was not the infinitely great but the arbitrary large to be the subject of mathematics. What about the infinitely small? Did, according to Bruno, mathematicians believe that there was a limit to the division of the mathematical continuum, or did they believe that it could go to infinity? We know that classical Euclidean geometry advocated an Aristotelian view of the continuum, insofar as the only allowed indivisibles were points, lines and planes conceived as ends and not as parts of the continuum. In fact, most of the medieval objections to atomism were inspired by geometric considerations. For example, critics of atomism noted that it was impossible to account for incommensurables if magnitudes were conceived as composed of a finite number of indivisible parts. Bruno himself had to reply to this objection in *De minimo*, but in the *Acrotismus* he adopted a different strategy, one that at first sight may appear rhetorical, but that retrospectively may be viewed as the inception of Bruno's project of mathematical reform:

> It is one thing to consider magnitude mathematically, quite another to consider magnitude physically. (...) If logic and mathematics want to assume the infinitely divisible regardless of any praxis and use for a vain consideration, let them have their way.[458]

Mathematics, for Bruno, had to conform to the natural order, lest it became a vain speculation. In Bruno's case, this meant that the infinite divisibility of the continuum had to be rejected both in physics and mathematics. With the passing of time, Bruno realized that it took more than a few adjustments to introduce atomism in mathematics. In fact, it required an entirely new theory, the development of which was carried out in *De minimo*. As for the *Acrotismus*, it seems to confirm that Bruno was far from being a mathematical realist. For in it, instead of saying that physical phenomena could be fully explained in mathematical terms (as the mathematical realist would do), Bruno claimed the opposite—that it was mathematics that had to be modelled after physics.

---

458 *Acrotismus camoeracensis*, 154: "Aliud sane est magnitudo mathematice, aliud magnitudo physice sumpta. (...) quod etiamsi ratio et mathesis citra praxin omnem et usum ad vanam tantum contemplationem velit infinite divisibile adsumere, faciat ad arbitrium."

4.3 *De minimo:* Where mathematics meets physics (and metaphysics)

In the spring of 1591, Giordano Bruno was in Frankfurt when he was invited to Venice by the patrician Giovanni Mocenigo.[459] This gave Bruno the opportunity to return to his home country, from where he had escaped more than ten years earlier under the suspicion of heresy. Bruno must have been aware that, once in Italy, the Inquisition could have pressed charges against him. Nevertheless, he accepted Mocenigo's invitation and, against his better judgement, set off to Venice. As is well known, this decision turned out to be unfortunate, since, after being denounced by Moceningo, Bruno was burned at the stake in Campo de' Fiori in Rome on February 17, 1600.

Bruno's activities had been brought to Moceningo's attention by the Latin poem entitled *De minimo*, which, together with *De immenso* and *De monade*, formed Bruno's so-called "Frankfurt trilogy." Indeed, all of these three works were published in Frankfurt shortly before Bruno left for Italy. For his part, Moceningo was interested in unlocking the secrets of the art of memory. More relevant to this study, *De minimo* marked the end of the mathematical journey started eight years earlier with the Italian dialogues, as it contained the ultimate version of Bruno's theory of minima. According to this theory, there were three different species of minima: a metaphysical minimum (the monad), a physical minimum (the atom), and a geometric minimum (the point):

> The minimum is the substance of things and, although it refers to a genus different from that of quantity, constitutes the principle of the quantity and magnitude of bodies. It is matter, that is, element, efficient cause, final cause, totality. It is a point in one- and two-dimensional magnitudes. It is properly named atom in the bodies which are the first parts; and less properly in those entities which are all in all and in the single parts (such as the voice, the soul, and similar

[459] Luigi Firpo, *Il processo di Giordano Bruno*, ed. Diego Quaglioni (Rome: Salerno Editrice, 1993), 154–55.

> entities). It is a monad rationally in numbers and essentially in all things.[460]

Bruno was adamant that both the physical and the mathematical continuum were composed of indivisibles, respectively, atoms and points. He also held that atoms and points had the same shape—the only difference was that atoms were spherical while points were circular because points had one dimension less than atoms.[461] Having a shape meant that both atoms and points were extended. It also implied that the number of atoms and points composing an object had to be finite, lest the object be less extended than the sum of its parts. For this reason, Bruno claimed that bodies were composed of a finite number of atoms.[462] However, the same could not be said of mathematical objects because, as already seen, this would have led to the denial of incommensurability. Bruno tried to turn this inconvenience on its head by presenting his theory as a solution to problem of incommensurability:

> Maybe, oh illustrious master, should I complain about the dissolution of incommensurability and irrationality, instead of being glad for the revival of rationality and measurability?[463]

Certainly, incommensurability posed a serious challenge to Bruno's theory of minima. So did Aristotle's objections against atomism, which Bruno had started

---

[460] *De minimo*, 139–40: "Minimum est substantia rerum, quatenus videlicet aliud a quantitatis genere significatur, corporearum vero magnitudinum prout est quantitatis principium. Est, inquam, materia seu elemen tum, efficiens, finis et totum, punctum in magnitudine unius et duarum dimensionum, atomus privative in corporibus quae sunt primae partes, atomus negative in iisce quae sunt tota in toto atque singulis, ut in voce, anima et huiusmodi genus, monas rationaliter in numeris, essentialiter in omnibus."

[461] *De minimo,* 177: "Minimi in plano propria figura est circulus, in solido sphaera."

[462] *De minimo*, 150: "Materies coram finita recepta finitis prorsus consistit partibus omnis."

[463] *De minimo*, 240: "Numquid, o a mplissime domine magister, pro interitu alogiae et incommensurabilitatis potius plorandum censebo, quam pro logiae et mensurae renascentia gaudendum?"

to deal with in the *Acrotismus*. For Aristotle, the problem with atomism was that if the parts of an object were indivisible, they could not touch each other, and thus the object to which they belonged could not be said to be continuous. [464] Indeed, two indivisibles touching each other with all of themselves overlapped, thus creating no extension. The only other way in which two things could touch each other was through one of their parts, but this was impossible in the case of indivisibles which by definition had no parts whatsoever. Bruno's solution to this problem was to claim that the contact between indivisibles occurred through their extremity (*terminus*). [465] The extremity was not a part of the indivisible because it could not be separated from it, but it existed only when two indivisibles touch each other. As already mentioned in the Introduction, Bruno was indebted to Epicurus for this aspect of his theory.

In his previous works, Bruno had criticized contemporary mathematical research for its failure to acknowledge the importance of the minimum: "The ignorance of the minimum makes the geometers of this century ge*a*meters, and the philosophers phil*a*sophers" Bruno wrote in *Articuli adversus mathematicos*.[466] This criticism continued in *De minimo,* where, building on the premise that physical reality had an atomic structure, the bone of contention became the infinite divisibility of the mathematical continuum:

> When infinitely dividing what has a precise measure, the geometer makes

---

[464] *Physics*, 861–62 (231a18–231a28).

[465] *De minimo*, 160: "Minimum non tangit se toto neque sui parte alterum minimum, sed suo fine plura potest attingere minima, sicut etiam nullum corpus se toto vel parte sui, sed vel tota vel extremitatis parte tangit alterum; [...] terminus ergo est qui nulla est pars, et per consequens neque minima pars." On Bruno's concept of extremity, see Barbara Amato, "Il concetto di 'termine' nel «De minimo»," in *Letture bruniane I-II del lessico intellettuale europeo (1996-1997)* (Pisa: Istituti Editoriali e Poligrafici Internazionali, 2002).

[466] *Articuli adversus mathematicos,* 21: "Ignorantia minimi facit geometras huius saeculi esse geametras, et philosophos esse philasophos."

> a mistake, he does not follow in the footsteps of nature which, being never reached, cannot be imitated by the geometer.[467]

If in the *Acrotismus* the acceptance by mathematicians of infinite divisibility was "tolerated" ("let them have their way"), in *De minimo* Bruno was less indulgent. This was probably due to the fact that, in the Latin poem, Bruno presented his own mathematical theory, which he viewed as an alternative to classical mathematics. As a matter of fact, Bruno's theory of minima was one of the first attempts to introduce indivisibles in geometry, an issue that would become central to seventeenth-century mathematics. On the other hand, there was the role (or lack thereof) of mathematics in the study of nature. As is evident from the above quotation, Bruno thought of mathematics a set of mental representations that had to mirror rather than explain the physical world. Once again, this suggests that Bruno was far removed from the Galilean metaphor of the book of nature and, more generally, from what historians of the Scientific Revolution have called the "mathematization of nature."[468] In fact, if anything, the project carried out by Bruno may be said to be a "naturalization of mathematics."

What was the cause of Bruno's mistrust in explanatory power of mathematical physics? We have seen that, in *La cena*, solid orbs (still employed by Copernicus) were rejected in favor of an universe populated by animated celestial bodies, the reason being that no perfect circular form or motion was to be found in nature. This was reaffirmed in the Frankfurt poems, especially in *De immenso*, where, in denying that the perfect circle—in fact, any perfect form—was

---

467 *De minimo*, 154–55: "Ergo errat mensor certum sine fine resolvens quantum, naturae nusquam vestigia lustrans, nusquam illa attingens, non ullis sortibus aequans."

468 A non-exhaustive list of studies on the mathematization of nature include: Michel Blay, *Reasoning with the Infinite: From the Closed World to the Mathematical Universe* (University of Chicago Press, 1999); Roux, "Forms of Mathematization (14th-17th Centuries)"; William R Shea, *Nature Mathematized: Historical and Philosophical Case Studies in Classical Modern Natural Philosophy* (Dordrecht: Springer Netherlands, 1983); Geoffrey Gorham et al., eds., *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century* (Minneapolis: University of Minnesota Press, 2016).

present in nature, Bruno appealed to the authority of the Platonists.[469] Hence, it seems safe to conclude that Bruno's skepticism towards mathematical physics was rooted in his opposition to mathematical realism, that is the view that mathematical objects were mind-independent. The Latin poems combined this rejection of mathematical realism—the seeds of which could already be found in Bruno's vernacular works (see § 4.1)—with an element that instead was the result of later researches: the concept of the minimum. However, there was an aspect of Bruno's theory of minima that contrasted with his anti-realism: the circular shape of the minimum. Bruno, indeed, claimed that the minimum was a circle in two-dimensional space and a sphere in three-dimensional space.[470] (That is to say, the point was a circle and the atom was sphere). But why did Bruno endow the most important entity in his ontology with a circular shape, if such a shape was nowhere to be found in the real world?

The problem of the circular shape of the Brunian minimum is even more compelling if we consider the importance that Bruno attributed to images, and more in general to visual thinking.[471] In the specific case of Bruno's mathematical works, images had a double application. First, images offered a means to promote mathematics as a practical discipline essential to social and economic development. An example is provided by the "temples" of Apollo, Minerva, and Venus that Bruno constructed in *De minimo,* by means of which he aimed to lay the foundations for a universal theory of measurement that would enable mathematical practitioners to measure everything (Figures 2a-2b-2c). In addition, Bruno used mathematical images in a symbolic fashion to express what was difficult to put into words, or to visualize what was invisible to the human eye. For instance, in *De minimo,* Bruno relied on circular constructions to show

---

[469] Giordano Bruno, "De immenso et innumberabilibus (Books 1-3)," in *Opera latine conscripta*, ed. F. Fiorentino, vol. I, pt. 1 (Neaples: Morano, 1879), 361: "Mathematice enim circularis motus non est in materia, quaecunque et qualiscunque sit, immo neque ullam formam vere in materia esse Platonici dixerunt (et non omnino male), neque hominem verum, neque verum equum."

[470] *De immenso*: "Minimi in plano propria figura est circulus, in solido sphaera."

[471] For the importance of images in Bruno's thought, see especially Rossi, *Art of Memory*, 81–96.

the patterns in which atoms were arranged to form geometric and physical figures (Figures 3a-3b-3c).

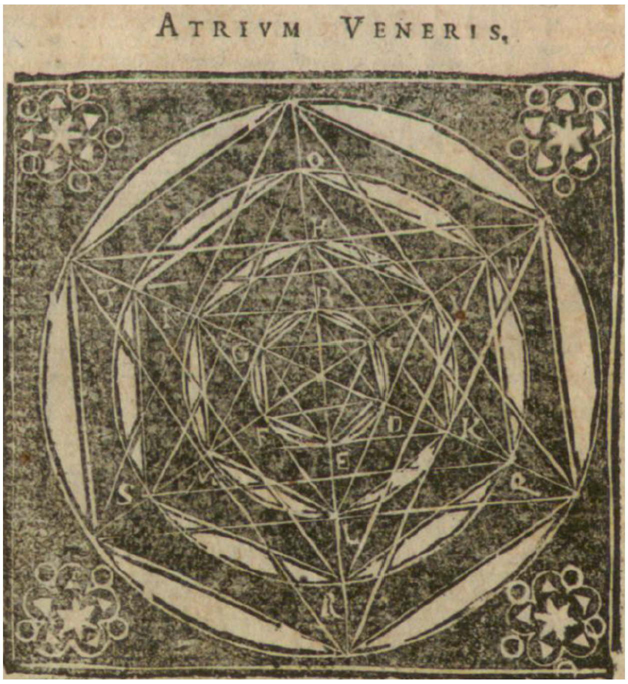

Figure 2a: *Atrium Veneris*

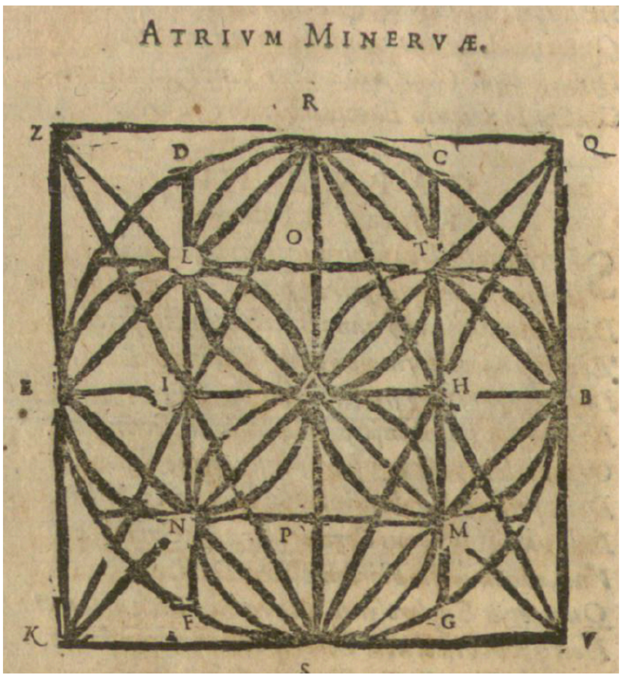

Figure 2b: *Atrium Minervae*

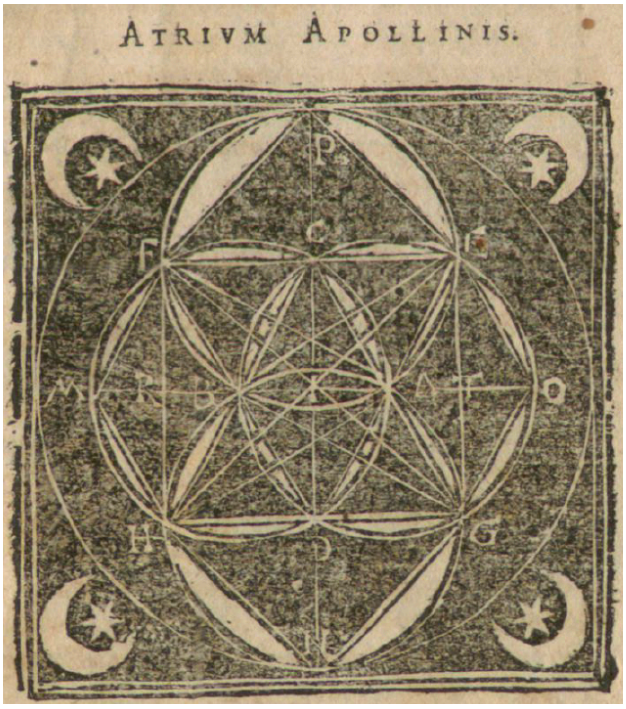

Figure 2c: *Atrium Apollinis*

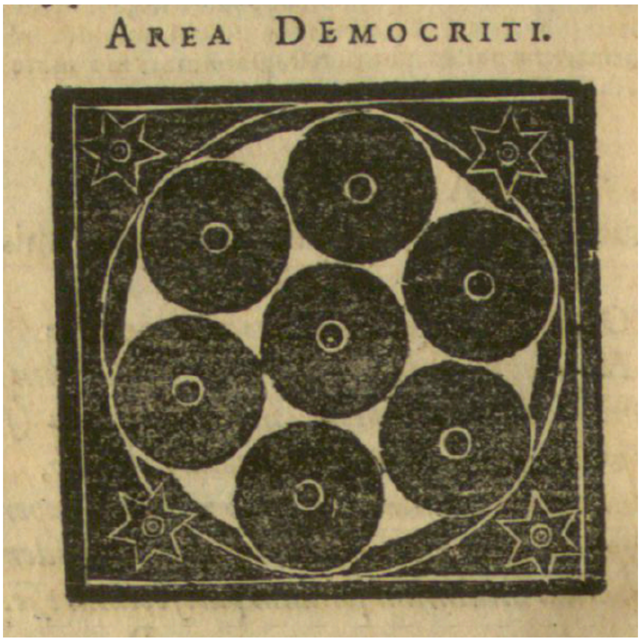

Figure 3a: *Area Democriti*

Figure 3b: *Campus Democriti*

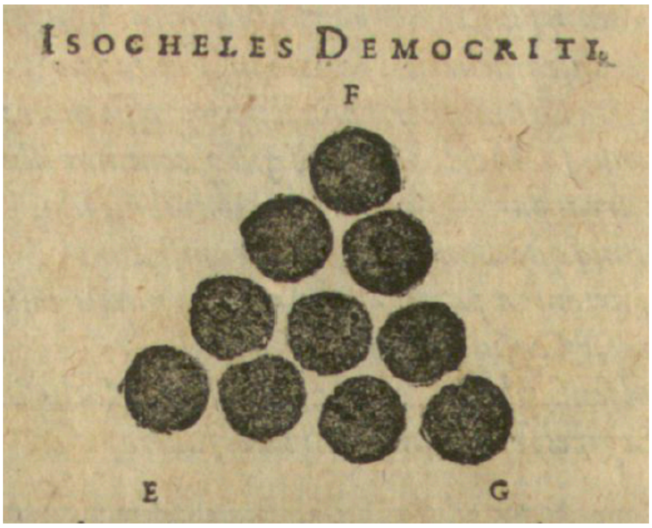

Figure 3c: *Isocheles Democriti*

Christoph Lüthy has demonstrated that Bruno inherited these circular diagrams from a centuries-old tradition, going all the way back to Augustine (354-430 AD) and based on the mathematical teachings of Boethius (477-524 AD).[472] What

[472] Christoph Lüthy, "Bruno's *Area Democriti* and the Origins of Atomist Imagery," *Bruniana & Campanelliana* 4, no. 1 (1998): 59–92.

distinguished Bruno from his predecessors was the meaning that he attached to those diagrams, as he was the first to use circles to represent atoms. In parallel, Bruno was the recipient of a metaphor which also had its origin in the Middle Ages, in particular in the anonymous *Liber XXIV philosophorum* (*Book of the 24 Philosophers*): the metaphor of the infinite sphere whose center was everywhere and whose circumference was nowhere.[473] Originally devised as a description of God, the metaphor of the infinite sphere took on different meanings throughout its long history, depending on the context in which it was understood. With Nicholas of Cusa, the infinite sphere came to represent the universe, the maximum in the realm of finite beings.[474] In keeping with the idea (also from Cusa) that the opposites coincided, Bruno took a step further and employed the metaphor of the infinite sphere to describe the minimum:

> It is evident to everybody that the centre, i.e. the circle, the chord, the area, the diameter, the arc and the radius are all without distinction, whether we refer to the minimum or to the maximum.[475]

---

473 This metaphor has been the subject of extensive studies: Dietrich Mahnke, *Unendliche Sphäre Und Allmittelpunkt Beiträge Zur Genealogie der Mathematischen Mystik* (Halle: M. Niemeyer, 1937); Karsten Harries, "The Infinite Sphere: Comments on the History of a Metaphor," *Journal of the History of Philosophy* 13, no. 1 (1975): 5–15.

474 Nicholas of Cusa, *De docta ignorantia,* ed. Paul Wilpert and Hans Gerhard Senger (Hamburg: Felix Meiner, 2002), bk. II, chap. 12, para. 162: "Unde erit machina mundi quasi habens undique centrum et nullibi circumferentiam, quoniam eius circumferentia et centrum est Deus, qui est undique et nullibi."

475 *De minimo,* 145: "Centrum, aio, cyclus, chord', area, dimetrus, arcus et radius nullo veniunt discrimine coram omnia, seu minimum seu maxima concipiantur." I understand that this statement seems to run counter to the view, which I hold, that the Brunian minimum was an extended point. Indeed, if the center and circumference of the minimum coincided, the minimum could not have been extended. However, there are other factors to be considered, which lead to think that Bruno regarded the minimum as an extended entity. First and foremost, there is his denial of incommensurability. If the minima were not extended, Bruno could have claimed that magnitudes were composed of an infinite number of them. This in

Hence, the circular shape of the Brunian minimum appeared to result from the interplay of two theoretical elements: the circular diagrams and the metaphor of the infinite sphere. Despite their geometric appearance, both these elements had a metaphysical rather than a strictly mathematical origin, as they were used to symbolize God.[476] It was Bruno who turned the circular diagrams and the infinite sphere into full-fledged mathematical notions. He did so by employing them as a visual aid to understanding the fundamental properties of the minimum (e.g. the way in which minima interact with one another). Nevertheless, the metaphysical 'heritage' of these two notions interfered with their new mathematical function, causing problems at the geometric level. For instance, we have seen that Bruno's theory was incapable of accounting for incommensurable magnitudes. Another problem was that since around one circle there was only room for six other circles with the same diameter (see *Area Democriti*, Figure 3a), and since each circle stood for a geometric point, Bruno concluded that in a circumference the center (the central circle) was reached by only six radii (whose extremities were the six peripherical circles).[477]

Thus, the circular shape of the minimum was at odds not only with Bruno's rejection of mathematical realism, but also with the basic tenets of Euclidean geometry. As seen above, these contradictions were generated by the introduction of metaphysical elements into a theory that was at once physical and mathematical. Nonetheless, Bruno was unwilling to abandon those metaphysical

---

turn would have enabled him to save incommensurability. Also, there is Bruno's claim that the center of the circumference was the end-point of only six radii. There would have been no need to claim so, had the points been not extended.

476 We have already said that the infinite sphere was originally conceived as a metaphor of God. As for the circle, Augustine attributed its shape to God by virtue of its being the most perfect form. See Christoph Lüthy, "*Entiae* & *sphaerae*: due aspetti dell'atomismo bruniano," in *La filosofia di Giordano Bruno. Problemi ermeneutici e storiografici*, ed. Eugenio Canone (Florence: Leo S. Olschki, 2003), 184.

477 *De minimo*, 247: "Ostendat minimum minimorum, cuius typus est circulus in quem omne resolvitur angulatum, a pluribus circumquaque possit attingi quam sex, et tunc concedemus eundem vel minimam partem vel nullam partem communem esse posse terminum omnium quae a peripheria descendere possunt lineae, non autem sex tantummodo radiorum terminum et trium communem partem diametrorum."

elements because its objective was precisely to integrate physics, metaphysics and mathematics into a single theory. In fact, the whole of Bruno's theory was built on the distinction of the three species of minima: physical (the atom), metaphysical (the monad) and mathematical (the point). This meant that all of these three aspects of reality could be described in terms of minima or, to put it otherwise, that in Bruno's understanding the minima provided a "theory of everything."

Bruno may or may not have been aware that his project was too ambitious, and that any attempt to unify physics, metaphysics and mathematics would have caused problems at one or more of these three levels. What is certain is that he was willing to face these problems to ensure the comprehensiveness of his theory. This does not alter the fact that Bruno's theory of minima was flawed, particularly with regard to its mathematical and physical applications. However, acknowledging that Bruno's theory served a 'higher purpose' may help to better assess its overall value.

4.4 The marriage of numerology and magic in *De monade*

More than fifty years have passed since the publication of Frances Yates's classical studies on *Giordano Bruno and the Hermetic Tradition* (1964). As Yates herself explained in the introduction to the book, Bruno had long been a problem for her ("masses of notes and manuscript accumulated but full understanding eluded me"[478]), until she realized that the solution lay in contemporary Renaissance scholarship, and more precisely in the studies that Kristeller, Garin and others were conducting on Renaissance Hermeticism. Although Yates' purpose in writing her book was "to do only what its title states, to place him [i.e. Bruno] in the Hermetic tradition,"[479] she went further than that, presenting the whole of Bruno's doctrine as the work of a Renaissance magus. More recent interpreters have objected that the "Yates thesis" was too simplistic, since its emphasis on magic overshadowed the importance of other aspects of Bruno's thought, above all its philosophical character.[480] Indeed, as we have seen throughout this chapter, Bruno's works tends to frustrate attempts to read them

[478] Yates, *Bruno and the Hermetic Tradition,* ix.

[479] Yates, x.

[480] See Ciliberto, "Introduction."

in a one-sided way. And yet, in spite of this, Yates' interpretation of Bruno continues to provide the key to his most obscure texts, as happens in the case of *De monade. numero et figura* (*On the Monad, Number and Figure*, 1591).[481]

Of the Latin poems that compose Bruno's Frankfurt Trilogy, *De monade* is arguably the most difficult to interpret. This difficulty arises from the fact that, at first sight, *De monade* appears to be a mere exercise in numerology. The poem was divided in ten chapters, each of which was devoted to describing the properties of the first ten numbers. Each number was associated with a geometric figure and a list of extra-mathematical realities (moral virtues, natural phenomena, philosophical concepts, and so on) in order to show that "in any species and number all things can be found, according to the disposition of the diverse elements."[482] For instance, the number one or "monad" was associated with the circle and was said to be found in both the Mind and Soul.[483] In this respect, Bruno's *De monade* was not unlike Iamblichus' *Theology of Arithmetics* (*Theologumena arithmeticae*, fourth century AD) and other late antique 'arithmological' treatises, with the only difference that Bruno's work was little concerned with theology.[484] Rather, Bruno's purpose was to show how mathematics—which, in his understanding, was not opposed to numerology—could provide a universal science through which to explore the world at all levels. In fact, it was precisely in that Neoplatonic and Neopythagorean tradition that Bruno placed his own poem, a tradition that in the Renaissance the likes of Marsilio Ficino and Giovanni Pico della Mirandola had renamed "ancient theology," *prisca theologia* or *perennis philosophia* (although the meaning of these terms do not exactly coincide).

---

[481] Giordano Bruno, "De monade, numero et figura," in *Opera latine conscripta*, ed. F. Fiorentino, vol. I, pt. 2 (Neaples: Morano, 1884).

[482] *De monade*, 330: "Qualibet in specie ac numero mox comperientur omnia, pro varia variorum conditione."

[483] *De monade*, 346.

[484] Iamblichus, *The Theology of Arithmetic: On the Mystical, Mathematical and Cosmological Symbolism of the First Ten Numbers*, trans. Robin Waterfield (Grand Rapids, Mich: Phanes Press, 1988).

Ancient theology has been the subject of extensive studies.[485] I will limit myself to an outline of this tradition to illuminate Bruno's *De monade* and the historical context in which it was written. The main reason why Renaissance thinkers invoked ancient theology was to integrate the original pagan Platonism into Christianity. As D. P. Walker explains, "that this integration could be successfully carried out was largely due to the mistaken belief that behind Plato stood Moses and the ancient theologians."[486] In other words, the underlying idea was that Plato had inherited a much older body of knowledge which was already possessed by Moses and the progenitors of the Judeo-Christian tradition. Besides Moses, a chief representative of ancient theology was Hermes Trismegistus, whose works, the *Corpus Hermeticum*, were translated in the Renaissance by Ficino. Hermes' works were considered to be as old as the ancient Egyptians until in 1614 Isaac Casaubon showed that, in fact, they dated to the first centuries of the Christian era.[487] Ficino's translation, which covered 14 of the 15 treatises traditionally attributed to Hermes (the last one was missing from the manuscript on which Ficino worked), was entitled *Pimander* and dedicated to Ficino's patron, Cosimo de' Medici. In the preface to his work, Ficino proposed the following genealogy of ancient theology:

> He [i.e. Hermes] was called the first author of theology, and Orpheus followed him, taking second place in the ancient theology. After Aglaophemus, Pythagoras came next in theological succession, having been initiated into the rites of Orpheus, and he was followed by

---

[485] Charles B. Schmitt, "Perrenial Philosophy: From Agostino Steuco to Leibniz," *Journal of the History of Ideas* 27, no. 4 (October 1966): 505; D. P. Walker, *The Ancient Theology: Studies in Christian Platonism from the Fifteenth to the Eighteenth Century* (London: Duckworth, 1972); Maria Muccillo, *Platonismo, ermetismo e "prisca theologia": ricerche di storiografia filosofica rinascimentale* (Florence: L.S. Olschki, 1996); Martin Mulsow, "Ambiguities of the Prisca Sapientia in Late Renaissance Humanism," *Journal of the History of Ideas* 65, no. 1 (2004): 1–13; Wilhelm Schmidt-Biggemann, *Philosophia Perennis: Historical Outlines of Western Spirituality in Ancient, Medieval and Early Modern Thought* (Dordrecht: Springer, 2004).

[486] Walker, *The Ancient Theology*, 12.

[487] Walker, 18.

> Philolaus, teacher of our divine Plato. In this way, from a wondrous line of six theologians emerged a single system of ancient theology, harmonious in every part.[488]

As already mentioned, it was Yates' contribution to draw a connection between Bruno and Renaissance Hermeticism. Surely, *De monade* was one of those texts in which this connection was more evident, since reference to the ancient theologians was made in the introductory chapter to the poem. With the only exception of Philolaus, Bruno mentioned the same ancient theologians as Ficino while adding to the list Zoroaster and Apollonius of Tyana.[489] However, it was neither Hermes nor Orpheus who inspired Bruno to write *De monade,* but rather Plato and even more so Pythagoras. Indeed, as already mentioned, researches on numerology and number mysticism had been carried out by the Pythagoreans even since the foundation of their school. In addition, Bruno entertained the idea of creating a bridge between arithmetic and geometry by associating numbers and figures. This idea had also a Pythagorean origin, since the ancient Pythagoreans had established a connection between the first four numbers and the sequence point-line-surface-solid in attempt to explain the origin of all things out of numbers. We find traces of this ancient Pythagorean theory in Sextus Empiricus' *Against the Physicists*:

> Thus the point, for example, is ranked under the head of the One; for as the One is an indivisible thing, so also is the point; and just as the One is a principle in numbers, so too the point is a principle in lines. So that the point comes under the head of the One, but the line is regarded as belonging to the class of the Dyad; for both the Dyad and

[488] Marsilio Ficino, *Opera omnia* (Basel, 1576), 1836, cited and translated in Brian P. Copenhaver and Charles B. Schmitt, *Renaissance Philosophy* (Oxford: Oxford University Press, 1992), 147.

[489] *De monade,* 334: "Sed nos propositum resumentes dicimus huiusce generis numeros Pythagorae, Aglaophemo, Zoroastrio, Hermetique Babylonio fuisse principia, quibus operanti naturae homines cooperatores esse possint. Huiusce generis figuras Platonem supra sensibilium specierum orbem extulisse constat; 'Apollonius propter numerorum virtutem, audito illius nomine, puellam suscitavit.'"

> the line are conceived by way of transition.—And again: the length without breadth conceived as lying between two points is a line. So then, the line will belong to the Dyad class, but the plane to the Triad since it is not merely regarded as length, as was the Dyad, but has also taken to itself a third dimension, breadth. Also when three points are set down, two at an interval opposite to each other, and a third midway in the line formed from the two, but at a different interval, a plane is constructed. And the solid form and the body, as also the pyramid, are classed under the Tetrad. For when the three points are placed, as I said before, and another point is placed upon them from above, there is constructed the pyramidal form of the solid body; for it now possesses the three dimensions length, breadth, and depth.[490]

Interestingly enough, Sextus distinguished this theory from the fluxion theory since, in his historical reconstruction, the ancient Pythagoreans charged both of them with a cosmogonic meaning, although the fluxion theory was developed at a later stage. As for Bruno, he claimed, on the one hand, that the line was generated by the flowing of a point, the plane by the flowing of a line, and the solid by the flowing of a plane.[491] On the other hand, he established the following one-to-one correspondence between numbers and figures: point–one, line–two, plane–three, solid–four.[492] Hence, we can conclude that Bruno accepted both Pythagorean theories reported by Sextus. Building on this, he went on to show how the derivation process of the first ten numbers unfolded. The starting point of this process was, of course, the monad.

As you may recall, in De *minimo,* Bruno gave the following definition of the monad as the metaphysical minimum: "it [i.e. the mininum] is a monad

---

[490] Sextus Empiricus, *Against the Physicists*, 345–47.

[491] *De minimo*, 148: "Ergo linea nihil est nisi punctus motus, superficies nisi linea mota, corpus nisi superficies mota, et consequenter punctus mobilis est substantia omnium, et punctus manens est totum".

[492] *De monade*, 380-81: "Et tetrade est primum solidi natura reperta, quando in corporeis rebus numeri esse priores aptati debent. Quia punctum dat monas, atque Dat puncti fluxum dias, haec extenditur inde in planum triadis; demum tetras esse reponit corporeum."

rationally in numbers and essentially in all things."[493] This definition laid the foundation for the numerological project carried out in *De monade*. Here, the monad was conceived as both the beginning of the numerical series and the origin of all things by virtue of the fact that numbers were found everywhere. In addition, the monad was associated with the circle, the same figure that was attributed to the other two species of the minimum, the point and the atom. In the previous section, we have seen that, in endowing the minimum with a circular figure, Bruno gave a new meaning to the circle, which in the Middle Ages was used to signify God. In *De monade,* Bruno explained why the circle was the best way to represent the monad by means of analogies. For instance, the circle was the source of all geometric figures, just like the monad was the source of all numbers, for "all figures, whatever is their substance, when divided in their magnitude, are not reduced to diverse species, but they find their indivisible minimum in the circle and sphere."[494] Another reason why all figures could be reduced to the circle was that, regardless of the number of their sides, they could always be inscribed in a circle.[495] Finally, Bruno noticed that the fin, wing and arm all moved following a circular trajectory—another reason to believe that the circle was the most primordial figure.[496] However, Bruno specified, this did not mean that perfect circles were found in nature, "for nothing [in nature] is pure."[497] In *De monade* as well in *De minimo*, mathematical realism was off the table.

The chapter on the monad ended with a *scala* or "ladder," divided in three "orders," listing the beings in which the monad manifested itself. For the most part, these were metaphysical entities—such as the Mind, the Soul, the Mover—which occupied a central place in Bruno's philosophical system. Carlo Monti suggests that Bruno borrowed the original Neoplatonic idea of the ladder of being from Agrippa's *De occulta philosophia* (*On the Occult Philosophy*, 1531 – 1533),

---

[493] *De minimo*, 139–40: "Minimum est ... monas rationaliter in numeris, essentialiter in omnibus."

[494] *De monade,* 336: "Nec variam in speciem quantum resecando resolvunt, dividuum ii minime cyclum sphaeramque capessunt."

[495] *De monade*, 337.

[496] *De monade*, 339.

[497] *De monade*, 340: "Quoniam purum nihil est."

more precisely from book II, chapter IV ("De unitate et eius scala," "The unity and its ladder"). Because of this ladder, a version of which was appended to each chapter of *De monade,* this work has been regarded as a numerological treatise. However, *De monade* was not just about numerology. In it, speculations about the symbolic and mystical powers of numbers were intertwined with considerations regarding the existence a world-soul which made all things alive, including those apparently inanimate. For instance, while speaking of the ontological priority of the monad over numbers and beings, Bruno mentioned the fact that every being unfolded from a single soul located at the center of the universe.[498] Likewise, he spoke of a single principle by which all things were originated and kept alive, and to which all things returned after their death.[499]

This panpsychist view of the world, which was already expressed in Bruno's vernacular works, opened the door to natural magic. In Bruno's eyes, natural magic was the view that natural beings could be used to act upon one another by virtue of their spiritual interconnectedness. For this reason, for example, stones were thought to have an influence on the human soul.[500] As highlighted by Walker, Renaissance invocations of ancient theology were often accompanied by experiments in the field of natural magic, astrology, and numerology.[501] This was also the case of Bruno, who wrote several magical treatises throughout his life[502] and combined magic and numerology in *De monade*.

---

[498] *De monade*, 338: "Nempe anima a medio cordis membrum explicat omne, principio, arcano de semine stamina mittens, inde iterum relegenda suis verso ordine fatis, ac certa rerum serie."

[499] *De monade*, 343: "Lex una est qua per naturam fluximus alto e principio, qua servamur, sensu, ingenioque donati vegetique sumus, quo deinde refluxu occidua e regione altos redeamus ad ortus."

[500] Bruno, *Cause, Principle, and Unity*, 63: "[...] The properties of many stones and gems which, broken, recut or set in irregular pieces, have certain virtues of altering the spirit or of engendering affections and passions in the soul, not only in the body."

[501] Walker, *The Ancient Theology*, 2.

[502] Giordano Bruno, *Opere magiche*, ed. Simonetta Bassi, Elisabetta Scapparone, and Nicoletta Tirinnanzi (Milan: Adelphi, 2000).

Taking a step further, it might have been the interplay of magic and numerology which drew Leibniz's attention to Bruno's *De monade*, and inspired his conception of the monad as a mind-like substance. There has been much discussion about Leibniz's monadology and his debt to Bruno. Indeed, the similarities between Bruno's and Leibniz's theory of monads have suggested that the latter might have borrowed aspects of his theory from the former. For Leibniz scholars, the question is when Leibniz's debt incurred, whether in his youth (as suggested by Stuart Brown) or later in his life (as advocated by earlier interpreters).[503] It is not my purpose to address this question, but to focus on the extent of Leibniz's debt to Bruno. For Brandon Look, Leibinz's monadology was the ultimate expression of his metaphysics of substance, which can be viewed as a synthesis of five philosophical views: idealism, panpsychism, perspectivism, divine emanation and monadic hierarchy.[504] Of these five views, only two—panpsychism and divine emanation—could also be ascribed to Bruno. As we have seen, Bruno's panpsychist view of the universe was an integral part of the infinist cosmology expounded in his vernacular and Latin works. As for divine emanation, suffice it to say that in *De minimo* Bruno called God the "monad of the monads" (*monas monadum*)[505] and defined him as "the monad source of all numbers, the simplicity of all magnitudes and the substance of all compounds."[506]

On the contrary, in Bruno's works there were no traces of the other three Leibnizian views. In fact, Bruno utterly rejected idealism, namely, the view that only minds and ideas were substances, while matter and motion were phenomena derived from those substances. More precisely, in *De la causa*, he took issue with

---

503 Stuart Brown, "Monadology and the Reception of Bruno in the Young Leibniz," in *Giordano Bruno: Philosopher of the Renaissance*, ed. Hilary Gatti (Aldershot: Ashgate, 2002), 381–404.

504 Brandon C. Look, "Gottfried Wilhelm Leibniz," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, Summer 2017 (Metaphysics Research Lab, Stanford University, 2017), para. 5, https://plato.stanford.edu/archives/sum2017/entries/leibniz/.

505 *De minimo*, 146.

506 *De minimo*, 136: "Deus est monas omnium numerorum fons, simplicitas omnis magnitudinis et compositionis substantia."

the Platonists (the idealists *par excellence*) and their "ideal signs, separate from matter, for if these are not monsters, they are assuredly worse than monsters, being chimeras and pointless fantasies."[507] This leads Alfonso Ingegno to conclude that, rather than being a mere phenomenon, the Brunian matter "is a principle which is neither passing nor transient, a principle which cannot be annihilated and which is identified with the substance of beings themselves."[508] In general terms, Bruno was also opposed to the concept of ontological hierarchy, although in *De monade* he adopted the Neoplatonic idea of the ladder of being but only for numerological purposes. Finally, unlike Leibniz, Bruno did not believe that each monad offered a unique perspective on the universe, nor that monads were capable of perception. This was a major difference with Leibniz's monadology, for although in *De monade* there was a connection between monads and world-soul (and numerology and magic), Bruno never came to conclusion that monads were mind-like substances.

In conclusion, one might well argue that there were more differences than similarities between Bruno's and Leibniz's theory of monads. In fact, we have seen that most of the distinctive features of Leibniz's monadology were views that Bruno rejected. On the other hand, we know that Leibniz read Bruno's *De monade* as well as other of his works and it is hard not to see a link between their theories of monads. Like Bruno, Leibniz had a panpsychist view of the universe and believed in the existence of an original unity from which all things were derived—a unity that was called either God or monad depending on the context. Hence, if it is true that Leibniz was indebted to Bruno for his theory of monads, this is what the German philosopher was more likely to have borrowed from his Italian predecessor. However, it should be noted that Bruno himself was indebted to a much older tradition for the ideas of monad and world-soul, since the Pythagoreans and the Stoics must be credited with the first formulation of those ideas. In this respect, I agree with Brown who argues that "the main reason for the similarities between the two philosophers is their debt to common sources

[507] *Cause, Principle, and Unity*, 85.

[508] Alfonso Ingegno, "Introduction," in *Cause, Principle, and Unity*, by Giordano Bruno, ed. Robert de Lucca and Richard J. Blackwell (Cambridge: Cambridge University Press, 1998), xviii.

and their membership of a common philosophical tradition."[509] The fact that Pythagoreanism was an important part of this tradition offers yet another example of how, in many aspects of his mathematical thought, Bruno was an heir of the philosopher of Samos.

[509] Brown, "Monadology and the Reception of Bruno," 384.

Conclusions

Constraints were imposed to finish this work within the time limit. For this reason, I will use this space to present the issues not covered by this work as well as its main results. First of all, this work does not contain an account of all the mathematical works written by Bruno. For the record, this includes eight works: the four dialogues on Mordente's compass (1586), the *Articuli adversus mathematicos* (1588), *De minimo* (1591), the *Praelectiones geometricae* (1591/2) and the *Ars deformationum* (1591/2). These last two works remained unpublished until 1964 when Aquilecchia discovered a copy of them contained in a manuscript of the University Library of Jena.[510] Aquilecchia believed that both the *Praelectiones* and the *Ars* dated back to Bruno's stay in Padua from August 1591 to May 1592, and that they contained the teaching material used by Bruno in his lectures to a group of German students. As for the content, the two works provided an explanation of the mathematical theory presented in *De minimo*. The same can be said of the *Articuli,* although this work was written at an earlier date than *De minimo* and in it the mathematical reform advocated by Bruno was discussed in greater detail. As such, the *Articuli*, the *Pralectiones* and the *Ars* can be useful to understand *De minimo*, of which however we already have a detailed account provided by Atanasijević.[511] After all, *De minimo* was the main mathematical work written by Bruno. For this reason, interpreters and, especially, critics of Bruno's mathematics have focused mainly on this work. However, as I have tried to show in Chapter 3, this has also led scholars to neglect important aspects of Bruno's mathematical thought, starting with the fact that, in purely mathematical terms, the theory presented in the dialogues on Mordente's compass was more coherent that the version of it presented in Bruno's later works. In light of this, I decided to give more visibility to the dialogues on Mordente's compass (although I could not addressed all the issues raised in them), while I studied *De minimo* and Bruno's mathematics in general from a

[510] Giordano Bruno, *Praelectiones geometricae e Ars deformationum. Testi indediti*, ed. Giovanni Aquilecchia (Rome: Edizioni di storia e letteratura, 1964).

[511] Ksenija Atanasijević, *The Metaphysical and Geometrical Doctrine of Bruno, as given in His Work De Triplici Minimo*, trans. G. V. Tomashevich (St. Louis, Missouri: W. H. Green, 1972).

thematic rather than analytic perspective.

Moreover, this work does not contain an account of the reception of Bruno's mathematics. This issue has been addressed especially with regard to England and the Northumberland circle, focusing on the extent to which English scholars were indebted to Bruno for their ideas about infinity and atomism.[512] On the contrary, no one has examined whether the transfer of ideas occurred in both directions, and thus whether Bruno was influenced by English authors such as John Dee and Thomas Harriot as well as the other way around. This investigation is left for future studies.

The account of Bruno's mathematics presented in this work is structured around three questions (this is what I mean when I say that my approach to Bruno's mathematics is more thematic than analytic):

(1) What are the sources of Bruno's atomistic geometry?
(2) Did Bruno anticipated the concept of infinitesimal?
(3) Was Bruno a mathematical realist?

I think that it is important to answer these questions because they were either left open (as in the case of the first question) or they were answered incorrectly (as in the case of the second and third question). In both cases, this had a negative effect because the lack of sources led to regard Bruno's atomistic geometry as an isolated episode in the history of mathematics, while refusing to acknowledge a connection between Bruno and the infinitesimals and viewing him as a mathematical realist made his atomistic geometry look outdated. What was at stake was Bruno's reputation as a mathematician.

Starting with the first question, I addressed it in terms of 'tradition.' More specifically, I tried to show that Bruno was in line with a group of medieval atomists who, drawing inspiration from Neopythagorean texts such as Boethius'

---

[512] Robert Kargon, "Thomas Hariot, the Northumberland Circle and Early Atomism in England," *Journal of the History of Ideas* 27, no. 1 (January 1966): 128–36; Daniel Massa, "Giordano Bruno's Ideas in Seventeenth-Century England," *Journal of the History of Ideas* 38, no. 2 (April 1977): 227–42; Hilary Gatti, "Minimum and Maximum, Finite and Infinite Bruno and the Northumberland Circle," *Journal of the Warburg and Courtauld Institutes* 48 (1985): 144–63; Saverio Ricci, "Giordano Bruno e Il 'Northumberland Circle' (1600-1630)," *Rinascimento* 25 (1985): 335–56.

Institutio arithmetica, defined the point as an atom or "unit having position." As demonstrated by Robert, this atomistic definition of the point became the starting point for explorations of non-Aristotelian conceptions of the continuum. Indeed, as is well known, Aristotle was a fierce opponent of atomism as he believed that the division of the continuum could go to infinity. Ramon Llull was among the medieval authors who, challenging Aristotle, claimed that the continuum was composed of indivisible points. Thus, like Bruno, Llull belonged to the tradition of Pythagorean atomism. Llull presented his atomistic view of mathematical objects in a little known work, the *Liber de geometria nova*, which to my knowledge has never been related to Bruno's mathematics. Since Bruno had an extensive knowledge of Llull's works, I assumed that the *Geometra nova* may have inspired Bruno to develop his atomistic geometry. To substantiate this claim, I examined whether Bruno could have read this work, concluding that there was evidence that this might have occurred.

If Llull was Bruno's source for his atomistic view of the continuum, Bruno himself acknowledged Cusanus as the privileged source of his mathematical ideas. The bridge between Cusan and Pythagorean atomism was provided by the concepts of enfolding and unfolding. Indeed, both these concepts were inherited from the Pythagorean tradition (in particular from Thierry of Chartres' commentary on Boethius' *Institutio*) and embedded in the atomistic theory presented by Cusanus in *De mente*. On the other hand, Cusanus was Bruno's source of his idea of the minimum. This did not mean that Bruno and Cusanus shared the same view of the minimum. In fact, in Cusanus' account the minimum was an ineffable reality, while in Bruno's account the minimum could be visualized using instruments such as Mordente's compass.

As for the second question—whether Bruno's idea of the minimum can be considered a forerunner of the modern concept of infinitesimal—I addressed it by analyzing Bruno's first mathematical writings, the four dialogues on Mordente's compass. The analysis of these dialogues revealed that Bruno tried to impose his interpretation of the compass, which, in his opinion but not in that of Mordente, confirmed the existence of minimum magnitudes of which mathematical objects were composed. More importantly, Bruno's concept of the minimum, as presented in the dialogues on Mordente's compass, was closer to the concept of infinitesimal than previously thought. Early interpreters of Bruno's mathematics such as Olschki have claimed that the Brunian minimum was far removed from the concept of infinitesimal on account of its having a specific form. On the contrary, in the dialogues on Mordente's compass, Bruno based his argument on the assumption that minimum magnitudes were shapeless. It was in *De minimo* that Bruno claimed that the mathematical minimum was an extended point having a circular shape, but this was the result of Bruno's attempt to integrate physics, metaphysics and mathematics into a single theory.

Finally, I dealt with the third question—on Bruno's mathematical realism—by tracing the development of his conception of mathematics from his vernacular works to his Latin poems. I claimed that Bruno opposed both the idea that mathematical objects exist independently of our mind (as attested by his rejection of circular devices such as celestial orbs) and the idea that mathematical models could be used to explain nature (as attested by his belief that natural philosophers and not mathematicians were able to understand the true causes of physical phenomena). In fact, Bruno's mistrust in mathematical physics and mathematical astronomy was a consequence of his unwillingness to accept the existence of mathematical objects in nature.

On the one hand, this provides a response to Védrine's objection that Bruno's realism was the cause of his mathematical failure. On the contrary, I believe that Bruno did not succeed in turning his atomistic geometry into a full-fledge mathematical theory because he was led astray by his philosophical agenda. On the other hand, one may argue that Bruno's rejection of mathematical realism, and in particular of the possibility of explaining nature in mathematical terms, opens a gap between Bruno and the Scientific Revolution. Indeed, the narrative of the Scientific Revolution, as told by the likes of Husserl and Koyré,[513] rested on the assumption that the rise of modern science coincided with the emergence of a new view of nature, one characterized by a quantitative and mathematical approach. The mathematization of nature has since become the epitome of the Scientific Revolution. But what if mathematicizing was not the

513 Edmund Husserl, *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy*, trans. David Carr (Evanston, IL: Northwestern University Press, 1970), 23–59; Alexandre Koyré, *From the Closed World to the Infinite Universe* (New York: Harper Torchbooks, 1958).

only approach to the study of nature? What if there were more "forms of mathematization" (as claimed by Roux[514]) or there were scholars who tried to "naturalize mathematics?"[515] The invention of the calculus provides evidence that, as it was, classical mathematics was unable to account for natural phenomena such as motion.[516] Bruno was aware of this, as proved by the fact that he argued against those who wanted to force nature into predetermined mathematical models (e.g. the Ptolemaic astronomers). What if the modernity of Bruno's mathematics consisted in this awareness? Will it lead to reconsider his mathematical abilities and, perhaps, the whole relationship between early modern mathematics and the Scientific Revolution? This study is a first step in this direction.

---

514 Sophie Roux, "Forms of Mathematization (14th-17th Centuries)," *Early Science and Medicine* 15, no. 4–5 (2010): 319–37.

515 For example, an attempt to naturalize mathematics was made by Francis Bacon. See Giuliano Mori, "Mathematical Subtleties and Scientific Knowledge: Francis Bacon and Mathematics, at the Crossing of Two Traditions," *The British Journal for the History of Science* 50, no. 01 (March 2017): 1–21.

516 David Berlinski, *A Tour of the Calculus* (New York: Pantheon Books, 1995).

Bibliography

Albanese, Luciano. “Bruno e le linee indivisibili.” *Bruniana & Campanelliana* 7, no. 1 (2001): 201–7.

Albertson, David. “*Boethius Noster*. Thierry of Chartres’s *Arithmetica* Commentary as a Missing Source of Nicholas of Cusa’s *De Docta Ignorantia*.” *Recherches de Théologie et Philosophie Médiévales* 83, no. 1 (2016): 143–99.

———. *Mathematical Theologies: Nicholas of Cusa and the Legacy of Thierry of Chartres*. Oxford: Oxford University Press, 2014.

Alexander, Amir. *Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World*. New York: Scientific American/Farrar, Straus and Giroux, 2014.

Alós y de Dou, Ramón de. *Los catalogos lulianos. Contribución al estudio de la obra de Ramón Lull*. Barcelona: Francisco J. Altés y Alabart, 1918.

Amato, Barbara. “Il concetto di ‘termine’ nel «De minimo».” In *Letture bruniane I-II del lessico intellettuale europeo (1996-1997)*. Pisa: Istituti Editoriali e Poligrafici Internazionali, 2002. http://digital.casalini.it/10.1400/42792.

———. “Introduction.” In *Acrotismo cameracense: le spiegazioni degli articoli di fisica contro i peripatetici*, by Giordano Bruno, 11–34. edited by Barbara Amato. Pisa: Fabrizio Serra, 2009.

Aquilecchia, Giovanni. “Bruno e la matematica a lui contemporanea.” In *Schede bruniane: 1950-1992*, 311–17. Milan: Vecchiarelli, 1993.

———. “Giordano Bruno in Inghilterra (1583-1585). Documenti e testimonianze.” *Bruniana & Campanelliana* 1, no. 1/2 (1995): 21–42.

———. “Introduction.” In *Due dialoghi sconosciuti e due dialoghi noti:* Idiota triumphans, De somnii interpretatione Mordentius, De mordentii circino, by Giordano Bruno, edited by Giovanni Aquilecchia. Rome: Edizioni di storia e letteratura, 1957.

———. “Introduction.” In *Praelectiones Geometricae e Ars Deformationum. Testi Inediti*, by Giordano Bruno, ix–xxvi. edited by Giovanni Aquilecchia. Roma: Edizioni di Storia e Letteratura, 1964.

Aristotle. “On Generation and Corruption.” In *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, edited by J. Barnes, 1122–1215. Princeton, NJ: Princeton University Press, 1995.

———. "On Indivisible Lines." In *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, edited by J. Barnes, 3288–3309. Princeton, NJ: Princeton University Press, 1995.

———. "On the Soul." In *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, edited by J. Barnes, 1405–1518. Princeton, NJ: Princeton University Press, 1995.

———. "Physics." In *The Complete Works of Aristotle. The Revised Oxford Translation. One Volume Digital Edition*, edited by J. Barnes, 699–983. Princeton, NJ: Princeton University Press, 1995.

Armitage, Angus. *Copernicus, the Founder of Modern Astronomy*. London: Allen and Unwin, 1938.

Artmann, Benno, and Lothar Schäfer. "On Plato's 'Fairest Triangles' (Timaeus 54a)." *Historia Mathematica* 20, no. 3 (August 1993): 255–64. https://doi.org/10.1006/hmat.1993.1022.

Atanasijević, Ksenija. *The Metaphysical and Geometrical Doctrine of Bruno, as given in His Work De Triplici Minimo*. Translated by G. V. Tomashevich. St. Louis, Missouri: W. H. Green, 1972.

Badia, Lola, Joan Santanach, and Albert Soler. *Ramon Llull as a Vernacular Writer. Communicating a New Kind of Knowledge*. Woodbridge: Tamesis, 2016.

Barker, Peter, and Bernard R Goldstein. "Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal." *Perspectives on Science* 6, no. 3 (1998): 232–258.

Beierwaltes, Werner. "'Centrum tocius vite'. Zur Bedeutung von Proklos' 'Theologia Platonis' im Denken des Cusanus." In *Proclus et la théologie platonicienne*, edited by A. Ph. Segonds and Carlos G. Steel, 629–651. Leuven: Leuven University Press, 2000.

Berryman, Sylvia. "Ancient Atomism." In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Winter 2016. Metaphysics Research Lab, Stanford University, 2016. https://plato.stanford.edu/archives/win2016/entries/atomism-ancient/.

Berti, Domenico. *Vita di Giordano Bruno da Nola*. Turin: Paravia, 1868.

Bignone, Ettore. "Per la fortuna di Lucrezio e dell'epicureismo nel medio evo." *Rivista di filologia e d'istruzione classica* 41 (1913): 230–62.

Biver, Paul, and Marie-Louise Biver. *Abbayes, Monastères et Couvents de Paris*. Paris: Editions d'histoire et d'art, 1970.

Blay, Michel. *Reasoning with the Infinite: From the Closed World to the Mathematical Universe*. University of Chicago Press, 1999.

Blum, Paul Richard. "'Saper trar il contrario dopo aver trovato il punto de l'unione': Bruno, Cusano e il platonismo." In *Letture bruniane I-II del lessico intellettuale europeo (1996-1997)*. Pisa: Istituti editoriali e poligrafici internazionali, 2002.

Blumenberg, Hans. *The Legitimacy of the Modern Age*. Translated by Robert M. Wallace. Cambridge, Massachusetts, and London, England: The MIT Press, 1999.

Blumenthal, Geoffrey. "Diplomacy, Patronage, and the Preface to *De Revolutionibus*." *Journal for the History of Astronomy* 44, no. 1 (February 2013): 75–92. https://doi.org/10.1177/002182861304400104.

Boethius. *In Categorias Aristotelis commentaria*. Edited by J.-P. Migne. Patrologia Latina 64. Paris, 1847.

———. *Institutio arithmetica*. Edited by Jean-Yves Guillaumin. Paris: Les Belles Lettres, 2002.

Boffito, Giuseppe. *Paolo dell'Abbaco e Fabrizio Mordente. Il primo compasso proporzionale costruito da Fabrizio Mordente e la Operatio Cilindri di Paolo dell'Abbaco*. Il Facsmile 6. Florence: Libreria internazionale, 1931.

Bönker-Vallon, Angelika. "Bruno e Proclo: connessioni e differenze tra la matematica neoplatonica e quella bruniana." In *La filosofia di Giordano Bruno. Problemi ermeneutici e storiografici*, edited by E. Canone. Florence: Leo S. Olschki, 2003.

———. "Giordano Bruno e la matematica." *Rinascimento* 39 (1999): 67–93.

———. *Metaphysik und Mathematik bei Giordano Bruno*. Berlin: Akademie Verlag, 1995.

Bonner, Anthony. *Doctor Illuminatus: A Ramón Llull Reader*. Mythos. Princeton, N.J: Princeton University Press, 1993.

———. *The Art and Logic of Ramon Llull: A User's Guide*. Leiden: Brill, 2007.

Bork, Robert Odell. *The Geometry of Creation: Architectural Drawing and the Dynamics of Gothic Design*. Farnham: Ashgate, 2011.

Bossy, John. *Giordano Bruno and the Embassy Affair*. New Haven: Yale University Press, 2002.

Boyer, Carl B. *The History of the Calculus and Its Conceptual Development*. New York: Dover, 1949.

Bredow, Gerda von. “Die Bedeutung des Minimum in der Coincidentia oppositorum.” In *Nicolò Cusano agli inizi del mondo moderno: atti del Congresso intemazionale in occasione de V centenario della morte di Nicolò Cusano, Bressanone, 6-10 Settembre, 1964*, 357–66. Florence: G.C. Sansoni, 1970.

Brown, Alison. *The Return of Lucretius to Renaissance Florence*. Cambridge, MA: Harvard University Press, 2010.

Brown, Stuart. “Monadology and the Reception of Bruno in the Young Leibniz.” In *Giordano Bruno: Philosopher of the Renaissance*, edited by Hilary Gatti, 381–404. Aldershot: Ashgate, 2002.

Bruno, Giordano. “Acrotismus camoeracensis.” In *Opera latine conscripta*, edited by F. Fiorentino, I, pt. 1:53–190. Neaples: Morano, 1879.

———. “Articuli adversus mathematicos.” In *Opera latine conscripta*, edited by F. Tocco and H. Vitelli, I, pt. 3:1–132. Florence: Le Monnier, 1889.

———. *Cause, Principle, and Unity*. Edited by Robert de Lucca and Richard J. Blackwell. Cambridge: Cambridge University Press, 1998.

———. “De immenso et innumberabilibus (Books 1-3).” In *Opera latine conscripta*, edited by F. Fiorentino, I, pt. 1:191–398. Neaples: Morano, 1879.

———. “De monade, numero et figura.” In *Opera latine conscripta*, edited by F. Fiorentino, Vol. I, pt. 2. Neaples: Morano, 1884.

———. “De triplici minimo et mensura.” In *Opera latine conscripta*, edited by F. Tocco and H. Vitelli, I, pt. 3:119–361. Florence: Le Monnier, 1889.

———. *Dialoghi filosofici italiani*. Edited by Michele Ciliberto. Milan: Arnoldo Mondadori, 2000.

———. *Due dialoghi sconosciuti e due dialoghi noti:* Idiota triumphans, De somnii interpretatione Mordentius, De Mordentii circino. Edited by Giovanni Aquilecchia. Rome: Edizioni di storia e letteratura, 1957.

———. “On the Infinite Universe and Worlds.” In *Giordano Bruno. His Life and Thought, with an Annotated Translation of His Work On the Infinite*

Universe and Worlds, by Dorothea Waley Singer. New York: H. Schuman, 1950.
———. *Opera latine conscripta*. Edited by F. Tocco and H. Vitelli. Vol. 3. Florence: Le Monnier, 1891.
———. *Opere latine*. Translated by Carlo Monti. Torino: UTET, 1980.
———. *Opere lulliane*. Edited by Marco Matteoli, Rita Sturlese, and Nicoletta Tirinnanzi. Milan: Adelphi, 2012.
———. *Opere magiche*. Edited by Simonetta Bassi, Elisabetta Scapparone, and Nicoletta Tirinnanzi. Milan: Adelphi, 2000.
———. *The Ash Wednesday Supper*. Translated by Hilary Gatti. Toronto: University of Toronto Press, 2018.
Bucciantini, Massimo. *Campo dei Fiori: storia di un monumento maledetto*. Torino: Einaudi, 2015.
Burkert, Walter. *Lore and Science in Ancient Pythagoreanism*. Cambridge, Massachusetts: Harvard University Press, 1972.
Busard, Hubert L. L. *The First Latin Translation of Euclid's Elements Commonly Ascribed to Adelard of Bath*. Toronto: Pontificial Institute of Medieval Studies, 1983.
Caiazzo, Irene. "Introduction." In *The Commentary on the* De Arithmetica *of Boethius*, by Thierry of Chartres, edited by Irene Caiazzo. Toronto: Pontificial Institute of Medieval Studies, 2015.
———. "The Four Elements in the Work of William of Conches." In *Guillaume de Conches: Philosophie et science au XIIe siècle*, 3–66. Florence: SISMEL Edizioni del Galluzzo, 2011.
Calderini De Marchi, Rita. *J. Corbinelli et les Erudits Francaise d'après la correspondance inédite Corbinelli-Pinelli (1566-1587)*. Milan: Hoepli, 1914.
Camerota, Filippo. *Il compasso di Fabrizio Mordente. Per la storia del compasso di proporzione*. Florence: Leo S. Olschki, 2000.
Campanella, Tommaso. *A Defense of Galileo, the Mathematician from Florence: Which Is an Inquiry as to Whether the Philosophical View Advocated by Galileo Is in Agreement with, or Is Opposed to, the Sacred Scriptures*. Edited by Richard J. Blackwell. Notre Dame: University of Notre Dame Press, 1994.

———. *La città del sole: e altri scritti*. Edited by Germana Ernst and Luigi Firpo. Bari: Laterza, 1997.

Canone, Eugenio. "Contributo per una ricostruzione dell'antica 'libraria' di S. Domenico Maggiore." In *Giordano Bruno: gli anni napoletani e la* peregrinatio *europea: immagini, testi, documenti*, edited by Eugenio Canone, 191–246. Cassino: Università degli studi di Cassino, 1992.

———. *Giordano Bruno: gli anni napoletani e la* peregrinatio *europea: immagini, testi, documenti*. Cassino: Università degli studi, 1992.

Cassirer, Ernest. "Il problema della conoscenza nella filosofia e nella scienza dall'Umanesimo alla scuola cartesiana." In *Storia della filosofia moderna*, translated by Pasquinelli, A., Vol. I. Turin: Einaudi, 1961.

Cassirer, Ernst. *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit*. Berlin: B. Cassirer, 1906.

———. *Individuum und Kosmos in der Philosophie der Renaissance*. Leipzig: Teubner, 1927.

———. *Substance and Function, and Einstein's Theory of Relativity*. Chicago: Open Court, 1953.

Cavalieri, Bonaventura. *Exercitationes geometricae sex*. Bologna: Monti, 1647.

———. *Geometria indivisibilibus continuorum nova quadam rations promota*. Bologna: Monti, 1635.

Ciliberto, Michele. "Giordano Bruno, angelo della luce tra furore e disincanto." In *Dialoghi filosofici italiani*, by Giordano Bruno, edited by Michele Ciliberto. Milan: Arnoldo Mondadori, 2000.

———. "Introduction." In *Opere magiche*, by Giordano Bruno, edited by Simonetta Bassi, Elisabetta Scapparone, and Nicoletta Tirinnanzi. Milan: Adelphi, 2000.

Çimen, Ünsal. "On Saving the Astronomical Phenomena: Physical Realism in Struggle with Mathematical Realism in Francis Bacon, Al-Bitruji, and Averroes." *HOPOS: The Journal of the International Society for the History of Philosophy of Science*, October 5, 2018. https://doi.org/10.1086/701058.

Coignet, Michel. *Della forma, et parti del compasso di Fabrizio Mordente Salernitano* ... Antwerp, 1608.

———. *La geometrie reduite en un facile et briefve pratique* … Paris: Charles Hulpeau, 1626.

———. *L'uso del compasso di Fabrizio Mordente Salernitano* … Antwerp, 1608.

Compagno, Carla. "Il *Liber de geometia noua et compendiosa* di Raimondo Lullo." *Ambitos* 31 (2014): 35–45.

Copenhaver, Brian P., and Charles B. Schmitt. *Renaissance Philosophy*. Oxford: Oxford University Press, 1992.

Cormack, Lesley B., Steven A. Walton, and John A. Schuster, eds. *Mathematical Practitioners and the Transformation of Natural Knowledge in Early Modern Europe*. Cham: Springer, 2017.

Cornelli, Gabriele. *In Search of Pythagoreanism: Pythagoreanism as an Historiographical Category*. Berlin: De Gruyter, 2013.

Cornford, Francis M. "Mysticism and Science in the Pythagorean Tradition." *The Classical Quarterly* 16, no. 3–4 (1922): 137–150.

———. *Plato and Parmenides. Parmenides' Way of Truth and Plato's Parmenides Translated with an Introduction and a Running Commentary*. London: Routledge, 1964.

Custurer, Jaume. *Disertaciones historicas del culto inmemorial del B. Raymundo Lulio*. Mallorca, 1700.

De Bernart, Luciana. *Cusano e i matematici*. Pisa: Scuola Normale Superiore, 1999.

———. Numerus quodammodo infinitus*: per un approccio storico-teorico al dilemma matematico nella filosofia di Giordano Bruno*. Rome: Edizioni di storia e letteratura, 2002.

De Pace, Anna. *Le matematiche e il mondo: ricerche su un dibattito in Italia nella seconda metà del Cinquecento*. Milan: Franco Angeli, 1993.

Delisle, Léopold. *Le cabinet des manuscripts de la Bibliothèque Nationale*. Vol. 2. Paris: Imprimerie impériale, 1874.

Descotes, Dominique. "Two Jesuits Against the Indivisibles." In *Seventeenth-Century Indivisibles Revisited*, edited by V. Jullien, 249–74. Basel: Birkhäuser, 2015.

Dhanani, Alnoor. *The Physical Theory of Kalām: Atoms, Space, and Void in Basrian Muʿtazilī Cosmology*. Leiden: Brill, 1994.

Dillon, John M. *The Heirs of Plato: A Study of the Old Academy, 347-274 B.C.* Oxford: Oxford University Press, 2003.

———. *The Middle Platonists, 80 B.C. to A.D. 220*. Ithaca, N.Y: Cornell University Press, 1996.

Duhem, Pierre Maurice Marie. *To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo*. Translated by Edmund Dolan and Chaninah Maschler. Chicago: University of Chicago Press, 1985.

Dupré, Louis. *Passage to Modernity: An Essay in the Hermeneutics of Nature and Culture*. New Haven: Yale University Press, 1993.

Edwards, C. Henry. *The Historical Development of the Calculus*. New York: Springer, 1994.

Epicurus. *Epistola a Erodoto*. Edited by Francesco Verde. Rome: Carocci, 2010.

Favaro, Antonio. "Per la storia del compasso di proporzione." *Atti del Reale Instituto Veneto di Scienze, Lettere e Arti* 67 (1907): 723–39.

Feingold, Mordechai. "Giordano Bruno in England, Revisited." *Huntington Library Quarterly* 67, no. 3 (September 2004): 329–46. https://doi.org/10.1525/hlq.2004.67.3.329.

Festa, Egidio. "Aspects de la controverse sur les invisibles." In *Geometria e atomismo nella scuola galileiana*, edited by Massimo Bucciantini and Maurizio Torrini. Florence: Leo S. Olschki, 1992.

Festugière, André-Jean. *La révélation d'Hermès Trismégiste*. Paris: Belles lettres, 2014.

Ficino, Marsilio. *Opera omnia*. Basel, 1576.

Firpo, Luigi. *Il processo di Giordano Bruno*. Edited by Diego Quaglioni. Rome: Salerno Editrice, 1993.

Flasch, Kurt. *Nikolaus von Kues: Geschichte einer Entwicklung*. Frankfurt am Main: Vittorio Klostermann, 1998.

Franklin, Alfred. *Les anciennes bibliothèques de Paris: églises, monastères, colléges, etc.* Vol. 1. Paris: Imprimerie impériale, 1867.

Froidmont, Libert. *Labyrinthus sive de compositione continui liber unus*. Antwerp: Balthasar Moretus, 1631.

Gaiffer, B. de, ed. *Vita Beati Raymundi Lulli*. Analecta Bollandiana, xlviii, 1930.

Galilei, Galileo. *Le operazioni del compasso geometrico e militare ...* Padoa: Pietro Marinelli, 1606.

Gandillac, M. De. "*Explicatio-Complicatio* chez Nicolas de Cues." In *Concordia discors. Studi su Niccolò Cusano e l'umanesimo europeo offerti a Giovanni Santinello*, 77–106. Padua: Antenore, 1993.

Garber, Daniel. "Descartes among the Novatores." *Res Philosophica* 92, no. 1 (2015): 1–19. https://doi.org/10.11612/resphil.2015.92.1.1.

Garin, Eugenio, ed. *Testi umanistici su l'ermetismo*. Roma: Bocca, 1955.

Gatti, Hilary. *Giordano Bruno and Renaissance Science*. Ithaca and London: Cornell University Press, 1999.

———. *The Renaissance Drama of Knowledge. Giordano Bruno in England*. London; New York: Routledge, 1989.

Gentile, Giovanni. *Giordano Bruno e il pensiero del Rinascimento*. Florence: Vallecchi, 1920.

Giardina, Giovanna R. *Erone di Alessandria: le radici filosofico-matematiche della tecnologia applicata*. Definitiones. *Testo, traduzione e commento*. Catania: CUECM, 2003.

Giusti, Enrico. *Bonaventura Cavalieri and the Theory of Indivisibles*. Cremona: Edizioni Cremonese, 1980.

Glasner, Ruth. *Averroes' Physics*. Oxford: Oxford University Press, 2009.

———. "Ibn Rushd's Theory of *Minima Naturalia*." *Arabic Sciences and Philosophy* 11, no. 1 (March 2001).

———. "Proclus' Commentary on Euclid's Definitions 1, 3 and 1, 6." *Hermes* 120, no. H. 3 (1992): 320–333.

Goddu, Andr□. "The Realism That Duhem Rejected in Copernicus." *Synthese* 83, no. 2 (May 1990): 301–15. https://doi.org/10.1007/BF00413763.

Gorham, Geoffrey, Benjamin Hill, Edward Slowik, and C. K. Waters, eds. *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century*. Minneapolis: University of Minnesota Press, 2016.

Goulding, Robert. "Five Versions of Ramus's *Geometry*." In *Et Amicorum: Essays on Renaissance Humanism and Philosophy in Honour of Jill Kraye*, edited by Anthony Ossa-Richardson and Margaret Meserve, 355–87. Leiden: Brill, 2018.

Granada, Miguel A. "Epicuro y Giordano Bruno: descubrimiento de la naturaleza y liberación moral (una confrontación a través de Lucrecio)." In *Historia,* *lenguaje, sociedad. Homenaje a Emilio Lledó*, edited by Manuel Cruz, Miguel A. Granada, and Anna Pappol, 125–41. Barcelona: Editorial Crítica, 1989.

———. "Il rifiuto della distinzione fra *potentia absoluta* e *potentia ordinata* di Dio e l'affermazione dell'universo infinito in Giordano Bruno." *Rivista di storia della filosofia* 49, no. 3 (1994): 495–532.

———. "L'héliocentrisme de Giordano Bruno entre 1584 et 1591: la disposition des planètes inférieures et les mouvements de la Terre." *Bruniana & Campanelliana* 16, no. 1 (2010): 31–50.

Granada, Miguel Ángel. "Introduction." In *La cena de las cenizas*, by Giordano Bruno, edited by Miguel A. Granada. Madrid: Tecnos, 2015.

Greenblatt, Stephen. *The Swerve: How the World Became Modern*. New York: W.W. Norton, 2011.

Grellard, Christophe, and Aurélien Robert, eds. *Atomism in Late Medieval Philosophy and Theology*. Leiden: Brill, 2009.

Guthrie, William K. C. *A History of Greek Philosophy. Vol. I: The Earlier Presocratics and the Pythagoreans*. Cambridge: Cambridge University Press, 1962.

Hall, A. Rupert. *The Revolution in Science, 1500-1750*. London: Longman, 1983.

Häring, N.M., ed. *Commentaries on Boethius by Thierry of Chartres and His School*. Toronto: Pontificial Institute of Medieval Studies, 1971.

Harries, Karsten. "The Infinite Sphere: Comments on the History of a Metaphor." *Journal of the History of Philosophy* 13, no. 1 (1975): 5–15. https://doi.org/10.1353/hph.2008.0143.

Haskell, Yasmin. "Religion and Enlightenment in the Neo-Latin Reception of Lucretius." In *The Cambridge Companion to Lucretius*, edited by Stuart Gillespie and Philip R. Hardie, 185–201. Cambridge: Cambridge University Press, 2007.

Henry, John. *The Scientific Revolution and the Origins of Modern Science*. Houndmills, Basingstoke, Hampshire: Palgrave, 2001.

———. "Void Space, Mathematical Realism, and Francesco Patrizi Da Cherso's Use of Atomistic Arguments." In *Late Medieval and Early Modern Matter Corpuscular Theories*, edited by C. H. Lüthy, J. E. Murdoch, and W. R. Newman, 133–61. Leiden: Brill, 2001.

Hero of Alexandria. *Opera quae supersunt omnia. Vol. 4: Definitiones*. Edited by J.L. Heiberg. Stuttgart: B.G. Teubner, 1903.

Higuera Rubio, José. “El ‘atomismo’ luliano y el problema del continuo: Una explicación lógico-geométrica de la constitución elemental de las sustancias.” *Scintilla* 10, no. 1 (2013): 19–36.

———. *Fisica y teologia (atomismo y movimiento en el Arte luliano)*. Madrid: Circulo Rojo, 2014.

Hillgarth, J. N. *Ramon Lull and Lullism in Fourteenth-Century France*. Oxford: Clarendon Press, 1971.

Hirai, Hiro. “Seed Concept.” In *Encyclopedia of Renaissance Philosophy*, edited by Marco Sgarbi, 1–4. Cham: Springer International Publishing, 2015. https://doi.org/10.1007/978-3-319-02848-4_404-1.

Hopkins, Jasper. *Nicholas of Cusa on Wisdom and Knowledge*. Minneapolis: Banning, 1996.

Horky, Philip Sidney. *Plato and Pythagoreanism*. New York: Oxford University Press, 2013.

Iamblichus. *The Theology of Arithmetic: On the Mystical, Mathematical and Cosmological Symbolism of the First Ten Numbers*. Translated by Robin Waterfield. Grand Rapids, Mich: Phanes Press, 1988.

Ingegno, Alfonso. *Cosmologia e filosofia nel pensiero di Giordano Bruno*. Florence: La nuova Italia, 1978.

———. “Introduction.” In *Cause, Principle, and Unity*, by Giordano Bruno, vii–xxix. edited by Robert de Lucca and Richard J. Blackwell. Cambridge: Cambridge University Press, 1998.

Jacquart, Danielle. “Minima in Twelfth-Century Medical Texts from Salerno.” In *Late Medieval and Early Modern Corpuscular Matter Theories*, edited by Christoph H. Lüthy, John E. Murdoch, and William R. Newman, 75–90. Leiden: Brill, 2001.

Jardine, Nicholas. “Epistemology of the Sciences.” In *The Cambridge History of Renaissance Philosophy*, edited by Charles B. Schmitt, Quentin Skinner, and Eckhard Kessler, 685–711. Cambridge: Cambridge University Press, 1988.

John E. Murdoch. “Beyond Aristotle: Indivisibles and Infinite Divisibility in the Later Middle Ages.” In *Atomism in Late Medieval Philosophy and*

Theology, edited by Christophe Grellard and Aurélien Robert, 15–38. Leiden; Boston: Brill, 2009.

Johnson, Monte, and Catherine Wilson. "Lucretius and the History of Science." In *The Cambridge Companion to Lucretius*, edited by Stuart Gillespie and Philip R. Hardie, 131–48. Cambridge: Cambridge University Press, 2007.

Jullien, V. "Explaining the Sudden Rise of Methods of Indivisibles." In *Seventeenth-Century Indivisibles Revisited*, edited by V. Jullien. Basel: Birkhäuser, 2015.

Klibansky, Raymond. "Ein Proklos-Fund und seine Bedeutung." In *Sitzungsberichte der Heidelberger Akademie der Wissenschaften*. Heidelberg: Carl Winter Universitätsverlag, 1929.

———. *The Continuity of the Platonic Tradition during the Middle Ages*. London: The Warburg Institute, 1981.

Konstan, David. "Chapter One: Points, Lines, and Infinity: Aristotle's Physics Zeta and Hellenistic Philosophy." *Proceedings of the Boston Area Colloquium of Ancient Philosophy* 3, no. 1 (January 1, 1987): 1–32. https://doi.org/10.1163/2213441787X00038.

Koyré, Alexandre. *From the Closed World to the Infinite Universe*. New York: Harper Torchbooks, 1958.

Kristeller, Paul Oskar. "Ancora per Giovanni Mercurio da Correggio." *La Bibliofilía* 43, no. 1/2 (1941): 23–28.

———. "Marsilio Ficino e Lodovico Lazzarelli. Contributo alla diffusione delle idee ermetiche nel rinascimento." *Annali della R. Scuola Normale Superiore di Pisa. Lettere, Storia e Filosofia*, 2, 7, no. 2/3 (1938): 237–62.

———. "The Modern System of the Arts: A Study in the History of Aesthetics Part I." *Journal of the History of Ideas* 12, no. 4 (October 1951): 496–527. https://doi.org/10.2307/2707484.

Lasswitz, Kurd. *Geschichte der Atomistik vom Mittelalter bis Newton*. Hamburg: Leopold Voss, 1890.

Lavinheta, Bernard de. *Explanatio compendiosaque applicatio artis Raymundi Lulli*. Lyon, 1523.

Leibniz, Gottfried Wilhelm. *Theodicy: Essays on the Goodness of God, the Freedom of Man, and the Origin of Evil*. Edited by Austin Farrer. Translated by E.M. Huggard. Eugene, OR: Wipf and Stock, 2007.

Lerner, Michel-Pierre, and Alain-Philippe Segonds. "Sur un '"advertisement"' célèbre: L'*Ad lectorem* du *De revolutionibus* de Nicolas Copernic." *Galilaeana* 5 (2008): 113–48.

Llull, Ramon. *Arbor scientiae. Volumen I. Libri I-VII*. Edited by P. Villalba. Vol. 24. Raimundi Lulli opera latina. Turnhout: Brepols, 2000.

———. *Die neue Logik. Logica nova*. Edited by Charles H. Lohr and Vittorio Hösle. Hamburg: Felix Meiner, 1985.

———. *El libro de la "Nova geometria."* Edited by José Maria Millás Vallicrosa. Barcelona: R. Torra, 1953.

———. *In monte pessulano anno mcccv composita*. Edited by Aloisius Madre. Vol. 9. Raimundi Lulli opera latina. Turhnolt: Brepols, 1981.

Lohr, Charles. "Mathematics and the Divine: Ramon Lull." In *Mathematics and the Divine: A Historical Study*, edited by T. Koetzier and L. Bergmans, 215–28. Amsterdam: Elsevier, 2005.

———. "Ramon Lull's Theory of the Continuous and Discrete." In *Late Medieval and Early Modern Corpuscular Matter Theories*, edited by C. H. Lüthy, J. E. Murdoch, and W. R. Newman, 75–90. Leiden: Brill, 2001.

Look, Brandon C. "Gottfried Wilhelm Leibniz." In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Summer 2017. Metaphysics Research Lab, Stanford University, 2017. https://plato.stanford.edu/archives/sum2017/entries/leibniz/.

Lovejoy, Arthur O. *The Great Chain of Being: A Study of the History of an Idea ; The William James Lectures Delivered at Harvard University, 1933*. Cambridge, MA: Harvard University Press, 1982.

Löwith, Karl. *Meaning in History*. Chicago: University of Chicago Press, 2006.

Lucretius Carus, Titus. *On the Nature of Things*. Translated by Martin Ferguson Smith. Indianapolis: Hackett Pub, 2001.

Lugt, Maaike van der. "Chronobiologie, combinatoire et conjonctions élémentaires dans le *De commixtionibus elementorum* d'Urso de Salerne (fin XIIe siècle)." *La misura – Measuring, Micrologus. Natura, scienze e società medievali*, no. 19 (2011): 277–323.

Lüthy, Christoph. "Bruno's *Area Democriti* and the Origins of Atomist Imagery." *Bruniana & Campanelliana* 4, no. 1 (1998): 59–92.

———. "*Entiae* & *sphaerae*: due aspetti dell'atomismo bruniano." In *La filosofia di Giordano Bruno. Problemi ermeneutici e storiografici*, edited by Eugenio Canone, 165–98. Florence: Leo S. Olschki, 2003.

Lüthy, Christoph, John E. Murdoch, and William R. Newman. "Introduction: Corpuscles, Atoms, Particles, and Minima." In *Late Medieval and Early Modern Corpuscular Matter Theories*, edited by Christoph Lüthy, John E. Murdoch, and William R. Newman, 1–38. Leiden: Brill, 2001.

Mach, Ernst. *Die Mechanik in ihrer Entwicklung: Historisch-Kritisch Dargestellt*. Leipzig: F.A. Brockhaus, 1883.

Mahnke, Dietrich. *Unendliche Sphäre Und Allmittelpunkt Beiträge Zur Genealogie der Mathematischen Mystik*. Halle: M. Niemeyer, 1937.

Maier, Anneliese. *Die Vorläufer Galileis im 14. Jahrhundert: Studien zur Naturphilosophie der Spätscholastik*. Rome: Edizioni di Storia e Letteratura, 1966.

Malet, Antoni. *From Indivisibles to Infinitesimals. Studies on Seventeenth-Century Mathematizations of Infinitely Small Quantities*. Bellaterra: Universitat Autònoma de Barcelona. Servei de Publicacions, 1996.

Mancosu, Paolo. *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*. New York: Oxford University Press, 1996.

Martin, Dennis D. *Fifteenth Century Carthusian Reform: The World of Nicholas Kempf*. Leiden: Brill, 1992.

Masi, Micheal. *Boethian Number Theory. A Translation of the* De Istitutione Arithmetica. Amsterdam: Rodopi, 1983.

Matteoli, Marco. "Materia, minimo e misura: la genesi dell'atomismo 'geometrico' in Giordano Bruno." *Rinascimento* 50 (2010): 425–49.

McMullin, Ernan. "Bruno and Copernicus." *Isis* 78, no. 1 (1987): 55–74.

McTighe, T. P. "The Meaning of the Couple '*Complicatio-Explicatio*' in the Philosophy of Nicholas of Cusa." *Proceedings of the American Catholic Philosophical Association* 32 (1958): 206–14.

Mendell, Henry. "Aristotle and Mathematics." In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Spring 2017. Metaphysics Research Lab, Stanford University, 2017. https://plato.stanford.edu/archives/spr2017/entries/aristotle-mathematics/.

Meskens, Ad. “Michel Coignet’s Contribution to the Development of the Sector.” *Annals of Science* 54 (1997): 143–60.

Mignini, Filippo. “La dottrina dell’individuo in Bruno e Cusano.” *Bruniana & Campanelliana* 6, no. 2 (2000): 325–49.

Millás Vallicrosa, José Maria. “Introduction.” In *El libro de la “Nova geometria,”* by Ramon Llull, 13–52. edited by José Maria Millás Vallicrosa. Barcelona: R. Torra, 1953.

Miller, Alexander. “Realism.” In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Winter 2016. Metaphysics Research Lab, Stanford University, 2016. https://plato.stanford.edu/archives/win2016/entries/realism/.

Miller, Clyde Lee. “Cusanus, Nicolaus [Nicolas of Cusa].” In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Summer 2017. Accessed April 17, 2018. https://plato.stanford.edu/archives/sum2017/entries/cusanus/.

Monti, Carlo. “Incidenza e significato della tradizione materialistica antica dei poemi latini di Giordano Bruno: la mediazione di Lucrezio.” *Nouvelles de la République des Lettres* 2 (1994): 75–87.

———. “Introduction.” In *Opere latine*, by Giordano Bruno, 9–62. translated by Carlo Monti. Torino: UTET, 1980.

Mordente, Fabrizio. *Il Compasso e Figura Di Fabrizio Mordente* ... Paris: Jean le Clerc, 1585.

———. *Modo di trovare con l’Astrolabio, ò Quadrante, ò altro instrumento, oltre gradi, intieri, i minuti, et secondi, et ognaltra particella*. Venice: Paolo Forlani, 1567.

Mordente, Fabrizio, and Gasparo Mordente. *La Quadratura del cerchio, la Scienza de’ residui, il Compasso et riga*. Antwerp: Philippe Galles, 1591.

Moritz, Arne. *Explizite Komplikationen: der radikale Holismus des Nikolaus von Kues*. Münster: Aschendorff, 2006.

Muccillo, Maria. *Platonismo, ermetismo e* prisca theologia*: ricerche di storiografia filosofica rinascimentale*. Florence: L.S. Olschki, 1996.

Mulsow, Martin. “Ambiguities of the *Prisca Sapientia* in Late Renaissance Humanism.” *Journal of the History of Ideas* 65, no. 1 (2004): 1–13.

Murdoch, John E. “Infinity and Continuity.” In *The Cambridge History of Later Medieval Philosophy*, edited by Norman Kretzmann, Jan Pinborg, and Anthony Kenny, 564–91. Cambridge: Cambridge University Press, 1982.

———. “Naissance et développement de l’atomisme au las moyen âge latin.” In *La science de la nature: théories et pratiques*, 11–32. Montreal: Bellarmin, 1974.

———. “The Medieval and Renaissance Tradition of *Minima Naturalia*.” In *Late Medieval and Early Modern Corpuscular Matter Theories*, edited by Christoph H. Lüthy, John E. Murdoch, and William R. Newman, 91–132. Leiden: Brill, 2001.

Nagel, Fritz. *Nicolaus Cusanus un die Entstehung der Exakten Wissenschaften*. Münster: Aschendorff, 1984.

Newman, William R. *Newton the Alchemist: Science, Enigma, and the Quest for Nature’s “Secret Fire.”* Princeton, N.J.: Princeton University Press, 2019. http://search.ebscohost.com/login.aspx?direct=true&scope=site&db=nlebk&db=nlabk&AN=1822246.

Newman, William R., and Lawrence Principe. *Alchemy Tried in the Fire: Starkey, Boyle, and the Fate of Helmontian Chemistry*. Chicago: University of Chicago Press, 2002.

Nicholas of Cusa. *De docta ignorantia*. Edited by Ernst Hoffmann and Raymond Klibansky. Nicolai de Cusa Opera Omnia, I. Leipzig: Felix Meiner, 1932.

———. *De docta ignorantia*. Edited by Paul Wilpert and Hans Gerhard Senger. Hamburg: Felix Meiner, 2002.

———. *De theologicis complementis*. Edited by Heide D. Riemann and Karl Bormann. Hamburg: Felix Meiner, 1994.

———. *Idiota de mente*. Edited by Renata Steiger and Ludwig Baur. Hamburg: Felix Meiner, 1983.

———. *Opuscula I: De deo abscondito, De quaerendo deum, De filiatione dei, De dato patris luminum, Coniectura de ultimis diebus, De genesi*. Edited by Paul Wilpert. Hamburg: Felix Meiner, 1959.

Nicoli, Elena. “The Earliest Renaissance Commentaries on Lucretius and the Issue of Atomism.” Unpublished dissertation, Radboud University, 2017.

Nicolle, Jean-Marie. “Introduction.” In *Les écrits mathématiques*, by Nicholas of Cusa, edited by Jean-Marie Nicolle. Paris: Honoré Champion, 2007.

Olschki, Leonardo. *Giordano Bruno*. Bari: Laterza, 1927.

Oosterhoff, Richard J. "*Idiotae*, Mathematics, and Artisans: The Untutored Mind and the Discovery of Nature in the Fabrist Circle." *Intellectual History Review* 24, no. 3 (July 3, 2014): 301–19. https://doi.org/10.1080/17496977.2014.891180.

Pabst, Bernhard. *Atomtheorien Des Lateinischen Mittelalters*. Darmstadt: Wissenschaftliche Buchgesellschaft, 1994.

Palmer, Ada. *Reading Lucretius in the Renaissance*. Cambridge, MA: Harvard University Press, 2014.

Palmerino, Carla Rita. "Geschichte Des Kontinuumproblems or Notes on Fromondus's Labyrinthus?: On the True Nature of LH XXXVII, IV, 57 R°-58v°." *Leibniz Society Review* 26 (2016): 63–98. https://doi.org/10.5840/leibniz2016264.

———. "Libertus Fromondus' Escape from the Labyrinth of the Continuum (1631)." *Lias: Journal of Early Modern Intellectual Culture and Its Sources* 42, no. 1 (2015): 3–36.

———. "Reading the Book of Nature: The Ontological and Epistemological Underpinnings of Galileo's Mathematical Realism." In *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century*, edited by Geoffrey Gorham, Benjamin Hill, Edward Slowik, and C. K. Waters, 29–50. Minneapolis: University of Minnesota Press, 2016.

Papi, Fulvio. *Antropologia e civiltà nel pensiero di Giordano Bruno*. Florence: La Nuova Italia, 1968.

Pereira, Michela. "Bernardo Lavinheta e la diffusione del Lullismo a Parigi nei primi anni del '500." *Interpres. Rivista di studi quattrocentesci* 5 (1984): 242–65.

Perfetti, Amalia. "Motivi lucreziani in Bruno: la Terra come 'madre delle cose' e la teoria dei 'semina.'" In *Letture bruniane 1-2*, 189–208. Pisa: Istituti editoriali e poligrafici internazionali, 2002.

———. "Un nuovo documento sul secondo soggiorno parigino di Giordano Bruno (1585-1586)." In *Giordano Bruno: gli anni napoletani e la* peregrinatio *europea: immagini, testi, documenti*, edited by Eugenio Canone. Cassino: Università degli studi di Cassino, 1992.

Philip, J. A. “The ‘Pythagorean’ Theory of the Derivation of Magnitudes.” *Phoenix* 20, no. 1 (1966): 32–50.

Philippe, Jean. *Lucrèce dans la théologie chrétienne du iiie au xiiie siècle et spécialement dans les écoles Carolingiennes*. Paris: E. Leroux, 1896.

Pineau, François. “Historiographie de Paul Tannery et receptions de son œuvre: sur l’invention du metier d’historien des sciences.” Doctoral dissertation, Université de Nantes, 2010.

Pirillo, Diego. *Filosofia ed eresia nell’Inghilterra del tardo Cinquecento: Bruno, Sidney e i dissidenti religiosi italiani*. Rome: Edizioni di storia e letteratura, 2010.

Pistolesi, Elena. “Quadrar el cercle després de Ramon Llull: el cas de Nicolau de Cusa.” In *2n Col·loqui Europeu d’Estudis Catalans. La recepció de la literatura catalana medieval a Europa*, edited by Alexander Fidora and Eliseu Trenc, 17–32. Péronnas: Tour Gile, 2007.

Plato. *The Republic*. Edited by G. R. F. Ferrari. Translated by Tom Griffith. Cambridge: Cambridge University Press, 2000.

———. *Timaeus and Critias*. Translated by Robin Waterfield. Oxford: Oxford University Press, 2008.

Pomaro, Gabriella. “La tradizione latina del *Liber contemplationis*: il manoscritto Paris, Bibliothèque Nationale de France, lat. 3348A.” In *Gottes Schau und Weltbetrachtung. Interpretationen zum »Liber contemplationis« des Raimundus Lullus*, edited by Fernando Domínguez Reboiras, Viola Tenge-Wolf, and Peter Walter, 21–77. Turnhout: Brepols, 2011.

Pompeo Faracovi, Ornella. “Tra Ficino e Bruno: Gli animali celesti e l’astrologia nel Rinascimento.” *Bruniana & Campanelliana* 8, no. 1 (2002): 197–232.

Powrie, Sarah. “The Importance of Fourteenth-Century Natural Philosophy for Nicholas of Cusa’s Infinite Universe.” *American Catholic Philosophical Quarterly* 87, no. 1 (2013): 33–53.

Principe, Lawrence. *The Aspiring Adept: Robert Boyle and His Alchemical Quest : Including Boyle’s “Lost” Dialogue on the Transmutation of Metals*. Princeton, N.J.: Princeton University Press, 1998.

Puccini, Francesca. “La ‘Geschichte der Atomistik’ di Kurd Lasswitz e la ricezione del materialismo di Bruno nella scienza tedesca del XIX secolo.” *Bruniana & Campanelliana* 8, no. 2 (2002): 399–430.

Pyle, Andrew. *Atomism and Its Critics: From Democritus to Newton*. Bristol: Thoemmes Press, 1997.

Raven, John Earle. *Pythagoreans and Eleatics. An Account of the Interaction between the Two Opposed Schools during the Fifth and Early Fourth Centuries B.C.* Cambridge: Cambridge University Press, 1948.

Reeve, Michael. "Lucretius in the Middle Ages and Early Renaissance: Transmission and Scholarship." In *The Cambridge Companion to Lucretius*, edited by Stuart Gillespie and Philip R. Hardie, 205–13. Cambridge: Cambridge University Press, 2007.

Ricatti, C. "Processio" *et* "explicatio"*: La doctrine de la création chez Jean Scot et Nicolas de Cues*. Neaples: Bibliopolis, 1983.

Ricci, Saverio. *Giordano Bruno nell'Europa del Cinquecento*. Rome: Salerno editrice, 2000.

Rice, Eugene F. "Jacques Lèfevre d'Étaples and the Medieval Christian Mystics." In *Florilegium Historiale: Essays Presented to Wallace K. Ferguson*, edited by J. G. Rowe and W. H. Stockdale, 90–124. Toronto: University of Toronto Press, 1971.

Robert, Aurélien. "Atomisme et géométrie à Oxford au XIVe siècle." In *Mathématiques et connaissances du réel avant Galilée*, edited by Sabine Rommevaux, 15–85. Paris: Omniscience, 2010.

———. "Atomisme pythagoricien et espace géométrique au Moyen Âge." In *Lieu, espace, mouvement: physique, métaphysique et cosmologie (XIIe-XVIe siècles). Actes du colloque international Université de Fribourg (Suisse), 12-14 mars 2015*, edited by Tiziana Suarez-Nani, Olivier Ribordy, and Antonio Petagine. Turnhout: Brepols, 2017.

———. "Space, Imagination, and Numbers in John Wyclif's Mathematical Theology." In *Space, Imagination and the Cosmos from Antiquity to the Early Modern Period*, edited by Frederik A. Bakker, Delphine Bellis, and Carla Rita Palmerino, 107–31. Cham: Springer, 2018.

Rose, Paul Lawrence. "The Origins of the Proportional Compass from Mordente to Galileo." *Physis* 10 (1968): 53–69.

Rosen, Edward. "The Invention of the Reduction Compass." *Physis* 10 (1968): 306–8.

———. “The Ramus-Rheticus Correspondence.” *Journal of the History of Ideas* 1, no. 3 (June 1940): 363. https://doi.org/10.2307/2707093.

Rossi, Paolo. *Logic and the Art of Memory: The Quest for a Universal Language*. Translated by Stephen Clucas. London: Continuum, 2006.

Rossini, Paolo. “Giordano Bruno and Bonaventura Cavalieri’s Theories of Indivisibles: A Case of Shared Knowledge.” *Intellectual History Review* 28, no. 04 (2018): 461–76. https://doi.org/10.1080/17496977.2018.1502979.

Roux, Sophie. “Forms of Mathematization (14th-17th Centuries).” *Early Science and Medicine* 15, no. 4–5 (2010): 319–37.

Rowland, Ingrid D. “Giordano Bruno e la geometria dell’infinitamente piccolo.” In *Aspetti della geometria nell’opera di Giordano Bruno*, edited by Ornella Pompeo Faracovi, 53–70. Pubblicazioni del Centro Studi Enriques 8. Lugano: Agorà, 2012.

———. *Giordano Bruno: Philosopher/Heretic*. Chicago: University of Chicago Press, 2009.

Schmidt-Biggemann, Wilhelm. Philosophia Perennis*: Historical Outlines of Western Spirituality in Ancient, Medieval and Early Modern Thought*. Dordrecht: Springer, 2004.

Schmitt, Charles B. “Perrenial Philosophy: From Agostino Steuco to Leibniz.” *Journal of the History of Ideas* 27, no. 4 (October 1966): 505. https://doi.org/10.2307/2708338.

Schneider, Ivo. *Der Proportionalzirkel: ein universelles Analogrecheninstrument der Vergangenheit*. Munich: R. Oldenburg, 1970.

Secchi, Pietro. *“Del mar più che del ciel amante.” Bruno e Cusano*. Rome: Edizioni di storia e letteratura, 2006.

Sedley, David. “Epicurean Physics.” In *The Cambridge History of Hellenistic Philosophy*, edited by Keimpe Algra, Barnes, Jonathan, Mansfled, Jaap, and Schofield, Malcolm, 362–82. Cambridge: Cambridge University Press, 1999.

Seidengart, Jean. “La metaphysique du minimum indivisible et la réforme des mathématiques chez Giordano Bruno.” In *Atomismo e continuo nel XVII secolo*, edited by E. Festa and R. Gatto, 55–86. Neaples: Vivarium, 2000.

Senger, Hans Gerhard. *Die Philosophie Des Nikolaus von Kues Dem Jahre 1440. Untersuchungen Zur Entwicklung Einer Philosophie in Der Fruhzeit Des Nikolaus (1430-1440)*. Münster: Aschendorff, 1971.

———. “Metaphysischer Atomismus.” In *Ludus Sapientiae: Studien Zum Werk Und Zur Wirkungsgeschichte Des Nikolaus von Kues*, 117–40. Leiden: Brill, 2002.

Sergio, Emilio. *Verità matematiche e forme della natura da Galileo a Newton*. Rome: Aracne, 2006.

Sextus Empiricus. *Against the Physicists. Against the Ethicists*. Translated by Robert Gregg Bury. Cambridge, MA: Harvard University Press, 1958.

Shank, J.B. “Special Issue: After the Scientific Revolution: Thinking Globally about the Histories of the Modern Sciences.” *Journal of Early Modern History* 21, no. 5 (October 27, 2017): 377–93. https://doi.org/10.1163/15700658-12342567.

Shea, William R. *Nature Mathematized: Historical and Philosophical Case Studies in Classical Modern Natural Philosophy*. Dordrecht: Springer Netherlands, 1983.

Silverstein, Theodore. “*Elementatum*: Its Appearance Among the Twelfth-Century Cosmogonists.” *Mediaeval Studies* 16 (January 1954): 156–62. https://doi.org/10.1484/J.MS.2.305939.

Simson, Otto Georg von. *The Gothic Cathedral: Origins of Gothic Architecture and the Medieval Concept of Order*. Princeton: Princeton University Press, 1988.

Singer, Dorothea Waley. “The Cosmology of Giordano Bruno (1548-1600).” *Isis* 33, no. 2 (1941): 187–96.

Sorabji, Richard. *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages*. London: Duckworth, 1983.

Spampanato, Vittorio. *Documenti della vita di Giordano Bruno*. Vol. 3. Florence: Leo S. Olschki, 1933.

Swerdlow, N., and O. Neugebauer. *Mathematical Astronomy in Copernicus’s* De Revolutionibus. New York: Springer, 1984.

Tannery, Paul. *Pour l’histoire de la science hellène*. Paris: Felix Alcan, 1887.

Taylor, Eva G. R. *The Mathematical Practitioners of Tudor and Stuart England*. Cambridge: Cambridge University Press, 1954.

Tessicini, Dario. *I dintorni dell'infinito: Giordano Bruno e l'astronomia del Cinquecento*. Pisa: Fabrizio Serra, 2007.

Thierry of Chartres. *The Commentary on the* De Arithmetica *of Boethius*. Edited by Irene Caiazzo. Toronto: Pontificial Institute of Medieval Studies, 2015.

Tocco, Felice. *Le opere inedite di Giordano Bruno: memoria letta all'Accademia di Scienze Morali e Politiche della Società Reale di Napoli*. Neaples: Tipografia della Regia Università, 1891.

Valleriani, Matteo. *Galileo Engineer*. Dordrecht: Springer Netherlands, 2010.

Védrine, Hélène. "L'obstacle réaliste en mathématique chez deux philosophes du XVI siècle: Bruno et Patrizzi." In *Platon et Aristote à la Renaissance (XVI Colloque International de Tours)*, edited by J.-C. Margolin and M. Gandillac, 239–48. Paris: J. Vrin, 1976.

Velthoven, Theo van. *Gottesschau und menschliche Kreativität. Studien zur Erkenntnislehredes Nikolaus von Kues*. Leiden: Brill, 1977.

Verde, Francesco. *Elachista: la dottrina dei minimi nell'epicureismo*. Leuven: Leuven University Press, 2013.

Vernon, Jean-Marie de. *L'histoire véritable du bienheureux Raymond Lulle, martyr du tiers ordre S. François et la réparation de son honneur*. Paris: Renat Guignard, 1668.

Victor, Joseph M. "The Revival of Lullism at Paris, 1499-1516." *Renaissance Quarterly* 28, no. 4 (December 1975): 504–34. https://doi.org/10.2307/2860171.

Visintainer, John. "A Potential Infinity of Triangle Types On the Chemistry of Plato's Timaeus." *HYLE: International Journal for Philosophy of Chemistry* 4, no. 2 (1998): 117–28.

Vlastos, Gregory. "Plato's Supposed Theory of Irregular Atomic Figures." *Isis* 58, no. 2 (July 1967): 204–9. https://doi.org/10.1086/350220.

———. "Zeno of Elea." In *The Encyclopedia of Philosophy*, edited by Paul Edwards. Vol. 8. New York: The Macmillan Company & The Free Press, 1967.

Walker, D. P. *The Ancient Theology: Studies in Christian Platonism from the Fifteenth to the Eighteenth Century*. London: Duckworth, 1972.

Watanabe, Morimichi. "The Origins of Modern Cusanus Research in Germany and the Foundation of the Heidelberg Opera Omnia." In *Nicholas of Cusa* *in Search of God and Wisdom. Essays in Honor of Morimichi Watanabe by the American Cusanus Society*, edited by Gerald Christianson and Thomas M. Izbicki, 17–44. Leiden: Brill, 1991.

Westfall, R. S. “Newton and the Fudge Factor.” *Science* 179, no. 4075 (February 23, 1973): 751–58. https://doi.org/10.1126/science.179.4075.751.

Westman, Robert S. “Magical Reform and Astronomical Reform: The Yates Thesis Reconsidered.” In *Hermeticism and the Scientific Revolution*, edited by Robert S. Westman and J. E. McGuire, 3–91. Los Angeles: University of California Press, 1977.

———. “Proof, Poetics, and Patronage: Copernicus Preface to *De Revolutionibus*.” In *Reappraisals of the Scientific Revolution*, edited by David C. Lindberg and Robert S. Westman, 167–205. Cambridge: Cambridge University Press, 1990.

———. “The Astronomer’s Role in the Sixteenth Century: A Preliminary Study.” *History of Science* 18, no. 2 (1980): 105–147.

———. “The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory.” *Isis* 66, no. 2 (June 1975): 165–93. https://doi.org/10.1086/351431.

Wrightsman, Bruce. “Andreas Osiander’s Contribution to the Copernican Achievement.” In *The Copernican Achievement*, edited by Robert S. Westman, 213–43. Berkeley: University of California Press, 1975.

Yates, Francis A. *Giordano Bruno and the Hermetic Tradition*. London: Routledge and Kegan Paul, 1964.

———. “Giordano Bruno: Some New Documents.” *Revue Internationale de Philosophie*, 1951, 174–99.

Zhmud, Leonid. *Pythagoras and the Early Pythagoreans*. Oxford: Oxford University Press, 2012.

Zinner, Ernst. *Die Geschichte der Sternkunde: Von den Ersten Anfängen bis zur Gegenwart*. Berlin: Springer, 1931.