

Scuola Normale Superiore

CLASSE DI LETTERE Corso di Perfezionamento in Discipline Filosofiche

TESI DI PERFEZIONAMENTO IN DISCIPLINE FILOSOFICHE

CONCEDERE, NEGARE, DUBITARE

Peter of Mantua's treatise on obligations

Candidate: Riccardo Strobino Advisor: Prof. Massimo Mugnai Πάλαι τοί σου ἀχρῶμαι, ῶ Σώχρατες, χαθομολογῶν, ἐνθυμούμενος ὅτι, κἄν παίζων τίς σοι ἐνδῷ ὀτιοῦν, τούτου ἄσμενος ἔχῃ ὥσπερ τὰ μειράχια. Plato, *Gorgias*, 499 B 4–6.

ταῦτα ἤμῖν ἄνω ἐκεῖ ἐν τοῖς πρόσθεν λόγοις οὕτω φανέντα, ὡς ἐγὼ λέγω, κατέχεται καὶ δέδεται, καὶ εἰ ἀγροικότερόν τι εἰπεῖν ἔστιν, σιδηροῖς καὶ ἀδαμαντίνοις λόγοις, ὡς γοῦν ἂν δόξειεν οὑτωσί, οὓς σὺ εἰ μὴ λύσεις ἢ σοῦ τις νεανικώτερος, οὐχ οἶόν τε ἄλλως λέγοντα ἢ ὡς ἐγὼ νῦν λέγω καλῶς λέγειν. Plato, *Gorgias*, 508 E 6–509 A 4.

Angélique:

- Mais la grande marque d'amour, c'est d'être soumis aux volontés de celle qu'on aime.

Thomas Diafoirus:

- *Distinguo*, mademoiselle: dans ce qui ne regarde point sa possession, *concedo*; mais dans ce qui la regarde, *nego*.

Molière, Le malade imaginaire, acte II, scène VI

Acknowledgements

The completion of this thesis brings to an end my years of Ph.D. at Scuola Normale Superiore, and I realize that mentioning all people to whom I am indebted would require a separate chapter.

However, I wish to take at least the opportunity to express my appreciation and gratitude, first and foremost, to professor Massimo Mugnai for his precious advice, understanding and extraordinary patience, not to mention the interest in my work that he has been showing since my very first day in Pisa. I have benefited greatly from his knowledge of the history of logic while attending his lectures and in countless individual talks.

For his constant concern about the development of my thesis and his strenuous encouragement, I also wish to mention professor Francesco Del Punta, who has generously shared with me his expertise in paleography. Besides, I should thank Dr. Mario Bertagna for kindly showing me his draft transcription of Peter's obligations.

Although it might sound inappropriate, I will leave aside the etiquette on the present occasion, to thank – deeply – professor Calvin Normore for the time spent with me in front of a blackboard that had to be erased over and over again; and professor Jennifer Ashworth for encouraging me, a couple of years ago, to keep pursuing the study of Peter of Mantua's obligations. The standard of scholarship they have been setting, over the years, in the study of medieval logic, has been a model of continuous inspiration to me. One that I have endeavoured to follow – as I will still try to do in the future – to the best of my ability.

Finally, my deepest appreciation to Chiara Pidatella for her loving patience and constant support, and to Andrea Campoleoni for showing me that a string physicist can have brilliant insights on medieval logic, let alone when it comes to IAT_EX .

Contents

Introd	luction	III
PAR	ГІ	1
Obliga	ations at a glance	2
1.1.	8	
1.2.		
1.3.		
1.4.	1 0	
1.5.		
1.6.	Survey of modern interpretations	34
PAR	Г II	47
Peter	of Mantua's obligations	48
	e and works	
	ucture and contents	
	ble of contents	
	la et abbreviationes	
0		
Tracta	atus obligationum	66
1.	Preliminary assumptions	67
2.	Rules	76
3.	On positio	83
4.	Further subdivision and examples	135
5.	On impositio	158
6.	On depositio	201

Index of sophisms 2	211
List of symbols 2	214
Bibliography 2	215
I. Obligations treatises	216
A. Manuscripts	216
B. Early printed texts	216
C. Editions	217
II. Other sources	218

Introduction

In the last five decades, increasing attention has been paid by scholarship interested in medieval logical literature to the genre of obligations, a chapter of the *logica moderna* whose understanding has proved to be a problem of very difficult solution. It is quite common, in the opening lines of most contributions devoted to obligations, to come accross quick characterizations made by means of adjectives such as 'puzzling', 'vague', 'obscure' or 'mysterious',¹ whose tone is generally meant to express the difficulty one should be prepared to face in the attempt to find satisfactory explanations or, maybe, even to warn the newcomer about the risk of sudden frustration prompted by the intricacy of the doctrine as opposed to the sparsity of feasible solutions.

The measure of this disappointment, I suspect, has been proportional to expectations. Scholars have experienced much trouble in the attempt to explain *all* aspects of the obligational literature in *one* single consistent framework; as a result, the only consensus that has arisen among them is centred on the fact that we are still in search for an answer, possibly the Answer, to the question as to what obligations really are. Curiously enough – though I can't help seeing pretty much of the irony of history having a hand in it – *What obligations really are* is the title of a paper by A. Perreiah that should have, but in fact has never,² appeared in *Medioevo*, 5 (1979). This odd circumstance might evocatively seem to imply that our search for the truth about obligations has been, so far, fatally doomed to encounter insurmountable obstacles to its own realization and that we should still be waiting for a definitive answer. The trustworthiness of this picture is, however,

¹ A few examples, among many others, of this kind of rhetorical *ouvertures* are found in [Spade, 1992, p. 171], [Martin, 2001, p. 64], [Yrjönsuuri, 2001a, p. 7], and [Dutilh Novaes, 2007, p. 145].

² The paper is quoted by Spade and King in two influential papers on the topic, cf. [Spade, 1982b, p. 2] and [King, 1991, p. 64], respectively. Anyway, no spell has been cast on the question: the mistery has a simple solution. I have come to know, thanks to the information prof. Perreiah provided me in a private comunication, that the paper was originally supposed to appear in Medioevo, but in fact did come out, later on and under a different (perhaps less emphatic) title, in *Analecta Augustiniana*, cf. [Perreiah, 1982]. Presumably, an earlier draft, circulated beforehand, is at the origin of both misquotations.

INTRODUCTION

highly questionable insofar as it strongly depends on how much importance we put on the real need of *one* answer, especially when it comes to weighing the achievements that have undoubtedly been made in the past, both in terms of historical research and in terms of conceptual analysis.

Let us start with a list of problems that have puzzled historians of logic working in this field:

- 1. genesis of the genre
- 2. aims
- 3. understanding of its historical development (with special attention to the revision of rules occurring in the first half of the XIV century)
- 4. the place of obligations in the context of medieval logic
- 5. concrete vs. fictitious character of disputations
- 6. application to other fields of medieval thought (e.g. theology, metaphysics, natural philosophy, moral philosophy, law)
- 7. attempts to formulate interpretations that can help identify and describe at best the logical structure of obligational disputations
- 8. attempts to formulate interpretations that can establish a connection to contemporary research in the philosophy of logic and language

My suggestion is that several of these issues have been often unsuitably conflated together, thereby generating a problem which, in such terms, turns out to be ill-formed and vitiated from the outset by a close relation of the fallacy of many questions. All the above issues, if taken individually, are indeed perfectly sensible (e.g. the origins of the obligational literature, its vicinity to modern, more or less overlapping, areas of research in logic or the philosophy of language, the actual purpose of medieval authors of this kind of treatises, namely what they might have had in mind or what they might have thought they were doing; or, again, the possible extension and application of techniques developed in the realm of obligations to other branches of medieval thought). Yet, out of many legitimate questions, that at least to some extent might well be investigated independently of one another, one single Question has been forged which is hardly possible to find an answer to. This attitude quite easily explains why dissatisfaction has been for long widespread and new interpretations have often conveyed the impression of being nothing more than additional hypotheses destined to miss the target: scholars have indeed answered, in some cases with brilliant solutions, over a span of almost fifty years, to a series of individual problems, but they have never been able

INTRODUCTION

to work out the Answer as such. The question has earned, in a sense, the status of a mythologized problem: instead of narrowing the focus on specific issues and on the clarification of individual aspects – historical or theoretical – there seem to have been, sometimes, a sort of general worry about the fact that we still do not know everything (or worse: anything) about obligations. But I think we are in a better condition than that. In the light of the difficulties in finding an answer to the Question (which should be reduced, in fact, to a plurality of questions), I suggest it might be reasonable, in terms of methodology, to forget about it and look at things the opposite way around, namely by taking the results that we have for what they can tell us (and I reckon they can tell us much in various respects) and start therefrom to pursue further research in the field. In this connection, it is worth recalling that we have now several editions of major texts, we know quite a lot about the logical structure of disputations, we have learned something about the application of obligational techniques in the field of theology or medieval 'science' and to some extent even the *vexata quaestio* of the historical origin of the genre (as a distinct issue from that of other aims that might have interevened at later phases of its historical development) has been the object of interesting insights. Not to speak of the variety of proposals that have been put forward with the aim of finding a plausible explanation for the purpose(s) of the genre. To cut a long story short, research has made great progress and all these efforts should

faithfulness to the texts, and explanatory value. Besides, it will be useful to single out specific problems or issues that still wait for deeper inspection. Just to mention one, an especially important further line for future research would be the investigation of the long lists of *sophismata* that feature in treatises on obligations. We have a relatively high amount of good analyses of what I would call the conceptual bulk of the theory (namely definitions, classification of types, general rules, specific rules and so on), but a study of the use of sophisms in a given treatise, the presence of some of them in a given tradition or group of authors, has been rarely undertaken in a systematic form.

be considered and evaluated, individually, in terms of their historical accuracy,

In addition to that, since the study of medieval logic is intrinsically bipedal, as long as our knowledge depends not only on the ability to elaborate good interpretations, but also on the general perception of a given tradition, which rests in turn on the availability of texts, I think there is a strong need to ampliate the domain of accessible sources. Although several good editions of major treatises are now available, in order to draw a fuller and richer picture of the context and of the network of influences, textual research should keep up with conceptual analysis. Yet, as far as I know, the last treatise belonging to the obligational tradition that has become available as an independent text provided with some sort of contextualization or comment³ is Brinkley's, which dates back to the mid 1990s.⁴

With this background in mind, in the present work, I intend to adopt an empirical approach to the matter and undertake the study of a text which is still inedited. I will be concerned with the treatise on obligations by Peter of Mantua (d. 1399) and propose a preliminary analysis of the main issues it raises. Although the aim is giving a contribution in the direction of both historical and conceptual understanding, I should say from the very beginning that I shall not come up with a new general interpretation of obligations. My hope, however, is that the materials deal with in this dissertation will contribute, in some way, to enrich our knowledge of the obligational tradition in the late XIV century, in the attempt to make the general picture, in the long run, as complete and accurate as possible.

Here is the *divisio textus*, i.e. how I intend to organize the materials of my research. The work is divided into two parts. The first part is an introduction to the theme of obligations, while in the second the focus shifts on Peter of Mantua and his treatise.

In Part I, I will deal with a number of general aspects. Since there are now several studies that provide good characterizations of the genre, in both historical and theoretical respect, I will selectively focus only on what I regard as the most problematic issues. I will first sketch a minimal account of obligations (Part I, sec. 1.1). I will then cursorily outline the evolution of the genre in historical perspective (Part I sec. 1.2). In the third section I will zoom into the basic logical features that come into play in such disputational structures (Part I, sec. 1.3) to set the stage for some developments, that will be the object of a separate analysis (Part I, sec. 1.4). I will then provide a sketchy example of how the model is supposed to work (Part I, sec. 1.5) and finally outline a survey of the literature on obligations, focusing especially on the question of the purpose (Part I, sec. 1.6).

All of that will be made with an eye to the object of the second part of the thesis. In the latter, which should count, at least in my intentions, as the relevant one in terms of original research, I will first provide some information about Peter's life and works and outline the structure of his treatise on obligations, before coming to the study of the text, which is accompanied by some notes. These do not have the form of a systematic commentary insofar as they are intended both as a tool for the clarification of the most intricate conceptual issues raised in the text and as an inventory of suggestions for the reconstruction of a network of historical influences. In this respect, I wish to stress that I have not been primarily concerned with a systematic identification of Peter's sources – that would have meant biting off

³ Save for a chapter on obligations featuring in Spruyt's edition of the *Logica Morelli*, a XV century Spanish logical tract, cf. [Spruyt, 2004] and Perreiah's edition of Paul of Venice's *Logica Parva*, cf. [Paul of Venice, 2002]. These two texts, however, do not contain articulated analyses devoted to obligations.

⁴ Cf. [Spade and Wilson, 1995].

INTRODUCTION

more than I could chew, on the present occasion. Rather I have tried to point out several textual connections, relying on the outcome of my independent research and, much more, on the significant results achieved by others, the most relevant example for the purposes of this work being Ashworth's commentary on Paul of Venice's obligations contained in the *Logica Magna*.

The main goal is making a dense and sophisticated text available to the community of scholars. It is a text that occasionally gives its reader a hard time. Its analysis does not modify the general framework of the obligational tradition in the late XIV century; Peter turns out to be pretty much adherent to the responsio antiqua account and does not put forward breakthrough views. Still, he is a refined logician capable of remarkable insights. A deeper assessment of Peter's contribution to the history of late medieval logic will require the study of the *Logica* in its entirety and would really be an advance in our knowledge of the context of Italian logic before Paul of Venice, also in connection with the reception of both the continental and the British tradition. In this connection Peter seems to be a figure of relevant stature. Some partial results have already been achieved in the last fourty years⁵ but in order for a reliable picture to emerge from this archipelago they will have to be put together and evaluated in a more extensive perspective; with respect to the last point, I want to recall, before concluding these introductory remarks, the words of Boh, who in a number of contributions over the past fifteen years, has repeatedly stressed the importance, in Peter of Mantua, of the theory of obligations as a background for the understanding of other logical doctrines:

The context in which suppositions, beliefs, and assumptions are most prominent is that of formal disputation. Understanding of the contents is not always there "in virtue of one's own insight", but it must be there in virtue of beliefs based on authority and especially in virtue of stipulations; these last are all-pervasive in obligational disputation and it appears as if Mantuani wrote his *Consequentie*, *De scire*, and in a sense his whole *Logica* with *obligatio* situations in the back of his mind.⁶

On the basis of such inspiration, it is now time to get to the heart of the matter.

 $^{^{5}}$ In chronological order, specific contributions related to Peter of Mantua's *Logica* are [Maierù, 1972] on various topics, [Maierù, 1974] on the meaning of propositions, [Bos, 1982] on *appellatio*, [Bos, 1985a] on *ampliatio* and *restrictio*, [Bos, 1985b] on the truth of propositions and, finally, [Bertagna, 2000] on the theory of consequences. In [Paul of Venice, 1978] Peter is identified as one of the sources that Paul of Venice quotes almost *verbatim* in his treatise on the meaning of propositions.

⁶ Cf. [Boh, 1990, p. 83].

PART I

1. Obligations at a glance

1.1. A minimal account of obligations

A problem it is reasonable to start with is the following: what are obligations about? A precise answer to this apparently simple question would require from the very beginning a number of distinctions and might turn out to be quite complicated, because there are many alternative accounts available over more than three-hundred years in the Middle Ages and not all of them reduce to a single pattern. Nevertheless, there is a safe point of departure which fits the texts on the topic without implying any preliminary strong theoretical commitment to this or that interpretation: obligations, in a rough, generic sense, are about disputations. More precisely, they are about a series of constraints that someone has to stick to in the context of a disputation. The nature and features of these constraints will be clarified later on; for the moment we just need to be acquainted with some structural elements that characterize the notion of a disputation in the technical sense which is presupposed in the obligational framework. The different accounts and types of obligations acknowledged by medieval authors, all include a bunch of invariant core-elements, whose mutual relationships I shall now briefly describe. In this section, I will proceed with a certain degree of extrapolation and abstraction with respect to actual theories of obligations and try to keep the description as schematic as possible.

Treatises on obligations, as we know them from the beginning of the XIII century to the end of the XIV century and beyond, all deal with one and the same problem: they codify a highly stylized and regimented format of dialectical disputation. The notion of a dialectical disputation which is at stake in this kind of literature has a technical meaning. It presupposes (i) a pair of agents who play two distinct roles, i.e. opposing and responding (whereby the denomination of opponent and respondent). The acts of opposing and responding are carried out by (ii) performing certain speech acts relative to (iii) sentences. The opponent picks out the moves that are available to him from the set {posit,⁷ propose}, whereas the respondent picks out his moves from the set {admit, reject, concede, deny,

⁷ In these terms the formulation might look equivocal: I do not intend the act of *positio* proper

doubt. The acts performed on sentences by the respondent might be translated into tantamount ascriptions of truth values to the sentences in question.

How to reply to a given sentence in a disputation depends on its logical status. Two mutually related notions are crucial in this respect, namely (iv) relevance and irrelevance. The identification of sentences as relevant or irrelevant is the basis for the formulation of (v) rules that govern the speech acts that must be performed by the respondent.

Relevance in a disputation can be intuitively characterized in terms of the notion of 'following from or being inconsistent with something'. What the 'something' occurring in this description is like depends on the obligational set of rules that we endorse.⁸ The idea of 'following', by contrast, is problematic since it depends on the underlying intuition of what we take a logical consequence to be. A precise characterization of the properties of this notion in the framework of medieval logic is far from being uncontroversial.⁹ Be this as it may, it makes sense at this preliminary stage to speak of relevance in such general terms: what at some point is relevant is what follows or what is incompatible with something that is already part of the disputation. On the other hand, the notion of irrelevance is generally understood in terms of logical independence and requires the introduction, in our model, of a last core element, i.e. (vi) a set of background assumptions that comes into play when the respondent has to reply to irrelevant sentences.

A disputation is a well-formed, ordered sequence of sentences that are submitted by the opponent and evaluated by the respondent, over a limited span of time, according to certain rules. The theory deals with the set of rules that are supposed to govern the respondent's replies to the sentences put forward by the opponent. What it specifically provides are norms, prescriptions or instructions¹⁰

but rather the assertion of a sentence as a thesis to be upheld in some way that is determined by the environment in which the two agents are operating (if it is a *positio* the initial hypothesis is supposed to be upheld as true, if it is a *dubitatio* the initial hypothesis is supposed to be upheld as doubtful and so on).

⁸ The main distinction is between two approaches that are commonly referred to in the literature with the labels *responsio antiqua* and *responsio nova*: on the former account the 'something' roughly coincides with the sequence of previously conceded and denied steps of the disputation, whereas on the latter account it coincides solely with the initial hypothesis.

⁹ Basically in virtue of two facts: first, the taxonomy is variable and it does usually allow for *several* notions of 'following from' even in the work of a single author (all the more so if we look at the development of the theory of consequences in historical perspective); second, it is not altogether clear what 'modern' notion(s) of logical consequence would be the most suitable candidate to capture the medieval notion(s) of *consequentia*. For an overview of the main issues related to this topic, cf. [Boh, 1982].

¹⁰ The nature and extent of this constraints is debated. Generally speaking, they seem to have a normative character that prevents any form of arbitrary choice. This interpretation seems to be largely predominant in the tradition. In some cases, however, a weaker interpretation that allows for some degree of freedom in the criteria of response to a particular class of sentences (irrelevant

telling the respondent how to answer to sentences that are submitted to his evaluation. A variety of types of obligations are accepted by medieval logicians, the classification usually depending on the kind of thing that is subject to the norms.¹¹ Nevertheless, two agents, a set of speech acts to be performed, in order, on set of sentences within a disputational context (to be understood as a temporal environment limited at one extreme by the assertion and acceptance of the initial sentence and at the other extreme by the utterance of the sentence 'let the time come to an end'), the distinction between relevance and irrelevance, as well as a shared background functional to the evaluation of irrelevant sentences – these are all structural ingredients that are always presupposed in any kind of disputation conducted in this framework.

¹¹ A popular classification, held for instance by Burley, distinguishes between six species of obligations, that are grouped in two classes according to whether the respondent is committed to a disposition or to an act. Obligations relative to dispositions are among the most commonly discussed. They focus on the type of reply that the respondent is supposed to give to an initial hypothesis (a sentence): at this level the classification usually further distinguishes between three types of commitments with respect to the truth value of that sentence, namely to its truth (positio), to its falsehood (depositio) or to neither (dubitatio). That amounts, in turn, to the requirement that the respondent concede, deny or doubt, throughout the disputation, the initial sentence, and reply accordingly to proposed sentences that are logically related to it (for instance, in the case of *depositio* the respondent should never concede a sentence if it implies the depositum). Institutio (also known as impositio) is the obligation relative to a disposition which requires the acceptance of the stipulation of a new meaning for a word or a sentence. Obligations relative to the performance of acts are *petitio* and *sit verum*. Petitio requires the performance of a given speech act relative to a sentence in a slightly different way which does not imply a commitment to its truth value (in other words, the respondent commits himself to the concession of sentences like 'I concede p', 'I deny p' or 'I am in doubt about p', without thereby conceding, denying or doubting p itself. Sit verum mainly involves the presence of propositional attitudes. It should be said that the last two types have received very little attention in contemporary literature, save for occasional exceptions like [Spade, 1997].

The taxonomy, however, is flexible and there are authors who admit a restricted number of types of obligations. Peter of Mantua, for instance, accepts only *positio* and *depositio*, even if in fact his classification turns out to be less rigid than it seems. Peter modifies the hierarchy and retrieve some traditionally independent types as further subdivisions of those he accepts. As a result, in his treatise, several examples of *petitio* and, above all, *impositio* turn out to be discussed at considerable length, even if they are not recognized as independent species. In this respect, cf. *infra*, Part II, sec. 4. and 5.

ones) seems to be fitting, too. In this connection cf. in particular [Dutilh Novaes, 2007, p. 157]. The idea of a non-deterministic interpretation of the rule for irrelevant sentences is intriguing, since it allows for strategic moves on the respondent's side. In Peter's treatise, however, it seems not to apply, since there are several occasions in which he clearly holds that when one grants a *falsum non sequens* or denies a *verum non repugnans*, he responds badly. On the other hand, there seems to be something like a ranking of the different types of outcomes, in some cases, as when one grants the sentence 'you respond badly', a move which does not make the respondent lose, but which is probably to be avoided as much as possible. In this respect, the idea of a point system might turn out to be very fruitful.

Let us now turn to a more detailed account of how these elements contribute to build up the structure of a disputation. According to a common practice in the literature,¹² I will switch from general considerations to a specific paradigmatic type of obligation, the so-called *positio*, in order to characterize and describe the development of a disputation and the application of rules in its context.

The opponent puts forward a series of sentences one by one. Among these, the first one has a prominent role since it is the hypothesis that influence most of what happens next. The first sentence is said to be *posited* as opposed to the subsequent ones which are said to be *proposed*. Before laying down the *positum* (i.e. outside the context of the disputation) additional information might be provided through the statement of a *casus*. If we assume that our background assumptions about the actual world have propositional content and thereby regard them collectively as a set of sentences, then the *casus* simply modifies the truth value of one or more elements in that set.

When a sentence has been posited, it is the respondent's turn to make his move. With respect to the *positum*, the respondent picks out his choice from a subset of the set of speech acts listed above. He can either admit or reject the *positum*, according to whether the latter is a possible sentence or not.¹³ If the respondent refuses to admit the *positum*, the disputation stops.¹⁴ If the respondent admits the *positum*, then the disputation goes along with a back-and-forth structure. At each subsequent round the opponent *proposes* a sentence to which the respondent replies by either *conceding*, *denying* or *doubting* it. The respondent's replies are not arbitrary, because they are always determined by the rules, in function of their logical status. The disputation thus takes the shape of a sequence of rounds each of which presupposes an act on the opponent's side, a sentence, and an act on the

¹² This is often done not only for the sake of clarity and simplification: it might be not entirely inappropriate to claim that, in a sense, medievals themselves tend to regard *positio*, if not as *the* format all kinds of obligations actually reduce to, at least as a strong paradigm to which they structurally conform.

¹³ Again, a *caveat* is in order: in the standard case of the so-called *positio falsa* (or *falsi*), the criterion is the consistency/possibility of the *positum*, but in another type of obligation, known as *positio impossibilis*, impossible *posita* can be admitted too, provided that they are not formal contradictions. In this connection, an interesting use of the distinction between 'possible' and 'possibly-true' is found in Peter's treatise, cf. *infra*, Part II, sec. 3.40, to the effect that a possible *positum*, even if it can never be true, should be admitted by the respondent. As for *positio impossibilis*, there seems to be good evidence that Peter utterly rejects it, cf. *infra*, Part II, sec. 2.7.

 $^{^{14}}$ It is not fully clear whether the disputation stops or does not even take place. At least in Peter of Mantua's treatise, the rejection of a given *positum* is not a matter of arbitrary choice but depends on the fact that it has been identified as a contradictory or as an impossible sentence. For this reason, I am more inclined to think that the rejection is indeed part of the disputation, and of one that the respondent wins because he exactly performs the act that he is required to perform according to the rule for admission, cf. *infra*, Part II, sec. 2.1.

respondent's side.

The outcome of a disputation, in terms of sentences, can be metaphorically regarded as a list or as a box. After each round, the sentence that has just been evaluated is recorded on a list or 'put in a box' with a label which tells us how the respondent has replied to it. At the end of the disputation the list (or the box) will contain the ordered sequence of all sentences put forward by the opponent and evaluated by the respondent.

So far we have still made no assumption as to what criteria the respondent must follow in providing his replies/evaluations to the proposed sentences. Two are the leading principles, and this is where the notions of relevance and irrelevance come into play. First, the respondent is required to concede any sentence that follows and deny any sentence that is repugnant. Second, he is required to reply to irrelevant sentences according to their own status (as determined by the aforementioned set of background assumptions). Now, one might reasonably ask at this point: anything that follows from or is repugnant to *what*? The obligational tradition of *positio* is basically divided into two camps over the correct answer that should be given to this question. In a nutshell, we may say that according to the most common account, often labelled by modern interpreters responsio antiqua,¹⁵ the notion of relevance should be understood with respect to the whole set of sentences that have been evaluated at the previous steps. According to the so-called *responsio nova*, on the other hand, relevance is defined only with respect to the *positum*. Thus, as regards the normative criteria of construction of the set of sentences that make up a disputation, the difference lies almost entirely in the extension of the classes of relevant and irrelevant sentences, respectively. In the first case, the two notions can (and usually do) evolve during a disputation and they can be be said to have a dynamic character. In the second case, the two notions are static and identify two classes of sentences that remain the same throughout the disputation.¹⁶ The first view is largely predominant (it is indeed the only one available before the second quarter of the XIV century and even thereafter it turns out to be held by a majority of authors), and is often associated with the presentation that Walter Burley gave of it, whereas the second view is the outcome of a revision of the rules introduced around 1335 by Roger Swyneshed.

When the disputation has come to an end, what we are left with is the list recording the ordered sequence of evaluated sentences (or the box cointaining all labelled sentences).¹⁷ At this stage, the two participants are supposed to assess

¹⁵ After a famous distinction found in [Robert Fland, 1980, pp. 45–46].

 $^{^{16}}$ The distinction is discussed in [Dutilh Novaes, 2007, pp. 164–167]. Of course there are some complications that would involve additional considerations (for instance about the role of time, or the presence of *multiple posita*), but for a very simplified reconstruction the fundamental difference is the one I have mentioned.

¹⁷ A disputation can end in several ways. The most tragic circumstance one can envisage is

these materials and establish who wins and who loses.¹⁸ The phase of assessment is driven by the following idea: the opponent wins, if in the list or in the box there is stuff that, according to the rules, should not be there. If, by contrast, we go through the sequence by a fine-tooth comb, and do not come accross anything undesired with regard to the chosen logical criteria (the test depending upon the theory we endorse), then it is the respondent who wins.¹⁹

probably the death of one of the two parties. The warning is in Paul of Venice "si opponens vel respondens infra disputationem desineret esse, et obligatio desineret esse" [Paul of Venice, 1988, p. 22]. Literally, Paul refers to the end of the *obligatio* as to the end of the logical constraint, but I think the remark can be obviously understood as referring to disputations as well.

In this connection [Dutilh Novaes, 2007, p.159] argues that to guarantee the practical feasibility of a disputation we need to presuppose a time stipulation. Although the suggestion appears to be very well grounded, there seems to be no explicit trace of this kind of background assumption or agreement between the two disputants, at least in Peter's case. My impression is that the disputations that are actually represented in his treatise simply go on until the opponent decides to call the whole thing off, usually because he thinks he has already made the respondent grant an inconsistency.

 18 The idea is intriguing: obligations might be seen as two-person zero-sum games. There are *always* a winner and a loser, and there are *exactly* one winner and one loser (who cannot be the same individual).

¹⁹ In both cases the test is intended to detect whether the respondent has granted some kind of inconsistency: with respect to what the inconsistency is evaluated, depends on the account we decide to follow. The inconsistency can be with one or more members of the whole set of sentences that have been granted, denied or doubted up to the final step, by the rules of *responsio antiqua*; or just with a more restricted class of sentences, i.e. the *positum* or *posita* and their logical consequences, regardless of whatever else has been put forward during the disputation, by the rules of *responsio nova*.

Finally, an additional aspect is worth noting. With respect to the notion of inconsistency, there seems to be a distinction between circumstances in which the respondent implicitly falls in contradiction and circumstances in which he is forced to grant a contradiction, cf. especially [Dutilh Novaes, 2007, p. 169]. The issue is extremely interesting and I suggest it lends itself quite well to be read in terms of a game metaphor (cf. also *infra*, sec. 1.6 for discussion and references). Take for example the game of chess and two players that know the rules but are not quite expert.



White to mate in three moves

There is a sense in which White has already won, in this position, but it might be not entirely clear to him *how* to mate in three. Conversely, Black has already lost the game, in principle, because it is entirely up to White to make the right moves (guess for the interested reader) and win. The above position stands to Black in the same way as a set of bad replies stands to the

Before taking up a more detailed analysis of the features of the rules of *responsio* antiqua, I will just briefly sketch an outline of the development of the obligational literature to set up the background.

1.2. Historical perspective

In the reconstruction of the historical development of the theory of obligations²⁰ there are two categories of things that should be considered: a number of serious questions that are still open and a nucleus of facts of which we can claim to have some solid knowledge.²¹ The history of obligations can be roughly divided into three periods:

1. an early phase (encompassing the developments in the XIII century)²²

²¹ In the first category, we may include, for instance, the problem of the historical origin of the genre or the explanation of the disappearance of *positio impossibilis* from the scene in the XIV century. As far as the second category is concerned, on the other hand, several elements are now available for a general reconstruction of the history of false *positio*, especially in the XIV century, some major authors have been thoroughly studied, and there is also a variety of interesting results on the evolution of the late literature.

²² In fact, the history "may be traced as far back as the late twelfth century" [Ashworth, 1988a, p. x] for "even in the twelfth century, there is some use of terminology associated with obligations in discussion of disputation, and in some treatises from this period either disputation or obligations themselves are connected with fallacies or sophismata" [Stump, 1982a, p. 315]. The relevant literature relative to this period includes a number of pioneering contributions by De Rijk with the edition of four short anonymous treatises: the *Obligationes Parisienses*, cf. [De Rijk, 1975b]; the *Tractatus Emmeranus de falsi positione* and the *Tractatus Emmeranus de impossibili positione*, cf. [De Rijk, 1974]; and finally, the *Tractatus Sorbonensis de petitionibus contrariorum*, cf. [De Rijk, 1976]. Again, De Rijk has contributed to cast some light on theoretically contiguous areas, involving the study of techniques of opposing and responding, in his edition of the treatises *De modo opponendi et respondendi*, cf. [De Rijk, 1980]. More recently, strong evidence has been

respondent. The opponent might be asked to force the latter to grant an explicit contradiction, exactly in the same way as White is supposed to bring the game to an end by making the appropriate moves. For both pairs of players – opponent and respondent, White and Black – the accomplishment of these tasks requires a good deal of expertise, in logic just as in chess.

²⁰ In the present section I am not aiming at a full reconstruction of the history of obligations. I simply wish to provide some coordinates that may come in useful to locate problems in their historical perspective and to highlight some controversial issues that still wait for an answer. General accounts of the history of the genre are [Stump, 1982a] and [Spade, 1982a] which cover the essential developments in the XIII and XIV centuries. It must be noted, however, that both papers are slightly outdated, since several editions of major treatises have come out in the subsequent years and many exceptical efforts have contributed to clarify the theoretical content even of the texts that are taken into account by Stump and Spade in those articles. A good way to update these contributions is looking at more recent specific studies of various sorts. An excellent general guide is [Yrjönsuuri, 1994, especially pp. 18–35]. Other specific studies will be mentioned in the following pages.

- 2. a phase of consolidation of the standard account and of significant revisions in the first half of the XIV century (prominent figures in this period being Walter Burley, Richard Kilvington and Roger Swyneshed)²³
- 3. a late phase from the second half of the XIV century on (the tradition will survive for a long time as is attested by the presence of treatises on obligations in logic textbooks well into the beginning of the XVI century).²⁴

The central phase is the one that has been studied in greatest detail. Despite some difficulties that remain, such as the clarification of the reasons behind the emergence of alternative account(s) in the first half of the XIV century,²⁵ we now have various characterizations of the theoretical bulk of Burley's and Swyneshed's theories that provide satisfactory explanations of their fundamental logical features.²⁶

Kilvington's views on obligations have been the object of some speculation since they do not amount to not a full-blown theory but rather are confined to a number of remarks in his *Sophismata*, cf. [Kretzmann and Kretzmann, 1990, pp. 126–137]. Nevertheless, on the basis of various attempts to reconstruct his theory, a wide consensus has arisen about the fact that it can be safely regarded as a kind of revision of the standard rules of *positio*, possibly the only one that might be really related to some form of counterfactual reasoning. The most systematic intepretation of Kilvington's hints remains [Spade, 1982b, pp. 19–28], but cf. also [Yrjönsuuri, 2001a, pp. 16–20] and [D'Ors, 1991a].

Swyneshed is the author of an important treatise on obligations, written between 1330 and 1335, that proposes a strong revision of the standard rules, cf. the edition in [Spade, 1977]. The impact of his reform is remarkable not much in terms of the number of people who actually became convert to the cause but rather for the discussion it prompted in the subsequent tradition, especially in the second half of the XIV century, even among supporters of the old account. Its relevance (and, to some extent, even the brute fact that what goes on in Swyneshed's treatise should actually count as a revision) has been put into question on a number of occasions by D'Ors, but the matter has been settled on the basis of decisive evidence in [Ashworth, 1996] which also contains the most detailed account of the influence of Swyneshed's views on the subsequent generation of logicians. His views are also specifically discussed in [Spade, 1982b, pp. 28–31], [Stump, 1981], [Yrjönsuuri, 2001a, pp. 20–24] and [Dutilh Novaes, 2006b].

²⁴ The persistence in the XV and XVI centuries is documented in [Ashworth, 1982, pp. 793–794], [Ashworth, 1988b, pp. 146–153], and especially [Ashworth, 1999].

²⁵ This difficulty is closely related to the obnoxious question of the aims of the genre.

 26 By contrast, much is still to be done to reach a systematic account of the history and the logical complications involved in the plethora of sophisms that are found even in these two

produced in support of the claim that the theory already had an articulated level of elaboration in the first half of the XIII century, cf. [Braakhuis, 1998].

²³ Burley's text has been edited in [Green, 1963] and has played for a long time an influential role also in the historiography on obligations, since it "presents the theory in a fully developed form, and [...] sets the stage for later developments" [Ashworth, 1988a, p. x]. Besides the fundamental presentation found in [Green, 1963, pp. 91–119], early accounts of Burley's obligations are [Spade, 1982b, pp. 4–9 and 14–19], and [Stump, 1985]. For a more recent and systematic discussion of the logical aspects, cf. [Yrjönsuuri, 2001a, pp. 12–16] and [Dutilh Novaes, 2005].

Both the early and the late phase are still waiting for a systematic characterization. The former, in particular, raises a number of problems. On the one hand, we have a relatively small amount of sources and, for a long time, there has been some controversy over the datation of the earlier texts. This issue has to do not only with the need to draw an accurate historical picture, but also with that of establishing a reliable *terminus post quem* for the existence of the doctrine in an elaborate form. The consensus used to converge on the beginning of the XIV century (Burley's treatise is dated 1302), but there are good arguments to claim that the threshold should be pushed back to the first half of the XIII century.²⁷

The major problem concerning the beginnings of this literature, however, is that at first sight it seems to have come out of nothing. There have been various attempts to identify prefigurations and early uses of the obligational vocabulary;²⁸

²⁸ It has been claimed that there is a strong link between the early development of obligations and the treatment of self-referential paradoxes such as the Liar. For instance cf. [Martin, 1993, p. 373] "I would thus suggest not only that twelfth century logicians first became aware of insolubles via obligations but also that they developed their first solution in terms of them." Martin devotes his analyses in particular to such early treatises as the *Tractatus Emmeranus de falsi positione*, the *Tractatus Emmeranus de impossibili positione*, and the *Obligationes Parisienses*. In the discussions on *positio* that are found in those texts, Martin recognizes a connection to what he calls 'Eudemian hypotheses and procedures', a method of reasoning described by Boethius in the *De hypotheticis syllogismis*, cf. [A. M. S. Boethius, 1969, I, ii, 5-6] "hypothesis namque, unde hypothetici syllogismi accepter vocabulum, duobus, ut Eudemo placet, dicetur modis: aut enim tale adquiescetur per quamdam inter se consentientum conditione $\langle m \rangle$ quae fieri nullo modo possit, ut ad suum terminum ratio preducatur; aut in conditione posita consequentia vi coniunctionis

important texts, not to speak of those featuring in the works on obligations of other less studied authors.

²⁷ Cf. the discussion in [Braakhuis, 1998]. The anonymous treatises edited by De Rijk show that, already in the first half of the XIII century, a rather mature form of these texts was available, but it is still far from the level of elaboration found Burley. After Braakhuis' edition of the putative Nicholas of Paris' treatise, the debated question on the form of obligational treatises in the XIII century seems to have reached a decisive turn. Before this edition, much depended on the position one would take with respect to the authorship of the treatise edited in [Green, 1963] and ascribed to William of Sherwood (for an analysis of its contents, cf. [Green, 1963, pp. 51–90] and [Stump, 1980]). This work shows a strong similarity to Burley's treatise and has indeed been regarded as a preliminary version by Burley himself rather than as an indipendent treatise by Sherwood, cf. [Spade and Stump, 1983]. As a result of the ascription of that treatise to Burley, the only obligational texts dating to the XIII century would be those edited by De Rijk, which actually present less articulated doctrines. Yet, if Braakhuis is correct in dating the treatise attributed to Nicholas of Paris, then no matter what decision we take about the Sherwood-Burley dilemma, we would turn out to have at least one articulated and structured treatise (in many respects not inferior to the one by Burley) as an independent witness of the obligational literature already in the first half of the XIII century (cf. [Braakhuis, 1998, pp. 166-167] "it is clear that our treatise provides substantial proof that the state of the art of the theory of obligations was already fully developed in the first half of the thirteenth century in Paris"). And that changes the picture in no small measure, because the following legitimate question becomes urgent: how come this literature is already so full-fledged at such an early stage of its history?

it has often been insisted upon a connection with the dialectical framework outlined especially in the eighth book of the *Topics* (the tradition of medieval commentaries on the latter has been explored too, with the aim of finding additional evidence to corroborate the idea of such a remote dependence).²⁹ Finally, if one looks at texts like the *Excerpta Norimbergensia*³⁰ or the Ars Emmerana,³¹ a link can be found also with the study of fallacies. But let us briefly consider the connection with Aristotle. In the secondary literature on obligations a handful of textual references to his works are often regarded, if not as direct sources, at least as a general background for understanding the origin of the genre. A relevant text, found in the eighth book of the *Topics*, insists on the conflicting tasks of the disputants and on the distinction between two different mistakes, which are not logically on a par, namely that of assuming a wrong thesis and that of being unable to defend it properly (the former being not just as bad as the latter).³² Besides, two additional aristotelian remarks would count, as it were, as a theoretical foundation for the theory, namely a passage featuring in the first book of the *Prior Analytics* and one in the ninth book *Metaphysics*.³³ Now, how to regard these elements

To what extent these materials can contribute to a reliable reconstruction of the origins of the obligational literature is still difficult to say, but they certainly enrich our knowledge of how this vocabulary was used in a much earlier phase of medieval logic than the one in which obligations actually flourished and took the shape that has come down to us.

²⁹ Cf. in particular [Yrjönsuuri, 1993a].

³⁰ Cf. [De Rijk, 1967, p. 131] "Disputationis sunt duae partes: alia scientia arguendi, alia scientia redarguendi, idest opponendi et respondendi". In this respect, it is notheworthy that a much later treatise such as Strode's sets out by saying what it means for a respondent to be *redargutum*, cf. [Ralph Strode, 1517, fol. 78^{ra}], i.e. reproached for being forced to change his response to a sentence, in virtue of the inability to conduct an argument in the appropriate way. Moreover, as [Ashworth, 1988a, pp. 34–35 f. 6] has pointed out, the word *redargutio* is used to translate 'refutation' in the Latin versions of Aristotle's *De Sophisticis Elenchis*.

³¹ Cf. especially [De Rijk, 1967, p. 148].

 32 Cf. Top. VIII, 4, 159a 17–24 "The business of the questioner is so to develop the argument as to make the answerer utter the most implausible of the necessary consequences of his thesis; while that of the answerer is to make it appear that it is not he who is responsible for the impossibility or paradox, but only his thesis; for one may, no doubt, distinguish between the mistake of taking up a wrong thesis to start with, and that of not maintaining it properly, when once taken up" (translation from [Aristotle, 1984, vol. 1, p. 268]).

³³ Cf. An. Pr. I, 13, 32a 18–20 "I use the terms 'to be possible' and 'the possible' of that which is not necessary but, being assumed, results in nothing impossible" (transl. from [Aristotle, 1984, vol. 1, p. 51]), and Met. IX, 4, 1047b 10–12 "if we actually suppose that which is not, but is

vel disiunctionis ostenditur". Besides, the application of this procedure is also witnessed in Boethius' *Quomodo substantiae*, in Garlandus Compotista's *Dialectica*, in Abelard's *Dialectica* (cf. [Peter Abelard, 1970, p. 471] "Per consensum et concessionem hypothesis propositio illa dicitur quae non in se vera recipitur, sed gratia argumentandi conceditur, ut quid ex ea possit extrahi videatur"), and in XII century texts by Gilbert of Poitiers, Clarembald of Arras and Thierry of Chartres who explicitly employs the expression 'positio impossibilis per consensum'. For more detailed references cf. [Martin, 1993, p. 360 f. 11].

is quite problematic. On the one hand, there is certainly a relation of some sort, especially with the disputational setting of the *Topics* and the *Elenchi*. But on the other hand, references in medieval obligations treatises to this kind of background are extremely rare and, for the most part, chronologically late, to the effect that the connection appears loose and looks more like a retroactive justification of the purpose of obligations rather than an explanation of their origin.³⁴

Although this problem should be kept distinct from the question of the aim (or of the different aims) that might have arisen or intervened later on in the historical development of the theory, one thing is worth noting. These aristotelian references point in two quite different directions, even if both can be called into play with equal legitimacy to suggest what obligations could serve for. Assuming the text from the *Topics* as a background, means being more oriented to regard obligations as a test of sound reasoning in a disputational context. The passages from the *Prior Analytics* and the *Metaphysics*, on the other hand, lend themselves to an interpretation that identifies the purpose of obligations in the examination of what follows from a given hypothesis. This fact, in itself, would not be particularly interesting, but I have the impression that the few remarks found in obligational treatises about the purpose of the art, are often oriented exactly in one of these two directions. And this is probably why the aristotelian background has attracted so much attention: it has offered a choice for modern interpreters to pick out a

capable of being, to be or to have come to be, there will be nothing impossible in this" (transl. from [Aristotle, 1984, vol. 2, p. 1654]).

³⁴ Cf. in this respect [Ashworth, 1988a, p. 33 f. 3] "commentators on Aristotle did not refer to the subject of *obligationes*, with the single exception of Boethius of Dacia in his commentary on Topics 8 [...] Nor did writers on *Obligationes* make reference to the relevant Aristotle texts". Ashworth's contention remains substantially true, even if an early text, that has been published in the meanwhile, may count as an exception, since in fact it contains an explicit reference to the eighth book of the *Topics* (including the passage quoted above), cf. [Nicholas of Paris (?), 1998, p. 171]. Among some late exceptions, we find a reference for instance in the Copulata super omnes tractatus parvorum logicalium Petri hispani ac super tres tractatus modernorum.([Cologne], 1493) fol. lxxxi^v (the quotation is taken from [Ashworth, 1986, p.25 f. 56]): "... principium ex quo iste tractatus obligatoriorum elicitur, sumitur ex nono Metaphysicae et primo Priorum, ut dicit Aristoteles quod possibili posito inesse, nullum sequitur impossibile. Et ideo tota scientia ista in illo principio fundatur quia scientia obligatoriorum in hoc consistit quod aliquis concedat multa possibilia secundum conditionem obligationis, et illa sic defendat ne cogatur ex illis concedere aliquod impossibile, licet aliquando et saepe cogatur concedere aliquod falsum". In the same place, Ashworth points out that an early reference (to the *Analytics*) is found in [John of Holland, 1985, p. 94]. Spade has rightly suggested that the connection may be dubious since the early development of the obligational literature is prior to the ready availability of the Latin translation of the *Metaphysics* and no mention of these passages is found in the most ancient treatises. Be this as it may, the presence of such remarks in later treatises is interesting in its own right because it tells us something about the reception of the tradition of obligations and possibly reveals that the need of finding a philosophical motivation for their purpose was felt already *then* as a relevant issue.

preferred reading.³⁵ There might be a problem, however. Among those explicit references to the purpose of obligations, there seems to be an asymmetry:³⁶ for the most part, they insist on the first of the two alternatives presented above, i.e. the idea that obligations are devised to test or to teach logical skills. That would correspond to the background of the first aristotelian reference from the *Topics* rather than to the more philosophically-oriented suggestion derived from the *Prior* Analytics (or from the *Metaphysics*). Why then explicit references are so rare? For instance in Peter of Mantua's treatise there are only two textual passages that might be regarded as 'mission statements'.³⁷ Both suggest, that he is more inclined to conceive of obligations in terms of the first interpretation rather that in terms of the second. In other words, the underlying idea is that obligations might be viewed as an environment designed to assess the respondent's ability to maintain consistency (or as an art devised to teach such an ability), rather than as a method employed to see what follows from an initial assumption. And in his case just as in that of several other authors, this standpoint is assumed without feeling the need to call into play an aristotelian justification that might have been after all quite easy to produce.

The variety of hints that have been proposed to account for the emergence of the theory undoubtedly seems to have a high degree of interest and some plausibility, but on the whole, I think a genuinely convincing solution to the problem of the origins has not been sorted out yet, because all these results are the outcome specific researches that still need to be put together and assessed in a more general perspective.³⁸ All in all what Spade wittily remarked a few years ago about

³⁵ Evidence of this influence is reflected especially in Spade's counterfactual interpretation and in Yrjönsuuri's account in terms of thought experiments who both base many of their considerations on some scattered remarks in the early treatises to stress the idea that obligations are set up to see what follows from a given assumption, cf. for instance [*Tractatus Emmeranus de falsi positione*, 1974, p. 103, ll. 16-23] "Et notandum quod hec obligatio non dicitur *falsi positio* propter hoc quod tantummodo falsa ponantur, sed quia sepius ponantur quam vera. Cum enim omnis positio fiat propter concessionem et vera propter sui veritatem habea $\langle n \rangle$ t concedi non indigent positione. Sed cum falsa non in se causam concessionis $\langle habeaat \rangle$, indigent positione ut concedantur et videatur quid inde accidat".

³⁶ A very interesting example of interplay between the two levels is found in [John Tarteys, 1992, p. 671] "sciendum quod finis principalis huius artis est scire connexionem veritatum, scilicet quid sequitur posito tali casu aut tali, et quid eidem repugnat et quid est eidem impertinens. Taliter enim homines communiter consiliantur. Finis secundarius est dirigere sermonem respondentis ne in eodem tamquam falsidicus capiatur".

 $^{^{37}}$ Cf. *infra*, Part II, sec. 4.1, "[...] sepe vult opponens obligare ad falsum sustinendum respondentem, ut videat qualiter sciat se a contradictione defendere" and, Part II, sec. 5.10, "iste conclusiones in hac arte non sunt sustinende, quia tota intentio est convenientem responsionem instruere respondentem".

 $^{^{38}}$ Take for instance the promising line of research that points to the tradition of the commentaries on the *Topics*. This is a place it seems very natural to look for attestations of the vocabu-

our inability to find a good explanation or compelling evidence for a satisfactory reconstruction of the roots of this genre remains still true.³⁹

As far as the late phase of the history of obligations is concerned, the terms of the problem seem to be inverted. The situation becomes more complicated both in quantitative terms (number of treatises and at least in some cases, like that of Paul of Venice, their remarkable length) and in qualitative terms (interaction and contamination between different logical doctrines, complexity of the collections of sophisms), but the stage for further research has been set by some fundamental contributions.⁴⁰ There is no space here to discuss the complexity of these devel-

Further research in this field might reveal earlier attestations, although Green Pedersen's work on this tradition seems not to be very encouraging as regards the possibility of finding substantial evidence to supplement Yrjönsuuri's reconstruction, cf. [Green-Pedersen, 1984, p. 342] " the doctrine of obligations (*de obligationibus*) must almost inevitably owe something to Aristotle's *Topics*, book VIII. As yet, however, we cannot say how much, and at any rate the development of the doctrine of obligations has left very few traces in the commentaries on Aristotle's book, with the anonymous commentary *Aristoteles distinguens* as the only exception". The text Green-Pedersen refers to is MS Cambridge, Gonville and Caius College, 344/540, fols. 216^{ra} – 219^{rb} "the entire text deals with the *obligationes*, a subject about which almost nothing is otherwise found in the commentaries on the *topics*. If the headline had not been there we should not have considered the text to be a commentary on the *Topics* at all. Anyway it is at best only a fragmentary text connected with book VIII". This work is also recorded by Ashworth in her comprehensive catalogue, cf. [Ashworth, 1994, p. 119].

³⁹ Cf. [Spade, 2000, pp. 13-14] "This is something I find extremely mysterious. Other branches of mediaeval philosophy still have their murky areas and unanswered questions – yes. That goes without saying, there as in any other branch of scholarship. But it is only here, in this one area of mediaeval philosophy, in late mediaeval logic and semantic theory, that I repeatedly find this fundamental murkiness about even the most basic starting points. This isn't the case with all the topics that come up in late mediaeval logic and semantics, to be sure. But it's true of an astonishing number of them. In case after case, these theories [obligations are one of the examples discussed in detail by Spade] seem to have sprung full-grown from the head of Zeus – without explanation, without motivation, without preparation, without anything to help us figure out what was really going on. The earliest texts we possess already presuppose a well-understood agenda that we are simply not privy to".

 40 I will just recall some cornerstones in this field. For the most general aspects, cf. [Ashworth, 1981] which provides a first characterization of the state of the art in the late XIV century. [Ashworth, 1985] outlines the British tradition after Swyneshed and some of the results are extended in an even more comprehensive picture in [Ashworth, 1996] which takes into account a whole lot of texts that in different ways have to do with the *responsio nova*. [Ashworth, 1994] remains the essential catalogue for orientation: a quick glance at it is sufficient to realize how much work is still to be done, especially in terms of textual research. Finally, for a reconstruction

lary, techniques or at least for embryonic progenitors of the obligational literature. Yrjönsuuri in a number of contributions, cf. especially [Yrjönsuuri, 1994] and [Yrjönsuuri, 1996], has provided quite interesting analyses of two texts that mention obligations, namely Boethius of Dacia's and Albert the Great's commentaries. The problem is that both of these texts are posterior to the early development of the theory. Therefore, they presumably cannot tell us much about its origin, but rather about its reception in a closely related context.

opments, which would lead us far beyond the purpose of this work. But at least two things must be recalled. First and foremost, a distinctive feature of treatises classified in this category is the standard presence of traces of Swyneshed's revision of the rules of *positio* (either to endorse or to reject it). Second, there are two main clusters of authors, grouped on the basis of geographical provenance. In connection to the English scene the following relevant sources should be recalled:⁴¹ Martinus Anglicus,⁴² Robert Fland,⁴³ Richard Lavenham,⁴⁴ Richard Billingham,⁴⁵ the *Logica Oxoniensis*,⁴⁶ (printed version in the *Libellus Sophistarum ad Usum Oxoniensium*⁴⁷), the *Logica Cantabrigiensis*,⁴⁸ (printed in the *Libellus Sophistarum ad Usum Cantabrigiensium*⁴⁹), John Tarteys,⁵⁰ John Wyclif,⁵¹ Richard Brinkley,⁵² and John of Holland.⁵³ Finally, a text that proves to be of greatest interest for the purpose of the present work is Strode's treatise.⁵⁴

⁴³ Fland's treatise is edited in [Spade, 1980]. To this author we owe the most renowned, albeit by no means unique, distinction between different accounts of *positio*.

⁴⁴ Cf. [Spade, 1978]. Lavenham's short treatise summarizes and accepts Swyneshed's views.

⁴⁵ Billingham, who was closely related to Merton College, Oxford, in the 1340s, turned out to have considerable influence especially on the English tradition since his "lengthy section on *positio* was reproduced almost without change in a series of obligations texts which belong to the *Logica Oxoniensis*" [Ashworth, 1988a, p. xi]. A printed version of the latter features in the *Libellus Sophistarum ad Usum Oxoniensium*. Billingham's set of definitions and rules are also employed in [John Wyclif, 1893] and feature in the section on obligations of the *Logica Cantabrigiensis* (printed version in the Cambridge counterpart of the Oxford text-book, namely the *Libellus Sophistarum ad Usum Cantabrigiensium*, cf. [Ashworth, 1988a, p. xi]. A general reconstruction of the history of these manuals is found in [Ashworth, 1979], whereas in [Ashworth, 1999] sharper focus is on obligations.

⁴⁶ Cf. [De Rijk, 1977].

⁴⁷ Cf. [Anonymous, 1524].

- ⁴⁹ Cf. [Anonymous, 1510].
- ⁵⁰ Cf. [Ashworth, 1992].
- ⁵¹ Cf. [John Wyclif, 1893].
- ⁵² Cf. [Spade and Wilson, 1995].
- ⁵³ Cf. [John of Holland, 1985].

⁵⁴ Cf. [Ralph Strode, 1517]. Two essential interpretations of this text are [Ashworth, 1993] and [Dutilh Novaes, 2006a]. I will come the influence of Strode on Peter of Mantua, in due course, namely in the commentary on Peter's treatise. Let it be said here that besides the textual evidence that will be produced below, there are probably also suggestive (and grounded) arguments to contend that copies of Strode's text were available in the early 1390s within the

of the developments of the obligational literature in the XV century and beyond, the point of departure are the three references quoted above, namely [Ashworth, 1982], [Ashworth, 1988b], and [Ashworth, 1999] (the latter is more specifically oriented to obligations texts from 1400 to 1530 in the English context).

⁴¹ I follow here the synthetic account given by [Ashworth, 1988a, pp. x-xii].

⁴² Cf. [Schupp, 1993]. This treatise is very similar to Fland's text, to the effect that "it is impossible to say who influenced whom" [Ashworth, 1988a, p. xi]. Martinus is sympathetic with Swyneshed's views.

⁴⁸ Cf. [De Rijk, 1975a].

As far as the Parisian tradition is concerned, the most outstanding figures are Albert of Saxony,⁵⁵ William Buser,⁵⁶ and Marsilius of Inghen.⁵⁷ Besides these central figures, interesting developments are found on the continent much later in the work of Josse Clitchove.⁵⁸

Both traditions turn out to be quite relevant for the Italian context of the late XIV century. Here we find at least four authors: Peter of Candia, Peter of Mantua, Paul of Venice⁵⁹ and Paul of Pergula.⁶⁰ As regards Peter of Mantua, the presence of a link with Albert of Saxony's and Marsilius of Inghen's logical doctrines had been already noticed by Prantl and has been confirmed by more recent studies devoted to specific chapters of the *Logica*.⁶¹ The relationship is attested in the treatise on obligations, too. In addition to that, several elements in his text suggest a direct acquaintance with Strode's work.⁶²

To conclude the remarks of this section, one final comment is in order. This sketchy presentation of some basic issues related to the history of obligations has been outlined from a very general perspective. However, this was by no means unavoidable. On the contrary, I think it should be stressed that, given the present

⁵⁷ Marsilius's work is strongly influenced by Buser. The treatise is still inedited and there is no specific study on it. The state of research on the Parisian line appears relatively poor in comparison with the English tradition, that has been investigated more extensively. As a compensation, an essential point of departure are the numerous remarks, referring to the Parisian triad, that can systematically be found in Ashworth's notes to Paul of Venice's treatise, cf. [Ashworth, 1988a, *passim*].

⁵⁸ A suggestive study of this figure is [Ashworth, 1986].

⁵⁹ Cf. the edition in [Ashworth, 1988a].

⁶⁰ Paul of Pergula was a pupil of Paul of Venice, cf. the edition of his treatise in [Paul of Pergula, 1961].

⁶¹ Cf. [Prantl, 1870, pp. 176–180], [Bos, 1985a], [Bos, 1982], and [Bos, 1985b].

 62 Cf. also [Ashworth, 1988a, p. xii and *passim* in the notes to the text]. The double connection, with Strode and some Parisian authors, is apparent not only in the work on obligations but also, for instance, in the treatise on consequences, cf. [Bertagna, 2000].

Paduan circle of students Peter used to belong to before moving to Bologna, at latest in 1392.

 $^{^{55}}$ Albert's obligations are still inedited. There are basically two versions of it, one in the *Perutilis Logica* and the other in the *Sophismata* (I have used the latter for some textual parallel with Peter's obligations). To the best of my knowledge, the only study explicitly devoted to this work is [Braakhuis, 1993].

⁵⁶ The two basic sources of information on Buser are [Federici Vescovini, 1981] and [Kneepkens, 1982]. An edition of Buser's treatise on obligations is found in [Pozzi, 1990]. In the light of the attribution of the treatise in Ms. Paris, B. N. lat., 11.412 to Nicholas of Paris it has been argued in favour of the existence of an independent Parisian tradition, cf. [Braakhuis, 1998, p. 167, f. 83] "In view [...] of the internal similarity with the *Obligationes Parisienses* on the one hand and some aspects of the *Obligationes* by William Buser on the other [...] our treatise [i.e. the treatise attributed to Nicholas of Paris, datable to the first half of the XIII century] may be considered as evidence for an autonomous continuous tradition in logical theory in the thirteenth and early fourteenth century at Paris" (cf. also, in this respect, [Braakhuis, 1993, pp. 325-6]).

state of research, 'local' accounts of the history of specific problems, or of the internal development of less studied subtypes of obligations (like *impositio* or *dubitatio*), or again of the geography of reception of a given doctrine, might turn out to be extremely relevant for an accurate reconstruction of the tradition as a whole.⁶³

1.3. The logical structure of a disputation

After the preliminary account introduced in sec. 1.1, we can now turn to the logical structure of a disputation as it is conceived of in the obligational framework. We will see more in detail how rules are supposed to govern the construction of the set of sentences of which a disputation is constituted. As has been said above, what we obtain at each step (and cumulatively at the end) of a disputation is an ordered sequence of sentences. The rules of both the *responsio antiqua* and the responsio nova specify what should or should not be in that sequence. In this section, I will outline the rules relative to the former account – which is the one endorsed by Peter – according to the formal presentation proposed in recent years by Dutilh Novaes, which represents a very straightforward way to characterize its logical features.⁶⁴ In the next section, I will propose an equivalent version which presents some notational differences and a provisional conservative extension, in order to account for an aspect of Peter's theory that would otherwise be left out of the picture.⁶⁵ The characterization of the *responsio nova* results very naturally from a simplification of the rules of the standard account, but I will just allude to its basic features in the notes to Peter's treatise.⁶⁶

A disputation can be regarded as a quadruple:⁶⁷

$$Ob = \langle K_c, \Phi, \Gamma, R(\phi_n) \rangle$$

⁶³ Parts of such local histories have begun to be written in recent times, cf. for instance [Ashworth, 2000] on the use of *dubie positio* in Domingo de Soto, [D'Ors, 2000] on Andrés Limos, or [Uckelman et al., 2008] which provides some interesting elements for a formal characterization of the properties of *dubitatio* in the *Obligationes Parisienses*.

 $^{^{64}}$ A first description of the logical properties of Burley-style obligations is already found in [Spade, 1982b] (I will give a sketch of it below), but I have decided to follow here the more recent account, because of its clear and systematic character which has also helped me figure out the extension required to describe some features of Peter's approach.

⁶⁵ Although the logical structure of a disputation played according to the *responsio antiqua* rules is roughly the same for most authors adopting this account, there are significant differences concerning singular aspects, and my focus here is on what is going on in Peter's treatise.

⁶⁶ Cf. in particular *infra*, Part II, sec. 3.20–3.25.

⁶⁷ Unless noted otherwise, the main content of what comes next in this section is due to [Dutilh Novaes, 2007].

 K_c is the set of background assumptions (a common state of knowledge) shared by opponent and respondent which comes into play for the evaluation of irrelevant sentences. It can be regarded as a set of sentences providing a partial description of the world (sentences which are neither known to be true nor known to be false are not included in K_c).

 Φ is the ordered set of sentences that are actually put forward by the opponent during the disputation. In general, ϕ_0 will always stand for the (first) positum, while ϕ_n will stand for the sentence put forward at step n.

 Γ is an ordered set whose elements are sets of sentences, such that the following relation holds: $\Gamma_i \supseteq \Gamma_{i-1}$. At each stage *i*, the corresponding set Γ_i displays the cumulative outcome of the disputation up to that point, that is the set of all sentences that have been conceded or denied by the respondent. As a result, Γ literally *is* the list or the box that I have metaphorically used above to describe the record of a disputation. The criteria of construction of this set depend on the rules and will be illustrated in a moment.

 $R(\phi_n)$ is a function that maps each proposed sentence ϕ_n onto a member of the set $\{1,0,?\}$. This function simulates, from the logical point of view, the speech acts that the respondent is supposed to perform, according to the rules, on the sentences that are submitted to his evaluation. 1, 0 and ? correspond to the acts of accepting (i.e. admitting or conceding), denying (or rejecting) and doubting a given sentence, respectively.

Within this framework, it is very easy to define the rules for admission and rejection of *posita* as well as the rules for concession, denial and doubt relative to *proposita*. Let us first define the former:

$$R(\phi_n) = 1 \quad \text{iff} \qquad \begin{cases} \phi_n = \phi_0 \text{ and } \phi_0 \not\vdash \bot \\ & \text{or} \\ \phi_n \neq \phi_0, \ \phi_n \not\vdash \bot \quad \text{and} \ \Gamma_{n-1} \not\vdash \neg \phi_n \end{cases}$$

 $R(\phi_n) = 0$ otherwise

The standard condition that the *positum* be consistent must be supplemented in Peter's theory, by the condition that, whenever at some step of a disputation the opponent posits a new sentence, this extra *positum* be consistent not only in itself, but also with the previous steps of that disputation.⁶⁸

As far as the rules for *proposita* are concerned, the criteria of response can be expressed as follows:

 $^{^{68}}$ Cf. infra, Part II, sec. 2.1.

$$R(\phi_n) = 1 \quad \text{iff} \qquad \begin{cases} \Gamma_{n-1} \vdash \phi_n \\ \text{or} \\ \Gamma_{n-1} \nvDash \phi_n, \Gamma_{n-1} \nvDash \neg \phi_n \text{ and } K_c \Vdash \phi_n \end{cases}$$
$$R(\phi_n) = 0 \quad \text{iff} \qquad \begin{cases} \Gamma_{n-1} \vdash \neg \phi_n \\ \text{or} \\ \Gamma_{n-1} \nvDash \phi_n, \Gamma_{n-1} \nvDash \neg \phi_n \text{ and } K_c \Vdash \neg \phi_n \end{cases}$$
$$R(\phi_n) = ? \quad \text{iff} \qquad \Gamma_{n-1} \nvDash \phi_n, \Gamma_{n-1} \nvDash \neg \phi_n, K_c \nvDash \phi_n \text{ and } K_c \nvDash \neg \phi_n \end{cases}$$

This formulation tells us on the basis of what conditions the respondent should concede, deny or doubt a sentence, when it is proposed. A sentence ought to be conceded either if it follows from the set of sentences previously evaluated or if it is irrelevant and known to be true; a sentence ought to be denied either if it is inconsistent with the set of sentences previously evaluated or if it is irrelevant and known to be false; a sentence ought to be doubted otherwise, i.e. if it is logically independent from the set of sentences previously evaluated and it is neither known to be true nor known to be false. Usually, in obligations treatises, rules are presented in a slightly different, albeit logically equivalent way, since they are laid down in function of the logical status of sentences.⁶⁹ In the present terms,

1. Let we first define the notion of relevance and irrelevance:

$$p_{n+1}$$
 is relevant at step $n+1$ iff
$$\begin{cases} X_n \text{ implies } p_{n+1} \\ \text{or} \\ X_n \text{ implies } \neg p_{n+1} \end{cases}$$

 p_{n+1} is *irrelevant* at step n+1 iff it is not relevant at step n+1

A sentence is sequentially (or incompatibly) relevant at step n + 1 if it follows (or if its negation follows) from a particular set of sentences X_n . How is this set constructed? In the following way:

- 2. X_n is the smallest set containing the *positum* such that, for all sentences p and for all i < n,
 - a) if p was proposed at step i and had to be conceded there according to the rules in (3) below, then $p \in X_n$, and

⁶⁹ Spade's reconstruction of the rules is meant to express in an explicit way exactly these relations. I give below a formulation with slight modifications of the notation that do not affect the general structure in any substantial way:

if $\Gamma_{n-1} \vdash \phi_n$, then ϕ_n is said to be sequentially relevant (*pertinens sequens*); if $\Gamma_{n-1} \vdash \neg \phi_n$, then ϕ_n is said to be incompatibly relevant (*pertinens repugnans*). Any relevant sentence is either sequentially relevant or incompatibly relevant. A sentence is irrelevant (*impertinens*) if and only if it is not relevant. It holds in general that any sequentially relevant sentence ought to be conceded, any incompatibly relevant sentence ought to be denied and any irrelevant sentence ought to be evaluated according to the external criteria that in our model are supplied by K_c .⁷⁰

On the basis of these notions, we can now see how the development of a disputation can be reconstructed. This reduces to laying down the criteria of formation for the cumulative set Γ which includes the ordered sequence of steps of the disputation. At the very beginning of the disputation, if $R(\phi_0) = 1$, i.e. if the *positum* is admitted, the following will hold:

$$\Gamma_0 = \{ \phi_0 \}$$

At an arbitrary step n, Γ_n is formed as follows:

if
$$R(\phi_n) = 1$$
 then $\Gamma_n = \Gamma_{n-1} \cup \{\phi_n\}$
if $R(\phi_n) = 0$ then $\Gamma_n = \Gamma_{n-1} \cup \{\neg \phi_n\}$
if $R(\phi_n) = ?$ then $\Gamma_n = \Gamma_{n-1}$

The reconstruction is thus complete and accounts for the progressive addition of sentences according to the qualitative description given in sec. 1.1 above. At

- 3. p_{n+1} is to be conceded, denied or doubted at step n+1 according to the following rules:
 - a) if p_{n+1} is sequentially relevant at n+1, it is to be conceded there;
 - b) if p_{n+1} is incompatibly relevant at n+1, it is to be denied there;
 - c) if p+1 is irrelevant at n+1 and
 - (i) the respondent knows p_{n+1} to be true, it is to be conceded at n+1
 - (ii) the respondent knows p_{n+1} to be false, it is to be denied at n+1
 - (iii) otherwise p_{n+1} is to be doubted at n+1

Cf. [Spade, 1982b, p. 6]. This formulation is altogether equivalent to the one proposed by Dutilh Novaes. As will be clear below, both accounts leave open the issue of how we are supposed to account for sentences about which the respondent should be in doubt.

⁷⁰ Cf. *infra*, Part II, sec. 2.2 and 2.3 for the rules governing the replies to relevant sentences, sec. 2.5 for the rule relative to irrelevant ones.

b) if p was proposed at step i and had to be denied there according to the rules in (3) below, then $\neg p \in X_n$.

each round, a sentence is put forward, evaluated and put in the record of the disputation. Round after round, the cumulative set gets bigger and determining the logical status of sentences proposed at later steps may become comparatively more complex.

At the very beginning, the respondent must simply check the status of the positum ϕ_0 and establish whether it is a consistent sentence or not. In the former case he should admit it. As a result, ϕ_0 ends up in the list. When the opponent puts forward his first propositum ϕ_1 , the respondent is supposed to proceed as follows: (i) first, he should check whether ϕ_1 or its negation follow from Γ_0 . If one of the two does, he will concede or deny ϕ_1 , respectively. If neither does, the respondent is supposed to (ii) have recourse to set K_c , pick out an evaluation for ϕ_1 and concede it or deny it accordingly. In case K_c does not provide any information about ϕ_1 , the sentence will have to be doubted. According to whether ϕ_1 is conceded, denied or doubted, it will be recorded, or its negation will be recorded, or neither will be recorded in the list. After this procedure has been completed, the disputation can go along. The opponent proposes another sentence ϕ_2 at the next step, and the respondent runs the same 'algorithm' with only one difference, namely that he must now check the logical dependence of ϕ_2 with respect to Γ_1 .

Two things must be noted. First, from the logical standpoint, real changes take place only when irrelevant sentences are added to Γ . Suppose the opponent puts forward throughout the disputation nothing but sequentially or incompatibly relevant sentences: the set with respect to which the respondent is required to check the logical status of a given *propositum* will be, at any step of the disputation, logically equivalent to Γ_0 . This means that the class of relevant sentences is ampliated only through the addition, from a given moment of the disputation on, of sentences that are logically independent with respect to the previous steps. The respective extensions of the classes of relevant and irrelevant sentences change only in this case.⁷¹

Second, in the present framework, it is as if doubtful sentences were discarded, since no record is kept of their being proposed and doubted in the course of the disputation. This might be a problem with respect to Peter's theory, because he allows for rules explicitly devoted the treatment of such sentences which by definition do not feature in any Γ_i . That will be the object of some remarks in the next section.

⁷¹ An elegant proof based on a model-theoretic approach is found in [Dutilh Novaes, 2007, p. 166]. Irrelevant sentences represent the only increments of information in a disputation, because when they are introduced, they are by definition logically independent with respect to the outcome of the disputation up to that point.

1.4. Glimpses beyond

The leading idea of this section is reworking some of the notions introduced above and add some elements to the picture in order to capture at least in a provisional way the idea of keeping track of doubted sentences. This will amount to a slightly different presentation of the results just discussed and to a conservative extension devised for that purpose.⁷²

Let us first define the set of logical consequences of an arbitrary sentence α , where α and β are metavariables standing for any well-formed sentences of Latin, and by well-formed sentence we mean (i) a categorical sentence or (ii) anything obtained by applying negation, conjunction, disjunction and conditional form to categorical sentences, and (iii) nothing else:

$$Cn(\alpha) = \{ \beta \mid \alpha \vdash_F \beta \}$$

The relation of logical consequence to be used in this context, as I recalled in sec.1.1, is highly problematic. I would like to avoid the problem, because it is not clear, even if we confine ourselves to the case of Peter, what an appropriate candidate would be to simulate his notion of *consequentia* in modern terms. The symbol \vdash_F is just a shortcut to express the fact that, whatever it may mean, according to Peter, that something *formally follows* from something else, this is the notion that is at stake in the definition of relevance which comes into play in the context of his obligations.⁷³

The only assumption that we need to make on the resulting set is that we

 $^{^{72}}$ The notational change is introduced with the aim of highlighting the notions of relevance and irrelevance in terms of sets of logical consequences. As for the difference with regard to the above results, it entirely depends on whether sentences that ought to be doubted are proposed or not. If they are not, the account I will present here works exactly in the same way as Dutilh Novaes' one. Otherwise, it is supposed to do a job that the other account seems not originally intended to do.

⁷³ I am inclined to favour, at least as a preliminary solution, the use of Lewis' strict conditional. The reason is that Peter's definition of a valid consequence explicitly requires that the conjunction of the negation of the consequent with the antecedent be an impossible sentence, cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii+3^{ra}]. Material conditional is therefore ruled out. Relevant entailment should be excluded too, because Peter explicitly accepts the validity of consequences having a formal contradiction as antecedent or a tautology as consequent, no matter whether there is no relation of meaning or containment between antecedent and consequent. This last notion seems to come in to play in several places as a condition of the formal character of a consequence can be consistent with the claim that anything formally follows from a contradiction (or that anything formally implies a sentence like $\alpha \vee \neg \alpha$). It would be too wholesale, however, to be content with these criteria: a more detailed analysis of Peter's theoretical claims and examples will be required to sort out the issues involved in his definition and use of the notion of consequentia.

are allowed to iterate the operation that generates it without gain or loss (i.e. the logical consequences of the logical consequences of α are, at least, already contained in the logical consequences of α):

$$Cn(\alpha) \supseteq Cn(Cn(\alpha))^{74}$$

Once we have introduced this basic notion for an arbitrary sentence, we can use it to define, more generally, the notion of relevance – which is most important for us in this context – at an arbitrary step of a disputation. The set of relevant sentences at step n is defined as the set of logical consequences of the cumulative sequence of previous steps. The latter, in turn, is equal to the set of logical consequences of the conjunction of all sentences that have been conceded and of the negations of all sentences that have been denied up to that point. Let us see more in detail how the model is supposed to work.

Consider the sequence $\alpha_0, \ldots, \alpha_{n-1}$ of n sentences that are put forward in a disputation (as above, α_0 is the *positum*). Let us call Δ_{n-1}^* the set containing all α_i , with $0 \leq i \leq n-1$, such that α_i is either conceded or denied in the disputation. Δ^* is, as a matter of fact, the same kind of set as Dutilh Novaes' Γ , i.e. an ordered set whose elements are sets of sentences such that $\Delta_i^* \supseteq \Delta_{i-1}^*$ holds by definition. If we represent the whole outcome of a disputation, i.e. the set of *all* evaluated sentences – including those that are doubted – by means of Δ (unstarred), then Δ^* represents the (proper) subset of sentences that are either conceded or denied.

As in the original model, it will hold in general that, if α_0 is the *positum* and it is admitted, then

$$\Delta_0^* = \{ \alpha_0 \}$$

Similarly, the criteria of formation for Δ_n^* mirror the process of progressive addition of sentences to the cumulative outcome of the disputation previously outlined (see below for the notation):

if
$$C(\alpha_n)$$
 then $\Delta_n^* = \Delta_{n-1}^* \cup \{\alpha_n\}$
if $N(\alpha_n)$ then $\Delta_n^* = \Delta_{n-1}^* \cup \{\neg \alpha_n\}$
if $D(\alpha_n)$ then $\Delta_n^* = \Delta_{n-1}^*$

Now, on this basis, we can define the set of relevant sentences at the first step of the disputation in the following way:

$$Rel_{\Delta_0^*} = Cn(\alpha_0)$$

⁷⁴ This reduces to the minimal requirement that the notion of 'formally following from something' be transitive: if $\alpha \vdash_F \beta$ and $\beta \vdash_F \gamma$, then $\alpha \vdash_F \gamma$.

Once the *positum* has been admitted, the set of relevant sentences – i.e. the set of logical consequences of the cumulative outcome of the disputation up to that point – coincides with the set of logical consequences of the *positum* alone. How can we generalize it? A straightforward way is assuming that the following relation holds, for all α_i such that $\alpha_i \in \Delta_{n-1}^*$:⁷⁵

$$Rel_{\Delta_{n-1}^*} = Cn\left(\bigwedge_{i=0}^{n-1} (\alpha_i)\right)$$

which means that the notion of relevance at an arbitrary step is defined in terms of the set of logical consequence of the whole conjunction of sentences that have been either conceded or denied at the previous steps.

 $Rel_{\Delta_{n-1}^*}$ is the set of sentences that are relevant at step n. In particular, it is the smallest set containing (a) the *positum*, (b) all conceded sentences and all the negations of denied sentences, (c) the logical consequences of each step taken alone, and (d) the logical consequences of all conjunctions of such steps. In this framework, it is very easy to figure out what it means for a sentence α_n to be relevant or irrelevant at step n:

$$\alpha_n \text{ is relevant at step } n \quad \text{iff} \quad \begin{cases} \alpha_n \in \operatorname{Rel}_{\Delta_{n-1}^*} \\ \text{or} \\ \neg \alpha_n \in \operatorname{Rel}_{\Delta_{n-1}^*} \end{cases}$$

 α_n is *irrelevant* at step n iff it is not *relevant* at n

In particular, α_n is said to be sequentially relevant at step n if and only if $\alpha_n \in \operatorname{Rel}_{\Delta_{n-1}^*}$, whereas α_n is said to be incompatibly relevant at step n if and only if $\neg \alpha_n \in \operatorname{Rel}_{\Delta_{n-1}^*}$. In addition to that, we obtain a characterization of the class of irrelevant sentences at step n in set-theoretical terms, since the latter can be regarded as the complement of relevant ones:

$$Irr_{\Delta_{n-1}^*} = \overline{Rel}_{\Delta_{n-1}^*}$$

Thus, α_n is said to be irrelevant at step *n* if and only if $\alpha_n \in Irr_{\Delta_{n-1}^*}$. On the basis of these definitions, we can now (re)formulate the rules for concession, denial

⁷⁵ I.e. for all conceded or denied sentences in the ordered sequence Δ_{n-1} of the *n* steps that make up our disputation. It must be noted that usually $\Delta_{n-1}^* \subset \Delta_{n-1}$ unless there are no doubted sentences at all in the disputation. If there are indeed no doubted sentences then $\Delta_{n-1}^* = \Delta_{n-1}$. Therefore the appropriate way to describe the relation between the two sets is $\Delta_{n-1}^* \subseteq \Delta_{n-1}$, i.e. the set of all evaluated sentences either properly includes or is equal to the set of conceded or denied sentences.

and doubt.⁷⁶ The three alternatives are available only in the case of irrelevant sentences, because relevant ones can only be either conceded or denied, according to whether they (or their negations) belong to $Rel_{\Delta_{n-1}^*}$. As for irrelevant sentences, moreover, we still need to include in our model set K_c , to guarantee the required independent informational basis for their evaluation.

Let us start, once again, with the rule governing the admission of the *positum*. As we have already seen, its eligibility depends on the fact that it is not a contradiction or an impossible *simpliciter* sentence, and in case of disputations with multiple *posita*, on its being compatible with the previous steps. That amounts to the following conditions:

$$OA(\alpha_n) \quad \text{iff} \qquad \begin{cases} \alpha_n = \alpha_0 \text{ and } \alpha_n \not\vdash \bot \\ \text{or} \\ \alpha_n \neq \alpha_0, \ \alpha_n \not\vdash \bot \text{ and } \neg \alpha_n \notin Rel_{\Delta_{n-1}^*} \end{cases}$$

 $O \neg A(\alpha_n)$ otherwise

As to the sentences that are proposed at subsequent steps, the corresponding rules for concession, denial and doubt can be formulated as follows:

 $^{^{76}}$ I have decided to adopt for the formulation of rules a different notation based on the operators A, C, N, D (standing for admission, concession, denial, and doubt) and the obligational operator O expressing the duty of responding in the way explicited by the operator that falls within its scope. The negation of O is understood as the duty not to perform a given speech act, rather than as the permission not to perform it, cf. infra, Part II, sec. 2.1. The resulting picture is substantially equivalent to the use of the function R introduced above which mapped sentences onto the values $\{1, 0, ?\}$. I think this approach lends itself to express more clearly the sharp distinction between the normative status of rules, that are such precisely insofar they formulate duties, and, on the other hand, the descriptive character of the record of responses that are actually given in the course of a disputation. This development falls beyond the scope of the present work, but I believe it may be useful to have a sightly more fine-grained language at our disposal in the attempt to reconstruct the structure of many sophisms discussed in obligational treatises, since very often they involve a complex interaction between the two levels which we should be able to express in our formalizations. To make it explicit: the use of expressions like $OC(\alpha)$ and $C(\alpha)$ enables us to render more clearly the important difference between sentences like ' α est concedenda' and ' α conceditur' or ' α est concessa'.

$$OC(\alpha_n) \quad \text{iff} \qquad \begin{cases} \alpha_n \in Rel_{\Delta_{n-1}^*} \\ \text{or} \\ \alpha_n \in Irr_{\Delta_{n-1}^*} \text{ and } K_c \Vdash \alpha_n \end{cases}$$
$$ON(\alpha_n) \quad \text{iff} \qquad \begin{cases} \neg \alpha_n \in Rel_{\Delta_{n-1}^*} \\ \text{or} \\ \alpha_n \in Irr_{\Delta_{n-1}^*} \text{ and } K_c \Vdash \neg \alpha_n \end{cases}$$
$$OD(\alpha_n) \quad \text{iff} \qquad \alpha_n \in Irr_{\Delta_{n-1}^*}, K_c \nvDash \alpha_n \text{ and } K_c \nvDash \neg \alpha_n \end{cases}$$

This formulation, save for notational variants, is substantially equivalent to the one presented in the previous section. It is fitting in all situations in which we discard doubted sentences and do not keep track of their presence. In other words, it is as though we had been considering, so far, only the 'portion' of a disputation in which conceded and denied sentences come into play to determine the correct responses to *proposita* put forward at subsequent steps. There are, to be sure, good reasons to do that, since the logical structure we have described has to do with mechanical aspects of the business of conceding, denying or doubting, to the effect that the algorithm unconditionally provides fully determined replies: at each step there always exists a correct answer and it is unique.⁷⁷

There is, however, something lacking in this picture, since we completely lose track of doubted sentences. Yet, their presence can in fact influence the correct replies to sentences that are proposed at subsequent steps in a disputation. On the one hand, there are negative duties, as it were, which determine only what replies the respondent *should not* give to certain *proposita*. Although such duties have a weaker character, if compared to the general structure outlined so far, they should be taken into account. In particular, if a sentence α_i has been doubted, then nothing that is formally entailed by α_i should be denied at any subsequent step.⁷⁸ And, on the other hand, there are also positive duties (i.e. determining *the* correct reply), such as the duty of doubting the contradictory of any sentence

⁷⁷ A very interesting issue is how to interpret the rules for irrelevant sentences, namely as deterministic or non-deterministic, cf. [Dutilh Novaes, 2007, p. 157-158] I am inclined to favour the first interpretation, at least in Peter's case, because in a few passages of his treatise he seems to argue in explicit terms that if the respondent grants an irrelevant falsehood or denies an irrelevant truth, he replies incorrectly (which would make him lose the disputation) cf. *infra*, Part II, sec. 3.28 and sec. 5.19.

⁷⁸ The reason becomes immediately apparent if one thinks that the denial of the antecedent α_i of a valid consequence follows by contraposition as long as we deny the consequent. If the antecedent is doubted, we can either concede or doubt the consequent, but we cannot deny it.

that has been already doubted at a previous step. I will now try to propose a way to account for these aspects and include them in our model. The solution I have tried to figure out is very simple and should count just as a provisional point of departure for further developments. Let us introduce an additional ordered set Θ whose members are sets of sentences, with the usual requirement that $\Theta_i \supseteq \Theta_{i-1}$. Θ is devised to supplement the information supplied by $\Delta *$, whenever doubtful sentences come into play.

Since we are discussing *positio*, the initial *obligatum*, say α_0 , provided that it fulfills the appropriate consistency requirements, is always admitted and recorded in Δ_0^* . Therefore, we can assume that the following relation holds by definition:

$$\Theta_0 = \emptyset$$

As far as the criteria of formation of Θ_n are concerned, they are exactly the mirror image of those governing the formation of Δ_n^* (or Dutilh Novaes's Γ_n):

if
$$C(\alpha_n)$$
 then $\Theta_n = \Theta_{n-1}$
if $N(\alpha_n)$ then $\Theta_n = \Theta_{n-1}$
if $D(\alpha_n)$ then $\Theta_n = \Theta_{n-1} \cup \{\alpha_n\}$

The idea is to have Θ perform the same function as Δ^* but in the case of doubtful sentences. In this way the picture becomes complete, because we have found a way to save from oblivion an entire class of sentences that do indeed have a role in the obligational framework. Take for instance a disputation consisting of six steps (i.e. *positum* plus five *proposita*). Let the *positum* α_0 be admitted, the first *propositum* α_1 be denied, the third and fourth *proposita* α_3 and α_4 be conceded and the second and fifth *proposita* α_2 and α_5 be doubted. The record of the disputation will thus take the following shape:

$$\begin{array}{c|c} \underline{\Delta^*} & \Theta \\ \hline \alpha_0 & - \\ \neg \alpha_1 & - \\ - & \alpha_2 \\ \alpha_3 & - \\ \alpha_4 & - \\ - & \alpha_5 \end{array}$$

It can be easily seen that this construction (a) allows for a complete record of all evaluated sentences (conceded, denied as well as doubted) and (b) preserves the ordering. The final stage of this extension consists in putting together the two sets and obtain a 'label' for the whole outcome of the disputation. Let us introduce an additional set Δ , which is again an ordered set whose elements are sets of sentences such that $\Delta_i \supseteq \Delta_{i-1}$. This set is simply defined as follows:

$$\Delta = \Delta^* \cup \Theta$$

By such definition, Δ contains all evaluated sentences of a disputation:⁷⁹ those that have been conceded or denied are recorded in Δ^* , whereas doubted sentences feature in Θ . That permits us, on the one hand, to be conservative with respect to the previous formulation (since the 'starred' portion of the disputation behaves exactly in the same way as has been described in sec. 1.3) but it also enables us to expand the original model. We can now characterize the additional duties that are relative to the set of doubted sentences, and this allows us to capture some moves of the game that in the model we started with were not considered.⁸⁰

(1) We need to understand what the appropriate semantics of doubted sentences is. Much depends on whether we consider them as sentences that are evaluated once and for all in a disputation, or as sentences that, after being doubted, can receive a different evaluation at a later step of the disputation. In the former case, doubting a sentence is likely to turn out to be equivalent to the ascription of a third, independent, truth value, whereas in the latter case, the resulting picture might be more similar to a procedure of revision (there seem to be more evidence in support of the first claim, cf. the obligational rules from Peter's treatise on consequences presented below). Be this as it may, there are at least two explicit things that we should include in our representation since they are firmly held by Peter: (i) a sentence that has been doubted is such that whenever something that follows from it is proposed at a later step, that 'something' should not be denied (negative duty that leaves open the possibility of conceding it or doubting it) because otherwise the denial of the the doubted sentence would immediately follow by contraposition; (ii) the contradictory of a doubted sentence should always be doubted too.

(2) The idea that I propose here (i.e. the addition of a set Θ to record doubted sentences) is a first step which lends itself to the characterization of additional rules involving *only* doubted sentences. Yet, to make sure that the model works, it is indispensable to verify what may result from combining doubted sentences with conceded or denied sentences, i.e. what happens if we pick out elements of Θ and Δ^* , combine them and put them forward as *proposita* in a disputation. Special care and attention is required also for the following situations: both a doubted sentence and its contradictory can be in Θ , but what happens if their disjunction or conjunction is in Θ as well? For instance, does the rule that prohibits the denial of the consequent of a valid consequence whose antecedent has been doubted hold even in case the antecedent is a contradiction?

Peter gives us a hint, I reckon, in his treatise (cf. *infra*, Part II, sec. 4.6, ORObj.6), where he seems to accept the idea of a valid consequence with a doubted contradiction as its antecedent. This situation might look weird, because (i) anything whatsoever formally follows from such an

⁷⁹ Notice that, in a disputation of n steps, for all steps i (with $0 \le i \le n-1$) it holds that Δ_i^* and Θ_i (1) have empty intersection and (2) their union coincides with the sequence of evaluated steps up to i, i.e. with set Δ_i .

⁸⁰ As I have already stressed, this extension is just an embryonic proposal. There are two important things that must be figured out to make it work at a general level and I am not sure about both points at the moment.

By applying the same the notion of a set of logical consequences of a given sentence introduced above, we can for example formulate such negative requirements as the duty not to deny anything that is proposed in a disputation if it follows from a sentence that has already been doubted in the same disputation. This will be expressed by the following weak rule:

$$O \neg N(\alpha_n)$$
 if $\alpha_n \in Cn(\alpha_i)$ and $\alpha_i \in \Theta_{n-1}^{81}$

Again, by allowing for a record of doubted sentences in our model, there is a straightforward way to express the duty of doubting the proposed contradictory of a sentence that has been doubted at a previous step:

$$OD(\alpha_n)$$
 if $\alpha_i \in \Theta_{n-1}$ and $\alpha_n = \neg \alpha_i$

antecedent, and (ii) whatever we plug into the consequence as its consequent, it should never be denied, even if it is an impossible sentence. Peter, however, proves to be aware of this fact and clearly seems to be willing to accept it. In any case, until these kind of restrictions have been fully spelled out on the basis of a closer inspection of the texts, it will be difficult to make entirely safe statements about the features that the underlying logic of doubtful sentences should have.

⁸¹ One might wonder whether it is possible to generalize this notion to the conjunction of doubted sentences, as in the case of conceded or denied sentences, by assuming that the set of logical consequences of the sequence of doubted steps is, for all $\alpha_i \in \Theta_{n-1}$:

$$Cn\left(\bigwedge_{j=0}^{n-1} (\alpha_j)\right)$$

This set would include everything that follows from the sequence of doubted sentences. Nevertheless, as I have just pointed out, there is a serious theoretical question about the constraints that should be put on specific cases (like for instance when contradictions are explicitly recorded in Θ).

In addition to that, one thing is worth noting. The negative condition that prohibits the denial of anything that follows from a doubted sentence might be supplemented by an additional rule requiring that anything from which a doubted sentence follows ought not to be conceded:

$$O \neg C(\alpha_n)$$
 if $\alpha_j \in \Theta_{n-1}$ and $\alpha_j \in Cn(\alpha_n)$

Notice that, as a result of this additional requirement, the class of doubted sentences would turn out to be characterized in terms of a third independent truth value, which is ascribed once and for all in the context of a disputation exactly in the same way as truth and falsehood are ascribed once and for all. The reason is the combined effect of the duty not to deny anything that follows from a doubted sentence and of the duty not to concede anything from which a doubted sentence follows: in this framework, when a sentence has been doubted then no matter what is put forward next, its evaluation will remain the same. The former duty prevents any negative revision (by contraposition) while the latter prevents any positive revision (by *modus ponens*) of any sentence that has been doubted.

The use of both rules is explicit in Peter's treatise on obligations⁸² and their relevance is also confirmed by the treatment found in his treatise on consequences. I will give below a sketch of the obligational rules found in the latter, since they will come into play quite often in the analysis of Peter's text on obligations.⁸³

On the basis of our definitions of the obligational duties, it follows immediately that the three rules below hold in the system as a minimal criterion of consistency:

$$\frac{OC p}{O \neg Np \land O \neg Dp} \\
\frac{ON p}{O \neg Cp \land O \neg Dp} \\
\frac{OD p}{O \neg Cp \land O \neg Np}$$

There can be no conflicting duties, like for instance the duty to concede and deny, or to doubt and deny the same sentence. For any sentence that 'ought-tobe-X', no matter whatever X stands for, it rules out the other alternatives.

The following rule and its corollary are simple applications of obligational concepts to the standard rule of *modus ponens*:

$$\frac{(p \Rightarrow q) \land OC p}{OC q}_{84}$$

⁸³ In this respect, cf. also [Boh, 1990], [Boh, 1993] and [Boh, 2000] which all stress the presence of epistemic concerns and draw an interesting parallel with Strode.

⁸⁴ This principle is the object of some discussion in Peter's treatise on consequences. Peter contends that it is not sufficient that the antecedent of a valid consequence ought to be granted in order to claim that its consequent ought to be granted too. This is true only if we adopt the general principle that from what ought to be granted nothing but what ought to be granted follows. The complication has to do with epistemic concerns. Cf. *Tractatus de consequentiis*, [Peter of Mantua, 1492a, sig. Eiiii+3^{va-vb}] "Alia regula est quod si consequentia est bona, affirmativa, denominata etc., significans etc., habens antecedens et consequens expressa, quorum nullum est multiplex, scita esse formalis, et antecedens est concedendum ab aliquo, et consequens est intellectum propositum, *et cum his bene scitur quod ex concedendo non sequitur nisi concedendum*, tunc consequens ab eodem est concedendum. Sed non sequitur 'hec consequentia est bona [scita] scita esse talis et antecedens est ab aliquo concedendum et consequens est intellectum, igitur consequens est concedendum', quia forte creditum est quod antecedens sit falsum aut forte creditum est quod ex concedendo sequatur non concedendum". [emphasis mine]

 $^{^{82}}$ Cf. *infra*, Part II, sec. 3.1 and 4.4 for the prohibition to deny the consequent of a consequence whose antecedent ought to be doubted; and sec. 3.5, 3.9 and 4.6 for the rule involving contradictories.

$$\frac{(p \Rightarrow q) \land OC p}{O \neg D p \land O \neg N p)}^{85}$$

The same extends also to the case of contraposition:

$$\frac{(p \Rightarrow q) \land ON q}{ON p}_{86}$$
$$\frac{(p \Rightarrow q) \land ON q}{O\neg C p \land O\neg D p}_{87}$$

Finally, three rules are supposed to govern doubtful responses:

$$\frac{(p \Rightarrow q) \land OD p}{O \neg Nq}_{88}$$
$$\frac{(p \Rightarrow q) \land OD q}{ON p \lor OD p}_{89}$$

⁸⁵ As Peter rightly points out, this rule follows from the former, cf. *Tractatus de consequentiis*, [Peter of Mantua, 1492a, sig. Eiiii+3^{vb}] "Ex qua regula sequitur alia, scilicet quod si sit aliqua consequentia bona, denominata etc., significans etc., habens antecedens etc., quorum nullum est multiplex, scita esse formalis, et antecedens est concedendum, et consequens $\langle est \rangle$ intellectum propositum, et cum his bene scitum est quod ex concedendo non sequitur nisi concedendum et quod ex vero non sequitur falsum, consequens non est dubitandum nec negandum".

⁸⁶ Cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii $+3^{vb}$] "Alia regula est quod si consequentia est bona, denominata etc., significans etc., habens antecedens etc., quorum nullum etc., scita esse formalis, et consequens est negandum ab aliquo, et antecedens $\langle est \rangle$ intellectum propositum, et cum his bene scitum est quod negandum non sequitur nisi ex negando, tunc est antecedens ab eodem negandum".

⁸⁷ Cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii $+3^{vb}$] "Ex qua sequitur alia, quod si consequentia est bona. affirmativa, denominata etc., significans etc., habens antecedens etc., quorum nullum etc., scita esse formalis, et consequens est intellectum propositum, et cum his bene scitur quod ex vero non sequitur falsum, et est consequens negandum, tunc antecedens non est ab eodem dubitandum nec concedendum. Et capitur hic dubitandum pro eo cui debet dubie responderi ab aliquo".

⁸⁸ Cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii+ 3^{vb}] "Alia regula est quod, si consequentia (est) bona etc., significans etc., habens etc., quorum nullum etc., scita esse formalis, et antecedens est dubitandum, et consequens (est) intellectum propositum, et cum his scitum est quod ex nullo vero sequitur falsum, tunc consequens non est negandum".

⁸⁹ Cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii $+3^{vb}$] "Alia regula est quod, si consequentia est bona etc., significans etc., habens etc., quorum nullum etc., scita esse formalis, et consequens est dubitandum, et antecedens est intellectum propositum, et cum his bene scitum est quod ex nullo vero sequitur falsum, tunc antecedens est negandum vel dubitandum". The presence of this rule speaks against positive revision in the context of obligational disputations, because we are not permitted to concede whatever entails a doubtful sentence.

$$\frac{OD p}{OD \neg p}_{90}$$

The application of these rules is ubiquitous in Peter's treatise, but we will come to this in due course. Now, to conclude this cursory analysis of the logical structure of obligational disputations, let us briefly see how these notions turn out to be applied in an oversimplified example.

1.5. An example of disputation

I shall illustrate in this section how the above reconstruction is supposed to work. Consider the following disputation: the opponent posits a sentence α_0 = 'every man is running' while the respondent is sitting. This apparently pointless remark about the respondent's being seated plays, in fact, an important role which is quite analogous to the statement of a *casus* in the sophismata literature, where the amount of information relevant for the discussion is passed on to the participants in a preliminary phase. This piece of information, in particular, will become from now on part of K_c . The sentence 'every man is running' will play the role of *positum* during the disputation. Since this sentence is not incompatible with any previous step (actually, there is none in this disputation since α_0 is the first and only *positum*) nor is it inconsistent in itself, the respondent must admit it according to the first rule given in Peter's treatise.

Once the positum α_0 has been admitted, the opponent proposes the sentence $\alpha_1 = \text{'you}$ are running'. The respondent must first determine the status of this sentence in terms of logical dependence or independence with respect to the previous steps of the disputation. At this stage, he should take set Rel_{Δ_0} and check whether α_1 or its negation $\neg \alpha_1$ belong to it.

This happens not to be the case since, whatever a precise account of the notion of 'following from' may turn out to be like, it will certainly have the property to invalidate both the inference 'every man is running, therefore you are running' and the inference 'every man is running, therefore you are not running'. In other words, $\alpha_1 \notin Rel_{\Delta_0}$ and $\neg \alpha_1 \notin Rel_{\Delta_0}$. The respondent must therefore classify α_1 as irrelevant. The rule relative to irrelevant sentences forces him to answer according to K_c . We have assumed, in the preamble, that the respondent was

Suppose that the consequent of a valid consequence has been doubted at some point and that later on the antecedent of that consequence is put forward. If we were allowed to concede it, then by *modus ponens* we would immediately infer that the previously doubted sentence (= the consequent) ought to be conceded too.

⁹⁰ For this last rule, which is not discussed in the treatise on consequences, cf. *infra*, Part II, sec. sec.3.5.

sitting, therefore K_c tells him that the *propositum* 'you are running' should be denied. The set that represents the cumulative outcome of the disputation thus becomes: $\Delta_1 = \Delta_1^* = \{ \alpha_0, \neg \alpha_1 \}.^{91}$

The opponent will then propose a second *propositum*, namely $\alpha_2 = \text{'you}$ are a man'. Now, what is the logical status of this sentence? Is it a relevant or an irrelevant one? If the respondent wants to answer correctly, again, he will have first to identify what kind of relation α_2 bears to the previous steps: the choice of the rule by which he must reply will be determined accordingly. In the present case, sentence $\alpha_2 = \text{'you}$ are a man' is a relevant one. That is because its negation follows from the conjunction of the *positum* and the negation of the first *propositum*. Let us see what that means in terms of the cumulative sets of logical consequences defined above. The disputation at this stage contains two sentences: $\alpha_0 = \text{'every}$ man is running', and $\neg \alpha_1 = \text{'you}$ are not running'. The first *propositum* had to be denied because it was irrelevant and false (which was equivalent, in turn, to the concession of its negation). At step 2, the respondent is expected to evaluate sentence α_2 , which means by the above stipulations that he needs to run through set $Rel_{\Delta_1^*}$ and see whether α_2 or $\neg \alpha_2$ belong to it. The algorithm requires him to assess relevance with respect to the following set:

$$Rel_{\Delta_1^*} = Cn\left(\bigwedge_{i=0}^1 (\alpha_i)\right) = Cn\left(\alpha_0 \land \neg \alpha_1\right)$$

Now, since α_0 = 'every man is running' and $\neg \alpha_1$ = 'you are not running', it is easy to see that $\neg \alpha_2$ (= 'you are not a man') follows by contraposition. On the basis of the above definitions, α_2 is incompatibly relevant at step 2, because its negation $\neg \alpha_2$ belongs to the set $Rel_{\Delta_1^*}$ of logical consequences generated by steps 0 and 1. We can display the structure of this simple disputation as follows:⁹²

Р	Every man is running	А	possible
1.	You are running	Ν	irrelevant and false
2.	You are not a man	Ν	incompatibly relevant

So much for a first assessment of the underlying logical structure of an obligational disputation conducted according to the *responsio antiqua* rules. A cursory

⁹¹ Recall that no sentence has been doubted so far, therefore $\Theta_1 = \emptyset$, i.e. the set of doubted sentences at this stage is still empty, to the effect that $\Delta_1 = \Delta_1^* \cup \Theta_1 = \Delta_1^*$.

⁹² From left to right: the first column labels each single step starting from the *positum* (=*P*); the second column is the list of sentences; the third one is the record of the respondent's reply; the fourth one provides the identification of the status of each sentence and the explanation of the respondent's replies (sometimes, in the analysis of complex sophisms, it also displays objections, remarks or the symbol \perp when at a given step an inconsistency arises.

look at any treatise on obligations immediately shows, however, that things almost never turn out to be so simple and the complications that may arise because of various factors (e.g. the complexity of proposed sentences, an intentionally ambiguous use of language on the opponent's part, the presence of pragmatic paradoxes, or the application of other problematic principles involving the use of obligational operators) put the respondent under considerable stress. The analysis of sophisms discussed in Peter's treatise will give a measure of this difficulty.

1.6. Survey of modern interpretations

As has been already recalled in the Introduction, there have been many attempts in the secondary literature on obligations to put forward interpretations with the aim of explaining various aspects of the genre, and the historiography on this tradition constitutes an interesting phenomenon in its own right. In this section, however, I will not be concerned with a general assessment of such developments but rather try to outline a pocket map for orientation. I shall briefly recall and describe a number of modern interpretations and point to their distinctive features, because I think that several extremely interesting and valuable insights are available and may contribute to the reconstruction of a comprehensive picture.

The greatest controversy is centred on the problem of determining the purpose of obligations. In this respect, two main families of interpretations can be identified: the first focuses on the practical and pedagogical function of the *ars obligatoria*, while the second embraces an articulated series of attempts to find in it some genuinely philosophical intent and to investigate its fundamental logical properties. The distinction of course has a merely heuristic value and is by no means intended to draw sharp boundaries. There are often borderline cases of exegetical proposals in which different motives add up in a composite picture.

The history of historiography on obligations can be roughly split into two periods, the dividing point being a controversial paper by Spade appeared in 1982.⁹³ In that paper, Spade proposed the idea that obligations might be seen as an example of counterfactual reasoning. This idea has been challenged, ever since, in

⁹³ Cf. [Spade, 1982b]. This paper provides a first systematic analysis of the three fundamental versions of the theory that are available in the first half of the XIV century (Burley, Kilvington and Swyneshed). Strictly speaking, Spade had proposed his reading in terms of counterfactuals already in [Spade, 1977], in the introduction to his edition of Swyneshed's tract. Nonetheless it is in 1982 that the proposal is grounded and develpoed in detail, and it is that paper that is mostly referred to in subsequent literature as the target of an impressive battery of objections. An interesting fact is that Spade himself, before working out the counterfactual interpretation, had suggested, in the attempt to identify one of the sources of Heytesbury's treatment of *insolubilia*, that the vocabulary and techniques developed in obligations treatises quite naturally qualify as a useful tool for that context too, cf. [Spade, 1976].

various ways and can be regarded as a kind of watershed in the literature: it has set the stage for further conceptual efforts to understand the philosophical import of obligations (fostering, in this sense, an interest in the topic that goes beyond purely historical motives) and most interpretations that have been put forward thereafter have felt the need to discuss Spade's views. We will turn to it in a moment.

The first explicit mention of obligations in a study on medieval logic is, to my knowledge, Boehner's claim that obligations are a rudimentary prefiguration of a theory of logical deduction.⁹⁴ This hypothesis has not received much appreciation and is usually dismissed on the basis of two main objections commonly accepted in the literature: first, a theory of logical deduction or inference, as it were, is presupposed rather than developed in treatises on obligations (the appropriate context being discussion of consequences)⁹⁵; second, this interpretation does not account in any plausible way for the need to introduce irrelevant sentences.⁹⁶

In 1960s the first editions of texts see the light⁹⁷ along with a prevailing sentiment that obligations fulfill primarily the pedagogical function of an exercise or a test.⁹⁸ This view seems to have strong historical plausibility for at least two rea-

⁹⁶ Curiously, if it is true that the objection holds if we regard obligations as an attempt to build an axiomatic method for, say, an ancestor of propositional logic, it is less evident that Boehner's suggestion is entirely misplaced if we consider the theory as providing an axiomatic basis governing the assertability of sentences in disputational contexts. In that sense obligational rules might be viewed as axioms governing the behaviour of certain speech acts, i.e. conceding, denying and doubting.

⁹⁷ Cf. [Paul of Pergula, 1961] and [Walter Burley, 1963].

 98 Cf. [Brown, 1966, p. 27] "Of all the mediaeval treatises on logic, none demanded more application on the part of the student than the treatises on obligations [...] each treatise had the same general purpose: to train the student to argue correctly within a stringently regulated framework of argumentation"; [Green, 1963, p. vii] "In general terms, *De Obligationibus* is a medieval logical treatise designed to initiate students into the practice of logical reasoning, especially with a view to taking part in disputations"; [Weisheipl, 1966, pp. 163-164] "This treatise of logic was designed to give the beginner basic rules of inference in purely logical exercises [..] These were strictly ingenious exercises among school boys and had little objective value". This view is also accepted by D'Ors, cf. for instance [D'Ors and García-Clavel, 1994, p. 54] "En razón de los fines didácticos de estos tratados, éstos se completan de ordinario con ejemplos – ejercicios resueltos –, mediante los que se llama la atención del lector, futuro *respondens*, sobre las dificultades más características con las que se puede tropezar en la práctica

 $^{^{94}}$ Cf. [Boehner, 1952, p. 14] "It would, however, be incorrect to see in these tracts only a collection of rules for school exercises, since they contain a nucleus of rules for an axiomatic method, though in a rather crude form".

⁹⁵ In this respect an interesting remark is found in [Paul of Venice, 1988, p. 32] "omnes regulae superius adsignatae in *Tractatu Consequentiarum* de consequentia bona vel non bona sunt hic fundamentaliter sustinendae. Et ratio quia materia obligationum non est nisi materia consequentiarum stilo subtiliori procedens, et an respondens sit sani capitis gressu deceptorio temptativa; nam per huiusmodi casus, sive veri sint sive falsi, stabilem sustentationem docetur infallibiliter et invariabiliter sustinere".

sons. The first is that obligations codify in a highly stylized form the ubiquitous disputational character of many medieval intellectual and academic activities.⁹⁹ Secondly, there are very few remarks in such treatises that tell us something about their purpose, and almost all insist on the idea of training or testing logical skills.¹⁰⁰

The main objection that is usually raised against this interpretation is that it might fail to be exhaustive. In particular, the function of obligations as a tool for teaching or examining logical abilities would not explain, on the one hand, the efforts of medieval logicians to develop doctrines that often reach a level of remarkable complexity and, on the other hand, the controversy arising in the XIV century about the right set of rules to be adopted. The latter, especially, would appear "entirely unmotivated if one thinks of obligational disputations merely as exercises or examinations and [...] suggests instead that the focus of this literature was on theoretical matters, not exclusively on pedagogical ones".¹⁰¹

A first philosophical interpretation, centred on the idea of obligations as a *game*, has been put forward, in the context of independent discussions, by Angelelli and Hamblin at the beginning of the 1970s.¹⁰² The former has insisted on the game-like character of obligations as a disputational technique. The latter, in particular, has suggested a link with the treatment of fallacies, arguing that the regimented

¹⁰¹ Cf. [Spade, 1982b, p. 2] Spade's claim is well grounded, and I agree that some theoretical interest certainly comes into play (though it would probably be advisable to investigate the emergence of theoretical interests author by author). Yet, I suspect the argument is not necessarily conclusive. Take for instance the case of *positio*: different sets of rules might well have been devised to test different logical abilities. In particular, in case Swyneshed's, the focus would be on the ability to recognize what follows from one single hypothesis and keep it distinct from the rest (i.e. inferences involving irrelevant sentences); in the case of the *responsio antiqua* instead, the focus would be on the ability to manage more complex interactions between an initial hypothesis and subsequently proposed sentences. This would also explain the role of irrelevant sentences. In Swyneshed's framework they would count as 'disturbing factors', put forward to generate confusion and divert the respondent's attention from the safe domain of the logical consequences of the *positum*, whereas on the *responsio antiqua* account, they would be regarded as auxiliary assumptions that make the identification of the logical status of proposed sentences increasingly difficult as the disputation goes along.

¹⁰² Cf. [Angelelli, 1970, pp. 800–806 and 813–815] and [Hamblin, 1970, pp. 260–263].

de tales diálogos". The idea of an even stronger form of entrenchment in practical contexts has been put forward by Perreiah who on several occasions has insisted that obligations where devised for oral examinations in logic, cf. especially [Perreiah, 1978, pp. 152-156], [Perreiah, 1982, p. 113–116], and [Perreiah, 1984].

⁹⁹ In this respect, cf. [Green, 1963, pp. 1–15] and bibliography therein.

 $^{^{100}}$ Cf. the passage quoted above from Paul of Venice, which refers to the art as 'temptativa', i.e. directed to test the logical skills of the respondent. A similar sentiment is found in Peter too, cf. *infra*, Part II, sec. 4.1 and 5.10, although his terms are slightly less explicit. Another relevant source in this respect is [Ralph Strode, 1517, fol. 78^{ra}]. According to him, the purpose of obligations is "both to provide exercise for beginning students in handling logical inferences; and to prepare them to reason from truths in real-life situations" [Ashworth, 1993, p. 362].

context of obligations could serve as an artificial environment for the assessment of invalid reasoning.

In more recent years, the interpretation of obligations as a *game* (of consistency maintenance) has been revisited with originality, in a strongly revised form which makes it much more elaborate and systematic, by Dutilh Novaes, especially in connection with the *responsio antiqua* account.

Appealing to the notion of a *game* has undoubtedly proved to be an interesting way to provide a philosophical characterization of obligations: in particular it singles out a distinctive feature, namely the fact that they can be viewed as regimented activities governed by rules that can be studied as *structures* with certain logical properties (regardless of actual applications). There are, however, some issues that remain open. First, if we understand the game metaphor in connection with game-theoretical semantics and, especially, dialogical logic,¹⁰³ the link with obligations seems problematic, because there seem to be quite different purposes at stake.¹⁰⁴ In the case of obligations, their formalization might well lend itself to a characterization in terms of game properties (like for instance the use of the notion of a winning strategy to describe the guarantee that, whenever the respondent complies with the rules, the resulting sets of sentences will always be consistent), but this being said, there still seems to be as little concern over foundational issues as there could be.

Second, a difficulty that the game interpretation is required to solve is the question of the degree of freedom with which the players make their moves.¹⁰⁵ In the context of obligations it seems that the only genuinely free actor is the opponent. In principle, the only thing the respondent is free to do is choosing whether or not to apply the rules. Yet, for the respondent there is always a winning strategy because the rules (at least in the *responsio antiqua*) are constructed in order to enable him to maintain consistency. This results in an odd conclusion: the one who seems to have the greatest freedom, i.e. the opponent, is the one who unconditionally loses the game, at least in an ideal world. On these grounds, one might wonder whether the sort of thing we are talking about here can still be

¹⁰³ Cf. [Dutilh Novaes, 2007, p. 145]. Relevant references, in this connection, are [Lorenzen and Lorenz, 1978], [Hintikka and Sandu, 1997], and [Rahman and Tulenheimo, 2006].

¹⁰⁴ This fact was already recognized by Angelelli – who by the way is the first to think of obligations in terms of games. I will quote his passage in full because it clearly expresses the fundamentally different nature of the two enterprises, cf. [Angelelli, 1970, p. 813] "in Lorenzen's approach, the dialogue serves to define the meaning of the connectives; the logical constants are introduced by attack and defense rules. [...] it seems safe to assert that there are no signs of using disputation as source of meaning for the logical particles. The *ars disputatoria* [...] is very important [...] but as an application or extension rather than as a foundation of logic".

¹⁰⁵ This issue is discussed at some length in [Yrjönsuuri, 1994]. The view that obligations can be understood in terms of games has been also strongly opposed by [Perreiah, 1982, pp.115–116]. Finally, some additional and more logically-oriented reservations are made by [Hodges, 2009].

called a game.¹⁰⁶ Be this as it may, this interpretation is at present presumably the most interesting one in terms of philosophical analysis and it will likely set the stage for further developments.

Let us now turn back for a moment to the historical reconstruction of the evolution of studies. In the 1970s several progresses are made also from the standpoint of textual knowledge. The first critical editions of texts referable to the early phase (such as the *Tractatus Emmeranus de falsi positione*, the *Tractatus Emmeranus de impossibili positione*, the *Obligationes Parisienses* and the *De petitionibus contrariorum*, which are all connected to the development of the literature in the XIII century) come out, along with much later productions, like Swyneshed's treatise, followed a year later by the edition of Lavenham's short tract. At the end of the decade, a first significant bulk of materials has become available and that is probably one of the reasons behind the great flourishing of studies in the 1980s. In this period, besides the edition of other fundamental texts,¹⁰⁷ great efforts start to be made with the aim of clarifying more systematically the conceptual framework of the theory in the central phase of its development (with special attention to Burley's and Swyneshed's doctrines which are the object of numerous contributions). In 1981 a fundamental article by Ashworth comes out in *Medioevo*, providing a first

In the case of obligations the situation is not dissimilar. Obligations are a determined game, because there is always a winning strategy for the respondent (i.e. there is always a way to maintain consistency) and they are also interesting to play since it often turns out that the winning strategy is difficult to figure out in factual situations (because sentences are complex or ambiguous, because as long as the disputation proceeds the recognition of inferential relations might gets more and more difficult, and the like).

¹⁰⁷ I will just mention here the treatises of the so-called anonymous Merton author and Paul of Venice, cf. [Kretzmann and Stump, 1985] and [Ashworth, 1988a], respectively. The first provides some original insights and is relevant for the developments of the theory in the second quarter of the XIV century in England. The second is of greatest importance for various reasons: it is a lengthy text characterized, on the whole, by a clear style of exposition (as opposed, for instance, to the somewhat elliptical language of Peter's treatise); it contains a considerable amount of sophisms that clarify how the rules should be applied and (sometimes) interpreted; finally it draws heavily on the tradition, thereby providing the basis for a reconstruction of a network of influences in the late phase of the history of obligations, cf. in this respect [Ashworth, 1990].

 $^{^{106}}$ The question is not whether the game is worth playing, nor does the fact that it is a determined game play any significant role. Chess might be a determined game (to the best of my knowledge, there is no proof at present of this being or not being the case). Now, suppose it actually *is* a determined game, i.e. a game such that there is a winning strategy for either white or black. For this reason the game would not lose any appeal, because it would still remain difficult to play. The reason is that when a game is determined, it becomes scarcely interesting to play only when the player for whom a winning strategy is theoretically available, also knows what that strategy actually is like. In other words, one would hardly accept to play chess with black pieces if one knew that (1) chess is a determined game with winning strategy for white, and (2) that there is a feasible and easy way for white to compute it. Since the second condition does not generally obtain, at least when players with similar abilities are involved and especially at the beginning of a match, chess preserves all of its interest and fun.

systematic outline of the relationships between the two crucial notions of relevance and order in several late XIV century authors.¹⁰⁸ The list could go on for several pages, but discussing every entry of this catalogue is not my intention here: suffice it to say that the history of studies, in the last three decades, is characterized by an increasing complexity in terms of interpretations and from the beginning of the 1980s onwards, we are confronted with a multi-voice dialogue with probably more than an entry per year that would deserve a separate discussion.¹⁰⁹ As I have said above, an important watershed in this story is [Spade, 1982b]. Among the arguments that Spade provides in support of the thesis that obligational disputations are to be viewed as a model of counterfactual reasoning,¹¹⁰ some are worth recalling because they focus on relevant issues that might serve as a test also for other interpretations. Let us briefly see them one by one.

(1) According to some passages found in the early literature, what is conceded or denied in an obligational disputation is said to be 'following' from the *positum*, in the sense that it expresses what would be the case if the *positum* were to be the case. Since this kind of connection cannot be interpreted as logical implication, the best candidate are counterfactual conditionals (subjunctive conditionals). The sense in which one should understand the notion of 'following from' is 'being the consequent of a true or assertable counterfactual conditional with the *positum* as antecedent'.

(2) The counterfactual interpretation would account for the seemingly odd treatment of irrelevant sentences, explaining their presence and their role ("obligational disputations are meant to explore what whould happen if the *positum* were true but everything else stayed as much as possible the same" [Spade, 1982b, p. 12]). Irrelevant sentences, that are evaluated according to their own status, i.e. according to their truth value in the actual world, are supposed to fulfill this condition, namely that of picking out the closest possible world with respect to the actual world.

(3) It would justify the admission, in certain authors, of the *positio* of impossible *per accidens* sentences or of true sentences that are not known to be true.

¹⁰⁸ Cf. [Ashworth, 1981]. This is an example of what, in the heuristic perspective suggested above, would turn out to be a case of mixed exceptical approach, motivated by both historical interests and the need of a systematic conceptual analysis of the logical relations involved in the theory.

¹⁰⁹ In this respect, the essential source remains [Ashworth, 1994] which presents a catalogue of (a) medieval authors, (b) manuscripts, (c) early printed editions, (d) edited texts and (e) studies and translations. With regard to the past fifteen years, the last section can be supplemented with a bibliography compiled by Uckelman which has recently become available on the web, cf. [Uckelman, 2009].

 $^{^{110}}$ The claim – that is partially reconsidered in [Spade, 1992] – is even stronger: disputations are sequences of sentences which can be represented in the form of counterfactual (subjunctive) conditionals with the *positum* as antecedent.

(4) Obligational disputations would share with counterfactual conditionals the failure of some important logical properties such as strengthening the antecedent, transitivity and contraposition.

(5) The counterfactual interpretation would provide an account of the revision of the rules that the theory undergoes in the first half of the XIV century, especially in Kilvington and Swyneshed.

All these arguments have been challenged strenuously – and I think with good reasons – on a number of occasions.¹¹¹ Yet, no matter whether or not one agrees with Spade's proposal, the relevant fact is that he has undoubtedly isolated a checklist of some important parameters, that should be taken into account by any serious attempt to understand the features and development of the genre (particularly, of *positio*).

A strong reaction to Spade's interpretation has been proposed already in the early 1980s by Stump, who has argued that different philosophical concerns are found in Burley's and Swyneshed's theories. In particular, as far as the latter is concerned, a recognition of the lists of *obligata* that are found in his treatise would show a concern with "epistemic logic, indexicals, propositional attitudes, and other issues in the philosophy of language".¹¹² In Burley's case, on the other hand, the focus would be on certain kind of "logical or semantic paradoxes based on some sort of reflexivity which resemble but are generally weaker than the self-referential

It should be noted, finally, that Spade, in addition to that, has proposed a parallel interpretation based on the idea that obligations are a prefiguration of the modern academic practice of thesis defense. The view is discussed in [Spade, 1993] and has been strongly opposed by [King, 2004].

¹¹² Cf. [Stump, 1981, p. 172].

¹¹¹ Cf. for instance [Stump, 1981, pp. 170–171, especially p. 171 f. 45], [Martin, 1993, pp. 363–367], and more recently, [Dutilh Novaes, 2005, pp. 142–148]. One point, in particular, is crucial. Spade appeals to possible world semantics to claim that the sequence of sentences evaluated in an obligational disputation picks out the closest possible world to the actual world and can be interpreted as a true or assertable counterfactual conditional statement having the positum as its antecedent and the last evaluated propositum as its consequent. It has been shown, however, that such sequences can be taken at most as might-counterfactuals, but not as would-counterfactuals, cf. [Martin, 1993, p. 363]. This depends on a distinctive feature of the responsio antiqua, namely that replies to the proposed sentences (and consequently the outcome of disputations) are determined in function of the order in which those sentences are put forward. It turns out, thus, that conceded and denied sentences progressively specify a *class* of possible worlds that make their conjunction true, but the theory is unable to pick out a single world as the closest to the actual world. Moreover, in this respect, the idea that Swyneshed's revision is prompted by the inadequacy of Burley's account as a theory of counterfactuals proves to be hard to defend, since Swyneshed's own account doesn't work any better, for this purpose, than the one it would supposedly be intended to correct. By contrast, there is some consensus about the possibility of regarding Kilvington's remarks in the light of a counterfactual reading, cf. for instance [Stump, 1981, p. 169] "Kilvington's innovation in the rules for obligations does appear to make obligations resemble a logic of counterfactuals".

paradoxes of insolubles in that they depend on the disputational context for their paradoxical character".¹¹³ According to Stump, moreover, these aspects are better understood if they are considered in the framework of a close connection to the literature of *sophismata* and *insolubilia*.

Besides prompting much controversy, the counterfactual interpretation has also loosely inspired two accounts that have been developed in the early 1990s. The first is Yrjönsuuri's view that obligations are devised for the construction of thought experiments.¹¹⁴ The second is King's suggestion that they should be regarded as a general metamethodology of medieval scientific discourse that finds applications in various fields of medieval thought (especially physics and theology).¹¹⁵ Both interpretations are to some extent related to the counterfactual reading, for they all share a basic assumption: reasoning in a disputation conducted within the obligational framework means assuming a hypothesis to see what – in some sense – 'follows' from it. According to Yrjönsuuri and King, this should be interpreted in terms of the construction of consistent sets of sentences describing (for the most part) contrary to fact state of affairs. In this connection, obligational disputations would serve very naturally as an environment to perform reasoning *secundum imaginationem*.

The problem with the thought experiment solution lies, I think, in the kind of examples and sophisms that are actually found in obligations treatises. One would maybe expect more substantive theoretical issues to test the theory, *if* the theory were intended to provide a tool for the construction of thought experiments. Yet we are rather confronted with typical examples of semantic conflicts, pragmatic paradoxes and the like, i.e. with issues related to the relations between logic and language. If it is true, on the one hand, that most practices of philosophical reflection and scientific reasoning, in the late Middle Ages, take place at this level,¹¹⁶ it still remains hard to escape the impression that what is primarily at stake, in obligations treatises, is not to be understood in terms of the development of a tool to be used for the investigation of what follows under a given assumption. Obligational disputations might well lend themselves to such a use, and the presence of their vocabulary in contexts other than that of logic text-books, speaks for this

¹¹³ Cf. [Stump, 1981, pp. 172–173].

¹¹⁴ Cf. especially [Yrjönsuuri, 1994, p. 13] "As I see it, rejection of Spade's interpretation has been too wholesale. Spade's interpretation has been taken as the only way of applying modal logic to obligations. [...] Indeed, even if it seems to be clear now that the issue is not counterfactual conditionals, there seems to be no way around the fact that the typical positum of an obligational disputation is an assumed contingently false sentence. In one way or another, obligational disputations consider non-actual possibilities". Another relevant source for this discussion is [Yrjönsuuri, 1996].

¹¹⁵ Cf. [King, 1991, especially pp. 49-56].

¹¹⁶ Classical arguments in support of this view are found, for instance, in [Murdoch, 1975] and [Murdoch, 1981].

reading. But I think we should keep the two issues distinct from each other. The application of disputational techniques *outside* the context of obligations treatises is probably not to be regarded as the only (or the main) key to the understanding of what is going on in those treatises.¹¹⁷ Apart from the general issue whether the frequency of *application* of a theory in a given context that is extrinsically related to it, is sufficient reason to infer that the theory is intentionally construed for that purpose, the question would still remain open: if obligations are a metamethodology or a theory devised for the construction of imaginary cases, why then are the typical examples and sophisms, that we encounter in the texts that are primarily concerned with obligations (i.e. the treatises where the theory is elaborated and tested) not of this sort, for the most part? Obligations seem to be more closely related to the idea of constructing a logic that governs disputational exchanges and the assertability of sentences in that setting. In standard examples of disputations that are to be found all over the place in such treatises, when the time of the obligation is over, and it comes down to assessing the sequence of proposed and evaluated sentences, it is much more frequent to come accross situations in which the disputants discuss whether one or more sentences have been correctly granted or denied, rather than situations in which they check what sentences have been granted or denied and infer, on this basis, how the actual world or an imagined situation would be like.¹¹⁸ In other words, the prevailing focus is on the logical relations between sentences rather than on the identification of particular sets of sentences endowed with some special philosophical interest. To some extent, no matter what sentences are proposed, the important thing is the underlying logical problem they exemplify: this suggests also a way to look at collections of sophisms as representing certain classes of problems that advocate in turn certain classes

¹¹⁷ The vocabulary of obligations is not uncommon in theology. Interesting examples of this kind of application are found for instance in Robert Holkot, cf. [Knuuttila, 1993]. An outline of the "emergence of an obligational theology" is offered in [Gelber, 2004, pp. 151–190] where several figures like Arnold of Strelley, Holkot again, and John Duns Scotus are discussed at some length. The latter's use of obligations is also investigated by [Vos, 1998]. Cf. also [Martin, 1990] for an analysis of Bradwardine's use of *positio impossibilis*. The opposite tendency, namely the presence of theological examples in treatises on obligations, is by contrast extremely rare, cf. [Ashworth, 1992, p. 654]. And I have the impression that physical examples tend to be quite a find, too.

¹¹⁸ It must said, however, that at least one strong piece of evidence in support of Yrjönsuuri's view is to be found in the so-called anonymous Merton treatise. Besides the purposes of testing ("ut temptetur respondens numquid artem habeat") and exercise ("ut dispensatio habeatur, ut accidit inter conferentes ob exercitii causam"), a third relevant purpose is assigned to the art, namely "ut falsum possibile admittamus inquirendo quid sequitur, ut cum res ita de facto se habeant ut falsus casus denotat, sciamus per prius suppositum et rationem quid sumus acturi et responsuri. Iuristae enim et morales philosophi per possibiliter casus antequam in re accidat de observandis inquirentes veritatem exercitiis explorant", [Anonymous Merton College MS. 306, 1985, pp. 243-244].

of stock-in-trade solutions. There is a restricted number of recurrent archetypes of logical issues that medieval authors on obligations seem to be interested in and there is a corresponding restricted number of techniques devised to resolve them.

Another related problem for the thought experiment account, is represented, in my opinion, by the kind of sentences that are often involved in the examples. Consider the following distinction: let us call sentences such as 'the king is seated', 'you are in Rome', 'every man is running' and the like, basic sentences; on the other hand, let sentences with a more complex structure – involving the presence of obligational operators such as 'concessum', 'concedendum', 'negandum' and the like – be called *higher-order* sentences. It seems that much of the discussion occurring in obligations treatises is devoted in great measure (if not actually for the most part) to explore the conflicts arising by virtue of the combination of basic sentences with higher-order sentences. For instance, take a case of a disputation in which it turns out that a sentence p ought to be conceded but also that the sentence 'p ought to be conceded' ought to be denied, which we can express by means of OCp and ON(OCp).¹¹⁹ It seems difficult to regard a case like that as being designed to perform any form of reasoning secundum imaginationem. Rather the point is to establish what should and what should not be asserted in a given disputational context.

And here, again, the question is in virtue of what mutual relations certain sentences should be *conceded*, *denied* or *doubted*, rather than a matter of determining what (states of affairs) can be consistently said to obtain in the imaginary situations described by those sentences. The two levels differ in no small measure. The insistence on the evaluation of the respondent's behaviour with respect to the speech acts he performs in a given disputation takes the shape of a struggle over sentences and the presence of so many cases in which the evaluation takes place *within* the context of the disputation, i.e. *infra tempus*, by means of iterated operations of concession, denial or doubt suggest little concern, as it were, with partial descriptions of possible worlds. That being said, if a disputation contains only sentences of the basic type, it lends itself very well to be read in Yrjösuuri's terms: in a disputation of such a sort conducted according to the rules of *responsio antiqua*, the conjunction of all evaluated sentences always turns out to be the description of a possible situation (or of a class of worlds in which that situation is true).¹²⁰

¹¹⁹ By the way, this often happens to be the case in obligational disputations. For an example of this kind of problem in Peter of Mantua, cf. *infra*, Part II, sec. 4.17 ff. This circumstance has to do with the failure of the principle $OCp \Leftrightarrow OC(OC(p))$ which, along with a number of other similar theses, is not universally accepted. An interesting discussion concerning Strode's treatment of such topics is found in [Dutilh Novaes, 2006a, pp. 368–373] and [Dutilh Novaes, 2007, pp. 209–213].

 $^{^{120}}$ That the conjunction of all evaluated steps be a possible sentence is Peter's fourth rule, cf. *infra*, Part II, sec. 2.4. I am not claiming that in disputations involving higher-order sentences

But if we consider more complex situations involving not only basic sentences but also application of the obligational operators as above, then it seems less plausible to claim that at stake is an assessment of co-tenability or the investigation of possible state of affairs. Usually it also looks like a way to make the exercise more difficult by *describing*, from the inside, what is going on in the disputation, like for instance by proposing sentences like 'tu concedis istam', 'ista est concedenda', 'tu male respondes'.¹²¹ In sum, these accounts are quite useful and illuminating when they deal with a restricted sub-class of disputations (involving only basic sentences), but I think they leave out those that are centred on semantic problems involving higher-order sentences.

To end, two other contributions that are both related to the notion of consistency should be finally mentioned. The first is Martin's suggestion that obligations be connected to the treatment of insolubles insofar as they are based on the idea of checking the co-tenability of sets of sentences.¹²² The second is a more recent interpretation of Burley's obligations as a prefiguration of modern theories of belief revision.¹²³ To sum up, here is a synoptical look at interpretations focusing on the question of the purpose:¹²⁴

are involved it is no longer required that the whole conjunction of steps be a possible sentence too (and hence a 'description' of a possible situation). I am *just* claiming that the kind of possible situations described by such disputations are not philosophically as appealing as the ones Yrjönsuuri's model relies on.

¹²¹ On the same basis, it might also be accounted for the existence and features of other subtypes of obligations, especially *petitio* which focuses on duties relative to the performance of certain speech acts (the *obligatum* directly involves the application of an operator to a basic sentence, to the effect that a *petitio* to concede p is often equivalent to a *positio* where the *positum* is Cp, i.e. a sentence like 'tu concedis p' or 'p est a te concessum') and *institutio* or *impositio*, where, again, the point is making stipulations on language (and not, primarily, the assessment of whether something might or might not be the case).

¹²² Martin insists especially on the connection with the procedure of *cassatio*, cf. [Martin, 1993, p. 373] "The solution to the problem of propositions ungrounded with respect to *positio* or *depositio* found in [*Obligationes Parisienses*] and [*Tractatus Emmerani*] is thus *cassatio*, apparently the earliest of the solutions to the Liar. [...] It seems to me that it makes much more sense to speak of cancellation or annulment with respect to a *positio* than it does apart from that context. I would thus suggest not only that twelfth century logicians first became aware of insolubles via obligations but also that they developed their first solutions in terms of them".

¹²³ Cf. [Lagerlund and Olsson, 2001]. Although the analogy might be at first sight suggestive, decisive objections to this view are presented in [Dutilh Novaes, 2007, pp. 148-149] on the basis of the fact that there is definitely no possibility for the respondent to revise the evaluation of sentences to which he has already replied and that the belief revision account would hardly be able to explain the competitive dimension of disputations; moreover, there seem to be no textual evidence in support of this view.

¹²⁴ The list has no pretensions to being exhaustive. When a proposal has been developed by the same author on a number of different occasions, I have selected only the most relevant contributions.

exercises or tests with peda-	[Green, 1963]
gogical goal	[Brown, 1966]
	[Weisheipl, 1966]
	[Perreiah, 1982]
	[Sinkler, 1992]
	[D'Ors and García-Clavel, 1994]
prefiguration of axiomatic	[Boehner, 1952]
method	
counterfactual reasoning	[Spade, 1982b]
	[Spade, 1992]
	[Olsson, 2006]
thesis defense	[Spade, 1993]
tool for the analysis and res-	[Stump, 1981]
olution of <i>sophismata</i> and	[Stump, 1989]
insolubilia	[Pironet, 2001]
regimentation of theological	[Gelber, 2004]
enquiry	
detection of fallacies	[Hamblin, 1970]
co-tenability, identification	[Martin, 1993]
of consistent sets of sen-	[Martin, 2001]
tences; insolubilia	
games of consistency mainte-	[Dutilh Novaes, 2005]
nance (Burley)	[Dutilh Novaes, 2007]
test of inference recognition	[Dutilh Novaes, 2006b]
(Swyneshed)	[Dutilh Novaes, 2007]
theory of belief revision	[Lagerlund and Olsson, 2001]
thought experiments	[Yrjönsuuri, 1994]
	[Yrjönsuuri, 2001a]
metamethodology of me-	[King, 1991]
dieval science	

This picture speaks for the coexistence of a variety of motives: the question of the purpose(s) of obligations is not an irredeemable problem as long as we accept that various needs, tendencies and aims might have prompted medieval authors to encompass in their treatises different issues and bring them together in a conceptual framework that, moreover, turns out to evolve over time. Even a superficial consideration of the variety of different classifications of subtypes of obligations, or of the historical development of *positio* alone, is sufficient to justify the claim that we need to be somehow eclectic.

Once we have picked out a preferred interpretation (on the grounds, for instance, that it fits at best a class of texts or problems), we need not exclude, in

principle, that an alternative reading might capture in a better way a distinct class of problems. My claim, to be sure, is not that anything goes. Nor does it reduce to a generic statement of methodological caution. Rather, I think we should be ready to take into account the hypothesis that there might have been indeed different purposes, say, in the treatment of *positio impossibilis* in the first half of the XIII century and in that of *impositio* in a late XIV century treatise like Peter of Mantua's.

In this literature there is an underiable practical component oriented to the aim of teaching or testing one's ability in handling logical inferences. This basic underlying thread should not be overlooked, especially because for the most part the scant textual evidence that medieval authors have left us with, confirm this hypothesis, if any. On the other hand, there are plausibly a lot of additional issues that come into play, but they should be contextualized when appropriate evidence speak for their presence. The most well-balanced judgement on these matters still remains the one pronounced by Ashworth twenty years ago in her introduction to Paul of Venice's edition.¹²⁵.

¹²⁵ Cf. [Ashworth, 1988a, pp. xiii-xiv]"A contentious and as yet unresolved issue has to do with the purpose of obligations treatises. The treatises themselves do not offer much discussion of this point being content to remark that the opponent in a disputation is to try to push the respondent into accepting a contradiction, whereas the respondent has to resist this [...]. In the process both participants would have their knowledge of valid inferences thouroughly tested $[\ldots]$. It should be also emphasized that the bulk of almost all treatises on obligations consisted of a series of sophisms which [...] formed an integral part, at least in fourteenth century Oxford, and were designed to develop a student's subtlety and skill in handling logical rules. These remarks suggest that obligational disputations (if such were ever in fact held) had the primary function of providing oral exercise in formal logic, and hence were of mainly pedagogical significance". In addition to that, after mentioning Spade's and Stump's views, Ashworth maintains that "there is probably something to be said for all these accounts and that remains true, I think, of most accounts that have been put forward thereafter]. Insofar as these treatises described a routine to be followed in class-room disputations, the purpose could only have been that of testing a student's skill in formal logic, since truth was explicitly not an issue; but the authors and readers of such treatises obviously welcomed the opportunity to discuss other matters in some depth [...] One must also bear in mind the often-noted link between treatises on obligations and treatises on insolubles. [...] This suggests a general interest in discussing all kinds of paradoxes, both semantic and non-semantic" cf. [Ashworth, 1988a, p. xiv].

PART II

Peter of Mantua's obligations

Life and works

Presumably, the first relevant reference to Peter of Mantua in modern times is due to Prantl, who devotes some space to the description of Peter's *Logica* in his *Geschichte der Logik im Abendlande*.¹²⁶ His name also features among those of a number of medieval authors considered by Duhem in his Études sur Léonard de *Vinci*.¹²⁷ Both Prantl and Duhem wrongly take him to be a XV century author.¹²⁸ More systematic attention to his figure began to arise, only in the mid XX century, within the environment of historians of Renaissance philosophy and intellectual *milieu*.¹²⁹ Although De Libera's claim that Peter is the only great Italian logician of the *Trecento*¹³⁰ might stand in the need of further corroboration on the basis of a deeper assessment of his logical doctrines, "Peter's influence on his contemporaries and succeeding generations of scholars seems to have exceeded any merely parochial limits".¹³¹ First and foremost, Peter's profile is that of a full-blown scholastic, his renown being justly due to the depth and length of his *Logica* as well as to the original ideas exposed in his treatise *De primo et ultimo instanti*.¹³² Among his

 131 Cf. [James, 1974, p. 167].

¹³² An unpublished study with the edition of this text is [James, 1968]. Cf. also [De Libera, 1982] for a discussion of Apollinare Offredi's reaction to Peter's doctrine of tem-

¹²⁶ Cf. [Prantl, 1870, pp. 176–180]. As far as other general histories of logic are concerned, [Kneale and Kneale, 1962] does not mention him at all, whereas a short reference is found in [Bochenski, 1961, p. 219] that cursorily brings up Peter's treatment of the fourth syllogistic figure.

¹²⁷ Cf. [Duhem, 1913, pp. 495 and 535].

¹²⁸ The first to identify the error was Kristeller, who suggested that both had confused the dates of publication of Peter's work (in the late XV century) with the dates of his life, cf. [Kristeller, 1956, p. 577 f. 67].

¹²⁹ Cf. [Garin, 1948], [Dionisotti, 1955] and [Vasoli, 1963].

¹³⁰ Cf. [De Libera, 1982, p. 254]. From the historical standpoint, moreover, it should be pointed out that recent research has cast light on a much earlier figure such as Franciscus de Prato, cf. [Amerini, 2006], and that Paul of Venice's most relevant logical works, the *Logica Parva* and *Logica Magna*, have both been dated to the last decade of the XIV century, cf. [Bottin, 1983, pp. 87–93].

connections, however, we also find a prominent humanist like Coluccio Salutati and there is enough evidence to claim that he was familiar with the intellectual tendencies that were emerging in his time.¹³³

The state of our knowledge about Peter's life is fragmentary, to say the least, and it is confined, in all likelihood, only to the last ten years that preceded his death. A bunch of facts are witnessed by historical sources that inform us about his name and geographical provenance.¹³⁴ What we know basically relies on the university records of Padua and Bologna, on two letters,¹³⁵ and on scattered sources of heterogeneous sorts that mention his name or his works. Peter is recorded among the 'Scolari della filosofia, logica dal 1355 al 1390' at the university of Padua, in September 1389, under the name of 'Pietro Alboini da Mantova, figlio di Giovanni'.¹³⁶ Unfortunately, however, we do not have any additional information

¹³³ The two essential contributions to the reconstruction of Peter's life are [Vasoli, 1963] and [James, 1974]. My exposition is mainly based on the latter, who has also published, on a separate occasion, a fragment of an exposition of the first letter of Seneca to Lucilius which is attributed to Peter, cf. [James, 1976] and also [Buonocore, 2000, p. 36]. Kristeller has also insisted on Peter's relevance to the curriculum of the Italian universities, cf. [Kristeller, 1996, p. 83] "[...]"the sophistic" logic which flourished in the fourteenth century at Oxford [...] began to spread in Italy and found its first influential representative in Peter of Mantua who studied at Padua but taught in his later years at Bologna and died before 1400", as well as on his connection with the humanist trends, cf. [Kristeller, 1985, p. 83–84].

 134 The connection with Mantua is confirmed by a curious example of profiteering and human greed. Novati mentions (unfortunately without providing the archive signature) a letter from the *Archivio storico Gonzaga* in Mantua , addressed to the secretary of the Lord of the Mantua from a law student called Rainerius de Fanctellinis, who pleaded with him that a benefit previously allotted to Peter be assigned to him after Peter's death: "cum sit quod excellentissimum artium doctor magister Petrus de Mantua decesserit, quo multum doleo et deflendum est quodlibet in morte tanti viri, qui habuerat Mantue unum beneficium aut clericam vel archisacerdatum (*sic*), considerata paupertate quam magna [...] instantissime vos deprecor, ut prefato eidem [...] domino nostro scribatis quod me vellit de hoc beneficio subvenire" [Novati, 1911, p. 319].

¹³⁵ The first letter is dated from Bologna, November 5, 1392 and is addressed by Peter to his former friend Pietro Tomasi, in Padua. The second letter is dated from Florence, August, 26 1398 and is addressed by Coluccio Salutati to Peter, in Bologna, cf. [Cessi, 1913, pp. 130–131] and [Novati, 1911, pp. 318–322], respectively. As far as the former is concerned, [James, 1974, p. 161 f. 2] points out that Cessi's article (along with another source) are based on a manuscript of the Archivio di Stato in Venice (Procurator di S. Marco di Citra, Atti Congregazione di Carità, Busta 120) but that according to Kristeller the letter is no longer to be found there.

¹³⁶ Cf. [Gloria, 1888, p. 512] "Pietro Alboini da Mantova figlio di Giovanni, (sett. 1389) a

poral limits. It should be noted, finally, that an intriguing reference found in [Garin, 1960, p. 389] might prompt the belief that to the record of Peter's work we should add a commentary on the *Physics*, possibly lost, which was mentioned in the *Catalogue de la Bibliothèque de M. le C.mte D. Bouturlin*, Florence 1831, p. 20. The codex to which the catalogue refers still exists in fact (Florence, Biblioteca Medicea Laurenziana, Cod. 1348, Fondo Ashburnham), but unfortunately it is not a commentary on the *Physics*. It is a composite manuscript containing Peter's *De instanti*, the *Questiones* on the latitude of forms by Blasius of Parma, and a long section with Buridan's commentary on the *Parva naturalia*.

about his studies there. After three years, in 1392, we find him in Bologna, where the official rolls of the university tell us that he was a teacher of natural philosophy in 1392-1393. His name appears in the same source also for the years 1395-1396, 1397-1398 as a teacher of moral philosophy, whereas in 1398-1399 he is appointed to teach both natural and moral philosophy.¹³⁷ As for the years 1393–1395, we have no information to rely on from the extant rolls. For this period, however, we can conjecture, that Peter was still teaching in that university on the basis of a reference found in the life of Ugo Benzi of Siena, a famous physician and commentator of Galen and Avicenna, who first studied and later taught at Bologna.¹³⁸ Peter seems therefore to have pursued a continuous academic activity in the prestigious *Studium* from 1392 to 1399.¹³⁹ As far as the date of his death is concerned, James has proposed convincing arguments in support of the claim that it is likely to have occurred before the end of 1399.¹⁴⁰

There are no elements for a solid datation of Peter's *Logica*. It has been conjectured, however, that since his teaching activity is uniquely concerned with natural and moral philosophy from 1392 onwards and, on the other hand, Peter was still engaged in the study of logic in 1389 at Padua, it is in that span of years that he might have written his treatise.¹⁴¹

Despite the brevity of Peter's academic career, which is all enclosed in a decade, his influence on the Italian university environment should not be underes-

¹³⁸ Ugo was in Bologna in the year 1393 and "became a devoted follower of the illustrious dialectician, Peter of Mantua, and attended him with such veneration that Ugo addressed him as 'father'" [Lockwood, 1951, p. 23].

¹³⁹ Cf. [James, 1974, p. 163].

¹⁴⁰ Cf. [James, 1974, pp. 166–167].

¹⁴¹ Cf. [James, 1974, p. 163]. In the same place, James proposes 1392-1395 as the years of composition of the *De instanti*. Caution is always in order when it comes to geographical references in medieval logic texts, but I think it is worth noting, at this point, that two manuscripts of the *Logica*, O and M, have 'credas regem esse Bononie' in a place where the other manuscripts have the more common 'credas papam esse Romae' (even if referring, by end of the XIV century, to the Pope's being in Rome as an example of uncontroversial truth looks pretty much like a mockery). The presence of such a reference might depend on various factors: the place where the manuscript was copied being a strong candidate. But it cannot be excluded, in principle, that the geographical reference was already in the original. In that case, one might wonder whether Peter had already in Bologna when he wrote the *Logica* (or at least the treatise on obligations).

S. Margherita nella casa di Giovanni da Marliano milanese abate di S. Cipriano di Murano e dottore dei decreti: n. 9 Mon. 1389, 3 Settembre".

¹³⁷ Along with Jacobus de Armis and Franciscus de Camerino, cf. [Dallari, 1889, pp. 16– 17]. Peter himself in the letter to Tomasi mentions the two colleagues and boasts about the number of students attending his classes and about his salary, cf. [Cessi, 1913, p. 131] "hic tamen me legentem scias naturalem philosophiam concurrentem cum magistro Jacobo de Armis et cum magistro Francesco de Camerino, quorum uterque succumbit. Magister vero Jacobus de Armis me minor nunquam maior in scolaribus fuit, nisi duobus primis diebus [...] Habeo etiam sesqualterum salarium ad illorum utrumque".

timated.¹⁴² The *Logica* is read and commented upon in the prominent intellectual environments of Padua, Bologna, and Pavia.¹⁴³ One of the extant witnesses of the work was copied by a student of the university of Ferrara, Johannes de Medallis of Brescia, in 1420.¹⁴⁴ Peter is explicitly referred to in the *Recollectae super sophis-matibus Hentisberi* by Caietan of Thiene and Simon of Lendenaria,¹⁴⁵ who both discuss some of his logical doctrines (with a stunning example being taken from the treatise on obligations).¹⁴⁶ Moreover, Peter's reputation in the XV and XVI centuries is witnessed by the number of manuscripts and printed editions of his *Logica* and *De primo et ultimo instanti*, by four commentaries on the latter and by numerous references made to those texts in the works of later authors.¹⁴⁷

¹⁴³ Cf. [Vasoli, 1963, p. 5].

¹⁴⁵ For detailed references cf. [James, 1974, p. 169 f. 41] which mentions the early printed versions of both texts. They are found also in a manuscript at Biblioteca Nazionale Marciana, L.VI.128 (2559). I have used this source to check the passages where Peter is mentioned and I will refer to it again below.

¹⁴⁶ Cf. *infra*, Part II, sec. 3.39–3.40. The source for the sophism seems to be quite definitely Heytesbury's *Regulae*.

¹⁴⁷ The *Logica* has survived in six manuscripts plus a seventh long fragment. I have already mentioned two of them, M and V, and the complete list will be given below. The fragment is in the Biblioteca Apostolica Vaticana, Vat. lat. 2189 and consists only of the chapters *De terminis officiabilibus* and *De equipollentiis*, cf. [Maier, 1961, p. 161]. Four printed editions in the XV century are known (not five, as [De Libera, 1982, p. 52] has claimed, on account of a misinterpretation of Copinger 4724 which is in fact now commonly recognized as the *editio princeps* printed in Padua, by Johannes Herbort, probably in 1477). The *De instanti* enjoyed an even greater fortune. There are fifteen extant manuscripts, and five manuscripts containing three commentaries on the work (three of the commentary by Apollinare Offredi, one of the commentary by Giovanni Marliani and one by an anonymous commentator. The last known commentary is the one by Mengo Bianchelli: it survives only in an edition printed in Venice in 1507. References to Peter's work are found also in Ermolao Barbaro (according to the identification made by Dionisotti, of a purported 'filosofastro mantovano' with Peter, cf. [Dionisotti, 1955, p. 232 and 234]). A dedicatory letter by Bassano Politi da Lodi to Rodrigo Carvajal in the 1505 edition of a collection of works of Swyneshed (the calculator), Bradwardine, Blasius Pelacani of Parma

¹⁴² A manuscript with Peter's *Logica* features in the will of Benedetto Greco, cf. [Garin, 1960, p. 204]. A copy of the text was also owned by Giovanni Marcanova, a XV century humanist and friend of Andrea Mantegna, cf. [Vitali, 1983, p. 151 f. 51]. Marcanova's library was bequested in 1443 to the convent of San Giovanni in Verdara in Padua, and became, eventually, part of the funds of the Biblioteca Marciana, where the manuscript is still preserved (it is the witness that I have called V).

¹⁴⁴ It is the manuscript now at the Biblioteca Comunale of Mantua that I have labelled M. Johannes also made a copy of Strode's obligations which is found in the same manuscript. A marginal note, possibly in a different hand, from the latter, explicitly points to a convergence of opinions between Peter and Strode on a certain technical problem: "Nota quod Petrus de Mantua insequitur istam opinionem quam hic insequitur Strot" [Ralph Strode, *Obl.*, Mantua, fol. $131^{\rm vb}$ in margine]. The note then goes on like this "scilicet quod quando modus finaliter subsequitur illa est distinguenda: si autem determinat totum dictum est in sensu composito; si autem li 'est' solum, est in sensu diviso, quod non" [the page is cut off at bottom edge].

2. PETER OF MANTUA'S OBLIGATIONS

To end, I would like to point out one thing that might be relevant to confirm the hypothesis of Peter's direct acquaintance with Strode's (and Buser's) obligations. It is an unfortunate circumstance that we know almost nothing about Peter's early education and about his university career as a student, apart from the solid link with Padua. That might be sufficient, however, if we combine it with a piece of information that comes, again, from the letter that Peter wrote in 1392 to Tomasi, to strongly argue in favour of a direct acquaintance with both authors, even besides the textual evidence that will be presented below.

There is one especially interesting fact about this letter.¹⁴⁸ Peter mentions a number of people of the Paduan circle to which he used to belong before moving to Bologna and sends his regards, among others, to a 'm. Almericus'.¹⁴⁹ It happens to be the case that two manuscripts in the Bodleian Library in Oxford contain the treatises on obligations of Strode and Buser, respectively: and the name of the copyist is in both cases Almerico da Serravalle, student of Arts at Padua in the early 1390s.¹⁵⁰ Moreover, it turns out that the Almericus mentioned by Peter in his letter to Tomasi certainly is an Almericus da Serravalle (his last name occurs several times in other letters transcribed and discussed in Cessi's article). It is highly plausible that they are in fact one and the same person. This hypothesis is corroborated by an additional piece of evidence, i.e. another name which occurs in both contexts. In his letter, Peter sends his regards also to a 'm. Anthonio de Monte'. And again, an Antonio da Monte¹⁵¹ is involved in the history of Canon. misc. 219, because he is the copyist of a number of slightly later treatises (referable to the year 1395) that are contained in that manuscript. Antonio the copyist is also more directly related to Strode's and Buser's obligations, because he comes into possession of the two manuscripts that are both passed on to him by Almerico. Now, it is possible, in principle, that there have existed two *distinct* pairs of people (a) with the same names, (b) coming from the same geographical area (where area has a precise meaning: Padua), (c) living and studying in the same span of years, i.e. the early 1390s, (d) having some kind of direct or indirect relation to two particular logical treatises: in one case via the two manuscripts and in the other case via the acquaintance with someone (= Peter) who knew

and others, mentions the *De instanti* and the commentary on it by Giovanni Marliani, cf. again [Dionisotti, 1955, p. 238].

¹⁴⁸ To my knowledge, the connection has been first proposed by [Maierù, 1982b, p. 90].

¹⁴⁹ Cf. [Cessi, 1913, p. 131].

¹⁵⁰ Cf. [Ashworth, 1988a, p. 393]. Strode's text is found in ms. *Canon. misc. 219*, fols. 37^{ra}–47^{ra}, there is no date, but Almerico copied the preceding tract on consequences in 1393. Buser's text is found in ms. *Canon. Class. Lat. 278*, fols. 72^{ra}–78^{rb} and the copy is dated 1391. ¹⁵¹ We apprehend, from two *explicit* in ms. *Canon. misc. 219*, that Antonio was a student of

Arts in the mid 1390s (*expl.* of Henricus Anglicus' *insolubilia* on fol. 11^{vb}), and that he was from Padua (*expl.* of Heytesbury's *De sensu composito et diviso* on fol. 6^{rb}); cf. also [Maierù, 1982b, pp. 98–99].

both texts. That these four conditions have simultaneously obtained is a possible circumstance, indeed; but it is very unlikely.

Whatever the exact terms of these interesting historical relations, the most striking fact is that the environment Peter had left at the beginning of the 1390s in Padua was remarkably up-to-date with respect to the developments of logic North of the Alps. Peter's *Logica*, in this connection, is another significant element for the history of the influence of English logic in Italy. If it is true that this history did not start as late as the end of the XIV century,¹⁵² it is also true that Italy is the area in which this tradition survived and flourished best, especially after the beginning of the XV century.¹⁵³

Whether Peter has been up to Salutati's expectations¹⁵⁴ and has succeeded in reviving the splendours of philosophy in Italy at the turn of the century is a question that the study of his obligations will not resolve once and for all. Yet, it looks much less relevant to the history of logic than it might be to the history of the humanist obsession with the spectre of the *barbari britanni*.¹⁵⁵

¹⁵² Cf. for instance the general survey in [Courtenay, 1982].

¹⁵³ Cf. [Ashworth and Spade, 1992, p. 60].

¹⁵⁴ In this respect it is worth quoting in full the opening passages of Salutati's letter to Peter, which should count "Iam pridem, vir insignis, te fama perceperam virum admirande scientie de physices altitudine inauditis speculationibus eminere. Quod quidem gaudebam et gaudeo, ne videretur hec, quam hodie philosophiam dicimus, a Grecis quondam, nunc ab Italis ad toto divisos orbe Britannos omnino fugisse. Nimis enim molestum est, quod tantum sua cum dissidentia Latium laboret, quod, cum armorum super omnes gentes gloria floruerit, hac nostra etate langueat et de vincente victum turpiter obtorpescat. Habemus enim in excusatione huiusce pudoris discordie nostre culpam; sed vinci litteris, quibus etiam vigente Grecia florebamus, adeo turpe iudico, quod nichil excusationis inveniam, nisi turpem desidiam, scilicet voluptates, et, radix malorum omnium, cupiditatem; nam discordia in maximis civitatibus et in regnis quoddam gentibus saltem in hoc palmam eriperet, qualem me tibi [read: te mihi] fama et multorum relatio promittit" [Novati, 1911, pp. 318-320].

¹⁵⁵ Cf. [Garin, 1960] and [Vasoli, 1952].

Structure and contents

Some comments are in order, before finally coming to the text. Since the basic logical features of Peter's account have already been described above and I will discuss below in the notes to the treatise the fundamental issues it raises, I shall confine myself, in this section, to a brief outline of its structure. Peter's treatise on obligations features in the *Logica* as its last or as its penultimate chapter (depending on the witness). In the 1492 printed edition on which my transcription is based,¹⁵⁶ the treatise on obligations is actually the penultimate, preceded by the treatise *De scire et dubitare* and followed by the treatise *De insolubilibus*.¹⁵⁷

¹⁵⁶ Cf. [Peter of Mantua, 1492a].

 157 Here is the comprehensive list of contents of the *Logica*, as is found in [Peter of Mantua, 1492a]:

1.	Tractatus de suppositionibus	sig.	${\rm Aii^{ra}-Aiiii^{ra}}$
2.	Tractatus de relativis	sig.	${\rm Aiiii}^{\rm ra} - {\rm Aiiii}_{+3}^{\rm vb}$
3.	Tractatus de ampliationibus	sig.	$Aiiiii{+}3^{\rm vb}-B^{\rm ra}$
4.	Tractatus de appellationibus	sig.	$B^{\rm rb}-B3^{\rm rb}$
5.	De probatione propositionis universalis	sig.	$B3^{va} - Biiii + 1^{ra}$
6.	De probatione propositionis exclusive	sig.	$Biiii+1^{\rm ra}-Biiii+3^{\rm vb}$
7.	De expositione reduplicative	sig.	$Biiii+3^{\rm vb}-C^{\rm ra}$
8.	De expositione illius termini 'infinitum'	sig.	$C^{\rm ra}-C^{\rm va}$
9.	De probatione propositionis exceptive	sig.	$C^{va} - C + 1^{vb}$
10.	De expositione de li 'totus', 'tota', 'totum'	sig.	$\rm C+1^{vb}-\rm Cii^{rb}$
11.	De expositione illius termini 'eternaliter', 'ab	sig.	${\rm Cii^{va}-Cii^{va}}$
	eterno' et 'semper'		
12.	De expositione illius termini 'differt', 'aliud' et	sig.	${ m Cii^{va}-C3^{rb}}$
	de ly 'non idem' et de terminis positivis		
13.	De comparativo	sig.	$\mathrm{C3^{va}-C3+1^{rb}}$
14.	De expositione gradus superlativi	sig.	$C3{+}1^{\rm rb}-C3{+}2^{\rm ra}$
15.	Tractatus de 'incipit' et 'desinit'	sig.	$\mathrm{C3+2^{ra}-C3+4^{va}}$
16.	De ly 'immediate'	sig.	$C3+4^{va} - C3+4^{vb}$
17.	Tractatus de veritate et falsitate sive de taliter	sig.	$C3+4^{vb}$ – $Diiii+1^{rb}$
	$et \ qualiter$		
18.	Tractatus de terminis privativis	sig.	$Diiii+1^{\rm rb}-Diiii+3^{\rm ra}$
19.	Tractatus syllogismorum	sig.	$Diiii+3^{\rm ra}-Diiii+4^{\rm vb}$
20.	De syllogismo expositorio	sig.	${\rm Diiii}{+}4^{\rm vb}-{\rm Eii}^{\rm ra}$
21.	De syllosigmo resolutorio	sig.	${\rm Eii^{ra}-Eiiii^{ra}}$
22.	De terminis officiabilibus et officiatis et de de-	sig.	${\rm Eiiii^{ra}-Eiiii^{va}}$
	scriptibilibus		
23.	De equipollentiis	sig.	$Eiiii^{va} - Eiiii + 1^{va}$
24.	De conversionibus	sig.	$Eiiii+1^{va}-Eiiii+2^{va}$
25.	Tractatus de consequentiis	sig.	$Eiiii+2^{\rm va}-Fiiii+1^{\rm rb}$
26.	Tractatus de scire et dubitare	sig.	$Fiiiii{+}1^{\rm rb}-Gii^{\rm ra}$
27.	Tractatus obligationum	sig.	${\rm Gii^{ra}}-{\rm Giiii}{+}4^{\rm vb}$
27.	Tractatus de insolubilibus	sig.	$Giiii+4^{\rm vb}-Hii^{\rm vb}$

2. PETER OF MANTUA'S OBLIGATIONS

Leaving aside independent treatises, i.e. treatises that do not feature in more comprehensive works on logic either because of some event in the transmission of the text that has make it come down to us separately or because of the original intention of the author, the location of obligations in the final part of logical works is by no means an unusual characteristic.¹⁵⁸ As to the internal structure, Peter's treatise can be divided into six main sections.

- 1. Preliminary assumptions
- 2. Rules
- 3. Sophisms on *positio*
- 4. Further subdivisions of *positio* and other sophisms
- 5. Impositio
- 6. Depositio

The distinction between the six sections (save in the case of the fourth one which might be a little far-fetched since it contains heterogeneous materials) is quite sharp. The first two sections provide the conceptual basis of the theory. In the first one Peter gives his definition of *obligatio*, a characterization of the notions of *positio* and *admissio*, he lays down the principle according to which outside the time of the disputation one is always supposed to tell the truth, and finally he supplies a clarification of the meaning of the crucial properties 'concedendum' and 'negandum' (= such that it ought to be granted or denied, respectively). In the second section, five obligational rules (for *positio*) are laid down. The first rule governs the admission of the *positum*; the second rule norms the concession and denial of what follows or whose negation follows from the *positum* alone; the third rule extends the regimentation to sentences that follow or whose negations follow from whatever belongs to the set of previously granted or denied sentences; the fourth rule stipulates that the conjunction of all members of the set of sentences that constitute the outcome of the disputation must be a possible sentence; eventually, the fifth rule provides instructions for replying to irrelevant sentences. The first two sections (especially the second) may be regarded as the theoretical bulk of the whole treatise and, in a sense, a great deal of what comes next is functional to an articulated assessment of their features and applications. The set of rules is complete, insofar as it provides in principle an algorithm to determine in

¹⁵⁸ As Angelelli has claimed, with regard to the position of the treatise in Paul of Venice's *Logica* Magna and Logica Parva, "this is not by way of exotic appendix; on the contrary, obligations are a sort of culmination of logic", cf. [Angelelli, 1970, p. 803].

a mechanical way the correct response to any sentence whatsoever that might be posited or proposed in a disputation.¹⁵⁹

The third section contains five groups of arguments, centred in turn on the analysis of several clusters of sophisms. Although, again, the distinction is drawn primarily for the sake of clarity and it is not my intention to claim that Peter *necessarily* had such a division in mind, it is hard to escape the impression that in the treatise there emerges an implicit agenda of problems in which he seems to be particularly interested.¹⁶⁰ All sophisms in the third section focus on *positio* and should hold as tantamount objections raised against the adopted set of rules (especially against the first two).

According to the type of logical issue that, each time, is being taken into account, I have grouped them as follows: (I) underdetermined sentences (they cause problems, when posited, because the respondent does not actually know what he is replying to); (II) contradictions;¹⁶¹ (III) time and context of utterance; (IV) pragmatic paradoxes; (V) other materials (two sophisms on the notion of *sequens* and the distinction between possible and possibly-true, respectively).

The fourth section opens with a classification of further subtypes of *positio* which cannot qualify as independent species of obligations (the only two that Peter acknowledges being *positio* and *depositio*). It then goes on with examples illustrating the features of such subtypes and finally displays a battery of sophisms dealing with more complex instances of *positio*.¹⁶²

¹⁵⁹ As I have already stressed above, the distinction between what I call *basic* sentences and *higher-order* sentences might prompt us to water down this claim. But still, when the theory is put on trial in sophisms, the practice suggests that Peter makes no real distinction between plain and complex cases, and regards the rules as applying at a general level and in a consistent way: for the most part, when something goes wrong (as only the construction of an appropriate tailor-made artificial example can show), the respondent has the duty to reject the *casus* and this move does not count as an easy escape, since it is built in the rules.

 $^{^{160}}$ I will just mention here some relevant issues that we will encounter in the treatise: (a) careful rejection of any objection that might be raised to charge his set of rules with even the slightest form of inconsistency, (b) the use of *certificatio* to remove cases of ambiguity or indeterminacy, (c) a detailed account of the properties of *impositio*, with a stunning number of sophisms on *suppositio* (involving anaphoric expressions, quantifiers and so on), (d) rejection of *positio impossibilis*, and (e) the use of a strategy of response *per ordinem* (especially when the respondent finds himself in troubled waters).

¹⁶¹ This group of sophisms is particularly interesting because Peter proves to be clearly dealing with the problem of Spade's (4f)-inconsistency, cf. [Spade, 1982b, p. 9]. The conclusion Peter draws coincides in a nutshell with the backbone of the relativized interpretation, cf. [Ashworth, 1981, pp. 183–186]. There is no inconsistency whatsoever in the rules of *responsio antiqua*, because, at most, contradictory sentences are admitted or conceded with respect to *different* disputations.

 $^{^{162}}$ In this context, Peter partially retrieves a traditional taxonomy involving for instance *positio* cadens and *positio renascens*. In the discussion of the last sophism of this section, he also proves to be aware of a standard corollary of *responsio antiqua* which is often referred to in the literature

2. PETER OF MANTUA'S OBLIGATIONS

The fifth section is extremely dense. The topic it deals with is *impositio* which Peter does not regard as an independent type of obligation, but that turns out to be of great interest to him, at least if we judge by the amount of space he devotes to its treatment and by the complexity of the sophisms he discusses. *Impositio* is the obligation by virtue of which a respondent commits himself to responding in conformity with the stipulation of a (new) meaning for a word or a sentence. The internal division of this part of the treatise includes two opening sections where Peter makes some cursory remarks about the distinction between mental language and spoken or written language and about the relation between stipulations, relevance (in the technical sense of obligations) and truth. After spelling out some his philosophical presuppositions, Peter seems to have four main points to make. He first deals with the case in which a spoken or written term that did not previously signify is assigned a meaning for the first time. In the central part of the section, he then turns to the case of real *variations* of meaning, which take place when the ascription of meaning is relative to a word or to a sentence that did previously signify. Third, he engages in a lengthy analysis of sophisms involving, in various non-standard ways, the standard modes of *suppositio* (which seems quite sensible, since the 'meaning' of syncategorematic terms are strictly related with the latter).¹⁶³ Fourth and last, he takes up the issue of logically (or metaphysically) incompatible sentences that are assumed to convert.

The treatise ends with a relatively short section on *depositio*.¹⁶⁴ Peter regards *depositio* as an independent species of obligation and introduces some basic rules that are the mirror image of those of *positio*, with the difference that in this case the *obligatum* must be denied whenever it is put forward during a disputation, as well as anything which entails it. There are two interesting facts about the kind of sophisms that Peter discusses in this section. First, it is only here that he makes use of a standard form of argument – based on the assumption that two or more sentences have the same vs. different truth values – which is otherwise very common in treatises on obligations.¹⁶⁵ Second, there is a prevailing recourse to

on obligations, namely the fact the respondent can be in principle forced to grant any false sentence which is logically compatible with a contingently false *positum*.

¹⁶³ I use inverted commas here, since it might be not entirely appropriate to speak of meaning when it comes to syncategorematic terms. In any case, the important fact is that Peter, in this discussion, plays with the logical properties of quantified terms or anaphoric expressions and modifies the inferences that are usually associated with them.

¹⁶⁴ This seems a common practice as well, cf. for instance [William Buser, 1990, pp. 234–237], [Paul of Venice, 1988, pp. 369–391].

¹⁶⁵ This technique, involving the so-called similars and dissimilars, is very well-suited, for instance, to prove the aforementioned corollary of the set of rules of *responsio antiqua*. Typically, one would posit a false sentence p and then propose that p and another arbitrary false sentence q are similar (= have the same truth value). Usually this step would be regarded as irrelevant and true, since p and q have indeed the same truth value (they are both false). But then, no

2. PETER OF MANTUA'S OBLIGATIONS

sophisms involving truth and falsehood combined with some kind of self-reference. If we take the ordering of chapters that is reflected in the incunable as the right one, this circumstance -I would venture to say - might be not entirely unconnected with the need of introducing in some way the imminent treatment of insolubles.

matter what q stands for, if it is proposed it would have to be granted, because it would follow from the two preceding steps.

TABLE OF CONTENTS

SECTION 1. PRELIMINARY ASSUMPTIONS

1.1	Definition of <i>obligatio</i>	p.	67
1.2	Species of obligations	р.	68
1.3	Remarks on admission	р.	70
1.4	Objections 1–4 to the preliminary assumptions	р.	70
1.5	Replies to objections $1, 2, 4$	р.	71
1.6	How to reply <i>extra tempus</i>	р.	73
1.7	Reply to objection 3	р.	74
1.8	On the terms 'concedendum' and 'negandum'	р.	74
1.9	Definition of <i>positio</i>	р.	75

SECTION 2. RULES

2.1	First rule (<i>admissio</i>)	р.	76
2.2	Second rule (relevance to the <i>positum</i>)	р.	77
2.3	Third rule (relevance to the <i>positum</i> and <i>proposita</i>)	p.	78
2.4	Fourth rule (consistency of the cumulative set of senten-	р.	79
	ces)		
2.5	Fifth rule (irrelevance)	р.	80
2.6	Objections 1–5 (against <i>positio impossibilis</i>)	р.	81
2.7	Replies to objections 1–5	р.	81

SECTION 3. ON positio

(I) Underdetermined sentences

3.1	Sophism 1	р.	83
3.2	First solution	р.	86
3.3	Second solution	р.	87
3.4	Sophism 2	р.	88
3.5	Sophism 3	р.	89
3.6	Solution to sophisms 2–3	р.	91
3.7	Sophism 4 and solution	р.	92
3.8	Further reply to sophism 2	р.	93
3.9	Sophism 5, first objection	р.	94
3.10	Reply to the first objection	р.	94
3.11	Sophism 5, second objection	р.	96
3.12	Reply to the second objection	р.	96
3.13	Sophism 6 and solution	р.	97

(II) On contradictions

(a) Co	ontradictory <i>posita</i>				
3.14 3.15 3.16 3.17 3.18 3.19	General objection against the adopted rules. Sophism 7 A relativizing remark. Sophism 8 Continuation of sophism 7 with three objections Solution to sophism 7 Sophism 9 Solution to sophism 9	р. р. р. р.	97 99 99 101 102 103		
	gainst responsio nova	p.	105		
$3.20 \\ 3.21$	Sophism 10 (based on Swyneshed's rule for conjunction) Sophism 10 (continuation). A defense of the rule for con-	р. р.	$\begin{array}{c} 105 \\ 107 \end{array}$		
3.22	junction Solution to sophism 10 (rejection of the rule for conjun- ction)	p.	109		
$3.23 \\ 3.24 \\ 3.25$	Objection against the proposed solution to sophism 10 First reply to the objection Second reply to the objection (further variation on sop- hism 10)	р. р. р.	110 110 111		
$3.26 \\ 3.27$	Third reply to the objection (sophism 11) Objection to the third reply (solution to sophism 11)	р. р.	112 113		
	(III) Time and context of utterance				
3.28 3.29 3.30 3.31 3.32	Sophism 13 General remarks on time-linked replying Solution to sophism 13 Objections 1–5 to time-linked replying Replies to objections 1–5	р. р. р. р.	114 116 117 119 120		
	(IV) Pragmatic paradoxes				
$3.33 \\ 3.34 \\ 3.35 \\ 3.36$	Sophism 14 Solution to sophism 14 Sophism 15 Solution to sophism 15	р. р. р.	121 122 123 124		
(V)					
3.37 3.38 3.39 3.40	Sophism 16 Solution to sophism 16 Sophism 17 Variation on sophism 17 and solution	р. р. р. р.	124 126 129 131		

SECTION 4. ON questio, certificatio, dubitatio, petitio, sit verum, suppositio AND OTHER SOPHISMS ON positio

4.1	Subtypes of <i>positio</i>	p.	135
4.2	Further characterization of the subtypes	р.	137
4.3	Arguments against the taxonomy. Sophism 18 (on <i>petitio</i>)	р.	138
4.4	Solution to sophism 18	р.	139
4.5	Sophism 19 (on <i>petitio</i>)	р.	140
4.6	Sophism 19 (continuation)	р.	141
4.7	Solution to sophism 19	р.	142
4.8	Sophism 20 (on the use of subjunctive and <i>certificatio</i>)	р.	143
	and solution		
4.9	Sophism 21 (impossible vs necessary <i>per accidens</i> senten-	р.	144
	$\cos)$		
4.10	Solution to sophism 21	р.	146
4.11	Sophism 22 (on conditional sentences)	р.	148
4.12	Solution to sophism 22	р.	149
4.13	Sophism 23 (on <i>positio cadens</i>)	р.	150
4.14	Sophism 24 (variation on sophism 23)	p.	151
4.15	Solution to sophisms 23–24	р.	152
4.16	Sophism 25 (on <i>positio renascens</i>) and solution	р.	152
4.17	Sophism 26 with three objections (on conjunctions and	р.	153
	disjunctions)		
4.18	Solution to sophism 26 and reply to the first objection	р.	154
4.19	Reply to the second objection	р.	156
4.20	Reply to the third objection	р.	156
4.21	Generalization: any false sentence, compossible with a	р.	157
	contingently false <i>positum</i> , can be proved		

SECTION 5. ON impositio

5.1	First assumption on <i>impositio</i>	р.	158
5.2	Second assumption on <i>impositio</i>	р.	158
5.3	Two arguments against the assumptions	p.	159
5.4	First type of <i>impositio</i> : on spoken and written terms that	р.	160
	are assigned a meaning for the first time		
5.5	Sophism 27 (on the first type of $impositio$)	p.	161
5.6	Sophism 28 (on the first type of <i>impositio</i>)	р.	163
5.7	Second type of <i>impositio</i> : on already signifying spoken	р.	164
	and written terms that are assigned a new meaning. Sop-		
	hism 29		
5.8	Five arguments against the first <i>responsio</i> to sophism 29	р.	165
	(including sophisms 30 and 31)		

5.9	Four arguments against the second <i>responsio</i> to sophism 29: untenable conclusions follow from its adoption	p.	168
5.10	Proof that the conclusions follow from the second <i>responsio</i> (including sophisms 32 and 33)	p.	168
5.11	Third <i>responsio</i> and solution to sophism 29	p.	171
5.12	Sophism 34	р. р.	172
5.13	Solution to sophism 34	р. р.	174
5.14	Sophism 35 (on <i>suppositio</i> and variation of meaning of	р. р.	176
0.11	syncategorematic terms)	P	110
5.15	Solution to sophism 35	p.	181
5.16	Sophism 36 (again on <i>suppositio</i>) and solution	р. р.	183
5.17	Sophism 37 and solution	р. р.	187
5.18	Reply to objection 4 in sophism 35	р. р.	187
5.19	Sophism 38	р. р.	188
5.20	Solution to sophism 38		190
5.20 5.21	Sophism 39 (on relative terms) and solution	p. p	191
5.21 5.22	Sophism 35 (on relative terms) and solution Sophism 40	р. р.	191
5.23	Solution to sophism 40	р. р.	193
5.23 5.24	Sophism 41 (on the interchangeability of meanings)		$195 \\ 195$
5.24 5.25	Solution to sophism 41	p. p	197
5.26	Sophism 42 and solution	p. p	197
5.20 5.27	Sophism 42 and solution Sophism 43	p. p	198
5.27	Solution to sophism 43	p. p	200
5.28 5.29	Variation on sophism 43 and solution	p. p	200
0.29	Variation on sophism 45 and solution	р.	200
	SECTION 6. ON depositio		
6.1	Definition of <i>depositio</i> and two rules (conjunction and dis-	р.	201
	junction)		
6.2	Sophism 44 (on semantic paradoxes)	р.	202
6.3	Solution to sophism 44	р.	203
6.4	Sophism 45 and 46 (variations on sophism 44). Two ob-	р.	204
	jections and replies		
6.5	Sophism 47	р.	204
6.6	Solution to sophism 47	р.	205
6.7	Sophism 48	р.	206
6.8	Solution to sophism 48	p.	207
6.9	Sophism 49 (on similars and dissimilars)	р.	208
6.10	Solution to sophism 49. Sophism 50	р.	208
6.11	Solution to sophism 50	p.	209

NOTE TO THE TEXT

The text presented here is a transcription of the treatise on obligations included in the typographical edition of Peter of Mantua's *Logica*, printed in Venice in April 1492 by Bonetus Locatellus (from the microfilm of the exemplar conserved in Oxford, Bodleian Library, Auct.2 Q V I). It goes without saying, therefore, that it is by no means a critical edition. Rather, it should count as a preliminary study in preparation of it. The reason of this choice is that an essential step toward the accomplishment of such a task is the need of reaching at least a first comprehensive understanding of the content of the treatise, which is quite dense and complex. Moreover, Peter's *Logica* has survived in six manuscripts (four of them are complete – M, O, P and V – while two – B and L – preserve a large portion of the work, including the treatise that is object of my study) and has been printed in four editions during the XV century: Padua 1477, Pavia 1483, Venice 1492 (April) and, finally, Venice 1492 (December). Due to the complexity of the work and to the obvious necessity of a rigorous establishment of the stemma (with a view to a future edition of the text) I have opted for a first presentation of it with a partial collation of the manuscripts. I have a rough, preliminary hypothesis on the relations among the witnesses and some hints are given by [James, 1968] with regard to the treatise *De instanti*. James's stemma, however, is not fully convincing, for at least in the case of the treatise on obligations, his conclusions appear to be not always consistent with the preliminary results of my collation. Be this as it may, a good deal of further research will be needed to reach a satisfactory starting point for the edition. For the moment I assume that the text of E, B and V are related to one another, while L, M, O, P possibly belong to a distinct family. In many respects, O seems to be a good witness and this is the reason why I have picked it out as control-text for my study. In absence of a critically established text, I have decided to adopt, at least, a criterion of strong homogeneity and internal coherence: the text presented here is therefore strictly adherent to that of the 1492 (Apr.) edition. This means in turn that, save for rare occasions, the transcription faithfully reproduces that text, including its errors and imprecisions. An awkward feature of it, for instance, is a certain stylistic inconsistency, which results particularly in a somehow loose use of the demonstratives 'ille' and 'iste', that are employed interchangeably, throughout the treatise, often within the context of the same sentence to refer to the same term or sequence of terms. Despite the sense of uneasiness that this may cause to the reader, I have opted for a non-intervention policy. On the other hand, in the apparatus, the outcome of a complete collation of manuscript O is found, and for some *loci critici*, variants documented in some or all of the witnesses are registered, especially in case of conceptual difficulties. Moreover, the text of the edition is sometimes mistaken or needs to be supplemented to become understandable: in those cases too, I have made no direct intervention in the text, but decided,

rather, to discuss all problems in the footnotes, where the necessary references are also given to the variants recorded in the apparatus. I have also kept the spelling of the edition, which for instance always has 'e' instead of the classical diphthong 'ae' in words like 'consequentie', 'ille' (for 'illae') and so on. On the other hand, manuscript O has often characteristic Italian spellings – e.g. '-x' for '-ss' like 'admixum' for 'admissum' – or the termination '-cio' for '-tio' like 'obligacio, sensacio' for 'obligatio, sensatio'. When a variant of such a kind is reported in the apparatus, I have always standardized the spelling by supplying the received classical form. Punctuation and the insertion of all inverted commas in the text for mentioning words and sentences are mine.

I am aware that this approach suffers from some defects, for a critical edition would have resolved many problems from the outset, but still, given the present state of textual research on Peter's *Logica*, I have tried at least to be homogeneous and consistent in providing a text with minimal interventions and discussing separately all textual issues.

Finally some remarks on the overall typographical layout of this thesis are in order. First, I have used a double vertical stroke || in the text to indicate the change of column or folio in the edition and in the manuscripts. The indication is always reported on the right-hand margin. In case there is more than one occurrence of || on the same line, the corresponding labels in the margin are ordered top-down. Second, I obviously take full responsibility for all typos that may have slipped into the text. But there is a number of things that I have literally not been able to do and which are both related to the LATEX packages that I have used to write the thesis. First, I have not found a sensible way to force the software to include all section titles (numbered and in uppercase body) and subsection numbers (in bold) between $\langle \ldots \rangle$ as I have done for all other 'external additions' that feature in the text. Second, there are cases of awkward pagebreaks which are due to the interaction between the text, the length of footnotes and the apparatus: I have tried to fix them all, but I have not always succeeded, disappointingly. Third, the software determines automatically the spacing between words on a line. This happens to conflict with the presence of several references on the same line, because they are generated independently. As a result, there may be odd blank spaces in the footnotes. To minimize this effect, I have sometimes been forced not to respect the chronological order of references (especially for medieval authors). Fourth and last, hyphenation may also turn out be incorrect on some occasions, because the software does not unconditionally recognize the difference between Latin and English rules and to do it manually was virtually unfeasible.

I apologize for these unpleasant flaws, which might look as signs of negligence.

SIGLA ET ABBREVIATIONES

- O = Codex manuscriptus, Oxford, Bodleian Library, Canon. misc. 219, fols. $116^{vb}-126^{rb}$ (saec. XV ?)
- B = Codex manuscriptus, Berlin, Staatsbibliothek Preussischer Kulturbesitz, Hamilton 525, fols. 95^{vb}-102^{ra} (saec. XV, a. 1419 ?)
- M = Codex manuscriptus, Mantova, Biblioteca Comunale, 76, A III 12, fols. $82^{rb}-94^{va}$ (a. 1420)
- $P = Codex manuscriptus, Venezia, Archivio dei Padri Redentoristi di Santa Maria della Fava, 457, fols. <math>66^{ra}-75^{ra}$ (saec. XV)
- V = Codex manuscriptus, Venezia, Biblioteca Nazionale Marciana, L.VI.128 (2559), fols. 71^{va} - 80^{va} (a. 1426)
- L = Codex manuscriptus, Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Lat. 2135, fols. $67^{vb}-75^{va}$ (a. 1416)

abbrv.	abbreviatio
add.	addidit
conf.	confusum
del.	delevit
om.	omisit
inv.	invertit
inc.	incertum
praep.	praeposuit
transp.	$\operatorname{transposuit}$
in marg.	in margine
scr.	scripsit

$\langle \dots \rangle$	includunt	verba	$^{\rm ab}$	editore	addita
[]	$\operatorname{includunt}$	verba	ab	editore	expuncta

Obj. Objection

RObj. Reply to objection

1. Preliminary assumptions

1.1. Volo, primo, accipere quod obligatio logica est actus obligantis secundum quem ipse est obligans. Ex quo sequitur, primo, quod si obligatio logica est, obligans est; || et si obligans est, aliquis obligatur; igitur, a primo ad ultimum, si obligatio est, aliquis obligatur. Patet consequentia quia sequitur 'calefactio est, igitur calefaciens est', 'visio est, igitur videns est'.¹

 $\begin{array}{l} M \,\, 82^{\rm rb} \\ P \,\, 66^{\rm ra} \\ V \,\, 71^{\rm va} \\ O \,\, 116^{\rm vb} \\ L \,\, 67^{\rm vb} \\ B \,\, 95^{\rm vb} \end{array}$

5

E Gii^{ra}

O 117^{ra}

¹ The first preliminary assumption provides Peter's definition of *obligatio*. An *obligatio* is, on his view, the *act* by virtue of which someone plays the role of *obligans* in a disputation, insofar as he is an *obligans*. Taken as such, the definition seems to be quite uninformative and conveys an impression of circularity. However, at least two features must be singled out, namely the use of the term of *actus* and the specification of its logical character.

⁽i) Definition patterns: looking at the historical development of obligations, it is possible to identify a range of alternative definitions for the notion of *obligatio* and its subtypes. It is worth pointing out from the very beginning, however, that despite the variety of patterns available, definition is, to some extent, not the right place (or at least not the most relevant one) to look for criteria of distinction between alternative approaches to obligations: this function is much better accomplished by rules, since rules are the core of the process that governs responses in a disputation. But rules can be – and in the actual practice definitely are – chosen regardless of the adopted definition of obligation which seems to have little influence on the logical properties of the theory (in support of the view that the two problems should be treated independently, cf. [Schupp, 1993, p. 53]). Several patterns of definition can be identified in the obligational literature, as is shown in the list below (I have neglected slight variations that can be found in different authors adopting the same pattern):

⁽¹⁾ Obligatio est praefixio alicuius enuntiabilis secundum aliquem statum:

<sup>cf. [Tractatus Emmeranus de falsi positione, 1974, p. 103], [Obligationes Parisienses, 1975, p. 27], [William of Sherwood (?), 1963, p. 1], [William of Ockham, 1974, pp. 731–732], [Walter Burley, 1963, p. 34], [Marsilius of Inghen, 1489, sig. Aii^{ra}], [William Buser, 1990, p. 68].
(2) Obligatio est [...] alicuius ad aliquid ex petitione opponentis et concessione respondentis astrictio. Et obligare est: aliquem ad aliquid concedendum vel negandum quod non concederet vel negaret, in petendo opponentem et respondentem consentiendo, astringere, voluntarie autem et remota coactione: cf. [Nicholas of Paris (?), 1998, p. 171].</sup>

⁽³⁾ Obligatio est oratio composita ex signis obligationis et obligato: cf. [Albert of Saxony, 1490, sig. Niiii^{ra}], [Roger Swyneshed, 1977, p. 252], [Paul of Venice, 2002, p. 104], and [Paul of Pergula, 1961, p. 102].

⁽⁴⁾ Obligatio est quaedam ars mediante qua aliquis opponens potest ligare respondentem ut ad suum bene placitum respondeat ad obligationem sibi positam; vel obligatio est oratio mediante qua aliquis obligatus tenetur affirmative vel negative ad obligationem respondere: cf. [Richard Lavenham, 1978, p. 226], [Martinus Anglicus, 1993, p. 2], [John Tarteys, 1992, p. 671]. In addition to these authors, Ashworth has identified a significant number of English treatises that hold this definition, which is formulated in its fullest version in Richard Billingham, cf.

¹ Volo] hic add. O 2 primo] om. O 3 ad ultimum] om. O 4 Patet] et praep. O 4 consequentia] prima praep. O 4 quia] sicut O 5 est₂] sensatio est igitur sentiens est add. O

1.2. Secundo, accipiamus quod obligatio logica habet istas species, scilicet positionem et depositionem. Et si plures videndum erit. || Ex quo || sequitur quod omnis positio, in hac arte logica, est obligatio et, consequenter, quod omne ponere est obligare. Ex quibus sequitur istam consequentiam valere 'ego po-||no tibi istam, igitur ego obligo te ad istam'. Et valet ista 'ego bene pono tibi istam, igitur tu obligaris ad istam'. Et ideo bene sequitur 'ego bene posui tibi istam, igitur tu fuisti obligatus ad istam'.²

(5) Obligatio est enuntiatio cui consentit se respondens esse obligatum vel se debere respondere qualitercumque penes usum communiter disputantium per notam exigitur praecedentem: [Ralph Strode, 1517, fol. 78^{ra}].

(6) Obligatio est actus obligantis: this is the definition that Peter adopts in his treatise. Cf. also [Marsilius of Inghen, 1489, sig. Aii^{ra}] (the full passage is reported a few lines below) and [William Buser, 1990, p. 68]. In addition to that, a significant discussion can be found in the first part of Paul of Venice's treatise (especially relevant to this point is the seventh *descriptio*, providing Paul's definition of *obligatio*), cf. [Paul of Venice, 1988, p. 6].

(7) Obligatio est relatio limitans ad aliquod enuntiabile vel sibi consimile aliqualiter sustinendum: cf. [Paul of Venice, 1988, p. 6]. The terms of the relation are explained as follows: "obligatio est in praedicamento relationis, et est formaliter relatio fundata in obligante et obligato: in obligante, ratione positionis vel depositionis; in obligato, vero ratione admissionis" [Paul of Venice, 1988, p. 10].

(ii) With respect to the qualification of the kind of obligation under consideration as *obligatio logica*, it is noteworthy that such a lexical accuracy is not common in medieval treatises on the topic, and suggests the existence of other kinds of obligations beyond logical ones. In this connection, scattered remarks are found in the literature, cf. for instance [Marsilius of Inghen, 1489, sig. Aii^{ra}] "Primo notandum est quod obligatio multis modis capitur; uno modo pro actu quo quis obligatur alteri in aliquo bono, quod multis modis contingit, puta ex pacto vel sine pacto, sicut habet videri sexto Ethicorum; de hac obligatione nihil est ad presens. Secundo capitur logicaliter, puta pro actu obligantis aliquem ad sustinendum pro certo tempore aliquam propositionem secundum aliquem statum et sic capitur hic". Another account is in [William Buser, 1990, p. 66] who mentions three characterizations of the notion of obligation beyond the domain of logic: *realis, iuridica* and *practica*. In connection with this topic, noteworthy remarks are also found in [Schupp, 1993, p. 51] who, in his commentary on Martinus' treatise, suggests that the term 'obligatio' might stem from the juridical context and refers to an anonymous text *Circa tractatum obligatoriorum*. Two additional sources are a passage found in [Anonymous Merton College MS. 306, 1985, pp. 243-244] and one in [John of Wesel, 1996, p. 11] (which probably depends on the latter).

As a final remark it should be recalled that the example of heat used by Peter to clarify the definition of obligation as an *actus obligantis* occurs also in [Paul of Venice, 1988, p. 10] and might be regarded as a source for the latter.

 2 The second preliminary assumption declares how many species of obligations are to be accepted.

(i) As to the number of subtypes discussed throughout the development of the obligational literature, suffice it to recall that the taxonomy, and sometimes the terminology, i.e. the labels

6 accipiamus] *inv.* accipiendum O **6** obligatio logica] hoc genus *praep.* O **6** istas] has O **6** scilicet] *om.* O **7** plures] habeat *add.* O **7** erit] est O **9** sequitur ... consequentiam] patet hanc formam O **9** ego] *om.* O **10** ego] *om.* O **10** valet ista] hanc O **11** bene] etiam O

68

10

[[]Ashworth, 1985, p. 2].

employed to refer to one and the same type, vary from author to author, cf. [Spade, 1982b, p. 1],

and also over time (with a certain tendency to simplification in the second half of the XIV century). A maximum of six species (positio, depositio, dubitatio, petitio, sit verum and institutio, alsocalled *impositio*, as in Peter's tract, among many others) are accepted by [Walter Burley, 1963, p. 35], [Albert of Saxony, 1490, sig. Niiii+2^{vb}], [Obligationes Parisienses, 1975, p. 27], and [William of Ockham, 1974, p. 732]. The same number is found in [William Buser, 1990, p. 116] and [Marsilius of Inghen, 1489, sig. Aiiii+2^{ra}] (the latter in fact accepts only the first three). [Roger Swyneshed, 1977, p. 250 and 253], [Robert Fland, 1980], [John of Holland, 1985, p. 91], [Richard Lavenham, 1978, p. 226] accept three species, namely positio, depositio and impositio (notice that the terminology for the type concerned with variations of meaning apparently shifts, roughly from the third decade of the XIV century onwards, particularly within the English tradition after Swyneshed, from institutio to impositio). [Paul of Venice, 1988, p. 36] gives a somehow original list of three, by substituting *impositio* with *suppositio* (a kind of objation in which the *obligatum* must be upheld as necessary or true *per se*). [Brinkley, 1995, pp. 15–16], too, regards these three subtypes as the most relevant ones, although he explicitly recognizes that they do not embrace the whole domain of obligations (in the very same place he makes the interesting remark that every case of insoluble is a species of obligation). Two species (*positio* and *depositio*) are accepted by [Paul of Venice, 2002, p. 105].[Paul of Pergula, 1961, p. 102], [John Wyclif, 1893, p. 69], [Peter of Candia, Obl., Mantua, fol. 163^{ra}], [Martinus Anglicus, 1993, p. 2], [Ralph Strode, 1517, fol. 78^{ra-rb}], and Peter of Mantua, but it must be kept in mind that, as in the case of definition, setting a limit to the number of species admitted does not prevent an author from inserting in his work a number of sophisms or discussions concerning a type that traditionally has an independent status and that the author in question reduces to one of the species he accepts. This is for example what Peter does in section 5. of his treatise, where he has a lengthy discussion on *impositio*, which deserves a significant independent treatment (with its own theoretical import, as we will see) although Peter regards it as a subtype of *positio*. The move is not dissimilar from what we find in [Ralph Strode, 1517, fol. 78^{ra}], who also seems to take *impositio* as a subtype of *positio*. Strode also mentions, in the same passage, two other types of obligation: *suppositio* and *certificatio*; both of them are discussed by Peter, cf. infra, sec. 4.2, whose list is probably influenced by the English author. (ii) A second aspect is worth mentioning: even within the context of a single subtype, differences of taxonomy can be registered in different authors and, more importantly, at different moments of time. Take the emblematic example of *positio impossibilis*. This subtype of *positio* in the XIII century has a certain fortune which fades away (a gentle way to say that it disappears almost abruptly) in the first decades of the XIV century, particularly after Ockham who seems to be the last XIV century author to encompass it (cf. [Spade, 1982b, p. 5 f. 15]; for some attempts to explain the phenomenon, cf. [Martin, 1992], [Martin, 1997] and [Gelber, 2004]). Other differences in the historical evolution can be found also in the case of *dubitatio*, cf. [Spade, 1982b, p. 5 f. 17] and [Uckelman et al., 2008]. In addition to that, differences of classification and vocabulary in obligations treatises can be found also within the standard paradigm of *positio*: for instance, authors like Burley, Albert of Saxony and Marsilius of Inghen lay down an articulated subdivision according to the type of positum: e.g. positio simplex and composita, positio cathegorica, hypothetica and dependens (cf. also infra, sec. 4.13). Peter includes a variety of these problems in his treatise, as when he discusses a number of subtypes of *positio* (questio, certificatio, dubitatio, sit verum, suppositio and presuppositio, cf. infra, sec. 4.1) or devotes an entire section of the work to the analysis of sophisms on *impositio*.

As to the corollaries to the second assumption, Peter appeals to a procedure which is reminiscent of a technique sometimes proposed by Aristotle in the *Topics*, consisting in the evaluation **1.3.** Tertio, ultra, accipitur quod sequitur 'tu bene admittis A propositionem, igitur obligaris ad A propositionem'. Et de preterito 'tu bene admisisti A propositionem, igitur tu fuisti obligatus ad A propositionem', quia admittere A propositionem non est nisi promittere se responsurum ad istam secundum exigentiam eius cui promittitur; sed promittere aliquid est obligare se ad illud; igitur etc.³

1.4. (Obj.1) Sed contra ista forte arguitur, quia aliqua est obligatio qua nullus obligatur: quia scribatur ista oratio 'pono tibi istam 'tu curris" || vel aliqua talis, nullo admittente; et patet quod est obligatio, quia illa est oratio composita ex 20 signis obligationis et obligato.

 $\langle \text{Obj.2} \rangle$ Item, possum ponere tibi istam 'tu curris' te inuito et non admittente, igitur potest esse quod ponatur tibi ista absque hoc quod obligem te ad istam. Patet consequentia. Et antecedens arguitur quia possum proferre tibi istam sic significantem 'pono tibi istam 'tu || curris", te non admittente; igitur potest esse quod ponam tibi illam.

 $\langle \text{Obj.3} \rangle$ Item, etiam tu potes admittere illam absque hoc quod obligeris, quia tu potes proferre istam sic significando 'ego admitto istam', igitur potest esse quod tu admittas istam absque hoc quod obligeris ad istam.

 $\langle \text{Obj.4} \rangle$ Item si ponere est obligare, igitur poni est obligari. Igitur solum quod 30 ponitur obligatur. Sed solum propositio ponitur. Igitur solum propositio obligatur. Consequens est falsum, igitur etc.⁴

³ The third preliminary assumption stipulates the validity of the inference from 'bene admittere A propositionem' to 'obligari ad A propositionem' and of its version *de preterito* (with a possible influence of the *Topics*, as has been just recalled). The significance of this move lies in that it explicitly states the role played by correct *admissio* in the context of an obligational disputation. As will be clear shortly, while examining the objections and replies that immediately come after this passage, *admissio* is a necessary step (along with *positio*, or to say it better, with the act by means of which something is actually posited), indispensably required in order for an obligation to arise.

⁴ A sequence of four objections is raised against the first three assumptions.

Obj.1 is based on an alternative definition of *obligatio*. If we adopt the definition of obligation as 'oratio', then the link between positing and admitting, which are two conditions that must always be fulfilled in order for an obligation to arise, is lost. Thus, a sequence of written signs, provided that it contains the right elements (obligational operators such as 'pono' or 'depono'

13 Tertio, ... quod] etiam O 16 istam] A propositionem O 17 igitur] ideo O 18 Sed] forte add. O 18 ista] om. O 18 est obligatio] inv. O 20 quod] illa add. O 22 non admittente] tu potes eam non admittere, igitur potest esse quod ponatur tibi illa praep. O 23 ponatur... ista] ponam tibi illam O 24 antecedens arguitur] inv. O 24 possum proferre] potest esse quod ego proferam O 26 ponam... illam] te non admittente add. O 27 Item] ita O 27 tu potes] inv. O 27 obligeris] obligaris O 27–28 tu potes] inv. O 28 proferre] perficere O 28 significando] significantem 28 istam] absque hoc quod obliger add. O 29 quod] tu add. O 29 ad istam] om. O 30–32 Item ... igitur etc.] BEV om. MOP 15

V 71^{vb}

25 L 68^{ra}

of the validity of an argument form, if it is invariant with respect to the tenses of the verbs employed in its formulation. Peter makes use of this approach twice in the treatise, the first time here, and the second one in the next section.

1.5. (RObj.4) Sed huic dicitur quod equivoce possunt sumi illi termini.⁵

 $\langle \text{RObj.1} \rangle$ Et ad primum illorum dicitur quod illi termini 'oratio', 'propositio' pro maiori parte apud logicum capiuntur prout non sunt nomina verbalia. Et ita conceditur quod propositio est absque proponente et oratio est absque orante. Sed capiendo ista nomina prout || verbalia sunt, valet hoc argumentum 'propositio est, igitur proponens est', 'oratio est, igitur orans est'. Et ita sequitur 'obligatio est, igitur obligans || est'. Et hoc modo in diffinitione locuti sumus. Unde, isto modo, obligatio est illud per quod responsalis obligatur. Et sic iste terminus 'obligatio' est de predicamento actionis.⁶ Et secundum istum modum patet responsio ad primum.

Finally, Obj.4 is a variation on the theme, once again directed against the corollary of the second assumption according to which every position is an obligation. The argument is based on the passage from the active to the passive form of the verbs 'ponere' and 'obligare' and runs as follows: only what is posited is obligated, but only sentences are posited, therefore only sentences are obligated. The conclusion is false: even if Peter does not explicitly point it out, he takes for granted that *someone* too (i.e. the respondent) is obligated in a obligation, therefore the assumption that every position is an obligation, from which we have drawn this false conclusion, should be rejected as well.

 5 Peter first replies, with a quick dismissal, to the last objection, which is fallacious since it hinges on the fact that the term *obligari* can be taken as referring equivocally either to someone, i.e. the respondent, or to something, i.e. a sentence.

⁶ Cf. [William Buser, 1990, p. 68].

33 Sed ... termini.] BEV om. MOP **34** Et ... illorum] ad horum primum O **34** dicitur] respondetur O **34** 'oratio', 'propositio'] post parte transp. O **35** logicum] loycam O **37** hoc] om. O **39** in diffinitione] om. O **40** responsalis] responsurus O

M 82^{vb}

35

40

O $117^{\rm rb}$

and *obligata*-sentences) would count as an obligation, even if there is no one who admits what is posited.

Obj.2 and 3 are directed against the arguments presented in sec. 1.2 and 1.3. The inferences in sec. 1.2 justify the conclusion (i) from 'ponere' to 'obligare' and (ii) from 'bene ponere' to 'obligate (i.e. if a posite p to b, then a obligates b and so on), whereas in sec. 1.3, likewise, the inference is assumed to hold as well in the case of 'admittere' (if b correctly admits p, then b is obligated). The two objections claim, on the contrary, that *positio* without *admissio*, and vice versa, can stand alone without giving rise to an obligation, which amounts to denying that the inference from 'ponere' to 'obligare', on the one hand, and from 'admittere' to 'obligari', on the other hand, are sound. The problem at stake is the relation between *positio*, *admissio* and obligatio. As is shown by the replies to these objections, the idea that seems to lead Peter in these passages is that of a strong link between the three notions, in such a way that correct position implies admission (properly speaking, nothing can be said to be posited, if it is not admitted; otherwise it is only an utterance that is performed, not an actual position), correct admission implies position (again, properly speaking, nothing can be said to be admitted if it is not posited by someone, cf. infra, sec. 1.7 "cum admittere sit promittere et non sit promissio sine utriusque partis consensu, non est admissio sine obligante"; otherwise we are, once again, left just with an act of utterance by means of which the respondent merely says that he admits something, even if nothing has been posited to him) and, last, the notion of 'obligatio' implies both correct position and correct admission.

Sumpto autem isto termino 'obligatio' primo modo – secundum quem modum non accipimus – non valet consequentia. Neque illo modo est de predicamento actionis, sicut neque iste terminus 'oratio' vel 'constructio'. Nec tamen, hoc modo sumpto termino, est illa diffinitio bona 'obligatio est oratio composita ex signis obligationis et obligato', quia diffinitum ponitur in diffinitione neque obligatio est illud aggregatum.⁷

(RObj.2) Ad secundam formam dicitur || negando assumptum illud, scilicet B 96^{ra} 'potest esse quod ponam tibi istam 'tu curris' absque hoc quod obligem te ad 50

A similar example, that clarifies how we should understand the term *verbaliter*, is also found in the Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Fii^{ra}] "Ad sextum dicitur quod iste terminus 'relatio' potest accipi verbaliter prout derivatur a 'refero, refers', et pro tunc dicit actualem relationem et non est genus generalissimum in predicamento relationis quo ad abstracta. Sed alio modo 'relatio' est idem quod 'respectus', non connotando aliquam actionem sed puram habitudinem respectivam alicuius ad aliquid; et illo modo est genus generalissimum. Et est sicut de istis terminis 'propositio', 'constructio', 'oratio'. Unde sicut est propositio licet nullus proponatur et constructio licet nullus construat et oratio licet nullus oret, sic est relatio licet nullus referat". Again, later on in the same text, [Peter of Mantua, 1492a, sig. Fii^{rb}] "Ad decimoquartum dicitur quod iste terminus 'corruptio' et iste terminus 'remissio' et 'perfectio' aliquando sunt nomina verbalia denotantia actuales actiones. Et isto modo sunt in predicamento actionis. Et secundum istum modum hoc argumentum tenet 'hoc est remissio, igitur hoc est actio', 'hoc est perfectio, igitur hoc est actio [...] Alio modo illi termini 'intensio' et 'remissio' sunt termini nominales entia non de predicamento actionis sed de predicamento habitus vel relationis significantia, vel secundum alios de predicamento qualitatis. Et isto modo non sequitur 'hoc est remissio, igitur hoc est actio'".

With respect to this set of problems, see also the objections put forward by [William Buser, 1990, p. 68] and [Marsilius of Inghen, 1489, sig. Aii^{rb}] against alternatives definitions of *obligatio*. The definition as 'oratio composita ex signis obligationis et obligato' is held by Albert of Saxony who may be reasonably seen as the target of these remarks. Albert says in his Sophismata: "Obligatio est oratio (composita) ex signis obligationis et obligato", cf. [Albert of Saxony, 1490, sig. Niii^{rb}]. As Ashworth has pointed out, cf. [Paul of Venice, 1988, p. 14 f. 13], this definition is rejected by Marsilius in virtue of its generality, while Buser claims that it does not provide the nature of the defined. Here Peter seems to give an additional reason by suggesting that the definition does not work because it is circular. To be noted is also the fact that the term 'aggregatum' used in connection with the definition of obligation as 'oratio composita' is found in [Roger Swyneshed, 1977, p. 252].

43 Sumpto] Supp praep. et del. O **44** consequentia] illa praep. O **45–48** Nec ... aggregatum] om. O

45

 $^{^{7}}$ The reply to the first objection is based on a distinction between two readings that are available for certain terms (as verbalia or non verbalia). In the first sense, which is connected to the definition of obligation as *actus* and the corollary, with the examples of heat and vision, presented in sec. 1.1, the alternative definition of obligation as *oratio* is automatically ruled out, since the presence of individual agents (one positing, the other admitting) is presupposed in the definition itself, which is formulated in terms of actus. Moreover, there is an additional reason to dismiss the alternative definition, even if we took the term 'obligatio' in a non-verbal sense: the definition, according to Peter, would still suffer from circularity, since the defined is part of the definition.

istam', quia ponere est obligare.⁸

 $\langle i \rangle$ Et negatur ista consequentia 'potest esse quod ego proferam istam propositionem 'pono tibi istam 'tu curris" absque hoc quod obligem te; igitur potest esse quod ponam tibi istam absque hoc quod obligem te', quia non sequitur 'profero istam 'pono tibi istam 'tu curris", igitur pono tibi istam 'tu curris", sicut non sequitur 'profero istam 'pono tibi istam tu 'curris", || igitur obligo te ad istam 'tu curris", quia numquam est ponere nisi sit admittere.⁹

 $\langle ii \rangle$ Et si arguitur, quia prius ponitur propositio quam admittatur, dicitur quod non, licet prius proferantur illi termini 'pono tibi istam 'tu curris" vel aliam, quia non est positio nisi quando obligatur respondens.

 $\langle iii \rangle$ Et si arguitur quod numquam sunt simul positio et admissio || et per consequens numquam est obligatio, quia, ut frequentius, prius est corrupta ista prolatio talis orationis 'pono tibi istam 'tu curris" quam respondens dicat 'admitto illam', huic dicitur quod in hac arte presupponimus, gratia argumenti, eandem propositionem || remanere vel unam sic significantem sicut significavit illa quam intendebat ponere ponens. Non enim posset bene de vocalibus disputari, quia ut frequenter quod ponebatur non est, quando respondens concedit vel negat illud.

1.6. Quarto, volo ponere quod extra tempus obligationis rei veritas est fatenda.¹⁰

⁹ A trace of this line of thought might have influenced Paul of Venice: as has been already pointed out by Ashworth, a sequence of arguments similar to what Peter is suggesting here and in the next two sections can be found in [Paul of Venice, 1988, p. 6].

55 P 66^{va}

V 72^{ra}

 $65 E Gii^{va}$

60

⁸ The replies to Obj.2 and Obj.3 point out that we cannot correctly speak of *positio* or *admissio* as stand-alone moments of a disputation since one logically presupposes the other. We are entitled to speak correctly of something being posited in a disputation if and only if it is posited to someone who, in turn, admits it (in this respect cf. par. $\langle i \rangle$ for the difference between 'I utter the sentence 'I posit something to you" and 'I posit something to you'; and par. $\langle ii \rangle$ for the temporal occurrence of position before admission). Conversely, no one can be said, properly speaking, to admit a sentence that has not been posited to him (cf. *infra*, sec. 1.7). Moreover, the fact that *positio* and *admissio* do not occur at one and the same time, since their utterances take place at different moments, is without effect: for it is assumed that what is signified by those utterances has some kind of persistence throughout the disputation (cf. par. $\langle ii \rangle$).

¹⁰ This assumption is universally accepted by authors of obligations treatises, insofar as it is a necessary condition for the assessment, when the time of the obligation has come to an end, of what has been going on during the disputation; in particular it is a condition for an objective evaluation of the responses given by the respondent. It is formulated in the same words, for instance, in [Ralph Strode, 1517, fol. 78^{va}], [Paul of Venice, 1988, p. 34], and [Walter Burley, 1963, p. 53].

^{52–53} propositionem] *om.* O **54** quod] ego *add.* O **54** ponam] pono O **56** pono ... 'curris'] *om.* O **59** aliam] tales O **60** respondens] *om.* O **62** ut frequentius] frequenter O **63** talis] illius O **63** admitto] illam ego *praep.* O **64** gratia argumenti] *om.* O **65** remanere] *om.* O **65** sic] *om.* O **65** significavit] *om.* O **66–67** Non ... illud] EV *om.* MO **68** Quarto, ... fatenda] *om.* MOP; volo adhuc ponere ... fatenda BV

1.7. (RObj.3) Et per istam respondetur ad ul-||timam formam¹¹ negando istam 70 M 83^{ra} consequentiam 'tu profers istam propositionem 'ego admitto istam 'tu curris"; igitur tu admittis istam 'tu curris" quia, cum admittere sit promittere et non sit promissio sine utriusque partis consensu, non est admissio sine obligante, igitur etc.

1.8. Quinto, postea, accipiendum est quod illi termini 'concedendum' et 'negan-75 dum' idem significant, in sequentibus, quod 'dignum concedi' et 'dignum negari'. || Et non participialiter, scilicet quod significet 'concedendum' quod concedetur et 'negandum' quod negabitur, quia isto modo necessarium simpliciter, bene respondendo, est negandum et impossibile simpliciter concedendum.¹²

The second problem is textual: the clause 'ad ultimam', which occurs in our incunable, is inconsistent with the rest of the text, for it should refer to Obj.4, not to Obj.3. But Obj.4 has already received a reply (cf. supra 1.5). The solution is provided by the manuscripts. In M, O and P the whole passages containing Obj.4 and RObj.4 are lacking and, therefore, the clause 'ad ultimam' correctly refers to Obj.3, which is actually the last objection dealt with in those witnesses. In manuscript V, Obj.4 and RObj.4 are witnessed, but so is also the clause 'ad ultimam': the text is thus inconsistent (possibly the inconsistency in E is derived from the inconsistency in the line of transmission of V). If we want to maintain Obj.4 and RObj.4, the best choice is therefore adopting the last reading available, namely that of manuscript B, which has the whole set of four objections and replies, as in E and V, but reads 'ad penultimam' instead of 'ad ultimam' in the passage we are considering.

 12 This treament of the basic obligational properties 'concedendum' and 'negandum' can be traced as far back as the first half of the XIII century. A very similar formulation, based on the distinction *nominaliter* vs *participialiter*, can be found in one of the first witnesses of the obligational literature, cf. [Nicholas of Paris (?), 1998, p. 191]. A characterization in the same terms (i.e. 'concedendum' = 'dignum concedi', 'negandum' = 'dignum negari') is outlined by [Ralph Strode, 1517, fol. 78^{va}]. Strode does not present the distinction in terms of the readings *nominaliter* vs *participialiter*. Peter's treatment is retrieved, afterwards, by Paul of Venice [Paul of Venice, 1988, p. 34] and [Paul of Pergula, 1961, p. 106].

 $L 68^{rb}$

¹¹ Two problems must be highlighted in connection with this passage. First, the target of this reply is clearly Obj.3, cf. supra 1.4, but it is not altogether clear in what sense the expression 'per istam' should be taken: apparently, it refers to the fourth assumption, but it is difficult to see what kind of role that assumption plays in justifying the reply to Obj.3. In other words, while the reply to the objection – denying the inference from 'a makes the utterance 'I admit p'' to 'a admits p' – may be seen as an acceptable move, it seems independent of the fact that outside the time of obligation the respondent must always tell the truth, while assessing the outcome of the disputation. The only plausible solution I see is that there is actually no logical link entitling us to draw, from the mere utterance of a sentence, the conclusion that what the uttered sentence says is true. For this reason, such an inference must be rejected outside the time, because it is invalid (or, to put it better, the corresponding conditional must be denied, because it is false).

⁷⁰ istam] illud etiam O 70 ul-||timam] EOPV penultimam B 72 tu curris] om. O 72–73 non ... promissio] promittere sive obligare non fit O **73–74** igitur etc.] om. Ο 75 Quinto] om. O 75 postea, ... est] inv. O 75 et] om. O 76 significant] significent O **76** et] vel O **77** scilicet quod] ut O **79** concedendum] bene respondendo est *praep*. O

1.9. Sexto, ulterius, dicitur quod positio est obligatio per quam respondens obligatur ad concecendum positam propositionem, cum sibi proposita fuerit.¹³

80 Sexto] *om.* O **80** dicitur] dicamus O **81** positam] positum O **81** propositionem] per positionem O **81** sibi] ibi *ante* per positionem *transp.* O **81** proposita] propositum

Peter makes use of the same argument in the *Tractatus de consequentiis*, [Peter of Mantua, 1492a, sig. $\text{Eiiii}+3^{\text{vb}}$] "Et accipiatur in dicta regula 'concedendum' nominaliter et non participialiter, quia accipiendo participialiter multa necessaria simpliciter sunt neganda et multa impossibilia simpliciter sunt concedenda bene respondendo".

 $^{^{13}}$ This is the specific definition of *positio*. Its distinctive feature is that when something is posited (and admitted) at the beginning of a disputation, it must be conceded, whenever proposed at any subsequent step of the disputation. The criterion adopted for determining the subdivision of obligations into their subtypes, therefore, appeals to the way the respondent must treat (i.e. respond to) the *obligatum* when it is put forward as a *propositum*. This idea is confirmed by the dual definition of *depositio*, cf. *infra*, sec. 6.1.

2. Rules

2.1. Quibus positis, pro regula teneatur quod omnis propositio intellecta, scita esse possibilis et non repugnans alicui bene admisso vel bene concesso, que petitur per signa positionis admitti, est admittenda.¹⁴ Ut si ponatur quod nulla propositio

Second, the rule states that the respondent is bound to admit a given sentence if four conditions are fulfilled: the sentence must be (1) understood, (2) known to be possible, (3) not repugnant to anything that has been already correctly admitted or granted and (4) posited. I shall adopt, with some modifications, the formalization first introduced by [Knuuttila and Yrjönsuuri, 1988], that I have also used above. But the focus here is on presenting the rules in thesis-form. There might be good reasons to maintain that rules should be viewed as tantamount conditionals: at least in Peter's treatise this seems to be how the author himself regards them (in this respect, cf. *infra* 3.35; and also, for an analogous treatment, [William Buser, 1990, pp. 108–110]). This means, in turn, that they should be put, as it were, at the object-language level. The first rule can be formalized as follows:

(R1)
$$\forall p \forall q ((U_a p \land K_a \Diamond p \land ((A_a q \lor C_a q) \Rightarrow \neg (q \Rightarrow \neg p)) \land Pp) \Leftrightarrow OA_a p)$$

The symbols should be read as follows (cf. also *infra*, 'List of symbols'): U_ap stands for 'a understands p', K_ap for 'a knows that p', A_ap for 'a admits p', C_ap for 'a grants (or has granted) p' while Pp stands for 'p is posited'; finally, the operator O stands for 'ought to' so that the expression OA_ap must be interpreted as 'a ought to admit p'. The same holds for the expressions that can be formed by applying the operator O to C (= 'grant'), N (= 'deny') and D (= 'doubt'). What we obtain are expressions of the form OC_ap , ON_ap and OD_ap that correspond to the fundamental operations described by the standard obligational vocabulary 'concedendum', 'negandum' and 'dubitandum'. It is worth noting that, although obligations have nothing to do with ethics or deontic logic (to my knowledge this view is endorsed only by [Dumitriu, 1977]), appealing to the basic notion of the latter can be fruitful to our purposes. For if we look at the dual notions representend by the box and diamond of deontic logic, it is easy to see that they can help us understand, by analogy, some properties that we want O (= ought to) to have. O can be regarded as a modal operator with a corresponding dual notion L (= it is permitted to) such that, by definition, O is equivalent to $\neg L \neg$. In particular, this approach is useful in

¹⁴ This is the first obligational rule put forward by Peter and it is meant to govern the very first step of a disputation, providing criteria for the admission of sentences.

Two facts must be noted, in this respect. First, it is very unusual to come accross an explicit rule that governs *admissio*. The requirement that a sentence be possible, in order for it to be admissible, is generally accepted throughout the development of the obligational literature (except for the case of *positio impossibilis*, where some kinds of impossible sentences can be admitted too). The duty of admitting (i.e. the explicit formulation of the principle that the respondent is obligated to admit) a possible sentence, on the other hand, is very seldom given (along with additional conditions) in the form of a rule. This is the case in Peter of Mantua, [Paul of Venice, 1988, p. 51] and another source, namely [Anon. Obligationes MS *Lat. misc.e 79*, fol. 18^{ra}] (the remark is due to Ashworth, cf. [Paul of Venice, 1988, p. 51, f. 2], I have not seen the manuscript); [Marsilius of Inghen, 1489, sig. Bi^{va}] gives it as a special rule of *positio* while it is found in [William Buser, 1990, p. 92] as a corollary to the second assumption. Again, the requirement is present, as the first assumption, in [Ralph Strode, 1517, fol. 77^{va}].

¹ regula] principio O 2 et] om. O 2 alicui] alteri O

sit aut quod nullus loquatur vel quod nulla obligatio fiat aut quod tu non sis, admittendus est casus.¹⁵

2.2. Secunda regula: omne bene admissum, propositum eodem modo significans, est concedendum. Et omne sequens ex eo et propositum est concedendum. Et omne sibi repugnans est negandum.¹⁶

the case of negation, because it enables us to correctly formulate expressions such as 'non est negandum', which should be read as $O \neg (= \neg L)$, it is not permitted to) rather than $\neg O$ (= $L \neg$, it is permitted not to). I have recalled above the opportunity to formulate explicitly certain negative duties that may arise in the context of a disputation. Interpreting O in the way just described can help us understand such negative obligations in terms of prohibitions (otherwise the denial of a duty to respond in a given way might ambiguously lend itself to be seen as a permission not to respond in that way. In this respect, cf. also [Knuuttila and Yrjönsuuri, 1988].

¹⁵ The kind of sentences provided here, as tantamount examples of *posita* that should be admitted according to the criteria established by (R1), belongs to an inventory of cases that are often referred to, in the secondary literature, as pragmatic paradoxes, i.e. problematic sentences that may cause trouble in the context of their utterance. For a discussion directly linked to the context of obligations, cf. especially [Ashworth, 1984]. It must be noted that the opinions about the admissibility of such sort of sentences are various in the obligational tradition, and do not depend on the adoption of any given set of rules. Within the framework of *responsio antiqua*, for instance, some authors openly reject posited sentences such as 'Nihil est tibi positum' or 'Tu non es', because they are considered incompatible with the very same fact that an obligation may actually arise. Other authors, like Peter, Buser, or Paul of Venice, on the other hand, admit this kind of sentences, their strategy of response focusing rather on the specific replies that must be given to the *proposita* that are put forward thereafter, in order to maintain consistency. For the peculiar case of Strode, cf. [Ashworth, 1993, especially pp. 366–365].

¹⁶ The second rule performs two tasks. The first clause (R2.1) determines the respondent's behaviour in the presence of the *positum*: whenever this is put forward by the opponent, during the disputation, with the same meaning, it must be granted. In this connection it is remarkable that medieval authors throughout the development of the obligational tradition are well aware of the risks of equivocation and opaque contexts. Peter's clause 'eodem modo significans' is very close, for instance, to Burley's requirement that the *positum* be admitted only when it is 'sub forma positi propositum'. If the opponent posits 'Marc is running', the respondent, once he has admitted it, will be bound to grant it whenever it is proposed during the disputation, in the same form, but he will be by no means required to grant 'Tully is running' as well, on these grounds alone. The second clause of the rule establishes how to deal with relevant sentences following from (cf. (R2.2)) or incompatible with (cf. (R2.3)) the positum alone: if a sentence follows from the *positum* it must be granted whenever proposed during the disputation, whereas if its negation follows from the *positum*, the sentence must be denied whenever proposed. As for the meaning of 'sequi', the crucial notion presupposed here is that of a formal consequence. As I have said above, I will not enter the question of the logical properties of this fundamental concept of medieval logic: it is problematic under various respects, for its historical development, for the differences of definition and rules adopted by medieval authors and for its theoretical import in connection with related notions in the contemporary philosophy of logic (e.g. entailment, relevance, conditionals, consequence relations). A good study devoted to the notion of

4 quod] om. O 4 quod] om. O 5 admittendus] admittenda O 5 casus] om. O 6 Secunda regula] item O 7 et] om. O 7 est concedendum] om. O 8 est] om. O 5 O 117^{va}

2.3. Tertia regula: omne sequens ex bene admisso cum bene concesso vel bene concessis et bene admissis, et opposito bene negati vel bene negatorum, est 10 concedendum propositum. Et quodlibet repugnans est negandum.¹⁷

consequence in Peter of Mantua is [Bertagna, 2000]. For an investigation of the logical features of formal consequences in connection with relevance logic, cf. [Read, 1993a] and bibliography therein. Finally, for more general accounts of the notion of consequence in medieval logic, cf. [Boh, 1982] and [Dutilh Novaes, 2008]. I will confine myself, in this context, to assume that the kind of consequence relation used by Peter may be tentatively described (or its logical behaviour may be described) by appeal to strict conditionals (indicated here by means of the symbol (\Rightarrow)), which will be therefore my candidate, in the following, to represent the notion in a modern perspective and to characterize the meaning of terms such as 'sequens', 'pertinens', 'repugnans' or 'impertinens' (in support of this choice, cf. also [Yrjönsuuri, 2001a] and [Schupp, 1993]). Peter's definition of a valid consequence is stated at the very beginning of his Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii+2^{va}] "Consequentia bona denominata a ly 'si' vel 'ergo' est necessaria habitudo duarum propositionum, quarum secunde non potest contradictorium stare cum prima sine nova impositione vel potest esse convertibilis cum una tali sine sua nova impositione". As to whether different types of consequences may come into play in the obligational framework, interesting remarks are found in [Peter of Candia, Obl., Mantua, fol. 164^{ra-vb}], who applies the distinction between formal and material consequence to the notion of 'following' in the context of obligations (for instance, a relevant sentence can be said to be materially or formally relevant according to the type of consequence in which it occurs as a conclusion, the premises being one or more already correctly granted or denied sentences). Be this as it may, the rule can be formalized as follows (the new symbol R is such that Rp stands for 'p is proposed'):

(R2.1)
$$\forall p ((A_a p \land R p) \Rightarrow OC_a p)$$

(R2.2)
$$\forall p \forall q ((A_a p \land (p \Rightarrow q) \land Rq) \Rightarrow OC_a q)$$

(R2.3)
$$\forall p \forall q ((A_a p \land (p \Rightarrow \neg q) \land Rq) \Rightarrow ON_a q)$$

¹⁷ The third rule instructs the respondent on how he is supposed to deal with sentences that are either sequentially or incompatibly relevant not only to the *positum* (which was the business of the preceding rule), but also with respect to the subsequent steps of the disputation. There is enough evidence, on these grounds alone, to conclude safely that Peter should be placed, beyond any reasonable doubt, within the framework of the responsio antiqua tradition. As is well known, the most distinctive feature of this line lies in that the relevance of sentences at any given step of a disputation is defined in terms of logical dependence on the cumulative outcome of the disputation up to that point, i.e. on the conjunction of the positum along with all correctly granted and (all the negations of) correctly denied sentences put forward in the disputation. Peter's adoption of this standard set of rules is reflected throughout the treatise by its application in the analysis of sophisms (strong evidence along these lines is to be found, especially, in the third section of the treatise, where he deals with a variety of issues concerning contradictions and definitely rejects *responsio nova*, i.e. the revised rules introduced by Swyneshed). I propose to rephrase the content of the present rule by means of two distinct clauses, namely (R3.1) and (R3.2), in order to better spell out the meaning of the text (the new symbol G is introduced here as a shortcut to indicate sentences that have been correctly admitted, granted or that are the

⁹ Tertia regula] item O 10 et] vel O 10 admissis] admisso O 10 vel] ex oppositis add. O 10-11 est ... propositum] *inv.* O

2.4. Quarto, pro regula teneatur quod omnes propositiones concesse et contradictorie negatorum debent facere copulativam possibilem. Et hoc est quod aliqui, voluerunt dicere || cum dixerunt quod omnes responsiones, in arte obligatoria, sunt retorquende ad idem instans.¹⁸

opposite of correctly denied sentences in a given disputation; Gq therefore stands for 'q is (i) a correctly granted or the opposite of a correctly denied sentence or (ii) q is a conjunction of such sentences'):

(R3.1)
$$\forall p \forall q \forall r ((A_a p \land G_a q \land ((p \land q) \Rightarrow r) \land Rr) \Rightarrow OC_a r)$$

(R3.2)
$$\forall p \forall q \forall r \left(\left(A_a p \land G_a q \land \left(\left(p \land q \right) \Rightarrow \neg r \right) \land R r \right) \Rightarrow ON_a r \right)$$

What (R3.1) says is that, at any given step of the disputation, whatever is sequentially relevant with respect to the sequence of sentences (including the *positum*) that have been put forward and correctly evaluated at the preceding steps, must be granted. Correspondingly, (R3.2) provides instructions for the duty of denying a sentence when its negation follows from (i.e. when it is incompatibly relevant with respect to) that very same set of sentences. If we take together (R2.1), (R2.2), (R2.3), (R3.1) and (R3.2) we have a partial set of rules to deal with (sequentially and incompatibly) relevant sentences (for the complete system, we still need to wait for the rule that governs the replies to irrelevant sentences, which is formulated below, cf. *infra*, sec. 2.5).

¹⁸ This rule should count as a test of consistency for the outcome of a disputation. If we regard a disputation as an ordered sequence of n evaluated sentences, with the *positum* as its first element, rule four requires that their conjunction be a possible sentence. This in turn, is equivalent, as Peter rightly points out, to the requirement that all responses be given as if they were referred to one and the same instant of time and this is indeed the form in which the statement is usually presented. Here is a way to formalize it (the operator $\bigwedge_{i=0}^{n-1}$ stands for the conjunction of n sentences):

(R4)
$$\forall p_i (G_a p_i \Rightarrow \Diamond \bigwedge_{i=0}^{n-1} (p_i))$$

The rule has a long tradition since it belongs to a nucleus of principles that are already discussed in the early phase of the theory's development. A standard feature in XIII century obligations is that disputations were taken to be led not over time but in an instant-like temporal environment. From a conceptual point of view, this assumption has a special relevance. It is easy to see that if we have this rule, there is little room even for 'harmless' forms of contradictions as in the case of Swyneshed's theory (if we accept that an account in terms of two-column book-keeping such as that given by [Yrjönsuuri, 2001a, pp. 22–23] solves all problems with Swyneshed's view). This principle becomes a matter of controversy at the turn of the century (a detailed discussion is found in [Gelber, 2004]). If time is allowed in the disputation, the evaluation of irrelevant sentences is subject to variations, since something that is true at the beginning of the disputation may become false after a lapse of time. If replies to irrelevant sentences depend on their truth value outside the disputation, it follows that variations in the states of affairs affect the way we are supposed to reply to the corresponding sentences. Now, to avoid this sort of clashes, the reference to a single instant of time was assumed in the *responsio antiqua*. With

12 Quarto] item O 12 regula] principio O 12–13 contradictorie] opposite O 13 negatorum] negatarum O 13 aliqui] alii O 14 quod] om. O 14 in ... obligatoria] post instans transp. O 14–15 sunt retorquende] inv. O 15

 $P 66^{vb}$

2.5. Quinta regula: ad omnem propositionem impertinentem concesso vel concessis, || respondendum est secundum sui qualitatem, scilicet si sit vera, concedenda; si sit falsa, neganda. Et istam || regulam ego extendo ad impositionem secundum opinionem hic sustinendam.¹⁹

Swyneshed the situation changes, since he assumes that disputations occur over time. For this reason, the importance of explicitly adopting the present rule becomes even greater in the late XIV century, since it amounts to an utter rejection of Swyneshed's views. Cf. *infra* 3.21 for a context where the rule applies, clearly in contrast with the latter's account of conjunctions.

For an analysis of the relationship between consistency, compossibility and cotenability (the latter resulting from contingency plus logical independence), cf. [Martin, 1993]. Peter's formulation of the rule is also briefly discussed in [Keffer, 2001, p. 148]. Finally, for a characterization of what it means for a conjunction to be possible, cf. *Tractatus de consequentiis* [Peter of Mantua, 1492a, sig. Fiii^{vb}] "Ad possibilitatem copulative requiritur quod quelibet pars sit compossibilis alteri et quod omnes collective sint compossibiles".

¹⁹ The fifth rule governs the treatment of irrelevant sentences. Peter never gives a definition of the notions of relevance and irrelevance in his treatise. Nevertheless, a close scrutiny of how rules are applied in the examples shows that it is reasonable to assume that his own interpretation coincides with the standard account of responsio antiqua. Relevant sentences are, thus, understood in terms of the relation of logical dependence (sequentiality or incompatibility) they bear towards the *positum* and the cumulative set of sentences that make up the disputation, whereas irrelevant sentences are to be understood as the complement of relevant ones, i.e. as those that are not relevant. Therefore, irrelevant sentences are logically independent with respect to anything that has been previously admitted, granted or denied during the disputation. Before providing a formalization of the rule, two things must be noted. First, there is no trace of an explicit indication of the treatment of doubtful sentences in Peter's text, although throughout the treatise it is clear that Peter assumes the last clause below to hold as well as the two explicitly stated clauses governing concession and denial, respectively. Second, the reference to the respondent's state of knowledge, as to a general criterion on the basis of which irrelevant sentences should be evaluated, does not enter the discussion here, but again the standard treatment of irrelevant sentences in the obligational literature assumes that the respondent does always have, as it were, a background to rely on when he has to establish the truth value of a given irrelevant sentence: this background is usually formulated in terms of knowing something to be true or false, or being in doubt about it. Moreover, this kind of approach is evident from the application of this rule throughout the treatise, although in this passage it is not fully and explicitly laid down in its components. The rule can be formalized as follows:

(R5)
$$\begin{array}{l} \forall p \,\forall q \,\forall r \left(\left(A_a \, p \,\land\, G_a \, q \,\land\, \neg \left(\left(\left(p \,\land\, q \right) \Rightarrow r \right) \,\lor\, \left(\left(p \,\land\, q \right) \Rightarrow \,\neg r \right) \right) \,\land\, Rr \right) \\ \Rightarrow \left(\left(K_a \, r \,\Rightarrow\, OC_a \, r \right) \,\lor\, \left(K_a \,\neg\, r \,\Rightarrow\, ON_a \, r \right) \,\lor\, \left(\left(\neg K_a \,r \,\land\, \neg K_a \,\neg\, r \right) \Rightarrow\, OD_a \,r \right) \right) \right) \end{array}$$

The antecedent of this conditional is an explicit formulation of the logical properties of being irrelevant: it could be replaced indeed by a shorter clause, if we assume that the definition of 'being irrelevant' coincides with the conditions that for a given a sentence p, neither p nor its negation $\neg p$ follow from the *positum*, either alone or in conjunction with correctly granted or the opposite of correctly denied *proposita*. For the extension of this rule to the case of *impositio* cf. *infra*, sec. 5.11.

16 Quinta regula] item O 17 scilicet] om. O 18 ego] etiam O 19 opinionem] abbrv. ap \overline{ne} O

V 72^{rb}

 $M 83^{rb}$

2.6. $\langle Obj, 1 \rangle$ Sed contra istam regulam forte arguitur, quia ex illa sequitur quod 20 non omnis positio est obligatio neque omne ponere est obligare. Consequens falsum. Et consequentia patet, quia regula ista innuit quod obligatur respondens solum ad bene admissum. Igitur, licet ponatur ista 'homo est asinus' et admittatur, adhuc non est obligatus respondens ad istam, quia non tenetur eam concedere quam non bene admisit. 25

(Obj.2) Ex isto sequitur quod non est possibile quod aliquis obligetur ad propositionem simpliciter impossibilem secundum positionem: quia non tenetur respondere, ut innuit regula, nisi ad licite promissum seu admissum; sed numquam licite admittitur impossibile simpliciter secundum positionem; igitur etc.

(Obj.3) Item, data ista regula, sequitur quod positum et admissum, || scitum 30 B $96^{\rm rb}$ esse tale,²⁰ est negandum et contradictorium concedendum respectu eiusdem.

(Obj.4) Item, sequitur quod, in aliqua consequentia bona et formali, denominata a li 'si' et significante ex compositione suorum terminorum etc., antecedens est concessum et consequens negandum. Et tunc sequitur quod non omne sequens ex concesso vel concessis est concedendum.

(Obj.5) Item, sequitur ultra quod non omne repugnans concesso vel concessis est negandum.²¹||

2.7. Sed pro istis advertatur quod admittere propositionem aliquam est promittere se responsurum ad eandem secundum exigentiam arguentis. Et cum non omne promissum sit servandum, sed solum licite || promissum, ideo non ad omne admissum repondendum est secundum quod est promissum, quia forte est non licite admissum vel promissum.²²

35

40 L 68^{va}

 $E \operatorname{Gii^{vb}}$

²⁰ I. e. *impossibile simpliciter*.

 $^{^{21}}$ A sequence of five objections – or, perhaps more precisely, of five remarks – is raised here with the aim of qualifying the second rule (concerning the reply to the *positum* and to *proposita* that are relevant to it). In particular, the focus is on the eventuality that an impossible simpliciter sentence (i.e. an explicit contradiction or a sentence such as 'homo est asinus', 'nullus deus est' and the like) is posited and then proposed in a disputation.

 $^{^{22}}$ This section displays the replies to the objections that have been put forward in the previous section. Cf. supra, sec. 2.6.

²⁰ istam] secundam O 21 Consequens] est add. O 22 Et] om. O 22 consequentia patet] inv. O 23 ad bene] ad b scr. et del. O 24 ad istam] om. O 24 eam] illam O 26 Ex] item praep. O 26 isto] illa O 28–29 licite admittitur] inv. O 29 simpliciter ... positionem] om. O 29 igitur] ideo O 31 contradictorium] eius add. O 32 consequentia] post formali transp. O 32 bona et] om. O 33 et] om. O 34 concessum] concedendum O 36 concessis] propositum add. O 41 est promissum] inv. O 41 est] post promissum transp. O 42 admissum vel] om. O

(RObj.1) Ex quibus patet quod non omnis positio obligat, sed solum licita positio. Neque omne ponere est obligare. Et non sequitur 'ego pono tibi istam; et tu admittis istam; igitur tu obligaris || ad illam'.²³

(RObj.2) Et tunc, ultra, conceditur quod non est possibile quod aliquis obligetur ad propositionem simpliciter impossibilem per positionem.²⁴

(RObj.3) Et per idem conceditur quod positum et admissum est negandum, ut si ipsum fuerit impossibile simpliciter.

 $\langle \text{RObj.4} \rangle$ Et ita conceditur quod, frequenter, antecedens est concessum et consequens non est concedendum, in consequentia denominata a li 'si' significante etc.²⁵ Sicut si posita fuerit propositio impossibilis, sequens ex ea est neganda, quamvis illa propositio posita et admissa fuerit concessa. Et causa tota istius est quia non oportet inconveniens pertinaciter multiplicare, sed solvere.²⁶

 $\langle \text{RObj.5} \rangle$ Ex quo patet responsio ad ultimum, quod oppositum male concessi et admissi || est concedendum. Negandum || tamen est quod contradictorium concedendi sit concedendum aut repugnans concedendo sit concedendum respectu eiusdem.

²⁴ Positio impossibilis is explicitly ruled out by this corollary.

 25 Cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii+4^{ra}] "Ex quibus patet non sequi 'antecedens istius consequentie scite esse bone etc., cum aliis particulis ibi positis, est concessum, igitur consequens est concedendum'. Sed oportet quod sit bene concessum".

²⁶ This somewhat unusual practical remark does not give precise instructions as to how the respondent should behave in this sort of situations, cf. [Ashworth, 1992, p. 666].

43 solum] omnis O 44 Et] ideo O 44–45 tu ... istam] eam admittis O 45 illam] eam O 49 ipsum] positum et admissum O 49 impossibile simpliciter] *inv.* O 50 ita] etiam *add.* O 51 non... concedendum] est negandum O 52 posita] positum O 52 sequens] tunc propositio impossibilis *praep.* O 53 propositio] impossibilis *add.* O 53 posita ... concessa] fuerit admissa et concessa O 53–54 Et ... solvere] *om.* O 55 quod] quia O

45 O $117^{\rm vb}$

M 83^{va}

55

 $\begin{array}{l} P \ 67^{ra} \\ V \ 72^{va} \end{array}$

²³ Again, the link between *positio*, *admissio* and *obligatio* must be understood according to the sense established above: an obligation arises only if and when something has been posited by the opponent and admitted by the respondent in compliance with all the requirements. As this reply and the following show, the five objections are more of qualifications of the rules laid down above than substantive objections directed against them. In particular, as far as I can see, they focus on a specific problem, namely that of ruling out impossibilities *per se*, like explicit contradictions of the form $p \land \neg p$, theses that are generally held to be metaphysically impossible or whereby a contradiction is implicitly embedded in the meaning of terms. Allowing this sort of sentences in an obligation would make the whole business trivial in virtue of the principle that from the impossible anything follows: there would be no way for the respondent to maintain consistency. 'Licita positio' thus coincides with what is commonly known under the name of 'falsi (or falsa) positio' or 'positio possibilis': only possible sentences – though often false and with controversial features – can be posited and admitted.

3. On positio

3.1. Sed, pro dictorum argumentorum praxi habenda, exemplariter disputemus.²⁷

A first group of sophisms (cf. sec. 3.1-3.14) mainly deals with the problem of qualifying the general notions of obligation and the first two rules (governing admission and the responses to the *positum* and to *proposita* logically related to it) so as to enable the respondent to face situations where he is asked to reply to a sentence that is underdetermined or whose reference is not precisely identified. Here the question at stake is how the criteria for admission are to be applied in problematic cases and how to treat the logical consequences of the *positum* alone, for example when it consists of a sentence with some sort of referential opacity.

A second group of sophisms (cf. sec. 3.15–3.27) is centred upon the issue of how the respondent is supposed to deal with various kinds of contradictions that may arise in the context of *positio*. This group may be further split into two parts. The first encompasses sec. 3.14 to 3.19 and contains a discussion of the treatment of contradictory sentences in special cases, as when they are, for instance, posited one after another. In this part, textual evidence can be found in support of the conclusion that Peter endorses a view (cf. infra the remark 'respectu diversarum obligationum') which is reminiscent of the so-called relativized interpretation of obligations, described in [Spade, 1982b] as a possible solution to explain away some problematic corollaries of the rules of *responsio antiqua*. This account has been also fruitfully explored by [Ashworth, 1981] in connection with late XIV century authors such as Strode and Paul of Venice. According to this interpretation, the circumstance that one and the same sentence ought to be conceded and denied, in two distinct disputations having the same *positum* (this feature depending on the fact that the evaluation of sentences is essentially related to the order in which they are proposed), does not represent a form of inconsistency for the theory, since it is with respect to different disputations that this happens to be the case, and therefore the concession of a *propositum* in one disputation is not inconsistent with its denial in another disputation. In addition to that, again in the second group of sophisms dealing with contradictions, there is an assessment (and rejection) of the reform of obligational rules, introduced by Swyneshed in the third decade of the XIV century, which is commonly known under the name of *responsio nova*. The contrast with this alternative view is strong, as is clearly visible in the discussions of sec. 3.20–3.27, where Swyneshed's rule for conjunctions is challenged and dismissed. Again, Peter approaches the problem in a way which is close to Strode's account, as long as he rejects the claim that two incompatible sentences are such only if they are conjoined (cf. infra, the arguments in sec. 3.22– 3.27; especially significant, in this respect, is the discussion of the role of the conjunction operator 'et', cf. also [Ashworth, 1996, pp. 360–365]).

In the final part of this section, Peter is concerned with a variety of other issues involving time and the context of utterance (cf. sec. 3.28–3.32), pragmatic paradoxes (cf. sec. 3.33–3.36), the properties of being relevant and irrelevant insofar as they are evaluated *infra tempus* (cf. sec. 3.37– 3.38) and, eventually, the distinction between the possibility of a given *casus* and the possibility for it to be true (cf. sec. 3.39–3.40).

 $^{^{27}}$ The third section of the treatise is devoted to the analysis of several difficulties arising from the rules that Peter has adopted. Although the chain of objections and replies – sometimes nested into one another – may appear, at first sight, quite entangled and confusing, it is possible to identify an underlying structure insofar as Peter seems to focus on a number of clustered problems connected to one another.

¹ pro ... habenda] dictarum regularum pro habenda practica O

Obligo te ad istam 'tu curris' posito quod demonstrem te qui sedes. Qua admissa, proponatur eadem. Qua concessa, negata vel dubitata, cedat tempus obligationis.²⁸

 $^{^{28}}$ The first sophism offers an example of the method used by Peter in the analysis of obligational disputations. A case is stated by means of which we are conveyed all information that is supposed to be relevant in the course of the disputation. Together with the statement of the case, a sentence is put forward to play the role of *obligatum* (I use, here, the general term *obligatum* because the description holds in the case of *positio* as well as in that of *depositio*). After the stage has been set, one or more *proposita* are put forward and submitted to the respondent's evaluation. Since the examples can usually be regarded as tantamount sophisms, the standard practice consists in displaying a number of objections that apparently leave the respondent with no solution. If we regard a disputation as a sequence of evaluated sentences, it is easy to see how sophisms are constructed. In the simplest case, they may be shaped with a linear structure, such that at some point, in the sequence starting with the *positum*, we run into a contradiction or into some other sort of incorrect answer. The linear structure obtains when, for each proposed sentence, only one type of reply is taken into account (i.e. concession, denial or doubt). In more complex cases, sophisms may be conceived of as having a tree structure. In such situations, we are actually in the presence of a plurality of linear disputations, that are considered together. Each point of the tree corresponds to a *propositum* with its evaluation (concession, denial or doubt). The tree always stems from a single point, the *positum*, but it branches into distinct paths whenever, for a given *propositum*, we want to take into account different outcomes depending on the way we reply to it. Each single disputation involved in such a structure, can be reconstructed by going backwards, along the path that starts with a termination of the tree, through the sequence of previous points, up to the *positum*. In the treatise, examples of both types of sophisms are found, as we will see shortly. I have made use of a schematic representation for a large number of disputations discussed in the sophisms, by means of numbered sequences of sentences, each of which is associated with the corresponding reply (by means of the letters A, for admission, C for concession, N for denial and D for doubt) and its justification (the latter always being on the right-hand column). Whenever the respondent replies incorrectly at a given step (i.e. he commits himself to the violation of a rule, he grants a contradiction or an impossible per se sentence such as 'homo est asinus' and so on), I use the symbol \perp . Generally speaking, if a sophism is constructed in such a way that the respondent actually has no way out, save for refusing to admit the *positum*, that will mean that the sequence ends up with \perp (if the disputation is linear) or all branches end up with \perp (if the disputation has a tree structure). After displaying the objections, usually a solution is picked out from a limited number of standard alternatives. The respondent has the following options at hand: either he will reject the *casus* refusing to admit the first sentence, or he will admit the first sentence and then modify one or more of the replies presented in the sophism, by offering an alternative reading. This, in turn, often means showing that a given sentence must be regarded as relevant rather than irrelevant or vice versa. Since the behaviour of the respondent in the obligational context is fixed once and for all when the set of proposition and the order in which they are proposed are given, the only room for manoeuvre consists in arguing in favour or against the logical status of sentences (occasionally, and only in the case of irrelevant sentences, the discussion may turn to truth and falsehood, cf. for instance infra, sec. 3.28–3.30. It will be soon clear that this pattern assumes the most various arrangements in Peter's treatise, although the structure is approximately invariant. Either Peter responds by rejecting the *casus* or he offers an alternative reading of one or more *proposita*. In

² Obligo] igitur add. O 2 posito ... sedes] om. O

 $\langle \text{Obj.1} \rangle$ Et arguitur quod ista non fuit concedenda, si concessa fuerit, quia ista fuit falsa et non fuisti obligatus, igitur etc. Patet consequentia. Et arguitur antecedens, quia in nulla specie obligationis fuisti obligatus a me, igitur etc. Probatur antecedens quia neque per positionem neque per depositionem et sic de aliis, quia qua ratione tu fuisti obligatus per unam speciem, eadem ratione quamlibet, igitur per nullam.²⁹

 $\langle \text{Obj.2} \rangle$ Si dubitatur vel negetur, arguitur tunc sic: ista 'tu curris' fuit tibi posita et a te bene admissa, igitur non fuit a te dubitanda neque neganda. Ista consequentia³⁰ est bona denominata etc., et antecedens est a te dubitandum, saltem si concedis te fuisse obligatum, igitur consequents non est a te negandum.³¹

addition to that, in some situations the solution may depend on a further qualification of a rule, if a case can be stipulated which puts that rule into question.

²⁹ The first sophism is meant to shed light on the following issue: the respondent must be always explicitly aware under what kind of obligation he is supposed to reply. The structure is quite simple, with only one level of objections. The strategy is proceeding by exclusion. Let us assume that the case has been admitted and the opponent has put forward his first *propositum*, namely 'Tu curris'. If the respondent grants it, the objection runs as follows: since it is actually the case that you are sitting and you are under no obligation (for no obligation has been specified), you should not have granted the *propositum* 'Tu curris'. The first available way of reply must be therefore ruled out.

³⁰ Cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii+ 3^{vb}] "Alia regula est quod, si consequentia (est) bona etc., significans etc., habens etc., quorum nullum etc., scita esse formalis, et antecedens est dubitandum, et consequens (est) intellectum propositum, et cum his scitum est quod ex nullo vero sequitur falsum, tunc consequens non est negandum".

³¹ If, on the other hand, the respondent chooses among the two alternatives that are left to him (doubting or denying the *propositum* 'Tu curris'), then the opponent will argue as follows: (*) "The sentence 'You are running' has been posited to you and correctly admitted by you, therefore you should neither have doubted nor denied it". The validity of inference (*) depends on a corollary of rule (R2.2) (if something has been correctly posited and admitted, it ought to be conceded; if it ought to be conceded, it ought not to be doubted nor to be denied), although Peter does not say it explicitly here. What happens next? The antecedent of (*) must be doubted, for we know that the respondent is under an obligation but ignore which obligation is at stake. On these grounds, we can at least infer, according to Peter, by appeal to the rule of consequence that governs responses to conditionals whose antecedent ought to be doubted (if the antecedent of a valid consequence ought to be doubted, then the consequent ought not to be denied) that the conclusion of (*) ought not to be denied. As a result, we have, on the one hand, a sentence p = 'Tu curris' that is either doubted or denied and, on the other hand, the consequent of (*). The difficulty is even more evident, if the situation is described in symbols as follows:

$(Dp \lor Np) \land O \neg N (O \neg Dp \land O \neg Np)$

This amounts to saying that no matter what decision is taken by the respondent – doubting or

10

5

⁵ fuit] sit O **5** si ... fuerit] om. O **6** falsa] ut pono add. O **6** fuisti] tu praep. O **6** consequentia] hec praep. O **7** in] om. O **7** a me] om. O **8** quia] om. O **9** ratione] nam add. O **9** quamlibet] per praep. BMOPV om. E; sed non per quamlibet add. MOPV om. BE **11** dubitatur] dubitetur O **11** tunc] om. O **12** neque] vel O **13** denominata etc.] om. O **13** et] eius add. O

3.2. $\langle R1Obj.1-2 \rangle$ Sed pro isto intelligendum est quod non obligatur aliquis nisi 15 per terminos specificos obligationis: quia cum obligatio fit circa certum officium exercendum – puta circa negare, concedere aut aliter respondere – ideo non potest respondens convenire cum arguente, nisi conveniat secum per terminos specificos. Et ideo per hoc genus 'obligo' non potest aliquis obligari ab arguente, quia nemo obligatur ad illud ad quod nescit se obligari.

Ideo, cum dicit arguens 'obligo te ad istam 'tu curris", dicat respondens querendo qua specie obligationis velit eum obligare, et non admittat aliquid antequam specificetur per quam || speciem obligationis || vult eum obligare. Et iste est modus regularis respondendi.³²

denying p – he will be compelled to accept at least that he is not allowed to deny that he should not have replied in either way. Strictly speaking, from the standpoint of the logic of obligations, this is not equivalent to the claim that the respondent is bound to grant that he should not have replied in either way, since, actually, from the prohibition to deny a sentence, apparently it just follows that the sentence should either be granted or doubted. Nevertheless, if the respondent doubts p and the opponent argues that p should not have been doubted, the respondent cannot dismiss the objection by rejecting it, since he ought not to deny that p should not have been doubted. For this reason, I reckon that here Peter is content with showing that the objection is sufficient to make both alternatives appear questionable. The sophism can be represented in the tabular form described above and which will be of frequent use in these notes:

0	Tu curris	А	possible
1.1	Tu curris	\mathbf{C}	\perp the respondent is under
			no obligation (neither <i>positio</i>
			nor <i>depositio</i>)
1.2		D	\perp cf. rule of consequen-
			ce (with an antecedent that
			ought to be doubted)
1.3		Ν	\perp cf.1.2
1.3		Ν	ought to be doubted)

The sophism has a very simple tree structure. A sentence is put forward (without specifying the type of obligation): it is admitted and then proposed in the first place. Three alternatives are taken into account, corresponding to the three type of replies available (conceding, denying, doubting). Each response is ruled out because it involves some kind of unacceptable conclusion. In the next section, Peter provides his solution.

 32 In Peter's treatise, solutions are often introduced, as this example shows, by clauses such as 'sed pro isto intelligendum', 'pro istis advertatur' or similar. The solution to the first sophism points out that no appeal to ambiguity must be left to the opponent, if we want the respondent not to be forced into a contradiction and, at the same time, comply with the obligational constraints, i.e. the rules he is subject to. Therefore, a case like this should not be admitted until it has been determined explicitly in the context of which kind of obligation (e.g. positio or depositio) the disputation is supposed to take place.

20

M 83^{vb} B 96^{va}

¹⁶ fit] sit O 16 circa] ce conf. scr. et del. ? O 17 aut ... respondere] vel dubitare O 17 ideo] et praep. O 18 arguente] arte O 18 specificos] speciales O 19 Et ideo] igitur O 19 arguente] arte O 20 ad illud] om. O 22 qua specie] per quam speciem O 22 velit] vult O 22 et non] nec O 22 admittat] admittet O 22 aliquid] respondens add. O

3.3. $\langle R2Obj.1-2 \rangle$ Aliter tamen posset responderi sophistice admittendo istam 'tu curris' cum dicitur 'obligo te ad istam 'tu curris". Et cum proponitur illam 'tu curris', dicitur 'respondeo ad istam'. Et si arguitur 'tu respondes ad istam, igitur concedendo, negando vel dubitando vel alio modo', conceditur consequentia. Et consequenter respondeatur || secundum || partes propositas. Et concesso antecedente, concedatur consequens. Et negatur prima pars antecedentis proposita et secunda. Et conceditur tertia, si per ordinem proponeretur. Et si queratur qua specie obligationis obligatur, dicat respondens quod non oportet certificare.

Unde diligenter advertatur quod quotienscumque est una disiunctiva sequens cuius || quelibet pars est falsa non sequens, quelibet pars est neganda preter ultimam propo-||sitam. Et ita per oppositum de copulativa: cum co-||pulativa fuerit vera sed repugnans, cuius nulla pars est repugnans per se, quelibet pars preter ultimam est concedenda.³³

 33 The argument is quite difficult. First, Peter proposes an alternative solution to the first sophism that would result in a sophistical (or captious) reply, as opposed to the preceding one which is said to be given in conformity with the rules, cf. *supra*, sec. 3.2 the remark 'et iste est modus regularis respondendi'. This alternative approach would require the following: the respondent should generically say that he is going to reply to the proposed sentence 'Tu curris'. The alternatives are, as above, concession, denial or doubt. I leave aside the clause 'alio modo' for it plays no effective role in the argument. If the respondent commits himself to replying (let us use the label *Resp*), he thereby commits himself to the disjunction of the three alternatives:

$Resp(p) \Rightarrow (Cp \lor Np \lor Dp)$

The idea behind the argument is that we might face a situation where a disjunction of nsentences is sequentially relevant and must therefore be granted, while each one of its disjuncts is irrelevant and false. In virtue of the fact that the disjunction is sequentially relevant, we must grant it, but we might also be willing to deny all of its disjuncts (since they are irrelevant and false). This is however not possible since we would obtain an inconsistent set of sentences. How are we suppose to act in such a case? Peter offers an account of what turns out to be a standard option that he adopts several times throughout his treatise. I will call it, according to Peter's use, responsio per ordinem. It consists in a sort of mechanical procedure aimed at maintaining consistency while minimizing the denial of true sentences or the concession of false ones. The procedure consists in denying the first n-1 proposed disjuncts of a sequentially relevant disjunction that has been previously granted, if they are put forward one after another, i.e. *per ordinem*: at step n the last disjunct will have become sequentially relevant and it will be granted accordingly. This procedure is nothing more than an application of disjunctive syllogism with n premises: from the rejection of the first n-1 premises, the last one follows (and at this stage, in the obligational framework, the reply to it is forced by the rules for relevant sentences). The converse of this principle plainly holds in the case of conjunction too, the only difference being that if a conjunction is incompatibly relevant, whose conjuncts are irrelevant and true, whenever they are proposed one after another, they must all be granted except for the last one.

87

25

 ${
m L}~68^{
m vb}$ 30 O 118^{ra}

 $\begin{array}{c} V \ 72^{\rm vb} \\ {\rm 35} \ P \ 67^{\rm rb} \\ E \ Giii^{\rm ra} \end{array}$

²⁵ responderi sophistice] *inv.* O 27 igitur] tu respondes *add.* O 28 conceditur consequentia] respondetur concedendo consequentiam O 28 Et] tunc *add.* O 29 respondeatur] respondetur O 29 Et] *om.* O 30 concedatur] conceditur O 31 proponeretur] proponatur O 32 obligatur] respondes O 32 dicat] dicatur O 32 respondens] *om.* O 34 non ... neganda] *om.* O 36–37 preter ultimam] *post* concedenda *transp.* O

3.4. Nunc, contra primam et secundam regulam, simul arguitur: quia utraque istarum est possibilis 'rex sedet', 'nullus rex sedet', ideo utraque illarum est admittenda, cum sit utraque intellecta a te. Pono igitur tibi utramque istarum.³⁴

It is noteworthy that a similar minimizing procedure is found in [Ralph Strode, 1517, fol. 88^{vb}], but in reverse order, in the case of an incompatibly relevant true conjunction: if none of its conjuncts is repugnant, the first that is put forward must be conceded, while any subsequently proposed conjunct must be denied. In the present sophism, I think Peter more or less takes up this line of reasoning, which can be represented as follows:

Ο	Tu curris	А	possible
1.	Tu curris	Resp	the respondent says 'Respon-
			deo ad istam'
2.	Tu respondes ad istam, igitur concedendo,	С	
	negando vel dubitando		
3.	Tu respondes ad istam	С	
4.	Tu concedis istam	Ν	irrelevant and false
5.	Tu dubitas istam	Ν	irrelevant and false
6.	Tu negas istam	\mathbf{C}	sequentially relevant by 25 .

This is the best solution I have been able to figure out in order to make sense of the text. To read it as I do, however, it is necessary to correct the text by substituting 'antecedentis' with 'consequentis' in the passage 'et negatur prima pars antecedentis proposita et secunda'. The antecedent is 'Respondes ad istam', while the compound sentence, whose parts can be granted or denied according to the order in which they are proposed can be, in my opinion, only the consequent. The correction is however supported only by the meaning of the text, since all manuscripts report 'antecedentis'. It goes without saying that at steps 4. to 6. the three options could also appear in a different order, the only invariant element being that whatever is proposed at step 4. and 5. must be denied, whereas step 6. must be granted as sequentially relevant.

³⁴ The second sophism addresses the problem of how to deal with sentences that are, to some extent, underdetermined.

Р	Utraque istarum	А	possible	
1.	Utraque istarum	С	sequentially relevant,	by
			(R1)	
2.	Ista sunt duo contradictoria	\mathbf{C}	\perp irrelevant and true	

Two alternatives are taken into account: either the *positum* 'utraque illarum' (where by 'illarum' two contradictory sentences are intended) is admitted by the respondent (who thereby commits himself to the concession of a contradiction) or the case is not admitted. If the case is not admitted, it can be argued that the respondent is violating rule (R1). The rule requires to admit any possible sentence whatsoever when it is posited; but both p and $\neg p$ are possible, therefore p and $\neg p$ must be admitted (the reply to this argument is given below (cf. *infra*, sec. 3.8 'Sed ad argumentum primum'). Moreover, if the respondent refuses to admit the case posited in this sophism (where the *positum* is 'utraque illarum'), the opponent can submit a variation on the theme, which is the object of the next sophism (the next *positum* being 'altera illarum').

38 Nunc] casus in marg. O **38** simul] om. O **38** quia] et praep. O **40** cum sit utraque] om. O **40** intellecta ... te] inv. et. ante ideo transp. O **40** utraque] utraque O

40

Si admittatur, proponitur utraque. Si conceditur, patet quod conceduntur duo contradictoria: quod enim sint duo contradictoria est verum non repugnans.³⁵

Sed forte dicitur non admittendo.

 $\langle {\rm Obj.1} \rangle$ Sed contra: tu teneris admittere omnem propositionem possibilem tibi positam; sed utraque istarum est possibilis et posita; igitur teneris admittere 45 utramque istarum.

3.5. $\langle \text{Obj.2} \rangle$ Item, si non admittitur utraque illarum quia altera alteri repugnat, saltem pono tibi alteram illarum 'rex sedet', 'nullus rex sedet'. Qua admissa, propono tibi istam 'rex sedet'. Et patet quod ista est dubia et impertinens, et ideo dubitanda. Qua dubitata, proponitur 'nullus rex sedet', que est etiam dubitanda, quia dubitato uno contradictoriorum, immediate proposito reliquo, dubitandum est. Quibus dubitatis, arguitur sic: utrumque illorum dubitas; sed alterum illorum est positum; igitur positum dubitas.³⁶

(RObj.2) Sed huic forte dicitur, cum || ponitur altera illarum, admittendo alteram illarum; et cum proponitur altera illarum, concedendo alteram illarum. Et 55 tamen utraque illarum dubitatur, cum proponitur.

³⁶ The third sophism is a variation on the same theme: here the *positum* is no longer 'Utraque istarum' but 'Altera illarum'. The problem, however, still lies in that the reference of the sentence is not fixed precisely enough so that the respondent can be aware of which sentence he is replying to. The structure of the disputation is the following:

Р	Altera illarum	А	possible
1.	Rex sedet	D	irrelevant and doubtful
2.	Nullus rex sedet	D	cf. the rule 'dubitato uno contradictoriorum, reliquum est dubitandum')
3.	Utrumque illorum dubitas; sed alterum illorum est positum; igitur positum dubitas	С	\perp

⁴¹ proponitur] proponatur O 41 utraque] et add. O 42 quod ... repugnans] om. O 43 Sed ... admittendo] Si non admittatur O 44 Sed] om. O 44-45 tibi positam] om. O 45 et posita] om. O 46 utramque] utraque O 47 si] omnino add. O 47 admittitur] admittatur O 47 altera alteri] inv. O 49 et] om. O 50 dubitanda] est add. O 50 proponitur] illa add. O 50 que ... dubitanda] qua dubitata quia dubitanda est O 51 immediate proposito] om. O 51 reliquo] et reliquum O 51-52 dubitandum est] dubitandum et inc.: post contradictoriorum transp. O 52 utrumque illorum] utramque illarum O 52 sed] om. O 52 alterum illorum] alteram illarum O 53 positum] tibi posita O 53 positum] tibi praep. O 54 ponitur] proponitur O 54 admittendo] admittenda O 55 altera ... Et] om. O

M 84^{ra}

 $^{^{35}}$ Here we have again a pair of dichotomic alternatives. Peter first assumes that the *positum* is admitted and in the next two-fold objection he assumes the opposite. In either case, the respondent apparently is committed to contradiction or to the violation of a rule.

Et ultra dicitur quod nulla illarum est mihi posita; et licet posueris mihi alteram illarum, tamen nullam illarum posuisti mihi; et licet utramque illarum dubitaverim, tamen non dubitavi mihi positum.³⁷

³⁷ This first reply to the third sophism is possibly taken from Strode (the whole sophism being reminiscent of his text), who discusses the very same difficulties in connection with his second assumption (the equivalent of Peter's first rule for the concession of the *positum*). I report in full Strode's passage with the aim of showing how close Peter's treatment is to arguments we can read in the text of the Englishman, cf. [Ralph Strode, 1517, fol. 82^{ra-rb}] "Super illud quod dicebatur in suppositione secunda, scilicet 'omne scitum esse positum etc.', pono tibi alterum illorum 'rex sedet' et 'nullus rex sedet', que scias esse contradictoria inter se contradicentia.

Sed diceres forte hic non admittendo, quia nescitur de qua fit mentio magis quam si diceretur 'pono tibi aliquam propositionem'.

Sed contra: igitur per idem non esset talis casus admittendus, si ponatur quod scias unam illarum, 'rex sedet' et 'nullus rex sedet', que sunt contradictoria, quia non fit mentio in tali casu de aliqua propositione. Nec iste casus est admittendus 'pono tibi quod aliquis homo currit', quia non fit mentio nec de isto nec de illo. Item sit rei veritas quod nulla propositio falsa sit. Tunc pono tibi omnem propositionem. Ille casus est admittendus, quia illum admittendo nullum sequitur inconveniens; ergo et sua particularis esset admittenda, quod fuit negatum.

Admittitur ergo forte casus. Deinde proponatur admissum [scil. the sentence 'alterum illorum']; conceditur secundum secundam suppositionem. Deinde proponitur 'rex sedet'. Sed si conceditur vel negatur, cedat tempus. Et constat quod concessisti vel negasti infra tempus propositionem que fuit tibi dubia; nec scivisti an sequebatur $\langle vel \rangle$ an repugnabat tibi posito; ergo nulla fuit causa quare illam debuisti concedere vel negare. Si tunc dubitatur illa 'rex sedet', cum proponitur infra tempus, proponatur illa 'nullus rex sedet'. Si conceditur vel negatur, contra: dubitato uno contradictoriorum, deberet aliud dubitari; sed scis illa esse contradictoria, per casum, quorum dubitas unum; ergo a te fuit aliud dubitandum.

Arguitur etiam extra tempus quod male respondebas infra, sicut arguebatur de alia. Dubitatur ergo forte 'nullus rex $\langle \text{sedet} \rangle$ ' cum proponitur. Sed contra: cedat tempus. Tunc, infra tempus, utrumque illorum dubitasti; et concessisti 'alterum illorum'; igitur tunc concessisti aliquam propositionem quam dubitasti. Et ideo male respondisti per quintam suppositionem [*scil.* the duty not to reply in two different ways to a sentence, in the same disputation]. Et minor probatur, quia concessisti propositionem sive casum tibi positum; sed nulla propositio erat tibi posita, vel nullus casus erat tibi positus nisi altera illarum; ergo concessisti alteram illarum. Et illo casu videtur concludi posse quod concessisti aliquam propositionem vel alterum illorum; et tamen nullam propositionem vel aliquam illarum concessisti nec admisisti; et sic a te concedebatur et a me proponebatur aliqua propositio; sed illa non fuit a me posita nec a te concessa.

Responderi ergo poterit, secundum viam communem probabilem, admittendo casum primum. Deinde consequenter, ut petunt argumenta, concedendo conclusiones adductas. Negatur tamen quod dubitavi propositionem quam concessi, quia tunc illa quam concessi fuisset tunc quando dubitavi [possibly an echo of this passage is found in another sophism, cf. *infra*, 4.19, RObj.2]. Quod negatur, sicut nec sequitur 'promitto tibi denarium et omnem denarium promitto Sor; ergo promitto tibi denarium quem promitto Sor".

In particular, Strode's text supplies a step that is missing in the sequence of sentences of Peter's own sophism, but which is presupposed in Peter's RObj.2. For the clause 'et cum proponitur altera illarum [presumably to be read as 'altera illarum', i.e. as the sentence 'altera illarum'], concedendo alteram illarum' hinges on the assumption that the first sentence proposed after the

⁵⁷ et ... posueris] posuisti enim O 58 utramque] utraque O 58 dubitaverim] dubitavi O 59 tamen non] *inv.* O

(ORObj.2) Sed contra: cedat tempus obligationis. Et arguitur quod in illo tempore aliqua obligatione fuisti obligatus, que fuerat B. Et arguitur sic: B obligatione fuisti obligatus; sed B obligatio fuit composita ex signis obligationis et obligato; igitur aliquid fuit obligatum quod fuit pars huius obligationis; et per consequens aliquid fuit tibi positum, cum non fuerit alia obligatio quam positio. Sic igitur aliqua istarum fuit tibi posita; sed utramque illarum dubitasti; igitur positum et 65 a te admissum dubitasti in tempore obligationis; igitur male respondisti.³⁸

3.6. Sed pro isto et pro similibus advertendum est quod non est a respondente respondendum 'admitto', nisi post || signum obligationis sequitur propositio expressa aut terminus certam propositionem admittendam declarans respondenti, ut cum ponitur illa 'tu curris' vel cum dicitur 'pono tibi istam', demonstrata certa propositione respondenti nota. Ita etiam a respondente non est respondendum ad aliquam propositionem, nisi illud verbum 'propono' determinet propositionem integram, ut 'propono tibi istam 'tu curris", aut terminum certam propositionem respondenti designantem, ut 'propono hanc', certa propositione || respondenti nota demonstrata, quam sciat ipse demonstrati.||

Et ideo, cum dicitur 'pono tibi alteram illarum', dicitur non || admittendo quousque certificeris que illarum ponitur. Et si ponitur || quod sis certificatus optime que sit ista que tibi ponitur, admittatur hoc. Et iterum cum dicitur 'pono tibi alteram illarum', dicitur non admittendo. Et si dicitur quare, dicitur quod non curet. Sed dicat que est ista quam vult ponere. Et si ponit ipse respondens quod ipse dicat, admittatur; et respondeatur ut prius cum ipse dicitur 'pono alteram

admission of the *positum* 'altera illarum' is the *positum* itself, i.e. the sentence 'altera illarum', rather than the sentence 'rex sedet' as in Peter's text. I cannot otherwise make sense of RObj.2, since the clause in question would not match the rest of the text. Moreover, I also see a link between the last part of Strode's text (from 'Responderi ergo poterit' to the end of the quotation) and the additional reason provided by Peter in support of RObj.2 (cf. 'Et ultra...'), namely the argument based on a distinction between readings *de sensu composito* and *de sensu diviso*, the connected issue of modes of supposition depending on it and the classical argument 'promitto tibi denarium, sed nullum denarium tibi promitto'.

³⁸ I assume that this additional counter-argument should count as an objection to the second part of RObj.2, aimed at showing that the solution was only apparent. Nonetheless, attributing this piece of reasoning to Peter, as if it represented his own point of view, leaves me perplexed because it makes use of a definition of obligation that he had previously rejected.

60 quod] sic O **60–61** in illo tempore] infra tempus O **62** signis] signo O **63** aliquid ... obligationis] aliquod obligatum fuit pars obligationis O **64** alia] aliqua *praep.* O **64** Sic] Et si *praep.* O **65** utramque] utraque O **65** positum] tibi *praep.* O **65–66** et ... admissum] om. O **66** in tempore] infra tempus O **66** obligationis] om. O **67** isto] istis O **68** admitto] om. O **68** post] om. O **68** sequitur] sequatur O **68–69** propositio expressa] *inv.* O **69** declarans respondenti] *inv.* O **70** cum ... vel] om. O **70** istam] hoc O **70** demonstrata] una add. O **72** determinet] terminet O **72–73** propositionem integram] propositio integra O **73** tibi istam] om. O **74** designantem] designans O **74** nota] om. O **75** sciat ipse] *inv.* O **77** certificeris] certificetur O **77–92.84** Et ... respondendum] om. LO

O $118^{\rm rb}$

 $P 67^{va}$

L 69^{ra}

B 96^{vb}

75 V 73^{ra}

70

illarum'. Sed cum hoc, dicat quod multum placeret admittere alteram illarum, si tibi velit certificare quam ponere vult, quamvis concedam quod non est opus certificare me. Et ita est obligatorie respondendum.³⁹

3.7. Consimiliter si dicitur 'pono propositionem possibilem et nulli repugnantem sa quam habeo in mente mea'. Et ita cum dicitur 'pono omnem propositionem possibilem', dicitur non admittendo, quia nulla propositio est sequens ad id verbum 'pono' ad quam debeat responderi.⁴⁰

Verumtamen, in certo casu, licet non sequitur aliqua propositio aut certus terminus discretus id verbum 'pono', per aliquas circumstantias potest respondens admittere, ut si respondens sciret quod nulla est propositio nisi || ista 'rex sedet' sic significans, et opponens diceret 'pono tibi omnem propositionem || possibilem que est in rerum natura', potest respondens admittere, quia sic esset certus quam propositionem admitteret.⁴¹

⁴⁰ Another variation on the same set of problems is given in the fourth sophism: if it is not welldefined what the posited sentence refers to, the case must not be admitted unless the sentence is specified in some additional way. In this case, the problem arises because the respondent ignores what sentence the opponent has (literally) in mind or, again, because of the presence of the term 'omnis' ranging over a set of sentences that is not furtherly qualified, except for the property that its elements are possible.

Again, the source is possibly [Ralph Strode, 1517, 80^{vb}-81^{ra}] where we find the sophisms 'pono tibi omne possibile' and 'pono tibi propositionem quam cogito'. A similar discussion is also in [Paul of Venice, 1988, p. 123].

⁴¹ The previous *caveat* is suspended in special cases, as for instance when the respondent knows that the class of possible sentences has only one element.

90

E Giii^{rb} M 84^{rb}

³⁹ The solution to the third sophism requires further specifications with respect to the posited sentence in case it contains elements of ambiguity or it is not well established what the respondent is actually going to reply to. In such situations the constraint of admitting the sentence fails. In particular, the sentence can be admitted only if it is explicitly expressed or if it is 'indicated', in an act of ostension, by means of a demonstrative pronoun. Thus, the sophism serves as a test to specify the limits of application of the first rule. A similar conclusion is found in [Ralph Strode, 1517, fol. 81^{ra}] "Nisi certificet non admittendo illam. Et si dicitur quare, dico quia nescio que sit illa [...] Et si arguitur contra: illa est a te admittenda per primam suppositionem, concedo non ut verum sed sic sequens ex casu. Si queritur tunc an admitto illum, respondeo quod non admitto istam nec respondeo ad illam. Et si queratur quare, dico quia sic mihi placet respondere, saltem nisi certificer aliter quam sim certificatus adhuc. Dico quod libenter obligarer ad illam, ut tecum conferrem, dum velles mihi proferre correspondentem quam audirem".

⁸⁵⁻⁸⁶ Consimiliter ... mea] om. LO
86 Et] om. O
86 pono] tibi add. O
87 est]
ibi add. O
87 id verbum] illud signum LO
89 sequitur] sequatur O
89 aliqua] certa
O
90 id] illud O
90 per] propter O
90 aliquas] alias O
91 ut] et O
91 est] esset O
92 diceret] dicat O
92 possibilem] que est possibilis O
93 que] aut praep. O
93 potest]
possit O
93 sic] ut praep. O
94 admitteret] admittit O

Et sicut dicitur de isto verbo 'pono', ita de illo verbo 'propono'. Unde, quamvis omne sequens ex bene concesso sit concedendum, non tamen cum propono 'omne sequens' est concedendum omne sequens, quia forte multa sequentia sunt non intellecta aut de quibus non est animadversum a respondente.⁴² Ita etiam, cum proponitur 'altera illarum' – 'deus est' vel 'homo est asinus' – non est respondendum concedendo vel negando alteram illarum, cum nesciat que propositio 100 proponatur concedenda vel neganda.⁴³

3.8. $\langle \text{RObj.1} \rangle$ Sed ad argumentum primum: cum arguitur quod utraque illarum est admittenda 'rex sedet', 'nullus rex sedet', dicitur concedendo et quod utramque illarum divisive teneor admittere, non tamen teneor admittere simul utramque illarum mihi positam neque plus probat argumentum.⁴⁴

⁴⁴ Cf. supra, sec. 3.4. Peter appeals to the distinction between the compound and divided senses in whose terms the sentence 'Utraque illarum est possibilis' can be read. In the compound sense, i.e. if we read it as $\Diamond (p \land \neg p)$, the sentence is false, since the scope of possibility extends over a contradiction. If, on the other hand, the sentence is taken in the divided sense, it will be equivalent to the conjunction $\Diamond p \land \Diamond \neg p$ which is true. In the context of the disputation of the second sophism, the opponent's strategy was trying to force the respondent into contradiction by positing 'Utraque illarum'. The term 'utraque' stands, here, for two contradictory sentences p and $\neg p$, that are both, separatedly, possible, and therefore should be admitted according to (R1). The point is that those sentences are possible in the divided but not in the compound sense. Hence, the respondent is by no means compelled, by (R1), to admit the contradictory conjunction $(p \land \neg p)$ for this sentence is, in fact, not possible (although its conjuncts are both possible, as long as they are taken separatedly).

95 ita] dicitur O **96** ex ... concesso] om. O **96** sit] est O **96** cum] dicitur add. P; conceditur O **96** propono] tibi add. O **97** est concedendum] respondendum est concedendo P; est respondendum concedendo O **97** sequentia sunt] inv. O **98** animadversum] advertens O **98** a respondente] om. O **99** proponitur] dicitur propono O **99** altera] alteram O **99** illarum] om. O **99** vel] om. O **99–100** est respondendum] respondetur O **100** nesciat] nesciatur O **101** concedenda] scilicet add. O **103** dicitur concedendo] illud conceditur O **103** et] om. O **104** divisive] per se mihi positam O **104** simul] om. O **105** mihi positam] om. O

⁴² The passage presents a textual problem: the correct reading is probably the one provided by manuscript P, whose text reads 'cum dicitur 'propono 'omne sequens" non est respondendum concedendo omne sequens". As for the spirit of the remark 'multa sequentia sunt non intellecta', an interesting discussion of the role of epistemic requirements in the obligational rules is found in [Boh, 1990, especially pp. 78–83] where Peter's views are examined in some detail.

⁴³ Generalization of the previous solution which is extended to the class of *proposita*: not only does the requirement of well-definedness hold with respect to the *positum*, but it also holds for any sentence that is put forward during the disputation. This is a further example of Peter's inclination to insert, in the discussion of objections, replies and solutions to the sophisms, a number of general considerations that can be regarded as refinements of the rules, if not as additional rules in their own right.

3.9. Si tamen ponitur quod omne possibile sit tibi positum et a te bene admissum, admittatur.⁴⁵ Deinde proponitur 'rex sedet'. Et cum ista sit dubia et impertinens, ideo ista est dubitanda. Et ita etiam dubitatur ista proposita 'nullus rex sedet' etc.

 $\langle \text{Obj.1} \rangle$ Sed contra: omne possibile est tibi positum et a te bene admissum; 110 sed utraque illarum est possibilis; igitur utraque || illarum est tibi posita et a te bene admissa. Et utraque illarum est tibi proposita; igitur utraque illarum est a te concedenda. Et utraque illarum dubitas; igitur male respondes.⁴⁶

3.10. Ideo, admisso casu, dicitur consequenter concedendo omnia, si per ordinem proponatur.

115

V 73^{rb}

 $\langle RObj.1 \rangle$ Et conceditur quod tu male respondes.⁴⁷

 45 The general strategy of answering to the sophisms without an explicit identification of the posited sentence that has been given above (for instance, cf. *supra*, sec. 3.7) consists in the rejection of the case, unless (or until) further specifications are provided, in precise terms, as to what the *positum*-sentence is going to be like. In the following two sophisms (cf. sec. 3.9–3.13), on the contrary, Peter takes into account cases that can be admitted, despite their superficial similarity with those of the preceding type.

 46 In the present case, we have the following disputation:

Р	Omne possibile est tibi positum et a te bene admissum	А	possible
1.	Rex sedet	D	irrelevant and doubtful
2.	Nullus rex sedet	D	cf. the rule 'dubitato uno contradictoriorum, reliquum est dubitandum')
3.	Omne possibile est tibi positum et a te bene admissum	С	sequentially relevant, (R1)
4.	Utraque illarum est possibilis [<i>scil.</i> 'rex sedet', 'nullus rex sedet']	С	irrelevant and true
5.	Utraque illarum est tibi posita et a te bene admissa	С	sequentially relevant by 3. and 4.
6.	Utraque illarum est tibi proposita	\mathbf{C}	irrelevant and true
7.	Utraque illarum est a te concedenda	\mathbf{C}	sequentially relevant by 5., 6. and (R3.1)
8.	Utramque illarum dubitas	\mathbf{C}	irrelevant and true
9.	Male respondes	С	sequentially relevant

⁴⁷ First, a textual remark: the reading 'proponatur' of the incunable is mistaken: it should be corrected in 'proponantur' (as in ms. O), since the verb refers to 'omnia'.

The solution to the fifth sophism consists in the admission of the *casus* and the concession of all *proposita* (save the first two), if they are put forward in the proposed order (to be noted is the application, once again, of the strategy of reply *per ordinem*). It must be recalled that the

106 Si tamen ponitur] tertius casus *in marg.* O 106 ponitur] ponatur O 106 a] ad O 107 proponitur] propono O 107 Et] *om.* O 108 ista est] *om.* O 108 Et] *om.* O 108 dubitatur] dubitetur O 109 etc.] *om.* O 115 proponatur] proponantur O 116 conceditur] etiam *add.* O 116 tu] *om.* O

Si tamen prius proponitur quod male respondes quam tu illarum utramque dubites, negandum est tamquam falsum et impertinens. Deinde negatur quod utramque illarum dubites, si proponitur, aliis concessis, per ordinem.⁴⁸

Et si arguitur, in priori ordine, quod tu non male respondes, || quia tu bene 120 O 118^{va} re-||spondes ad aliquam propositionem tibi positam, igitur tu non male respondes, negatur consequentia.⁴⁹

respondent is allowed to grant the sentence 'tu male respondes' if it is sequentially relevant in a disputation, since this does not amount to a violation of the rules. Obligational mistakes are other kinds of misbehaviours like, for instance, the concession and denial of the same sentence within one and the same disputation, or the concession of something irrelevant and false or the denial of something irrelevant and true. With respect to the sentence 'tu male respondes', a mistake occurs if the respondent grants it, without being compelled to do so, within the time of the obligation, or if he grants it outside the period of the obligation, since then he is committed only to truth. The issue has interesting implications and is discussed at some length in [Ralph Strode, 1517, fols. $78^{\nu b}$ and $82^{\nu b}$. The general point to be kept in mind is that there is a sharp distinction between the *internal* development of the obligation (with its stock of sentences, logical relations, commitments to the certain evaluations governed by the rules) and what goes on *outside* the disputation, when its development must be assessed in objective terms (which is expressed, by medieval authors, by means of the clause 'extra tempus est rei veritas fatenda'). Difficulties often arise because obligations are constructed in such a way as to make the two levels interact. It has been argued (cf. for instance [D'Ors, 1991b] on dialogue and meta-dialogue) that such attempts often result in confusing outcomes, where the distinction between the environments infra tempus and extra tempus collapses. Yet, I think it is the opposite way around. Although in a sometimes rough or even unprecise way, medieval authors of obligations treatises are generally well aware that they are conflating the two levels, and they do it on purpose. As I have stressed above, a typical example is the elaboration of a plethora of sophisticated *casus* based on the distinction between (i) granting p within the time of the obligation, (ii) granting – outside the time – that p ought to be granted within the time, and (iii) granting within the time that p ought to be granted. Now, to the best of my knowledge, a theory of these logical relations has never been sistematically developed, but there is much more to it than simple confusion. This kind of questions might be relevant to a better understanding of obligations both as a theory (with its logical features) and as a *historical* fact (since an accurate picture of their evolution cannot prescind from what we actually find in the texts).

 48 A variation in the order such as the one described here, would suspend the duty of granting the sentence 'tu male respondes', since it would no longer be sequentially relevant.

 49 By contrast, if the order remains the same as before, and it is argued that, since the respondent has replied correctly to some sentence, he has not replied incorrectly, then according to Peter the inference must be rejected. A justification for this move is not provided, but I assume it relies on the presupposition that in the obligational context the expression 'bene respondere' can only refer to a full compliance with the rules. In other words, it is not sufficient to answer correctly to a sentence, to infer that the respondent has done his job properly; or – which is the same – breaking the rules once, immediately leaves the him open to the charge of responding badly.

117 proponitur] proponatur O 117 quod] post quam transp. O **117** tu] om. 0 118 negandum est] negando O 119 si] postea add. O 119 proponitur] proponatur O 119 aliis concessis *inv.* O 120 si *om.* O 120 respondes et quod tu bene respondes add. O 121 positam] propositam O 121 tu] om. O 122 consequentia] argumentum O

 $P 67^{vb}$

3.11. (Obj.2) Item, admisso casu isto quod omne possibile sit tibi positum et a te bene admissum, arguitur quod antecedens ad impossibile simpliciter sit a te bene admissum. Consequens repugnat dictis. Et probatur consequentia: quia 125 admissa ista, proponitur tibi 'utraque istarum contradictoriarum est propositio possibilis', quod, quia verum et impertinens, est concedendum. Et deinde propono tibi 'utraque illarum est tibi posita et a te bene admissa': ista sequitur ex illis, ideo concedenda. Et per || consequens ista duo invicem contradictoria sunt a te sene admissa et simul. Et per consequens, antecedens ad illam copulativam 'rex 130 sedet, nullus rex sedet' simpliciter impossibilem est a te bene admissum.⁵⁰

3.12. Sed huic dicitur admittendo quod omne possibile sit tibi positum et a te bene admissum, idest quod omnis propositio possibilis sit tibi posita et a te bene admissa.

 $\langle \text{RObj.2} \rangle$ Deinde, cum proponitur quod utraque illarum duarum contradicto- 135 riarum est possibilis, negatur tamquam repugnans, quia repugnans est quod bene admittas duo invicem repugnantia, || quod tamen sequitur illo concesso.⁵¹||

Sed si ponitur quod omne possibile sit tibi positum et a te admissum absque li bene et proponatur quod utraque illarum sit possibilis, conceditur tamquam

 $^{^{50}}$ The argument can be represented as follows (note that the *positum* here is no longer 'omne possibile', but 'omne possibile est tibi positum et a te bene admissum'):

Р	Omne possibile est tibi positum et a te bene	Α	possible
	admissum		
1.	Utraque istarum contradictoriarum est pro-	\mathbf{C}	irrelevant and true
	positio possibilis [<i>scil.</i> 'rex sedet', 'nullus		
	rex sedet']		
2.	Utraque illarum est tibi posita et a te bene	\mathbf{C}	by P and 1.
	admissa		

⁵¹ The solution to the objection presented in the previous section consists in denying step 1. as incompatibly relevant. The argument has the following form (let p stand for the *positum*, q for the sentence proposed at step 1. and r for the sentence proposed at step 2.): $(p \land q) \Rightarrow r$, but $\neg r$ (because one cannot correctly admit contradictory sentences); therefore $\neg (p \land q)$; this is in turn equivalent to $(\neg p \lor \neg q)$; but p (= positum); therefore $\neg q$. This means that the negation of q follows. Hence, q is not irrelevant and true, but rather incompatibly relevant and should be denied accordingly.

123 a] ad O 124 a] ad O 125 probatur] arguitur O 126 admissa ista] admisso illo O 126 proponitur] proponatur O 126 tibi] om. O 126 contradictoriarum] om. O 126 propositio] om. O 127 est] ideo O 127 Et] om. O 127 propono] proponatur O 128 illarum] istarum contradictoriarum O 128 a] ad O 128 ista] ideo O 129 ideo concedenda] om. O 129 invicem] om. O 129 a] ad O 130 et simul] om. O 130 Et per consequens] igitur O 130–131 rex ... bene] om. O 132 admittendo] admisso O 132 a] ad O 133 quod] om. O 135 quod] om. O 135 duarum] om. O 137 duo] contradictoria add. O 137 invicem] ad invicem O 137 tamen] om. O 137 concesso] admisso O 138 ponitur] ponatur O 138 a] ad O 138–139 absque ... bene] om. O 139 quod] om. O 139 illarum] contradictoriarum add. O 139 sit] est O M 84^{va}

verum non repugnans et quod utraque istarum est a te admissa. Et si inferatur 140 tunc quod utraque illarum est a te concedenda, negatur consequentia.⁵²

3.13. Unde si ponitur quod propositio illa simpliciter impossibilis 'homo est asinus' sit tibi posita et a te admissa, admittendum est. Et cum proponitur quod ista 'homo est asinus' simpliciter impossibilis est a te concedenda, negatur quia non valet ista 'hec propositio impossibilis simpliciter est tibi posita et a te admis- 145 sa, igitur hec est a te concedenda', quia ad hoc quod ista foret a te concedenda, oportet quod in isto casu foret bene admissa, quod non est verum, quia bonum est non multiplicare sed solvere.⁵³

3.14. Sed contra responsionem arguitur probando quod eadem propositio est concedenda et neganda ab eodem in || eodem tempore obligationis, quod tamen 150 E Giiiva repugnat responsioni. Quia si unum contradictoriorum est concedendum, reliquum est negandum; ideo, si eadem propositio esset concedenda et || neganda et contradictorium esset propositio, igitur esset concedendum et negandum.⁵⁴

 53 The sophism is variation on the last remark. The *positum* is 'Hec propositio simpliciter impossibilis 'homo est asinus' est tibi posita et a te admissa' without the clause 'bene' (notice that in the present case the reading of the incunable is correct, as opposed to that of ms. O). The case can be admitted and when the *propositum* 'hec [= 'homo est asinus'] est concedenda' is put forward, it can be denied, because in order for it to be *concedendum* it should have been correctly admitted, contrary to the assumptions.

 54 In this section the focus shifts on a number of problems concerning contradictions. The crucial point is defending the consistency of the set of rules laid down above. As usual, this issue is spelled out at some length through series of objections and replies. Peter takes into account a number of arguments against his own view (cf. 'contra responsionem') that an imaginary opponent may come up with in the attempt to show ('probando') that one and the same sentence must be conceded and denied in the same disputation, a conclusion that would be inconsistent with the general set of assumptions and rules that Peter endorses ('quod tamen repugnat responsioni'). He considers, in particular, two classes of objections. First, in sec. 3.14–3.19, the problem of

140 inferatur] infertur O 142 ponitur] ponatur O 142 propositio] om. O 142 illa] **142** simpliciter] om. 0 **143** a] ad O 143 admissa] bene praep. hec sit O Ο 143 admittendum] admittenda O 143 quod] om. O 144 impossibilis] om. O 144 a] ad O 145 ista] consequentia O 145 propositio] om. O 145 tibi ... et] om. O 145 a] ad O 146 hec] om. O 146 foret] esset O 146 a te] om. O 147 oportet] oporteret O 147 in ... casu] illa O 147 foret] esset O 147 bene] a te praep. O 147–148 quia ... solvere] om. O 149 Sed contra] casus in marg. O 149 responsionem] EV; istam add. M; istam positionem O 149 est] sit O 150 in] et praep. O 153 esset] foret O 153 igitur] contradictorium etiam O 153 esset] foret O

97

V 73^{va}

 $^{^{52}}$ If the clause 'bene' is removed, the situation changes: the *positum* is still to be admitted (it may well be the case that any possible sentence is posited to you and that you admit it, without stipulating that your admission is correct), and step 1. is no longer incompatibly relevant but, rather, irrelevant and true. Both contradictories are admitted by you. Fair enough, this is just a true description of a fact. It might be unreasonable of you to admit them, but still, the fact that you do is not inconsistent in itself. In the previous case, by contrast, the assumption was that you *correctly* admitted two contradictories, i.e. in conformity with the rules, and this in principle can never be the case.

Sed arguitur consequentia: ponat Sor istam tibi 'homo currit', adequate significantem hominem currere; et ponat Plato tibi istam 'non homo currit', adequate 155 significantem non hominem currere, contradictorio modo prime. Et sis tu obligatus ad ambas istas per aliquod tempus. Deinde proponat Sor tibi istam 'homo currit': ista est concedenda quia posita etc. Deinde proponat || Plato tibi istam P 68^{ra} 'non homo currit': et patet quod est concedenda per idem. Ideo || sequitur quod eadem propositio sit concedenda et neganda, quod fuit probandum.⁵⁵ 160

 $^{^{55}}$ The seventh sophism is the first alleged proof against the opinion supported by Peter. It takes the following shape:

\mathbf{P}_{Sor}	Homo currit et ista [<i>scil.</i> 'homo currit']	А	possible
	adequate significat hominem currere		
\mathbf{P}_{Plato}	Non homo currit et ista [$scil.$ 'non homo	А	possible
	currit'] adequate significat non hominem		
	currere		
$1{Sor}$	Homo currit	\mathbf{C}	by $R1$
$1{Plato}$	Non homo currit	\mathbf{C}	by R1

The same sentence is granted and denied; but the respondent has just strictly played by the standard rules. The latter, therefore, seem to entail an undesired conclusion. What has gone wrong? Peter immediately recognizes the trivial mistake behind the argument: it is with respect to two distinct obligations that the same sentence is granted and denied, which means that it is in fact granted in one disputation and denied in the other.

98

granting and denying one and the same sentence, is examined in a number of sophisms where, for instance, contradictory *posita* are posited and then proposed to the same respondent by different people. In this group of sophisms, Peter focuses on of the relative character of the 'provability' of sentences which should be understood, each time, with respect to distinct disputations. Second, in sec. 3.20–3.27, the issue is discussed in connection with Swyneshed's controversial rule for conjunction.

¹⁵⁴ consequentia] illa *praep.* O 154 ponat] nam *add.* O 157 istas] *om.* O 157 aliquod] a O 158 quia ... etc.] *om.* O 158 tibi] *om.* O 159 per idem] quia posita tibi ab eo O 159 Ideo] et O 160 eadem propositio] *om.* O 160 sit] est O

3.15. Sed huic forte dicitur quod non est inconveniens quod eadem propositio sit ab eodem in eodem tempore obligationis || concedenda et neganda, quia illud est respectu diversarum obligationum.⁵⁶

(Obj.1) Sed contra: ponatur quod Sor teneat locum Platonis et ponat ipse utrumque.⁵⁷

(RObj.1) Sed adhuc forte dicitur quod non est inconveniens, quia est respectu Sor qui locum tenet duorum respondentium.⁵⁸

3.16. Deinde proponat tibi Sor 'hec est vera 'homo currit": quia id est sequens, ideo concedendum. Sequitur enim 'homo currit; et ista adequate significat hominem currere; igitur hec est vera 'homo currit". Antecedens enim est positum in 170 casu illo, igitur etc. Quo concesso, ponat Sor illam 'non homo currit', que contradicit illi, que per idem est concedenda. Deinde proponat Sor quod utraque illarum contradictoriarum est vera. Et patet quod illud est sequens. Quod si conceditur, cedat tempus obligationis.⁵⁹

⁵⁷ A cursory, harmless, objection to the previous remark (which insisted on relativizing contradictory outcomes to different disputations): what if we no longer have two people as opponents, but just one person who posits contradictory sentences?

⁵⁸ Again, the objection is quickly dismissed, because the person in question, as Peter says, must be regarded as actually playing the role of two opponents. The incunable text is mistaken, since it reports the phrase 'respondentium' where, clearly, 'opponentium' must be understood (cf. also the previous objection, reading 'Sor teneat locum Platonis et ponat ipse utrumque': it is only the opponent who is supposed to perform the act of positing something, not the respondent.

⁵⁹ The passage is a continuation of the sophism presentend in sec. 3.14. Two textual remarks are in order: the text 'ponat Sor illam 'non homo currit" does not properly fit in the symmetrical structure of the sophism. It must therefore be corrected, as is confirmed by the readings of ms.

162 in] et praep. O 164 contra] om. O 165 utrumque] utramque O 166 Sed adhuc] Huic O 166 est] adhuc O 167 locum tenet] inv. O 167 respondentium] om. O 168 quia] et praep. O 170 enim] om. O 170 est] tibi add. O 171 illo] om. O 171 Quo] Illa igitur O 171 concesso] concessa O 171 ponat] proponat O 171 illam] hec est vera O 171 homo] Sor O 172 quod] om. O 172 illarum] om. O 173 contradictoriarum] contradictoriam inc. O

165

M $84^{\rm vb}$

⁵⁶ Cf. also *Tractatus de consequentiis*, [Peter of Mantua, 1492a, sig. Eiiii+4^{ra}] "numquam est eadem propositio concedenda et neganda respectu eiusdem respondentis et respectu eiusdem obligationis et sensu". A similar line is taken by [Ralph Strode, 1517, fol. 87^{vb}]. A discussion of relevance and order in connection with the problem of granting contradictory sentences is found, again, in [Ashworth, 1981, especially pp. 183–186]. There is no problem of consistency because it may well be the case that contradictory outcomes are brought about by *different* disputations. Nevertheless, given the standard set of rules that Peter adopts, this can never happen to be the case in the context of one and the same disputation. Here Peter is facing the issue not from the point of view of outcomes, i.e. in terms of consistency with respect to sequences of *proposita*, which in turn amounts to the issue of determining either (1) whether it might be the case that a given sentence, put forward in the course of a disputation, must be granted and denied within the context of that disputation or (2) whether it must be granted in one disputation and denied in another disputation having the same *positum*. Rather, he takes into account an objection of consistency at the very first step of a disputation, i.e. at the level of *posita*.

(Obj.1) Et arguitur quod infra tempus obligationis concessisti impossibile sim- 175 pliciter, in quocumque tempore vel in quacumque obligatione fuerit non est cura; igitur in tempore male respondisti.

 $\langle \text{Obj.2} \rangle$ Item, in tempore concessisti repugnans tibi posito et a te bene admisso; igitur in tempore male respondisti. Patet consequentia quia tam inconveniens est concedere repugnans propositum ab uno quam ab alio, cum sit idem et eodem 180 modo repugnans.

 $\langle \text{Obj.3} \rangle$ Fiat autem ista forma extra tempus 'tu concessisti infra aliquod tempus duo contradictoria esse vera, igitur infra aliquod tempus male respondisti'. Patet consequentia quia neque respectu unius || neque respectu diversorum potest esse quod duo contradictoria sint simul vera in sensu et ad sensum iam expressum.⁶⁰ 185

O, at two points: first 'proponat' should replace 'ponat'; second 'non homo currit' should be supplemented with 'hec est vera'. This version of the sophism absorbs the objection of the previous section. Here Socrates is the only opponent, but with the two-fold task of positing a sentence and its contradictory along with some additional *proposita*:

$P.1_{Sor}$	Homo currit et ista [<i>scil.</i> 'homo currit'] adequate significat hominem currere	А	possible
$P.2_{Sor}$	Non homo currit et ista [<i>scil.</i> 'non homo currit'] adequate significat non hominem currere	А	possible
1.	Hec est vera 'homo currit'	С	sequentially relevant (the consequence 'homo currit et ista adequate significat hominem currere; igitur hec est vera 'homo currit" is valid, and the antecedent is P.1)
2.	Hec est vera 'non homo currit'	С	sequentially relevant (same valid inference, with P.2 as antecedent)
3.	Utraque illarum contradictoriarum est vera	\mathbf{C}	sequentially relevant

⁶⁰ The sequence of these three objections, put forward *extra tempus* (cf. the immediately preceding clause 'cedat tempus obligationis') is meant to point out that, apparently, further difficulties (beyond the fact that one and the same sentence ought to be granted and denied in the same disputation) derive from the *responsio* characterized by the set of assumptions and rules presented in the first two sections of the treatise. The conclusion of the three objections is the same: the respondent replies incorrectly. First, because he would grant an *impossibile simpliciter*;

 $L 69^{va}$

¹⁷⁶ obligatione] EO propositione BV 176 fuerit] om. O 177 in tempore] infra tempus obligationis OV 178–179 Item, ... respondisti] BE om. V 178 in tempore] infra repugnans tempus O 179 in tempore] infra tempus O 179 inconveniens] inc. forte conveniens aut consequens O 180 propositum] BEO om. V 182 Fiat] EO ante fiat add. item B 182 infra aliquod tempus] infra tempus a V 182 aliquod] om. O 183 aliquod] om. O; idem BV 183–184 Patet consequentia] Consequentia hec tenet O 185 in sensu] EO add. ad sensum V 185 et ad sensum] om. O 185 expressum] expresso O

3.17. Ideo dicitur, in principio, negando istam conclusionem, scilicet quod eadem propositio sit concedenda et neganda ab eodem infra idem tempus. Et admittitur propositio illa 'homo currit' posita primo mihi a Sor. Et cum ponitur sua contradictoria a Platone non admittitur, quia repugnat posito et bene admisso, quia numquam repugnantia possunt simul bene stare nec respectu unius neque respectu 190 diversorum, ut argutum est, sicut neque umquam possunt plures opiniones esse vere circa eandem materiam non convertibiles. Unde cum sunt plures opiniones circa eandem materiam non convertibiles, aut unica earum || est vera || aut omnes false, quamvis, ut sepe contingit, || multe videantur a contradictione defendibiles, quod accidit propter defectum speculationis et eius ignorantiam. Et ideo convenienter $195 B 97^{rb}$ respondendo || numquam debet idem respondens diversas opiniones circa eandem materiam || sustinere, nisi velit contradictoria concedere. Potest tamen unam post aliam gratia disputationis admittere et sustinere.⁶¹

 61 A solution is provided here to the first group of arguments that should have counted, allegedly, as tantamount proofs of the bad consequences (namely the duty of granting and denying the same sentence in a disputation) of the *responsio* that Peter is willing to endorse in his treatise. The solution consists in admitting the first *positum* and rejecting the second one in the context of the seventh sophism (in its original version involving two disputants: though the same would hold as well if we considered only one agent, as opposing 'on behalf of many'). I take the clause 'neque respectu unius neque respectu diversorum' as going along the same lines taken in sec. 3.15. No matter how many opponents there are, one must never admit two contradictory *posita* in the same disputation, while it is perfectly consistent with the rules to admit contradictory posita in different disputations: which makes the claim consistent with the conclusion of this paragraph 'potest tamen unam post aliam [...] admittere et sustinere'. The remark 'aut unica earum est vera aut omnes false' is problematic. It cannot certainly fit the square of opposition, at least as far as contradictories are concerned, since it is a logical law that when one contradictory is true, the other is false. One way to take the text as it stands, and make sense of it, is suggesting that Peter might be referring here, though in a rather generic and loose way, to different opinions formulated in the form of pairs of contrary (not contradictory) universal sentences which are the V 73^{vb} M 85^{ra} $P 68^{rb}$ O 119^{ra}

second, because he would grant something inconsistent with a posited sentence that has been correctly admitted; third, because he would grant, within the time of the obligation, that two contradictory sentences are both true. The objections have a feature in common, namely that in various ways, the attempt to solve the problem by relativizing responses to the context of different disputations is dismissed (cf. the remarks 'in quocumque tempore vel in quacumque obligatione fuerit non est cura', 'tam inconveniens est concedere repugnans propositum ab uno quam ab alio' and 'neque respectu unius neque respectu diversorum potest esse quod contradictoria sint simul vera').

¹⁸⁶ conclusionem] EO consequentiam BM **186** scilicet quod] om. O **187** sit] est O 187 idem] om. O 188 mihi] om. O 188–189 sua contradictoria] illa 'non homo currit' posita O 189 posito] priori O 189 et] om. O 190 repugnantia possunt] inv. O 191 neque umquam] numquam O 191 opiniones] EB positiones O 192 non convertibiles] om. BMOV **192** cum] numquam O **192** sunt] circa eandem materiam add. O **192** opiniones] vere add. O 193 unica] EO una BMV 193 est] EO erat V 193 omnes] sunt add. O 194 ut ... contingit] post videantur transp. O 195 speculationis] speculantis O 195 convenienter] consequenter OV 196 numquam] om. O 197 concedere] sustinere vel praep. O 197 tamen] om. O 198 gratia disputationis] om. V

3.18. Sed adhuc, forte, arguitur quod eadem propositio est concedenda et neganda ab eodem in eadem obligatione: quia ponatur tibi hec copulativa 'hec 'homo currit' 200 est tibi posita et a te bene admissa adequate significans hominem currere, et hec 'risibile currit' adequate significans risibile currere est tibi deposita et a te admissa'.

Deinde arguitur quod hec est a te concedenda 'homo currit', quia est tibi posita et a te bene admissa et est tibi proposita, igitur est a te concedenda: ponatur enim cum toto casu quod hec sit tibi proposita.

 $\langle i \rangle$ Et quod hec sit a te neganda arguitur: quia hec consequentia est bona 'homo currit, igitur risibile currit';

 $\langle ii \rangle$ et eius consequens est a te negandum, quia est de-||positum; igitur eius E Giii^{vb} antecedens est a te negandum.

Et sic ista 'homo currit' est a te concedenda et neganda.⁶²

210

205

only sort of sentences that can be together false but not true. My understanding of the passage for the moment does not reach further than this.

One last aspect is worth mentioning. The solution gives us a hint as to how to understand and apply the clause 'non repugnans alicui bene admisso' built in rule (R1). The clause is operating especially in the case of disputations with multiple *posita* such as the present one and it is meant to rule out all *posita* that are inconsistent with the previous steps.

 62 The sophism has the following structure:

- P 'Homo currit' est a tibi posita et a te bene admissa, et est tibi proposita, adequate significans hominem currere, et hec 'risibile currit', adequate significans risibile currere, est tibi deposita et a te admissa
- 1. 'Homo currit' est tibi posita et a te bene admissa et est tibi proposita, igitur est a te concedenda
- 2. Hec est a te concedenda 'homo currit'
- 3. Hec consequentia est bona 'Homo currit, igitur risibile currit'; et eius consequens est a te negandum quia est depositum; igitur eius antecedens est a te negandum'
- 4. Hec est a te neganda 'homo currit'

- C irrelevant and true; valid inference (by (R1))
- C sequentially relevant by P and 1.
- C irrelevant and true (valid inference)
- C sequentially relevant (by 3. and the second conjunct of P, which entails that 'risibile currit' must be denied, since it is deposited)

The sophism should count as an additional proof of the fact that the set of rules forces the respondent, in some cases, to grant and deny one and the same sentence in the same disputation

199 Sed adhuc] casus *in marg.* O **200** copulativa] BELMOP *om.* V **200** hec] E *om.* O propositio *add.* M **201** a] ad O **201** hec] propositio *add.* M **202** currere] currit O **202** deposita] posita OV **202** a] ad O **202** te] EP bene *add.* LMOV **203–204** Deinde ... admissa] *om.* V **203** quia] illa *add.* O **204** a] ad O **204** et ... proposita] *om.* O **206** a te] *om.* O **206** quia] quod O **208** est] tibi *add.* LO **210** Et] si *add.* MO **210** concedenda et] *om.* M

3.19. Sed huic dicitur, ut prius, negando quod eadem propositio est concedenda et neganda ad sensum iam dictum.

Et admittitur ille casus et quod ista 'homo currit' est a te concedenda, quia posita et bene admissa, ut ponit casus.

 $\langle Ad i \rangle$ Et cum arguitur quod illa sit neganda 'homo currit' quia illa consequentia 215 est bona 'homo currit, igitur risibile currit', et consequens eius est negandum, igitur et eius antecedens est negandum, negatur antecedens, scilicet quod consequens sit negandum.

 $\langle Ad | ii \rangle$ Et cum arguitur quod illud est depositum et admissum, igitur negandum, negatur consequentia.

 $\langle Obj.1 \rangle$ Sed illa consequentia est bona: hec 'risibile currit' est tibi deposita et a te bene admissa, igitur est neganda.

 $\langle \text{RObj.1} \rangle$ Sed eius antecedens est repugnans in illo casu, ideo etc.⁶³

(as is shown, in particular, by steps 2. and 4.). The *positum* is a conjunction of the form "p' is posited to you and correctly admitted by you and 'q' is deposited to you and admitted by you', where ($p \Leftrightarrow q$) holds. The *casus*, therefore, is a variation on the theme already discussed in the previous sophism. Instead of having multiple, contradictory, *posita*, here we have a compound sentence, whose conjuncts are, apparently, incompatible (the formulation is equivalent to positing the antecedent and the negation of the consequent of a true biconditional sentence). The plurality of incompatible sentences we were confronted with in the previous situation, is thus now absorbed within the structure of a single compound sentence.

⁶³ The strategy of reply suggested by Peter is, in this case, extremely subtle, at least as far as I have been able to reconstruct the argument. Some difficulties may be raised on account of textual problems that I will not face in detail here: much depending, for the sense of the argument, on the presence or omission in the manuscripts of two occurrences of the term 'bene' in connection with the admission of the *depositum* 'risibile currit'. I am confident that the reconstruction I will provide is sufficient to justify the textual choice that I am inclined to favour.

The general point conforms to the whole sequence of arguments presented in the previous sections. The purported conclusion endorsed by our imaginary objector is always the same, namely that one and the same sentence must be granted and denied in the same disputation on the assumption that we accept Peter's set of obligational rules. The structure of the example, too, is similar to that of the seventh sophism, where the key elements involved were multiple incompatible sentences posited one after another. Here, by contrast, we are confronted with another kind of incompatibility coming in the form of a posited conjunction composed of a posited sentence and a deposited one, the latter of which is entailed by the former. Yet, despite the strong superficial similarity of the two sophism, their respective solutions diverge. In the seventh sophism, the solution amounted to an explicit rejection of the second *positum* in virtue

²¹¹ est] sit O 213 et] om. O consequenter add. M 213 a te] om. MP 215 quia] igitur O 216 est bona] ante consequentia transp. O 216 consequens eius] inv. O 217 et eius] om. O 217 negatur] dicitur negando MO 217 quod] eius add. MO 217 sit] EM est O 219 illud] sit negandum quia add. O consequens scilicet 'risibile currit' sit negandum add. M 219 igitur] est add. O 220 negatur] dicitur negando MO 221 hec] om. O propositio add. M 221 deposita] EM depositum O posita V 221–222 a te] om. MO 222 admissa] admissum O 222 neganda] EM negandum O 223 Sed ... casu] EV om. O 223 est repugnans] repugnat M

of its incompatibility with the first one, on account of the fact that the latter had already been correctly admitted, prior to the *positio* of the second. Here in the ninth sophism, on the other hand, Peter opts for the admission of the casus (and of the whole compound positum) and for the denial of one of the two conjuncts occurring in the antecedent of the conditional sentence proposed at step 3. The *casus* is thus admitted, which in turn lead us to think that it must be possible. As we will see shortly, that is exactly what Peter is willing to maintain here, and it happens to be the case in virtue of the omission of the adverb 'bene' in the *depositio*-clause. The crucial point Peter's solution depends upon is precisely the way we regard step 3. of the disputation. Apparently, the inference is sound, hence the mechanism of contraposition holds: for this reason, in order to prevent the denial of the antecedent (which would amount to the denial of the posited sentence 'homo currit') and escape the undesired conclusion that, again, a sentence is both to be granted and denied in the same disputation, the only option left to the respondent is rejecting the clause that the consequent of this conditional ought to be denied (cf. the passage 'negatur antecedens, scilicet quod consequence eius est negandum'). In this way, a part of the premise which is essentially required in order for the argument to conclude that 'homo currit' must be denied is no longer available. So far, so good: Peter is asking us to deny the component of 3. which says that the sentence 'risibile currit' ought to be denied because it is deposited. One might wonder, now, on what grounds we could claim to be entitled to make such a move, since after all the sentence has been deposited and admitted. What makes the difference, here, is whether that sentence has been correctly admitted or simply admitted. In the former case, there is no way out: if we assume by hypothesis, i.e. in the statement of our casus, that the sentence 'risibile currit' has been deposited and correctly admitted, once we have already correctly admitted, at a previous stage, the *positum* 'homo currit', then we are going to be in trouble, because we could by no means claim that, at step 3., the solution is denying that the sentence 'risibile currit' ought to be denied. This would simply be inconsistent with the first rule of *depositio* (cf. *infra*, sec. 6.1). If that were the case (as might be suggested by the presence, in some witnesses, of the term 'bene' in connection with the admission of the *depositum* 'risibile currit') then Peter's solution seems to be incorrect: the only way for the respondent to maintain consistency, in that situation, would have been rejecting the case in the same way as the second positum of the seventh sophism had been rejected. The denial of the antecedent of 3. would not be an option.

This line of reasoning seems to be consistent with the last contracted remarks of Peter's reply. If it is argued that the denial of the antecedent of 3. (namely that the sentence 'risibile currit' ought to be denied) is illegitimate, because it follows from the assumption that the sentence has been deposited and (simply) admitted, we can reply that this is a non sequitur. Why? Because the rule for the denial of a proposed *depositum* is the mirror image of the rule for the concession of a proposed *positum*: in both cases, in order for the rule to apply, a deposited or posited sentence must have been *correctly* admitted by the respondent; if it has not, then the rule does not put any constraint or duty on his shoulders. I think this interpretation is supported by the fact that Peter makes a similar point, as we have already seen, in the treatise (cf. supra, sec. 3.13 when, to the claim that a posited and (simply) admitted sentence should be granted if it is proposed, he replies "ad hoc quod ista foret a te concedenda, oportet quod in isto casu foret bene admissa", i.e. correctly admitted, and not admitted *tout court*. One could still argue by appeal to the correct formulation of the rule of *depositio* (cf. infra, sec. 6.1), with the addition of the essential clause 'bene' (cf. Obj.1): in that case, there would be no way to escape the duty of denying the sentence 'risibile currit'. Peter's reply to the objection (cf. RObj.1) proves the subtlety of his reasoning and, most of all, that he is well aware of its logical implications: the antecedent of the rule, insofar as it is applied here, is inconsistent with the context of our *casus*,

3.20. Sed adhuc forte arguitur quod eadem propositio sit concedenda et neganda ad sensum iam dictum ab eodem in eodem tempore, quia aliqua copulativa est 225 neganda cuius quelibet pars est concedenda ab eodem in eodem tempore, igitur etc. Patet consequentia. Et arguitur antecedens quia ponatur tibi sedenti illa 'omnis homo currit'. Deinde proponatur tibi ista 'tu curris'. Qua negata quia falsa et impertinens, proponatur illa 'tu es homo'. Et patet || quod ista est vera non repugnans bene admisso, igitur est concedenda. Patet consequentia. Et arguitur 230 tunc sic: illa copulativa est neganda 'omnis homo currit tu es homo', quia ipsa est antecedens ad propositionem bene negatam, igitur neganda; et utraque eius pars est concedenda; igitur etc.⁶⁴

because it can never be the case that the deposited sentence 'risibile currit' is correctly admitted, on the assumption that the posited sentence 'homo currit' has already been correctly admitted at a previous step (this fact is expressed well by the clause 'eius antecedens [*scil.* hec 'risibile currit' est tibi deposita et a te *bene* admissa'] est repugnans in casu illo'). Therefore, the rule simply does not apply and no appeal can be made to it in order to rule out the denial of the antecedent of 3. In sum, this is I think the best way to reconstruct the argument. The textual assumptions that need to be made in order to justify it are, in the end, conservative with respect to the text of the incunable, despite some inconsistencies with the readings of several witnesses. Further external evidence in support of this interpretation is found in Peter's treatise on consequences, where he states the rule for conditionals with obligational operators and presents the same argument which is found here, cf. *Tractatus de consequentiis*, [Peter of Mantua, 1492a, sig. Eiiii+3^{vb}-Eiiii+4^{ra}] "Alia regula est quod si consequentia est bona, denominata etc., significans etc., habens antecedens etc., quorum nullum etc., scita esse formalis, et consequens est negandum ab aliquo, et antecedens (est) intellectum propositum, et cum his bene scitum est quod negandum non sequitur nisi ex negando, tunc est antecedens ab eodem negandum.

[...] Sed contra, forte, arguitur: quia ponatur quod antecedens istius consequentie bone 'homo currit, igitur risibile currit' sit tibi positum, scitum esse tale et a te bene admissum, et consequens tibi depositum; tunc ista consequentia est bona et antecedens est concedendum et tamen consequens est negandum.

Sed ad primum dicitur quod, stante illo casu, antecedens illius consequentie est concedendum, quia positum et bene admissum. Et consequens est concedendum, et non negandum, quia non sequitur 'est depositum et admissum, igitur est negandum'. Sed oportet plus assumere ad concludendum quod con-||sequens sit negandum, scilicet quod sit bene admissum scitum esse tale etc.; sed hoc ipsi posito repugnat, quia numquam est eadem propositio concedenda et neganda respectu eiusdem respondentis et respectu eiusdem obligationis et sensu".

⁶⁴ In this section, Peter goes on to argue from the standpoint of someone who is willing to challenge his views, but now under a different perspective or, to put it better, by appeal to an additional type of argument. In the previous sections, we have seen him face, successfully,

M $85^{\rm rb}$

²²⁴ Sed] casus *in marg.* O 224 sit] est O 225 ad... dictum] *om.* O 225] Et admittitur casus iste quod 'homo currit' est a te concedenda quia posita et bene admissa ut ponit casus, et tunc arguitur quod illa sit neganda 'homo currit, ergo risibile currit' *add.* V 225 in] et *praep.* O 226 quelibet] utraque O 226 ab eodem] *om.* O 226 igitur] ideo O 227 Patet] hec *add.* O 227 illa] *ante* tibi *transp.* O 228 tibi ista] *inv.* O 228 quia] est *add.* O 228 et] *om.* O 229 impertinens] non sequens BMOV 230 est] *om.* O 230 Patet] hec *add.* O 230 arguitur] antecedens *add.* O 231 currit] et *add.* OMV 232 propositionem ... negatam] oppositum bene negati MOV 232 igitur] est *add.* O 233 est] *om.* O O

various attempts directed against the *responsio* he endorses. Here the focus shifts, in particular, on the role played by Swyneshed's rule for conjunctions. The general strategy is always that of taking into account (in order to dismiss it) an alleged proof of the fact that one and the same sentence must be granted and denied in the same disputation. It is difficult to determine precisely whether Peter knew directly Swyneshed's text or not. What is unquestionable, though, is that Swyneshed's doctrine is largely discussed by a number of authors who are likely to be considered among Peter's sources, namely William Buser (perhaps *via* Marsilius of Inghen) and Ralph Strode. This material might therefore have been drawn by Peter indirectly from their treatises and brought in his own discussions not without a certain degree of freedom.

I will now outline the basic elements in the strategy of argument that we find in sec. 3.20-3.27. Two points of view are represented: Peter's own doctrine (cf. sec. 3.22, 3.24 and 3.25 to 3.27) and Swyneshed's doctrine (cf. sec. 3.21 and 3.23). Two theses and the logical relationship between them are at stake. For the sake of brevity, let us call the two theses (Contr) (= 'eadem propositio sit concedenda et neganda ad sensum iam dictum ab eodem in eodem tempore') and (SW) (= 'aliqua copulativa est neganda cuius quelibet pars est concedenda ab eodem in eodem tempore'), the latter coinciding approximately with Swyneshed's rule for conjunctions. A general introductory point is made, in sec. 3.20, by putting together the two theses. The objection, however, seems to be naive, since the aim is proving that (Contr), along the same lines taken in the previous sections, is a bad consequence of Peter's own rules; but this is done by assuming (SW), which is not part of Peter's original set and is indeed incompatible with it (possibly the whole point of the argument is giving a *proof* of this fact, namely that (SW) is actually inconsistent with Peter's rules).

Be this as it may, in sec. 3.20 it is argued as follows: (SW) entails (Contr) and an argument in support of (SW) can be put forward; therefore, (Contr) is proved, i.e. the conclusion we have become familiar with in this part of the treatise, namely that one and the same sentence must be granted and denied in the same disputation. The idea is that, if (SW) entails (Contr) and (SW) holds, then we are forced to accept (Contr), in virtue of an application of *modus ponens*. Yet, no one – not even a follower of Swyneshed – wants to accept (Contr), because it is contrary to the general purpose of obligations. The argument is therefore aimed at showing the unacceptability of (SW) which can be ruled out by contraposition plus the rejection of (Contr), once we have assumed that the entailment holds.

Thus, everything in the argument depends in the end on determining whether (SW) actually entails (Contr) or not, i.e. whether from the adoption of the special rule for conjunctions it follows that one and the same sentence must be granted and denied in the same disputation. If this is proved to be the case, then the rejection of (Contr), a thesis that we surely want to reject in order to preserve the consistency of the system, will entail by contraposition the rejection of (SW) as well.

What will a counter-argument on Swyneshed's side be like? The best option would be denying that (SW) entails (Contr). But this is exactly what is going on in sec. 3.21, where such a point of view is well represented by appeal to an argument based on the assumption that inconsistent sentences are such only when they are conjoined.

In sum, Swyneshed's followers would deny the entailment in order to keep (SW) and not to be forced to accept (Contr). Peter, on the other hand, accepts the entailment and denies (Contr), thereby obtaining, by contraposition, the denial of (SW). The argument he provides against the denial of the entailment (which is close to [Ralph Strode, 1517, fol. 79^{va}]) focuses on the role of the conjunction operator (= 'et') and the fourth rule, cf. *supra*, sec. 2.4 (= if all evaluated sentences that make up a disputation are conjoined, the resulting conjunction must be a possible sentence).

 $3.21. \langle Obj.1 \rangle$ Ideo forte dicitur, in principio, negando illam consequentiam 'aliqua copulativa est neganda cuius utraque || pars est concedenda, igitur eadem 235 O 119^{rb} propositio est concedenda et neganda': || quia dicitur quod ille due 'omnis homo currit' et 'tu es homo' non repugnant illi 'tu non curris' nisi copulentur; || ideo, cum copulantur, negatur copulativa, scilicet 'omnis homo currit et tu es homo',

V 74^{ra} L $69^{\rm vb}$

²³⁴ in principio] om. O 235 est] om. O 237 nisi] EMV ille due add. O; cum add. V 238 copulantur] copulentur O

quia est copulativa repugnans.⁶⁵

⁶⁵ In this passage, an argument is provided to reject the assumption that (SW) entails (Contr). The doctrine is a standard tenet of *responsio nova*, where the logical notions of relevance and irrelevance are defined with respect to the relation that any *propositum* bears to the *positum* alone. On this account, a *propositum* is sequentially (or incompatibly) relevant if and only if it (or its negation) follows from the *positum* (no other sentence proposed after the *positum* comes into play to determine its logical status). In particular, in contrast with the set of rules adopted by Peter and the responsio antiqua tradition, if a propositum is incompatible with a correctly granted or denied sentence (different from the *positum*) it must be treated as irrelevant, and evaluated, accordingly, with respect to its actual truth or falsehood. A corollary of this approach is that if a conjunction is put forward, whose conjuncts are a (false) positum and a true irrelevant sentence, it must be treated as irrelevant and false, and consequently denied, even if both of its conjuncts, when they are proposed separatedly, must be granted (the *positum* because it must always been granted; the other sentence because it is irrelevant and true, by hypothesis). Let us see how this is applied in the present case. In the previous section the tenth sophism was sketched through the following set of sentences and replies (the *casus* being 'ponatur tibi sedenti illa 'omnis homo currit"):

Р	Omnis homo currit	А	possible
1.	Tu curris	Ν	irrelevant and false
2.	Tu es homo	\mathbf{C}	irrelevant (on Swyneshed's
			account, because it does not
			follow from, nor is it in-
			compatible with the <i>positum</i>
			alone) and true
3.	Omnis homo currit et tu es homo	Ν	incompatibly relevant (it im-
			plies 1., which has been
			denied)
-			

Two features of this sophism must be pointed out. First, at step 2. there is an application of Swyneshed's approach. The sentence 'tu es homo' is irrelevant on his account, because it is not implied, nor is its negation implied by the *positum*. The sentence is true in the actual world, and therefore it ought to be granted, even if its negation is implied by the positum in conjunction with step 1. (by contraposition). The mechanism however is prevented in principle, because proposita have no influence on the logical status of what is put forward at subsequent steps of the disputation. Second, it might be argued that the evaluation of step 3., as it goes in the example, is a kind of confusing move which mixes up the approaches of responsio nova and antiqua. The reason is that the conjunction 'omnis homo currit et tu es homo' should certainly be denied, on Swyneshed's view, but not because it is incompatibly relevant (as it would turn out to be, according to the *responsio antiqua*, by virtue of its inconsistency with the already correctly denied *propositum* 'tu curris'). Rather, it would be denied as an *irrelevant and false* sentence (false because the *positum* is false, on the assumption that you are sitting; and irrelevant because a conjunction does not follow from one of its conjuncts alone, and relevance is indeed, for Swyneshed, a matter of logical dependence on the *positum* alone). I will not enter this specific issue here, but just mention that the confusion might stem from a miscast representation of Swyneshed's position some traces of which are already found in Strode, cf. [Ashworth, 1993, 381 - 383].

Be this as it may, the point made in this objection – representing Swyneshed's view – is that (Contr) does not follow from (SW), i.e. it does not follow that one and the same sentence must

239 est copulativa] *inv.* O

3.22. $||\langle R1Obj,1\rangle$ Sed contra: quia tunc posita et bene admissa ista 'omnis homo 240 P 68^{va} currit', proponatur illa 'homo non currit'. Et arguitur quod ista est concedenda, quia illa est vera non repugnans bene admisso, igitur est concedenda. Patet consequentia. Et arguitur antecedens quia iste propositiones non repugnant nisi sint copulate; sed iste non sunt copulate, ut pono; igitur neutra alteri repugnat. Patet consequentia et maior etiam ex responsione.⁶⁶ 245

be granted and denied in the same disputation. The most plausible candidate to play the role of the sentence that in this example would undergo such a double, contradictory evaluation is 'tu curris'. The argument seems to run as follows: the *positum* and step 2. must be granted. Until they are granted separatedly, nothing happens, because in order for an inconsistency to arise the two sentences should actually be conjoined: that would enable us to derive in the end, by modus ponens, the conclusion that the sentence 'tu curris', that has already be denied at step 1., must be now conceded. The question then boils down to whether it can be plausibly claimed that, once we have granted two sentences, we can still deny their conjunction. In the argument, Peter claims that if the *positum* and the *propositum* of step 2. are conjoined, the conjunction must be denied as incompatibly relevant. The confusion, I reckon, rests on the fact that Peter is probably assuming Swyneshed's point of view in a slightly distorted way. The reply actually is the one that Swyneshed would give: the conjunction is to be denied. The reason behind it, however, is different from the reason that Swyneshed would provide: according to him, the conjunction is simply irrelevant and false, and that is the rationale for its denial.

Moreover, even if the conjunction 'omnis homo currit et tu es homo' were granted, I venture that this would be no big deal in this context, since from the logical standpoint, the concession of 3. is sufficient to conclude that the sentence 'tu curris' ought to be granted only according to the responsio antiqua, which maintains that whatever follows from a correctly granted propositum must be conceded. On the other account, by contrast, nothing must be conceded by virtue of the fact that it follows from a propositum (alone, in conjunction with the positum or with other proposita): the sentence 'tu curris' after all would still remain an irrelevant and false sentence which would have to be denied (recall that the respondent is actually sitting, in virtue of the original stipulation). The same holds, by generalization, in the case of all inferences involving anything more than the *positum* in their premises and involving, as their conclusions, sentences which are irrelevant to the *positum* alone, the idea behind this picture being basically that no propositum should be allowed to feature in the derivation of a any sentence whatsoever, cf. in this respect [Ashworth, 1996, p. 364–365].

⁶⁶ In this section, Peter offers the first of three arguments (cf. *infra*, sec. 3.25–3.27, for the second and third one) against the attempt, based on the requirement of sentence conjunction for relevance, to reject the entailment from (SW) to (Contr). Peter's reply to the argument of the previous section, which is grounded on the assumption that inconsistency, in obligational disputations, derives only from actually conjoining sentences with one another (and from actually proposing such conjunctions) is subtle. If this were the case, then the idea of 'following from' or 'being incompatible with' the *positum* itself would be spoiled of its very same meaning, because by generalizing the previous move to the denial of any form of incompatibility, unless sentences are actually conjoined, the basic criterion to establish the logical status of a sentence as relevant or irrelevant would be thrown away. That this is what Peter has in mind in his reply is clear from the argument:

242 quia ... vera] om. O **244** copulate | EO cathegorice (!) V **244** Patet | hec O **245** etiam] patet O

3.23. (OR1Obj.1) Sed huic forte dicitur quod iste due bene repugnant, licet non copulentur aut sint copulate.⁶⁷

3.24. (ROR1Obj.1) Sed contra: igitur per idem iste tres repugnant 'omnis homo currit', 'tu es homo', 'tu non curris', aut saltem stat istas tres repugnare sic significando, licet non sint copulate, quod repugnat positioni.⁶⁸ 250

Р	Omnis homo currit	Α	possible
1.	Homo non currit	\mathbf{C}	irrelevant and true

If sentences cannot be said to be inconsistent with one another unless they are conjoined, as has been claimed in sec. 3.21, then also in this example we must take 1. to be irrelevant to the *positum*, as a generalization of the point made above. The *positum* is not conjoined with 1., therefore, even if 1. contradicts it (i.e. even if its negation is implied by the *positum*, which is pretty much the definition of incompatible relevance, also on the *responsio nova* account) it ought not to be denied, but rather it should be regarded as irrelevant (cf. 'non repugnans bene admisso') and evaluated accordingly as true (the respondent is assumed to be still sitting, therefore 'homo non currit' is true).

⁶⁷ In this section, again, it is the point of view of one who argues from the standpoint of Swyneshed's account that Peter takes into consideration and represents. The crucial point is that no one, not even the strongest supporter of the *responsio nova* would accept an extreme conclusion such as the one presented in the previous section: the *positum* 'omnis homo currit' and the *propositum* 'non homo currit' *are* definitely incompatible, even if they are not conjoined. But, as the next remark will show (cf. *infra*, sec. 3.24), the requirement that sentences be actually conjoined as a necessary condition for their incompatibility turns out to be hardly sensible, if not utterly untenable. In other words, if we were to concede, with the aim of keeping intact the notion of logical consequence, that in sec. 3.22 the conjunction requirement must not be operating, how could we ever deny, then, that the objection that prompted us to make an exception in that particular case, can be extended and generalized to any other situation whatsoever: 'per idem' any three or more appropriately chosen sentences would turn out to be incompatible with one another, no matter whether they come in the form of a conjunction or not.

⁶⁸ In this section Peter replies to (and argues against) the argument of sec. 3.23. His own idea is that if we assume that incompatible sentences are such only when they are conjoined, then the notion of relevance itself loses its content and effectiveness. Consider the argument in sec. 3.23: a sentence that contradicts the *positum* is proposed and it is argued that it should not be considered incompatible to it, because it is not conjoined with it. To this patently unacceptable conclusion, even a supporter of Swyneshed's view would reply that in a case like that, the two sentences of sec. 3.23 are incompatible, even if they are not actually conjoined. But here is exactly where the underlying assumptions of this approach come home to roost: if one is forced to say that the two sentences 'omnis homo currit' and 'homo non currit' are incompatible, in order to preserve a minimal distinction between relevant and irrelevant sentences as well as the very same notion of logical consequence, then how could one still deny that the triple presented in sec. 3.24 consists of sentences that are incompatible, even if they are not conjoined?

246 iste due] EMO ille regule V 246 licet] sed forte O 246–247 non copulentur] om. O
247 aut] seu non BV 247 sint] non sic O 248 contra] inc. scr. et del. O 248 igitur]
om. O 248 idem] BEV reponsionem MOP 250 positioni] responsioni BLOV

3.25. $\langle R2Obj.1 \rangle$ Item, si propositiones non repugnant nisi sint copulate, bene admissa illa 'omnis homo currit', proponitur ista 'tu es non currens' que, quia vera non repugnans, concedenda est.

Qua concessa, arguitur: tu es non currens, igitur homo est non currens, igitur homo non currit, quod || est oppositum positi.

Ideo, si negatur ista 'tu es non currens', proponitur illa 'tu es currens', que neganda est quia falsa non sequens. Deinde proponitur 'tu es homo': et etiam ista est concedenda ex responsione.

Et tunc sequitur quod non de quolibet termino singulari, pro aliquo supponente simpliciter sumpto, predicatur alterum contradictoriorum incomplexorum, quia 260 utraque istarum negatur 'tu es currens', 'tu es non currens', concesso quod tu es homo.⁶⁹

 69 The second argument comes in the form of a more elaborate sophism, with the following structure:

Р	Omnis homo currit	Α	possible
1.1	Tu es non currens	\mathbf{C}	irrelevant and true
2.	Tu es non currens; igitur homo est non	\mathbf{C}	\perp (it is unclear, though,
	currens; igitur homo non currit		why the inference should be
			granted, as Peter assumes)
1.2	Tu es non currens	Ν	
3.	Tu es currens	Ν	irrelevant (on the <i>responsio</i>
			nova account) and false
4.	Tu es homo	\mathbf{C}	irrelevant (on the <i>responsio</i>
			<i>nova</i> account) and true

As in sec. 3.22 above, Peter argues here against the conjunction requirement for relevance. If the requirement is adopted, as it happens to be the case on Swyneshed's account, the notions of relevance and irrelevance as well as the corresponding classes of sentences differ with respect to those that are involved on the *responsio antiqua* account. Consequently, the answers that we are supposed to give to *proposita* must be changed as well, according to the new criteria. For instance, the set of replies to *proposita* like those presented in this sophism is such that undesired consequences obtain, from the logical point of view. In particular, it can be shown that, for a given predicate term, neither it nor its opposite can be predicated of a given singular subject term (cf. 'non de quolibet termino singulari, pro aliquo supponente simpliciter sumpto, predicatur alterum contradictoriorum incomplexorum'), which eventually results in a rejection of the Law of the Excluded Middle.

The structure of the argument is straightforward, although there is some unclarity it is difficult to make sense of, in the first part. The idea is proposing in the first place the sentence 'tu es non currens'. If it is granted, as irrelevant and true, then sentence 2. 'tu es non currens; igitur homo

251 nisi] ipse add. L 251 bene] EV tunc O 252 currit] tibi add. O 252 proponitur] proponatur O 252 tu ... currens] add. homo V 252 es non] *inv.* O 252 quia] est add. O 254 arguitur] om. O sic add. M 254 es] EMO homo add. B 254 igitur] sequitur quod O 254 homo] om. V 254 igitur] et sic sequitur quod O et tunc sequitur homo est non currens ergo M 255 oppositum] contradictorium MO 256 que] om. O 257 et etiam] om. O 258 ex responsione] EMO te existente V; quia vera non repugnans add. M 259 pro aliquo] post supponente transp. O 260 predicatur] potest O 260 incomplexorum] verificari add. O

255 B 97^{va}

3.26. $\langle R3Obj.1 \rangle$ Item, ponatur tibi sedenti || 'omne sedens est Sor'. Deinde proponatur 'tu es Sor'. Qua negata, quia falsa non sequens, ut pono, proponitur ista 'tu es sedens'. Patet quod ista est vera non repugnans et per consequens concedenda. 265 Qua concessa, arguitur sic: tu es sedens, igitur aliquod sedens non est Sor, quod est oppositum positi. Si negatur consequentia, ponatur oppositum consequentis cum antecedente. Et stabunt ista simul 'tu es sedens et omne sedens est Sor et ultra, igitur tu es Sor. Patet sillogismus in quarta figura.⁷⁰ Et antecedens est concedendum, quia ista est negata 'tu es sedens, igitur aliquod sedens non est Sor'. 270 E Giiii^{ra} Igitur oportet concedere copulativam compositam ex antecedente et oppositum⁷¹

est non currens; igitur homo non currit', whose consequent is the contradictory of the *positum*, is put forward. If it is granted, then we have proved something inconsistent with the *positum* on the hypothesis that 'tu es non currens' has been granted (1.1). We could then switch to the second part of the sophism and deny 'tu es non currens', to obtain, in the end, the rejection of the Excluded Middle. At this stage, the pieces would still fit. The problem is that it is not clear why the respondent should be bound to grant sentence 2.: the inference, as it stands, is by no means valid, unless the additional assumption 'tu es homo' is made by the opponent, in the form of a *propositum*, and granted by the respondent at a previous step. Even the manuscripts do not help, in this sense: it would have been useful to find a passage dropped off the text with the required supplement (proposal and concession of 'tu es homo' not only in the second, but also in the first part of the sophism); a situation like that obtains sometimes in the tradition of this treatise, but unfortunately not in this case.

⁷⁰ Cf. *Tractatus syllogismorum*, under the rubric 'quarta figura', [Peter of Mantua, 1492a, sig. Dviii^{ra}] "Sequens modus 'Dirami' nominatur indirecte concludens, ex maiori particulari affirmativa et minori universali affirmativa, particularem affirmativam, ut 'Tu es homo; et omnis homo currit; igitur tu curris' ".

⁷¹ The correct reading, confirmed by mss. O, P and V, is indeed the ablative 'opposito'.
⁷² The sophism has the following structure:

Р	Ponatur tibi sedenti 'omne sedens est Sor'	А	possible
1.	Tu es Sor	Ν	irrelevant and false
2.	Tu es sedens	С	irrelevant (according to the <i>responsio nova</i>) and true
3.	Tu es sedens, igitur aliquod sedens non est Sor	Ν	
4.	Tu es sedens et omne sedens est Sor; igitur tu es Sor	С	valid inference (syllogism in the fourth figure I-A-I)
5.	Tu es sedens et omne sedens est Sor	С	sequentially relevant (negation of 3.)

263 Item] casus in marg. O 263–264 proponatur] proponitur O 264 tu es] om. O
264 quia ... pono] BEV om. MO 264–266 proponitur ... sic] om. V 264 proponitur]
proponatur O 265 Patet] et praep. O 265 et ... concedenda] om. MO 266 aliquod]
aliquid O 268–269 et ultra] BE om. OMPV 269 Patet] ille est add. O 269 figura] om.
O 270 ista] consequentia add. BMP 270 est] om. O 270 negata] post quia transp. O
270 aliquod] aliquid O 271 oppositum] opposito OPV 272 etc.] om. O

3.27. Huic dicitur admittendo casum. Et quando proponitur 'tu es sedens', negatur tamquam repugnans posito et opposito bene negati. Et licet sequatur ex obligatione tali, non tamen est concedenda, quia ad sequens vel repugnans obligationi 275 non est pertinenter respondendum, immo obligato.⁷³

The solution to this sophism consists in the admission of the case and the denial of 2. which is not irrelevant as one who endorses Swyneshed's view would claim, but rather incompatible with the *positum* and the correct denial of 1., i.e. with the sentences 'omne sedens est Sor' (= *positum*) and 'tu non es Sor' (= *opposito bene negati*). The denial of 2. follows, by contraposition, from those two sentences, therefore the sentence 'tu es sedens' is incompatibly relevant.

A sophism of similar structure (the triple of sentences involved being: 'omne stans in ista domo est asinus', 'tu stas in ista domo', 'tu es asinus') is discussed by Strode, albeit with a different treatment, in connection with the same set of issues concerning Sywneshed's rules and the role of the conjunction requirement, cf. [Ralph Strode, 1517, fol. 79^{va}]. In Peter there is an additional element that enters the discussion, namely the problem of relevance to the obligation (i.e. not only to the posited sentence, but to the whole *casus*; in this respect, cf. also *infra*, sec. 3.37–3.38). This issue has a well-established tradition reflected for instance, on the Parisian scene, by elaborate distinctions between different notions of relevance, which are defined with respect to the *obligatum*, to the *admissio*, to the *obligatio* and so forth, cf. [Albert of Saxony, 1490, sig. Niii^{rb}-Nv^{ra}], [William Buser, 1990, pp. 74-80], [Marsilius of Inghen, 1489, sig. Aii^{vb}-Aiii^{vb}]. Despite the apparently clear structure of the argument, it must be said that the treatment is not fully unproblematic, since although the logical status of 'tu es sedens' is rightly identified by Peter with that of an incompatibly relevant sentence, the role of the last remark (cf. 'quia ad sequens vel repugnans obligationi non est pertinenter respondendum, immo obligato') is difficult to understand. This characterization has a precise meaning in the tradition, which in all likelihood is not, however, the one intended here. The notion of being relevant to the obligation is employed for the most part in connection with sentences that may cause pragmatic paradoxes to arise (cf. infra, sec. 3.33–3.34). A hint about what Peter might have had in mind here is to be found, perhaps, a few sections below (cf. infra, sec. 3.37–3.38) where he returns on these matters. The present passage, however, remains puzzling, probably as a result of the presence of a number of overlapping issues that are in fact unrelated and should theoretically be kept distinct. Until a stemma will have been fully established, it is hard to draw conclusions and argue in favour of the expunction of a passage, solely on the basis of the frequency with which it occurs in the manuscripts but it should be noted, for the sake of completeness, that this problematic passage is omitted, in its entirety, by five out of six witnesses, and is reported only by ms. B.

On the whole, I think the passage is inconsistent with the general point that is being made here. If the sophism should count as an additional argument against the *responsio nova*, there would be no need to provide a solution by explaining how one should respond according to the *responsio antiqua*. It would have sufficed to prove, as Peter does in sec. 3.27, that, on the account that we want to reject, some problems happen to arise: in other words, the conclusion of sec. 3.26 would have been enough to make the point).

⁷³ The final part of the text is contracted. I assume that the clause 'immo obligato' is an adversative counterpart of 'obligationi'. The meaning of the sentence, therefore, is the following: 'it should not be appropriately replied to something which follows from or is incompatible with the obligation (= 'ad sequens vel repugnans obligationi'), but rather (to something which follows from or is incompatible with) the *obligatum* (= 'immo obligato')'.

^{273–276} Huic ... obligato] BE om. LMOPV

3.28. Item, ponatur quod Sor non obligato proponatur illa 'deus est' quam ipse concedat. Et proponatur Platoni eadem quam neget necessariam male respondendo. Et Cicero dubitet illam male etiam respondendo. || Quo posito, proponitur illa 'tu respondes sicut ille illorum qui male respondet' demonstrando per ly 'illorum' 280 Sor et Platonem et Ciceronem. Et non respondeat aliquis illorum || nisi ad istam P 68^{vb} et modo predicto.⁷⁴

 74 Let us try to see, first, what the structure of the argument is like:

Р	Ponatur quod Sor non obligato proponatur illa 'deus est' quam ipse concedat. Et pro- ponatur Platoni eadem quam neget necessa- riam male respondendo. Et Cicero dubitet illam male etiam respondendo	А	possible
1.1	Tu respondes sicut ille illorum qui male re- spondet, demonstrando per ly 'illorum' Sor et Platonem et Ciceronem, et non respon- deat aliquis illorum nisi ad istam [<i>scil.</i> 'deus est'] et modo predicto	С	
1.1.1	Tu concedis falsum non sequens, igitur tu male respondes	\mathbf{C}	\perp if understood with the correct time reference
1.1.2	Si tu respondes sicut iste illorum qui male respondet, et ille illorum qui male respon- det respondet negative, igitur tu respondes negative	С	\perp valid inference: the conse- quent ought to be granted in contrast with the reply given at 1.1
1.2		D	
1.2.1	Si dubitas illam 'tu respondes sicut iste isto- rum qui male respondet' et Cicero etiam du- bitat illam [<i>scil.</i> 'deus est'] male responden- do, igitur tu respondes sicut ille illorum qui male respondet	С	\perp valid inference: the conse- quent ought to be granted, in contrast with the reply given at 1.2
1.3	1	Ν	
1.3.1	Tu negas illam [<i>scil.</i> 'tu respondes si- cut ille illorum qui male respondet'], igi- tur respondes negative; et iste illorum qui male respondet respondet negative; igitur tu respondes sicut ille illorum qui male	C	\perp valid inference: the consequent ought to be granted in contrast with the reply given at 1.3
1.3.1.1	respondet Tu respondes ad istam et non respon- des affirmative neque dubitative neque alia responsione, igitur etc.	С	\perp
(T) (1000 0	

The sophism – which is found also in [Paul of Venice, 1988, p. 298] and [Robert Fland, 1980, p. 46] – is quite difficult, especially because it involves a number of remarks, in the reply, that cover different topics without an explicit statement of what the aim of the discussion is. Despite

278 proponatur Platoni] inv. O
278 neget necessariam] inv. O
279 male etiam] inv.
O
279 proponitur] proponatur tibi O
281 et ... Et] om. O
281 respondeat] nam add.
O
281 illorum] ad aliquam aliam propositionem add. O
281 ad istam] deus est add. O
282 et ... predicto] om. O

 $\langle i \rangle$ Si concedis illam, tu concedis falsum non sequens, igitur tu male respondes, posito quod tu non respondeas ad aliquam propositionem nisi ad istam quam nunc tibi propono.

Item, si tu respondes sicut iste illorum qui male respondet, et ille illorum qui male respondet respondet negative, igitur tu respondes negative. Voco enim hic unum ita respondere sicut alius quando uterque respondet affirmative aut negative etc.

 $\langle ii \rangle$ Si dubitas illam 'tu respondes sicut iste istorum || qui male respondet' et 290 V 74^{rb} Cicero etiam dubitat illam male respondendo, igitur tu respondes sicut ille illorum qui male respondet. || Illa consequentia est bona etc.; et antecedens est L 71^{ra} concedendum; igitur et consequens.

 $\langle iii \rangle$ Si negas istam et respondes negative, et iste illorum qui male respondet respondet negative, igitur tu respondes sicut ille illorum qui male respondet.⁷⁵

 75 (i) Two arguments are provided to rule out the first alternative, namely the concession of 1. The first argument runs as follows: if the respondent grants 1., he thereby violates the fifth rule (R5) governing the replies to irrelevant sentences. The reason is that 1. is in fact irrelevant and false, because the respondent is actually replying as the one who is replying correctly (=Socrates) and therefore he (= the respondent) is replying correctly as well. In other words, if the respondent grants the sentence 'tu respondes sicut ille illorum qui male respondet' and Socrates grants the sentence 'deus est', they both reply in the same way; but we have assumed that Socrates replies correctly, therefore the respondent too replies correctly. It makes no difference at all that this happens to be the case with respect to different sentences: the point is just the two of them perform the same speech act, namely conceding, denying or doubting. An additional problem would be that the sentence proposed at step 1. says that the respondent replies like the one who replies incorrectly, therefore, if he replies correctly, by granting it, that would imply that he is replying incorrectly. Peter does not mention anyway the risk represented by such a purported self-referential paradox. I think the reason of this absence is that the paradox is only apparent, because in fact if one grants that he is replying incorrectly, it does not follow that what he has granted is the case, namely that he is replying incorrectly.

The second reason why granting 1. must be ruled out is that afterwards step. 1.1.2 might be proposed. Now, 1.1.2 actually is a valid inference, provided that we supply the additional assumption 'et ille illorum qui male respondet respondet negative' (the case would hold as well if all occurrences of 'negative' were replaced by 'dubitative'). But its antecedent, namely step 1., has been granted by hypothesis, therefore its consequent ought to be granted too. But again, the consequent that ought to be granted is precisely the sentence 'tu respondes negative'. Thus we come to the paradoxical conclusion that if the respondent replies affirmatively, he ought to

285

this fact, however, a unifying common thread is likely to be identifiable in the treatment of issues that may arise as long as we introduce in the obligational context considerations concerning the time and context of utterance of certain types of *proposita*. Peter takes into account three different outcomes, depending on how the respondent decides to reply to the first *propositum* at step 1, namely to the sentence 'tu respondes sicut ille illorum qui male respondet'. The idea, as usual, is showing that no matter what kind of response is given, it will be incorrect. Cf. also [Ashworth, 1984] for discussion of related kinds of inconsistency.

²⁸³ tu ... respondes] etc. O 284 posito] pono enim O 288 unum] ante sicut transp. O 288 alius] alium O 290 Si] tu add. O 292 et] om. O 293 et] om. O 294 et] EO ergo M

3.29. Sed pro isto dicitur quod ad propositionem propositam est respondendum secundum qualitatem illius mensure pro qua proponitur, nisi alia fuerit conven-llio inter opponentem et respondentem huic repugnans aut nisi repugnet ei quod est obligatum. Ex quo sequitur quod non valet consequentia ista 'hec propositio est vera et impertinens tibi proposita, igitur hec est a te concedenda'. Ut si tibi tacenti 300 proponatur illa 'tu loqueris', dum tu neges eam, bene respondes; et illa est vera et impertinens tunc; sed non negas illam pro tempore pro quo respondes. Ita etiam non sequitur 'tu concedis falsum non obligatus, igitur tu male respondes'. Sed bene sequitur 'tu concedis falsum non obligatus, pro tempore pro quo proponitur, igitur male respondes'.⁷⁶ 305

grant that he replies negatively.

(ii) If the respondent replies by doubting 1., he replies exactly in the same way as Cicero does (albeit to a different sentence). But Cicero replies incorrectly, therefore the respondent as well replies incorrectly as Cicero does. The inference at step. 1.2.1 is valid and, again, the antecedent ought to be granted; therefore the consequent ought to be granted too. But the consequent is exactly the sentence that has just been doubted, hence respondent will have granted and doubted 1. at one and the same time, because the duty of conceding of 1. is derivable from a valid inference plus the assumption that 1. has been doubted.

(iii) An analogous piece of reasoning is supposed to hold also in the case of the denial of 1. The text requires a correction, whose plausibility is confirmed by O: the reading to adopt is 'si negas istam, igitur (instead of 'et') respondes negative'.

⁷⁶ These general remarks are, supposedly, meant to introduce the solution discussed in the next section (cf. infra sec. 3.30 'Et ita intelligantur quedam ex dictis' where 'dictis' obviously refers to the remarks made in the present section). Here we might find some clue to figure out the role of this sophism: the point is how to interpret the general duty of replying in a given way to a sentence. This must be done, in standard cases, in accordance with the status the *propositum* has with respect to the time when it is put forward (this is what I take the clause 'secundum qualitatem illius mensure pro qua proponitur' to mean). As a consequence of this assumption, it does not follow *simpliciter*, i.e. without any further qualification, that an irrelevant true sentence ought to be granted, if it is proposed in a disputation. The duty arises only if the rule is understood with the correct reference to time. From a more general point of view, that might be seen as a further qualification of the rule for irrelevant sentences, at least if we are to take at face value the particular example that is under consideration here, but it cannot be excluded that Peter might have upheld it as a more general characterization, valid also for the the other rules. Moreover, it strikes me that this sophism, which is found besides Peter's treatise, only in one case before him, namely in Fland, and one case after him, Paul of Venice, is discussed by the latter, albeit with a different treatment, in connection with his tenth rule for irrelevant sentences (the sophism in Paul hinges on the attempt to prove that there exists an irrelevant sentence that should neither be granted, denied nor doubted). Be it as it may, this is not the only problem that we are confronted with in the attempt to identify the right co-ordinates of this sophism, since the sequence of corollaries of sec. 3.32 also raises many difficulties, insofar as they seem to address, once more, the problem of granting and denying one and the same sentence in a

116

M 85^{vb}

²⁹⁸ opponentem et respondentem] *inv.* O **298** nisi] hoc *add.* O **298** repugnet] repugnat O 298–299 est obligatum] obligatur O 299 consequentia ista] hoc argumentum O **300** hec] om. O **301** proponatur] tibi add. O **301** neges] negas O **302** illam] eam O 303 tu] om. O 305 igitur] sup. lin. conf. O

3.30. Et ita intelligantur quedam ex dictis. Ex quibus patet responsio ad casum: admisso enim casu, negatur illa propositio 'tu respondes sicut ille illorum qui male respondet' tamquam falsa pro tempore pro quo proponebatur non sequens. Et capitur hic iste terminus 'ita respondere' pro similitudine responsionis in concessione, negatione vel dubitatione etc. De virtute enim sermonis, qualiter respondet 310 Sor taliter et Plato respondet ad illam 'deus est'.

 $\langle Ad \, iii \rangle$ Et quando, ulterius, arguitur 'tu negas illam, igitur tu respondes negative; et iste illorum qui male respondet respondet negative; igitur tu respondes sicut ille illorum qui male respondet', negatur antecedens || tamquam repugnans.

Et si proponitur 'tu respondes ad istam et non respondes affirmative neque 315 dubitative neque alia responsione nisi negative, igitur etc.', negatur antecedens.

Et si queritur qua responsione respondes, non certificetur || donec fiat sequens per ordinem proponendi.

Et ex hoc patet responsio ad tales propositiones etiam extra tempus: cum proponuntur, tu || negas propositionem quam tibi propono; illa enim neganda 320 P 69^{ra} est.

Et quando tu negas, si proponitur 'tu negas istam tibi propositam' conceditur, quia facta est vera pro tempore prolationis. 77

given disputation. In any case, a point that can safely be made at this preliminary stage, is that different issues are conflated in this discussion, and that they all have to do, roughly speaking, with the relation between certain *proposita*, their respective replies and the time or the context of their utterance. In the following sections the point will be spelled out more in detail.

⁷⁷ In this section, Peter provides his solution to the sophism. The case must be admitted and the correct way to reply is denying step 1., namely the sentence 'tu respondes sicut ille illorum qui male respondet'. The reason that justifies this move is related to the remarks of the previous section: the sentence is irrelevant and false *with respect to the time* when it is put forward. The irrelevance depends on the fact that 'tu respondes sicut ille illorum qui male respondet' does not follow from the *positum*. The reason for its falsehood is less evident. Peter does not say much about it. Basically, I think the problem is that we lack some essential information in order to evaluate its status in terms of truth or falsehood, because its truth value depends on an event (namely, the respondent's reply) that has not taken place yet, at the time when the sentence is put forward. It is difficult to assess the truth value of a sentence involving a reference to an act that has not been performed: the respondent has not replied to anything yet, when the sentence is put forward. How can we determine its correct truth value? It is pretty clear that Peter is willing to regard the sentence as false. My explanation thus is that the sentence is false exactly by virtue of the fact that the respondent has not replied yet: therefore, *a fortiori*, he cannot have replied 'sicut ille illorum qui male respondet'.

Let us return to the argument. We have decided to deny 1. as irrelevant and false *pro tempore pro quo proponebatur*. Now, how are we supposed to deal with the subsequent objection, put

307 propositio] proposita O **308** proponebatur] proponitur O **309** capitur] capiatur O **310** qualiter] qualitercumque O **311** et] om. O **311** Plato respondet] inv. O **312** tu] om. O **315** respondes] ad illam add. O **315** affirmative] nec praep. O **316** negatur] dicitur negando O **317** queritur] queratur O **317** fiat] fiet O **319** Et] om. O **320** proponuntur] proponitur O **322** tu] om. O **322** proponitur] proponatur O **322** conceditur] concedatur O **323** prolationis] propositionis O

O 119^{vb}

B 97^{vb}

forward at step 1.3.1? The following sentence is proposed '[antecedent =] tu negas illam [*scil.* 'tu respondes sicut ille illorum qui male respondet'], igitur respondes negative; et iste illorum qui male respondet respondet negative; igitur [consequent =] tu respondes sicut ille illorum qui male respondet'. It contains a valid inference, therefore the whole sentence should be granted. If we grant the antecedent 'tu negas illam' (where 'illam' always refers to the first *propositum* of step 1.), we have the problem already described, because by iterated detachment, we can derive the duty of granting the consequent 'tu respondes sicut ille illorum qui male respondet', i.e. the very same sentence that we have just denied, at the preceding step, as irrelevant and false *pro tempore pro quo proponebatur*. The solution is denying the antecedent of 1.3.1, namely the sentence 'tu negas illam', as incompatibly relevant (for its concession would entail the concession of a sentence that has already been correctly denied).

After this move, one might still be willing to raise the objection that, after all, the respondent does not reply but negatively, therefore the same conclusion is supposed to follow, i.e. that he replies as the one who replies incorrectly. Yet, on Peter's view, the antecedent of this consequence would still have to be denied, again – presumably – as incompatibly relevant (cf. 'et si proponitur 'tu respondes ad eam et non respondes affirmative, neque negative, neque dubitative, neque alia responsione nisi negative; igitur etc.' negatur antecedens').

Moreover, if it is argued that it can be shown by exclusion that the respondent replies negatively (it can be easily done by disjunctive syllogism), then Peter adopts the familiar strategy of reply *per ordinem*: let all alternative ways of replying (affirmatively, dubitatively and so on) be put forward one after another; and let them all be answered according to their status, until we come in the end to the last one, which will have become relevant, in the meanwhile (cf. 'et si queritur qua responsione respondes, non certificetur donec fiat sequens per ordinem proponendi').

Two concluding remarks. First, reference to time might also be needed in order to have the first part of the argument work properly. Step 1.1 (granting the first *propositum*) is incorrect, because it is argued that if the respondent grants something irrelevant and false, then he replies incorrectly. But the point made in sec. 3.29 seems to go the opposite way around (cf. 'Ita etiam non sequitur 'tu concedis falsum non obligatus, igitur tu male respondes'. Sed bene sequitur 'tu concedis falsum non obligatus, pro tempore pro quo proponitur, igitur male respondes") by calling for the fulfillment of an additional condition. If the condition is added to 1.1.1 above, then there is a different reason to maintain that 'tu respondes sicut ille illorum qui male respondet' is false. Above I have argued that the falsehood depends on the fact that the respondent actually replies as the one, in the triad, who replies correctly, i.e. Socrates; and therefore if he grants that he replies as the one who replies incorrectly, he is going to grant something false. Now, if we add the reference to time, the reason is the same as in case 1.2: the respondent a fortiori does not reply incorrectly, because at the time when the sentence 'tu respondes sicut ille illorum qui male respondet' is put forward, he is still not replying at all. Second, the solution is generalized, in the concluding part of sec. 3.28, to the reply that is supposed to be given *extra tempus*. In this connection, Peter makes a very intriguing, albeit contracted, example involving a self-referential statement. Take the sentence 'tu negas propositionem quam tibi propono'. If it is put forward (I propose to follow the text of manuscript O in emending 'proponuntur' with 'proponitur', as referring to the sentence which comes next rather than to the preceding 'propositiones'), one might argue that, since the clause 'propositionem quam tibi propono' refers to the very same sentence in which it occurs, then whatever reply is given to it, it is bound to be incorrect. If the sentence is granted or doubted, then it should have been denied, because the speech acts that are performed in replying to it are sufficient to falsify it (the sentence says of itself that you deny it, but you do not, therefore what it says to be the case, turns out not to be the case; but if what it says to be the case turns out not to be the case, the sentence is false and in the environment

3.31. Sed forte arguitur || probando quod non pro tempore pro quo proponitur M 86^{ra} || propositio est respondendum ad eam secundum sui qualitatem,⁷⁸ quia ex illo 325 E Giiii^{rb} (Obj.1) sequitur quod Sor et Plato respondent ad eandem propositionem unus concedendo, alter alio modo respondendo in eodem sensu || extra tempus obligationis, et tamen uterque non male respondet ad istam propositionem.⁷⁹

(Obj.2) Item, sequitur quod Sor concedit A propositionem et Plato concedit contradictorium A, utroque bene respondente et non obligato.

(Obj.3) Item, sequitur quod Sor concedit et negat eandem propositionem non male respondendo: quia ponatur quia Sor eadem propositio proponatur a diversis, sed unus ante alium proponat eam, sic quod sit falsa quando unus proponit et vera quando alter proponit.

(Obj.4) Item, potest Sor ponere A propositionem Platoni et per horam aut sal- 335 L 71^{rb} tem per tempus notabile || potest Plato differre responsionem; et interea forte propositio proposita est facta impossibilis; et tunc sequitur quod Sor bene respondet concedendo impossibile. Que omnia predictis videntur repugnare.⁸⁰

in which we are discussing, namely *extra tempus*, this entails that it should be denied. On the other hand, if the sentence is denied, what it says to be the case, actually happens to be the case, therefore the sentence should have been granted instead of being denied. What are we supposed to do, then? Peter's conclusion is that the sentence 'tu negas propositionem quam tibi propono' must be denied. And the reason is the same as in the solution to the sophism: it is false, because no one has replied to anything yet, and *extra tempus* one must always respond according to the assumption that rei veritas est fatenda (cf. supra, sec. 1.5 where the fourth assumption is laid down). After the sentence 'tu negas propositionem quam tibi propono' has been denied, if the sentence 'tu negas istam tibi propositam' is put forward, then and only then it must be granted, since in the meantime it has become true (cf. 'quia facta est vera pro tempore prolationis'). So the point is to deny sentences such that the states of affairs that would make them true have not obtained yet.

 78 In this section, Peter presents a group of five objections that might be raised against the requirement of replying to a sentence with respect to the time when it is put forward and uttered by the opponent.

 79 The first two objections concern the effects of the assumption in question *extra tempus* but the structure of both arguments is not perpicuous.

⁸⁰ The text of the incunable makes some confusion: the correct readings are supplied by ms. M, according to which the role of opponent is played by Plato, whereas the role of respondent is played by Socrates. Moreover, since at stake, in this set of objections, are the effects of the assumption of time reference outside the period of an obligation (sentences must be therefore evaluated according to their actual truth values), I think that 'ponere' should be replaced by 'proponere', as suggested by mss. M and O.

324 forte] contra *add*. O **325** est respondendum] *inv. et post* ad eam *transp.* Ο 325 secundum ... qualitatem] om. O 327 alio ... respondendo] negando O 328 istam] eandem O 331 sequitur] om. O 332 respondendo] ad illam add. O 333 sed ... eam] om. O 333 sit falsa] transp. O 333 proponit] eam ipsam add. O 333 et] om. O 333 vera] transp. O **334** quando] vero add. O **334** proponit] propositionis et add. ipsa sit vera O 335 potest] posset MO 335 Sor] Plato M 335 ponere] proponere MO 335 Platoni] transp. O; Sorti M 335 et] om. O 335–336 aut ... forte] om. O 336 Plato] Sor M 337 proposita] Sortis O

V 74^{va}

(Obj.5) Item, proponatur hec Sor tacenti et non obligato 'tu negabis vel dubitabis propositionem quam tibi propono'.

 $\langle i\rangle$ Si Sor neget e
am, tunc ista propositio pro tempore prolationis fuit vera: igitur concedenda.

(ii) Si concedat, tunc ista fuit falsa: igitur neganda.

(iii) Si dubitat, tunc ista fuit vera pro tempore prolationis: igitur concedenda.⁸¹

3.32. Sed pro his dicitur quod conclusiones antedicte non sunt impossibiles, cum $_{345}$ ad diversa tempora referantur.⁸²

Item, hic certa regula poni non potest nisi secundum quod opponens et respondens conveniunt, cum pro eadem mensura non possit responsurus respondere bene respondendo pro qua proponit opponens, quia ante responderet quam intelligeret illud ad quod responsurus erat.⁸³

The idea here is that Plato may propose to Socrates a sentence p, which is possible at time t_1 . Socrates takes his time, before replying; then, after a lapse of time, say at t_2 , Socrates replies to p, according to its status at t_1 , i.e. when the sentence was possible. But in the meanwhile, the sentence has become impossible for some change that has taken place in actual reality. If Socrates replies to p at t_2 according to its status with respect to the time of the original utterance, he will thereby grant an impossible sentence.

⁸¹ The fifth and last objection is interesting. Let us assume that, at time t_1 , the following sentence is proposed to Socrates, while he is silent and under no obligation: 'tu negabis vel dubitabis propositionem quam tibi propono' (= p). It plainly contains a self-referential clause, because the only sentence that is put forward actually is p itself; therefore, we could replace the clause in question by rephrasing the original sentence as follows: 'tu negabis vel dubitabis p'(= p). Now, let us assume that at some time t_2 Socrates replies to p. If he denies or doubts pat t_2 , then the sentence 'tu negabis vel dubitabis p', proposed at t_1 was true (cf. 'et tunc ista propositio pro tempore prolationis fuit vera, igitur concedenda': recall the duty of answering truthfully *extra tempus*). If, on the other hand, Socrates grants p at t_2 , then p at t_1 was false and should have been denied accordingly (cf. 'ista fuit falsa, igitur neganda'). In conclusion, if Socrates (i) denies or (iii) doubts p, he should have granted it, whereas if he (ii) grants p, he should have denied it.

⁸² The remark is especially appropriate if we look at the last objection of the previous section. There was no duty to grant sentence p, at time t_1 , because p becomes true only at time t_2 . Therefore, the duty of granting p is referred to t_2 , not to t_1 , i.e. to the time when the sentence is actually denied or doubted: the two inconsistent replies are referring to different times. What one could claim, at most, is that p should be granted at time t_2 , because it is true for or with respect to time t_1 .

⁸³ The sense of this remark is not fully clear. At first sight, it is meant to narrow down the scope of the time reference assumption, if we are allowed to take the term 'hic' as meaning 'in such cases', i.e. in cases like those presented in the objections, which should count as exceptions to the routine, when the assumption applies. This would be consistent, I reckon, with the remark made in sec. 3.29 after presenting the assumption, which is supposed to apply 'nisi alia fuerit conventio

341 pro ... prolationis] *post* vera *transp.* O **343** concedat] concedis O **343** ista] illa propositio O **344** dubitat] dubitas O **344** igitur concedenda] *om.* O **345** antedicte] antecedentes O **346** ad] *om.* O **347** hic] hec tamen O **347** certa regula] *inv.* O **347–348** opponens et respondens] *inv. et transp.* O **348** responsurus respondere] *inv.* O **350** illud] *om.* O **350** erat] est O

340

Sed de hoc postea dicetur.⁸⁴ Ad ultimum infra dicendum est.

3.33. Item, si ponatur quod ad nihil sis obligatus, deinde proponitur 'tu es obligatus'.⁸⁵

(Obj.1) Si conceditur, contra: nulla specie obligationis es obligatus, igitur non

 84 Possibly, the *locus* that is referred to, in this passage, is sec. 5.13, where Peter, in the context of a discussion on *impositio*, rejects the case posited in the sophism 'omne nesciens se esse A est A' because it is not understood from the very beginning what the signification of A is.

85 This sophism and the next one belong to a very popular class of arguments that are quite often discussed in obligational treatises. Sometimes this kind of sentences are referred to in modern literature by the label 'pragmatic paradoxes', cf. [Ashworth, 1984], [Ashworth, 1985], [Dutilh Novaes, 2006a] and [Schupp, 1993, especially pp. 103–115]. Their discussion is generally meant to establish whether possibility, as a criterion for the admission of *posita*, should apply in a restricted or in an unrestricted manner (cf. [Ashworth, 1985, p. 314]; an example of restricted application is provided, for instance, by [Albert of Saxony, 1490, sig. Niiii+4^{ra-rb}] "talis positio [scil. 'nihil est tibi positum'] non est admittenda, quia positum repugnat obligationi [...] Talis positio [scil. 'tu nihil admittis'] non est admittenda, quia repugnat admissioni". Other authors like [Ralph Strode, 1517, fol. 79^{ra}] "Ex quibus patet quod tales casus negare non est nisi fuga miserorum nescientium huius artis vigorem", [William Buser, 1990, pp. 178] "respondent aliqui faciliter unica responsione, scilicet non admittendo positum [...] Sed istae responsiones sunt tampuam fugae miserorum fugentium difficultatem" and Peter himself opt for unrestricted application because they see no reason to rule out sentences that are, after all, possible in their own right although they may cause problems with respect to the context of their utterance. Billingham also holds a similar view, cf. [Ashworth, 1985, p. 316]: "Billingham and his followers clearly did not see this sophism as posing any problems of pragmatic inconsistency". Peter has already quickly mentioned the problem (cf. supra, sec. 2.1). It is probably not a matter of mere coincidence that the examples he provides to clarify how the rule for admission must be understood, are picked out from a class of sentences (such as 'nulla propositio sit' and the like) that one might be inclined to reject because of the pragmatic inconsistency they may generate. This type of sentences, despite their apparent paradoxical character, must be admitted whenever posited, since they are possible. The idea is confirmed, here, by the solution provided to both sophisms.

The first one is a variation on a theme that commonly comes in the more familiar form of the sentences 'nihil est tibi positum' or 'tu non es obligatus', cf. for instance [Walter Burley, 1963, p. 49], [Marsilius of Inghen, 1489, sig. Biiii+2^{rb-va}], [Albert of Saxony, 1490, sig. Niiii+3^{vb}-Niiii+4^{va}], [Ralph Strode, 1517, fol. 79^{ra}], [William Buser, 1990, pp. 176–180], [Paul of Venice, 1988, p. 170], [Roger Swyneshed, 1977, p. 256].

351 hoc] hac O **351** postea] plus add. O **351** dicetur] om. O **351** Ad ... est] om. O **352** Item] alt- forte pro alter casus in marg. O **352** si] om. O **352** sis] conf. forte species O **352** proponitur] proponatur O **354** obligationis] tu add. O

M 86^{rb}

inter opponentem et respondentem huic repugnans [i.e. incompatible with the circumstance that a sentence is answered with respect to the time of its utterance] aut nisi repugnet ei quod est obligatum'. Cases like those presented in the above objections would be precisely the outcome of this kind of exceptional 'conventio', that make it difficult to establish a rule (cf. 'certa regula poni non potest'). In such cases, some additional hypothesis is needed and the assumption does not hold *simpliciter*.

es obligatus. Patet consequentia. Et antecedens quia ad nihil es obligatus.

 $\langle Obj.2 \rangle$ Item, hec est impertinens 'tu es obligatus', que extra tempus esset neganda, igitur nunc est neganda.

3.34. Sed huic breviter dicitur, admisso casu, concedendo istam 'tu es obligatus'.

(RObj.1) Et negatur ista consequentia 'ad nihil es obligatus, igitur nulla specie obligationis es obligatus', quia stat quod ista propositio ad quam sis obligatus non 360 sit aliquid sed aliqua.

(RObj.2) Secunda etiam forma non valet: quia, per obligationem factam, est ista vera 'tu es obligatus', que extra tempus erat falsa.⁸⁶

⁸⁶ The sophism has the following structure:

P	Ad nihil es obligatus	Α	possible
1.	Tu es obligatus	\mathbf{C}	
2.	Nulla specie obligationis es obligatus, igitur non es obligatus	С	\perp the inference is valid, and the antecedent holds because it is implied by P ('ad nihil es obligatus, igitur nulla spe- cie obligationis es obligatus' is valid too)
3.	Hec est impertinens 'tu es obligatus', que extra tempus esset neganda, igitur nunc est neganda	С	

Solution: admission of the *positum*, concession of 1., denial of the inference 'ad nihil es obligatus, igitur nulla specie obligationis es obligatus'. The argument against this consequence appears pretty sophistical, but Peter is not the only author to endorse such a view (cf. [William Buser, 1990, p. 180] "ista propositio 'nihil est tibi positum', sicut nec quaecumque alia, non est aliquid, sed est aliqua, scilicet subjectum, praedicatum et copula". This fact alone, I reckon, would speak against the claim that "we do not find direct traces of Buser's work in the writings of influential authors like [...] Peter of Mantua", [Kneepkens, 1982, p. 160]). The posited sentence 'ad nihil es obligatus' is equivalent to 'non ad aliquid es obligatus'. The respondent, however, is obligated to a sentence. But a sentence is not just *something* (= aliquid), but rather *several things* (= aliqua). One could also understand Peter's use of 'aliqua' as referring to the feminine gender of the latin term 'propositio' but I am more inclined to think that the former solution is the right one on the basis of the textual parallel with Buser.

The reason for the denial of the second argument ('hec est impertinens 'tu es obligatus', que extra tempus esset neganda, igitur nunc est neganda') is interesting. Apart for the role it plays in the argument, which is quite clear, it might prompt the impression of an unusual counterfactual flavour in the assertion that a sentence that was false (and therefore ought to be denied) outside the period of the obligation has become true *per obligationem factam*. At first sight one might be tempted to say that nothing like that is going on here, because we would need to have a clause such as 'per obligatum' or 'per positum' instead of the clause 'per obligationem factam'. The latter remark might suggest that we are in the presence of a familiar example of relevance to the obligation which fits pretty well in the context of a pragmatic paradox (we will see a different

355 Patet] hec O **355** Et] arguitur *add.* O **355** nihil] tu *add.* O **357** nunc] adhuc O **359** nihil] tu *add.* O **359** nulla specie] per nullam speciem O **360** obligationis] tu *add.* O **360** quam] tu *add.* O **362–363** etiam ... vera] *om.* O

355

O 120^{ra}

3.35. Item, ponatur quod nulla propositio sit. Deinde proponitur 'omne positum et bene admissum propositum est concedendum'.⁸⁷

 $\langle Obj.1 \rangle$ Si conceditur, igitur aliqua propositio est.

 $\langle \text{Obj.2} \rangle$ Si negatur ista, tu negas regu-lam; igitur male respondes. Si conceditur totum, inferatur quod aliqua propositio est, quia bene sequitur 'tu negas regulam, igitur aliqua propositio est', cum omnis regula sit propositio. Ideo forte dicitur negando antecedens.⁸⁸ 370

 $\langle Obj.3 \rangle$ Sed tunc, forte, queritur quare negatur antecedens.

 $\langle i \rangle$ Cui si dicitur quia repugnat, arguitur antecedens sic, quia bene sequitur 'id repugnat, igitur illud est'.

 $\langle ii\rangle$ Vel, si forte dicitur quod tu negas ipsum qui
a tibi placet negare ipsum, igitur ipsum est.

 $\langle {\rm Obj.4}\rangle$ Item, cedat tempus obligationis, negata ista 'omne positum et bene admissum est concedendum'. Et arguitur: infra tempus negasti regulam, igitur male.⁸⁹

story in the section on *impositio*, where Peter really seems to be claiming something that can be assimilated to a kind of counterfactual reasoning). Yet, Obj.2 claims that the sentence 'tu es obligatus' is irrelevant and true. Peter does not explicitly declares what logical status he ascribes to the sentence, but it is reasonable to assume that he regards it as irrelevant too (otherwise, rather than saying that the sentence is *true* in virtue of the obligation, he would have probably said that the sentence had become *relevant*, as opposed to the claim made in the objection). The point is that the sentence is not false, but true, and this is how the objection is dismissed. It is no big deal to realize that the truth value of an irrelevant sentence is often influenced by the casus and that the latter often happens to be inconsistent with the *positum*. The unusual thing is, by contrast, that the *casus* takes over the *positum* in this sophism. It is a matter of granting a sentence because it follows from the *casus* (the expression might be misleading: following from the casus simply means being true according to it) and is incompatible not with the positum but with the very same fact that an obligation arises (a common way of arguing against pragmatic paradoxes). It is basically not a matter of relevance at all: here all the weight of the argument is put on the shoulders the *casus*, i.e. the circumstance that something is actually posited to the respondent. Normally, the two levels are kept sharply distinct, but here the *casus* comes into play as the only independent variable.

⁸⁷ This is a contracted version of the first obligational rule (R1), cf. *supra*, sec. 2.1.

⁸⁸ Namely, the sentence 'tu negas regulam'.

⁸⁹ The first three objections claim, in various ways, that no matter how the respondent replies to the *propositum* that is put forward in the first place (i.e. the first rule in thesis-form), it follows, in any case, that at least a sentence exists, contrary to what is posited. The fourth objection, on the other hand, insists rather on the fact that the respondent replies incorrectly, if he denies a rule.

364 Item] casus *in marg.* O **365** admissum] infra tempus obligationis *add.* O **365** propositum] *om.* O **366** conceditur] contra *add.* O **366** propositio] *om.* O **367** ista] contra O **367** conceditur] concedatur O **369** cum] quia O **369** sit] est O **371** forte] *om.* O **372** Cui] *om.* O **372** antecedens sic] quod illud antecedens sit O **372** id] illud antecedens O **373** illud] antecedens *add.* O **374** tibi placet] *inv.* O **377** admissum] propositum *add.* O **377** arguitur] sic *add.* O **377** tempus] tu *add.* O **378** male] respondisti *add.* O

P 69^{rb}

V 74^{vb}

375

3.36. Ideo dicitur, admisso posito, negando quod omne positum et bene admissum propositum est concedendum.

 $\langle \mathrm{RObj.2}\rangle$ Et cum arguitur 'tu negas regulam, igitur male respondes', negatur antecedens.

 $\langle \text{RObj.3} \rangle$ Et cum queritur quare negatur, dicitur quod nulla propositio negatur neque regula negatur $\langle Ad \rangle$ quia repugnans, neque negatur $\langle Ad \rangle$ quia placet negare.

(RObj.4) Et cum arguitur ulterius 'cedat tempus obligationis' et arguitur quod in tempore negasti regulam || igitur male respondisti, dicitur negando consequentiam, quia illa regula erat contingens repugnans posito.

Unde generaliter nulla regula hic posita est concedenda, quando positum bene admissum repugnat ei. 90 390 M 86^{va}

3.37. Item, pono tibi sedenti istam 'omnis homo currit'. Deinde propono 'tu $L 71^{va}$ sedes'.

⁹⁰ The sophism has the following structure:

P N	Nulla propositio est	А	possible
1.1 (Omne positum et bene admissum proposi-	С	\perp (cf. Obj.1: aliqua
\mathbf{t}	um est concedendum		propositio est)
1.2		Ν	
2. T	Fu negas regulam, igitur male respondes	С	
3. Т	Fu negas regulam, igitur aliqua propositio	С	\perp (cf. Obj.2: omnis regula
e	est		est propositio, igitur aliqua
			propositio est)
4. T	Fu negas regulam	Ν	\perp (cf. Obj.3 i–ii: aliqua
			propositio est)

The solution to the sophism consists in admitting the *positum* and denying the first *propositum*, i.e. the sentence 'omne positum et bene admissum propositum est concedendum' (the replies to the objections are intended to prove that, once this sentence has been denied, we are by no means forced into contradiction by the structure of sentences and replies proposed the-reafter). In particular, the solution hinges on the assumption that a rule can be denied, if it is put forward in a disputation, in virtue of its contingency. Remarkably, Peter suggests here that rules must be regarded as non-necessary sentences whose negation is therefore possible (and sometimes acceptable as in the present case). The respondent can have recourse to this characterization, whenever the rule is repugnant to the *positum*. There seems to be a clash between posited sentences must be discussed. In any case, the suspension of validity holds only for incompatibility with the *positum*. An analogous treatment is found in [William Buser, 1990, p. 108–109] and [Paul of Venice, 1988, p. 314] "Omnes regule obligationis habent intelligi sine obligatione contraria".

379 posito] casu O **384** regula negatur] *inv.* O **384** negatur] *om.* O **384** placet] mihi add. O **385** negare] regulam add. O **386** ulterius] *transp.* O **387** in tempore] infra tempus O **387** dicitur negando] negatur O **387–388** consequentiam] consequentia O **388** erat] est O **388** contingens] et add. O **389** concedenda] est add. O **389** quando] quare O **389–390** bene admissum] *om.* O **391** Item] alius casus *in marg.* O

380

385

E Giiii^{va}

 $B 98^{ra}$

 $\langle \text{Obj.1} \rangle$ Si negatur, contra: tu negas sequens, igitur male respondes, quia bene sequitur 'pono tibi sedenti istam 'tu es homo', igitur tu sedes'.⁹¹

(Obj.2) Item, in eodem casu proponitur 'hec 'tu curris' non est sequens neque 395 repugnans sed impertinens".

Qua concessa, proponitur ista 'tu es homo'.

Qua concessa, quia vera non repugnans, proponitur 'tu curris'. Si negatur, cedat tempus. Et arguitur quod in tempore male respondisti, quia negasti sequens ex bene admisso et concesso. Si igitur conceditur ista 'tu curris', proponitur 'hec 'tu 400 O 120^{rb} curris' non est sequens neque repugnans sed impertinens'.

Si negatur, negatur a te concessum, igitur male.

Si conceditur, ista consequentia est bona 'omnis homo currit; tu es homo; igitur tu curris' et significat ex compositione suorum terminorum; igitur illa 'tu curris' sequitur ex illis; igitur est sequens.⁹²

 $^{^{91}}$ The correct reading is 'pono tibi sedenti istam 'omnis homo currit', as supplied by ms. O. 92 Here is the structure of the sophism:

P Omnis l	nomo currit	А	possible
1. Tu sede		N	possible
	s sequens, igitur male respondes	C	\perp (cf. Obj.1: bene sequi- tur 'pono tibi sedenti 'omnis homo currit', igitur tu sedes')
	a curris' non est sequens neque uns sed impertinens'	С	
4. Tu es h	-	С	irrelevant and true
5.1 Tu curr	is	Ν	\perp in tempore male respondi- sti, quia negasti sequens ex bene admisso et concesso (cf. P and 4.)
5.2		\mathbf{C}	,
	a curris' non est sequens neque uns sed impertinens'	Ν	\perp negatur a te concessum (cf. 3.), igitur male
6.2	-	С	<i>,,,</i> , , , , , , , , , , , , , , , , ,
currit; t ficat ex	nsequentia est bona 'omnis homo u es homo; igitur tu curris' et signi- compositione suorum terminorum; la 'tu curris' sequitur ex illis; igitur sequens	С	\perp the consequent of this valid inference is inconsistent with 6.2

The two objections, that are resolved in the next section, concern the proposed replies to 1. (first part of the argument) and the inconsistency of 6.2 and the consequent of 7. (second part of the argument).

394 tu ... homo] omnis homo currit O **395** proponitur] proponatur O **397** proponitur] proponatur O **397** ista] om. O **398** quia ... negatur] om. O **399** in tempore] infra tempus O **400** proponitur] proponatur O **401** repugnans] repugnat O **401** sed] quia O **402** negatur₂] om. O **403** conceditur] contra add. O **404** et] illa add. O **404** igitur] ex add. O

3.38. Sed pro his dicitur, admisso posito, negando istam 'tu sedes'.

 $\langle \text{RObj.1} \rangle$ Et negatur ista consequentia 'ista est sequens, igitur est a te concedenda'. Omnis enim propositio, de virtute sermonis, est sequens ex aliquo saltem in casu, licet ipsa sequens sit impossibilis. Respondens enim non obligatur ad concedendum aliquod falsum, nisi sequens ex bene admisso et bene concesso etc. 410 Neque obligatur respondens ad obligationem, sed obligatur ad obligatum per obligationem.⁹³

 93 The sophism is quite articulated and deals with how to understand properly the notion of 'sequens'. The first objection addresses the problem of whether anything else, apart from the positum (and the subsequent steps, as long as they become part of the cumulative 'history' of a disputation), has some role in determining the logical status of sentences. The problem concerns in particular the information conveyed right before the beginning of a disputation while stating the *casus*. Peter resolves the issue by denying the proposed answer of step 1. It does not follow that, if the opponent posits a sentence to the respondent while the latter is sitting, then the respondent's being seated should count in any way (i) to determine the logical status of sentences that come thereafter or (ii), worse, as a part of the *positum* itself. At most, a piece of information like that will play a role when it comes down to assigning a truth value to a sentence that has already been identified, on independent grounds, as irrelevant. Unfortunately, here we have a first big problem with the argument, because in the end Peter rightly denies that the sentence 'tu sedes' is relevant but he also claims that it should be denied. I do not see how he could justify this conclusion. The sentence is to be regarded as irrelevant, and it is actually true, according to the casus (cf. 'pono tibi sedenti istam 'omnis homo currit'). But the fifth rule (R5) requires us to grant sentences with these two features (we may forget about the condition of knowing that the sentence is true because it does not enter into the argument). What then? Is there any room for choice in our responses? The issue remains open.

The solution to the first part of the sophism consists thus in admitting the *positum* and denying the sentence 'tu sedes', proposed in the first place. Once we have denied 1., how are we supposed to deal with the objection put forward at step 2? And then what about the second, more elaborate, part of the sophism, from step 3. on? Peter makes a rather complicated point (cf. RObj.1). The argument is quite interesting in its own right, because it sheds some light on the notions of relevance and irrelevance, that turn out to be the outcome of a sophisticated conceptual effort.

I am not fully convinced by all the details of the replies (particularly those addressing the sequence of sentences proposed in the second part of the disputation), but the main point is quite straightforward as long as it consists in a qualification of the notions (i.e. the properties of being relevant vs irrelevant) that come into play in the sentences put forward at steps 2., 3. and 6.

Peter suggests, as a reply to the objection proposed at step 2., to reject the inference 'ista est sequens, igitur est a te concedenda', because it rests on the unreliable generalization that the respondent replies incorrectly, if he denies a sentence that *follows*. Note that I intentionally use this generic expression and not the technical phrase 'sequentially relevant'. The objection holds only on the assumption that, if a sentence *follows*, then it ought always to be granted. Is that true? On the face of it, in the obligational framework, one would be inclined to answer affirmatively. But in fact it depends on what we exactly mean by 'following'. If following means to be sequentially relevant (according to the definition resulting from the rules) then the answer

407 igitur] illa *add.* O **410** falsum] sequens BMOV **410** et] vel ex *add.* O **410** etc.] vel huiusmodi *add.* O

10 V 75^{ra}

Et per hoc est concedendum, de virtute sermonis, quod nulla propositio est impertinens, quia omnis propositio est pertinens saltem sibi ipsi, quia sequens ad seipsam.⁹⁴

(RObj.2) Et ideo, cum proponitur in illo casu quod hec non est sequens 'tu curris', neganda est; sed conceditur quod ipsa est impertinens admisso et adhuc non est sequens ex aliquo concesso neque ex aliquibus concessis.

Et ultra, si per ordinem formetur argumentum, conceditur ista 'tu es homo' et iterum conceditur quod ista 'tu curris' non est sequens ex aliquibus concessis neque 420 ex aliquo concesso. Et conceditur quod ista consequentia est bona, scilicet 'omnis homo currit; tu es homo; igitur tu curris', et quod iste due sunt concedende 'omnis

The point, therefore, is not that Peter is willing to deny the rule according to which sequentially relevant sentences ought to be granted. I think his aim, here, is showing that 'following' can be said in many ways. In a sense, any sentence, of which it is said that it follows, is said to be following *from something*. In the context of obligations, more specifically, this characterization is equivalent to what we indicate by means of the expression 'sequentially relevant at step n', i.e. anything that is logically entailed or derivable from something that has been previously admitted or granted, according to specific rules.

In sum, the sentence 'tu sedes' is not 'sequens' in the right sense, it is irrelevant and true; still it is claimed that there is no obligation to grant it (cf. 'Respondens enim non obligatur ad concedendum aliquod falsum [here the sentence 'tu sedes'], nisi sequens *ex bene admisso et bene concesso*. Neque obligatur respondens ad obligationem [i.e., I assume, to background assumptions made by stating the *casus*], sed obligatur ad obligatum per obligationem'). I am puzzled by this conclusion, but it is worth adding that it seems to be utterly in conflict with a claim made above, cf. *supra*, sec. 3.34. On that occasion, Peter argued that the sentence 'tu es obligatus' (in a *positio* where the *positum* was 'ad nihil es obligatus') should have been granted as true *per obligationem factam*. Some further reflections are needed to better understand this point.

⁹⁴ The remark, as interesting as it could be in its own right, looks quite threatening in the context of obligations: would it not make the very same distinction between relevant and irrelevant sentences collapse? The answer is negative, for again, as above, it is all about distinguishing the different senses in which the term can be taken. If it is taken literally, then nothing can be said to be irrelevant, because everything is at least relevant to itself, insofar as everything is entailed by itself. This is, however, not the standard sense of the term in the obligational context, where it is employed, by contrast, to characterize in a much more restrictive way the class of sentences falling under it. For it is always implicitly presupposed that it is *with respect to* a given sentence or set of sentences that something is (or is not) irrelevant.

413 per hoc] iuxta hoc O **413** est concedendum] *inv. et transp.* O **414** saltem] *transp.* O **415** seipsam] ipsam O **418** neque] vel O **420** quod] *om.* O **420** tu curris] *transp.* O **420** concessis] concedendis O **421** concesso] *om.* O **421–422** scilicet ... curris] *om.* O

415

 $P 69^{va}$

M 86^{vb}

is indeed affirmative: any sequentially relevant sentence ought to be granted. If, on the other hand, we say of a given sentence that it follows from the *casus*, then we are by no means required to grant it, when it is proposed. As a matter of fact, for the most part, in the construction of sophisms, the *casus* and the *positum* are supposed to conflict in some way, because the falsehood of the latter is determined with respect to the explicit assumptions laid down by means of the former.

homo currit', 'tu es homo'. Et negatur quod ista sequatur ex illis, scilicet 'tu curris', quia dicitur forte quod iste non significant ex compositione suorum terminorum. Et, breviter, semper respondeatur secundum ordinem, semper stando in ultima 425 repugnante ipsam negando.⁹⁵

 95 Here we come to the solution to the second part of the argument. It requires the denial of the sentence proposed at step 3. – 'hec 'tu curris' non est sequens neque repugnans sed impertinens' – that in the above reconstruction of the sophism was granted by hypothesis. If we deny 3. it follows that (i) 'tu curris' is either 'sequens' or 'repugnans' and (ii) it is not 'impertinens'. But Peter has just claimed that no sentence is 'impertinens' at least *de virtute sermonis*. How can we accomodate this? Peter claims that it might be granted that 'tu curris' is irrelevant *to* the admitted sentence, i.e. the *positum*, and that, on the other hand, it does not follow yet *from any other granted sentence* ('cf. 'adhuc non est sequens ex aliquo concesso neque ex aliquibus concessis'). This is based on a distinction between being irrelevant with respect to something as opposed to being irrelevant *simpliciter* and again on the idea that 'following' has a very precise sense in the oligational context: it means following from certain peculiar sentences.

The two notions (relevance and irrelevance) are, to some extent, qualified. A sentence is not irrelevant simpliciter but it may be said to be irrelevant to the positum; it cannot be denied that a sentence is relevant (for it is at least relevant to itself, on the literal reading), but that sentence can be said not to be relevant at a given time with respect to something that has been previously evaluated in a disputation. As a result, it is plausible to regard the two notions according to the characterization outlined above in Part I, sec. 1.3–1.4, i.e. as always involving an index: relevant at step n means relevant with respect to a particular set of sentences, while relevant at, say, step n+7 may well refer to a different set (which includes the former). All the more so, if we think of these notions in dynamic terms as it happens to be the case in the *responsio antiqua*. For any given disputation (i.e. once we have set up the *casus* and posited a sentence), there are no such things as the class of relevant vs the class of irrelevant sentences, fixed once and for all. The two classes evolve according to what is proposed and evaluated at each step. A sentence that is not relevant at, say, step n might become relevant at step n + 7, depending on what has been granted or denied in the meanwhile. If we think of these notions as being indexed to the steps of a disputation, so that we would have a sequence of predicates, each of which is linked to a single step, we may have a way to understand the structure of this sophism. Peter is claiming, I suggest, that the respondent is actually responding to different sentences (involving, as it were, different notions of being relevant and irrelevant) at the two steps 3. and 6.

The final part of the reply concerns the argument 'per ordinem' and is devoted to the sequence of *proposita* 4. to 7. Step 4. 'tu es homo' should be granted, as irrelevant and true; step 5. 'tu curris' should be granted too as sequentially relevant (if follows from the *positum* and 4.). With step 6., by contrast, we have a serious problem. Let us look at it closely. The original argument of the sophism is clear: the sentence 'hec 'tu curris' non est sequens neque repugnans sed impertinens' was proposed twice (at step 3. and at step 6.). The first occurrence (step 3.) was put forward before 'tu curris' (step 4., sequentially relevant) and apparently it was correctly granted, because *at that point* of the disputation 'tu curris' actually was irrelevant. In the evaluation of the second occurrence (step 6.) we must keep in mind that, in the meantime, 'tu curris' has become sequentially relevant and therefore it is no longer irrelevant. We can deny the *propositum* (step 6.1) or we can grant it (step 6.2). In either case we reply incorrectly, because 6.1 is inconsistent with the reply given at step 3., whereas 6.2 is inconsistent with the consequent

423 scilicet ... curris] om. O **424** iste] illa O **424** significant] significat O **425** semper] consequenter O **425** semper] om. O **425** in] illa add. O **426** repugnante] repugnanti O

3.39. Item, ponatur quod omnis homo qui est albus currat et nullus istorum potest currere. Et per ly 'illorum', in ista propositione 'nullus istorum potest currere' adequate significante quod nullus istorum potest currere, demonstrentur omnes homines qui sunt albi. Quo admisso, proponitur 'nullus istorum potest currere'. 430 Et illud est concedendum, quia sequens ex copulativa que est casus.

 $\langle Obj.1 \rangle$ Deinde proponitur 'aliquis istorum potest currere', demonstratis omnibus hominibus qui sunt albi per ly 'illorum'.

 $\langle i \rangle$ Si conceditur, contra: quia concedi
s repugnans, quia contradictorium concessi.

And this is not the end of the story, because Peter claims that the correct reply to the second occurrence of our problematic *propositum* is granting it. But how can this be compatible with the denial suggested as the correct reply to the first occurrence? We would have therefore a symmetric and reversed situation: the denial of step 3. would be incompatible with 6.2, not with 6.1. The problem is that 6.1 is not inconsistent with anything: we could simply deny both 3. and 6.1 and there would be no need to go on, as Peter does, to look for a solution. What has gone wrong? Is Peter right in thinking that there is still some form of inconsistency between the two occurrences, provided that denying the first occurrence is the correct reply to it?

I am not sure that I have a satisfactory answer to that question, but let us try to figure out a possible way out of the problem. Peter claims that at step 6. we should grant the sentence 'ista 'tu curris' non est sequens ex aliquibus concessis neque ex aliquo concesso' (let us call it 6^*), which is pretty similar to the sentence he is willing to grant at step 3. In the solution he proposes to deny the original occurrence 'hec 'tu curris' non est sequens neque repugnans sed impertinens', but he also suggests that the respondent grants the sentence 'hec 'tu curris' est impertinens admisso et adhuc non est sequens ex aliquo concesso neque ex aliquibus concessis' (let us call it 3^*), which, save for the clause 'impertinens admisso', is the same as 6^*).

Now, what happens if we read the argument with 3^* and 6^* instead of 3. and 6., respectively? 3^* is granted, therefore 6^* cannot be denied (as in the original argument, where we had an inconsistency between 3. and 6.1). Let therefore 6^* be granted. Here we still have an inconsistency with the rest of the argument (in particular with the consequent of 7.), because 6^* roughly says that the sentence 'tu curris' does not follow from other *proposita* and 7. proves that it is the opposite way around. Hence the need, for Peter, to search for a solution, that he finds in the usual strategy of replying *per ordinem*. This time, the only candidate to be denied at the last step as incompatibly relevant is the sentence 'iste non significant ex compositione suorum terminorum'. In the context of obligations thus, the need of maintaining consistency can force the respondent even to deny that words have their usual meaning.

of 7. The problem arises with respect to the discussion made above about the answer to be given to step 3. As we have seen, the sentence 'hec 'tu curris' non est sequens neque repugnans sed impertinens' should be denied and not granted in its first occurrence. But if this is the case, what happens to the rest of the argument? For instance, the inconsistency between the concession of 3. and 6.1 does not arise any longer: it is not granted that 'negatur a te concessum', since 3. must in fact be denied.

⁴²⁷ Item] alius casus *in marg.* O 427 nullus] nullorum O 428–429 Et ... currere] *om.* O 429 demonstrentur] *conf. forte* demonstratis O 429–430 omnes homines] omnibus hominibus O 430 proponitur] proponatur O 432 proponitur] proponatur O 434 contra: quia] *om.* O 434 concedis] conceditur O

(ii) Si negatur, contra: aliqui homines albi currunt. Sint igitur Sor et Plato albi qui currunt. Igitur aliquis illorum potest currere, illis albis demonstratis.

(Obj.2) Item, si nullus istorum potest currere et per ly 'illorum', in ista propositione significante adequate quod nullus illorum potest currere, sunt demonstrati homines qui sunt albi, igitur ista est vera 'nullus istorum potest currere' per cuius 440 partem sunt demonstrati homines albi.

(i) Et tunc arguitur sic: nullus homo demonstratus || per ly 'illorum' in ista $O \ 120^{va}$ propositione potest currere; sed omnis homo albus est demonstratus per ly 'illorum' in ista propositione; igitur nullus homo albus potest currere.

 $\langle ii \rangle$ Et alia ex parte, aliquis homo albus potest currere, || quia omnes homines 445 L 71^{vb} albi currunt. Igitur ex casu seguitur contradictio.

(iii) Et arguitur antecedens, scilicet quod nullus homo demonstratus per ly 'illorum' in ista propositione potest currere: quia nullus homo pro || quo verificatur ista propositio potest currere; sed omnis homo albus est homo pro quo verificatur ista propositio, ex quo per ly 'illorum' demonstrantur omnes homines qui sunt albi; 450 igitur nullus || homo albus demonstratus per ly 'illorum' potest currere. Ideo forte M 87^{ra} || dicitur, in principio, negando casum.⁹⁶ $B 98^{rb}$

⁹⁶ The sophism has the following structure:

	1 0		
Р	Omnis homo qui est albus currit et nul-	Α	possible
	lus istorum potest currere. Et per ly 'il-		
	lorum', in ista propositione adequate signi-		
	ficante quod nullus istorum potest currere,		
	demonstrentur omnes homines qui sunt albi		
1.	Nullus istorum potest currere	\mathbf{C}	sequentially relevant (it is
			part of the <i>positum</i>)
2.1	Aliquis istorum potest currere (demonstra-	\mathbf{C}	\perp incompatibly relevant (it is
	tis omnibus hominibus qui sunt albis, per		the negation of 1.)
	ly 'illorum')		
2.2		Ν	\perp aliqui homines albi cur-
			runt, igitur aliquis istorum
			potest currere

A similar sophism is in Heytesbury, cf. [Wilson, 1966, p. 155]. The point, here, is calling attention to cases whose stipulation may contain a contradiction. A quick look at the *casus* is sufficient to see immediately the contradiction, since the posited sentence is a conjunction of the following form (where H, B, C stand for 'being a man', 'being white' and 'run', respectively):

(Case 1)
$$\forall x ((Hx \land Bx) \Rightarrow Cx) \land \forall x ((Hx \land Bx) \Rightarrow \neg \Diamond Cx)$$

The argument for the rejection is sound: seemingly there is no way to defend the case of this sophism, as it stands. Nonetheless, there are interesting features, in its formulation, that Peter dwells on in the next section. In particular two aspects are worth mentioning which both concern

436 albi] om. O **437** currunt] currant O **437** Igitur] et arguitur sic: Sor et Plato albi currunt praep. O 437 illis] ipsis O 439 sunt demonstrati] demonstrantur O 440 homines] omnes praep. O 440 qui sunt] om. O 445 alia ex] inv. O 445 aliquis] omnis O 447 arguitur] illud add. O 447 scilicet quod] quia O

E Giiii^{vb}

3.40. $\langle i \rangle$ Sed contra, iste casus est possibilis: omnis homo qui est albus currit et Sor et Plato non possunt currere et per ly 'illorum' in ista propositione 'nullus istorum potest currere' adequate significante quod nullus illorum potest currere 455 demonstrantur omnes || homines qui sunt albi, et ex isto casu sequitur ille primus, igitur primus est possibilis.⁹⁷ Patet consequentia.

(ii) Et arguitur antecedens: quia Sor et Plato existentibus hodie omnibus hominibus albis, pro crastina die illis non existentibus, est hodie iste casus possibilis.

 $\langle iii \rangle$ Sed arguitur quod iste sequitur ex illo quia bene sequitur || 'omnis homo 460 V 75^{rb} qui est albus currit; et Sor et Plato non possunt currere; et per ly 'illorum' demonstrantur omnes homines qui sunt albi in ista propositione 'nullus istorum potest currere' adequate sic significante; igitur omnis homo qui est albus currit et nullus illorum potest currere; et per ly 'illorum' in ista propositione 'nullus istorum potest currere' demonstrantur omnes homines qui sunt albi'. Tenet consequentia quia 465 prima pars consequentis sequitur ex prima antecedentis, et secunda ex secunda, et tertia ex tertia.⁹⁸

the second clause of the posited conjunction. First, the presence, of a demonstrative term such as 'illorum'; second, the fact that the sentence is modalized. On the basis of these two elements, Peter conjures up an objection to the effect that the case should be regarded as possible (cf. sec. 3.40) and, accordingly, no longer rejected. The brief discussion below offers the occasion to introduce a distinction between the fact that a sentence is possible and the possibility for it to be true. Peter's conclusion is that some sentence are possible even if they cannot be true.

 97 The point of the argument is clear. In the situation just described in the previous section, one would be tempted to respond by rejecting the posited case (let us call C1) since it is contradictory. The strategy of the present objection is showing that (a) C1 is entailed by another case, say C2; that (b) C2 is, by contrast, possible; and, therefore, that (c) C1 is possible too.

 98 This part of the argument is intended to prove (b), i.e. that C2 is possible. It is, however, not quite straightforward to understand what Peter has in mind here. There are at least two problems that must be faced: first, a model is provided that should count as an imaginary situation in which the case is possible (i. e. a situation in which C2 can be realized), but it is not fully clear how such a model is supposed to do his job. Second, a distinction between the notions of 'possible' and 'possibly-true', not unusual in the obligational tradition (cf. [William Buser, 1990, p. 82]), is envisaged a few lines below by Peter in order to figure out a viable solution to the sophism (the conclusion is presumably that Case 1 is indeed possible although it cannot be true: it must be therefore admitted and then one should reply *per ordinem*). I will try to outline a tentative reconstruction. Let us follow the order of the text.

First, cf. par. (i) sec. 3.40, Peter lays down an inference, which he regards as a valid one and which is the basis of the whole argument: 'iste casus est possibilis [the statement of C2 follows], et ex iste casus sequitur ille primus [i.e. C1], igitur primus est possibilis'. Now, we need to see more closely what this new *casus* C2, that Peter regards as possible, is like. It consists of a conjunction of three sentences: 'omnis homo qui est albus currit (= C2.1) et Sor et Plato non

453 contra] quia add. Ο 453 possibilis] quod add. Ο 453 currit] currat O **456** demonstrantur] demonstrentur O **457** igitur] ille *add*. O 457 Patet] hec add. O 458 Plato] Platone OM 460 Sed] et O 461–462 demonstrantur] conf. forte demonstrentur O 463 sic] om. O 463 significante] quod nullus istorum potest currere add. O 463 qui est] om. O 464-465 in ... currere] om. O 466 prima] parte add. O 466 secunda] sequitur add. O

 $P 69^{vb}$

Third, cf. par. (iii) sec. 3.40, it is argued that C1 follows from C2. The argument, here, is simpler. Not only C2 but also C1 does have a three-fold structure: it consists of a conjunction of three sentences 'omnis homo qui est albus currit (= C1.1) et nullus illorum potest currere (= C1.2) et per ly 'illorum', in ista propositione 'nullus illorum potest currere', demonstrantur omnes homines qui sunt albi (= C1.3)'. Let us then regard C2 as the three-fold antecedent and C1 as the three-fold consequent of the entailment we want to prove. Now, C2 entails C1 because C2.1 entails C1.1, C2.2 entails C1.2 and C2.3 entails C1.3, respectively.

It is far too easy to see why the first and third inference are sound: C2.1 and C1.1 are the exactly one and the same sentence, namely 'omnis homo qui est albus currit', and the same holds for C2.3 and C1.3 (= 'per ly 'illorum' in ista propositione 'nullus illorum potest currere' demonstrantur omnes homines qui sunt albi').

The fact that C2.2 (= 'Sor et Plato non possunt currere') entails C1.2 (= 'nullus illorum potest currere') is, by contrast, less evident. Since the reference of 'illorum' in C1.2 is fixed by C1.3 on the set of all white men, the entailment must depend on the assumption, made in par. (ii) sec.

¹³²

possunt currere (= C2.2) et per ly 'illorum' in ista propositione 'nullus illorum' potest currere, adequate significante quod nullus illorum potest currere, demonstrantur omnes homines qui sunt albi (= C2.3)'.

Second, cf. par. (ii) sec. 3.40, Peter proves that C2 is possible. We have to imagine the following situation: let us consider two moments of time, today and tomorrow, and assume that today the set of white men contains, as its only members, Socrates and Plato. Moreover, let us assume, that tomorrow the set of white men will no longer contain Socrates and Plato, because they will have ceased to exist overnight. It might be necessary to make an additional assumption, namely that some other white have come into existence by tomorrow, but let us leave this aside for a moment. Peter's conclusion is that *today* the *casus* is possible. Unfortunately, he does not say anything more than this, therefore I think that it may be useful to spell out the underlying reasons of his claim. Let us consider the conjuncts one by one. The sentence C2.1, namely 'omnis homo qui est albus currit', will be true tomorrow if some white man exists tomorrow and no matter how many white men there are, they all run tomorrow. The sentence C2.2 will be true tomorrow if Socrates and Plato will not be able to run tomorrow (and indeed they will not, since we have assumed that they won't be there any longer). The sentence C2.3 will be true tomorrow if the term 'illorum' picks out all white men (it will turn out that the occurrence of 'illorum' tomorrow will have to pick out the individuals that are white men today, namely Socrates and Plato). The question arises as to what the status of the three components of C2 is, with respect to the model we have built up, i.e. with respect to the two moments of time. We know for sure, because Peter makes it explicit, that today Socrates and Plato are the only white men. And we also know for sure that tomorrow Socrates and Plato (= all white men existing today) will no longer exist. Let us assume, finally, that Socrates and Plato are both running today. What happens if we put all these pieces together? In particular what happens of the two conditions expressed by the other two conjuncts? C2.2 says that Socrates and Plato cannot run. On the assumption that Socrates and Plato are the only elements in the set of white men, C2.2 entails the modalized sentence C1.2 containing the demonstrative term 'illorum', as long as the reference of the latter is fixed by its occurrence in sentence C2.3 which claims that the term picks out all white men. Both sentences, needless to say, refer to Socrates and Plato. Thus, in sum, tomorrow there will be some white men who will running (by the additional hypothesis; they will not be Socrates and Plato, though), the sentence 'Sor et Plato non possunt currere' will be true, because the individuals it refers to will be no longer existent (Socrates and Plato), while the last sentence C2.3 will be true, because we assume that the demonstrative it contains will still be referring to the class of white men today, i.e. Socrates and Plato.

 $\langle iv \rangle$ Et sic arguitur quod iste casus est possibilis, scilicet quod tantum Sor demonstratur et tamen hoc est Sor et hoc non est Sor, quia sequitur: tantum Sor demonstratur, et Sor est Sor et Plato non est Sor, igitur tantum Sor demonstratur 470 et hoc est Sor et hoc non est Sor. Tenet consequentia et prima pars consequentis sequitur ex prima antecedentis, secunda ex secunda, et tertia ex tertia.⁹⁹

⁹⁹ So far, we have proved that C2 entails C1 and that C2 is possible; we may therefore expect Peter to draw the conclusion that C1 is possible too. In this paragraph, though, Peter sets up an analogy with a similar case, involving the presence of demonstratives, and in the next paragraph he will make his final point to the effect that the conclusion is qualified in a very peculiar sense.

It is not fully clear what the analogy is meant to show, because the terms involved are not completely symmetric with the former case of C1 and C2. Still, let us see what is going on here. Take two sentences, T1 and T2. T1 reads 'Tantum Sor demonstratur (= T1.1) et Sor est Sor (= T1.2) et Plato non est Sor (= T1.3)' whereas T2 is the sentence 'Tantum Sor demonstratur (= T2.1) et tamen hoc est hoc (= T2.2) et hoc non est hoc (= T2.3)'.

The first sentence, T1, is obviously possible because it simply says that only Socrates is indicated and, then, it contains two identities that are both true. But T1 entails T2, therefore T2 should be possible too. The entailment, again, is proved on the basis of the respective entailments of the atomic components (T1.1 entails T2.1, T1.2 entails T2.2 and T1.3 entails T2.3). The first entailment is clearly sound, since one and the same sentence is at stake. But what of the other two? Here the situation gets more complicated. In a sense, it is true that T1.2 entails T2.2 and T1.3 entails T2.3, for it would suffice to make the opportune substitutions of proper names with demonstratives. In other words, we need to index the demonstratives in an appropriate way in order to disambiguate the occurrences of the term 'hoc'. For instance, if h_1 stands for the demonstrative used to indicate Socrates and h_2 for the demonstrative used to indicate Plato, then the sentences 'Sor est Sor' obviously entails ' h_1 est h_1 ', whereas 'Plato non est Sor' entails ' h_2 non est h_1 '. Now what is the point of it all? It is the fact that, under the assumption that only Socrates is indicated, we are by no means entitled to make use of two distinct demonstratives in rephrasing the initial identities, and therefore we are left with the impossibility of uttering the sentence T2 and having all three conditions of T2 verified, at one and the same time. Under the hypothesis of T1.1/T2.1 (= 'Tantum Sor demonstratur') T2 cannot be true. The circumstance that is envisaged in T2 is indeed possible and it also follows from the possible case T1, but as it stands it cannot be true, because whenever we utter T2, we are committing ourselves to the use of only one demonstrative, and therefore we lack an additional demonstrative to pick out the right reference for T1.3 (= 'hoc non est Sor'). It is as though all demonstratives were to collapse onto one single demonstrative, used to indicate Socrates, the only individual who can be the object of an act of ostension in such a *casus*.

The situation of T1.3 is similar to a case where a demonstrative is used to refer to a nonexistent object: having a demonstrative that does not work properly because its natural reference (Plato, here) cannot be indicated (because of a stipulation) is more or less the same as having a demonstrative by means of which we want to try to indicate something that does not exist. And here we come to the point of the whole argument and of this analogy.

Again, unfortunately, Peter does not spell out the argument in detail, therefore some speculation is needed to fill in the gaps. The point is showing that a sentence might well be possible

468 sic] ita O **468** scilicet quod] om. O **471** et] quia O **472** prima] parte add. O **472** secunda] sequitur add. O **472** et] om. O **472** tertia] sequitur add. O

^{3.40,} that this set contains only Socrates and Plato. If the set of all white men = {Socrates, Plato}, then C2.2 entails C1.2. The condition is fulfilled by the stipulation of our model.

 $\langle v \rangle$ Tamen verum est quod, licet iste casus sit possibilis, tamen non potest esse verus, sicut quamvis illa propositio sit possibilis 'hoc non demonstratur' demonstrato Adam, tamen ipsa non potest esse vera sic significando. Ideo in principio 475 dicitur ad argumentum per ordinem, ut patet ex predictis.¹⁰⁰

and, still, it might be the case that it is not possibly-true. What sentence is most likely to have these logical properties in the above argument: C1 or C2? I am inclined to think that it is C1, since it is with respect to this sentence that the situation just described obtains. In particular, the problem is represented by the presence in C1.2 and C1.3 of demonstratives that lack a referent with respect to the time when the sentences should be true. The circumstance is possible, namely that none of *those that are indicated* through the demonstrative 'those' can run, because there is *another* circumstance (described by C2) in which the situation obtains. But in such a circumstance (i.e. tomorrow when Socrates and Plato no longer exist), the expression 'those that are indicated' is supposed to have empty reference.

Therefore, when we utter the sentences C1.2 and C1.3 today, we are thereby implying at one and the same time that the demonstrative occurring in it picks out something and that it does not. In the situation in which the sentences are supposed to be possible, the demonstrative must not pick out anything, whereas in any situation in which the sentences are actually uttered, they assert that something is indeed indicated. They can never be true when they are asserted, because the only situations in which they would be true are situations that in fact would falsify what these sentences say. We simply cannot truthfully express their content in that form.

The situation is not very different from more familiar examples. For instance, the sentence 'hoc non demonstratur' is possible, if the demonstrative 'hoc' picks out an individual like Adam who does not exist. It is possible because it describes a possible state of affairs, namely that a non-existent object is not indicated and, since Adam does not exist, *a fortiori* he will not be indicated. But the sentence as such cannot be true, if it is asserted, because, no negative sentence of this sort, containing a demonstrative, can be directly falsified. Consider the difference between the sentences 'Adam non demonstratur' and 'hoc non demonstratur' (wher 'hoc' still refers to Adam). The former is possible and possibly-true (trivially, because it is also true, as a matter of fact), while the latter is possible, but not possibly-true. The reason is that its being true would entail a contradiction, since 'hoc non demonstratur' can be rephrased as 'it is not the case that what is indicated (= hoc) is indicated'.

¹⁰⁰ As I have already recalled, a discussion of these notions in the context of obligations is found in [William Buser, 1990, pp. 82-88], where it features in the section devoted to the statement of preliminary assumptions and definitions, in connection with the characterization of the notion of a possible sentence. The distinction between 'possible' and 'possibly-true' is used by Buridan, cf. [Buridan, 1966, p. 183], whose example, which would perfectly fit in the above discussion, is the famous 'No sentence is negative'. The problem is extensively discussed in [Prior, 1969].

Finally, the distinction is also called into play in Peter's treatise on consequences, where we find a very similar case to the one presented here, cf. Tractatus de consequentiis, [Peter of Mantua, 1492a, sig. Eiiii+3^{rb-va}] "Secunda regula: si consequentia est bona, affirmativa, denominata a ly 'si' vel 'ergo' vel 'igitur', significans ex compositione suorum terminorum, et consequentia est falsum, antecedens etiam est falsum. [...] $\langle Obj.2 \rangle$ Secundo arguitur sic: ista consequentia est bona 'Sor non demonstratur, igitur hoc non demonstratur' et antecedens [correxi: the text has 'consequens', cf. ms. L] illius est verum in casu et consequentian. [...] $\langle RObj.2 \rangle$ Ad secundam dicitur concedendo illam consequentiam.

⁴⁷³ Tamen] transp. O 473 quod] om. O 473 tamen non] inv. O 476 ut... predictis] etc. O

4. On questio, certificatio, dubitatio, petitio, sit verum and other sophisms on positio

4.1. Questio, certificatio, petitio, dubitatio et suppositio non sunt species distincte a positione.¹⁰¹

Advertendum est preterea quod obli-||gatur respondens ad respondendum per hoc verbum 'certifico' seu per hoc complexum 'sit verum' vel 'sit dubium' aut || 'non sit dubium' vel 'sit scitum' et respondere debet obligatus per istam notam 'certifico' aut 'sit verum' sicut obligatus per illud signum 'pono', in hoc quod ubi fiat per illud signum 'pono', cedit obligatio facta per illud signum 'certifico' si obligatum repugnat obligato,¹⁰² quod est sic usitatum ex beneplacito disputantium, ex eo quod sepe vult opponens obligare ad falsum sustinendum respondentem, ut videat qualiter sciat se a contradictione defendere.¹⁰³ Sicut posito et bene admisso

¹⁰¹ This section of the treatise is divided into two parts. The first part, cf. sec. 4.1–4.10, is devoted to the analysis of a number of sophisms concerning specific subtypes of *positio*. Peter's list is similar to the one that is found in [Ralph Strode, 1517, fol. 78^{ra} et passim] except for the case of *dubitatio*. The second part, cf. sec. 4.11–4.21, contains a number of sophisms focusing on more elaborate types of positio, according to the complexity and the structure of posita (conditional sentences, temporal sentences, disjunctions, conjunctions and so on).

¹⁰² The point is made by [Ralph Strode, 1517, fol. 78^{ra}] "Et solet talis certificatio reputari vel sustineri ut positio [*correxi*: the printed edition has 'posito' instead], dum non sit altera species obligationis iam ducta in contrarium. Quando autem ponitur aliqua propositio repugnans certificationi, sustinetur positio et cedit [the printed text has 'conceditur'; I should thank prof. Ashworth for pointing out to me that the correct reading, as is reported by mss., is 'cedit'] obligatio certificationis".

¹⁰³ The clause provides a clue to Peter's understanding of the purpose of obligations.

 $\begin{array}{l} O \ 120^{\rm vb} \\ M \ 87^{\rm rb} \end{array}$

Et dicitur quod antecedens potest esse verum, sed consequens non potest esse verum sic significando [...] Sicut sepe accidit quod multe sunt propositiones possibiles que non possunt esse vere, sicut hec copulativa 'solus Sor qui est demonstratur et hoc est Sor et hoc non est Sor', quia ista sequitur ex ista 'solus Sor qui $\langle est \rangle$ demonstratur $\langle et Sor \rangle$ est Sor et Plato non est Sor'. Et ita conceditur quod hec universalis potest esse vera 'nihil demonstratur' quamvis eius nulla singularis possit esse vera sic significando; sed ista numquam est vera aliqua eius singulari existente. Item alique sunt propositiones vere que non possunt esse false sic significando, ut puta 'hoc demonstratur' demonstrato aliquo [...] Numquam tamen [...] est concedendum quod antecedens illius consequentie 'Sor non demonstratur, igitur hoc non demonstratur' sit verum et consequens falsum."

¹⁻² Questio, ... positione] om. O **3** est preterea] inv. O **3** ad respondendum] concedendo add. O **4-5** aut ... dubium'] om. O **6** pono] nisi add. O **6** hoc] hac O **6-7** ubi fiat] obligatio facta O **7** obligatio facta] obligationi facte O **8** obligato] obligationi O **8** est] om. O **9** ex] om. O **9** respondentem] respondente O **10** qualiter] quomodo O **10** Sicut] unde praep. BMOPV

quod Sor currat, non est hec concedenda 'hec est vera 'Sor currit", ita obligato respondente per certificationem sic dicendo 'sit verum quod Sor currat', non est hec concedenda primo loco proposita 'hec est vera 'Sor currit".¹⁰⁴

Solemus etiam certificationem facere per verba adiectiva coniunctivi vel preceptivi modi, sicut dicendo 'cedat tempus obligationis', 'concedatur a te ista 'homo 15 est asinus".¹⁰⁵

104 On the whole, the sense of the passage is clear. The respondent is always supposed to reply as in a disputation governed by the rules of *positio*. There is no structural difference, since all these variations fall within the scope of the latter. Yet, there are some interesting aspects that are worth pointing out. First, the remark that a *certificatio* is taken over by a *positio*, whenever the *obligatum* of the first is inconsistent with the *obligatum* of the second. The sense of that is not clear, in particular if we look at the examples provided to clarify the point. Second, there is a textual problem, because ms. O has a quite different text, suggesting that things might be read the opposite way around: 'respondere debet obligatus per illam notam 'certifico' aut 'sit verum' sicut obligatus per illud signum 'pono', nisi in hoc [correxi the ms. has 'hac'] quod obligatio facta per illud signum 'pono' cedit obligationi facte per illud signum 'certifico' si obligatum repugnat obligationi'. Here the idea is rather that *certificatio* has a priority over *positio* and not vice versa. This happens in those problematic cases where there is a latent inconsistency between the *obligatum* and the *obligatio* (not, as in the incunable text, between the two *obligata*). As far as I can see, I think the circumstance envisaged by O is preferable, because we have seen elsewhere in the treatise that *certificatio* is often introduced at a later step, when the work is already in progress, in a disputation with a problematic *casus*, and when it is introduced, to some extent, the specific information it conveys replaces or at least qualifies in a relevant way the original *obligatum*; cf. also *infra*, sec. 4.2 where a similar priority is ascribed to *suppositio* (not the property of terms, but a type of obligation such that the *suppositum* must be upheld not only as true but also as necessary) over *positio* and *certificatio*, when the latter are inconsistent with the former.

The third interesting point is the remark 'quod est sic usitatum ex beneplacito disputantium, ex eo quod sepe vult opponens obligare ad falsum sustinendum respondentem, ut videat qualiter sciat se a contradictione defendere'. In the obligational literature there are not many explicit descriptions or references to the purposes or to the applications of the 'art', and when they feature in the texts, they often happen to be overlooked (in this respect, a good point about the necessity of balancing historical accuracy and explanatory value is made by [Sinkler, 1992]). I do not want to emphasize too much the importance of these lines. I just want to single out the clause 'ut videat qualiter sciat se a contradictione defendere' since it gives us at least an idea of what the opponent might be willing to do when he conjures up a difficult case, containing a false *positum* (possibly with some sort of hidden inconsistency to the obligation, or between multiple *posita*): no doubt there might be further purposes, but the one thing that is *explicitly* witnessed in this text is the opponent's intention to see whether and how the respondent proves to be capable of defending a thesis and how (in a good or in a bad way, one may venture to ask?) he manages to avoid contradictions. In a sense, therefore, the oppondent is willing to test the respondent's logical skills; and this is done in a regimented context where the ability to maintain consistency is the relevant criterion for their assessment.

¹⁰⁵ The remark is found in [Ralph Strode, 1517, fol. 78^{rb}] "Sed certificatio fit communiter per verba coniunctivi modi ut cum dicimus 'stet oppositum consequentis cum antecedente' vel

14–15 adiectiva ... modi] om. O 15 sicut] sic O 15 cedat tempus obligationis] om. O 15 concedatur ... ista] peto te concedere istam O

4.2. Ideo, secundum veritatem, iste non sunt di-l|stincte species obligationis: 'du-L 70^{ra} bitatur a te ista 'rex sedet", 'peto te concedere istam 'homo est asinus", sed continentur sub certificatione vel positione. Quapropter, si obligetur respondens sic: 'dubitetur || a te deum esse', eo admisso, conceditur propositum, scilicet quod tu 20 P 70^{ra} dubites deum esse, quia positum. Deinde, si proponitur 'deus est', conceditur et negatur quod tu concedas.

Et capitur hic 'dubitare' pro 'dubie respondere'. Si enim 'dubitare' pro 'dubie respondere' esset species distincta obligationis, || sic 'concedere', 'negare', 'credere' essent species distin-||cte obligationis, ut cum dicitur 'concedatur a te Sor esse', 'credas papam esse Rome', 'negetur deum esse', quod non est verum.¹⁰⁶

Ita etiam dicitur quod dicendo 'peto quod concedas hominem esse asinum' non plus est dicere quam 'concedas'; || neque aliter respondendum est ab obligato per hunc modum 'peto quod concedas hominem esse asinum' quam ab obligato isto modo 'concedas hominem esse asinum' aut 'sit verum quod concedas hominem esse asinum'.

Et sicut predicti modi obligandi continentur sub certificatione vel positione, ita modus obligandi per ly 'suppono' aut 'presuppono' || continetur sub positione. Unde suppositio vel presuppositio est positio de qua hic intendimus. Unde per istam notam 'presuppono' solemus obligare respondentem ad sustinendum suppositum 35 non solum tampuam verum sed frequenter tampuam necessarium.¹⁰⁷ Et frequenter reputamus nos non esse obligatos ad illud suppositum, quia concederemus extra tempus tampuam necessarium.¹⁰⁸ Et cedit etiam in presentia suppositionis quelibet alia positio vel certificatio per quam ponitur repugnans supposito. Et hoc modo supponendi sepe usi sumus superius.¹⁰⁹ 40

 109 I have not identified a specific locus in the treatise.

V 75^{va} 25 E Giiii $+1^{ra}$

M 87^{va}

B 98^{va}

30

^{&#}x27;transeat A punctus B lineam' vel 'alteratur Sor sic vel sic'".

¹⁰⁶ The point is made by [Albert of Saxony, 1490, Niiii+2^{vb}] sig. and Aiiii $+2^{ra}$, when they determine the number of species to [Marsilius of Inghen, 1489, sig.] be admitted.

¹⁰⁷ Cf. [Ralph Strode, 1517, fol. 78^{rb}] "Sed utimur tali termino 'suppono' disputando non solum ut suppositam propositionem sustineat respondens concedendo, sicut conceditur aliquod contingens, sed sicut necessarium vel verum de se notum".

¹⁰⁸ Typically, the object of an obligation is a contingent falsehood, that we require someone to uphold as true during the period of the obligation. Nothing prevents us, however, from positing true or even necessary sentences, although that is by no means the standard practice in this kind of texts.

¹⁷ Ideo] et O 17–18 dubitatur] dubitetur O 19 Quapropter] om. O 20 esse] et non conceditur add. O 20-21 eo ... esse] deinde proponatur tu dubitas illam O 21 quia] concedatur praep. O **21** si] cum O **22** concedas] concedis O **23** capitur] capiatur O **26** papam] regem LMO 26 Rome] Bononie LMO 27 concedas] concedis O 27–30 non ... asinum'] om. O **34** vel] et O **34** de qua] ut O **37** esse] om. O **37–38** concederemus extra tempus] inv. et illud add. O **38** etiam] om. O **40** superius] in precedentibus O

4.3. Sed forte contra iam dicta arguitur.¹¹⁰ Peto te concedere quod nullus deus est. || Deinde propono 'deus est'.

 $\langle \text{Obj.1} \rangle$ Si concedis, tu concedis contradictorium obligati affirmative sustinendi, igitur male.

 $\langle Obj.2 \rangle$ Item, proponatur 'nullus deus est'.

 $\langle i \rangle$ Si negatur, cedat tempus obligationis et arguitur sic: in tempore tu fecisti contra petitum, igitur male fecisti. Patet consequentia. Et arguitur antecedens: quia petitum fuit te concedere nullum deum esse; et negasti nullum deum esse; igitur male respondisti.

(ii) Si conceditur, in tempore obligationis, nullum deum esse, arguitur sic: in 50 tempore concessisti impossibile simpliciter per obligatum possibile, igitur etc. Non enim debet respondens in disputatione ita ignavis verbis opponentis assentire quod concedat impossibile simpliciter.

Item, concessa ista 'nullus deus est', proponitur illa 'homo est asinus'. Et patet quod illa est sequens ex concesso, igitur concedenda. Et ita quodlibet aliud sequens ad illam esset concedendum. Et quia omnis propositio de mundo esset sequens ad illam, formata consequentia, igitur omnis propositio esset concedenda, concessa illa || 'nullus deus est'. Et, per consequens, repugnans concesso esset concedendum.

138

45

O 121^{ra}

M $87^{\rm vb}$

 $^{^{110}}$ In this section, Peter presents a sophism and a sequence of objections against an example of petitio.

⁴¹ Sed] casus in marg. O **41** arguitur] sic add. O **41–42** nullus ... est] significet scr. et del. O **43** concedis] conceditur et contra add. O **43** tu concedis] conceditur O **43** obligati] petiti O **44** male] om. O **45** Item, ... est] om. O **46** Si negatur] om. O **46** obligationis] om. O **46** in tempore] infra tempus O **47** fecisti] respondisti O **48** te] om. O **49** respondisti] om. O **50** in tempore] infra tempus O **50–51** in tempore] infra tempus O **51** etc.] male O **52** ita] om. LO **52** ignavis] ignarus et tantum add. O **52** opponentis] tantum add. L **54** proponitur] proponatur O **55** quodlibet aliud] quelibet alia O **56** concedendum] concedenda O **56–58** Et ... est] om. L

4.4. Ideo dicitur, admisso posito, concedendo illam 'deus est'.

(RObj.1) Et ista non est oppositum obligati, quia obligatum est totum illud || 'tu concedis illam 'nullum deum esse".

(RObj.2) Sed ad secundam formam, cum arguitur quia in tempore male respondisti, quia tu fecisti contra petitum, dicitur negando, quia non feci contra petitum, quia non concessi repugnans posito neque negavi sequens ex eo neque aliquid huiusmodi.¹¹¹

¹¹¹ The sophism has the following structure:

А	possible
\mathbf{C}	\perp tu concedis contradic-
	torium obligati affirmative sustinendi, igitur male
Ν	\perp tu fecisti contra peti-
	tum, igitur male respondi-
	sti (the antecedent is proved
	as follows: petitum fuit te
a	concedere nullum deum esse)
С	\perp in tempore obligationis
	concessisti impossibile sim-
	pliciter per obligatum possi-
	bile
\mathbf{C}	sequentially relevant (in vir-
	tue of the principle <i>ex impos-</i> <i>sibili quodlibet</i>)

The solution consists in the admission of the *casus*. To the first *propositum*, at step 1., one should reply by granting it as irrelevant and true. From this reply it does not follow that the respondent is granting the contradictory of the *obligatum*, because the *obligatum* is not the sentence 'nullus deus est' but the sentence 'tu concedis illam 'nullum deum esse". The respondent by admitting the present case of *petitio* is thereby committing himself only to the act of granting that he will grant the sentence 'nullus deus est' whenever it is proposed.

The second objection is two-fold. If the sentence 'nullus deus est' is put forward then either the respondent denies it, cf. par. (i), or he grants it, cf. par. (ii)-(iii). In the former case, he can be charged of replying incorrectly, because the *petitio* required him to grant the sentence 'nullus deus est'. In the latter case, i.e. if the respondent grants 'nullus deus est', Peter has two arguments: first, an impossible *simplciter* sentence is being granted, but the *obligatum* was possible (the respondent was only required to grant an impossible sentence, which is not per se an impossible circumstance, it is simply a fact that may be truthfully and consistently described by a sentence saving that it is the case), therefore something must have gone wrong on the respondent's side. Second, by granting 'nullus deus est', in virtue of the principle ex impossibili quodlibet and of the fact that 'nullus deus est' actually is an impossible sentence, the respondent is committing himself to grant whatever may be submitted to his evaluation from that point on, including sentences such as 'homo est asinus' and the like.

60 P 70^{rb}

65

⁶⁰ oppositum obligati] contradictorium positi O **60** obligatum] positum O **60** est totum] inv. O 60 illud] om. O 61 illam] om. O 62 secundam] illam O 62 quia] om. O 62 in tempore] infra tempus O **63** petitum] seu contra positum add. O **63** dicitur] illud add. O **64** petitum] positum O **64** negavi] unum *add*. O **64–65** neque aliquid] vel O

4.5. Et sicut responsum est ad illud obligatum, ita respondendum est si obligetur || per istum modum 'dubitetur a te ista 'deus est".¹¹² Deinde proponitur ista 'deus est'.¹¹³

Peter's reply to the second objection is less straightforward than the reply to the first one. He makes no point to (ii)-(iii), which I think he must consider as sound objections. Thus we are entitled to draw the conclusion that the respondent should not grant 'nullus deus est'. The argument then (cf. RObj.2) must be directed against the point made in par. (i). On the assumption that the respondent denies the *propositum* 'nullus deus est', the objection runs as follows: the inference 'in tempore fecisti contra petitum, igitur male fecisti' is sound and its antecedent (i.e. 'in tempore fecisti contra petitum') is proved by means of the following additional inference: 'petitum fuit te concedere nullum deum esse; et negasti nullum deum esse; igitur male respondisti'. Peter's reply consists in the denial of the antecedent of the first inference, namely 'in tempore fecisti contra petitum'. How does he make his point? He seems to argue that in order for the responent to be liable to the charge of taking a wrong line against the *petitum*, he should have granted something inconsistent with the *petitum* or denied something that follows from it; but the *petitum* is 'tu concedis illam 'nullus deus est" and 'nullus deus est' (i.e. the *propositum* denied by the respondent at step 2.1) neither follows nor is inconsistent with 'tu concedis illam 'nullus deus est". Examples of inconsistent senteces, in the present case, would probably be one might guess - 'tu non concedis illam 'nullus deus est", 'tu non respondes ad illam' and the like.

¹¹² This section presents a peculiar sophism that might generate some confusion. Although at first sight it appears to be a case of *dubitatio*, it is in fact another example of *petitio*, where instead of requiring the respondent to grant a sentence, he is asked to doubt it. The aim is, I reckon, to instruct the reader to keep distinct *positio* proper and the subtype that is in question here, namely *petitio*, i.e. the obligation by means of which one is required to perform certain second-order speech acts, and to warn him against the risk of confusing the two levels. The general point is that a *petitio* involving the requirement of granting the sentence p is equivalent to a *positio* where the *positum* is not p but rather 'you grant 'p''. This implies in turn that if p is put forward, there is no need to grant it (the need arises, by contrast, if the sentence 'you grant 'p'' is put forward). The rationale behind this is that the duty ranges over a 'higher-order' sentence involving a speech act that is supposed to take place *relative to a speech act* on p, not over the basic sentence itself.

¹¹³ The structure is similar to that of the previous sophism and the solution, too, conforms to an analogous pattern of reasoning:

PT Dubitetur a te ista 'deus est'	А	possible
1.1 Deus est	\mathbf{C}	\perp (cf. Obj.1–3)
1.2	D	
2.1 Nullus deus est	Ν	\perp (cf. Obj.4–5)
2.2.	D	\perp (cf. Obj.6–7)

The argument is not difficult but it has a complex development, therefore I have decided to split it into several sections. The *obligatum* is 'dubitetur a te ista 'deus est". Once it is admitted, the sentence 'deus est' is put forward in the first place. Two alternatives are taken into account: either 'deus est' is granted or it is doubted.

The former case is deal with in sec. 4.5, where three objections (Obj.1–3) are raised against step 1.1.

66 Et sicut] casus in marg. O 66 obligatum] om. O 67 proponitur] proponatur O 67 ista] om. O

 $L 70^{rb}$

 $\langle \text{Obj.1} \rangle$ Si conceditur, contra: tu concedis quod habes dubitare, igitur male || respondisti.

 $\langle \text{Obj.2} \rangle$ Item, tu fuisti obligatus ad dubitandum istam et non $\langle \text{du} \rangle$ bitasti, igitur male respondisti. Tenet consequentia quia, si tu fuisti obligatus ad dubitandum istam, tu tenebaris dubitare quam concessisti, igitur male.

 $\langle \text{Obj.3} \rangle$ Item, cedat tempus obligationis. Et arguitur, ut prius¹¹⁴, quia positum fuit quod dubitares istam 'deus est' et tu non dubitasti, igitur male respondisti.¹¹⁵ 75

4.6. Ideo, forte, dicitur dubitando illam 'deus est', sicut argumenta petunt. Qua dubitata, proponitur 'nullus deus est'.

 $\langle \text{Obj.4} \rangle$ Si negatur, et suum contradictorium est propositum, igitur suum contradictorium est concedendum. Et tu dubitas, igitur male.¹¹⁶

 $\langle \text{Obj.5} \rangle$ Item, dubitato uno contradictoriorum, reliquum immediate propositum so in eodem tempore obligationis est dubitandum.¹¹⁷

Ideo forte dubitatur 'nullus deus est'.

(Obj.6) Sed contra: quia ista 'homo est asinus' non esset neganda, quia ista consequen-||tia est bona 'nullus deus est, igitur homo est asinus' denominata etc., et antecedens est dubitandum a te, igitur consequens || non est a te negandum.¹¹⁸ Et ita arguitur quod nullum aliud impossibile simpliciter tibi propositum

¹¹⁴ Cf. *supra*, sec. 4.3, Obj.2 par. (i).

¹¹⁵ All three objections are based on the same type of confusion that we have seen above in the previous sophism, for it is argued, here, that the respondent should have doubted the sentence 'deus est'; but, since he has not, he replies incorrectly. Cf. *infra*, sec. 4.7 for the solution.

¹¹⁶ Cf. *supra*, sec. 2.3, where Peter states his third rule: if the contradictory of a correctly denied sentence is put forward, in a disputation, it ought to be granted.

¹¹⁷ This is not the first time we come accross this rule: Peter has already used it twice in this treatise (cf. *supra*, sec. 3.5 and 3.9). In both objections, the argument is that if the respondent denies 'nullus deus est' he replies incorrectly, because he should have doubted it.

¹¹⁸ Again, Peter makes use of this rule of *dubitatio* in connection with valid consequences.

70

V 75^{vb}

E Giiii+1^{rb} 85 O 121^{rb}

The latter case, on the other hand, is the topic of sec. 4.6, where the additional *propositum* 'nullus deus est' is put forward. This, in turn, is either denied or doubted. Each alternative is ruled out by a pair of objections: the first one (i.e. denying 'nullus deus est') is dismissed by Obj.4–5, while the second is rejected on the basis of Obj.6–7. Apparently, every examined reply has some problem. The solution, presented in sec. 4.7 below, consists in admitting the *casus*, granting the first *propositum* 'deus est' and replying to the three objections (Obj.1–3) that were raised against it.

⁶⁹ concedis] illud add. O 69 habes dubitare] dubitas O 71-72 Item, ... respondisti] om. O 72-73 Tenet ... male] ante Ideo forte dicitur transp. O 72 si tu] om. O 73 tu ... concessisti] et non dubitasti illam O 74 obligationis] om. O 74 quia] infra tempus add. O 75 quod dubitares] te dubitare O 75 tu] om. O 75 dubitasti] illam add. O 75 respondisti] om. O 77 proponitur] illa add. O 78-79 Si ... male] Si est concedendum et dubitandum igitur etc. O 80 reliquum] reliquo O 80 propositum] post posito O 81 obligationis] ipsum add. O 83 quia] tunc add. O 84 denominata] demonstrata inc. O 85 etc.] om. O 85 dubitandum a te] inv. O 86 tibi propositum] om. O

est negandum a te inferendo ipsum || ex ista 'nullus deus est'.¹¹⁹

 $\langle \rm RObj.6\rangle$ Sed forte dicitur quod non valet consequentia ista, quia non arguitur de consequentia formali.^{120}

 $\langle ORObj.6 \rangle$ Sed hec responsio non valet, quia neque in se est vera neque tollit argumentum aliquantulum mutatis terminis obligati. Dubitetur enim a te ista copulativa 'tu es et tu non es', contradictionem formaliter includens, ex qua quelibet propositio est apta formaliter sequi sic significando. Et tunc patet quod ista consequentia est formalis 'tu es et tu non es, igitur homo est asinus'; et antecedens est a te dubitandum; igitur consequens non est a te negandum.¹²¹

 $\langle Obj.7 \rangle$ Item, cedat tempus obligationis et arguitur quod in tempore dubitasti impossibile simpliciter scitum esse tale, igitur male respondisti.

4.7. Ideo dicitur, in principio, admittendo positum. Et cum proponitur 'deus est', conceditur.

(RObj.1) Deinde, cum proponitur 'tu concedis deum esse', conceditur quia verum non repugnans. Et si infertur quod tu male respondes, conceditur tam consequentia quam consequens, et negatur illa consequentia, si fiat, 'tu dubitas deum esse, igitur tu non concedis deum esse'.¹²²

¹²⁰ Here we have a purported reply to the previous objection, relying on the fact that 'homo est asinus', allegedly, does not follow *ex vi terminorum* from 'nullus deus est'. The point is interesting since, according to Peter's doctrine of consequences, a conditional sentence having 'nullus deus est' as its antecedent, is indeed valid, but only materially, whereas if we replace the antecedent with an explicit contradiction, the consequence becomes formally valid.

¹²¹ The reply is dismissed, therefore the original objection still holds. The idea is that the argument does not depend on the choice of the particular sentence 'nullus deus est', because it would preserve its strength even if we were to replace that sentence with a formal explicit contradiction.

 122 This reply is supposed to address the first objection that is raised against the respondent, if he grants the sentence 'deus est' proposed in the first place. For the sake of brevity, I have not included the content of this objection in my schematic reconstruction of the disputation. But properly speaking, the objection is raised *infra tempus*. This is why, presumably, Peter makes use of the locution '*cum proponitur* 'tu concedis deum esse'. The problem is, though, that in the text there is no step at which the sentence 'tu concedis deum esse' is put forward in this form.

87 negandum a te] *inv.* O 87 inferendo ipsum] *inv.* O 88 Sed] si O 88 non valet] *transp.* O 88 consequentia ista] *inv.* O 89 de] in O 91 mutatis] mutantis O 91 obligati] *om.* O 92 quelibet] alia *add.* O 93 apta] nata O 93 formaliter] *om.* O 96 et ... tempore] *om.* O 99 conceditur] concedatur O 101 non repugnans] et impertinens O 101 infertur] inferatur O 101 quod] *om.* O 101 conceditur] concedatur O 101–102 tam... consequens] tamquam sequens O 102 tu dubitas] dubitatis? O

M 88^{ra}

B 98^{vb}

95

90

 $^{^{119}}$ If an impossible sentence such as 'nullus deus est' is doubted, then by application of the rule of consequences, in virtue of which if the antecedent is doubted, then the consequent ought not to be denied, we draw the conclusion that an impossible sentence such as 'homo est asinus' ought not to be denied. The consequence 'nullus deus est, igitur homo est asinus' is valid because its antecedent is impossible and because the principle *ex impossibili quodlibet* holds. But 'nullus deus est' is doubted by hypothesis, therefore 'homo est asinus' ought not to be denied.

(RObj.2-3) Et cum arguitur quod infra tempus male respondisti, quia tu fuisti obligatus ad dubitandum istam 'deus est' et tu non dubitasti illam igitur etc., 105 respondetur negando antecedens, quod tu fuisti || obligatus ad dubitandum istam, quia tu fuisti obligatus ad concedendum istam 'tu dubitas deum esse'.¹²³

4.8. Item, si obligatio fiat per tales modos loquendi 'currat Sor', 'disputet Plato', arguitur quod aliqua est obligatio que non est obligatio.¹²⁴

Cedat enim omne tempus obligationis per quod est aut fuit obligatus aliquis. 110 Quo admisso, arguitur quod tu es obligatus et non es obligatus.

(i) Sequitur nam quod tu non es obligatus quia cessit omne tempus obligationis per quod est vel fuit obligatus aliquis, || igitur tu non es oligatus.¹²⁵

(ii) Et sequitur quod es obligatus, quia sequitur 'omne tempus obligationis cedit, igitur omne tempus obligationis est cedens'. Et ultra sequitur quod omne tempus 115 || obligationis est, igitur aliquis est obligatus et maxime tu qui admisisti.¹²⁶

 123 Again, the point is that the respondent has not committed himself to doubting the sentence 'deus est' but, rather, to granting the sentence 'tu dubitas deum esse'.

¹²⁴ The argument is supposed to show that if an obligation is made by means of expressions containing subjunctive verbs (it is not, by the way, a matter of grammatical form but rather, more generally, of underlying logical properties) then a contradiciton follows. It is interesting that Peter uses as an example the expression 'cedat tempus' which normally occurs at the end of a disputation to fix its temporal limit. After the sentence has been uttered, the opponent and the respondent no longer find themselves in the 'artificial' environment of the obligation and are bound to assess its development objectively. Outside the time, nothing but truth is allowed.

The example may also count as *casus* of *certificatio* (cf. *supra*, sec. 4.1, where Peter says "solemus etiam certificationem facere per verba adiectiva coniunctivi vel preceptivi modi, sicut dicendo 'cedat tempus obligationis'"). The argument should hold as an objection against the claim that *certificatio* is a subtype of *positio* along the lines of what Peter has said to introduce the present series of sophisms (cf. supra, sec. 4.3 the remark "sec forte contra iam dicta arguitur", i.e. against the list of accepted subtypes): if *certificatio* is made by means of such verbs, then it follows that 'alique est obligatio que non est obligatio'. Peter is certainly willing to reject this conclusion and retain *certificatio* in own his list.

¹²⁵ If the time has come to an end, then nobody is any longer under an obligation; therefore, a fortiori, the respondent involved in the present disputation is under no obligation either.

¹²⁶ The objection relies on the assumption that the obligation must be still existing, in some way, if we rephrase the expression 'cedit' with 'est cedens'. It all depends on whether we take the ascription of the property of 'being *cedens*' to the time of the obligation inclusively or exclusively

104 arguitur] ultra add. O **105** deus est] om. O **105** tu] om. O **106** quod] scilicet add. O 106 dubitandum istam] inv. O 107 quia] sed O 110 est aut] om. O 111 et ... obligatus] om. O 112 Sequitur ... obligatus] om. O 112 quia] sequitur add. O 112 cessit] cedit transp. O 113 obligatus aliquis] inv. O 114 quod] tu add. O 115 quod] igitur O

 $P 70^{va}$

V 76^{ra}

M 88^{rb}

The closest thing that we may find is, I suppose, the antecedent of the inference 'tu concedis quod habes dubitare, igitur male respondisti' (cf. Obj.1). The reply could have easily run along the same lines of the previous sophism, but here Peter seems to be implying that there is more to it (cf. the remark 'et si infertur quod tu male respondes, conceditur tam consequentia quam consequens et negatur illa consequentia, si fiat, 'tu dubitas deum esse, igitur tu non concedis deum esse').

Huic dicitur quod ista obligatio cedit ex convenientia facta aut presupposita inter opponentem et respondentem. Ideo respondeatur ibi secundum exigentiam habite significationis istius termini 'cedat' inter eos.¹²⁷

4.9. Item, presupponatur quod quelibet consequentia, denominata a ly 'si' vel 120 'ergo', significans ex compositione suorum terminorum, cuius antecedens est impossibile per accidens, est bona. $||^{128}$

Item, quod omnis consequentia, cuius consequens est necessarium, sit bona.¹²⁹

Quo admisso, sit rei veritas quod numquam respondisti ad istam 'celum movetur'. Deinde ponatur ista 'tu respondisti ad istam 'celum movetur".¹³⁰

(like in the case of terms such as 'incipit' and 'desinit'). If the time of the obligation has the property of 'being *cedens*', then it still is and the obligation in turn still is. But then someone is obligated (cf. *supra*, sec. 1.1 'si obligatio est, aliquis obligatur'): who is the most plausible candidate? Of course it is the respondent, i.e. the one who has admitted the original stipulation. It may sound a little weird, but this is how the argument actually runs.

¹²⁷ The reply depends on an agreement or, to put it better, on a stipulation about the meaning of the term 'cedat', that opponent and respondent should make before the disputation. I suggest that the only matter of controversy that may arise is related to the problem of determining, in advance, whether the obligation is still operating or not when the sentence 'cedat tempus' is uttered.

¹²⁸ Cf. Tractatus de consequentiis [Peter of Mantua, 1492a, sig. Eiiii+ 2^{vb}] "De qua consequentia 'ut nunc' ponitur ista regula, quod quelibet propositio impossibilis nunc seu per accidens est apta inferre quamlibet aliam propositionem. Unde bene sequitur 'Adam non fuit, igitur chymera est', quia per nullam potentiam posset oppositum consequentis stare cum antecedente".

¹²⁹ Cf. *Tractatus de consequentiis* [Peter of Mantua, 1492a, sig. Eiiii $+2^{vb}$] "Dicitur secundo quod quelibet propositio nunc seu per accidens necessaria est apta sequi ad quamlibet propositionem. Unde bene sequitur 'deus est, igitur Adam fuit".

¹³⁰ There is no way to make sense of this sophism according to the text of the incunable, as it stands. The needed information, however, is supplied by ms. O. In the incunable a fundamental passage is lacking, namely the proposal of the sentence 'celum movetur' and the corresponding reply to it. The sophism hinges on the fact that the respondent admits a posited sentence p which says that he has never replied to another sentence q. Immediately thereafter, q is put forward and answered. Then again, sentence $\neg p$ is proposed. Now, the latter is, on the one hand, the negation of the *positum*, but on the other hand, it truly says what happens to be the case, once the respondent has replied to q. The notions involved here are those of necessity *per accidens* vs impossibility *per accidens* which in turn call into play considerations about the truth and falsehood of sentences over time.

The sophism is also a variation on a standard theme. A useful guide is the treatment that is found in [William Buser, 1990, p. 106], where the posited sentence is the more familiar 'tu numquam respondisti ad deum esse'. In particular, Buser explicitly lays down a rule to allow for the concession and denial of this class of sentences. A discussion of earlier accounts of this type of problem is found in [Stump, 1980, p. 259-60]. The argument features also in Paul of Venice's

117 obligatio] certificatio O 119 termini] verbi O 120 Item] casus in marg. O 120 quod ... denominata] iter. O 120 quelibet] omnis O 120 a] da O 123 Item, ... bona] om. O 124 respondisti] responderis O 125 ponatur] proponatur tibi O 125 celum movetur] Qua posita, propono 'celum movetur'. Qua concessa, proponatur [celum movetur scr. et del.] 'tu respondisti ad istam 'celum movetur' O

L 70^{va}

125

(Obj.1) Si conceditur, conceditur contradictorium positi, igitur male.

 $\langle Obj.2 \rangle$ Si negatur, contra:

 $\langle i \rangle$ tu negas sequens ex tibi posito et a te bene || admisso, igitur male respondes. Patet consequentia. Et arguitur antecedens: quia ista consequentia est bona 'tu non respondisti ad istam 'celum movetur', igitur respondisti ad istam 'ce- 130 lum movetur', quia tam antecedens est impossibile per accidens quam consequents necessarium. Tenet consequentia.¹³¹

 $\langle ii \rangle$ Vel cedat tempus obligationis et arguitur quod in tempore negasti sequens ex tibi posito et bene admisso, igitur male respondisti.¹³²

treatise, cf. [Paul of Venice, 1988, p. 67, especially f. 13 for further references].

¹³¹ Peter takes into account the following inference: 'the antecedent of this consequence is impossible *per accidens* and the consequent is necessary, *therefore* this consequence (i.e. 'tu numquam respondisti ad istam 'celum movetur', igitur tu respondisti ad istam 'celum movetur') is sound', cf. the two assumptions stipulated in the *casus*.

 132 Here is the structure of the sophism:

	-		
Pr.1	Quelibet consequentia, denominata a ly 'si' vel 'ergo, significans ex compositione suorum terminorum, cuius antecedens est	А	possible
	impossibile per accidens, est bona		
Pr.2	Omnis consequentia, cuius consequens est	Α	possible
	necessarium, est bona		
Р	Tu numquam respondisti ad istam 'celum	А	possible
-	movetur'		Possible
1	Celum movetur	С	irrelevant and true
1.		-	
2.1	Tu respondisti ad istam 'celum movetur'	\mathbf{C}	\perp inconsistent with P
2.2		Ν	\perp
3.	Tu negas sequens ex tibi posito et a te bene	\mathbf{C}	valid inference
	admisso, igitur male respondes		
4.	Ista consequentia est bona 'tu non re-	\mathbf{C}	sequentially relevant (it fol-
	spondisti ad istam 'celum movetur', igitur		lows from Pr.1 and Pr.2)
			10.00 Hom 1 1.1 and 1 1.2)
	respondisti ad istam 'celum movetur"		

According to the solution provided in the next section, the situation might be more complicated than this, because it is not fully clear, on the basis of Peter's account, how exactly the respondent is supposed to reply. Anyway, there are some things that may safely pointed out. The *casus* encompasses two preliminary presuppositions concerning the validity of tantamount types of consequences. The first consequence at stake is valid *ut nunc*, since it involves an impossible *per accidens* sentence as its antecedent. Apparently, the second consequence is not *ut nunc*, but the development of the whole argument makes me think that it, too, should regarded as such (cf. *infra* Obj.2 where Peter says that 'tu respondisti ad istam 'celum movetur' is necessary: what kind of necessity, if not necessity *per accidens*, can be ascribed to such a sentence, provided that it is a true sentence *de preterito*, as in the present case?). Be this as it may, let the *positum* be 'tu numquam respondisti ad istam 'celum movetur'. Since it is possible, the respondent admits it. At step 1. the sentence 'celum movetur' is put forward and correctly granted, as irrelevant

126 conceditur₂] et *praep.* O 129 Patet] illa *add.* O 130–131 igitur ... movetur] *om.* O 131 impossibile per accidens] possibile O 131 consequens] est *add.* O 133 in tempore] infra tempus O 133 negasti] tu *praep.* O 134 et] ad te *add.* O O 121^{va}

4.10. (RObj.1) Sed huic dicitur negando istam consequentiam 'tu negasti sequens 135 ex tibi posito et a te bene admisso, igitur male respondisti'. Sed bene sequitur 'tu negasti in tempore sequens ex tibi posito et a te bene admisso in consequentia formali, igitur male respondisti'.

 $(OROb_{1})$ Sed contra hoc arguitur quia ex isto sequitur quod omnis propositio impossibilis per accidens est admittenda. Et sequi-||tur tunc quod concedendum 140 E Giii+1^{va} est quod Adam potest esse et quod Adam non fuit et quod ad preteritum sit potentia.¹³³

(RORObj.1) Sed huic forte dicitur negando istam ultimam consequentiam.

make no sense).

Negandum tamen est in tempore quod ista sit impossibilis 'tu respondisti ad istam 'celum movetur". Immo concedendum est quod illa est vera 'tu respondisti 145 || ad istam' saltem concesso in tempore obligationis quod ista est sic significans. Immo negandum est quod tu concedas || impossibile.¹³⁴

and true (as I have said, it is unavoidable to supplement the text, otherwise the argument would

Then, the sentence 'tu respondisti ad istam 'celum movetur" is put forward. How should the respondent reply to it? Peter, as usual, takes into account two alternatives, concession and denial. Suppose, first, that the respondent grants it, presumably on the grounds that it should be regarded as an irrelevant sentence which is also actually true, as of now, i.e. after the respondent has replied to 'celum movetur' at step 1. Then Obj.1 can be raised against this move: the sentence is incompatibly relevant to the *positum*. If on the contrary the respondent denies 'tu respondisti ad istam 'celum movetur", this time regarding it as incompatibly relevant, it can be argued to the opposite that the sentence is rather sequentially relevant and should be granted, accordingly. This fact is proved on the basis of the two presuppositions laid down in the initial statement of the casus. Take the consequence 'tu non respondisti ad istam 'celum movetur' (= p), igitur tu respondisti ad istam 'celum movetur' $(= \neg p)$ '. The antecedent is impossible per accidens as long as the respondent has actually replied to the sentence 'celum movetur'. For the same reason the consequent is necessary (per accident, although the condition is not stated explicitly). But that is exactly what the initial pair of presuppositions was concerned with. Every consequence of either sort (i.e. with an impossible per accidens antecedent or with a necessary *per accidens* consequent) is sound and in the present case both conditions are fulfilled. Therefore, the consequent follows from the antecedent, i.e. the sentence 'tu respondisti ad istam 'celum movetur' is sequentially relevant. The objection can be raised *infra tempus*, cf. par. (i), or *extra tempus*, cf. par. (ii).

¹³³ Cf. Tractatus de consequentiis. [Peter of Mantua, 1492a, sig. Eiiii+2^{vb}] "Apud eum, vero, qui poneret potentiam esse ad preteritum et quod deus potest verificare istam propositionem 'Adam non fuit', nulla foret consequentia ut nunc sed omnis consequentia foret ut semper".

 134 The solution is not entirely perspicuous. Supposedly, the *casus* must be admitted, since Peter focuses on alternative ways to reply to the proposita. The point is denying the inference 'tu negasti sequens ex tibi posito et a te bene admisso, igitur male respondisti', i.e. giving an alternative reply to step 3. of the disputation. But it is more likely that the remark refers not

135 tu negasti] infra tempus praep. O 136 bene] om. O 137 tu negasti] transp. O 137 in tempore intra tempus O 137 a ad O 137 consequentia bona add. L 139 hoc forte add. O 140 impossibilis] possibilis O 140 sequi-||tur] om. O 140 quod] om. O 143 forte] om. O 144 tamen] conf. O 144 est] quod add. O 144 in tempore] infra tempus obligationis O 146 est] sit O 147 concedas] concedis O

 $M 88^{va}$ $P 70^{vb}$ to the inference proposed *infra tempus*, cf. par (i), sec. 4.9, but to the occurrence of the very same inference outside the period of the obligation, cf. *ibid.*, par. (ii). Now, Peter qualifies the inference in question, by adding two conditions, namely (a) that the denial of a sequentially relevant sentence (which ought to be granted) takes place within the time of the obligation and (b) that the sentence is sequentially relevant according to a formal consequence (i.e. not materially). If the inference is specified by (a) and (b), then its denial is no longer a solution to the sophism, because it would be strictly against the rules.

However, the last requirement, namely that a sentence be sequentially relevant only in the sense of being the consequent of a formal consequence, is problematic. Peter argues that, on this assumption, it follows that every impossible *per accidens* sentence would have to be admitted (if it is posited). Moreover, it follows that there would be a *potentia ad preteritum*, and that sentences like 'Adam potest esse' or 'Adam non fuit' should be granted. Possibly, the reason is that the move would amount to obliterating the distinction between impossibility *simpliciter* vs *per accidens*, in the context of obligations. If there is a *potentia ad preteritum*, then no sentence is impossible *per accidens* (because a false sentence *de preterito* could be verified by a change in the course of events that have actually taken place in the past): this in turn implies that every impossible *per accidens* sentence, in the new framework, would have actually to be regarded as possible and to be admitted accordingly. Moreover, sentences such as 'Adam potest esse' or 'Adam non fuit', that are impossible *per accidens*, would have to be granted, if they are put forward in a disputation. Peter seems to reject this view (cf. 'sed huic forte dicitur negando istam ultimam consequentiam').

The final part of the solution is even less clear. Peter argues in support of the reply, within the time of the obligation, to proposita that are in fact not to be found in the original formulation of the sophism. In particular, the respondent ought to deny that the sentence 'tu respondisti ad istam 'celum movetur" is impossible and, by contrast, grant that it is true, if he has formerly granted that this is what it signifies; he ought to deny, finally, that he grants an impossible sentence. A crucial emendation to the text is needed. The only candidate to be regarded as an impossible sentence in the sophism is 'tu non respondisti ad istam 'celum movetur', therefore a 'non' must be supplied in the occurrence of this sentence, when Peter says 'negandum tamen est in tempore quod ista sit impossibilis 'tu (non) respondisti ad istam 'celum movetur'. The correction is required by the sense of the argument and it is confirmed by only one manuscript, namely M, whereas all other witnesses report the sentence without 'non'. A problem still remains, because apparently, when Peter says, immediately thereafter, that the respondent should grant that the sentence 'tu respondisti ad istam 'celum movetur" is true, all manuscript, including M, omit the 'non'. Either Peter is in fact referring to the other sentence involved in the sophism (i.e. 'tu respondisti ad istam 'celum movetur') or, if he is referring to the same as before, the correction must be supplied in this sentence too. I am inclined to opt for the first alternative: Peter is referring to one sentence in the former case, to the other in the latter case. The general solution to the sophism remains, in any case, uncertain.

A similar argument, involving impossible and necessary *per accidens* sentences, is found again in Peter's treatise on consequences, where the example clearly stems from the context of obligations, cf. *Tractatus de consequentiis* [Peter of Mantua, 1492a, sig. Eiiii+ 3^{vb}] "Et etiam posita ista 'hoc instans non fuit' et admissa, deinde durante tempore obligationis proposita ista 'hoc instans fuit', neganda est. Et ista est facta vera tunc 'hoc instans fuit', tamen adhuc ista est neganda 'hoc instans fuit', quare cedat tempus obligationis. Et tunc patet quod ista forma non valet 'hoc erat sequens ex posito et bene admisso, et antecedens fuit concedendum, igitur et consequens'. Immo antecedens fuit concedendum et consequens non: iam enim hec consequentia est bona 'hoc instans non fuit, igitur hoc instans fuit', quia per nullam potentiam posset oppositum

4.11. Item, ponatur quod, si rex sedet, tu scias regem sedere et, si nullus rex sedet, tu scias nullum regem sedere. Deinde proponitur ista 'rex sedet'. Qua dubitata, quia est impertinens quam extra tempus dubitares, non facta alia mutatione, proponitur 'nullus rex sedet'. Qua etiam dubitata, proponitur 'tu scis regem sedere vel tu scis nullum regem sedere'. Qua concessa, quia sequitur ex casu, proponitur 'tu scis quod rex sedet'. Qua negata, quia falsa est non sequens, proponitur 'tu scis quod nullus rex sedet'. Qua concessa, quia sequens ex concesso cum opposito bene negati, arguitur quod tu scis nullum regem sedere et est tibi dubium nullum 155 regem sedere, quod est impossibile.¹³⁵

This sophism was very popular, in the obligational tradition. Let us see its structure:

Р	Si rex sedet, tu scis regem sedere et si nullus rex sedet, tu scis nullum regem sedere	А	possible
1.	Rex sedet	D	irrelevant and doubtful
2.	Nullus rex sedet	D	
3.	Tu scis regem sedere vel tu scis nullum regem sedere	С	sequentially relevant (by P and the Law of the Excluded Middle)
4.	Tu scis quod rex sedet	Ν	irrelevant and false
5.	Tu scis quod nullus rex sedet	С	sequentially relevant (it fol- lows from 3. and the de- nial of 4. by disjunctive syllogism)

Replies 2. and 5. are incompatible. Moreover, as Peter notes in the treatise on consequences, such a case also entails both 'rex sedet' and 'nullus rex sedet', since we have $K p \vee K \neg p$; but then, depending on which one of the two disjuncts is proposed first (and denied as false), the second can be inferred by disjunctive syllogism. It is even easier to see the structure of this piece of propositional/obligational logic in the following way:

- P. $(p \Rightarrow Kp) \land (\neg p \Rightarrow K\neg p)$
- 1. Dp (irrelevant and doubtful)
- 2. $D \neg p$ (irrelevant and doubtful)
- 3. $C(Kp \lor K \neg p)$ (sequentially relevant by (1) and the Law of the Excluded Middle)
- 4. N(Kp) (irrelevant and false)
- 5. $C(K \neg p)$ (by (3) and (4))

But $K \neg p$ and $D \neg p$ are inconsistent. The sophism is found, in the context of a lengthy discussion involving several crucial issues pertaining to the fundamental obligational notions (relevance, irrelevance, criteria for the evaluation of sentences), in Kilvington, cf. [Kretzmann and Kretzmann, 1990, pp. 126–135]. A first systematic attempt to reconstruct Kilvington's account of obligations, on the basis of the arguments developed in his sophism, is

148 Item] casus in marg. O 148 si] ante ponatur transp. O 148 scias] scis O 149 Deinde] om. O 149 proponitur] proponatur O 150 quam] inc. forte et O 150 dubitares] dubitaretur O 150–151 proponitur] proponatur O 152 proponitur] quod add. O 153 proponitur] quod add. O 154 concesso] posito O

consequentis stare cum antecedente compossibiliter (sic significando) [add. L]."

¹³⁵ After covering a number of issues related to the proposed list of subtypes of *positio*, with this section, the focus shifts back on the topic of *positio* proper.

4.12. Ideo dicitur in principio negando casum: quia ponitur casus et conditionalis impossibilis.¹³⁶ Et ita negatur casus temporaliter positus, si ponatur quod, quandocumque rex sedet, || scias regem sedere et, quandocumque nullus rex sedet, scias nullum regem sedere: quia a temporali cuius utrumque verbum est affirmativum sequitur cathegorica tam de primo verbo quam de secundo. Et ex illo casu manifeste sequitur contradictio.¹³⁷ Et ita dicitur de aliis.¹³⁸

due to [Spade, 1982b, pp. 19–28], whose counterfactual interpretation is largely inspired by what can be grasped of Kilvington's solution. For a specific and detailed account of the sophism cf. also [D'Ors, 1991a].

¹³⁶ The topic is covered extensively in Peter's treatise on consequences, cf. Tractatus de consequentiis [Peter of Mantua, 1492a, sig. Fii^{va-vb}] "Accipiamus, primo, quod conditionalis bona est consequentia denominata a ly '[ni]si' vel a ly 'quando', quod aliquando sumitur pro '[ni]si', cuius contradictorium consequentis (non) potest repugnare antecedenti, aut potest esse synonima cum una tali sine nova impositione dum fuit de terminis expressis. Secundo, accipiatur quod omnis conditionalis bona est propositio necessaria, quod patet ex primo accepto. Ex quibus seguitur tertium, quod quelibet conditionalis infert conditionalem cum nota necessitatis consequentie de similibus terminis, ut sequitur 'si tu curris, tu disputas; igitur necessario, si tu curris, disputas', quia non potest esse quod antecedens sit verum sic significans et non sit necessarium, cum omnis conditionalis vera sit necessaria, per primam suppositionem. Item, quod omnis simpliciter mala est impossibilis, quia propositio necessaria numquam potest esse falsa sic significando, propositio tamen falsa potest incipere esse necessaria sic significando, ideo aliqua consequentia falsa non simpliciter potest esse vera sic significando. Ex quibus sequitur quod hi casus non sunt possibiles cum ponitur quod si rex sedet, scias regem sedere et si nullus rex sedet, scias nullum regem sedere. Similiter, non est possibile quod Sor erit liber si obviavit servo et quod Sor erit servus si obviabit libero, quia omnes iste conditionales sunt impossibiles".

¹³⁷ Cf. again *Tractatus de consequentiis* [Peter of Mantua, 1492a, sig. Fiii^{vb}] "Ex temporali cuius utraque pars est affirmativa sequitur utraque pars. Unde sequitur 'tu moveris, quando tu curris; ergo tu curris et tu moveris'. Ex illa regula sequitur quod omnes tales casus non sunt possibiles quibus ponitur quod, quando rex sedet, scias regem (sedere) et, quando nullus rex sedet, scias nullum regem sedere, quia ex illo casu sequitur quod rex sedet et quod nullus rex sedet".

¹³⁸ The solution to this sophism is rejecting the case posited since it contains an implicit contraditcion, under the form of what Peter refers to as an impossible conditional. The reason behind the rejection is that the *positum* has the following structure: $(p \to K_a p \land \neg p \to K_a \neg p)$. By the law of the excluded middle, this yields $(K_a p \lor K_a \neg p)$ which is inconsistent with the obligational duty of doubting p and $\neg p$ when put forward at previous steps of the disputation. The job of the respondent in such cases is to detect in advance the contradiction that is implicitly embedded in the sentence posited by the opponent and refuse to admit it. The rejection of a case, on the grounds that it contains a contradictory conditional sentence, is also found in Peter's treatise on insolubles, cf. *Tractatus de insolubilibus*, [Peter of Mantua, 1492a, sig. Hii^{vb}] "Item, ponatur quod talis sit conventio inter Sor et Platonem quod, si Sor doceat filium Platonis, debeat a Platone habere centum, si filius Platonis vincet primam questionem quam unquam habebit. Et ultra ponatur quod interea labatur tempus quousque ipse sit doctus et Sor requirente pecuniam habeat questionem cum filio Platonis et non habuerit iste filius unquam aliam questionem. Et queritur an Sor debeat habere istam pecuniam vel non. Si sic, igitur ipse perdet primam

157 casus et] una O 158 Et] *om.* O 159 sedet] tu *add.* O 160 utrumque] *om.* O 160–161 affirmativum] affirmatum O 161 Et] *om.* O

V $76^{\rm rb}$

4.13. Item, pono || tibi istam 'tu es obligatus' donec proponitur tibi aliquod B $99^{\rm ra}$ negandum.¹³⁹

questionem et per consequens Sor non debet habere istam pecuniam. Ideo, in principio, dicitur quod Sor non debet habere istam pecuniam. Et arguitur tunc sic: Sor non debet habere istam pecuniam ex ista lite; et hoc est solum ex eo quod discipulus suus vicit istam questionem; et ista est prima questio; igitur iste vicit primam questionem; et si sic, igitur Sor debet habere istam pecuniam. Item, ponatur quod Sor habeat duos servos et utrumque manumittat sub ista conditione, quod si unus eorum qui debet ire per viam obviavit libero, iste sit servus et alter, si obviavit servo, sit liber. Et vadant ambo et gratia argumenti obvient sibi ipsis solis. Et queritur utrum ambo sint liberi vel ambo servi vel unus servus et alter liber. Et patet quod quocumque dato sequitur illum esse servum et esse liberum. Ideo dicitur, in principio, negando omnes istos casus. Primum enim est impossibilis, ut clarum est. Secundus etiam et similes sunt impossibiles quia in eis clauduntur conditionales impossibiles". Moreover, in this reply Peter also points out another feature of this type of conditional, namely the fact that they contain a reference to time. This is the topic of the sophism discussed in the next sections.

¹³⁹ This type of sophism belongs to a well-established tradition in the development of *positio*. The taxonomy of *positio* embraces a first subdivision between *positio falsa* and *positio impossibilis*. The former is often divided, in turn, into several subtypes according to the structure of the posited sentence and by the connectives occurring in them. Very generally, the subdivision conforms to the following pattern:

- *positio simplex*: the *positum* is a categorical sentence
- positio composita: the positum is a compound sentence (i.e. a sentence with one or more connectives. It can be subject to a further subdivision in disiunctiva and coniunctiva, according to the presence of a disjunction or a conjunction as a positum (cf. infra, sec. 4.17-4.21)
- *positio dependens*: the *positum* must be upheld as true, under a given condition (many sophisms whose case is rejected are of this form, in virtue of the easiness of constructing cases with impossible conditionals, cf. sec. 4.11–4.12)
- *positio cadens*: this a subtype of the former, the condition of the obligation being that the obligation ceases when the given condition is fulfilled. It involves temporal terms such as 'donec', 'quamdiu' or 'usque' (cf. sec. 4.13–4.15)
- *positio renascens*: this category falls within the scope of *positio cadens* with the addition of a condition that, when fulfilled, implies the re-arising of the obligation. Again temporal terms such as 'quamdiu' are involved, for instance with the stipulation that the obligation is operating vs not operating according to the alternate truth of a given sentence (cf. sec. 4.16)

In many treatises, especially of the XIV century, these topics are introduced by means of rubrics explicitly subdividing the analysis according to a precise taxonomy. Peter proves to be aware of all this materials and discussions, which are substantially covered in their entirety in his treatise, although apparently without great intents of systematicity. The only red line that can be recognized is perhaps the order of presentation of sophisms. As far as the taxonomy of *positio* is concerned, we see that the materials which were dealt with in section 3., focused especially on *positio simplex*: the *posita* in the sophisms of that section are all categorical sentences of various sorts. In section four, after the discussion, in the first part, devoted to the subtypes of *positio* that are shown not to be '*species distincte*' (*petitio, dubitatio* etc.), the focus shifts not much on a different topic, but rather on *other* subtypes always belonging to this very same type

¹⁶³ tu ... obligatus] ELM currens est homo OP tu non es homo BV 163 proponitur] proponetur O

Qua posita et bene admissa, proponitur 'tu es obligatus'. Qua concessa, propo- 165 nitur 'tu es homo'. Qua concessa, proponitur iterum 'tu es obligatus'.||

 $\langle Obj.1 \rangle$ Si negatur, contra quia ista est concedenda, $\langle i \rangle$ quia tu es obligatus donec proponitur tibi aliquod negandum, sed adhuc non est propositum aliquod negandum, igitur adhuc tu es obligatus.

 $\langle ii \rangle$ Item, tu es obligatus donec proponitur tibi aliquod negandum, || igitur tu 170 L 70^{vb} es obligatus. Tenet consequentia a temporali || ad alteram eius partem. M 88^{vb}

Deinde proponitur 'tu non es homo'. Qua negata proponitur iterum 'tu es obligatus'. $\langle Obj.2 \rangle \langle i \rangle$ Si negatur, et prius concessisti, igitur male. $\langle ii \rangle$ Si conceditur, contra: tamcito tu non es obligatus, quamcito proponitur aliquod negandum; sed proponitur aliquod negandum; igitur iam tu non es obligatus. Consequentia 175 est concedenda et antecedens est concedendum, igitur et consequens, quod est oppositum concessi, igitur etc.¹⁴⁰

4.14. Idem fiat argumentum si ponatur quod tu scis obligatus quamdiu non proponitur tibi aliquod negandum aut quamdiu non est propositum aliquod negandum. 141

180

of obligation. This is the only reason, if any, that can be found at the origin of the order of treatment that he adopts.

¹⁴⁰ The present sophism, which is an example of *positio cadens*, has the following structure:

Р	Tu es obligatus, donec proponitur tibi	Α	possible
	aliquod negandum		
1.	Tu es obligatus	\mathbf{C}	irrelevant and true
2.	Tu es homo	\mathbf{C}	irrelevant and true
3.	1 Tu es obligatus	Ν	\perp (cf. Obj.1 i–ii)
4.	Tu non es homo	Ν	incompatibly relevant (it is
			inconsistent with the conces-
			sion of $2.$)
5.	1 Tu es obligatus	Ν	\perp inconsistent with 1. (cf.
			Obj.2 i)
5.	2	\mathbf{C}	
6.	Tamcito tu non es obligatus, quamcito pro-	\mathbf{C}	\perp (the consequent is incon-
	ponitur aliquod negandum; sed iam propo-		sistent with 5.2; cf. Obj.2
	nitur aliquod negandum; igitur tu non es		ii)
	obligatus		

¹⁴¹ These are nothing but variations on the same theme. The condition expressed by 'donec' is the same as that expressed by 'quamdiu non'. In the obligational tradition, a connected issue is often discussed, namely the question whether these temporal limits are to be taken inclusively or exclusively. In Peter, though, we find no trace of the debate. In this passage, obviously 'scis'

165 Qua ... admissa,] deinde O
166 Qua concessa,] si conceditur O
166 iterum] om. O
167 Si negatur, contra quia] et patet quod O
168 proponitur] proponetur O
168 est] tibi
add. O
168 propositum] positum O
170 tu] si praep. O
170 proponitur] proponetur O
172–177 Deinde ... etc.] om. O
178 Idem] Item praep. O
178 scis] es O
178 obligatus]
donec proponetur tibi aliquod negandum vel add. O
178–179 proponitur] proponetur O
179–180 aut ... negandum] om. O

O 121^{vb}

4.15. Ideo ad argumentum respondetur concedendo totum usque ad illam 'tu non es homo', que negatur. Et conceditur quod tu es obligatus.

 $\langle \text{RObj.2} \rangle \langle Ad \text{ ii} \rangle$ Et cum arguitur: ita cito tu non es obligatus quamcito proponitur tibi aliquod negandum; sed iam proponitur tibi aliquod negandum, igitur iam tu non es obligatus, huic dicitur quod hoc consequens bene stat cum illo antecedente, scilicet quod iam non es obligatus. Et tamen tu es obligatus. Sed de hoc non est cura, ideo negatur minor, scilicet quod iam proponitur aliquod negandum; et hoc admissa illa cathegorica temporali.¹⁴²

4.16. Sed si ponatur quod tu sis obligatus quamdiu preponetur¹⁴³ tibi aliquod negandum et non sis obligatus quamdiu non proponetur tibi aliquod negandum, 190 dicitur quod casus est impossibilis.¹⁴⁴

 142 The solution consists in the admission of the *casus* and in the acceptance of all replies proposed in the reconstruction up to step 4. The final *propositum* 'tu es obligatus' must be granted. The objection against this reply hinges on the fact that the sentence 'tu non es homo' has been correctly denied at step 4., which in turn implies that something that ought to be denied has been put forward, therefore the obligation should have come to an end. By putting forward the *propositum* at step 6., apparently, we can force the respondent to grant the sentence 'tu non es obligatus' which is an oppositum concessi (cf. step. 5.2). Peter seems not to give much importance to this fact and suggests to deny the sentence 'iam proponitur aliquod negandum', presumably on the grounds that it is the only factual sentence we have no logical commitment to (as when we are responding *per ordinem*): even if its denial is contrary to fact (because *actually*) a sentence that ought to be denied has been put forward) it is perfectly legitimate. Indeed, it becomes compelling by virtue of its logical relation to the previously evaluated sentences; and we know that logical dependence has always priority over factual truth in the context of obligations. The argument, however, is not fully convincing, because it strongly appeals to the order in which sentences are proposed which never depends on the respondent's choice. If the sentence 'iam propositur aliquod negandum' is proposed as the last *propositum*, nothing goes wrong, because, although it is factually true, it can be denied as incompatibly relevant (if it were granted, the negation of 'tu es obligatus' would follow; therefore 6. and 5.2 entail its denial). But what happens if that sentence is put forward *before* the whole consequence of step 6.? It would probably have to be granted as irrelevant and true. But then, if the respondent does not want to grant, in the end, the sentence 'tu non es obligatus', he would have to reject the consequence. Biting the bullet in this case might be not that easy.

¹⁴³ Obviously, the correct reading is 'proponitur', as is confirmed by O.

¹⁴⁴ A case like that must be rejected because it contains an impossible conditional. Peter does not fully spell out the argument, but his suggestion seems correct. For the *positum* is equivalent to the conditional sentence 'si aliquod negandum est tibi propositum, tu es obligatus; et si aliquod negandum non est tibi propositum, tu non es obligatus'. Let then the sentence 'tu es obligatus' be put forward in the first place. If it is granted, the condition that 'aliquod negandum non est

is a typographical error of the incunable, that must be emended with 'sis'.

¹⁸¹ ad ... respondetur] admisso casu dicitur O 181–182 totum ... obligatus] quod tu es obligatus et quod tu non es homo O 185 tu] *om.* O 185 illo] *om.* O 187 negatur] negetur O 187 minor] maior O 187 quod] *om.* O 188 illa] *om.* O 189 Sed] *om.* O 189 preponetur] proponitur O 189 tibi] *transp.* O 190 et] *om.* O 190 non sis] *inv.* O 190 non] *supra lin.* O 190 proponetur] proponitur O 190 proponitur O 190 m. O 190 tibi] *om.* O

Sed si velit opponens respondentem obligare non ponendo totam illam propositionem 'rex sedet donec proponitur tibi aliquod negandum' sed ponat sibi illam et || certificet respondentem quod sit rei veritas quod ipse sit obligatus ad illam donec proponetur sibi aliquod negandum vel non proponetur sibi aliquod negandum vel 'quamdiu' sive 'usque quo' etc., videat respondens quomodo debeat intelligere illum terminum 'donec' et illud verbum 'propono', quia optima regula respondentis est non respon-||dere ad aliquid nisi optime noverit terminorum vires.¹⁴⁵

4.17. Item, ponatur hec copulativa 'tu es papa || et rex sedet vel tu curris'. Deinde M 89^{ra} proponitur 'rex sedet'.¹⁴⁶ Qua dubitata quia impertinens est, proponitur 'nullus ²⁰⁰ rex sedet'. Qua dubitata, proponitur 'utraque illarum est dubitanda a te'. Qua concessa quia vera non repugnans, proponitur 'nulla illarum est concedenda'. Qua concessa, proponitur 'tu curris'. Et patet quod ista est falsa et non sequens, et ideo neganda. Deinde proponitur 'rex sedet'. Que si dubitetur, contra: tu dubitas sequens ex una parte positi cum opposito bene negati, igitur male respondes. Ideo ²⁰⁵ si conceditur illa, proponitur 'tu concedis illam'. Qua concessa proponitur 'tu non male respondes'. Quo iterum concesso proponitur 'illa est concedenda'. Et patet quod illa est sequens.

 $\langle Obj.1 \rangle$ Et sic eadem propositio est concedenda et non concedenda.

 $\langle \text{Obj.2} \rangle$ Item, cedat tempus obligationis: in tempore obligationis concessisti || 210 V 76^{va} et dubitasti eandem propositionem, igitur male || respondisti. O 122^{ra}

153

 $P 71^{ra}$

E Giiii+1^{vb}

tibi propositum' is fulfilled, therefore 'tu non es obligatus' should be granted too, which results in an inconsistency. The same holds if 'tu es obligatus' is denied, because then in turn it is the condition that 'aliquod negandum est tibi propositum' that will have been fulfilled; therefore the sentence 'tu es obligatus' should have been granted, contrary to the hypothesis.

¹⁴⁵ The passage is unclear. There might be a textual problem that I have not been able to fix. All manuscripts have the sentence 'rex sedet donec proponitur aliquod negandum' but this seems to make little sense.

¹⁴⁶ This is a common pattern of sophism that exemplifies another sybtype of *positio* known as *positio composita*. Here at stake are the effects of turning the crank of propositional logic in the context of an obligational disputation. Needless to say, albeit in the absence of brackets, the conjunction here is '(tu es papa) et (rex sedet vel tu curris)', i.e. a conjunction composed of a categorical sentence, as its first element, and of a disjunction, as its second element.

¹⁹² velit] vult O
192 respondentem] aliter add. O
193 proponitur] proponetur O
193 sed] om. O
193 illam] rex sedet add. O
194 quod ... veritas] om. O
194 obligatus ad illam] inv. O
196 sive] om. O
196 quo] om. O
196 debeat] debet O
198 noverit]
om. O
199 Item] inc. in marg. forte casus no-O
200 proponitur] proponatur O
202 concedenda]
a te add. O
203 proponitur] proponatur O
203 patet quod] cum O
203 est] sit O
203 et] om. O
204 proponitur] proponatur O
206 proponitur] proponatur O
207 concedenda] a te add. O
210 in tempore]
infra tempus O
210 obligationis] om. O
210 concessisti] tu praep. O

 $\langle \text{Obj.3} \rangle$ Item, in eodem casu proponitur 'hec est a te concedenda 'papa currit". Si conceditur, conceditur falsum non sequens. Si negatur, proponitur 'tu non es papa vel papa currit'. Qua concessa quia vera non repugnans, arguitur sic: tu non es papa vel papa currit; sed tu es papa; igitur papa currit. Illa consequentia 215 est bona etc.; et antecedens est concedendum; igitur et consequens. Et consequens est illa propositio 'papa currit'. Igitur illa propositio 'papa currit' est a te concedenda.¹⁴⁷

4.18. (RObj.1) Ideo dicitur in principio admisso casu dubitando illas propositiones 'rex sedet', 'nullus rex sedet' cum proponuntur. Et dicitur quod utraque illarum 220 est dubitanda. Et cum proponitur 'tu curris', negatur. Et conceditur iterum quod

147	The sequence of sentences can be displayed as f	ollows:	
Р	Tu es papa et rex sedet vel tu curris	А	possible
1.	Rex sedet	D	irrelevant and doubtful
2.	Nullus rex sedet	D	irrelevant and doubtful
3.	Utraque illarum est dubitanda a te	\mathbf{C}	irrelevant and true
4.	Nulla illarum est concedenda	\mathbf{C}	sequentially relevant (by 3.)
5.	Tu curris	Ν	irrelevant and false
6.1	Rex sedet	D	
7.	Tu dubitas sequens ex una parte positi cum	С	\perp
	opposito bene negati, igitur male respondes		
6.2	Rex sedet	\mathbf{C}	
8.	Tu concedis illam	\mathbf{C}	
9.	Tu non male respondes	\mathbf{C}	
10.	Illa est concedenda	\mathbf{C}	\perp
11.	Illa est sequens ex una parte tibi positi	\mathbf{C}	
	cum opposito bene negati, igitur illa est		
	concedenda		
12.1	Hec est concedenda 'papa currit'	\mathbf{C}	\perp conceditur falsum non
			sequens
12.2		Ν	
13.	Tu non es papa vel papa currit	С	irrelevant and true
14.	Tu non es papa vel papa currit; sed tu es	С	valid inference
	papa; igitur papa currit		
15.	Illa consequentia est bona etc.; et antece-	\mathbf{C}	
	dens est concedendum; igitur et consequens		
16.	Antecedens est concedendum	\mathbf{C}	
17.	Consequens est illa propositio 'papa currit'	\mathbf{C}	
18.	Illa propositio 'papa currit' est concedenda	\mathbf{C}	\perp inconsistent with 12.2
A s	similar type of sophism is discussed, for insta	ance, in	[Ralph Strode, 1517, fol. 84 ^{va}],

A similar type of sophism is discussed, for instance, in [Ralph Strode, 1517, fol. 84^{va}], [Paul of Venice, 1988, p. 336].

213 sequens] igitur ideo add. O **213** negatur] negetur O **213** proponitur] proponatur O **215** tu] non add. O **216** etc.] om. O **216** est] a te add. O **216–218** et ... concedenda] consequens illud 'papa currit' est a te concedendum, quod fuit probandum O **219** dubitando] dicendo O **219** propositiones] primo propositas O **220** cum proponuntur] om. O **220** dicitur] conceditur O **221** conceditur] cum dicitur O

rex sedet. Et cum proponitur 'tu concedis illam', conceditur. Et negatur quod non male respondeam, quia negatur quod illa sit concedenda.¹⁴⁸ Et si arguitur quod illa sit sequens || ex una parte tibi positi cum opposito bene negati, igitur illa L 72^{ra} est a te concedenda, dicitur de virtute sermonis quod illa consequentia non valet. $225 \text{ M } 89^{\text{rb}}$ sicut nec ista valet 'hec sequitur ex parte tibi positi et admissi, igitur hec est a te concedenda'. Sicut posita tibi ista 'nullus homo est asinus', hec 'homo est rudibilis' sequitur ex parte positi et tamen non est concedenda. Sed quia illud non est ad propositum, ideo negatur illud antecedens, scilicet quod illud sequatur ex parte tibi positi et a te bene admissi cum opposito bene negati. Et si arguitur quod sequitur 230 ex illis, et illa sunt partes admissi et oppositum bene negati, igitur etc., concessa prima parte ultima negatur, si per illum ordinem proponatur. In || hac enim $P 71^{rb}$ arte, maxime hec regula servanda est quod ad nullam propositionem respondeatur concedendo aut negando, nisi ipsa proponatur. Et cum fit consequentia, negetur vel concedatur, et non dicatur ad antecedens vel || ad partes antecedentis vel ad 235 B 99^{rb} consequens nisi secundum ordinem proponendi.¹⁴⁹

 $^{^{148}}$ The text of the incunable must be corrected according to manuscript O, where 'non' is omitted and the reading 'respondeam' replaced by 'respondes'.

 $^{^{149}}$ The solution to the sophism consists in the admission of the *casus* and the acceptance of the proposed replies up to step 5. At step 6, of the two alternatives available, namely doubt (= 6.1)or concession (= 6.2), the respondent should pick out the second. After granting 'rex sedet', he will have then to deny both 8. and 9. (cf. 'et negatur quod non male respondeam, quia negatur quod illa sit concedenda'). Step 11. must be rejected too. As far as the argument in support of the denial of 11. is concerned, namely that it is not always to be granted that if something follows from 'a part' of the *positum*, then it ought to be granted, the objection is rather sophistical, since 'homo est rudibilis' is taken to follow from 'nullus homo est asinus' by replacing 'asinus' with 'rudibilis' and simply removing the term 'nullus'. Obviously, the move is by no means legitimate. I think Peter realizes that this would be too far-fetched an argument, when he claims 'illud non est ad propositum' (i.e. it is not relevant to the point that is being made here). The consequence therefore holds, and the reply focuses on the denial of its antecedent, namely the sentence 'illa est sequens ex una parte tibi positi cum opposito bene negati'. How can this denial be justified? Once again with recourse to the strategy of reply *per ordinem*, which looks, as it were, like a spare solution in Peter's obligations. I have the impression, to some extent, that replies can be classified into types of decreasing 'value'. The top level is reached when (i) the respondent admits the *casus* and resolves all problems, by identifying the right status of every sentence submitted to his evaluation and providing the correct reply to each, according to the rules. The second choice, when (ii) there is no way to fix all the objections, is trying to save as much as possible by replying *per ordinem*. This often requires the respondent to concede an awkward sentence at the very last step of a disputation, but still it enables him to maintain consistency. The third alternative is that (iii) the respondent grants that he has replied incorrectly (i.e. that within the time, he grants the sentence 'tu male respondes'). Last, (iv) if he detects a contradiction

²²² conceditur] concedatur O 222 non] om. O 223 respondeam] respondes O 224 sit] est O 225 a te concedenda] inv. O 225 dicitur] quod add. O 226 nec ista] hec non O **226** et] ad te add. O **227** hec] om. O **230** a] ad O **230** Et] ut et se scr. et del. O 232 ultima negatur] inv. O 234 fit] sit O 234–235 negetur vel concedatur] negata vel concessa O 235 et] om. O 235 vel... antecedentis] om. O

4.19. (RObj.2) Et ad aliam formam, cum arguitur quod in tempore eandem propositionem concessisti et dubitasti, igitur male respondisti, dicitur negando consequentiam. Sed bene sequitur: infra tempus eandem propositionem concessisti et dubitasti ita se habendam ad concedendum et dubitandum quando concessisti 240 sicut quando dubitasti, igitur male respondisti.¹⁵⁰

Et si arguitur contra: omnes responsiones sunt retorquende ad idem instans, dicitur quod illud est impossibile: non enim due responsiones possunt fieri ab eodem in eodem instanti, immo nulla responsio.

Sed verum est quod concessa et opposita bene negatorum debent esse talia quod 245 copulativa ex illis composita sit possibilis, || sicut a principio dictum est.¹⁵¹

4.20. (RObj.3) Et cum proponitur || ulterius in illo casu 'hec est concedenda 'papa currit", negatur illa. Et conceditur illa disiunctiva proposita 'tu non es papa vel papa currit', quia || est vera non repugnans, quia una eius pars est vera scita esse talis. Et consequenter conceditur quod papa currit tamquam sequens ex concesso 250

at the very beginning, he should not admit the *casus* (nor, consequently, the *positum*). There is no explicit definition of this sort of ranking nor is the strategy of its application ever spelled out. Nonetheless, the common practice seems to conform to the principle that the respondent has to admit as many *casus* or *posita* as possible (after all there is a rule to govern the *duty*) of *admissio*); to grant as seldom as possible that he replies incorrectly (or to put it better, to grant as seldom as possible *infra tempus* the sentence 'you reply incorrectly'); to appeal to the argument per ordinem again as seldom as possible; and finally, to defend himself, as often as possible, by identifying all the incorrect ascriptions of relevance and irrelevance, truth and falsehood, that are proposed by the opponent. The last solution when combined with the first is the best outcome for the respondent.

An interesting attempt to interpret Burley-style obligations not simply in terms of fully regimented and determined games, but allowing also for non-deterministic aspects has been made by [Dutilh Novaes, 2005]. On that occasion, it has been argued that certain replies can be evaluated according to a 'point-system', in such a way that the respondent has a certain degree of freedom in his replies and can be guided by strategic reasons to opt for one reply rather than another in order to score as many points as possible. I think the idea of ranking various types of responses, according to the criteria briefly outlined here, might well fit in that context.

 150 Cf. supra, sec. 4.17. It is well known that in the context of obligations governed by the rules of *responsio antiqua*, whenever a disjunction is put forward involving a doubtful disjunct, the other being false, then by proposing sentences in an appropriate order, it can be shown that one and the same sentence is first doubted and afterwards granted in the same disputation (=Spade's (4d)-inconsistency, cf. [Spade, 1982b, p. 8]). Peter seems to accept here this soft kind of inconsistency (at least in the sense that a sentence can be replied in two different ways in the same disputation). The point is also interesting with respect to the discussion of the properties of sentences that ought to be doubted that I have taken up above. This suggestion may be an additional element for the reconstruction of Peter's views on the topic.

 151 Cf. supra, sec. 2.4.

237 in tempore] intra tempus O 238 dubitasti] negasti O 240-241 ita ... dubitasti] eandem propositionem concedendam quando dubitasti vel dubitanda quando concessisti O 243 non] nulle O 245 negatorum] vel oppositum bene negati add. O 246 copulativa] transp. O 246 illis] ipsis O 248 Et] cum add. O 249 quia] illa add. O

O 122^{rb}

M 89^{va}

V 76^{vb}

et prima parte positi, quia sequitur 'tu non es papa vel papa currit; sed tu es papa; igitur papa currit'. Ista consequentia est bona, quia ar-||guitur ad disiunctivam¹⁵² cum destructione unius partis super alteram partem eius.¹⁵³ Et cum ulterius arguitur quod illa consequentia est bona scita esse talis etc., illud conceditur. Et conceditur quod prima pars antecedentis est concessa et concedenda. Et negatur 255 quod secunda sit concedenda tamquam repugnans.¹⁵⁴

4.21. Et est hic advertendum quod sicut in illo casu conceditur illa falsa 'papa currit' quia sequens secundum illum modum proponendi, ita etiam quocumque casu contingenti falso et bene admisso secundum istam speciem fieri potest, quia quecumque propositio falsa contingens concedatur bene respondendo a respondente disiungendo illam cum opposito illius falsi admissi. Ut posita et admissa ista falsa 'tu curris', faciam concedere consequenter quod tu es papa proponendo tibi istam 'tu non curris vel tu es papa' que, quia vera non tibi repugnans, concedenda est. Qua concessa arguitur sic: tu non curris vel tu es papa; sed tu curris; igitur tu es papa. Et sic de aliis.|| 265

The issue here is the interaction between basic sentences and higher-order sentences. Now, the end of this section seems to imply that 'papa currit' (= p) ought to be granted, as sequentially relevant (cf. 'conceditur quod papa currit tamquam sequens ex concesso et prima parte positi, quia sequitur 'tu non es papa vel papa currit; sed tu es papa; igitur papa currit"). But "papa currit' est concedenda' (= OCp) cannot be granted because it is the opposite of step 12.2. Peter argues that step 15. should be granted but 16. should be denied (the second part of the antecedent, i.e. supposedly the sentence "tu es papa' est concedenda') as incompatibly relevant. The move is perfectly legitimate, because the following principle does not hold: $OCp \Leftrightarrow$ OC(OCp). In this way, Peter obtains a solution to the objection and maintains the principle of provability of arbitrary falsehoods.

251 positi] *inc. forte pro* presenti positi O 252 ad disiunctivam] E ab una disiunctiva O 253 super] ad O 253 eius] *om.* O 255 quod] *om.* O 257 illo casu] *inv.* O 258 etiam] *om.* O 258 quocumque] in *add.* O 259 contingenti] *correxi* contingente E; contingenti O 259 et] *om.* O 259 potest] *correxi* postet E; potest O 259 quia] quod O 261 disiungendo] distinguendo O 262 faciam] te *add.* O 262 consequenter] convenientem *et* respondere *add.* O 263 tibi] *om.* O

E Giiii $+2^{ra}$

265 L $72^{\rm rb}$

 $^{^{152}}$ The correct reading is 'a disiunctiva'.

¹⁵³ Cf. *Tractatus de consequentiis* [Peter of Mantua, 1492a, sig. Fiii^{va}] "Item a disiunctiva cum destructione unius partis super alteram partem valet consequentia".

¹⁵⁴ Peter proves to be aware of one of the most widely known consequences of the standard obligational rules, namely the fact that any false conclusion compossible with any contingently false *positum* whatsoever can be proved in a disputation. If p is a false *positum*, then any false sentence q, compossible with p, can be proved by proposing in the first place the disjunction $\neg p \lor q$, which ought to be granted because it is irrelevant and true (in virtue of the presence of $\neg p$, which is true, if p is false). Then by disjunctive syllogism (p is posited, hence $\neg p$ must be denied) we can conclude that q ought to be granted too. As is confirmed by remark of the next section, Peter is well aware of this feature of the theory. This thesis is discussed, among others, by [Ralph Strode, 1517, fol. 84^{ra}], [Paul of Venice, 1988, p. 336], and [Walter Burley, 1963, 3.21– 3.22].

5. On impositio

5.1. Quamvis omnis impositio sit positio, tamen aliqua exempla specialiter ponemus de impositione.¹⁵⁵

Et accipiamus primo quod impositio de qua hic intendimus est || obligatio qua instituitur terminus vel oratio ad significandum quod vel que vel quomodo prius non significabat. Que impositio maxime convenit terminis vocalibus aut scriptis. Mentalibus etiam convenire potest qui tunc fiunt equivoci cum imponuntur, quia absolvi non possunt ab eo quod naturaliter significant. Et in hoc refert inter vocalia aut scripta ex una parte et mentalia ex altera, quia vocalia et scripta absolvi possunt simpliciter ab imponente ab eorum significatione || priori, mentalia vero non.¹⁵⁶

5.2. Item, accipiendum quod quotienscumque imponitur ad significandum terminus qui ante non significabat aliquid, propter cuius impositionem fit propositio vera que non ante erat propositio, concedenda est simpliciter illa propositio non quia

¹⁵⁶ Impositio is a subtype of positio dealing with variations of meaning. According to Peter, it pertains, properly speaking, to spoken and written terms or sentences, but not to their mental counterparts, unless the latter are taken to be equivocal. The reason is that spoken and written signs signify ad placitum, i.e. conventionally, whereas their mental correlates have a natural signification that cannot be suppressed or replaced on the basis of new stipulations (the rationale is not hard to figure out: mental terms do not derive their signification from stipulations, therefore no stipulation can change it).

The distinction is used in a similar context by [Ralph Strode, 1517, fol. 89^{va-vb}]. The effects of this assumption are visible, for instance, in sec. 5.25 below, where Peter, in replying to an objection, recalls the distinction between those who maintain that mental terms, besides their natural signification, can also signify *ad placitum*, and those who hold the opposite view.

Some remarks on this issue are also found in Peter's treatise on supposition, in the context of a discussion intended to reject material and simple supposition, cf. *Tractatus de suppositionibus*, [Peter of Mantua, 1492a, sig. Ai^{rb}-Aii^{vb}].

M $89^{\rm vb}$

5

10

 $P 71^{va}$

 $^{^{155}}$ The fifth section of Peter's treatise is devoted to the analysis of a number of problems and sophisms concerning *impositio*, a further subtype of *positio* which focuses on the variation of meaning of words and sentences. A variety of topics is dealt with: sec. 5.1–5.3 lay down and discuss some preliminary assumptions specifically related to *impositio*; sec. 5.4–5.6 focus on a first type of *impositio* whose distinctive feature is the attribution of a meaning to terms that did not previously signify; the rest of the section concerns the variation of meaning of terms and sentences whose original meaning is modified by virtue of a new stipulation. Three groups of arguments are identifiable: the first is covered in sec. 5.7–5.11 where we find a lengthy discussion of a very popular sophism on the ascription to the sentence 'homo est asinus' of the meaning of the sentence 'deus est'; in sec. 5.14–5.23 the focus shifts on problems related to the *suppositio* of terms; finally, sec. 5.24–5.29 deal with incompatible sentences that are assumed to convert (with the exception of 5.26 which takes up the issue of ill-formed expressions).

¹ Quamvis] unde *praep.* O **3** accipiamus] accipiemus O **4** vel quomodo] .om. O **5** significabat] signivicavit O **5** convenit] contingit O **6** convenire] concedere O **6** qui] quia O **6** equivoci] equivoca O **7** ab eo] *om.* O **8** aut] et O **8** ex ... parte] *om.* O **8** ex altera] *om.* O **9** vero] autem O **11** ad significandum] *om.* O **13** erat] fuisset

sequens, sed quia vera.¹⁵⁷ Et in hoc omnes disputantes convenire oportet. Aliter enim nulla esset propositio vocalis concedenda vel scripta: quia si est concedenda, non est concedenda nisi quia impositor imposuit terminos ad significandum per quorum impositionem facte sunt propositiones vere. Nec || aliquis esset sillogismus aut consequentia vocalis vel scripta concedenda nisi fuisset casus de impositione factus.¹⁵⁸||

5.3. Sed contra illud, forte, arguitur quod ex illo sequitur quod omnis homo est 20 obligatus et cuilibet homini est facta impositio || aut presuppositum per quam concedit propositiones vocales aut scriptas.

¹⁵⁸ The idea is intriguing. In the context of a disputation, the agents involved must agree on a basic fact, which is a sort of prerequisite of meaningfulness (cf. 'omnes disputantes convenire oportet'): the concession of a sentence, as well as the concession of a sequence of sentences logically related to one another, both depend on a stipulation of meaning made by means of a *casus* of *impositio*, at least at the token-level (i.e. the level at which *proposita* are put forward in virtue of their being uttered by the opponent).

159

15

 $O \ 122^{va}$

B 99^{va}

V 77^{ra}

 $^{^{157}}$ This passage is of crucial importance. It seems that in the case of *impositio*, the act by means of which a meaning is assigned to a word or to a sentence modifies the implicit background - actual reality or our set of beliefs related to it - we need to turn to, whenever we are going to evaluate a sentence in the context of a disputation. The clause 'non quia sequens sed quia vera' explicitly gives a hint along these lines: if the duty of granting a sentence arises – according to a given modification of meaning stipulated in the *casus* – not in virtue of its being sequentially relevant but rather in virtue of its being irrelevant and *true*, then the act by means of which a new meaning is given to the sentence in question is effective *outside* the context of the disputation. Every new stipulation affects (our way to describe) the states of affairs that are to be looked at in the evaluation of irrelevant sentences. In other words, acts of baptizing are operating at the background level of obligational disputations and become functional to the evaluation of irrelevant sentences, since it is exactly to that level that we must go back to pick out the information needed to determine their truth value. This approach seems to be quite original, since, to my knowledge, the standard response in the obligational framework is regarding sentences affected by the *impositio* as irrelevant to it and then replying to them exactly as one would reply outside the disputation (this is often expressed by the clauses 'respondere infra tempus sicut extra' or 'propter impositionem non est responsio varianda'). Peter contends, on the contrary, that within the context of an obligational disputation, we must reply to irrelevant sentences with modified meanings according to actual reality insofar as our linguistic characterizations of the latter are modified by the new stipulations. The mechanism is the same but the information we get from the outside is influenced by the *impositio*. In this connection, Peter seems to distinguish between the general case of *positio* and the specific case of *impositio*, which is endowed with a peculiar property. On what grounds is such a different treatment justified? I think that one might be tempted to see the following intuition behind Peter's choice: it is only in case of stipulations about meaning that we must assume that the consequences of a counterfactual hypothesis must enter the otherwise water-proof context of a disputation, since if this were not the case, opponent and respondent would not even be able to understand each other.

¹⁴ omnes disputantes] omnis disputans O
15 propositio] om. O
15 si] ipsa add. O
16 impositor] imponitur O
16 ad significandum] om. O
19 factus] facta O
21 et] quia
O
21 presuppositum] presupposita O
21 per quam] pro qua O

Item, arguitur quod nullus terminus unquam fuit impositum ad significandum a primo impositore, quia nullus fuit terminus primus ab eo impositus nec esse potuit, igitur etc. Patet consequentia. Et arguitur antecedens: quia si aliquis fuit primus terminus, ponamus quod fuerit iste terminus 'homo'; tunc sit Sor, gratia argumenti, qui fuit primus impositor. Et imposuit illum terminum ad significandum non solum sibi sed alteri. Tunc oportuit quod, per propositionem vocalem vel scriptam, exprimeret quod iste terminus sic esset significativus. Et si sic, aliqua fuit propositio vocalis antequam ille terminus sic significaret.¹⁵⁹

Sed pro his formis iam supra dictum est.¹⁶⁰

5.4. De vocibus, autem, vel scriptis que ante non significabant et nuper imponuntur ad discrete significandum, intelligatur quod si ista vox 'Sor' imponeretur ad hunc hominem mere substantialiter et discrete significandum, qui homo ante fuit per magnum tempus, tunc concedendum est quod iste homo est Sor, posito quod illa vox nec alium significet nec ante significaverit. Et conceditur quod iste homo fuit || Sor etiam || antequam || ipse vocaretur 'Sor' et fuisset Sor, licet numquam ipse fuisset vocatus 'Sor'. Et sciebas istum hominem esse Sor quando ipse non vocabatur 'Sor', nec aliquis homo potuit unquam esse Sor, nisi iste homo.¹⁶¹ Et si queratur quare conceditur quod ille est Sor et non est Buff vel Plato vel ||

Strode in his treatise makes a similar point, explicitly mentioning the problem of an infinite regress, cf. [Ralph Strode, 1517, fol. 90^{ra}].

¹⁶⁰ I have not found a specific passage in this treatise or elsewhere in the *Logica* that could count as a reply to the two objections, but I am inclined to think that Peter is willing to reject both of them, and maintain the core of his doctrine unaltered.

¹⁶¹ A similar point is made in Peter's treatise on *appellatio*, cf. *Tractatus de appellationibus*, [Peter of Mantua, 1492a, sig. Bii^{vb}].

24 terminus] om. O 26 quod] ille add. O 26 fuerit] fuit O 27 fuit] sit O 28 quod, per] om. O 29 exprimeret] exprimere O 32 et nuper] sed O 33 discrete] om. O 33 ista] om. O 34 mere substantialiter] modo singulari O 36 ante] umquam O 36 conceditur quod] om. O 37 ipse] om. O 37 Sor] om. O 38 ipse] om. O 38 quando] ante quando O 40 est] om. O 40-161.41 vel || huiusmodi] om. O 25

30

35 L 72^{va}

E Giiii $+2^{rb}$ M 90^{ra}

¹⁵⁹ Here Peter presents two objections to the assumptions made in sec. 5.2. First, it can be argued, by generalization, that the idea that sentences are granted in virtue of stipulations of meaning implies that the scope of *impositio* should extend far too much beyond the context of obligational disputations, to the effect that it would in fact embrace the whole business of language. Second, the idea of a term acquiring its meaning for the first time in virtue of an act performed by a *primus impositor*, who makes the stipulation required in order for a term to be capable of signifying, is problematic, not only *de facto* but also in principle (cf. the remark 'nullus fuit terminus primus ab eo impositus nec esse potuit'). The argument is based on the idea of an infinite regress. If we assume that in order for a term to signify, there must have been a preliminary stipulation, which in turn is formulated through words or sentences, then we are bound to the ineluctable conclusion that there must also have been a first stipulation at the beginning of the chain. But the first stipulation, too, will have had to be made by means of some written or spoken expression, already provided of a meaning (cf. 'tunc oportuit quod, per propositionem vocalem vel scriptam, exprimeret quod iste terminus sic esset significativus').

⁴⁰ P 71^{vb}

huiusmodi, dicitur quod iste terminus 'Sor' modo mihi significat istum hominem et ista est modo mihi propositio 'iste est Sor' et non ista 'iste est Buff'.

Propter impositionem concedo quod iste est Sor; nego tamen quod iste sit Sortes propter impositionem. Sicut, quia vocatur 'Sor', concedo quod iste est Sor; sed non concedo quod iste est Sor quia vocatur 'Sor'; et quia non vocatur 'Buff' vel 45 'Plato', ideo non concedo quod sit Buff vel Plato.¹⁶²

5.5. Posito, autem, quod per totum istum diem iste terminus 'Sor' ita istum hominem significabit quod nullum alium, sed cras imponetur ad significandum alium hominem, est adhuc concedendum quod ille secundus non potest esse Sor nec poterit esse Sor neque unquam erit Sor, ita quod ille secundus est Sor, quia adhuc convertuntur et convertentur per totam istam diem iste propositiones 'ille secundus poterit esse Sor' et 'ille secundus poterit esse iste homo'. Cras tamen, facta illa secunda impositione, concedam illam 'iste est Sor'; illum tamen numquam

¹⁶² I suggest to formalize the situation described in this passage as follows: let I be a two-place operator that takes an individual constant as its first argument and a term standing for a proper name as its second argument, in such a way that $I(t, \mathbf{S})$ must be read 't is called \mathbf{S} ', where \mathbf{S} stands for the proper name S. This operator corresponds to the assignment of meaning to a term by means of an act of *impositio*. Then let expressions of the form t = S stand for 't is S'. Thus, to clarify the point made above, let t stand for 'iste', \mathbf{S} for the proper name 'Sor' and, since identity statements are at stake here, let the copula be represented through the symbol of equality: the predication simply says that the individual denoted by t is the same individual denoted by S; finally, C is the familiar operator for concession. Now, Peter's claim is that the following holds:

(1) $I(t, \mathbf{S}) \Rightarrow C(t = S)$	(= quia vocatur 'Sor', concedo quod iste est Sor)
$(2) \neg C(I(t, \mathbf{S})) \Rightarrow t = S)$	(= non concedo quod iste est Sor, quia vocatur 'Sor')

This formulation is meant to capture Peter's insight. I think the difference between the two claims lies in the relative position of the operator that stands for the stipulation of meaning, I, and the concession operator, C. In (1), the concession is made under the condition that t has been assigned a given name; if the condition is fulfilled, then, on account of that, we can grant the sentence 't is S'. But at the same time we are not willing to grant, more generally as in (2), that t is S on account of the impositio. Here the symbolic rephrasing tries to reflect the wording (and conceptual order) of Peter's point: (1) corresponds to the first claim 'propter impositionem (= 'quia vocatur') concedo quod iste est Sor', whereas (2) corresponds to the second claim 'nego tamen quod iste sit Sor propter impositionem (= 'quia vocatur 'Sor')'. In the second case, the negation, i.e. the refusal to grant, ranges over the whole implication. In (2) what is denied is that a factual circumstance (this individual being Socrates) is implied by an act of impositio. In (1), on the other hand, what the act of impositio implies is only the concession that something is the case. I am not completely sure that this works, but I am pretty confident that it is what Peter has in mind here.

50

⁴² iste] add. homo O 42 iste] add. homo O 43 concedo ... Sor] om. O 43 sit] est O 44 concedo ... Sor] om. O 45 non ... et] om. O 47 totum istum] totam istam O 48 significabit] significet O 49 hominem] et add. O 50 Sor] om. O 51 ille] om. O 52 poterit] potest O 52 poterit] potest O 53 illa secunda] illi secundo O

concedam esse Sor, et ille terminus 'Sor' cras non significabit istum esse Sor, demonstrato illo secundo. Sed tamen concedam cras illam non male respondendo || 'ista significat quod iste est Sor', quia tunc ille terminus aliter significabit quam nunc significet.¹⁶³

¹⁶³ The sophism presented here and the one in the next section constitute, to some extent, a complementary pair. Here Peter assumes that one and the same term, or better proper name, is assigned in turn to two different individuals at different moments of time. In the next one, by contrast, the idea is that the same name is assigned at the same time to two different individuals. The outcomes are different, as one would expect. Let us see what is going on here, first. The text is somewhat problematic because of a loose use of the demonstrative pronouns 'iste' and 'ille': it is sometimes unclear to which of the two individuals the pronoun is supposed to refer. Peter briefly shows what sentences should be granted at the two moments of time. I shall indicate the two individuals involved by h_1 (= the individual that is called Socrates today) and h_2 (= the individual that will be called Socrates tomorrow). Let then the indexes t_1 and t_2 stand for 'today' and 'tomorrow', respectively. Now, the situation, I think, can be depicted in the following way:

(1) $(I_{t_1}(h_1, \mathbf{S}) \land \forall x (I_{t_1}(x, \mathbf{S}) \Rightarrow x = h_1)) \Rightarrow C_{t_1}(h_1 = S) \land \forall x (x = S \Rightarrow x = h_1))$

This sentence is just a rephrasing of the assumption that there is only one individual, today, who is called Socrates. Now, Peter first proves that, today, the following sentence ought to be granted: 'ille secundus $[= h_2]$ non potest esse Sor nec poterit esse Sor neque umquam erit Sor'. Formally, it will turn out to be like this:

(2)
$$C_{t_1} (\neg \Diamond (h_2 =_{t_i} S)) \quad \forall t_i \ge t_1$$

The reason is presumably that every occurrence of 'Sor' in the above sentence refers, today, to h_1 , i.e. to the only bearer of that name. The prove is therefore based on the fact that the following also holds (cf. 'adhuc convertuntur et convertentur per totam istam diem iste propositiones 'ille secundus $[= h_2]$ poterit esse Sor $[= h_1$ as of today]' et 'ille secundus $[= h_2]$ poterit esse iste homo $[= h_1]$ ', because the term 'Sor', today, is convertible with 'iste homo $[= h_1]$ ':

(3)
$$\Diamond(h_2 =_{t_1} S) \Leftrightarrow \Diamond(h_2 =_{t_1} h_1)$$

The right-hand component of (3) is false, therefore the left-hand component is false, too. This entails (2), i.e. the concession of its negation. So far, so good. We have come to know that under the assumption that one individual has been assigned a proper name, today, for the first time, its reference is fixed in such a way that, today, a modal sentence, involving the ascription of that very same name to a different individual at a later moment of time, is false. I do not want to evoke anachronistic parallels, but there seems to be some kind of kripkean intuition operating here: at least the focus is somehow on the relationship between the attribution of proper names and the related behaviour of modalities over time, when the assumptions are modified.

What happens next? Or, better, what happens tomorrow, i.e. what sentences are we entitled to grant, tomorrow? If the reconstruction of the logical relation between *impositio* and concession that I have proposed (cf. footnote to the previous sec. 5.4) is correct, it is reasonable to read Peter's claim 'cras $[= t_2]$ tamen, facta illa secunda impositione $[= I_{t_2}]$, concedam illam 'iste [= ?] est Sor' as follows:

(4) $I_{t_2}(h_2, \mathbf{S}) \Rightarrow C_{t_2}(?=S)$

54–55 et ... secundo] om. O 57 significet] significavit O

55 O 122^{vb}

5.6. Posito, autem, quod aliquis alius fuerit vocatus 'Sor' aut quod nunc de presenti vocetur 'Sor', concedendum est quod uterque illorum est Sor et uterque istorum

But here a problem arises, namely that it is not altogheter clear whether it is h_1 or h_2 that should fill the gap. It looks as though Peter saw a conflict between the need of maintaining that the term 'Sor' is, properly speaking, tied to its original reference (i.e. h_1) as if it were a sort of rigid designator; and the need to cope with the new stipulation of meaning that the term has undergone in the meantime (according to which, from a given time on, the new denotation of 'Sor' should be h_2). The solution – claims Peter – is granting 'iste est Sor' on the one hand, and then, of a given individual, never granting that he is Socrates (cf. 'illum tamen numquam concedam esse Sor'). Besides the problem of how to understand the first reply, we also need to determine the right denotation of 'illum'. As to the latter point, I am inclined to think that Peter, by means of the demonstrative 'illum', is referring to h_2 . Hence, in the end, the sense of the observation 'illum numquam concedam esse Sor' is simply that one should never grant that the individual who will be called Socrates tomorrow is the same individual who is called Socrates now. As to the first problem, then, one might tentatively suggest to read 'concedam illa 'iste est Sor" by referring 'iste' to h_2 as well.

But this is not the end of the story. Immediately thereafter, Peter makes two further claims. First, the term 'Sor' at time t_2 (= 'cras') will not signify 'istum esse Sor, demonstrato illo secundo'; second, by contrast (cf. 'sed tamen'), at time t_2 the sentence 'ista significat quod iste est Sor', can be granted with no bad consequences. Again, what refers to what here? In the first passage, perhaps Peter is meaning to say that, at time t_2 , the term 'Sor' will no longer refer to h_1 so that, by indicating h_2 and uttering the sentence 'iste est Sor' we are no longer referring to the individual that was previously signified by the term 'Sor'. The problem with this interpretation is that, in the first sentence, there seems to be a strong link between 'iste' and 'demonstrato illo secundo'. Moreover, the possibility that Peter is actually referring, in both occurrences, to h_2 (and not, in the first occurrence, to h_1) when he says 'et ille terminus 'Sor' cras non significabit istum $[= h_2]$ esse Sor, demonstrato illo secundo $[= h_2]$ ' is supported by the fact that the subsequent step of the argument is put forward in the form of an adversative. What is the sense of the claim? Peter argues that tomorrow the sentence 'ista significat quod iste est Sor' will (possibly will have to) be granted. On what grounds? The alleged reason is that the term 'Sor' (cf. 'ille terminus') tomorrow will signify otherwise than it now signifies, i.e. the denotation the term will have tomorrow $[= h_2]$ will be different from the denotation the term has now [= h_1]. If this is the justification, then the term 'iste' in the sentence 'ista significat quod iste est Sor', can only refer to h_2 . What is then the last adversative opposed to? Either it refers to the immediately preceding claim that the term 'Sor' will not signify tomorrow 'istum esse Sor', or to the claim that tomorrow 'iste est Sor' will be conceded. The latter case, consisting of sentences (6) and (7) below, is to my opinion more plausible. This is therefore how I would reconstruct the argument, in the end (all replies being indexed at t_2):

- (5) Concedam illam 'iste $[= h_2]$ est Sor'
- (6) Illum $[= h_2]$ numquam concedam esse Sor $[= h_1]$

(7) Ille terminus 'Sor' cras non significabit istum $[= h_2]$ esse Sor $[= h_1]$

(8) Concedam cras illam non male respondendo 'ista significat quod iste $[= h_2]$ est Sor $[= h_2]$ '.

It seems to me that this is a viable way to find an interpretation that fits the text and preserves consistency: the problem remains as to whether this is exactly what Peter is maintaining. Be it as it may, the passage is on the whole destined to give the reader a hard time: part of the obscurity maybe cannot just be explained away.

Et si contra arguitur quod ille terminus 'Sor' est equivocus in illa propositione 'uterque illorum est Sor', igitur illa non est propositio: huic dicitur quod hic non petitur illa difficultas, ideo transeat. Ponatur enim gratia argumenti quod univoce significet illos duos et stat quod intendimus.¹⁶⁴

5.7. Cum vero imponuntur termini qui ex grammatica solent auctentice significare ad significandum aliter quam prius, respondere solent quidam in tempore obligationis sicut extra ad propositionem cuius termini variarunt significationem. Ut illam 'homo est asinus' significantem adequate || deum esse in tempore propositam negant, quia extra tempus etiam negarent, ut dicunt.¹⁶⁵

Sed isti tamen dividuntur, quia $\langle i \rangle$ eorum quidam negant illam in tempore et concedunt quod illa est concedenda quia necessaria. (ii) Alii autem negant illam et dicunt quod est neganda. Et refert iterum inter eos extra tempus, quia cum arguitur prius quod negasti necessarium, igitur male respondisti, primi negant antecedens et dicunt quod illa non erat necessaria, licet ipsi fuerunt obligati ad concedendum quod illa esset necessaria. Eorum autem reliqui negant consequentiam infra tempus obligationis || 'tu negasti necessarium simpliciter, igitur male respondisti'.¹⁶⁶

¹⁶⁵ This sophism can be found quite often in treatises on obligations. The reference to authors who deny the sentence 'homo est asinus' when its signification is turned into the signification of the sentence 'deus est' is not very indicative, since most medieval authors adopt this solution. In this case, Peter seems to hold a minority view. The idea, as will be clear shortly, cf. infra, sec. 5.11, is to grant the sentence as irrelevant and true.

¹⁶⁶ It is quite difficult to identify the followers of the views in question. General references to the tradition of this sophism are found in Paul of Venice, 1988, p. 195 f. 39]. Cf. also [Roger Swyneshed, 1977, pp. 264–265], [John of Holland, 1985, pp. 94 - 95], [Marsilius of Inghen, 1489, sig. Bii^{va}–Biiii+2^{rb}] and [William Buser, 1990, pp. 132-154]. The last two authors should probably be regarded as a general background, if not as the direct source, of Peter's presentation. As Ashworth has pointed out they are the only two authors who offer an elaborate treatment on this topic. Nonetheless the details of Peter's reconstruction, and

60 ultra dicitur *praep.* O **62** est fuit O **68** solent consuever of **69** in tempore intra tempus O **69–70** obligationis] om. O **70** extra] add. tempus O **71** in tempore] intra tempus O **72** quia] etiam add. O **73** in tempore] intra tempus O **73** et] supra lin. O 75 eos] eorum O 75 extra] intra O 76 quod] om. O 76 primi] om. O 77 fuerunt] forent O 78 quod ... necessaria] illam fore necessariam O 79 obligationis] om. O 79 tu] om. O

M $90^{\rm rb}$

65

B 99^{vb}

70

75

 $P 72^{ra}$



 $^{^{164}}$ The circumstance envisaged in this sophism, which supplements the discussion started in the previous one, is less hard to understand. The point here is that, if we drop the constraint of unicity from the above assumptions, it just follows that when more individuals are assigned a proper name in virtue of an act of *impositio*, then the name ceases to be a proper name and thereby acquires the status of a full-blown common term.

5.8. Sed contra primam illorum responsionem arguitur:¹⁶⁷ quia negata ista in tempore 'homo est asinus', cedat tempus obligationis.

(Obj.1) Et arguitur sic: tu negasti illam || que, in tempore obligationis, erat necessaria simpliciter, igitur male respondisti. Patet consequentia. Et arguitur antecedens quod illa erat necessaria: quia in tempore significavit principaliter deum esse, igitur in tempore fuit necessaria. Patet consequentia. Et arguitur

Be this as it may, I will opt for a analysis of the argument, leaving aside the problem of the identification of sources. This is my suggestion for a reconstruction of the disputation:

Ι	'Homo est asinus' adequate significat deum	Α	possible
	esse		
1.	Homo est asinus	Ν	(cf. 'quia extra tempus etiam
			negarent)
2.	1 Illa est concedenda	С	
2.2	2 Illa est neganda	\mathbf{C}	
3.	Negasti necessarium simpliciter, igitur male	Ν	
	respondisti		
4.	Negasti necessarium [simpliciter]	Ν	

Some remarks are in order. First, the assignment of a new meaning, as far as I can see, is an asymmetrical operation. If we stipulate that 'homo est asinus' signifies adequately as 'deus est', then 'homo est asinus' actually means that there is a God, or that God exists, but not the other way around (i.e. 'deus est' does not signify that a human being is a donkey). This is why the objection 'negasti necessarium simpliciter' can be put forward if 'homo est asinus' is denied. If, by contrast, the sentence 'deus est' had been assigned the meaning of 'homo est asinus', put forward and granted, the right objection would have been 'concessisti impossibile simpliciter' (this seems to be the case for instance in [Roger Swyneshed, 1977, p. 264] who devotes some space to the sophism in this inverted formulation). Second, the point of the whole discussion for Peter is challenging the background assumption according to which 'respondere solent quidam in tempore obligationis sicut extra ad propositionem cuius termini variarunt significationem'. Third, the views among these people are various, and they must be ruled out with specific arguments. There are basically two categories: (i) those who deny 'homo est asinus' infra tempus but grant the sentence 'illa est concedenda' (because 'illa' stands for a necessary sentence); and (ii) those who deny infra tempus 'homo est asinus' and grant 'illa est neganda'. The subdivision also conforms to another criterion. Let us assume that the following objection has been put forward (it is not clear whether the objection is made *infra* or *extra tempus*): 'negasti necessarium simpliciter, igitur male respondisti'. Now, the first group (i) would deny the antecedent of this consequence, i.e. they would deny that, by denying 'homo est asinus', they have thereby denied a necessary *simpliciter* sentence. The second group (ii), on the other hand, would deny the whole consequence. In the following two sections Peter argues against both views.

¹⁶⁷ The following objections are directed against the first reply (i): denial of 'homo est asinus' and of the antecedent of the consequence 'negasti necessarium simpliciter, igitur male respondisti'.

L $72^{\rm vb}$

especially, the kind of objections he raises seem to be different at some decisive points. In Buser the sophism is taken to be an example of absolute imposition, i.e. such that a 'complexum congruum complexe imponitur ad significandum' (a sentence that already has a meaning is assigned the meaning of another sentence).

^{81–82} in tempore | infra tempus O 84–86 Patet ... necessaria.] om. O

antecedens: quia illa fuit imposita ad significandum deum esse principaliter, igitur eius impositio fuit facta per quam significavit principaliter deum esse.¹⁶⁸

(Obj.2) Item, impositio illius fuit facta per quam significavit principaliter || deum esse; igitur illa variavit significationem, cum ante significaverit hominem esse asinum. Et ultra: variatio significationis facta fuit, et non nisi ad significandum deum esse principaliter; igitur illa principaliter significavit deum esse. Primum antecedens arguitur: quia tu fuisti obligatus per impositionem illius, || igitur || impositio illius fuit facta.¹⁶⁹

(Obj.3) Item, in illa responsione data, ponatur quod respondebis ad illam 'homo est asinus'. Deinde arguitur quod qualitercumque respondebis ad illam 'homo est asinus', tu male respondebis: quia vel concedes illam vel tu negabis vel dubitabis; sed sive sic sive sic, tu male respondebis ad illam; igitur qualitercumque tu respondebis ad illam, tu male respondebis ad illam. Et sic aliqua esset propositio ad quam non posses bene respondere infra tempus que tamen est scita a te esse 100 necessaria.¹⁷⁰

(Obj.4) Item, per idem, data illa responsione, posita et admissa illa 'tu es obligatus', cedente tempore obligationis, negandum esset quod tu fuisti obligatus. Consequens est falsum manifeste: quia ego posui tibi illam et tu bene admisisti eam, igitur tu fuisti obligatus. Consequentia patet, quia illa 'tu fuisti obligatus' 105 sequebatur ex illa admissione, sicut quod illa significavit sequebatur ex illa impositione. Ubi, autem, non sequitur obligatum ex obligatione vel admissione, non sic est; ideo non sequitur 'ego posui quod Sor curreret; igitur Sortes currebat'.¹⁷¹

 171 The argument presented here, by contrast, seems to be against the idea that *impositio* comes into play in the evaluation of irrelevant sentences; rather, it is argued, some sentences follow from (i.e. are relevant to) a given *impositio*, as when we say that a sentence is relevant to the *positio* and *admissio* of something. The difference may appear slight, but it is of great importance, because it affetcs the whole picture: if a sentence ought to be granted in virtue of its being relevant to the *impositio*, it does not mean that the *impositio* in question actually modifies $M 90^{va}$

90

95

O 123^{ra} $E Giiii+2^{va}$

V 77^{va}

¹⁶⁸ Here the fundamental tenet (cf. *supra*, sec. 5.2) of Peter's account of *impositio* is operating: the stipulation actually makes a given sentence signify otherwise than it previously did (cf. 'eius impositio fuit facta *per quam* significavit principaliter deum esse'). Accordingly, not only does a false sentence become true in virtue of the stipulation; it may also become necessary, if it is imposed the meaning of a necessary sentence, like in the present case.

¹⁶⁹ In support of the hypothesis that the signification of a sentence is actually modified by an act of *impositio*, in this passage we find the explicit remark 'illa variavit significationem'.

¹⁷⁰ This line of argumentation is not uncommon in the obligational literature, cf. for instance [Paul of Venice, 1988, p. 298]. It is part of a general strategy in constructing objections: if we can prove that there is a sentence that should neither be granted, nor doubted, nor denied, then we may cause no little trouble to the respondent. Here, however, Peter just brings up the possibility of this objection, without going through the details.

^{89–90} Item, ... esse] om. O **92** Primum] et praep. O **93** illius] om. O **95** in] om. O 96–97 homo...tu] om. O 97 concedes] concedis O 98 tu] om. O 99 tu] om. O 99 ad illam] om. O 99 esset] est O 103 esset] foret O 104 ego] om. O 105-108 Consequentia ... currebat] om. O

 $\langle \text{Obj.5} \rangle$ Item, per idem, data illa responsione, sequitur quod ille terminus 'homo' non significaret hominem: quia, licet primus impositor posuerit, non tamen 110 sequitur quod significavit. Vel si propter eius impositionem factam de illo termino 'homo' concedendum est quod iste est homo, consimiliter imponas tu aut ego: et non erit causa quare, ex impositione illa, ille terminus 'homo' significavit hominem quin ex impositione mea vel tua posset significare asinum; aliter termini vocales vel scripti non essent ad placitum significativi. Sed non est plus in istis sermonibus 115 morandum.¹⁷²

Finally, as regards the claim 'ubi non sequitur ex obligatione vel admissione, non sic est', cf. *supra*, sec. 1.3.

¹⁷² The point of the last objection to the first *responsio* is quite interesting. If *impositio* is without factual influence, i.e. if the actual meaning of words and sentences does not depend on it, this should also hold in the case of the very first assignments of meaning: then how would terms and sentences become capable of signifying? The existence of a stipulation, and the assumption that the stipulation has a cogent character on speakers, are the conditions of the fact that written and spoken terms are *ad placitum significativi*. The fact that they are the outcome of a stipulation is the reason of their being *ad placitum*, while the fact that they are *significativi* depends *tout court* on the normative nature of stipulations as such (once the latter are made, we are committed to upholding that, at least for a given lapse of time and in a given context – like in an obligation – words and sentences signify according to the agreed convention).

110 non] om. O 110 posuerit] imposuit O 112–113 imponas ... 'homo'] om. O 114 quin] qui O 114 mea ... tua] om. O 116 morandum] permorandum O

the meaning of that sentence because, as in the case of *positio*, some of the sentences that we are bound to grant in a disputation may actually be false. On the other hand, if a sentence, whose meaning is modified in virtue of an *impositio*, ought to be granted as irrelevant and true (cf. *supra*, 5.2 'non qiua sequens, sed quia vera'), this implies that in the case of *impositio* we are adopting an approach that is structurally different with respect to standard paradigm of *positio*. Either the point made here is inconsistent with the rest of Peter's doctrine, or, reluctantly, we should take the claim 'sicut quod illa significavit *sequebatur* ex illa impositione' in a loose and non-technical sense, i.e. as meaning not 'following from' but something like 'depending on' the *impositio*. Despite this difficulty, it should be pointed out that after all the notions of 'relevant to the obligation' and 'relevant to the admission', that Peter somehow recalls here, contain a factual element: if I must grant something because it is relevant to the admission (sentences like 'tu es obligatus'), that happens in virtue of the *fact* that something has been posited to me and I have correctly admitted it. Perhaps, it may be argued that this is in the end not that far from the factual import of *impositio* that Peter calls into play here.

5.9. Contra secundam responsionem arguitur¹⁷³ quia $\langle \text{Obj.1} \rangle$ ex illa sequitur quod antecedens ad contradictoria est concedendum non male respondendo.¹⁷⁴

 $\langle \text{Obj.2} \rangle$ Item, quod repugnans bene concesso est concedendum.¹⁷⁵

(Obj.3) Item, quod non omne possibile scitum esse possibile et intellectum non 120 repugnans, cum petitur per signa positionis admitti, est admittendum.¹⁷⁶

 $\langle \text{Obj.4} \rangle$ Item, quod in consequentia bona et formali, denominata etc., significante || etc., consequens est negandum et antecedens [non]¹⁷⁷ est concedendum, et L 73^{ra} per consequens non omne sequens ex bene con-||cesso propositum in tempore est concedendum. 125 P 72^{rb}

5.10. Iste conclusiones in hac arte || non sunt sustinende, quia tota intentio est M 90^{vb} convenientem responsionem instruere respondentem.¹⁷⁸

¹⁷⁴ The first objection claims that, on account of the second *responsio*, a sentence implying two contradictories would have to be granted, yet we would not be entitled to conclude on these grounds that the respondent is replying incorrectly. Why is this a bad consequence? The reason is that if something implies a contradiction, then it is by definition an incompatibly relevant sentence, which ought to be denied. If it is granted, the corresponding rule is violated and the respondent replies incorrectly. Cf. *infra*, sec. 5.10, PObj.1 (= proof of the first objection)

¹⁷⁵ Cf. *infra*, sec. 5.10, PObj.2.

¹⁷⁶ This objection relies on the claim that the denial of the first rule (cf. *supra*, sec. 2.1) would follow. The reason of this fact is explained below, cf. *infra*, sec. 5.10, (PObj.3).

¹⁷⁷ As the meaning of the sentence requires, the 'non' must be expuncted: the reading is confirmed below, when the objection is proved, since the same sequence and ordering of words occurs with the omission of 'non'. Moreover, this is a rule that we find also in Peter's treatise on consequences, cf. *Tractatus de consequentiis*, [Peter of Mantua, 1492a, sig. Eiiii+3^{ra}]. If the consequent ought to be denied, then no problem at all arises in case we claim that the antecedens ought not to be granted: that is exactly what one would expect. But the sense of the objection goes rather the opposite way around: if we hold the *secunda responsio*, then we obtain, as a result, a sound formal consequence whose consequent ought to be denied and whose antecedent *ought* to be granted, which is problematic, because it would amount to a rejection of contraposition.

¹⁷⁸ Another remark about the purpose of obligations, in this respect cf. also *supra*, sec. 4.1, where Peter says that false sentences are posited (often to the effect that some kind of inconsistency arises) "ut [opponens] videat qualiter [respondens] sciat se a contradictione defendere".

117 arguitur] sic add. O 117 quia] om. O 118 non ... respondendo] om. O 120 scitum ... intellectum] bene intellectum esse possibile O 121 per ... positionis] om. O 122 et formali] om. O 122 etc.] da ly si O 123 etc.] ex compositione suorum terminorum O 123 non] om. BO 123–125 et ... concedendum] om. O

¹⁷³ The following objections are directed against the second *responsio* (ii): denial of 'homo est asinus' and of the consequence 'negasti necessarium simpliciter, igitur male respondisti'. In contrast with the previous five objections against (i), all of which were more or less directly connected with the nature of *impositio*, the next five arguments raised here against (ii) are based on the idea that if we deny the consequence 'negasti necessarium simpliciter, igitur male respondisti' we are bound to deny also the basic set of obligational rules. In the present section the objections are simply stated, in the form of corollaries of (ii). In the next section Peter provides the proof that they actually follow from (ii).

Sed quod iste sequantur ex illa responsione arguitur quia ponatur quod hec 'homo est asinus' fiat¹⁷⁹ necessaria adequate significans deum esse convertibiliter cum illa 'deus est'. Cum proponitur illa infra tempus 'homo est asinus', negatur. Qua 130 negata proponitur suum contradictorium preponendo negationem toti, || scilicet 'non homo est asinus': sit enim illa sua contradictoria 'non homo est asinus'. Patet quod hec copulativa est concedenda secundum illos 'deus ||est et non homo est asinus', quia extra tempus concederetur.¹⁸⁰

 $\langle {\rm PObj.1}\rangle$ Et tamen ipsa est antecedens ad illa duo contradictoria 'deus est' et $_{135}$ 'nullus deus est'. 181

 $\langle PObj.2 \rangle$ Ex isto sequitur secunda conclusio: proposita autem et concessa ista 'deus est', concedenda est adhuc illa 'non homo est asinus' ex responsione, que tamen repugnat sibi; igitur repugnans bene concesso est concedendum.¹⁸²

The idea here is that, if one holds the view that is being put into question, then one is bound to reject the fundamental rules of obligations. But the rules are the basis of the discipline and they are set out to teach the respondent how to reply in the appropriate manner; therefore what gets thrown away, in the end, is the view along with its unwanted side effects.

¹⁷⁹ Possibly the reading 'sit', that is found in O, is preferable.

¹⁸⁰ In order to understand the arguments provided below as tantamount proofs of the corollaries to the second reply, we should see them in the framework of the following obligation (the *positum* should be taken to be equivalent to that of the original sophism "homo est asinus" adequate significat deum esse'):

Ι	'Homo est asinus' est necessaria adequa-	А	possible
	te significans deum esse convertibiliter cum		
	illa 'deus est'		
1.	Homo est asinus	Ν	
2.	Non homo est asinus	\mathbf{C}	
3.	Deus est et non homo est asinus	\mathbf{C}	(cf. 'hec copulativa est con-
			cedenda [] quia extra
			tempus concederetur'

¹⁸¹ The sentence proposed at step 3. and granted according to the assumption that *propter impositionem non est responsio varianda* is a conjunction and hence it implies both of its conjuncts, namely 'deus est' and 'non homo est asinus'. But the latter is nothing but the negation of 'deus est', provided that we make the right substitution in virtue of the assumption that 'homo est asinus' converts with 'deus est'. Therefore a sentence (= 'deus est et non homo est asinus') that implies two contradictories (= 'deus est', 'nullus deus est') ought to be granted.

¹⁸² The line is the same as before: if both 'deus est' and 'non homo est asinus' are put forward after each other, they should be granted according to the assumption that the reply to a sentence must not vary in function of new stipulations of meaning. On this view (cf. 'ex responsione') both sentences, that would have to be granted *extra tempus*, ought to be granted *infra tempus* as well. But once we have granted 'deus est', if we grant, afterwards, 'non homo est asinus', we will thereby grant a sentence that is incompatibly relevant to an already correctly granted sentence

128 iste sequantur] illa sequantur O 128 arguitur] sic add. O 129 fiat] sit O 129 convertibiliter] et convertatur O 130 Cum proponitur] Tunc proposita O 130 homo ... asinus] om. O 131 proponitur] proponatur O 132 sua] om. O 135 et] om. O 137 Ex] et praep. O 139 concesso] admisso O B 100^{ra}

O 123^{rb}

(PObj.3) Tertia etiam conclusio sequitur: quia, stante illo casu, hec est possibilis 140 'homo est asinus' scita esse talis etc. Et tamen illa non est admittenda, si petatur admitti per signa positionis: quia si admittitur illa 'homo est asinus' cum toto casu, pono igitur tibi illam 'homo est asinus' cum toto casu || et propono eandem. Qua negata, arguitur quod illa est concedenda, quia illa est posita et bene admissa.

 $\langle i \rangle$ Et si sic, igitur non eodem modo respondendum est extra tempus obligationis 145 et intra, quia extra tempus negaretur et tamen in tempore conceditur.

 $\langle ii\rangle$ Vel cedat tempus obligationis et arguitur quod infra tempus illa fuit concedenda, quia posita et bene admissa; et extra tempus est neganda; igitur responsio nulla.¹⁸³

 $\langle PObj.4 \rangle$ Alie etiam due conclusiones sequentur, quia illa est consequentia simpliciter bona infra tempus 'deus est, || igitur homo est asinus' denominata a ly 'si' etc., et antecedens est concedendum et consequens negandum.¹⁸⁴

 $\langle \text{Obj.5} \rangle$ Item, quod non eodem modo respondendum est intra tempus et extra ad propositionem impositam arguitur sic: quia ponatur quod ista 'homo est asinus' sit ad placitum significativa sicut una propositio greca, sic quod non significat 155 tibi latine; et sit propositio nec tu intelligas grecum.¹⁸⁵ Dato illo, proponitur illa

(= 'deus est'), which is against the rules.

¹⁸³ The second reply (ii) implies the rejection or, at least, a restricted version of the first rule for admission. On the other hand, if the rule holds, as a result one ought not to reply in the same way within the time of the obligation as outside the time. But we want to keep the rule in its unrestricted formulation, therefore it is this last claim that must be rejected. The argument hinges on the fact that, according to Peter, the sentence 'homo est asinus', if it is assigned the meaning of the sentence 'deus est', becomes necessary and therefore, *a fortiori*, possible. If it is possible, in turn, it will fall within the class of sentences to which the first rule applies, namely the domain of all possible sentences. But then, if 'homo est asinus' is posited, it will have to be granted *infra tempus*.

¹⁸⁴ Cf. Tractatus de consequentiis [Peter of Mantua, 1492a, sig. Eiiii+4^{ra}] "Apud eos qui, eodem modo respondendum, dicunt infra tempus et extra ad propositionem quam ponimus suam significationem variare, et dicunt istam esse negandam 'homo est asinus' – quamvis sit necessaria – quia ex communi modo negaretur, addendum fore, in regula, quod consequentia non esset bona propter aliquam impositionem a priori factam [as it is, by contrast, in the present case; cf. also *infra*, sec. 5.25]. Sed apud eos qui respondent ad propositiones secundum earum qualitatem, sicut infra dicetur, antecedens istius consequentie est concedendum et consequens, quia secundum istum modum omne simpliciter necessarium est concedendum." .

¹⁸⁵ This objection adds to the list of four presented above. Most notably, right after the passage quoted in the previous footnote, in the treatise on consequences, Peter has a similar example involving Greek terms, [Peter of Mantua, 1492a, sig. Eiiii+4^{ra}] "Ex quibus patet illum non esse bonum modum arguendi, scilicet 'hec consequentia est bona etc., denominata etc.,

141 etc.] om. O 142 admittitur] admittatur O 144 negata] concessa O 144 illa] om. O 144 est] tibi add. O 145 si] om. O 146 tempus] om. O 146 negaretur] negatur O 146 in tempore] intra tempus O 148 quia] fuit add. O 148 tempus] ipsa add. O 148 est neganda] negaretur O 150 etiam] autem O 151 deus... asinus] om. O 152 etc.] vel ergo O 155 sit... significativa] significet ad placitum O 155 significat] significet O 156 tibi] sibi O

170

V 77^{vb}

150 E Giiii $+2^{vb}$

'homo est asinus'. Si conceditur aut dubitatur, peccatur secundum illam responsionem. Si negatur, cedat tempus obligationis et arguitur quod in tempore negasti propositionem || quam non intellexisti, igitur male respondisti.¹⁸⁶ Patet consequentia. Et arguitur antecedens quia ista significavit solum grece in tempore, ut 160 habet concedere illa responsio et concedit.¹⁸⁷

5.11. Ideo, sequitur tertia responsio respondens ad propositiones secundum quod ille sibi significant vel non significant. Unde sicut facta impositione istius vocis 'bub' vel huius vocis 'Sor' || sit¹⁸⁸ nomen singulare istius hominis, certo homine demonstrato, conceditur quod iste homo est bub aut Sor, non quia illa sequatur 165 ex ista impositione sed quia est vera et impertinens, ita etiam, posito quod iste terminus 'homo' significet unum asinum, conceditur quod asinus est homo non quia ista 'asinus est homo' sequatur, sed quia est vera.¹⁸⁹||

Unde iste consequentie non valent 'tu vocaris 'Sor', igitur tu es Sor' capiendo illum terminum 'Sor' mere substantialiter, 'iste 'deus est' et 'homo est asinus' 170 convertuntur; sed deus est; igitur homo est asinus', quia antecedens || nunc forte

¹⁸⁷ Let us assume that the sentence 'homo est asinus' has the meaning of a Greek sentence that we do not understand, and let it be put forward in a disputation. If it is either doubted or denied, our opponent will claim that we have replied to 'homo est asinus' in a way that is different from the way we would usually reply to that sentence *extra tempus*. If we abide by his requirement and deny 'homo est asinus', then the reply comes home to roost, because we are denying a sentence that we do not understand.

¹⁸⁸ The syntax is awkward: supplying a 'quod' before 'sit', as in O, might be of some help.

¹⁸⁹ The general solution to the original sophism presented in sec. 5.7, then, simply amounts to conforming to the assumption that *impositio* modifies the background against which we are supposed to evaluate irrelevant sentences and that, obviously, if a sentence is the object of an *impositio*, it is indeed irrelevant and must be evaluated according to its own status. The point is that this status changes in function of the new meaning stipulated by means of the *casus*. In that way, irrelevance plus modification of the background imply that a newly imposed sentence should be be granted 'non quia sequens, sed quia vera'.

M 91^{ra}

 $P 72^{va}$

L $73^{\rm rb}$

O 123^{va}

significans etc., habens etc., quorum etc., et eius antecedens est intellectum a te, igitur eius consequens est intellectum a te', quia si fieret consequentia, cuius antecedens esset latinum et consequens grecum, que sciretur esse bona, antecedens in casu est intellectum et consequens non. Potes enim credere tu, per veram relationem cui firmiter assentires, quod illa consequentia esset bona, ut 'homo currit, igitur antropos trechi"'. If we turn back to the treatise on obligations, later in this work, Peter appeals again to a similar argument, cf. *infra*, sec. 5.13.

¹⁸⁶ The same inference is used later, cf. *infra*, sec. 5.26, where the point in question is that the respondent replies to a sentence involving a sequences of sounds with no signification such as 'bub' or 'buff'.

¹⁵⁸ Si] nam add. O
158 negatur] concedatur O
158 in tempore] infra tempus O
158 negasti] concessisti O
160 in tempore] intra tempus O
160 ut] EO et BV
163 sibi]
om. O
163 vel... significant] om. O
164 bub] buf O
164 vocis] termini O
164 sit]
quod praep. O
165 bub] buf O
166 et impertinens] om. O
166 ita] item O
167 significet] convenienter add. O
167 unum] om. O
168 ista ... homo'] om. O
168 sequatur] ante sequatur scr. et del. sequit O
168 quia] illa add. O

est possibile et consequens impossibile.¹⁹⁰

Et addo generaliter ad istam responsionem quod si consequentia sit bona et formalis denominata a ly 'si' etc. et antecedens est concedendum, consequens etiam est concedendum.¹⁹¹

Item, quod ad propositionem impertinentem concessis aut negatis vel eorum oppositis respondendum est secundum sui qualitatem. Et intelligatur ut dictum est supra.¹⁹²

5.12. Quibus acceptis, pono quod omne nesciens se esse A sit A. Deinde propono 'tu es homo'. Quo concesso, quia verum non repugnans, proponitur 'tu es A'. Si negatur, contra: omne nesciens se esse A est A; tu nescis te esse A; igitur tu es A. Si negatur minor, contra stet oppositum, scilicet 'tu scis te esse A, igitur tu es A'.

Ideo forte dicitur negando illam consequentiam, quia minor est negativa et per hoc non est forma in Darii.

¹⁹⁰ These observations are in accordance with the general assumptions made in sec. 5.4. The two consequences mentioned here are not unconditionally sound, and that is the reason of their denial; nothing goes wrong, however, if we hold that the antecedent entails the duty of granting the consequent in a *casus* of *impositio*. The remarks, therefore, are only apparently in contrast with the claim that is found a few sections below, cf. *infra*, sec. 5.26. There is a passage in [William Buser, 1990, p. 90] which shows a strong similarity with the point made here by Peter: "Omnes tales consequentiae sunt defectuosae et malae, scilicet [...] 'tu vere vocaris 'Sortes', ergo tu es Sortes', "deus est' et 'homo est asinus' convertuntur, sed deus est, ergo homo est asinus' [...] quoniam in istis omnibus consequentiis, aliquo casu possibili posito, stat sic esse, pro nunc, sicut per eorum antecedentia significatur; et tamen non erit ita, pro nunc, sicut per eorum consequentia significatur, quare sequitur quod praedictae consequentiae et sibi similes non valent [punctuation mine]".

¹⁹¹ In fact the rule has a more complex structure, cf. *Tractatus de consequentiis*, [Peter of Mantua, 1492a, sig. Eiiii $+3^{va-vb}$] "Alia regula est quod si consequentia est bona, affirmativa, denominata etc., significans etc., habens antecedens et consequens expressa, quorum nullum est multiplex, scita esse formalis, et antecedens est concedendum ab aliquo, et consequens est intellectum propositum, et cum his bene scitur quod ex concedendo non sequitur nisi concedendum, tunc consequens est ab eodem concedendum. Sed non sequitur hec consequentia est bona [scita] scita esse talis et antecedens est ab aliquo concedendum et consequents est intellectum, igitur consequens est concedendum, quia forte creditum erit quod antecedens sit falsum aut forte creditum est quod ex concedendo sequatur non concedendum".

 192 Cf. supra, 2.5 where Peter says that the fifth rule, concerning irrelevant sentences, must be extended to the case of *impositio*. As has been already pointed out, the characteristic feature of *impositio* is that the stipulation made in the *casus* affects the background from which we pick out the information needed to evaluate irrelevant sentences. The rule, therefore, is extended to this type obligation, since it always holds that irrelevant sentences should be answered according to their own status. The peculiarity in the case of *impositio* is that the latter is affected by the assumption to which one is obligated.

172 est] *om.* O **173** addo] accipio O **173** si] illa O **174** a] da O **174** etc.] vel ergo O **174** concedendum] et *add.* O **175** etiam] eius O **176** quod] *om.* O **177** Et] *om.* O **178** supra] superius O **182** contra] *om.* O **184** per] propter O

175

V 78^{ra}

Sed contra quia sequitur: tu nescis te esse A, igitur tu es nesciens te esse A. 193

¹⁹³ It is difficult to make sense of the presence of this sophism at this point, right after the lengthy discussion of the previous one. The only link I see is the need of pointing out the role of understanding sentences before replying also in such cases where variations of signification are at stake.

The ordering of Buser's treatise might also help us. In that text, after an interlocutory appendix to the sophism involving the sentences 'homo est asinus' and 'deus est', Buser inserts a discussion on the so-called *impositio dependens* (where the assignment of a new meaning depends on the fulfillment a given condition). In particular he takes into account the case where an 'incomplexum incomplexe imponitur ad significandum', i.e. a term is assigned the meaning of another term. The sophism there is 'Imponatur A ad significandum quod sit nomen tuum, si primum quod tibi proponitur sit falsum et non aliter, deinde propnatur tibi ista 'tu es A". Again, in a similar order, and even more similar in terms and treatment is [Marsilius of Inghen, 1489, sig. Biiii+1^{vb}]. What has called my attention is just the fact that after the same sophism, all three authors have a sophism involving a letter standing for something which is not known or understood. The insertion of this example here might have been suggested to Peter by the arrangement of materials on *impositio* in Buser and Marsilius, but I am in no position to produce stronger evidence than this at the moment.

The structure of the sophism is the following:

P Omne nesciens se esse A est A	А	possible
1. Tu es homo	\mathbf{C}	irrelevant and true
2.1 Tu es A	Ν	
3.1 Omne nesciens se esse A est A; tu nescis te	\mathbf{C}	\perp (the consequent of this va-
esse A; igitur tu es A		lid inference is inconsistent
		with the denial of 2.1)
4. Tu nescis te esse A	Ν	
5. Tu scis te esse A; igitur tu es A	\mathbf{C}	valid inference, whose ante-
		cedent is sequentially rele-
		vant (by denial of 4.)
3.2 Omne nesciens se esse A est A; tu nescis te	Ν	\perp cf. Obj.
esse A; igitur tu es A		

The argument focuses on the consequence put forward in the third place, i.e. 'omne nesciens se esse A est A; tu nescis te esse A; igitur tu es A'. If it is granted, it may be inferred, by detachment, that the consequent 'tu es A' ought to be granted too. But that will be inconsistent with its former denial, at step 2.1.

An alternative might be denying the minor premise, 'tu nescis te esse A', since the major is the *positum* itself. If the minor premise is denied, then its contradictory is put forward, 'tu scis te esse A'. This implies in turn the duty of granting the corresponding categorical sentence without the epistemic operator, i.e. the sentence 'tu es A'. This solution, again, is inconsistent with step 2.1. Here the question might be raised whether Peter correctly understands the denial of 'tu nescis te esse A'. Properly speaking, it might be argued that the sentence is best expressed as a conjunction of the following form: $(\neg K_a p \land \neg K_a \neg p)$, where p stands for the sentence 'tu es A', a is the respondent and K is the standard epistemic operator for knowledge. Its negation, therefore, will be $(K_a p \lor K_a \neg p)$. In that case, the passage from step 4. to 5. above would be prevented. I assume therefore, that Peter does not understand step 4. in terms of a conjunction, but rather as simply corresponding to $\neg K_a p$: once the latter is denied, it follows indeed that $K_a p$ holds.

186 contra] om. O

(i) Tenet consequentia quia illud est participium alicuius verbi et non nisi huius verbi 'nescio', igitur etc.

(ii) Item, tenet consequentia ista sicut hec 'tu non curris, igitur tu es non currens' tenendo ly 'non' infinite, ita quod illud sit unum verbum infinitum 'non curris'.

(iii) Item, tenet consequentia illa [consequentia] sicut hec 'ego volo currere, igitur ego sum volens currere'. Et tenet consequentia ultima ex tertio Ethicorum.¹⁹⁴ Nec illa contradicunt 'tu scis te esse hominem' et 'tu nescis te || esse hominem' tenendo ly 'non' infinite, sicut iste non contradicunt 'tu curris' et 'tu non curris' tenendo ly 'non' infinite, || qua utraque illarum est [in] affirmativa.

(iv) Item, illa responsio non tollit argumentum, quia si fiat illa forma 'omne nesciens se esse A est A; tu es nesciens te esse A; igitur tu es A', si negatur minor, contra: tu es homo, igitur es nesciens te esse A. Et antecedens est concessum. Igitur etc. ¹⁹⁵

5.13. In principio, igitur, dicitur quod casus non est admittendus, quia adhuc non 200 intelligitur quid significet || ly A. Sed tamen, gratia disputationis, volo admittere casum illum et volo admisisse nisi argumenta petant talem difficultatem. Deinde, cum proponitur 'tu es homo', conceditur. Et consequenter conceditur quod tu es A. Si tamen primo proponatur quod tu es A, negatur. Et negatur postea quod tu sis homo.

Sed forte contra illam responsionem arguitur, quia numquam concedenda est illa || in casu illo, scilicet 'tu es A': quia cedat tempus et arguitur quod infra tempus concessisti || propositionem quam non scivisti esse veram et quam non

The third an final alternative is the denial of the whole consequence, based on the fact that it should not count as a valid inference since it is not a syllogistic mood of the first figure (Darii) because it has a negative minor premise. The point is unclear. Still, in the following lines, Peter rejects the denial of the consequence by arguing that the minor should be rephrased into the categorical affirmative sentence 'tu es nesciens'. In order to prove that the inference is sound, Peter provides four arguments (i-iv).

¹⁹⁴ I have not found a passage in Aristotle's *Nicomachean Ethics* with such an argument. Apparently, the reference seems to be good, because a large part of the third book is devoted to the analysis of the notions of 'voluntary' vs 'involuntary'. Still, the argument seems to be lacking in Aristotle's text.

 195 The first step of the disputation, i.e. the concession of the sentence 'tu es homo', comes into play here: unfortunately the argument could hardly be less straightforward than this. In particular, I do not see what reason, if any, can justify the claim that the inference 'tu es homo; igitur tu es nesciens te esse A' is sound. It might depend on what 'A' means; but since we do not know the meaning of 'A', we are not entitled to claim that someone's being a man entails that he does not know to be A.

190 illud] totum add. O 191 [consequentia]] om. B 193 illa] ille O 193–194 tenendo ... infinite] om. O **195** qua] quia O **195** [in]] E om. O **196** si fiat] EO; om. B; fiat V 197 si] om. O 198 Et] Tenet consequentia praep. O 199 Igitur etc.] om. O 200 quod] ille add. O 202 et ... admisisse] om. O 202 nisi ... petant] ubi argumenta non petant O 203 conceditur] concedatur O 204 quod tu es A] om. O 206 forte] om. O 207 illa] om. O 207 scilicet] om. O 208 quam ... et] om. O

174

M $91^{\rm rb}$

190

195 B 100^{rb}

E Giiii+3^{ra}

205

 $P 72^{vb}$ O 123^{vb} intellexisti, igitur male respondisti.¹⁹⁶ Patet consequentia. Et arguitur antecedens quia predicatum huius quam concessisti nihil tibi significabat infra tempus, quia 210 nec hominem nec asinum nec aliquid aliud apprehendebas per illum terminum.

Sed huic forte dicitur quod iste terminus A significavit tibi aliquid et tamen nihil tibi significavit. Et apprehendebas per A aliquid et nihil per A apprehendebas.¹⁹⁷

Sed contra: per idem per hoc participium grecum 'on' tu apprehendis aliquid et nihil per ipsum apprehendis. Et sic quilibet alter terminus grecus, pari ratione, 215 significat tibi aliquid et nihil tibi significat, || posito quod de istis terminis advertas. Quod si concedatur, concedendum est etiam quod tu sis multum sapiens et nescis te esse sapientem.¹⁹⁸

Ideo dicitur ad argumentum quod ille casus non fuit admittendus, sicut probat argumentum. Et fuit excusatio secunda, in principio, quod volebat admisisse 220 casum illum nisi argumenta petant difficultatem de intellectione A.¹⁹⁹

A similar point is made below with the well-known example involving the promise of a penny: cf. *infra*, sec. 5.23.

¹⁹⁸ There seems to be a bit of irony in this remark which is presumably based on the fact that if one grants the sentence 'apprehendo aliquid', it does not follow that there is anything in particular of which one is entitled to say that he has apprehended it. Consequently, no matter how many things are said to be apprehended or known in the compound sense, unawareness of one's being in such a state of knowledge is always round the corner in the divided sense.

¹⁹⁹ The solution to this sophism, in the end, consists in a rejection of the *casus*, despite the attempt, made for the sake of argument, to evaluate a range of alternative paths.

L 73^{va}

¹⁹⁶ In this respect, cf. *supra*, sec. 5.10, Obj.5.

¹⁹⁷ The observation hinges on the distinction between the compound and the divided sense of a sentence. The two sentences 'A significavit tibi aliquid' and 'nihil tibi significavit' are perfectly compatible, because the inference 'A significavit tibi aliquid; igitur A aliquid tibi significavit' is not sound. Therefore, to put it in the standard terms of medieval logic, the opposite (i.e. the contradictory) of the consequent is consistent with the antecedent, and they can be both asserted together. Depending on the mutual position of the verbs 'significare', 'apprehendere' and of the terms 'aliquid', 'nihil', what changes, in such kind of examples, is the supposition of the latter which is determined by the scope of the verb. If the term falls within the scope of the verb, then it has merely confused supposition, which in turn implies that the only descent allowed is nominal disjunctive (the so-called descent *de disiuncto extremo*) to the effect that reference indeterminacy is preserved. If, on the other hand, the term falls outside the scope of the verb, it has determinate supposition, which implies that only propositional disjunctive descent is allowed, i.e. a conclusion in the form of a disjunction of sentences instantiating the term in question (in the instantiating sentences the term can be replaced by demonstrative pronouns in discrete supposition).

²⁰⁹ igitur ... respondisti] om. O **211** apprehendebas] comprehendebas O **214** per] supra lin. O **214** hoc participium] istum terminum O **215** apprehendis] et significat tibi aliquid et nihil tibi significat add. O **215** sic] ita O **217** concedatur] conceditur O **217** tu] om. O **217** sis] es O **217** et] sed O **217** nescis] tu praep. O **218** esse] add. ita O **220** secunda] facta O **220** volebat admisisse] volebam admittere O **221** nisi ... petant] ubi argumenta non peterent O

5.14. Item, ponatur quod predicatum in ista 'omnis homo est animal' supponat determinate, cuiuslibet alterius termini manente significatione, preter hoc quod ly 'omnis' non confundat predicatum confuse tantum.²⁰⁰ Et significet illa propositio

In order to facilitate a better understanding of the chain of arguments presented in the following sections, I deem it useful to recall, at this point, a number of (somewhat long) excerpts taken from Peter's treatise on supposition, in particular his definitions of the various modes and the corresponding rules of ascent and descent. A preliminary disclaimer is in order: Peter accepts only personal supposition (and rejects both simple and material supposition) Here is how he defines *suppositio personalis* and its subtypes, cf. *Tractatus de suppositionibus*, [Peter of Mantua, 1492a, Aiij^{vb}-Aiij^{ra}]:

"[Definition of personal supposition] Suppositio personalis est statio termini in oratione connexi significative sumpti, pro supposito aut suppositis pro quo vel pro quibus transit vis termini in ipsum a quo habet ut supponat, et quia nulla suppositio est nisi personalis, ideo potest dici pro regula quod omnis terminus supponens supponit personaliter.

[Division of personal supposition] Suppositionum autem personalium alia communis, alia discreta.

[Definition of discrete supposition] Suppositio discreta est statio termini discreti aut termini communis cum pronomine demonstrativo discreto pro supposito vel suppositis pro quo vel pro

222 in ista] istius O 222 omnis] quilibet O 223 manente] remanente O

 $^{^{200}}$ This is the first of a sequence of sophisms concerning the application of supposition theory within the obligational context. In these examples, stipulations are relative to the modes of supposition of certain terms. A term that is usually assumed to have a given mode of supposition, in the context of a sentence, is assigned another mode in the initial statement of the *casus*.

As a result of such variations, the standard application of rules of ascensus and descensus fails or, at least, turns out to be profoundly different from what we are accustomed to. In the present sophism, for instance, it is assumed that the predicate term occurring in the sentence 'omnis homo est animal' is in determinate supposition, which is contrary to the standard practice since (cf. *infra* the first rule governing the supposition of universal signs), terms such as 'omnis' usually cause the predicate of a universal affirmative sentence to be in merely confused supposition (the subject having, in turn, suppositio confusa distributiva). A similar set of problems is also dealt with in the final part of Peter's treatise on consequences, cf. Tractatus de consequentiis [Peter of Mantua, 1492a, sig. Fiv^{va-vb}], where he takes into account the question whether "a singularibus sufficienter enumeratis ad suam universalem valet consequentia". Peter's answer to the question is negative, cf. [Peter of Mantua, 1492a, sig. Fiv^{vb}] "Et ideo ad dubium dicitur quod a singularibus sufficienter enumeratis non adducitur sua universalis de forma, nec a singularibus sufficienter enumeratis cum ' $\langle et \rangle$ sic de omnibus singulis' vel cum tali constantia 'et isti sunt omnes homines' vel 'omnia animalia', quia fallit in exclusivis, in exceptivis, in destruentibus se, in propositionibus cum terminis ad actum mentis pertinentibus precedentibus – ut 'promittitur', 'cognoscitur' etc. – et modalibus. Et non valet [et non valet] generaliter precedente termino compositionem importantem, ut sunt tales 'incipit et 'desinit'. Nec valet de forma ubi fuerit propositio universalis in qua predicatum supponat non confuse tantum". In support of the conclusion that, in some cases, the inference from all singular instances of a sentence to the corresponding universally quantified sentence is not valid, Peter provides, as usual, a number of arguments. Some of them closely recall what we find in this part of the section devoted to *impo*sitio. This approach confirms the suggestion of a connection, in Peter's Logica, between various logical doctrines and the frequent use of an obligational environment to test their admissibility (if you claim that this is the case; you will have to defend it in an *obligatio*).

ex compositione suorum terminorum. Deinde proponatur illa 'omnis homo est 225 animal'.

(Obj.1) Que si conceditur, arguitur sic: omnis homo est animal; et ista sunt omnia || animalia, demonstratis omnibus animalibus; igitur omnis homo est illud animal vel omnis homo est illud animal et sic de || aliis. Consequens est falsum. Et illa consequentia est bona, quia ly 'animal' supponit determinate, sub quo 230 licet descendere ad omnia sua supposita cum debito medio disiunctive.²⁰¹ Igitur

quibus transit in ipsum vis termini a quo habet ut supponat. Et dico cum pronomine demonstrativo discreto propter tales terminos 'taliter', 'talis', 'tantus' etc. qui non faciunt propositiones singulares. Unde illa est indefinita, demonstrato te albo, 'talis homo currit' quamvis subiciatur terminus communis cum signo demonstrativo.

[Definition of common supposition] Suppositio personalis communis est statio termini communis pro supposito vel suppositis pro quo vel pro quibus in ipsum transit vis termini a quo habet ut supponat. Suppositionum communium personalium alia confusa tantum, alia confusa et distributiva, alia determinata.

[Definition of merely confused supposition] Suppositio confusa tantum est statio termini communis pro supposito vel suppositis pro quo vel pro quibus in ipsum transit vis termini a quo habet ut supponat, sic quod sub illo virtute illius suppositionis licet descendere cum debita constantia disiunctim aut copulatim ad sua singularia, si plura supposita habet, et non semper disiunctim nec semper copulatim. Unde non sequitur 'omnis homo differt ab omni homine; et isti sunt omnes homines; ergo omnis homo differt ab illo vel ab illo etc.' Consequens enim est falsum, quia omnis homo est iste vel iste et sic de singulis, sed bene concluditur ex illo antecedente quod omnis homo differt ab illo et ab illo homine etc.

[Definition of confuse and distributed supposition] Suppositio confusa et distributiva est statio termini communis pro supposito vel pro suppositis pro quo vel pro quibus transit vis termini in ipsum a quo habet ut supponat ille terminus super quem cadit vis termini ipsum habentis distribuere. [...]

[Definition of determinate supposition] Suppositio determinata est statio termini communis pro supposito vel suppositis pro quo vel pro quibus in ipsum transit vis termini a quo habet ut supponat, sic quod sub illo termino licet descendere disiunctive ad omnia sua singularia si plura supposita habuerit – et non aliunde fuerit impeditus sicut sit in exceptivis – ut Sor est hoc". Besides definitions, it will come in useful to recall the rule that comes into play in the present sophism. Peter's first rule of suppositio reads: "Prima regula: omne signum universale affirmativum, ut ly 'quilibet', ly 'omnis' etc., non equivalens orationi, distribuit terminum sequentem cui immediate additur et eius determinationem nisi impediatur, ut 'quilibet asinus hominis currit'. Tam ly 'asinus' quam ly ''hominis' supponit confuse distributive. Sed terminos mediate sequentes ab illo termino rectos, a quo illud signum regitur, confundit confuse tantum, dummodo illi termini sint confundibiles et supponentes", cf. [Peter of Mantua, 1492a, sig. Aiii^{ra}].

For an extensive discussion of several historical and conceptual issues connected with supposition theory (along with an analysis of the rules of ascent and descent) cf. [Spade, 1996, pp. 245–303].

²⁰¹ Notice that here, in virtue of the stipulation made above, the type of descent at work is the one which is allowed in the case of determinate supposition, namely propositional disjunctive descent (if this were a standard situation one would expect the term 'animal' to be in merely confused supposition which allows only for nominal disjunctive descent, i.e. the so-called inference M 91^{va}

V 78^{rb}

²²⁷ conceditur] concedatur O 227 sic] *om.* O 231 ad ... medio] debito modo O 231 Igitur] et O

antecedens est falsum: et non pro secunda parte, igitur pro prima parte.

 $\langle \text{Obj.2} \rangle$ Vel brevius arguitur: concesso primo illam in tempore obligationis, cedat tempus obligationis. Et arguitur: in tempore concessisti illam falsam non sequentem, igitur male respondisti.

 $\langle \text{Obj.3} \rangle$ Ideo si primo negatur ista 'omnis homo est animal', arguitur sic: iste homo est animal et iste homo est animal et sic de singulis; et isti sunt omnes homines; igitur omnis homo est animal. Illa consequentia patet a singularibus ad suam universalem cum debito medio; et antecedens est verum; igitur et consequens.

de disiuncto extremo where the disjunction operator ranges over a set of terms that constitute, disjunctively, the predicate).

²³² parte] om. O 233 arguitur] sic add. O 233 illam] om. O 233 in tempore] intra tempus O 234 Et] sic add. O 234 in tempore] intra tempus O 238 patet] satis praep. O 239 suam] om. O 239 et] eius add. O

 $\langle \text{Obj.4} \rangle$ Item, homo est animal; et nihil est homo non animal; igitur omnis 240 homo est animal. Consequentia patet a copulativa exponente ad suam expositam; et antecedens est verum; igitur et consequens. || Et non est repugnans quod negas, ergo male respondes.²⁰²

А

 202 The sophism has the following structure:

- P Predicatum in ista 'omnis homo est animal' supponit determinate, cuiuslibet alterius termini manente significatione, preter hoc quod ly 'omnis' non confundit predicatum confuse tantum
- 1.1 Omnis homo est animal
- 2. Omnis homo est animal; et ista sunt omnia animalia, demonstratis omnibus animalibus; igitur omnis homo est illud animal vel omnis homo est illud animal et sic de aliis
- 1.2 Omnis homo est animal
- 3. Iste homo est animal et iste homo est animal et sic de singulis; et isti sunt omnes homines; igitur omnis homo est animal
- 4. Homo est animal et nihil est homo non animal; igitur omnis homo est animal

possible

- \mathbf{C} irrelevant and true \mathbf{C} \perp valid inference (the descent is justified on the assumption that 'animal' has determinate supposition); false consequent; by contraposition, denial of 'omnis homo est animal', which is inconsistent with 1.1 Ν
- pulativa exponente ad suam expositam'); true antecedent; the consequent is incompatibly relevant to 1.2

The first two objections, Obj.1 and Obj.2, argue against the concession of the sentence 'omnis homo est animal'. Note that the former objection is supposed to take place within the time of the obligation through the proposal of one or more sentences (it depends on whether we decide to split step 2. into several steps or not; for the sake of brevity I have decided to present it in the shorter form, although probably it would be more appropriate to propose each sentence in turn). The latter objection, by contrast, is raised outside the time, immediately after the concession of 'omnis homo est animal'. They are both meant to show that granting this sentence raises difficulties, but it is always important to keep in mind that there is a structural difference between arguments that are put forward in the form of sequences of sentences within the time of the obligation and those that come along outside the time (for example sentences like 'tu male respondes' may be well granted in the former case, but must never be granted in the latter). Let us have a quick look at the content of Peter's argument. Let h stand for the common term 'homo', a for the common term 'animal'; then let h_1, h_2, \ldots, h_n stand for individual men and a_1, a_2, \ldots, a_n for individual animals. In the standard case, the sentence 'omnis homo est animal' can be rephrased as follows:

240 omnis] non praep. O 241 patet] tenet O

179

O 124^{ra}

(1) Omnis homo est $(a_1 \lor a_2 \lor \ldots \lor a_n)$

This is a rephrasing of the sentence 'omnis homo est animal', where the term 'animal' which was in merely confused supposition in the original, is now replaced by a disjoint predicate, according to the only inference we are entitled to make, namely to a sentence of the same structure, in which the predicate term is replaced by the disjunction of singular terms that instantiate it (the logical status of a disjoint predicate of such a sort is not completely unproblematic, but I will not face the problem here). In the present sophism, however, on account of the stipulation it is not (1) the sentence that comes into play, but rather the following one:

(2) (Omnis homo est a_1) \lor (Omnis homo est a_2) $\lor \ldots \lor$ (Omnis homo est a_n)

This is how 'omnis homo est animal' should be read if 'animal' has determinate supposition: the inference allows for descent to a disjunction of sentences in which the term in question is replaced by all of its individual instances. Just to make things clearer, it may be useful to recall in what sense the situation represented in the sophism is counter-intuitive and differs from the standard assumptions. Normally, the reading of the sentence 'omnis homo est animal' would imply that the following holds:

(3) $(h_1 \text{ est animal}) \land (h_2 \text{ est animal}) \land \ldots \land (h_n \text{ est animal})$

Now, (3) is a conjunction that can be inferred on the grounds that the term 'homo' in the original sentence 'omnis homo est animal' stands in confused and distributive supposition, which allows for descent to a conjunction of sentences where h is replaced by its individual instances. What is the status of the term 'animal' in each conjunct of (3)? In this sentence, its mode of supposition changes: the predicate is no longer (as in 'omnis homo est animal') in merely confused supposition, because the term 'omnis' has been distributed onto the whole conjunction, and now 'animal' occurs as a predicate in sentences of a different logical form. In the conjuncts of (3), it is in determinate supposition. This entails, in turn, that *each* conjunct of (3) is equivalent to the following disjunction:

$$(3.i) (h_i \text{ est } a_1) \lor (h_i \text{ est } a_2) \lor \dots (h_i \text{ est } a_n)$$

where we finally have a disjunction of identities (here the copula 'est' actually stands for =) with the guarantee that one of them is true, provided that the sequence a_1, \ldots, a_n coincides with the whole set of individual animals. In the end, therefore, (3) is equivalent to:

$$(3.1) \qquad (h_1 \text{ est } a_1) \lor (h_1 \text{ est } a_2) \lor \ldots \lor (h_1 \text{ est } a_n)$$

Λ

$$(3.2) \qquad (h_2 \text{ est } a_1) \lor (h_2 \text{ est } a_2) \lor \ldots \lor (h_2 \text{ est } a_n)$$

:

$$\vdots \qquad \qquad \stackrel{\wedge}{\vdots} \\ (3.m) \qquad (h_m \text{ est } a_1) \lor (h_m \text{ est } a_2) \lor \ldots \lor (h_m \text{ est } a_n)$$

Let us now turn back to the objections. Obj.1 argues that the consequence proposed at step 2. of the disputation outlined above is valid, on the assumption that 'animal' has determinate supposition; hence one can draw a conclusion of the form of (2). But this is obviously false, because the resulting reading would be untenable (all men would be a single individual animal, no matter what instance we pick out). Therefore, if we deny this conclusion, it turns out that the antecedent, too, should be rejected by contraposition, and in particular its first part (i.e. **5.15.** Ideo dicitur in principio, quando ponitur²⁰³ illa 'omnis homo est animal', negando illam.

 $\langle \text{RObj.3} \rangle$ Et negatur ista consequentia 'iste homo est animal et iste homo est animal et sic de singulis; et isti sunt omnes homines; igitur omis homo est animal'. Nec illa regula 'a singularibus ad suam universalem valet consequentia' est universalis: et unus de casibus in quibus || fallit est quando predicatum supponit communiter non confuse.

Et si queratur que universalis sequitur ex illis singularibus cum illo debito medio, dicitur quod illa questio est multum impertinens. Dicitur tamen quod multe sunt universales que sequuntur ex illis singularibus cum illo medio, quia ex illis singularibus sequitur quod omnis homo est ens et quod omnis homo est substantia et quod omnis homo sit ipsemet.²⁰⁴

The second pair of objections, Obj.3 and Obj.4, is supposed to do the same job with respect to the denial of 'omnis homo est animal', proposed in the first place, with the aim of showing that this reply is incorrect as well as the first one. In this case, as we will see shortly, the arguments are only apparently sound, because they in fact rely on the standard rules of descent and ascent, as if no stipulation had been made. I shall discuss the point in the analysis of the solution provided in the next section.

²⁰³ Like on other occasions, the incunable makes some confusion between 'pono' and 'propono'. The correct reading here, as in O, is 'proponitur', for the sentence is put forward in the course of the disputation (and it is not the *positum*).

 204 The solution to the sophism consists in admitting the *casus* and then replying negatively to the first *propositum* 'omnis homo est animal'. As has been just recalled, two objections are raised against this reply: the first one is dealt with and dismissed here, while the second will be the object of analysis a few sections below, cf. *infra*, sec. 5.18. In that place, Peter uses the words 'ad secundam formam', but he is obviously referring to what in my reconstruction of the argument turns out to be the *fourth* objection. The explanation is easy: Peter in fact regards the arguments of Obj.1 and Obj.2 as good points against the concession of 'omnis homo est animal'. Therefore, the only two arguments he feels the need to take into account, as real objections, are the last two arguments of my reconstruction (Obj.3 and 4.), i.e. the two objections against the solution he is willing to support (denial of 'omnis homo est animal').

As far as Obj.3 is concerned, the point is that an inference, which would normally be accepted as sound, turns out to be invalid, under certain conditions such as the modification of the properties of a term. In particular, the conjunction of sentences containing all singular instances

244 ponitur] proponitur O 247 et ... homines] om. O 248 suam] om. O 248–249 valet ... universalis] om. O 249 in quibus] quando O 249 predicatum] add. universalis O 251 debito] om. O 252 dicitur] dicatur O 253 sunt] om. O 253 que] om. O 253–254 cum ... sequitur] om. O 254 quod] scilicet praep. O 254 omnis ... et] om. O

P 73^{ra}

245

255

the sentence 'omnis homo currit') since the second part is just a true factual claim about the enumeration of the individuals falling under the term 'animal'. As to Obj.2, the idea is that 'omnis homo est animal' is also false in itself, at least as long as the underlying interpretation of it depends on the assumption that the term 'animal' has determinate supposition: it is false because in fact it would mean either that every man is this animal or that every man is that animal and so on, but it is simply false that every man is, for instance, Socrates or that every man is the ass Brunellus and so forth. The first pair of objections, therefore hits the target: 'omnis homo est animal' should not be granted.

of a given common term does not entail the corresponding universally quantified sentence (i.e. the sentence containing the term 'omnis'). Peter does not dwell on it, but I think that his claim 'nec illa regula 'a singularibus ad suam universalem valet consequentia' est universalis: et unus de casibus in quibus fallit est quando predicatum supponit communiter non confuse' gives a hint to work out the right interpretation of this passage. Let us recall the array-like development of the original sentence 'omnis homo est animal' introduced above (I will label it, for the sake of brevity, (Exp)):

(3.1)	$(h_1 \text{ est } a_1) \lor (h_1 \text{ est } a_2) \lor \ldots \lor (h_1 \text{ est } a_n)$
(3.2)	$(h_2 \text{ est } a_1) \lor (h_2 \text{ est } a_2) \lor \ldots \lor (h_2 \text{ est } a_n)$
()	
:	:
	\wedge

$$(3.m) \qquad (h_m \text{ est } a_1) \lor (h_m \text{ est } a_2) \lor \ldots \lor (h_m \text{ est } a_n)$$

This a conjunction of disjunctions (each member of the disjunctions being an identity statement) which is equivalent to the sentence 'omnis homo est animal' when the subject term and the predicate term, in the latter, have their standard mode of supposition (confuse and distributive and merely confused supposition, respectively). The inference 'a singularibus ad suam universalem' fits in this structure because it consists in the reverse process: from the conjuction of all instances to the original universally quantified sentence, with the predicate in merely confused supposition. Yet, in the present case, we have stipulated that the universally quantified sentence 'omnis homo est animal' must not be read as if its predicate were in merely confused supposition, but rather as if it were in determinate supposition: therefore the conjunction of all singular instances cannot yield the sentence 'omnis homo est animal' and, at the same, time fulfill the requirement that 'animal', in that sentence, is in determinate supposition. The inference would just yield the sentence 'omnis homo est animal' with the standard modes of supposition, which is not the sentence we are supposed to be replying to in this disputation.

Moreover, there is an extremely interesting fact to note about (Exp). It can be proved that this conjunction of disjunctions does not entail the sentence 'omnis homo est animal' with the determinate supposition reading either. Let us see how the proof works. First, let us take the conjunction of all the first disjuncts of each row, then the conjunction of all the second disjuncts of each row and so forth. What we get is, basically, a sequence of conjunctions, each of which corresponds to a column of the array. The conjunction of all members of an arbitrary column iis a sentence of the following form:

(i)
$$(h_1 \text{ est } a_i) \land (h_2 \text{ est } a_i) \land \ldots \land (h_m \text{ est } a_i)$$

which in turn is equivalent to the sentence 'omnis homo est a_i '. Now take the disjunction of all *i*-columns. What we obtain is the following sentence (I will label it (Det) after determinate supposition):

(Omnis homo est a_1) \lor (Omnis homo est est a_2) $\lor \ldots \lor$ (Omnis homo est est a_n)

which is in fact equivalent to the sentence 'omnis homo est animal' with the predicate term in determinate supposition. Now, if we can prove that there is a model which verifies (Exp) but does not verify (Det), then we will have a guarantee that the former does not entail the latter. In order for (Exp) to be true, we just need a true disjunct for each row (every row will then be true, thereby guaranteeing the truth of the whole conjunction). In order for (Det) to be true, on the **5.16.** Et sicut dictum est de ista 'omnis homo est animal' || in casu illo, ita dicendum est de ista 'aliquis homo est [animal] et quilibet homo est ille' posito quod ly 'ille' suppont determinate. Unde secunda eius pars copulative est falsa et tamen quelibet singularis illius universalis est vera, quia quelibet singularis illius,

But then, what is going on here? Against the objection that the commonly accepted inference from the conjunction of all singular instances of a sentence to the corresponding universally quantified sentence is valid, we are told that this is the case only when the term 'omnis' has its usual meaning (I will take 'meaning' in a very loose sense here and in the following: strictly speaking, syncategorematic terms do not signify as categorematic terms do; we could also say that it does not have its usual properties). And this depends in turn on whether or not the mechanism of *suppositio* works in the standard way. Now, the interesting fact is, in my opinion, that this type of example is discussed within the context of sophisms on *impositio*. In the case of syncategorematic terms such as 'omnis', a modification of meaning is equivalent to redefining the behaviour of their logical properties. The ordinary interpretation of 'omnis' implies that in any sentence in which the term occurs, the predicate term has merely confused supposition. If I want to shuffle the cards and make things a little more complicated, I can therefore stipulate that the mode of supposition of the predicate term differs with respect to the ordinary usage. At this point, familiar inferences turn out to fail. If a new *impositio* is made that modifies the ordinary meaning of 'omnis', then the corresponding logical properties associated with the original meaning will be modified as well. The new *impositio* stipulates that the predicate in a universal affirmative sentence falling within the scope of 'omnis' has, contrary to the standard practice, determinate supposition. This is the variation of meaning that takes place here, and this is why Peter inserts this kind of discussions in his section on *impositio*. It is all about determining how to react when a change in the meaning of something is made: in the case of sentences or nouns that are assigned a new signification, the process normally requires that we know how to adjust the attribution of truth values according to the stipulations; in the case of syncategorematic terms, it also requires us to take into account the modifications of the logical properties (e.g. inferences that were previously valid and that now become invalid or the other way around) that the terms in question are supposed to convey to the sentences in which they occur.

257 animal] E om. OV 258 eius pars] pars illius O

B 100^{va}

other hand, we would need the truth of at least one column. But each column is a conjunction, therefore, we need that *all* members of at least one column be true at one and the same time. There is an easy way to find a model that fulfills the requirement for (Exp) but does not fulfill the requirement for (Det). It goes by diagonalization. Let us assume that the arbitrary sentence $(h_i \text{ est } a_j)$ is true if and only if i = j. Now, the truth of (Exp) is easy to ascertain: the first row is true in virtue of its first element, the second row in virtue of its second element and so on and so forth. As far as (Det) is concerned, by contrast, it will never be true in this model, beacuse the model is not sufficient to make any of its disjuncts true. Recall that (Det) is a disjunction of conjunctions, therefore in order for it to be true, the truth of at least one conjunction would be required (i.e. the truth of all singular instances (the truth of all members of an *i*-column). The model however guarantees the truth of only *one* element for each column, which is by far insufficient. Therefore, (Exp) is true and (Det) false in the model, which proves that the latter is not logically entaild by the former.

divisim capta, est vera cum prima parte et sequitur bene: quelibet singularis illius 260 divisim capta est vera, igitur quelibet singularis istius est vera.²⁰⁵

²⁰⁵ The sophism is supposed to go along the same lines of the previous one, the only difference being here the presence of a relative term (as is well known, the doctrine of relatives involves specific rules to govern the supposition of anaphoric expressions in function of the supposition of their antecedents). Peter makes a few cursory remarks that do not explain in full all the details of the parallel with the previous case. I shall just point out some issues. First, let the sentence 'aliquis homo est et quilibet homo est ille' be posited, along with the assumption that 'ille' has determinate supposition. Peter claims that the second conjunct, consisting in the universal affirmative 'quilibet homo est ille', is false although all of its singular instances are true *divisim* (i.e. if taken in turn one after another) in conjunction with 'aliquis homo est'. The universal sentence, however, cannot be true jointly with 'aliquis homo est' because the relative term 'ille' has determinate supposition. This implies that the intended interpretation of 'quilibet homo est ille' is:

(Quilibet homo est ille₁) \lor (Quilibet homo est ille₂) $\lor \ldots \lor$ (Quilibet homo est ille_n)

On this reading, however, the sentence is obviously false. What Peter claims to be true, on the other hand, are all singular sentences instantiating 'quilibet homo est ille' together with 'aliquis homo est'. What is the justification of this view? I think that Peter takes all elements of the following sequence of conjunctions:

- (1) (Aliquis homo est) \land (h_1 est ille)
- (2) (Aliquis homo est) \wedge (h_2 est ille)
- :
- (n) (Aliquis homo est) \wedge (h_n est ille)

to be true. To understand why, it is sufficient to consider the first one. In the second conjunct $(= h_1 \text{ est ille'})$, the relative term 'ille' stands in determinate supposition; therefore the sentence is further analyzable into the disjunction:

 $(h_1 \text{ est ille}_1) \lor (h_1 \text{ est ille}_2) \lor \ldots \lor (h_1 \text{ est ille}_m)$

Now, if there is a man (and there must be at least one, since the sentence 'aliquis homo est' is assumed to be true) we can be sure that one of the above disjuncts is verified. By going through the list of the m individuals that are indicated through 'ille', we will sooner or later come accross the one that we need, or in other words, we will pick out in the sequence ille₁,..., ille_m the relative that stands for the same individual that verifies 'aliquis homo est' (this is just a generalization of the example with the three singular sentences A, B, C that Peter mentions in this passage).

If that were the end of the story we might be content with this solution. There are, however, two problems. The first one is that this interpretation does not completely fit the text. I really see no way to make sense of the claim that 'quattuor vera divisim capta vel quecumque non componunt copulativam falsam significantem ex compositione illorum'. The insertion seems to be out of place in the context of this argument. Second, it is also difficult to maintain, on the one hand, the claim that all singular instances of 'quilibet homo est ille' are separately true with the sentence 'aliquis homo est' and, on the other hand, the assumption that 'ille' in the universal sentence is in determinate supposition. The point is that in the standard case, the

260–261 et ... vera] om. O

term 'ille' in each of the *i*-conjunctions laid down above (cf. 'aliquis homo est $\wedge h_i$ est ille') can refer to a different individual. But even if, at each row, 'ille' picks out a different individual, this is perfectly sufficient to make both members of any *i*-conjunction true. The casus of this sophism, by contrast, implies that the reference of 'ille' is one and the same individual for all *i*-conjunctions at one and the same time. I suspect that if we were to take the assumption at face value, therefore, we would no longer be entitled to claim that the singular instances of 'quilibet homo est ille' are all true divisim with the conjunct 'aliquis homo est'. The difficulty rests on that, presumably, what Peter passes off as singular instances of the sentence 'quilibet homo est ille', in fact are not its singular instances. The reason is that the term 'ille' is assumed to occur in determinate supposition, which implies in turn that we can first descend to a sentence of the following form:

(Quilibet homo est ille₁) \lor (Quilibet homo est ille₂) $\lor \ldots \lor$ (Quilibet homo est ille_m)

and secondly to the following *singulares* (the procedure holds, again, for each disjunct):

 $(h_1 \text{ est ille}_i) \land (h_2 \text{ est ille}_i) \land \ldots \land (h_n \text{ est ille}_i)$

Now, these are the singular instances of the sentence 'quilibet homo est ille' which is convertible with their conjunction for an appropriately chosen i. If we turn back to the argument, though, a problem immediately arises. The possibility for all sentences of the form ' h_j est ille' to be separately true in conjunction with 'aliquis homo est' depended upon the fact that the reference of 'ille' was not fixed once and for all and that the demonstrative could pick out a *different* individual *at each step*. And this in turn depended on the fact that, at each step j, the occurrence of 'ille' in sentences of the form ' h_j est ille' did not refer to an individual fixed once and for all, but could still be analyzed by means of a disjunction of sentences. Yet, on the assumption of the *casus*, the situation is rather like this:

- (1) (Aliquis homo est) \wedge (h_1 est ille_i)
- (2) (Aliquis homo est) \land (h_2 est ille_i)

:

(n) (Aliquis homo est) \wedge (h_n est ille_i)

I venture that, on this reading, there is at most one sentence of the form $(h_j \text{ est ille}_i)$, for a fixed i, which is true; but the claim that all singular instances are true together with 'aliquis homo est' is plainly untenable. What are we to do then? I am inclined to think that Peter is aware of the problem and therefore requires us to assume that the singular instances of 'quilibet homo est ille' remain the same as if no change of the logical properties of the terms involved in the original sentence had been made. I am tempted to regard the concluding remark of the argument as a piece of evidence in support of this claim, cf. 'pono enim quod ita significant ille singulares, dum sunt conjuncte cum illa particulari [scil. 'aliquis homo est'], sicut significabant: aliter enim non essent vel ante non fuissent singulares illius [scil. of the universal sentence 'quilibet homo est ille']'.

Finally, it must be noted, that an elaborate discussion of these kind of sentences is also found, as one might expect, in Peter's treatise on relatives, cf. *Tractatus de relativis*, [Peter of Mantua, 1492a, sig. Av^{va}]. Several sentences that are discussed here, including the present sophism, are analyzed in that context too; nontheless the text on relatives is not of much help because the discussion is quite intricate in that place as well.

|| Item si quelibet singularis il-||lius divisim capta est vera, habeat igitur illa universalis 'quilibet homo est ille' supposita tria et tres singulares, puta A, B, C. Tunc A est vera cum || prima parte divisim; et B vera cum prima parte divisim; et etiam C est vera cum prima parte divisim. Sed quattuor vera divisim capta 265 vel quecumque non componunt copulativam falsam significantem ex compositione illorum, quia verum numquam repugnat vero. Igitur illa copulativa facta ex illis singularibus cum prima parte erit vera. Pono nam quod ita significent ille singulares, dum sunt coniuncte cum illa particulari, sicut significabant: aliter nam iam non essent vel ante non fuissent singulares illius.

186

270

M 91^{vb} E Giiii+3^{rb} L $73^{\rm vb}$

²⁶² Item ... vera] om. O 263 universalis] om. O 264 divisim] om. O 264 vera] est praep. O 264 divisim] om. O 265–266 et ... quecumque] sequitur igitur, cum verum non repugnat vero, quod ille cum prima parte O **267** illorum] suorum et terminorum add. O 267 quia ... vero] transp. O 267–268 Igitur ... vera] om. O 268 nam] igitur O 268 quod] om. O 269 dum] ille add. O 269 significabant] significabat et ex. add. O **269–270** aliter ... illius] om. O

5.17. Et sicut dictum est de illa copulativa 'aliquis homo est et quilibet homo est ille', || ita dicendum est de illa 'aliquid est et nihil est illud' dato quod ly 'illud' supponat determinate. Et tunc sequitur quod illa est falsa pro secunda parte; neque secunda pars sequitur ex singularibus, quarum quelibet est vera cum prima parte.²⁰⁶

5.18. $\langle \text{RObj.4} \rangle$ Sed ad secundam formam²⁰⁷ dicitur negando illam consequentiam 'homo est animal; et nihil est homo non animal; igitur omnis homo est animal'. Et negatur quod illa sit copulativa exponens illam.²⁰⁸ Et si queratur que sit copulativa exponens illam, dicitur quod illa potest poni²⁰⁹ per hunc modum: homo est animal; et nihil est animal quin illud sit omnis homo; igitur omnis homo est animal.²¹⁰

Et si iterum arguitur quod ista vocalis sit vera 'omnis homo est animal', quia mentalis cui illa subordinatur est vera, igitur ista est vera, quia ista mentalis est

 207 Cf. supra 5.14, reply to Obj.3.

 208 I.e. it is denied that the conjunction 'homo est animal et nihil est homo non animal' is a correct *expositio* of the sentence 'omnis homo est animal', under the assumption that the predicate 'animal', occurring in the latter, is in determinate supposition.

²⁰⁹ Presumably, the correct reading is 'exponi', as is confirmed by O.

 210 This is a reply to the fourth objection to the sophism discussed previously, cf. *supra*, sec. 5.14. The objection, that we had left aside for a moment, runs as follows: there is a sound inference from the conjunction of a pair of sentences that are the *exponentes* of 'omnis homo est animal'; and the two sentences are true, therefore 'omnis homo est animal' is true as well; but the respondent has denied it, therefore he has replied incorrectly (cf. the remark at the end of sec. 5.14, Obj.4 'et non est repugnans [*scil.* the consequent 'omnis homo est animal'] quod negas, ergo male respondes').

Once more, the fulcrum of the argument, in the reply, is the non-admissibility of a standard inference, as a result of the new stipulations of meaning and logical properties of a term. Here the point is the relation that a sentence bears to its *exponentes* (i.e. logically equivalent sentences where, for instance, a given categorematic term, occurring in the original sentence, is explained away). The inference from a sentence p to another sentence q, that in a standard situation would count as equivalent to the former, fails under specific assumptions such as the variations of the familiar modes of supposition of the terms involved are. In the present case, for instance, the *expositio* proposed in Obj.4 is incorrect, because it rests on the hypothesis that everything has its usual logical properties. But in virtue of the new *impositio*, the correct *expositio* presupposes a different reading, such as the one provided in this reply.

273 Et... sequitur] om. O 274 singularibus] suis praep.O 274–275 quarum ... parte] om. O 276 Sed... formam] om. O 276 illam] om. O 279 poni] exponi O 280 omnis] BEOP om. V 280 igitur ... animal] om. O 282 illa] om. O 282 mentalis] in mente O

275

V 78^{va}

²⁰⁶ This is a variation on the same theme of the last two sophisms, in particular of the immediately preceding one. Again, it is the universal sentence (in this case 'nihil est illud') that is false, on the assumption that the relative term has determinate supposition. And, again, it cannot be claimed that the sentence can be proved to be true because all of its singular instantiating sentences are true, for the universal sentence in which the relative term features in determinate supposition, simply does not follow from the conjunction of all of its singular instances (presumably, the term would have to be in merely confused supposition in order for the inference to be sound).

vera 'omnis homo est animal' cui subordinatur ista, dicitur quod illa vocalis non subordinatur illi sed isti 'animal omnis homo est', || intelligendo tamen ly 'animal' a parte predicati et ly 'homo' a parte subiecti.²¹¹ 285

5.19. Item, ponatur quod predicatum illius 'nullus homo est animal' stet determinate, cuiuslibet alterius termini significatione remanente et illa propositione significante ex compositione suorum terminorum. || Deinde proponitur 'nullus homo est animal'.

Si negas, negas verum non repugnans, igitur male respondes. Hec consequentia 290 patet. Et arguitur antecedens, quia bene sequitur 'nullus homo est illud animal vel nullus homo est illud animal; et illa sunt omnia animalia; igitur nullus homo est animal'. Consequentia patet a descensu termini ad eius ascensum.²¹²

Item, illa significat omnino sicut illa 'animal nullus homo est'; sed hec est vera; igitur et illa.²¹³

Ideo forte conceditur illa 'nullus homo est animal'.

(Obj.1) Sed contra: aliquis homo est animal, igitur non nullus homo est animal. Tenet consequentia ab uno equipollenti ad aliud, quia ly 'aliquis' et 'non nullus' equivalent.²¹⁴

(Obj.2) Item, contraria illius est vera, scilicet 'quilibet homo est animal'; igitur 300 illa non est vera 'nullus homo est animal. Vel seguitur quod duo contraria sunt vera.²¹⁵

 213 Again, assuming that the predicate term has determinate supposition implies that the original sentence can be replaced by a disjunction of universal negative sentences having the same subject (i.e. 'homo') and such that in each of them the original predicate term is replaced by an individual instance. This in turn implies that for the whole disjunction to be true, there need to be one true disjunct. But each disjunct has the form 'nullus homo est a_i '. This sentence is true if and only if there is one individual animal such that no man is that animal, which is expressed by Peter by reversing the order of words to produce the sentence 'animal nullus homo est'.

188

 $O \ 124^{rb}$

 $P 73^{rb}$

M 92^{ra}

 $^{^{211}}$ A similar use of the term 'subordinatur', in connection with mental propositions in the context of *impositio*, is found in [Brinkley, 1995, p. 34].

²¹² The inference relies on the assumption, contrary to the standard doctrine, that in a sentence such as 'nullus homo est animal' the predicate term has determinate supposition instead of confused and distributive supposition.

²¹⁴ The objection relies on a standard inference based on the interchangeability of 'non nullus' and 'aliquis'. Cf. Tractatus de equipollentiis, [Peter of Mantua, 1492a, sig. Eiv^{vb}].

²¹⁵ This objection and the following one are based on the square of opposition.

²⁸³ ista] in voce 'omnis homo est animal' add. O **283** vocalis] om. O **284** sed isti] om. O 284 intelligendo] intelligo O 284 ly 'animal'] om. O 287 termini] om. O 288 proponitur] proponatur O 293 patet | tenet O 295 et] om. O 298 aliud | reliquum O 299 equivalent | equipollent **301** sequitur quod] aliter O **301** sunt] simul add. O

 $\langle \text{Obj.3} \rangle$ Item, si nullus homo est animal, aliquis homo non est animal. Tenet consequentia ab universali ad particularem. Et ex alia parte quilibet homo est animal. Igitur contradictio.

Item, si aliquis homo non est animal, igitur non quilibet homo est animal. Tenet consequentia ab equipollenti ad aliud.²¹⁶

 216 This is the structure of the sophism:

Ι	Predicatum illius 'nullus homo est animal'	А	possible
	stet determinate, cuiuslibet alterius termi-		
	ni significatione remanente et illa proposi-		
	tione significante ex compositione suorum		
	terminorum		
1.1	Nullus homo est animal	Ν	
2.	Negas verum non repugnans, igitur male	\mathbf{C}	\perp valid inference with true
	respondes		antecedent
3.	Illa significat omnino sicut illa 'animal nul-	\mathbf{C}	\perp valid inference with true
	lus homo est'; sed hec est vera; igitur et		antecedent
	illa		
1.2	Nullus homo est animal	\mathbf{C}	
4.	Aliquis homo est animal; igitur non nullus	\mathbf{C}	\perp valid inference, whose con-
	homo est animal		sequent is inconsistent with
			1.2 (cf. Obj.1)
5.	Contraria illius est vera, scilicet 'quilibet	\mathbf{C}	\perp valid inference, whose con-
	homo est animal'; igitur illa non est vera		sequent is inconsistent with
	'nullus homo est animal'		1.2 (cf. Obj.2)
6.	Si nullus homo est animal, aliquis homo non	\mathbf{C}	\perp inconsistent with 'quilibet
	est animal		homo est animal (cf. Obj.3)

The sophism consists of two groups of arguments focusing on the denial and concession of the first *propositum* 'nullus homo est animal', put forward in the first place. The first two arguments are raised against its denial and are regarded by Peter as sound. The sentence therefore is presumably to be conceded. In this connection, three objections are raised (cf. Obj.1–3). In the next section Peter dismisses them all and provides his solution to the whole argument.

The interesting feature of this sophism and of the one presented in sec. 5.21 is that the kind of situations they represent is complementary with respect to those analyzed in the sophisms of sec. 5.14–5.16. In those cases, the sentences that were affected by a variation of the standard modes of supposition were universal affirmative (merely confused supposition of the predicate term was substituted by determinate supposition). Here, on the other hand, Peter takes into account two examples of universal negative sentences, whose predicate normally is in confused and distributive supposition, and again replaces the standard mode with determinate supposition (cf. *infra*, sec. 5.20 RObj.1 'vis istius termini 'nullus' est distribuere terminum communem supponentem rectum ab eodem verbo sequente in eadem cathegorica; sed non est sic in illo casu'). The correct reading of 'nullus homo est animal' therefore is no longer:

(Nullus homo est a_1) \wedge (Nullus homo est a_2) $\wedge \ldots \wedge$ (Nullus homo est a_n)

but rather becomes,

303 aliquis] igitur *praep.* O **304** consequentia] illa *add.* O **304** ad particularem] et cetera O **307** ab] uno *add.* O **307** aliud] reliquum O

5.20. Sed pro isto dicitur admisso casu concedendo illam 'nullus homo est animal'.²¹⁷

 $\langle \text{RObj.1} \rangle$ Et cum arguitur quod aliquis homo est animal, igitur non nullus homo 310 est animal, dicitur negando consequentiam. Et cum arguitur quod illa est bona quia est ab || uno equipollenti ad reliquum, dicitur quod ly 'aliquis' et 'non nullus' non equipollent nisi utriusque termini remaneat vis sua; sed non remanet utriusque termini vis sua, quia vis istius || termini 'nullus' est distribuere terminum communem supponentem rectum ab eodem verbo sequente in eadem cathegorica;²¹⁸ sed 315 non est sic in illo casu; igitur etc.||

 $\langle \text{RObj.2} \rangle$ Et per hoc etiam dicitur quod iste non contradicunt 'aliquis homo est animal' et 'nullus homo est animal' dato illo casu, sed ista || 'nullus homo est animal', 'non nullus homo est animal'. Nec iste sunt contrarie: 'nullus homo est animal', 'quilibet homo est animal'; sed iste: 'nullus homo est animal' et 'quilibet 320 homo omne animal est'.²¹⁹

(RObj.3)Et negatur illa consequentia 'nullus homo est animal, igitur aliquis homo non est animal', sed sequitur solum ex illa, data illius termini significatione, quod aliquis homo aliquod animal non est.

(Nullus homo est a_1) \vee (Nullus homo est a_2) $\vee \ldots \vee$ (Nullus homo est a_n)

 217 The solution to the sophism is admitting the *casus* and granting the first *propositum*. All subsequent objections can be dismissed. Once again, the general strategy is showing how the change of modes of supposition invalidates several familiar inferences on which the objections are based.

²¹⁸ Cf. Tractatus de suppositionibus, [Peter of Mantua, 1492a, sig. Aiii^{ra-rb}].

²¹⁹ The last remark of this reply is particularly interesting, for the relation of contrariety must be adjusted according to the assumption that 'animal' has determinate supposition in the universal negative sentence 'nullus homo est animal'. The latter can be represented as follows (where H is stands for 'homo' and, as usual, a_1, \ldots, a_n stand for individual animals):

(1) $\forall x(Hx \Rightarrow (\neg(x = a_1) \lor \neg(x = a_2) \lor \ldots \lor \neg(x = a_n)))$

Now the contrary of (1) is the following sentence:

(2) $\forall x(Hx \Rightarrow \neg(\neg(x = a_1) \lor \neg(x = a_2) \lor \ldots \lor \neg(x = a_n)))$

which is equivalent, by De Morgan's law, to the sentence

(3) $\forall x(Hx \Rightarrow ((x = a_1) \land (x = a_2) \land \ldots \land (x = a_n)))$

Now, (3) is a close candidate to capture Peter's suggestion, namely that the contrary of the sentence 'nullus homo est animal' (on the determinate supposition reading) is the sentence 'quilibet homo omne animal est'.

190

 $L 74^{ra}$

B $100^{\rm vb}$

V $78^{\rm vb}$

E Giiii+3^{va}

³¹⁰ quod] om. O **311** illa] add. consequentia O **312** quia est] om. O **312** et] ly add. O **313** non] om. O **313** nisi] ubi O **313** sua] om. O **314** sua] om. O **316** igitur] ideo O **317** quod... non] negando quod iste O **321** est] om. O **323** ex... significatione] om. O

5.21. Et sicut dictum est de istis, ita dicendum est de istis relativis, si ponatur 325 quod in secunda parte illius 'aliquod animal est et nullus homo est id' ly 'id' supponat communiter non distributive, quod ille non contradicunt || 'aliquis homo est id' || et 'nullus homo est id', cum utraque sit vera, dato quod in utraque supponat determinate: quia illa significatione data, ille convertuntur 'aliquid est et nullus homo est illud' et 'aliquid est et id nullus homo est'.²²⁰ 330

Unde credendum est quod non potest bene contradictorium unius propositionis cathegorice signari nisi terminus in uno eorum communiter et disiunctive vel disiunctim supponens in alio copulative supponat. Neque valet consequentia illa, retenta illa significatione et relatione illius termini 'id': 'aliquis homo est illud seu id, igitur non nullus homo est id'. Sed de hoc amplius dictum est.²²¹

5.22. Item, ponatur quod subiectum in ista 'aliquis homo est || quilibet homo' supponat confuse tantum, manente cuiuslibet alterius termini significatione et significante illa propositione ex compositione suorum terminorum etc. Deinde proponitur illa 'aliquis homo est quilibet homo'. Que si negatur, arguitur sic: quilibet homo aliquis homo est, igitur aliquis homo quilibet homo est.²²² Consequentia pa-340 tet, quia ille convertuntur, cum omnes termini earum convertantur et pro eisdem

325 est] om. O **325** istis] ista O **325** est] om. O **325** istis] terminis O **325** si ponatur] supponatur O **326** id] illud O **326** id] ille **327** quod] et O **328** id] illud **328** id] illud **328** cum ... vera] ante unde credendum est transp. O **328** in utraque] utriusque predicatum O **330** id] aliquid O **331** unius] om. O **332** signari] significari O **332–333** eorum ... copulative] distributus et copulative O **333** valet consequentia illa] om. O **334** retenta] retentis transp. O **334** et] illa add. O **334** illius termini 'id'] om. O **334–335** seu id] om. O **335** id] illud **335** Sed] om. O **336** in ista] istius O **336** quilibet homo] EMOP est ille add. BV **337** manente] remanente O **337** termini] om. O **338** illa] adequate add. O **338** propositione] om. O **339** homo est] EMOP et B **339** quilibet homo] EMOP est ille add. B **339** negatur] negetur O **339–340** quilibet ... est] EV aliquis et quilibet homo est ille B; aliquis homo quilibet homo est, ergo aliquis homo est quilibet homo M; quilibet homo est aliquis homo et aliquis homo est quilibet homo O **340–341** pate1] tenet **341** ille] propositiones add. BMO **341** cum] cuius O **341** earum] eorum O **341** convertantur]

M 92^{rb} O 124^{va}

P 73^{va}

....

²²⁰ The argument is quite analogous to the line presented in the context of the previous sophism: modifying the properties of a syncategorematic term affects the mutual logical relationships between sentences whose status is usually taken for granted.

 $^{^{221}}$ I have not found a corresponding *locus* in the treatise.

 $^{^{222}}$ Since the whole argument hinges on the mutual relations between the two terms, and such relations depend on their positions relative to each other, it is reasonable to rephrase the consequent to keep the structure of the sentence invariant throughout the sophism (I will disregard minor variations that are found in the manuscripts, since the structure of the argument is rigid and requires us to make the most plausible choice from the logical standpoint). The most appropriate ordering of terms presumably is 'aliquis homo est quilibet homo': this is the sentence whose concession is inconsistent with the *propositum* that has been previously denied.

et eodem modo supponant. Ideo forte dicitur concedendo illam.

 $\langle \text{Obj.1} \rangle$ Sed contra: non aliquis homo est quilibet homo, quia nec ille homo est quilibet homo nec ille et sic de singulis; et isti sunt omnes homines; igitur non aliquis homo est quilibet homo, quod est oppositum concessis.²²³ Consequentia 345 patet.

 $\langle \text{Obj.2} \rangle$ Item, si aliquis homo est quilibet homo, igitur aliquod animal est quilibet homo. Consequentia patet: contradictorium consequentis repugnat antecedenti. Ista enim repugnant 'aliquis homo est quilibet homo' et 'nullum animal est quilibet homo'.

 $\langle \text{Obj.3} \rangle$ Item, si aliquis || homo est quilibet homo et isti sunt omnes homines, igitur ille vel ille et sic de aliis est quilibet homo. Consequentia patet, quia ille terminus 'homo' supponit confuse tantum in illa 'aliquis homo est quilibet homo' per positum. Et sequitur: iste vel ille homo et sic de aliis est quilibet homo, igitur istud vel illud animal et sic de aliis est quilibet homo. Quo dato, sequitur 355 quod aliquod animal est quilibet homo tamquam ab inferiori ad suum superius. Consequens falsum, || ut prius.

 $\begin{array}{l} {\rm L} ~ 74^{\rm rb} \\ {\rm M} ~ 92^{\rm va} \end{array}$

V 79^{ra}

 $^{^{223}}$ The right reading is 'concessi', as in O, because the sentence is the contradictory of the first *propositum*, which in this part of the argument is supposed to be granted.

³⁴² supponant] supponunt O; Item quilibet homo est iste homo vel iste homo, demonstrando omnes homines, ergo iste homo vel iste homo et sic de aliis est quilibet homo. Oportet illa consequentia quia concessa omnino supponant termini antecedentis sicut termini consequentis cum partes illius disiunctive supponant discrete manente sic in consequente cum illud disiunctim non supponat sed parte illius disiunctim supponant add. BV **343** quilibet] aliquis O **343** quia] om. O **343** homo] om. O **344** nec ille] om. O **344** nec ille] E add. est quilibet homo M **344** singulis] aliis O **344** isti] homines add. O **345** oppositum] contradictorium MO **345** concessis] concessi MO **345**-346 Consequentia patet] om. MO **348** Consequentia] illa add. O **348** patet] tenet O **348** contradictorium] quia praep. O **351** homo] igitur iste vel iste est quilibet homo add. O **352** aliis] singulis O **352** patet] tenet O **353** aliquis] om. O **355** istud] illud et animal add. O **357** Consequens] est add. O

5.23. Ideo dicitur, in principio, admittendo casum. Et nego illam propositionem 'aliquis homo est quilibet homo'.

 $\langle \text{RObj.1} \rangle$ Et negatur ista consequentia 'quilibet homo aliquis homo est, igitur 360 aliquis homo est quilibet homo'. Unde licet in ista 'aliquis homo est quilibet homo' ly 'homo' supponat confuse tantum, tamen quia ex ipsa sequitur falsum, ut quod homo est omnis homo, ideo est neganda. Ita etiam hec est neganda || 'semper O 124^{vb} homo omnis homo est', quia ex illa sequitur quod aliquando homo omnis homo est, et hec est concedenda 'semper omnis homo || homo est', quamvis termini pro 365 B 101^{ra} eisdem et eodem modo supponant et termini unius correspondentis cum terminis alterius convertantur.²²⁴

 224 The sophism has the following structure:

- *I* Subjectum in ista 'aliquis homo est quilibet homo' supponit confuse tantum, manente cuiuslibet alterius termini significatione et significante illa propositione ex compositione suorum terminorum
- 1.1 Aliquis homo est quilibet homo
- 2.1 Quilibet homo aliquis homo est; igitur aliquis homo est quilibet homo
- 3. Non aliquis homo est quilibet homo, quia nec ille homo est quilibet homo nec ille et sic de singulis; et isti sunt omnes homines; igitur non aliquis homo est quilibet homo
- 4. Si aliquis homo est quilibet homo, igitur aliquod animal est quilibet homo
- 5. Si aliquis homo est quilibet homo et isti sunt omnes homines, igitur ille vel ille et sic de aliis est quilibet homo
- 6. Iste vel ille homo et sic de aliis est quilibet homo; igitur aliquod animal est quilibet homo

A possible

Ν

С

valid inference

- C \perp valid inference; the consequent is inconsistent with 1.1
- C \perp (cf. Obj.2)

The three objections focus on step 3–4 and 5–6 of the disputation, respectively. Either reply (concession or denial) to the first *propositum* 'aliquis homo est quilibet homo' is apparently ruled out. The solution to the sophism consists in the admission of the *casus* and the denial of the first *propositum* 'aliquis homo est quilibet homo' put forward in the first place. There are some problems, however. First, it seems that at some point of the argument some confusion arises as to what is supposed to be the subject and what is supposed to be the predicate of the sentence

358 nego] negatur O **358** illam] illa O **358** propositionem] *om.* O **359** aliquis ... homo] aliquis est omnis homo O **360** aliquis ... est] est aliquis homo O **361** est ... homo] omnis homo est O **361** quilibet] omnis O **362** supponat] stet O **362** falsum, ut] *om.* O **363** ideo ... neganda] *om.*O **363** neganda] quod *add.* O **363** semper] *om.* O **365** et ... est'] *om.* O **366** supponant] supponunt O **366** termini] terminis O **366** correspondentis] *om.* O **366** terminis] contento *sic* O Et per hoc patet A et B propositiones esse similis quantitatis et qualitatis, et subiecta et predicata et copulas converti, et pro eisdem et eodem modo supponere, et significata esse convertibilia illarum propositionum significantium ex compositione 370 illorum terminorum adequate, et unam esse necessariam et aliam impossibilem. Ultra enim illa omnia, requiritur omnino similis ordo terminorum.

(RObj.3) Ad aliam etiam formam dicitur negando, sicut non sequitur 'promitto tibi istum vel istum denarium, igitur istum vel istum denarium tibi promitto'.²²⁵

²²⁵ This remark should seemingly count as a reply to the second objection. Cf. *supra*, sec. 5.22. The incorrect inference presented here, however, is not the one upon which the argument that Peter wants to reject is based. In Obj.3 it is rather the converse inference which comes into play, i.e. from the conjunction of singular terms (cf. above 'istud vel illud animal') to the corresponding universally quantified affirmative sentence. It is unclear therefore, how the counter-objection of this paragraph is supposed to hit the target.

368 patet] *inc.* add. O 369 supponere] supponunt O 370 significata] *inc.* O 370 significantium] *transp.* O 371 illorum] suorum O 371 aliam] alteram O 372 illa omnia] *om.* O 374 denarium] *om.* O 374 denarium] *om.* O

in question. The reply is based on the denial of the consequence 'quilibet homo aliquis homo est, igitur aliquis homo est quilibet homo' because the consequent is false and we want to reject it. But why is the consequent false? It seems that Peter's claim is that it would imply that there is at least one man such that every man is that man. In this case, however, 'aliquis homo' would have become the predicate and 'quilibet homo' the subject of the sentence. Moreover, it is unclear, how Peter can claim that the respondent is allowed to make such a move. The antecedent of this consequence is reasonably true, since it just claims that every man is a man (whereby no individual is specified). The consequent, as we have seen, is false, therefore there must be something wrong with the inference, and this is what Peter seems to have in mind when he says 'et negatur ista consequentia'. The problem is that this is apparently incompatible with the assumption that, in 'aliquis homo est quilibet homo', the first occurrence of 'homo' has merely confused supposition, because from this assumption it just follows that there is a disjoint term $h_1 \vee h_2, \vee \ldots \vee h_n$ such that the expression 'every man (= 'quilibet homo')' collectively refers to what is meant by that disjoint term (it would simply be a statement about the identity of two classes). But Peter seems to draw the conclusion that from this we can infer that there is one individual man such that every man is that one.

The same holds for the other example which is based on the confusion that may arise between the properties and the 'roles' (as subject or predicate) of the terms involved. The sentence 'semper homo omnis homo est' should be denied, because it implies the sentence 'aliquando homo omnis homo est' which is false if it means that sometimes there is *one* man such that every man is *that* man. By the same token, however, the sentence 'semper homo omnis homo est' would be also eligible for denial, since it all depends on how we interpret the first occurrence of 'homo' and in what kind of supposition. If the interpretation is the same for both sentences (and I see no reason why it should apply in the second case but not in the first), then Peter's argument does not make perfect sense to me. Moreover, he claims that the sentence 'semper omnis homo homo est' should be granted. This holds, I assume, if the sentence means that it is always the case that every given man is *a* man (again, no individual being specified thereby). In sum, therefore, some obscurity remains for the reader to pore over.

5.24. Item, ponatur quod iste convertuntur 'deus est', 'nullus deus est' una illarum 375 significante primarie sicut solet significare prima-||rie, puta quod deus est vel quod nullus deus est.²²⁶ Deinde proponitur 'deus est'. Qua concessa, quia vera non repugnans, proponitur 'nullus deus est'.

 $\langle Obj.1 \rangle$ Si conceditur, contra: illa est || impertinens que extra tempus negaretur, igitur et nunc est neganda. Patet consequentia.

 $\langle i \rangle$ Et arguitur antecedens quia non sequitur 'illa convertuntur 'deus est' et 'nullus deus est'; sed deus est; igitur nullus deus est': quia extra tempus illa consequentia non erat bona, quia antecedens fuisset possibile et consequents impossibile; et casus non facit [ad] consequentiam; igitur et nunc non est bona. Ideo forte dicitur quod illa consequentia non est bona.

 $\langle Ad \ i \rangle$ Contra: consequents sequitur ex secunda parte antecedentis, igitur illa consequentia est bona. Patet consequentia. Et antecedens arguitur, quia ille convertuntur 'deus est' et 'nullus deus est'.²²⁷

 $\langle \text{Obj.2} \rangle$ Item, ex tibi dubio illa duo|| non convertuntur, igitur illa non est a te concedenda 'nullus deus est'. Patet consequentia. Et arguitur antecedens, quia 390 ex tibi dubio tu admisisti impossibile, quia forte demonstrantur iste due mentales 'deus est', 'nullus deus est', que invicem converti non possunt.²²⁸

 227 The passage needs some clarification. Take the consequence 'iste convertuntur 'deus est' et 'nullus deus est'; sed deus est (= second part of the antecedent); igitur nullus deus est (= consequent)'. It is argued that the following holds: *if* the consequent follows from the second part of the antecedent, *then* the consequence is sound. But in this case the consequent does follow from the second part of the antecedent, since 'deus est' entails 'nullus deus est' on the assumption that the two sentences mutually convert.

 228 It is unclear (as in other cases, in the treatise, where the reconstruction of the disputations becomes tentative) whether this objection is supposed to be raised *infra* or *extra tempus*. I have opted for the second alternative, since in the reply supplied in the next section there is no unquestionable proof of the contrary (one could argue, though, that the claim 'debet respondes certificari que propositiones demonstrentur per ly 'iste' actually *is* a proof of this sort, to the effect that the objection must be faced *infra tempus*. I will leave the issue open). Still, it must be always kept in mind that it is very hard to establish beyond any reasonable doubt, in Peter's text, whether the occurrence of a sentence, or even of an argument, is included within the disputation's time boundaries or if it takes place beyond them. Caution is in order in such

375 convertuntur] convertantur O **375** una] illa *add.* O **376** primarie] *om.* O **376** puta] scilicet O **377** proponitur] proponatur O **378** proponitur] proponatur O **379** que] quam O **379** negaretur] negares O **380** igitur et] *om.* O **381** quia] iam *add.* O **381** illa] ille O **383** erat] fuisset O **384** et] sed O **384** ad] *om.* O **389** illa ... non] nulla duo O **391** demonstrantur] demonstrabantur O **392** invicem] ad invicem O

P 73^{vb}

E Giiii+3^{vb}

380

385

M 92^{vb}

 $^{^{226}}$ In the development of the argument it turns out that Peter assumes that it is the first one to be proposed, namely 'deus est' with its original meaning (cf. 'sicut solet significare primarie'), that in the end affects the meaning of the other, as if one were to say that if William of Ockham and Peter of Mantua were fellow countrymen, then Ockham would be Italian on the assumption that (i.e. if we have already granted that) Peter is Italian. In support of this interpretation, cf. infra sec. 5.25, where the relevance of the relative order, in which sentences of such a sort are put forward, is explicitly regarded as the reason that justifies two different pairs of replies.

 $\langle \text{Obj.3} \rangle$ Item, una illarum propositionum est affirmativa et alia negativa, igitur ille non convertuntur. || Patet consequentia et maior. Sed minor arguitur quia illa 'nullus deus est' est una propositio cuius verbum principale negatur, igitur illa est 395 negativa.²²⁹

assessments because, as I have recalled, there is a fundamental conceptual difference, in the theory of obligations, between what the respondent says (and how he replies to arguments) within the time, and what he says oustide the time. The weight of an objection and of the corresponding reply may result in quite different outcomes, according to the context in which they occur: just recall the profoundly different impact of granting *infra tempus* that one has replied incorrectly (allowed move), and granting the same *extra tempus* (prohibited move which would entail defeat for the respondent).

Generally speaking, there are at least two rough criteria that may help us find a way through the problem. First, the occurrence of the explicit declaration 'cedat tempus', which goes the whole hog. This is a decisive piece of evidence, but we cannot infer that whenever the declaration is lacking, everything ends up to be taking place in the *infra tempus*-environment: this is simply contrary to the factual practice, since sometimes Peter seems to be reasoning (by raising objections or setting up arguments) outside the time, despite the omission of the clause 'cedat tempus'. A second type of evidence may be a clue to the definition of time boundaries, namely the presence – normally to be looked for in those textual passages that contain instructions for the respondent's replies to the objections – of technical phrases such as 'concedatur', 'negetur' and the like, that may refer to sentences that can be identified as *proposita*. In these situations we can be reasonably confident that the replies refer to a step of the argument that takes place *infra tempus*.

 229 Here is the structure of the disputation:

- *I* Iste convertuntur 'deus est', 'nullus deus est', una illarum significante primarie sicut solet significare primarie, puta quod deus est vel quod nullus deus est
- 1. Deus est
- 2. Nullus deus est
- 3. Illa est impertinens que extra tempus negaretur; igitur et nunc est neganda
- 4. Non sequitur 'ille convertuntur 'deus est' et 'nullus deus est'; sed deus est; igitur nullus deus est'
- 5. Una illarum propositionum est affirmativa et alia negativa; igitur ille non convertuntur

- A possible
- C irrelevant and true
- $\begin{array}{cc} C & \perp \\ C & valid \mbox{ inference} \end{array}$
- C irrelevant and true (cf. 'extra tempus illa consequentia non erat bona, quia antece-
- non erat bona, quia antecedens fuisset possibile et consequens impossibile; et casus non facit consequentiam; igitur et nunc non est bona') C \perp

The arguments are against the concession of the sentence 'nullus deus est'. The first objection hinges on the claim that 'nullus deus est' is irrelevant and outside the time it should be denied (according to the standard account that Peter rejects), therefore it should be denied within the time as well, since no reply should be modified on account of the fact that we now are under an obligation. From a general point of view, if one claims that the sentence should be granted, it is

196

V $79^{\rm rb}$

5.25. Ideo dicitur in principio admittendo casum. Et conceditur illa 'deus est'. Deinde conceditur illa 'nullus deus est'. Et si variaretur ordo proponendi, negaretur utraque.

 $\langle \text{RObj.1} \rangle$ Et ad formam, cum arguitur, dicitur $\langle \text{non} \rangle$ negando illam de forma.²³⁰ 400 Sed negatur eius antecedens: illa enim non est || impertinens, sed sequens ex concesso. Et conceditur quod illa est bona consequentia 'ille convertuntur 'deus est' et 'nullus deus est'; sed deus est; igitur nullus deus est' quia iam verum est quod consequens est necessarium. Et cum arguitur quod non, quia ante casum illa consequentia non valebat, sed casus non facit consequentiam, igitur sequitur 405 quod et nunc illa consequentia non valet, dicitur quod sic, et precipue casus de impositione qualis est iste.||

(RObj.2) Ad aliam formam dicitur quod apud illum qui poneret etiam quod mentalia, ultra id quod naturaliter significant, possunt ad placitum significare, quod casus non est impossibilis. Sed apud illum qui poneret quod mentales 410 non possunt ad placitum significare ultra id quod naturaliter significant, debet respondens certificari que propositiones demonstrentur per ly 'iste'.

(RObj.3) Ad aliam formam dicitur negando consequentiam, scilicet 'principalis copula istius propositionis est negata, igitur ista est negativa': sic etiam pars istius propositionis 'nullus deus est' esset negativa. Sed bene sequitur 'ista est propositio 415 cathegorica, et principalis copula huius negatur per signum quod est pars istius, et ista adequate significat ex compositione terminorum, igitur ista est negativa', quod est negandum in illo casu, concesso antecedente illius consequentie.

397 Et] om. O **398** Et] om. O **398** variaretur] varietur O **398** negaretur] negatur O **400** illam] ipsam O **403** et] om. O **403** sed] et O **404–405** quia ... consequentiam] quia casus non facit consequentiam et non valebat ante O **406** et] om. O **406** illa consequentia] om. O **408** illum] eum O **408** etiam quod] om. O **409** id quod] ea que O **409** significare] aliud add. O **410** quod] om. O **410** non ... impossibilis] esset possibilis O **410–411** apud ... significant] tenendo oppositum O **411** debet] deberet O **412** demonstrentur] demonstrarentur O **413** dicitur] istam add. O **413** negando] transp. O **413** scilicet] om. O **414** etiam] nam O **415** Sed] et O **416** et ... negatur] cuius principalis copulativa est negata O **416** signum] aliquid O **417** compositione] suorum add. O

O 125^{ra}

L 74^{va}

because of its convertibility with 'deus est' (that has been already granted). The objector would appeal to the fact that no *casus* of *impositio* makes a consequence sound. Unfortunately, the discussion, from Peter's viewpoint is fatally destined to stop here, because he exactly claims that this is the case. It is noteworthy, in this respect, that an essential clause of Peter's definition of a valid consequence refers to *impositio*.

²³⁰ Presumably, an emendation is needed here. A 'non' should be supplied before 'negando', as required by the sense of the argument, because the point is not denying the validity of the inference, but rather the truth of the antecedent. The sentence 'nullus deus est' is in fact sequentially relevant once we have granted the sentence 'deus est'.

5.26. Item, ponatur quod 'buff' sit dictio bissillaba.²³¹ Si negatur casus, contra: possibile est quod 'bub' sit dictio monosillaba, igitur possibile est quod 'bub' sit dic-420 tio bisillaba. || Patet consequentia, || quia non stat contradictorium consequentis cum antecedente.²³²

Vel cedat tempus obligationis et arguitur quod in tempore negasti quod non intellexisti, igitur male respondisti: quia in tempore non fuit ista propositio 'bub est dictio bisillaba',²³³ || cum id quod ponitur pro subiecto illius verbi 'est' non 425 P 74^{ra} sit nec erat pars orationis, igitur non reddebat suppositum illi verbo 'est'. Aliter enim non posset dari vox litterata que non esset pars orationis: quia si proferatur 'babu', adhuc id est pars orationis, ut sequitur ex responsione, quia statim ista est propositio 'babu est vox bisillaba'.²³⁴

Ideo dicitur in principio non admittendo casum donec sciat respondens quid illa 430 vox 'bub' debeat sibi significare. Et ita de aliis.

5.27. Item, ponatur quod omne antecedens alicuius consequentie et oppositum consequentis eiusdem sint similia, et omne consequents et oppositum antecedentis eiusdem sint similia. Et sint gratia argumenti iste due consequentie 'homo est, igitur risibile est', 'non homo est, igitur nullum risibile est', et || significent ille ex 435 O 125^{rb}

²³² One could be tempted to reply to this objection that the consequence is sound (with modalized antecedent and consequent) but the problem is that the consequent is not the posited sentence, i.e. 'buff est dictio bisillaba', but rather 'possibile est quod "bub' sit dictio bisillaba'. Is there anything that we have missed? I think the opponent would argue that this conclusion forces the respondent to concede that the posited sentence "bub' est dictio bisillaba' is possible, which is enough to say that it should have been admitted.

Be this as it may, the general point of the sophism is, once again, pointing out that one should respond only to what one understands and nothing should be admitted unless (or until) this condition has been satisfied.

²³³ A better wording would probably have been 'quia in tempore ista 'bub est dictio bisillaba' non fuit propositio'. Properly speaking, the term 'bub' is ill-formed: as a result any sequence of written or spoken signs that contains it, cannot count as a sentence. It may be useful to recall once more, in this respect, that Peter also rejects material supposition and that he does not regard strings of sounds or signs as tantamount parts of speech.

²³⁴ Cf. *Tractatus de suppositionibus*, [Peter of Mantua, 1492a, sig. Aii^{rb}] "Tertia suppositio: nulla propositio est, que habeat aliquam partem propinquam que non sit pars orationis grammatice. Patet illa, quia illa non intelligeretur ab aliquo ut propositio".

419 buff] EO bub BMV buf P **419** sit] est O **419** bissillaba] disillaba O **419–420** Si ... monosillaba] om. M **420** bub] buff O **420–421** igitur ... bisillaba] om. MO **423** in tempore] intra tempus O **424** in tempore] infra tempus O **424** non ... propositio] hec non fuit propositio O **424** bub] buff O **425** id] illud O **425** verbi] om. O **426** illi verbo] illius verbi O **427–428** quia ... orationis] om. O **428** babu] BMOV; ba buf P **428** est] iam add. O **429** bisillaba] disillaba O **431** bub] om. O **431** debeat ... significare] significet O **433** eiusdem] om. O **433** consequens] alicuius consequentie add. O **433** oppositum] contradictorium O

198

M 93^{ra}

B $101^{\rm rb}$

 $^{^{231}}$ The term 'buff' is to be emended with 'bub' here, or 'bub' is to be emended with 'buff' in the rest of the sophism. In the manuscripts there is some confusion between the two terms, but the argument is not affected by this fact.

compositione || suorum terminorum. Deinde proponitur 'homo est'. Quo concesso, proponitur 'nullum risibile est'. Si conceditur, cedat tempus et || arguitur: tu concessisti repugnans concesso in tempore, igitur male respondisti.

Item proponitur 'hec est vera 'homo est". Qua concessa, quia vera non repugnans, proponitur 'hec est vera 'nullum risibile est". Qua concessa tunc sic: hec 440 est vera 'homo est' et hec est vera 'nullum risibile est'; et iste adequate significant secundum primarias significationes; igitur homo est et nullum risibile est.

Et si forte negatur illa propositio 'homo est', proponitur 'illa est falsa'. Qua concessa, proponitur 'hec est falsa 'nullum risibile est". Quibus concessis, proponitur 'he sunt vere 'risibile est', 'non homo est" que contradicunt primis. Qua conces- 445 sa, arguitur sic: iste sunt vere 'risibile est', 'non homo est', adequate et primarie significantes etc.; igitur risibile est et non homo est.²³⁵

А

Ν

 \mathbf{C} С

 \mathbf{C}

 \mathbf{C}

 235 The sophism has the following structure:

- Omne antecedens alicuius consequentie et oppositum consequentis eiusdem sunt similia, et omne consequens et oppositum antecedentis eiusdem sunt similia. Et sint gratia argumenti, iste due consequentie 'homo est, igitur risibile est', 'non homo est, igitur nullum risibile est', et significent ex compositione suorum terminorum
- 1.1 Homo est
- Nullum risibile est 2
- 3. Hec est vera 'homo est'
- Hec est vera 'nullum risibile est' 4.
- Hec est vera 'homo est' et hec est vera 'nul-5.lum risibile est'; et iste adequate significant secundum primarias significationes; igitur homo est et nullum risibile est
- 1.2 Homo est
- 6. Illa est falsa [scil. 'homo est']
- 7. Hec est falsa 'nullum risibile est'
- He sunt vere 'risibile est', 'non homo est' 8.
- 9. Iste sunt vere 'risibile est', 'non homo est', adequate et primarie significantes etc.; igitur risibile est et non homo est

possible

- \mathbf{C} irrelevant and true
- \mathbf{C} \perp inconsistent with 1.
- С irrelevant and true
- \mathbf{C} irrelevant and true \mathbf{C}
- irrelevant and true

V 79^{va} $E Giiii+4^{ra}$

⁴³⁶ proponitur] proponatur O **437** proponitur] proponatur O **437** et || arguitur] om. O **438** in tempore infra tempus *et* obligationis *add.* O **439** proponitur proponatur O 439–440 non repugnans] et impertinens O 440 proponitur] proponatur O 440 Qua concessa] om. O 440 sic] si O 443 Et... propositio] primo illa O 444 proponitur] proponatur O 445 he ... vere] hec sunt vera O 445 que ... primis] quia contra dictorium scr. et del. dicunt primis et est verum add. O 445–446 Qua concessa] om. O 446 arguitur] et praep. O 446–447 adequate ... significantes] que adequate significant primarie O 447 etc.] om. O

5.28. Ideo dicitur in principio, admisso casu, concedendo illam 'homo est' cum proponitur. Et negatur illa 'nullum risibile est'. Et cum proponitur 'illa est vera 'homo est'', conceditur. Et conceditur quod illa || est falsa 'nullum risibile est'.

Et cum arguitur: illa est vera'homo est' || que est antecedens, igitur et consequens eius est verum, scilicet 'risibile est'; et ultra arguitur: omne antecedens alicuius consequentie et oppositum consequentis eiusdem sunt similia; sed iste 'homo est' et 'nullum risibile est' sunt antecedens et oppositum consequentis eiusdem consequentie; igitur ista sunt similia; sed illa est vera 'homo est'; igitur et illa 'nullum risibile est'; et ex alia parte illa est vera 'risibile est'; igitur duo contradictoria sunt simul vera, ideo huic dicitur negando illam consequentiam.

Sed oportet minorem sic sumi: ille due sunt vere 'homo est' et 'nullum risibile est'; et illa adequate significant quod homo est et nullum risibile est; igitur homo est et nullum risibile est.

Sed negatur quod ista sit primaria significatio tampuam repugnans.

5.29. Si tamen in casu illo adderetur quod ille consequentie essent adequate significantes ex compositione suorum terminorum, et quod illa 'homo est' adequate significaret hominem esse, et illa 'nullum risibile est' adequate significaret nullum risibile esse, negandus est casus.

460

450 M 93^{rb}

L 74^{vb}

⁴⁶⁵

⁴⁴⁹ Et] om. O **450** homo est] om. O **450** conceditur] om. O **451** arguitur] tunc add. O **451** antecedens] eius add. O **451–452** et ... est'] illa est vera 'risibile est' que est consequens O **452** et] tunc add. O **453** alicuius consequentie] om. O **453** iste] illa O **454** est] currit O **454** est] currit O **454–455** eiusdem consequentie] om. O **455** est] ante est scr. et del. currit O **455** et illa] illa est vera O **458** sumi] sumere O **458** et] om. O **459** illa] ille O **459** significant] et primarie add. O **459** et] quod add. O **459–460** igitur ... est] om. O **461** Sed] et O **462** adequate] sic add. O

6. **ON** depositio

6.1. Sed nunc dicamus quod licita || depositio est obligatio qua obligatur respondens ad negandum sibi depositam propositionem et quodlibet antecedens ad illam.

Ex descriptione patet, primo, quod deposita et bene admissa aliqua disiunctiva et proposita aliqua eius parte principali, ipsa est neganda. Deposita tamen copulativa et bene admissa non continue, quelibet eius pars est neganda.²³⁶

 236 The opening lines of the sixth – and last – section of the treatise are devoted to the definition of *depositio* and to the formulation of a pair of special rules, in the form of corollaries, that apply only to this subtype of obligation. The definition of *depositio*, as one may reasonably expect, on the grounds of the duality between the two notions, is a mirror image of the corresponding definition of *positio* given above, in sec. 1.9. In the present case, the respondent commits himself to upholding the *depositum* as false and denying it, throughout the development of the disputation, whenever it is put forward. In addition to that, it is explicitly required that the respondent commit himself also to the denial of any sentence that logically entails the *depositum*, a clause that was missing in the characterization of *positio*. At least explicitly: because, after all, an analogous element is embedded in the first rule of *positio* which states that anything that is entailed by the *positum* must be conceded, whenever put forward. To be pointed out, again, is the duality between the two notions, now with respect to entailment: in the former case, we must uphold the *positum* as true and grant it along with anything that follows from it, in the latter case, we must uphold the *depositum* as false and deny it, along with anything from which it follows.

There is a problem with the rules formulated after the definition. It may be reasonable to suppose that, in the text of the edition, the terms 'disjunctiva' and 'copulativa' are mistakenly inverted. From the logical point of view, what is said to hold of disjunction should rather hold of conjunction and vice versa. Here are my reasons in support of the claim. The first special rule for *depositio* is supposed to govern the behaviour of the respondent in the presence of disjunctions. It says that if a disjunction is deposited, then whenever one (one, at least one or some = 'aliqua eius parte principali') of its disjuncts is put forward, it must be denied. Likewise in the case of conjunctions: if a conjunction is deposited, then any of its conjuncts must be denied, whenever put forward in a disputation. Now, the former seems to be too weak a requirement for the denial of disjunctions, whereas the latter seems to be too strong a requirement for the denial of conjunctions. On a classical reading, depositing $p \lor q$ is equivalent to positing $\neg p \land \neg q$, therefore, if either p or q is put forward, it must be denied, because the falsehood of a disjunction entails the falsehood of each of its disjuncts (it entails the falsehood of one, or some, of its disjuncts only a fortiori, i.e. because it entails the falsehood of all of them). Therefore, as a rule, it should be formulated with the requirement that any disjunct be denied, whenever it is proposed. In the case of conjunctions, it is the other way around, since the falsehood of a conjunction simply entails the falsity of at least one of its conjuncts, not the falsehood of all of them (in other words, if $\neg (p \land q)$ it just follows that $\neg p \lor \neg q$. But Peter's argument seems to require that $\neg p \land \neg q$. when he says that any counjunct (= 'quelibet eius pars') must be denied, whenever it is put forward. A first suggestion therefore might be keeping the rules as they stand and invert the labels: what is ascribed to disjunctions should, in fact, count as a property of conjunctions and vice versa.

There is however another way to look at these requirements, by strengthening the role of

 $P 74^{rb}$

² negandum] negandam O 2-3 et ... illam] om. O 4-6 Ex ... neganda] EL om. BMOPV

6.2. De qua primo depono tibi illam 'alique propositiones non sunt vere'; et significet illa adequate || quod alique propositiones non sunt vere. Deinde propono illam 'alique propositiones non sunt vere'. || Qua negata, quia deposita, proponitur 'omnes propositiones [non] sunt vere divisive sumendo'. Qua concessa, quia contradictorium depositi, proponitur etiam 'quelibet propositio est vera'. Qua concessa tamquam sequente, quia bene sequitur: omnes propositiones sunt vere divisive, igitur quelibet propositio est vera, et arguitur sic ultra: quelibet propositio est vera; et illa est propositio 'alique propositiones non sunt vere'; igitur illa est vera. Et ultra: illa est vera; et illa adequate significat || quod alique propositiones non sunt vere; igitur alique propositiones non sunt vere. Et ex alia parte, omnes propositiones sunt vere. || Igitur contradictio.²³⁷

the *depositio* operator when it ranges over a compound sentence. The interpretation I have proposed relies on the assumption that if a conjunction is deposited, what must be upheld as false throughout the disputation is the conjunction itself and, in order for this to obtain, it is sufficient that one of its conjuncts is false. But one might argue that, when a conjunction is deposited, this must be taken in fact as equivalent to depositing both conjuncts at one and the same time. In that case, the requirement that any conjunct be denied when it is proposed would be correct (the same reasoning holds, *mutatis mutandis*, in the case of disjunctions, too, because depositing a disjunction would actually be equivalent to depositing one conjunct or depositing the the other, but not the disjunction as a whole). On this account the text would make sense as it stands. Yet, there is a conceptual problem with this interpretation and it would get Peter into much trouble (this is why I am also inclined to prefer the first explanation). Assume that on this latter account a contradiction is deposited. It is an admissible type of sentence, for it is not necessary (necessary sentences in the case of *depositio* have the same role that impossible sentences play in the case of *positio*: both classes are non-admissible in principle). Now, if $p \wedge \neg p$ is deposited, on the stronger reading, both of its conjuncts are *deposited* and consequently they must be both denied, whenever proposed. As a result, one and the same sentence would have to be granted and denied.

Finally, unclear is also the role of the clause 'non continue'. It must be noted by the way that the whole passage containing the rule for disjunctions and conjunctions is reported by only one manuscript, namely L, besides the edition. We cannot exclude that some kind of problem in the transmission of the text might have occurred at this point.

A treatment of rules for conjunctions and disjunctions in the same context is found in Strode, who describes a variety of relations between members of conjunctions and disjunctions along with the corresponding criteria of response. Moreover, in discussing the rule for disjunction, Strode proves to be aware of the point made above in connection with the denial of disjunctions since he claims "admissa ergo una disjunctiva, debet quelibet eius pars principalis negari, quia quelibet antecedit ad ipsam depositam" [Ralph Strode, 1517, fols. 89^{va}]. The printed text of Strode has 'ad ipsam depositum' which is incorrect. I wish to thank prof. Ashworth for the suggestion and for pointing out to me that six mss. have the variant "ad depositum vel ad ipsam depositam".

²³⁷ The text must be emended, removing the 'non', according to manuscripts B and O, that both consistently have 'omnes propositiones sunt vere' (the edition itself, at later occurrences of the sentence in the same sophism, reports the correct reading). The sophism is also found in

202

B 101^{va} O 125^{va}

10

15 V $79^{\rm vb}$

M 93^{va}

^{9–10} proponitur] proponatur O **10** omnes] *ante* omnes *scr. et del.* alique propositiones O **10** non] *om.* BO **11** contradictorium] contradictoria O **11** proponitur] proponatur O **11** etiam] *om.* O **12** sequente] sequentem O sequens MV **13** divisive] sumendo *add.* O

6.3. Ideo dicitur in principio non admittendo casum: repugnat enim quod omnes propositiones sint vere divisim et quod illa 'alique propositiones non sunt vere' adequate significet quod alique propositiones non sunt vere. Verumtamen non valet consequentia illa 'omnes propositiones sunt vere, collective sumendo; igitur quelibet propositio || est vera', quia stat quod omnes propositiones sunt vere, collective, et quod aliqua sit impossibilis, et quod omnes sint false, collective sumendo, et quod aliqua sit necessaria. Ut posito quod iste sint omnes propositiones, collective, 'deus est', 'nullus homo est asinus', sic primarie significantes, tunc, quia ille ambe sunt vere et ille sunt omnes propositiones, collective, igitur omnes propositiones, collective, sunt vere. Et tamen aliqua est impossibilis, puta pars illius, scilicet illa 'homo est asinus' que est pars illius || 'nullus homo est asinus'. Et si ex illo arguitur quod illa propositio 'nullus homo est asinus' est propositio necessaria et impossibilis, conceditur de copulato extremo.²³⁸

[Ralph Strode, 1517, fol. 89^{rb}], albeit with a different treatment.

 238 The first sophism on *depositio* has the following structure:

		ing ser as	04101
D	Alique propositiones non sunt vere	А	possible
1.	Alique propositiones non sunt vere	Ν	by D
2.	Omnes propositiones sunt vere divisive	\mathbf{C}	contradictory of D
3.	Quelibet propositio est vera	\mathbf{C}	sequentially relevant (follows
			from 2.)
4.	Quelibet propositio est vera; et illa est pro-	\mathbf{C}	sequentially relevant (valid
	positio 'alique propositiones non sunt vere';		inference)
	igitur illa est vera		
5.	Illa est vera; et illa adequate significat quod	\mathbf{C}	sequentially relevant (valid
	alique propositiones non sunt vere; igitur		inference)
	alique propositiones non sunt vere		
6.	Alique propositiones non sunt vere	\mathbf{C}	\perp (incompatible with 2.)

On the basis of the replies given at steps 3–5, the conclusion put forward at step 6., namely 'alique propositiones non sunt vere' can be drawn, by iterated detachment. But it is incompatible with 2., namely with the sentence 'omnes propositiones sunt vere divisive', that has already been granted. Therefore the case is contradictory. As to the clause 'divisive', I assume that if p, q, r, ..., stand for sentences and 'T' is a truth predicate, then the fact that all sentences are true divisim can be expressed as follows: $Tp \wedge Tq \wedge Tr$. By contrast, the truth of all sentences taken collectively is probably to be read, even if Peter does not make it explicit in this passage, as $T(p \wedge q \wedge r)$. According to Peter, 2. is incompatible with the depositum if 2. is taken divisively. The rest of the argument appears to be intended at pointing out that the case is contradictory only on this assumption, but not if we take 2. collectively. The grounds that Peter provides to justify this claim, however, do not seem entirely convincing, from the logical standpoint. This is the line he adopts: let us take two sentences p and q, both true and necessary, and assume that q has the form $\neg r$ (in Peter's words, r is a part of q). Now, $T(p \wedge q)$ holds because p and q are true collectively. This implies that the truth predicate applies to their conjunction in an unanalyzed way. But, with an awkward move, Peter extracts r from q, by removing the

19 sint] sunt O 19 divisim] om. O 22 sunt] sint O 22–23 collective] sumendo add. O 23 sumendo] om. O 24 et] tamen add. O 24 quod] om. O 25 sic] om. O 28–29 scilicet ... propositio] om. O 29 est] iter. O 30 conceditur] concedatur O

203

 $E Giiii+4^{rb}$

25

20

L 75^{ra} 30 6.4. $\langle \text{Obj.1} \rangle$ Sed forte arguitur removendo illam particulam, scilicet quod ista 'alique propositiones non sunt vere' adequate significet quod alique propositiones non sunt vere et deponatur simpliciter ista 'alique propositiones non sunt vere'. Et sit A propositio una – quecumque sit non curo – que significet principaliter quod alique propositiones non sunt vere. Et arguitur sic: A propositio est vera; 35 et A propositio significat adequate quod alique propositiones non sunt vere; igitur alique propositiones non sunt vere. Et probatur quod A est verum, sicut prius arguebatur de illa 'alique propositiones non sunt vere', quia quelibet propositio est vera.

(RObj.1) Sed huic dicitur negando istum secundum casum sicut primum.²³⁹

 $\langle \text{Obj.2} \rangle$ Sed forte contra arguitur deponendo illam 'alique propositiones non sunt vere' et ponendo illam 'omnes propositiones sunt vere divisive', que est contradictoria prime adequate significans sic, scilicet quod omnes propositiones sunt vere.

 $\langle \text{RObj.2} \rangle$ Sed huic dicitur adhuc non admittendo istum casum, quia est impossibilis.²⁴⁰

6.5. Item, depono tibi quod propositiones tam ad invicem repugnantes || quam que non sunt repugnantes non sunt similes, et quod || illa propositio 'aliqua propositio est sibi similis' significet adequate quod aliqua propositio est sibi similis. Deinde propono 'omnes propositiones sunt similes divisive'. Que, quia contradictoria 50

 239 The solution to the previous sophism amounted to a rejection of the case and it still holds even if the terms involved are modified by removing the clause according to which the former *depositum* meant that some sentences are not true. For it is sufficient to take any sentence p, stipulate that its meaning is that of the removed clause and plug it into the same disputational structure as above. It can be shown that p, too, is incompatible with the *depositum* 'Alique propositiones non sunt vere' precisely in the same way, because from that *depositum*, the assertion that all sentences are true divisively still follows; hence the contradiction is not ruled out.

²⁴⁰ Another variation on the same theme. Peter's idea is that whenever the opponent presents a sentence that in different ways implies that all sentences are true, the respondent must refuse to admit the case, since otherwise he would somehow be forced into a contradiction, as the examples show.

31 illam particulam] illa particula O **33** et] quia O **34** sit ... que] illa propositio O **35** arguitur] tunc add. O **36** adequate] principaliter O **42–43** est contradictoria] sunt contradictorie O **43** prime] om. O **43** sic, scilicet] om. O **45** quia] ille add. O **47–48** quam ... repugnantes] om. O **48** aliqua propositio] om. O **50** divisive] divisim O **50** Que] om. O **50** contradictoria] contradictorium O

40

45

O 125^{vb}

M 93^{vb}

 $P 74^{va}$

negation and makes it a member of the conjunction; then we would have $T(p \land q \land r)$. But r is impossible, since in the example it stands for the sentence 'homo est asinus'. Therefore, in virtue of this curious procedure, we obtain that the impossible sentence r, as a part of the content of the conjunction taken in a collective opacity, would turn out to true. I have some difficulty in understanding how exactly Peter can claim that this procedure has some logical plausibility. The same holds also in the case of the final remark, when he contends that the sentence must be considered as necessary and impossible 'de copulato extremo', i.e. it is to be seen as 'necessary and impossible', where the predicate is a conjoint term.

depositi, concedenda est. Deinde proponitur 'quelibet propositio cuilibet propositioni est similis'. Que, quia sequens, est et $\langle iam \rangle$ concedenda. Deinde proponitur 'omnes propositiones sunt false'. Si negatur, contra: omnes propositiones sunt similes; et alique propositiones sunt repugnantes; igitur omnes propositiones sunt false. Et tenet consequentia illa, quia verum non repugnat vero. || Deinde proponitur 'alique propositiones non sunt false'. Si negatur, contra: aliqua propositio est vera, igitur alique propositiones non sunt false || divisim. Patet consequentia. Et arguitur antecedens, quia hec propositio est vera 'aliqua propositio est sibi similis', quia aliqua propositio est sibi similis et ista adequate significat quod aliqua propositio est sibi similis, igitur ista est vera. ²⁴¹

6.6. Ideo dicitur in principio non admittendo casum.²⁴² Pro quo est notandum quod, cum deponitur aliqua propositio, pro faciliori responsione habenda, imaginetur respondens quod sibi ponatur contradictorium depositi nec advertat ad aliud nisi ad propositionem quam imaginatur esse sibi positam. Ideo imaginandum est – cum deponitur illa, quod propositiones tam invicem repugnantes quam non repugnantes non sunt similes, et ponitur quod hec 'aliqua propositio est sibi similis' adequate significet quod aliqua propositio est sibi similis – quod hec copulativa ponatur 'omnes propositiones tam invicem repugnantes quam non repugnantes sunt invicem similes, et hec propositio 'aliqua propositio est sibi similis' significet adequate quod aliqua propositio est sibi similis'. Et patet quod illa copulativa est ro impossibilis, quia ex prima parte sequitur quod omnes propositiones sunt false, quia bene sequitur 'omnes propositiones sunt similes et alique invicem repugnant,

 242 The sophism has the following structure:

205

55 B $101^{\rm vb}$

 $V 80^{ra}$

²⁴¹ This sophism covers another familiar topic in the obligational literature, namely the socalled 'de similibus' and 'de dissimilibus'. It is another important rubric belonging to the genre and has attracted attention to various extents. Strode, for example, dismisses it, as well as Buser does, because he thinks that it is some sort of puerile matter for those who master the skills of the *ars obligatoria*; others, like Burley or Marsilius, or again Paul of Venice devote more space to this subtype of obligation (or to put it better, of *positio* and *depositio*). Being similar or dissimilar means in the obligational jargon having the same vs different truth value(s). Similarity is often used in the tradition to show some characteristic features of the standard approach of *responsio antiqua* such as the provability of any false sentence compossible with the *positum*. For instance if, immediately after positing a false sentence p, the opponent proposes 'p and q are similar [i.e. have the same truth value]' where q is a false sentence compossible with the *positum*, the respondent should grant it because it is irrelevant and actually true (p and q are similar, since they both are false). But then, if q is put forward it will have to be granted as well.

⁵¹ concedenda] concedendum O **51** proponitur] proponatur O **52** $et\langle iam \rangle$] om. O **52** $et\langle iam \rangle$] et E **52** proponitur] proponatur O **54–55** et ... false] om. O **55** non] numquam O **55–56** proponitur] proponatur O **56** false] om. O **57** Patet] hec add. O **60** vera] aliqua propositio est sibi similis add. O **61** notandum] advertendum O **64** nisi ... propositionem] om. O **64** positam] positum O **65** invicem] ad praep. O **66** quod] ante ponitur transp. O **66** aliqua propositio] inv. O **67–70** quod ... Et] om. O **72** similes] false O **72** alique] propositiones add. O **72** invicem] ad praep. O

igitur omnes || propositiones sunt false'. Et ex alia parte se-||quitur quod || aliqua propositio est vera; igitur alique propositiones non sunt false. Et antecedens illius consequentie sequitur ex casu. Ideo ex illo casu sequitur contradictio: ideo non est admittendus.²⁴³

6.7. Sed forte, non ponendo secundam particulam²⁴⁴, deponitur illa solum 'tam propositiones invicem repugnantes quam non repugnantes non sunt similes'. Quo admisso, proponitur 'deus est'.

D	Tam propositiones ad invicem repugnantes quam que non sunt repugnantes non sunt similes	А	possible
Ρ	Illa propositio 'aliqua propositio est si- bi similis' significat adequate quod aliqua propositio est sibi similis	А	possible
1.	Omnes propositiones sunt similes divisive	\mathbf{C}	contradictory of D
2.	Quelibet propositio cuilibet propositioni est similis	С	sequentially relevant
3.1	Omnes propositiones sunt false	Ν	
4.	Omnes propositiones sunt similes; et ali- que propositiones sunt repugnantes; igitur omnes propositiones sunt false	С	\perp valid inference (cf. 'quia verum non repugnat vero') whose consequent is inconsi- stent with 3.1
5.	Alique propositiones non sunt false	Ν	
6.	Aliqua propositio est vera, igitur alique propositiones non sunt false divisim	С	valid inference
7.	Hec propositio est vera 'aliqua propositio est sibi similis', quia aliqua propositio est sibi similis et ista adequate significat quod aliqua propositio est sibi similis, igitur ista	С	

est vera

It must be noted that in order for the argument to work properly, we need to understand the sophism as being actually composed of a *depositio* (= 'omnes propositiones tam invicem repugnantes quam non repugnantes non sunt similes') together with a *positio* (= "aliqua propositio est sibi similis' significat adequate quod aliqua propositio est sibi similis'). In addition to that, two intriguing facts must be pointed out. First, the deposited sentence implies that all sentences are false, only in conjunction with the background presupposition that a truth is never inconsistent with another truth. Second, the point made at step 7. is also quite remarkable, since it rests on the semantic principle: p and 'p' signifies that p, therefore 'p' is true.

 243 The disputation is apparently bound to end up in a contradiction, since from the deposited sentence it can be argued that all sentences are false, while from the posited sentence it follows that at least one sentence is true. On account of this, one might be inclined to reject the *casus*.

²⁴⁴ Here is a variation on the theme: since in the previous ramified structure the contradiction depended on the incompatibility between a deposited and a posited sentence, Peter explores the consequences of removing the second clause. It will turn out that in this case the *casus* can be admitted.

73 Et] om. O **75** est] om. O **77** deponitur] deponatur O **77** tam] om. O **78** invicem] ad praep. O **78** non₂] om. O **79** proponitur] proponatur et hec add.O

 $\begin{array}{c} {\rm M} \ 94^{\rm ra} \\ {\rm L} \ 75^{\rm rb} \\ {\rm 75} \ {\rm E} \ {\rm Giiii}{\rm +}4^{\rm va} \end{array}$

 $\langle \text{Obj.1} \rangle$ Que, si conceditur, arguitur sic: tu concedis istam; et ista est falsa nec so est sequens; igitur male etc.

Sed forte dicitur concedendo primam partem antecedentis propositam. Deinde proponitur secunda.

(i) Que, si negatur, arguitur sic: illa est propositio et non est falsa, igitur est vera;
igitur alique propositiones non sunt false. Et ex alia parte, omnes propositiones sunt false, ut sequitur ex contradictorio depositi. Igitur contradictio.

 $\langle ii \rangle$ Item etiam arguitur, si forte negatur quod illa non est sequens, [si forte dicitur quod illa est sequens] || quia necessaria.²⁴⁵

6.8. Ideo || dicitur, admisso illo secundo casu, concedendo illam 'deus est'.

(RObj.1) Et cum proponitur 'tu concedis illam, et illa est falsa non sequens, 90 igitur male respondes', conceditur consequentia. Neque dicatur ad antecedens neque ad aliquam eius partem nisi secundum quod proponuntur propositiones per ordinem: quia si primo proponitur prima pars antecedentis, conceditur et conceditur etiam secunda, secundo loco proposita. Deinde si postponatur consequens, negetur ipsum. Quo negato, negetur alia pars antecedentis, et conceditur quod illa est sequens, non tamen quia necessaria, quia hoc repugnat. Et concedendum est quod est falsa tamquam sequens.||

 $^{^{\}rm 245}$ The sophism has the following structure:

D	Tam propositiones ad invicem repugnantes quam que non sunt repugnantes non sunt	А	possible
	similes		
1.	Deus est	\mathbf{C}	
2.	Tu concedis istam; et ista est falsa nec est	\mathbf{C}	sequentially relevant valid in-
	sequens; igitur male		ference
2.1	Tu concedis istam	С	(cf. 'sed forte dicitur concedendo primam partem antecedentis propositam')
2.2	Ista est falsa	Ν	/
3.	Illa est propositio et non est falsa, igitur est	\mathbf{C}	\perp valid inference whose con-
	vera		sequent is inconsistent with a corollary of the denial of D
2.3	Ista non est sequens	Ν	

⁸⁰ nec] illa add. O
81 etc.] respondes O
83 proponitur] proponatur O
84 igitur ...
vera] om. O
86 ut] et O
87-88 si ... sequens] EP om. MO in marg. L
88 quia] illa est
add. O
89 secundo] om. O
91 conceditur] concedo O
91 consequentia] consequentia
O
92 proponuntur] proponantur O
93 quia] et O
93 conceditur] concedatur O
93 et] om.
O
93-94 conceditur] concedatur O
94 postponatur] proponatur O
95 negetur] negatur O
95 alia] tertia O
95 conceditur] concedatur O

207

O 126^{ra}

 $V 80^{rb}$

 $P 74^{vb}$

TRACTATUS OBLIGATIONUM

6.9. Item, depono quod alique propositiones – tres vel quattuor – non sint dissimiles. Quo admisso, proponitur 'omnes tres aut quattuor propositiones sunt dissimiles'. Quo concesso, quia oppositum depositi, arguitur sic: omnes tres vel 100 quattuor propositiones divisim sunt dissimiles; sed A, B, C, D sunt quattuor propositiones, gratia argumenti; igitur A, B, C, D sunt dissimiles propositiones. Et appello dissimiles propositiones quarum una est vera et alia non est vera; similes vero quarum utraque est vera vel utraque est falsa. Consequens || autem est impossibile, quia vel omnes ille sunt vere vel omnes ille sunt false vel due sunt vere et 105 due false vel tres false et una vera vel econtra. Sed sive sic sive sic, sequitur quod non omnes propositiones sunt dissimiles in illo casu, immo in nullo casu non plures quam due possunt esse propositiones dissimiles ad invicem, sic quod quelibet cuilibet alteri sit dissimilis.²⁴⁶

6.10. Ideo dicitur in principio non admittendo || casum. Admittatur tamen ca- 110 B 102^{ra} sus, remota illa particula 'tres vel quattuor'.²⁴⁷ Deinde proponatur 'ille invicem contradicunt 'rex sedet', 'nullus rex sedet', 'deus est', 'nullus deus est".

²⁴⁶ This sophism in turn is discussed in two versions: the first one which is in question here contains two clauses, the second one is obtained by removing the second clause, namely the explicit assumption that there are more than two dissimilar sentences. The structure of the first version is the following:

D	Alique propositiones, tres vel quattuor, non	Α	possible
	sunt dissimiles		
1.	Omnes tres aut quattuor propositiones sunt	\mathbf{C}	Contradictory of D
	dissimiles		
2.	Omnes tres aut quattuor propositiones sunt	\mathbf{C}	sequentially relevant (valid
	dissimiles; sed A, B, C, D sunt quat-		inference)
	tuor propositiones; igitur A, B, C, D sunt		,

The consequent of 2., namely the sentence 'A, B, C, D sunt dissimiles propositiones' is impossible, since we have just two truth values at our disposal and there is no way to distribute four elements (sentences in this case) in two classes (of truth values) without putting at least two of them together in the same class. Since it can never be the case that three or four (in general more than two sentences, if the number of truth values is set to two) sentences are all different in truth value with respect to one another, the original claim – that was deposited – is in fact necessary. Consequently, it should be rejected. As in the case of *positio* one should never admit an impossible sentence, conversely in the case of *depositio* it is the class of necessary sentences that one should avoid to admit. From the logical standpoint, the two types of obligation are symmetrical.

 247 The second version, on the other hand, runs as follows:

dissimiles propositiones

98 depono] tibi add. O **98** tres] ante propositiones transp. O **98** vel quattuor] om. O; non est cura add. B **98** sint] sunt O **99** omnes] ante omnes scr. et del. alique O **100** Quo] om. O **100** oppositum] contradictorium O **100** tres vel] om. O **103** alia] reliqua O **103** non est vera] falsa O **104** vero] autem O **104** utraque ... falsa] una est vera et reliqua vera O **106** sequitur] om. O **107** omnes] inc. add. O **108** propositiones] om. O **111** invicem] ad invicem O **112** deus ... est] om. O

208

Si conceditur, tunc sequitur quod alique non sunt dissimiles, quia sequitur quod illarum due sunt false et due sunt vere.

Si negatur, proponitur 'alique illarum quattuor contradicunt'. Si conceditur, ad- 115 huc sequitur quod alique propositiones non sunt dissimiles. Si negatur, proponitur 'ille due contradicunt 'deus est', 'nullus deus est".

(Obj.1) Si conceditur, arguitur sic, quod tres propositiones sunt: quia illa 'deus est' et 'nullus deus est' et iterum illa 'deus est' que est pars illius 'nullus deus est'. Et per consequens alique propositiones non sunt dissimiles, quia bene sequitur: 120 tres propositiones sunt, igitur alique propositiones non sunt dissimiles.

(Obj.2) Item in illo casu, || concesso quod ille contradicunt 'deus est', 'nullus deus est', proponitur 'tu es obligatus ad istam 'alique propositiones non sunt dissimiles". Si conceditur, arguitur quod tres propositiones sunt et per consequent alique propositiones non sunt dissimiles. 125

6.11. Ideo dicitur admisso casu concedendo quod ille || contradicunt 'deus||est', 'nullus deus est'.

(RObj.1) Et cum arguitur quod tres propositiones sunt, quia illa 'deus est' et illa 'nullus deus est' et etiam illa 'deus est', que est pars eius, est propositio, negatur quod pars eius sit propositio tamquam repugnans. Et si proponeretur in illo casu 130 'hec est vera 'iste contradicunt 'deus est' et 'nullus deus est"', negatur tamquam repugnans. Et si proponatur 'hec est oratio 'iste contradicunt 'deus est' et 'nullus deus est", conceditur; et negatur quod sit propositio tamquam repugnans.

D 1.1 1.2	Alique propositiones non sunt dissimiles Ille invicem contradicunt 'deus est', 'nullus deus est', 'rex sedet', 'nullus rex sedet'	A C N	possible \perp (it would follow that some sentences are dissimilar)
1.2.1	Alique illarum quattuor contradicunt	С	\perp (some sentences are not dissimilar, contradictory <i>D</i> , cf. sophism 49)
1.2.2		Ν	
1.2.2.1	Ille due contradicunt 'deus est', 'nullus deus est'	С	\perp (there are three sentences: 'deus est', 'nullus deus est' and again 'deus est' as a part of the second sentence)
1.2.2.2		С	,
1.2.2.2.1	Tu es obligatus ad istam 'alique propositio- nes non sunt dissimiles'	С	\perp (there are three sentences)

113 tunc] om. O 114 sunt] om. O 115 proponitur] proponatur O 116 proponitur] proponatur O **117** due] *om.* O **118** arguitur] tunc *add.* O **118** quod] *om*. 0 120-121 quia ... dissimiles] om. O 123 proponitur] proponatur O 124-125 Si ... dissimiles] om. O **128** cum] om. O **129** etiam] iterum O **129** eius] illius et 'nullus deus est' add. O 129 est propositio] om. O 130 proponeretur] proponatur O 132–133 deus \dots est] om. O **133** et] sed O

209

L 75^{va}

E Giiii $+4^{vb}$

O $126^{\rm rb}$



M 94^{va}

TRACTATUS OBLIGATIONUM

 $\langle \text{RObj.2} \rangle$ Et tunc ad aliud argumentum, cum proponitur, concesso quod ille contradicunt 'deus est', 'nullus deus est', quod tu es obligatus ad illam 'alique 135 propositiones non sunt dissimiles', negatur tamquam repugnans concessis etc.²⁴⁸

 $^{^{248}}$ The text of E, M, O and P break off at this point. Mss. B, L and V have a short additional part.

¹³⁴ concesso] concedendo O 136 concessis etc.] Queritur utrum tantum propositio sit obligabilis vel res ad ? Prima suppositio: non sequitur 'nomino Sor, ergo Sor nominatur'. Secunda: non sequitur 'nomino Sor currere, ergo Sortem currens est'. Tertio non sequitur 'dico Sortem currere ergo Sor curret/currens est' et 'dico Sor currere quia dicere Sor currere est nuntiare Sortem currere. Quarto non sequitur 'dico vel profero illam propositionem Sor currit, igitur profero Sortem currere'. Quinto hec consequentia est bona terminis? 'dico vel profero illam propositionem Sortes currit que adequate scio significare quod Sor currit, ergo dico 'Sor currere'. Sexto non sequitur 'concedo, nego aut dubito Sor currere, ergo Sor currens est. Septimo non sequitur pono vel depono Sor currere ergo Sor currens est Octavo non sequitur 'pono illam Sor currit, ergo pono Sor currere et consequenter sumantur termini personaliter. Nono non sequitur profero Sor currere ergo Sor currens est. Sicut per propositionem res ad extra complexe cognoscitur complexioni distincta ita per propositionem res extra conceditur aut assentitur Item concedo 'sequitur Sor currere, ergo aliqua propositio est' quia non est concessio nisi per propositionem. Queritur utrum omnis propositio sit concedenda neganda vel dubitanda, respondeatur quod non, quia negando, concedendo, dubitando *add.* BLV: *om.* EMOP

Index of sophisms

Section 3

- Obligo te ad istam 'tu curris' posito quod demonstrem te qui sedes. (Sec. 3.1, p. 84)
- 2. Pono tibi utramque istarum [*scil.* 'rex sedet', 'nullus rex sedet']. (Sec. 3.4, p. 88)
- 3. Pono tibi alteram illarum [*scil.* 'rex sedet', 'nullus rex sedet']. (Sec. 3.5, p. 89)
- 4. Pono propositionem possibilem et nulli repugnantem quam habeo in mente mea. (Sec. 3.7, p. 92)
- 5. Ponitur quod omne possibile sit tibi positum et a te bene admissum. (Sec. 3.9, p. 94)
- 6. Ponitur quod propositio illa simpliciter impossibilis 'homo est asinus' sit tibi posita et a te admissa. (Sec. 3.13, p. 97)
- 7. Ponat Sor istam tibi 'homo currit' adequate significantem hominem currere; et ponat Plato tibi istam 'non homo currit' adequate significantem non hominem currere contradictorio modo prime. (Sec. 3.14, p. 98)
- Ponatur quod Sor teneat locum Platonis et ponat ipse utrumque. (Sec. 3.15, p. 99)
- 9. Ponatur tibi haec copulativa 'haec 'homo currit' est tibi posita et a te bene admissa adequate significans hominem currere, et haec 'risibile currit' adequate significans risibile currit est tibi deposita et a te admissa'. (Sec. 3.18, p. 102)
- 10. Ponatur tibi sedenti illa 'omnis homo currit'. (Sec. 3.20, p. 105)
- 11. Ponatur tibi sedenti 'omne sedens est Sor'. (Sec. 3.26, p. 112)
- Ponatur quod Sor non obligato proponatur illa 'deus est' quam tempore concedat. Et proponatur Platoni eadem quam neget necessariam male respondendo. Et Cicero dubitet illam male etiam respondendo. (Sec. 3.28, p. 114)
- 13. Ponatur quod Sor eadem propositio proponatur a diversis, sed unus ante alium proponat eam, sic quod sit falsa quando unus proponit et vera quando alter proponit. (Sec. 3.31, p. 119)
- 14. Ponatur quod ad nihil sis obligatus. (Sec. 3.33, p. 121)
- 15. Ponatur quod nulla propositio sit. (Sec. 3.35, p. 123)
- 16. Pono tibi sedenti istam 'omnis homo currit' [bis]. (Sec. 3.37, p. 124)
- 17. Ponatur quod omnis homo qui est albus currat et nullus istorum [*scil.* omnes homines qui sunt albi] potest currere. (Sec. 3.39, p. 129)

Section 4

- 18. Peto te concedere quod nullus deus est. (Sec. 4.3, p. 138)
- 19. Dubitetur a te ista 'deus est'. (Sec. 4.5, p. 140)
- 20. Cedat enim omne tempus obligationis per quod est aut fuit obligatus aliquis. (Sec. 4.8, p. 143)
- 21. Sit rei veritas quod numquam respondisti ad istam 'celum movetur'. (Sec. 4.9, p. 144)
- 22. Ponatur quod si rex sedet tu scias regem sedere et si nullus rex sedet tu scias nullum regem sedere. (Sec. 4.11, p. 148)
- 23. Tu es obligatus donec proponitur tibi aliquod negandum. (Sec. 4.13, p. 150)
- 24. Ponatur quod tu sis obligatus quamdiu non proponitur tibi aliquod negandum aut quamdiu non est propositum aliquod negandum. (Sec. 4.14, p. 151)
- 25. Tu sis obligatus quamdiu proponetur tibi aliquod negandum et non sis obligatus quamdiu non proponetur tibi aliquod negandum. (Sec. 4.16, p. 152)
- 26. Tu es papa vel rex sedet et tu curris. (Sec. 4.17, p. 153)

Section 5

- 27. Posito autem quod per totam istam diem iste terminus 'Sor' ita istum hominem significabit quod nullum alium, sed cras imponetur ad significandum alium hominem, est adhuc concedendum quod ille secundus non potest esse Sor nec poterit esse Sor neque unquam erit Sor, ita quod ille secundus est Sor. (Sec. 5.5, p. 161)
- 28. Posito autem quod aliquis alius fuerit vocatus 'Sor' aut quod nunc de presenti vocetur 'Sor', concedendum est quod uterque illorum est Sor et uterque istorum fuit Sor, illis duobus demonstratis. (Sec. 5.6, p. 163)
- 29. 'Homo est asinus' significans adequate deum esse. (Sec. 5.7, p. 164)
- 30. Ponatur quod respondebis ad illam 'homo est asinus'. (Sec. 5.8, p. 166)
- Posita et admissa illa 'tu es obligatus', cedente tempore obligationis, negandum esset quod tu fuisti obligatus. (Sec. 5.8, p. 166)
- 32. Ponatur quod hec 'homo est asinus' fiat necessaria adequate significans deum esse convertibiliter cum illa 'deus est'. (Sec. 5.10, p. 169)
- 33. Ponatur quod ista 'homo est asinus' sit ad placitum significativa sicut una propositio greca, sic quod non significat tibi latine; et sit propositio nec tu intelligas grecum. (Sec. 5.10, p. 170)
- 34. Pono quod omne nesciens se esse A sit A. (Sec. 5.12, p. 172)
- 35. Ponatur quod predicatum in ista 'omnis homo est animal' supponat determinate, cuiuslibet alteri termini manente significatione preter hoc quod ly 'omnis' non confundat predicatum confuse tantum. Et significet illa propositione ex compositione suorum terminorum. (Sec. 5.14, p. 176)
- 36. [Et sicut dictum est de ista 'omnis homo est animal' in casu illo, ita dicendum est de ista] 'aliquis homo est et quilibet homo est ille', posito quod ly 'ille' supponat determinate. (Sec. 5.16, p. 182)

- 37. [Et sicut dictum est de illa copulativa 'aliquis homo est et quilibet homo est ille', ita dicendum est de illa] 'aliquid est et nihil est illud', posito quod ly 'illud' supponat determinate. (Sec. 5.17, p. 187)
- 38. Ponatur quod predicatum illius 'nullus homo est animal' stet determinate cuiuslibet alteri termini significatione remanente et illa propositione significante ex compositione suorum terminorum. (Sec. 5.19, p. 188)
- 39. Ponatur quod in secunda parte illius 'aliquod animal est et nullus homo est id' ly 'id' supponat communiter et non distributive. (Sec. 5.21, p. 191)
- Ponatur quod subiectum in ista 'aliquis homo est quilibet homo' supponat confuse tantum manente cuiuslibet alteri termini significatione et significante illa propositione ex compositione suorum terminorum. (Sec. 5.22, p. 191)
- 41. Ponatur quod iste convertuntur 'deus est', 'nullus deus est', una illarum significante primarie sicut solet significare primarie, puta quod deus est vel quod nullus deus est. (Sec. 5.24, p. 195)
- 42. Ponatur quod 'bub' sit dictio bisillaba. (Sec. 5.26, p. 198)
- 43. Ponatur quod omne antecedens alicuius consequentiae et oppositum consequentis eiusdem sint similia, et omne consequens et oppositum antecedentis eiusdem sint similia. Et sint gratia argumenti iste due consequentie 'homo est, igitur risibile est', 'non homo est, igitur nullum risibile est', et significent ille ex compositione suorum terminorum. (Sec. 5.27, p. 198)

Section 6

- 44. Depono tibi illam 'alique propositiones non sunt vere'; et significet illa adequate quod alique propositiones non sunt vere. (Sec. 6.2, p. 202)
- 45. Deponatur simpliciter ista 'alique propositiones non sunt vere'. Et sit A propositio una que significet principaliter quod alique propositiones non sunt vere. (Sec. 6.4, p. 204)
- 46. Arguitur deponendo illam 'alique propositiones non sunt vere' et ponendo illam 'omnes propositiones sunt vere' divisive. (Sec. 6.4, p. 204)
- 47. Depono tibi quod propositiones tam adinvicem repugnantes quam que non sunt repugnantes non sunt similes, et quod illa propositio 'aliqua propositio est sibi similis' significet adequate quod aliqua propositio est sibi similis. (Sec. 6.5, p. 204)
- 48. Deponitur illa solum 'tam propositiones invicem repugnantes quam non repugnantes non sunt similes'. Quo admisso proponitur 'deus est'. (Sec. 6.7, p. 206)
- 49. Depono quod alique propositiones tres vel quattuor non sint dissimiles. (Sec. 6.9, p. 208)
- 50. Depono quod alique propositiones non sint dissimiles remota illa particula 'tres vel quattuor'. (Sec. 6.10, p. 208)

OBLIGATIONAL LOGIC SYMBOL LIST

- $A_a = \text{admission} (a \text{ admits} \dots)$
- $C_a = \operatorname{concession}(a \operatorname{grants} \dots)$
- $D_a = \text{doubt} (a \text{ doubts} \dots)$
- $N_a = \text{denial} (a \text{ denies} \dots)$
- G = being a (i) correctly admitted, granted or the negation of a correctly denied sentence or (ii) a conjunction of such sentences
- P = being posited
- R = being put forward
- I = being imposed
- O_a = obligation (a ought to ...)
- U_a = epistemic operator (a understands ...)
- K_a = epistemic operator (a knows ...)

I use the subscript a to indicate the agent who is supposed to perform the act of admitting, granting and so forth. The subscript is often omitted where the context permits. All operators apply to sentences. O has the peculiarity that it applies to sentences of the form Cp, Dp and the like in order to express the duty to perform the corresponding act.

PROPOSITIONAL AND PREDICATE LOGIC SYMBOL LIST

p, q, r, \ldots	=	propositional variables
\Rightarrow	=	formal (strict) conditional
\Leftrightarrow	=	formal biconditional
\wedge	=	conjunction
\vee	=	disjunction
-	=	negation
T	=	truth predicate
\diamond	=	possibility
	=	necessity
\perp	=	contradiction or impossibility
\forall	=	universal quantifier
Ξ	=	existential quantifier
$\bigwedge_{i=0}^{n-1}$	=	conjunction of n sentences

The symbols are also introduced in the text before they are used for the first time. The same holds for other local notations that I may employ in the discussion of sophisms. Bibliography

I. OBLIGATIONS TREATISES

A. MANUSCRIPTS

- [Anon. Obligationes MS Lat. misc.e 79] Anonymous. [Obligationes]. Oxford, Bodleian Library, Lat. misc.e 79, fols. 18^{ra}-24^{ra}.
- [Anon. Obligationes, Angelica 1053] Anonymous. [Obligationes]. Roma, Biblioteca Angelica, 1053, fols. $36^{v}-39^{v}$ (the text is largely the same as that ascribed to Richard Billingham).
- [Peter of Candia, *Obl.*, Mantua] Peter of Candia. *Obligationes*. Mantova, Biblioteca Comunale, 76 (A III 12), fols. 163^{ra}–171^{vb}.
- [Ralph Strode, Obl., Mantua] Ralph Strode. Obligationes. Mantova, Biblioteca Comunale, 76 (A III 12), fols. 116^r-133r.
- [Ralph Strode, Obl. Oxford] Ralph Strode. Obligationes. Oxford, Bodleian Library, Canon. misc. 219, fols. 37^{ra}-47^{ra}.
- [Peter of Mantua, Obl., ms. O] Peter of Mantua. Obligationes. Oxford, Bodleian Library, Canon. misc. 219, fols. 116^{vb}-126^{rb}.
- [Peter of Mantua, Obl., ms. M] Peter of Mantua. Obligationes. Mantova, Biblioteca Comunale, 76 (A III 12), fols. 82^{ra}–94^{va}.
- [Peter of Mantua, Obl., ms. B] Peter of Mantua. Obligationes. Berlin, Staatsbibliothek Preussischer Kulturbesitzt, Hamilton 525, fols. 95^{vb}-102^{ra}.
- [Peter of Mantua, Obl., ms. V] Peter of Mantua. Obligationes. Venezia, Biblioteca Nazionale Marciana, L.VI.128 (2559), fols. 71^{va}-80^{va}.
- [Peter of Mantua, Obl., ms. P] Peter of Mantua. Obligationes. Venezia, Archivio dei Padri Redentoristi di Santa Maria della Fava, 457, fols. 66^{ra}–75^{ra}
- [Peter of Mantua, Obl., ms. L] Peter of Mantua. Obligationes. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Lat. 2135, fols. $67^{vb}-75^{va}$ (fols. 70 and 71 in inverted order).
- [Johannes Venator (?), Obligationes] Johannes Venator (?). [Obligationes]. Erfurt, Wissenschaftliche Allgemeine Bibliothek, Amploniana, Q 270, fols. 39^r-42^v.

B. Early printed texts

- [Albert of Saxony, 1490] Albert of Saxony (1490). *Sophismata*. Georgius Wolff Badensis, Parisius.
- [Anonymous, 1510] Anonymous (1510). De Obligationibus in Libellus Sophistarum ad Usum Cantabrigiensium. Wynandum de Worde, Londoniis. (Oxford, Bodleian Library, 4°S 38 Art. Seld.).

- [Anonymous, 1524] Anonymous (1524). De Obligationibus in Libellus Sophistarum ad Usum Oxoniensium. Richard Pynson, [London]. (Oxford, Bodleian Library, Vet.A I e 60).
- [Marsilius of Inghen, 1489] Marsilius of Inghen (1489). Tractatus de arte obligandi editus a magistro Petro de Alliaco Sacre theologie doctor. Georgium Mittelsbach, Parisius.
- [Peter of Mantua, 1477] Peter of Mantua (1477). Logica. Johannes Herbort, Padua. (London, British Library, IB. 29939).
- [Peter of Mantua, 1483] Peter of Mantua (1483). *Logica*. [Antonius Carcanus for] Hieronymus De Durantibus, Papie. (San Marino, California, Huntington Library, Shelfmark 93567).
- [Peter of Mantua, 1492a] Peter of Mantua (1492a). *Logica*. Bonetus Locatellus, Venetiis. (Oxford, Bodleian Library Auct.2 Q V I).
- [Peter of Mantua, 1492b] Peter of Mantua (1492b). *Logica*. Simon Bevilaqua, Venetiis. (New Haven, Conneticut, Yale University Library, Zi 5380.5).
- [Ralph Strode, 1517] Ralph Strode (1517). Contenta in hoc volumine hec sunt. Consequentie Strodi cum commento Alexandri Sermonete. Declarationes Gaetani in easdem consequentias. Dubia magistri Pauli pergulensis. Obligationes eiusdem Strodi. Consequentie Ricardi de Ferabrich. Expositio Gaetani super easdem. Consequentie subtiles Hentisbari. Questiones in consequentias Strodi perutiles eximii artium doctoris domini Antonii Frachantiani vicentini. Lazarum de Soardis, Venetiis. (Perugia, Biblioteca Comunale Augusta, I, 985, 1).

C. Editions

- [Anonymous Merton College MS. 306, 1985] Anonymous Merton College MS. 306 (1985). De arte obligatoria, in [Kretzmann and Stump, 1985] pp. 239–280.
- [Brinkley, 1995] Brinkley, R. (1995). Obligationes, in [Spade and Wilson, 1995] pp. 12-92.
- [John of Holland, 1985] John of Holland (1985). *Four Tracts on Logic* edited by E. P. Bos. Ingenium, Nijmegen.
- [John of Wesel, 1996] John of Wesel (1996). Three Questions by John of Wesel on *Obligatio*nes and *Insolubilia*. Latin edition with introduction and notes by P. V. Spade. Available at http://pvspade.com/Logic/docs/wesel.pdf.
- [John Tarteys, 1992] John Tarteys (1992). Obligationes, in [Ashworth, 1992] pp. 653–703.
- [John Wyclif, 1893] John Wyclif (1893). *Tractatus de Logica* edited by M. H. Dziewicki, Vol. I. Trübner & Co., London.
- [Martinus Anglicus, 1993] Martinus Anglicus (1993). *De obligationibus*, in [Schupp, 1993] pp. 1–49.
- [Nicholas of Paris (?), 1998] Nicholas of Paris (?) (1998). Obligationes, in [Braakhuis, 1998] pp. 170–233.
- [Obligationes Parisienses, 1975] Obligationes Parisienses (1975). In [De Rijk, 1975b] pp. 22–54.
 [Paul of Pergula, 1961] Paul of Pergula (1961). Logica and Tractatus de Sensu Composito et Diviso, edited by Sister M. A. Brown. The Franciscan Institute, St. Bonaventure, N.Y.

- [Paul of Venice, 1988] Paul of Venice (1988). Logica magna, Part. II, Fascicule 8: Tractatus de obligationibus, edited with an English Translation and Notes by E. J. Ashworth. Published for the British Academy by the Oxford University Press, Oxford.
- [Paul of Venice, 2002] Paul of Venice (2002). *Logica Parva* First Critical Edition of the Latin Text from the Manuscripts with Introduction and Commentary by A. Perreiah. Brill, Leiden.
- [Richard Lavenham, 1978] Richard Lavenham (1978). Obligationes, in [Spade, 1978] pp. 226–241.[Robert Fland, 1980] Robert Fland (1980). Obligationes, in [Spade, 1980] pp. 41–60.
- [Roger Swyneshed, 1977] Roger Swyneshed (1977). Obligationes, in [Spade, 1977] pp. 249–285.
- [Tractatus Emmeranus de falsi positione, 1974] Tractatus Emmeranus de falsi positione (1974). in [De Rijk, 1974] pp. 103–117; an English translation by M. Yrjönsuuri is found in [Yrjönsuuri 2001b] pp. 199–205.
- [Tractatus Emmeranus de impossibili positione, 1974] Tractatus Emmeranus de impossibili positione (1974). In [De Rijk, 1974] pp. 117–123; an English translation by M. Yrjönsuuri is found in [Yrjönsuuri 2001b] pp. 199–205.
- [Walter Burley, 1963] Walter Burley (1963). *Tractatus de Obligationibus* in [Green, 1963], vol. II, pp. 34–96.
- [William Buser, 1990] William Buser (1990). Obligationes in [Pozzi, 1990] pp. 66–248.
- [William Heytesbury, 1994] William Heytesbury (1994). Sophismata Asinina. Une introduction aux disputes médiévales. Présentation, édition critique et analyse par Fabienne Pironet. Vrin, Paris.
- [William of Ockham, 1974] William of Ockham (1974). Summa Logicae edited by P. Boehner, G. Gál and S. Brown. Opera Philosophica I. The Franciscan Institute, St. Bonaventure, N.Y.
 [William of Sherwood (?), 1963] William of Sherwood (?) (1963). Tractatus de Obligationibus, in [Green, 1963], vol. II, pp. 1–33.

II. OTHER SOURCES

- [A. M. S. Boethius, 1969] A. M. S. Boethius (1969). De hypotheticis syllogismis. ed. L. Orbetello. Paideia, Brescia.
- [Amerini, 2006] Amerini, F. (2006). La logica di Francesco da Prato con l'edizione critica della Loyca e del Tractatus de voce univoca. SISMEL Edizioni del Galluzzo, Firenze.
- [Angelelli, 1970] Angelelli, I. (1970). The Techniques of Disputation in the History of Logic. Journal of Philosophy, 67:800–815.
- [Angelelli, 1983] Angelelli, I. (1983). Review of L. M. De Rijk. Die mittelalterliche Traktate De modo opponendi et respondendi. Journal of the History of Philosophy, 21:249–250.
- [Aristotle, 1984] Aristotle (1984). *The complete works of Aristotle*. The revised Oxford translation, ed. J. Barnes. 2 Vols. Princeton University Press, Princeton, NJ.

- [Ashworth, 1972] Ashworth, E. J. (1972). The Treatment of Semantic Paradoxes from 1400 to 1700. Notre Dame Journal of Formal Logic, 13:34–52.
- [Ashworth, 1974a] Ashworth, E. J. (1974a). For Riding is Required A Horse: A Problem of Meaning and Reference in Late Fifteenth and Early Sixteenth Century Logic. Vivarium, 12:146–172.
- [Ashworth, 1974b] Ashworth, E. J. (1974b). Language and Logic in the Post-Medieval Period. Reidel, Dordrecht.
- [Ashworth, 1974c] Ashworth, E. J. (1974c). Some Additions to Risse's *Bibliographia Logica*. Journal of the History of Philosophy, 12(3):361–365.
- [Ashworth, 1976a] Ashworth, E. J. (1976a). I Promise You A Horse: A Second Problem of Meaning and Reference in Late Fifteenth and Early Sixteenth Century Logic (first part). Vivarium, 14:62–79.
- [Ashworth, 1976b] Ashworth, E. J. (1976b). I Promise You A Horse: A Second Problem of Meaning and Reference in Late Fifteenth and Early Sixteenth Century Logic (second part). Vivarium, 14:139–155.
- [Ashworth, 1979] Ashworth, E. J. (1979). The "Libelli Sophistarum" and the Use of Medieval Logic Texts at Oxford and Cambridge in the Early Sixteenth Century. *Vivarium*, 17(2):134–158.
- [Ashworth, 1981] Ashworth, E. J. (1981). The Problems of Relevance and Order in Obligational Disputations: Some Late Fourteenth Century Views. *Medioevo*, 7:175–193.
- [Ashworth, 1982] Ashworth, E. J. (1982). The Eclipse of Medieval Logic. In [Kretzmann et al., 1982], pages 787–796.
- [Ashworth, 1984] Ashworth, E. J. (1984). Inconsistency and Paradox in Medieval Disputations:
 A Development of some Hints in Ockham. *Franciscan Studies*, 44:129–139.
- [Ashworth, 1985] Ashworth, E. J. (1985). English *Obligationes* texts after Roger Swyneshed. The Tracts beginning *Obligatio est quaedam ars.* In [Lewry, 1985], pages 309–333.
- [Ashworth, 1986] Ashworth, E. J. (1986). Renaissance Man as Logician: Josse Clichtove (1472-1543) on Disputations. *History and Philosophy of Logic*, 7:15–29.
- [Ashworth, 1988a] Ashworth, E. J. (1988a). Paul of Venice, *Logica magna, Part. II, Fascicule 8: Tractatus de obligationibus*, edited with an English Translation and Notes by E. J. Ashworth. Published for the British Academy by the Oxford University Press, Oxford.
- [Ashworth, 1988b] Ashworth, E. J. (1988b). Traditional Logic. In Skinner, Q., Kessler, E., and Kraye, J., editors, The Cambridge History of Renaissance Philosophy, pages 143–172. Cambridge University Press, Cambridge.
- [Ashworth, 1989] Ashworth, E. J. (1989). La sémantique du XIV^e siècle vue à travers cinq traités oxoniens sur les *Obligationes*. Université du Québec à Montréal. *Cahiers d'épistémologie* no. 8915.
- [Ashworth, 1990] Ashworth, E. J. (1990). Paul of Venice on Obligations. The sources for both the *Logica Magna* and the *Logica Parva* versions. In [Knuuttila et al., 1990], pages 407–415.

- [Ashworth, 1992] Ashworth, E. J. (1992). The *Obligationes* of John Tarteys: Edition and Introduction. *Documenti e Studi sulla Tradizione Filosofica Medievale*, 3:653–703.
- [Ashworth, 1993] Ashworth, E. J. (1993). Ralph Strode on Inconsistency in Obligational Disputations. In [Jacobi, 1993], pages 363–386.
- [Ashworth, 1994] Ashworth, E. J. (1994). Obligationes Treatises: A Catalogue of Manuscripts, Editions and Studies. Bulletin de philosophie médiévale, 36:118–147.
- [Ashworth, 1996] Ashworth, E. J. (1996). Autour des Obligationes de Roger Swineshed: la Nova responsio. Les Études Philosophiques, 3:341–360.
- [Ashworth, 1999] Ashworth, E. J. (1999). Text-books: a case study logic. In Hellinga, L. and Trapp, J. B., editors, *The Cambridge History of the Book in Britain (Vol.III 1400–1557)*, pages 380–386. Cambridge University Press, Cambridge.
- [Ashworth, 2000] Ashworth, E. J. (2000). Domingo de Soto on Obligationes: His Use of Dubie Positio. In Angelelli, I. and Perez-Ilzarbe, P., editors, Medieval and Renaissance Logic in Spain. Acts of the 12th European Symposium on Medieval Logic and Semantics, Pamplona, 26-30 May 1997, pages 291–307, Hildesheim. Georg Olms.
- [Ashworth and Spade, 1992] Ashworth, E. J. and Spade, P. V. (1992). Logic in Late Medieval Oxford. In Catto, J. I. and Evans, R., editors, *The History of the University of Oxford*, Vol. II *Late Medieval Oxford*, pages 35–64. Clarendon Press, Oxford.
- [Baccin, 1977] Baccin, N. A. (1977). Supposizione confusa tantum e descensus. Medioevo, 3:285– 300.
- [Bertagna, 2000] Bertagna, M. (2000). La dottrina delle conseguenze nella *Logica* di Pietro da Mantova. *Documenti e Studi sulla Tradizione Filosofica Medievale*, 11:459–496.
- [Beuchot, 1988] Beuchot, M. (1988). El tratado de las obligaciones dialógica en la Edad Media: el caso de Robert Fland. *Diánoia*, 11:169–179.
- [Bochenski, 1961] Bochenski, J. M. (1961). *History of Formal Logic*. University of Notre Dame Press, Notre Dame.
- [Boehner, 1952] Boehner, P. (1952). Medieval Logic: An Outline of the Development from 1250 to c. 1400. Manchester University Press, Manchester.
- [Boethius of Dacia, 1976] Boethius of Dacia (1976). Quaestiones super librum Topicorum edited by N. J. Green-Pedersen and J. Pinborg (Corpus Philosophorum Danicorum Medii Aevi, vol. VI). Gad, København.
- [Boh, 1982] Boh, I. (1982). Consequences. In [Kretzmann et al., 1982], pages 300-314.
- [Boh, 1990] Boh, I. (1990). On Medieval Rules of Obligation and Rules of Consequence. In Angelelli, I. and D'Ors, A., editors, *Estudios de Historia de Lógica. Actas del II Simposio de Historia de Lógica, Universidad de Navarra, Pamplona, 25-27 de Mayo de 1987*, pages 39–102, Pamplona. Eunate.
- [Boh, 1993] Boh, I. (1993). Epistemic Logic in Late Middle Ages. Routledge, London.
- [Boh, 2000] Boh, I. (2000). Four Phases of Medieval Epistemic Logic. Theoria, 56(2):129-144.

[Bos, 1981] Bos, E. P. (1981). Review of L. M. De Rijk. Die mittelalterliche Traktate De modo opponendi et respondendi. Vivarium, 19:152–155.

[Bos, 1982] Bos, E. P. (1982). Peter of Mantua's Tract on Appellatio and his Interpretation of Immanent Forms. In [Maierù, 1982a], pages 231–252.

- [Bos, 1985a] Bos, E. P. (1985a). Peter of Mantua and his Rejection of Ampliatio and Restrictio. In [Lewry, 1985], pages 381–399.
- [Bos, 1985b] Bos, E. P. (1985b). Peter of Mantua's Treatise De veritate et falsitate, sive De taliter et qualiter. In E. P. Bos, editor, Medieval Semantics and Metaphysics: Studies Dedicated to L. M. de Rijk, Ph.D., Professor of Ancient and Mediaeval Philosophy at the University of Leiden on the Occasion of His 60th Birthday, pages 291–312. Ingenium, Nijmegen.
- [Bottin, 1983] Bottin, F. (1983). Logica e filosofia naturale nelle opere di Paolo Veneto. In Poppi,
 A., editor, Scienza e Filosofia all'Università di Padova nel Quattrocento. Lint, Trieste.
- [Braakhuis, 1982] Braakhuis, H. A. G. (1982). Paul of Pergula's Commentary on the *Sophismata* of William Heytesbury. In [Maierù, 1982a], pages 343–357.
- [Braakhuis, 1993] Braakhuis, H. A. G. (1993). Albert of Saxony's *De obligationibus*: Its Place in the Development of Fourteenth Century Obligational Theory. In [Jacobi, 1993], pages 323–341.
- [Braakhuis, 1998] Braakhuis, H. A. G. (1998). Obligations in Early Thirteenth Century Paris: The Obligationes of Nicholas of Paris (?) (Ms. Paris B.N.lat., 11.412). *Vivarium*, 36:152–233.
- [Brown, 1966] Brown, M. A. (1966). The Role of the *Tractatus de obligationibus* in Medieval Logic. *Franciscan Studies*, 26:26–55.
- [Buonocore, 2000] Buonocore, M. (2000). Per un *iter* tra i codici di Seneca alla Biblioteca Apostolica Vaticana: primi traguardi. *Giornale Italiano di Filologia*, 52:17–100.
- [Buridan, 1966] Buridan, J. (1966). Sophisms on Meaning and Truth. Translated and with an introduction by T. K. Scott. Appleton Century Crofts, New York.
- [Buzzetti and Tabarroni, 1992] Buzzetti, D. and Tabarroni, A., editors (1992). L'insegnamento della Logica a Bologna nel XIV Secolo, Bologna. Istituto per la Storia dell'Università.
- [Courtenay, 1982] Courtenay, W. J. (1982). The Early Stages in the Introduction of Oxford Logic into Italy. In [Maierù, 1982a], pages 13–32.
- [Dallari, 1889] Dallari, U. (1888–1889). I rotuli dei lettori e artisti dello Studio bolognese dal 1384 al 1799. Regia Tip. fratelli Merlani, Bologna.
- [De Libera, 1982] De Libera, A. (1982). Apollinaire Offredi critique de Pierre de Mantoue: le *Tractatus de instanti* et la logique du changement. In [Maierù, 1982a], pages 253–292.
- [De Libera, 1990] De Libera, A. (1990). La logique de la discussion dans l'université médiévale. In Meyer, M. and Lempereur, A., editors, *Figures et conflits rhétoriques*, pages 59–81. Éditions de l'Université de Bruxelles, Bruxelles.
- [De Rijk, 1962] De Rijk, L. M. (1962). Logica Modernorum. A Contribution to the History of Early Terminist Logic. Vol. 1: On the Twelfth Century Theory of Fallacies. Van Gorcum, Assen.

- [De Rijk, 1967] De Rijk, L. M. (1967). Logica Modernorum. A Contribution to the History of Early Terminist Logic. Vol. 2: The Origin and Early Development of the Theory of Supposition. Van Gorcum, Assen.
- [De Rijk, 1974] De Rijk, L. M. (1974). Some Thirteenth Century Tracts on the Game of Obligations I. *Vivarium*, 12:94–123.
- [De Rijk, 1975a] De Rijk, L. M. (1975a). Logica Cantabrigiensis a fifteenth century Cambridge manual of logic. Revue internationale de philosophie, 29:297–315.
- [De Rijk, 1975b] De Rijk, L. M. (1975b). Some Thirteenth Century Tracts on the Game of Obligations II. *Vivarium*, 13:22–54.
- [De Rijk, 1976] De Rijk, L. M. (1976). Some Thirteenth Century Tracts on the Game of Obligations III. *Vivarium*, 14:26–49.
- [De Rijk, 1977] De Rijk, L. M. (1977). Logica Oxoniensis. An attempt to Reconstruct a Fifteenth Century Oxford Manual of Logic. Medioevo, 3:121–164.
- [De Rijk, 1980] De Rijk, L. M. (1980). *Die mittelalterliche Traktate* De modo opponendi et respondendi: *Einleitung und Ausgabe der einschlägigen Texte*. Aschendorff, Münster.
- [Dionisotti, 1955] Dionisotti, C. (1955). Ermolao Barbaro e la fortuna di Suisseth. In *Medioevo e Rinascimento, Studi in onore di Bruno Nardi*, volume I, pages 218–253. Sansoni, Firenze.
- [D'Ors, 1988a] D'Ors, A. (1988a). Sobre el tratado anónimo De Arte Obligatoria recogido en el MS. 306 del Merton College. Philosophica, 11:169–185.
- [D'Ors, 1988b] D'Ors, A. (1988b). Sobre las *Obligationes* de Juan de Holanda. *Anuario Filosófico*, 21:33–70.
- [D'Ors, 1990a] D'Ors, A. (1990a). Ex impossibili quodlibet sequitur (Walter Burley). Archives d'Histoire doctrinale et litéraire du moyen âge, 57:121–154.
- [D'Ors, 1990b] D'Ors, A. (1990b). On Stump's Interpretation of Burley's De Obligationibus. In [Knuuttila et al., 1990], pages 468–478.
- [D'Ors, 1991a] D'Ors, A. (1991a). Tu scis regem sedere (Kilvington, S47 [48]). Anuario Filosófico, 24:49–74.
- [D'Ors, 1991b] D'Ors, A. (1991b). Sobre la *Obligationes* de Richard Lavenham. Archives d'Histoire doctrinale et litéraire du moyen âge, 58:253–278.
- [D'Ors, 1991c] D'Ors, A. (1991c). Un problema de interpretación en el *De Obligationibus* de Walter Burley. *Revista de Filosofía* (Universidad de Zulia, Maracaibo, Venezuela), 15:33–42.
- [D'Ors, 1993] D'Ors, A. (1993). Sortes non currit vel Sortes movetur (Roger Swyneshed, Obligationes, 137–138). Archives d'Histoire doctrinale et litéraire du moyen âge, 60:165–172.
- [D'Ors, 2000] D'Ors, A. (2000). "Dubium Proponitur: Andrés Limos and the Treatise on Obligations. In Angelelli, I. and Perez Ilzarbe, P., editors, Medieval and Renaissance Logic in Spain. Acts of the 12th European Symposium on Medieval Logic and Semantics, Pamplona, 26-30 May 1997, pages 224–249, Hildesheim. Georg Olms.

- [D'Ors and García-Clavel, 1994] D'Ors, A. and García-Clavel, M. (1994). Sobre las Obligationes de Robert Fland. Antiqua et nova responsio. Revista de Filosofía, 7:51–58.
- [Duhem, 1913] Duhem, P. (1913). Études sur Léonard de Vinci (Vol. III). Hermann, Paris.
- [Dukes, 1973] Dukes, E. N. (1973). Alexander Sermoneta's commentary on Heytesbury and Strode: a contribution to late medieval logic in England and Italy. Ph. D. Thesis – Columbia University; Ann Arbor, Michigan: University Microfilms.
- [Dumitriu, 1977] Dumitriu, A. (1977). History of Logic. Abacus Press, Tunbridge Wells, Kent.
- [Dutilh Novaes, 2005] Dutilh Novaes, C. (2005). Medieval *Obligationes* as Logical Games of Consistency Maintenance. *Synthese*, 145(3):371–395.
- [Dutilh Novaes, 2006a] Dutilh Novaes, C. (2006a). Ralph Strode's *obligationes*: the return of consistency and the epistemic turn. *Vivarium*, 44(2–3):338–374.
- [Dutilh Novaes, 2006b] Dutilh Novaes, C. (2006b). Roger Swyneshed's *obligationes*: a logical game of inference recognition? *Synthese*, 151(1):127–155.
- [Dutilh Novaes, 2007] Dutilh Novaes, C. (2007). Formalizing Medieval Logical Theories: Suppositio, Consequentiae and Obligationes. Springer, Dordrecht.
- [Dutilh Novaes, 2008] Dutilh Novaes, C. (2008). Logic in the 14th Century after Ockham. In Gabbay, D. M. and Woods, J., editors, *Handbook of the History of Logic (Vol. II Medieval and Renaissance Logic)*, pages 434–500. Brill, Leiden.
- [Ekenberg, 2002] Ekenberg, T. (2002). Order in Obligational Disputations. Disputatio: An International Transdisciplinary Journal of the Late Middle Ages, 5:23–29.
- [Etchemendy, 1990] Etchemendy, J. (1990). *The Concept of Logical Consequence*. Harvard University Press, Cambridge, Mass.
- [Federici Vescovini, 1981] Federici Vescovini, G. (1981). À la recherche du mystérieux Buser. À propos de trois copies manuscrites de la même œuvre: le *De obligationibus* de "Buser" ou "Busel". In Braakhuis, H. A. G., Kneepkens, C. H., and De Rijk, L. M., editors, *English Logic* and Semantics from the End of the 12th Century to the Time of Ockham and Burleigh, pages 443–457, Nijmegen. Ingenium.
- [Garin, 1948] Garin, E. (1948). [Note on P. O. Kristeller Humanism and Scholasticism in the Italian Renaissance]. Giornale critico della filosofia italiana, 27:203–204 and 389–390.
- [Garin, 1960] Garin, E. (1960). La cultura fiorentina nella seconda metà del '300 e i 'barbari britanni'. La rassegna della letteratura italiana, 64:181–195.
- [Gelber, 2004] Gelber, H. G. (2004). It Could Have Been Otherwise. Contingency and Necessity in Dominican Theology at Oxford 1300-1350. Brill, Leiden.
- [Gloria, 1888] Gloria, A. (1888). Monumenti dell'Università di Padova (1318–1405) Vol. II. Antonelli, Padova.
- [Green, 1963] Green, R. (1963). An Introduction to the Logical Treatise *De Obligationibus*: with critical Texts of William of Sherwood (?) and Walter Burley (2 Vols.). Unpublished Ph. D. Thesis, Université Catholique de Louvain.

- [Green-Pedersen, 1984] Green-Pedersen, N. J. (1984). The Tradition of the Topics in the Middle Ages. Philosophia Verlag, München.
- [Hain, 1838] Hain, L. (1826–1838). Repertorium Bibliographicum in quo libri omnes ab arte typographica inventa usque ad annum MD typis expressi ordine alphabetico vel simpliciter enumerantur vel adcuratius recensentur (2 Vols.). J. G. Cotta, Stuttgartiae et Lutetiae.
- [Hamblin, 1970] Hamblin, C. L. (1970). Fallacies. Methuen, London.
- [Heinzer, 1983] Heinzer, F. (1983). Textkritisches zu den sog. Obligationes Parisienses. *Vivarium*, 21:127–135.
- [Hintikka and Sandu, 1997] Hintikka, J. and Sandu, G. (1997). Game-Theoretical Semantics. In van Benthem, J. and ter Meulen, A., editors, *Handbook of Logic and Language*, pages 361–410. Elsevier, Amsterdam.
- [Hodges, 2009] Hodges, W. (2009). Logic and Games. Available at http://plato.stanford.edu/entries/logic-games/.
- [Jacobi, 1993] Jacobi, K., editor (1993). Argumentationstheorie. Scholastische Forschungen zu den logischen und semantischen Regeln korrekten Folgerns, Leiden. Brill.
- [James, 1968] James, T. E. (1968). *De primo et ultimo instanti Petri Alboini Mantuani*. Edited with an introduction, analysis and notes. Ph. D. Thesis Columbia University; Ann Arbor, Michigan: University Microfilms.
- [James, 1974] James, T. E. (1974). Peter Alboini of Mantua: Philosopher–Humanist. Journal of the History of Philosophy, 12(2):161–170.
- [James, 1976] James, T. E. (1976). A fragment of An exposition of the first letter of Seneca to Lucilius attributed to Peter of Mantua. In Mahoney, E. P., editor, Philosophy and Humanism: Renaissance essays in honor of P. O. Kristeller, pages 531–541. Brill, Leiden.
- [Johannes Venator, 1999] Johannes Venator (1999). Johannes Venator Anglicus. Logica. Edited by L. M. De Rijk. Frommann-Holzboog, Stuttgart, Bad Cannstatt.
- [Keffer, 2001] Keffer, H. (2001). De Obligationibus Rekonstruktion einer spätmittelalterlicher Disputationstheorie. Brill, Leiden.
- [Keiff, 2009] Keiff, L. (2009). Dialogical Logic. Available at http://plato.stanford.edu/entries/logic-dialogical/.
- [King, 1991] King, P. (1991). Mediaeval Thought Experiments. The Metamethodology of Mediaeval Science. In Horowitz, T. and Massey, G. J., editors, *Thought Experiments in Science* and *Philosophy*, pages 43–64. Rowman and Littlefield, Savage.
- [King, 2004] King, P. (2004). Opposing and Responding: Comments on Paul Spade. Available at http://individual.utoronto.ca/pking/presentations/Spade_Comments.pdf.
- [Kneale and Kneale, 1962] Kneale, W. and Kneale, M. (1962). *The Development of Logic*. Clarendon Press, Oxford.
- [Kneepkens, 1982] Kneepkens, C. H. (1982). The Mysterious Buser Again: William Buser of Heusden and the *Obligationes* Tract *Ob rogatum*. In [Maierù, 1982a], pages 147–166.

- [Kneepkens, 1993] Kneepkens, C. H. (1993). William Buser of Heusden's Obligationes-Treatise: Ob rogatum: A Ressourcement in the Doctrine of Logical Obligation? In [Jacobi, 1993], pages 344–362.
- [Knuuttila, 1989] Knuuttila, S. (1989). Modalities in Obligational Disputations. In Corsi, G., Mangione, C., and Mugnai, M., editors, Atti del Convegno Internazionale di Storia della Logica. Le Teorie delle modalità, San Gimignano, 5-8 dicembre 1987, pages 79–92, Bologna. CLUEB.
- [Knuuttila, 1993] Knuuttila, S. (1993). Trinitarian Sophisms in Robert Holcot's Theology. In [Read, 1993b], pages 348–356.
- [Knuuttila, 1997] Knuuttila, S. (1997). Positio impossibilis in medieval discussions of the Trinity. In Marmo, C., editor, Vestigia, Imagines, Verba: Semiotics and Logic in Medieval Theological Texts (XIIth-XIVth Century), pages 277–288. Brepols, Turnhout.
- [Knuuttila et al., 1990] Knuuttila, S., Työrinoja, R., and Ebbesen, S., editors (1990). Knowledge and the Sciences in Medieval Philosophy. Proceedings of the 8th International Congress of Medieval Philosophy, Helsinki, 24-29 August 1987, Helsinki. Yliopistopaino.
- [Knuuttila and Yrjönsuuri, 1988] Knuuttila, S. and Yrjönsuuri, M. (1988). Norms and Actions in Obligational Disputations. In Pluta, O., editor, *Die Philosophie im 14. und 15. Jahrhundert*, pages 191–202. Grüner, Amsterdam.
- [Kretzmann, 1982] Kretzmann, N. (1982). Syncathegoremata, sophismata, exponibilia. In [Kretzmann et al., 1982], pages 211–245.
- [Kretzmann et al., 1982] Kretzmann, N., Kenny, A., and Pinborg, J., editors (1982). *The Cambridge History of Later Medieval Philosophy*, Cambridge. Cambridge University Press.
- [Kretzmann and Kretzmann, 1990] Kretzmann, N. and Kretzmann, B. E. (1990). *The Sophismata of Richard Kilvington*. Cambridge University Press, Cambridge.
- [Kretzmann and Stump, 1985] Kretzmann, N. and Stump, E. (1985). The Anonymous De arte obligatoria in Merton College MS. 306. In E. P. Bos, editor, Medieval Semantics and Metaphysics: Studies Dedicated to L. M. de Rijk, Ph.D., Professor of Ancient and Mediaeval Philosophy at the University of Leiden on the Occasion of His 60th Birthday, pages 239–280. Ingenium, Nijmegen.
- [Kretzmann and Stump, 1988] Kretzmann, N. and Stump, E. (1988). Translation of Walter Burley, Obligations (selections) and William Heytesbury, Excursus on the Art of Obligation. In The Cambridge Translation of Medieval Philosophical Texts (Vol. I, Logic and the Philosophy of Language, pages 370–412 and 432–434. Cambridge University Press, Cambridge.
- [Kristeller, 1956] Kristeller, P. O. (1956). Humanism and Scholasticism in the Italian Renaissance. In *Studies in Renaissance Thought and Letters*, pages 553–583. Edizioni di Storia e Letteratura, Roma.
- [Kristeller, 1963] Kristeller, P. O. (1963). Iter Italicum: A Finding List of Uncatalogued or Incompletely Catalogued Humanistic Manuscripts of the Renaissance in Italian and Other Libraries Vol. I, Italy: Agrigento to Novara. The Warburg Institute, London.

- [Kristeller, 1967] Kristeller, P. O. (1967). Iter Italicum: A Finding List of Uncatalogued or Incompletely Catalogued Humanistic Manuscripts of the Renaissance in Italian and Other Libraries Vol. II, Italy: Orvieto to Volterra, Vatican City. The Warburg Institute, London.
- [Kristeller, 1985a] Kristeller, P. O. (1985a). The Impact of Early Italian Humanism on Thought and Learning. In *Studies in Renaissance Thought and Letters. Vol. II*, pages 65–91. Edizioni di Storia e Letteratura, Roma.
- [Kristeller, 1985b] Kristeller, P. O. (1985b). The University of Bologna and the Renaissance. In Studies in Renaissance Thought and Letters. Vol. II, pages 135–146. Edizioni di Storia e Letteratura, Roma.
- [Kristeller, 1996a] Kristeller, P. O. (1996a). In Search of Renaissance Manuscripts. In Studies in Renaissance Thought and Letters. Vol. IV, pages 465–480. Edizioni di Storia e Letteratura, Roma.
- [Kristeller, 1996b] Kristeller, P. O. (1996b). The Curriculum of the Italian Universities from the Middle Ages to the Renaissance. In *Studies in Renaissance Thought and Letters. Vol. IV*, pages 75–98. Edizioni di Storia e Letteratura, Roma.
- [Lagerlund and Olsson, 2001] Lagerlund, H. and Olsson, E. (2001). Disputation and Change of Belief – Burley's Theory of *Obligationes* as a Theory of Belief-Revision. In [Yrjönsuuri, 2001b], pages 35–62.
- [Lewis, 1973] Lewis, D. K. (1973). Counterfactuals. Harvard University Press, Cambridge, Mass.
- [Lewry, 1985] Lewry, O., editor (1985). The rise of British Logic. Acts of the 6th European Symposium on Medieval Logic and Semantics, Balliol College, Oxford, 19-24 June 1983, Toronto. Pontifical Institute of Mediaeval Studies.
- [Lockwood, 1951] Lockwood, D. P. (1951). Ugo Benzi Medieval Philosopher and Physician (1376-1439). The University of Chicago Press, Chicago, Ill.
- [Lorenzen and Lorenz, 1978] Lorenzen, P. and Lorenz, K. (1978). *Dialogische Logik*. Wissenschaftliche Buchgesellschaft, Darmstadt.
- [Maierù, 1972] Maierù, A. (1972). *Terminologia logica della tarda scolastica*. Edizioni dell'Ateneo, Roma.
- [Maierù, 1974] Maierù, A. (1974). Il problema del significato nella logica di Pietro da Mantova. In Zimmermann, A., editor, *Miscellanea Medievalia Bd. 9*. Antiqui und Moderni. Traditionsbewusstsein und Fortschrittsbewusstsein im späten Mittelalter, pages 155–170. de Gruyter, Berlin.
- [Maierù, 1982a] Maierù, A., editor (1982a). English Logic in Italy in the 14th and 15th Centuries. Acts of the 5th European Symposium on Medieval Logic and Semantics, Rome 10-14 November 1980, Napoli. Bibliopolis.
- [Maierù, 1982b] Maierù, A. (1982b). Le ms. Oxford, Canonici misc. 219 et la *Logica* de Strode. In [Maierù, 1982a], pages 87–110.

[Marenbom, 1987] Marenbom, J. (1987). Later Medieval Philosophy (1150–1350) – An Introduction. Routledge, London.

[Martin, 1990] Martin, C. J. (1990). Bradwardine and the Use of *Positio* as a test of possibility. In [Knuuttila et al., 1990], pages 574–585.

- [Martin, 1992] Martin, C. J. (1992). The Logic of the *Nominales*, or The Rise and Fall of Impossible *Positio*. *Vivarium*, 30:110–126.
- [Martin, 1993] Martin, C. J. (1993). Obligations and Liars. In [Read, 1993b], pages 357–381.
- [Martin, 1997] Martin, C. J. (1997). Impossible positio as the foundation of metaphysics or, logic on the scotist plan? In Marmo, C., editor, Vestigia, Imagines, Verba: Semiotics and Logic in Medieval Theological Texts (XIIth-XIVth Century), pages 255–276, Turnhout. Brepols.
- [Martin, 2001] Martin, C. J. (2001). Obligations and Liars. In [Yrjönsuuri, 2001b], pages 63–94.
- [Moody, 1953] Moody, E. (1953). Truth and Consequence in Medieval Logic. North Holland, Amsterdam.
- [Mullally, 1964] Mullally, J. P. (1964). Peter of Spain: Tractatus syncategorematum and selected anonymous treatises. Marquette University Press, Milwaukee.
- [Müller, 1994] Müller, P. (1994). Le Obligationes nella Summa logicae di Guglielmo di Ockham. In Bianchi, L., editor, Filosofia e teologia nel trecento: Studi in ricordo di Eugenio Randi, pages 85–104. FIDEM, Louvain-la-Neuve.
- [Murdoch, 1975] Murdoch, J. E. (1975). From Social into Intellectual Factors: An Aspect of the Unitary Character of Late Medieval Learning. In Murdoch, J. E. and Sylla, E. D., editors, *The Cultural Context of Medieval Learning*, pages 271–339. Reidel, Dordrecht.
- [Murdoch, 1981] Murdoch, J. E. (1981). Scientia mediantibus vocibus: Metalinguistic Analysis in Late Medieval Natural Philosophy. In Zimmermann, A., editor, Miscellanea Medievalia Bd. 13. Sprache und Erkenntniss im Mittelalter, pages 73–106. de Gruyter, Berlin.
- [Novati, 1911] Novati, F., editor (1891–1911). *Epistolario di Coluccio Salutati* (Vol. III), Roma. Forzani e C.
- [Nuchelmans, 1982] Nuchelmans, G. (1982). The semantics of propositions. In [Kretzmann et al., 1982], pages 197–210.
- [Olsson, 2006] Olsson, E. J. (2006). Walter Burley's *Obligationes* als Theorie kontrafaktischen Räsonierens. Available at http://www.fil.lu.se/publicationfiles/pp67.pdf.
- [Paul of Venice, 1978] Paul of Venice (1978). Logica magna, Part. II, Fascicule 6: Tractatus de veritate et falsitate propositionis et Tractatus de significato propositionis, edited with notes on the sources by F. Del Punta. Translated into English with explanatory notes by M. McCord Adams. Published for the British Academy by the Oxford University Press, Oxford.
- [Perreiah, 1978] Perreiah, A. (1978). Insolubilia in the Logica Parva of Paul of Venice. Medioevo, 4:145–171.
- [Perreiah, 1982] Perreiah, A. (1982). *Obligationes* in Paul of Venice's *Logica Parva. Analecta Augustiniana*, 45:89–116.

- [Perreiah, 1984] Perreiah, A. (1984). Logic examinations in Padua *circa* 1400. *History of Education*, 13:85–103.
- [Perreiah, 1986] Perreiah, A. (1986). *Paul of Venice: A Bibliographical guide*. Philosophy Documentation Center, Bowling Green State University, Bowling Green, Ohio.
- [Peter Abelard, 1970] Peter Abelard (1970). Dialectica. ed. L. M. De Rijk. Van Gorcum, Assen. [Pironet, 1994] Pironet, F. (1994). Guillaume Heytesbury: Sophismata Asinina. Une introduction aux disputes médiévales. Présentation, édition critique et analyse. J. Vrin, Paris.
- [Pironet, 2001] Pironet, F. (2001). The Relations between Insolubles and Obligations in Medieval Disputations. In [Yrjönsuuri, 2001b], pages 95–114.
- [Pozzi, 1978] Pozzi, L. (1978). Le consequentiae nella logica medievale. Liviana, Padova.
- [Pozzi, 1990] Pozzi, L. (1990). La coerenza logica nella teoria medievale delle obbligazioni. Con l'edizione del trattato 'Obligationes' di Guglielmo Buser. Zara, Parma.
- [Pozzi, 1991] Pozzi, L. (1991). Il tempo e il valore di verità delle proposizioni possibili nella teoria medievale delle obbligazioni. *Medioevo*, 17:281–308.
- [Pozzi, 1994] Pozzi, L. (1994). Coerenza logica e realtà nella teoria delle obligationes medievali. Philo-logica. Rassegna di analisi linguistica ed ironia culturale, 3:19–27.
- [Prantl, 1870] Prantl, C. (1870). Geschichte der Logik im Abendlande. Bd. IV. Verlag Von S. Hirzel, Leipzig.
- [Prior, 1969] Prior, A. (1969). The possibly-true and the possible. Mind, 78:481–492.
- [Pütz, 1997] Pütz, C. (1997). Die Obligationenlehre in der Scholastischen Logik: Untersuchungen zum Einfluß der Stoischen Logik auf die Lehre von den Verpflichtungen (de obligationibus). Unpublished Ph. D. Thesis, Universität Düsseldorf.
- [Quinto, 2006] Quinto, R. (2006). Manoscritti medievali nella Biblioteca dei Redentoristi a Venezia. Il Poligrafo, Padova.
- [Rahman and Tulenheimo, 2006] Rahman, S. and Tulenheimo, T. (2006). From games to dialogues and back: towards a general frame for validity. In Majer, O., Pietarinen, A., and Tulenheimo, T., editors, *Games: Unifying Logic, Language, and Philosophy*, pages 153–208. Springer, Dordrecht.
- [Read, 1993a] Read, S. (1993a). Formal and Material Consequence. In [Jacobi, 1993], pages 233– 259.
- [Read, 1993b] Read, S., editor (1993b). Sophisms in Medieval Logic and Grammar. Acts of the 9th European Symposium on Medieval Logic and Semantics held at St Andrews, June 1990, Dordrecht. Kluwer.
- [Read, 1995] Read, S. (1995). Thinking About Logic. An Introduction to the Philosophy of Logic. Oxford University Press, Oxford.
- [Restori, 1904] Restori, M. C. (1904). Libreria del maestro Agostino Santucci. *Rivista delle Biblioteche e degli Archivi*, 15:35–37.

- [Richter, 1990] Richter, V. (1990). Zu De obligationibus in der Summa Logicae. In Vossenkuhl,
 W. and Schönberger, R., editors, Die Gegenwart Ockhams, pages 256–261. VHC, Weinheim.
- [Risse, 1965] Risse, W. (1965). Bibliographia Logica I: Verzeichnis der Druckschriften zur Logik mit Angabe ihrer Fundorte (1472-1800). Georg Olms, Hildesheim.
- [Risse, 1979] Risse, W. (1979). Bibliographia Logica IV: Verzeichnis der Handschriften zur Logik. Georg Olms, Hildesheim.
- [Roncaglia, 1992] Roncaglia, G. (1992). Le *Questiones* di Mesino de Codronchi sul *De Interpretatione*. In [Buzzetti and Tabarroni, 1992], pages 545–577.
- [Schepers, 1984] Schepers, H. (1984). s. v. Obligatio, Ars Obligatoria. In Historisches Wörterbuch der Philosophie, 6. Mo-O, pages 1068–1072. Schwabe Verlag, Basel.
- [Schupp, 1993] Schupp, F. (1993). Martinus Anglicus De obligationibus, Textkritisch herausgegeben, übersetzt, eingeleitet und kommentiert von Franz Schupp. Felix Meiner Verlag, Hamburg.
- [Seaton, 1973] Seaton, W. K. (1973). An Edition and Translation of the "Tractatus de Consequentiis" by Ralph Strode, Fourteenth Century Logician and Friend of Geoffrey Chaucer. Ph. D. Thesis – University of California, Berkeley.
- [Sinkler, 1992] Sinkler, G. (1992). Paul of Venice on Obligations. Dialogue, 31:475-493.
- [Spade, 1975] Spade, P. V. (1975). The Medieval Liar: a catalogue of the insolubilia-literature. The Pontifical Institute of Mediaeval Studies, Toronto.
- [Spade, 1976] Spade, P. V. (1976). William Heytesbury's Position on 'insolubles': One Possible Source. Vivarium, 14:114–120.
- [Spade, 1977] Spade, P. V. (1977). Roger Swyneshed's Obligationes: Edition and Comments. Archives d'Histoire doctrinale et litéraire du moyen âge, 44:243–285.
- [Spade, 1978] Spade, P. V. (1978). Richard Lavenham's Obligationes: Edition and Comments. Rivista critica di storia della filosofia, 33:225–242.
- [Spade, 1980] Spade, P. V. (1980). Robert Fland's Obligationes: An Edition. Mediaeval Studies, 42:41–60.
- [Spade, 1982a] Spade, P. V. (1982a). Obligations: B. Developments in the Fourteenth Century. In [Kretzmann et al., 1982], pages 335–341.
- [Spade, 1982b] Spade, P. V. (1982b). Three Theories of Obligationes: Burley, Kilvington and Swyneshed on Counterfactual Reasoning. *History and Philosophy of Logic*, 3:1–32.
- [Spade, 1992] Spade, P. V. (1992). If obligationes were counterfactuals. Philosophical Topics, 21:171–188.
- [Spade, 1993] Spade, P. V. (1993). Opposing and Responding: A New Look at positio. Medioevo, 19:233–270.
- [Spade, 1997] Spade, P. V. (1994–1997). The Logic of Sit verum in Richard Brinkley and William of Ockham. Franciscan Studies, 54:227–250.

- [Spade, 1996] Spade, Ρ. ν. (1996).Thoughts, Words and Things: In-AntroductionMedieval andtoLate Logic Semantic Theory. Available at http://pvspade.com/Logic/docs/thoughts.pdf.
- [Spade, 2000] Spade, P. V. (2000). Why Don't Mediaeval Logicians Ever Tell Us What They're Doing? Or, What Is This, A Conspiracy? Available at http://pvspade.com/Logic/docs/Conspiracy.pdf.
- [Spade, 2003] Spade, P. V. (2003). *Medieval Theories of* Obligationes, in E. Zalta, *Stanford Encyclopedia of Philosophy*. Available at http://plato.stanford.edu/entries/obligationes/.
- [Spade and Stump, 1983] Spade, P. V. and Stump, E. (1983). Walter Burley and the *Obligationes* attributed to William of Sherwood. *History and Philosophy of Logic*, 4:9–26.
- [Spade and Wilson, 1995] Spade, P. V. and Wilson, G. (1995). Richard Brinkley's Obligationes. A Late Fourteenth Century Treatise on the Logic of Disputation. Aschendorff, Münster.

[Spruyt, 2004] Spruyt, J. (2004). Logica Morelli. Brepols, Turnhout.

- [Stump, 1980] Stump, E. (1980). William of Sherwood's Treatise on Obligations. *Historiographia Linguistica*, 7:249–264.
- [Stump, 1981] Stump, E. (1981). Roger Swyneshed's Theory of Obligations. Medioevo, 7:135– 174.
- [Stump, 1982a] Stump, E. (1982a). Obligations: A. From the Beginning to the Early Fourteenth Century. In [Kretzmann et al., 1982], pages 315–334.
- [Stump, 1982b] Stump, E. (1982b). review of L. M. De Rijk. Die mittelalterliche Traktate De modo opponendi et respondendi. History and Philosophy of Logic, 3:213–216.
- [Stump, 1985] Stump, E. (1985). The Logic of Disputation in Walter Burley's Treatise on Obligations. *Synthese*, 63:335–374.
- [Stump, 1989] Stump, E. (1989). Dialectic and its Place in the Development of Medieval Logic. Cornell University Press, Ithaca.
- [Sylla, 1982] Sylla, E. D. (1982). The Oxford calculators. In [Kretzmann et al., 1982], pages 540– 563.
- [Uckelman, 2009] Uckelman, S. L. (2009). Bibliography of *obligationes* literature. Available at http://staff.science.uva.nl/ suckelma/obbib.pdf.
- [Uckelman et al., 2008] Uckelman, S. L., Maat, J., and Rybalko, K. (2008). The Art of Doubting in *Obligationes Parisienses*. Available at http://www.illc.uva.nl/
- Publications/ResearchReports/PP-2008-50.text.pdf.
- [Vasoli, 1952] Vasoli, C. (1952). Polemiche occamiste. Rinascimento, 3(1):119–141.
- [Vasoli, 1963] Vasoli, C. (1963). Pietro degli Alboini da Mantova 'scolastico' della fine del Trecento e un'epistola di Coluccio Salutati. *Rinascimento* 2nd. ser., 3:3–21.
- [Vasoli, 1974] Vasoli, C. (1974). Intorno al Petrarca ed ai logici 'moderni'. In Zimmermann, A., editor, *Miscellanea Medievalia Bd. 9*. Antiqui und Moderni. Traditionsbewusstsein und Fortschrittsbewusstsein im späten Mittelalter, pages 142–155. de Gruyter, Berlin.

- [Vitali, 1983] Vitali, M. C. (1983). La biblioteca dell'umanista padovano Giovanni Marcanova. Ateneo Veneto, 21(2):123–161.
- [Vos, 1998] Vos, A. (1998). Moments of the Ars Obligatoria According to John Duns. Franciscan Studies, 56:383–419.
- [Weisheipl, 1964] Weisheipl, J. A. (1964). Curriculum of the Faculty of Arts at Oxford in the Early Fourteenth Century. *Mediaeval Studies*, 26:143–185.
- [Weisheipl, 1966] Weisheipl, J. A. (1966). Developments in the Arts Curriculum at Oxford in the Early Fourteenth Century. *Mediaeval Studies*, 28:151–175.
- [Wilson, 1966] Wilson, C. (1966). William Heytesbury. Medieval Logic and the Rise of Mathematical Physics. Wisconsin University Press, Madison.
- [Yrjönsuuri, 1990] Yrjönsuuri, M. (1990). *Obligationes, Sophismata* and Oxford Calculators. In [Knuuttila et al., 1990], pages 645–654.
- [Yrjönsuuri, 1993a] Yrjönsuuri, M. (1993a). Aristotle's Topics and Medieval Obligational Disputations. *Synthese*, 96:59–82.
- [Yrjönsuuri, 1993b] Yrjönsuuri, M. (1993b). The Role of Casus in Some Fourteenth Century Treatises on Sophismata and Obligations. In [Jacobi, 1993], pages 301–321.
- [Yrjönsuuri, 1994] Yrjönsuuri, M. (1994). Obligations: 14th Century Logic of Disputational Duties. Acta Philosophica Fennica, 55:1–9.
- [Yrjönsuuri, 1996] Yrjönsuuri, M. (1996). Obligations as Thought Experiments. In Angelelli, I. and Cerezo, M., editors, *Studies on The History of Logic*, pages 79–96. de Gruyter, Berlin.
- [Yrjönsuuri, 1998] Yrjönsuuri, M. (1998). The compossibility of Impossibilities and Ars Obligatoria. History and Philosophy of Logic, 19:235–248.
- [Yrjönsuuri, 2000a] Yrjönsuuri, M. (2000a). Disputations, Obligations and Logical Coherence. *Theoria*, 66:205–223.
- [Yrjönsuuri, 2000b] Yrjönsuuri, M. (2000b). The Trinity and Positio Impossibilis: Some Remarks on Inconsistence. In Holmström, G. and Hintikka, J., editors, Medieval Philosophy and Modern Times, pages 59–68. Kluwer, Dordrecht.
- [Yrjönsuuri, 2001a] Yrjönsuuri, M. (2001a). Duties, Rules and Interpretations in Obligational Disputations. In [Yrjönsuuri, 2001b], pages 3–34.
- [Yrjönsuuri, 2001b] Yrjönsuuri, M., editor (2001b). Medieval Formal Logic, Dordrecht. Kluwer.