

DYNAMICAL EFFECTS OF SELF-GENERATED MAGNETIC FIELDS IN COSMIC-RAY-MODIFIED SHOCKS

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ABSTRACT

Recent observations of greatly amplified magnetic fields ($dB/B \sim 100$) around supernova shocks are consistent with the predictions of the nonlinear theory of particle acceleration (NLT), if the field is generated upstream of the shock by cosmic-ray-induced streaming instability. The high acceleration efficiencies and large shock modifications predicted by NLT need however to be mitigated to confront observations, and this is usually assumed to be accomplished by some form of turbulent heating. We show here that magnetic fields with the strength inferred from observations have an important dynamical role on the shock, and imply a shock modification substantially reduced with respect to the naive unmagnetized case. The effect appears as soon as the pressure in the turbulent magnetic field becomes comparable with the pressure of the thermal gas. The relative importance of this unavoidable effect and of the poorly known turbulent heating is assessed. More specifically we conclude that even in the cases in which turbulent heating may be of some importance, the dynamical reaction of the field cannot be neglected, as is usually done in most current calculations.

Subject headings: acceleration of particles — magnetic fields — shock waves

1. INTRODUCTION

The supernova remnant (SNR) paradigm for the origin of Galactic cosmic rays is based on the assumption that at least $\sim 10\%$ – 20% of the kinetic energy of the expanding shell is converted into cosmic rays (CRs). Moreover, as recent observations have proven, the magnetic field (MF) in the shock vicinity is amplified by a large amount (e.g., Ballet 2006) as would be expected if cosmic rays induce streaming instability (SI) upstream of the shock. We stress that such MF amplification is required to accelerate protons up to $\sim 10^6$ GeV. The need for a satisfactory and self-consistent description of these points is sufficient to justify the development of a NLT of particle acceleration.

The developments of the theory are summarized in Drury (1983), Jones & Ellison (1991), and Malkov & Drury (2001). The kinetic theory for arbitrary diffusion coefficients (Amato & Blasi 2005) and even in the case of self-generated MFs (Amato & Blasi 2006; Vladimirov et al. 2006) has been recently developed.

All approaches to NL shock acceleration find that the large pressure of the accelerated particles decelerates the incoming gas, and leads to total compression factors that scale with the Mach number of the shock as $R_t \sim M_0^{3/4} \sim 20$ – 50 . Such large shock modifications however are at odds with observations which are better fit by $R_t \sim 7$ – 10 . The problem with larger values of R_t resides in both the estimated distance between the forward and reverse shocks (Warren et al. 2005) and in the fit to multifrequency observations with concave spectra (Völk et al. 2005 [hereafter VBK05] and references therein). The reduction in the compression factor is almost invariably attributed to turbulent heating (TH) in the precursor (Berezhko & Völk 1997 and later) as due to damping of waves on the background plasma (McKenzie & Völk 1982, hereafter MKV82). In fact this mechanism was originally proposed in order to keep the MF amplification in the linear regime (e.g., $dB/B \ll 1$), but is now commonly applied to cases in which $dB/B \gg 1$. Unfortunately, the heating process is quite model dependent and even

its applicability to situations of interest for SNRs can and should be seriously questioned. The effectiveness of the heating process can easily be reduced to negligible levels or artificially amplified to unphysical levels.

As mentioned above, a breakthrough in the field has recently been provided by X-ray observations: the detection of X-ray-bright filaments in the outskirts of some SNRs allows one to infer the strength of the MF in these filaments, found to be $B \sim 100$ – 500 mG. Such strong fields are generally attributed to the SI induced by CRs efficiently accelerated at the shock front, although alternative explanations have been proposed (Giacalone & Jokipii 2007). In Table 1 we list some SNRs with estimated MFs: u_0 is the shock velocity, B_2 is the MF downstream of the shock as inferred from the X-ray brightness profile, and $P_{w2} = B_2^2/(8\pi u_0^2)$ is the downstream magnetic pressure normalized to the bulk one. The values of the parameters are from Parizot et al. (2006) and from VBK05 (in parentheses in Table 1).

We show below that for the field strengths inferred for SNRs, the magnetic pressure is comparable to or even in excess of the thermal pressure of the background plasma and that whenever this happens the dynamical reaction of the field on the fluid is such that the compression factors are substantially reduced and fall in the range suggested by observations. It is crucial to keep in mind that, contrary to the TH, which can be either suppressed or amplified by changing parameters on which there is little or no control, the feedback of the self-generated turbulent MF on the plasma is not model dependent and must be included.

2. DYNAMICS OF A MAGNETIZED CR-MODIFIED SHOCK

The reaction of accelerated particles upstream of the shock leads to the formation of a precursor, in which the fluid speed decreases while approaching the shock. One can describe this effect by introducing two compression factors $R_t = u_0/u_2$ and $R_s = u_1/u_2$, where u is the fluid speed and the indexes “0,” “1,” and “2” refer to quantities at upstream infinity, upstream, and downstream of the subshock, respectively.

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TABLE 1
PARAMETERS FOR FIVE WELL-KNOWN SNRS

SNR	u_0 (km s ⁻¹)	B_2 (mG)	$P_{w2} \times 10^3$
Cas A	5200 (2500)	250–390	32 (36)
Kepler	5400 (4500)	210–340	23 (25)
Tycho	4600 (3100)	300–530	27 (31)
SN 1006	2900 (3200)	91–110	40 (42)
RCW 86	(800)	75–145	14–35 (16–42)

NOTES.—For SNRs discussed by Parizot et al. (2006) we used $r_0 = 0.1 m_p \text{ cm}^{-3}$ in the case of SN 1006 and $r_0 = 0.5 m_p \text{ cm}^{-3}$ in the other cases, while VBK05 (values in parentheses) provide directly P_{w2} .

The most general equations of conservation of mass and momentum in the stationary case for a parallel shock are

$$\frac{\partial}{\partial x}(ru) = 0, \quad \frac{\partial}{\partial x}(ru^2 + p + p_c + p_w) = 0. \quad (1)$$

The equation of energy conservation will only be used at the subshock, where the jump reads

$$\left[\frac{1}{2} ru^3 + \frac{g pu}{g-1} + F_w \right]_1^2 = 0, \quad (2)$$

and the brackets stand for $[x]_1^2 = x_2 - x_1$. As usual, r , u , p , and g represent density, velocity, pressure, and the ratio of specific heats of the gas, while p_w and F_w are the pressure and energy flux in the form of Alfvén waves; p_c is the CR pressure. The form of equation (2) is due to the continuity of the distribution function of accelerated particles through the subshock, which implies that the CR pressure is also continuous ($p_{c1} = p_{c2}$). All these equations at the subshock read as the usual Rankine-Hugoniot relations at a magnetized gaseous shock.

In order to treat the presence of Alfvén waves correctly, we use the approach of Vainio & Schlickeiser (1999, hereafter VS99), considering two upstream wave trains with helicities $H_c = \pm 1$, and their respective downstream counterparts. If $d\mathbf{B}_i$ is a mode of the MF perturbation, we write the velocity perturbation as $d\mathbf{u} = -H_c(d\mathbf{B}/\sqrt{4\pi r})$. Neglecting the electric field contribution, which is of order $(u/c)^2$, the magnetic pressure and the energy flux, which for Alfvén waves is the sum of the normal component of Poynting vector $\mathbf{u} \times d\mathbf{B} \times d\mathbf{B}/4\pi$ and the transverse kinetic energy flux $r d\mathbf{u}^2/2$, are

$$p_w = \frac{1}{8\pi} \left(\sum_i d\mathbf{B}_i \right)^2, \quad F_w = \sum_i \frac{d\mathbf{B}_i^2}{4\pi} \tilde{u}_i + p_w u, \quad (3)$$

having posed $\tilde{u}_i = u + H_c v_A$. The upstream magnetic turbulence typically shows two opposite helicities (Bell & Lucek 2001), each of which splits into a reflected and a transmitted wave crossing the subshock. According to VS99, the transmission and reflection coefficients, in the limit $M_A^2 \gg R_s$ (large Alfvénic Mach number), do not depend on H_c and read

$$T \approx (R_s + \sqrt{R_s})/2, \quad R \approx (R_s - \sqrt{R_s})/2. \quad (4)$$

For a typical supernova shock, the Alfvénic Mach number is $M_{A,1} = u_1/v_A \geq 100$; hence in the following we adopt these coefficients and neglect v_A with respect to the fluid velocity in equation (3). For each H_c we therefore have $d\mathbf{B}_2/d\mathbf{B}_1 = T + R = R_s$ and thus $p_{w2} = p_{w1} R_s^2$.

As pointed out above, the subshock can be viewed as a

simple shock in a magnetized gas; therefore the pressure jump is (VS99)

$$\frac{p_2}{p_1} = \frac{(g+1)R_s - (g-1) + (g-1)(R_s-1)D}{g+1 - (g-1)R_s}, \quad (5)$$

with D defined as

$$D = \frac{R_s+1}{R_s-1} \frac{[p_{w1}]^2}{p_1} - \frac{2R_s}{R_s-1} \frac{[F_{w1}]^2}{p_1 u_1}. \quad (6)$$

Using the expressions for T and R (eq. [4]) we get

$$D = (R_s-1)^2 \frac{p_{w1}}{p_1} + R_s \frac{\mathbf{B}_- \cdot \mathbf{B}_+}{2p p_1}. \quad (7)$$

Following VS99, we assume that the two opposite-propagating waves carry MFs \mathbf{B}_\pm displaced in such a way that $\mathbf{B}_- \cdot \mathbf{B}_+ = 0$. This is not the most general configuration, but it is nevertheless rather common since it occurs (1) when there is only one wave train, (2) when the two fields are orthogonal, and (3) on average, when the relative phase between the wave trains is arbitrary.

At this point we normalize all quantities to the ones at upstream infinity: $U(x) = u(x)/u_0$, $P_w(x) = p_w/r_0 u_0^2$, and $P(x) = p(x)/r_0 u_0^2 = U(x)^{-g}/gM_0^2$. In the latter, we used the assumption of adiabatic heating in the precursor and the conservation of mass.

Substituting equation (5), equation (7), and the above expression for $P(x)$ in the equation for momentum conservation, the compression factors R_s and R_t are related through the equation

$$R_t^{g+1} = \frac{M_0^2 R_s^g}{2} \left[\frac{g+1 - R_s(g-1)}{1 + L_B} \right], \quad (8)$$

which is the same as the standard relation apart from the factor $(1 + L_B)$, with

$$L_B = W[1 + R_s(2/g - 1)], \quad (9)$$

and $W = P_{w1}/P_t$. It is clear that the net effect of the magnetic turbulence is to make the fluid less compressible, noticeably reducing R_t if $W = P_{w1}/P_t$ is of order 1. Moreover, the pressure and temperature jumps at the subshock are enhanced (eq. [5]).

We should note that if one naively assumed that $F_w = 3up_w$ everywhere, neglecting the T and R coefficients needed to satisfy Maxwell equations at the subshock, one would obtain $D' = [(R_s-1)^2 - 2R_s]W < D$, leading to an incorrect pressure jump. This approach, adopted by Vladimirov et al. (2006), also leads to a less marked decrease of R_t since $L'_B = W[1 + R_s(3/g - 2)]$.

3. CONFRONTING OBSERVATIONS

Here we show that the magnetization levels estimated in SNRs as reported in Table 1 imply that $W \geq 1$, so that the dynamical feedback of the amplified MF needs to be taken into account.

In Figure 1 we plot R_t versus R_s for $M_0 = 100$. The three shadowed regions represent the relation between R_s and R_t for fixed $P_{w2} \in [0.02, 0.04]$ and $W = 1, 3, 10$; $P_{w2}/R_s^2 = P_{w1} = WP_{g1} = W(R_t/R_s)^g/(gM_0^2)$.

The three solid lines represent the relation R_t - R_s for the three

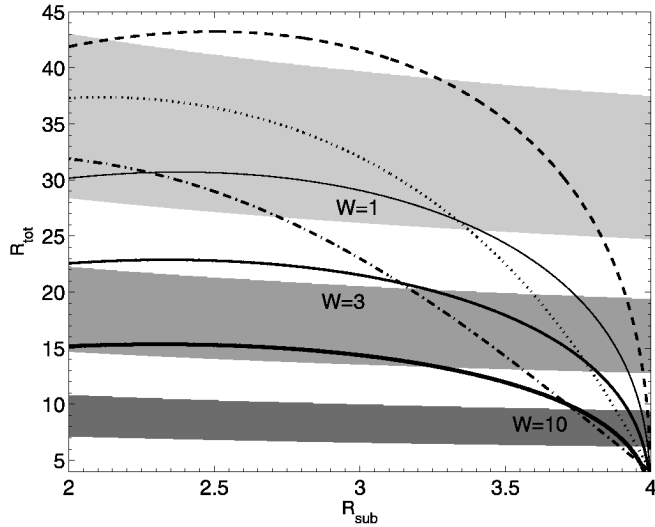


FIG. 1.— $R_t R_s$ for $W = 1, 3$, and 10 (intersections of solid lines with corresponding shadowed regions) and for $x = 200$ and 500 (dash-dotted and dotted lines). The dashed line represents the case $W = 0$ (see text for details).

given values of W as given by equation (8); the dashed line refers to $W = 0$, when p_w is not included.

The compression factor lies at the intersection between the curve and the shadowed region for a given value of W . If $W < 0.7$ there are no intersections. This implies that the values of the magnetic pressure inferred from observations require substantial MF amplification upstream, and that the conservation equations are affected by the dynamical reaction of the field. Only values $W \geq 3$ are compatible with the whole range $0.02 \leq P_{w2} \leq 0.04$ inferred from observations. This means that in order to account for the inferred values of B_2 , the magnetic pressure must be *at least* comparable to the gas pressure, and thus its dynamical role cannot be neglected. From Figure 1 one also sees that the magnetic reaction leads to values of R_t lower by roughly a factor ~ 2 compared with the case $W = 0$. We comment further on this point below.

Up until this point we never used the physically crucial point that the observed fields may be generated through a CR-induced SI upstream of the shock. The instability may operate in the resonant (Bell 1978) and in the nonresonant (Bell 2004) regime.

The growth rates of these different modes can be easily estimated only in the context of quasi-linear theory. Given the difficulty in deriving this information in the general nonlinear case, here we assume the following general relation between the pressures of CRs and wave pressure upstream of the subshock:

$$y_1 = x P_{w1}, \quad (10)$$

where $y(x) = p_{cr}(x)/r_0 u_0^2$ is the normalized CR pressure. For resonant SI, one has $x \approx M_A = u_0/v_A$, while for nonresonant modes, $x \sim 4c/u_0$. In both cases, for typical values of the parameters, one obtains $200 \leq x \leq 500$.

From equation (1) applied to the precursor, namely between upstream of the subshock ($x = 0^-$) and upstream infinity, we can write

$$\frac{R_s}{R_t} + \frac{1}{g M_0^2} \left[\left(\frac{R_t}{R_s} \right)^g - 1 \right] + P_{w1}(1+x) = 1. \quad (11)$$

The physical values of R_s and R_t for a given x are obtained

by determining the intersection of the corresponding curve with that obtained for a given value of W at the subshock. Whether the solution reproduces the estimated value of P_{w2} depends on whether the intersection falls within or outside the shadowed region for the same W in Figure 1. The dash-dotted and the dotted lines show the results for $x = 200$ and 500 , respectively; it is evident that the chosen values of x allow for a consistent explanation of the downstream magnetic pressures as inferred from observations and, equally important, lead to compression factors which are much lower than those predicted by the standard NLT (Berezhko & Völk 1997 and papers that followed).

4. HEATING IN THE PRECURSOR

The strong shock modification predicted by NLT when the magnetic pressure is ignored is usually assumed to be somewhat mitigated by heating of the precursor as a result of damping of Alfvén waves (Völk & McKenzie 1981, hereafter VMK81; MKV82) on the background gas. Other phenomena (for instance acoustic instability; Drury & Falle 1986) may also lead to heating of the precursor. In the original description, which remained basically unchanged to the present time, VMK81 assumed that the rate of damping (G) equals the rate of growth (j) of Alfvén waves. The main implication of this assumption is that the growth of the waves never reaches the nonlinear regime, which is in fact the very reason why the mechanism was invoked in the 1980s. The recent observations prove that waves can grow to $dB/B \gg 1$. It is therefore at least not self-consistent to apply the standard treatment for TH to situations in which MF amplification to the nonlinear regime takes place. In a minimal attempt to include faster growth one may assume that $G = \alpha j$, with $\alpha < 1$. Following MKV82 and Berezhko & Ellison (1999) one can then obtain a generalized relation between R_t and R_s in the form

$$R_t^{g+1} = \frac{M_0^2 R_s^g}{2} \left[\frac{g+1 - R_s(g-1)}{(1+L_B)(1+L_{TH})} \right], \quad (12)$$

where

$$L_{TH} = \alpha(g-1) \frac{M_0^2}{M_A} \left[1 - \left(\frac{R_s}{R_t} \right)^g \right], \quad (13)$$

which becomes equivalent to the standard equation (50) of Berezhko & Ellison (1999) only for $\alpha = 1$. Now it is easy to check that, for typical values of R_s and R_t , $L_{TH} > L_B$ if $\alpha \gtrsim 3W(M_A/M_0^2)$. For instance, for $M_A \sim 10^3$ and $M_0 \sim 100$ one requires α to be of order unity. In this case, however, it is not easy to amplify the MF to $dB \gg B_0$. If α is appreciably smaller than unity, the main process for the smoothening of the precursor is the dynamical reaction of the self-generated MF. In both cases the role of TH can be seriously questioned.

A deeper look at the physical processes that may result in the heating of the precursor make the role of TH even more uncertain; in the original papers of VMK81 and MKV82 the Alfvén heating was considered to be the result of nonlinear Landau damping in a gas in the hot coronal phase. The authors reached the conclusion that the damping is important if $u_0 \ll 4000 \text{ km s}^{-1} (T_0/5 \times 10^5 \text{ K})^{1/2}$, where u_0 is the shock velocity and T_0 is the temperature of the unshocked gas. It is all but clear whether for the velocities and temperatures that apply to the SNRs in Table 1, nonlinear Landau damping is such that it leads to $\alpha \sim 1$. We stress that at the same time, α cannot be too close to unity, otherwise TH inhibits the growth of dB to the observed levels.

Other types of turbulent heating may be at work but a quantitative analysis of these phenomena is lacking at the present time. The expression for L_{TH} is however rather general, in that we did not specify the mathematical form of the growth and damping rates. Therefore we expect to draw similar conclusions in terms of the parameter a .

This section strongly suggests that, contrary to the common wisdom, the most likely reason for the smoothening of the precursor is the dynamical reaction of the generated MFs rather than some form of nonadiabatic heating in the precursor.

5. CONCLUSIONS

It is well known that the effect of a MF is in general that of reducing the plasma compressibility. We showed here that when applied to a CR-modified shock, (1) this finding implies that CR-induced SI is adequate to explain the magnetization inferred from X-ray observations; (2) the downstream MFs imply that $W \sim 1-10$, so that the field becomes dynamically important, since this happens whenever the magnetic pressure upstream becomes comparable with the gas pressure, namely when $W > 0.7-1$; (3) the dynamical reaction of the MF reduces the compression in the precursor, leading to smaller (larger) values of R_r (R_s) in agreement with the values required to explain the distance between forward and reverse shock and the multifrequency observations of several SNRs; (4) this effect comes from first principles, although in our calculations we restricted our attention to the case of Alfvén waves, and is not affected by the huge uncertainties typical of TH; (5) an efficient TH may smoothen the precursor if a is close to unity, but in this case it is likely to inhibit the growth of the field to $\delta B \gg B_0$.

Although the underlying physics is well known, the dynamical

effect of the magnetic pressure has not been included in the calculations of multifrequency emission from SNRs (Berezhko & Völk 1997 and successive papers), so that the strong modifications predicted by NLT had to be compensated by assuming TH. The only exception that we are aware of is the recent work by Vladimirov et al. (2006) in which the authors perform Monte Carlo simulations of the particle acceleration process including the pressure of self-generated MFs. In such simulations, which represent the state of the art in the field, however, thermal and accelerated particles are treated in the same way; therefore the condition $W \sim 1$ could not be tested. We suspect that for this reason the smoothening of the precursor was attributed mostly to the back-reaction of the accelerated particles on the field through injection. This effect is certainly present but as we showed here by using only a hydrodynamical approach, the smoothening is in fact mainly due to the reaction of the magnetic pressure on the background plasma.

The smoothening of the precursor also results in two important effects: (1) the spectra of accelerated particles are closer to power laws, although the concavity which is peculiar to NL diffusive shock acceleration remains evident; (2) the maximum momentum of accelerated particles for a given Mach number is predicted to be somewhat larger (see Blasi et al. 2007 for a detailed discussion). Both of these effects will be discussed in detail in a forthcoming paper.

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