

# Phase-dependent topological interface state and spatial adiabatic passage in a generalized Su-Schrieffer-Heeger model

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(Dated: May 10, 2019)

We extend the Su-Schrieffer-Heeger model to include both an additional real intercell coupling and a complex intracell coupling whose phase can be interpreted as the Peierls phase associated with a synthetic gauge field. Using different Peierls phases for the two halves of a sample, we can realize an heterostructure supporting a topologically protected interface state, depending on the existence of one trivial and two nontrivial phases of distinct winding numbers. The spatial adiabatic passage of this localized state from the inner interface to either open boundary can be attained simply by modulating the corresponding Peierls phase, while its decay or growth rate is controlled by the on-site parity-time symmetric loss and gain terms. Our results provide a novel scheme to achieve dynamical control of topologically protected states while being of potential interest to topological lasers with an adjustable spatial profile.

PACS numbers: 03.65.Vf, 42.60.Da, 73.20.At, 11.30.Er

## I. INTRODUCTION

As a new phase of matter, topological insulators (TI) are characterized by an insulating bulk and a conducting surface [1, 2]. According to the conventional bulk-edge correspondence, one can observe gapless edge modes near the interface between a TI and a trivial medium. While in two-dimensional TIs, waves typically propagate along the edges as chiral states, in one-dimensional TIs edge states are simply localized at the boundaries, decaying rapidly into the bulk. One crucial advantage of topologically protected edge states is that they are robust against fabrication imperfections and disorders. For Hermitian TIs, the bulk-edge correspondence is well explored and understood, and various topological invariants have been proposed to characterize distinct topological phases [3]. However, many realistic physical systems, such as open systems [4–7] and photonic systems with loss and/or gain [8–12], are described by non-Hermitian Hamiltonians, the topological properties of which are intensively being investigated [13]. Topological photonics, in particular, has attracted a great deal of interest accompanied with rapid progress [14, 15]. The experimental realization of topological edge states in a lossy waveguide array [16] has revived the debate about the bulk-edge correspondence for non-Hermitian TIs, a notion that is reconsidered in systems exhibiting an anomalous localization or non-Hermitian skin effect, whereas in other instances

holds true as typically described by topological invariants [17–29]. Compared with condensed matter electronic systems, photonic systems exhibit three major advantages in studying topological effects: first, in photonics, loss and gain are ubiquitous and can be controlled [30]; second, optical nonlinearities enable richer phenomena in topological photonics [31]; third, due to various internal degrees of freedom, it is possible to realize synthetic dimensions in photonic systems [32]. To date, a lot of breakthroughs have been achieved based on such properties, such as optical delay lines with enhanced transport properties [33], backscattering-free edge states [34, 35] and topological polaritons [36], to name a few. In particular, the concept of topological lasing has been put forth and demonstrated by exploiting the confinement of light to a topological protected edge mode in an active TI to achieve single-mode laser operation [37–42].

The tight-binding Su-Schrieffer-Heeger (SSH) model [43], which in its most basic version only includes a real intracell and a real nearest-neighbor intercell hopping terms, has been widely studied as a one-dimensional prototype allowing for a nontrivial topological phase [44, 45]. This model has been extended in a variety of Hermitian [46–49] and non-Hermitian forms [16, 20, 21, 27, 39–41, 50–58]. In particular, the non-Hermitian extensions, which are also called complex SSH (cSSH) models, are a powerful platform for studying interactions of topological properties with non-Hermiticity and many breakthroughs have been made based on them, such as anomalous edge states [21], non-Bloch bulk-edge correspondence [27], topological lasing [39–41], and spontaneous topological pumping [58]. Here, we consider a further extension of the non-Hermitian SSH model to include for: (i) a complex intracell hopping term; (ii) an additional

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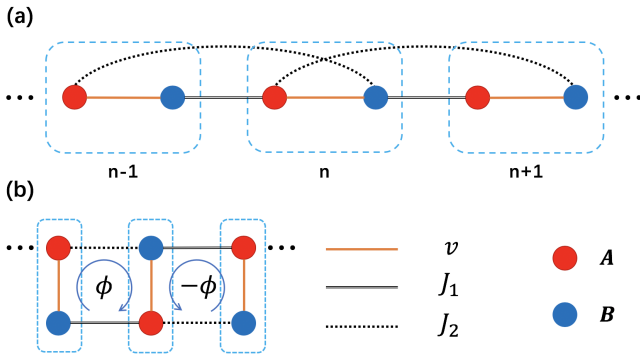


FIG. 1: (Color online) (a) The schematic illustration of a generalized SSH model. (b) The equivalent ladder lattice of the SSH model in (a). While the intercell hopping terms  $J_1$  and  $J_2$  are real-valued, the intracell hopping term is complex with modulus  $v$  (which will be taken as units of energies) and Peierls phase  $\phi$ . On-site loss and gain terms can be introduced in a balanced fashion on sites A and sites B, respectively, to realize a non-Hermitian PT-symmetric system.

intercell hopping term beyond the nearest-neighbor one; (iii) non-Hermitian on-site loss and gain terms. The phase of the complex intracell hopping term can be tuned via a synthetic gauge field (Peierls phase) [32, 59–66]. While loss and gain are needed for lasing action, the Peierls phase degree of freedom would be immaterial in the absence of the additional intercell coupling as discussed below. Thus, our extension represents the minimal SSH model apt to implement a topologically protected mode the spatial profile of which can be controlled via the Peierls phase: by dividing a finite sample into two parts with different Peierls phases, we can observe a topologically protected mode at the interface which can be adiabatically transferred between the interface and one open boundary by adjusting the synthetic gauge field. During the process, this mode remains topologically protected because its eigenvalue does not merge into the bulk bands; moreover, in the presence of suitable loss and gain terms, we show that this adiabatic process also holds when the mode is amplified so that it is of potential interest in topological lasing with a tunable spatial profile, a highly sought-after feature for signal spatial encoding and broad-area emission [38, 53].

The focus of the work is to elucidate the main features of the proposed extension of the Su-Schrieffer-Heeger model through a specific single-particle tight-binding Hamiltonian. Hence we are not committed here to any particular platform for exploring the new topological physics associated with such an extension though possible settings will be briefly suggested in Sect. IV. A summary is also given there, while details of our extended model are presented in Sect. II with the main results be-

ing discussed in Sect. III.

## II. THE GENERALIZED SSH MODEL

We consider the generalized SSH model shown in Fig. 1(a) having two sites per unit cell, sites A (red) and B (blue), and three distinct hopping terms, the intracell coupling (orange single line), the nearest-neighbor intercell coupling ( $J_1$ , gray thick line) and an additional next-nearest-neighbor intercell coupling ( $J_2$ , gray dotted line). While we assume both intercell hopping terms  $J_1$  and  $J_2$  to be real-valued, we consider a complex-valued intracell coupling of modulus  $v$  and a tunable Peierls phase  $\phi$ , i.e., the intracell hopping term is  $v_{AB} = v_{BA}^* = v e^{i\phi}$ . In fact, this generalized SSH model is equivalent to a ladder lattice with sites A and B arranged alternately around each plaquette as shown in Fig. 1(b), and a tunable synthetic gauge field enables the control of the phase  $\phi$ . Here, the synthetic gauge field should be opposite in adjacent plaquettes in order to introduce uniform Peierls phase in Fig. 1(a). Unlike other SSH models [27, 53, 55], we note that all hopping terms preserve Hermiticity. Since a nonvanishing average value of gain (or loss) would not affect the spatial profile of the edge mode we are interested in [40], we choose to introduce loss on every A sites and gain on every B sites in a balanced fashion. In this way, our model respects either chiral or parity-time (PT) symmetry when the loss and gain rate is taken to be, respectively, vanishing or nonvanishing.

Since the homogeneous model is translationally invariant, using the Bloch theorem, the k-space Hamiltonian is obtained as

$$H(k) = h_x(k)\sigma_x + h_y(k)\sigma_y + h_z\sigma_z \quad (1)$$

with

$$\begin{aligned} h_x(k) &= v \cos \phi + (J_1 + J_2) \cos k, \\ h_y(k) &= -v \sin \phi + (J_1 - J_2) \sin k, \\ h_z &= -i\gamma. \end{aligned}$$

Here,  $\sigma_j$  ( $j = x, y, z$ ) are the Pauli matrices and  $k$  is the dimensionless scaled Bloch wave number in the first Brillouin zone ( $-\pi \leq k < \pi$ ). While  $h_x$  and  $h_y$  are real and  $k$  dependent,  $h_z$  is purely imaginary and constant, with a positive value of  $\gamma$  corresponding to loss on the A sites and gain on the B sites. Then, the symmetries mentioned above lead to the pseudo-anti-Hermiticity of  $H(k)$  with  $\sigma_z H(k)^\dagger \sigma_z = -H(k)$  [18, 52], as well as to the so-called chiral-time symmetry [40] of  $H(k)$ , which can be represented by the anti-unitary operator  $\sigma_x \mathcal{K}$ , being  $\mathcal{K}$  the complex conjugation, with  $[\sigma_x \mathcal{K}, H(k)] = 0$ . The eigenvalues of  $H(k)$  are given by

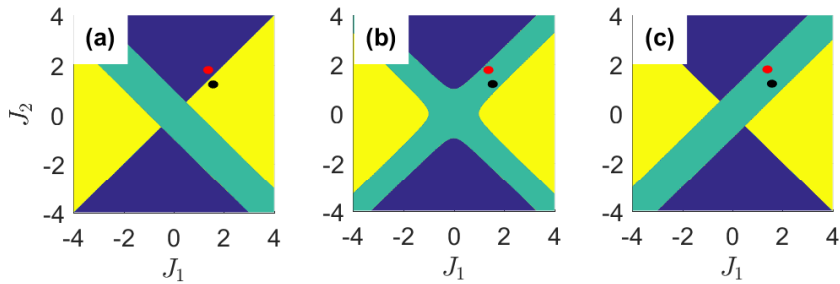


FIG. 2: (Color online) Topological phase diagrams of the generalized SSH model with (a)  $\phi = 0$ , (b)  $\phi = \pi/4$ , (c)  $\phi = \pi/2$ . In each panel, the yellow, blue and green regions correspond to the winding numbers  $w = 1$ ,  $w = -1$  and  $w = 0$ , respectively. The red (black) dots in (a)-(c) correspond to  $J_1 = 1.4$  and  $J_2 = 1.8$  ( $J_1 = 1.5$  and  $J_2 = 1.2$ ),  $v = 1$  being taken as unit of energies.

$$E_{\pm}(k) = \pm \sqrt{-\gamma^2 + v^2 + J_1^2 + J_2^2 + 2vJ_1 \cos(k + \phi) + 2vJ_2 \cos(k - \phi) + 2J_1J_2 \cos(2k)}, \quad (2)$$

which satisfy  $E_+(k) = -E_-(k)$  and are either real or purely imaginary. Eq. (2) also shows that the energy bands have a nontrivial dependence on the Peierls phase  $\phi$  only when both  $J_1$  and  $J_2$  are present: if  $J_2$  vanishes, the only effect of  $\phi$  is a rigid displacement of the bands in the  $k$  space. As a matter of fact, in the absence of  $J_2$ , the phase  $\phi$  could always be gauged away via a redefinition of the basis states [45], as also evident from the fact that in this case there are no closed plaquettes in Fig. 1(b).

Differently from some non-Hermitian models [21, 23, 27, 53] while similarly to others [16, 40, 50, 52, 57], the present generalized SSH model does not exhibit the non-Hermitian skin effect. Thus, the characterization of its nontrivial topological properties, in terms of the winding number corresponding to the non-Hermitian extension [19, 67] of the Zak phase [44, 68], can provide guidance for the realization of heterostructures with spatially tunable and robust localized modes suitable for topological lasing [40]. We indicate with  $\langle \varphi_{\alpha}(k) |$  and  $|\psi_{\alpha}(k) \rangle$  the left and right eigenvectors of  $H(k)$  corresponding to the eigenvalue  $E_{\alpha}(k)$ , where  $\alpha = \pm$  labels the energy bands as in Eq. (2), with the biorthogonal normalization  $\langle \varphi_{\alpha}(k) | \psi_{\beta}(k) \rangle = \delta_{\alpha,\beta}$  [69]. Then, the complex single-band Zak phase (in units of  $\pi$ ) is given by

$$w_{\alpha} = \frac{1}{\pi} \int_{-\pi}^{\pi} dk \langle \varphi_{\alpha} | i \partial_k | \psi_{\alpha} \rangle. \quad (3)$$

For a model of the form of Eq. (1), the above formula can be analytically rewritten as [55]

$$w_{\alpha} = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk \frac{h_x \partial_k h_y - h_y \partial_k h_x}{E_{\alpha}(k)(E_{\alpha}(k) - h_z)}, \quad (4)$$

and the global Zak phase  $w_{tot} = w_+ + w_-$ , a proper topological invariant for our non-Hermitian system [16, 19, 52, 57], making use of Eq. (2) becomes simply

$$w_{tot} = \frac{1}{\pi} \int_{-\pi}^{\pi} dk \frac{h_x \partial_k h_y - h_y \partial_k h_x}{h_x^2 + h_y^2}. \quad (5)$$

It is noticeable that the value of  $w_{tot}$  equals to twice the single-band Zak phase of the parent Hermitian model (i.e., the one obtained by setting  $h_z = 0$  in Eq. (1)) which is the same for both bands and takes integer values. Thus, the topologically distinct phases of the present generalized SSH model are inherited from its parent Hermitian one, similarly to other instances of non-Hermitian systems [19, 50, 57]. In particular, the value of  $\gamma$ , which characterizes the amount of loss and gain introduced in the Hamiltonian and determines its PT-symmetry transition [70], does not affect the topological phase transitions. In the following, our strategy is: (A) to harness the degree of freedom provided by the Peierls phase  $\phi$  in order to realize a heterostructure between topologically distinct phases; (B) to assess how the heterostructure is affected by loss and gain allowing for topological lasing; (C) to implement the control of the spatial profile of the topologically protected mode and, finally, (D) to show that the energy of this mode and its spatial adiabatic passage are robust against disorder.

### III. RESULTS AND DISCUSSION

#### A. Topological heterostructure

We start by assessing how the Peierls phase  $\phi$  affects the topological properties of the generalized SSH model which, as discussed above, do not depend on the loss and gain terms proportional to  $\gamma$ . We plot in Fig. 2 the winding number  $w \equiv w_{tot}/2$  for different parameter values. As shown in Fig. 2(a), one can observe in a wide range of parameters three topological phases, characterized by  $w = \pm 1$  and  $w = 0$ , respectively. While  $w = 0$  corresponds to a topologically trivial phase with no edge states,  $w = \pm 1$  correspond to two distinct topologically nontrivial phases with different types of edge states.

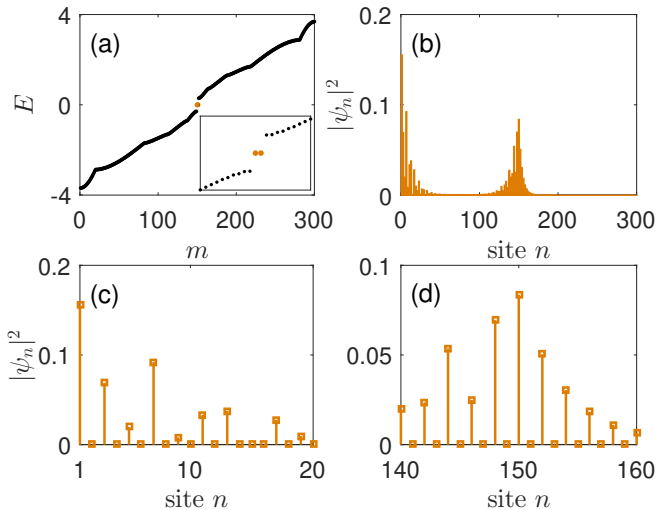


FIG. 3: (Color online) (a) The energy spectrum of a finite heterostructure based on the Hermitian ( $\gamma = 0$ ) generalized SSH model with  $v = 1$ ,  $J_1 = 1.5$ ,  $J_2 = 1.2$ ,  $\phi_1 = 0$  and  $\phi_2 = \pi/2$ ; the inset shows in detail the two orange dots correspond to the degenerate left edge and interface states;  $m$  labels the eigenvalues in order of increasing energy. (b) The spatial profiles of the edge and interface states in this case. (c) and (d) The zoom-in images of the edge and interface states, respectively.

In fact, similar results have already been discussed in Refs. [47, 48]. Here, we show in Fig. 2(b) and Fig. 2(c) that the topological phase diagram can be changed by adjusting  $\phi$ , being the parameter range of the topological trivial phase with  $w = 0$  most sensitive to  $\phi$ . We conclude that in the present extension of the SSH model ( $J_2 \neq 0$ ) the topological phase can be controlled by adjusting the value of  $\phi$ .

Next, we consider a heterostructure between two aforementioned generalized SSH models having different values of  $\phi$ , which can be obtained by dividing the whole sample into two halves subject to distinct synthetic gauge fields. To be definite, here we consider a finite generalized SSH model with 150 unit cells (300 sites), assuming that the left part contains the first 75 unit cells with the Peierls phase  $\phi_1$ , while the right part contains the last 75 unit cells with the Peierls phase  $\phi_2$ . If  $\phi_1 = \phi_2$ , the model recovers a (finite) homogeneous structure.

According to the specific results in Fig. 2, if  $\phi_1 \neq \phi_2$ , the left and right parts of the model may be topologically distinct, and thereby we expect a topologically protected state localized at the interface. In particular, choosing the parameters corresponding to the black dots in Fig. 2(a) and Fig. 2(b) and letting  $\phi_1 = 0$  and  $\phi_2 = \pi/2$ , the left part is topologically nontrivial with  $w = 1$  while the right part is topologically trivial with  $w = 0$ .

Considering first the Hermitian case with  $\gamma = 0$ , the presence of a zero mode localized at the interface is shown in Fig. 3. In Fig. 3(a), we plot the energy spectrum of a finite heterostructure with open boundary conditions at the outer edges, and find that there are two zero modes

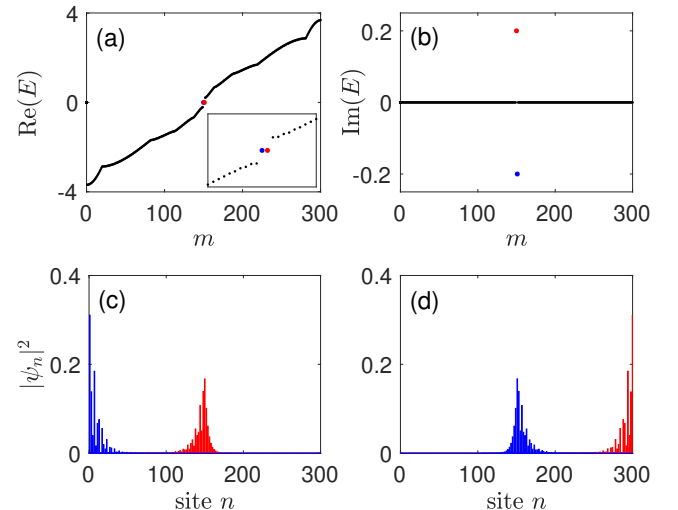


FIG. 4: (Color online) The real (a) and imaginary (b) parts of the energy spectrum of the PT-symmetric heterostructure, respectively, with  $\phi_1 = 0$  and  $\phi_2 = \pi/2$ . The spatial profiles of the edge and interface states are depicted in (c) with  $\phi_1 = 0$  and  $\phi_2 = \pi/2$ , and in (d) with  $\phi_1 = \pi/2$  and  $\phi_2 = 0$ . Here, the profiles depicted by red lines correspond to growing states, while those depicted by blue lines correspond to damped states. In particular, the edge (blue) and interface (red) states in (c) correspond to the blue and red dots in (a) and (b), respectively. Other parameters are the same as in Fig. 3 except  $\gamma = 0.2$ .

residing within the band gap (see the two orange dot), as typical of topological edge states. Indeed, as shown in Fig. 3(b), the two degenerate modes correspond to a localized state at the left edge and a localized state at the middle interface. In Fig. 3(c) and Fig. 3(d), we further demonstrate the zoom-in images of the edge and interface states, respectively. They show that the left edge state resides only on  $A$  sites (odd number of sites), while the interface state resides only on  $B$  sites (even number of sites). This is much similar to the case of a standard SSH model, in which the two edge states reside on different sublattices, respectively. In contrast, if we choose the coupling parameters corresponding to the red dots in Fig. 2, the winding number of the left part becomes  $w = -1$  while that of the right part remains  $w = 0$ . In this case, the left edge (interface) state resides only on  $B$  ( $A$ ) sites (not shown here).

## B. Topological modes with gain and loss

We further extend the above results to the non-Hermitian regime by introducing balanced loss and gain terms alternately on the  $A$  and  $B$  sites. According to Eq. (3)- Eq. (5) and related discussions, such an introduction does not alter the topological phase transitions, so it is still viable to find an interface state with its spatial profile controlled by the Peierls phase. We start showing the complex energy spectrum of the heterostructure in

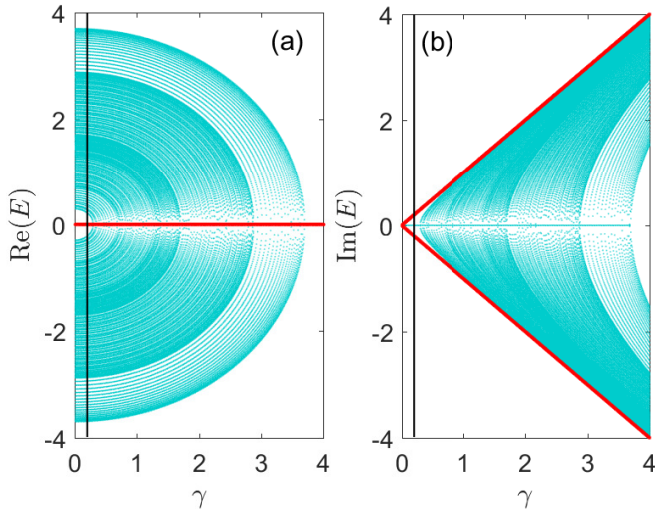


FIG. 5: (Color online) The real (a) and imaginary (b) parts of the energy spectrum of the PT-symmetric heterostructure as functions of  $\gamma$  with  $\phi_1 = 0$  and  $\phi_2 = \pi/2$ . The red lines depict the eigenvalues of the edge and interface states. Other parameters are the same as in Fig. 4.

Fig. 4(a) and Fig. 4(b). Clearly, there are two midgap modes, the eigenvalues of which have zero-valued real parts and opposite imaginary parts ( $\pm\gamma$ ). As shown in Fig. 4(c), the two modes correspond, respectively, to a damped left edge state with a negative imaginary eigenvalue and a growing interface state with a positive imaginary eigenvalue. Recently, it has been proved that an edge mode whose eigenvalue has a positive imaginary part can be used for realizing single-mode topological lasing [37–42, 57] robust against local perturbations. Generally, an edge (interface) state can be set to be either damped or growing by a suitable choice of the loss and gain terms [20, 24]. However, we anticipate here that this can also be accomplished via a dynamic modulation of the Peierls phases. For instance, if we exchange the values of  $\phi_1$  and  $\phi_2$  in Fig. 4(c) to make the left (right) half topologically trivial (nontrivial) instead, the interface state becomes damped and a growing edge state arises at the right open boundary, as shown in Fig. 4(d). In the latter case, the interface state resides only on the *A* sites with loss, while the right edge states resides only on the *B* sites with gain.

To investigate the PT-symmetry transitions of the non-Hermitian heterostructure, we plot in Fig. 5 its energy spectrum as a function of  $\gamma$ . The vertical lines in Fig. 5 correspond to the case shown in Fig. 4, the different shading along this line in Fig. 5(a) representing the varying density of states of the spectrum in Fig. 4(a). Since the introduction of  $\gamma$  does not affect the topological phases, no matter how large  $\gamma$  is, there are always only two topologically protected modes (one edge and one interface) with imaginary eigenvalues  $\pm i\gamma$ . It is also clear that as  $\gamma$  increases, the real part of the spectrum as well as the band gap will shrink, while the imaginary part

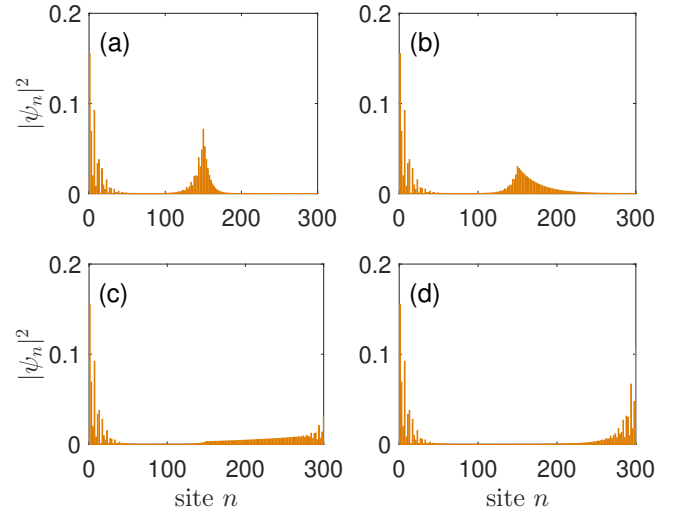


FIG. 6: (Color online) The spatial profiles of the edge and interface states for the finite heterostructure based on the Hermitian ( $\gamma = 0$ ) generalized SSH model with (a)  $\phi_2 = \pi/4$ , (b)  $\phi_2 = \pi/8$ , (c)  $\phi_2 = \pi/12$  and (d)  $\phi_2 = \pi/20$ . Other parameters are the same as in Fig. 3.

widens gradually. We find in particular two critical values  $\gamma_{c1} = 0.30$  and  $\gamma_{c2} = 3.65$  characterizing the PT-symmetry transitions in this figure. For  $\gamma < \gamma_{c1}$ , the eigenvalues of all bulk states are real while those of the edge and interface states are imaginary and conjugate. For  $\gamma_{c1} < \gamma < \gamma_{c2}$ , a part of the bulk states become PT-symmetry-broken (i.e., their eigenvalues become imaginary) and the number of these bulk states increases with  $\gamma$ . This is consistent with previous works [80, 81], where relevant systems undergo PT-symmetry breaking immediately once a nonvanishing  $\gamma$  is introduced because the eigenvalues of the edge modes acquire imaginary parts. Further increasing  $\gamma$ , we can see a second phase transition at  $\gamma = \gamma_{c2}$ , above which all eigenvalues become imaginary. It is worth noting that the PT-symmetry breaking will induce a change from single edge-mode lasing to multimode lasing [40].

### C. Spatial adiabatic passage

In this section we show that the spatial profile of the interface state can be controlled via the Peierls phase. As shown in Fig. 6(a)-Fig. 6(c), if we reduce  $\phi_2$  from  $\pi/2$  gradually, the interface state becomes more and more extended at first. With small enough  $\phi_2$ , its wavefunction extends into the bulk of the right part, as shown in Fig. 6(c). If  $\phi_2$  further decreases, we find from Fig. 6(d) that the interface state eventually becomes localized to the right open boundary. In particular, when  $\phi_2$  vanishes, the heterostructure reduces to a homogeneous sample with winding number  $w = 1$ , and thereby one can observe two chiral edge states localized at the left and right boundaries, respectively. Notice that as  $\phi_2$  changes, the

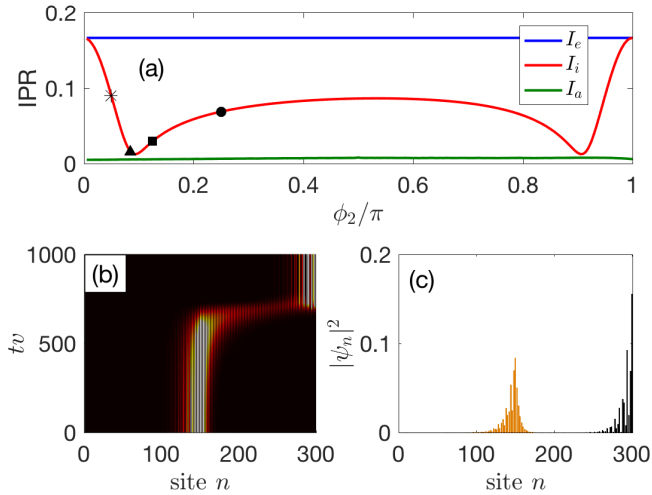


FIG. 7: (Color online) (a) The IPRs of the edge and interface states, and the averaged IPR of all bulk states of a finite heterostructure based on the Hermitian ( $\gamma = 0$ ) generalized SSH. The black dot, square, triangle and asterisk correspond to  $\phi_2 = \pi/4, \pi/8, \pi/12, \pi/20$ , respectively. (b) A dynamic SAP of the interface state shown in Fig. 3(b). (c) The intensity profiles of the initial (orange) and the final (black) states in (b). Other parameters are the same as in Fig. 3.

interface state remains on the same sublattice (i.e., it always resides on  $B$  sites here), even when it becomes extended during the process. At the same time, the left edge state is insensitive to  $\phi_2$ .

We gain further insight by introducing the inverse participation ratio (IPR) [71, 72] of an eigenstate  $\psi = \sum_n \psi_n$ ,

$$I = \frac{\sum_n |\psi_n|^4}{(\sum_n |\psi_n|^2)^2}, \quad (6)$$

which is a measure of the state's localization whereby larger IPRs correspond to stronger localization. In one-dimensional systems, the IPR of an extended state roughly equals the inverse of the system length. In Fig. 7(a), we plot the IPRs of the edge and interface states denoted by  $I_e$  and  $I_i$  respectively, as well as the average of the IPRs of all bulk states, denoted by  $I_a$ . The variation of  $I_i$  indicates that the interface state becomes at first more and more extended as  $\phi_2$  decreases from  $\pi/2$  until  $I_i$  reaches its minimum at around  $\phi_2 = \pi/10$ . With  $\phi_2$  further decreasing, the interface state rapidly becomes localized again. In particular, if  $\phi_2 = 0$ , the heterostructure reduces to a homogeneous sample and thereby the interface state becomes the standard right edge state, possessing the identical IPR with the left edge state. The black dot, square, triangle and asterisk correspond to  $I_i$  of the interface states in Fig. 6(a)-Fig. 6(d), respectively. We also find that the left edge state is completely insensitive to  $\phi_2$  with a large  $I_e$  while the bulk states are almost insensitive to  $\phi_2$  with a roughly vanishing  $I_a$ .

According to the adiabatic theorem [73, 74], a system will remain in its instantaneous eigenstate for (1.) slow

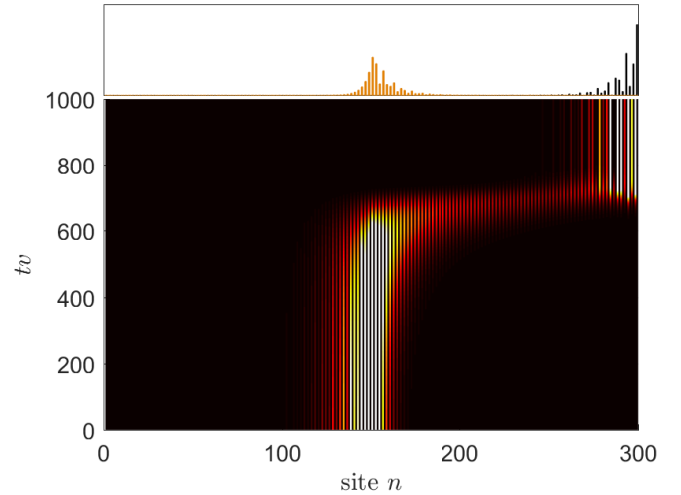


FIG. 8: (Color online) A dynamic SAP of the growing interface state in Fig. 6(c) for the PT-symmetric heterostructure. The top inset depicts the spatial profiles of the initial (orange) and the final (black) states. Other parameters are the same as in Fig. 4(c).

enough perturbations and for (2.) large enough gap between the corresponding eigenvalue and the rest of the spectrum. Thus we foresee as viable the spatial adiabatic passage (SAP) of a topologically protected mode [75, 76] through slow changes of the Peierls phase  $\phi$ . In this case excitations can be coherently transferred between spatially separated localized states with high efficiency and strong robustness. SAP was initially proposed following the idea of stimulated Raman adiabatic passage (STIRAP) in three-level atomic systems, and has been extended to optical systems with more than three coupled waveguides [77–79]. In our model, SAP is achieved by initially exciting a zero-energy mode and then, by adiabatically changing the Peierls phase of the half part of the sample through which it is to be transferred. Here we demonstrate this dynamic process in Fig. 7(b) by using the interface state shown in Fig. 3(b) as the initial state and assuming  $\phi_2(t) = \pi/2 - \pi tv/1600$  for  $tv < 800$  and  $\phi_2(t) \equiv 0$  for  $tv \geq 800$ . It is clear that the interface state is adiabatically transferred to the right open boundary and thereby evolves into the right edge state. In Fig. 7(c), we plot the spatial profiles of the initial interface and the final right edge states. These results are consistent with those in Fig. 4 and confirm that the SAP is highly efficient.

Moreover, an effective SAP can be realized also for the non-Hermitian case. We demonstrate in Fig. 8 how the growing interface state shown in Fig. 4(c) can be adiabatically transferred to the right open boundary in the same way as in Fig. 7(b). Note that the intensity profiles here have been normalized by  $\sum_n |\psi_n(t)|^2$ . The spatial profiles of the initial and final states in the top inset indicates that the SAP in this case maintains high efficiency. It is worth noting that the SAP scheme does not always hold in the non-Hermitian case as there are limitations

FIG. 9: (Color online) The real (a) and imaginary (b) parts of the energy spectrum of the PT-symmetric heterostructure as functions of  $\phi_2$  without disorder. The real (c) and imaginary (d) parts of the energy spectrum of the PT-symmetric heterostructure as functions of  $\phi_2$  with disorder described by  $\{\delta v_n, \delta J_{1,n}, \delta J_{2,n}\} \in [-0.2, 0.2]$ . The red lines depict the eigenvalues of the edge and interface states. Other parameters are the same as in Fig. 3 except  $\gamma = 0.2$ .

to the observation of adiabatic transport [82–84]. In our case, the SAP of damped states is unfeasible because they vanish with time while non-adiabatic processes populate other states which are amplified, similarly to what observed in Ref. [58]. In the case shown in Fig. 8, the initial state is actually amplified and adiabatically transferred to the final state with high efficiency for any positive value of  $\gamma$  (provided  $\gamma < \gamma_{c1}$ ). As we are not concerned here with the actual determination of the threshold for a potential lasing action based on such state, we do not consider gain saturation or nonlinear effects which would eventually affect the propagation of the amplified mode. The relevant message conveyed by the above results is that the spatial profile of the mode potentially suitable for topological lasing can be dynamically controlled via the Peierls phase.

#### D. Robustness against disorder

Finally, we aim to show that both the edge and the interface states remain topologically protected during a SAP so that they are robust against certain types of disorder. In Fig. 9(a) and Fig. 9(b), we plot the complex energy spectrum as a function of  $\phi_2$  with its real part only shown around the energy gap. Fig. 9(b) shows that the eigenvalues of some bulk states become imaginary in the vicinities of  $\phi_2 = 0.1\pi$  and  $0.9\pi$ , while all of them are real elsewhere. So changing  $\phi_2$  may lead to PT-symmetry transitions while the PT-broken regions may shrink or even disappear if we reduce  $\gamma$ . Most importantly, though the interface state becomes extended in its wavefunction during a SAP, its eigenvalue remains invariant (see the red lines) so that the SAP in our model is topologically protected. This is quite different from other instances of adiabatic pumping [85], whereby during the process the edge state acquires a bulk-like character in terms of both its wavefunction and eigenvalue, the latter shifting and tending to merge into the bulk spectrum. One of the most intriguing characteristics of the topologically protected states is that they are robust against imperfections. To prove this, we introduce disorders in the couplings, i.e.,  $v \rightarrow 1 + \delta v_n$  and  $J_\beta \rightarrow J_\beta + \delta J_{\beta n}$  ( $\beta = 1, 2$ ), with  $-0.2 \leq \delta v_n, \delta J_{1n}, \delta J_{2n} \leq 0.2$  being three independent stochastic (real) variables. In Fig. 9(c) and Fig. 9(d), we can find that the edge and interface states are immune to disorder, i.e., their eigenvalues are essentially identical to

those in Fig. 9(a) and Fig. 9(b). Moreover, we have numerically verified that the edge and interface states also resist real-valued on-site disorders, i.e.,  $\gamma \rightarrow \gamma + i\Delta_n$  with  $-0.2 \leq \Delta_n \leq 0.2$ . If we introduce complex-valued on-site disorders instead, the real parts of their eigenvalues do not change, while the imaginary parts undergo limited random shifts (not shown here).

## IV. CONCLUSIONS

The one-dimensional Su-Schrieffer-Heeger model, one among the simplest to host topological properties, has long served as an archetype of a topological structure [45]. Our extended SSH model, with two distinct real intercell couplings yet a *complex* intracell coupling, is found to exhibit a trivial and two nontrivial topological phases characterized by the non-Hermitian winding numbers  $w = \{0, \pm 1\}$ , respectively. Although these are not affected by the degree of non-Hermiticity, yet they are sensitive to the phase of the hopping term (Peierls phase). A topologically protected mode, lying at the interface between the two halves of a structure bearing different Peierls phases, is seen to be robust against disorder and of potential interest to topological lasing. Dynamic control of the spatial profile of this zero mode can be efficiently achieved through spatial adiabatic passage.

The emergence of topological phenomena in a simple one-dimensional setting has clearly inspired a number of experimental investigations and remarkably using rather different platforms such as, *e.g.*, ultracold atoms [86, 87], superconducting qubits [64] and RLC circuits [88]. One-dimensional periodic [51] and quasiperiodic [85] waveguide arrays have recently provided a complementary window into the physics of topological photonic structures. It is within such a framework that aspects of the model here proposed may be tested. In particular, structures based on coupled ring resonators subject to synthetic gauge fields control are envisaged as promising. Synthetic gauge fields can in fact be realized via dynamically modulated couplers [62, 63] and via thermal or electro-optic modulators [33, 65, 66].

## ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (No.s 10534002 and 11674049), the Cooperative Program by the Italian Ministry of Foreign Affairs and International Cooperation (No. PGR00960) and the National Natural Science Foundation of China (No. 11861131001), and the Program of State Key Laboratory of Quantum Optics and Quantum Optics Devices (No. KF201807). We also thank S. Chesi, Y. D. Wang and Y. Zhang for helpful discussions.

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