

Electron g factor anisotropy in asymmetric III-V semiconductor quantum wells

M. A. Toloza Sandoval¹, E. A. de Andrada e Silva¹, A. Ferreira da Silva² and G. C. La Rocca³

¹ *Instituto Nacional de Pesquisas Espaciais, C.P. 515,
12201-970 São José dos Campos, São Paulo, Brazil*

² *Instituto de Física, Universidade Federal da Bahia 40210-340, Salvador, Bahia, Brazil and*

³ *Scuola Normale Superiore and CNISM,
Piazza dei Cavalieri 7, I-56126 Pisa, Italy*

Abstract

The electron effective g factor tensor in asymmetric III-V semiconductor quantum wells (AQWs) and its tuning with the structure parameters and composition are investigated with envelope-function theory and $8 \times 8 \mathbf{k} \cdot \mathbf{p}$ Kane model. The spin-dependent terms in the electron effective Hamiltonian in the presence of an external magnetic-field are treated as a perturbation and the g factors g_{\perp}^* and g_{\parallel}^* , for the magnetic field in the QW plane and along the growth direction, are obtained analytically as a function of the well width L . The effects of the structure inversion asymmetry (SIA) on the electron g factor are analyzed. For the g-factor main anisotropy $\Delta g = g_{\perp}^* - g_{\parallel}^*$ in AQWs, a sign change is predicted in the narrow well limit due to SIA, which can explain recent measurements and be useful in spintronic applications. Specific results for narrow-gap $AlSb/InAs/GaSb$ and $Al_xGa_{1-x}As/GaAs/Al_yGa_{1-y}As$ AQWs are presented and discussed with the available experimental data; in particular $InAs$ QWs are shown to not only present much larger g factors but also a larger g-factor anisotropy, and with the opposite sign with respect to $GaAs$ QWs.

I. INTRODUCTION

Electron g-factor engineering, i.e. tuning the effective g factor (g^*) with quantum confinement effects in semiconductor nanostructures is of great interest to semiconductor spintronics [1–3]. g^* is a fundamental parameter that determines the Zeeman splitting of the electronic states and depends on different quantum effects. For the most basic example, in *GaAs/AlGaAs* like quantum wells (QWs) the g factor is determined by the confined wavefunction and by the mesoscopic (Rashba type) spin-orbit (SO) interaction at the interfaces [4–7]. Breaking of translation symmetry along the QW growth direction (\hat{z}) leads to an electron (leading-order) g-factor tensor in the following form:

$$g_{QW}^* = \begin{pmatrix} g_{\perp}^* & 0 & 0 \\ 0 & g_{\perp}^* & 0 \\ 0 & 0 & g_{\parallel}^* \end{pmatrix}, \quad (1)$$

where g_{\perp}^* gives the Zeeman splitting for magnetic-field in the QW (i.e. xy) plane and g_{\parallel}^* for $\vec{B} \parallel \hat{z}$. The difference $\Delta g = g_{\perp}^* - g_{\parallel}^*$ is the QW g-factor anisotropy, which in first order perturbation theory and for symmetric QWs with barriers at $z = \pm L/2$, reads [6]

$$\Delta g = \frac{4m_e}{\hbar^2} (\beta_w - \beta_b) L |f^{(0)}(L/2)|^2, \quad (2)$$

$f^{(0)}$ being the unperturbed confined wave-function and β the (energy-dependent) Rashba SO coupling parameter, discussed below; with $g_{\parallel}^* = \bar{g}_{bulk} = \langle f^{(0)} | g_{bulk} | f^{(0)} \rangle$ (i.e. equals to the bulk average). The g-factor anisotropy is then proportional to the difference between the β s in the well and in the barrier, and to the amplitude squared of the wave-function at the interface; and as a function of L (as shown later in Figures 3 and 4), Δg is seen to start equals to zero at $L = 0$, to reach then an extremum (e.g. a maximum at $L \sim 4 \text{ nm}$ for *GaAs* QWs) and then to return slowly to zero as L goes to infinity.

These theoretical results for $g_{QW}^*(L)$ give a simple picture for the renormalization of the electron g factor due to quantum confinement effects in III-V semiconductor QWs, are in good agreement with the experimental data for *GaAs* QWs [8–13], but are limited to symmetric QWs. The well known Ivchenko and Kiselev framework for the g-factor calculation [4, 14, 15] which is also based on the Kane model and envelope function approximation, presents an accurate solution for a general confinement (i.e. for both symmetric and asymmetric QWs) and was explicitly applied in biased *GaAs* and *InGaAs* triangular QWs [16].

However, the above perturbative solution is much simpler, gives an intuitive and useful physical interpretation for the g-factor renormalization, derives from long used and tested approximations in similar problems [17], more recently the g-factor solution has been applied also to PbTe QWs [7], GaN QWs [18] and GaAs nanodisks [3] and therefore it would be highly desirable to have such a solution for a general nanostructure. In particular, asymmetric quantum wells (AQWs), i.e. quantum wells with structure inversion asymmetry (SIA), are important candidates for structures with large g-factor variation. It is still not clear, for example, how the SIA affects the above results for the g factor in typical III-V AQWs. In this work we consider square AQWs (i.e. QWs with different left and right barriers), focus on the small L limit where both the quantum and SIA effects are larger, and extend the above solution for $g_{QW}^*(L)$ to the case of AQWs. Such low field perturbation solution has shown to be accurate in the $L \rightarrow 0$ limit [6, 9–11], and reveal here an interesting g-factor anisotropy sign change in narrow AQWs that can be useful in different spintronic applications; a detailed derivation of the main results is provided.

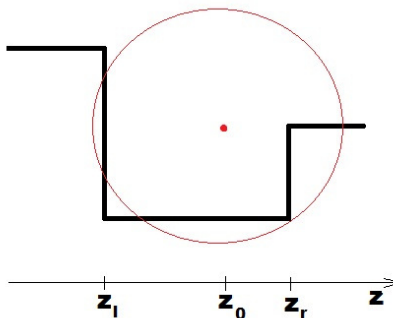


FIG. 1. Schematic illustration of a square asymmetric QW grown along z with different interfaces at z_l and z_r , and of the classical cyclotron orbit in real space for in-plane magnetic-fields, centered at z_0 .

It is also interesting to investigate the electron g factor in narrow-gap III-V semiconductor QWs with larger bulk g factors, stronger SO interaction and smaller remote-bands contribution than *GaAs* QWs. Among the narrow gap III-V semiconductors, *InAs* presents large SO interaction, high electron mobilities, small Shottky barriers and is therefore particularly attractive for spintronic applications [19, 20]. The gate-controlled electron g factor has been studied using *InAs* AQWs [2]. Here we consider the g_{QW}^* for electrons confined in

undoped *AlSb/InAs/GaSb* AQWs. These are QW structures with SIA, analytical spin-split electronic structure and therefore of interest to the physics of Rashba coupling in semiconductor 2DEGs; the special possibilities connected with the type II band-alignment in one interface and type III in the other make them of interest also to the topological insulator physics [21, 22]. Here expressions for the electron $g_{QW}^*(L)$ in general III-V square AQWs are obtained and used to calculate the electron g factor in specific *InAs* and *GaAs* AQWs. Several differences are found between them and between symmetric and asymmetric QWs; for example, the g-factor anisotropy Δg is seen to have opposite signs in *InAs* and *GaAs* QWs and, differently to the symmetric case, to change sign in narrow AQWs due to SIA.

Next we present the multi-band envelope-function model, then the AQW g-factor calculation, the results for *GaAs* and *InAs* AQWs, the comparison with the experiments and finally, in the conclusions, the summary of the results.

II. MULTI-BAND ENVELOPE-FUNCTION MODEL

Using standard envelope-function method based on the 8×8 $\mathbf{k} \cdot \mathbf{p}$ Kane model for the bulk, the set of equations for the envelope functions can be written as an effective Hamiltonian for the electron (i.e. conduction band) envelope-functions, with energy dependent effective-mass and explicit Rashba SO coupling [23–25]. In the presence of an external magnetic field we follow Ref. [6], add the bare Zeeman interaction, make the fundamental substitution $\mathbf{k} \rightarrow \mathbf{k} + \frac{e}{\hbar} \mathbf{A}$ ($-e$ being the electron charge) and, as further explained in the Appendix, obtain the following effective Hamiltonian for the QW electronic states in an in-plane magnetic field $\mathbf{B} = (0, B, 0)$:

$$H_{eff}^{(\sigma)} = -\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m(z, \varepsilon_\sigma)} \frac{d}{dz} + \frac{\hbar^2}{2} \frac{[(z - z_0)/\ell^2]^2}{m(z, \varepsilon_\sigma)} + E_c(z) \mp \frac{2m_e}{\hbar^2} \left[\alpha_R(z, \varepsilon_\sigma) (z - z_0) + \beta(z, \varepsilon_\sigma) \right] \mu_B B, \quad (3)$$

where the conduction band (CB) edge profile $E_c(z)$ is now a general one. The Landau gauge is used with vector potential $\mathbf{A} = (zB, 0, 0)$ and the signs \mp stand for spin down or up along y . Note that with this gauge the envelope function can be written as $\Psi = e^{i(k_x x + k_y y)} f(z)$, with $f(z)$ satisfying $H_{eff}^{(\sigma)} f = \varepsilon_\sigma f$, k_x being a quantum number that gives the center of the cyclotron orbit, i.e. $z_0 = -\ell^2 k_x$, where $\ell = \sqrt{\hbar/eB}$ is the magnetic length; and k_y is the wave-vector for the free motion along the B-field direction which is zero for the ground-state.

Finally ε_σ is the electron energy with spin σ ($= \pm$), the Bohr magneton $\mu_B = e\hbar/2m_e$ (m_e being the free-electron mass), $\alpha_R(z, \varepsilon_\sigma) = \frac{d}{dz}\beta(z, \varepsilon_\sigma)$ and the effective mass $m(z, \varepsilon_\sigma)$ and Rashba SO parameter $\beta(z, \varepsilon_\sigma)$ are given by:

$$\frac{1}{m(z, \varepsilon_\sigma)} = \frac{P^2}{\hbar^2} \left[\frac{2}{\varepsilon_\sigma - E_v(z)} + \frac{1}{\varepsilon_\sigma - E_v(z) + \Delta(z)} \right] \quad (4)$$

and

$$\beta(z, \varepsilon_\sigma) = \frac{P^2}{2} \left[\frac{1}{\varepsilon_\sigma - E_v(z)} - \frac{1}{\varepsilon_\sigma - E_v(z) + \Delta(z)} \right], \quad (5)$$

where E_v and Δ stand for the material valence band (VB) edge and spin-orbit splitting respectively; $P = -i(\hbar/m_e)\sqrt{\frac{2}{3}}\langle S|p_x|X\rangle$ is the momentum matrix element, assumed constant along the structure (as a fundamental assumption of the envelope-function approximation) and determined by the measured CB edge effective-mass in the well m^* , i.e. $P = \sqrt{\frac{\hbar^2}{m^*} \frac{E_g(E_g + \Delta)}{3E_g + 2\Delta}}$. It is easy to test and verify that the above effective Hamiltonian reduces exactly to well-known Hamiltonians in three limits: 1) in the zero magnetic field limit, giving the usual model for the Rashba effect [23–25], 2) with no SO interaction, giving the regular Landau level quantization in a QW [26] and 3) in the bulk limit, reducing to the theory of Roth et al. [27] for the energy-dependent bulk g factor .

III. G FACTOR IN AQWS

The ground state g_{QW}^* can be calculated in the small magnetic-field limit considering the spin dependent terms in $H_{eff}^{(\sigma)}$ as a perturbation. The zeroth order wave-function $f^{(0)}(z)$ and energy ε_0 are solutions of the unperturbed problem, i.e. with $B = 0$ and $k_y = k_x = 0$, $\left(-\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m(z, \varepsilon_\sigma)} \frac{d}{dz} + E_c(z)\right) f^{(0)}(z) = \varepsilon_0 f^{(0)}(z)$, which corresponds to the Kane AQW problem, which in turn can be easily solved exactly [25]. However, differently to the case of a symmetric QW, the expectation value \bar{z} does not coincide in general with the center of the well. For symmetric QWs with barriers at $z = \pm L/2$, $\bar{z} = 0$ and thus the lowest energy state also has $z_0 = 0$ (i.e., $k_x = 0$). For AQWs instead, z_0 has to be calculated minimizing the term $\frac{\hbar^2}{2m(z, \varepsilon_\sigma)} [(z - z_0)/\ell^2]^2$ in $H_{eff}^{(\sigma)}$ which simply leads to $z_0 = \bar{z}$, independent on B (while $k_x \propto B z_0$ still vanishes when $B \rightarrow 0$). We note that an alternative approach would be to change both coordinates, using $z' = z - \bar{z}$, and gauge, using $\mathbf{A}' = (z' B, 0, 0)$, in which case the wave function changes as $\Psi \rightarrow \Psi' = e^{ie\bar{z}Bx/\hbar} \Psi$ and the lowest energy Ψ' would

correspond to $z'_0 = 0$ and $k'_x \equiv 0$. Focussing here on AQWs, we prefer to leave the role played by \bar{z} explicit, which accounts for the diamagnetic shift [14–16].

In first order perturbation theory, one can simply calculate the g factor from Eq.(3) as:

$$g_{\perp}^* = \bar{g}_{bulk} - \frac{4m_e}{\hbar^2} \langle f^{(0)} | \alpha_R(z, \varepsilon_0) (z - \bar{z}) | f^{(0)} \rangle. \quad (6)$$

Note that in flat-band wells $\alpha_R (= \frac{d}{dz}\beta)$ is different from zero only at the interfaces where β changes abruptly. With $\vec{B} \parallel \hat{z}$, the system recovers the rotation symmetry around to growth direction, and in the same approximation (see Appendix) g_{\parallel}^* is given by the bulk average, i.e.

$$g_{\parallel}^* = \bar{g}_{bulk} = g_e + \langle f^{(0)} | -\frac{4m_e}{\hbar^2} \beta(z, \varepsilon_0) + \delta g_{rem}(z) | f^{(0)} \rangle, \quad (7)$$

δg_{rem} being the difference between the g-factor measured experimentally and that given by the Roth formula, i.e. the remote-bands contribution [27], and g_e the free electron g factor equals to 2. The QW g-factor anisotropy is then given simply by:

$$\Delta g = g_{\perp}^* - g_{\parallel}^* = -\frac{4m_e}{\hbar^2} \langle f^{(0)} | \alpha_R(z, \varepsilon_0) (z - \bar{z}) | f^{(0)} \rangle. \quad (8)$$

As illustrated in Fig. 1, for a general III-V square AQW we set the two non-equivalent interfaces at $z = z_l$ and $z = z_r$, with $z_r - z_l = L$. The expectation values above can be easily calculated and one gets:

$$g_{\parallel}^* = g_l(\varepsilon_0)P_l + g_w(\varepsilon_0)P_w + g_r(\varepsilon_0)P_r \quad (9)$$

and

$$\Delta g = (\Delta g)_r - (\Delta g)_l \quad (10)$$

where $P_i (= \int_i |f^{(0)}(z)|^2 dz)$ is the probability to find the electron in the region i ($i = l$ for $z \leq z_l$, $i = w$ for $z_l < z < z_r$ and $i = r$ for $z \geq z_r$); the bulk g factors $g_i = g_e - \frac{4m_e}{\hbar^2} \beta_i(\varepsilon_0) + \delta g_{rem}^{(i)}$ and

$$(\Delta g)_j = \frac{4m_e}{\hbar^2} \delta \beta_j (z_j - \bar{z}) |f^{(0)}(z_j)|^2 \quad (11)$$

with $j = l, r$ and $\delta \beta_j = \beta_w - \beta_j$.

The g-factor anisotropy Δg in AQWs is then seen to be determined by the differences $\delta \beta$ and by the wave-function amplitudes (squared) at each interface, weighted however by their distance to the center of the ground state orbit $z_0 = \bar{z}$. Note that in symmetric QWs $\delta \beta_l = \delta \beta_r$, $\bar{z} - z_l = z_r - \bar{z} = L/2$ and the symmetrical result in Eq. (2) is recovered. Recall

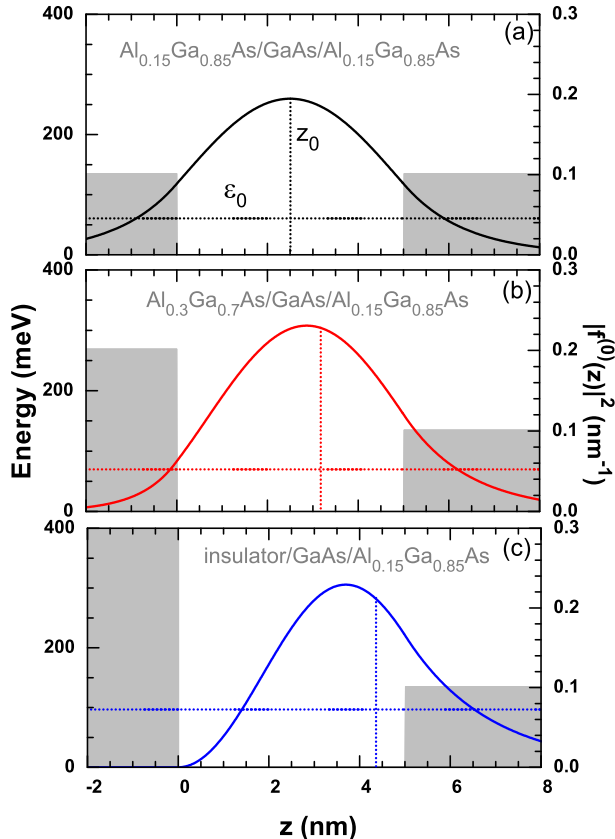


FIG. 2. (Color online) Conduction band-edge profile $E_c(z)$, ground state energy ε_0 and probability density $|f^{(0)}(z)|^2$ (with axis on the right) for $L = 5 \text{ nm}$ (a) $Al_{0.15}Ga_{0.85}As/GaAs/Al_{0.15}Ga_{0.85}As$ SQW, (b) $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.15}Ga_{0.85}As$ AQW (red lines) and (c) $insulator/GaAs/Al_{0.15}Ga_{0.85}As$ AQW (blue lines). The respective ground state orbit center $z_0 = \bar{z}$ are also shown. The parameters used were $E_g = (1.519 + 1.247x) \text{ eV}$, $\Delta = (1.859 + 1.115x + 0.37x^2 - E_g) \text{ eV}$ (x being the Al concentration) and $0.067m_e$ for the $GaAs$ conduction-band edge effective mass. For the conduction band-offset the 72% rule was used.

also that for symmetric QWs, the ground-state corresponds to $\bar{z} = (z_l + z_r)/2$ at the center of the well and the contributions from the two interfaces are equal: $-(\Delta g)_l = (\Delta g)_r = \Delta g/2$. The specific contribution of the SIA to Δg increases with $|(z_l + z_r)/2 - \bar{z}|$ and with $|\delta\beta_r|f^{(0)}(z_r)|^2 - \delta\beta_l|f^{(0)}(z_l)|^2$. It is interesting to consider also the limit case of an infinite high barrier (a perfect insulator) in one of the two sides (say the l side); in this case, the g factor anisotropy is simply given by $\Delta g = \frac{4m_e}{\hbar^2} \delta\beta_r (z_r - \bar{z}) |f^{(0)}(z_r)|^2$.

Contrary to the symmetric QW case, the sign of Δg in AQWs is not uniquely determined by the sign of $\delta\beta$ but depends also on the sign of $(z_{interf} - \bar{z})$. In practice, to calculate

$g_{QW}^*(L)$ one solves the unperturbed problem and in the above equations just plug in the obtained $f_L^{(0)}(z)$, $\varepsilon_0(L)$ and also $\bar{z}(L)$. Next we discuss the results for specific *GaAs* and *InAs* AQWs.

IV. $Al_xGa_{1-x}As/GaAs/Al_yGa_{1-y}As$ AQW

As a first example we consider $Al_xGa_{1-x}As/GaAs/Al_yGa_{1-y}As$ (with $x \neq y$) AQWs. The results for $g_{QW}^*(L)$ in these wells are compared to those in similar *GaAs* symmetric wells (SQWs) and in *insulator/GaAs/Al_yGa_{1-y}As* AQWs. Typical ground-state unperturbed solutions for these three types of QWs are shown in Figure 2 with the conduction band profile and $L = 5$ nm, showing that the wave-function is deformed and z_0 is pushed away from the barrier on the left as the barrier height increases. Similarly, for a fixed AQW profile, z_0 is pushed away from the higher barrier as the well width L is decreased.

In Figure 3, the obtained g_{\perp}^* , g_{\parallel}^* and Δg for these three wells are plotted as a function of L . First it is interesting to see that the break of specular symmetry (or SIA) has only small quantitative effects in the *GaAs* QW electron g factor except for narrow wells when the g-factor anisotropy changes sign and starts to increase rapidly. In these *GaAs* square AQWs there is always a critical well width below which g_{\perp}^* becomes smaller than g_{\parallel}^* , i.e. Δg becomes negative, while in symmetric QWs Δg is always positive. Such anisotropy sign change in thin AQWs is due to the \bar{z} dependence. For example, in *insulator/GaAs/GaAlAs* AQWs it happens when $\bar{z} > z_r$, i.e. when the expectation value of the electron position along the growth direction lies outside the QW or *GaAs* region. Note also that for large well widths, the anisotropy of the $Al_xGa_{1-x}As/GaAs/Al_yGa_{1-y}As$ AQWs tend to that of the *GaAs* SQW, and when one of the barriers is infinitely high, the anisotropy is a factor of 2 smaller, as due to one interface only.

V. $AlSb/InAs/GaSb$ AQW

The present 8×8 $\mathbf{k} \cdot \mathbf{p}$ Kane model is much more precise for *InAs* than it is for *GaAs*, as indicated by a much smaller $\delta g_{rem}/g$, which is ~ 0.03 for *InAs* and ~ 1.1 for *GaAs*. Here we consider the electron g factor in *InAs* QWs similar to the *GaAs* ones discussed above, namely thin *InAs/GaSb* symmetric QWs, *AlSb/InAs/GaSb* AQWs and *insulator/InAs/GaSb*

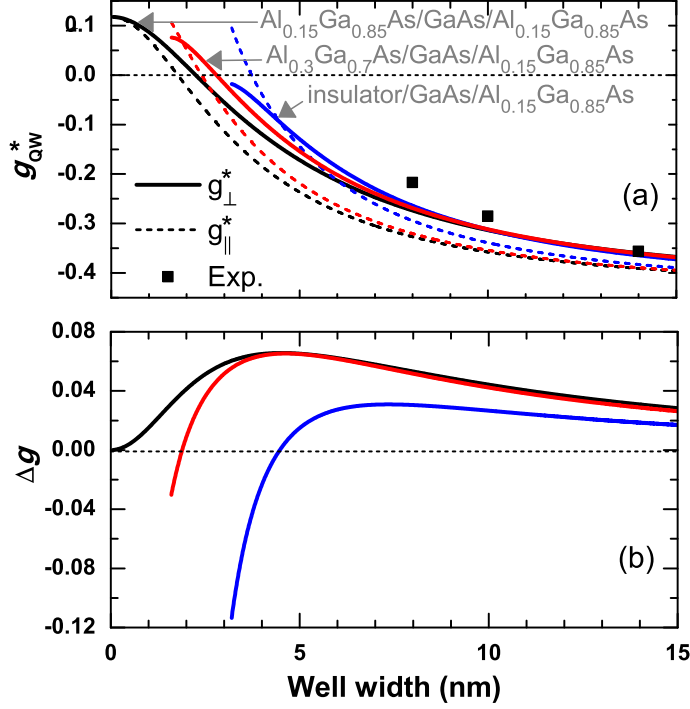


FIG. 3. (Color online) Effective electron g factors and the anisotropy Δg as a function of the well width, in panels (a) and (b) respectively, for the same structures in Figure 2. In AQWs, Δg is seen to change sign for narrow wells; note that the curves stop at the corresponding critical well width for a bound state. A bulk conduction-band edge $g^* = -0.44 + 4.25x - 3.9x^2$ (x being the Al concentration) was used [28]. The experimental points are for g_{\parallel}^* in asymmetrically doped $GaAs/Al_{0.15}Ga_{0.85}As$ AQWs as reported in Ref. [29].

AQWs. Due to the broken-gap band alignment, there are $InAs$ confined electron states in these QWs with $GaSb$ barriers only when the well width $L < L_c \sim 9 \text{ nm}$. This is because in $InAs/GaSb$ QWs with $L > L_c$, the electron energy ε_0 gets below E_v^{GaSb} and the state is not confined in the $InAs$ layer anymore [30, 31]. Note that when $\varepsilon_0 = E_v^{GaSb}$, the $GaSb$ (bulk) g factor diverges (see expression for β in Eq. (5)) and $g_{QW}^*(L \sim L_c)$ is therefore expected to be quite large since the barrier penetration is also expected to increase when $L \rightarrow L_c$.

In Figure 4(a) it is plotted the calculated g_{\perp}^* (continuous lines) and g_{\parallel}^* (dashed lines) for the three types of wells as a function of L ; and in Figure 4(b), the corresponding Δg . The SIA can be clearly seen to have a much stronger effect in $InAs$ QW g factors than it has in $GaAs$ wells, due to the stronger SO interaction. Compared to $GaAs$ QWs, the electron g factor in $InAs$ QWs is seen to be two-orders of magnitude larger, with Δg also much larger

and of the opposite sign. Note that the anisotropy Δg in these *InAs* AQWs changes sign at a smaller well width and that in *AlSb/InAs/GaSb* wells, in the large L limit, Δg tend to that of *insulator/InAs/GaSb* AQWs, instead to that of SQWs as in *GaAs* wells, due to the large conduction (Γ) *AlSb/InAs* band-offset. In these *InAs* AQWs, g_{\parallel}^* is seen to present a maximum as a function of L .

It is also interesting to note the obtained large (in absolute value) electron g factor (~ 30) in *InAs/GaSb* QWs when $L \rightarrow L_c$, which as just discussed, is due to barrier penetration and to a divergence in the energy dependence of the bulk g factor in *GaSb*. We now compare these calculations with the available experimental data.

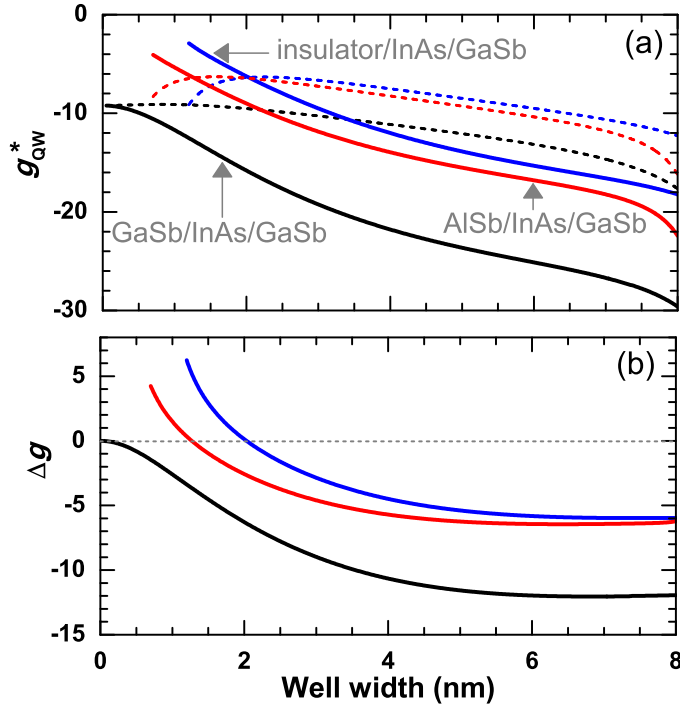


FIG. 4. (Color online) Effective electron g factors (g_{\perp}^* , g_{\parallel}^*) and the corresponding anisotropy Δg (panels (a) and (b) respectively) as a function of the well width, for three different *InAs* QWs: *GaSb/InAs/GaSb* symmetric QW (black lines), *insulator/InAs/GaSb* asymmetric QW (blue lines) and *AlSb/InAs/GaSb* asymmetric QW (red lines). As before, dashed lines give g_{\parallel}^* and continuous g_{\perp}^* . Well known bulk low temperature parameters [32] and 0.96 eV and 1.98 eV for *InAs/GaSb* and *InAs/AlSb* conduction-band offsets respectively, were used.

VI. COMPARISON WITH THE EXPERIMENTS

The electron g factor in $GaAs/AlGaAs$ symmetric QWs has been measured by different groups using both optical and transport techniques, involving time-resolved photoluminescence and magnetoresistance measurements [8–13]. In particular g_{\parallel}^* is now well known; with good accuracy, it is given by the above QW average of the bulk g factors, which includes effects from the energy dependence of the bulk g factors and from the wave-function barrier penetration. Shown in Fig. 3(a), as L goes from very large values to near zero, g_{\parallel}^* interpolates from the g factor in the well material to that in the barrier, with a sign change and, therefore, $g_{\parallel}^* = 0$ for a certain (narrow) well [4, 13].

The anisotropy Δg and its well width dependence are less well known. With spin-quantum beats in the time-resolved photoluminescence [9], Le Jeune et al. [10] and Malinowski and Harley [11] have measured $\Delta g(L)$ in $GaAs$ symmetric QWs, and as shown in ref. [6] it is simply and accurately described by Eq. (2) above. In $Al_xGa_{1-x}As/GaAs/Al_yGa_{1-y}As$ square AQWs, Ye et al. [33] have measured the in-plane Zeeman splitting anisotropy which is allowed by the SIA but is due to higher order terms (in k and in B) [34].

More recently, $g_{QW}^*(L)$ in asymmetrically doped $GaAs/AlGaAs$ AQWs was studied by Shchepetilnikov et al. [29] using electron spin resonance detected with magnetoresistance measurements. These data with doped AQWs can not be precisely compared with our undoped square AQW results; nevertheless, as shown in Fig. 3(a), the observed values and well width dependence of g_{\parallel}^* agree fairly well with the theory. As expected the experimental g_{\parallel}^* (which is for doped, $\varepsilon_F \neq 0$, AQWs) is a little larger than the calculated one (which is for undoped wells); more interesting, the measured g -factor anisotropy $\Delta g \sim 0.08$ for the 8 nm AQW (see ref. [29] Fig. 3) is not far from the calculated $\Delta g = 0.05$ in Fig. 3(b) above (note that in such well width range, both g_{\parallel}^* and Δg increase with electron energy). However, measurements of Δg in thin undoped $GaAs$ square AQWs are still needed in order to confirm the above obtained effect of anisotropy sign change.

Regarding such negative anisotropy in narrow $GaAs$ AQWs, Tomimoto et al. [35] have measured a negative Δg in a single sample with very narrow ($L = 0.32\text{ nm}$) $CdTe/ZnTe$ QW and attributed it to a change in the sign of $\delta\beta$. It is interesting to note however that the g factor in such wells behaves as in $GaAs$ QWs, with positive Δg and no change in the sign of $\delta\beta$ as a function of L when it is symmetric; this observed negative Δg is then likely to

have another explanation. The growth of homogeneous and symmetric *CdTe/ZnTe* QWs is difficult due to the large lattice mismatch and the sample studied may well present some specular asymmetry which can account for the observed negative Δg .

For *InAs* QWs there are much fewer experimental data. Smith and Fang [36] measured an electron $g_{QW}^* \sim -8$, in 10 nm *GaSb/InAs/GaSb* QWs, which however is a too wide QW, i.e. beyond the bound state regime considered here; and can not be compare with our results also due to the large magnetic-fields employed in the coincidence (between Zeeman and Landau-level splittings) method used. The g factor in wide *InAs* QWs was determined with similar methods also in references [37, 38]. The electron g factor in thin *InAs* AQWs was studied by Nitta et al. [2] and an $|g_{\parallel}^*|$ of the order of 3.5 was measured in biased 4 nm *InAs/InGaAs* AQWs, which is not far from the present results, however the structures are quite different and can not be directly compared. In general, a self-consistent treatment of the band-edge profile and/or a fine control of the structure parameters (including temperature dependence, precise band-offset and electron effective mass etc.) are needed for a quantitative precise description of the experimental data. The anisotropy Δg in *InAs* QWs does not seem to have been measured yet.

VII. CONCLUSIONS

Enough ground has ben given to believe that the electron effective g factor in III-V semiconductor AQWs can be tuned within a wide range of values by controlling the well width and composition, including a change of sign in both g factor and g-factor anisotropy. The obtained effect of g-factor anisotropy sign change in narrow wells is shown to be due to SIA and determined by the electron average position in the AQW, and can explain recent observations. Results for the electron effective g factor tensor in different *GaAs* and *InAs* QWs have been presented in fairly good agreement with the available experimental data. With respect to *GaAs* QWs, *InAs* QWs are seen to not only have a much larger g factor and g-factor anisotropy but also opposite anisotropy sign. The analytical expressions derived apply to general III-V square AQWs. These results for the g-factor renormalization by the mesoscopic quantum confinement in semiconductor nanostructures should be of importance not only for the development of spintronic devices but also for the spin manipulation with external electric or magnetic fields in spin-based qubits made with semiconductor nanos-

structures.

Appendix: Effective Hamiltonian and g^* for III-V QWs

The effective Hamiltonian used for the QW electronic states in both transverse (Eq. 3) and longitudinal magnetic fields, is obtained projecting the 8×8 Kane Hamiltonian into the 2×2 conduction-band space. Considering QWs grown along \hat{z} and using the same basis states as in Ref. [24] after some simple algebra one obtains the following Schroedinger-Pauli-like effective Hamiltonian:

$$H_{eff} = H_0 \mathbf{1} + i [\beta(z, \varepsilon_\sigma) \hat{k}_x, \hat{k}_z] \sigma_y + i \beta(z, \varepsilon_\sigma) [\hat{k}_x, \hat{k}_y] \sigma_z + i [\beta(z, \varepsilon_\sigma) \hat{k}_y, \hat{k}_z] \sigma_x, \quad (\text{A.1})$$

where H_0 is the usual spin-independent part, i.e. kinetic energy plus confining potential, with the energy dependent effective mass in Eq. (4). The components \hat{k}_i are the momentum $(\mathbf{k} + \frac{e}{\hbar} \mathbf{A})$ operators, which for zero magnetic field lead to $\hat{k}_z = -i \frac{d}{dz}$ plus $\hat{k}_x = k_x$ and $\hat{k}_y = k_y$ (i.e. good quantum numbers). Note that in this case one then has $[k_x, k_y] = 0$, $[\beta k_x, \hat{k}_z] = i \alpha k_x$ and $[\beta k_y, \hat{k}_z] = i \alpha k_y$ (recall that $\alpha = \frac{d\beta}{dz}$) which substituting above give the well known Rashba effective Hamiltonian [23–25] (with energy dependent SO coupling parameter α ; note that β is sometimes also called Rashba coupling parameter, but clearly should not be confused with its derivative α).

In the presence of a longitudinal homogeneous magnetic field, one can use the Landau gauge $\mathbf{A} = (-By, 0, 0)$ and after the fundamental substitution finds $[\hat{k}_x, \hat{k}_y] = -i \frac{e}{\hbar} B$, $[\beta \hat{k}_y, \hat{k}_z] = i \alpha \hat{k}_y$ and $[\beta \hat{k}_x, \hat{k}_z] = i \alpha (\hat{k}_x - \frac{e}{\hbar} By)$. Since in this case H_{eff} does not depend on x , one can write the envelope spinor as $\psi = e^{ik_x x} \psi(y, z)$, where k_x sets the center of the orbit and can be chosen equal to zero, and one then has:

$$H_{eff} = H_0 \mathbf{1} + \alpha (\hat{k}_y \sigma_x + \frac{e}{\hbar} By \sigma_y) + \beta \frac{e}{\hbar} B \sigma_z. \quad (\text{A.2})$$

Note that independently of the known divergence of the vector potential in infinite systems (which can be treated with a modulated vector potential [15, 39] or by considering finite systems [14]), the off-diagonal terms above, i.e. those proportional to α are much smaller than the diagonal ones (proportional to β) and in a good first approximation can be neglected, so that g_{\parallel}^* is simply given by Eq.(7). The accuracy of this approximation was verified in Ref. [4] where it is shown to be in close agreement with the full solution of the Kiselev-Ivchenko

equations (see curves 1 and 3 in Figure 3 there); the present approximation corresponds to neglecting the Kiselev-Ivchenko auxiliary function h [14, 15], which is indeed several orders of magnitude smaller than the main function. Note also that in the flat-band QWs considered here α is different from zero only at the two interfaces, where it presents opposite signs, so that the corresponding expectation values should indeed be very small compared with the main term.

Similarly for a transverse magnetic-field, as already discussed, one can chose $\mathbf{A} = (Bz, 0, 0)$ and has $[\hat{k}_x, \hat{k}_y] = 0$, $[\beta\hat{k}_y, \hat{k}_z] = i\alpha k_y$ and $[\beta\hat{k}_x, \hat{k}_z] = i(\alpha(k_x + \frac{e}{\hbar}Bz) + \frac{e}{\hbar}\beta B)$, which substituting above give the effective Hamiltonian in Equation (3).

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