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## Anharmonic Effects on Vibrational Spectra Intensities: Infrared, Raman, Vibrational Circular Dichroism and Raman Optical Activity

Julien Bloino<sup>\*,†</sup>, Malgorzata Biczysko<sup>\*,‡,†</sup>, and Vincenzo Barone<sup>¶</sup>

<sup>†</sup>Consiglio Nazionale delle Ricerche, Istituto di Chimica dei Composti OrganoMetallici (ICCOM-CNR), UOS di Pisa, Area della Ricerca CNR, Via G. Moruzzi 1, I-56124 Pisa, Italy

<sup>‡</sup>International Center of Quantum and Molecular Structures, College of Sciences, Shanghai University, 99 Shangda Road, Shanghai, 200444 China

<sup>¶</sup>Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy

### Abstract

The aim of this paper is twofold. First, we want to report the extension of our virtual multifrequency spectrometer (VMS) to anharmonic intensities for Raman Optical Activity (ROA) with the full inclusion of first- and second-order resonances for both frequencies and intensities in the framework of the generalized second-order vibrational perturbation theory (GVPT2) for all kinds of vibrational spectroscopies. Then, from a more general point of view, we want to present and validate the performance of VMS for the parallel analysis of different vibrational spectra for medium-sized molecules (IR, Raman, VCD, ROA) including both mechanical and electric/magnetic anharmonicity. For the well-known methyloxirane benchmark, careful selection of density functional, basis set, and resonance thresholds permitted to reach qualitative and quantitative vis-à-vis comparison between experimental and computed band positions and shapes. Next, the whole series of halogenated azetidinones is analyzed, showing that it is now possible to interpret different spectra in terms of electronegativity, polarizability, and hindrance variation between closely related substituents, chiral spectroscopies being particularly effective in this connection.

### Keywords

anharmonicity; VPT2; intensities; IR; Raman; VCD; ROA

## 1 Introduction

Contemporary chemical research relies more and more on sophisticated experimental techniques, mainly based on rotational, vibrational, electronic, and resonance spectroscopies<sup>1–13</sup>, allowing the investigation of systems of increasing complexity, e.g. in biology, to identify molecular life mechanisms,<sup>14</sup> and in materials science, to search for new

\* julien.bloino@pi.iccom.cnr.it; biczysko@shu.edu.cn.

materials with tailored/tuneable functionalities.<sup>15</sup> Particularly effective routes are established when several spectroscopic techniques are combined together and are further supported by integrated computational approaches. As a matter of fact, spectra do not provide direct access to molecular structure and dynamics, and interpretation of the indirect information that can be inferred from the analysis of the experimental data is seldom straightforward. These complications arise from the inherent complexity caused by thermal or environmental effects and/or from intrinsic properties of the system itself, whose specific roles are not easy to separate and evaluate. In this context, computational spectroscopy is becoming a powerful and reliable tool to help unravel the various contributions to the spectroscopic signal, allowing a deeper understanding of the underlying phenomena.  
2,7,9,16–25

Till now vibrational computations for all but the smallest molecules<sup>26–31</sup> are still mainly performed within the double harmonic approximation, possibly employing simple scaling factors<sup>32–37</sup> or more sophisticated scaling methods<sup>32,38,39</sup> to improve the agreement with anharmonic fundamental transitions. However, such approaches can correct at best the band positions, but have little effect on the general pattern of the harmonic spectrum. Only the explicit inclusion of anharmonic contributions can provide more realistic spectral patterns and either variational<sup>40–43</sup> or perturbative<sup>44–53</sup> routes can be followed to this end. However, all computations which do not take into account anharmonic effects on the transition moments do not provide any information about the intensities of overtones and combination bands, or the intensity re-distribution of fundamental transitions, which might be necessary to correctly analyze experimental outcomes. The most obvious cases are spectral ranges where only non-fundamental transitions are present, either in mid-IR range (i.e. the 1700–2800  $\text{cm}^{-1}$  range for naphthalene<sup>54</sup> or methyloxirane<sup>55</sup>) or in the near-infrared (NIR) region.<sup>54,56–58</sup> However, fully anharmonic computations are also needed to distinguish low-intensity features related to non-fundamental transitions of the most populated species present in experimental mixtures from fundamental transitions of the less abundant ones.<sup>59–62</sup> The situation becomes even more involved for chiroptical spectra, where anharmonic effects might modulate the observed overall band-shape. However, in most of the contemporary computational studies the anharmonic effects are still at best applied to the energies.

Direct vis-à-vis comparison between simulated and computed spectra can be greatly facilitated by the development of computational models for anharmonic vibrational computations, along with their implementation into computational packages.<sup>40,41,43–49</sup> In particular, in our group we are actively developing a virtual multi-frequency spectrometer (VMS),<sup>23,63,64</sup> providing user-friendly access to the latest developments of computational spectroscopy.<sup>54,65–71</sup> The section of VMS in charge of vibrational spectroscopy<sup>54,65–67</sup> has been already successfully applied to the study of Infrared (IR), Raman and vibrational circular dichroism (VCD) spectra of molecular systems of different sizes and complexities, in gas phase, and in other environments (see for instance Refs.<sup>25,55,58,62,72–77</sup>). The present work is devoted to the extension of the VMS capabilities to the simulation of fully anharmonic Raman Optical Activity (ROA) spectra with a proper account of first- (1-2) and second- (1-1, 2-2, 1-3) order resonances in a perturbative/variational evaluation of both frequencies and intensities.

In this context, the vibrational spectra of (*R*)-methyloxirane (MeOx, see Figure 1) recorded in gas phase<sup>78,79</sup> or low-temperature matrices<sup>76,80</sup> permit a direct comparison with theoretical results, without any strong environmental perturbation within the fingerprint region. For infrared spectra, it has been shown that a very good agreement with experiment, for both band positions and intensities, can be obtained by coupling harmonic calculations at the Coupled Cluster (CC) level with anharmonic effects computed by density functional theory (DFT).<sup>55</sup> However, obtaining accurate VCD spectra, which require both electric and magnetic dipole moments, has proven more challenging. Indeed, extensions of CC to compute the atomic axial tensor have been proposed<sup>81,82</sup> but not implemented to the best of our knowledge. As a result, simulations of VCD spectra are commonly performed at the DFT level,<sup>76</sup> with some uncertainty on the reliability of the methodology. In the present work we will apply the highly accurate CC/DFT force field along with the DFT-based property potential to simulate ROA spectra of methyloxirane in gas phase.

These new developments allow an extensive characterisation through multiple spectroscopies (e.g. IR, Raman, VCD and ROA) in a way similar to what is now commonly done experimentally. In this framework, an effective computational strategy will be applied to analyze substituent effects in  $\beta$ -lactams, a class of molecules showing remarkable biological activity.<sup>83</sup> The (*R*)-4-*X*-2-azetidinone derivatives (*X*=F, Cl, Br, see Figure 1), the simplest chiral compounds from this family, have been already selected as model systems in systematic computational investigations of structural, IR, Raman, VCD, and resonance ROA features<sup>84,85</sup> within the rigid-rotor / harmonic-oscillator (RRHO) model, pointing out a significant tuning of spectral properties by the halogen substituent. Here, we will extend this analysis to a more refined model including both mechanical and electric/magnetic anharmonicities for IR, Raman and VCD together with far-from-resonance Raman Optical Activity.

The paper is organized as follows: we start from a brief description of the theoretical framework underlying the anharmonic vibrational computations, with particular focus on ROA intensities (section 2), followed by the computational details (section 3). Then, the fully anharmonic IR, Raman, VCD and ROA spectra of methyloxirane are presented and compared with experimental results in section 4.1, while anharmonic effects on the spectral line-shape, considering both band positions and intensities, are discussed taking (*R*)-4-fluoro-2-azetidinone as a test case. Finally fully anharmonic spectra computed for (*R*)-4-*X*-2-azetidinones (*X*=F, Cl, Br) are discussed in section 4.3, highlighting advantages of our integrated vibrational model for the analysis of vibrational features. General conclusions and perspectives are outlined in the last section.

## 2 Theory

### 2.1 ROA intensities

In order to simulate a broad range of spectroscopies at the VPT2 level, the general framework presented in Refs<sup>54,66</sup> has been used here. The basic principle is to construct a generic property,  $\mathbf{P}$ , defined in such a way as to represent either properties functions of the normal coordinates ( $\mathbf{q}$ ) or their conjugate momenta ( $\mathbf{p}$ ),

$$P = P^{(0)} + P^{(1)} + P^{(2)} \quad (1)$$

$$P^{(0)} = P^{\text{eq}} + s_0 \sum_{i=1}^N P_i (a_i^\dagger + S a_i) \quad (2)$$

$$P^{(1)} = s_1 \sum_{i=1}^N \sum_{j=1}^N P_{ji} q_j (a_i^\dagger + S a_i) \quad (3)$$

$$P^{(2)} = s_2 \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N P_{jki} q_j q_k (a_i^\dagger + S a_i) \quad (4)$$

where  $a_i^\dagger$  and  $a_i$  are respectively the creation and annihilation operators.  $s_0$ ,  $s_1$  and  $s_2$  are constant factors and  $S$  corresponds to a sign, represented as either +1 or -1.

This function is then used to obtain analytic formulas for the transition moments up to 3 quanta<sup>54</sup> and can be simply related to the property of interest by identifying the variables in eqs. 2–4 with the actual quantities. The case of IR, VCD and Raman has already been discussed before,<sup>25,54,66</sup> so we will focus here on the extension of our framework to ROA.

Raman optical activity is measured as the difference between Raman scatterings associated to right (R) and left (L) circularly polarized lights. However, at variance with VCD for instance, the measurement setup is not unique, as either the incident (incident circular polarization, ICP) or the scattered (SCP) radiation beams can be modulated to get the right and left circular polarization states. Alternatively, both beams can be simultaneously modulated (dual circular polarization, DCP), either in-phase (DCP<sub>I</sub>) or out-of-phase (DCP<sub>II</sub>). Furthermore, several scattering geometries and polarizations can be chosen for the measurements. Indeed, the scattered beam can be measured at different angles with respect to the incident beam, along the same direction (0°, forward scattering), in the opposite direction (180°, backward scattering or backscattering), or at right angle (90°, right-angle scattering) for instance, and different orientations of the linear polarization analyzer can be chosen. The most common scattering setups are summarized in table 1, as well as the symbol used to represent them.

An important simplification to the theoretical problem, used here as well, is to consider that the measurement is done far from resonance. Hence, the present formulas and the relative discussions should only be applied to such cases. Near-resonance conditions require a different development (see for instance Refs.85,86 for discussions on this matter).

ROA was first predicted theoretically by Barron and Buckingham.<sup>87</sup> Successive works contributed to a comprehensive definition of the physical bases underlying the various kinds of measurements of ROA intensities (see Refs.6,88–92 for reviews on the subject). Of particular interest in the present work, ROA and Raman intensities have been derived for a large number of measurement setups by Hecht and Nafie.<sup>93</sup> Those formulas can be significantly simplified in the far-from-resonance regime. In this context, ICP and SCP provide equivalent information, which differ from DCP. An interesting feature of the latter, in the in-phase, backscattering configuration, is that it can be used to eliminate all isotropic configurations.<sup>90,93</sup>

Based on these simplifications, it is more convenient to use the notation of Barron and coworkers<sup>90</sup> than the general one employed in Ref.93. The formulas for the most common types of scatterings and measurements contributions are reported in table 2.

Hence, the Raman and ROA intensities can be directly related to the isotropic invariants,  $\alpha$  and  $G'$ , and the symmetric anisotropic invariants,  $\beta(\alpha)^2$ ,  $\beta(G')^2$ ,  $\beta(A)^2$ , given by,

$$\alpha = \frac{1}{3} \langle \alpha_{\tau\tau} \rangle_{I,F} = \frac{1}{3} \sum_{\tau=x,y,z} \langle \alpha_{\tau\tau} \rangle_{I,F} \quad (5)$$

$$G' = \frac{1}{3} \langle G'_{\tau\tau} \rangle_{I,F} = \frac{1}{3} \sum_{\tau=x,y,z} \langle G'_{\tau\tau} \rangle_{I,F} \quad (6)$$

$$\beta(\alpha)^2 = \frac{1}{2} \sum_{\tau,\eta=x,y,z} \left( 3 \langle \alpha_{\tau\eta} \rangle_{I,F} \langle \alpha_{\tau\eta} \rangle_{I,F} - \langle \alpha_{\tau\tau} \rangle_{I,F} \langle \alpha_{\eta\eta} \rangle_{I,F} \right) \quad (7)$$

$$\beta(G')^2 = \frac{1}{2} \sum_{\tau,\eta=x,y,z} \left( 3 \langle \alpha_{\tau\eta} \rangle_{I,F} \langle G'_{\tau\eta} \rangle_{I,F} - \langle \alpha_{\tau\tau} \rangle_{I,F} \langle G'_{\eta\eta} \rangle_{I,F} \right) \quad (8)$$

$$\beta(A)^2 = \frac{1}{2} \sum_{\tau,\eta,\varsigma,\rho=x,y,z} \omega_0 \langle \alpha_{\tau\eta} \rangle_{I,F} \epsilon_{\tau\varsigma\rho} \langle A_{\varsigma\rho\eta} \rangle_{I,F} \quad (9)$$

$\omega_0$  is the incident wavenumber, and  $\epsilon_{\tau\eta\varsigma}$  is the alternating (or Levi-Civita) tensor, defined by the following relations:

$$\begin{aligned} \epsilon_{xyz} &= \epsilon_{zxy} = \epsilon_{yzx} = 1 \\ \epsilon_{zyx} &= \epsilon_{xzy} = \epsilon_{yxz} = -1 \\ \text{all other components} &\text{ are null.} \end{aligned}$$

Hence, the definition of the ROA invariants requires the calculation of the transition moments of 3 quantities,

- $\alpha$ : the electric dipole-electric dipole tensor, i.e. the polarizability tensor
- $G'$ : the electric dipole-magnetic dipole optical activity tensor
- $A$ : the electric dipole-electric quadrupole tensor

As stated before, thanks to the general formulation used to compute the transition moment  $\langle P \rangle_{I,F}$  at the VPT2 level, extension to the previous quantities requires simply to build a list of equivalence relations, as given in table 3,

## 2.2 Resonances

As extensively discussed in the literature,<sup>44,47,48,52,65,66</sup> VPT2 energies and transition moments can be plagued by resonances. The main issue for energies is the potential presence of Fermi resonances (FR), also labeled 1-2 resonances since they involve the creation of 1 quantum and the annihilation of 2 quanta, or conversely. Several strategies have been proposed to deal with this problem, also by our group.<sup>48,50,65,95,96</sup> The most commonly used approach, referred to here as deperturbed VPT2 (DVPT2), is to first identify and remove the resonance. In this work, this is done through a double test, first on the magnitude of the wavenumber difference ( $|\Delta\omega| \leq \Delta\omega^{1-2}$ , with “ $\omega = \omega_j - 2\omega_k$ ” for type I resonances and “ $\omega = \omega_i - (\omega_j + \omega_k)$ ” for type II), then on the difference between the VPT2 term and the result from a model variational calculation, the so-called Martin test.<sup>95</sup> Successively, the resonant terms, removed from the VPT2 treatment are reintroduced through variational computations, leading to the generalized VPT2 (GVPT2).

This approach is often insufficient to properly account for all anharmonic effects. Indeed, other types of resonances might occur, collectively called Darling-Dennison resonances (DDR) in reference to the first extensive study on one of those types.<sup>97</sup> At variance with FR, their presence does not directly impact the calculation of the vibrational energies, and the resonant terms need to be included as a variational treatment. As a result, their study has been scarcer. They are generally classified and labeled based on the number of quanta created and annihilated between the states involved in the resonance, “1-1”, “1-3” and “2-2”. The actual definition of the different DDR has been studied by different authors.<sup>52,97-102</sup> The revised equations proposed by Rosnik and Polik<sup>52</sup> have been used here to identify the resonant terms. An important difficulty to their correct evaluations is the potential presence of Fermi resonances in those equations. In order to treat consistently those occurrences already noted by others,<sup>99,102</sup> an adapted version of our hybrid degeneracy-corrected PT2/VPT2 method (HDCPT2), introduced for the calculation of vibrational energies<sup>65</sup> was chosen. The idea behind the degeneracy-corrected approach (DCPT2) is to replace all potentially resonant terms by a non-divergent form, following the transformation,

$$\frac{Sk^2}{2\Delta\omega} \rightarrow S(\sqrt{\Delta\omega^2+k^2} - \Delta\omega) \quad (10)$$

where  $S$  is the sign of “ $k^2 \omega$ ”,  $k^2$  is a term containing a product of cubic force constants, and  $\omega$  is a frequency difference, which can be very small in case of Fermi resonances. Since the relation in eq. 10 can be also derived from the Taylor expansion of  $\sqrt{\Delta\omega^2 + k^2}$ , its application can be extended to the calculation of Darling-Dennison resonances as well. HDCPT2 compensates the error of DCPT2 far from resonance by introducing a transition function between DCPT2 and VPT2 (see Ref.65 for details). It should be noted that Krasnoshchekov and collaborators have proposed an alternative approach directly embedded in their on-the-fly canonical Van Vleck perturbation calculations.<sup>102,103</sup>

Similarly to FR, the test to identify DDR terms is done sequentially in two steps, first on the frequency difference, then on the magnitude of the off-diagonal term,

1.  $|\Delta\omega| \leq \Delta_{\omega}^{\text{DD}}$
2.  $|\langle v' | \hat{\mathcal{H}} | v'' \rangle| \leq K^{\text{DD}}$

where the superscript “DD,  $v'$ ,  $v''$ ” refers to the type of Darling-Dennison resonance, with  $\hat{\mathcal{H}}$  the appropriate vibrational Hamiltonian.

Like vibrational energies, the formulas used for the transition moments are directly impacted by FR, but also by DDR, namely 1-1 for fundamental bands and 1-3 for 3-quanta transitions. While the procedure described above is able to treat satisfactorily most resonant terms for the transition intensities as well, provided the thresholds have been properly set, it may be insufficient in some cases. Indeed, an off-diagonal term (variational correction) of low magnitude would have a small-to-negligible impact on the energies, and so may not pass the second step of the previous test. However, in some critical cases, where the frequency difference is very small, sometimes below  $1 \text{ cm}^{-1}$ , such a term can have a significant impact on transition intensities due to the differences in the forms of the terms involved in energies and intensities equations. In order to catch such situations as well, a complementary test is added to the second step, where the off-diagonal term is scaled down by the inverse squared wavenumber difference,

$$\left| \frac{\langle v' | \hat{\mathcal{H}} | v'' \rangle}{\Delta\omega^2} \right| \geq K_I^{\text{A-B}}$$

where “A-B” in superscript represents the type of resonance, which can be either “1-1” or “1-3”, which are the two kinds of DDR directly involved in intensity calculations. Tests on the molecule studied in this work have shown that  $K_I^{\text{A-B}}$  can be set to be one order of magnitude lower than  $K^{\text{A-B}}$  ( $K_I^{\text{A-B}} = K^{\text{A-B}}/10$ ). A more systematic and technical discussion on resonances will be deferred to a future work.

Once the resonant terms are identified, variational correction to the VPT2 energies and intensities within the GVPT2 approach is done as follows. A symmetric (or hermitian if degenerate modes are present) matrix is built, containing all states of interest. The deperturbed energies are computed and placed on the diagonal of this matrix. Next, off-diagonal terms corresponding to the Fermi or Darling-Dennison resonant terms are added. It



should be noted that in this configuration, only couplings between the states of interest in the simulation are included and interactions with higher-quanta states are ignored. The matrix is then diagonalized and the eigenvalues correspond to the final, GVPT2 vibrational energies. The eigenvectors ( $\mathbf{L}_E$ ) are used to project the deperturbed transition moments on the variationally corrected states, following the procedure described in Ref.47

$$\langle \mathbf{P} \rangle_{I,F}^{\text{var}} = \mathbf{L}_E^T \langle \mathbf{P} \rangle_{I,F}^{\text{VPT2}} \quad (11)$$

### 3 Computational details

Density Functional Theory (DFT) was employed to compute equilibrium geometries, quadratic, cubic and semi-diagonal quartic force fields together with first, second, and semi-diagonal third derivatives of the electric field, magnetic field and mixed properties, needed for the computation of fully anharmonic IR, Raman, VCD and ROA spectra at the VPT2 level. For Raman and ROA spectra the chosen configuration will be backscattering with modulation of the scattered beam ( $\theta_{\text{SCP}}(180^\circ)$ ). We also note that all studied systems satisfy the far-from-resonance regime, with the lowest excited electronic states lying above 5 eV (250 nm),<sup>69,85</sup> to be compared with incident frequency of 532 nm. In view of its efficiency and reliability for prediction of structural parameters,<sup>60,104,105</sup> vibrational wavenumbers<sup>55,62,105–107</sup> along with IR and Raman intensities,<sup>25,60,72</sup> the standard functional B3LYP108 has been employed in conjunction with the SNSD polarized basis set of double- $\zeta$  quality supplemented by diffuse functions,<sup>25,72,109</sup> and including Stuttgart-Dresden core pseudopotentials<sup>110,111</sup> for bromine. All equilibrium structures have been obtained using tight convergence criteria (maximum forces and displacements smaller than  $1.5 \times 10^{-5}$  Hartree/Bohr and  $6 \times 10^{-5}$  Å, respectively) for geometry optimisation. The energy and property derivatives at energy minima including mechanical and electric/magnetic anharmonicities, were determined by numerical differentiations of analytic force constants and first derivatives of the electric and magnetic dipoles, and the frequency-dependent electric dipole-electric dipole, electric dipole-magnetic dipole optical activity and electric dipole-electric quadrupole tensors, at displaced geometries along the normal coordinates (with a 0.01 Å step). To get accurate results, all computations were carried out with an ultrafine integration grid (99 radial shells and 590 angular points per shell) to integrate the exchange-correlation kernel. Basis set effects were investigated for methyloxirane by employing the aug-cc-pVTZ basis set.<sup>112</sup> For all computations the following thresholds have been employed for the Fermi and Darling-Dennison type resonances:  $\Delta_\omega^{1-2} = 200 \text{ cm}^{-1}$  and  $K^{1-2} = 3 \text{ cm}^{-1}$ ;  $\Delta_\omega^{\text{DD}} = 100 \text{ cm}^{-1}$  and  $K^{\text{DD}} = 10 \text{ cm}^{-1}$ , respectively.

The force field of methyloxirane has been improved by employing a hybrid scheme, which assumes that the differences between vibrational frequencies computed at two different levels of theory are mainly due to the harmonic terms (see, for instance, Refs. 25,57,65,106,113,114). In the present case the hybrid force field was obtained in a normal-coordinate representation by adding the cubic and semi-diagonal quartic B3LYP/SNSD force constants to the best-estimated theoretical harmonic frequencies<sup>55</sup> obtained by the so

called “cheap”<sup>55,60</sup> composite scheme based on the coupled-cluster computations (at the CC singles and doubles augmented by a perturbative treatment of triple excitations, CCSD(T)<sup>115</sup>) in conjunction with the cc-pVTZ<sup>112,116,117</sup> basis set.

All calculations have been carried out employing a development version of the GAUSSIAN suite of programs.<sup>118</sup> Assignments of vibrational modes were performed by means of visual inspection of the atomic displacements along normal modes and by comparison with the assignments reported in the literature. A graphical user interface (VMS-Draw)<sup>119</sup> has been used to visualize normal modes and analyze the outcome of vibrational computations.

## 4 Results and Discussion

### 4.1 Theoretical and experimental IR, Raman, VCD and ROA spectra of (R)-methyloxirane

Benchmark studies for a large number of molecules have shown that the B3LYP/SNSD model provides reliable harmonic values and accurate anharmonic contributions for vibrational energies, IR intensities and Raman Activities.<sup>25,65,72,106</sup> For the specific case of methyloxirane<sup>55</sup> the B3LYP/SNSD harmonic wavenumbers and IR intensities show mean absolute errors (MAE) with respect to the best theoretical estimates (CCSD(T)/cc-pVTZ computations corrected for basis-set extrapolation and core correlation by means of the “cheap” scheme<sup>55</sup>) of about  $12\text{ cm}^{-1}$  and  $2\text{ km mol}^{-1}$ , respectively. The B3LYP/SNSD anharmonic wavenumbers show MAE and maximum deviations ( $|\text{MAX}|$ ) with respect to experiment of  $12\text{ cm}^{-1}$  and  $32\text{ cm}^{-1}$ , while improved agreement, with MAE and  $|\text{MAX}|$  of  $5\text{ cm}^{-1}$  and  $18\text{ cm}^{-1}$ , respectively, has been obtained by coupling the harmonic coupled cluster (“cheapCC”) values with anharmonic contributions computed at the B3LYP/SNSD level.<sup>55</sup> The CC/B3LYP force-field, validated by comparison with the well resolved IR spectra,<sup>76</sup> ensures a high accuracy for the band positions of Raman, VCD and ROA spectra. The situation is more complex for intensities, which in addition to the electric dipoles require, depending on the specific spectroscopy, also magnetic moments, frequency-dependent polarizabilities, frequency-dependent optical rotations, and frequency-dependent dipole-quadrupole tensors. Unfortunately, in most cases, reference high-level post-Hartree-Fock computations (i.e. at the CCSD(T) level with extended basis sets) are not yet feasible for these properties. In fact only recently the first calculations of harmonic ROA spectra using coupled-cluster theory have been reported by Crawford and Ruud,<sup>120</sup> resorting to the CC singles and doubles excitations (CCSD) and double- $\zeta$  basis set. Furthermore, some spectra (like VCD) require not only the values, but also the relative orientations of the derivatives of electric and magnetic moments. Considering that the very few studies available on Raman and ROA intensities have shown that the effects due to the electron correlation are less pronounced than those originating from the truncation of the basis set,<sup>121</sup> DFT approaches are more effective than their MP2 counterparts,<sup>122</sup> and the B3LYP functional performs well with comparison to CCSD,<sup>120,123</sup> in the present work we will resort to B3LYP computations for all properties, performing a test on basis set convergence of line-shapes for IR, Raman, VCD, and ROA spectra.

The most accurate CC/B3LYP force fields, are obtained combining the best theoretical estimates of harmonic frequencies from Ref.<sup>55</sup> with B3LYP/SNSD and B3LYP/aug-cc-pVTZ anharmonic corrections, which are further combined with property surfaces computed

at the B3LYP level with the same basis sets. The fully anharmonic IR, Raman, VCD and ROA spectra presented in Figure 2 show a very good agreement with respect to experiment<sup>76,79</sup> for both basis sets, confirming the effectiveness of the relatively small SNSD basis set. Moreover, the results reported in the present work, which involve variational contributions to intensities together with modified resonance criteria, fully confirm the results and analysis already presented in previous works for IR<sup>55</sup> and VCD<sup>76</sup> spectra. However, some improvements are also observed, as an example, the variational correction due to the 1-1 resonance between ring C–C stretching ( $\nu_{20}$  at  $834\text{ cm}^{-1}$ ) and  $\text{CH}_2/\text{CH}_3$  rocking ( $\nu_{19}$  at  $894\text{ cm}^{-1}$ ) leads to the redistribution of the VCD signal, in agreement with experiment (see Figure 3). The increased number of variational contributions, which can be now taken into account (6 Fermi and 80 Darling-Denison (6 (2-2), 12 (1-1) and 62 (1-3)) resonances) leads also to a remarkably robust procedure with a strongly reduced dependence on specific threshold values.

The ROA spectra presented in Figure 4, show that anharmonic computations convoluted with the larger band-width yield spectra pattern in good agreement with the experimental spectrum recorded in the gas phase,<sup>79</sup> which is characterized by rather broad bands and is complicated by low signal-to-noise ratio and a not-fully resolved rotational structure.<sup>79</sup> On the contrary, harmonic computations show larger discrepancies, with bands noticeably shifted and several transitions missing, while clearly visible in the anharmonic ROA spectrum at higher resolution. Considering the experimental challenge of ROA measurements in the gas phase due to the low sensitivity of the scattering technique,<sup>79</sup> the availability of fully anharmonic theoretical spectra represents a viable alternative for obtaining reference ROA spectra of isolated systems, which, in turn, allow to put in evidence the environmental effects present in condensed phases.

## 4.2 Anharmonic effects on IR, Raman, VCD and ROA spectra: the case of (R)-4-fluoro-2-azetidinone

As mentioned in the introduction, anharmonic effects on the transition intensities can have an important impact on the overall spectral band-shape, in particular for the chiroptical spectroscopies. In this and the following section these aspects will be analyzed using azetidinone derivatives as test cases. This class of molecules is attracting increasing interest in view of the remarkable pharmacological properties of some of its members and their semi-rigid structure ensures the reliability of perturbative treatments based on polynomial representations of potential energy and property surfaces in terms of cartesian normal modes.

Figure 5 compares different models to simulate IR, Raman, VCD and ROA spectra of (R)-4-F-2-azetidinone in the spectral range  $300\text{-}1600\text{ cm}^{-1}$ , where both fundamental and non-fundamental transitions are present. It is clear that within the harmonic approximation, all bands are shifted and many transitions are missing with respect to fully anharmonic computations already for IR and Raman spectra, such deviations being further enhanced for VCD and ROA spectra. The situation is quite similar also once the anharmonic wavenumbers are combined with harmonic intensities since the positions of the fundamental transitions are significantly improved, but overtones and combination bands have still null

intensities and intensities of some fundamental bands are incorrect. As a result, in the present case fully harmonic and 'mixed' (anharmonic wavenumbers and harmonic intensities) simulations show quite similar band patterns for all spectroscopies. The situation is very different for the fully anharmonic spectra, which show several new transitions due to the overtones and combination bands together with significant intensity changes for some fundamental bands. In particular, all fully anharmonic spectra show a much richer structure within the 900-1200  $\text{cm}^{-1}$  range with several non-fundamental bands (combinations involving  $\gamma(\text{R}^{\text{pucker}})$ ,  $\gamma(\text{N-H})+\rho(\text{CH}_2)$ ,  $\gamma(\text{C4})+\beta(\text{C=O})$  and  $\gamma(\text{C=O})+\gamma(\text{NH})+\rho(\text{CH}_2)$  vibrations<sup>124</sup>) of similar intensity to the fundamental transitions. Those effects might be less pronounced for low-resolution IR and Raman spectra, leading just to band broadening, but they modify also qualitatively the band shapes of VCD and ROA spectra due to positive and negative contributions of non-fundamental transitions. Moreover, for the ROA spectra the interaction between  $\gamma(\text{R}^{\text{pucker}})$  ( $\nu_{24}$ ) and  $\gamma(\text{N-H})+\rho(\text{CH}_2)$  ( $\nu_{22}$ ) leads to a non-negligible intensity of the corresponding combination band and further increases the intensity of the  $\nu_{22}$  fundamental, resulting in two new distinct bands at 490  $\text{cm}^{-1}$  (positive) and 399  $\text{cm}^{-1}$  (negative), respectively.

These examples suggest that reliable comparative analysis of substituent or environmental effects on chiroptical spectra need to be performed at fully anharmonic level, and this will be analyzed in deeper detail in the next section for the series of (R)-4-X-2-azetidinone derivatives (X=F, Cl, Br).

### 4.3 Fully anharmonic IR, Raman, VCD and ROA spectra of (R)-4-X-2-azetidinones (X=F,Cl,Br)

Integrated studies combining several spectroscopic techniques allow to get a more complete and accurate picture of vibrational properties, along with their tuning by substituent and/or environmental effects. In fact, all vibrational spectroscopies share the same energy levels (hence the vibrational band positions), but the intensities of the different transitions change often strongly for IR, Raman, VCD, and ROA spectra. As a result, each of these different techniques shows different "fingerprint" regions and gives enhanced information for specific vibrational features. The consequent advantage of combining different spectroscopies is clearly demonstrated by comparison of IR, Raman, VCD and ROA spectra for the series of (R)-4-X-2-azetidinone derivatives (X-2az, X=F, Cl, Br), presented in Figure 6. The most intense IR features of all (R)-4-X-2-azetidinones are related to the C2=O6 stretching vibrations, at about 1838-1844  $\text{cm}^{-1}$  and to the closely-lying non-fundamental transitions gaining intensity through anharmonic resonances. All IR spectra are very rich and show few distinct bands in the 300-1500  $\text{cm}^{-1}$  and 2900-3500  $\text{cm}^{-1}$  ranges. In the high-energy wing the position of the N1-H5 stretching vibration at 3426  $\text{cm}^{-1}$  is not influenced by the halogen substituent, while the C4-H8 stretchings are shifted to higher energies and decrease in intensity with the increasing mass and electronegativity of the halogen. For Raman spectra, the most intense bands fall in the higher energy range, and are related to the N1-H5 and C-H stretchings, with the position and intensity of  $\nu(\text{C4-H8})$  showing a dependence on the halogen substituent analogous to that found in the IR spectrum. Moreover, the 300-600  $\text{cm}^{-1}$  range, corresponding to the ring deformation vibrations involving halogen atoms present very different band patterns and the overall intensity increase from F to Cl and Br. The VCD

spectra show several intense features in the 300–1500  $\text{cm}^{-1}$  range, with all other regions showing only very weak bands for all molecules. Moreover, F-2az shows many bands of similar intensity in the 300–1500  $\text{cm}^{-1}$  range, while the VCD spectrum of Cl-2az shows two very strong transitions of opposite sign (negative at 608  $\text{cm}^{-1}$  and positive at 979  $\text{cm}^{-1}$ ), with the latter band present also for Br-2az at 970  $\text{cm}^{-1}$ . At variance with VCD, ROA spectra permit to analyze chiroptical properties also for the higher energy region, showing several very intense features in this spectral range. In particular, all spectra show a doublet of bands with opposite signs (positive+negative) at about 2900  $\text{cm}^{-1}$  and single positive bands at about 3430  $\text{cm}^{-1}$ . Interestingly, in this case the positive bands corresponding to the  $\nu(\text{C4-H8})$  stretching show essentially the same intensities for all halogens, whereas the negative  $\nu(\text{C3H2})_{as}$  and positive  $\nu(\text{N1-H5})$  bands are influenced by the nature of the halogen atom, showing increasing intensities when going from F to Br. As a specific example, let us consider the close-lying bands of (R)-4-Br-2-azetidinone in the spectral range 470–540  $\text{cm}^{-1}$ , marked by asterisks in Figure 7. In the IR spectrum, both  $\beta(\text{C=O})$  ( $\nu_{20}$ ) at 477  $\text{cm}^{-1}$  and  $\gamma(\text{C=O})+\gamma(\text{NH})$  ( $\nu_{19}$ ) at 537  $\text{cm}^{-1}$  are rather weak, whereas their intensities increase in the Raman spectra, and thus can be more easily detected. These two vibrations show strong signals in both VCD and ROA spectra, which however differ by the sign, showing negative/positive and positive/negative patterns for VCD and ROA, respectively.

In our opinion, the comparative study sketched above demonstrates how several spectroscopic techniques can be analyzed within the same computational framework, also thanks to the integration of all tools in a virtual multifrequency spectrometer. It is noteworthy that spectral tuning by different halogens changes significantly for specific spectroscopic techniques, allowing a very detailed analysis of substituent effects by combining IR, Raman, VCD and ROA measurements with their theoretical fully anharmonic counterparts.

## 5 Conclusion

This paper reports the first full implementation of ROA spectra including both mechanical and electric/magnetic anharmonicities in the framework of generalized second-order vibrational perturbation theory taking full account of all first- and second-order resonances. Implementation of these new features in our virtual multifrequency spectrometer (VMS) allows comprehensive studies and vis-a-vis comparisons of simulated and experimental spectra for all the vibrational spectroscopies (IR, Raman, VCD, ROA). Although the present paper focuses on medium-size molecules in the gas phase, VMS can deal also with much larger systems thanks to the availability of reduced-dimensionality approaches and of effective continuum or discrete-continuum polarizable solvent models. After validation of the new tool for the well-studied methyloxirane benchmark, a comparative study of halogen-substituted azetidinones has been performed with the aim of revealing spectral tuning by closely related substituents with different mass, electronegativity, polarizability and/or steric hindrance. Our results point out the remarkable sensitivity to those parameters of all vibrational spectroscopies, and in particular of the chiral ones, thus paving the route to multi-spectroscopic characterization of subtle structural, stereo-electronic and, possibly, environmental effects. Although further developments are surely needed to extend such kind of studies from semirigid to flexible molecules, the results presented in this paper and in

parallel studies show, in our opinion, that we now possess of a robust tool for complementing experimental spectroscopic studies from both assignment and prediction points of view by means of direct vis-a-vis comparison between experimental spectra and those issuing from numerical simulations in which different effects can be selectively switched on and off.

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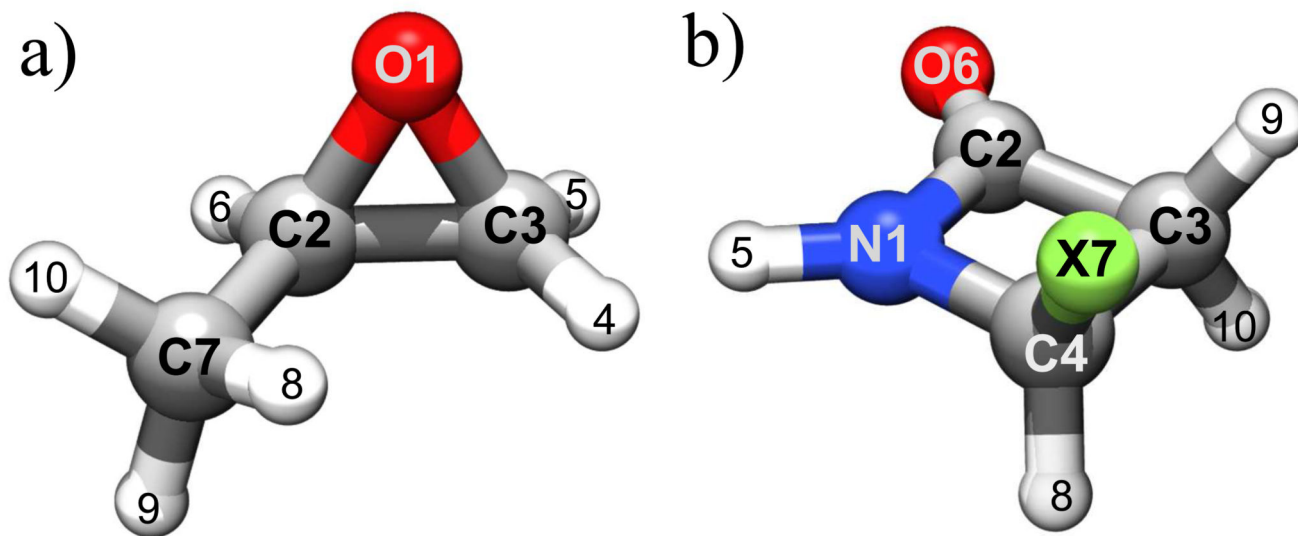


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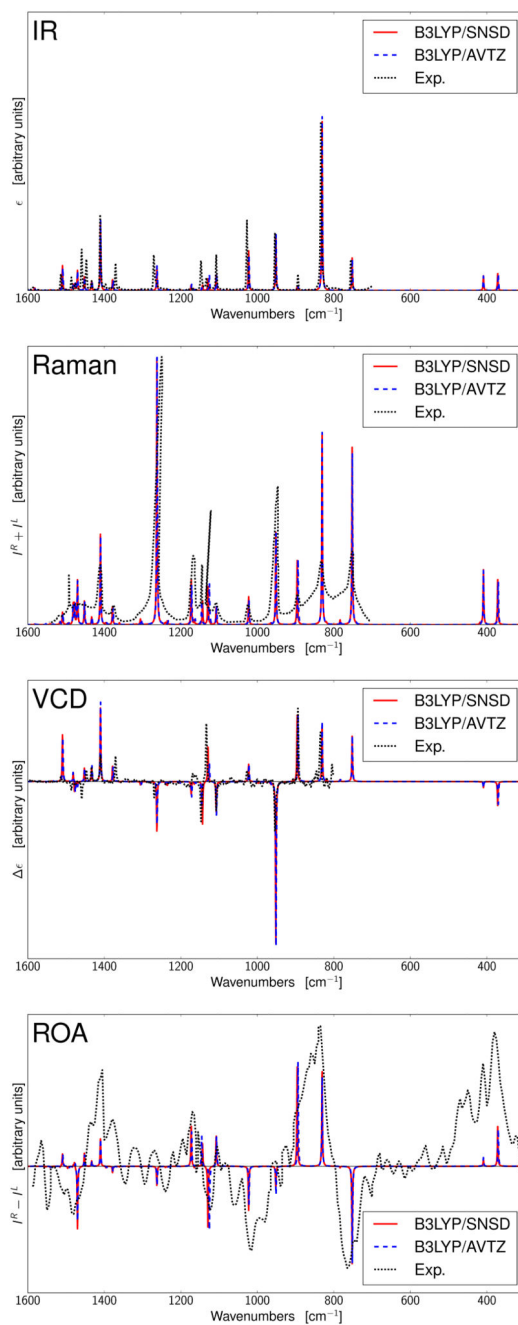
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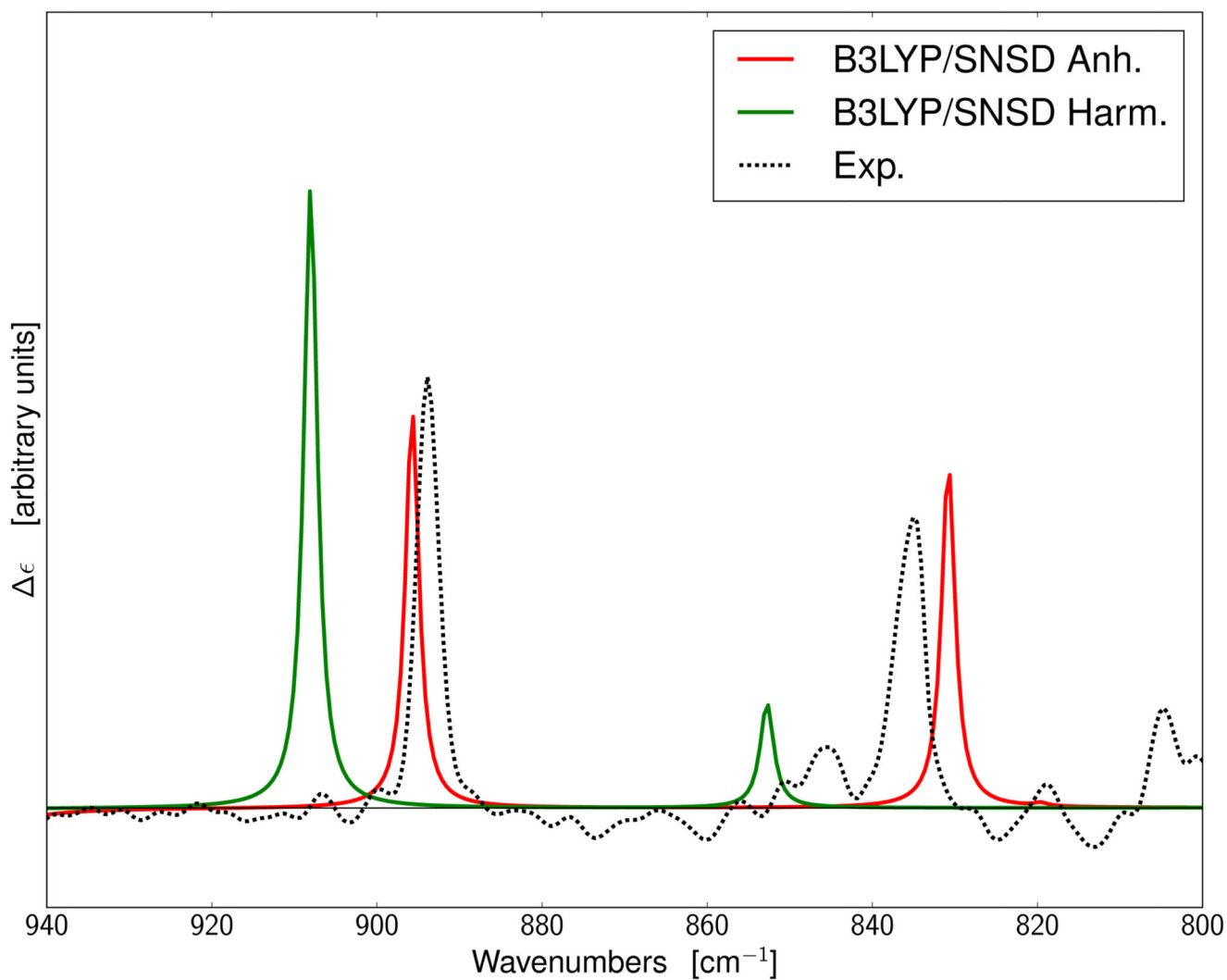


**Figure 1.** Molecular structures of methyloxirane (a) and 4-X-2-azetidinones (X=F, Cl, Br) (b) along with the atom labelling in line with references 55 and, 84 respectively.

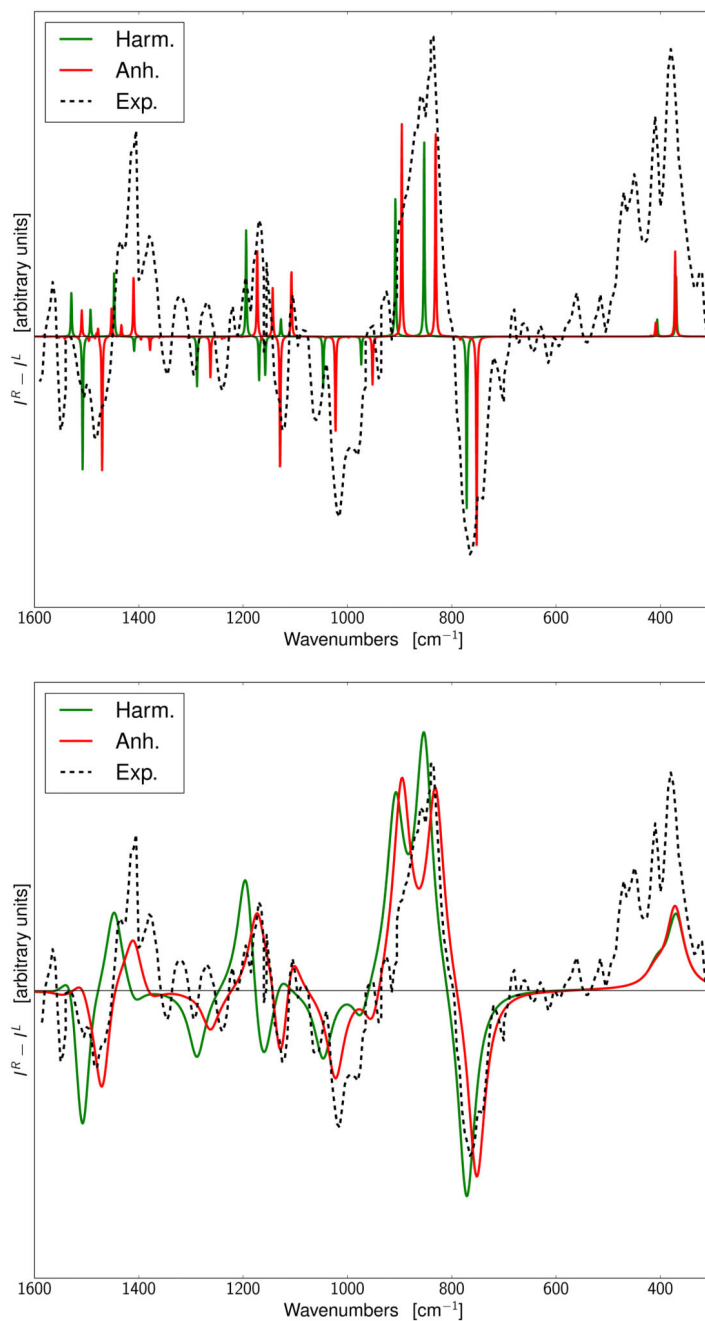


**Figure 2.**

Fully anharmonic IR, Raman, VCD and ROA spectra of (R)-methyloxirane compared to their experimental counterparts measured in low-temperature Ar Matrix (IR, VCD76) or gas phase (Raman, ROA79). Vibrational wavenumbers have been computed at the “cheapCC”/B3LYP level55 in conjunction with B3LYP intensities. B3LYP computations have been performed with the SNSD and aug-cc-pVTZ (AVTZ) basis sets. All spectra have been convoluted by means of Lorentzian distribution functions with FWHM of  $2 \text{ cm}^{-1}$ . 36



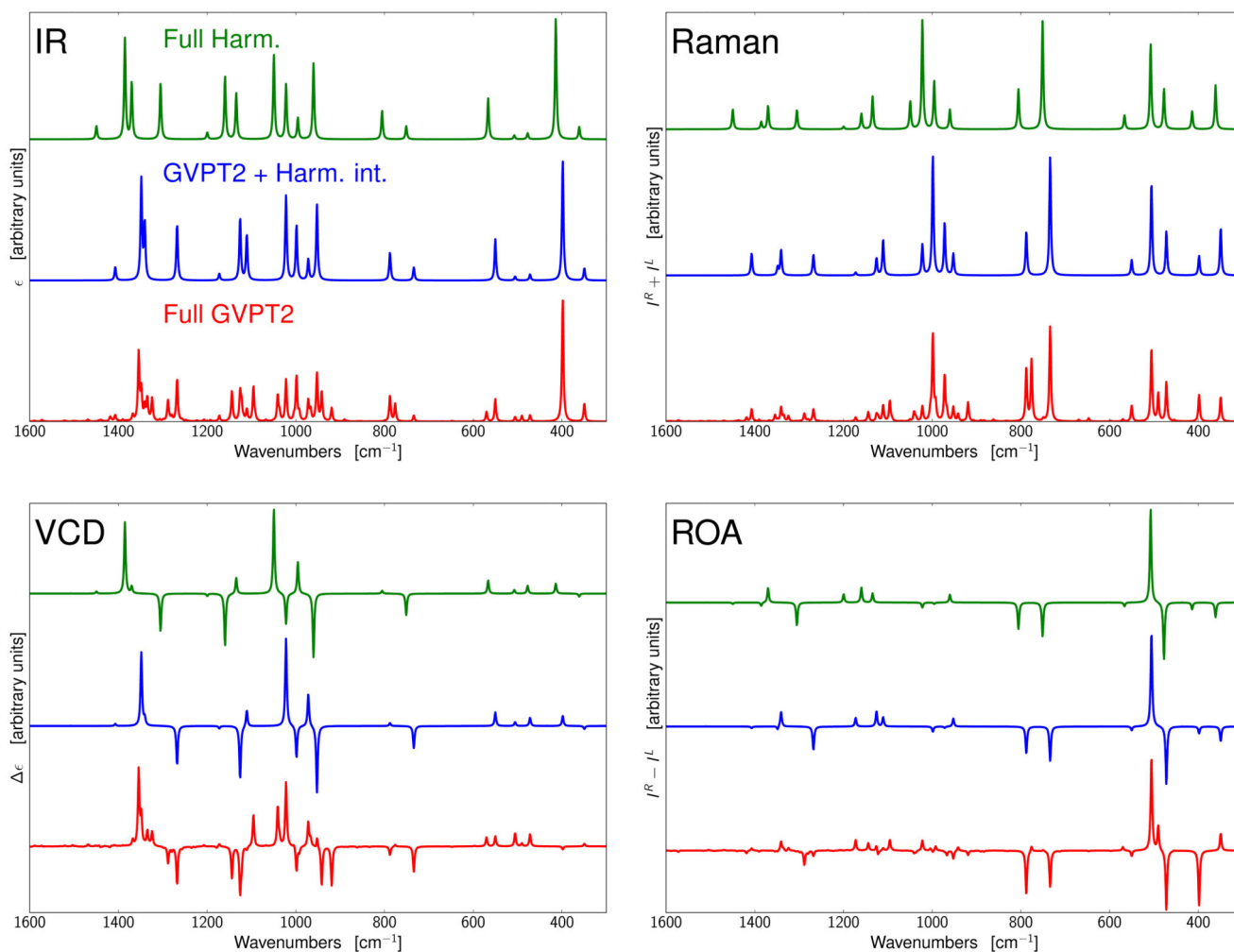
**Figure 3.** Harmonic and fully anharmonic VCD spectra of (R)-methyloxirane in the 800–940  $\text{cm}^{-1}$  range compared to experiment.<sup>76</sup> Harmonic wavenumbers have been computed at the “cheapCC” level<sup>55</sup> and combined with anharmonic corrections at the B3LYP/SNSD level. VCD harmonic and anharmonic intensities are computed at B3LYP/SNSD level. All spectra have been convoluted by means of Lorentzian distribution functions with FWHM of 2  $\text{cm}^{-1}$ .



**Figure 4.**

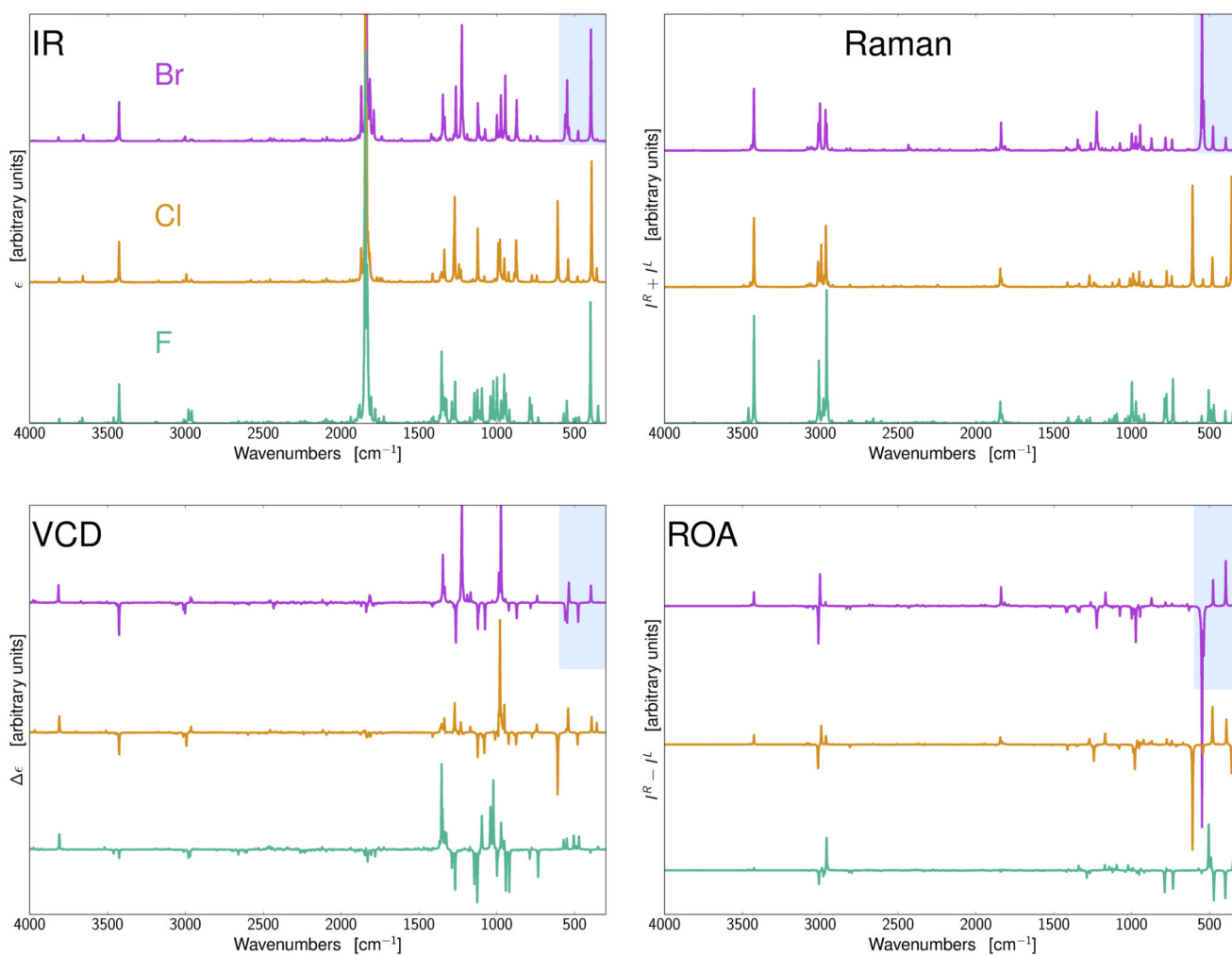
Harmonic and fully anharmonic ROA spectra of (R)-methyloxirane compared to its experimental counterpart (in the gas phase<sup>79</sup>). Harmonic wavenumbers have been computed at the “cheapCC” level<sup>55</sup> and combined with anharmonic corrections at the B3LYP/SNSD level. ROA harmonic and anharmonic intensities are computed at B3LYP/SNSD level. Theoretical spectra have been convoluted by means of Lorentzian distribution functions with FWHM of  $2\text{ cm}^{-1}$  (upper panel) and of  $40\text{ cm}^{-1}$  (lower panel).



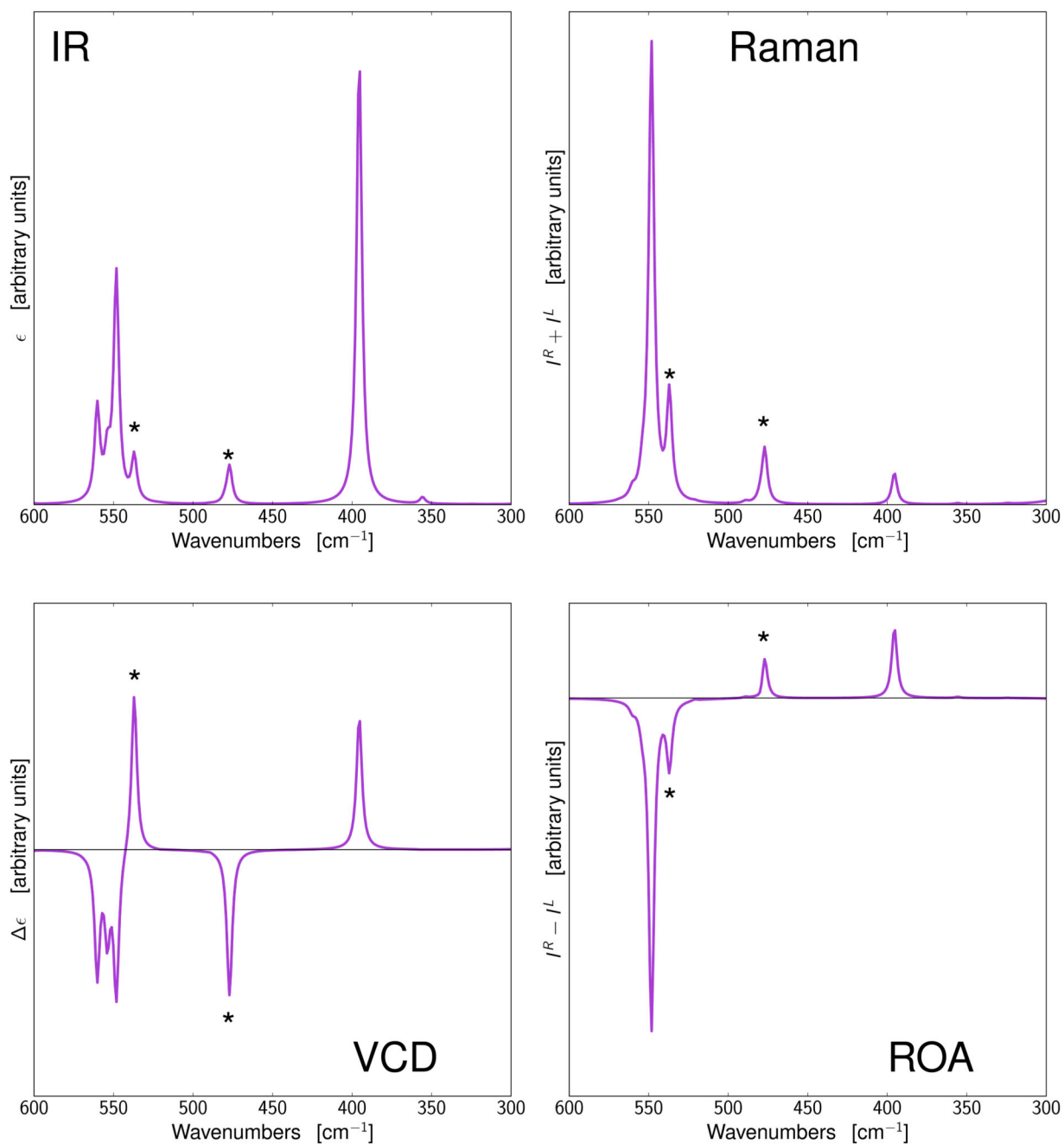


**Figure 5.**

Theoretical IR, VCD, Raman and ROA spectra of (R)-4-fluoro-2-azetidinone in the 300–1600  $\text{cm}^{-1}$  range. Fully harmonic (harmonic energies and intensities) spectra (red, upper traces), spectra obtained by combining anharmonic wavenumbers with harmonic intensities (blue, middle traces) and fully anharmonic (anharmonic energies and intensities) spectra (green, lower traces). All computations have been performed at the B3LYP/SNSD level, and the computed lines have been convoluted by means of Lorentzian distribution functions with FWHM of 4  $\text{cm}^{-1}$ .



**Figure 6.** Fully anharmonic IR, VCD, Raman and ROA spectra of (R)-4-X-2-azetidinones (X=F, Cl, Br) in the 300–4000  $\text{cm}^{-1}$  range. All computations have been performed at the B3LYP/SNSD level, and the computed lines have been convoluted by means of Lorentzian distribution functions with FWHM of 4  $\text{cm}^{-1}$ .



**Figure 7.**

Fully anharmonic IR, VCD, Raman and ROA spectra of (R)-4-Br-azetidinone in the 300–600  $\text{cm}^{-1}$  range, highlighted in light blue in Figure 6.  $\nu_{19}$  and  $\nu_{20}$  vibrations are marked by asterisks. All computations have been performed at the B3LYP/SNSD level, and the computed lines have been convoluted by means of Lorentzian distribution functions with FWHM of 4  $\text{cm}^{-1}$ .

**Table 1**

Description of the most common type of scattering geometries and polarizations.

Symbol	Description
(0°)	forward scattering
(180°)	backward scattering
$^z(90^\circ)$	depolarized right-angle scattering, where the scattering direction is at right angle with respect to the incident beam and the transmission axis of the linear polarization analyzer, placed on the path of the scattered beam (for ICP) or the incident beam (for SCP), is parallel to the scattering plane ( $yz$ )
$^x(90^\circ)$	polarized right-angle scattering, same as $^z(90^\circ)$ but the transmission axis of the linear polarization analyzer is perpendicular to the scattering plane
$^*(90^\circ)$	magic-angle right-angle scattering, same as $^z(90^\circ)$ but the transmission axis of the linear polarization analyzer is set at the "magic angle" of $\pm \sin^{-1}(2/3)^{1/2} \approx \pm 54.74^\circ$ to the scattering plane to remove the contribution from the electric dipole-electric quadrupole ( <i>magic angle</i> CID)

**Table 2**  
**Equations for the most common types of scattering in ROA.**

$K$  is a term related to the experimental setup, proportional to the fourth power of the scattered energy,  $c$  is the speed of light.

Scattering	$I^R - I^L$	$I^R + I^L$
scp(0°)	$\frac{8K}{c}[90\alpha G' + 2\beta(G')^2 - 2\beta(A)^2]$	$4K[45a^2 + 7\beta(a)^2]$
$\Delta_{\text{SCP}}^x(90^\circ)$	$\frac{4K}{c}[45\alpha G' + 7\beta(G')^2 + \beta(A)^2]$	$2K[45a^2 + 7\beta(a)^2]$
$\Delta_{\text{SCP}}^*(90^\circ)$	$\frac{40K}{3c}[9\alpha G' + 2\beta(G')^2]$	$\frac{20K}{3}[9a^2 + 2\beta(a)^2]$
$\Delta_{\text{SCP}}^z(90^\circ)$	$\frac{8K}{c}[3\beta(G')^2 - \beta(A)^2]$	$4K[3\beta(a)^2]$
scp(180°)	$\frac{8K}{c}[12\beta(G')^2 + 4\beta(A)^2]$	$4K[45a^2 + 7\beta(a)^2]$
DCP <sub>1</sub> (180°)	$\frac{16K}{c}[6\beta(G')^2 + 2\beta(A)^2]$	$24K\beta(a)^2$

**Table 3**

Equivalence relations between the model property  $\mathbf{P}$  and actual properties  $\boldsymbol{\alpha}$ ,  $\mathbf{G}'$  and  $\mathbf{A}$ . The electric ( $\boldsymbol{\mu}$ ) and magnetic ( $\mathbf{m}$ ) dipoles, used in IR and VCD intensities, are also reported.  $\mathbf{M}$  is the atomic axial tensor (AAT).<sup>94</sup>

$\mathbf{P}$	$\mathbf{P}_0$	$\mathbf{P}_i$	$\mathbf{P}_{ji}$	$\mathbf{P}_{jki}$	$s_0$	$s_1$	$s_2$	$\mathbf{S}$
$\boldsymbol{\mu}$	$\boldsymbol{\mu}^{\text{eq}}$	$\frac{\partial \boldsymbol{\mu}}{\partial q_i}$	$\frac{\partial^2 \boldsymbol{\mu}}{\partial q_i \partial q_j}$	$\frac{\partial^3 \boldsymbol{\mu}}{\partial q_i \partial q_j \partial q_k}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{6\sqrt{2}}$	+1
$\mathbf{m}$	$\mathbf{0}$	$\mathbf{M}_i$	$\frac{\partial \mathbf{M}_i}{\partial q_j}$	$\frac{\partial^2 \mathbf{M}_i}{\partial q_j \partial q_k}$	$\frac{i \hbar}{\sqrt{2}}$	$\frac{i \hbar}{\sqrt{2}}$	$\frac{i \hbar}{\sqrt{2}}$	-1
$\boldsymbol{\alpha}$	$\boldsymbol{\alpha}^{\text{eq}}$	$\frac{\partial \boldsymbol{\alpha}}{\partial q_i}$	$\frac{\partial^2 \boldsymbol{\alpha}}{\partial q_i \partial q_j}$	$\frac{\partial^3 \boldsymbol{\alpha}}{\partial q_i \partial q_j \partial q_k}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{6\sqrt{2}}$	+1
$\mathbf{G}'$	$\mathbf{G}'^{\text{eq}}$	$\frac{\partial \mathbf{G}'}{\partial q_i}$	$\frac{\partial^2 \mathbf{G}'}{\partial q_i \partial q_j}$	$\frac{\partial^3 \mathbf{G}'}{\partial q_i \partial q_j \partial q_k}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{6\sqrt{2}}$	+1
$\mathbf{A}$	$\mathbf{A}^{\text{eq}}$	$\frac{\partial \mathbf{A}}{\partial q_i}$	$\frac{\partial^2 \mathbf{A}}{\partial q_i \partial q_j}$	$\frac{\partial^3 \mathbf{A}}{\partial q_i \partial q_j \partial q_k}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{6\sqrt{2}}$	+1