

Radio signals from early direct collapse black holes

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ABSTRACT

We explore the possibility to detect the continuum radio signal from direct collapse black holes (DCBHs) by upcoming radio telescopes such as the SKA and ngVLA, assuming that after formation they can launch and sustain powerful jets at the accretion stage. We assume that the high- z DCBHs have similar jet properties as the observed radio-loud AGNs, then use a jet model to predict their radio flux detectability. If the jet power $P_{\text{jet}} \gtrsim 10^{42} - 10^{43} \text{ erg s}^{-1}$, it can be detectable by SKA/ngVLA, depending on the jet inclination angle. Considering the relation between jet power and black hole mass and spin, generally, jetted DCBHs with mass $\gtrsim 10^5 M_{\odot}$ can be detected. For a total jetted DCBH number density of $\sim 2.5 \times 10^{-3} \text{ Mpc}^{-3}$ at $z = 10$, about $100 \text{ deg}^{-2} z^{-1}$ DCBHs are expected to be above the detection threshold of SKA1-mid (100-h integration). If the jet ‘blob’ emitting most of the radio signal is dense and highly relativistic, then the DCBH would only feebly emit in the SKA-low band, because of self-synchrotron absorption (SSA) and blueshift. Moreover, the free-free absorption in the DCBH envelope may further reduce the signal in the SKA-low band. Thus, combining SKA-low and SKA-mid observations might provide a potential tool to distinguish a DCBH from a normal star-forming galaxy.

Key words: galaxies: jet.

1 INTRODUCTION

Supermassive black holes (SMBHs) routinely observed at redshift $z \simeq 6$ must have collected their mass, $\sim 10^8 - 10^{10} M_{\odot}$, within ~ 900 million years from the big bang (e.g. Wu et al. 2015; Bañados et al. 2018), starting from much smaller black hole seeds. The origin and nature of the seeds are still very uncertain, although several hypotheses have been proposed, ranging from stellar-mass black holes formed after the death of Pop III stars (Heger et al. 2003), to the collapse of nuclear star clusters, and even direct collapse black holes (DCBHs) that form directly in pristine haloes in which H_2 cooling is suppressed (e.g. Omukai 2001; Oh & Haiman 2002; Begelman, Volonteri & Rees 2006; Begelman, Rossi & Armitage 2008; Begelman 2010a; Umeda et al. 2016). Usually this is achieved by a supercritical radiation field that dissociates the H_2 , and/or detaches the H^- that is crucial for H_2 formation. The critical value J_{21} , which is the specific intensity of the radiation field in units of $10^{-21} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$ at 13.6 eV, is still not yet confirmed. It ranges between ~ 30 and 10^5 , depending on spectrum shape of the radiation source; how to model the self-shielding and hydrodynamical effects; and so on. These alternatives are reviewed in Volonteri (2010) and Latif & Ferrara (2016). If the seed is a stellar-mass black hole, then it must always keep accreting material at the highest rate (Eddington limit) throughout the Hubble time. This is quite unlikely (Valiante et al. 2016). Numerical simulations show that, at the early growth stage, radiative feedback is efficient and the accretion rate is therefore limited to $\simeq 10^{-5} - 10^{-3}$ of the Eddington limit (Alvarez, Wise &

Abel 2009). Also Smith et al. (2018) found that for 15 000 stellar-mass black holes hosted by minihaloes in simulations, the growth is negligible.

The DCBH scenario, at least in principle, solves this tension. Such kind of black holes could be as massive as $\sim 10^4 - 10^6 M_{\odot}$ at birth, when they find themselves surrounded by pristine gas with high temperature and density. This is an ideal situation to sustain the high accretion rate required by SMBH growth (Pacucci, Volonteri & Ferrara 2015a; Latif & Khochfar 2020; Regan et al. 2020).

DCBHs have not been detected until now, either because they are too faint and/or too rare. Potentially promising detections techniques have been proposed in the literature. They include, among others, the cosmic infrared background (Yue et al. 2013b), multicolour sampling of the spectral energy distribution (SED) combined with X-ray surveys (Pacucci et al. 2016), a specific Ly α signature (Dijkstra, Gronke & Sobral 2016a), and the neutral hydrogen $\lambda = 3\text{-cm}$ maser line (Dijkstra, Sethi & Loeb 2016b). These methods are summarized in Dijkstra (2019).

Here we introduce a new strategy based on the detection of the radio continuum signal from early DCBHs. We consider a DCBH population featuring a powerful jet, similar to what observed in the radio-loud AGN (or blazars, if the jet beam happens to point towards the observer). Such a DCBH must be at the accretion stage, the collapse stage before the DCBH formation, including the lifetime of possible supermassive star, is much shorter (e.g. Begelman 2010b). Launching the jet relies on the presence of a very strong magnetic field close to the black hole surface. It is found that indeed the magnetic field amplification mechanism works during the accretion process around a DCBH (Latif, Schleicher & Schmidt 2014). And, Sun et al. (2017) simulated the collapse of a supermassive star (the

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progenitor of a DCBH), showing that the end result of such process is a high spin black hole with an incipient jet. Moreover, Matsumoto et al. (2015, 2016) showed that the jet can indeed break out the stellar envelope of the supermassive star. Based on these theoretical predictions, we consider the possible existence of jetted DCBHs at high redshifts, and use this assumption in this paper.

For jetted DCBHs, we first use an empirical jet power-radio power relation to estimate the strength of radio flux at the bands of upcoming radio telescopes SKA-low, SKA-mid, and ngVLA in Section 2.1; then, we combine the spectral radiation model (Section 2.2) and the jet total power model (Section 2.4) to make detailed predictions on the expected radio flux and the DCBH detectability. In Section 2.3, we show the free-free absorption of the DCBH envelope. We give summary in Section 4.

Appendix A gives the synchrotron radiation formulas. In Appendix B, we estimate the angular momentum supply in the accretion disc, to see if it is enough to support a black hole with high-spin. Appendix C contains the results for radio-quiet DCBHs. For radio-quiet DCBHs, we follow Yue et al. (2013b), who developed a radiation model. The primary radiation includes the contribution from a disc, a hot corona, and a reflected component; it is re-processed by the envelope material. In Yue et al. (2013b), the radio emission is mainly produced by free-free radiation; however, we add some synchrotron contribution that is not considered by the previous work. In Appendix D, we estimate the propagation of a jet in different DCBH envelopes, to see in which case the jet can break out the envelope. If a jet can break out the envelope, then it does not suffer the free-free absorption.

2 JETTED DCBHs

2.1 Radio flux from the jet

We start by obtaining a rough estimate of the jet radio flux using the known empirical relation between the jet power and the radio power. The relation is approximated by a power-law form and the coefficients obtained from a number of observations are consistent with each other (e.g. Birzan et al. 2008; Cavagnolo et al. 2010; O’Sullivan et al. 2011; Daly et al. 2012). However, Godfrey & Shabala (2016) pointed out that this apparent consistence is due to the bias in measuring jet power of samples that span large distance range, and found different relations for different sample classifications.

Nevertheless, we adopt the relation for radio luminosity at 1.4 GHz in Birzan et al. (2008), which is actually close to that is found in Godfrey & Shabala (2016),

$$\log \left(\frac{P_{\text{jet}}}{10^{42} \text{ erg s}^{-1}} \right) = p_0 \log \left(\frac{L_{1.4}}{10^{24} \text{ W Hz}^{-1}} \right) + p_1, \quad (1)$$

where $L_{1.4}$ is the luminosity at 1.4 GHz (rest frame), and $p_0 = 0.35$, $p_1 = 1.85$. The observed flux is then

$$S(\nu_{\text{obs}}) = \frac{L_\nu(\nu')(1+z)}{4\pi d_L^2}, \quad (2)$$

if the energy is isotropically distributed in space, where $\nu' = \nu_{\text{obs}}(1+z)$ and d_L is the luminosity distance up to z . Assuming that $L_\nu \propto \nu^{-0.8}$, for $z = 10$ and $P_{\text{jet}} = 10^{43.5} \text{ erg s}^{-1}$, the observed flux is shown by a thin dashed line in Fig. 1. Using Cavagnolo et al. (2010) and O’Sullivan et al. (2011) relations gives similar flux for $P_{\text{jet}} = 10^{43.5} \text{ erg s}^{-1}$. However, since their $p_0 \sim 0.7$, they will give much lower flux for $P_{\text{jet}} \gtrsim 10^{43.5} \text{ erg s}^{-1}$, and much higher flux for $P_{\text{jet}} \lesssim 10^{43.5} \text{ erg s}^{-1}$.

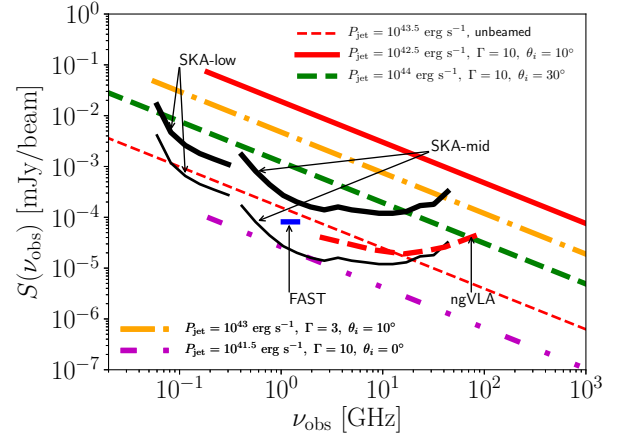


Figure 1. Observed radio flux from a jetted DCBH, for different jet powers, Lorentz factors and inclination angles, as indicated in the label. Sensitivities of different telescopes are given. For all sensitivities the integration time is 100 h, and bandwidth is given in the text.

For a highly relativistic jet, the beaming effect can boost/reduce the signal (Ghisellini 2000),

$$S(\nu_{\text{obs}}) = \delta_D^3 \frac{L'_\nu(\nu')(1+z)}{4\pi d_L^2}, \quad (3)$$

here L'_ν is the radio luminosity at the jet comoving frame. The Doppler factor can be written as

$$\delta_D = \frac{1}{\Gamma(1 - \beta_j \cos \theta_i)}, \quad (4)$$

where Γ is the Lorentz factor, and β_j is the jet velocity in units of the light speed; θ_i is the inclination angle of the jet axis with respect to the line of sight. In this case, the relation between the observed frequency ν_{obs} , and the jet comoving frame frequency ν' is

$$\nu_{\text{obs}} = \frac{\nu' \delta_D}{1+z}. \quad (5)$$

However, the radio luminosity in equation (1) is derived by assuming that the radio power is isotropically distributed, without beaming correction. Here we treat $L_{1.4}$ as in the jet comoving frame, with the warning that a beaming-induced bias might be present in the observed samples.

In Fig. 1, we plot the observed radio flux versus ν_{obs} for different jet power, inclination angle, and Lorentz factor. We cut the intrinsic luminosity below $\nu' = 0.1$ GHz, so that the redshift/blueshift effects can be observed from the cut-frequency of the observed flux. We also plot the sensitivity for SKA1-low/SKA2-low and SKA1-mid/SKA2-mid for fractional bandwidth 0.3 (Braun et al. 2019). We assume the SKA2-low has sensitivity four times higher than SKA1-low, while SKA2-mid has sensitivity ten times higher than SKA1-mid (Braun 2014). We also plot the sensitivity of the ngVLA; see Selina et al. (2018), where the bandwidth is also given, and the sensitivity of FAST telescope for a 400-MHz bandwidth (Li et al. 2018). For all telescopes, we adopt a 100-h integration time. Note that FAST has low angular resolution, with a beam size ~ 2.5 arcmin at 1.4 GHz. So one must take into account the confusion noise as well (Condon et al. 2012) when observing distant objects.

From these results in Fig. 1, we conclude that if a $z \simeq 10$ DCBH can launch a jet with $P_{\text{jet}} \sim 10^{42} \text{ erg s}^{-1}$, and if the scaling relation equation (1) holds, it will be possible to detect it at radio frequencies with both SKA and ngVLA in about 100 h of observation, depending

Table 1. The jet parameters for the observed flux shown in Fig. 2.

(0)No.	(1) Γ —	(2) r_{blob} [r_s]	(3) B [G]	(4) γ_{min} —	(5) γ_{break} —	(6) γ_{max} —	(7) s_1 —	(8) s_2 —
1	10	50	1	1	3×10^2	5×10^3	1.0	3.0
2	3	50	1	1	3×10^2	5×10^3	1.0	3.0
3	10	50	10	1	3×10^2	5×10^3	1.0	3.0
4	10	2000	1	1	1×10^4	1×10^6	2.0	3.0

Notes: Col. (0): jet No.; Col. (1): Lorentz factor of the blob bulk motion; Col. (2): physical radius of the blob; Col. (3): magnetic field in the blob; Col. (4): minimum Lorentz factor of relativistic electrons; Col. (5): Lorentz factor at the break; Col. (6): maximum Lorentz factor; Col. (7): spectrum index of the electron distribution between γ_{min} and γ_{break} ; Col. (8): spectrum index of the electron distribution between γ_{break} and γ_{max} .

on the inclination angle θ_i and Γ . If the jet is powerful enough, lower Γ values improve detection chances as the emission is less beamed. However, considering the large scatters in observations on the $P_{\text{jet}}-L_{1.4}$ relation, and possible bias in measuring the jet power, the above is just to present a very rough estimate.

2.2 Jet radiation spectrum

To refine our previous analysis, we need to discuss more in depth the details of the jet synchrotron emission. Luckily, a considerable amount of works are present in the literature. We will follow, in particular, the study by Ghisellini & Tavecchio (2009, hereafter GT09), which concentrates on blazars; see also Ghisellini et al. (2010) and Gardner & Done (2018). Their code is also publicly available.¹ We will use GT09 results to assess whether the physical parameters required for a jet to produce an observed radio flux similar to Section 2.1 are reasonable. Below we briefly summarize the model main features, referring the reader to the original papers for details.

GT09 is a one-zone leptonic jet model. Most of the jet radiation originates from a spherical ‘blob’ that forms when the jet dissipates. The blob contains relativistic electrons and has a bulk motion along the jet propagation direction. Given the energy spectrum of injected relativistic electrons in the blob, the magnetic field B , the Lorentz factor of the bulk motion Γ , the physical size of the blob r_{blob} , and the power of the injected relativistic electrons in jet comoving frame P'_i , the model returns the power of the jet in each component: relativistic electrons, P_e ; protons P_p ; magnetic field P_B ; and radiation P_r . The jet power is the sum of these four components: $P_{\text{jet}} = P_e + P_p + P_B + P_r$. The radiation power P_r is mainly produced through two mechanisms: synchrotron and Compton scattering. The observed radio flux at SKA and ngVLA band is from the synchrotron part. Finally, it gives the observed radiation spectrum of the jet and of the whole system.

Regarding the jet properties, Ghisellini et al. (2010) obtained the properties of 85 blazars with $z \lesssim 3$, including flat spectrum radio quasars (FSRQs) and BL Lacs, by fitting their observed SEDs using the GT09 model. They found that the mean physical radius of the blob is $r_{\text{blob}} \simeq 50r_s$, where r_s is the Schwarzschild radius of the central black hole. Blobs have a bulk motion with a mean Lorentz factor $\Gamma \sim 10$, and mean magnetic field $B \sim 1$ G. The energy spectrum of injected relativistic electrons in the blob is assumed to be a broken power law, with mean break Lorentz factor $\gamma_{\text{break}} \sim 300$, and mean maximum Lorentz factor $\gamma_{\text{max}} \sim 3 \times 10^3$ for FSRQs; and $\gamma_{\text{break}} \sim 1.5 \times 10^4$, $\gamma_{\text{max}} \sim 8 \times 10^5$ for BL Lacs. The slopes of the energy spectrum are uncertain, but typically below the break point the spectral slope is s_1

~ 1 and above it is $s_2 \sim 3$. They also found that the ratio P_r/P_{jet} is in the range $\sim 0.01-0.5$, and that the jet power is generally larger than the disc luminosity. Ghisellini et al. (2015) found similar properties for several $z \gtrsim 5$ blazars: $r_{\text{blob}} \sim 50r_s$, $B \sim 1$ G, $\Gamma \sim 10$, $\gamma_{\text{break}} \sim 70-300$, $\gamma_{\text{max}} \sim 3000$, and $P_r/P_{\text{jet}} \sim 0.02-0.13$. Paliya et al. (2017, 2019, 2020) analysed a sample of blazars and found $r_{\text{blob}} \sim 50-200r_s$, $B \sim 1$ G, $\Gamma \sim 10$, the spectrum index of the electrons distribution below γ_{break} is ~ 2 where $\gamma_{\text{break}} \sim 100$, and $P_r/P_{\text{jet}} \sim 0.02-0.13$.

Motivated by the above observations, we adopt some jet parameter sets for the following investigation, see the jet 1, 2, and 3 listed in Table 1. These parameters are typical ones, i.e. not rare among the above observed samples. We assume the black hole mass $M_{\text{BH}} = 10^6 M_\odot$ (Eddington luminosity is $L_E = 1.4 \times 10^{44}$ erg s $^{-1}$) at redshift $z = 10$, then calculate the synchrotron radiation from the jet of a high- z DCBH with these parameters. We ignore the effects of other parameters that are less relevant to synchrotron. GT09 uses a δ -function to approximate synchrotron from electrons with a given Lorentz factor. We slightly improve their code to allow for a more realistic electron energy distribution, as described in Appendix A.

We plot the observed radio spectrum in Fig. 2. For each model, we plot the curve for a total jet power equal to the Eddington luminosity. Around the curve, the filled colour shows the range corresponding the jet power from $0.1L_E$ to $10L_E$. We show the results for inclination angles of 0° and 10° , respectively. For large inclinations, the flux is further reduced. Since r_{blob} is given in units of Schwarzschild radius and the jet power is given in units of L_E , their actual physical values are scaled to match DCBHs.

For jet 1, 2, and 3 in Table 1, we confirm that the jet emission is detectable by SKA-mid or ngVLA as long as the jet power is $\gtrsim 0.1$ of the Eddington luminosity, and the jet has a small inclination angle. In particular, Γ , r_{blob} , and P'_i are important for generating strong synchrotron emission, while the inclination angle crucially determines whether such emission is detectable. For the SKA-low, the signal is not detectable. This is because if the blob is luminous and has small size as jet 1, 2, and 3, then it must be dense. The synchrotron self-absorption (SSA) would suppress low-frequency radiation, making the signal hard to detect by SKA-low. For this reason, we consider a large blob size $2000r_s$, and also increase the Lorentz factors of relativistic electrons therein; see jet 4 in Table 1. A large blob size up to several thousand Schwarzschild radius is physically conceivable. For example, M87 has a jet that starts to become conical when the width is $\sim 2000r_s$, as observed by Asada & Nakamura (2012) and supported by numerical models (Nakamura et al. 2018). We plot these results in Fig. 2 as well. We find that in this case, as long as the inclination angle is not too small, then the SSA effect is weaker, and the signal is detectable even at frequencies as low as 100 MHz. Quite interestingly, the SSA frequency-dependent

¹<https://starchild.gsfc.nasa.gov/xanadu/xspec/models/jet.html>.

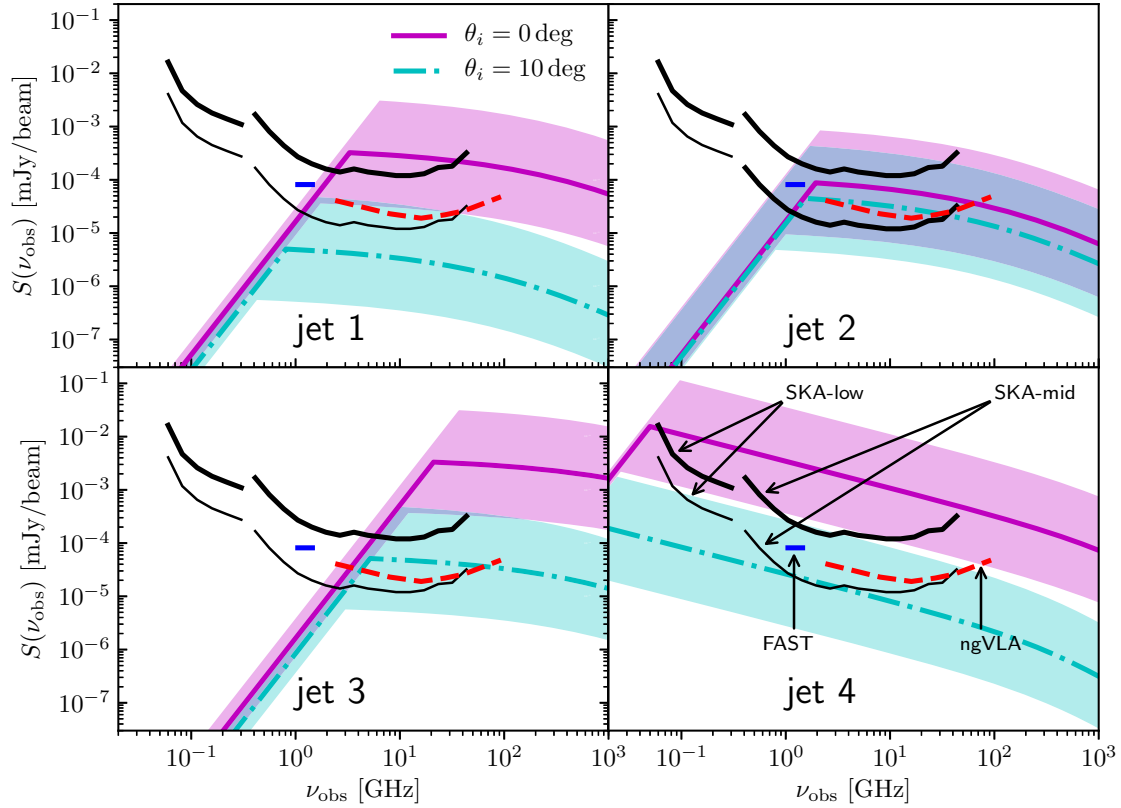


Figure 2. Observed radio flux from a jetted DCBH at $z = 10$ with emission parameters listed in Table 1. Sensitivities of various radio telescopes are the same as in Fig. 1. In each panel, we show the results for inclination angle 0° and 10° (solid and dash-dotted lines), and jet power from 0.1 to 10 Eddington luminosity (shaded regions around each line). The lines are for jet power equal to the Eddington luminosity.

signal suppression effect offers, at least in a certain parameter range, a direct way to discriminate between a radio source powered by a jetted DCBH with respect to, e.g. normal star-forming galaxies in which SSA should be essentially negligible.

2.3 The envelope surrounding DCBH

DCBH may be embedded into a warm and ionized envelope whose radius is much larger than the Bondi radius; its column number density can reach up to $\sim 10^{25} \text{ cm}^{-2}$. If the jet can break out the envelope and the radio blob forms outside it, then radio signal is not influenced. However, in the opposite case the radio blob forms within the envelope and the radio signal would be absorbed due to the free-free absorption. Here we estimate the absorption level. The envelope also produces free-free emission. This is the main radio signal in case of no synchrotron radiation; see Appendix C.

From numerical simulations, the radial density profile for the envelope is (Latif et al. 2013)

$$n_a(r) = n_0 \frac{1}{1 + (r/r_0)^\alpha}, \quad (6)$$

where n_0 , r_0 , and α are free parameters. Simulations show that $\alpha = 2$ and we adopt this value in this paper. We assume that both hydrogen and free electrons follow this profile. This profile evolves as the accretion proceeds. A final steady profile is likely $r_0 \sim$ Bondi radius $\sim 1 \text{ pc}$ (Pacucci, Volonteri & Ferrara 2015b).

The free-free absorption optical depth is (Draine 2011)

$$\tau_{\text{ff}}(\nu) = 0.05468 T_e^{-1.5} \nu_9^{-2} \text{EM} g_{\text{ff}}, \quad (7)$$

where ν_9 is the frequency in units 10^9 Hz and T_e is the electron temperature; the emission measure (in units of pc cm^{-6}),

$$\text{EM} = \int_{z_{\text{blob}}} n_a^2(r) dr, \quad (8)$$

where z_{blob} is the distance of the dissipation blob from the central black hole; and the Gaunt factor,

$$g_{\text{ff}}(\nu) = \ln \left\{ \exp \left[5.960 - \frac{\sqrt{3}}{\pi} \ln(\nu_9 T_4^{-1.5}) \right] + \exp(1) \right\}, \quad (9)$$

where $T_4 = T_e/10^4 \text{ K}$ is the electron temperature in units 10^4 K .

We plot the curve $\exp[-\tau_{\text{ff}}(\nu)]$ in top panel of Fig. 3 for some n_0 and r_0 . We always assume $z_{\text{blob}} = r_0$, implicitly assuming that this is the maximum propagation distance of the jet trapped in the envelope. Indeed, we see that if the jet blob is within the envelope, free-free absorption significantly reduces the radio signal. This represents a severe problem for DCBH detection by SKA or ngVLA because the radio signal below $\sim 100 \text{ GHz}$ would be absorbed ($n_0 \gtrsim 10^8 \text{ cm}^{-2}$ and/or $r_0 \gtrsim 0.1 \text{ pc}$).

In the bottom panel of Fig. 3, we plot the $\exp(-\tau_{\text{ff}})$ at $\nu_{\text{obs}} = 100 \text{ GHz}$ and the log of column number density, $\log(N_{\text{H}})$, as a function of n_0 and r_0 . From Fig. 3, to make sure that the radio signal at 100 GHz is less absorbed, the column number density of the envelope should be $N_{\text{H}} \lesssim 10^{24} - 10^{25} \text{ cm}^{-2}$. We also checked that for signal detectable at 17.09 GHz , the column density should be $N_{\text{H}} \lesssim 10^{23} - 10^{24} \text{ cm}^{-2}$. These are all reasonable values for typical DCBHs.

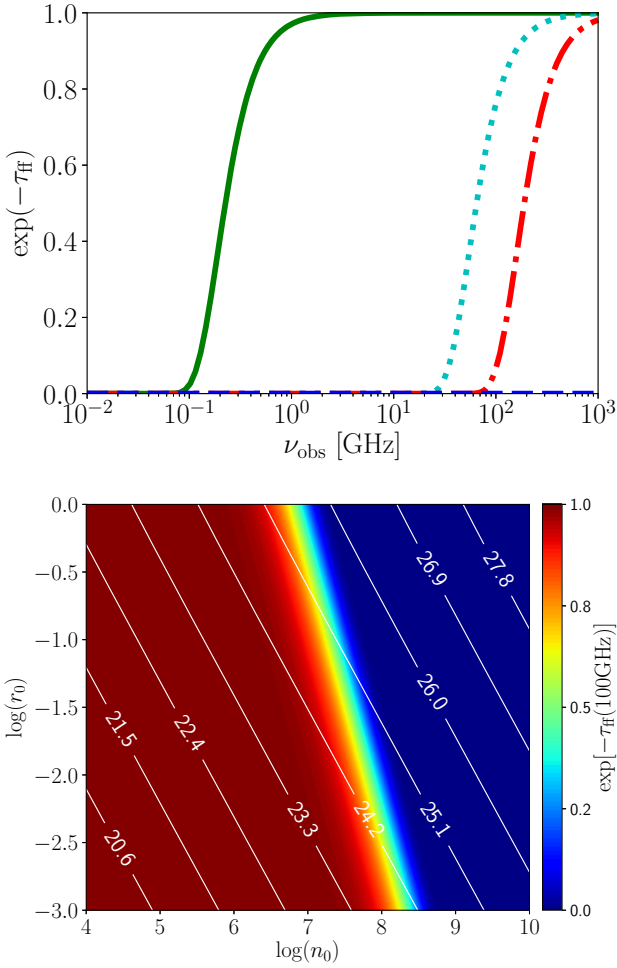


Figure 3. Top panel: the $\exp(-\tau_{\text{ff}})$ curve for different envelope profiles. In the panel from the left- to right-hand side, the curves correspond to $n_0 = 10^4 \text{ cm}^{-3}$, $r_0 = 1 \text{ pc}$; $n_0 = 10^6 \text{ cm}^{-3}$, $r_0 = 10^{-1} \text{ pc}$; $n_0 = 10^8 \text{ cm}^{-3}$, $r_0 = 10^{-2} \text{ pc}$, and $n_0 = 10^{10} \text{ cm}^{-3}$, $r_0 = 10^{-3} \text{ pc}$, respectively. Bottom panel: the $\exp(-\tau_{\text{ff}})$ at $\nu_{\text{obs}} = 100 \text{ GHz}$ as a function of n_0 and r_0 . We mark the $\log(N_{\text{H}})$ as well by lines.

Using the jet propagation model theory in Bromberg et al. (2011; see Appendix D), we estimate whether the jet can break out of the envelope. We find that, for shallow density profile ($\alpha \sim 2$), this does not occur unless the column density is very small. However, if the envelope has a steeper density profile, i.e. $\alpha \sim 3$, the jet can easily break out even if the column density is large.

Because of the uncertainties on n_0 and r_0 , we will not consider absorption by the envelope when estimating the radio detectability. However, from now on, we will concentrate on the SKA-mid and ngVLA bands that are less influenced by such effect.

2.4 Jet power supply

In Sections 2.1 and 2.2, we note that a radio detection requires $P_{\text{jet}} \gtrsim 10^{42} - 10^{43} \text{ erg s}^{-1}$, so the next key question is to verify whether such power can be physically extracted from the rotational energy of the system.

Jets are usually described in the framework of the Blandford & Znajek (1977, hereafter **BZ**) or Blandford & Payne (1982, hereafter **BP**) popular models. In the **BZ** model, the power of jet is originally

from the spin of black hole, while in the **BP** model the jet is powered by the spin of the accretion disc. Here we adopt the **BZ** model because it is considered more suitable for relativistic jets (McKinney 2005; Yuan & Narayan 2014), and it is supported by some observations, e.g. Ghisellini et al. 2014 (for a different interpretation, however, see, e.g. Fender, Gallo & Russell 2010).

In the **BZ** model, the jet power originates from the black hole spin energy and it is extracted via a magnetic field. The relation between the jet power, black hole mass, spin, and the magnetic flux is

$$P_{\text{jet}} = \frac{\kappa}{4\pi c} \Phi_{\text{BH}}^2 \Omega_{\text{H}}^2 f(\Omega_{\text{H}}), \quad (10)$$

where $f(\Omega_{\text{H}}) \approx 1 + 1.38(\Omega_{\text{H}} r_{\text{g}}/c)^2 - 9.2(\Omega_{\text{H}} r_{\text{g}}/c)^4$ (Tchekhovskoy, Narayan & McKinney 2010, 2011; Tchekhovskoy & McKinney 2012; Tchekhovskoy, McKinney & Narayan 2012); $\kappa \approx 0.05$; $\Omega_{\text{H}} = ac/2r_{\text{H}}$ is the angular frequency. Furthermore, $r_{\text{H}} = r_{\text{g}}[1 + (1 - a^2)^{1/2}]$, $r_{\text{g}} = GM_{\text{BH}}/c^2$, and $a = Jc/GM_{\text{BH}}^2$ is the dimensionless spin parameter; Φ_{BH} is the magnetic flux threading the black hole. To launch a powerful jet, a rather strong magnetic flux is necessary. The magnetically arrested disc (MAD) model predicts that the poloidal magnetic flux can reach a maximum saturation value $\Phi_{\text{MAD}} \approx 50(\dot{M}_{\text{BH}} r_{\text{g}}^2 c)^{1/2}$ (Narayan, Igumenshchev & Abramowicz 2003; Tchekhovskoy et al. 2011). With this maximum magnetic flux, assuming radiative efficiency $\epsilon \gtrsim 0.1$, the jet power could be larger than the Eddington luminosity when $a \gtrsim 0.4$. When $a \gtrsim 0.9$ it could be even larger than the net inward power $P_{\text{in}} = \dot{M}_{\text{BH}} c^2 = 1.26 \times 10^{38} M_{\text{BH}} (1 - \epsilon)/\epsilon \text{ erg s}^{-1}$. So if such a jet lasts for a long time, the accretion must be radiatively inefficient. This was already pointed out by Tchekhovskoy et al. (2010).

From the above discussion, it is clear that to supply the jet power a high-spin central black hole is required. In Appendix B, we estimate the angular momentum in the dark matter host halo of a DCBH, and in the innermost part of the accretion disc. We show there that these reservoirs contain enough angular momentum to support a high-spin black hole.

As a final comment, we warn that it is unclear whether the poloidal magnetic flux can reach the MAD saturation level. In fact, this large field amplification might be prevented by, e.g. the magnetic flux Eddington limit set by equipartition between B -field, and radiation field energy density near the BH surface (Dermer, Finke & Menon 2008). In spite of these uncertainties, we will assume that the MAD value can be nevertheless achieved, as an optimistic forecast.

3 SIGNAL DETECTABILITY

For a given black hole, the **BZ** jet model provides the total jet power P_{jet} , while the **GT09** model gives the power distribution in each component: relativistic electrons P_{e} , protons P_{p} , magnetic field P_{B} , and radiation P_{r} . Hence, $P_{\text{jet}} = P_{\text{e}} + P_{\text{p}} + P_{\text{B}} + P_{\text{r}}$ must hold. In the **GT09** model, in addition to the energy spectrum of relativistic electrons in the blob, other input parameters that are most relevant to the synchrotron radiation are: B , Γ , r_{blob} , and the total injected power of relativistic electrons in comoving frame, P'_{e} (Ghisellini & Tavecchio 2010; Gardner & Done 2014). Since Ghisellini et al. (2010) found that B , Γ and r_{blob} depend only weakly on P_{jet} , also to simplify the treatment, in this work, we assume they are all independent of P_{jet} .

Given the black hole mass and spin, we first get its jet power P_{jet} from equation (10). In the **GT09** model, given the energy spectrum of relativistic electrons in the blob, and B , Γ , r_{blob} , and P'_{e} , one can obtain the P_{e} , P_{p} , P_{B} , and P_{r} . However, by letting $P_{\text{e}} + P_{\text{p}} + P_{\text{B}} + P_{\text{r}} = P_{\text{jet}}$ we can reduce the freedom by one, i.e. by iteration, we

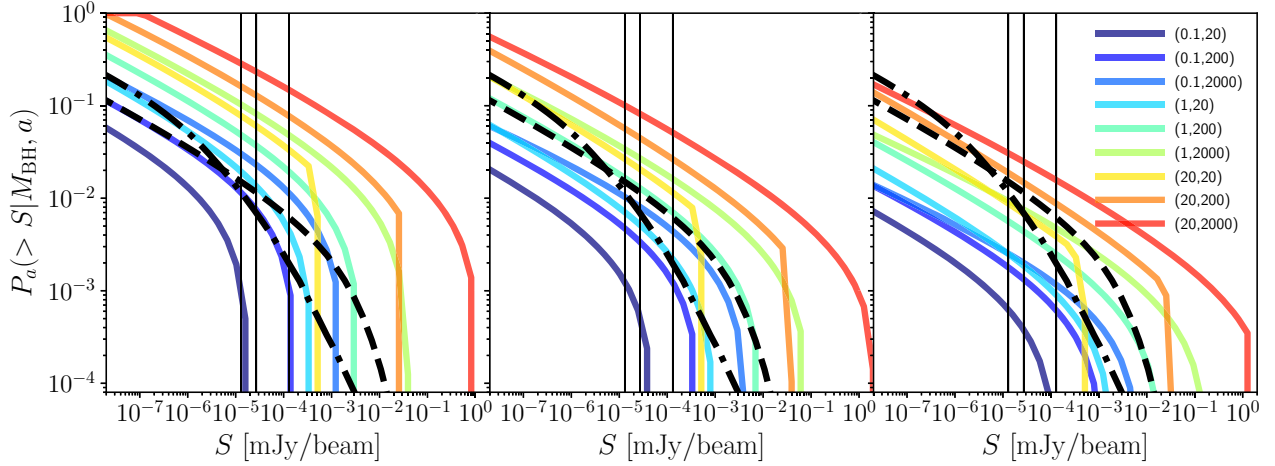


Figure 4. The probability to observe a DCBH with flux $>S$ at 17.09 GHz. In each row, from the left- to right-hand side, the panels correspond to $\Gamma = 5.0$, 10.0, and 20.0, respectively. For each panel, the B and r_{blob} of each curve are given in parentheses in the right-hand panel. The dashed and dash-dotted lines are probabilities after marginalizing the B , Γ , and r_{blob} for d1 distribution model and d2 distribution model, respectively. In each panel, vertical lines refer to sensitivities of SKA2-mid (left-hand panel), ngVLA (middle panel), and SKA1-mid (right-hand panel) respectively.

derive a P'_i for which $P_e + P_p + P_B + P_r = P_{\text{jet}}$ holds. We then obtain the final observed synchrotron flux of this black hole for different θ_i .

We further assume that the jet pointing direction is an uniform random distribution. Then the cumulative probability to observe a DCBH (given the mass, spin, redshift, and jet parameters) with flux $>S$ writes

$$P_a(> S | M_{\text{BH}}, a, B, \Gamma, r_{\text{blob}}) = \frac{\{1 - \cos[\theta_{i,S}(M_{\text{BH}}, a, B, \Gamma, r_{\text{blob}})]\}}{2}, \quad (11)$$

where $\theta_{i,S}$ is the critical inclination angle at which the observed flux is S ; clearly, for $\theta_i < \theta_{i,S}$ the observed flux is $>S$.

The probability distribution given by equation (11) depends on jet properties that are unfortunately not known for DCBHs. In Fig. 4, we first plot $P_a(> S)$ given jet parameters: $B = (0.1, 1, 20)$ G, $\Gamma = (5, 10, 20)$ and $r_{\text{blob}} = (20, 200, 2000) r_s$, to show how it changes with varying jet properties. We set $M_{\text{BH}} = 10^6 M_\odot$, $a = 0.9$ and $z = 10$, and observed frequency $\nu_{\text{obs}} = 17.06$ GHz. For simplicity, we fix the form of injected electrons distribution within the blob, say $\gamma_{\text{min}} = 1$, $\gamma_{\text{break}} = 3 \times 10^2$ and $\gamma_{\text{max}} = 5 \times 10^3$, $s_1 = 1.0$, and $s_2 = 3.0$, only adjust its normalization according to the P'_i that is derived from P_{jet} .

In each panel of Fig. 4, we mark the SKA sensitivity at 17.06 GHz and ngVLA sensitivity at 16 GHz. For all sensitivities we assume 100 integration hours and, for SKA we have bandwidth $\Delta\nu/\nu_c = 0.3$, and for ngVLA $\Delta\nu/\nu_c = 0.5$. From this figure, for SKA-mid and ngVLA, the black hole is detectable in a large parameter space, i.e. as long as $B \gtrsim 0.1$ G, $\Gamma \gtrsim 5$, and $r_{\text{blob}} \gtrsim 20 r_s$. We also check that for SKA-low (not shown in the figure) the black hole is not detectable for all above parameters. This is because when the black hole has large flux, generally it has small inclination angle. As shown in Fig. 2, for a small inclination angle, the low-frequency radiation is heavily absorbed due to the SSA effect. In particular, for higher Γ , the probability to detect a high flux is higher; however, in this case, the energy is more concentrated on a narrow beam; therefore, the probability to detect low-frequency flux is even smaller. The combined SKA-low and SKA-mid measurements could be useful for revealing the jet properties.

If the probability distributions of the three jet parameters are known, then we can marginalize them to get a probability that only

depends on black hole mass and spin,

$$P_a(> S | M_{\text{BH}}, a) = \int dB d\Gamma dr_{\text{blob}} f(B) f(\Gamma) f(r_{\text{blob}}) P_a(> S | M_{\text{BH}}, a, B, \Gamma, r_{\text{blob}}). \quad (12)$$

Unfortunately, for DCBHs, $f(B)$, $f(\Gamma)$, and $f(r_{\text{blob}})$ are only vaguely known; lacking a better argument, we make some assumptions. The assumed distributions do not have physical motivations, and they are rather empirically inspired; however, we hope that they can provide at least a semiquantitative estimate of the detectability.

First, the combination of the Ghisellini et al. (2010, 2011, 2015), Ghisellini (2013), and Paliya et al. (2017, 2019, 2020) observed samples shows that the $\log B$, Γ , and $\log r_{\text{blob}}$ follow distributions similar to Gaussian shape, with $\langle \log B \rangle = 0.2$, $\sigma_{\log B} = 0.2$, $\langle \Gamma \rangle = 11$, $\sigma_\Gamma = 2.5$, and $\langle \log r_{\text{blob}} \rangle = 1.9$, $\sigma_{\log r_{\text{blob}}} = 0.3$. Since there is no any prior knowledge about jets of DCBHs, we borrow these statistics directly. This is model d1. The results for $\nu_{\text{obs}} = 17.09$ GHz are added to the panel of Fig. 4 as a dashed black curve.

Model d1 is based on properties of observed samples. Since in principle weaker jets are more difficult to detect, it is possible that compared with the intrinsic distributions the observed jets are biased to stronger samples. On the other hand, power-law distribution $f(\Gamma) \propto \Gamma^{-2}$ for Γ is found for some observed samples (e.g. Saikia, K rding & Falcke 2016). So we also adopt another assumption that B , Γ and r_{blob} follow power-law distributions with index -2 . This is model d2. The results for $\nu_{\text{obs}} = 17.09$ GHz are shown by dash-dotted curves in Fig. 4.

We limit our parameters in the ranges: $0.1 \leq B \leq 20$ G, $2 \leq \Gamma \leq 30$, and $10 \leq r_{\text{blob}} \leq 4000 r_s$, roughly the boundaries of the above observed samples. At the lower limits, the jet is already very weak for radio detection, so it is not necessary to investigate parameters even smaller than these lower limits. Beyond the upper limits, objects would be too rare in both d1 and d2 distributions. The results of the probabilities are actually not sensitive to the upper limits.

We further calculate the detectability by SKA1-mid at 17.09 GHz for varying the black hole mass and spins. The results for model d1 (top panel) and model d2 (bottom panel) are plotted in Fig. 5. We find that black holes with $M_{\text{BH}} \gtrsim 10^5$ or $\gtrsim 3 \times 10^5 M_\odot$ are detectable in this case. Of course, the results depend on the jet properties

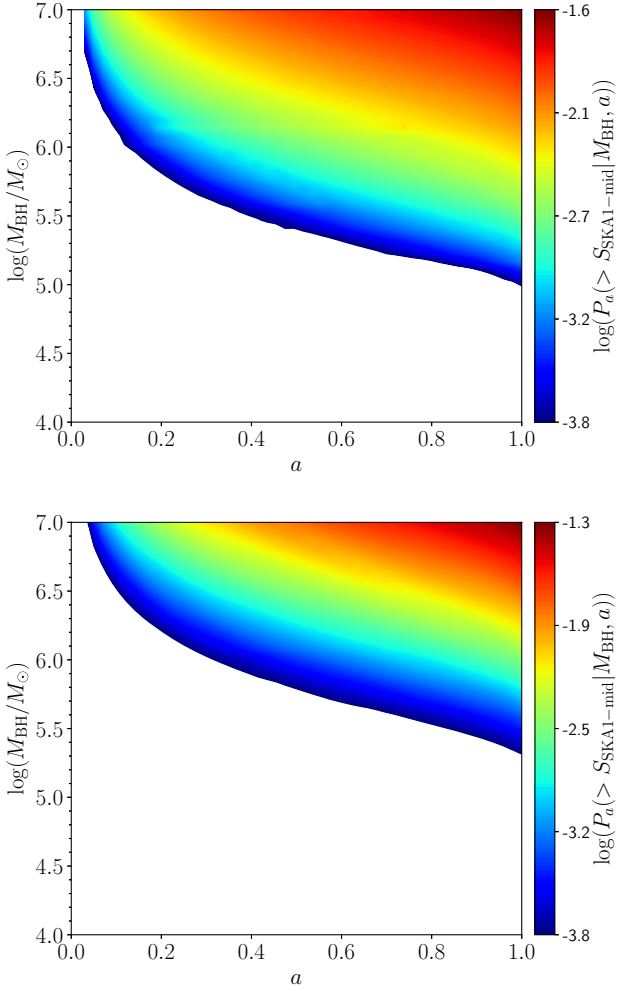


Figure 5. The probability of a BH with flux above SKA1-mid sensitivity with 100-h integration time (at 17.09 GHz with $\Delta\nu/\nu_c = 0.3$), as a function of BH mass and spin. *Top*: for model d1. *Bottom*: for model d2.

distributions and currently the distribution model d1 and d2 are pure assumptions.

We next consider marginalizing the spin distribution $f_{\text{spin}}(a)$,

$$\begin{aligned} P(> S | M_{\text{BH}}) &= \int da f_{\text{spin}}(a) P_a(> S | M_{\text{BH}}, a) \\ &= \int da f_{\text{spin}}(a) \frac{[1 - \cos\theta_i(S, M_{\text{BH}}, a)]}{2}, \end{aligned} \quad (13)$$

to get the possibility just depends on black hole mass, where $f_{\text{spin}}(a)$ is the normalized spin distribution.

Observations support the idea that most black holes have high spins, see Reynolds (2019). However, as high spin black holes are more easily selected, this evidence might be affected by a bias. On the other hand, the intrinsic spin distribution is still poorly known. Tiwari, Fairhurst & Hannam (2018) suggested three possible spin distributions:

$$f_{\text{spin}}(a) = \begin{cases} 2(1-a) & \text{favouring low spin} \\ 1 & \text{flat spin distribution} \\ 2a & \text{favouring high spin} \end{cases} \quad (14)$$

For the above spin distribution, the observed flux probability at 17.09 GHz from a 10^6 - M_{\odot} black hole is plotted in the top panel of Fig. 6, assuming jet properties distribution as model d1 and model

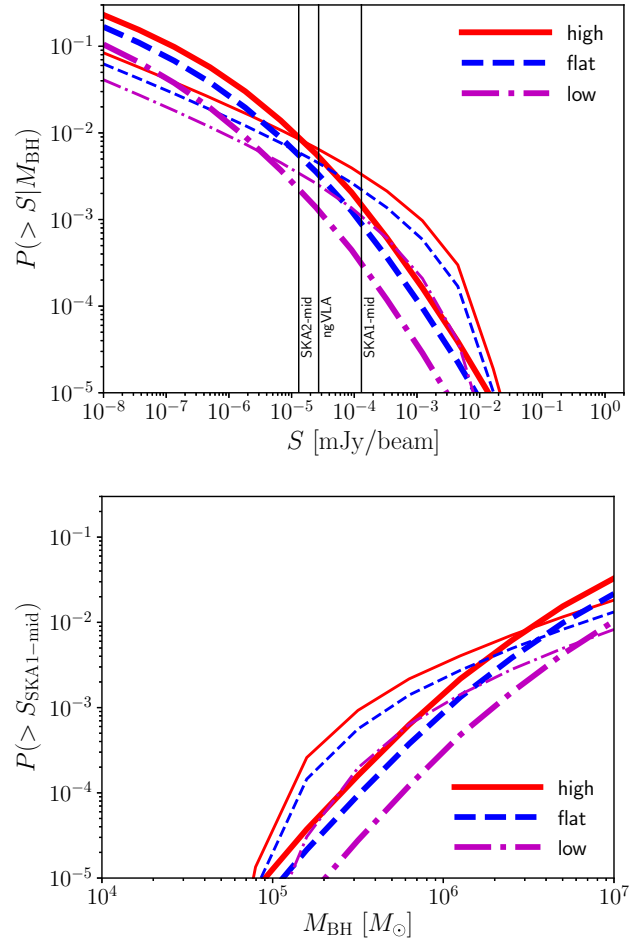


Figure 6. Top panel: the detection probability for a $10^6 M_{\odot}$ black hole with observed flux $> S$ for 17.09 GHz, given the spin distribution models indicated in the panel. Thin lines are for $(B, \Gamma, r_{\text{blob}})$ distribution model d1 and thick lines are for model d2. Vertical lines show the sensitivity for SKA and ngVLA, as in Fig. 4. Bottom panel: the detection probability by SKA1-mid at 17.09 GHz, as a function of black hole mass.

d2. In the bottom panel of Fig. 6, we plot the probability for a DCBH to be detected by SKA1-mid at 17.09 GHz in 100 integration hours, as a function of black hole mass. We find that, considering the spin distribution, roughly fraction $\sim 10^{-3}$ of DCBHs with mass $10^6 M_{\odot}$ is detectable by SKA1-mid, if they all launch jets.

Finally, for DCBHs, if the mass function is known, then the number density above a flux S is

$$n_{\text{BH}}(> S) = \int d \log M_{\text{BH}} \frac{dn}{d \log M_{\text{BH}}} P(> S | M_{\text{BH}}), \quad (15)$$

Until now there are two theoretical investigations about the DCBH mass function in literatures, see Lodato & Natarajan (2007), Ferrara et al. (2014). Here we follow the results in Ferrara et al. (2014), who show that the mass function is a bimodal Gaussian form,² i.e. $\propto \exp[-(\log M_{\text{BH}} - x_1)^2/2\sigma_1^2] + \exp[-(\log M_{\text{BH}} - x_2)^2/2\sigma_2^2]$, where $x_1 = 5.1$, $\sigma_1 = 0.3$, $x_2 = 5.9$, and $\sigma_2 = 0.1$.

²In Ferrara et al. (2014), the mass function evolves as the accretion proceeds, and it depends on whether stars can form in minihalo or not. This formula is for the mass function at the end of accretion, in the model where stars can form in minihalo.

Regarding the normalizations for the total DCBH number density, from Dijkstra, Ferrara & Mesinger (2014), if the critical radiation $J_{21} = 30$, then at $z \sim 10$ the total number density of DCBHs is $\sim 2.5 \times 10^{-3} \text{ Mpc}^{-3}$. This is close to Habouzit et al. (2016) where for critical radiation $J_{21} = 30$ the DCBH number density is $\sim 1.0 \times 10^{-3} \text{ Mpc}^{-3}$. If however the critical radiation is 300, then the total number density is $\sim 1.7 \times 10^{-7} \text{ Mpc}^{-3}$ in Dijkstra et al. (2014).

DCBHs could be even rarer than found above, since there are feedback effects impact their formation. For example, for a realistic spectrum of external radiation from normal star-forming galaxies, the critical J_{21} could be as high as $\sim 10^3$ – 10^4 (Sugimura, Omukai & Inoue 2014; Latif et al. 2015). During the formation of DCBHs the negative effects, such as the X-ray ionization/heating (Inayoshi & Tanaka 2015; Regan, Johansson & Wise 2016) and the tidal disruption (Chon et al. 2016), would significantly reduce the formation rate.

However, there are also positive effects, such as dynamical heating (Wise et al. 2019), H^- detachment by $\text{Ly}\alpha$ (Johnson & Dijkstra 2017), and cooling suppression by radiation from other previously formed DCBHs (Yue et al. 2017), which can enhance the DCBH formation. Moreover, if indeed DCBHs are seeds of SMBHs, their number density should be at least \gtrsim of the number density of SMBHs. From some observations in recent years, at $z \sim 6$ the observed QSOs number density is $\sim 10^{-7} \text{ Mpc}^{-3}$ (e.g. Willott et al. 2010; Kashikawa et al. 2015; Faisst et al. 2021). Theoretical models based on these observations predict that the QSOs number density reaches $\sim 10^{-3}$ – 10^{-4} Mpc^{-3} down to -15 absolute UV magnitude (Manti et al. 2017; Kulkarni, Worbeck & Hennawi 2019).

To predict the DCBH number count, one should specify a total number density as normalization. Currently, this normalization is quite uncertain, as explained above. In Fig. 7, we plot the $n_{\text{BH}}(> S)$ at $z = 10$, for total DCBH number density normalization of 2.5×10^{-3} and $1.7 \times 10^{-7} \text{ Mpc}^{-3}$, respectively. We convert the number density into surface number density in the right y-axis. For other number density of DCBHs, the curves should be re-normalized accordingly.

Generally for low-luminosity radio galaxies (classified as ‘FR I’), the jet lifetime could be as long as the Hubble time because the central black hole is fueled at a low rate (Blandford, Meier & Readhead 2019). while for high-luminosity FR IIs the duty cycle is $\simeq 2$ per cent, and each jet episode lasts only $\simeq 15$ million years (Bird, Martini & Kaiser 2008). Motivated by this, in Fig. 7, we also plot $n_{\text{BH}}(> S)$ assuming a 2 per cent duty cycle by thin lines. In this case, the number counts are considerably reduced.

We remind that in Fig. 7 it is assumed that all DCBHs are jetted. If, however, only a fraction of them can launch the jet, then the number count or luminosity function should be re-normalized according to the jetted fraction.

We also estimate the radio flux from star-forming galaxies, to check if it would confuse the DCBH observation. For a star-forming galaxy, the synchrotron luminosity is related to the star formation rate (SFR) because the electrons are accelerated by shocks from supernova explosions whose rate is proportional to the SFR. A relation is given as (see Bonato et al. 2017 and references therein)

$$\frac{\bar{L}_{\text{sync}}(\nu)}{\text{erg s}^{-1}\text{Hz}^{-1}} = 1.9 \times 10^{28} \left(\frac{\text{SFR}}{\text{M}_{\odot}\text{yr}^{-1}} \right) \left(\frac{\nu}{\text{GHz}} \right)^{-0.85} \left[1 + \left(\frac{\nu}{20\text{GHz}} \right)^{0.5} \right]^{-1}, \quad (16)$$

and after the low-luminosity correction,

$$L_{\text{syn}}(\nu) = \frac{L_{\text{syn}}^*}{(L_{\text{syn}}^*/\bar{L}_{\text{syn}})^3 + (L_{\text{syn}}^*/\bar{L}_{\text{syn}})}, \quad (17)$$

where $L_{\text{syn}}^* = 0.886 \bar{L}_{\text{syn}} (\text{SFR} = 1 \text{ M}_{\odot}\text{yr}^{-1})$.

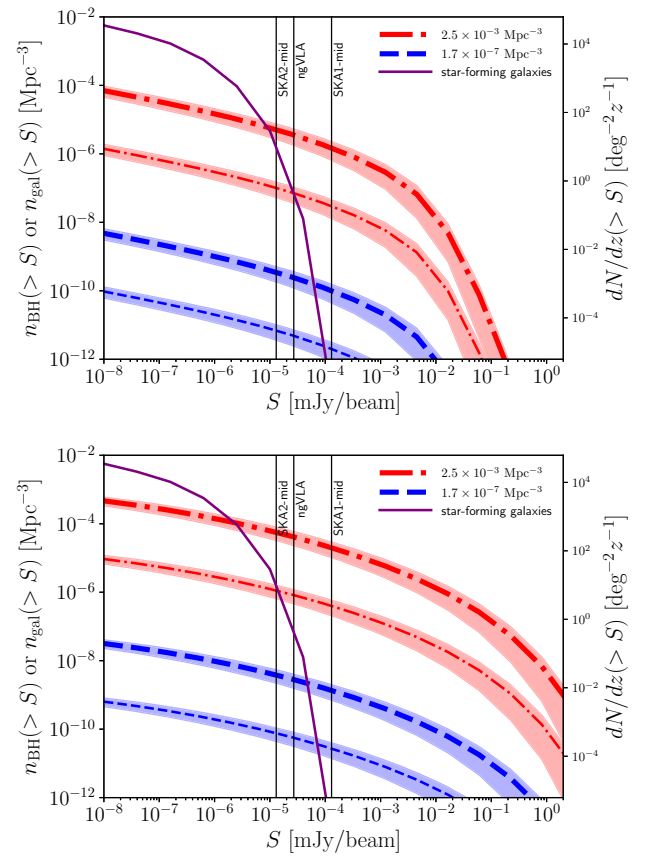


Figure 7. The number density of DCBHs with flux $> S$ at $z = 10$, for model d1 (top panel) and model d2 (bottom panel), respectively, for total DCBH number density 2.5×10^{-3} (dash-dotted) and $1.7 \times 10^{-7} \text{ Mpc}^{-3}$ (dashed), respectively. We assume the black hole mass follows the Ferrara et al. (2014) distribution. Thick curves are for a duty cycle 100 per cent, while thin curves are for duty cycle 2 per cent. Curves are for flat spin distribution, around each line, the filled regions mark the ranges from favouring low-spin to favouring high-spin distribution. We also plot the number density of star-forming galaxies with synchrotron flux $> S$ at $z = 6$ (solid). Right y-axis is the surface number density.

On the other hand, the SFR is also proportional to the UV luminosity (Yue & Ferrara 2019),

$$\frac{\text{SFR}}{\text{M}_{\odot}\text{yr}^{-1}} \approx 0.7 \times 10^{-28} \frac{L_{\text{UV}}}{\text{erg s}^{-1}\text{Hz}^{-1}}, \quad (18)$$

we therefore can construct the radio luminosity function from the UV luminosity function via

$$\frac{dn}{d \log L_{\text{syn}}} = \frac{dn}{d \log L_{\text{UV}}} \frac{d \log L_{\text{UV}}}{d \log L_{\text{syn}}}. \quad (19)$$

The dust-corrected UV luminosity function $dn/d \log L_{\text{UV}}$ is given by Bouwens et al. (2014, 2015).

In Fig. 7 we also plot the number density of the star-forming galaxies at $z = 6$. We conclude that, although DCBH might be rarer than star-forming galaxies, they still dominate the brightest-end of the flux distribution. Restricting to sources with continuum flux $\gtrsim 10^{-4} \text{ mJy}$ would prevent confusion between DCBH and star-forming galaxies.

4 SUMMARY AND DISCUSSION

We have explored the possibility to detect the radio signal from DCBHs by upcoming radio telescopes, such as the SKA and ngVLA. We just consider the mechanisms that produce radio signal at the accretion stage after DCBH formation. DCBHs may also produce radio signal during the collapse stage. However this stage is shorter therefore the detection probability is smaller. Our model supposes that DCBHs can launch powerful jets similar to those observed in the radio-loud AGNs. We applied the jet properties of the observed blazars to the jetted high- z DCBHs, and used the publicly available GT09 model to predict the radio signal from the jet. We found the following:

- (i) If the jet power $P_{\text{jet}} \gtrsim 10^{42} - 10^{43} \text{ erg s}^{-1}$, the continuum radio flux can be detected by SKA and ngVLA with 100 integration hours, depending on the inclination angle with respect to the line of sight.
- (ii) However, as the jet power depends on both black hole mass and spin, if the black hole mass $M_{\text{BH}} \lesssim 1 \times 10^5 M_{\odot}$, even maximally spinning DCBH can be hardly detected by SKA1.
- (iii) By considering the spin distribution, about 10^{-3} of DCBHs with $M_{\text{BH}} = 10^6 M_{\odot}$, if they are all jetted, are detectable.
- (iv) Moreover, if the blob at the jet head is dense ($r_{\text{blob}} \sim 50 r_{\text{S}}$), and has large bulk motion velocity ($\Gamma \sim 10$), then it may only feebly emit in the SKA-low band, because of the SSA and radiation blueshift. These events are more suitable for study with SKA-mid. However, by combing the observations of SKA-low and SKA-mid, this may also provide a potential to distinguish a jetted DCBH from a normal star-forming galaxy.
- (v) If all DCBHs are jetted, for the most optimistic case $\sim 100 \text{ deg}^{-2} \text{ z}^{-1}$ at $z = 10$ would be detected by SKA1-mid with 100-hour integration time.

We have predicted that some DCBHs can be detected by upcoming radio telescopes if their jets have small inclination angles. Radio flux from other unresolved DCBHs forms a diffuse radiation field that is part of the cosmic radio background (CRB). This might be connected with previous claims of an ‘excess’ on the CRB (e.g. Seiffert et al. 2011), whose origin has been related with undetected black holes (Ewall-Wice et al. 2018). If confirmed, future CRB observations will provide an alternative tool for study DCBHs.

Our study inevitably contains some uncertainties. We list them below and discuss their impact briefly:

- (i) Jetted fraction:
Although DCBH may launch a jet, it is not clear how many of DCBHs can actually do this. Our predicted probabilities are for jetted DCBHs. So if some DCBHs do not show jets, or their jets are weaker than our lower boundaries (i.e. $B < 0.1 \text{ G}$, $\Gamma < 2$ and $r_{\text{blob}} < 10 r_{\text{S}}$), then the denominator is higher and our probabilities should be re-normalized. However, the probability-flux shape at the high-flux end is not influenced by normalization.

- (ii) Jet properties:
To predict the observed radio flux from individual jetted DCBH, we have used the simple empirical $P_{\text{jet}} - L_{1.4}$ relation, with reasonable assumed parameters, for example the jet power is \lesssim the Eddington luminosity, and a typical Lorentz factor for the jet head is ~ 10 , see equation (1) and Fig. 1. This prediction is not quite sensitive to the details of the jet mechanism.
We then predicted the observed flux expanding the GT09 jet model. The results are consistent with the above simple empirical relation, see Fig. 2. We conclude that without using extreme assumptions, some jetted DCBHs can be detected by SKA or ngVLA.

Further predictions on the detection probability suffer from additional uncertainties. These predictions are based on jet property parameters borrowed from observed blazars, and the assumed distribution forms (Gaussian or power law). Currently, we cannot further improve such predictions. However, our estimations are at least useful as optimistic references.

- (iii) Number density of the DCBHs and their mass function:
This is also an uncertain aspect, although its study is beyond the scope of this paper. The number density of DCBHs predicted in literature span approximately eight orders of magnitude. The lower limit is close to the number density of the observed high- z SMBHs, which is $\sim 10^{-9} \text{ Mpc}^{-3}$; the upper limit is close to the number density of newly formed atomic-cooling haloes, which is $\sim 0.01 - 0.1 \text{ Mpc}^{-3}$, depending on the critical radiation field, and how negative feedback mechanisms work. In Fig. 7, we show the number counts corresponding to total number $\sim 10^{-3}$ and $\sim 10^{-7} \text{ Mpc}^{-3}$. The former would be close to upper limits. Regarding the mass function, although there is no observational support, it is generally believed that the mass distribution peaks at $10^5 - 10^6 M_{\odot}$. We have made our choice accordingly.

In this work, we attempt to present some tentative investigations, we hope it can draw attention to observing these first black holes by SKA and ngVLA. Future theoretical investigations, e.g. the numerical simulations, would help to improve our predictions. However, the uncertainties could only be reduced by observations themselves. For example, if the radio signal from DCBHs is much lower than our predictions, then it is likely that most DCBH cannot launch strong jet. This will force us to investigate the difference between DCBHs and low- z black holes, e.g. on the magnetic field, the density of the envelope, or the spin.

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DATA AVAILABILITY

The data underlying this paper can be shared on reasonable request to the corresponding author.

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APPENDIX A: THE SYNCHROTRON RADIATION CALCULATION DETAILS

The synchrotron power per frequency emitted by a single particle with Lorentz factor γ (Strong, Orlando & Jaffe 2011; Di Bernardo et al. 2013, 2015; Ghisellini 2013) is

$$G_{\text{syn}}(\nu, \gamma) = \frac{\sqrt{3}e^3 B}{m_e c^2} F(\nu/\nu_c), \quad (\text{A1})$$

where B is the magnetic field, e is the electron charge and m_e is the electron mass, and c is the speed of light. The function

$$F(\nu/\nu_c) = \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(y) dy, \quad (\text{A2})$$

with peak frequency

$$\nu_c = \frac{3e}{4\pi m_e c} B \gamma^2, \quad (\text{A3})$$

K_l is the modified Bessel function of order l of the second kind. F is called synchrotron function, we used the fittings given by Fouka & Ouichaoui (2013).

Suppose the electron number density distribution is $dN_e/d\gamma$, then the synchrotron emissivity is

$$j_{\text{syn}}(\nu) = \frac{1}{4\pi} \int G_{\text{syn}}(\nu, \gamma) \frac{dN_e}{d\gamma}(\gamma) d\gamma, \quad (\text{A4})$$

where $dN_e/d\gamma$ is provided in the GT09 code.

APPENDIX B: THE BLACK HOLE SPIN

As the total jet power depends on spin, it is important to check whether the host dark matter halo or the accretion disc contain enough angular momentum to support a high-spin central black hole but still allows a DCBH to form.

The angular momentum of a dark matter halo is (Lodato & Natarajan 2007)

$$J_h = \lambda_h \frac{GM_h^{5/2}}{|E_h|^{1/2}}, \quad (\text{B1})$$

where λ_h is the dimensionless spin parameter of the halo, $E_h = (1/2)W_h$ is the total energy of the halo, and W_h is the gravitational potential energy. For an NFW profile (Łokas & Mamon 2001),

$$W_h(s) = -W_\infty \left[1 - \frac{1}{(1+C_s)^2} - \frac{2\ln(1+C_s)}{1+C_s} \right], \quad (\text{B2})$$

where $s = r/r_{\text{vir}}$ is the halo radius in units of virial radius r_{vir} , C is the concentration parameter,

$$W_\infty = \frac{cg^2(C)GM_h^2}{2r_v} \quad (\text{B3})$$

is the potential energy when $s \rightarrow \infty$, and

$$g(C) = \frac{1}{\ln(1+C) - C/(1+C)}. \quad (\text{B4})$$

For reference, a dark matter halo with $M_h = 10^8 M_\odot$ at $z = 10$ has $J_h \approx 2 \times 10^{66} (\lambda_h/0.001) \text{ erg s}$.

In Lodato & Natarajan (2006), Lodato & Natarajan (2007), they gave the disc surface density profile $\Sigma(R) \propto R^{-1}$, both the disc size and the surface density profile normalization are derived by let the mass equal fraction m_d of the halo and angular momentum equal fraction j_d of the dark matter halo angular momentum. We determine

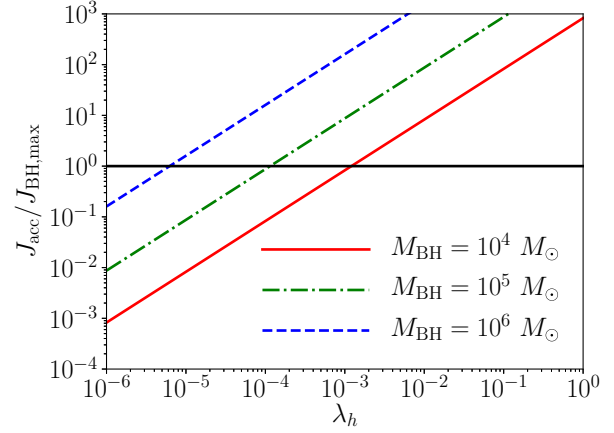


Figure B1. The $J_{\text{acc}}/J_{\text{BH,max}}$ as a function of λ_h for different black hole mass. To guide the eye, we plot a horizontal line corresponding to $J_{\text{acc}} = J_{\text{BH,max}}$.

an inner radius of the accretion disc, letting

$$M_{\text{BH}} = \int_0^{R_{\text{in}}} \Sigma(R) 2\pi R dR, \quad (\text{B5})$$

therefore, the maximum angular momentum available for BH accretion is

$$J_{\text{acc}} \sim \int_0^{R_{\text{in}}} \Sigma(R) 2\pi R R^2 \omega_d dR, \quad (\text{B6})$$

where ω_d is the angular velocity of the disc derived from

$$j_d J_h = \int_0^{R_d} \Sigma(R) 2\pi R R^2 \omega_d dR, \quad (\text{B7})$$

where R_d is the size of the accretion disc.

In Fig. B1, we plot the $J_{\text{acc}}/J_{\text{BH,max}} = J_{\text{acc}}/(GM_{\text{BH}}^2/c^2)$ as a function of λ_h , given the halo mass $M_h = 10^8 M_\odot$, $z = 10$, and $m_d = j_d = 0.05$. We find that, as long as the $\lambda_h \gtrsim 10^{-3}$ – 10^{-5} , the accreted angular momentum is more than enough to support the maximum black hole spin. Since the spin parameter distribution of dark matter haloes follows a ln-normal distribution, with central value $\ln(0.05)$ and standard deviation 0.5, most dark matter haloes have $\lambda_h \gtrsim 10^{-3}$ – 10^{-5} . A disc with high angular momentum is stable and supported by rotation, leading to a reduced central DCBH mass. Lodato & Natarajan (2006), Lodato & Natarajan (2007) also give the relation between λ_h and the central DCBH mass. We check that, $\lambda_h \sim 10^{-3}$ – 10^{-5} is small enough so that the formation of a massive central DCBH is possible.

Finally, theoretical calculations predict that if the BH spins up by acquiring the angular momentum of the innermost disc boundary throughout the accretion phase, for a thin disc, the spin increases from 0 to ≈ 1 in a short time-scale, roughly $\epsilon \times t_{\text{Edd}}$, where $t_{\text{Edd}} = 45 \text{ Myr}$ is the Eddington time; generally, the radiative efficiency $\epsilon \sim 0.1$. Within this time-scale, the black hole doubles its mass (e.g. Li 2012; Zhang, Lu & Liu 2019).

APPENDIX C: RADIO SIGNAL OF RADIO-QUIET DCBHs

Generally, radio-loud AGNs have higher spin than radio-quiet ones (Wilson & Colbert 1995; Sikora, Stawarz & Lasota 2007), and they are usually hosted by elliptical galaxies; radio-quiet AGNs are instead mostly found in spiral galaxies. However, even in radio-

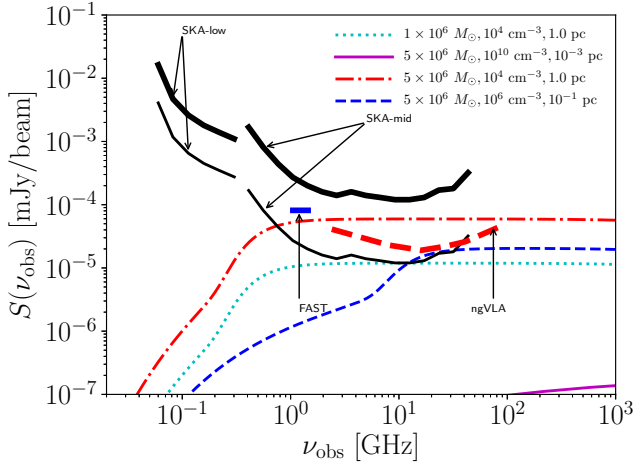


Figure C1. The flux from a $z = 10$ DCBH with mass and N_H given in the legend, we assume the total luminosity is the Eddington luminosity. We show the sensitivities of SKA1(2)-low, SKA1(2)-mid, and ngVLA as well, for integration time 100 h.

quiet AGNs, there are mechanisms that produce some synchrotron radiation, such as, e.g. weaker jets, relativistic electrons in the corona, or winds/outflows (see the review; e.g. Panessa et al. 2019). Shocks can also accelerate relativistic electrons and this process represents an additional possibility for synchrotron (Ishibashi & Courvoisier 2011). Here, we do not model the complex radiative processes occurring in the vicinity of the black hole. Instead, we just assume some radio loudness parameter R , and, unlike the jet, now the synchrotron radiation is isotropic and we do not need to consider the beaming effect.

In Yue et al. (2013b), we calculated the DCBH SED analytically. In the radio band, the radiation is mainly free-free emission. Free-free emission has flat spectrum when the energy of photons is much smaller than the kinetic energy of the thermal electrons. At a low frequency, free-free absorption works, as described in Section 2.3.

For Compton-thick DCBH, free-free absorption becomes important even at frequency as high as $\sim 10^2$ – 10^3 GHz. As a result, the radio signal from such a DCBH is difficult to detect. We add the free-free absorption to Yue et al. (2013b), assuming that the density profile of the medium surrounding the central black hole is equation (6).

In Fig. C1, we plot the flux of DCBH at $z = 10$ with various black hole mass and envelope profile (we assume $N_H \sim N_e$). We find that, without the synchrotron radiation, the radio-quiet DCBH is not able to detect by SKA-low and SKA1-mid. However, by SKA2-mid or ngVLA, it is marginally detectable if $M_{BH} \gtrsim 5 \times 10^6 M_\odot$ and $N_H \lesssim 10^{23} \text{ cm}^{-2}$.

We then assume that such radio-quiet DCBH may also produce some synchrotron by assuming a radio loudness parameter R . Here R is the ratio between the flux observed at 5 GHz and at 4400 Å. The results are shown in Fig. C2. For a black hole with mass $\sim 5 \times 10^6 M_\odot$ and $R = 10$, the signal is marginally detectable by SKA-low. Note that by the definition of the terminology ‘radio-quiet’, $R = 10$ is the maximum radio loudness parameter (Kellermann et al. 1989). So the latter is considered as an optimistic extreme. Again, detecting the synchrotron emission by SKA-low typically requires $N_H \lesssim 10^{22} \text{ cm}^{-2}$.

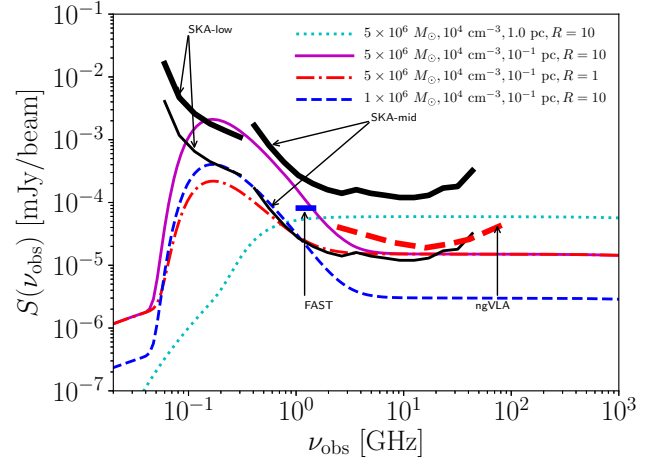


Figure C2. Similar to Fig. C1; however, here we assume the radio-quiet DCBH also produce some synchrotron radiation.

APPENDIX D: JET BREAK OUT AND HIGHLY RELATIVISTIC BLOB FORMATION

Generally, the DCBH accretion disc is surrounded by an envelope that could be Compton-thick with column number densities up to $\sim 10^{25} \text{ cm}^{-2}$ (Yue et al. 2013a). Before investigating the DCBH radio detectability, it is necessary to study the jet propagation inside this envelope and its eventual break-out. We follow the theory of jet propagation given in Bromberg et al. (2011) (see also Matzner 2003; Matsumoto et al. 2015).

If the jet energy loss is negligible, the speed of the jet head (in units of c) is

$$\beta_h = \frac{\beta_j}{1 + \tilde{L}^{-1/2}}, \quad (\text{D1})$$

where β_j is the jet speed, and

$$\tilde{L} \approx \frac{P_{\text{jet}}}{\Sigma_h \rho_a c^3}, \quad (\text{D2})$$

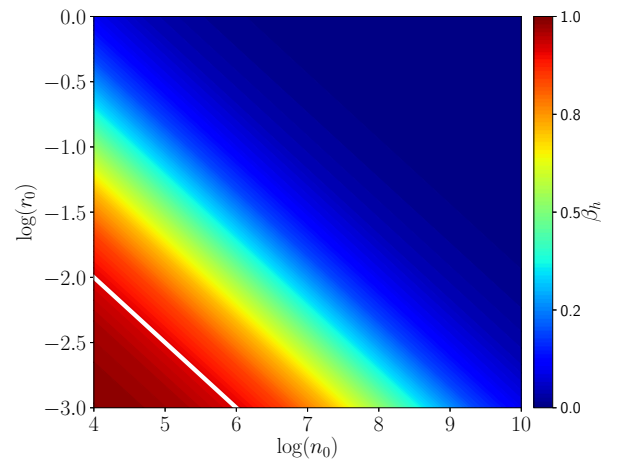


Figure D1. Top panel: The β_h as a function of n_0 and r_0 . Here β_h is calculated either at the time when half of the jet power is deposited into the cocoon, or at the distance > 10 pc from the central black hole. We take density profile slope $\alpha = 2$. A white curve describes the relation $\log r_0 = -0.5(\log n_0 - 4) - 2$.

here Σ_h is the cross-section of the jet head; $\rho_a = m_H n_a$ is the envelope mass density at the jet head, with m_H being the hydrogen mass (for simplicity we assume a pure H gas), for which is given by equation (6).

A jet could be either ‘collimate’ or ‘un-collimated’ (conical), depending on the competition between the jet pressure and the cocoon pressure. If the jet is un-collimated and the opening angle is θ_0 , then

$$\Sigma_h = r_h^2 \pi \theta_0^2, \quad (D3)$$

where the propagation distance of the jet head ,

$$r_h(t) = \int_0^t c \beta_h dt. \quad (D4)$$

During jet propagation, a cocoon is produced, and the jet loses energy due to the cocoon expansion. The energy going into the cocoon is

$$E_c = \eta P_{\text{jet}} \int_0^t (1 - \beta_h) dt, \quad (D5)$$

where $\eta \lesssim 1$ is an efficiency. One writes the volume, pressure and expansion velocity of the cocoon as

$$V_c = \pi c^3 \int \beta_h dt \left(\int \beta_c dt \right)^2, \quad (D6)$$

$$P_c = \frac{E_c}{3V_c}, \quad (D7)$$

$$\beta_c = \sqrt{\frac{P_c}{\bar{\rho}_c c^2}}, \quad (D8)$$

respectively, where $\bar{\rho}_c$ is the mean density of the full cocoon.

We consider a black hole with mass $M_{\text{BH}} = 10^6 M_\odot$ and $a = 0.9$, and solve the above equations numerically for different density profiles and black hole parameters, assuming that the jet is always conical. We stop the calculation when the ratio between the energy loss rate due to cocoon expansion and the jet power, $P_c \dot{V}_c / P_{\text{jet}}$, reaches ~ 50 per cent, since, in principle, our adopted formulas are only valid when this ratio is very small. We assume an opening angle $\theta_0 = 5^\circ$, $\beta_j \approx 1$.

We show the β_h as a function of n_0 and r_0 in Fig. D1. Here β_h is either at the point when $P_c \dot{V}_c / P_{\text{jet}} = 0.5$ (for the jet that fails to break out the envelope), or at $r = 10$ pc that is $\gg r_0$ (for the jet that is considered to break out the envelope successfully). We find that, indeed, just in a small parameter space, say roughly below the curve $\log r_0 = -0.5(\log n_0 - 4) - 2$ (the white curve), the jet is able to break out the envelope. However, we check that if the density profile is more steeper, e.g. the slope $\alpha = 3$, then for more profiles (roughly below the curve $\log r_0 = -1/3(\log n_0 - 4) - 1$) the jet can break out the envelope.

Here we conclude that, if indeed the density profile has slope $\alpha \sim 2$, as given by numerical simulations, lots of DCBHs may suffer from the free-free absorption by their envelopes at SKA-low band unless the $N_H \lesssim 10^{22} \text{ cm}^{-2}$, and at SKA-mid band unless the $N_H \lesssim 10^{24} \text{ cm}^{-2}$.

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