Mixing and CP violation in $D^0 \to K^-\pi^+$ decays

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ABSTRACT: We review the experimental methods to measure mixing and time-dependent CP violation in D^0 decays into two hadrons. While these phenomena are usually neglected for $D^0 \to K^-\pi^+$ decays, this approximation is not always justified. In particular, it produces a bias on the measurement of the parameter y_{CP}, when this is performed by relying on $D^0 \to K^-\pi^+$ decays as a normalisation channel, whose size is around 40% of the precision of the current world average. Finally, we estimate the sensitivity to the weak mixing phases achievable by studying $D^0 \to K^-\pi^+$ and untagged $D \to K^-\pi^+$ decays, where D stands for either of the D^0 and $\overline{D}^0$ mesons. Contrary to Cabibbo-suppressed $D^0 \to h^+h^-$ decays, these decay channels allow to measure these phases without final-state dependent nuisance contributions from the decay amplitudes, but their sensitivity is lower by a factor of six.

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Contents

1 Introduction

Charm hadrons are the only hadrons made up of solely up-type quarks where mixing and CP violation can be studied. Thus, they provide a unique opportunity to detect new interactions beyond the Standard Model (SM) that leave down-type quarks unaffected [1]. Both mixing and CP violation are highly suppressed in charm by the Glashow-Iliopoulos-Maiani mechanism and by the hierarchical structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and have been observed only in recent years [2–17]. The mass and decay-width differences of the D^0 mass eigenstates are known with relative precision of around 10% [16]; while CP violation in the decay amplitudes has been observed [17], evidence of CP violation in D^0 mixing is still missing [4, 8, 12, 14–16, 18–22]. The upgrades of the LHCb experiment and the Belle II experiment are expected to improve the precision on these observables by about a factor of 10 within the next two decades [23, 24], allowing to attain per-cent precision on the mixing parameters, and a precision on CP violation in the mixing comparable to the size of the SM predictions [25–29].

The most precise measurements of these phenomena employ multi-body decays such as $D^0 \to K_S^0 \pi^+ \pi^-$ [14, 15], or D^0 decays into either of the Cabibbo-suppressed (CS) final states

K^+K^- and $\pi^+\pi^-$ [4, 8, 13, 20–22], or into the wrong-sign (WS) final state $K^+\pi^-$ [12].[1] Measurements of two-body decays rely on right-sign (RS) $D^0 \to K^-\pi^+$ decays as a normalisation channel. Since RS decays mainly consist of Cabibbo-favoured (CF) decays without D^0 flavour oscillation, while the decay amplitude following flavour oscillation is doubly Cabibbo-suppressed (DCS), the effect of mixing on their decay rate is usually neglected in the literature. However, it can no longer be ignored at the current level of experimental precision, and its size needs to be carefully assessed.

This is the aim of the present article, which extends the discussion in refs. [27, 30] by drawing the experimental consequences of the results presented therein. The article is structured as follows. Section 2 summarises the theoretical formalism introduced in refs. [27, 31, 32] to parametrise mixing and time-dependent CP violation in D^0 decays, limiting the discussion to decays into two hadrons. The corresponding time-dependent decay rates are presented in section 3, where the observables $y_{CP}^{K^-\pi^+}$ and $\Delta Y_{K^-\pi^+}$, analogues of the observables y_{CP}^f and ΔY_f defined for the CS final states $f = K^+K^-$ and $\pi^+\pi^-$, are introduced to parametrise the first-order contributions of mixing and time-dependent CP violation to the RS decay rate. The parameters $y_{CP}^{K\pi,\mathrm{untag}}$ and $\Delta Y_{K\pi}^{\mathrm{untag}}$ are similarly defined for untagged $D \to K^-\pi^+$ decays, where the flavour at production of the D^0 meson is unknown. Section 4 reviews the experimental approaches to measure the time-dependent decay rates, and pinpoints a bias equal to $-y_{CP}^{K^-\pi^+}$ which affects most of the measurements of y_{CP}^f performed to date and has been missed in the literature. The improvement in precision that might be achieved by including the $\Delta Y_{K^-\pi^+}$ and $\Delta Y_{K\pi}^{\mathrm{untag}}$ observables in the experimental program of the LHCb experiment, and by measuring y_{CP}^f with untagged D decays, are discussed as well. Section 5 concludes by summarising the main findings. Finally, appendix A quantifies the contribution of mixing in RS decays to the parameters measured in the analysis of the time-dependent ratio of the WS to RS decays rates, and of the terms quadratic in the mixing parameters to the expansion of the time-dependent asymmetry of the decay rates of D^0 and $\overline{D}^0$ mesons into the final states K^+K^- or $\pi^+\pi^-$. The former contribution might become relevant at the LHCb upgrades.

2 Formalism

This section reviews the theoretical formalism introduced in refs. [27, 31, 32] to parametrise mixing and time-dependent CP violation in D^0 decays.

2.1 D^0 mixing

An arbitrary linear combination of D^0 and $\overline{D}^0$ flavour eigenstates,

$$a(t)|D^0\rangle + b(t)|\overline{D}^0\rangle, \tag{2.1}$$

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[1]The $D^0 \to K^+\pi^-$ decay is named wrong-sign since, when the D^0 flavour at production is determined by relying on the $D^*(2010)^+ \to D^0\pi^+$ strong decay, the charges of the pions from the $D^*(2010)^+$ and D^0 decays have opposite signs.

evolves in time according to the Schrödinger equation,

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \boldsymbol{H}\begin{pmatrix} a(0) \\ b(0) \end{pmatrix}, \qquad (2.2)$$

where $\boldsymbol{H}$ is the 2×2 effective Hamiltonian governing the dynamics of the D^0–$\overline{D}^0$ system.[2] This Hamiltonian can be expressed in terms of two Hermitian matrices as $\boldsymbol{H} \equiv \boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma}$. The *CPT* symmetry, which is assumed hereafter, implies $H_{11} = H_{22} = M - \frac{i}{2}\Gamma$, with M and Γ the mass and decay width of the D^0 meson. The transition amplitudes between D^0 and $\overline{D}^0$ mesons,

$$\langle D^0|H|\overline{D}^0\rangle = M_{12} - \frac{i}{2}\Gamma_{12}, \qquad \langle \overline{D}^0|H|D^0\rangle = M_{12}^* - \frac{i}{2}\Gamma_{12}^*, \qquad (2.3)$$

depend on the dispersive and absorptive mixing amplitudes, M_{12} and Γ_{12}, which correspond to off-shell and on-shell transitions, respectively. They are parametrised in terms of two *CP*-even mixing parameters,

$$x_{12} \equiv \frac{2|M_{12}|}{\Gamma}, \qquad y_{12} \equiv \frac{|\Gamma_{12}|}{\Gamma}, \qquad (2.4)$$

and one *CP*-odd weak phase,

$$\phi_{12} \equiv \arg\left(\frac{M_{12}}{\Gamma_{12}}\right). \qquad (2.5)$$

The additional global phase shared by M_{12} and Γ_{12} depends on the phase conventions for quarks and mesons and is unobservable. The parameters x_{12} and y_{12} are equal to the absolute value of normalised mass and decay-width differences of the eigenstates of $\boldsymbol{H}$, $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/(2\Gamma)$, up to corrections quadratic in $\sin\phi_{12}$.

2.2 Time-dependent *CP* violation

The D^0 and $\overline{D}^0$ decay amplitudes into the *CP*-conjugate final states f and $\bar{f}$ are denoted as

$$\begin{aligned} A_f &\equiv \langle f|\mathcal{H}|D^0\rangle, \quad \bar{A}_f \equiv \langle f|\mathcal{H}|\overline{D}^0\rangle, \\ A_{\bar{f}} &\equiv \langle \bar{f}|\mathcal{H}|D^0\rangle, \quad \bar{A}_{\bar{f}} \equiv \langle \bar{f}|\mathcal{H}|\overline{D}^0\rangle, \end{aligned} \qquad (2.6)$$

where $\mathcal{H}$ is the $|\Delta C| = 1$ effective Hamiltonian.

For CS decays into *CP* eigenstates, *CP* violation in the interference of the decay amplitudes with and without flavour oscillation is parametrised in terms of the following observables [27],

$$\begin{aligned} \lambda_f^M &\equiv \frac{M_{12}}{|M_{12}|}\frac{A_f}{\bar{A}_f} \equiv \eta_f^{CP}\left|\frac{A_f}{\bar{A}_f}\right|e^{i\phi_f^M}, \\ \lambda_f^\Gamma &\equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_f}{\bar{A}_f} \equiv \eta_f^{CP}\left|\frac{A_f}{\bar{A}_f}\right|e^{i\phi_f^\Gamma}, \end{aligned} \qquad (2.7)$$

where η_f^{CP} equals $+1$ for *CP*-even final states such as K^+K^- and $\pi^+\pi^-$, and -1 for *CP*-odd final states such as $K_S^0\phi$ and $K_S^0\omega$, and the weak phases ϕ_f^M and ϕ_f^Γ are quark- and meson-rephasing invariant and satisfy $\phi_{12} = \phi_f^M - \phi_f^\Gamma$.

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[2]Natural units are employed throughout, $\hbar = c = 1$.

The analogous parameters for the decays into the RS and WS final states $f = K^-\pi^+$ and $\bar{f} = K^+\pi^-$ are

$$
\lambda_f^M \equiv \frac{M_{12}}{|M_{12}|}\frac{A_f}{\bar{A}_f} \equiv -\left|\frac{A_f}{\bar{A}_f}\right|e^{i(\phi_f^M - \Delta_f)}, \quad \lambda_{\bar{f}}^M \equiv \frac{M_{12}}{|M_{12}|}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} \equiv -\left|\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}\right|e^{i(\phi_f^M + \Delta_f)},
$$
$$
\lambda_f^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_f}{\bar{A}_f} \equiv -\left|\frac{A_f}{\bar{A}_f}\right|e^{i(\phi_f^\Gamma - \Delta_f)}, \quad \lambda_{\bar{f}}^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} \equiv -\left|\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}\right|e^{i(\phi_f^\Gamma + \Delta_f)},
$$

$$(2.8)$$

where Δ_f, the strong-phase difference between the doubly Cabibbo-suppressed (DCS) and CF decay amplitudes, equals zero in the limit of U-spin symmetry,[3] and the weak phases ϕ_f^M and ϕ_f^Γ again satisfy $\phi_{12} = \phi_f^M - \phi_f^\Gamma$, but in general differ from those of eq. (2.7). Finally, the minus sign in the right-hand side of the definitions accounts for the overall minus sign of the CKM matrix elements involved in the tree-level decay amplitudes, with respect to CS decays.

The chosen conventions imply that all of the phases ϕ_f^M and ϕ_f^Γ in eqs. (2.7) and (2.8) equal the intrinsic dispersive and absorptive mixing phases ϕ_2^M and ϕ_2^Γ, defined as the phases of M_{12} and Γ_{12} with respect to their dominant $\Delta U = 2$ contributions (hence the subscript "2") [27], up to subleading corrections due to CP violation in the decay. These corrections depend on the final state, but are shared by the dispersive and absorptive weak phases: $\delta\phi_f \equiv \phi_f^M - \phi_2^M = \phi_f^\Gamma - \phi_2^\Gamma$. In the SM, the phases ϕ_2^M and ϕ_2^Γ are naively predicted to be of the order of $2\,\mathrm{mrad}$, even though enhancements up to one order of magnitude are not excluded [25–28]. An upper bound of $5\,\mathrm{mrad}$ on the size of ϕ_2^Γ has been proposed recently [29]. The final-state dependent corrections, $\delta\phi_f$, are smaller than 10^{-6} for CF and DCS decays, and are suppressed with respect to ϕ_2^M and ϕ_2^Γ by one order in the U-spin breaking parameter $\epsilon \sim 0.4$ for CS decays [27]. Since no evidence of time-dependent CP violation has been reported to date, they are neglected in the current combinations of experimental results to maximise the achieved precision [16, 27, 33]. Once an evidence is found, measurements of CS decays may be dropped from the combinations or replaced by the arithmetic average of the experimental observables measured in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays, which allows to reduce final-state dependent contributions by a further factor of ϵ [27].

3 Time-dependent decay rates

The time-dependent decay rates for D^0 and $\overline{D}^0$ mesons produced in their flavour eigenstates at time zero to decay into the final state f at time t, indicated with $\Gamma(D^0(t) \to f)$ and $\Gamma(\overline{D}^0(t) \to f)$, respectively, can be obtained by substituting the definitions above in eq. (50) of ref. [27]. Since both mixing parameters are smaller than 1% [16], the rates are expanded up to quadratic order in x_{12} and y_{12}. This approximation is expected to hold with excellent precision also at the LHCb and Belle II upgrades as, owing to the exponential decay of the rates, candidates with $\Gamma t \lesssim 10$ do not contribute significantly to the experimental precision

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[3]Note that this convention for the strong phase differs from those adopted in refs. [33] and [16], respectively: $\Delta_{K^-\pi^+} = -\delta_{K\pi} = -\delta_D^{K\pi} - \pi$.

and are often discarded to ease the estimation of systematic uncertainties [13, 15, 22]. In the following, decay time is expressed in D^0-lifetime units, $\tau \equiv \Gamma t$, to simplify the notation. For the same reason, the normalisation factors from phase-space integration, which are common to D^0 and $\overline{D}^0$ decay rates for each final state f and are additionally equal for the $K^-\pi^+$ and $K^+\pi^-$ final states, are omitted.

3.1 Cabibbo-suppressed decays

For CS final states f that are CP eigenstates, the time-dependent decay rates are parametrised up to second order in the mixing parameters as

$$
\begin{aligned}
\Gamma(D^0(\tau) \to f) &\equiv e^{-\tau}|A_f|^2 \left(1 + c_f^+ \tau + c_f'^+ \tau^2\right), \\
\Gamma(\overline{D}^0(\tau) \to f) &\equiv e^{-\tau}|\bar{A}_f|^2 \left(1 + c_f^- \tau + c_f'^- \tau^2\right),
\end{aligned}
\tag{3.1}
$$

where the coefficients $c_f^\pm$ and $c_f'^\pm$ are equal to

$$
c_f^\pm \approx \eta_f^{CP} \left[\mp x_{12}\sin\phi_f^M - y_{12}\cos\phi_f^\Gamma(1 \mp a_f^d)\right],
\tag{3.2a}
$$

$$
c_f'^\pm \approx \frac{1}{2}\left[y_{12}^2 \pm x_{12}y_{12}\sin\phi_{12} \mp (x_{12}^2 + y_{12}^2)a_f^d\right].
\tag{3.2b}
$$

In eq. (3.2), the parameter

$$
a_f^d \equiv \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2} \approx 1 - \left|\frac{\bar{A}_f}{A_f}\right|
\tag{3.3}
$$

is the CP asymmetry in the decay, and terms proportional to a_f^d have been expanded to first order in the CP-violation parameters a_f^d, $\sin\phi_f^M$ and $\sin\phi_f^\Gamma$.

The following CP-even and CP-odd combinations of c_f^+ and c_f^- are more convenient as experimental observables,

$$
y_{CP}^f \equiv -\frac{c_f^+ + c_f^-}{2} \approx \eta_f^{CP} y_{12}\cos\phi_f^\Gamma,
\tag{3.4a}
$$

$$
\Delta Y_f \equiv \frac{c_f^+ - c_f^-}{2} \approx \eta_f^{CP}(-x_{12}\sin\phi_f^M + y_{12}a_f^d).
\tag{3.4b}
$$

In the limit of CP symmetry, the parameter y_{CP}^f equals y_{12} ($-y_{12}$) for CP-even (CP-odd) final states, and ΔY_f equals zero.

3.2 Wrong-sign and right-sign decays

In this section, f ($\bar{f}$) denotes the RS final state $K^-\pi^+$ (the WS final state $K^+\pi^-$), and the notation follows ref. [34] rather than ref. [27]. The ratios of the DCS to CF branching fractions of D^0 and $\overline{D}^0$ mesons are denoted as

$$
R_f^+ \equiv |A_{\bar{f}}/A_f|^2, \qquad R_f^- \equiv |\bar{A}_f/\bar{A}_{\bar{f}}|^2;
\tag{3.5}
$$

their average is denoted as

$$
R_f \equiv \frac{R_f^+ + R_f^-}{2}
\tag{3.6}
$$

and is equal to $(3.43 \pm 0.02) \times 10^{-3}$ [16]. Finally, the *CP* asymmetry in the decay for CF decays is denoted as

$$a_f^d \equiv \frac{|A_f|^2 - |\bar{A}_{\bar{f}}|^2}{|A_f|^2 + |\bar{A}_{\bar{f}}|^2} \approx 1 - \left| \frac{\bar{A}_{\bar{f}}}{A_f} \right|, \qquad (3.7)$$

and its analogue for DCS decays as

$$a_{\bar{f}}^d \equiv \frac{|A_{\bar{f}}|^2 - |\bar{A}_f|^2}{|A_{\bar{f}}|^2 + |\bar{A}_f|^2} \approx 1 - \left| \frac{\bar{A}_f}{A_{\bar{f}}} \right|. \qquad (3.8)$$

Both of these asymmetries are expected to be beyond experimental reach in the SM [1].[4]

The time-dependent decay rates of RS decays are parametrised as

$$\Gamma(D^0(\tau) \to f) \equiv e^{-\tau}|A_f|^2 \left(1 + \sqrt{R_f}c_f^+\tau + c_f'^+\tau^2 \right),$$
$$\Gamma(\bar{D}^0(\tau) \to \bar{f}) \equiv e^{-\tau}|\bar{A}_{\bar{f}}|^2 \left(1 + \sqrt{R_f}c_f^-\tau + c_f'^-\tau^2 \right), \qquad (3.9)$$

up to second order in the mixing parameters, where the coefficients $c_f^{\pm}$ and $c_f'^{\pm}$ are equal to

$$c_f^{\pm} \approx \left[1 \mp \frac{1}{2}(a_{\bar{f}}^d + a_f^d) \right] (-x_{12}\cos\phi_f^M \sin\Delta_f + y_{12}\cos\phi_f^\Gamma \cos\Delta_f)$$
$$\pm x_{12}\sin\phi_f^M \cos\Delta_f \pm y_{12}\sin\phi_f^\Gamma \sin\Delta_f, \qquad (3.10)$$
$$c_f'^{\pm} \approx \frac{1}{4}(y_{12}^2 - x_{12}^2) + \frac{1}{4}R_f(1 \mp a_{\bar{f}}^d \mp a_f^d)(x_{12}^2 + y_{12}^2) \pm \frac{1}{2}R_f x_{12}y_{12}\sin\phi_{12}.$$

In eq. (3.10), terms multiplying a_f^d or $a_{\bar{f}}^d$ have been expanded to first order in the *CP*-violation parameters a_f^d, $a_{\bar{f}}^d$, $\sin\phi_f^M$ and $\sin\phi_f^\Gamma$.

Employing the same approximations as above, the time-dependent decay rates of WS decays are parametrised as

$$\Gamma(D^0(\tau) \to \bar{f}) \equiv e^{-\tau}|A_f|^2 \left(R_f^+ + \sqrt{R_f^+}c_{\bar{f}}^+\tau + c_{\bar{f}}'^+\tau^2 \right),$$
$$\Gamma(\bar{D}^0(\tau) \to f) \equiv e^{-\tau}|\bar{A}_{\bar{f}}|^2 \left(R_f^- + \sqrt{R_f^-}c_{\bar{f}}^-\tau + c_{\bar{f}}'^-\tau^2 \right), \qquad (3.11)$$

where the coefficients $c_{\bar{f}}^{\pm}$ and $c_{\bar{f}}'^{\pm}$ are approximately equal to

$$c_{\bar{f}}^{\pm} \approx (1 \mp a_f^d)(x_{12}\cos\phi_f^M \sin\Delta_f + y_{12}\cos\phi_f^\Gamma \cos\Delta_f)$$
$$\pm x_{12}\sin\phi_f^M \cos\Delta_f \mp y_{12}\sin\phi_f^\Gamma \sin\Delta_f, \qquad (3.12)$$
$$c_{\bar{f}}'^{\pm} \approx \frac{1}{4}[(x_{12}^2 + y_{12}^2)(1 \mp 2a_f^d) \pm 2x_{12}y_{12}\sin\phi_{12}] + \frac{1}{4}R_f^{\pm}(y_{12}^2 - x_{12}^2).$$

The choice of the parametrisation in eqs. (3.9) and (3.11) ensures that the coefficients $c_{f,\bar{f}}^{\pm}$ and $c_{f,\bar{f}}'^{\pm}$ are of similar size to those in eq. (3.1). The suppression (enhancement) of

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[4]As a consequence, the following equality is expected to hold, $R_f^+ = R_f^- = R_f$. Allowing for *CP* violation in the decay, instead, these parameters are related as $R_f^{\pm} \approx R_f[1 \pm (a_{\bar{f}}^d - a_f^d)]$ up to first order in $a_{\bar{f}}^d$ and a_f^d. The parameter $A_D \equiv (R_f^+ - R_f^-)/(R_f^+ + R_f^-)$ employed in the fits performed by the HFLAV collaboration [33] is equal to $a_{\bar{f}}^d - a_f^d$ up to first order in $a_{\bar{f}}^d$ and a_f^d.

the terms linear (and quadratic) in the mixing parameters with respect to the constant ones for RS (WS) decays, due to the different size of the CF and DCS decay amplitudes contributing to the decays without and following mixing (or vice versa), are taken into account by the $\sqrt{R_f^{(\pm)}}$ factors. These factors equal unity up to corrections smaller than 10^{-3} for CS decays, where the same amplitude contributes to both the decays without and following mixing in the limit of no *CP* violation.

Contrary to CS decays, where the only strong-phase difference between the amplitudes without and following mixing is given by the imaginary unit that multiplies $M_{12}^{(*)}$ in the solution of eq. (2.2), WS and RS decays receive a nontrivial contribution also from the strong-phase difference, Δ_f, between the DCS and CF decay amplitudes. Thus, WS and RS decays are sensitive to the weak phase ϕ_f^Γ through sine functions, unlike for CS decays, which are sensitive only to $\sin\phi_f^M$ and to $\cos\phi_f^\Gamma$. However, this sensitivity is limited by smallness of Δ_f, which is equal to $-0.17 \pm 0.07\,\mathrm{rad}$ [16].

In analogy with eqs. (3.4a) and (3.4b), the following parameters are defined for RS decays,

$$y_{CP}^f \equiv -\sqrt{R_f}\,\frac{c_f^+ + c_f^-}{2} \tag{3.13a}$$
$$\approx \sqrt{R_f}\left(x_{12}\cos\phi_f^M\sin\Delta_f - y_{12}\cos\phi_f^\Gamma\cos\Delta_f\right),$$

$$\Delta Y_f \equiv \sqrt{R_f}\,\frac{c_f^+ - c_f^-}{2}$$
$$\approx \sqrt{R_f}\left[x_{12}\sin\phi_f^M\cos\Delta_f + y_{12}\sin\phi_f^\Gamma\sin\Delta_f + \frac{1}{2}(a_f^d + a_f^d)(x_{12}\sin\Delta_f - y_{12}\cos\Delta_f)\right]. \tag{3.13b}$$

In the limit of no *CP* violation in the decay and of small strong phase Δ_f, these parameters are equal to the negative of the parameters y_{CP}^f and ΔY_f of CS decays, multiplied by a suppression factor of $\sqrt{R_f}$.

Analogous parameters are defined also for untagged decays, for which the D^0 flavour at production is unknown and the final states f and $\bar{f}$ are analysed independently of the meson from which they originate,

$$y_{CP}^{K\pi,\mathrm{untag}} \equiv -\frac{1}{1+R_f}\left(\sqrt{R_f}\,\frac{c_f^+ + c_f^-}{2} + \frac{\sqrt{R_f^+}\,c_{\bar{f}}^+ + \sqrt{R_f^-}\,c_{\bar{f}}^-}{2}\right) \tag{3.14a}$$

$$\approx -\frac{2\sqrt{R_f}}{1+R_f}\,y_{12}\cos\phi_f^\Gamma\cos\Delta_f,$$

$$\Delta Y_{K\pi}^{\mathrm{untag}} \equiv \frac{\sqrt{R_f}\,c_f^+ + \sqrt{R_f^-}\,c_{\bar{f}}^- - \sqrt{R_f}\,c_f^- - \sqrt{R_f^+}\,c_{\bar{f}}^+}{2(1+R_f)} \tag{3.14b}$$

$$\approx \frac{\sqrt{R_f}}{1+R_f}\,y_{12}\left[2\sin\phi_f^\Gamma\sin\Delta_f + (a_{\bar{f}}^d - a_f^d)\cos\Delta_f\right].$$

In the limit of no CP violation in the decay, ΔY_f and $\Delta Y_{K\pi}$ are equal to the time-integrated asymmetries of eqs. (A1) and (A2) of ref. [27].[5]

4 Experimental methods

The following sections discuss how the observables introduced so far can be measured.

4.1 Cabibbo-suppressed decays

The $c_f^{\pm}$ coefficients in eq. (3.2a) have been measured at the B factories by modelling the CS decay rates with an exponential function [4, 8],

$$
\begin{aligned}
\Gamma(D^0(\tau) \to f) &= |A_f|^2 \exp(-\hat{\Gamma}_{D^0 \to f}\tau), \\
\Gamma(\overline{D}^0(\tau) \to f) &= |\bar{A}_f|^2 \exp(-\hat{\Gamma}_{\overline{D}^0 \to f}\tau).
\end{aligned}
\tag{4.1}
$$

Neglecting terms quadratic in the mixing parameters, the effective decay widths defined in eq. (4.1) satisfy

$$
\hat{\Gamma}_{D^0/\overline{D}^0 \to f} \approx 1 - c_f^{\pm}.
\tag{4.2}
$$

Under this approximation, the observables y_{CP}^f and ΔY_f defined in eqs. (3.4a) and (3.4b) can be measured as

$$
y_{CP}^f \approx \frac{\hat{\Gamma}_{D^0 \to f} + \hat{\Gamma}_{\overline{D}^0 \to f}}{2} - 1,
\tag{4.3a}
$$

$$
\Delta Y_f \approx \frac{\hat{\Gamma}_{\overline{D}^0 \to f} - \hat{\Gamma}_{D^0 \to f}}{2}.
\tag{4.3b}
$$

Precision measurements of the effective decay widths require measuring precisely both the decay time and the decay width Γ, as they enter their definition in eq. (4.1) through τ. Relative errors on the measurement of decay time or Γ provoke relative errors of the same size on the measurement of the asymmetry parameter ΔY_f (unless the bias on the measurement of decay time differs for D^0 and $\overline{D}^0$ mesons), and thus affect it only marginally, as its value is still compatible with zero. On the contrary, they are added to the value of y_{CP}^f, which quantifies deviations of the CP-averaged effective decay width from unity. Therefore, they can cause large biases on this small-sized parameter. To reduce the size of possible biases, determinations of ΔY_f and y_{CP}^f based on the measurement of the effective decay widths are usually normalised to the measurement of the effective decay width of RS decays, whose deviation from unity is smaller approximately by a factor of $\sqrt{R_{K^-\pi^+}} = (5.87 \pm 0.02)\%$ [16] with respect to that of CS decays:

$$
\hat{\Gamma}_{D^0 \to K^-\pi^+/\overline{D}^0 \to K^+\pi^-} \approx 1 - \sqrt{R_{K^-\pi^+}}\, c_{K^-\pi^+}^{\pm}.
\tag{4.4}
$$

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[5]The same formulas had been previously presented in the phenomenological parametrisation in section V.A of ref. [30].

The eqs. (4.3a) and (4.3b) are modified accordingly, yielding

$$y_{CP}^f - y_{CP}^{K^-\pi^+} \approx \frac{\hat{\Gamma}_{D^0 \to f} + \hat{\Gamma}_{\overline{D}^0 \to f}}{\hat{\Gamma}_{D^0 \to K^-\pi^+} + \hat{\Gamma}_{\overline{D}^0 \to K^+\pi^-}} - 1, \tag{4.5a}$$

$$\frac{\Delta Y_f}{1 + y_{CP}^{K^-\pi^+}} \approx \frac{\hat{\Gamma}_{\overline{D}^0 \to f} - \hat{\Gamma}_{D^0 \to f}}{\hat{\Gamma}_{D^0 \to K^-\pi^+} + \hat{\Gamma}_{\overline{D}^0 \to K^+\pi^-}}, \tag{4.5b}$$

where the dependence on Γ and on the time scale cancels out in the ratio of the effective decay widths. A similar cancellation has been exploited by the LHCb and Belle collaborations, which measured the asymmetry of the effective lifetimes of D^0 and $\overline{D}^0$ mesons [8, 35],

$$A_\Gamma^f \equiv \frac{\hat{\Gamma}_{D^0 \to f} - \hat{\Gamma}_{\overline{D}^0 \to f}}{\hat{\Gamma}_{D^0 \to f} + \hat{\Gamma}_{\overline{D}^0 \to f}} \approx -\frac{\Delta Y_f}{1 + y_{CP}^f}. \tag{4.6}$$

This observable differs from that of eq. (4.5b) for the different calibration channel at denominator as well as for an overall minus sign.

Given the smallness of y_{CP}^f [33], the observable A_Γ^f is equal to the negative of ΔY_f within 1% precision. The observable defined in eq. (4.5b) is equal to ΔY_f with even better precision, as $y_{CP}^{K^-\pi^+}$ is smaller than y_{CP}^f approximately by a factor of $\sqrt{R_{K^-\pi^+}}$. On the contrary, the approximation that $\hat{\Gamma}_{D^0 \to K^-\pi^+/\overline{D}^0 \to K^+\pi^-}$ is equal to unity cannot be used effectively in the measurement of y_{CP}^f through eq. (4.5a), since the contribution from $y_{CP}^{K^-\pi^+}$ in the left-hand side yields relative corrections to y_{CP}^f of the order of $\sqrt{R_{K^-\pi^+}} \approx 6\%$ [16].

In recent years, alternative methods have been preferred to measure ΔY_f and y_{CP}^f. Measuring the effective decay widths from the decay time distributions requires knowing with high precision the reconstruction efficiency as a function of decay time, even when a normalisation channel such as $D^0 \to K^-\pi^+$ is available. This task is challenging in case of stringent trigger requirements on D^0 displacement-related variables, such as those set at hadron colliders. However, it can be mitigated by measuring the parameter ΔY_f from the slope of the time-dependent asymmetry of the decay rates of D^0 and $\overline{D}^0$ mesons into the final state f [18–22],

$$A_{CP}(f, \tau) \equiv \frac{\Gamma(D^0(\tau) \to f) - \Gamma(\overline{D}^0(\tau) \to f)}{\Gamma(D^0(\tau) \to f) + \Gamma(\overline{D}^0(\tau) \to f)} \approx a_f^d + \Delta Y_f \tau, \tag{4.7}$$

where terms quadratic in the mixing parameters or in a_f^d have been neglected. The average reconstruction efficiency of D^0 and $\overline{D}^0$ decays cancels out in the ratio, and only charge-dependent efficiency effects must be corrected for. In addition, the CP-even terms quadratic in the mixing parameters in eq. (3.2b) cancel out in the numerator and do not bias the results, while the CP-odd quadratic terms are negligible within the current precision as well as that attainable at the LHCb upgrades and at the Belle II experiment, as shown in appendix A. The measurements based on the effective decay widths do not benefit from this feature: the CP-even terms of the $c_f'^\pm$ coefficients in eq. (3.2b) can be of the same order of the CP-odd terms of the $c_f^\pm$ coefficients in eq. (3.2a), thus limiting the validity of the assumption in eq. (4.2).

The y_{CP}^f parameter has analogously been measured from the time-dependent ratio of the decay rates of CS to RS decays [13],

$$\frac{\Gamma(D^0(\tau) \to f) + \Gamma(\overline{D}^0(\tau) \to f)}{\Gamma(D^0(\tau) \to K^-\pi^+) + \Gamma(\overline{D}^0(\tau) \to K^+\pi^-)} \approx \text{const.} \times \left\{ 1 - (y_{CP}^f - y_{CP}^{K^-\pi^+})\tau \right.$$
$$\left. + \left[\frac{1}{4}(x_{12}^2 + y_{12}^2)(1 - R_{K^-\pi^+}) + y_{CP}^{K^-\pi^+}(y_{CP}^{K^-\pi^+} - y_{CP}^f) \right] \tau^2 \right\},$$
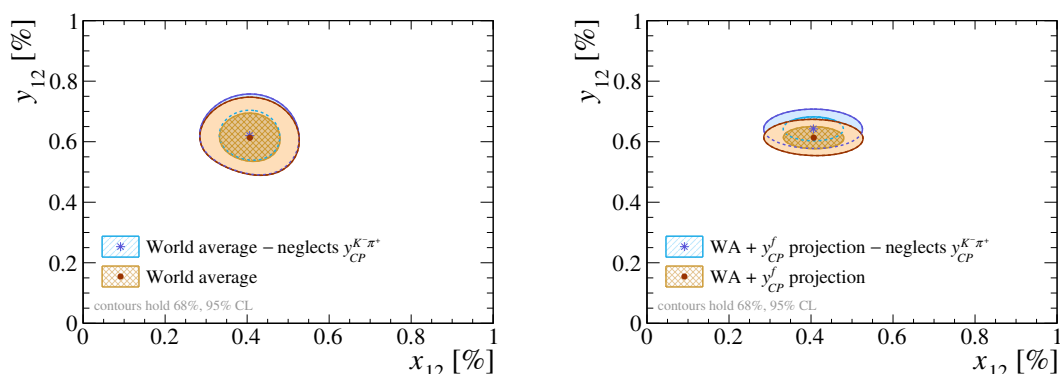$$(4.8)$$

where terms cubic in the mixing parameters have been neglected and terms proportional to a_f^d and $a_{\bar{f}}^d$ have been expanded to first order in the *CP*-violation parameters a_f^d, $a_{\bar{f}}^d$, $\sin \phi_f^M$ and $\sin \phi_f^\Gamma$. The negative of the slope differs from y_{CP}^f by the same quantity, $-y_{CP}^{K^-\pi^+}$, as the ratio of the effective decay widths in eq. (4.5a), and cannot be neglected. The term quadratic in the mixing parameters is negligible at the current level of experimental precision, but will become relevant at large values of τ in the expected measurements employing the LHCb Run 2 data sample (2015–2018) and beyond. When this happens, approximating the time evolution of the ratio with an exponential, as done in ref. [13], might provoke biases to the measurement, as the quadratic term is independent of the linear one, though accidentally its numerical value is not so different from that expected from the expansion of the exponential, $\frac{1}{2}(y_{CP}^f - y_{CP}^{K^-\pi^+})^2$ [16].

The term $y_{CP}^{K^-\pi^+}$ in the left-hand side of eq. (4.5a) and in the linear term on the right-hand side of eq. (4.8) has been neglected in all combinations of charm mixing and *CP*-violation measurements [27, 33] except for that in ref. [16], but amounts to about 40% of the precision of the current world average of $y_{CP} = (7.19 \pm 1.13) \times 10^{-3}$ [33], where the convention $\eta_f^{CP} = 1$ is employed and measurements of *CP*-odd final states have been multiplied by -1. Since the average is driven by measurements of *CP*-even final states, it is biased towards larger values. While its current value contributes marginally to the results of the combinations, the statistical uncertainty attainable with the data collected during the Run 2 of the LHCb experiment is smaller by a factor of 5. When this precision is achieved, correcting the expressions used in the fits according to eqs. (4.5a) and (4.8) will be indispensable to avoid large biases, as shown in figure 1.

4.2 Wrong-sign decays

In the measurements of WS decays into the final state $\bar{f} = K^+\pi^-$, modelling the exponential factor $e^{-\tau}$ in the decay rates of eq. (3.11), as well as the time dependence of the efficiency due to trigger and reconstruction effects, the *D*-meson production asymmetry and most of the detection asymmetries, is usually avoided by analysing the ratio of the WS to RS decay rates [2, 5, 6, 11, 12],

$$\frac{\Gamma(D^0(\tau) \to \bar{f})}{\Gamma(D^0(\tau) \to f)} \approx R_f^+ + \sqrt{R_f^+} c_{\bar{f}}^+ \tau + c_{\bar{f}}'^+ \tau^2,$$
$$\frac{\Gamma(\overline{D}^0(\tau) \to f)}{\Gamma(\overline{D}^0(\tau) \to \bar{f})} \approx R_f^- + \sqrt{R_f^-} c_{\bar{f}}^- \tau + c_{\bar{f}}'^- \tau^2.$$
$$(4.9)$$

Figure 1. Impact of neglecting the $y_{CP}^{K^-\pi^+}$ terms in eqs. (4.5a) and (4.8) on the world average of the mixing parameters, (left) as of today and (right) after including a new measurement of y_{CP}^f, where f equals K^+K^- or $\pi^+\pi^-$, with the precision achievable with the $D^*(2010)^+$-tagged data sample collected by the LHCb experiment during its Run 2 data taking (2015–2018). In the right plot, the value of y_{CP}^f is fixed to its point estimate based on the results of the fit in ref. [33], and its uncertainty is set to 0.3×10^{-4}. Measurements of the γ angle of the CKM unitary triangle relying on two-body D^0 decays, which improve the precision on the strong-phase $\Delta_{K^-\pi^+}$ and indirectly on the current world average of y_{12} by around 40% [16], are not included for simplicity. The code employed to perform the fits is available on GitHub [36].

Here, the contribution of RS decays to the linear and quadratic terms is suppressed by a relative factor of R_f and is neglected. It might become relevant at the future upgrades of the LHCb experiment, but only for the *CP*-even observables, as detailed in appendix A.

An alternative approach consists in taking the ratio of the WS decay rates for a given D^0 initial flavour, over RS decay rates from D^0 mesons with opposite initial flavour. This new observable, $\Gamma(D^0(\tau) \to \bar{f})/\Gamma(\overline{D}^0(\tau) \to \bar{f})$, is indistinguishable from that in eq. (4.9) up to a multiplicative factor of $(1 \pm a_f^d)/(1 \mp a_f^d) \approx 1 \pm 2a_f^d$, where the top (bottom) sign applies to WS decays of D^0 ($\overline{D}^0$) mesons, but presents different experimental advantages and challenges. The approximate cancellation in the ratio of the production asymmetry and of the detection asymmetry of the accompanying particles employed to infer the D^0 flavour at production (typically a π^+ meson from a $D^*(2010)^+ \to D^0\pi^+$ decay or a muon from a $\overline{B} \to D^0\mu^- X$ decay, where X stands for an arbitrary number of unreconstructed particles) does not take place any longer. On the other hand, the detection asymmetry of the $K^{\pm}\pi^{\mp}$ final state approximately cancels out in the ratio, whereas it needs to be corrected for when using the standard observable of eq. (4.9). Depending on the strategy adopted to cancel the asymmetries, one or other of the two observables could be more convenient.

4.3 Right-sign decays

All aforementioned measurements employ RS $D^0 \to f$ decays, where f stands for $K^-\pi^+$, merely as a normalisation channel, whose decay width is used as a proxy for Γ. However, RS decays are sensitive to *CP* violation in the mixing, although with lower sensitivity with respect to CS and WS decays. A convenient observable to measure this effect is the

time-dependent asymmetry between the D^0 and $\overline{D}^0$ RS decay rates,

$$A_{CP}(f,\tau) \equiv \frac{\Gamma(D^0(\tau) \to f) - \Gamma(\overline{D}^0(\tau) \to \bar{f})}{\Gamma(D^0(\tau) \to f) + \Gamma(\overline{D}^0(\tau) \to \bar{f})} \approx a_f^d + \Delta Y_f \tau, \qquad (4.10)$$

where terms quadratic in the mixing parameters and in a_f^d have been neglected.

The relative experimental sensitivity to CP violation in the mixing of measurements of $\Delta Y_{K^- \pi^+}$ and $\Delta Y_{\rm CS}$ is estimated as follows. The sensitivity of $\Delta Y_{K^- \pi^+}$ to this phenomenon is suppressed roughly by a factor of $\sqrt{R_{K^- \pi^+}}$ with respect to $\Delta Y_{\rm CS}$; see eqs. (3.4b) and (3.13b). On the other hand, the branching fraction of $D^0 \to K^- \pi^+$ decays is larger by a factor of about 9.7 (27.1) than that of $D^0 \to K^+ K^-$ ($D^0 \to \pi^+ \pi^-$) decays [37]. Since the precision of the measured asymmetries is proportional to $N^{-1/2}$, where N is the number of reconstructed decays, the statistical uncertainty achievable for $\Delta Y_{K^- \pi^+}$ is around 37% of that of the weighted average of $\Delta Y_{K^+ K^-}$ and $\Delta Y_{\pi^+ \pi^-}$, where the reconstruction efficiency and background contamination are assumed to be the same for all three decay channels. This estimate is confirmed by the latest measurements of $\Delta Y_{K^+ K^-/\pi^+ \pi^-}$, where the compatibility of the measurement of $\Delta Y_{K^- \pi^+}$ with zero is used to cross-check the effectiveness of the analysis procedure in removing nuisance asymmetries [18–22]. The relative sensitivity to CP violation in the mixing achievable with $D^0 \to K^- \pi^+$ with respect to $D^0 \to K^+ K^-/\pi^+ \pi^-$ decays is thus $\sqrt{R_{K^- \pi^+}}/37\% \approx 16\%$. While currently it is not competitive with that of CS decays, it might become of interest once a nonzero value of $\Delta Y_{K^+ K^-/\pi^+ \pi^-}$ is measured, since it provides cleaner access to the mixing phase ϕ_2^M [27].

The measurement of $\Delta Y_{K^- \pi^+}$ is effectively uncorrelated with that of the WS-to-RS ratio, in which CP-violating contributions from RS decays are expected to be negligible at all foreseen upgrades of the LHCb and Belle II experiments; see appendix A. Therefore, measuring the WS-to-RS ratio and the $\Delta Y_{K^- \pi^+}$ parameter is equivalent to measuring the time-integrated asymmetries of WS and RS decays, as proposed in section V.A of ref. [30] and in appendix A of ref. [27]. However, it is more convenient experimentally, since the former observables are by construction less sensitive to time-independent nuisance asymmetries. Subleading nuisance asymmetries can be corrected for by relying on control channels for the WS-to-RS ratio [11, 12], and by building on the methods developed in refs. [20, 22, 38] for the $\Delta Y_{K^- \pi^+}$ observable. Neither of these approaches degrades significantly the statistical uncertainty of the measurement, contrary to the more complex corrections needed to measure time-integrated asymmetries; see for example ref. [39].

A direct consequence of the considerations above is that the compatibility of the measurement of $\Delta Y_{K^- \pi^+}$ with zero will not be utilisable any longer to check the effectiveness of the analysis method in the measurements of $\Delta Y_{K^+ K^-}$ and $\Delta Y_{\pi^+ \pi^-}$, once these are measured to differ significantly from zero. This problem might be bypassed by calculating the confidence interval for $\Delta Y_{K^- \pi^+}$ based on the previous measurements of $\Delta Y_{K^+ K^-}$, $\Delta Y_{\pi^+ \pi^-}$ and other available charm measurements, which allow to determine the CP-violation parameters ϕ_2^M and ϕ_2^Γ with better precision.[6] Issues may arise if the size of the analysed

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[6]This relies on the assumption that the shifts of ϕ_f^M and ϕ_f^Γ with respect to ϕ_2^M and ϕ_2^Γ are small. While this condition is not necessarily verified for the $K^+ K^-$ and $\pi^+ \pi^-$ final states, the size of the final-state dependent shifts can be reduced by utilising the arithmetic average of $\Delta Y_{K^+ K^-}$ and $\Delta Y_{\pi^+ \pi^-}$; see ref. [27].

sample increases considerably from one measurement to the subsequent, such that the precision achieved with the new sample of RS decays is comparable to that of the measurements performed with the previous data samples of CS and WS decays. This would require an increase of the data sample size by a factor of approximately $(16\%)^{-2} \approx 40$, where 16% is the relative sensitivity of RS and CS measurements at equal number of produced D^0 mesons, as estimated above.

4.4 Untagged decays

The *CP* asymmetry of untagged *D* decays into the final state $f = K^-\pi^+$ is defined, in analogy with eq. (4.10), as

$$
\begin{aligned}
A_{CP}^{\text{untag}}(f, \tau) &\equiv \frac{[\Gamma(D^0(\tau) \to f) + \Gamma(\overline{D}^0(\tau) \to f)] - [\Gamma(\overline{D}^0(\tau) \to \bar{f}) + \Gamma(D^0(\tau) \to \bar{f})]}{[\Gamma(D^0(\tau) \to f) + \Gamma(\overline{D}^0(\tau) \to f)] + [\Gamma(\overline{D}^0(\tau) \to \bar{f}) + \Gamma(D^0(\tau) \to \bar{f})]} \\
&\approx a_{K\pi}^{d,\text{untag}} + \Delta Y_{K\pi}^{\text{untag}} \tau,
\end{aligned}
\tag{4.11}
$$

where the *CP* asymmetry in the decay is equal to

$$
a_{K\pi}^{d,\text{untag}} \approx a_f^d - R_f a_{\bar{f}}^d,
\tag{4.12}
$$

$\Delta Y_{K\pi}^{\text{untag}}$ has been defined in eq. (3.14b), and terms quadratic in the mixing parameters or in the *CP* asymmetries a_f^d and $a_{\bar{f}}^d$ have been neglected.

These parameters can be measured with better precision than their analogues for RS decays. The LHCb experiment currently provides the largest yields of D^0 decays into two charged hadrons, with the largest yields coming from mesons produced promptly in proton-proton collisions. Since the production cross-section of D^0 mesons is larger than that of $D^*(2010)^+$ mesons by a factor of 2.6 [40, 41], the branching fraction of the $D^*(2010)^+$ meson into the $D^0\pi^+$ final state is 68% [37], and the reconstruction efficiency of the low-momentum pion from the $D^*(2010)^+$ decay is relatively low, the ratio of the yields of untagged and tagged decays is close to 5 [42].

Measuring $a_{K\pi}^{d,\text{untag}}$ is very challenging due to the need to determine the D^0 production asymmetry and to correct for the detection asymmetry of the $K^-\pi^+$ final state. On the other hand, the slope $\Delta Y_{K\pi}^{\text{untag}}$ might be measured analogously to $\Delta Y_{K^-\pi^+}$, by building on the methods developed in refs. [20, 22, 38] to correct for the time-dependent component of the nuisance asymmetries. Since the tagged sample is a subset of the untagged sample, the measurement of $\Delta Y_{K\pi}^{\text{untag}}$ would be correlated with those of the WS-to-RS ratio and of $\Delta Y_{K^-\pi^+}$. However, the correlation coefficient can be measured and is expected to be small due to the very different size of the two samples. While the precision achievable for $\Delta Y_{K\pi}^{\text{untag}}$ is better by a factor of $\sqrt{5} \approx 2.2$ with respect to that of $\Delta Y_{K^-\pi^+}$, where 5 is the yields ratio estimated above, the sensitivity of $\Delta Y_{K\pi}^{\text{untag}}$ to ϕ_f^Γ is suppressed by a factor of $2(y_{12}/x_{12}) \tan \Delta_f \approx 0.54 \pm 0.26$ [16] with respect to the sensitivity of $\Delta Y_{K^-\pi^+}$ to ϕ_f^M; see eqs. (3.13b) and (3.14b). The larger combinatorial background of untagged decays, which is largely rejected in tagged decays thanks to the low Q value of the $D^*(2010)^+$ decay, might decrease the relative precision between the two samples, but all in all RS and untagged decays might provide similar sensitivity to the two weak mixing phases.

Finally, untagged $D \to K^+K^-/\pi^+\pi^-$ and $D \to K\pi$ decays could be used also to measure $y_{CP} - y_{CP}^{K\pi,\text{untag}}$ with the same methods detailed in section 4.1. Again, this would allow to collect larger yields and to avoid the reconstruction effects of the low-momentum pion from the $D^*(2010)^+$ decay, at the cost of increased combinatorial background. The shift of the measured observable from y_{CP} due to the normalisation $D \to K\pi$ channel would be around twice as large as that in eq. (4.8); see eqs. (3.13a) and (3.14a).

5 Summary

The theoretical formalism to describe mixing and CP violation in D^0 decays into two charged hadrons (K^+K^-, $\pi^+\pi^-$, $K^+\pi^-$ or $K^-\pi^+$) has been reviewed, along with the experimental methods to measure them. An analogous discussion employing the phenomenological parametrisation can be found in appendices A and B of ref. [34]. The impact of these phenomena on the RS decay rates, which has usually been neglected in the literature, has been quantified in sections 3.2 and 4.3.[7]

Mixing in RS decays has been shown to affect the measurements of the parameter y_{CP} that rely on RS decays as normalisation channel, shifting the results by $-y_{CP}^{K^-\pi^+}$, where the latter observable is defined in eq. (3.13a); see eqs. (4.5a) and (4.8). The size of the shift is about 40% of the uncertainty of the current world average. This effect has been missed in the literature and is not accounted for in the combinations of the time-dependent measurements of charm decays in refs. [27, 33]. Its inclusion in the next iterations of the combinations, as first done in ref. [16], will be crucial to avoid inconsistencies and biases in the fits results.

The y_{CP} parameter could be measured also by relying on untagged decays as in ref. [43], achieving larger yields at the cost of slightly increased background. The deviation of the observable, $y_{CP} - y_{CP}^{K\pi,\text{untag}}$, from y_{CP} would be larger than that of RS decays by around a factor of two; see eqs. (3.13a) and (3.14a).

The sensitivity of RS and untagged $D \to K^-\pi^+$ decays to the weak mixing phases has been estimated in sections 4.3 and 4.4. While it is worse than that achievable by analysing the K^+K^- and $\pi^+\pi^-$ final states by approximately a factor of 6, it might become interesting once time-dependent CP violation is observed. In fact, final-state dependent corrections to the weak mixing phases ϕ_2^M and ϕ_2^Γ are negligible for RS and WS decays, contrary to what happens for CS decays. The two decay channels would be complementary, as RS and $D \to K^-\pi^+$ decays mostly provide sensitivity to ϕ_2^M and ϕ_2^Γ, respectively, with similar experimental precision, and are nearly statistically independent.

Attention should be paid to the usage of $K^-\pi^+$ decays as a control channel in the measurements of the ΔY_f parameter for CS final states f, as the compatibility of $\Delta Y_{K^-\pi^+}$ with zero will no longer be guaranteed once ΔY_f is measured to differ significantly from zero, or if the size of the collected sample of D^0 decays into two charged hadrons in-

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[7]References [30] and [27] have discussed the effect of CP violation in section V.A and in appendix A, respectively. Here, their discussion has been extended by including possible contributions from CP violation in the decay in addition to those from CP violation in the mixing and in the interference between mixing and decay, and by employing a different parametrisation for RS decays.

creases significantly with respect to that employed in previous measurements, as shown in section 4.3.

Acknowledgments

A Subleading corrections

This appendix reports subleading corrections to eqs. (4.7) and (4.9), which might become relevant at the precision foreseen at the end of the LHCb Upgrade II data taking [23].

The contribution of mixing and *CP* violation in RS decays has been neglected in eq. (4.9) for the WS-to-RS ratios. The complete expressions up to quadratic order in the mixing parameters and to linear order in the *CP* asymmetries a_f^d and $a_{\bar{f}}^d$ are

$$
\frac{\Gamma(D^0(\tau) \to \bar{f})}{\Gamma(D^0(\tau) \to f)} \left[\frac{\Gamma(\overline{D}^0(\tau) \to f)}{\Gamma(\overline{D}^0(\tau) \to \bar{f})} \right] \approx R_f \left[1 \pm (a_{\bar{f}}^d - a_f^d) \right]
$$
$$
+ \sqrt{R_f} \left[x_{12} \cos \phi_f^M \sin \Delta_f \left(1 + R_f \pm \frac{a_{\bar{f}}^d - 3a_f^d}{2} \right) + y_{12} \cos \phi_f^\Gamma \cos \Delta_f \left(1 - R_f \pm \frac{a_{\bar{f}}^d - 3a_f^d}{2} \right) \right.
$$
$$
\left. \pm x_{12} \sin \phi_f^M \cos \Delta_f \mp y_{12} \sin \phi_f^\Gamma \sin \Delta_f \right] \tau
$$
$$
+ \left[\frac{1}{4}(x_{12}^2 + y_{12}^2)(1 \mp 2a_f^d) \pm \frac{1}{2} x_{12} y_{12}(\sin \phi_f^M - \sin \phi_f^\Gamma) \right] \tau^2,
$$
$$
\tag{A.1}
$$

where the following terms are additionally neglected: terms proportional to the *CP* asymmetries a_f^d and $a_{\bar{f}}^d$ and to either of the parameters $\sin \phi_f^M$, $\sin \phi_f^\Gamma$ or R_f; terms proportional to $\sin \phi_f^M$ or $\sin \phi_f^\Gamma$ and to R_f; and terms proportional to the second power of the mixing parameters and to R_f. This approximation will have a negligible impact on the theoretical interpretation of the observables even at the end of the LHCb Upgrade II, since its target relative precision on the terms linear and quadratic in decay time is about 2% and 0.5%, respectively [23].[8] Terms of third order in the mixing parameters will be equally negligible, as they are proportional to $\sqrt{R_f}$. The contribution of RS decays to the time dependence of the ratios corresponds to the terms proportional to R_f in the second line of eq. (A.1), and if neglected will cause a bias on the linear coefficient of the time-dependent ratio of the same order of the statistical precision targeted by the LHCb Upgrade II.

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[8]This estimate assumes that the correlation between the measured parameters will be the same as in past measurements [12].

The analogue of eq. (A.1) for the new experimental observable proposed in section 4.2 is just the same up to a multiplicative factor of $1+2a_f^d$ (of $1-2a_f^d$) for $\Gamma(D^0(\tau) \to \bar{f})/\Gamma(\overline{D}^0(\tau) \to \bar{f})$ (for $\Gamma(\overline{D}^0(\tau) \to f)/\Gamma(D^0(\tau) \to f)$), where the same approximations as above are employed.

The expansion up to second order in the mixing parameters of the time-dependent *CP* asymmetry defined in eq. (4.7) for D^0 decays into the CS final state f is

$$A_{CP}(f,\tau) \approx a_f^d + \Delta Y_f \tau + \frac{1}{2}\Big[x_{12}y_{12}(\sin\phi_f^M - \sin\phi_f^\Gamma) - (x_{12}^2 + y_{12}^2)a_f^d\Big]\tau^2, \qquad (A.2)$$

where terms multiplying a_f^d have been expanded to first order in the *CP*-violation parameters a_f^d, $\sin\phi_f^M$ and $\sin\phi_f^\Gamma$, and terms of order higher than two in $\sin\phi_f^M$ and $\sin\phi_f^\Gamma$ have been neglected. Since both mixing parameters are of the order of 5×10^{-3} [16, 33], a_f^d is of the order of 1×10^{-3} [17, 39] and the effective maximum value of τ tested at experiments is of the order of 6 [22], the term quadratic in the mixing parameters might become relevant only at large decay times and at the precision targeted for ΔY_f by the LHCb Upgrade II, 1×10^{-5} [23], and if the magnitude of ϕ_2^M or ϕ_2^Γ is of the same order of the current experimental limits, $\mathcal{O}(0.05\,\mathrm{rad})$ [16]. The latter condition is unlikely in the SM [27, 29].

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