

Metadata of the chapter that will be visualized in SpringerLink

Book Title	Essays in Economic Dynamics	
Series Title		
Chapter Title	A Stylized Model for Long-Run Index Return Dynamics	
Copyright Year	2016	
Copyright HolderName	Springer Science+Business Media Singapore	
Author	Family Name	Angelini
	Particle	
	Given Name	Natascia
	Prefix	
	Suffix	
	Division	School of Economics, Management and Statistics
	Organization	University of Bologna
	Address	Via Angherà 22, 40127, Rimini, Italy
	Email	natascia.angelini@unibo.it
Author	Family Name	Bormetti
	Particle	
	Given Name	Giacomo
	Prefix	
	Suffix	
	Division	Department of Mathematics
	Organization	University of Bologna
	Address	Viale Filopanti 5, 40126, Bologna, Italy
	Email	giacomo.bormetti@unibo.it
Author	Family Name	Marmi
	Particle	
	Given Name	Stefano
	Prefix	
	Suffix	
	Division	Scuola Normale Superiore and C.N.R.S. UMI 3483
	Organization	Laboratorio Fibonacci
	Address	Piazza Dei Cavalieri 7, 56126, Pisa, Italy
	Email	stefano.marmi@sns.it
Corresponding Author	Family Name	Nardini
	Particle	
	Given Name	Franco
	Prefix	
	Suffix	
	Division	Department of Mathematics
	Organization	University of Bologna
	Address	Viale Filopanti 5, 40126, Bologna, Italy

Email

franco.nardini@unibo.it

Abstract

We introduce a discrete-time model of stock index return dynamics grounded on the ability of Shiller's Cyclically Adjusted Price-to-Earning ratio to predict long-horizon market performances. Specifically, we discuss a model in which returns are driven by a fundamental term and an autoregressive component perturbed by external random disturbances. The autoregressive component arises from the agents' belief that expected returns are higher in bullish markets than in bearish markets. The fundamental term, driven by the value towards which fundamentalists expect the current price should revert, varies in time and depends on the initial averaged price-to-earnings ratio. The actual stock price may deviate from the perceived reference level as a combined effect of an idiosyncratic noise component and local trends due to trading strategies. We demonstrate both analytically and by means of numerical experiments that the long-run behavior of our stylized dynamics agrees with empirical evidences reported in literature.

Keywords
(separated by '-')

Fundamental value - Shiller's cape

A Stylized Model for Long-Run Index Return Dynamics

Natascia Angelini, Giacomo Bormetti, Stefano Marmi and Franco Nardini

1 **Abstract** We introduce a discrete-time model of stock index return dynamics
 2 grounded on the ability of Shiller's Cyclically Adjusted Price-to-Earning ratio to pre-
 3 dict long-horizon market performances. Specifically, we discuss a model in which
 4 returns are driven by a fundamental term and an autoregressive component per-
 5 turbed by external random disturbances. The autoregressive component arises from
 6 the agents' belief that expected returns are higher in bullish markets than in bearish
 7 markets. The fundamental term, driven by the value towards which fundamentalists
 8 expect the current price should revert, varies in time and depends on the initial aver-
 9 aged price-to-earnings ratio. The actual stock price may deviate from the perceived
 10 reference level as a combined effect of an idiosyncratic noise component and local
 11 trends due to trading strategies. We demonstrate both analytically and by means of
 12 numerical experiments that the long-run behavior of our stylized dynamics agrees
 13 with empirical evidences reported in literature.

14 **Keywords** Fundamental value · Shiller's cape

N. Angelini
 School of Economics, Management and Statistics, University of Bologna,
 Via Angherà 22, 40127 Rimini, Italy
 e-mail: natascia.angelini@unibo.it

G. Bormetti · F. Nardini (✉)
 Department of Mathematics, University of Bologna, Viale Filopanti 5,
 40126 Bologna, Italy
 e-mail: franco.nardini@unibo.it

G. Bormetti
 e-mail: giacomo.bormetti@unibo.it

S. Marmi
 Scuola Normale Superiore and C.N.R.S. UMI 3483, Laboratorio Fibonacci,
 Piazza Dei Cavalieri 7, 56126 Pisa, Italy
 e-mail: stefano.marmi@sns.it

1 Introduction

“I never have the faintest idea what the stock market is going to do in the next six months, or the next year, or the next two. But I think it is very easy to see what is likely to happen over the long term”, Buffet (2001).

The first part of Buffett’s statement clearly explains why nowadays it is widely accepted that stock prices and stock market indexes behave like stochastic processes. Such a long lived popularity is further supported by two different arguments. The first one is the argument put forth by Fama that financial markets are “informationally efficient”. One can not achieve returns in excess of average market returns on a risk-adjusted basis, given the information available at the time the investment is made since the instantaneous adjustment property of an efficient market implies that successive price changes in individual securities may be assumed independent for any practical purpose, see Fama (1965) and Samuelson (1965). The second argument is the possibility—within the formal framework of stochastic processes—to develop pricing models.

Despite the second part of the statement may be tracked back to Buffett’s mentor Benjamin Graham, it has been obscured by the efficient market hypothesis for decades. Campbell and Shiller are two of the few scholars long skeptical about the latter; as early as 1988 they found statistical evidence that “the present value of future dividends is, for each year, roughly a weighted average of moving-average earnings and current real price” which has implication for the present-value model of stock prices and for recent results that long-horizon stock returns are highly predictable. At the very beginning of 2000 Robert Shiller wrote “we do not know whether the market level makes any sense, or whether they are indeed the result of some human tendency that might be called irrational exuberance”, Shiller (2000). He reached his conclusion through an innovative test of the appropriateness of prices in the stock market: the Cyclically Adjusted Price-to-Earning ratio (CAPE), which he proved to be a powerful predictor of future long run performances of the market. The performance of the test is quite satisfactory in the case of the US market from the end of 19th century up to today. For a detailed discussion refer to Campbell and Yogo (2006) and references therein.

It is clear that modeling a Shiller-type price dynamics requires a time scale completely different from those considered when pricing options. The latter scales range from several days to several months (see Cont and Tankov (2004) p. 3), time scales for which “the full effects of new information on intrinsic values to be reflected “instantaneously” in actual prices” (see Fama 1965 p. 56). On the other hand, introducing in the model some mean reverting mechanism would not be enough to generate stock prices which “have a life of their own; they are not simply responding to earnings or dividends. Nor does it appear that they are determined only by information about future” earnings or dividends, see Shiller (2000) p. 183 and Zhong et al. (2003).

To cope with this evidence a model of stock price dynamics should be able

- i. to generate a significant transitory component around the rationally expected equilibrium value of the asset. This component requires the action of at least two

- 58 different contrasting forces: One pushing the price towards its equilibrium and
 59 the other pointing at the opposite direction;
- 60 ii. to determine whether the trajectory is wandering far from the fundamentals. To do
 61 so the model should explicitly take into account macroeconomic variables such
 62 as the CAPE.

63 Surprisingly enough to our knowledge very few such models have been so far put
 64 forth. Boswijk et al. (2007) propose a model in which agents have different beliefs
 65 about the persistence of deviations of stock prices from the publicly known funda-
 66 mental value. Quite recently, following Kojien et al. (2009), He et al. (2014) propose
 67 an asset pricing model which incorporates a mean reversion process and a mov-
 68 ing average momentum component into the drift of a standard geometric Brownian
 69 motion. They prove that the profitability of different investment strategies depends on
 70 different time horizons and on the market state. In all these models the fundamental
 71 value is constant at its (very) long-run historical mean.

72 Obviously how the fundamental price is determined is a very delicate issue: The
 73 initial assumption of a known constant fundamental price may be regarded as a pre-
 74 liminary simplifying hypothesis. A more realistic assumption is that the fundamen-
 75 tal value follows itself a random walk (see Lux and Marchesi 1999; Chiarella et al.
 76 2008) and agents know it only approximately due to their bounded rationality. In
 77 Westerhoff (2004) agents make estimates by starting from an initial value that is
 78 adjusted as time goes on. Thanks to this assumption the model can exhibit prolonged
 79 phases of under and over valuation.

80 Here we choose to follow the approach suggested in a similar context by Biagini
 81 et al. (2013), who describe the effects at an aggregate level of the interaction at a
 82 micro-level of different types of agents. In particular they assume that “the perceived
 83 fundamental value” shifts in time because of the varying share of optimists in the
 84 market.¹ Differently from all the above cited papers, we do not try to a priori guess
 85 how the mood of the market dictates “the perceived fundamental value”. Instead, we
 86 allow the fundamental value, towards which fundamentalists expect that the current
 87 price should revert, to vary in time and to depend on the initial averaged price-to-
 88 earnings ratio as on an initial anchor (see Tversky and Kahneman 1974).

89 In our model the price growth depends on three components

- 90 1. an autoregressive component, naturally justified in terms of agents’ expectation
 91 that expected returns are higher in bullish markets than in bearish ones;
- 92 2. a fundamental component, proportional to the level of the logarithmic averaged
 93 Earnings-to-Price ratio (for brevity log EP ratio) and the perceived fundamental
 94 value;
- 95 3. a stochastic component ensuring the diffusive behavior of stock prices.

¹A similar assumption of possible shifts of the perceived fundamental value is proposed in Lengnick and Wohltmann (2010) where financial and real markets are taken into account. In De Grauwe and Kaltwasser (2012) traders switch between optimistic and pessimistic views about the fundamental value.

We show that with a suitable choice of the parameters the assumptions of Lengnick and Wohltmann (2010) are in some sense corroborated by our model. Initially the fundamentalists' perception of the fundamental value is biased in the direction of the most recent performance of the market, i.e., if prices are high (low) the fundamental stock price is perceived to lie above (below) its true counterpart. However optimism (pessimism) does not last for ever, as in Biagini et al. (2013) (see p. 10), and within approximately 11 or 12 years it reverts to a value independent of the initial one and compatible with the long-run mean observed by Shiller.

Moreover, we are able to prove that, if we consider a sufficiently large number of periods, the expected rate of return and the expected gross return are linear in the initial time value of log EP, and their variance converges to zero with rate of convergence consistent with a diffusive behavior. This means that, in our model, the stock prices dynamics may exhibit significant and persistent upwards and downwards deviations from the long run mean value of the averaged earning-to-price ratio, nevertheless the averaged earning-to-price ratio is a good predictor of future long-run returns, as claimed by Campbell and Shiller (1988a), Shiller (2000). The result holds for both returns and gross returns; in the latter case we assume that the log dividend-to-price ratio follows a stationary stochastic process as in Campbell and Shiller (1988a, b). Our results are also in keeping with Hodrick (1992), who "demonstrates that a relatively large amount of long-run predictability is consistent with only a small amount of short-run predictability".

2 The Model

We refer to the inflation adjusted price of the stock index measured at the beginning of time period t with P_t , while D_t denotes the real dividend paid between t and $t + 1$. Accordingly, we write the real log gross return on the index held from time t until time $t + 1$ as

$$H_t = \log(P_{t+1} + D_t) - \log P_t.$$

The description we provide of the return dynamics is on a monthly basis. Thus, the notation $t + 1$ refers to time t increased by one month and the real gross yield over a period of length h months corresponds to

$$y_{t,h} = \frac{1}{h} \sum_{i=0}^{h-1} H_{t+i}. \quad (1)$$

We also introduce the index log price $p_t = \log P_t$, in terms of which the gross yield can be rewritten as

$$y_{t,h} = \frac{1}{h} \sum_{i=0}^{h-1} (p_{t+i+1} - p_{t+i}) + \frac{1}{h} \sum_{i=0}^{h-1} \log \left(1 + \frac{D_{t+i}}{P_{t+1+i}} \right), \quad (2)$$

where the telescopic sum is equivalent to $(p_{t+h} - p_t)/h$. The latter term on the right hand side represents a non linear function of the logarithmic dividend-to-price ratio. Campbell and Shiller argue that the log dividend-to-price ratio $d_t - p_{t+1} \doteq \log D_t - \log P_{t+1}$ follows a stationary stochastic process (see page 666 of Campbell and Shiller 1988b). In light of this evidence the dynamics of the log dividend-to-price is given by

$$\Delta(d_{t-1} - p_t) = -\theta(d_{t-1} - p_t - \log \mathcal{G}) + \sigma_d W_t^d, \quad (3)$$

with initial time condition equal to $d_{-1} = \log D_{-1}$. The AR(1) coefficient is given by $1 - \theta$, σ_d is a positive volatility constant, $\{W_t^d\}$ are independent identically distributed (i.i.d.) Gaussian increments with zero mean and unit variance, and $\log \mathcal{G}$ is the fixed mean. By means of a first-order Taylor expansion centred in $\log \mathcal{G}$, the quantity $\log(1 + D_t/P_{t+1})$ appearing in Eq. 2 can be replaced by a linear function of the log dividend-to-price ratio.

The dependent variable dealt with throughout the paper is the gross return of the stock index, while as a predictive quantity we consider the log price-to-earnings ratio $cape_t \doteq p_t - \log \langle e \rangle_t^{10}$. The symbol $\langle e \rangle_t^{10}$ refers to the moving average of real earnings over a time window of ten years. The use of an average of earnings in computing the price ratios has been strongly pushed by the literature in recognition of the cyclical variability of earnings.

In Campbell and Shiller (1988a, b) the regression of real and excess stock returns on explanatory variables which are known at the start of the year t shows that the log dividend-to-price ratio and the log earnings-to-price ratio have good predictive capabilities. The ratio variables are used as indicators of fundamental value relative to price. The basic idea is that if stocks are under-priced relative to fundamental value, returns tend to be high subsequently, while the converse holds if stocks are overpriced. Consistently, we describe the dynamics of the log price assuming the existence of an exogenous fundamental component given by a mean-reverting term whose long-run target level depends linearly on the current value of the earnings-to-price ratio.

We model the dynamics of the log price by means of the linear system of stochastic difference equations

$$\begin{cases} p_{t+1} = p_t + \mu_t + \xi_t, \\ \mu_{t+1} = \gamma \mu_t + \kappa (\mathcal{F} + \mathcal{F}t - cape_t) + \sigma_\mu W_t^\mu, \\ \xi_{t+1} = \xi_t + \sigma_\xi W_t^\xi, \end{cases} \quad (4)$$

with initial time conditions equal to $p_0 = \log P_0$, and μ_0 . The quantities $\{W_t^\mu\}$, and $\{W_t^\xi\}$ for $t = 0, \dots, h$ are i.i.d. Gaussian increments with zero mean and unit variance, and σ_μ , and σ_ξ are positive volatility constants. The system of equations (4) determines the evolution of log prices as a superposition of a local drift μ_t and a noise component ξ_t . The latter is a zero mean process originating from ξ_0 which ensures the diffusive behavior of stock prices. The most relevant component corresponds to the equation driving the local drift

$$\mu_{t+1} = \gamma\mu_t + \kappa (\mathcal{H} + \mathcal{F}_t - \text{cape}_t) + \sigma_\mu W_t^\mu. \quad (5)$$

The dependence of the future level of μ_{t+1} on the value μ_t prevailing at the previous time step is expressed in terms of an autoregressive component whose intensity is determined by the agents' sensitivity to the market trend, γ . This effect can be justified in terms of the expectation that returns are higher in bullish markets than in bearish markets. Competing with the latter effect we add a second mechanism which affects the drift from a fundamental perspective. The second term in the right hand side of Eq. (5) represents the exogenous "fundamental" component given in terms of a mean reverting term. The actual stock price may deviate from the long-run behavior as a combined effect of both random external disturbances and short-term speculative component. Eventually this disequilibrium becomes apparent causing stock prices to move in the direction that reduces the deviation. In modeling the fundamental effect we bear in mind that "in reality it is very difficult (if not impossible) to identify the true fundamental value of any stock" (see Lengnick and Wohltmann 2010). Consistently we allow the mean reversion target to vary in time. Finally, we assume that the evolution of the averaged earnings is exogenous and follows an exponential law, i.e. $\langle e \rangle_t^{10} = \langle e \rangle_0^{10} \exp(gt)$.

The main theoretical result of this paper characterises the asymptotic behavior of the first and second moment of the log-price gross returns.

Proposition 1 *The expected gross yield over h months is asymptotically linear in \mathcal{F} and \mathcal{G}*

$$\mathbb{E}_0 [y_{0,h}] = g + \mathcal{F} + \mathcal{G} + O\left(\frac{1}{h}\right), \quad (6)$$

while the variance converges to zero as predicted by a diffusive model

$$\text{Var}_0 [y_{0,h}] = \frac{\sigma_p^2}{h} + \frac{1}{h} \frac{\mathcal{G}^2 \sigma_d^2}{\theta(2-\theta)} + o\left(\frac{1}{h}\right), \quad (7)$$

with $\sigma_p = \sigma_\xi(1 - \gamma)/\kappa$.

Equation (6) provides an insightful decomposition of the return growth in three components: the growth of earnings, g , a fundamental term, \mathcal{F} , ascribable to the price-over-earnings ratio, and the long-run level of the dividend-to-price ratio, \mathcal{G} . In this respect Proposition 1 sheds light on the economic constituents of the expected gross yield and matches John Bogle's suggestion for forecasting the long-term performance of stock markets. At the beginning of the 1990s in an article entitled "Investing in the 1990s" he propose to forecast long-run behavior on the basis of three variables: The initial dividend yield, the expected growth of earnings, and the expected change in the price-to-earnings ratio, Bogle (1991). More recently, Estrada (2007) extends Bogle's proposal including a fourth variable, the expected growth of dividends, providing a simple framework for the decomposition of returns similar in spirit to our findings. Proposition 1 also clarifies the long-run behavior of the gross

206 yield's variance. Equation 7 states that it asymptotically reduces to zero with a rate
 207 of convergence which is coherent with the diffusive behavior of stock returns.

208 3 Numerical Computations

209 In light of the result Eq. (6) and supported by the evidence provided in Campbell and
 210 Shiller (1988a, b) that the long-run expected gross yield is a linear function of the
 211 initial CAPE, we assume the following

$$\begin{aligned} \mathcal{F} &= \alpha_{\mathcal{F}} - \beta_{\mathcal{F}} \text{cape}_0, \\ \mathcal{G} &= \alpha_{\mathcal{G}} - \beta_{\mathcal{G}} \text{cape}_0. \end{aligned}$$

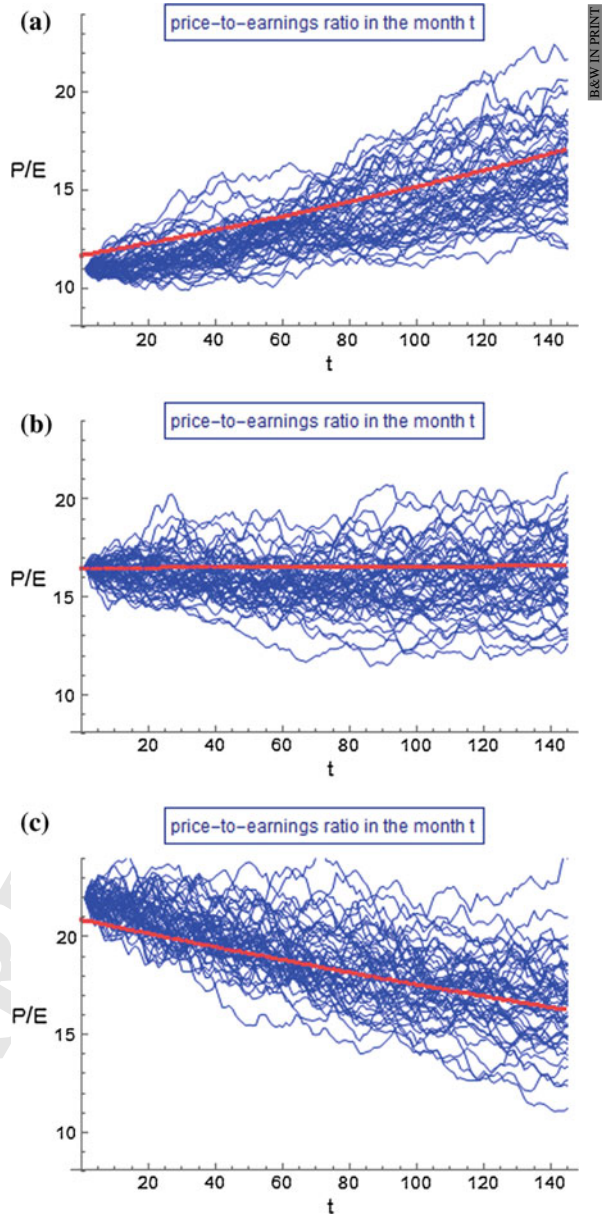
213 Coherently, we also assume that \mathcal{H} depends linearly on the initial ratio

$$\mathcal{H} = \alpha_{\mathcal{H}} - \beta_{\mathcal{H}} \text{cape}_0.$$

215 It is interesting to comment how the perceived fundamental value $\mathcal{H} + \mathcal{F}t$ evolves
 216 in time. If $\beta_{\mathcal{F}} > 0$ and $\beta_{\mathcal{H}} < 0$, its initial value is smaller (larger) the lower (higher)
 217 cape_0 , but it gradually reverts toward larger and larger (smaller and smaller) values
 218 as time elapses and within $-\beta_{\mathcal{H}}/\beta_{\mathcal{F}}$ months reaches a value independent of the
 219 initial level of the value ratio. This behavior is confirmed by Fig. 1 where we plot
 220 the evolution of the price-to-earnings ratio over 12 years computed by means of
 221 fifty Monte Carlo simulations with different initial values. Figure 1a corresponds
 222 to an initial price-to-earnings equal to 11, Fig. 1b–16.6 and 1c–22. The red line is
 223 the target of the mean reversion. All paths are sampled with $\alpha_{\mathcal{F}} = 0.033$, $\alpha_{\mathcal{H}} =$
 224 0.84 , $\beta_{\mathcal{F}} = 0.006$, $\beta_{\mathcal{H}} = -0.84$, $g = 0.0012$, and $\kappa = 0.037$. In all figures there
 225 are paths which exhibit long transients wandering away from the long run value of
 226 the price-to-earnings, but finally most of the paths end in the same interval around
 227 the long run value of 16.6, irrespective of the initial ratio. These values are close to
 228 those considered by Campbell and Shiller (1988a, b) to prove the forecasting ability
 229 of long-term stock returns. Coherently with their findings our model captures the
 230 mechanism for which an initially under-priced market is driven to the higher long-
 231 run level by means of the fundamental anchor. Conversely, keeping fixed all the
 232 parameter values, an initially over-priced market is deflated to the long-run price-to-
 233 earnings ratio of 16.6 within a transient period of nearly 12 years ($-\beta_{\mathcal{H}}/\beta_{\mathcal{F}} \simeq 141$
 234 months).

235 Figure 2 is obtained using a data sample consisting of prices, earnings, and divi-
 236 dends for the Standard and Poor Composite Stock Price Index (S&P) on a monthly
 237 basis. The data are discussed in Campbell and Shiller (1987, 1988a, b), and are freely
 238 available from Robert J. Shiller's webpage <http://www.econ.yale.edu/>. These time
 239 series cover the entire period from January 1871 until December 2012. Figure 2a
 240 shows the empirical yields for a time horizon of two years. The dashed line cor-
 241 responds to a linear regression on the logarithmic CAPE. Figure 2b–h report the

Fig. 1 Fifty simulated paths of price-to-earnings ratio for initial CAPE equal to 11 (Fig. 1a), 16.6 (Fig. 1b), and 22 (Fig. 1c). The *solid lines* correspond to the target of mean reversion



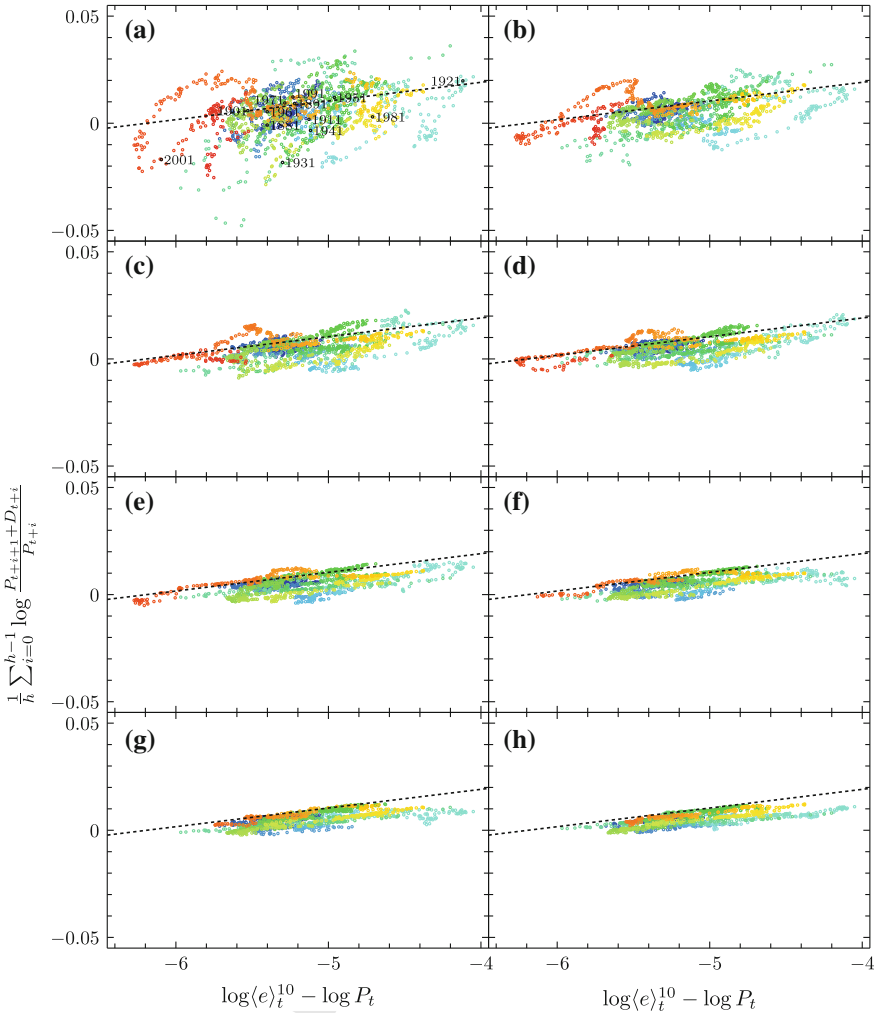


Fig. 2 Empirical yields for the Standard & Poor's 500 from January 1871 until December 2012 for a 2, 4, 6, 8, 10, 12, 14, and 16 year time horizon (figures a, b, c, d, e, f, g, and h, respectively) (color figure online)

242 same as Fig. 2a with time horizons increasing from 4 to 16 years. Points are given
 243 in chronological order according to the color scale ranging from dark blue to red
 244 passing through light blue, green, yellow, and orange; labels in the top left figure
 245 refer to points which correspond to the first month of the specified year. In Fig. 3 we
 246 present a Monte Carlo simulation of the model given by Eqs. (3) and (4). The dashed
 247 line corresponds to the long-run behavior predicted by the Eq. (6) and the dotted
 248 lines to the boundaries of the 95 % confidence level region. The parameter values
 249 chosen for the simulation are given in Table 1. Values reported in the second column

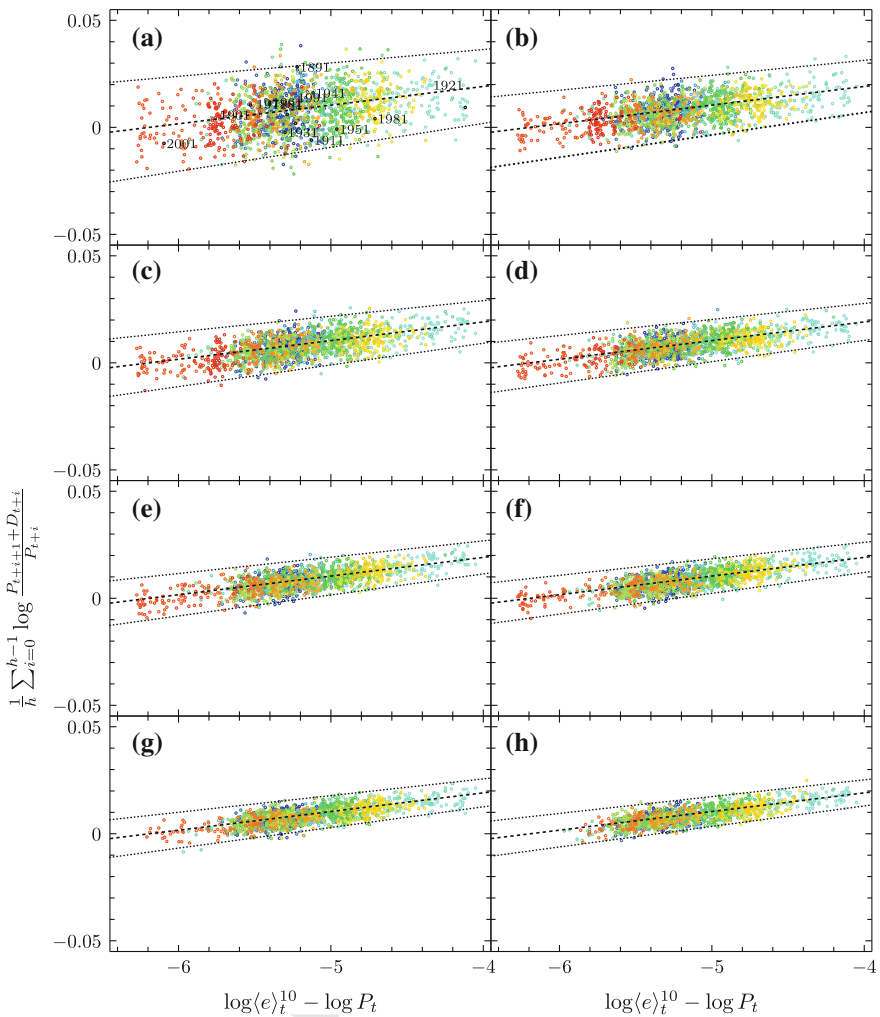


Fig. 3 Monte Carlo scenarios generated with parameter values given in Table 1 and initial time conditions as in Fig. 2

250 are obtained from the linear regressions displayed in Fig. 2. The monthly earning
 251 growth, g , is consistent with the historical long-run growth, while κ provides the
 252 typical scale of mean reversion of the fundamental component consistent with the
 253 results discussed in Campbell and Shiller (1988a, b). The autoregressive coefficient
 254 γ reflects the positive empirical autocorrelation measured from equity index monthly
 255 returns. In line with the strong evidence that the log dividend-to-price ratio follows
 256 a near unit root process, we set θ equal to 0.025. Finally, all values of the variance
 257 coefficients are set equal to 18 bps yielding 15% yearly volatility for the market
 258 index. The color scale determines the initial time condition prevailing historically at

Table 1 Common set of parameter values used in the numerical analysis

$\alpha_{\mathcal{F}}$ ($month^{-1}$)	0.033	g ($month^{-1}$)	0.0012
$\beta_{\mathcal{F}}$ ($month^{-1}$)	0.006	θ ($month^{-1}$)	0.025
$\alpha_{\mathcal{G}}$ ($month^{-1}$)	0.021	γ	0.25
$\beta_{\mathcal{G}}$ ($month^{-1}$)	0.003	κ ($month^{-1}$)	0.037
$\alpha_{\mathcal{J}\kappa}$	0.84	σ_d^2 ($month$)	18 (bps)
$\beta_{\mathcal{J}\kappa}$	-0.84	σ_μ^2 ($month$)	18 (bps)
		σ_p^2 ($month$)	18 (bps)

the beginning of each month. Since Monte Carlo scenarios are generated under the same initial time conditions, the remarkable agreement of the color distributions in Figs. 3 and 2 confirms the ability of the model to capture the long-run behavior of the market index.

4 Conclusions and Perspectives

This paper proposes a simple dynamic model for the long-run behavior of stock index returns for the U.S. market. The log price dynamics depend on two market forces: A positive autoregressive component typical for stock index returns and a mean-reverting term whose long-run level is fixed exogenously on the basis of the predictive ability of Shiller's CAPE. Accordingly, we show that the long-run expected growth of the market index can be decomposed in three components: The earning growth, the log dividend-to-price ratio long-run level, and a fundamental term ascribable to the price-over-earnings ratio.

Substantial evidence of the importance of fundamentals in the valuation of international stock markets has been accumulated by the proponents of fundamental indexation e.g. Arnott et al. (2005). Practitioners and academicians alike have been using several valuation measures for estimating the intrinsic value of a stock index. For example, in Table 2 of Poterba and Samwick (1995) the ratio of market value of corporate stock to GDP, the year-end price-to-earnings ratio, the year-end price-to-dividend ratio and Tobin's q are reported from 1947 to 1995 in an effort of alerting the reader on the possible overvaluation of the index. In particular Tobin's q has been proposed as another efficient method of measuring the value of the stock market, with an efficiency comparable to the CAPE (see Smithers 2009). The q ratio is the ratio of price to net worth at replacement cost rather than the historic or book cost of companies. It therefore allows for the impact of inflation, much alike the CAPE which averages real earnings over a ten year span. It would be interesting to carry out an empirical analysis of the relationship between Tobin's q and future stock index returns as far as to extend the present approach to countries other than the U.S. Both perspectives are worth to be followed but require high quality long-term time series. As a possible future extension to model the emergence of explosive bubbles, we

289 plan to relax the assumption of stationarity of the log dividend price ratio process
 290 following the approach recently investigated by Engsted et al. (2012).

291 References

- 292 Arnott RD, Hsu J, Moore P (2005) Fundamental indexation. *Financ Anal J* 61(2):83–99
- 293 Biagini F, Föllmer H, Nedelcu S (2013) Shifting martingale measures and the birth of a bubble as
 294 a submartingale. To appear in *Finance and Stochastics*
- 295 Bogle J (1991) Investing in the, (1990s). *J Portf Manag Spring* 5–14
- 296 Boswijk HP, Hommes CH, Manzan S (2007) Behavioral heterogeneity in stock prices. *J Econ Dyn*
 297 *Control* 31(6):1938–1970
- 298 Buffet W (2001) *Fortune magazine*
- 299 Campbell JY, Shiller RJ (1987) Cointegration and tests of present value models. *J Political Econ*
 300 95(5):1062–1088
- 301 Campbell JY, Shiller RJ (1988a) The dividend-price ratio and expectations of future dividends and
 302 discount factors. *Rev Financ Stud* 1(3):195–228
- 303 Campbell JY, Shiller RJ (1988b) Stock prices, earnings and expected dividends. *J Financ*
 304 *XLIII*(3):661–676
- 305 Campbell JY, Yogo M (2006) Efficient tests of stock return predictability. *J Financ Econ* 81(1):27–60
- 306 Chiarella C, He X-Z, Wang D, Zheng M (2008) The stochastic bifurcation behaviour of speculative
 307 financial markets. *Phys A: Stat Mech Appl* 387(15):3837–3846
- 308 Cont R, Tankov P (2004) *Financial modelling with jump processes*. Chapman & Hall/CRC, London
- 309 De Grauwe P, Kaltwasser PR (2012) Animal spirits in the foreign exchange market. *J Econ Dyn*
 310 *Control* 36(8):1176–1192
- 311 Engsted T, Pedersen TQ, Tanggaard C (2012) The log-linear return approximation, bubbles, and
 312 predictability. *J Financ Quant Anal* 47:643–665
- 313 Estrada J (2007) Investing in the twenty-first century: with Occam’s razor and Bogle’s wit. *Corp*
 314 *Financ Rev*
- 315 Fama EF (1965) The behavior of stock-market prices. *J Bus* 38:34–105
- 316 He X, Li K, Li Y (2014) Optimality of momentum and reversal. SSRN 2467392
- 317 Hodrick RJ (1992) Dividend yields and expected stock returns: alternative procedures for inference
 318 and measurement. *Rev Financ Stud* 5(3):357–386
- 319 Koijen RS, Rodriguez JC, Sbuels A (2009) Momentum and mean reversion in strategic asset allo-
 320 cation. *Manag Sci* 55(7):1199–1213
- 321 Lengnick M, Wohltmann HW (2010) Agent-based financial markets and new keynesian macroeco-
 322 nomics: a synthesis. *J Econ Interact Coord* pages 1–32
- 323 Lux T, Marchesi M (1999) Scaling and criticality in a stochastic multi-agent model of a financial
 324 market. *Nature* 397(6719):498–500
- 325 Poterba JM, Samwick AA (1995) Stock ownership patterns, stock market fluctuations, and con-
 326 sumption. *Brook Pap Econ Act* 1995(2):295–372
- 327 Samuelson P (1965) Proof that properly anticipated prices fluctuate randomly. *Ind Manag Rev*
 328 6(2):41–49
- 329 Shiller RJ (2000) *Irrational Exuberance*. Princeton University Press, New Jersey
- 330 Smithers A (2009) *Wall street revalued imperfect markets and inept central bankers*. Wiley, Hoboken
- 331 Tversky A, Kahneman D (1974) Judgment under uncertainty: heuristics and biases. *Science*
 332 185(4157):1124–1131
- 333 Westerhoff FH (2004) Multiasset market dynamics. *Macrocon Dyn* 8(05):596–616
- 334 Zhong M, Darrat AF, Anderson DC (2003) Do US stock prices deviate from their fundamental
 335 values? some new evidence. *J Bank Financ* 27(4):673–697

Author Queries

Chapter 7

Query Refs.	Details Required	Author's response
	No queries.	

UNCORRECTED PROOF

MARKED PROOF

Please correct and return this set

Please use the proof correction marks shown below for all alterations and corrections. If you wish to return your proof by fax you should ensure that all amendments are written clearly in dark ink and are made well within the page margins.

<i>Instruction to printer</i>	<i>Textual mark</i>	<i>Marginal mark</i>
Leave unchanged	... under matter to remain	Ⓟ
Insert in text the matter indicated in the margin	∧	New matter followed by ∧ or ∧ [Ⓢ]
Delete	/ through single character, rule or underline or ┌───┐ through all characters to be deleted	Ⓞ or Ⓞ [Ⓢ]
Substitute character or substitute part of one or more word(s)	/ through letter or ┌───┐ through characters	new character / or new characters /
Change to italics	— under matter to be changed	↙
Change to capitals	≡ under matter to be changed	≡
Change to small capitals	≡ under matter to be changed	≡
Change to bold type	~ under matter to be changed	~
Change to bold italic	≈ under matter to be changed	≈
Change to lower case	Encircle matter to be changed	≡
Change italic to upright type	(As above)	⊕
Change bold to non-bold type	(As above)	⊖
Insert 'superior' character	/ through character or ∧ where required	Υ or Υ under character e.g. Υ or Υ
Insert 'inferior' character	(As above)	∧ over character e.g. ∧
Insert full stop	(As above)	⊙
Insert comma	(As above)	,
Insert single quotation marks	(As above)	ʹ or ʸ and/or ʹ or ʸ
Insert double quotation marks	(As above)	“ or ” and/or ” or ”
Insert hyphen	(As above)	⊞
Start new paragraph	┌	┌
No new paragraph	┐	┐
Transpose	└┘	└┘
Close up	linking ○ characters	○
Insert or substitute space between characters or words	/ through character or ∧ where required	Υ
Reduce space between characters or words		↑