Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **419,** 1480–1488 (2012) [redacted] 04.x

Joint quasar–cosmic microwave background con [redacted] history

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Accepted 2011 September 10. Received 2011 September 6; in original form 2011 June 21

ABSTRACT

Based on the work by Mitra, Choudhury & Ferr [redacted] nts
on reionization from cosmic microwave backgro [redacted] by
decomposing the function $N_{\rm ion}(z)$ (the number o [redacted] um
per baryon in collapsed objects) into its princi [redacted] his
work is that for the CMB data set, we explicit [redacted] for
TT, TE and EE modes in our analysis which se [redacted] ion
than taking the electron scattering optical depth [redacted] kov
chain Monte Carlo methods, we find that all th [redacted] be
severely constrained at $z < 6$ whereas a broad ran [redacted] till
permitted by the current data sets. With currently [redacted] ain
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complete for $9.0 < z(Q_{\rm H\,II} = 0.5) < 11.8$ (95 p [redacted] ete
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on $z(Q_{\rm H\,II} = 0.5)$ and $z(Q_{\rm H\,II} = 0.99)$ will be ∼1 a [redacted] For
more stringent constraints on reionization at $z > 6$, [redacted] han
the CMB. Our method will be useful in such a case [redacted] tric
reconstruction of the reionization history with arbitra [redacted]

Key words: intergalactic medium – cosmology: theo [redacted] rs –
large-scale structure of Universe.

1 INTRODUCTION

In the past few years, the understanding of the reionization process has become increasingly sophisticated in both the observational and theoretical communities (for reviews, see Barkana & Loeb 2001; Loeb & Barkana 2001; Choudhury & Ferrara 2006a; Fan, Carilli & Keating 2006; Furlanetto, Oh & Briggs 2006; Choudhury 2009, and references therein), thanks to the availability of good-quality data related to reionization. Mainly, the observations by the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) satellite of the cosmic microwave background (CMB) and highest redshift QSOs put very tight constraints on the reionization history of the Universe. The *WMAP* seven-year (*WMAP*7) observation manifests the Thomson scattering optical depth $\tau_{\rm el} = 0.088 \pm 0.015$ (Larson et al. 2011) with the [redacted] was
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so complex that neither a [redacted] one
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★E-mail: smitra@hri.res.in (SM); tirth@hri.res.in (TRC); andrea.ferrara@sns.it (AF) using principal component analysis (PCA) (Mitra et al. 2010, hereafter Paper I).

The principal component method has been applied to study the constraints on reionization from large-scale CMB polarization (Hu & Holder 2003; Mortonson & Hu 2008). It is well established that the inhomogeneity signature of reionization is expected to contribute to the CMB temperature and polarization anisotropies (Hu 2000; Salvaterra et al. 2005; Iliev et al. 2006; Mortonson & Hu 2007). In fact, the CMB power spectra contain more information than the optical depth integrated over the whole ionization history (Hu & Holder 2003). So it is worth asking what we can ultimately expect to learn about the reionization model with the principal component technique from the current CMB data sets instead of single optical depth data.

In our previous work, we made a preliminary attempt to constrain $N_{\rm ion}(z)$ using PCA and estimated the uncertainties in the reionization history. The main difference of our work from other PCA studies of the reionization history using CMB data (Hu & Holder 2003; Mortonson & Hu 2008) is that we use a self-consistent model of reionization and include data sets other than the CMB (e.g. QSO absorption lines) in the analysis. Such an analysis should give us a handle not only in constraining the evolution of the electron fraction $x_{\rm e}(z)$ (as is done in usual reionization related studies using CMB data) but also in constraining the evolution of source properties like the galactic initial mass function (IMF), star formation history and escape fraction of ionizing photons.

In Paper I, we found that to model $N_{\rm ion}(z)$ over the range $2 < z < 14$ one should include the first five principal components with smaller uncertainties. We concluded that a wide range of reionization scenarios are allowed by the data sets of photoionization rates, the redshift distribution of Lyman-limit systems and the electron scattering optical depth from *WMAP*7. In this paper, we extend our previous work to study the effect of inclusion of the angular power spectra C_l of the CMB temperature (T) and polarization (E) modes. Using the available *WMAP*7 data on $C_l^{\rm TT,TE,EE}$, we study the present constraints on reionization history. We also forecast the errors on reionization history as would be determined by future observations of the large-scale polarization signal by *Planck*.[1]

The paper is organized as follows. In Section 2, we discuss the features of the semi-analytical model of reionization and its modifications for inclusion of CMB data. We also outline the basic theory of PCA in this section. We describe our results of the principal component approach to the reionization model with large-scale E-mode data in Section 3. In this section, using Markov chain Monte Carlo (MCMC) methods, we examine our model for both *WMAP7* data and simulated *Planck* data. Finally we summarize our main findings and conclude in Section 4.

2 MODEL AND METHOD OF STATISTICAL ANALYSIS

2.1 Semi-analytical model of reionization with PCA

We first describe the method used in our previous work (Paper I) which was based on the semi-analytical model of reionization developed in Choudhury & Ferrara (2006b) and Choudhury & Ferrara (2005). The main features of the model are as follows.

(i) The model follows the ionization and thermal histories of neutral, H II and He III regions simultaneously and self-consistently taking the IGM inhomogeneities by adopting a lognormal distribution according to the method outlined in Miralda-Escudé, Haehnelt & Rees (2000).

(ii) Given the collapsed fraction $f_{\rm coll}$ of dark matter haloes, this model calculates the production rate of ionizing photons in the IGM as

$$\dot{n}_{\rm ph}(z) = n_{\rm b} N_{\rm ion} \frac{{\rm d}f_{\rm coll}}{{\rm d}t}, \tag{1}$$

where $n_{\rm b}$ is the total baryonic number density in the IGM, and $N_{\rm ion}$ is the number of photons entering the IGM per baryon in collapsed objects. The parameter $N_{\rm ion}$ can actually be written as a combination of various other parameters which characterize the star-forming efficiency (fraction of baryons within collapsed haloes going into stars), the fraction of photons escaping into the IGM, and the number of photons emitted per frequency range per unit mass of stars (which depends on the stellar IMF and the corresponding stellar spectrum).

(iii) The minimum mass of star-forming haloes is assumed to be determined by atomic cooling in neutral regions (i.e. we neglect cooling through molecular hydrogen). However, the minimum mass will be larger in ionized regions because of *radiative feedback*. The model computes radiative feedback (suppressing star formation in low-mass haloes using a Jeans mass prescription) self-consistently from the evolution of the thermal properties of the IGM. The corresponding filtering scale, which depends on the temperature evolution of the IGM, is found to be typically around $\sim$30 km s^{-1}.

(iv) In Paper I, we assumed $N_{\rm ion}$ to be an unknown function of z and decomposed it into principal components. These principal components essentially filter out components of the model that are most sensitive to the data and thus they are the ones that can be constrained most accurately. We carry out our analysis assuming that only one population of stars contribute to the ionizing radiation; any change in the characteristics of these stars over time or the chemical feedback prescription would be accounted for indirectly by the evolution of $N_{\rm ion}$. We also include the contribution of quasars at $z < 6$ assuming that they have negligible effects on the IGM at higher redshifts, but are significant sources of photons at $z \lesssim 4$.

(v) Usually, the model is constrained by comparing with a variety of observational data, but to keep the analysis simple, we used the three main data sets in our earlier work, namely, the photoionization rates $\Gamma_{\rm PI}$ obtained using Lyα forest Gunn–Peterson optical depth observations and a large set of hydrodynamical simulations (Bolton & Haehnelt 2007), the redshift distribution of Lyman-limit systems (LLS) ${\rm d}N_{\rm LL}/{\rm d}z$ at $z \sim 3.5$ (Prochaska, O'Meara & Worseck 2010), and the *WMAP*7 data on electron scattering optical depth $\tau_{\rm el}$ (Larson et al. 2011). It should be mentioned that in this work, we have used the data on C_l values rather than the constraints on $\tau_{\rm el}$, which will be described in the next subsection.

(vi) The free parameters used in the model are the coefficients related to the principal components of $N_{\rm ion}$ and λ_0 (the normalization which determines the mean free path of photons). The constraints on $N_{\rm ion}$ were obtained by marginalizing over λ_0. The cosmological parameters were taken to be fixed (given by the best-fitting *WMAP*7 values) and not varied at all.

2.2 Data sets and free parameters

The major modifications made in this work compared with our previous study are related to how we treat the CMB data sets. Note that in Paper I, the $\tau_{\rm el}$ constraint was treated as a single data point which can be thought of as a simplification of the CMB polarization

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[1] http://www.esa.int/SPECIALS/Planck/index.html

observations at low multipole moments (Burigana et al. 2008). We know that the amplitude of fluctuations in the large-scale (low-l) E-mode component of CMB polarization provides the current best constraint on τ_{el}. Using the data from *WMAP7* and the assumption of instantaneous reionization, Larson et al. (2011) find $\tau_{\mathrm{el}} = 0.088 \pm 0.015$. However, recent theoretical and numerical studies suggest that reionization is a fairly complex process. In that case, the low-l E-mode spectrum depends not just on τ_{el} but also on the detailed redshift evolution of the number density of free electrons in the IGM, $x_{\mathrm{e}}(z)$. For fixed values of τ_{el} and all other relevant cosmological parameters, differences in $x_{\mathrm{e}}(z)$ can affect the shape of the large-scale E-mode angular power spectrum up to multipoles $l \simeq 40$–50. Because of this dependence, measurements of the low-l C_l^{EE} should place at least weak constraints on the overall reionization history in addition to the constraint on the total optical depth.

Now, in our model, the change in the parameter $N_{\mathrm{ion}}(z)$ directly corresponds to the change in $x_{\mathrm{e}}(z)$, i.e., in other words, changes in N_{ion} can affect the shape of the low-l C_l^{EE}. So, incorporating the data sets for the large-scale EE polarization signal in our model can provide important information about the evolution of N_{ion} at $z > 6$ beyond the information about τ_{el}. Our hope is that this may be most useful for distinguishing the models of reionization with different ionization histories but the same optical depth. Keeping this in mind, it would be more prudent to work with the actual data related to the angular power spectra C_l and obtain constraints on reionization parameters; the constraint on τ_{el} will be determined a posteriori.

The moment we include the C_l values (TT+TE+EE) in our analysis, we realize that parameters related to reionization may have strong degeneracies with (some of) the cosmological parameters and hence constraints on reionization without varying cosmological parameters would be misleading. On the other hand, including all the cosmological parameters in the analysis would increase the number of free parameters to a large number. Usually, it is found that τ_{el} is strongly degenerate with the normalization of the matter power spectrum σ_8 and also with the slope n_{s} (Spergel et al. 2003). Hence, it may be worthwhile to verify whether we can carry out our analysis by varying only these two parameters (in addition to the parameters related to the reionization model) and keeping all the other cosmological parameters fixed to their mean value.

To verify the viability of this method, we re-do the analysis of *WMAP*7 data with instantaneous reionization history (as in Larson et al. 2011). We assume the universe to be described by a flat cold dark matter model with a cosmological constant (ΛCDM) which is parametrized by six parameters ($\Omega_{\mathrm{b}}\,h^2$, $\Omega_{\mathrm{DM}}\,h^2$, H_0, n_{s}, σ_8, τ_{el}). We then carry out the standard MCMC analysis (Verde et al. 2003), first varying all six parameters and then keeping all but σ_8, n_{s} and τ_{el} fixed to their best-fitting values. The results are shown in Table 1. It is clear that though the uncertainties on n_{s} and σ_8 are reduced considerably because of not varying the other three parameters, the constraints on τ_{el} are relatively unchanged. There is only a slight ($\lesssim$15 per cent) decrease in the error-bars, thus indicating that the parameters related to reionization are only moderately degenerate with the other cosmological parameters. Hence, we can carry out our analysis with the other cosmological parameters fixed, keeping in mind that the uncertainties in reionization history would possibly be slightly underestimated. This approach is similar to what is adopted by Mortonson & Hu (2007).

In addition to the CMB data, we have also included the more recent measurements of $\mathrm{d}N_{\mathrm{LL}}/\mathrm{d}z$ by Songaila & Cowie (2010) instead of the previous data by Prochaska et al. (2010). The new data set includes observations over a wide redshift range ($0.36 < z < 6$) and is well suited for studying the evolution of reionization.

Table 1. Mean values of parameters and the corresponding errors for a flat ΛCDM cosmological model with instantaneous reionization. The results are shown when all six parameters are varied and when all but n_{s}, σ_8 and τ_{el} are kept fixed to their mean values.

Parameters	Mean value and 1σ errors	
	Varying all six parameters	Varying only three parameters
$\Omega_{\mathrm{b}}\,h^2 \times 10^2$	$2.249^{+0.056}_{-0.057}$	2.249 (fixed)
$\Omega_{\mathrm{DM}}\,h^2$	$0.1120^{+0.0056}_{-0.0056}$	0.1120 (fixed)
H_0	$70.4^{+2.5}_{-2.5}$	70.4 (fixed)
n_{s}	$0.967^{+0.014}_{-0.014}$	$0.969^{+0.007}_{-0.007}$
σ_8	$0.811^{+0.030}_{-0.031}$	$0.816^{+0.013}_{-0.013}$
τ_{el}	$0.088^{+0.007}_{-0.008}$	$0.088^{+0.006}_{-0.007}$

The likelihood function used in our calculations is given by

$$L \propto \exp(-\mathcal{L}), \tag{2}$$

where $\mathcal{L}$ is the negative of the log-likelihood and estimated using the relation

$$\mathcal{L} = \frac{1}{2}\sum_{\alpha=1}^{N_{\mathrm{obs}}}\left[\frac{\mathcal{J}_\alpha^{\mathrm{obs}} - \mathcal{J}_\alpha^{\mathrm{th}}}{\sigma_\alpha}\right]^2 + \mathcal{L}', \tag{3}$$

where $\mathcal{J}_\alpha$ represents the set of N_{obs} observational data points related to photoionization rate and distribution of Lyman-limit systems, i.e. $\mathcal{J}_\alpha = \{\log(\Gamma_{\mathrm{PI}}), \mathrm{d}N_{\mathrm{LL}}/\mathrm{d}z\}$, σ_α are the corresponding observational error-bars and $\mathcal{L}'$ is the negative of the *WMAP*7 or *Planck* log-likelihood function for C_l^{TT}, C_l^{TE} and C_l^{EE} up to $l = 2000$. We constrain the free parameters by maximizing the likelihood function with a prior that reionization should be completed by $z = 5.8$, otherwise it will not match Lyα and Lyβ forest transmitted flux data.

In this work, we calculate likelihoods using the code described in Paper I which is essentially based on the publicly available COSMOMC[2] (Lewis & Bridle 2002) code. Besides this, throughout we work in a flat cold dark matter model with a cosmological constant (ΛCDM) cosmology with the cosmological parameters given by the current *WMAP*7 [based on RECFAST 1.5 (Seager, Sasselov & Scott 1996, 2000; Wong, Moss & Scott 2008)] and version 4.1 of the *WMAP* likelihood] best-fitting values: $\Omega_{\mathrm{m}} = \Omega_{\mathrm{DM}} + \Omega_{\mathrm{b}} = 0.27$, $\Omega_\Lambda = 1 - \Omega_{\mathrm{m}}$, $\Omega_{\mathrm{b}}\,h^2 = 0.022\,49$, $h = 0.704$ and $\mathrm{d}n_{\mathrm{s}}/\mathrm{d}\ln k = 0$ (Larson et al. 2011). Note that here in all cases, τ_{el} is a derived parameter and the error on obtaining this quantity is slightly underestimated because of the neglect of the degeneracies between τ_{el} and other cosmological parameters.

2.3 Brief theory of PCA

In this section, we outline the principal component method and introduce the notation that we will use throughout the paper. As has been described in Paper I, the principal components filter out components of the model that are most sensitive to the data. In order to determine the principal components of $N_{\mathrm{ion}}(z)$, we consider the data for the photoionization rate Γ_{PI}, the redshift distribution of Lyman-limit systems $\mathrm{d}N_{\mathrm{LL}}/\mathrm{d}z$ and the large-scale E-mode polarization angular power spectrum C_l^{EE} ($l \leq 23$).

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[2] http://cosmologist.info/cosmomc/

We represent the unknown function $N_{\rm ion}(z)$ by a set of $n_{\rm bin}$ discrete free parameters with the bin width

$$\Delta z = \frac{z_{\rm max} - z_{\rm min}}{n_{\rm bin} - 1}. \tag{4}$$

We have taken a redshift range $[z_{\rm min}\colon z_{\rm max}] = [0\colon 30]$ and

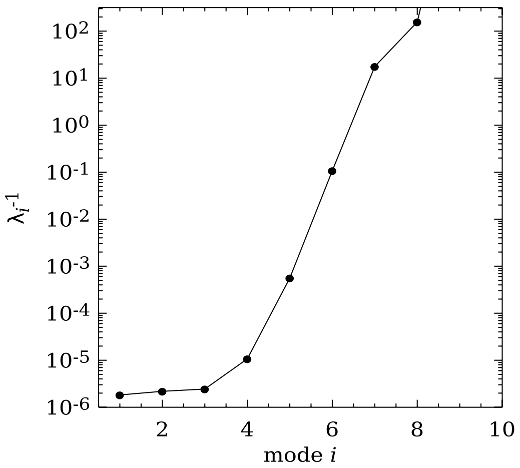

Figure 1. The inverse of eigenvalues of the Fisher matrix F_{ij} which essentially measures the variance on the corresponding coefficient.

one needs to check first whether we can simply neglect these higher modes, because neglecting modes seven and eight may introduce large biases in the recovered quantities. In order to determine this, we have used more than one method to fix M (as described in the earlier section and in Paper I), and each method suggests that we should keep up to $M = 8$ modes in our analysis unlike the case for Paper I, where we got the optimum value of M to be 5. This is not surprising given that the C_l values contain somewhat more information than what is contained within a single data point $\tau_{\rm el}$. This fact can be noted from the plot of the first eight eigenmodes (i.e. those that have the lowest variances) in Fig. 2. The first five modes are similar to what was obtained in Paper I. However, the modes 6 to 8 in Paper I did not contain any information, while in this case they show the sensitivity of $N_{\rm ion}(z)$ on different angular scales l. We find that all the eigenmodes tend to vanish at $z > 15$, which is obvious because of F_{ij} being negligible at these redshifts. We can see a number of spikes and troughs in the first four modes whose positions correspond to the presence of data points for $\Gamma_{\rm PI}$ and $dN_{\rm LL}/dz$ at $2 < z < 6$. The last four modes contain the information about the sensitivity of $C_l^{\rm EE}$. This sensitivity is maximum around $z \approx 7$–8 and decreases at $z > 8$ due to unavailability of free electrons; it also decreases at $z < 7$ because of the fact that reionization is mostly completed at these redshifts ($x_{\rm e} \rightarrow 1$) and hence changing $N_{\rm ion}$ does not affect the value of $C_l^{\rm EE}$ significantly at this redshift range. The modes (>8) with smaller eigenvalues, i.e., large variances introduce huge uncertainties in the determination of $N_{\rm ion}$ and hence do not contain any meaningful information about the reionization history.

3.2 MCMC constraints from *WMAP*7 data

The constraints on reionization are obtained by performing a MCMC analysis over the parameter space of the optimum number of PCA amplitudes, λ_0, $n_{\rm s}$ and σ_8. Other cosmological parameters are kept fixed to the *WMAP*7 best-fitting values (see Section 2.2). To avoid the confusion about the correct choice of number of modes, we perform the MCMC analysis for PCA amplitudes taking from $M = 2$ to 8, all of which obey the AIC criterion (equation 10). We then weight each choice of M equally and fold the corresponding errors together to reproduce $N_{\rm ion}$ and other related quantities along with their effective errors. In order to carry out the analysis,

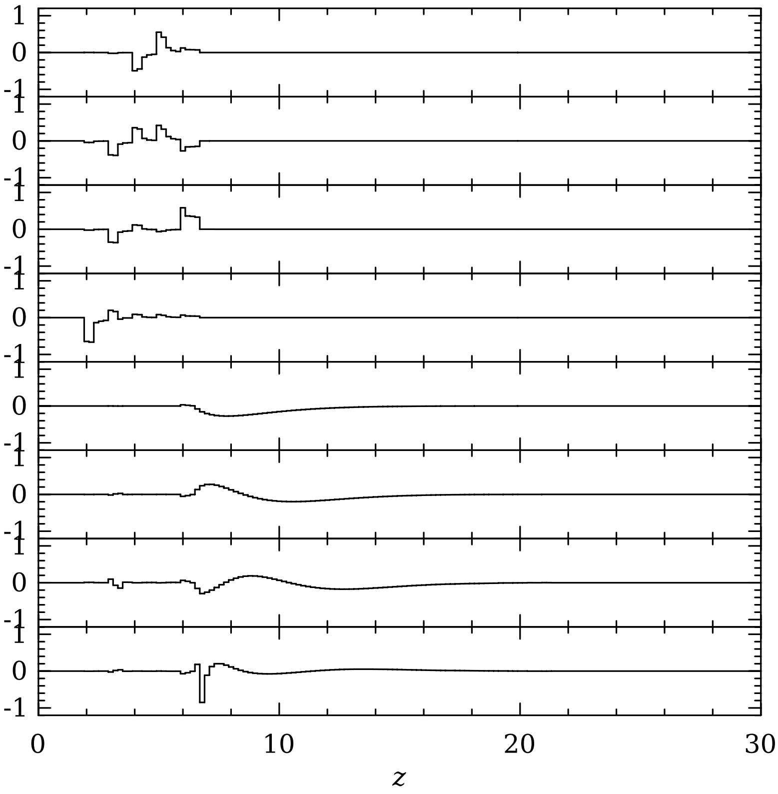

Figure 2. The first eight eigenmodes of the Fisher matrix.

Table 2. The marginalized posterior probabilities with 95 per cent CL errors of all the derived parameters for the reionization model obtained from the current analysis using the AIC criterion for *WMAP* data.

Parameters	Mean value	95 per cent confidence limits
$\tau_{\rm el}$	0.093	[0.080, 0.112]
$z(Q_{\rm H\,II} = 0.5)$	10.206	[8.952, 11.814]
$z(Q_{\rm H\,II} = 0.99)$	7.791	[5.800, 10.427]

prominent peak around $z \sim 6.5$. The prominent peak-like structure is also present in the plot of $dN_{\rm LL}/dz$ (top-right panel).

From the plot of $Q_{\rm H\,II}(z)$ (bottom-left panel), we see that the growth of $Q_{\rm H\,II}(z)$ for the mean model is much faster than that of the fiducial model at the initial stages, though the completion of reionization takes place only at $z \approx 6$. One can also find that reionization can be completed as early as $z \approx 10.4$ (95 per cent CL). Similarly, $x_{\rm H\,I}(z)$ (bottom-middle panel) decreases much faster than the fiducial one at $6 < z < 12$ and then smoothly matches the Lyα forest data.

Finally, we have shown the values of (a) $C_l^{\rm TT}$, (b) $C_l^{\rm TE}$ and (c) $C_l^{\rm EE}$ for the mean model in the bottom-right panel of this figure, which is almost the same as the fiducial model. So the current *WMAP*7 EE polarization data alone cannot distinguish between the various models of reionization. One can see that our mean model includes most of the current *WMAP*7 best-fitting CMB data within the error bars, except for a few $C_l^{\rm EE}$ data points. Note that these discrepant points at $l \gtrsim 15$ cannot be reconciled by any *physical* reionization model, implying that the spectral contribution might come from some other cosmological process, such as e.g. gravitational lensing.

The mean values and the 95 per cent CL on the parameters obtained from our analysis are shown in Table 2. We have checked that our fiducial model, which is characterized by $m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = 0$ and the best-fitting values of λ_0, n_s and σ_8, is included within the 95 per cent CL of those parameters corresponding to our current analyses using the AIC criterion. We find that reionization is 50 per cent complete between redshifts 9.0 and 11.8 (95 per cent CL), while it is almost (99 per cent) complete between redshifts 5.8 and 10.4 (95 per cent CL). These values are similar to what was obtained in Paper I. Note that the lower limit on the redshift of reionization (5.8) is imposed as a prior on the parameters. Here the mean model for $\tau_{\rm el}$ shows a higher value than the best-fitting *WMAP*7 value which arises from relatively complex reionization histories giving non-zero ionized fractions at high redshifts. The value of $\tau_{\rm el}$ obtained is slightly lower than what we got in our earlier work, where we included $\tau_{\rm el}$ as a single data point instead of considering CMB large-scale EE polarization data, which is because many models with very high $N_{\rm ion}$ are ruled out in this work.

We have checked that if we take any particular choice of M, say $M = 7$ or 8, our main findings are almost the same as the above results, except that with the help of the AIC criterion we have reduced the inherent bias which is present for that specific choice of M and got a mean model which matches the current data sets quite reasonably.

To summarize, we find that using $C_l^{\rm EE}$ data set instead of $\tau_{\rm el}$, we can get a relatively smaller error for $N_{\rm ion}(z)$ (see fig. 7 of Paper I) but get a $\tau_{\rm el}$ which is higher than the current *WMAP* value. So a wide range of reionization histories is still allowed by the data we have used. Reionization can be quite early or can be gradual and late, depending on the behaviour of $N_{\rm ion}(z)$. Hence, using these data, it is somewhat difficult to put strong constraints on chemical feedback and/or the evolution of star-forming efficiencies and/or escape fractions.

3.3 MCMC constraints from simulated *Planck* forecast data

Given that the current data allow a large range of reionization models, it is worthwhile computing the level of constraints expected from future large-scale polarization measurements like those obtained from *Planck*. To forecast the errors for parameters related to the reionization history, we first generate the simulated *Planck* data of CMB power spectra for our fiducial model up to $l \le 2000$ using the exact full-sky likelihood function at *Planck*-like sensitivity (Perotto et al. 2006; Galli et al. 2010). We assume that beam uncertainties are small and that uncertainties due to foreground removal are smaller than statistical errors. More sensitive observations will also require an exact analysis of the non-Gaussian likelihood function; here for simplicity we assume isotropic Gaussian noise and neglect non-Gaussianity of the full sky (Lewis 2005) and try to see what we can learn about the global reionization history from *Planck*-like sensitivity. We then repeat the MCMC analysis over the same parameter space of Section 3.2 using these simulated data. Like the previous case, here we have also varied the number of modes included in the analysis from two to eight using the AIC criterion in order to study the effect of truncating the PCA expansion for the recovery of various quantities related to reionization.

In Table 3, we have shown the comparison of the 2σ errors on the derived parameters obtained for currently available *WMAP*7 data and the same for forecasts from simulated *Planck* data. It is clear that the uncertainties on all the parameters related to reionization would be reduced considerably. In particular, we find that we should be able to constrain the redshift range at which reionization was 99 per cent (50 per cent) completed to about 3 (1). This is clearly a significant improvement over what can be achieved through current data sets.

In Fig. 4, we have illustrated the recovery of the same quantities as mentioned in the earlier section using the AIC criterion taking up to eight eigenmodes for the simulated *Planck* data. For comparison, here also we have plotted the results for the fiducial model (short-dashed lines) along with the mean results (solid lines) from MCMC analysis with shaded 2σ limits. We find that our main results are in quite reasonable agreement with those obtained from the *WMAP* data (Section 3.2), except that all the 2σ (95 per cent) limits are reduced remarkably for all redshift ranges.

We thus find that we can constrain the global reionization history better using the *Planck* forecast data sets; in particular the 2σ limits for $Q_{\rm H\,II}$ reduce significantly for this case. However there is no room to substantially improve the constraints using large-scale E-modes for *WMAP*7 data sets, and one still has to rely on other types of data for understanding reionization.

Table 3. The 95 per cent CL errors of derived parameters for the reionization model obtained from the current analyses using the AIC criterion for *WMAP*7 and simulated *Planck* data.

Parameters	2σ errors	
	*WMAP*7	*Planck* (forecast)
$\tau_{\rm el}$	0.032	0.009
$z(Q_{\rm H\,II} = 0.5)$	2.862	1.117
$z(Q_{\rm H\,II} = 0.99)$	4.627	3.013

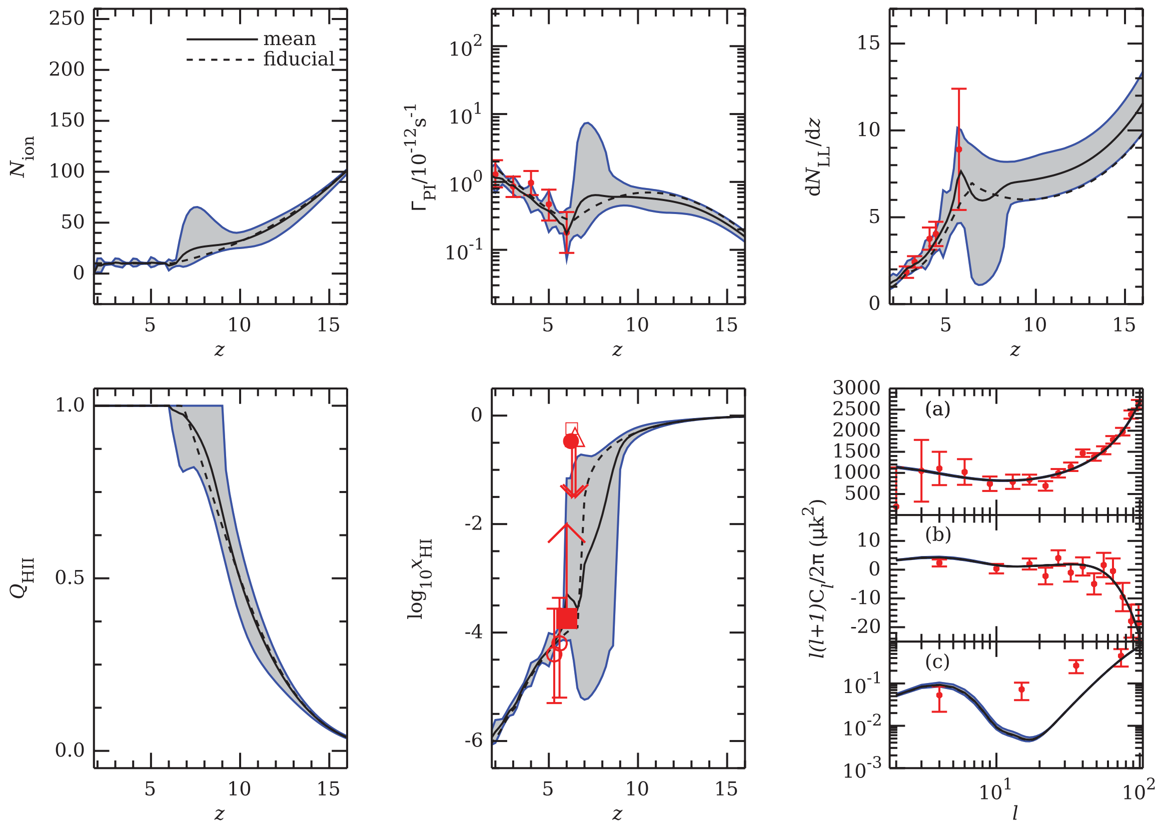

Figure 4. Same as Fig. 3 but for *Planck* likelihood.

4 DISCUSSION AND SUMMARY

Based on the work of Mitra et al. (2010) on PCA of reionization models, we have studied constraints on reionization history using non-parametric methods. To model the unknown function $N_{\rm ion}(z)$, we have applied the principal component method using three different sets of data points – the photoionization rate $\Gamma_{\rm PI}$

We also now have observations of Lyman-break galaxies up to $z \sim 10$ (Bouwens et al. 2007, 2011a,b). The luminosity function of such galaxies would be helpful in constraining properties of the galaxies like the IMF and/or the star-forming efficiency. Unfortunately, that would still leave out the escape fraction of ionizing photons, which remains an uncertain parameter to date.

Other indirect observations that could help in constraining reionization are the temperature measurements at $z < 6$ (Ricotti, Gnedin & Shull 2000; Schaye et al. 2000; McDonald et al. 2001; Zaldarriaga, Hui & Tegmark 2001; Cen et al. 2009). The temperature evolution can retain a memory of how and when the IGM was reionized and thus could provide additional constraints on reionization. Whatever the case, the principal component method described in this paper could be a promising tool for extracting information from future data sets in a model-independent manner.

ACKNOWLEDGMENTS

Computational work for this study was carried out at the cluster computing facility in the Harish-Chandra Research Institute.[3] TRC would like to thank the Regular Associateship Programmes of the ICTP (Trieste) during which the work was completed.

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