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The ionizing background at the end of reionization

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ABSTRACT

One of the most sought-after signatures of reionization is a rapid increase in the ionizing background (usually measured through the Lyα optical depth towards distant quasars). Conventional wisdom associates this with the 'overlap' phase when ionized bubbles merge, allowing each source to affect a much larger volume. We argue that this picture fails to describe the transition to the post-overlap Universe, where Lyman-limit systems (LLSs) absorb ionizing photons over moderate length-scales ($\lesssim$20–100 Mpc). Using an analytic model, we compute the probability distribution of the amplitude of the ionizing background throughout reionization, including both discrete ionized bubbles and LLSs (parametrized by an attenuation length, which we impose rather than attempt to model self-consistently). We show that the overlap does not by itself cause a rapid increase in the ionizing background or a rapid decrease in the mean Lyα transmission towards distant quasars. More detailed seminumeric models support these conclusions. We argue that the rapid changes should instead be interpreted as evolution in the attenuation length itself, which may or may not be directly related to overlap.

Key words: intergalactic medium – cosmology: theory – diffuse radiation.

1 INTRODUCTION

In the last several years, the cosmological community has made an enormous effort to measure the reionization history of the intergalactic medium (IGM). But the picture remains murky: some evidence (principally the cosmic microwave background, or CMB) points towards a mostly ionized universe at $z \gtrsim 9$ (Page et al. 2007; Dunkley et al. 2008; Komatsu et al. 2008), but other observations (principally from quasar absorption) are usually taken to imply that the reionization ends only at $z \approx 6$ (Fan et al. 2002; Mesinger & Haiman 2004; Fan et al. 2006; Mesinger & Haiman 2007). Many other techniques cannot yet distinguish between early and late reionization scenarios (e.g. Kashikawa et al. 2006; Totani et al. 2006; McQuinn et al. 2007a,c, 2008; Ota et al. 2007; see also Fan, Carilli & Keating 2006 for a review).

While there is not necessarily a contradiction between the CMB and quasar measurements – reionization may simply be relatively extended, which would hardly be surprising from a theoretical standpoint (e.g. Cen 2003; Wyithe & Loeb 2003; Furlanetto & Loeb 2005; Iliev et al. 2007) – the tension does call for a critical examination of the existing evidence. The most widely recognized point in favour of late reionization is the apparent rapid decrease in the mean Lyα transmission towards quasars at $z \gtrsim 6$ (Fan et al. 2001, 2002; White et al. 2003; Fan et al. 2006). There are two aspects to this. First, a complete Gunn & Peterson (1965) absorption trough appears towards some quasars, although the enormous optical depth of that line means that this still only requires a small neutral fraction. Second, there appears to be a substantial steepening of the amount of absorption beyond $z \sim 6$ (Fan et al. 2006). However, the latter conclusion has been challenged on empirical grounds (Songaila & Cowie 2002; Songaila 2004; Oh & Furlanetto 2005; Becker, Rauch & Sargent 2007); unfortunately, known lines of sight are sparse enough that no clear resolution has emerged (e.g. Lidz, Oh & Furlanetto 2006).

Here, we take a complementary approach and examine the theoretical underpinnings of the conclusion that such a rapid change implies a detection of 'reionization'. The crux of this argument is that the moment of 'overlap' (i.e. the point at which ionized bubbles merge into much larger units, usually considered to be the 'end' of reionization) must be accompanied by a sudden, rapid increase in the amplitude Γ of the ionization rate. Qualitatively, this expectation comes from percolation models of the reionization process: when ionized bubbles (presumed to be transparent to ionizing photons) overlap, many more sources illuminate any given patch, so Γ should increase rapidly. Quantitatively, the first self-consistent simulation of reionization showed precisely such a jump (Gnedin 2000). In that simulation, Γ evolved much more sluggishly both before overlap (when ionized regions grew slowly because the photons had to ionize fresh material) and afterwards (when the ionizing photons were already able to propagate large distances), so overlap appeared to be clearly defined.

However, this picture does not match properly on to the well-understood post-reionization Universe, where dense 'Lyman-limit systems' (LLSs) absorb ionizing photons over relatively small

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distances (e.g. ∼110 Mpc at $z = 4$; Storrie-Lombardi et al. 1994; Miralda-Escudé 2003; probably falling to $\lesssim$30 Mpc by $z = 6$; Lidz et al. 2007). Once ionized regions grew beyond the mean separation of these systems, LLSs (rather than the edges of ionized bubbles) controlled the mean free path of ionizing photons (Furlanetto & Oh 2005; Gnedin & Fan 2006) and by extension the effective horizon to which sources could be seen. Because reionization is so inhomogeneous, many ionized regions will reach this size ($\gtrsim$20 Mpc) well before complete overlap (Barkana & Loeb 2004; Furlanetto, Zaldarriaga & Hernquist 2004); there must in fact be a gradual transition from the 'bubble-dominated' ionization topology characteristic of reionization to the 'web-dominated' topology characteristic of the post-reionization Universe (Furlanetto & Oh 2005).

Here, we ask whether overlap must be accompanied by a rapid increase in Γ and, conversely, whether such an increase is a robust signal of overlap. We also examine the implications of recent reionization models for the Lyα forest observations conventionally used to argue for late reionization. Unfortunately, numerical simulations do not yet have the dynamic range to sample the large (∼100 Mpc) scales necessary for reionization and simultaneously predict detailed properties of the Lyα forest [though, see Gnedin & Fan (2006) for a detailed study of LLSs and the Lyα forest during reionization in an 8 h^{-1} Mpc box]. Thus, we will use a simple analytic model that incorporates both the discrete ionized bubbles and intervening absorption by LLSs to show that, during the end stages of reionization, Γ is primarily controlled by the mean separation of LLSs, which may not evolve rapidly at the moment of overlap – and, more importantly, may continue evolving long afterwards.

Before beginning, we note explicitly that our model is not 'complete' in the sense of self-consistently predicting the evolution of LLSs during and immediately after reionization; in fact, no theoretical model is able to describe these features adequately. We cannot therefore make predictions about the evolution of the mean free path of ionizing photons in the very final stages of reionization (when the ionized fraction exceeds ∼0.99). Rather, we are interested in the moment of 'overlap' when the discrete ionized bubbles merge and effect a transition in the mean free path from the bubble size to the LLS spacing, a distinct (but possibly related) issue from the evolution of the post-overlap attenuation length. We defer a discussion of this latter possibility to the final section of this paper.

In our numerical calculations, we assume a cosmology with $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, $\Omega_b = 0.044$, $H_0 = 100\,h$ km s^{-1} Mpc^{-1} (with $h = 0.74$), $n = 0.95$ and $\sigma_8 = 0.8$, consistent with the most recent measurements (Dunkley et al. 2008; Komatsu et al. 2008). Unless otherwise specified, we use comoving units for all distances.

2 METHOD

We wish to compute the probability distribution $f(J)$ of the angle-averaged specific intensity of the radiation background at the H I ionization edge, J. Its value at a given point depends on four basic parameters: the number density of ionizing sources, n_i; the characteristic luminosity of each source, $L_\star$; the size of the local ionized bubble, R; and the attenuation length for ionizing photons within each bubble, r_0. This last quantity is determined by the spacing of LLSs in the mostly ionized IGM (Miralda-Escudé 2003); again, we will impose its value rather than trying to compute it self-consistently.

We first suppose that R is prescribed and compute $f(J)$ within that single ionized region. For the sources, we assume that every dark matter halo in the region with a virial temperature above the minimum level for atomic cooling ($T_{\rm vir} > 10^4$ K; Barkana & Loeb 2001 and references therein) hosts a single galaxy with luminosity proportional to its mass, although the mass function is steep enough that the luminosity–mass relation makes little difference to our results. Our qualitative results are also unaffected if only a fraction of galaxies actively form stars (and hence do not produce ionizing photons) at any given time; as we will see below, random fluctuations are small, even if most galaxies are quiescent.

For our purposes, we can divide the ionized bubbles into two limiting regimes. Small bubbles (usually appearing early in reionization) have $R \ll r_0$. In this case, we can let $r_0 \to \infty$ (or, equivalently, the optical depth $\tau = r/r_0$ experienced by a photon within that bubble vanishes). We then compute $f_{\tau=0}(J)$ following Zuo (1992), with a simple extension to account for the luminosity function of the sources (as in Meiksin & White 2003). Assuming that the total number of sources in the bubble is Poisson-distributed with mean $\bar{N} = (4\pi/3)n_i R^3$ (a reasonable approximation according to N-body simulations; Casas-Miranda et al. 2002),

$$f_{\tau=0}(j) = \frac{1}{\pi}\int_0^\infty {\rm d}s\, \exp\left[\bar{N}\int {\rm d}x\, \phi(x)\kappa_1(sx)\right] \times \cos\left[-sj + \bar{N}\int {\rm d}x\, \phi(x)\kappa_2(sx)\right]. \quad (1)$$

Here, $j = J/J_\star^{\tau=0}$, $J_\star^{\tau=0} = L_\star/(4\pi R)^2$, $L_\star$ is the mean luminosity of the sources, $x = L/L_\star$, $\phi(x)$ is the source luminosity function normalized so that $\int dx \phi(x) = 1$,

$$\kappa_1(t) = \cos(t) - 1 - 2t\,{\rm Im}\,g(t), \quad (2)$$

$$\kappa_2(t) = \sin(t) + 2t\,{\rm Re}\,g(t) \quad (3)$$

and

$$g(t) = \int_0^1 {\rm d}u\, {\rm e}^{{\rm i}t/u^2}. \quad (4)$$

In this limit, the mean background is $\langle j\rangle_{\tau=0} = 3\bar{N}$, or $\langle J\rangle \propto \bar{N} J_\star^{\tau=0} \propto L_\star R$.

In the later stages, most bubbles become much larger than the attenuation length set by embedded LLSs ($r_0 \ll R$), which then absorb most of the ionizing photons (Furlanetto & Oh 2005). In this case, we can approximate the ionizing background by taking the limit $R \to \infty$ (Meiksin & White 2003), which gives

$$f_{R=\infty}(j') = \frac{1}{\pi}\int_0^\infty {\rm d}s\, \exp\left[-s\bar{N}_0\int {\rm d}x\, x\phi(x)\,{\rm Im}\,G(sx)\right] \times \cos\left[-sj' + s\bar{N}_0\int {\rm d}x\, x\phi(x){\rm Re}\,G(sx)\right], \quad (5)$$

where $\bar{N}_0 = (4\pi/3)n_i r_0^3$, $j' = J/J_\star^{R=\infty}$, $J_\star^{R=\infty} = L_\star/(4\pi r_0)^2$,

$$G(t) = \int_0^\infty {\rm d}u\, \tau^3(u){\rm e}^{{\rm i}tu} \quad (6)$$

and $u = {\rm e}^{-\tau}/\tau^2$. In this limit, the mean background is $\langle j'\rangle_{R=\infty} = 3\bar{N}_0$, or $\langle J\rangle \propto \bar{N}_0 J_\star^{R=\infty} \propto L_\star r_0$.

Meiksin & White (2003) describe how to obtain the full distribution for arbitrary R and r_0; however, we find by explicit computation of the full distributions that the two limiting cases above are reasonable approximations to the true one (and much simpler to compute), provided that they are renormalized to have the proper mean value that includes both attenuation and the finite bubble size (Meiksin & White 2003):

$$\langle j'\rangle = 3\bar{N}_0(1 - {\rm e}^{-R/r_0}). \quad (7)$$

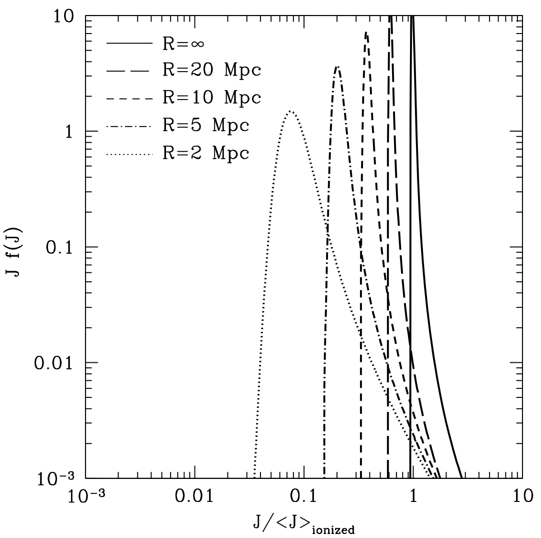

Figure 1. Distribution of J relative to its mean value in a fully ionized IGM, $\langle J\rangle_{\rm ionized}$. All curves assume $z = 6.5$ and $r_0 = 20$ Mpc. The solid curve is for a fully ionized universe; the others assume discrete bubbles with $R = 2,\ 5,$ 10 and 20 Mpc, from left to right, or $\bar{N} = (0.155,\ 0.241,\ 1.93,\ 15.4) \times 10^4$.

The approximation is effective because reasonable attenuation lengths at the end of reionization ($r_0 \gtrsim 1$ Mpc; see Lidz et al. 2007) contain huge numbers of sources, for which the limiting forms converge to each other. For a given bubble, we therefore use equation (1) if $R < r_0$ or equation (5) otherwise, rescaled by a constant factor so that $f(j)$ has the proper mean.

Fig. 1 shows some example distributions for bubbles at $z = 6.5$ [with $n_{\rm i}$ computed from the Press & Schechter (1974) distribution]. Note that we have normalized J to its mean value in a fully ionized IGM (taking $R \to \infty$). Clearly, the distributions narrow rapidly as more sources become visible (i.e. as R increases); this is because the fractional variation in the source counts goes like $1/\sqrt{\bar{N}} \sim 1/R^{3/2}$ (ignoring absorption and including only Poisson fluctuations; see below for a discussion of cosmic variance). However, the peak (and the mean) evolves much less rapidly. When $R \ll r_0$, $\langle J\rangle \propto R$, but the evolution slows once $R \sim r_0$. Indeed, for a constant r_0, the mean amplitude increases by only a factor of ~1.6 from $R = r_0$ to $R = \infty$, as the additional sources are already significantly attenuated.

There are three shortcomings of this simple model. First, our rescaling via equation (7) does not exactly preserve the shape of the $J \gg \langle J\rangle$ tail, which corresponds to points very near to a single source; in this limit $f(J) \propto J^{-5/2}$ and all the curves should converge to the solid one. Fortunately, precisely because this tail describes the immediate neighbourhood of sources, it is not important for our argument: we are only concerned with the ionizing background in the diffuse IGM, where transmission in the Lyα forest appears. In practice, at the ionized fractions of interest most of the volume is filled by large bubbles ($R > r_0$) for which the rescaling is quite accurate.

More seriously, these curves assume that the galaxy density is Poisson-distributed around the universal mean value; in reality, galaxy clustering causes variations in the number counts above and beyond the Poissonian fluctuations in the halo number counts that we include (cf. Wyithe & Loeb 2006; Mesinger & Dijkstra 2008). These have fractional amplitude $\sim\bar{b}_{\rm i}\sigma(r_0)$ in the fully ionized limit, where $\bar{b}_{\rm i}$ is the mean bias of ionizing sources and $\sigma^2(r_0)$ is the variance in the dark matter field smoothed on a scale r_0; for $z = 6$ and $r_0 = 20$ Mpc, we have $\bar{b}_{\rm i}\sigma(r_0) \sim 0.25$. To obtain $f(J)$ throughout the universe, the solid curve in Fig. 1 must be convolved with the underlying distribution of $\bar{N}$ determined by clustering, which broadens it substantially (Mesinger & Dijkstra 2008). Fortunately, this only strengthens our conclusions (see the discussion in Section 4), so we defer a detailed model of it to future work.

Finally, we also assume a spatially constant r_0 throughout the Universe. This is certainly an oversimplification, because r_0 must itself depend on J. However, the largest variations in J are across bubbles with $R \ll r_0$, for which the attenuation length does not strongly affect our calculations. When r_0 is important, the variations in J are relatively modest anyway, so our assumption seems a reasonable one.

3 THE IONIZING BACKGROUND DURING REIONIZATION

To obtain $f(J)$ during reionization, we must convolve the distributions for single bubbles in Fig. 1 with the distribution of ionized bubble sizes, $n_{\rm b}(R)$. We use the analytic formulation of Furlanetto et al. (2004) to compute the latter; note that it is determined primarily by the ionized fraction and has only a weak dependence on redshift (Furlanetto, McQuinn & Hernquist 2006; McQuinn et al. 2007b; Mesinger & Furlanetto 2008), so our results are fairly generic.[1] We also use this formalism to compute the mass function of dark matter haloes inside each bubble (Furlanetto, Hernquist & Zaldarriaga 2004); note that this procedure does properly capture clustering's effects on $\bar{N}$ in discrete bubbles, because it is precisely this galaxy clustering which determines $n_{\rm b}(R)$. In other words, overdense regions with more galaxies are already part of larger ionized bubbles, with correspondingly more galaxies (Furlanetto et al. 2004).

Fig. 2 shows $f(J)$ at $z = 6.5$ for a variety of ionized fractions [normalized so that $\int {\rm d}J\, f(J) = \bar{x}_{\rm i}$, the mean ionized fraction]. In all cases, we assume that $r_0 = 20$ Mpc, consistent with the extrapolations of Lidz et al. (2007) from models of the Lyα forest at lower redshifts. The dot–dashed curve shows $f(J)$ once overlap is complete. Again, the most important result is the relatively slow evolution of $f(J)$ throughout the last half of reionization; $\langle J\rangle$ increases primarily because the tail towards small J shrinks as more and more of the Universe is incorporated into large bubbles.

Because the ionized bubbles reach such large sizes early in reionization (typically ~4.5 Mpc when $\bar{x}_{\rm i} = 0.5$), the radiation background incident on a typical ionized patch increases by only a factor of ~5 throughout the last half of reionization: although the size of each ionized region continues to increase until $R \to \infty$, the fixed attenuation length prevents a corresponding rapid increase in $\langle J\rangle$. Moreover, regardless of $\bar{x}_{\rm i}$, some regions have $J \approx \langle J\rangle_{\rm ionized}$ even though they are not particularly near any individual source. This broad range occurs because $n_{\rm b}(R)$ always contains some large bubbles, even early in reionization, and is not due to random Poisson fluctuations in the galaxy counts inside each bubble. To see this, the dotted curve shows what happens if we ignore fluctuations within each bubble at $\bar{x}_{\rm i} = 0.4$ (i.e. we set J equal to its mean value for

[1] Furlanetto & Oh (2005) also computed the 'bubble' size distribution during reionization, but including the effects of attenuation. We use the simpler model of Furlanetto et al. (2004) because that more accurately describes the distribution of R, the total size of ionized regions: by construction, Furlanetto & Oh (2005) always returns the smaller of R and r_0. In order to be more transparent about the imposed attenuation, we leave r_0 a free parameter.

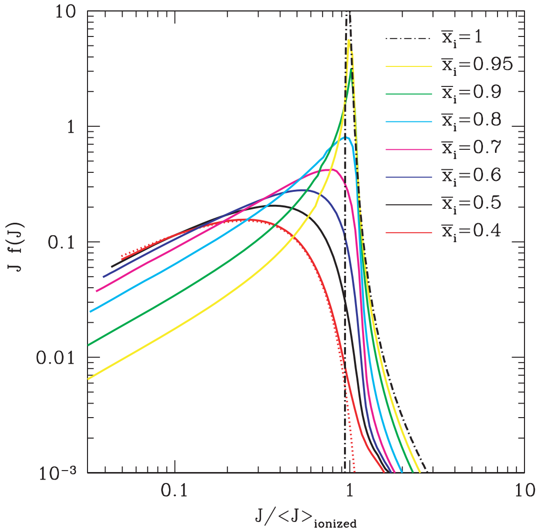

Figure 2. Distribution of J during reionization relative to its mean value in a fully ionized IGM, $\langle J\rangle_{\rm ionized}$. The solid curves take $\bar{x}_i = 0.4$, 0.5, 0.6, 0.7, 0.8, 0.9 and 0.95, from left to right. The dot–dashed curve is for a fully ionized universe. The dotted curve shows $f(J)$ when $\bar{x}_i = 0.4$ if we ignore fluctuations inside the discrete bubbles. All assume $r_0 = 20$ Mpc and $z = 6.5$.

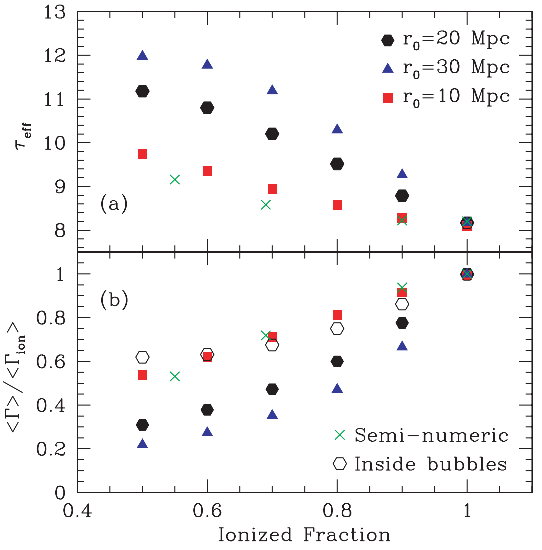

Figure 3. Effective optical depth (a) and mean ionizing background (b) as a function of $\bar{x}_i$ at a constant redshift. The filled squares, hexagons and triangles assume $r_0 = 10$, 20 and 30 Mpc, respectively, at $z = 6.5$. The open hexagons show the mean ionizing background inside ionized regions, $\langle\Gamma_{-12}\rangle/\bar{x}_i$, for $r_0 = 20$ Mpc at $z = 6.5$. The crosses use $f(J)$ taken from the seminumeric simulations of Mesinger & Dijkstra (2008) (see text).

that R); it traces the full curve closely except in the high-J tail that is due to the proximity to a single source.[2]

4 OBSERVABLE IMPLICATIONS

We now examine how the slow evolution of $f(J)$ affects the Lyα forest. The filled hexagons in Fig. 3(b) show the mean ionization rate $\langle\Gamma\rangle$ for the models in Fig. 2;[3] the filled squares and triangles show the same sequence of ionized fractions, but with $r_0 = 10$ and 30 Mpc, respectively (approximately spanning the range of values expected from extrapolating Lyα forest measurements at lower redshifts; Lidz et al. 2007). Note that these values are averaged across the entire Universe; the mean amplitudes within ionized regions are shown for the $r_0 = 20$ Mpc model by the open hexagons. Again, it is obvious that $\langle\Gamma\rangle$ evolves only slowly during reionization. The rate of evolution decreases with r_0, because bubbles reach the 'saturation radius' $R = r_0$ quicker (Furlanetto & Oh 2005); beyond that point, $\langle J\rangle$ can only increase by a factor $(1 - 1/e)^{-1}$ so long as r_0 remains fixed.

The most easily observed property of quasar Lyα forest spectra is the mean transmission $\mathcal{T} \equiv \exp(-\tau_{\rm eff})$; we show the effective optical depth in Fig. 3(a) for the same set of models. To compute $\tau_{\rm eff}$, we convolve the (volume-averaged) IGM density distribution of Miralda-Escudé, Haehnelt & Rees (2000) with $f(J)$. We assume that the local density is uncorrelated with J; while this is certainly not correct in detail, it is a reasonable first step because Γ is typically dominated by distant galaxies (Olber's paradox). Note that the absolute value of $\tau_{\rm eff}$ is somewhat sensitive to the high-J tail; we truncate all of our calculations at $10\langle J\rangle_{R=\infty}$ (this essentially prevents the low-density voids that provide transmission from being extremely close to galaxies). For concreteness, we assume that $\langle\Gamma_{-12}\rangle \equiv \langle\Gamma\rangle/(10^{-12}\,{\rm s}^{-1}) = 0.05$ when $\bar{x}_i = 1$, consistent with limits from Lyα forest measurements in quasar spectra at $z \sim 6$ (Fan et al. 2002, 2006; Bolton & Haehnelt 2007). This yields a reasonable $\tau_{\rm eff} \approx 8$ at $z = 6.5$ (cf. Fan et al. 2001, 2006).

Fig. 3 shows that there need not be a discontinuity in $\tau_{\rm eff}$ due simply to the overlap of ionized bubbles: in all of our models, the transmission evolves smoothly and (relatively) slowly as a function of ionized fraction, especially when r_0 is small (so that bubbles enter the saturated regime earlier).[4] However, the observational relevance of these results is questionable, because such large $\tau_{\rm eff}$ are probably unobservable. Fortunately, the higher Lyman-series transitions are easier to observe; for example, Lyβ has an optical depth 6.24 times smaller than Lyα (although the ratio $\tau_{{\rm eff},\beta}/\tau_{\rm eff}$ is closer to unity because of the convolution with the density field; Songaila & Cowie 2002; Songaila 2004; Oh & Furlanetto 2005). Fig. 4 shows $\tau_{{\rm eff},\beta}$ for the same models as Fig. 3(a). This shows the same qualitative trends as for Lyα, although the evolution is somewhat steeper, and again illustrates that no strong break is necessary at overlap.

As described above, the most important shortcoming of our model is that it ignores galaxy clustering when $R > r_0$; clustering substantially broadens $f(J)$ (but does not affect $\langle J\rangle$ much). To check the importance of this effect, we have drawn $f(J)$ from the $z = 7$ seminumeric simulations of Mesinger & Dijkstra (2008) (in turn based on the reionization model of Mesinger & Furlanetto 2007). These simulations include attenuation (with $r_0 = 20$ Mpc), clustering

[2] Imposing a duty cycle $f_{\rm d}$ on star formation (and emission of UV photons), or suppressing star formation in small haloes, also has no substantial effect. Even if $f_{\rm d} \sim 0.01$, significantly smaller than one would expect from the dynamical times of galaxies, the shape of $f(J)$ changes only slightly, except in the high-J tail (which increases in amplitude, because each galaxy dominates a larger region around itself).

[3] Here, we assume $\Gamma \propto J$, neglecting the variations in the mean free path for higher-energy photons because they do not contribute strongly to the total ionization rate.

[4] Note, however, that because we do not calculate r_0 self-consistently, there may still be a discontinuity if overlap is accompanied by a rapid increase in that quantity. See Section 5 for a discussion of this possibility.

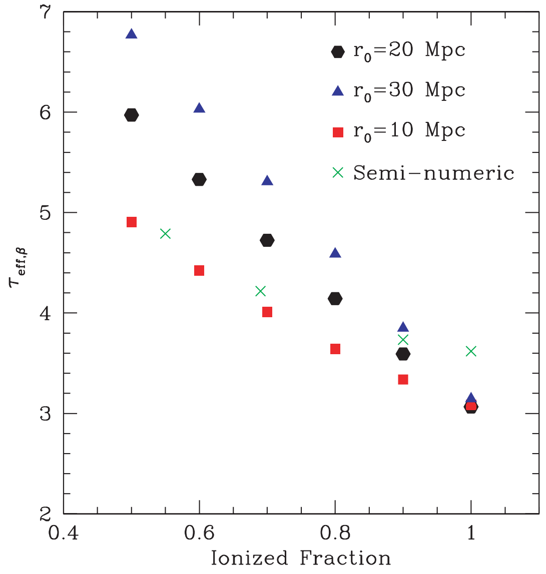

Figure 4. Same as Fig. 3*a*, except for $\tau_{\mathrm{eff},\beta}$.

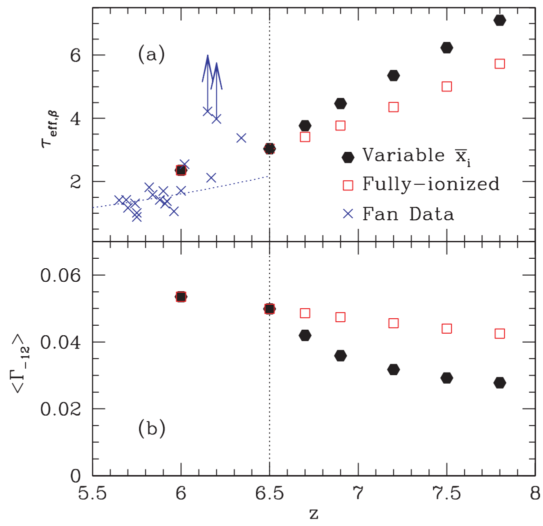

Figure 5. Evolution of the effective optical depth in the Lyβ transition (a) and the mean ionizing background (b) for a scenario in which reionization ends at $z \le 6.5$ (marked by a vertical dotted line) and $r_0 \propto (1+z)^{-3}$ (filled hexagons). Above $z = 6.5$, adjacent hexagons have $\Delta\bar{x}_{\rm i} = 0.1$ between them (with $\bar{x}_{\rm i} = 0.5$ at $z = 7.8$). The open squares show an alternate model with a constant comoving emissivity and $\bar{x}_{\rm i} = 1$ throughout this range. In (a), the crosses show the observed Lyβ effective optical depth along several quasar lines of sight from Fan et al. (2006). The dotted line shows the extrapolation from data at lower redshifts ($z < 5.5$).

and more accurate bubble sizes. In principle, we can also use them to compute correlations between the flux field and the underlying density field; however, the finite box size (100 Mpc) means that rare voids (which account for the bulk of the transmission) may be missed and the quasi-linear treatment may not capture the full density distribution. We therefore draw $f(J)$ from the seminumeric simulations and convolve it with the Miralda-Escudé et al. (2000) density distribution to compute $\tau_{\rm eff}$, shown by the crosses in Figs 3 and 4.[5] These show even slower evolution throughout reionization, partly because the simulated ionized regions are somewhat larger than the analytic model predicts and partly due to the inclusion of clustering when $R > r_0$. Thus, the seminumeric approach confirms our conclusion that the overlap need not be accompanied by a rapid increase in $\langle\Gamma\rangle$, unless r_0 also increases.

To this point, we have fixed the mean free path and redshift to isolate how $\bar{x}_{\rm i}$ affects $f(J)$. With the filled hexagons in Fig. 5, we specialize to a particular reionization scenario where $\bar{x}_{\rm i}$ is proportional to the fraction of gas in galaxies [more specifically, the so-called collapse fraction of gas inside haloes large enough for efficient atomic cooling or with virial temperatures $>10^4$ K according to the Press & Schechter (1974) mass function; Barkana & Loeb 2001] and where we set $\bar{x}_{\rm i} = 1$ at $z \le 6.5$. We further assume that $r_0 = 15\,[(1+z)/7.5]^{-3}$ Mpc, consistent with extrapolations from the lower redshift forest (Lidz et al. 2007). Again, we normalize so that $\langle\Gamma_{-12}\rangle = 0.05$ at $z = 6.5$. Even with this evolution in r_0, the mean transmission evolves smoothly across this entire range, and there is only a modest break in $\langle\Gamma\rangle$ at $z = 6.5$.

In contrast, the open squares assume $\bar{x}_{\rm i} = 1$ at all redshifts, with an identical $r_0(z)$ and a constant comoving emissivity. The latter does give more transmission at all redshifts, but both models evolve quite smoothly – there is only a slight break in the reionization model's slope at overlap compared to the fully ionized one. Thus, overlap does not, in general, cause an obvious feature in the transmission, even for the higher-series transitions. In fact, slow evolution is guaranteed to occur unless the mean free path itself evolves more rapidly during reionization than we have assumed.

For comparison, Fig. 5(a) also shows the absorption from 19 observed segments of the Lyβ forest from Fan et al. (2006), corrected for the mean level of foreground Lyα absorption (blue crosses; the two points with arrows show 3σ lower limits). The dotted line shows the Lyβ absorption extrapolated from lines of sight with $z < 5.5$; this is qualitatively similar to the extrapolation of Becker et al. (2007) as well. Note that we have made no attempt to fit our models to these data, other than choosing a normalization such that $\tau_{\rm eff}, \beta$ is roughly similar to the data at $z \sim 6$–6.5. The data show a sharper break than either of our models can accommodate, so additional physics must be at play, as we discuss below – perhaps due to stronger evolution in r_0 [over and above the $(1+z)^{-3}$ we have already assumed], or else effects like an evolving emissivity or IGM temperature.

5 DISCUSSION

We have shown that, because attenuation due to LLSs must become important before overlap, the late stages of reionization need not be accompanied by a rapid increase in $\langle\Gamma\rangle$ (or $\tau_{\rm eff}$). Thus, the conventional wisdom that a sharp increase in the ionizing background is a robust indicator of overlap must be modified: namely such a feature indicates that only the attenuation length r_0 is evolving rapidly.

Of course, it may be that r_0 does evolve quickly at the end of reionization (and some theoretical calculations suggest that this does occur; Choudhury, Haehnelt & Regan 2008; Wyithe, Bolton & Haehnelt 2008). For example, $\langle\Gamma\rangle$ must (at least to some degree) control the ionization structure of LLSs and hence r_0 (Miralda-Escudé 2005; Schaye 2006). As $\langle\Gamma\rangle$ increases, LLSs will shrink,

[5] We actually take $z = 7$ $f(J)$ from the simulations and apply them to the $z = 7$ density field. We then choose $\langle\Gamma_{-12}\rangle = 0.032$ at $\bar{x}_{\rm i} = 1$ so that $\tau_{\rm eff}$ is identical to that in the analytic model. Note that, by widening $f(J)$, clustering decreases the required mean ionizing background for a given mean transmission level. Interestingly, it also changes the relative ratio of Lyα and Lyβ transmission. We have verified that the slow evolution persists even if we use the seminumeric density field.

increasing r_0 and hence $\langle\Gamma\rangle$, creating a positive feedback loop. Thus, even a slow initial increase in the emissivity and/or mean free path at the end of reionization may spiral into a relatively rapid change in $\langle\Gamma\rangle$. Unfortunately, our understanding of LLSs is not yet advanced enough to determine the effectiveness of such a feedback loop; for now, we can only say that the effective feedback requires that the LLSs have rather shallow density profiles (see e.g. the appendix to Furlanetto & Oh 2005).

More importantly, there is no obvious reason that the mean free path can *only* change during overlap: so long as (i) the emissivity continues to increase and (ii) $\langle\Gamma\rangle$ controls the properties of LLSs, the feedback loop will continue – and with the rapid increase in the collapsed fraction at $z \gtrsim 6$, such a scenario seems entirely plausible. Thus, even if $\langle\Gamma\rangle$ is evolving strongly at $z \sim 6$ (Fan et al. 2002, 2006), we do not have a robust indicator of overlap.

Indeed, it may be better to regard the final stage of reionization as the disappearance of LLSs after overlap. This 'post-overlap' phase, which consumes just a few per cent of the neutral gas, is typically thought to be indistinguishable from the cosmic-web-dominated Universe at later times. We have shown instead that $\langle\Gamma\rangle$ may evolve slowly during overlap but rapidly afterwards, making quasar absorption spectra a useful probe of this tail end of reionization (which matches smoothly on to the post-reionization Universe) but not necessarily of overlap itself. This contrasts with the conventional picture in which $\langle\Gamma\rangle$ evolves rapidly only during overlap; that intuition came from reionization simulations that were too small to include the full inhomogeneity of reionization and so exaggerated the importance of overlap (e.g. Gnedin 2000). The recent seminumeric calculations of Choudhury et al. (2008), which included an approximate prescription for self-shielding gas, present a similar picture to ours, in which these LLSs are burned off over the (rather extended) final stages of reionization that follow overlap.

Beyond an increase in Γ, there are other reasons that r_0 may evolve throughout (and after) reionization. For example, photoheating can evaporate loosely bound structures, substantially modifying the IGM gas distribution by eliminating dense systems (Haiman, Abel & Madau 2001; Shapiro, Iliev & Raga 2004; Pawlik, Schaye & van Scherpenzeel 2008). This will increase r_0 as well; however, the process should happen gradually throughout reionization as more and more of the volume is illuminated (and so will be particularly slow if reionization is extended). Evaporation also occurs over the sound-crossing time, which is relatively long for the moderate overdensities of most interest at these high redshifts (Pawlik et al. 2008), so that LLSs probably continue to evolve past the 'end' of reionization.

Given the challenges we have raised to the conventional interpretation, it is worth asking three further questions about the data. First, are there other effects that can cause a rapid increase in $\tau_{\rm eff}$ without a significant change in Γ? One possibility is the density distribution itself: at least according to current models, it is extremely steep in the low-density tail that is responsible for Lyα forest transmission, so it is possible that a small increase in the ionizing background renders a relatively large fraction of the IGM visible (Oh & Furlanetto 2005). This must await more detailed calculations of the evolving density field at the end of reionization.

Second, how certain can we be that overlap has actually occurred by $z \sim 6$? Is it possible that, although most of the IGM is highly ionized before that point, a small fraction far from ionizing sources could still be completely neutral (Lidz et al. 2007)? In the conventional picture, in which $\langle\Gamma\rangle$ evolves rapidly at overlap, such a conclusion can be easily dismissed. However, our results suggest that overlap itself may be buried inside the smoothly evolving transmission at $z \lesssim 6$ – or it may have occurred at much higher redshifts.

Third, if bubble overlap itself does not necessarily imply that the effective optical depth should increase dramatically, are there other possible reasons for such a jump? One plausible explanation is the emissivity, which may evolve relatively rapidly at high redshifts. For example, the (observed, dust-corrected) star formation rate density falls by a factor of 4 from $z = 4$–6 (Bouwens et al. 2007) and another factor of 2.5 from $z = 6$ to 7 (Bouwens et al. 2008). Such a decline could at least help to account for the evolution in the Lyα forest. For example, Fan et al. (2006) find that $\tau_{\rm eff} \approx (2.1, 7.1)$ at $z \sim (5, 6.1)$. Assuming a uniform ionizing background with no evolution in r_0, and using the Miralda-Escudé et al. (2000) density distribution, this would require that the comoving emissivity decline by a factor of $\sim$3 over this range, not too much larger than the observed decline at $z = 6$–7. However, the observational estimates are likely increasingly incomplete at high redshifts, because the characteristic luminosity is falling and the shape of the luminosity function may be changing (e.g. Bouwens et al. 2008). Thus, it is not clear that the total emissivity does indeed fall by this large factor.

Another complication is thermal evolution in the IGM, which affects the hydrogen recombination rate and hence the conversion from effective optical depth to ionization rate. Reionization is accompanied by rapid photoheating in the IGM, increasing its temperature by an order of magnitude or more (Hui & Gnedin 1997; Hui & Haiman 2003). However, once the IGM is fully ionized, this photo-ionization heating becomes much less efficient and so the IGM begins to cool rapidly as the Universe expands. The post-reionization equation of state of the IGM is quite complex (Trac, Cen & Loeb 2008) and must be included in any in-depth evaluation of the IGM optical depth (Furlanetto & Oh 2009).

Finally, we expect our conclusions to hold with even more force during helium reionization: the clumpy IGM and enhanced recombination rate during that era make attenuation more important (Bolton, Oh & Furlanetto 2008; Furlanetto & Oh 2008; McQuinn et al. 2008) and high-energy photons can more easily create a nearly uniform ionizing background, so the transition from bubble-domination to web-domination will be even smoother.

In summary, with our present knowledge of high-z LLSs, there is no compelling reason to associate evolution in r_0 or Lyα forest transmission exclusively with overlap. If indeed the quasar data show a more rapid evolution in $\tau_{\rm eff}$ at $z > 6$ (Fan et al. 2006), this may indicate that we are seeing a rapid increase in r_0 that followed or even preceded overlap (note, however, that this measurement is itself controversial; Songaila & Cowie 2002; Songaila 2004; Becker et al. 2007). More detailed studies of the coupling between the ionizing background and the dense, neutral blobs that trap ionizing photons are needed before the next step – associating such a jump with overlap itself – can be taken securely.

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